Message Authentication Codes and Collision-Resistant Hash Functions

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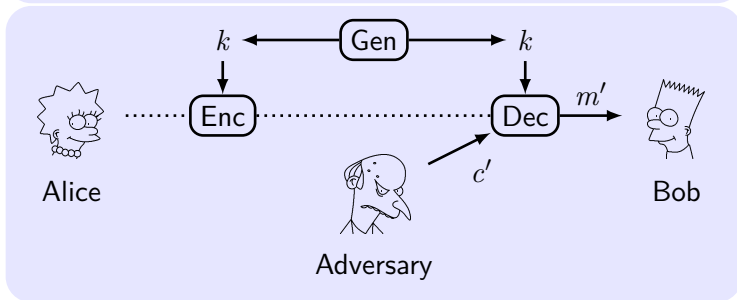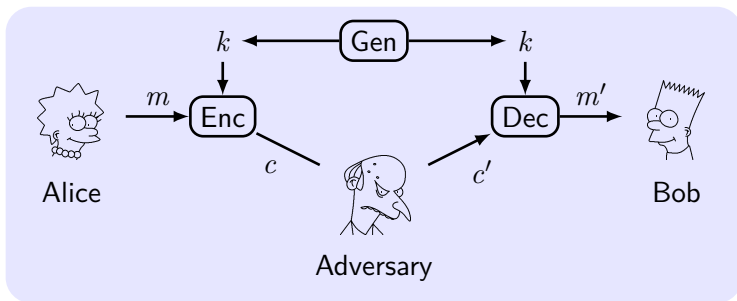
HIT/CST/NIS

Cryptography, Autumn, 2014
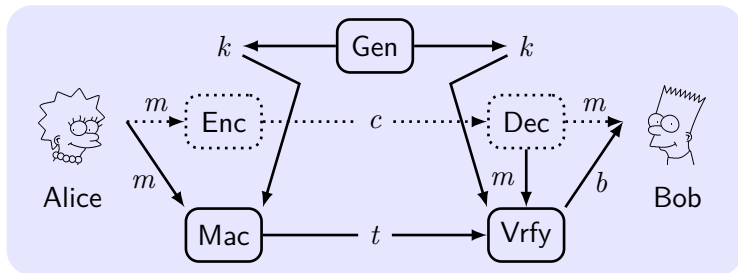
Outline

Content

Integrity and Authentication

The Syntax of MAC

- key k, tag t, a bit b means valid if $b = 1$; invalid if $b = 0$.
- **Key-generation** algorithm $k \leftarrow \mathsf{Gen}(1^n), |k| \geq n$.
- **Tag-generation** algorithm $t \leftarrow \mathsf{Mac}_k(m)$.
- **Verification** algorithm $b := \mathsf{Vrfy}_k(m, t)$.
- **Message authentication code**: $\Pi = (\mathsf{Gen}, \mathsf{Mac}, \mathsf{Vrfy})$.
- **Basic correctness requirement**: $\mathsf{Vrfy}_k(m, \mathsf{Mac}_k(m)) = 1$.
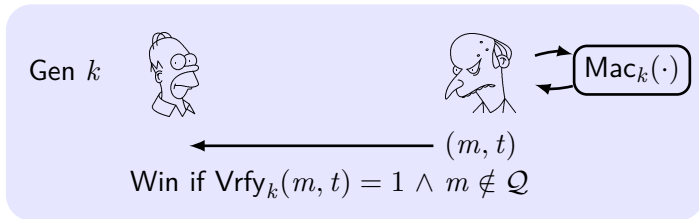
Security of MAC

- **Intuition**: No adversary should be able to generate a **valid** tag on any "**new**" message[1] that was not previously sent.
- **Replay attack**: Copy a message and tag previously sent. (**excluded by only considering "new" message**)
    - Sequence numbers: receiver must store the previous ones.
    - Time-Stamps: sender/receiver maintain synchronized clocks.
- **Existential unforgeability**: **Not** be able to forge a valid tag on **any** message.
    - **Existential forgery**: *at least one* message.
    - **Selective forgery**: message chosen *prior* to the attack.
    - **Universal forgery**: *any* given message.
- **Adaptive chosen-message attack (CMA)**: be able to obtain tags on *any* message chosen adaptively *during* its attack.

