Explaining Ships Dynamic and Handling Using MATLAB & SIMULINK as Simulation Tool in Teaching Maritime Students

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Abstract

The understanding of basic mechanical problems is vital to form a sound basis for maritime students to be prepared for the specific parts of the syllabus in later semesters in order to explain Ships Dynamic and Handling. Normally simulators are used for familiarisation with the practical aspect of ship manoeuvrability. However, for providing knowledge on university level it is necessary to explain the dynamics of ship motion characteristic in more detail and based on the mathematical equation of ships motion.

In the paper and presentation some examples will be given on how to use the software package MATLAB & SIMULINK as simulation Tool in teaching maritime students and even to build up more sophisticated programs for use in teaching, simulation and on board ships.

The introduction starts with a simplified approach of the Equations of ships motion and samples for manoeuvres on straight track like coasting from full ahead to stop. Extending the equation for steering forces gives access to discuss steering motion of ships.

Simplified Modelling of ships forces acting on the vessel like thrust and steering forces complete the set of equations. The graphical representation of the dynamic system by blocks in SIMULINK allows for an overview easy to understand to students as a sort of flow diagram.

Results from fast time simulation of several manoeuvres using a more enhanced program version SIMOPT allow for a discussion of yaw stability of ships and its impact on steering quality.

A simplified model for rolling motion of ships is given and some examples are simulated to explain the general effects of stability and wave parameters. The relation of these results to the more practical use for decision making to avoid resonance effects of ships in waves by loading operations or speed/course changes is demonstrated by means of a sophisticated software tool ARROW interfaced to a voyage planning program.

Keywords: Manoeuvring of ships, Simulation, Modelling of Ships' Dynamic, MATLAB / SIMULINK Tools

1. Model Approach for Manoeuvres on Straight Track

1.1 Equilibrium of forces - Equation of Motion

The introduction into ships dynamic starts with a simple example: we limit the discussion to a simplified approach for the equations of ships motion and samples for manoeuvres on straight track in one degree of freedom (1 DOF) like coasting from full ahead to stop. The equation of motion has the following general format, representing the balance of forces due to inertia on the left side and the external forces on the right:

$$m \cdot \frac{dv}{dt} = \sum F$$

For this basic ship motion on straight track this equation represents the equilibrium of forces on the ship: the forces of inertia on the left side due to the acceleration of the ship (consisting of ships mass and added mass, multiplied by acceleration from changing ships speed per time) are balanced by the forces on the right side due to constant motion (consisting of the thrust T of the propeller and the ships resistance R).
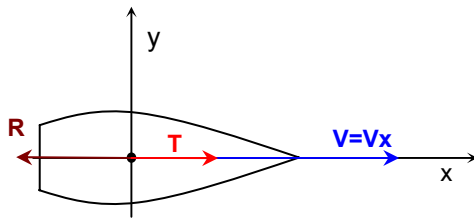
This leads to the following equation:

$$(m + m_x) \cdot \frac{dv}{dt} = T(1 - t) + R$$

where: m - ships mass

m_x - added mass due to inertia of the water when accelerating in x-direction

R - Resistance for constant speed

T -Thrust of the propulsion system

t - Thrust deduction coefficient due to the suction t of the propellers at the ships stern

For modelling of the forces resistance and thrust several approaches can be made. This normally reveals an equation of motion which has the character of an Ordinary Differential Equation (ODE) of first order, normally non-linear due to the character of the forces T and R.

The following two chapters will describe the effect of these forces in a simplified way for two specific manoeuvres.

1.2 Coasting Stop Manoeuvre

During the coasting stop manoeuvre the engine will be brought to STOP when the ship has a certain initial speed Vo. Due to the resistance the ship will slow down, the propeller is assumed to generate no thrust anymore, T=0.

For the resistance the following formula will be taken into account

$$R = C_R * \frac{\rho}{2} * V^2 * A$$

This leads to a simple equation of motion:

$$(m + m_x) \cdot \frac{dv}{dt} = R = C_R * \frac{\rho}{2} * V^2 * A$$

If the resistance coefficient C_R and the ships lateral area are taken as constants we can put them together into the constant C and get

$$\frac{dV}{dt} = V' = \frac{C \cdot V^2}{(m + m_x)}$$

There are two possible types as solution of the equation:

a) Exact Solution: For this simplified equation it is possible to get an exact solution which is for the initial speed V_0:

$$V(t) = \frac{m + m_x}{C \cdot t + (m + m_x)/V_0}$$

b) Numerical Solution: Generally a numerical solution can be achieved using methods like EULER-Cauchy or RUNGE-KUTTA to integrate the differential equation step-by-step. Programming languages and software like MATLAB or SIMULINK are excellent tools to model and to solve those tasks - there are even more methods available which can be selected and activated by a mouse click.

The following Figure 1 shows this example of a coasting stop modelling in SIMULINK:

SIMULINK is a block oriented modelling method following a strict Input-Output relation of the block structure: The function block "Equation of Motion: V'=Resistance/(mass)" contains the elements of the right hand side of the equation of motion where the speed V is represented by the input signal u(1). The output is the derivative V' which will be integrated in the next block "Integrator V' " (for the first step of the solution process the initial value V0 is used from the respective constant block "Initial value: Speed").

The output of this integrator block is then the speed V which will be transferred via the feedback line to the input of the function block to be available for the next step of the solution process. At the same time this speed signal V can be used to calculate the distance S in another block "Integrator S'=V" (for the first step of the solution process the initial value S0 is used from the respective constant block "Initial value: Distance"). The results of both integrators can be visualised by means of scope blocks like "Speed versus time" and "Distance versus time", the respective characteristic graphs of the time history of this manoeuvre are displayed. The data for the variables in the function and other blocks can be loaded into the workspace of the system via so called m.-files or alternatively easily changed in the blocks directly. This enables the user to easily vary the data to demonstrate several effects like mass of the ship or resistance coefficient on the speed and distance results.

Generally the graphical representation of the dynamic system by blocks in SIMULINK allows for an overview easy to understand to students as a sort of flow diagram. Introduction of solvers for these differential equations and explanation of use for simulation can be understood from the help system and easily selected from pull down windows. There are even some features to understand the solution process through a step by step approach.
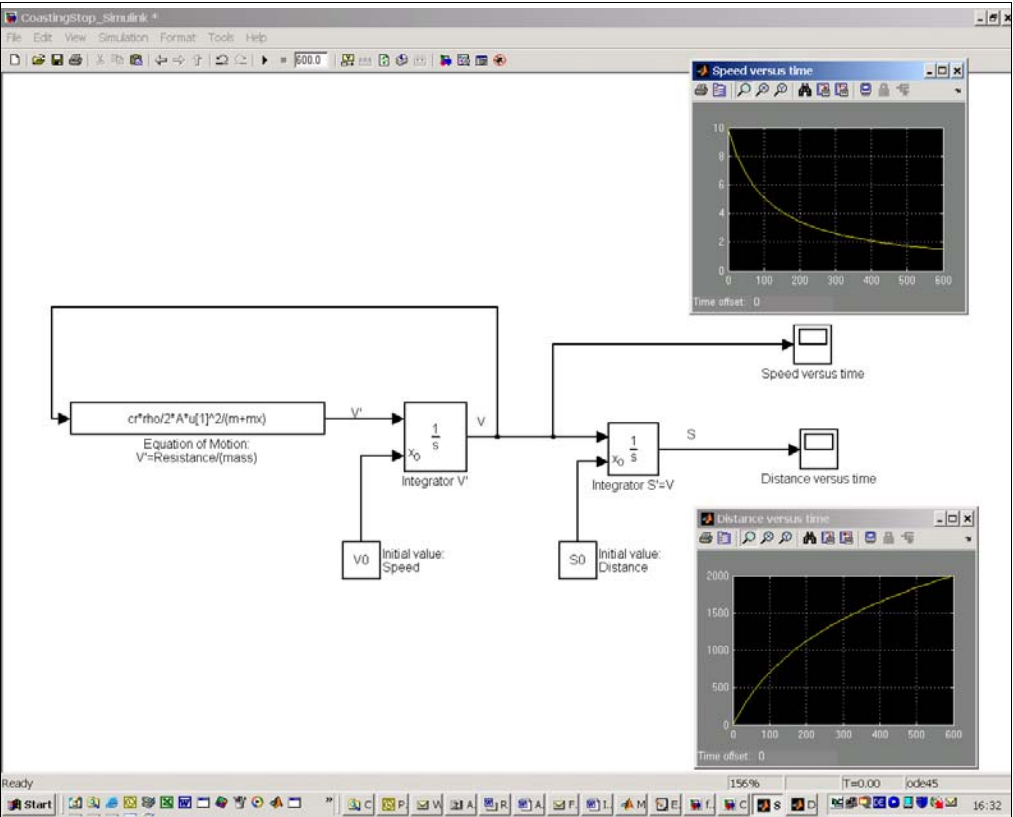

Figure 1: Graphical presentation of blocks in SIMULINK modelling a Coasting Stop Manoeuvre with the results for graphs of speed and distance versus time

1.3 Acceleration and Crash stop manoeuvre

For the complete speed behaviour the thrust T has to be added to the equation of motion:

$$\left(m + m_x\right) \cdot \frac{dv}{dt} = T(1 - t) + R$$

There are two options for a Thrust model:

a) Using the equation $T = K_T(J) * \rho * D^4 * n^2$

where D is the propeller diameter, n revolutions, ρ water density;

K_T – Thrust coefficient versus advance coefficient $J = V_A /(n*D)$;

The thrust coefficient $K_T(J)$ can taken from experiments as open water propeller diagram which is suitable to represent manoeuvring conditions for constant speed and small accelerations/decelerations between speed ranges from zero to service speed.

b) Using the equation $T = A_T(\beta_P) * \rho * \left[V_A^{\,2} + V_U^{\,2}\right] * D^2$ where

$V_A = V * (1 - w)$ - Propeller advance speed in the ships wake

$V_U = 0.7 * \pi * n * D$ the circular speed of the blade at 0.7 of the radius

$\beta_P = \arctan(V_A / V_U)$ - Propeller advance angle related to the speed components

The thrust coefficient AT(ß) can also taken from experiments which are suitable to represent all manoeuvring conditions for constant speed between speed ranges from zero to service speed and even astern motion and acceleration/deceleration manoeuvres up to crash stop manoeuvres.

