Impact of the Anti-Aliasing Pre-Filtering on the Measurement of Maximum Time Interval Error

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Abstract

Latest ETSI and ITU-T standards recommend to evaluate MTIE basing on Time Error data measured, for observation intervals in the range from 0.1 s to 1000 s, "through an equivalent 10 Hz, first-order, low-pass measurement filter". *In this paper, the impact of such pre-filtering is thoroughly studied by simulating periodical and power-law noise types. Moreover, some results of measurements on telecommunications clocks are provided in order to support the conclusions stemming from simulation with experimental evidence. The results shown point out that the impact that such anti-aliasing pre-filtering can have in practice may be substantial and even misleading: measurements accomplished through such filter may produce too optimistic results, hiding the true amplitude of phase fluctuations at timing interfaces.*

1. Introduction

A major topic of discussion in standard bodies dealing with network synchronization [1]-[3] is clock noise characterization and measurement. Among the quantities considered in latest international standards for specification of phase and frequency stability requirements [4][5], the Maximum Time Interval Error (MTIE) has played historically a major role for characterizing time and frequency performance in digital telecommunications networks [4]–[8], as specifications in terms of MTIE are well suited to support the design of equipment buffer size.

As a matter of fact, historically, MTIE has been considered in telecommunications standards (as well as the other main standard quantity Time Deviation or TDEV [4][5][7][8]) oriented mostly to the measure of wander (i.e. slow phase fluctuations, at Fourier frequencies less than 10 Hz). Therefore, the range of the observation interval over which such standards specify the maximum values of MTIE and TDEV allowed at timing interfaces is limited to values greater than 0.1 s.

Besides this, latest ETSI and ITU-T standards [5][8] recommend to evaluate MTIE and TDEV basing on Time Error (TE) data measured, for observation intervals in the range from 0.1 s to 1000 s, *"through an equivalent 10 Hz, first-order, low-pass measurement filter"*. The rationale of this choice has been the wish of getting rid of the spectrum aliasing which may occur by subsampling the TE function, in the presence of broadband noise, and it has been adopted for MTIE mainly for uniformity with the measurement procedure adopted previously for TDEV. Nevertheless, the actual impact and the rationale itself of such an anti-aliasing pre-filtering should be investigated thoroughly, in order to support this choice or not.

In this paper, MTIE is first introduced according to its formal definition. The principle of operation of elastic buffering is recalled shortly, pointing out why specifications in terms of MTIE are well suited to buffer size design. Then, the impact on MTIE of filtering the TE measurement data through such low-pass filter is thoroughly studied through a simulation approach, by generating TE sequences affected by periodical noise (sinusoidal TE) and by power-law noise. Finally, some results of measurements on telecommunications clocks are provided, in order to support the main conclusions drawn by simulation with experimental evidence and to point out the substantial impact that such anti-aliasing pre-filtering can have in practice.

2. Definition of MTIE

A thorough treatment of MTIE and of its properties can be found in [9]. A further specific analysis is reported in [10]. Here, solely the main definitions are summarized for the sake of understanding and to provide the reader with the background concepts.

A general expression describing a pseudo-periodic waveform which models the timing signal $s(t)$ at the clock output is given by [11]—[13]

$$s(t) = A\sin\Phi(t) \qquad (1)$$

where A is the peak amplitude and $\Phi(t)$ is the *total instantaneous phase*, expressing the ideal linear phase increasing with t and any frequency drift or random phase fluctuation.