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[1]A stronger requirement is concerning *new message/tag pair*.

Definition of MAC Security

The message authentication experiment $\text{Macforge}_{\mathcal{A},\Pi}(n)$:

1. $k \leftarrow \text{Gen}(1^n)$.
2. $\mathcal{A}$ is given input 1^n and oracle access to $\text{Mac}_k(\cdot)$, and outputs (m, t). $\mathcal{Q}$ is the set of queries to its oracle.
3. $\text{Macforge}_{\mathcal{A},\Pi}(n) = 1 \iff \text{Vrfy}_k(m, t) = 1 \land m \notin \mathcal{Q}$.

Gen k

$\text{Mac}_k(\cdot)$

(m, t)

Win if $\text{Vrfy}_k(m, t) = 1 \land m \notin \mathcal{Q}$

Definition 1

A MAC Π is **existentially unforgeable under an adaptive CMA** if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that:
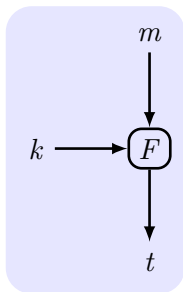
$$\Pr[\text{Macforge}_{\mathcal{A},\Pi}(n) = 1] \leq \text{negl}(n).$$

Examples

Suppose $\langle S, V \rangle$ are CMA-secure, are $\langle S', V' \rangle$ secure?

- $S'_{k_1, k_2}(m) = (S_{k1}(m), S_{k_2}(m))$
  $V'_{k_1, k_2}(m, (t_1, t_2)) = V_{k1}(m, t_1)$ and $V_{k_2}(m, t_2)$
- $S'_k(m) = (S_k(m), S_k(m))$
  $V'_k(m, (t_1, t_2)) = \left\{ \begin{array}{ll} V_k(m, t_1) & \text{if } t_1 = t_2 \\ 0 & \text{otherwise} \end{array} \right.$
- $S'_k(m) = (S_k(m), S_k(0^n))$
  $V'_k(m, (t_1, t_2)) = V_k(m, t_1)$ and $V_k(0^n, t_2)$
- $S'_k(m) = S_k(m)$
  $V'_k(m, t) = \left\{ \begin{array}{ll} V_k(m, t) & \text{if } m \neq 0^n \\ 1 & \text{otherwise} \end{array} \right.$
- $S'_k(m) = S_k(m)$ without the LSB
  $V'_k(m, t) = V_k(m, t\|0)$ or $V_k(m, t\|1)$

Content

Constructing Secure MACs

Construction 2

- F is PRF. $|m| = n$.
- Gen(1^n): $k \leftarrow \{0,1\}^n$ u.a.r.
- Mac$_k(m)$: $t := F_k(m)$.
- Vrfy$_k(m, t)$: $1 \iff t \overset{?}{=} F_k(m)$.

Theorem 3

If F is a PRF, Construction is a secure fixed-length MAC.
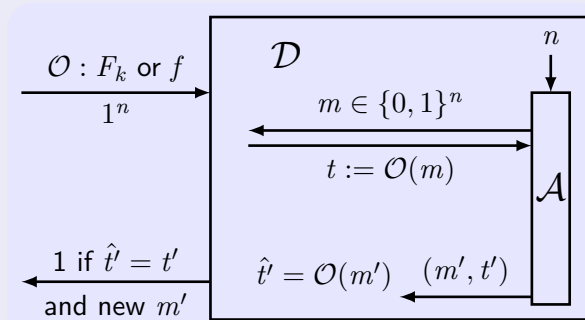
Lemma 4

Truncating MACs based on PRFs: If F is a PRF, so is
$F_k^t(m) = F_k(m)[1, \ldots, t]$.

Proof of Secure MAC from PRF

Idea: Show Π is secure unless F_k is not PRF by reduction.

Proof.

D distinguishes F_k; $\mathcal{A}$ attacks Π.

Proof.

