

# 强化学习

*ML24*

---



礼欣

[www.python123.org](http://www.python123.org)

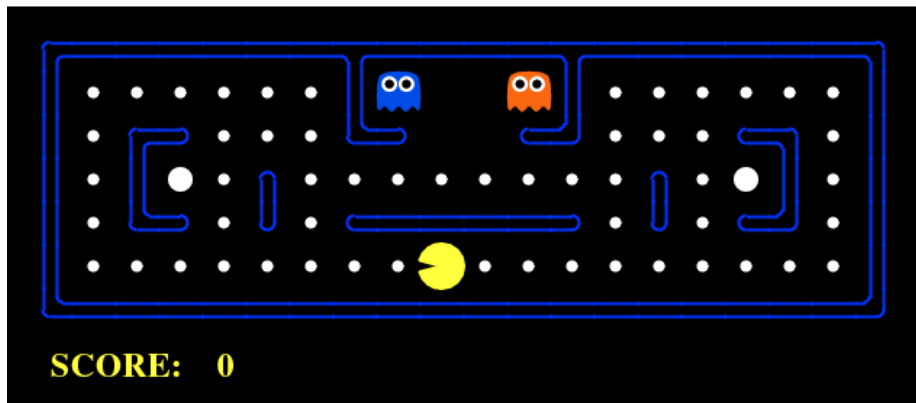


# 强化学习简介

# 强化学习

- 强化学习就是程序或智能体（agent）通过与环境不断地进行交互学习一个从环境到动作的映射，学习的目标就是使累计回报最大化。
- 强化学习是一种试错学习，因其在各种状态（环境）下需要尽量尝试所有可以选择的动作，通过环境给出的反馈（即奖励）来判断动作的优劣，最终获得环境和最优动作的映射关系（即策略）。

