

Message Authentication Codes and Collision-Resistant Hash Functions

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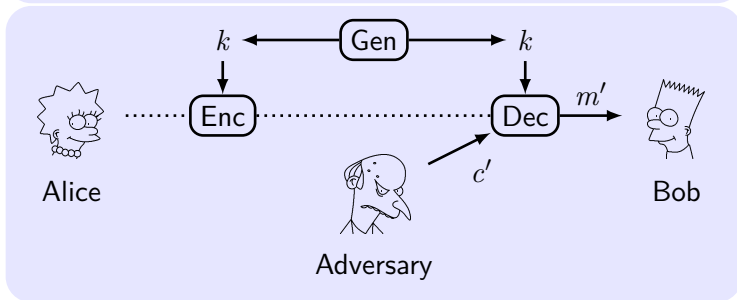
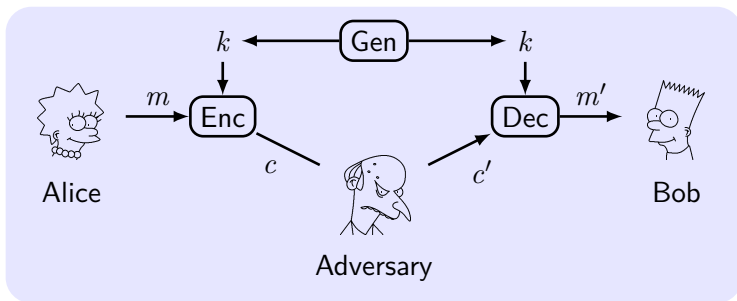
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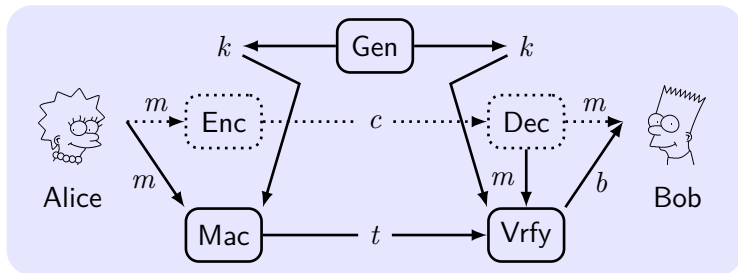
- 1 Message Authentication Codes (MAC) – Definitions**
- 2 Constructing Secure Message Authentication Codes**
- 3 CBC-MAC**
- 4 Collision-Resistant Hash Functions**
- 5 HMAC**

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Integrity and Authentication



The Syntax of MAC



- key k , tag t , a bit b means valid if $b = 1$; invalid if $b = 0$.
- **Key-generation** algorithm $k \leftarrow \text{Gen}(1^n)$, $|k| \geq n$.
- **Tag-generation** algorithm $t \leftarrow \text{Mac}_k(m)$.
- **Verification** algorithm $b := \text{Vrfy}_k(m, t)$.
- **Message authentication code:** $\Pi = (\text{Gen}, \text{Mac}, \text{Vrfy})$.
- **Basic correctness requirement:** $\text{Vrfy}_k(m, \text{Mac}_k(m)) = 1$.

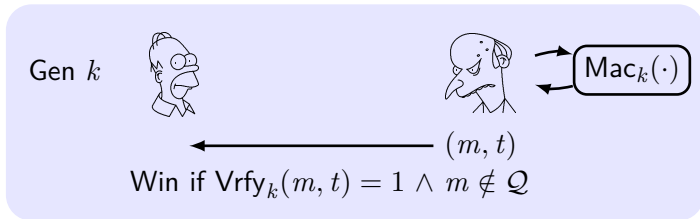
- **Intuition:** No adversary should be able to generate a **valid** tag on any “**new**” message¹ that was not previously sent.
- **Replay attack:** Copy a message and tag previously sent. (excluded by only considering “**new**” message)
 - Sequence numbers: receiver must store the previous ones.
 - Time-Stamps: sender/receiver maintain synchronized clocks.
- **Existential unforgeability:** **Not** be able to forge a valid tag on **any** message.
 - **Existential forgery:** *at least one* message.
 - **Selective forgery:** message chosen *prior* to the attack.
 - **Universal forgery:** *any* given message.
- **Adaptive chosen-message attack (CMA):** be able to obtain tags on *any* message chosen adaptively *during* its attack.

