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Modeling and Trajectory Control of an Autonomous Sailboat for Path Planning

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Abstract

Payload directed flight research focuses on closing control loops around sensor data to optimize mission objectives for manned and unmanned vehicle systems. A critical element in optimized sensor based control is the ability to plan trajectories based on expected sensor return. This paper describes the modeling and simulation of a sailboat for the implementation of a new trajectory planning algorithm. The trajectory planning algorithm is a closed loop system, based on probabilistic roadmaps and rapidly expanding root tree algorithm. The sailboat is modeled as a 5 Degree of Freedom (DOF) system. The control system design describes the implementation of a sail, heading command and waypoint tracking system. Simulation and testing of the model and controller are also discussed.

Nomenclature

α = angle of attack
ϕ = heel/ roll angle
ψ = heading/ yaw angle
β = relative wind angle
δ_r = rudder deflection angle
δ_s = sail angle
τ = torque
ρ = density
m = mass
g = gravitational acceleration
l_1 = horizontal distance between CP of sail and COE of hull
l_2 = distance between center of gravity and COE
l_3 = distance between center of buoyancy and COE
l_4 = lateral horizontal distance between CP of sail and COE of hull
B = buoyant forces
CP = center of pressure
COE = center of efforts
C_L = lift coefficient
C_D = drag coefficient
F = force
WA = world axis
BA = body axis
N_r = Yaw rotational dynamic resistance
N_u = Rotation due to rudder
N_v = Rotation due to lee helm
N_p = Roll rotational dynamic resistance
$\boldsymbol{P}$ = position vector
RW = relative wind
R = rotation matrix
TW = true wind
$\boldsymbol{V}$ = Velocity
X_u = Forward velocity lumped drag
Y_v = Tracking/Sideslip lumped drag
x, y, z = body axis North, East, Down positions
u, v, w = body axis North, East, Down velocities
X, Y, Z = world axis North, East, Down positions
p, q, r = body axis angular rates along *x, y, z*, respectively

1 Introduction

Payload Directed Flight (PDF) research at NASA Ames Research Center focuses on maximizing payload sensor feedback data for satisfaction of mission objectives. A major focus of sensor based control is determining optimal trajectory planning algorithms that can take into account sensor positioning in the optimization. Though PDF is focused on research that directly applies to subsonic fixed wing aircraft, the algorithms need to be broad enough to be applied to other types of systems. This paper focuses on development of a test platform for validation of trajectory planning algorithms in support of PDF. The vehicle platform chosen for this is a radio control sailboat. Unlike fixed-wing aircraft, a sailboat is completely dependent on the environment for propulsion. Changes in the external conditions pose challenges for trajectory planning, and hence make the sailboat an ideal platform for validation of the trajectory planning algorithm.

2 Modeling

2.1 Axis Systems Definitions

The sailboat model assumes an orthogonal world axis (*WA*) system that is earth fixed and inertial (earth rotation is neglected), with a flat-earth assumption. The *WA* system is aligned with *X*-north, *Y*-east/right, *Z*-down. The sailboat dynamics are modeled with a 5 Degree of Freedom (DOF) model that allows for translation and rotation (roll) along the world *X* axis, translation along world *Y* axis, translation and rotation (yaw) about the world *Z* axis. The sailboat's body axis (*BA*) system's origin is fixed (translates and rotates) to the center of rotation of the sailboat, and moves and translates with the vehicle. The body axis system is aligned along the boat with *X*-forward, *Y*-right, and *Z*-down. The relative wind axis (*RW*) system is defined by an *X*-axis vector pointing opposite of the incoming wind vector. Figure (1) describes the axes systems and force definition