The generated *Time* function $\mathrm{T}(t)$ of a clock is defined, in terms of its total instantaneous phase, as

$$\mathrm{T}(t) = \frac{\Phi(t)}{2\pi\nu_{\mathrm{nom}}} \qquad (2)$$

where ν_{nom} represents the oscillator nominal frequency. It is worthwhile noticing that for an ideal clock $\mathrm{T}_{\mathrm{id}}(t)=t$ holds, as expected. For a given clock, the *Time Error* function $\mathrm{TE}(t)$ (in standards also called $x(t)$) between its time $\mathrm{T}(t)$ and a reference time $\mathrm{T}_{\mathrm{ref}}(t)$ is defined as

$$x(t) \equiv \mathrm{TE}(t) = \mathrm{T}(t)-\mathrm{T}_{\mathrm{ref}}(t) \qquad (3).$$

The *Maximum Time Interval Error* function $\mathrm{MTIE}(\tau,T)$ is the maximum peak-to-peak variation of TE in all the possible observation intervals τ (in former standards [6][7] denoted as S) within a measurement period T (see Fig. 1) and is defined as

$$\mathrm{MTIE}(\tau,T) = \max_{0\le t_0\le T-\tau}\left\{\max_{t_0\le t\le t_0+\tau}\left[\mathrm{TE}(t)\right] - \min_{t_0\le t\le t_0+\tau}\left[\mathrm{TE}(t)\right]\right\} \qquad (4).$$

It should be noted, however, that the standards in force specify the MTIE limits simply as a function of τ (or S), thus implicitly assuming

$$\mathrm{MTIE}(\tau) = \lim_{T\to\infty}\mathrm{MTIE}(\tau,T) \qquad (5).$$

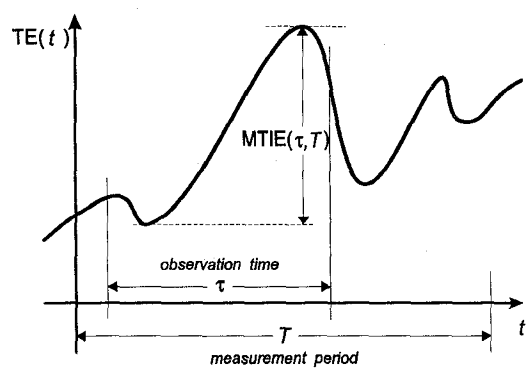

Fig. 1 - Definition of MTIE(τ,T)

3. Elastic Buffering and MTIE

Elastic buffering is a very common technique adopted in digital transmission and switching equipment. The expression *bit* *synchronization* is used to denote the synchronization of an asynchronous bit stream according to an equipment local clock. This is accomplished, according to the scheme of principle in Fig. 2, by writing the bits of the asynchronous bit stream into an elastic store (buffer) at their own instantaneous arrival rate f_w and by reading them out with the frequency f_r of the equipment local clock. The elastic store absorbs limited random zero-mean frequency fluctuations between the write and read clocks. Nevertheless, larger fluctuations or any frequency offset $|f_w-f_r|$ will make the buffer empty or overflow sooner or later. If the buffer empties, some bytes are repeated in transmission; if the buffer overflows, some are lost. Such events are called *slips*. A larger buffer size allows reducing the slip rate, for any given clock accuracy [14].

Bit synchronization is accomplished, for instance, on PCM signals at the inputs of digital switching exchanges and in Plesiochronous Digital Hierarchy (PDH) [15] or Synchronous Digital Hierarchy (SDH) [16][17] digital multiplexers to map (bit-synchronize) tributaries into the multiplex signal. Moreover, a similar mechanism is applied in SDH pointer processors (pointer justification).

Now, it is important to point out that in elastic stores the buffer fill level is proportional to the TE between the input digital signal and the local equipment clock. Therefore, ensuring the clock compliance with timing specifications given in terms of MTIE (expressing the peak-to-peak TE deviation over an observation interval) guarantees that certain buffer thresholds are never exceeded. If MTIE limits are low enough, this may mean that no slips or pointer justifications should ever take place. This fact accounts for the MTIE's appeal in supporting the design of equipment buffer size.

However, in a bit synchronizer or a SDH pointer processor as described above, TE fast fluctuations (jitter) are obviously as important as the slower ones (wander) and *may cause threshold overflows as well*, depending on their peak amplitude: of course, no 10 Hz low-pass filtering is performed on the TE in the equipment. Therefore, it is possible that measuring MTIE on the timing interfaces through a 10 Hz low-pass filter may yield too optimistic results, which seem to ensure that the buffer thresholds cannot be exceeded, while the true peak-to-peak TE fluctuations are much larger and thus causing slips or pointer justifications. Hence the need of investigating thoroughly the impact of filtering the TE measurement data through such an anti-aliasing low-pass filter, ascertaining if the MTIE values measured may change substantially and under what conditions.

