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Theory and Methodology

# An analytical model for the container loading problem

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**Abstract**

This paper considers the problem of loading containers with cartons of non-uniform size and presents an analytical model to capture the mathematical essence of the problem. The container loading problem is formulated as a zero–one mixed integer programming model. It includes the consideration of multiple containers, multiple carton sizes, carton orientations, and the overlapping of cartons in a container. This model is then extended to formulate some special container loading problems. Numerical examples are used to validate the model.

*Keywords:* Cutting stock; Storage; Container loading; Three-dimensional palletization; Mathematical modeling

## 1. Introduction

Packing cartons into a container is an important material handling activity in the manufacturing and distribution industries. A container is defined in this paper as a rectangular box used to hold smaller cartons. The cartons can be any rectangular stackable object. All kinds of goods can be and are packaged in cartons for easy handling. Cartons are then packed into a container (or a box) for transportation and warehouse storage. The packing process may be performed manually or by a numerically controlled device. We consider the general packing problem that determines the optimal number of containers to pack a given set of rectangular cartons of different dimensions. The objective is to minimize the total unused space.

Container loading is classified as the three-dimensional (3D) rectangular packing problem in the general cutting and packing problem. A survey and classification of the cutting and packing problem is available in Dyckhoff (1990). The container loading problem is related to the two-dimensional (2D) stock cutting and pallet packing problem. In the stock-cutting problem, the emphasis is on cutting a single planer rectangular stock into several smaller pieces of known dimensions. This problem arises in the cutting of fabric, glass, paper, or wood sheets in the manufacturing industry. The objective is to minimize waste. A typical 2D cutting stock problem is the one that involves guillotine cuts, i.e., cuts that start from

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one edge of the stock and run parallel to the other two edges. Many papers have appeared in the area of the guillotine cutting-stock problem. These include the ones by Gilmore and Gomory (1965, 1966), Herz (1972), Christofides and Whitlock (1977), Wang (1983), Dagli (1988), Farley (1988), and Vasko (1989).

The 2D pallet loading problem is the other related problem that does not require guillotine cuts. Hodgson (1982) divides the problem into two palletization problems: the distributor's pallet packing problem and the manufacturer's pallet packing problem. Hodgson's paper focused on the distributor's problem in which customer orders are to be packed in cartons of varying dimensions and finally to be stacked on the pallet. The model developed by Hodgson considers a batch environment in which the distribution of the different carton sizes is predetermined. The problem is to select the carton and pallet dimensions so that the volume packed on the pallet is a maximum. The manufacturer's problem involves producing and packing products in identical cartons and stacking the cartons on identical pallets. Steudel (1979), Smith and DeCani (1980), and Bischoff and Dowsland (1982) presented procedures for solving this problem.

The studies referred above emphasize the development of a good heuristic solution procedure for the 2D packing and cutting problem. Some papers published offer exact algorithm. One, proposed by Dowsland (1987), considers the manufacturer's pallet loading problem. Dowsland's approach is based on a graph-theoretic model of the layout problem. An extensive computational study of her algorithm indicates that it may be appropriate for up to 50 cartons on a pallet. The second, by Beasley (1985), considers the 2D cutting stock problem formulated as an integer programming problem. Beasley's approach determines an optimal cutting pattern that maximizes the value of rectangles cut from a large stock rectangle. If the values of the rectangles are set equal to their areas, then the problem assumes the form of the distributor's pallet packing problem. A Lagrangian relaxation based tree search procedure produces optimal solutions within reasonable computer time. More recently, Chen, Sarin and Ram (1991) developed a mathematical model for the distributor's problem that is guaranteed to lead to an optimal solution. The objective is to place a given set of cartons on minimal number of uniform pallets.

The 3D container loading problem has attracted some research interest in the cutting and packing research community. Several works have been published in this area. Some studies focused on the practical aspects of loading a container and developed heuristic solutions based on the concept of filling out the container with cartons organized in layers, walls, and columns; while others applied 2D pallet packing heuristics to the general 3D container loading problem. George and Robinson (1980) first studied the 3D container loading problem and developed a heuristic procedure for orderly loading boxes of non-uniform size in a container. The procedure fills the container by building layers across its width and combines spare spaces between layers to increase space utilization. Based on the work by George and Robinson and the 2D pallet packing procedure by Bischoff and Dowsland (1982), Bischoff and Marriott (1990) enumerated fourteen container loading heuristics and evaluated their performance in terms of the length of the container required to pack all given cartons. It suggested that the performance of these heuristics depend on the mix of cartons. Gehring, Menschner and Meyer (1990) developed a heuristic that allows packing only a subset of given cartons to maximize the container's space utilization. The development was based on the loading procedures presented in George and Robinson (1980) and in Haessler and Talbot (1990). In addition to the concern of space utilization, practical aspects of the container loading problem were considered in Gilmore et al. (1989). Rules and constraints for manually loading non-identical boxes into a container were enumerated and organized as an expert system.

