# Private-Key Management and the Public-Key Revolution

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# Outline

# Content

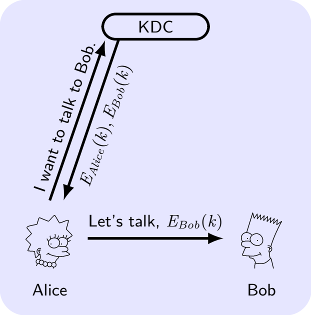
# Limitations of Private-Key Cryptography

- The key-distribution need physically meeting.
- The number of keys for $U$ users is $\Theta(U^2)$.
- Secure communication in open system:

Solutions that are based on private-key cryptography are not sufficient to deal with the problem of secure communication in open systems where parties cannot physically meet, or where parties have transient interactions.

# Needham-Schroeder Protocol

- Key Distribution Center (KDC) as Trusted Third Party (TTP).
- $E_{Bob}(k)$ is a **ticket** to access $Bob$, $k$ is **session key**.
- Used in MIT's Kerberos protocol (in Windows).




**Strength**:

- each one stores one key.
- no updates.

**Weakness**:

- all-or-nothing.
- single-point-of-failure.

# Merkle Puzzles (Key Exchange W/O TTP)

**Alice** prepares $2^{32}$ puzzles $\text{Puzzle}_i$, and sends to Bob.

$$\text{Puzzle}_i \leftarrow \text{Enc}_{(0^{96}\|p_i)}(\text{"Puzzle \#"}x_i\|k_i),$$

where Enc is 128-bit, $p_i \leftarrow \{0,1\}^{32}$ and $x_i, k_i \leftarrow \{0,1\}^{128}$.

**Bob** chooses $\text{Puzzle}_j$ randomly, guesses $p_j$ in $2^{32}$ time, obtains $x_j, k_j$ and sends $x_j$ to Alice.

**Alice** lookups puzzle with $x_j$, and uses $k_j$ as secret key.

- **Adversary** needs $2^{32+32}$ time.

**Better Gap?**

Quadratic gap is best possible if we treat cipher as a black box oracle.

# Content

# Public-Key Revolution

- In 1976, Whitfield Diffie and Martin Hellman published "*New Directions in Cryptography*".
- **Asymmetric** or **public-key** encryption schemes:
  - **Public key** as the encryption key.
  - **Private key** as the decryption key.
- **Public-key primitives**:
  - Public-key encryption.
  - Digital signatures. (non-repudiation)
  - Interactive key exchange.
- **Strength**:
  - Key distribution over public channels.
  - Reduce the need to store many keys.
  - Enable security in open system.
- **Weakness**: slow, active attack on public key distribution.

# Content

# The Setting and Definition of Security

The key-exchange experiment $\mathsf{KE}^{\mathsf{eav}}_{\mathcal{A},\Pi}(n)$:

1. Two parties holding $1^n$ execute protocol $\Pi$. $\Pi$ results in a **transcript** trans containing all the messages sent by the parties, and a **key** $k$ that is output by each of the parties.
2. A random bit $b \leftarrow \{0,1\}$ is chosen. If $b = 0$ then choose $\hat{k} \leftarrow \{0,1\}^n$ *u.a.r*, and if $b = 1$ then set $\hat{k} := k$.
3. $\mathcal{A}$ is given trans and $\hat{k}$, and outputs a bit $b'$.
4. $\mathsf{KE}^{\mathsf{eav}}_{\mathcal{A},\Pi}(n) = 1$ if $b' = b$, and 0 otherwise.

**Definition 1**

A key-exchange protocol $\Pi$ is secure in the presence of an eavesdropper if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that

$$\Pr[\mathsf{KE}^{\mathsf{eav}}_{\mathcal{A},\Pi}(n) = 1] < \frac{1}{2} + \mathsf{negl}(n).$$

# Diffie-Hellman Key-Exchange Protocol

$(\mathbb{G}, q, g) \leftarrow \mathcal{G}$

$x \leftarrow \mathbb{Z}_q$

$h_1 := g^x \xrightarrow{\mathbb{G}, q, g, h_1}$

$y \leftarrow \mathbb{Z}_q$

$\xleftarrow{h_2} h_2 := g^y$

$k_A := h_2^x \qquad k_B := h_1^y$

$$k_A = k_B = k = g^{xy}.$$

$\widehat{\mathsf{KE}}^{\mathsf{eav}}_{\mathcal{A},\Pi}$ denote an experiment where if $b = 0$ the adversary is given $\hat{k} \leftarrow \mathbb{G}$.

**Theorem 2**

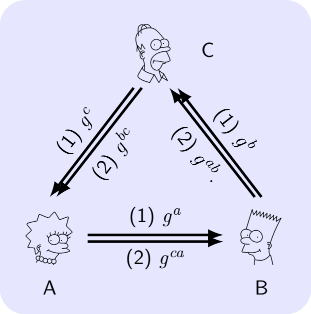
*If DDH problem is hard relative to $\mathcal{G}$, then DH key-exchange protocol $\Pi$ is secure in the presence of an eavesdropper (with respect to the modified experiment $\widehat{\mathsf{KE}}^{\mathsf{eav}}_{\mathcal{A},\Pi}$).*

**Security**

Insecurity against active adversaries (Man-In-The-Middle).

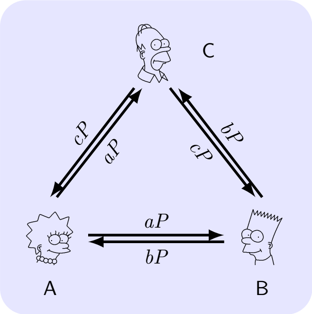
# Triparties Key Exchange

DH-based KE in 2 rounds:




Key= $g^{abc}$.

Joux's KE in 1 round:




Key= $e(P, P)^{abc}$ in bilinear map.

### Open Problem

How to exchange keys between 4 parties in one round?

# Summary

- Merkle, Diffie, Hellman, Rivest, Shamir, Adleman and Rabin.
- Needham-Schroeder protocol, Diffie-Hellman key-exchange protocol.