# 基本组件



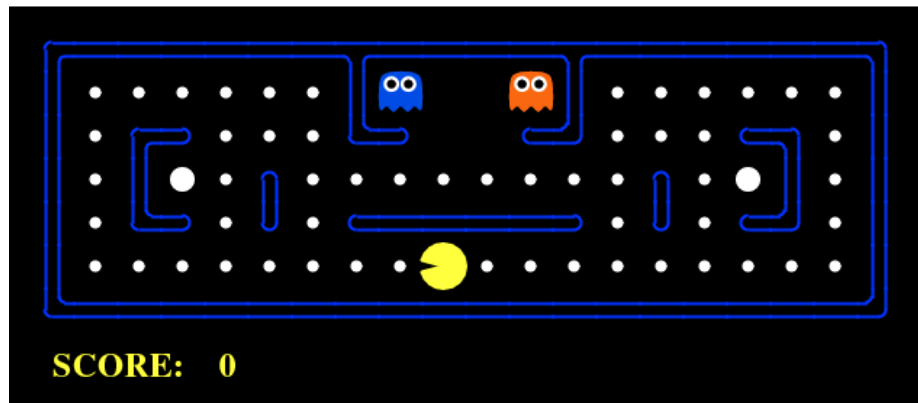
agent : 🍷 大嘴小怪物

环境 : 整个迷宫中的所有信息

奖励 : agent每走一步，需要扣除1分，吃掉小球得10分，吃掉敌人得200分，被吃掉游戏结束

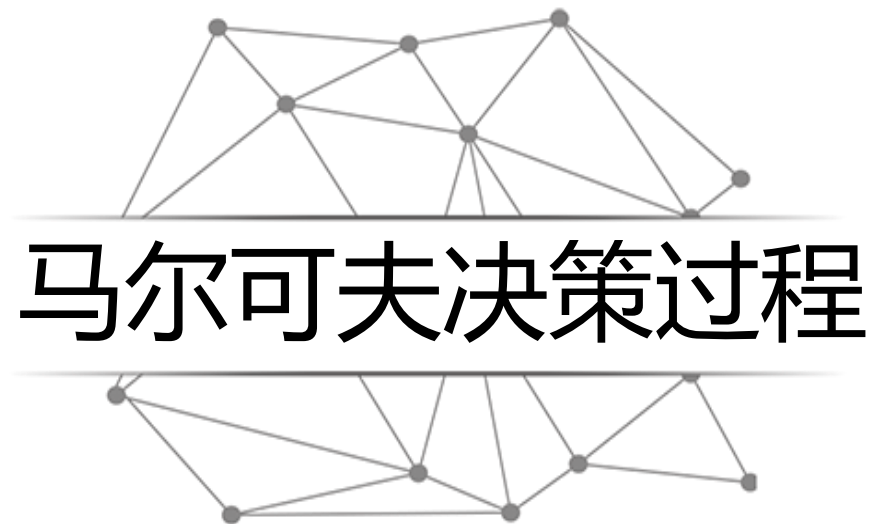
动作 : 在每种状态下，agent能够采用的动作，比如上下左右移动

# 目标



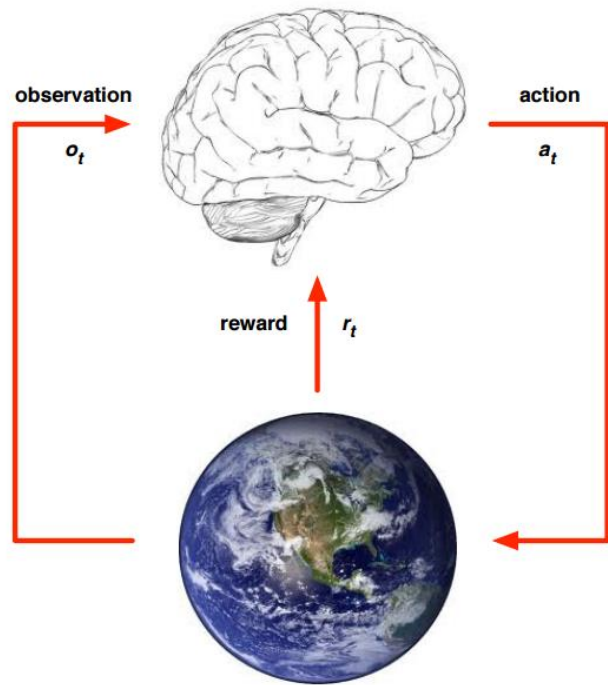
**策略：**在每种状态下，采取最优的动作

**学习目标：**获得最优的策略，以使累计奖励最大（即Score）



# 马尔可夫决策过程 ( MDP )

- 马尔可夫决策过程 ( Markov Decision Process ) 通常用来描述一个强化学习问题
- 智能体agent根据当前对环境的观察采取动作获得环境的反馈，并使环境发生改变的循环过程。



# MDP 基本元素

$s \in S$ : 有限状态state集合,  $s$ 表示某个特定状态;

$a \in A$ : 有限动作action集合,  $a$ 表示某个特定动作;

$T(S, a, S') \sim P_r(s' | s, a)$ : 状态转移模型, 根据当前状态 $s$ 和动作 $a$ 预测下一个状态 $s'$ , 这里的 $P_r$ 表示从 $s$ 采取行动 $a$ 转移到 $s'$ 的概率;

$R(s, a)$ : 表示agent采取某个动作后的即时奖励, 它还有  $R(s, a, s')$ ,  $R(s)$  等表现形式;

Policy  $\pi(s) \rightarrow a$ : 根据当前state来产生action, 可表现为 $a = \pi(s)$ 或 $\pi(a | s) = P(a | s)$ , 后者表示某种状态下执行某个动作的概率。



# 值函数

状态值函数 $V$ 表示执行策略 $\pi$ 能得到的累计折扣奖励：

$$V^\pi(s) = E[R(s_0, a_0) + \gamma R(s_1, a_1) + \gamma^2 R(s_2, a_2) + \gamma^3 R(s_3, a_3) + \dots | s = s_0]$$