For packing a container with uniform cartons, Han et al. (1989) proposed a heuristic procedure that attempts to optimize the space utilization by forming L-shaped layers and maximizing the use of the edge length of the container. Dowsland (1991) applied the knowledge of the 2D pallet loading problem to the 3D container loading problem and summarized the dimensional relationship between a container and equal-sized cartons in 2D graphic charts. All the works referred to above, however, are heuristic solution procedures. There exists no analytical model for the 3D container loading problem. In this paper, we will address the analytical aspect of the problem and present a mathematical model for this problem.

## 2. A general model of the container loading problem

We first consider the general container loading problem; that is, selecting a number of containers to pack a given set of cartons. Both containers and cartons can be different in their size. The primary objective is to minimize the unused space. Based on the general model, special cases of the container loading problem are then discussed.

For the general container loading problem, we consider a mathematical model that is guaranteed to lead to an optimal solution. It is assumed that the dimensions of each container type are known; and the quantity and dimensions of each carton type are given. Each carton is treated as an independent entity in this model and is orthogonally placed in a container. That is, the edges of a carton are either parallel to or perpendicular to the axes of the container. The longest dimension of a carton (or container) is referred to as its length; the shortest dimension is the height; and the middle one is the width. The parameters and the variables used in the model are defined as below:

| | |
|---|---|
| $N$ | Total number of cartons to be packed. |
| $m$ | Total number of containers available. |
| $M$ | An arbitrarily large number. |
| $s_{ij}$ | A binary variable which is equal to 1 if carton number $i$ is placed in container $j$; otherwise it is equal to 0. |
| $n_j$ | A binary variable which is equal to 1 if the container $j$ is used; otherwise it is equal to 0. |
| $(p_i, q_i, r_i)$ | Parameters indicating the length, width, and height of carton $i$. |
| $(L_j, W_j, H_j)$ | Parameters indicating the length, width, and height of container $j$. |
| $(x_i, y_i, z_i)$ | Continuous variables (for location) indicating the coordinates of the front-left bottom (FLB) corner of carton $i$. |
| $(l_{xi}, l_{yi}, l_{zi})$ | Binary variables indicating whether the length of carton $i$ is parallel to the $X$-, $Y$-, or $Z$-axis. For example, the value of $l_{xi}$ is equal to 1 if the length of carton $i$ is parallel to the $X$-axis; otherwise it is equal to 0. |
| $(w_{xi}, w_{yi}, w_{zi})$ | Binary variables indicating whether the width of carton $i$ is parallel to the $X$-, $Y$-, or $Z$-axis. For example, the value of $w_{xi}$ equals to 1 if the width of carton $i$ is parallel to the $X$-axis; otherwise it is equal to 0. |
| $(h_{xi}, h_{yi}, h_{zi})$ | Binary variable indicating whether the height of carton $i$ is parallel to the $X$-, $Y$-, or $Z$-axis. For example, the value of $h_{xi}$ is equal to 1 if the height of carton $i$ is parallel to the $X$-axis; otherwise it is equal to 0. |

Binary variables $a_{ik}$, $b_{ik}$, $c_{ik}$, $d_{ik}$, $e_{ik}$, and $f_{ik}$ are defined to indicate the placement of cartons relative to each other. The $a_{ik}$ is equal to 1 if box $i$ is on the left side of carton $k$. Similarly, the variables $b_{ik}$, $c_{ik}$, $d_{ik}$, $e_{ik}$, *and* $f_{ik}$ represent whether carton $i$ is on the right of, behind, in front of, below, or above carton $k$, respectively. These variables are needed and defined only when $i < k$. The interpretation of these variables is illustrated in Fig. 1. Without loss of its generality, we assume that each container is placed with its length along the $X$-axis and its width along the $Y$-axis. The FLB corner of the container is fixed at the origin. The container $(j)$ in the figure is loaded with two cartons ($i$ and $k$). Therefore the assignment indicators $s_{ij}$ and $s_{kj}$ are equal to 1. Since the carton $i$ is located on the left-hand side of, and behind carton $k$, $a_{ik}$ and $d_{ik}$ are equal to 1. Other indicators for relative location of cartons $i$ and $k$ are set to 0 in this instance. Note that carton $k$ is located with its length along the $Z$-axis and the width parallel to the $X$-axis. Therefore, the orientation indicators for carton $k$, $l_{zk}$, $w_{xk}$, and $h_{yk}$ are equal to 1.