Now the structure of the model in SIMULINK will be described and the results of an acceleration manoeuvre will be explained as on example.

Figure 2 shows the overview of the block model which is clearly arranged according to the SIMULINK principle: the input signals (commanded propeller revolutions) controls the block for ships dynamic which is modelled as a so called subsystem, the output of this block is send to be displayed on a scope block.
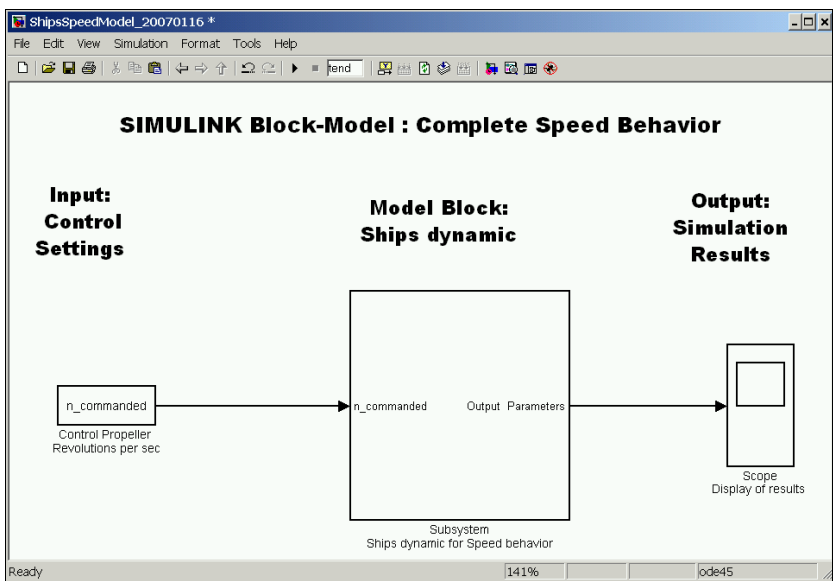

Figure 2: Graphical presentation of blocks in SIMULINK modelling manoeuvring behaviour for complete speed characteristic – Overview model

By clicking onto the subsystem block a window opens representing the details of the dynamic model which looks very similar to the model of the coasting stop, but this time the equation of motion is represented by the respective subsystem. The two inputs state/variable V and the

control n are combined in a multiplexer to be available in both the formula for resistance and the subsystem for thrust as inputs u(1) and u(3), the x coordinate is u(2).

The subsystem for calculation of thrust represents the formulas given for the option b) above. The data for the AT curve are modelled by a look-up table which shows the characteristic graph; alternatively a more simplified approach as a sinus curve can be used by a simple exchange of both block models.
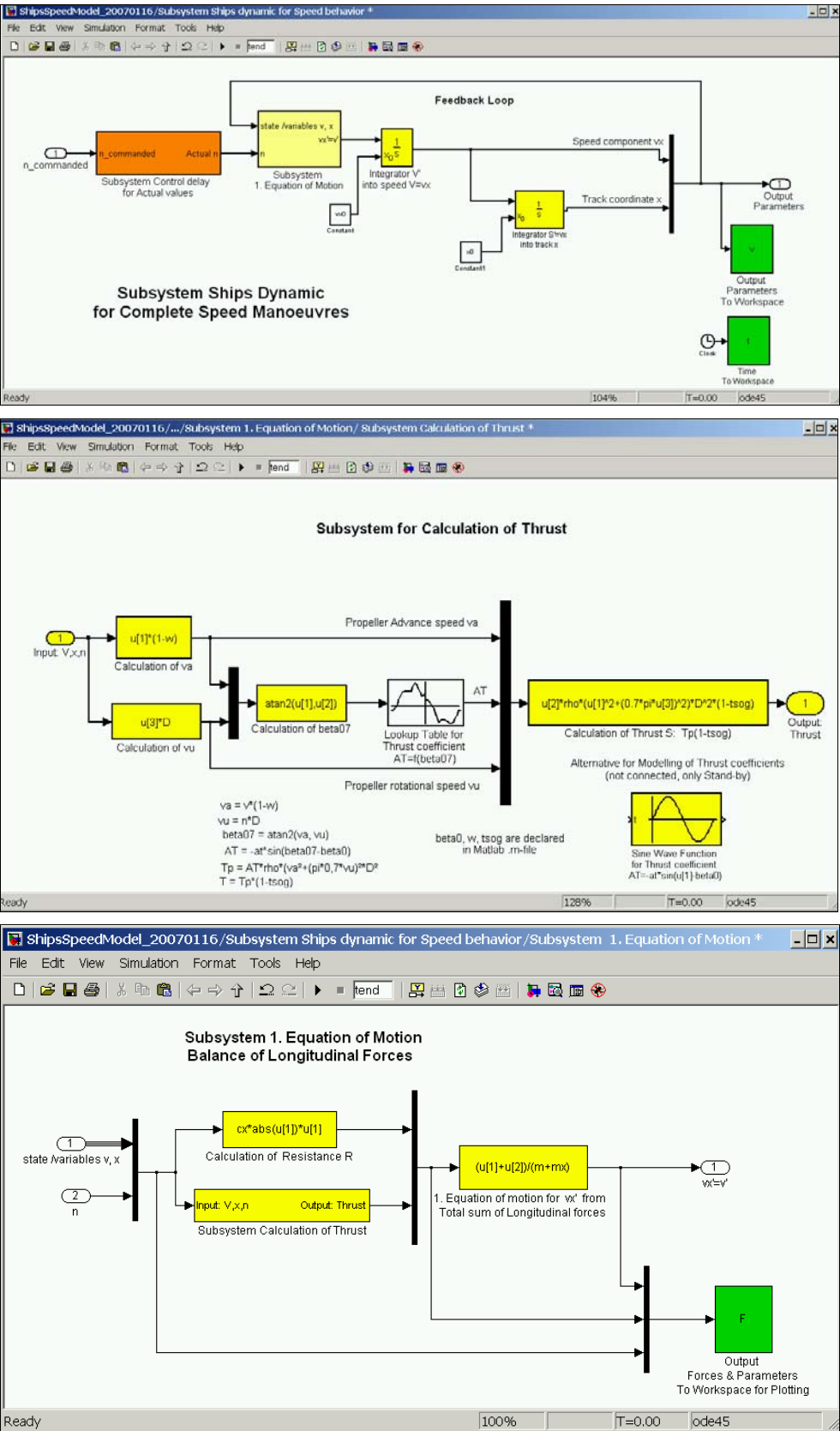

Figure 3: Graphical presentation of blocks in SIMULINK modelling the manoeuvring behaviour for complete speed characteristic – examples for subsystems