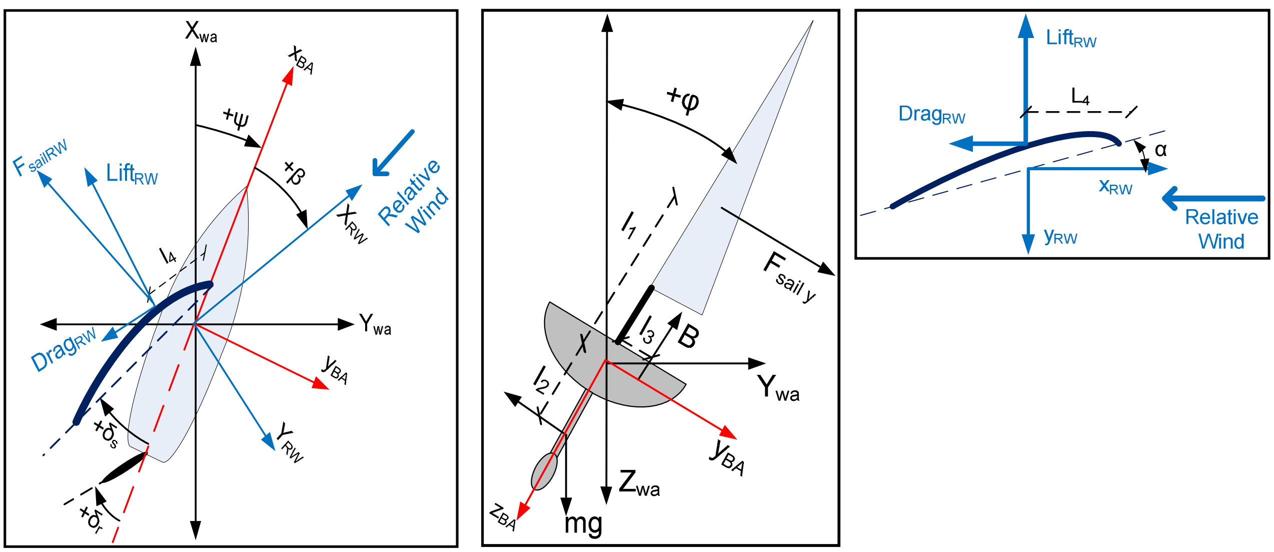

Figure 1: Axis System and Definitions

The transformation matrices between the body and world axis systems shown in [2] are given by equation (1)

$$R_{\phi_{WA\rightarrow BA}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \qquad R_{\psi_{WA\rightarrow BA}} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

The rotation matrices are orthogonal and hence the transformation matrices between world and body axis systems are described by equations (2) and (3)

$$R_{WA\to BA} = R_{\phi_{WA\to BA}} \cdot R_{\psi_{WA\to BA}} \tag{2}$$

$$R_{BA\to WA} = R^{T}_{WA\to BA} \tag{3}$$

The transformation matrix between the relative wind and body axis systems is given by equation (4) as

$$R_{RW\to BA} = \begin{bmatrix} \cos(\pi/2-\beta) & sin(\pi/2-\beta) & 0 \\ -sin(\pi/2-\beta) & cos(\pi/2-\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

The true wind vector in body co-ordinates is calculated by transforming the world axis relative wind vector into body axis co-ordinates using the world to body axis rotation matrix.

$$\boldsymbol{V}_{RW_{WA}} = \boldsymbol{V}_{TW} - \boldsymbol{V}_{WA} \quad \& \quad \boldsymbol{V}_{RW_{BA}} = R_{WA\to BA} \cdot \boldsymbol{V}_{RW_{WA}} \tag{5}$$

The wind vector relationships between the relative wind angle and the angle of attack in body axis system, are given by equations (6) and (7) respectively.

$$\beta = \tan^{-1}\left(-\frac{v_{RW_{BA}}}{-u_{RA_{BA}}}\right) \tag{6}$$

$$\alpha = \beta - \delta_s \tag{7}$$

2.2 Sailboat Model

The position, velocity, and angular velocities of the vehicle are represented in terms of vectors aligned with the world axis system. The control input vector $\boldsymbol{u}$ and state vector $\boldsymbol{x}$ are defined in equation (8).