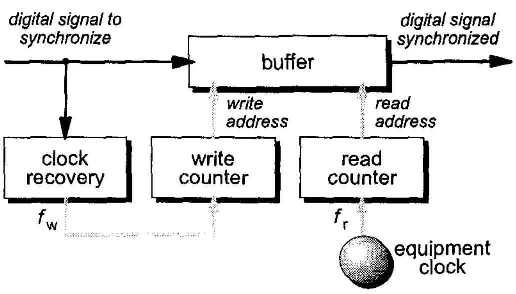

Fig. 2 - Scheme of principle of a bit synchronizer

4. Simulation Results

To this purpose, a systematic simulation study was carried out. Sequences of N TE samples $\{x_i\}$, defined as

$$x_i = x(t_0 + (i-1)\tau_0) \quad i = 1,2,\ldots,N \tag{6}$$

where t_0 is the initial observation time and τ_0 is the sampling period, were generated. As detailed in the next subsections, periodical noise (sinusoidal TE) and power-law noise were simulated. Then, the TE data were filtered in the Fourier domain [18] through a first-order, low-pass, anti-aliasing filter with cut-off frequency f_c=10 Hz, having transfer function

$$\mathrm{H}(f) = \frac{1}{1 + jf/f_c} \tag{7}$$

Curves of MTIE were finally evaluated, basing on both the original and filtered TE data for the sake of comparison, by means of the following standard estimator [4][5][9] derived directly by the def. (4)

$$\mathrm{MTIE}(\tau,T) = \max_{j=1}^{N-n}\left[\max_{i=j}^{n+j}(x_i) - \min_{i=j}^{n+j}(x_i)\right] \tag{8}$$

where $n=\tau/\tau_0$, $N=T/\tau_0+1$ and thus n=1, 2, ..., N-1. To cut down the computation time, MTIE was computed only for $n=2^0, 2^1, 2^2, 2^3$, etc.

The following two subsections provide the most significant results obtained, together with further details on the simulation technique, for both the noise types considered.

4.1. Sinusoidal Noise

In order to study the impact of the anti-aliasing pre-filtering in the presence of periodical noise, nine sequences of sinusoidal TE

$$x_k(t) = A\sin 2\pi f_k t \tag{9}$$

each made of N=65536 samples (6), with unitary peak amplitude A=1 ns, increasing jitter frequency f_1=0.1 Hz, f_2=0.3 Hz, f_3=1 Hz, f_4=3 Hz, f_5=10 Hz, f_6=30 Hz, f_7=100 Hz, f_8=300 Hz and f_9=1000 Hz were generated. The sampling period was set to τ_0=50 μs (T=3.3 s) for 30 Hz $\le f_k \le$ 1 kHz and to τ_0=500 μs (T=33 s) for 0.1 Hz $\le f_k \le$ 10 Hz. The nine sequences $\{x_i\}_k$ were then processed through the filter (7), yielding other nine sequences of 10 Hz low-pass filtered noise.

The MTIE curves evaluated basing on the two sets of sequences are plotted in Fig. 3. The former graph (original TE) shows that MTIE(τ) reaches its maximum value in a semi-period of the underlying sinusoidal noise. In the latter, the same MTIE(τ) curves are affected by the 10 Hz low-pass filter: as obvious, the higher is the noise frequency f_k, the lower is the residual noise level according to the H(f) of Eq. (7).