(1) If true random f is used, $t = f(m)$ is uniformly distributed.

$$\Pr[D^{f(\cdot)}(1^n) = 1] = \Pr[\mathsf{Macforge}_{\mathcal{A},\tilde{\Pi}}(n) = 1] \le 2^{-n}.$$

(2) If F_k is used, conduct the experiment $\mathsf{Macforge}_{\mathcal{A},\Pi}(n)$.

$$\Pr[D^{F_k(\cdot)}(1^n) = 1] = \Pr[\mathsf{Macforge}_{\mathcal{A},\Pi}(n) = 1] = \varepsilon(n).$$

According to the definition of PGF,

$$\left| \Pr[D^{F_k(\cdot)}(1^n) = 1] - \Pr[D^{f(\cdot)}(1^n) = 1] \right| \ge \varepsilon(n) - 2^{-n}.$$

$\square$

Extension to Variable-Length Messages

For variable-Length Messages, would the following suggestions be secure?

- **Suggestion 1**: XOR all the blocks together and authenticate the result. $t := \mathsf{Mac}'_k(\oplus_i m_i)$.
- **Suggestion 2**: Authenticate each block separately. $t_i := \mathsf{Mac}'_k(m_i)$.
- **Suggestion 3**: Authenticate each block along with a sequence number. $t_i := \mathsf{Mac}'_k(i \| m_i)$.

Content

Constructing Fixed-Length CBC-MAC
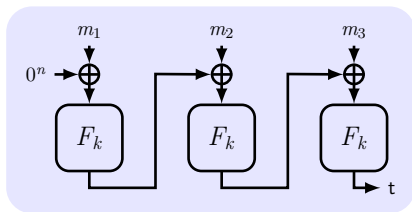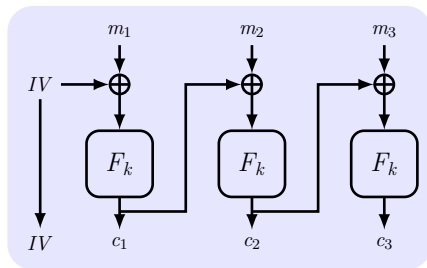
Modify CBC encryption into CBC-MAC:

- Change random IV to encrypted fixed 0^n, *otherwise*:
  Q: query m_1 and get (IV, t_1); output $m_1' = IV' \oplus IV \oplus m_1$ and $t' = \underline{\quad}$.

- Tag only includes the output of the final block, *otherwise*:
  Q: query m_i and get t_i; output $m_i' = t_{i-1}' \oplus t_{i-1} \oplus m_i$ and $t_i' = \underline{\quad}$.

Constructing Fixed-Length CBC-MAC (Cont.)

Construction 5

- *a PRF F and a length function ℓ. $|m| = \ell(n) \cdot n$. $\ell = \ell(n)$. $m = m_1, \ldots, m_\ell$.*
- $\mathsf{Gen}(1^n)$: $k \leftarrow \{0,1\}^n$ *u.a.r.*
- $\mathsf{Mac}_k(m)$: $t_i := F_k(t_{i-1} \oplus m_i), t_0 = 0^n$. *Output $t = t_\ell$.*
- $\mathsf{Vrfy}_k(m, t)$: $1 \iff t \overset{?}{=} \mathsf{Mac}_k(m)$.
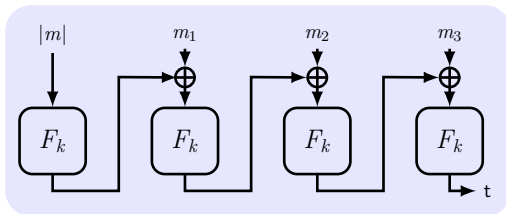
Theorem 6

*If F is a PRF, Construction is a secure **fixed-length** MAC.*

Not for **variable-length** message:
Q: For one-block message m with tag t, adversary can append a block ____ and output tag t.

Secure Variable-Length MAC

- **Input-length key separation**: $k_\ell := F_k(\ell)$, use k_ℓ for CBC-MAC.

- **Length-prepending**: Prepend m with $|m|$, then use CBC-MAC.