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{P}_{WA} \\ \boldsymbol{V}_{WA} \\ \psi \\ \phi \end{bmatrix} \qquad \boldsymbol{u} = [\delta_r \quad \delta_s] \tag{8}$$

The linear derivatives will be based on the body axis angular and linear velocities, so for convenience, we define the body axis velocity components in equation (9).

$$\boldsymbol{V}_{BA} = R_{WA\to BA} \cdot \boldsymbol{V}_{WA} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{9}$$

The position of the vehicle is governed by the kinematic relationship given in equation (10)

$$\dot{\boldsymbol{P}}_{WA} = \boldsymbol{V}_{WA} \tag{10}$$

The orientation of the boat is defined by the yaw angle which by our convention, aligns with the compass heading of the vehicle and the angular velocity component about the Z axis. The resulting kinematic relation is given by (11)

$$\dot{\psi} = r \tag{11}$$

The translational dynamics of the vehicle are modeled in the body axis system, as the sum of forces acting on the hull and the forces exerted onto the boat by the sail. This is described in equation (12)

$$\dot{\boldsymbol{V}}_{BA} = \frac{1}{m}(F_{hull_{BA}} + F_{sail_{BA}}) \tag{12}$$

The hull and sail force components of the equation are decomposed into x and y direction components given equations (13) and 14)

$$\dot{u} = -\mathrm{X_u} \cdot u^2 \tag{13}$$

$$\dot{v} = -\mathrm{Y_v} \cdot v^2 \tag{14}$$

X_u and Y_v are lumped constant parameters, representing various drag terms caused by translational motion of the boat. This term includes fluid constants such as density of water and wetted area of the hull and keel. The other contributing factors are hydrodynamic coefficients of the hull, coefficient of skin friction and lift and drag coefficients of the hull and keel [2]. Considering a very small area of the hull above the waterline, the aerodynamic drag forces on the hull alone are very small compared to the drag forces acting on the entire sailboat and hence are neglected.

The yaw rotational dynamics of the sailboat are modeled along the z axis in the body co-ordinate system and are represented by equation (15).

$$\dot{r} = \frac{1}{I_{zz}}\left(\tau_{hull_z} + \tau_{rudder} + \tau_{sail}\right) \tag{15}$$

Where,

$$\tau_{hull_z} = -N_r \cdot r \tag{16}$$

$$\tau_{rudder} = sgn(u) \cdot \mathrm{N_u} \cdot \sin\delta_r \cdot u^2 + sign(v) \cdot \mathrm{N_v} \cdot \cos\delta_r \cdot v^2 \tag{17}$$

$$\tau_{sail} = F_{sail_xBA} \cdot l_4 \cdot \sin(\delta_s) + F_{sail_yBA} \cdot l_4 \cdot \cos(\delta_s) \tag{18}$$

Equation (16) represents the viscous resistance to angular velocity of the sailboat. N_r is a lumped constant parameter representing various retardation coefficients caused by rotation of the boat. N_u and N_v are lumped constant terms representing drag force coefficients of the rudder in the x and y directions respectively. Equation (17) represents the overall torque generated by the rudder. The first term of the equation models the torque generated due to rudder deflection. The second term of equation (17) represents lee helm of the boat. The lee helm is defined as the tendency of the sailboat to turn into the direction of the sideslip velocity or head away from the wind, and is caused by the hydrodynamic side forces acting on the rudder, thus creating a moment. Equation (18) represents the weather helm effect of the sailboat. Weather helm is defined as the tendency of the sailboat to head into the wind and is caused by the moment produced by the sail force components [4].

The roll/heel rotational dynamics of the sailboat are modeled along the x axis in the body coordinates. The effect of waves being neglected, the heeling angle is obtained from the simple kinematic relationship defined by equation (19)

$$\dot{\phi} = p \tag{19}$$

The roll rotational dynamics are modeled as

$$\dot{p} = \frac{1}{I_{xx}}\left(\tau_{hull_x} + \tau_{heeling_x} - \tau_{correcting}\right) \tag{20}$$

Where,

$$\tau_{hull_x} = -\mathrm{N_p} \cdot p \tag{21}$$

$$\tau_{heeling_x} = F_{sail_yBA} \cdot l_1 + \dot{y} \cdot l_2 \tag{22}$$

$$\tau_{correcting} = mgl_2 \cdot \sin\phi + \mathrm{B}l_3 \sin\,\phi \tag{23}$$

Equation (20) represents the heeling torque along the x axis. The first term, detailed in equation (21) is the viscous resistance to the roll rate. Equation (22) describes the heeling torque acting on the boat. The first component of the equation represents the heeling moment due to sail force and the second term models heeling force due to the sideslip of the boat, in water. Equation (23) models the righting moments acting on the boat. The first term models the righting moment due to the keel while the second term models the righting moment due to the shift in center of buoyancy. The effect of heel is generally to increase the sideslip resistance, but this effect is nominal for the shape and scale of hull and hence is neglected [3].

