Cryptographic Protocols

Yu Zhang

HIT/CST/NIS

Cryptography, Autumn, 2014

Outline

Content

Protocols

- **Communications protocol** is a formal description of digital message formats and the rules for exchanging those messages for a specific purpose.
- Protocols are to communications what algorithms are to computations.
- Everyone involved in the protocol must know the protocol and all of the steps to follow in advance.
- Everyone involved in the protocol must agree to follow it.
- The protocol must be unambiguous; each step must be well defined and there must be no chance of a misunderstanding.
- The protocol must be complete; there must be a specified action for every possible situation.
- It should not be possible to do more or learn more than what is specified in the protocol.

Protocol Types

- **Arbitrated protocols**: An arbitrator is a disinterested third party trusted to complete a protocol.
- **Adjudicated protocols**: An adjudicator is also a disinterested and trusted third party. Unlike an arbitrator, he is not directly involved in every protocol.
- **Self-enforcing protocols**: the best type of protocol. The protocol itself guarantees fairness.

Attacks against Protocols

- **Passive attacks**: the attacker does not affect the protocol.
- **Active attacks**: the attacker alters the protocol to his own advantage.

Cheater: the attacker could be one of the parties involved in the protocol.

- **Passive cheaters**: follow the protocol, but try to obtain more information than the protocol intends them to.
- **Active cheaters**: disrupt the protocol in progress in an attempt to cheat.

Content

Three-Pass Protocol

Purpose: communication without shared keys.
Requirement: $\mathsf{Dec}_{k_1}(\mathsf{Enc}_{k_2}(\mathsf{Enc}_{k_1}(m))) = \mathsf{Enc}_{k_2}(m)$.
Shamir Protocol: p is a prime, find e, d with $\gcd(e, p-1) = 1$ and $ed \equiv 1 \pmod{p-1}$.

p, e_A, d_A $\qquad$ p, e_B, d_B

$c = [m^{e_A} \bmod p] \longrightarrow$

$\longleftarrow c_1 = [c^{e_B} \bmod p]$

$c_2 = [c_1^{d_A} \bmod p] \longrightarrow$

$m = [c_2^{d_B} \bmod p]$

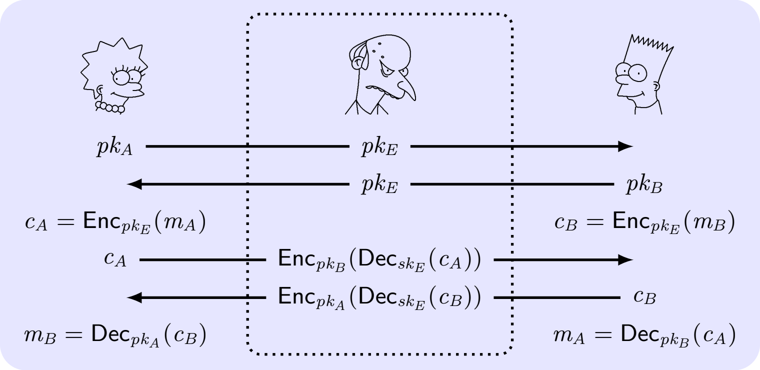
$$c_2^{d_B} = c_1^{d_A \cdot d_B} = c^{e_B \cdot d_A \cdot d_B} = m^{e_A \cdot e_B \cdot d_A \cdot d_B} = m^{e_A d_A \cdot e_B d_B} = m.$$

Weakness: insecurity under the man-in-the-middle attack.

Content

The Man-In-The-Middle Attack

Also called **bucket-brigade attack**: A form of active eavesdropping in which the attacker makes independent connections with the victims and relays messages between them, making them believe that they are talking directly to each other.