4.2. Power-Law Noise

In the frequency domain, the model most frequently used to represent the clock output phase noise is the so-called *power-law model* [12]. In terms of the one-sided Power Spectral Density (PSD) of $x(t)$ such model is expressed by

$$S_x(f) = \begin{cases} \dfrac{1}{(2\pi)^2}\displaystyle\sum_{\alpha=-4}^{0} h_{\alpha+2} f^{\alpha} & 0 \le f \le f_h \\ 0 & f > f_h \end{cases} \tag{10}$$

where the h_{-2}, h_{-1}, h_0, h_{+1} and h_{+2} coefficients are device-dependent parameters[1] and f_h is an upper cut-off frequency, mainly depending on low-pass filtering in the oscillator and in its output buffer amplifier. This clock upper cut-off frequency is usually in the range 10-100 kHz in precision frequency sources [19]. The noise types of the model (10) are: White Phase Modulation (WPM) for α=0, Flicker Phase Modulation (FPM) for α=-1, White Frequency Modulation (WFM) for α=-2, Flicker Frequency Modulation (FFM) for α=-3 and Random Walk Frequency Modulation (RWFM) for α=-4.

First, in order to simulate WPM (α=0) noise, two white and uniformly distributed pseudo-random sequences of length N=65536 were generated. Then, applying a well-known transformation formula [18][20], one white Gaussian pseudo-random sequence was obtained, thus approximating a WPM noise. Spectral shaping was accomplished by filtering in the Fourier domain the WPM (α=0) noise sequence through integrators of fractional order $-\alpha/2$ [21], having transfer function $\mathrm{H}_{-\alpha/2}(f)=K(j2\pi f)^{\alpha/2}$, to generate the FPM ($\alpha$=-1), WFM ($\alpha$=-2), FFM ($\alpha$=-3) and RWFM ($\alpha$=-4) noise sequences according to the

[1] The reason of the subscript $\alpha+2$ (α=-4,-3,-2,-1,0) is that, historically, the coefficients h_{-2}, h_{-1}, h_0, h_{+1}, h_{+2} have been used in the power-law model definition in terms of the PSD $S_y(f)$ of the random fractional frequency deviation $y(t)=dx(t)/dt$. The relationship $S_y(f)=(2\pi f)^2 S_x(f)$ holds [12].

power-law model (10). The sampling period was set to τ_0=5 ms (T=327 s) and the noise coefficients to h_{+2}=1 ns^2/Hz (WPM), h_{+1}=10^{-1} ns^2 (FPM), h_0=10^{-2} ns^2·Hz (WFM), h_{-1}=10^{-3} ns^2·Hz^2 (FFM) and h_{-2}=10^{-4} ns^2·Hz^3 (RWFM). Finally, as in the previous case, the five TE sequences obtained were processed through the filter (7), yielding the sequences of 10 Hz low-pass filtered noise, and MTIE was computed.

The graphs in Figs. 4 through 8 plot, for each noise type, the original and filtered TE sequences simulated and the corresponding MTIE curves. It appears that the impact of the anti-aliasing low-pass filtering is substantial for WPM and FPM noises, owing to their broadband spectrum.

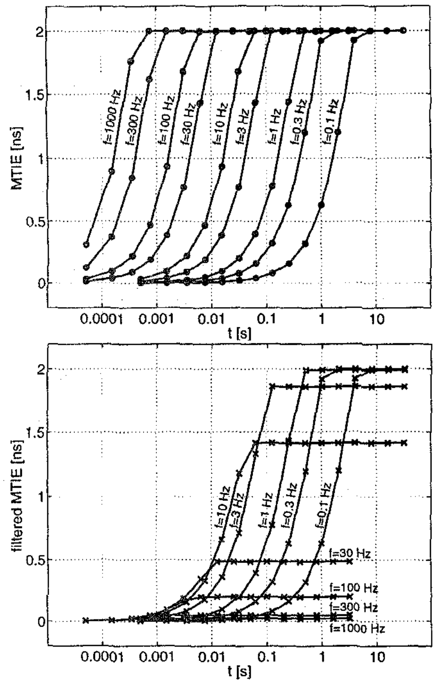

Fig. 3 - Set of nine MTIE curves evaluated from the original (up) and filtered (down) sinusoidal TE sequences (A=1 ns, 0.1 Hz ≤ f_k ≤ 1 kHz)