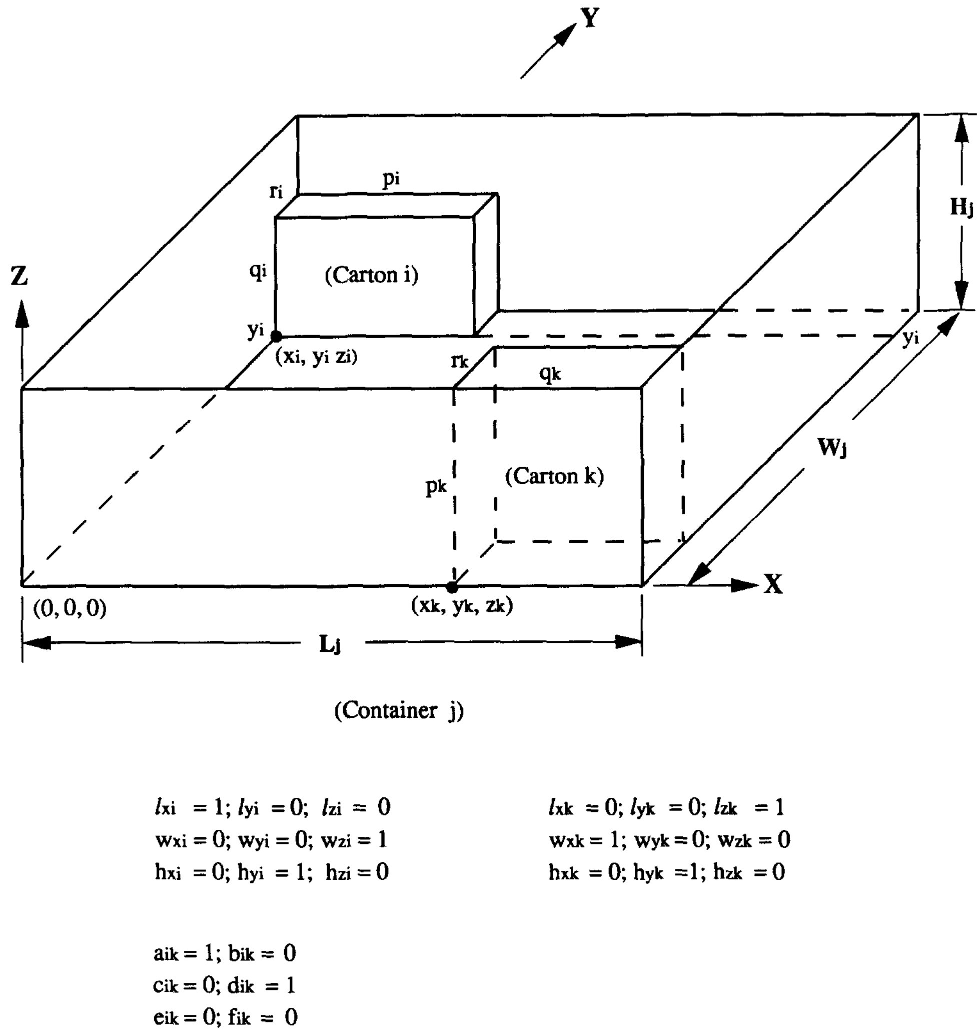


Fig. 1. Variable definition.