¹A stronger requirement is concerning *new message/tag pair*.

Definition of MAC Security

The message authentication experiment $\text{Macforge}_{\mathcal{A}, \Pi}(n)$:

- 1 $k \leftarrow \text{Gen}(1^n)$.
- 2 \mathcal{A} is given input 1^n and oracle access to $\text{Mac}_k(\cdot)$, and outputs (m, t) . \mathcal{Q} is the set of queries to its oracle.
- 3 $\text{Macforge}_{\mathcal{A}, \Pi}(n) = 1 \iff \text{Vrfy}_k(m, t) = 1 \wedge m \notin \mathcal{Q}$.



Definition 1

A MAC Π is **existentially unforgeable under an adaptive CMA** if \forall PPT \mathcal{A} , \exists negl such that:

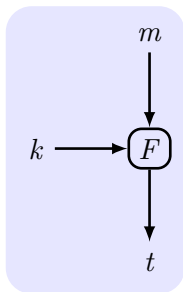
$$\Pr[\text{Macforge}_{\mathcal{A}, \Pi}(n) = 1] \leq \text{negl}(n).$$

Suppose $\langle S, V \rangle$ are CMA-secure, are $\langle S', V' \rangle$ secure?

- $S'_{k_1, k_2}(m) = (S_{k_1}(m), S_{k_2}(m))$
 $V'_{k_1, k_2}(m, (t_1, t_2)) = V_{k_1}(m, t_1)$ and $V_{k_2}(m, t_2)$
- $S'_k(m) = (S_k(m), S_k(m))$
 $V'_k(m, (t_1, t_2)) = \begin{cases} V_k(m, t_1) & \text{if } t_1 = t_2 \\ 0 & \text{otherwise} \end{cases}$
- $S'_k(m) = (S_k(m), S_k(0^n))$
 $V'_k(m, (t_1, t_2)) = V_k(m, t_1)$ and $V_k(0^n, t_2)$
- $S'_k(m) = S_k(m)$
 $V'_k(m, t) = \begin{cases} V_k(m, t) & \text{if } m \neq 0^n \\ 1 & \text{otherwise} \end{cases}$
- $S'_k(m) = S_k(m)[0, \dots, 126]$
 $V'_k(m, t) = V_k(m, t||0)$ or $V_k(m, t||1)$

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Constructing Secure MACs



Construction 2

- F is PRF. $|m| = n$.
- $\text{Gen}(1^n)$: $k \leftarrow \{0, 1\}^n$ u.a.r.
- $\text{Mac}_k(m)$: $t := F_k(m)$.
- $\text{Vrfy}_k(m, t)$: $1 \iff t \stackrel{?}{=} F_k(m)$.

Theorem 3

If F is a PRF, Construction is a secure fixed-length MAC.

Lemma 4

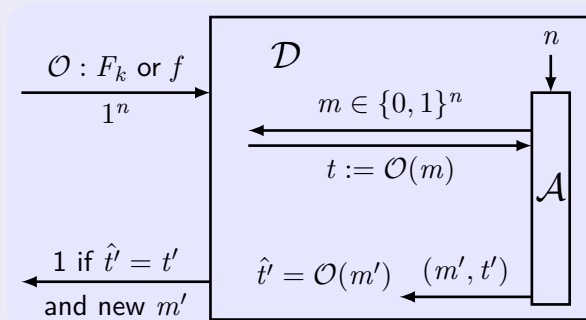
Truncating MACs based on PRFs: *If F is a PRF, so is $F_k^t(m) = F_k(m)[1, \dots, t]$.*

Proof of Secure MAC from PRF

Idea: Show Π is secure unless F_k is not PRF by reduction.

Proof.

D distinguishes F_k ; \mathcal{A} attacks Π .