The results of the simulation can be displayed either in the scope blocks directly or presented via a MATLAB figure. In Figure 4 the characteristic shape of an acceleration manoeuvre can be seen. After setting a certain revolution n the thrust will increase and therefore acceleration will start. With

increasing speed the resistance will increase and when both the thrust and the resistance have equal magnitudes the ship will proceed with constant speed in steady state motion.
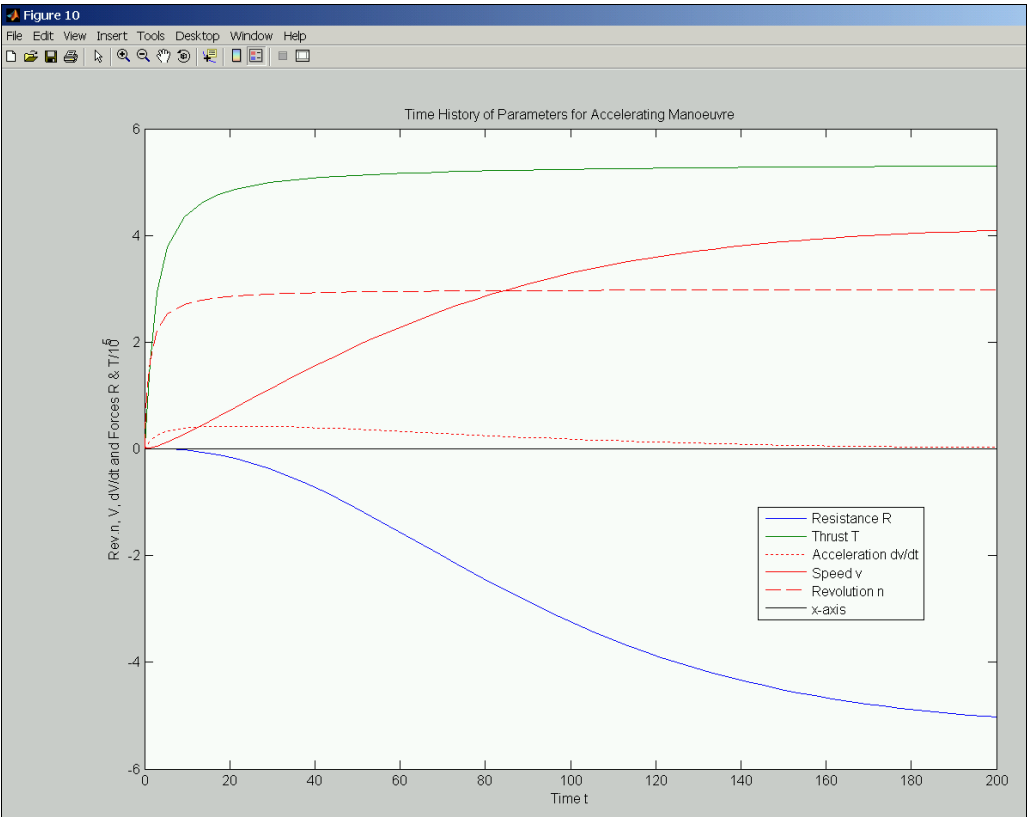

*Figure 4:* Results of SIMULINK modelling for complete speed characteristic

– Acceleration manoeuvre from stop -

2. Model Approach for Steering Manoeuvres

2.1 Equilibrium of forces - Equations of Motion

Extending the equation for three degrees of freedom (3 DOF) gives access to discuss steering motion of ships. Simplified modelling of ships forces acting on the vessel additionally to resistance and thrust, namely steering forces at rudder and transverse forces and moments on ships hull completes the set of equations.

Here are the definitions of motion parameters:
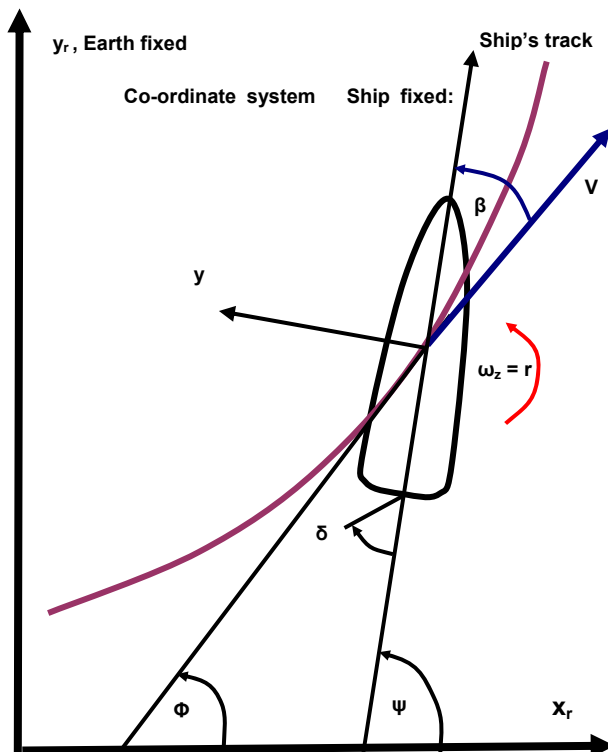
δ Rudder angle

β - Drift angle ($\beta= \psi-\Phi$)

ψ - Heading

Φ - Course

r - rate of turn (r=ω_z=dψ / dt)

Types of motion:

Translation: vector of ships translation with components:

$$V = \begin{bmatrix} v_x = surge \\ v_y = sway \\ v_z = heave \end{bmatrix} = \begin{bmatrix} V \cdot \cos \beta \\ -V \cdot \sin \beta \\ 0 \end{bmatrix}$$

Rotation: vector of ships rotation with components

$$\omega = \begin{bmatrix} \omega_x = roll \\ \omega_y = pitch \\ \omega_z = yaw \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{d\psi}{dt} \end{bmatrix}$$

The equations of motion read as follows

Longitudinal Forces in x-direction:

$$(m + m_x) \cdot \frac{dv_x}{dt} - (m + m_y) \cdot v_y \cdot \omega_z = F_x$$

Longitudinal Forces in y- direction:

$$(m + m_y) \cdot \frac{dv_y}{dt} + (m + m_x) \cdot v_x \cdot \omega_z = F_y$$

Yawing moments around z-axis:

$$(I + I_z) \cdot \frac{d\omega_z}{dt} + (m_y - m_x) \cdot v_x \cdot v_y = M_z$$

In the last equation it is common to represent the unstable moment term together with the moment on the right side. To model this equation in MATLAB/SIMULINK it is suitable to arrange the terms in the so called Cauchy - Form where the first derivatives are at the left side only:

$$x' = f(x,t)$$

This means for the full set of equations of motion:

$$v_x' = \frac{m + m_y}{m + m_x} * v_y * \omega_z + \frac{F_x}{m + m_x}$$

$$v_y' = -\frac{m + m_x}{m + m_y} * v_x * \omega_z + \frac{F_y}{m + m_y}$$

$$\omega_z' = \frac{M_z}{I + I_z} = r'$$

Additionally the formulas for heading and the track coordinates are necessary

$$\psi' = \omega_z = r$$

$$x_r' = \sqrt{(v_x^2 + v_y^2)} * \cos\phi$$

$$y_r' = \sqrt{(v_x^2 + v_y^2)} * \sin\phi$$

$$\phi = \psi - \beta$$

where

with $\beta = \arcsin(v_y / V)$

and $v_y = -V * \sin\beta$ with $V = \sqrt{v_x^2 + v_y^2}$

The external forces F_x and F_y as well as the yawing moment M_z depend on the variables v_x, v_y, β, r and on the control parameters rudder δ and propeller revolution n.

The modelling of these equations follows the same principles as in the foregoing example for speed characteristics, but additionally we have two more equations.
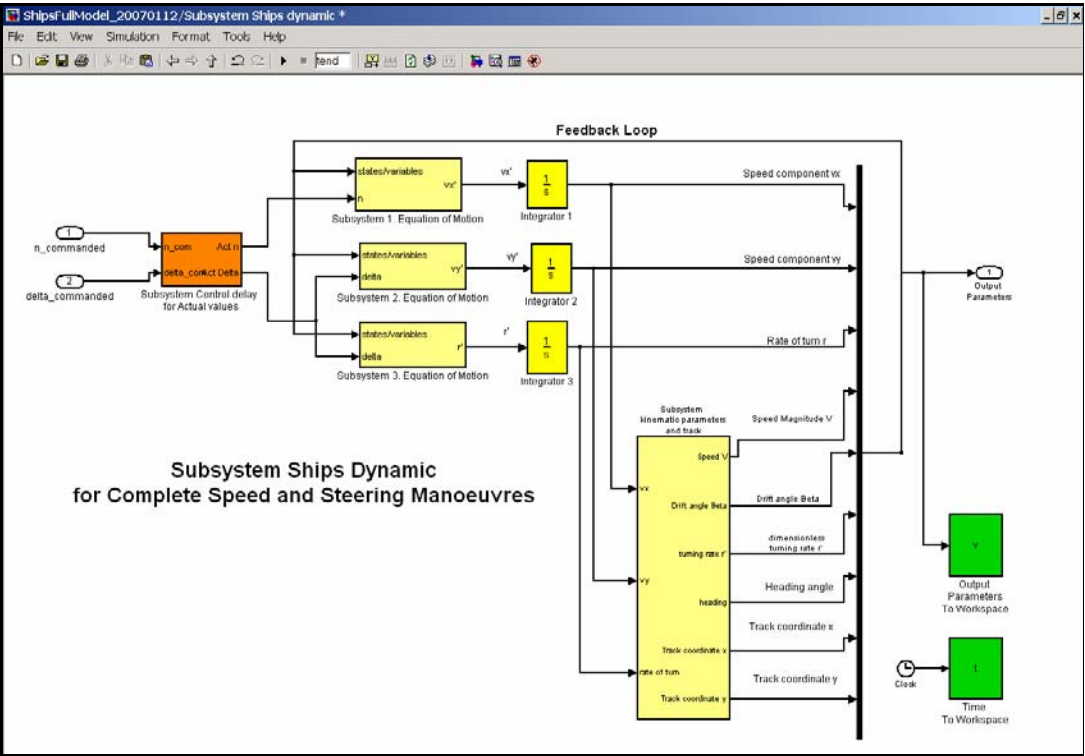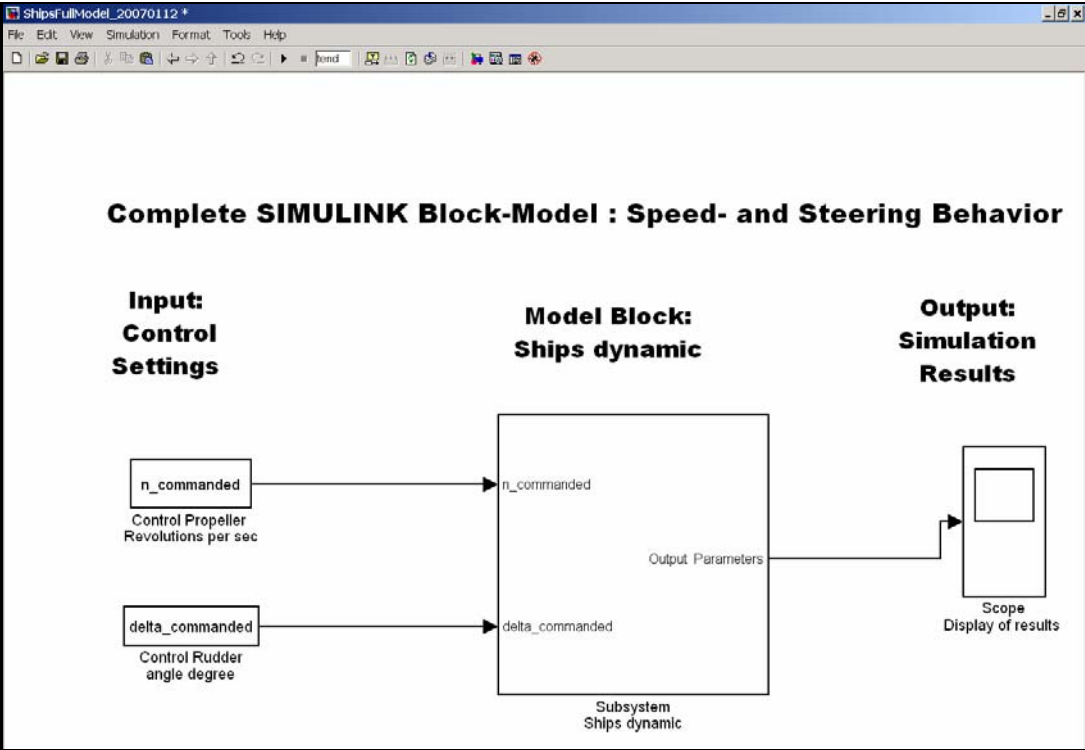