Interlock Protocol

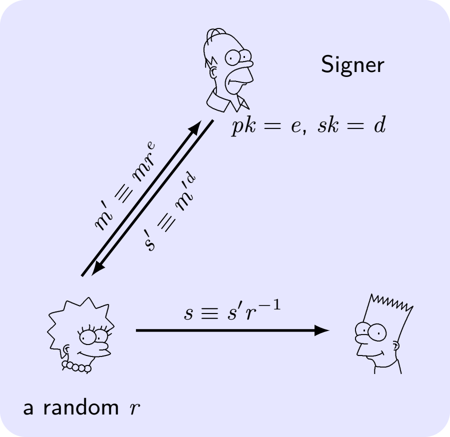
Purpose: foil the man-in-the-middle attack.

		
pk_A	$\longrightarrow$	
	$\longleftarrow$	pk_B
$c_A = \mathsf{Enc}_{pk_B}(m_A)$		$c_B = \mathsf{Enc}_{pk_A}(m_B)$
$(c_{A1} \| c_{A2}) = c_A$		$(c_{B1} \| c_{B2}) = c_B$
c_{A1}	$\longrightarrow$	
	$\longleftarrow$	c_{B1}
c_{A2}	$\longrightarrow$	
	$\longleftarrow$	c_{B2}
$c_B = (c_{B1} \| c_{B2})$		$c_A = (c_{A1} \| c_{A2})$
$m_B = \mathsf{Dec}_{pk_A}(c_B)$		$m_A = \mathsf{Dec}_{pk_B}(c_A)$

Content

Blind Signature

Blind signature is a form of digital signature in which the message is blinded before it is signed.
Alice asks for Signer to sign m blindly and then sends to Bob.



$$s \equiv s' r^{-1} \equiv m'^d r^{-1} \equiv (mr^e)^d r^{-1} \equiv m^d.$$

Group Signature

Group Signature: allowing a member of a group to anonymously sign a message on behalf of the group (with a group manager).

- **Soundness**: valid sigs by members verify correctly
- **Unforaeable**: only members can create valid sigs
- **Anonymity**: signer can be determined only by manager.
- **Traceability**: manager can trace which member signed.
- **Unlinkability**: cannot tell if two sigs were from same signer.
- **Exculpability**: cannot forge a sig for other/non members.

A trivial group signature with trusted manager GM:

- **KeyGen**: GM generates a secret key list for each member and publishes all of public keys.
- **Sign**: sign with an unused secret key.
- **Verify**: try all of public keys.

Ring Signature

Ring Signature: Group signature without group manager, and:

- cannot revoke the anonymity of an individual signature
- any group of users can be a group without additional setup

A ring signature by Boneh et al. 2003:

$q-$order cyclic groups: G_1 with $+$ and generator P, G_2 with $\times$, bilinear map $e : G_1 \times G_1 \to G_2$ such that $e(aP, bP) = e(P, P)^{ab}$, hash function $H : \{0, 1\}^* \to G_1$.

- **KeyGen**: for member U_i: $sk = x_i \leftarrow Z_q, pk = Y_i = x_i P$.
- **Sign**: message m with $(\sigma_i), i = 1, \cdots, n$ by U_k:

$$\text{for } i \neq k, a_i \leftarrow Z_q, \sigma_i = a_i P; \quad \sigma_k = \frac{1}{x_k}(H(m) - \Sigma_{j \neq k} a_j Y_j)$$

- **Verify**:

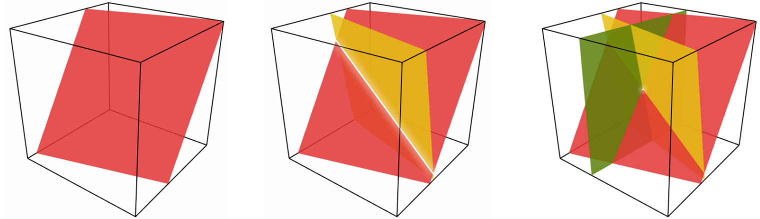
$$e(H(m), P) = \Pi_i e(Y_i, \sigma_i)$$

Content

Secret Sharing

Purpose: distribute a secret amongst a group of n participants, each of whom is allocated a share of the secret. The secret can be reconstructed only when a sufficient number of shares t are combined together. It is called (t, n)-**threshold scheme**.

Blakley's scheme: any n nonparallel n-dimensional hyperplanes intersect at a specific point.



Chinese remainder theorem: the shares of secret are generated by reduction modulo some relatively prime integers, and the secret is recovered by solving the system of congruences using the CRT.

(t, n)-**threshold scheme**: at least t of parties can efficiently decrypt/sign the ciphertext, while less than t have no useful information.