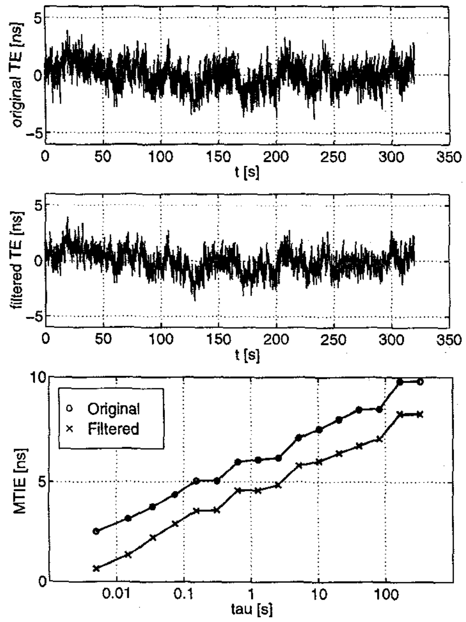

Fig. 5 - Simulation of FPM (α=-1) noise: original TE(t), filtered TE(t) and corresponding MTIE(τ)

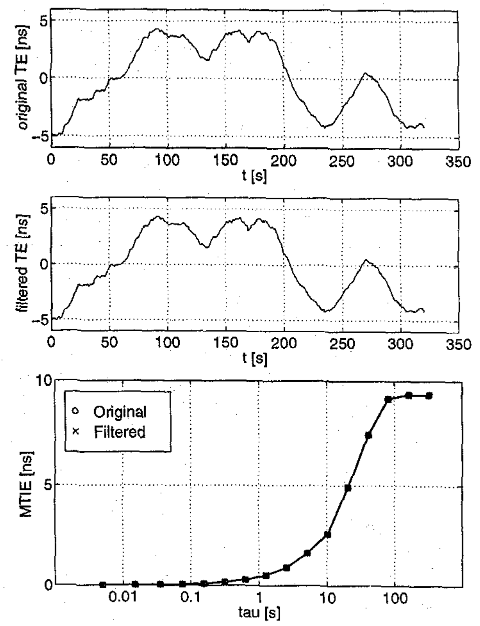

Fig. 7 - Simulation of FFM (α=-3) noise: original TE(t), filtered TE(t) and corresponding MTIE(τ)

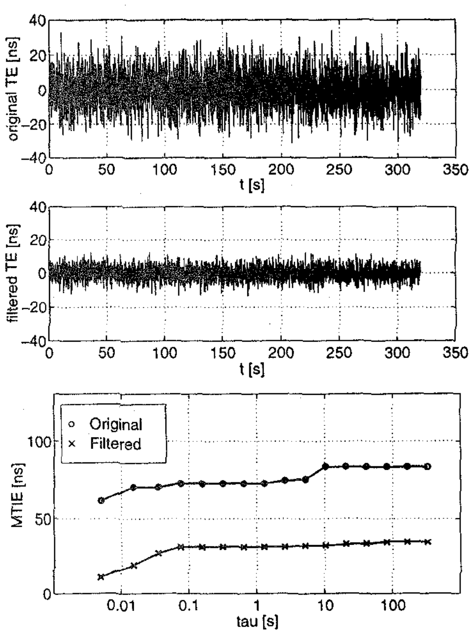

Fig. 4 - Simulation of WPM (α=0) noise: original TE(t), filtered TE(t) and corresponding MTIE(τ)