Figure 5: Graphical presentation of blocks in SIMULINK modelling complete steering characteristic
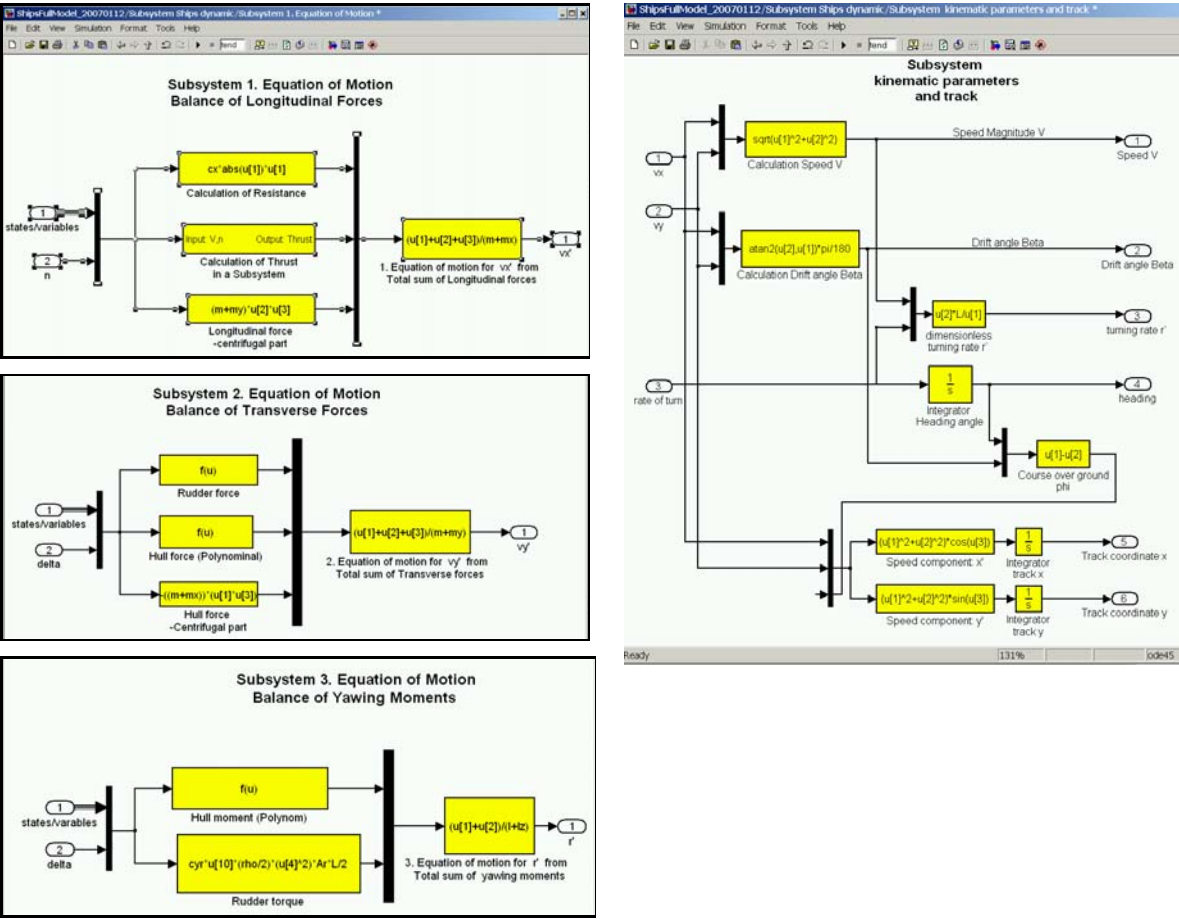
–Model Overview and main subsystem

Figure 6: Graphical presentation of blocks in SIMULINK modelling complete steering characteristic –Subsystems for the three equations of motion and kinematical calculations
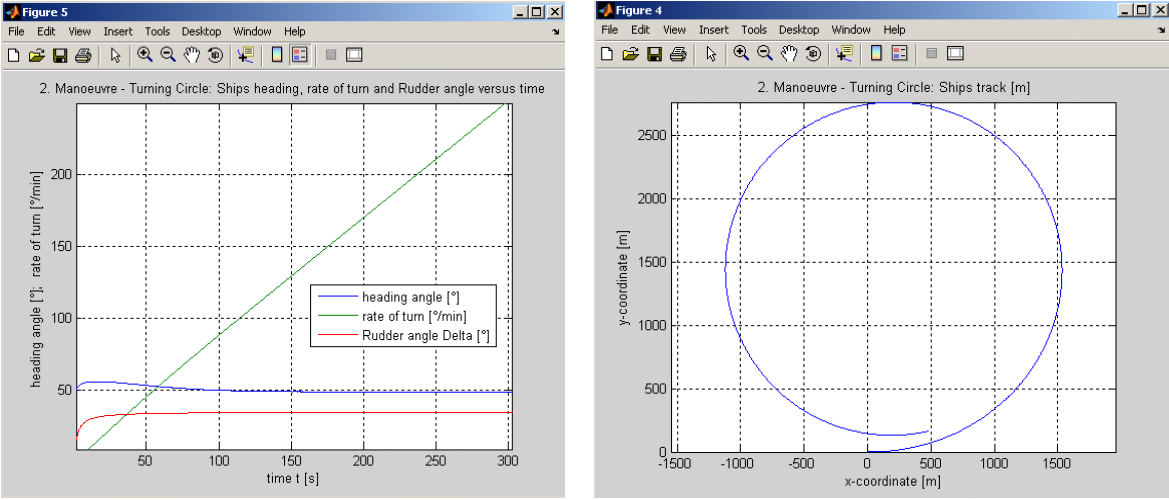

Figure 7: Results of SIMULINK modelling for complete steering characteristic – Turning circle manoeuvre with full speed: Time history of rudder, heading and rate of turn (left) and Turning circle track - Path of ships centre of gravity (right)