Threshold Elgamal Cryptosystem:

- **Key sharing**: $sk = s, pk = h = g^s$. Party i obtains a share s_i with Shamir's scheme ((t, n)-threshold secret sharing) such that $s = \Sigma_i s_i \cdot \lambda_i$ with public info λ_i and publishes $h_i = g^{s_i}$.
- **Enc**: $y \leftarrow \mathbb{Z}_q$, $\langle c_1, c_2 \rangle = \langle g^y, h^y \cdot m \rangle$.
- **Dec**: Party i outputs $d_i = c_1^{s_i}$ and ZKP of $\log_g h_i = \log_{c_1} d_i$.

$$m = c_2 / \Pi_i d_i^{\lambda_i}.$$

$$c_2 / \Pi_i d_i^{\lambda_i} = c_2 / \Pi_i c_1^{s_i \cdot \lambda_i} = c_2 / c_1^{\Sigma_i s_i \cdot \lambda_i} = c_2 / c_1^s = m.$$

Content

Commitment Scheme

Commitment scheme allows one to commit to a value while keeping it hidden, with the ability to reveal the committed value.

Coin flipping over Internet: Alice flips the coin, Bob guesses.

$b \leftarrow \{H, T\}$

a random r

$m = b \| r$

$h = \mathsf{Hash}(m) \longrightarrow$

$\longleftarrow b' \in \{H, T\}$

$m \longrightarrow$

$h \stackrel{?}{=} \mathsf{Hash}(m)$

Win if $b = b'$

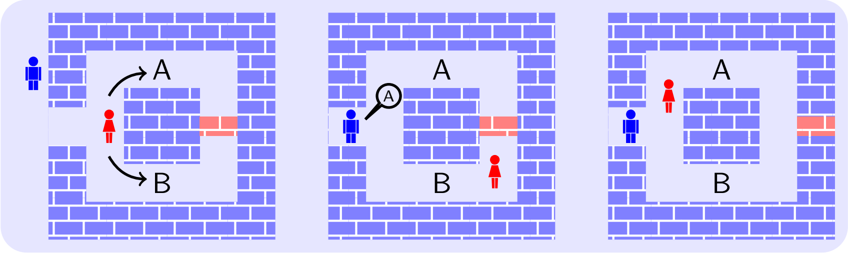
Content

Zero-Knowledge Proof

- **Interactive proof system** is an abstract machine that models computation as the exchange of messages between two parties: verifier and prover.
- **Proof of knowledge**: an interactive proof in which **prover** succeeds convincing **verifier** that it knows something.
- **Zero-knowledge proof (ZKP)**: an interactive proof *without revealing anything other than the veracity of the statement*.
  - **Completeness**: if the statement is true, the honest "verifier" will be convinced by an honest prover.
  - **Soundness**: if the statement is false, no cheating prover can convince the honest verifier.
  - **Existence**: If OWF exists, ZKP exists for any NP-set.

A Toy Example of ZKP

Alice proves to Bob that she knows the secret word used to open a magic door in a cave.



Schnorr protocol: Alice proves to Bob the knowledge of $x = \log_g y$ in the discrete log problem.

a random r

$t = g^r \longrightarrow$

$\longleftarrow$ a random bit $c \leftarrow \{0, 1\}$

$s = r + cx \longrightarrow$

$g^s \stackrel{?}{=} ty^c$

If Alice can foresee c, Alice can cheat with $t = g^s/y$ when $c = 1$.

ZKP of the Ability to Break RSA

Purpose: Alice convinces Bob that she knows Charlie's private key d for RSA problem $\langle N, e, d \rangle$, but she doesn't want to tell Bob d.

random $k, s > 3$

$\longleftrightarrow$

$ks \equiv e \pmod{N}$

random c

$\longleftrightarrow$

$m = c^d$

$x = m^k \longrightarrow$

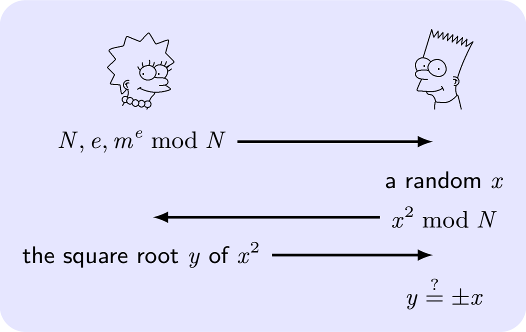
$x^s \stackrel{?}{=} c$

If Alice can manipulate c, Alice can cheat with $c = m^e$.