Proof of Secure MAC from PRF (Cont.)

Proof.

(1) If true random f is used, $t = f(m)$ is uniformly distributed.

$$\Pr[D^{f(\cdot)}(1^n) = 1] = \Pr[\text{Macforge}_{\mathcal{A}, \tilde{\Pi}}(n) = 1] \leq 2^{-n}.$$

(2) If F_k is used, conduct the experiment $\text{Macforge}_{\mathcal{A}, \Pi}(n)$.

$$\Pr[D^{F_k(\cdot)}(1^n) = 1] = \Pr[\text{Macforge}_{\mathcal{A}, \Pi}(n) = 1] = \varepsilon(n).$$

According to the definition of PGF,

$$\left| \Pr[D^{F_k(\cdot)}(1^n) = 1] - \Pr[D^{f(\cdot)}(1^n) = 1] \right| \geq \varepsilon(n) - 2^{-n}.$$



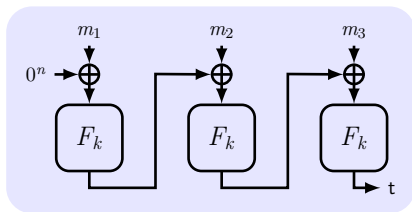
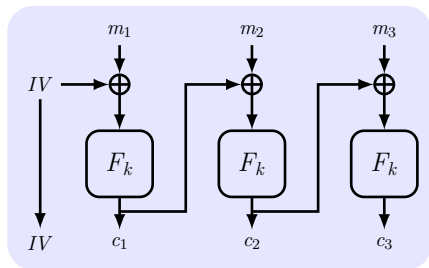
Extension to Variable-Length Messages

For variable-Length Messages, would the following suggestions be secure?

- **Suggestion 1:** XOR all the blocks together and authenticate the result. $t := \text{Mac}'_k(\oplus_i m_i)$.
- **Suggestion 2:** Authenticate each block separately. $t_i := \text{Mac}'_k(m_i)$.
- **Suggestion 3:** Authenticate each block along with a sequence number. $t_i := \text{Mac}'_k(i \| m_i)$.

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Constructing Fixed-Length CBC-MAC



Modify CBC encryption into CBC-MAC:

- Change random IV to encrypted fixed 0^n , otherwise:
Q: query m_1 and get (IV, t_1) ; output $m'_1 = IV' \oplus IV \oplus m_1$ and $t' = \underline{\hspace{1cm}}$.
- Tag only includes the output of the final block, otherwise:
Q: query m_i and get t_i ; output $m'_i = t'_{i-1} \oplus t_{i-1} \oplus m_i$ and $t'_i = \underline{\hspace{1cm}}$.

Constructing Fixed-Length CBC-MAC (Cont.)

Construction 5

- a PRF F and a length function ℓ . $|m| = \ell(n) \cdot n$. $\ell = \ell(n)$.
 $m = m_1, \dots, m_\ell$.
- $\text{Gen}(1^n)$: $k \leftarrow \{0, 1\}^n$ u.a.r.
- $\text{Mac}_k(m)$: $t_i := F_k(t_{i-1} \oplus m_i)$, $t_0 = 0^n$. Output $t = t_\ell$.
- $\text{Vrfy}_k(m, t)$: $1 \iff t \stackrel{?}{=} \text{Mac}_k(m)$.

Theorem 6

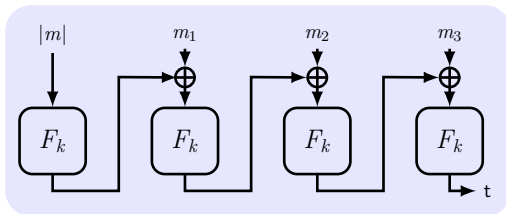
If F is a PRF, Construction is a secure **fixed-length** MAC.

Not for **variable-length** message:

Q: For one-block message m with tag t , adversary can append a block ____ and output tag t .

Secure Variable-Length MAC

- **Input-length key separation:** $k_\ell := F_k(\ell)$, use k_ℓ for CBC-MAC.
- **Length-prepend:** Prepend m with $|m|$, then use CBC-MAC.