2.2   Fast time manoeuvring Simulation program

Whereas the foregoing programs are dedicated to the basic explanation of mechanical formulas and the detailed insight of effect of forces and learning about the programming of dynamical systems with a high degree of students' involvement - the following enhanced program package SIMOPT & SIMDAT is a professional tool to investigate and demonstrate high level Ship handling characteristics. SIMOPT & SIMDAT are used in parallel to the Ship handling simulator to optimise

the simulator ship models dynamic in order to tune them as good as possible to the real vessels. Additional demonstrations are very helpful and allow for a discussion of yaw stability of ships and its impact on steering quality as well as optimization for different types of manoeuvres by means of the results from fast time Simulation from several manoeuvres.
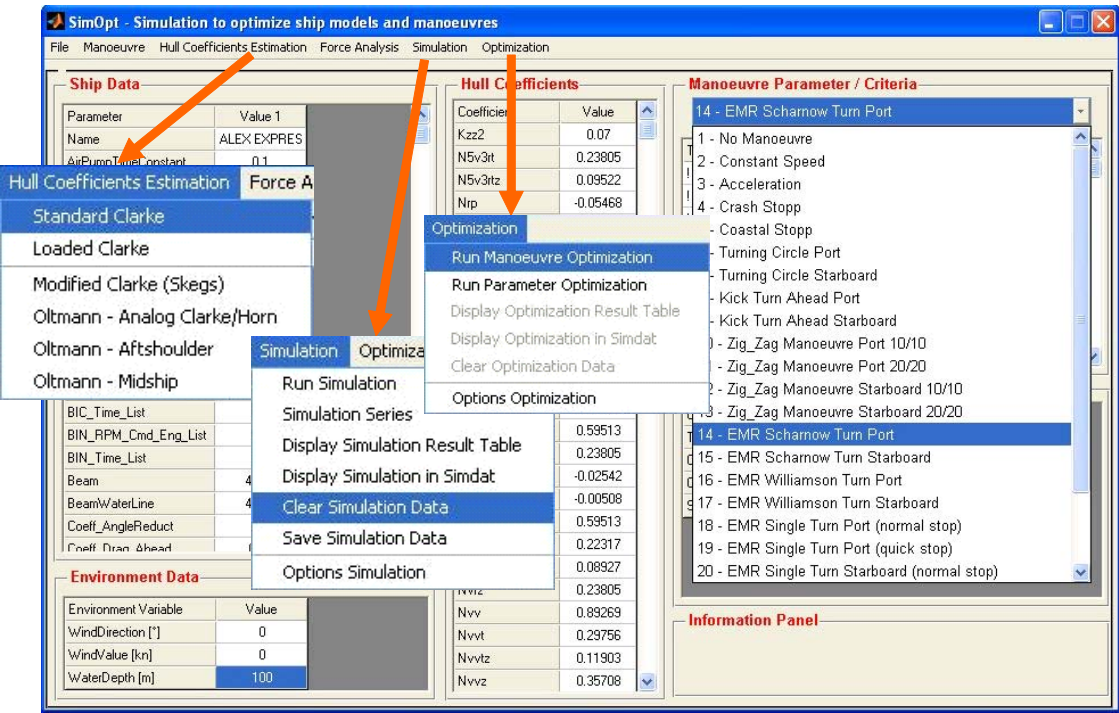

Figure 8: SIMOPT Interface Elements – Top Menus: Detailed Selection of Simulation and Analysis Elements from several menus

Figure 8 gives an overview about the main menu indicating all the options to load ship data, calculating the parameters to models the ships motion characteristics and to run single manoeuvres or series of manoeuvres from a pop up window table. This will be used e.g. to create a set of turning circles for the ships manoeuvring booklet or to optimise the procedure and parameters of a person over board manoeuvre, see Figure 9.
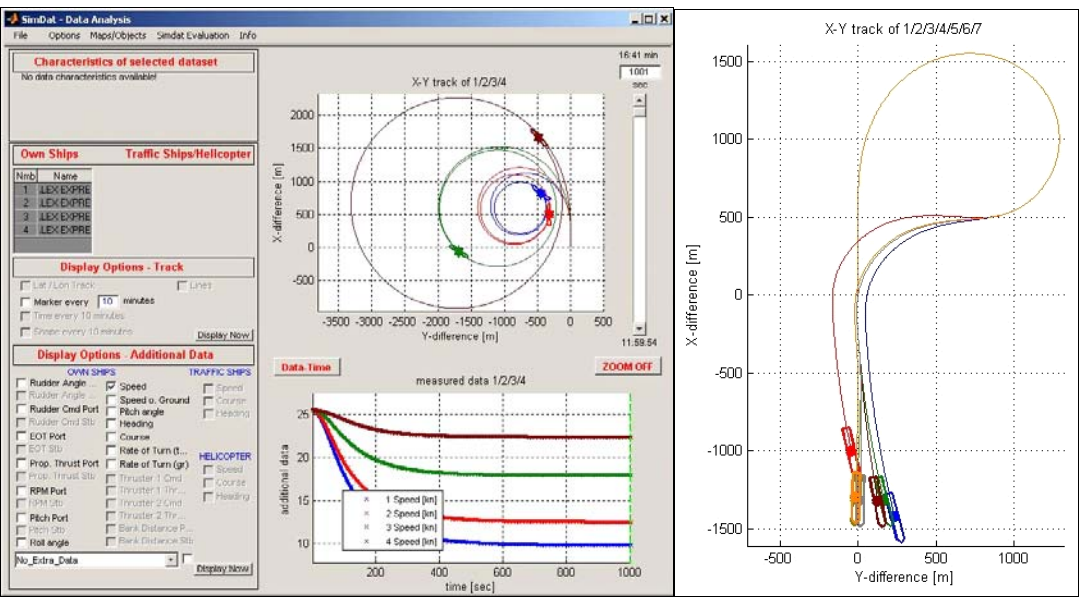

Figure 9: SIMOPT Simulation result analysis in plots:

Left: Main interface and results for turning circle series varying rudder angles

Right: SIMOPT to optimise Emergency Return Manoeuvre „Scharnow-Turn"-Optimisation (Series for different heading changes for counter rudder)

Zig-Zag-Tests (Figure 10) are a common method for investigation of overswing characteristics in order to check for compliance with the IMO standards for manoeuvring (IMO, 2002). In case of yaw instability a spiral test could be necessary as a mean to find out the potential range of yaw instability. In Figure 11 there are shown the results of a spiral test indicating that the ship is stable.
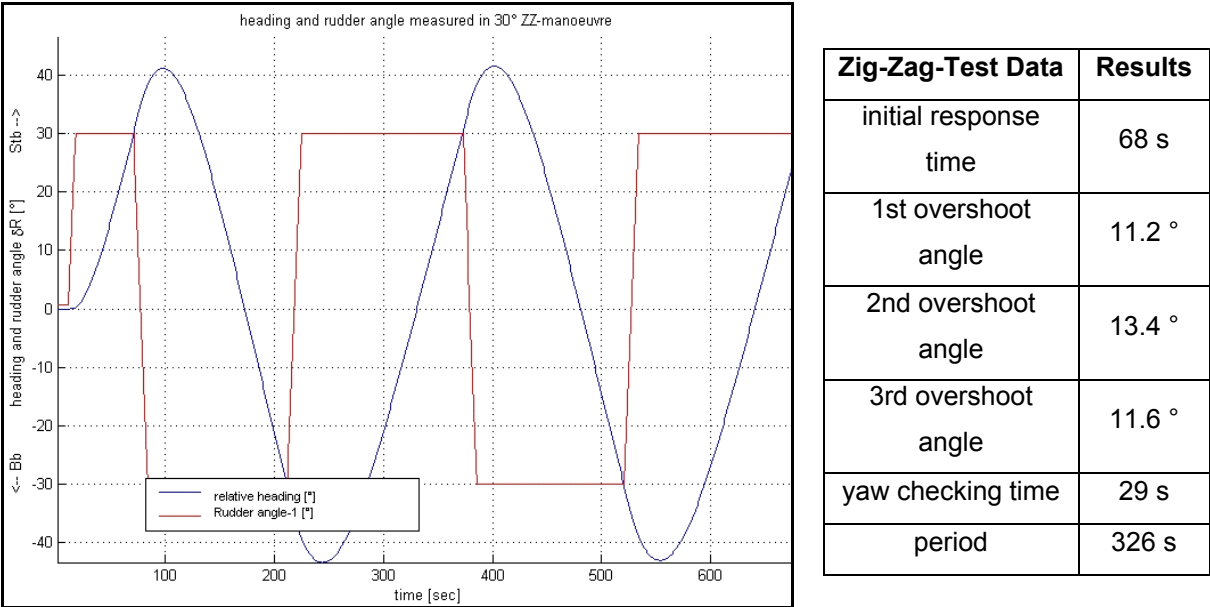

Zig-Zag-Test Data	Results
initial response time	68 s
1st overshoot angle	11.2 °
2nd overshoot angle	13.4 °
3rd overshoot angle	11.6 °
yaw checking time	29 s
period	326 s

Figure 10: SIMOPT Simulation result analysis in plots: Time history of rudder and heading change for Zig-Zag-Test and result table

