CCA-Secure and Authentication Encryption

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Outline

1. Constructing CCA-Secure Encryption Schemes
2. Obtaining Privacy and Message Authentication
3. Key Derivation Function (FYI)
4. Deterministic Encryption (FYI)

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1 Constructing CCA-Secure Encryption Schemes

2 Obtaining Privacy and Message Authentication

3 Key Derivation Function (FYI)

4 Deterministic Encryption (FYI)

Recall Security Against CCA

The CCA indistinguishability experiment $\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi}(n)$:

1. $k \leftarrow \mathsf{Gen}(1^n)$.
2. $\mathcal{A}$ is given input 1^n and oracle access $\mathcal{A}^{\mathsf{Enc}_k(\cdot)}$ and $\mathcal{A}^{\mathsf{Dec}_k(\cdot)}$, outputs m_0, m_1 of the same length.
3. a random bit $b \leftarrow \{0,1\}$ is chosen. Then $c \leftarrow \mathsf{Enc}_k(m_b)$ is given to $\mathcal{A}$.
4. $\mathcal{A}$ continues to have oracle access **except for** c, outputs b'.
5. If $b' = b$, $\mathcal{A}$ succeeded $\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi} = 1$, otherwise 0.

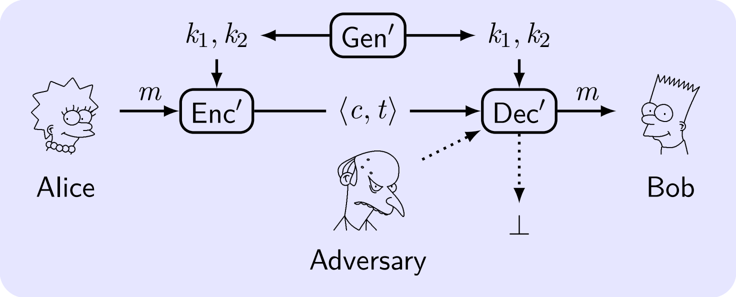
Definition 1

Π has **indistinguishable encryptions under a CCA (CCA-secure)** if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that

$$\Pr\left[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi}(n) = 1\right] \leq \frac{1}{2} + \mathsf{negl}(n).$$

Constructing CCA-Secure Encryption Schemes

Idea: Make decryption oracle useless. CCA = CPA-then-MAC.



Construction 2

$\Pi_E = (\mathsf{Gen}_E, \mathsf{Enc}, \mathsf{Dec})$, $\Pi_M = (\mathsf{Gen}_M, \mathsf{Mac}, \mathsf{Vrfy})$. Π':

- $\mathsf{Gen}'(1^n)$: $k_1 \leftarrow \mathsf{Gen}_E(1^n)$ *and* $k_2 \leftarrow \mathsf{Gen}_M(1^n)$
- $\mathsf{Enc}'_{k_1,k_2}(m)$: $c \leftarrow \mathsf{Enc}_{k_1}(m)$, $t \leftarrow \mathsf{Mac}_{k_2}(c)$ *and output* $\langle c, t\rangle$
- $\mathsf{Dec}'_{k_1,k_2}(\langle c, t\rangle) = \mathsf{Dec}_{k_1}(c)$ *if* $\mathsf{Vrfy}_{k_2}(c, t) \overset{?}{=} 1$; *otherwise* $\perp$

Proof of CCA-Secure Encryption Schemes

Theorem 3

If Π_E is a CPA-secure private-key encryption scheme and Π_M is a secure MAC with unique tags, then Construction Π' is CCA-secure.

Proof.

VQ: $\mathcal{A}$ submits a "new" query to oracle Dec' and $\mathsf{Vrfy} = 1$.

$$\Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1] \leq \Pr[\mathsf{VQ}] + \Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1 \wedge \overline{\mathsf{VQ}}]$$

We need to prove the following claims.

1. $\Pr[\mathsf{VQ}]$ is negligible.
2. $\Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1 \wedge \overline{\mathsf{VQ}}] \leq \frac{1}{2} + \mathsf{negl}(n)$.

□

Proof of "$\Pr[\text{VQ}]$ is negligible"

Idea: Reduce $\mathcal{A}_M$ (attacking Π_M with an oracle $\mathsf{Mac}_{k_2}(\cdot)$) to $\mathcal{A}$.

Proof.