The sail is modeled as a wing with a thin airfoil section. The lift and drag forces generated by the sail are calculated based on the aerodynamic equations listed below

$$Lift_{RW} = \frac{1}{2}\rho_{air} C_L |V_{Rw}|^2 \qquad\qquad Drag_{RW} = \frac{1}{2}\rho_{air} C_D\,|V_{Rw}|^2 \tag{24}$$

Where,

$$C_L = C_{L_0} + \frac{\delta C_L}{\delta\alpha}\alpha \qquad\qquad C_D = C_{D_0} + K C_L^2 \tag{25}$$

Based on thin airfoil assumption for the sail a C_{L0} of 0 was chosen and an approximate value of π was used for the lift curve slope [4]. A zero angle of attack drag C_{D0}, of 0.02 was used and a K value of 0.1 was used.

Converting the sail forces to body coordinates,

$$F_{sail_{BA}} = R_{RW\rightarrow BA} \cdot \begin{bmatrix} Lift_{RW} \\ -Drag_{RW} \\ 0 \end{bmatrix} \tag{26}$$

The negative Drag force in equation (26) maintains directionality. The sail force converted into the body axis system is added to the translation dynamic equation (12).

3 Controller Design

Various methods for rudder control have been previously published and implemented on sailboat systems. These include fuzzy logic type controllers [5]. The disadvantage of using such control system is the absence of the mathematical model of the plant being controlled, and hence is not a very useful for our purpose. The control system used on this boat is a simple Proportional-Integral-Derivative (PID) controller. Three main control systems are used on the boat. The sail controller maintains an optimum sail angle, based on relative wind. The heading controller maintains a desired course and the waypoint tracking system follows assigned linear trajectories based on initial and final waypoints, and existing position. All control commands also consider linear first order dynamic responses of servo motors and actuation limits. The controllers were hand tuned by trial and error method [6]. The proportional gains were adjusted until a realistic yet quick response of the system was observed. The integral gains were then adjusted to minimize steady state error in the desired system output.

3.1 Sail Controller

The sail angle controller is a conditional type controller which maintains a maximum lift coefficient angle of attack. The output sail angle is calculated based on relative wind angle. The offset from the relative wind or the angle of attack, is defined based on the design lift curve of the sail [4]. The sum of this angle of attack and relative wind angle in the body axis is the maintained sail angle.

3.2 Heading Controller

The heading controller maintains commanded compass heading of the vehicle. A hand tuned Proportional Integral (PI) controller converts heading commands into rudder inputs to the model. Use of a simple PI controller permits faster tuning of the controller in real-time. The heading control system has a switch to toggle between pilot heading or course commands or the course defined by the waypoint tracking controller.

3.3 Waypoint Tracking Controller

The waypoint tracking controller maintains a trajectory calculated using the final and initial waypoints. The course to be followed is calculated as a vector between the initial and final waypoint inputs. The cross track error is calculated as the geometric distance between the position of the boat and the heading vector. The cross track error is translated into a course heading error by a hand tuned PI controller. The corrected course heading is passed to the heading controller for path following.

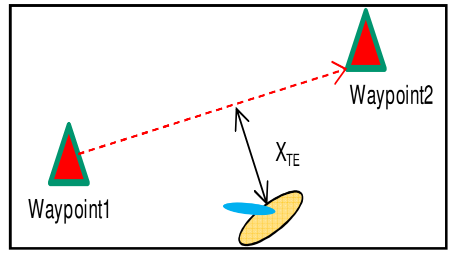

Figure 2: Waypoint Tracking

4 Simulation and Testing

4.1 MATLAB/Simulink models

The model (Fig 3) and the control systems (Fig 4) were created as embedded MATLAB functions in Simulink. The Runge-Kutta fourth order solver is used for numerical solutions. The model coefficients were approximated by comparison of plots of the model states with responses of the real sailboat platform. While qualitatively the reactions of linearalized sailboat model correlates well with the real sailboat, it is yet to be demonstrated with quantitative sensor data from a real sailboat instrumented appropriately. The initial states of the Simulink model are set by the initial conditions of the state integrators. The Simulink model was converted to C++ code using MATLAB's Real-Time Workshop Embedded Coder for real-time simulation.