The container loading problem is therefore formulated as the following linear mixed integer programming model:

$$\text{Minimize} \sum_{j=1}^{m} L_j \cdot W_j \cdot H_j \cdot n_j - \sum_{i=1}^{N} p_i \cdot q_i \cdot r_i$$

subject to

$$x_i + p_i \cdot l_{xi} + q_i \cdot w_{xi} + r_i \cdot h_{xi} \le x_k + (1 - a_{ik}) \cdot M \quad \text{for all } i, k, \quad i < k, \tag{1}$$

$$x_k + p_k \cdot l_{xk} + q_k \cdot w_{xk} + r_k \cdot h_{xk} \le x_i + (1 - b_{ik}) \cdot M \quad \text{for all } i, k, \quad i < k, \tag{2}$$

$$y_i + q_i \cdot w_{yi} + p_i \cdot l_{yi} + r_i \cdot h_{yi} \le y_k + (1 - c_{ik}) \cdot M \quad \text{for all } i, k, \quad i < k, \tag{3}$$

$$y_k + q_k \cdot w_{yk} + p_k \cdot l_{yk} + r_k \cdot h_{yk} \le y_i + (1 - d_{ik}) \cdot M \quad \text{for all } i, k, \quad i < k, \tag{4}$$

$$z_i + r_i \cdot h_{zi} + q_i \cdot w_{zi} + p_i \cdot l_{zi} \le z_k + (1 - e_{ik}) \cdot M \quad \text{for all } i, k, \quad i < k, \tag{5}$$

$$z_k + r_k \cdot h_{zk} + q_k \cdot w_{zk} + p_k \cdot l_{zk} \leq z_i + (1 - f_{ik}) \cdot M \quad \text{for all } i, k, \ i < k, \tag{6}$$

$$a_{ik} + b_{ik} + c_{ik} + d_{ik} + e_{ik} + f_{ik} \geq s_{ij} + s_{kj} - 1 \quad \text{for all } i, k, j, \quad i < k, \tag{7}$$

$$\sum_{j=1}^{m} s_{ij} = 1 \quad \text{for all } i, \tag{8}$$

$$\sum_{i=1}^{N} s_{ij} \leq M \cdot n_j \quad \text{for all } j, \tag{9}$$

$$x_i + p_i \cdot l_{xi} + q_i \cdot w_{xi} + r_i \cdot h_{xi} \leq L_j + (1 - s_{ij}) \cdot M \quad \text{for all } i, j, \tag{10}$$

$$y_i + q_i \cdot w_{yi} + p_i \cdot l_{yi} + r_i \cdot h_{yi} \leq W_j + (1 - s_{ij}) \cdot M \quad \text{for all } i, j, \tag{11}$$

$$z_i + r_i \cdot h_{zi} + q_i \cdot w_{zi} + p_i \cdot l_{zi} \leq H_j + (1 - s_{ij}) \cdot M \quad \text{for all } i, j, \tag{12}$$

$$l_{xi}, l_{yi}, l_{zi}, w_{xi}, w_{yi}, w_{zi}, h_{xi}, h_{yi}, h_{zi}, a_{ik}, b_{ik}, c_{ik}, d_{ik}, e_{ik}, f_{ik}, s_{ij}, n_j = 0 \text{ or } 1,$$

$$x_i, y_i, z_i \geq 0.$$

The solution to the model provides an optimal pattern for packing a given set of cartons in selected container(s). The objective of this model is to minimize the total unused space of the container(s) selected. The constraints (1)–(6) ensure that cartons do not overlap each other. This check for overlap is necessary only if a pair of cartons are placed in the same container. This is taken care of by constraint (7). Constraint (8) guarantees that each carton will be placed in exactly one container. If any carton is assigned to a container, the container is considered used. This requirement is handled by constraint (9). Constraints (10)–(12) ensure that all the cartons placed in a container fit within the physical dimensions of the container.

It is observed that the binary variables, $l_{xi}$, $l_{yi}$, $l_{zi}$, $w_{xi}$, $w_{yi}$, $w_{zi}$, $h_{xi}$, $h_{yi}$, and $h_{zi}$, are dependent and there exist the following relationships:

$$l_{xi} + l_{yi} + l_{zi} = 1,$$

$$w_{xi} + w_{yi} + w_{zi} = 1,$$

$$h_{xi} + h_{yi} + h_{zi} = 1,$$

$$l_{xi} + w_{xi} + h_{xi} = 1,$$

$$l_{yi} + w_{yi} + h_{yi} = 1,$$

$$l_{zi} + w_{zi} + h_{zi} = 1.$$

Using the above relationships, we can eliminate from the model the following five variables: $l_{yi}$, $w_{xi}$, $w_{zi}$, $h_{xi}$, and $h_{yi}$, resulting in a significant reduction in the model size. The affected constraints are modified as below:

$$x_i + p_i \cdot l_{xi} + q_i \cdot (l_{zi} - w_{yi} + h_{zi}) + r_i \cdot (l - l_{xi} - l_{zi} + w_{yi} - h_{zi}) \leq x_k + (1 - a_{ik}) \cdot M, \tag{1'}$$

$$x_k + p_k \cdot l_{xk} + q_k \cdot (l_{zk} - w_{yk} + h_{zk}) + r_k \cdot (1 - l_{xk} - l_{zk} + w_{yk} - h_{zk}) \leq x_i + (1 - b_{ik}) \cdot M, \tag{2'}$$

$$y_i + q_i \cdot w_{yi} + p_i (1 - l_{xi} - l_{zi}) + r_i \cdot (l_{xi} + l_{zi} - w_{yi}) \leq y_k + (1 - c_{ik}) \cdot M, \tag{3'}$$

$$y_k + q_k \cdot w_{yk} + p_k \cdot (1 - l_{xk} - l_{zk}) + r_k \cdot (l_{xk} + l_{zk} - w_{yk}) \leq y_i + (1 - d_{ik}) \cdot M, \tag{4'}$$

$$z_i + r_i \cdot h_{zi} + q_i \cdot (1 - l_{zi} - h_{zi}) + p_i \cdot l_{zi} \leq z_k + (1 - e_{ik}) \cdot M, \tag{5'}$$

$$z_k + r_k \cdot h_{zk} + q_k \cdot (1 - l_{zk} - h_{zk}) + p_k \cdot l_{zk} \leq z_i + (1 - f_{ik}) \cdot M, \tag{6'}$$

$$x_i + p_i \cdot l_{xi} + q_i \cdot (l_{zi} - w_{yi} + h_{zi}) + r_i \cdot (1 - l_{xi} - l_{zi} + w_{yi} - h_{zi}) \le L_j + (1 - s_{ij}) \cdot M, \tag{10'}$$

$$y_i + q_i \cdot w_{yi} + p_i \cdot (1 - l_{xi} - l_{zi}) + r_i \cdot (l_{xi} + l_{zi} - w_{yi}) \le W_j + (1 - s_{ij}) \cdot M, \tag{11'}$$

$$z_i + r_i \cdot h_{zi} + q_i \cdot (1 - l_{zi} - h_{zi}) + p_i \cdot l_{zi} \le H_j + (1 - s_{ij}) \cdot M. \tag{12'}$$

The revised model maintains the form of a zero-one mixed integer linear program. It can be solved with a commercial mathematical programming package such as LINGO, MathPro/XPRESS-MP, or MPS/MIP. The computer time required to solve the model, though, depends on both the size of the problem and the computer used. The revised model contains $\frac{1}{2}mN(N-1) + 3N(N+m-1) + N + m$ constraints and $(3N+m+4)N+m$ variables. Among the variables, there are $3N$ continuous and $(3N+m+1)N+m$ binary variables.

To validate the above model and to explore its computational time, an example problem of three unequal-sized containers and six non-uniform cartons was solved with a general-purpose mathematical programming package. The model for this example problem contains 198 constraints and 153 (18 continuous and 135 binary) variables. It took fifteen minutes to solve the model to its optimum with a LINGO package on a DEC5000/P200 computer.

## 3. Application of the model

Under the heading of 3D container loading problem, there exist a range of container loading sub-problems; each represents a special application in the industry. For a special container loading problem, it is possible to reduce the number of variables and tighten the constraints of the above model. Some of these special cases are discussed in this section. Required modifications to the general model for these special cases are also provided.

As in the 2D cutting and packing problem, space utilization is the primary objective of the container loading problem. In addition to space utilization, however, there exist other concerns which usually require additional constraints and sometimes a different packing approach as well. Dyckhoff (1990) suggested the following three criteria: type of assignment, assortment of the container, and assortment of cartons, to classify a 3D container loading problem. Dowsland (1991), on the other hand, enumerated several specific container loading problems. The above general model can be conveniently extended to formulate many of these special situations. The general model can readily be applied to the first five issues raised by Dowsland. For example, if the concern is the selection of one container for a given set of cartons to minimize wasted space, the general model can be used with the following modifications: (a) delete constraints (8) and (9); (b) delete variable, $s_{ij}$; (c) set the value of the right-hand side of the constraint (7) to 1; (d) replace the right-hand side of the constraints (10), (11), and (12) with $\sum_{j=1}^{m} L_j \cdot n_j$, $\sum_{j=1}^{m} W_j \cdot n_j$, and $\sum_{j=1}^{m} H_j \cdot n_j$, respectively and modify their constraint range to be *for all* $i$; and (e) add the following constraint to the model: $\sum_{j=1}^{m} n_j = 1$.