- $\mathcal{A}_M$ chooses $i \leftarrow \{1, \ldots, q(n)\}$ *u.a.r.*
- Run $\mathcal{A}$ with the encryption/decryption oracles.
- If the ith decryption oracle query from $\mathcal{A}$ uses a "new" c, output (c, t) and stop.
- $\mathsf{Macforge}_{\mathcal{A}_M,\Pi_M}(n) = 1$ only if VQ occurs.
- $\mathcal{A}_M$ correctly guesses i with probability $1/q(n)$.

$$\Pr[\mathsf{Macforge}_{\mathcal{A}_M,\Pi_M}(n) = 1] \geq \Pr[\mathsf{VQ}]/q(n).$$

□

Proof of "$\Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1 \wedge \overline{\mathsf{VQ}}] \leq \frac{1}{2} + \mathsf{negl}(n)$"

Idea: Reduce $\mathcal{A}_E$ (attacking Π_E with an oracle $\mathsf{Enc}_{k_1}(\cdot)$) to $\mathcal{A}$.

Proof.

- Run $\mathcal{A}$ with the encryption/decryption oracles.
- Run $\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A}_E,\Pi_E}$ as $\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}$.
- $\mathcal{A}_E$ outputs the same b' that is output by $\mathcal{A}$.
- $\Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A}_E,\Pi_E}(n) = 1 \wedge \overline{\mathsf{VQ}}] = \Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1 \wedge \overline{\mathsf{VQ}}]$ unless VQ occurs.

$$\Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A}_E,\Pi_E}(n) = 1] \geq \Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1 \wedge \overline{\mathsf{VQ}}].$$

□

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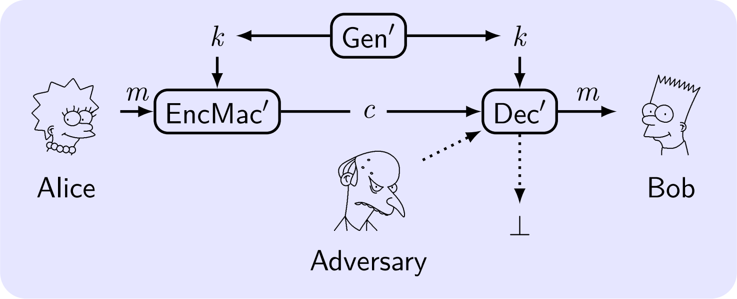
Message Transmission Scheme



- **Key-generation** algorithm outputs $k \leftarrow \mathsf{Gen}'(1^n)$.
  $k = (k_1, k_2)$. $k_1 \leftarrow \mathsf{Gen}_E(1^n)$, $k_2 \leftarrow \mathsf{Gen}_M(1^n)$.
- **Message transmission** algorithm is derived from $\mathsf{Enc}_{k_1}(\cdot)$ and $\mathsf{Mac}_{k_2}(\cdot)$, outputs $c \leftarrow \mathsf{EncMac}'_{k_1,k_2}(m)$.
- **Decryption** algorithm is derived from $\mathsf{Dec}_{k_1}(\cdot)$ and $\mathsf{Vrfy}_{k_2}(\cdot)$, outputs $m \leftarrow \mathsf{Dec}'_{k_1,k_2}(c)$ or $\perp$.
- **Correctness requirement**: $\mathsf{Dec}'_{k_1,k_2}(\mathsf{EncMac}'_{k_1,k_2}(m)) = m$.

Defining Secure Message Transmission

The secure message transmission experiment $\mathsf{Auth}_{\mathcal{A},\Pi'}(n)$:

1. $k = (k_1, k_2) \leftarrow \mathsf{Gen}'(1^n)$.
2. $\mathcal{A}$ is given input 1^n and oracle access to EncMac'_k, and outputs $c \leftarrow \mathsf{EncMac}'_k(m)$.
3. $m := \mathsf{Dec}'_k(c)$. $\mathsf{Auth}_{\mathcal{A},\Pi'}(n) = 1 \iff m \neq \perp \wedge\ m \notin \mathcal{Q}$.

Definition 4

Π' achieves **authenticated communication** if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that

$$\Pr[\mathsf{Auth}_{\mathcal{A},\Pi'}(n) = 1] \leq \mathsf{negl}(n).$$

Definition 5

Π' is **secure Authenticated Encryption (A.E.)** if it is both CCA-secure and also achieves authenticated communication.

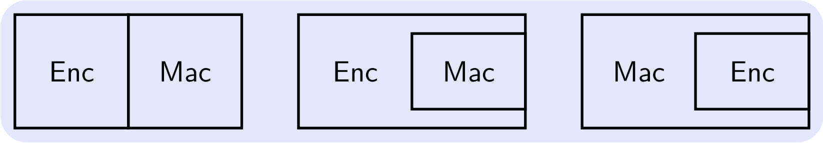
Does CCA-security imply A.E.? (homework)

Questions

Suppose (E, D) provides A.E. Which of the following systems provide A.E.?