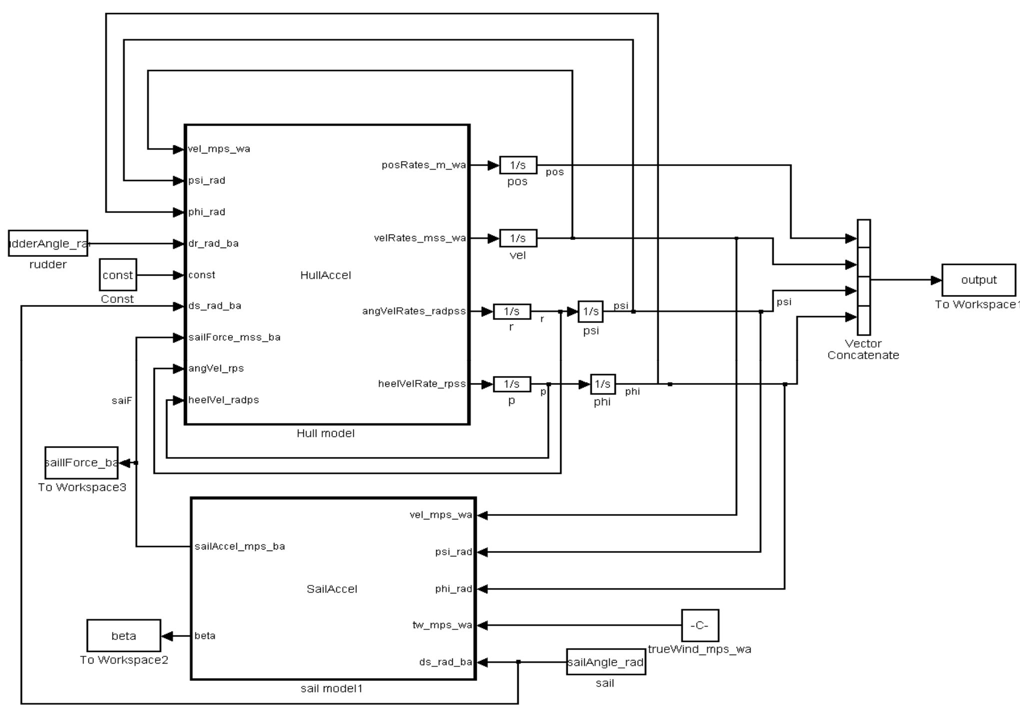

Figure 3: Simulink Model of the Sailboat

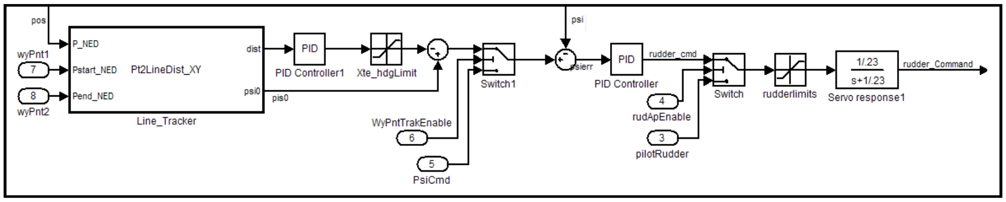

Figure 4: Simulink Model of Waypoint Tracker and Heading Controller

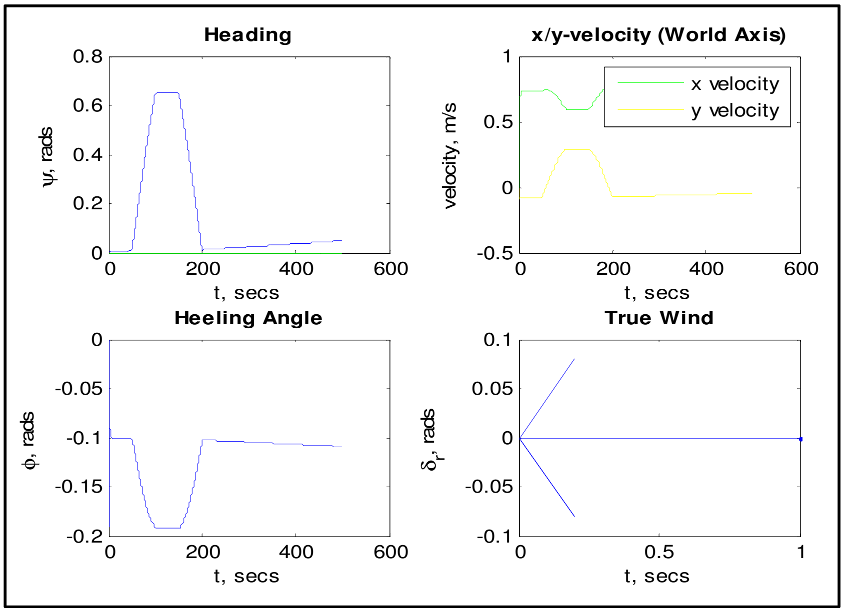

