Assignment 3: Database Design (Spring 2019)

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Name: ____________ Student ID: ____________ Grade: ______

Question	1	2(a)	2(b)	2(c)	2(d)	2(e)	3(a)	3(b)	3(c)	3(d)	3(e)	4	Total
Score													

Notes

- **Print the assignment on A4 paper and answer the questions.**
- **Assignment due date: April 3, 2019.**

Questions

1. (2 Points) Given the database schema $R(A, B, C)$, and a relation r on the schema R, write an SQL query to test whether the functional dependency $B \to C$ holds on relation r (the query returns 1 if $B \to C$ holds on r and 0 otherwise). Hint: the following SQL query returns 1 if its result is non-empty; otherwise, it returns 0.

```
SELECT IF(COUNT(*), 1, 0) FROM ... WHERE ... LIMIT 1;
```

2. (10 Points) Prove or disprove the following inference rules for functional dependencies. A proof can be made by using Armstrong's axioms. A disproof should be performed by demonstrating a relation instance that satisfies the conditions and functional dependencies in the left-hand side of the inference rule but does not satisfy the dependencies in the right-hand side.

   (a) (2 Points) $\{W \to Y, X \to Z\} \vDash \{WX \to Y\}$

   (b) (2 Points) $\{X \to Y, X \to W, WY \to Z\} \vDash \{X \to Z\}$

   (c) (2 Points) $\{X \to Z, Y \to Z\} \vDash \{X \to Y\}$

   (d) (2 Points) $\{X \to Y, Z \to W\} \vDash \{XZ \to YW\}$

   (e) (2 Points) $\{X \to Y, Y \to Z\} \vDash \{X \to YZ\}$

3. (15 Points) Consider the following set F of functional dependencies on the relation schema $R(A, B, C, D, E)$:

$$
\begin{aligned}
A &\to B \\
A &\to C \\
BC &\to A \\
CD &\to E \\
B &\to D \\
E &\to A
\end{aligned}
$$

   Answer the following questions:

(a) (3 Points) Compute $(BC)^+_F$.

(b) (3 Points) List the candidate keys for R. Prove that they are candidate keys using Armstrong's axioms.

(c) (3 Points) Compute a canonical cover for F; give each step of your derivation with an explanation.

(d) (3 Points) Suppose that R is decomposed into $R_1(A, B, C)$ and $R_2(C, D, E)$. Show that this decomposition is not a lossless decomposition. Hint: Give an example of a relation r on schema R such that

$$\Pi_{A,B,C}(r) \bowtie \Pi_{C,D,E}(r) \neq r.$$

(e) (3 Points) Give a 3NF decomposition of R based on the canonical cover for F.

4. (3 Points) Let $R_1(U_1), R_2(U_2), \ldots, R_n(U_n)$ be a decomposition of schema $R(U)$. Let r be a relation on schema R, and let $r_i = \Pi_{U_i}(r)$. Show that

$$r \subseteq r_1 \bowtie r_2 \bowtie \cdots \bowtie r_n.$$

Answers

1.
```
SELECT IF(COUNT(*), 1, 0)
FROM R R1 JOIN R R2 ON (R1.B = R2.B AND R1.C != R2.C)
LIMIT 1;
```

2. (a) Correct.
   - It is certain that $WX \to W$.
   - $\{WX \to W, W \to Y\} \models \{WX \to Y\}$.

   (b) Correct.
   - $\{X \to Y, X \to W\} \models \{X \to WY\}$.
   - $\{X \to WY, WY \to Z\} \models \{X \to Z\}$.

   (c) Incorrect. Suppose a relation $R(X, Y, Z)$ contians two tupes $\{(x, y, z), (x, y', z)\}$, where $y \neq y'$.

   (d) Correct.
   - $\{X \to Y\} \models \{XZ \to YZ\}$.
   - $\{Z \to W\} \models \{YZ \to YW\}$.
   - $\{XZ \to YZ, YZ \to YW\} \models \{XZ \to YW\}$.

   (e) Correct.
   - $\{Y \to Z\} \models \{Y \to YZ\}$.
   - $\{X \to Y, Y \to YZ\} \models \{X \to YZ\}$.

3. (a)
   i. $X^{(0)} = BC$.
   ii. $X^{(1)} = X^{(0)} \cup AD = ABCD$.
   iii. $X^{(2)} = X^{(1)} \cup BCE = ABCDE$. Because $X^{(2)}$ contains all attributes in R, we have $(BC)^+_F = ABCDE$.

   (b)
   - A is a candidate key.
   - BC is a candidate key.
   - CD is a candidate key.
   - E is a candidate key.

   The proofs are omitted.

   (c)
   - $BC \to A$ is a redundant FD because $F - \{BC \to A\} \models \{BC \to A\}$. Thus, $F = \{A \to B, A \to C, CD \to E, B \to D, E \to A\}$.
   - No redundant attribute is found on the left-hand side of each FD left in F.

- $A \to B$ and $A \to C$ should be merged, so $F = \{A \to BC, CD \to E, B \to D, E \to A\}$.
- No more redundant FDs, redundant attributes, and mergable FDs can be fould, so the canonical cover for F is $\{A \to BC, CD \to E, B \to D, E \to A\}$.

(d) Let $r = \{(a, b, c, d, e), (a', b', c, d', e')\}$. We have

$$\Pi_{A,B,C}(r) \bowtie \Pi_{C,D,E}(r) = \{(a, b, c, d, e), (a', b', c, d', e'), (a, b, c, d', e'), (a', b', c, d, e)\}.$$

(e) $\{R_1(A, B, C), R_2(C, D, E), R_3(B, D), R_4(A, E)\}$ is a 3NF decomposition of R.

4. The proof is omitted.