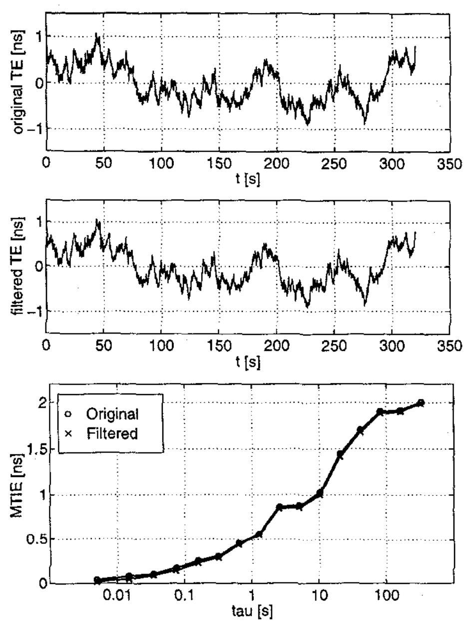

Fig. 6 - Simulation of WFM (α=-2) noise: original TE(t), filtered TE(t) and corresponding MTIE(τ)

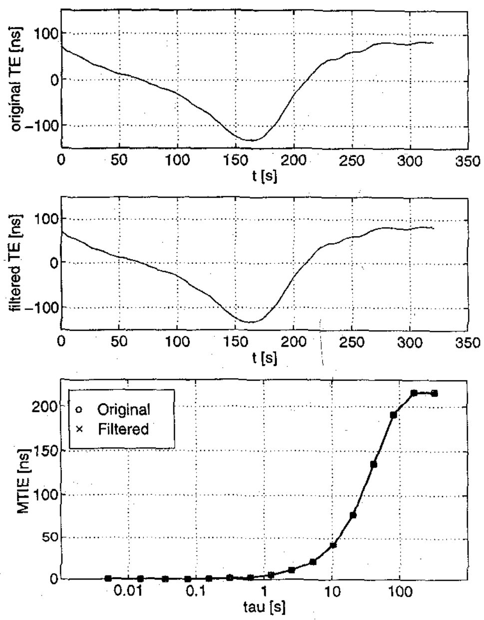

Fig. 8 - Simulation of RWFM (α=-4) noise: original TE(t), filtered TE(t) and corresponding MTIE(τ)

5. Measurement Results

In order to support the conclusions stemming from simulation with experimental evidence and to point out the impact that such anti-aliasing pre-filtering can have in practice, some measurement results are eventually provided in this section. These results have been chosen among those measured on telecommunications clocks of different types throughout the last few years.

Measurements were accomplished in the standard *synchronized clocks configuration* [4][5][13]. As shown in Fig. 9, a digital time counter with resolution 200 ps measured the TE between the output timing signal of the clock under test operating in slave mode and its input reference (both 2.048 MHz G.703/10 [22] signals). The latter was synthesized from the 10 MHz output signal of the reference clock, a rubidium frequency standard, which also synchronized the time counter. The counter was driven via a GPIB IEEE488.2 interface by a laptop PC controlling data acquisition. The TE sequences were then low-pass filtered (7) and MTIE was computed, as done for simulation.

The measurement results provided in Figs. 10 through 14 were obtained by testing four telecommunications clocks: two pieces of Stand-Alone Synchronization Equipment (SASE) (suppliers A and B) and two SDH Equipment Clocks (SECs), one of a Digital Cross-Connect (DXC) 4/3/1 (supplier C) and one of an Add-Drop Multiplexer STM-1 (ADM-1) of first generation (supplier D). The sequence length N and duration T and the sampling period τ_0 are specified in the respective figure captions.