Figure 5: Sailboat response to applied doublet input to rudder (sail controller engaged)

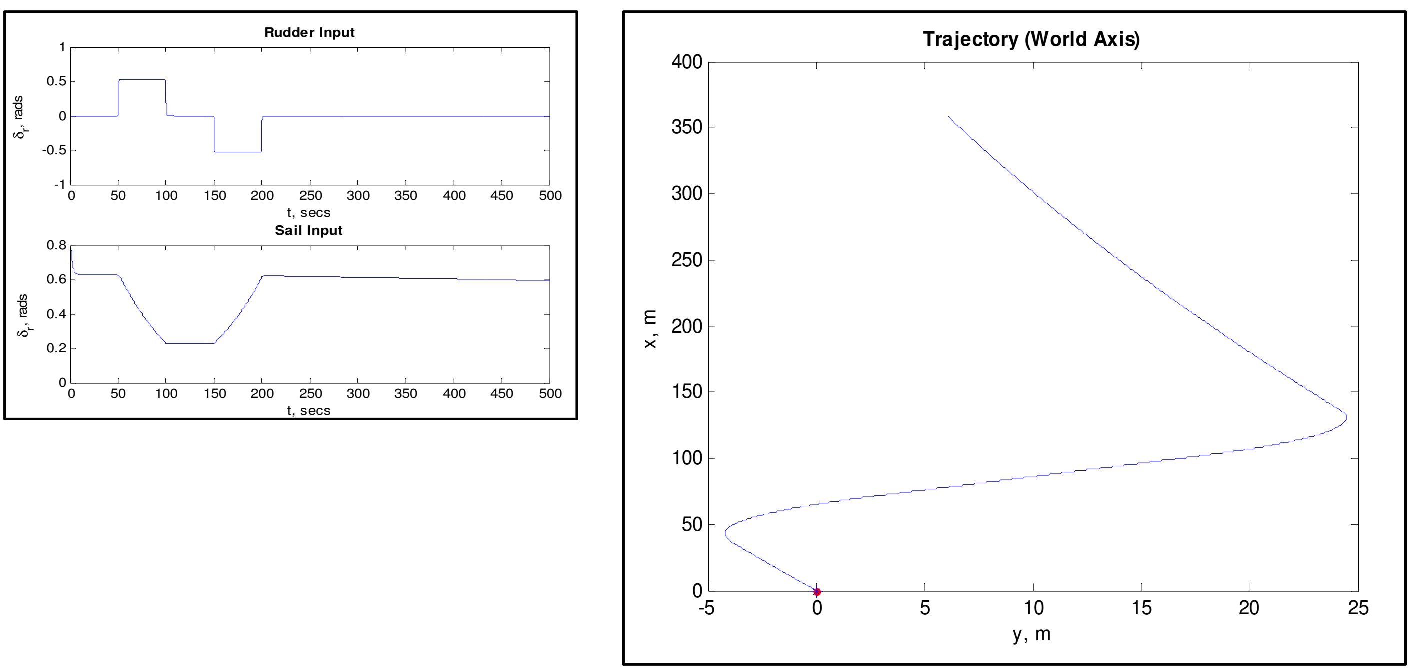

Figure 6: Trajectory of sailboat for applied doublet input to rudder (sail controller engaged)