- **Encrypt last block (ECBC-MAC)**: Use two keys k_1, k_2. Get t with k_1 by CBC-MAC, then output $\hat{t} := F_{k_2}(t)$.

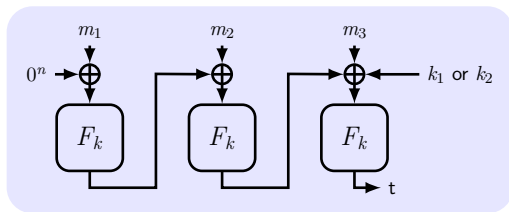Q: to authenticate a voice stream, which approach do you prefer?

MAC Padding

Padding must be invertible!

$$m_0 \neq m_1 \Rightarrow \mathsf{pad}(m_0) \neq \mathsf{pad}(m_1).$$

ISO: pad with "100...00". Add dummy block if needed.
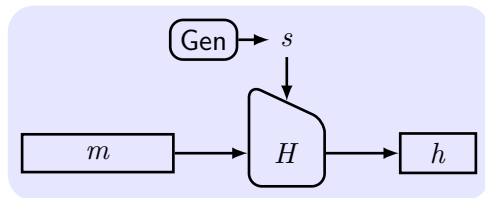Q: what if no dummy block?
CMAC (Cipher-based MAC from NIST): key= (k, k_1, k_2).

- No final encryption: extension attack thwarted by keyed XOR.
- No dummy block: ambiguity resolved by use of k_1 or k_2.

Content

Defining Hash Function

Definition 7

A **hash function (compression function)** is a pair of PPT algorithms (Gen, H) satisfying:

- a key $s \leftarrow \text{Gen}(1^n)$, s is **not kept secret**.
- $H^s(x) \in \{0,1\}^{\ell(n)}$, where $x \in \{0,1\}^*$ and ℓ is polynomial.

If H^s is defined only for $x \in \{0,1\}^{\ell'(n)}$ and $\ell'(n) > \ell(n)$, then (Gen, H) is a **fixed-length** hash function.

Defining Collision Resistance

- **Collision** in H: $x \neq x'$ and $H(x) = H(x')$.
- **Collision Resistance**: infeasible for any PPT alg. to find.

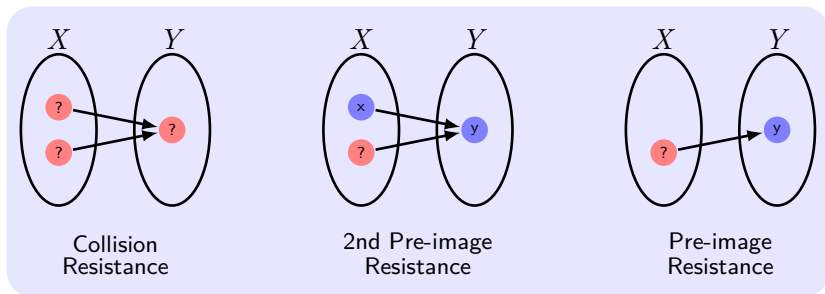The collision-finding experiment $\mathsf{Hashcoll}_{\mathcal{A},\Pi}(n)$:

1. $s \leftarrow \mathsf{Gen}(1^n)$.
2. $\mathcal{A}$ is given s and outputs x, x'.
3. $\mathsf{Hashcoll}_{\mathcal{A},\Pi}(n) = 1 \iff x \neq x' \land H^s(x) = H^s(x')$.

Definition 8

Π (H, H^s) is **collision resistant** if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that

$$\Pr[\mathsf{Hashcoll}_{\mathcal{A},\Pi}(n) = 1] \leq \mathsf{negl}(n).$$

Weaker Notions of Security for Hash Functions

- **Collision resistance**: It is hard to find $(x, x'), x' \neq x$ such that $H(x) = H(x')$.
- **Second pre-image resistance**: Given s and x, it is hard to find $x' \neq x$ such that $H^s(x') = H^s(x)$.
- **Pre-image resistance**: Given s and $y = H^s(x)$, it is hard to find x' such that $H^s(x') = y$.