- **Encrypt last block (ECBC-MAC):** Use two keys k_1, k_2 . Get t with k_1 by CBC-MAC, then output $\hat{t} := F_{k_2}(t)$.

Q: to authenticate a voice stream, which approach do you prefer?

MAC Padding

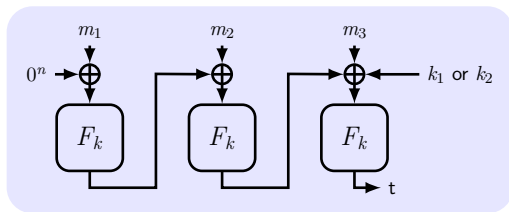
Padding must be invertible!

$$m_0 \neq m_1 \Rightarrow \text{pad}(m_0) \neq \text{pad}(m_1).$$

ISO: pad with “100...00”. Add dummy block if needed.

Q: what if no dummy block?

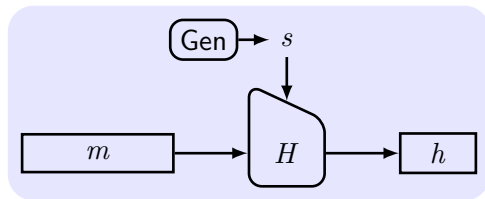
CMAC (Cipher-based MAC from NIST): key = (k, k_1, k_2) .



- No final encryption: extension attack thwarted by keyed XOR.
- No dummy block: ambiguity resolved by use of k_1 or k_2 .

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Defining Hash Function



Definition 7

A **hash function (compression function)** is a pair of PPT algorithms (Gen, H) satisfying:

- a key $s \leftarrow \text{Gen}(1^n)$, s is **not kept secret**.
- $H^s(x) \in \{0, 1\}^{\ell(n)}$, where $x \in \{0, 1\}^*$ and ℓ is polynomial.

If H^s is defined only for $x \in \{0, 1\}^{\ell'(n)}$ and $\ell'(n) > \ell(n)$, then (Gen, H) is a **fixed-length** hash function.

Defining Collision Resistance

- **Collision** in H : $x \neq x'$ and $H(x) = H(x')$.
- **Collision Resistance**: infeasible for any PPT alg. to find.

The collision-finding experiment $\text{Hashcoll}_{\mathcal{A}, \Pi}(n)$:

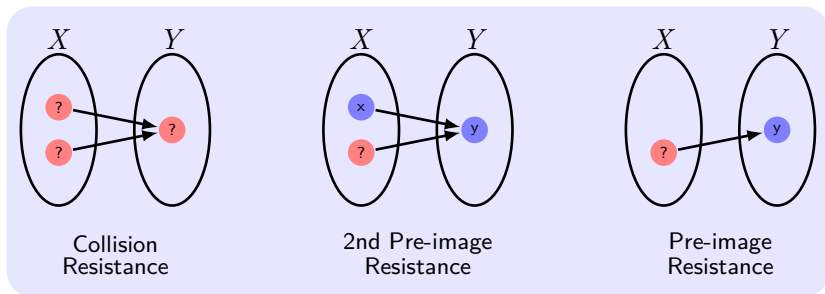
- 1 $s \leftarrow \text{Gen}(1^n)$.
- 2 \mathcal{A} is given s and outputs x, x' .
- 3 $\text{Hashcoll}_{\mathcal{A}, \Pi}(n) = 1 \iff x \neq x' \wedge H^s(x) = H^s(x')$.

Definition 8

$\Pi(H, H^s)$ is **collision resistant** if \forall PPT \mathcal{A} , $\exists \text{negl}$ such that

$$\Pr[\text{Hashcoll}_{\mathcal{A}, \Pi}(n) = 1] \leq \text{negl}(n).$$

Weaker Notions of Security for Hash Functions



- **Collision resistance:** It is hard to find (x, x') , $x' \neq x$ such that $H(x) = H(x')$.
- **Second pre-image resistance:** Given s and x , it is hard to find $x' \neq x$ such that $H^s(x') = H^s(x)$.
- **Pre-image resistance:** Given s and $y = H^s(x)$, it is hard to find x' such that $H^s(x') = y$.