If weight distribution of cartons in the container is also a concern in the above situation, the following two constraints, for example, can be added to the model to control the weight imbalance along the $X$-axis:

$$-c_x \le \sum_{i=1}^{N} \frac{w_i}{2} \cdot \left[ \sum_{j=1}^{m} L_j \cdot n_j - 2x_i - p_i \cdot l_{xi} - q_i \cdot (l_{zi} - w_{yi} + h_{zi}) - r_i \cdot (1 - l_{xi} - l_{zi} + w_{yi} - h_{zi}) \right] \le c_x$$

where $w_i$ is the weight of carton $i$ and $c_x$ is the weight imbalance limit along the $X$-axis.

In the situation where a given set of cartons are to be packed in a container, each carton must be loaded right side up, and the objective is to minimize the length of the container required to pack the entire cargo, the general model can be applied with the following changes: (a) define the length ($L$) of the container as a continuous variable; (b) replace $w_{yi}$ with $l_{xi}$, replace $w_{xi}$ with $1 - l_{xi}$, replace $l_{yi}$ with $1 - l_{xi}$, set the value of the variables, $h_{xi}$, $h_{yi}$, $l_{zi}$, and $w_{zi}$ to 0, and set the value of the variable $h_{zi}$ to 1 (as the up side of carton i is always placed along the $Z$-axis); (c) delete variables $s_{ij}$, $s_{kj}$, and $n_j$ from the model; (d) replace the objective function with the function Minimize $L$; (e) delete constraints (8) and (9); (f) set the value of the right-hand side of constraint (7) to be 1; and (g) replace the right-hand side of the constraints (10)–(12) with $L$, $W$ and $H$, respectively. The modifications result in a relatively smaller model with $\frac{7}{2}N(N-1) + 3N$ constraints, $3N$ continuous variables, and $N + 3N(N-1)$ binary variables.

To validate the above modifications, an example problem of six cartons was used and solved with the LINGO package on the same computer. In this special case, the objective is to pack the cartons such that the required length of the container is minimized. Hence, the length of the container in this special case is a variable. The width and height are arbitrarily fixed at 20″ and 10″, respectively, for this problem. The dimensions of the six cartons are given as follows:

| Carton number | Dimension (inches) | | |
|---|---|---|---|
| | Up side | Length | Width |
| 1 | 6 | 25 | 8 |
| 2 | 5 | 20 | 10 |
| 3 | 3 | 16 | 7 |
| 4 | 6 | 15 | 12 |
| 5 | 3 | 22 | 8 |
| 6 | 4 | 20 | 10 |

Note that each of the cartons must be loaded with its up side up (i.e., placed along the $Z$-axis). Therefore in this special case, the longer side of the two remaining dimensions of a carton is defined as its length and the other is the width. The model for this problem contains 123 constraints and 114 (18 continuous and 96 binary) variables. The optimal solution to the problem is summarized in Table 1. It includes the