H **is CRHF. Is** H' **CRHF?**

- $H'(m) = H(m) \oplus H(m \oplus 1^{|m|})$
- $H'(m) = H(m) \| H(0)$
- $H'(m) = H(m) \| H(m)$
- $H'(m) = H(m) \oplus H(m)$
- $H'(m) = H(m[0, \ldots, |m| - 2])$
- $H'(m) = H(m \| 0)$

The "Birthday" Problem

The "Birthday" Problem

Q: "*What size group of people do we need to take such that with probability $1/2$ some pair of people in the group share a birthday?*"
A: 23.

Lemma 9

Choose q elements $y_1, \ldots, y_q$ u.a.r from a set of size N, the probability that $\exists\, i \neq j$ with $y_i = y_j$ is $\mathsf{coll}(q, N)$, then

$$\mathsf{coll}(q, N) \leq \frac{q^2}{2N}.$$

$$\mathsf{coll}(q, N) \geq \frac{q(q-1)}{4N} \quad \text{if } q \leq \sqrt{2N}.$$

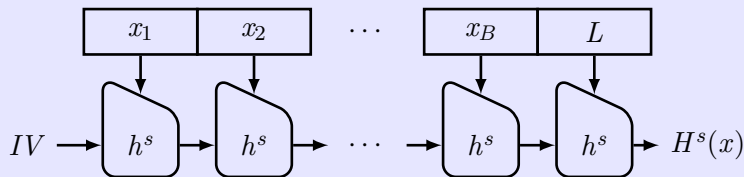$$\mathsf{coll}(q, N) = \Theta(q^2/N) \quad \text{if } q < \sqrt{N}.$$

The length of hash value should be long enough.

How many different meaningful sentences are in the below paragraph?

It is **hard/difficult/challenging/impossible** to **imagine/believe** that we will **find/locate/hire** another **employee/person** having similar **abilities/skills/character** as Alice. She has done a **great/super** job.

The Merkle-Damgård Transform

Construction 10

*Construct **variable-length** CRHF* (Gen, H) *from fixed-length*
(Gen, h) *(2ℓ bits $\to \ell$ bits, $\ell = \ell(n)$):*

- Gen*: remains unchanged.*
- H*: key s and string $x \in \{0,1\}^*$, $L = |x| < 2^\ell$:*
  - $B := \lceil \frac{L}{\ell} \rceil$ *(# blocks).* **Pad x with 0s**. *ℓ-bit blocks $x_1, \ldots, x_B$.*
    $x_{B+1} := L$, L is encoded using ℓ bits.
  - $z_0 := IV = 0^\ell$. *For $i = 1, \ldots, B+1$, compute*
    $z_i := h^s(z_{i-1}\|x_i)$.

Security of the Merkle-Damgård Transform

Theorem 11

If (Gen, h) is a fixed-length CRHF, then (Gen, H) is a CRHF.

Proof.

Idea: a collision in H^s yields a collision in h^s.

Two messages $x \neq x'$ of respective lengths L and L' such that $H^s(x) = H^s(x')$. # blocks are B and B'.

$x_{B+1} := L$ is necessary since **Padding with 0s** will lead to the same input with different messages.
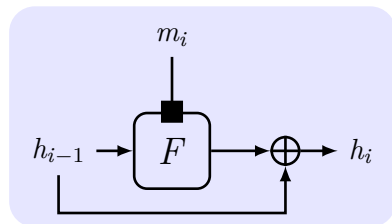
1. $L \neq L'$: $z_B \| L \neq z_{B'} \| L'$.
2. $L = L'$: $z_{i^*-1} \| x_{i^*} \neq z'_{i^*-1} \| x'_{i^*}$.