Content

Oblivious Transfer

Oblivious transfer (OT) protocol: a sender remains oblivious as to whether or which info has been transferred.
Rabin's OT protocol: RSA problem $\langle N, e, d \rangle$.



If $y \neq \pm x$, then Bob can factorize N with $\gcd(y - x, N)$ and find d. Since every quadratic residue modulo N has four square roots, Bob can learn m with probability $\frac{1}{2}$.

1-out-of-2 Oblivious Transfer

1-out-of-2 OT: the sender has two messages m_0 and m_1, and the receiver wishes to receive m_b, without the sender learning b, while the sender ensures that the receiver receive only one message.

RSA: N, e $\longrightarrow$

random x_0, x_1 $\longrightarrow$

random b, k

$\longleftarrow$ $v = x_b + k^e$

$k_0 = (v - x_0)^d,\ k_1 = (v - x_1)^d$

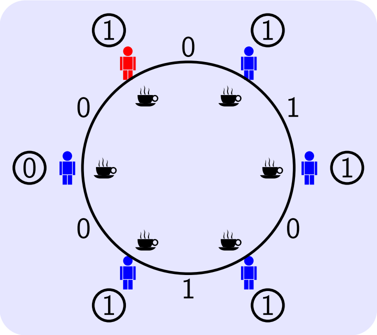
$m_0' = m_0 + k_0, m_1' = m_1 + k_1$ $\longrightarrow$

$m_b = m_b' - k$

Content

Secure Multi-Party Computation

Secure multi-party computation (MPC): enable parties to jointly compute a function over their inputs, while at the same time keeping these inputs private.
Dining Cryptographers Problem: how to perform a secure MPC of the boolean-OR function.



- at most one  (1), other  (0).
- every two adjacent people establish a shared one-bit secret.
- everyone shouts the XOR of two shared secrets and its own bit.
- output the XOR of all of what everyone shouts. If 1, there is a , otherwise there is none.

Content

Why Quantum Cryptography?

Quantum cryptography taps the natural uncertainty of the quantum world.

- **Superposition**: object doesn't have definite properties (location, speed) but has probabilities over them.
- **Interference**: probabilities can be negative.
- **Entanglement**: properties of many particles can be correlated.
- **Measurement**: object's properties collapse to definite value when measured, collapsing also properties of other entangled objects.

State-of-the-Art of Quantum Cryptography

- (Unsurprisingly) there is **no proof** that quantum computers are more powerful than classical computers/Boolean circuits/Turing machines.
- There are **polynomial** algorithms for quantum computers solving problems unknown to be solvable classically in poly-time: factoring and discrete logs.
- There are **hard** problem with no quantum poly-time algorithm: NPC, inverting many candidate OWF, private key encryption and signature schemes.

Quantum Key Distribution

Purpose: Using photon polarization states to transmit the information in a public channel against eavesdroppers.

BB84 protocol: C. H. Bennett and G. Brassard (1984)

<table>
<tr><td>Basis</td><td>0</td><td>1</td></tr>
<tr><td>+</td><td>|</td><td>–</td></tr>
<tr><td>x</td><td>/</td><td>\</td></tr>
</table>

<table>
<tr><td>Alice’s random bits</td><td>01101001</td></tr>
<tr><td>Alice’s random sending basis</td><td>++x+xxx+</td></tr>
<tr><td>Photon polarization Alice sends</td><td>|–\|\//–</td></tr>
<tr><td>Bob’s random measuring basis</td><td>+xxx+x++</td></tr>
<tr><td>Photon polarization Bob measures</td><td>|/\/–/––</td></tr>
<tr><td>Shared secret key</td><td>0 1 0 1</td></tr>
</table>

- Two bases are public.
- Eavesdropping would change the photon polarization states.
- Check for the presence of eavesdropping by comparing a subset of shared bit string.

Summary

- Shamir three-pass protocol
- interlock protocol, man-in-the-middle attack
- blind/group/ring signature
- secret sharing, threshold cryptography
- commitment scheme
- zero knowledge proofs, Schnorr protocol
- Rabin's/1-out-of-2 oblivious transfer
- multi-party computation, dining cryptographers problem
- quantum cryptography, BB84