- $E'_k(m) = (E_k(m), E_k(m))$ and $D'_k(c_1, c_2) = D_k(c_1)$
- $E'_k(m) = (E_k(m), 0)$ and $D'_k(c, b) = \begin{cases} D_k(c) & \text{if } b = 0 \\ \perp & \text{otherwise} \end{cases}$
- $E'_k(m) = E_k(m) \oplus 1^*$ and $D'_k(c) = D_k(c \oplus 1^*)$
- $E'_k(m) = (E_k(m), E_k(m))$ and
  $D'_k(c_1, c_2) = \begin{cases} D_k(c_1) & \text{if } D_k(c_1) = D_k(c_2) \\ \perp & \text{otherwise} \end{cases}$
- $E'_k(m) = (E_k(m), H(m))$ (H is a CRHF) and
  $D'_k(c, h) = \begin{cases} D_k(c) & \text{if } H(D_k(c)) = h \\ \perp & \text{otherwise} \end{cases}$

Combining Encryption and Authentication



- **Encrypt-and-authenticate** (e.g., SSH):

$$c \leftarrow \mathsf{Enc}_{k_1}(m),\ t \leftarrow \mathsf{Mac}_{k_2}(m).$$

- **Authenticate-then-encrypt** (e.g, SSL):

$$t \leftarrow \mathsf{Mac}_{k_2}(m),\ c \leftarrow \mathsf{Enc}_{k_1}(m\|t).$$

- **Encrypt-then-authenticate** (e.g, IPsec):

$$c \leftarrow \mathsf{Enc}_{k_1}(m),\ t \leftarrow \mathsf{Mac}_{k_2}(c).$$

Analyzing Security of Combinations

All-or-nothing: Reject any combination for which there exists even a single counterexample is insecure.

- **Encrypt-and-authenticate**: $\mathsf{Mac}'_k(m) = (m, \mathsf{Mac}_k(m))$.
- **Authenticate-then-encrypt**:
  - $\mathsf{Trans} : 0 \to 00; 1 \to 10/01$; Enc' uses CTR mode; $c = \mathsf{Enc}'(\mathsf{Trans}(m\|\mathsf{Mac}(m)))$.
  - Flip the first two bits of c and verify whether the ciphertext is valid. $10/01 \to 01/10 \to 1$, $00 \to 11 \to \perp$.
  - If valid, the first bit of message is 1; otherwise 0.
  - For any MAC, this is not CCA-secure.
- **Encrypt-then-authenticate**:
  Decryption: If $\mathsf{Vrfy}(\cdot) = 1$, then $\mathsf{Dec}(\cdot)$; otherwise output $\perp$.

Authenticated Encryption Theory and Practice

Theorem 6

Π_E is CPA-secure and Π_E is a secure MAC with unique tages, Π' deriving from encrypt-then-authenticate approach is secure.

GCM(Galois/Counter Mode): CTR encryption then Galois MAC. (RFC4106/4543/5647/5288 on IPsec/SSH/TLS)
EAX: CTR encryption then CMAC.

Proposition 7

Authenticate-then-encrypt approach is secure if Π_E is rand-CTR mode or rand-CBC mode.

CCM (Counter with CBC-MAC): CBC-MAC then CTR encryption. (802.11i, RFC3610)
OCB (Offset Codebook Mode): integrating MAC into ENC. (two times fast as CCM, EAX)
All support AEAD (A.E. with associated data): part of message is in clear, and all is authenticated

Remarks on Secure Message Transmission

- Authentication may leak the message.
- Secure message transmission implies CCA-security. The opposite direction is not necessarily true.
- Different security goals should always use different keys.
  - otherwise, the message may be leaked if $\mathsf{Mac}_k(c) = \mathsf{Dec}_k(c)$.
- Implementation may destroy the security proved by theory.
  - **Attack with padding oracle** (in TLS 1.0):
    Dec return two types of error: padding error, MAC error.
    Adv. learns last bytes if no padding error with guessed bytes.
  - **Attack non-atomic dec.** (in SSH Binary Packet Protocol):
    Dec (1)decrypt length field; (2)read packets as specified by the length; (3)check MAC.
    Adv. (1)send c; (2)send l packets until "MAC error" occurs; (3)learn $l = \mathsf{Dec}(c)$.

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Key Derivation Function (KDF)

Key Derivation Function (KDF) generates many keys from a secret source key sk.

For uniformly random sk: F is PRF, ctx is a unique string identifying application,

$$\mathsf{KDF}(sk, ctx, l) = \langle F_{sk}(ctx\|0), F_{sk}(ctx\|1) \cdots , F_{sk}(ctx\|l)\rangle .$$

For not-uniform sk: extract-then-expand paradigm.

extract: HKDF $k \leftarrow \mathsf{HMAC}(salt, sk)$. $salt$ is a random number.

expand: as the above.