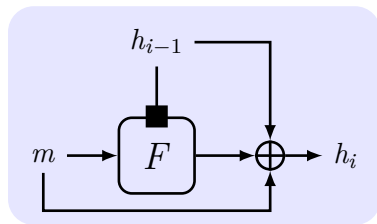So there must be $x \neq x'$ such that $h^s(x) = h^s(x')$. $\qquad\qquad\square$

CRHF from Block Cipher

Davies-Meyer:

Miyaguchi-Preneel:

Used by SHA-1/2, MD5
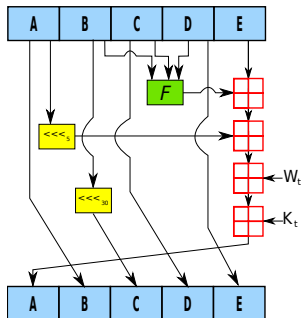
$h_i = F_{m_i}(h_{i-1}) \oplus h_{i-1}$

Used by Whirlpool

$h_i = F_{h_{i-1}}(m_i) \oplus h_{i-1} \oplus m$
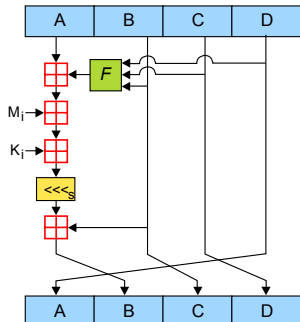
Q: what if $h_i = F_{m_i}(h_{i-1})$ without XOR with h_{i-1}?

# Cryptographic Hash Functions: SHA-1 and MD5

SHA-1:

MD5:

A, B, C, D and E are 32-bit words of the state; F is a nonlinear function that varies; $\lll n$ denotes a left bit rotation by n places; W_t/M_t is the expanded message word of round t; K_t is the round constant of round t; $\boxplus$ denotes addition modulo 2^{32}.

- Finding a collision in 128-bit MD5 requires time $2^{20.96}$.
- Finding a collision in 160-bit SHA-1 requires time 2^{51}.

Content

Hash-based MAC (HMAC)
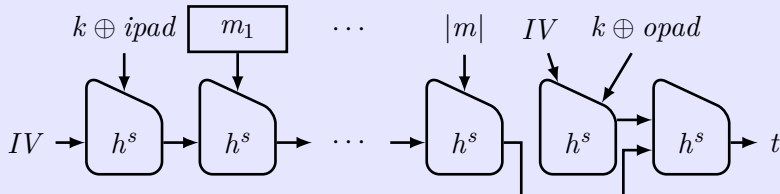
Construction 12

$(\widetilde{\mathsf{Gen}}, h)$ is a fixed-length CRHF. $(\widetilde{\mathsf{Gen}}, H)$ is the Merkle-Damgård transform. IV, opad *(0x36)*, ipad *(0x5C)* are fixed constants of length n. HMAC:

- $\mathsf{Gen}(1^n)$: Output (s, k). $s \leftarrow \widetilde{\mathsf{Gen}}, k \leftarrow \{0,1\}^n$ *u.a.r.*
- $\mathsf{Mac}_{s,k}(m)$: $t := H_{IV}^s\Big((k \oplus \mathsf{opad}) \| H_{IV}^s((k \oplus \mathsf{ipad}) \| m)\Big)$.
- $\mathsf{Vrfy}_{s,k}(m, t)$: $1 \iff t \overset{?}{=} \mathsf{Mac}_{s,k}(m)$.

Security of HMAC

Theorem 13

$$G(k) \stackrel{\textit{def}}{=} h^s(IV\|(k \oplus \mathsf{opad}))\|h^s(IV\|(k \oplus \mathsf{ipad})) = k_1\|k_2.$$

$(\widetilde{\mathsf{Gen}}, h)$ *is CRHF. If G is a PRG, then HMAC is secure.*

- HMAC is an industry standard (RFC2104)
- HMAC is faster than CBC-MAC
- Before HMAC, a common mistake was to use $H^s(k\|x)$
- Verification timing attacks: (Keyczar crypto library (Python))
  def Verify(key, msg, sig_bytes):
       return HMAC(key, msg) $==$ sig_bytes
  The problem: implemented as a byte-by-byte comparison
- *Don't implement it yourself.*

- adaptive CMA, replay attack, birthday attack.
- existential unforgeability, collision resistance.
- CBC-MAC, CRHF, Merkle-Damgård transform, NMAC, HMAC.