Simulation/Optimization Results

Maneouvre Type

Turning Cirlce

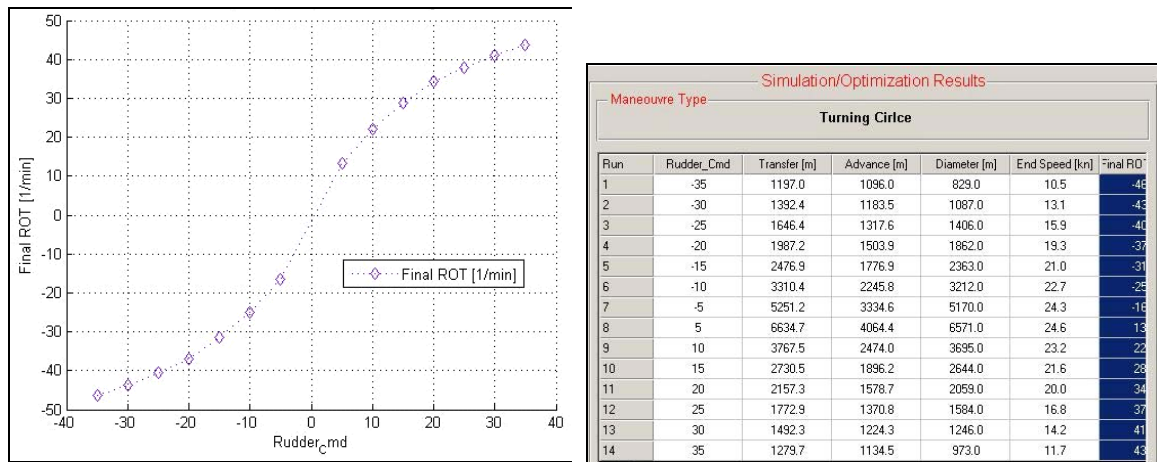| Run	Rudder_Cmd	Transfer [m]	Advance [m]	Diameter [m]	End Speed [kn]	Final ROT
1	-35	1197.0	1096.0	829.0	10.5	-46
2	-30	1392.4	1183.5	1087.0	13.1	-43
3	-25	1646.4	1317.6	1406.0	15.9	-40
4	-20	1987.2	1503.9	1862.0	19.3	-37
5	-15	2476.9	1776.9	2363.0	21.0	-31
6	-10	3310.4	2245.8	3212.0	22.7	-25
7	-5	5251.2	3334.6	5170.0	24.3	-18
8	5	6634.7	4064.4	6571.0	24.6	13
9	10	3767.5	2474.0	3695.0	23.2	22
10	15	2730.5	1896.2	2644.0	21.6	28
11	20	2157.3	1578.7	2059.0	20.0	34
12	25	1772.9	1370.8	1584.0	16.8	37
13	30	1492.3	1224.3	1246.0	14.2	41
14	35	1279.7	1134.5	973.0	11.7	43

Figure 11: SIMOPT Simulation result analysis in plots: Spiral test plot (final rate of turn versus rudder angle) for a yaw stable ship and extract of Turning circle data

3. Model Approach for Ships Rolling Motion

3.1 Equilibrium of Moments - Equations of Motion

The same tools are now used to investigate rolling motion of ships:

(a) The rolling motion dynamic will be modelled in a simplified way to get some insight into the motion dynamics and the effect of ships

(b) A professional software ARROW was developed for education and teaching maritime students and for practical use in decision making on board to avoid synchronous or parametric resonance of ships in waves by loading and ballasting operations for changing ships stability or using speed and course changes.

The equation of motion for the ships rolling represented by the balance of moments around the longitudinal x-axis of the ship reads

$$\left(I_x + I_{xx}\right) \cdot \frac{d^2\Phi}{dt^2} + d \cdot \frac{d\Phi}{dt} + D \cdot g \cdot GZ(\Phi) = P(\alpha, t)$$

where:

I_x, I_{xx} is the moment of inertia around x-axis of ships mass and added mass due to angular acceleration respectively

d – damping coefficient

D – displacement mass of ship

G – gravity constant

GZ – uprighting lever of restoring moments

P(α, t) – external moment, e.g. due to wave slope angle α (wave steepness) and time t

Division by I_x+I_{xx} produces a result in the more suitable form:

$$\Phi'' + \delta \cdot \Phi' + \frac{g}{k^2} \cdot GZ(\Phi) = P(\alpha, t)$$

with k - radius of inertia ≈ 0,37 to 0,5 • B, normally represented by k=Cr • B, where Cr is the inertia coefficient for rolling motion and B is the ships beam.

The result of this differential equation specifically for the steady state motion with small rolling angles Φ is well known to ships officers: For small rolling angles we can replace GZ by using the metacentric height GM: GZ(Φ)=GM• sinΦ, which can be further simplified by sinΦ ≈ Φ:

$$Tr = \frac{\pi \cdot C_r \cdot B}{\sqrt{g \cdot GM}}$$

and with $\pi \approx \sqrt{g}$ we get the final equation for the natural roll time period Tr of the ship:

$$Tr = \frac{C_r \cdot B}{\sqrt{GM}}$$

For explaining the rolling dynamics to students the process of rolling motion as time history and to demonstrate the effects of wave excitation the equation has to be arranged in the following format:

$$\Phi'' = -\delta \cdot \Phi' - \frac{g}{k^2} \cdot GZ(\Phi) + P(\alpha, t)$$

In order to simplify the calculation the exiting effects from the waves are assumed as heeling moment due to the wave slope angle α with the same size as the uprighting moment GZ from the roll angle Φ. Thus, the equation reads:

$$\Phi'' = -\delta \cdot \Phi' - \frac{g}{k^2} \cdot GZ((\alpha(t) - \Phi)$$

This equation is used for the SIMULINK-model. Additionally the effect of linearization will be demonstrated by using non-linear uprighting lever curves for GZ. All of the curves have the same data for the linear range -10°<Φ <+10°:
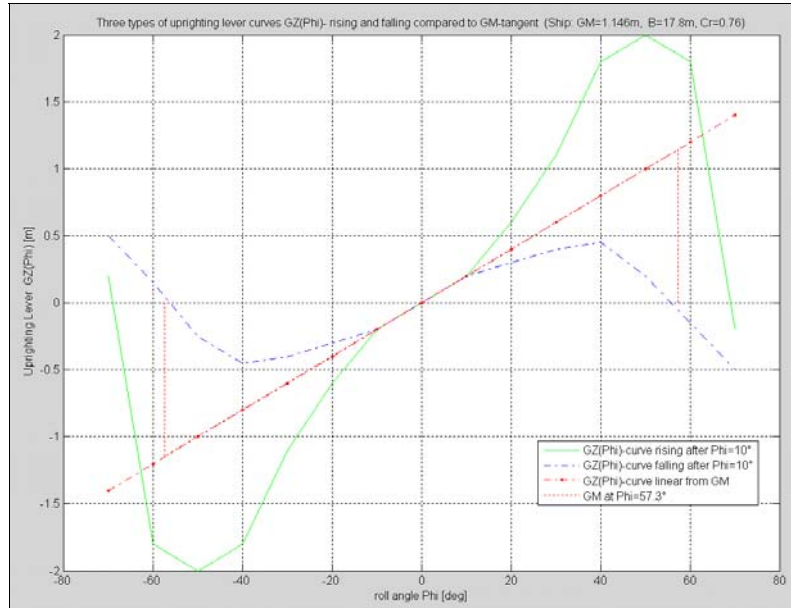

Figure 12: Three types of up righting lever curves GZ(Phi) versus roll angle - rising and falling compared to GM-tangent for SIMULINK modelling

3.2   Modelling of Roll Equation in SIMULINK

The modelling of the equation follows the same principles as in the foregoing example for speed characteristics:

Figure 13 gives an the overview of the block model which is clearly arranged according to the SIMULINK principle: the input signals (wave excitation from wave slope angle) triggers the block for ships roll dynamic which is modelled as a subsystem, the output of this block is send to another subsystem to be displayed and stored.

The subsystem block for ships roll dynamic represents the equation above with the two elements of damping term and the restoring/exiting term modelled by function blocks. The outputs of both blocks are combined by a multiplexer to be available in the function "Equation of Motion" as inputs u(1) and u(2). The result / output of this equation will be integrated two times in the following integrators. The constant block "Initial roll angle" allows for the generation of an initial roll angle. As a result we get the roll angle, to be scaled by the gain block from radiant to degree. The roll angle is used as feedback loop to the input on the left side where in the sum block the signal will be generated as input to the look up table block for the GZ curves. By means of the variable sel_curve the respective GZ curve will be used.