Table 1
The optimal solution

| Carton number $i$ | Coordinates of carton $i$ | | | Orientation indicator $l_{xi}$ | Pair of cart. $i$ $k$ | Position indicators | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $z_i$ | | | $a_{ik}$ | $b_{ik}$ | $c_{ik}$ | $d_{ik}$ | $e_{ik}$ | $f_{ik}$ |
| 1 | 7 | 12 | 4 | 1 | 2 | 0 | 0 | 0 | 1 | 0 | 0 |
| | | | | | 3 | 0 | 1 | 0 | 0 | 0 | 1 |
| | | | | | 4 | 0 | 0 | 0 | 1 | 0 | 0 |
| | | | | | 5 | 0 | 0 | 0 | 1 | 0 | 1 |
| | | | | | 6 | 0 | 0 | 0 | 0 | 0 | 1 |
| 2 | 0 | 0 | 5 | 1 | 3 | 0 | 0 | 0 | 0 | 0 | 1 |
| | | | | | 4 | 1 | 0 | 0 | 0 | 0 | 0 |
| | | | | | 5 | 0 | 0 | 0 | 0 | 0 | 1 |
| | | | | | 6 | 0 | 0 | 0 | 0 | 0 | 1 |
| 3 | 0 | 0 | 0 | 0 | 4 | 1 | 0 | 0 | 0 | 1 | 0 |
| | | | | | 5 | 1 | 0 | 0 | 0 | 0 | 0 |
| | | | | | 6 | 1 | 0 | 0 | 0 | 0 | 0 |
| 4 | 20 | 0 | 4 | 1 | 5 | 0 | 0 | 0 | 0 | 0 | 1 |
| | | | | | 6 | 0 | 0 | 0 | 0 | 0 | 1 |
| 5 | 7 | 0 | 1 | 1 | 6 | 0 | 0 | 1 | 0 | 0 | 0 |
| 6 | 7 | 10 | 0 | 1 | – | – | – | – | – | – | – |

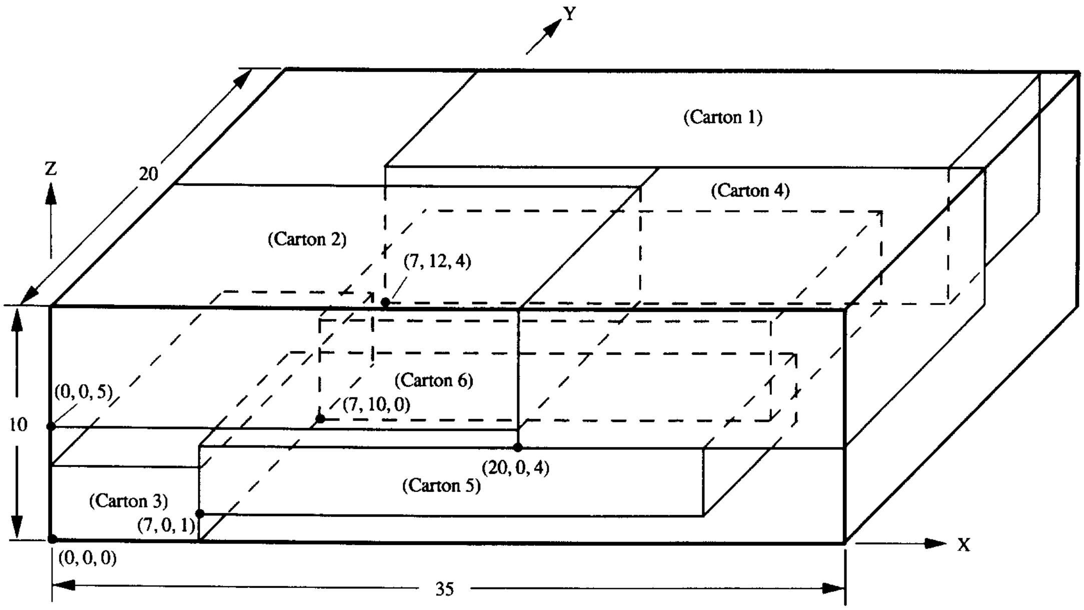


Fig. 2. The graphical representation.

location and the orientation of each carton, and the relationship between cartons in the container. A graphical representation of the loading pattern for this problem is given in Fig. 2. The location of each carton in the container is defined by its FLB corner. The FLB corner of the third carton is at the origin with its length placed along the $Y$-axis. On its right-hand side are cartons 5 and 6. On the top of these two cartons are cartons 2 and 4. Behind cartons 2 and 4 is the first carton. The minimum container length required to pack the six cartons is 35 inches.

## 4. Conclusion

In this paper, we presented a zero-one mixed integer linear programming model for the general 3D container loading problem. The problem involves packing a set of non-uniform cartons into unequal-sized containers. The model considers the issues of carton orientations, multiple carton sizes, multiple container sizes, avoidance of carton overlapping, and space utilization. Several special container loading problems such as selecting one container from several alternatives, weight balance, and variable container length were addressed. The modifications to the general model needed for these situations were also provided. Example problems were illustrated to validate the models. For further development, additional constraints can be introduced to the models to include other concerns in the container loading problem such as the stability of the packing pattern, stackability, the integrity of each carton type, and weight restriction. A more efficient solution procedure is needed to solve large scale container loading problems.

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