CCA-Secure and Authentication Encryption

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Outline

1. Constructing CCA-Secure Encryption Schemes
2. Obtaining Privacy and Message Authentication
3. Key Derivation Function (FYI)
4. Deterministic Encryption (FYI)

Content

Recall Security Against CCA

The CCA indistinguishability experiment $\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi}(n)$:

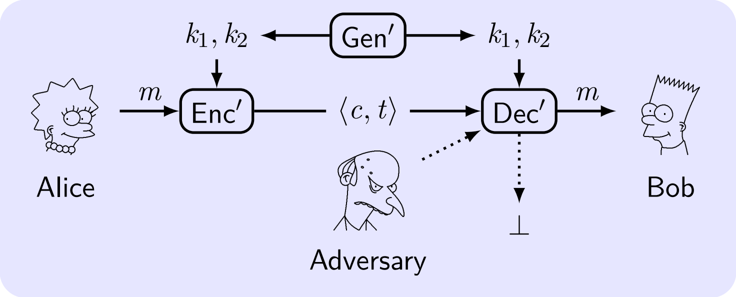
1. $k \leftarrow \mathsf{Gen}(1^n)$.
2. $\mathcal{A}$ is given input 1^n and oracle access $\mathcal{A}^{\mathsf{Enc}_k(\cdot)}$ and $\mathcal{A}^{\mathsf{Dec}_k(\cdot)}$, outputs m_0, m_1 of the same length.
3. a random bit $b \leftarrow \{0,1\}$ is chosen. Then $c \leftarrow \mathsf{Enc}_k(m_b)$ is given to $\mathcal{A}$.
4. $\mathcal{A}$ continues to have oracle access **except for** c, outputs b'.
5. If $b' = b$, $\mathcal{A}$ succeeded $\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi} = 1$, otherwise 0.

Definition 1

Π has **indistinguishable encryptions under a CCA (CCA-secure)** if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that

$$\Pr\left[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi}(n) = 1\right] \leq \frac{1}{2} + \mathsf{negl}(n).$$

Constructing CCA-Secure Encryption Schemes



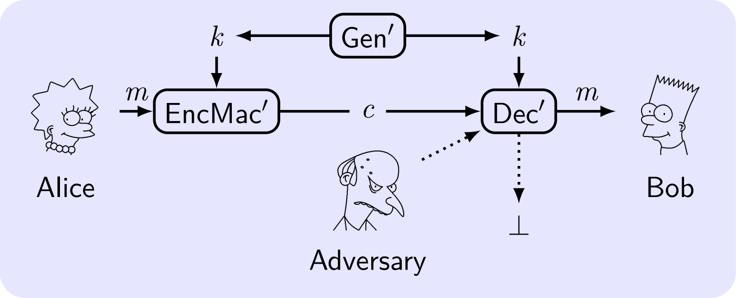
Construction 2

$\Pi_E = (\mathsf{Gen}_E, \mathsf{Enc}, \mathsf{Dec})$, $\Pi_M = (\mathsf{Gen}_M, \mathsf{Mac}, \mathsf{Vrfy})$. Π':

- $\mathsf{Gen}'(1^n)$: $k_1 \leftarrow \mathsf{Gen}_E(1^n)$ *and* $k_2 \leftarrow \mathsf{Gen}_M(1^n)$.
- $\mathsf{Enc}'_{k_1,k_2}(m)$: $c \leftarrow \mathsf{Enc}_{k_1}(m)$, $t \leftarrow \mathsf{Mac}_{k_2}(c)$ *and output* $\langle c, t \rangle$.
- $\mathsf{Dec}'_{k_1,k_2}(\langle c, t \rangle)$: *If* $\mathsf{Vrfy}_{k_2}(c, t) \stackrel{?}{=} 1$, *output* $\mathsf{Dec}_{k_1}(c)$; *otherwise output "failure"* $\perp$.

Content

Message Transmission Scheme



- **Key-generation** algorithm outputs $k \leftarrow \mathsf{Gen}'(1^n)$.
  $k = (k_1, k_2)$. $k_1 \leftarrow \mathsf{Gen}_E(1^n)$, $k_2 \leftarrow \mathsf{Gen}_M(1^n)$.
- **Message transmission** algorithm is derived from $\mathsf{Enc}_{k_1}(\cdot)$ and $\mathsf{Mac}_{k_2}(\cdot)$, outputs $c \leftarrow \mathsf{EncMac}'_{k_1,k_2}(m)$.
- **Decryption** algorithm is derived from $\mathsf{Dec}_{k_1}(\cdot)$ and $\mathsf{Vrfy}_{k_2}(\cdot)$, outputs $m \leftarrow \mathsf{Dec}'_{k_1,k_2}(c)$ or $\perp$.
- **Correctness requirement**: $\mathsf{Dec}'_{k_1,k_2}(\mathsf{EncMac}'_{k_1,k_2}(m)) = m$.