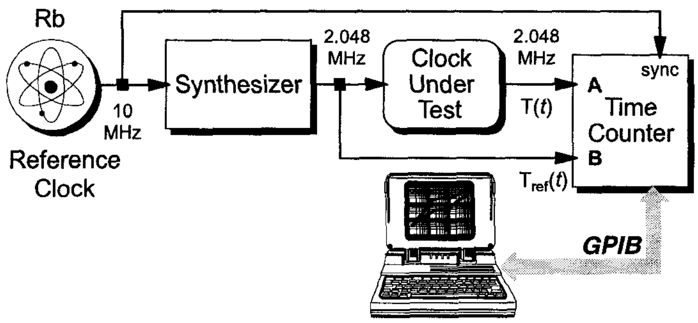

Fig. 9 - Measurement test bench

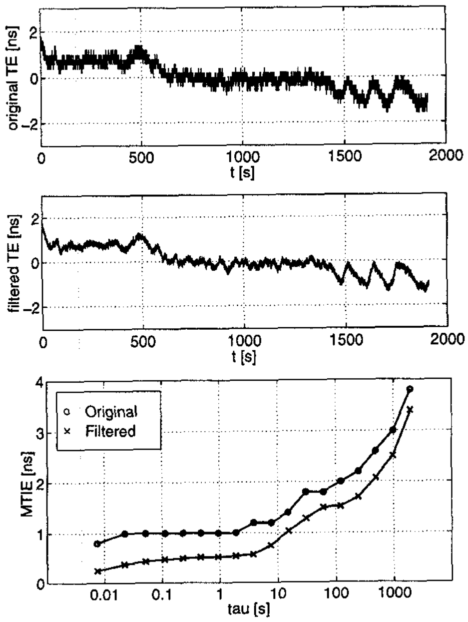

Fig. 10 - Original TE(t), filtered TE(t) and corresponding MTIE(τ) curves measured on the SASE A (N=262144, τ_0≅7.5 ms, T=1966 s)

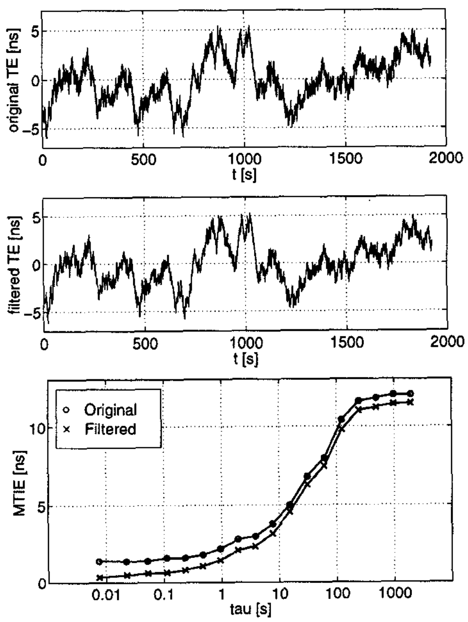

Fig. 11 - Original TE(t), filtered TE(t) and corresponding MTIE(τ) curves measured on the SASE B (N=262144, τ_0≅7.5 ms, T=1966 s)

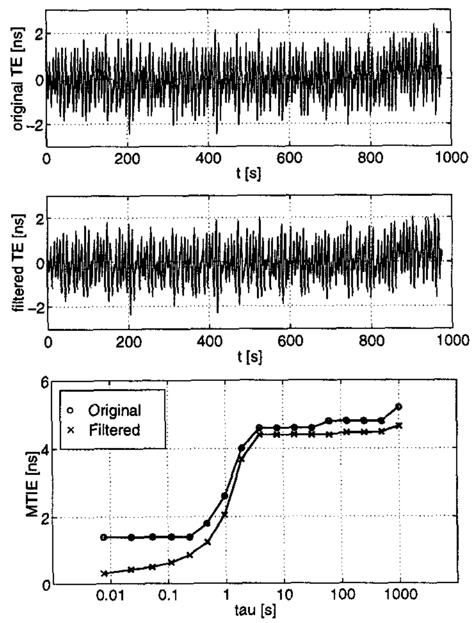

Fig. 12 - Original TE(t), filtered TE(t) and corresponding MTIE(τ) curves measured on the SEC C (N=131072, τ_0≅7.6 ms, T=998 s)