H is CRHF. Is *H'* CRHF?

- $H'(m) = H(m) \oplus H(m \oplus 1^{|m|})$
- $H'(m) = H(m) \| H(0)$
- $H'(m) = H(m) \| H(m)$
- $H'(m) = H(m) \oplus H(m)$
- $H'(m) = H(m[0, \dots, |m| - 2])$
- $H'(m) = H(m \| 0)$

The “Birthday” Problem

The “Birthday” Problem

Q: “What size group of people do we need to take such that with probability $1/2$ some pair of people in the group share a birthday?”

A: 23.

Lemma 9

Choose q elements y_1, \dots, y_q u.a.r from a set of size N , the probability that $\exists i \neq j$ with $y_i = y_j$ is $\text{coll}(q, N)$, then

$$\text{coll}(q, N) \leq \frac{q^2}{2N}.$$

$$\text{coll}(q, N) \geq \frac{q(q-1)}{4N} \quad \text{if } q \leq \sqrt{2N}.$$

$$\text{coll}(q, N) = \Theta(q^2/N) \quad \text{if } q < \sqrt{N}.$$

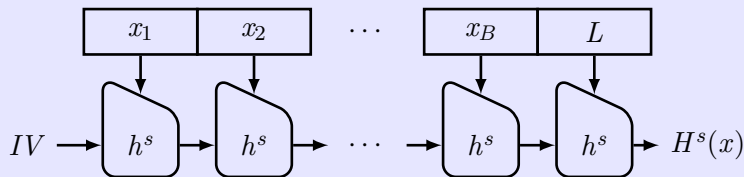
The length of hash value should be long enough.

Constructing “Meaningful” Collisions

How many different meaningful sentences are in the below paragraph?

It is **hard/difficult/challenging/impossible** to **imagine/believe** that we will **find/locate/hire** another **employee/person** having similar **abilities/skills/character** as Alice. She has done a **great/super** job.

The Merkle-Damgård Transform



Construction 10

Construct **variable-length** CRHF (Gen, H) from fixed-length (Gen, h) (2ℓ bits $\rightarrow \ell$ bits, $\ell = \ell(n)$):

- Gen : remains unchanged.
- H : key s and string $x \in \{0, 1\}^*$, $L = |x| < 2^\ell$:
 - $B := \lceil \frac{L}{\ell} \rceil$ (# blocks). **Pad x with 0s.** ℓ -bit blocks x_1, \dots, x_B .
 $x_{B+1} := L$, L is encoded using ℓ bits.
 - $z_0 := IV = 0^\ell$. For $i = 1, \dots, B + 1$, compute
 $z_i := h^s(z_{i-1} \| x_i)$.

Security of the Merkle-Damgård Transform

Theorem 11

If (Gen, h) is a fixed-length CRHF, then (Gen, H) is a CRHF.

Proof.

Idea: a collision in H^s yields a collision in h^s .

Two messages $x \neq x'$ of respective lengths L and L' such that $H^s(x) = H^s(x')$. # blocks are B and B' .

$x_{B+1} := L$ is necessary since **Padding with 0s** will lead to the same input with different messages.

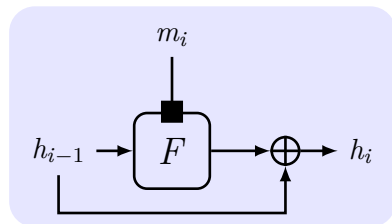
1 $L \neq L'$: $z_B \| L \neq z_{B'} \| L'$.

2 $L = L'$: $z_{i^*-1} \| x_{i^*} \neq z'_{i^*-1} \| x'_{i^*}$.