Defining Secure Message Transmission

The secure message transmission experiment $\mathsf{Auth}_{\mathcal{A},\Pi'}(n)$:

1. $k = (k_1, k_2) \leftarrow \mathsf{Gen}'(1^n)$.
2. $\mathcal{A}$ is given input 1^n and oracle access to EncMac'_k, and outputs $c \leftarrow \mathsf{EncMac}'_k(m)$.
3. $m := \mathsf{Dec}'_k(c)$. $\mathsf{Auth}_{\mathcal{A},\Pi'}(n) = 1 \iff m \neq \perp \wedge\ m \notin \mathcal{Q}$.

Definition 3

Π' achieves **authenticated communication** if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that

$$\Pr[\mathsf{Auth}_{\mathcal{A},\Pi'}(n) = 1] \leq \mathsf{negl}(n).$$

Definition 4

Π' is **secure (authenticated encryption)** if it is both CCA-secure and also achieves authenticated communication.[1]

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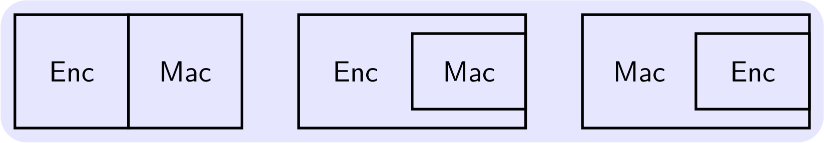
[1] CPA security and integrity imply CCA security.

Combining Encryption and Authentication



- **Encrypt-and-authenticate** (e.g., SSH):

$$c \leftarrow \mathsf{Enc}_{k_1}(m), \ t \leftarrow \mathsf{Mac}_{k_2}(m).$$

- **Authenticate-then-encrypt** (e.g, SSL):

$$t \leftarrow \mathsf{Mac}_{k_2}(m), \ c \leftarrow \mathsf{Enc}_{k_1}(m\|t).$$

- **Encrypt-then-authenticate** (e.g, IPsec):

$$c \leftarrow \mathsf{Enc}_{k_1}(m), \ t \leftarrow \mathsf{Mac}_{k_2}(c).$$

Analyzing Security of Combinations

All-or-nothing: Reject any combination for which there exists even a single counterexample is insecure.

- **Encrypt-and-authenticate**: $\mathsf{Mac}'_k(m) = (m, \mathsf{Mac}_k(m))$.
- **Authenticate-then-encrypt**:
  - $\mathsf{Trans} : 0 \to 00; 1 \to 10/01$; Enc' uses CTR mode; $c = \mathsf{Enc}'(\mathsf{Trans}(m \| \mathsf{Mac}(m)))$.
  - Flip the first two bits of c and verify whether the ciphertext is valid. $10/01 \to 01/10 \to 1$, $00 \to 11 \to \perp$.
  - If valid, the first bit of message is 1; otherwise 0.
  - For any MAC, this is not CCA-secure.
- **Encrypt-then-authenticate**:
  Decryption: If $\mathsf{Vrfy}(\cdot) = 1$, then $\mathsf{Dec}(\cdot)$; otherwise output $\perp$.

Remarks on Secure Message Transmission

- Authentication may leak the message.
- Secure message transmission implies CCA-security. The opposite direction is not necessarily true.
- Different security goals should always use different keys.
  - otherwise, the message may be leaked if $\mathsf{Mac}_k(c) = \mathsf{Dec}_k(c)$.
- Implementation may destroy the security proved by theory.
  - **Attack with padding oracle** (in TLS 1.0):
    Dec return two types of error: padding error, MAC error.
    Adv. learns last bytes if no padding error with guessed bytes.
  - **Attack non-atomic dec.** (in SSH Binary Packet Protocol):
    Dec (1)decrypt length field; (2)read packets as specified by the length; (3)check MAC.
    Adv. (1)send c; (2)send l packets until "MAC error" occurs; (3)learn $l = \mathsf{Dec}(c)$.

Content

Key Derivation Function (KDF)

Key Derivation Function (KDF) generates many keys from a secret source key sk.

For uniformly random sk: F is PRF, ctx is a unique string identifying application,

$$\mathsf{KDF}(sk, ctx, l) = \langle F_{sk}(ctx\|0), F_{sk}(ctx\|1) \cdots, F_{sk}(ctx\|l) \rangle .$$

For not-uniform sk: extract-then-expand paradigm.

extract: HKDF $k \leftarrow \mathsf{HMAC}(salt, sk)$. $salt$ is a random number.

expand: as the above.

Password-Based KDF (PBKDF)

Key stretching increases the time of testing key (with slow hash function).

Key strengthening increases the length/randomness of key (with salt).