Very efficient for evaluating the oscillations with respect to the roll periods are the blocks for "hit zero crossing detection" in the subsystem "Display and storing of results"
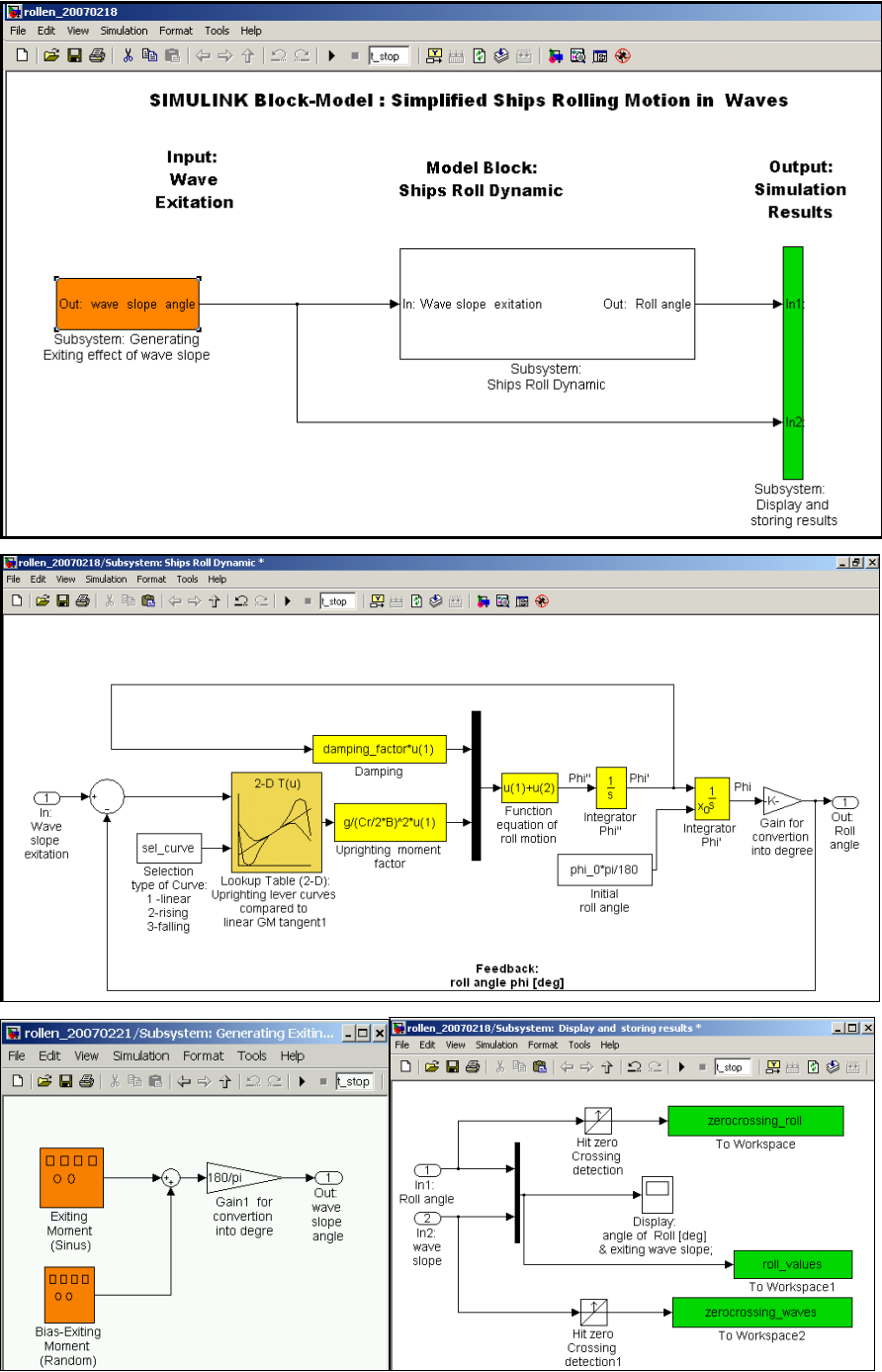
Figure 13: Graphical presentation of blocks in SIMULINK modelling the rolling motion behaviour - Overview block and examples for subsystems -

3.3 Results for Roll Experiments

Results for ships rolling motion without waves:

For demonstrating the dynamic characteristic of the ships rolling motion the behaviour after a certain initial disturbance was calculated: the ship starts from an initial heeling and then it was released - the time history of rolling is displayed in Figure 14, additionally the roll time periods Tr_sim were measured during the oscillations:

- Due to the damping effect the amplitudes of the oscillation decreases.
- For the GZ curves with rising characteristic according to Figure 12 the roll time period Tr_sim is getting smaller because of the larger restoring moments for higher roll angles
- For the GZ curves with falling characteristic the roll time period Tr_sim is getting higher.
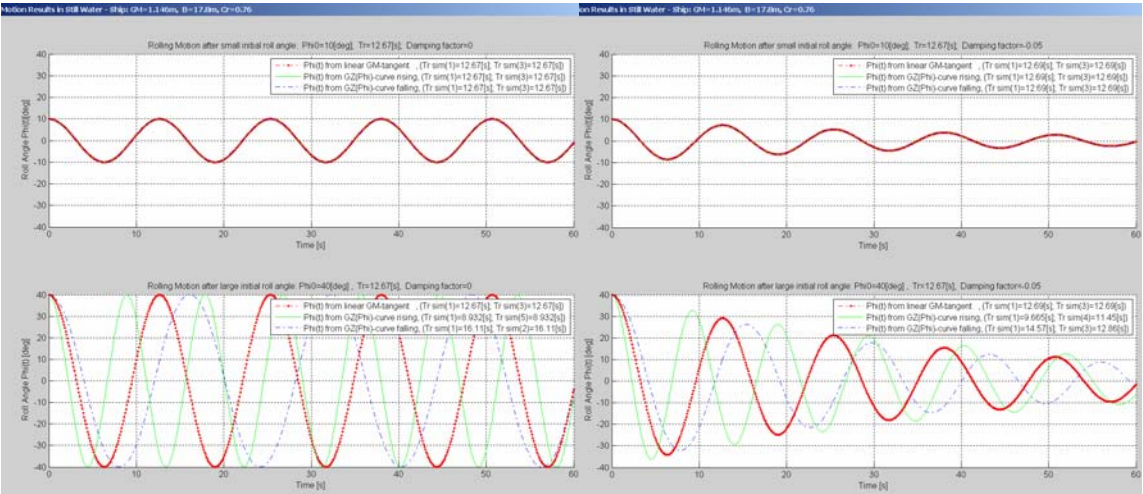
Figure 14: Results for ships rolling motion without waves in still water with small and high initial roll angle – without damping (above) and with damping (below)

Results for ships rolling motion in waves

If the ship is in rest at Φ=0 and then exited by waves the oscillations can be seen in the following figures indicating the effect of exciting amplitude, damping and natural period:

If the ship is in resonance condition i.e. the ships natural period is in coincidence with the exciting period the rolling amplitudes will increase.

In Figure 15 the results for ships rolling motion in waves are compared for conditions with two different wave periods and small damping:

If the ship is in resonance in "linear conditions" (upper part of the figures) where the period Tr(10°)=12.67s for linear GZ curve is equal to the wave period Tw then the amplitude for a small excitation of wave slope angle 2.865° is rising up to 24°. It is limited because for larger roll angles the ship has the roll time period of Tr(40°)=9s and so it leaves the resonance ratio=1. If the ship is excited by Tw=9s then for smaller wave amplitudes the responses of the ship are not high enough to get into resonance.

The higher excitation with wave slope angle 5.73° in figure b) below normally leads to respective larger roll angles but for the ship with roll time period of Tr(40°)=9s it ends up in a capsizing!
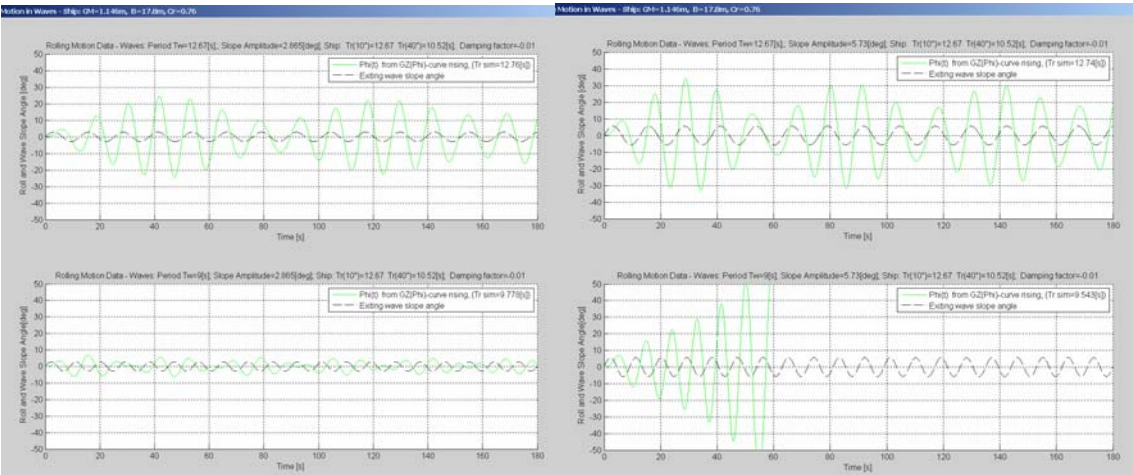

Figure 15: Results for ships rolling motion in waves with two different wave periods: a) small excitation / slope amplitudes (above) and b) high excitation/slope amplitudes) (below)