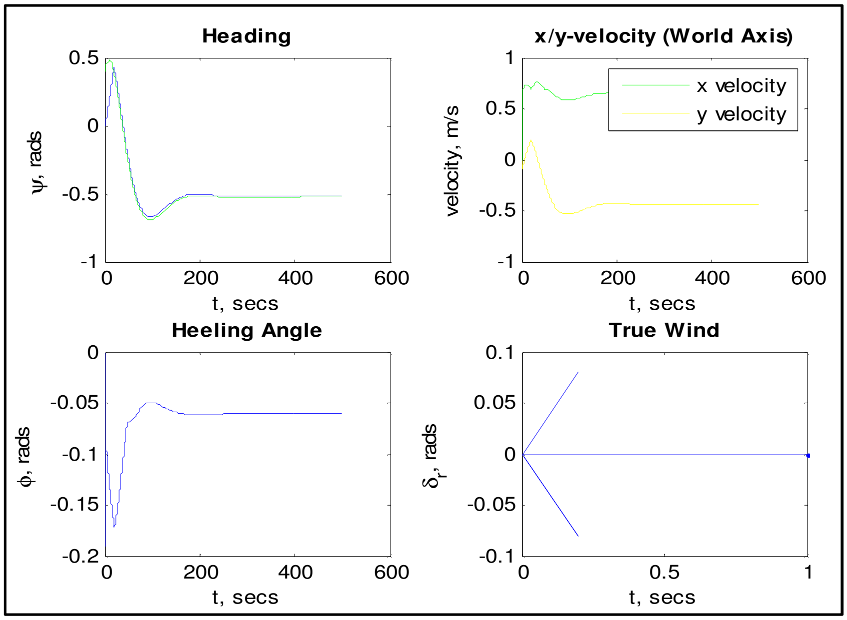

Figure 7: Sailboat response to step input heading command (heading & sail controllers engaged)