So there must be $x \neq x'$ such that $h^s(x) = h^s(x')$. □

CRHF from Block Cipher

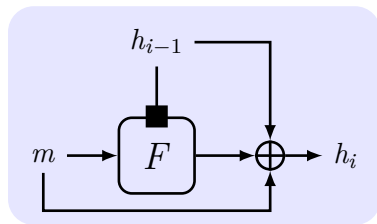
Davies-Meyer:



Used by SHA-1/2, MD5

$$h_i = F_{m_i}(h_{i-1}) \oplus h_{i-1}$$

Miyaguchi-Preneel:



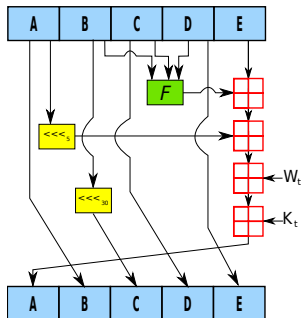
Used by Whirlpool

$$h_i = F_{h_{i-1}}(m_i) \oplus h_{i-1} \oplus m$$

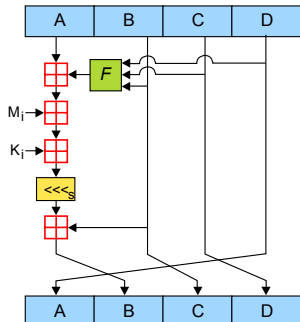
Q: what if $h_i = F_{m_i}(h_{i-1})$ without XOR with h_{i-1} ?

Cryptographic Hash Functions: SHA-1 and MD5

SHA-1:



MD5:

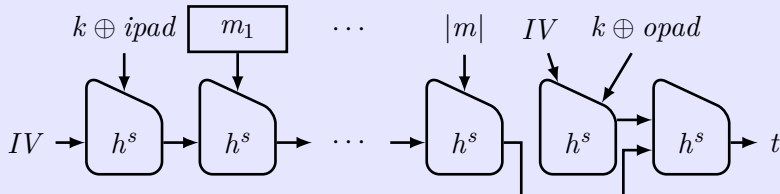


A, B, C, D and E are 32-bit words of the state; F is a nonlinear function that varies; $\lll n$ denotes a left bit rotation by n places; W_t/M_t is the expanded message word of round t ; K_t is the round constant of round t ; \boxplus denotes addition modulo 2^{32} .

- Finding a collision in 128-bit MD5 requires time $2^{20.96}$.
- Finding a collision in 160-bit SHA-1 requires time 2^{51} .

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Hash-based MAC (HMAC)



Construction 12

$(\widetilde{\text{Gen}}, h)$ is a fixed-length CRHF. $(\widetilde{\text{Gen}}, H)$ is the Merkle-Damgård transform. IV , $opad$ ($0x36$), $ipad$ ($0x5C$) are fixed constants of length n . HMAC:

- $\text{Gen}(1^n)$: Output (s, k) . $s \leftarrow \widetilde{\text{Gen}}, k \leftarrow \{0, 1\}^n$ u.a.r.
- $\text{Mac}_{s,k}(m)$: $t := H_{IV}^s((k \oplus opad) \| H_{IV}^s((k \oplus ipad) \| m))$.
- $\text{Vrfy}_{s,k}(m, t)$: $1 \iff t \stackrel{?}{=} \text{Mac}_{s,k}(m)$.

Theorem 13

$$G(k) \stackrel{\text{def}}{=} h^s(IV \parallel (k \oplus \text{opad})) \parallel h^s(IV \parallel (k \oplus \text{ipad})) = k_1 \parallel k_2.$$

$(\widetilde{\text{Gen}}, h)$ is CRHF. If G is a PRG, then HMAC is secure.

- HMAC is an industry standard (RFC2104)
- HMAC is faster than CBC-MAC
- Before HMAC, a common mistake was to use $H^s(k \parallel x)$
- Verification timing attacks: (Keyczar crypto library (Python))

```
def Verify(key, msg, sig_bytes):  
    return HMAC(key, msg) == sig_bytes
```

The problem: implemented as a byte-by-byte comparison
- *Don't implement it yourself.*

- adaptive CMA, replay attack, birthday attack.
- existential unforgeability, collision resistance.
- CBC-MAC, CRHF, Merkle-Damgård transform, NMAC, HMAC.