PKCS#5 (PBKDF1): $H^{(c)}(pwd \| salt)$, iterate hash function c times.

Attack: either try the enhanced key (larger key space), or else try the initial key (longer time per key).

IV, Nonce, Counter and Salt

IV an input to a cryptographic primitive, providing randomness.

nonce a number used only once to sign a communication.

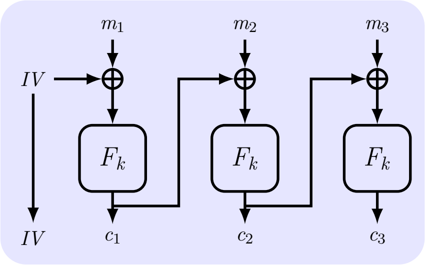
counter a sequence number used as nonce.

salt consists of random bits, creating the input to a function.

Content

Deterministic CPA Security

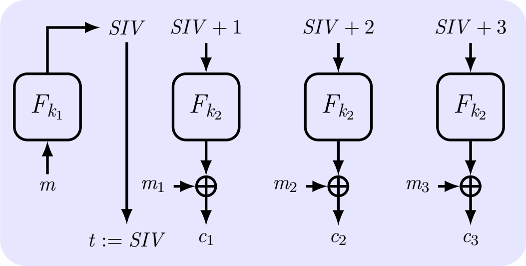
- **Deterministic encryption**: the same message is encrypted to the same ciphertext under the same key.
- **Application**: encrypted database index, disk encryption.
- **But no deterministic encryption is CPA-secure.**
- **Deterministic CPA Security**: CPA-secure if *never encrypt same message twice* using same key. The pair $\langle k, m \rangle$ is unique.
- **Common Mistake**: CBC/CTR with **fixed** IV.



Adversary can query (m_{q1}, m_{q2}) and get (c_{q1}, c_{q2}); then output PT: $IV \oplus c_{q1} \oplus m_{q2}$ and expect CT: c_{q2}.

Synthetic IV (SIV) for Det. Encryption

- **Synthetic IV (SIV)**:
  If F is PRF, $\Pi : (\mathsf{Enc}_k(r, m), \mathsf{Dec}_k(r, s))$ is CPA-secure, then Π_{det} is det. CPA-secure scheme:
  $(k_1, k_2) \leftarrow \mathsf{Gen}$; $SIV \leftarrow F_{k_1}(m)$; $s \leftarrow \mathsf{Enc}_{k_2}(SIV, m)$;
  output $c = \langle SIV, s \rangle$.
- **Deterministic Authenticated Encryption (DAE)**: det. CPA-security and integrity.
- **DAE for free with SIV-CTR**: Tag $t := SIV \leftarrow F_{k_1}(m)$ then CTR_{k_2} encryption.

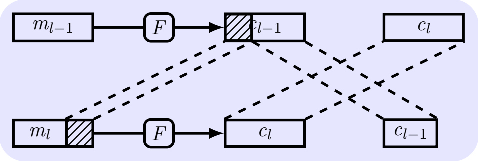


Wide Block PRP for Det. Encryption

- **Just PRP**: If F is PRP, then F is also det. CPA-secure.
- **PRP-based DAE**: $\mathsf{Enc}_k(m\|0^\ell)$. In Dec, if $\neq 0^\ell$, output $\perp$.
- **Narrow block** may leak info. as some blocks are the same.
- **Wide block PRP**: PRP with longer block length (e.g. a sector on disk) from PRP with short block length (e.g. AES).
- **Standards**: CBC-mask-CBC (CMC) and ECB-mask-ECB (EME) in IEEE P1619.2.
- **Cost**: 2x slower than SIV due to two-pass encryption.

Tweakable Encryption

- **Encryption without expansion**: $\mathcal{M} = \mathcal{C}$ implies det. encryption without integrity (e.g., disk encryption).
- **Trivial solution**: $k_t = F_k(t), t = 1, \ldots, \ell$.
- **Tweakable block ciphers**: many PRPs from one key $\mathcal{K} \times \mathcal{T} \times \mathcal{X} \to \mathcal{X}$, $\mathcal{T}$ is the set of tweaks.
- **XTS**: XEX(Xor-Encrypt-Xor)-based tweaked-codebook mode with ciphertext stealing. (XTS-AES, NIST SP 800-38E)
- **XEX**: To encrypt block j in sector I, $c = F_k(m \oplus x) \oplus x$, where $x = F_k(I) \otimes 2^j$ in Galois field, (I, j) is tweak.
- **Ciphertext stealing (CTS)**: no padding, **no expansion**.



Summary

- CCA-secure, AE, det. enc., det. CPA-secure, DAE.
- Enc-then-auth, KDF, SIV, wide block cipher, tweakable encryption.
- SIV-CTR, PBKDF, salt, enc. w/o expansion, CTS.