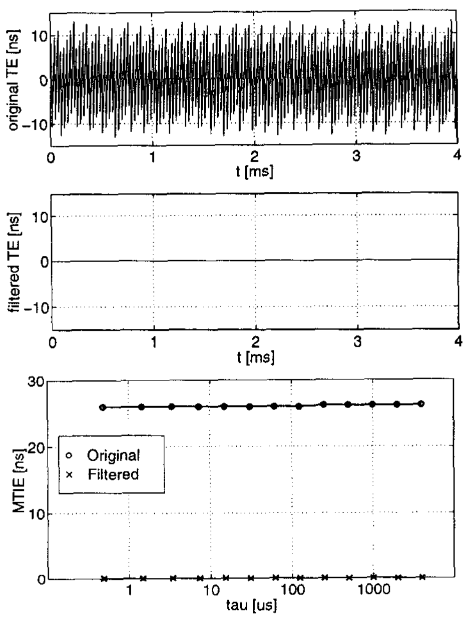

Fig. 13 - Original TE(t), filtered TE(t) and corresponding MTIE(τ) curves measured on the SEC D (N=8196, τ_0≅488 ns, T=4 ms)

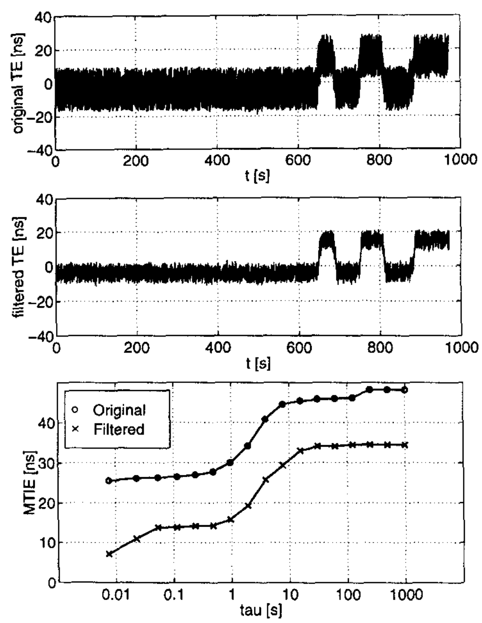

Fig. 14 - Original TE(t), filtered TE(t) and corresponding MTIE(τ) curves measured on the SEC D (N=131072, τ_0≅7.6 ms, T=998 s)

The graphs shown exhibit a different behaviour according to the underlying noise spectrum. The impact of the 10 Hz low-pass pre-filtering is always evident, but is substantial in particular in the case of the SEC D (Figs. 13 and 14), a Digital Phase-Locked Loop (DPLL) which deserves a closer look. This clock exhibits a wide and very fast periodic noise, a saw-toothed TE waveform due to frequency quantization error of peak-to-peak amplitude 25 ns and period 32 µs. In the measurement accomplished with sampling period τ_0=488 ns (Fig. 13), this noise gets completely cancelled by the 10 Hz low-pass filter, so that, while the filtered MTIE reports a reassuring 0 ns, the true MTIE is over 25 ns.

Considering the measurement accomplished with sampling period τ_0=7.6 ms (Fig. 14), on the other hand, it is interesting to notice that the filtered MTIE curve does not start from the 0 ns level for $\tau=\tau_0$, as it might be expected on the basis of Fig. 13. The reason lies in the spectrum aliasing due to subsampling the saw-toothed noise: the aliased noise is not fully filtered out due to its lower frequency. In spite of this, the filtered MTIE still reports a noise level three times smaller than the actual one.

6. Conclusions

The impact on MTIE of filtering the TE measurement data through a first-order, low-pass, anti-aliasing filter with cut-off frequency f_c=10 Hz was thoroughly studied through a simulation approach, by generating TE sequences affected by periodical noise (sinusoidal TE) and by power-law noise. The simulation results showed in particular that the impact of anti-aliasing pre-filtering may be substantial for some types of noise (namely, WPM and FPM), owing to their broadband spectrum.

Moreover, some results of measurements on various telecommunications clocks have been provided. The results shown point out that the impact that such anti-aliasing pre-filtering can have in practice on clocks may be substantial and even misleading: measurements accomplished through such filter may produce too optimistic results, hiding the *true* amplitude of TE fluctuations at timing interfaces, and therefore may yield errors in buffer size design or in assessing the pointer justification rate in SDH transmission chains.

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