Password-Based KDF (PBKDF)

Key stretching increases the time of testing key (with slow hash function).

Key strengthening increases the length/randomness of key (with salt).

PKCS#5 (PBKDF1): $H^{(c)}(pwd \| salt)$, iterate hash function c times.

Attack: either try the enhanced key (larger key space), or else try the initial key (longer time per key).

IV, Nonce, Counter and Salt

IV an input to a cryptographic primitive, providing randomness.

nonce a number used only once to sign a communication.

counter a sequence number used as nonce.

salt consists of random bits, creating the input to a function.

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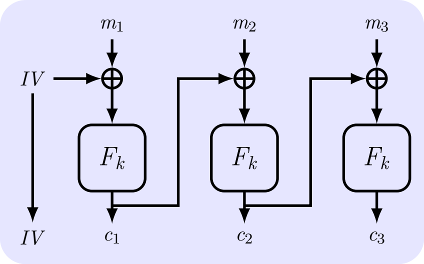
Deterministic CPA Security

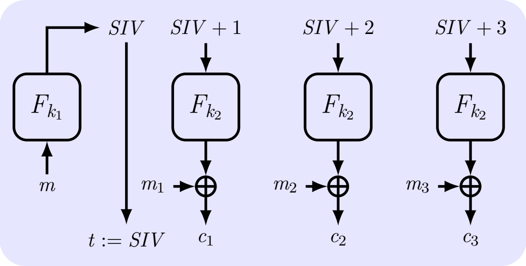
- **Application**: encrypted database index, disk encryption
- **Deterministic encryption**: the same message is encrypted to the same ciphertext under the same key.
  Q: would it be CPA-secure?
- **Deterministic CPA Security**: CPA-secure if *never encrypt same message twice* using same key. The pair $\langle k, m \rangle$ is unique.
- **Deterministic Authenticated Encryption (DAE)**
- **Common Mistake**: CBC/CTR with **fixed** IV.



Adversary can query (m_{q1}, m_{q2}) and get (c_{q1}, c_{q2}); then output PT: $IV \oplus c_{q1} \oplus m_{q2}$ and expect CT: c_{q2}.

Synthetic IV (SIV) for Det. Encryption

- **SIV** (fixed IV for the same $\langle k, m \rangle$):
  PRF F, CPA-secure $\Pi : (\mathsf{Enc}_k(r, m), \mathsf{Dec}_k(r, s))$
  $(k_1, k_2) \leftarrow \mathsf{Gen}$; $SIV \leftarrow F_{k_1}(m)$
  $c = \langle SIV, \mathsf{Enc}_{k_2}(SIV, m) \rangle$.
- **DAE for free with SIV-CTR**:

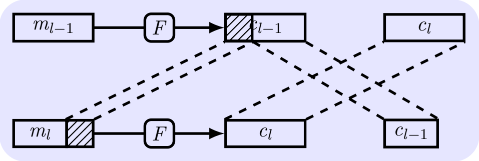


Wide Block PRP for Det. Encryption

- **Just PRP**: If F is PRP, then F is also det. CPA-secure.
- **PRP-based DAE**: $\mathsf{Enc}_k(m\|0^\ell)$. In Dec, if $\neq 0^\ell$, output $\perp$.
- **Narrow block** may leak info. as some blocks are the same.
- **Wide block PRP**: PRP with longer block length (e.g. a sector on disk) from PRP with short block length (e.g. AES).
- **Standards**: CBC-mask-CBC (CMC) and ECB-mask-ECB (EME) in IEEE P1619.2.
- **Cost**: 2x slower than SIV due to two-pass encryption.

Tweakable Encryption

- **Encryption without expansion**: $\mathcal{M} = \mathcal{C}$ implies det. encryption without integrity (e.g., disk encryption).
- **Trivial solution**: $k_t = F_k(t), t = 1, \ldots, \ell$.
- **Tweakable block ciphers**: many PRPs from one key $\mathcal{K} \times \mathcal{T} \times \mathcal{X} \to \mathcal{X}$, $\mathcal{T}$ is the set of tweaks.
- **XTS**: XEX(Xor-Encrypt-Xor)-based tweaked-codebook mode with ciphertext stealing. (XTS-AES, NIST SP 800-38E)
- **XEX**: To encrypt block j in sector I, $c = F_k(m \oplus x) \oplus x$, where $x = F_k(I) \otimes 2^j$ in Galois field, (I, j) is tweak.
- **Ciphertext stealing (CTS)**: no padding, **no expansion**.



Summary

- CCA-secure, AE, det. enc., det. CPA-secure, DAE.
- Enc-then-auth, KDF, SIV, wide block cipher, tweakable encryption.
- SIV-CTR, PBKDF, salt, enc. w/o expansion, CTS.