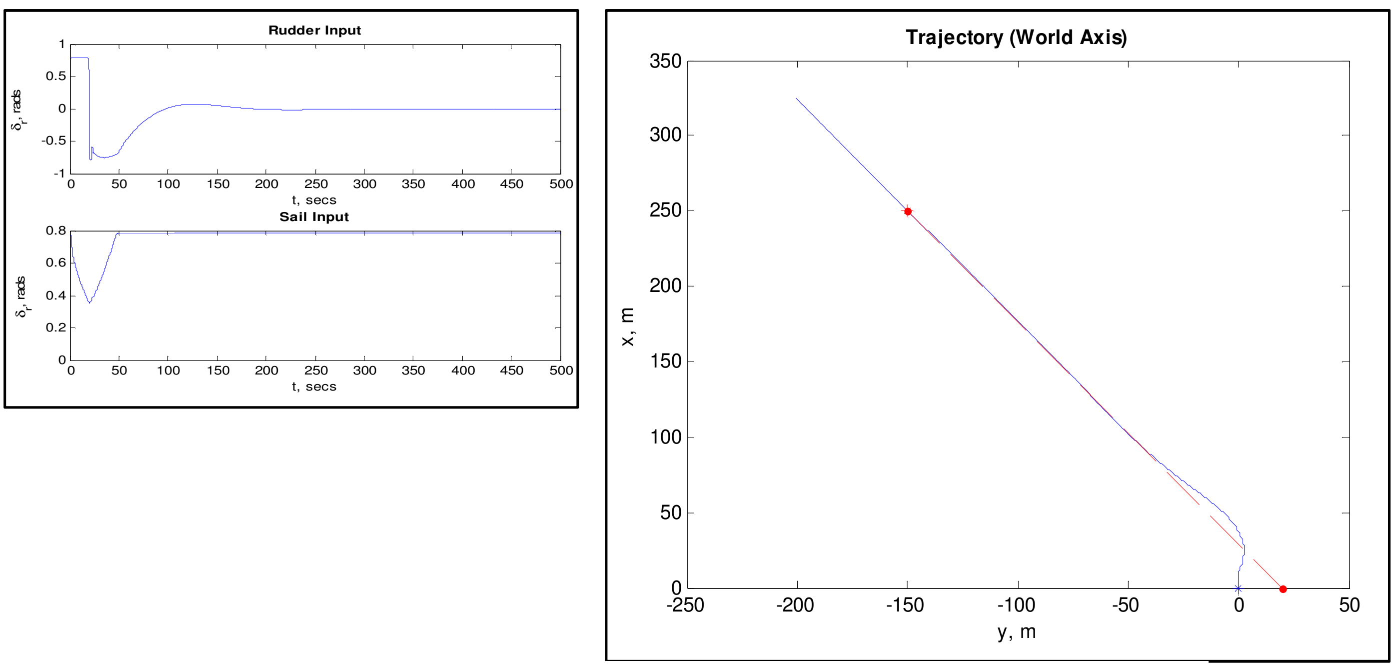

Figure 8: Sailboat response to waypoint commands (waypoint tracking system engaged)

4.2 Simulation Testing in Reflection

The Reflection Architecture is an environment for assembly of embedded control systems and simulations. Reflection allows the user to link different modules together through the output and input parameters of each module. The MATLAB generated C++ code was introduced into this architecture as a module, and linked to a 3D visual simulation environment of a sailboat (Fig. 9). The inputs to this system can be accessed through command line as variables. Simulation testing of the model in conditions such as upwind sailing, results in quite realistic responses. When the boat model is sailed directly upwind, the sail stalls or in sailing terms, goes 'in irons', and slowly begins to reverse due to drag buildup. This is a complex condition observed in the real sailboat platform, and hence shows close correlation of the described model with the real sailboat platform.

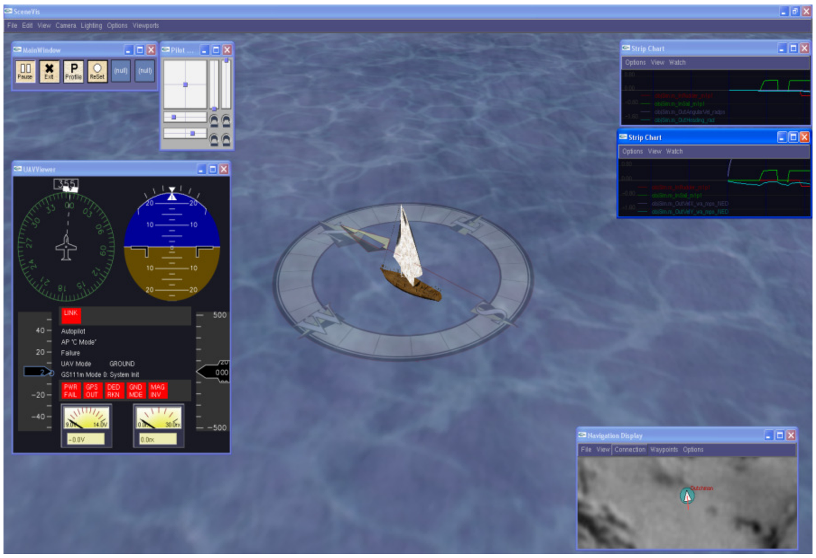

Figure 9: Real-time simulation in Reflection Architecture

5 Conclusions & Future Work

The 5 DOF mathematical model of the sailboat, outlined in this paper, sufficiently models most of the dynamics of a real sailboat. Since the vehicle platform is highly dependent on environmental conditions, capturing all the dynamic effects of the sailboat is important for trajectory planning. The development of a trajectory planner using the described mathematical model is in progress. This trajectory planning algorithm is the one of the key features of the Payload Directed Flight Project. Future research would be to integrate this outer loop trajectory planner with the discussed control system and model for validation and testing.

A Chesapeake Yacht CR914 radio control sailboat will be used as the test platform (Fig 10). System Identification using real-time data from the sailboats sensors can be used to tune the coefficients in the model, to achieve higher fidelity.

Figure 10: CR914 Yacht

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