3.4 ARROW – SOFTWARE PROGRAM

OVERVIEW

The ARROW program is a software tool to estimate and display the potential conditions and countermeasures to <u>A</u>void <u>R</u>olling <u>R</u>esonances <u>O</u>r <u>W</u>ave impacts on ships due to specific wave encounter situations. By means of the Main user interfaces (Figure 16) the a small amount of data needs to be entered into the areas of the Ship Parameter Input (top left side) and Wave Parameter Input (lower left side) to provide the qualitative results in the Result Display Area (right hand side):
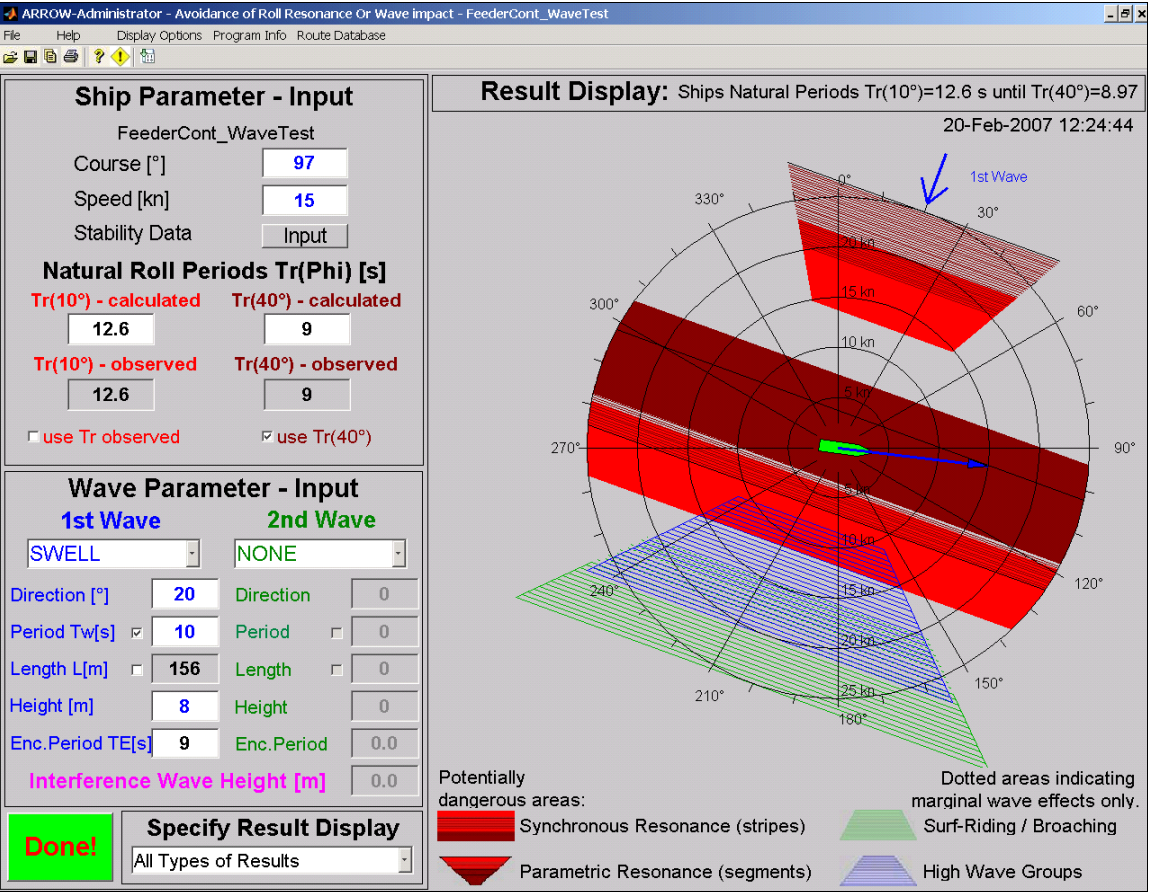

Figure 16: ARROW program window – overview on main user interface elements

PARAMETER INPUT

Ships course and speed can be entered in the respective data fields. The heading direction of the ships contour and speed vector in the Result Display will immediately change according to changes in the data fields. Alternatively these values can be set by mouse click (right button) into the ARROW - Result Display Area.

Natural Roll periods of the ship can be either calculated (a) by using stability data or alternatively (b) by entering observed roll periods directly.

For Wave Parameter Input the ARROW program accepts the input of two different wave systems. Only a few wave input parameters, taken either from observations on the ship or from weather reports and forecasts, have to be entered in the respective fields. The direction of the first wave system is drawn as a blue arrow outside of the polar diagram in the ARROW - Display Area (s. Figure 16), the second system as green arrow. Another arrow shows the direction of the interference wave.
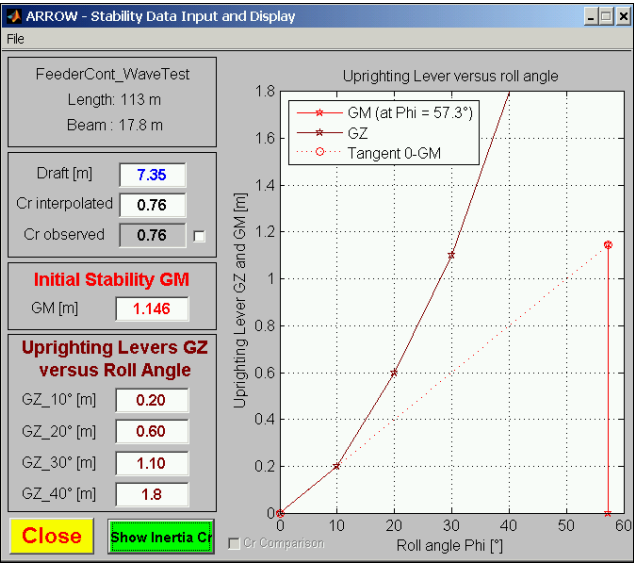
Figure 17: ARROW program stability input window – with the same ship stability data as for the roll simulation experiments (see Figure 12)

Results and Connection to Weather Routing Program

In the ARROW program direct methods are used to calculate the rolling periods Tr(10°) for small and Tr(40°) for high roll angles (Benedict, 2003). By using the stability data of the ship (see Figure 12) the same natural roll periods are calculated in ARROW with direct methods both for small and high roll angles as measured from the simulation experiments in Figure 14.

The result display in Figure 16 shows the potential danger of the wave effects: it is indicating synchronous resonance for encounter period equal to ships natural period Tphi(40)=9s (for the example in use: stability curve rising): on course at speed we experience an encounter period of 9 s which is equal to the ships natural period.

The data for the sea state conditions can be entered manually for any situations possible, but it is more effective to directly transfer the data from voyage planning and ships routing programs to ARROW evaluations. Therefore an interface was built to the AWT software program "Bon Voyage" (Figure 18) which can be efficiently used to check the route plan or to optimise the ships voyage with respect to avoid dangerous wave encounters (Benedict et al. 2006, August).
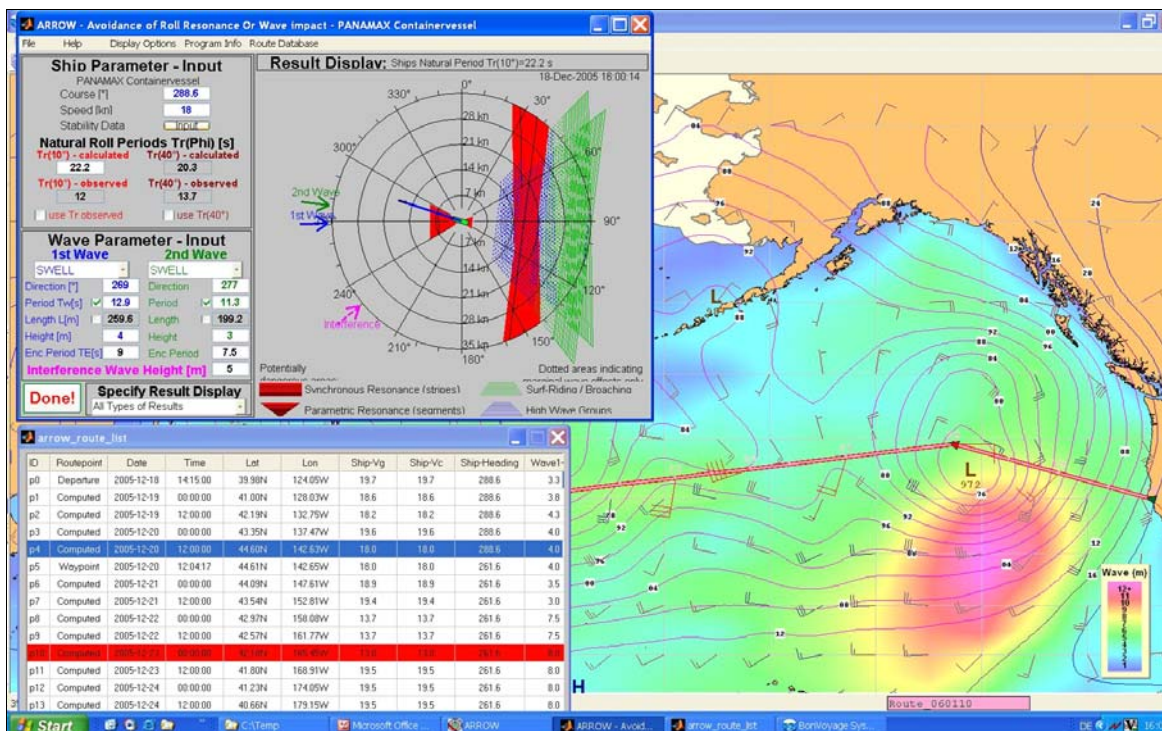
Figure 18: Use of ARROW program together with onboard routing system "Bon Voyage" (AWT): Avoiding Resonance due to change of the ships route by shifting of one waypoint on Rhumb line and by Changing Stability from rolling period Tr=18.2s to Tr=22.2s

4. References

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