整理之后可得：

$$V^\pi(s) = R(s, a) + \gamma \sum_{s' \in S} p(s' | s, \pi(s)) V^\pi(s')$$

# 值函数

状态动作值函数 $Q(s, a)$ 表示在状态 $s$ 下执行动作 $a$ 能得到的累计折扣奖励：

$$Q^\pi(s, a) =$$

$$E[R(s_0, a_0) + \gamma R(s_1, a_1) + \gamma^2 R(s_2, a_2) + \gamma^3 R(s_3, a_3) + \dots \mid s = s_0, a = a_0]$$

整理之后可得：

$$Q^\pi(s, a) = R(s, a) + \gamma \sum_{s' \in S} p(s' | s, \pi(s)) Q^\pi(s', \pi(s'))$$

# 最优值函数

最优值函数：

$$V^*(s) = \max_{a \in A} [R(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) V^*(s')]$$

$$Q^*(s, a) = R(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) \max_{b \in A} Q^*(s', b)$$

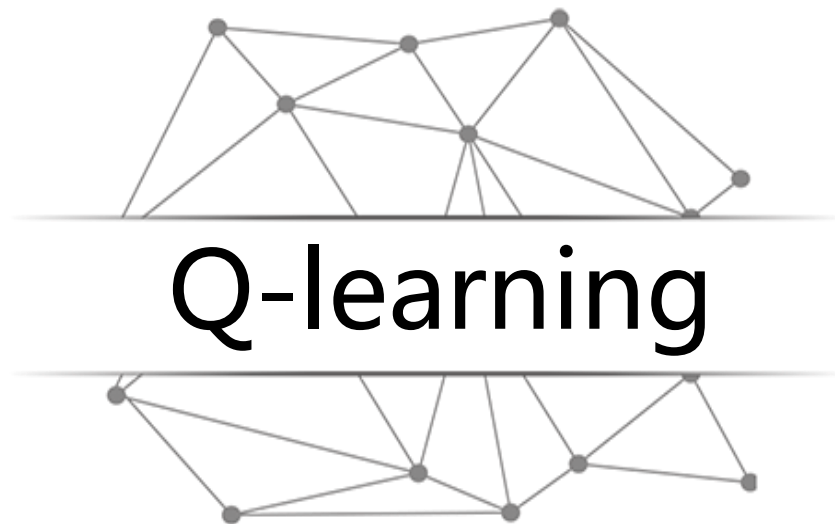
# 最优控制

在得到最优值函数之后，可以通过值函数的值得到状态s时应该采取的动作a：

$$\pi(s) = \operatorname{argmax}_{a \in A} [R(s, a) + \gamma \sum_{s' \in S} p(s'|s, a) V^*(s')]$$

$$\pi(s) = \operatorname{argmax}_{a \in A} Q^*(s, a)$$

$$V^*(s) = \max_{a \in A} Q^*(s, a)$$



# 蒙特卡洛强化学习

- 在现实的强化学习任务中，环境的转移概率、奖励函数往往很难得知，甚至很难得知环境中有多少状态。若学习算法不再依赖于环境建模，则称为免模型学习，蒙特卡洛强化学习就是其中的一种。
- 蒙特卡洛强化学习使用多次采样，然后求取平均累计奖赏作为期望累计奖赏的近似。

$$\langle s_0, a_0, r_1, s_1, a_1, r_2, \dots, s_{T-1}, a_{T-1}, r_T, s_T \rangle$$

# 蒙特卡洛强化学习

蒙特卡洛强化学习：直接对状态动作值函数 $Q(s, a)$ 进行估计，每采样一条轨迹，就根据轨迹中的所有“状态-动作”利用下面的公式对值函数进行更新。

$$Q(s, a) = \frac{Q(s, a) * \text{count}(s, a) + R}{\text{count}(s, a) + 1}$$

# 蒙特卡洛强化学习

每次采样更新完所有的“状态-动作”对所对应的 $Q(s, a)$ ，就需要更新采样策略 $\pi$ 。但由于策略可能是确定性的，即一个状态对应一个动作，多次采样可能获得相同的采样轨迹，因此需要借助 $\epsilon$ 贪心策略：

$$\pi(s, a) = \begin{cases} \mathit{argmax}_a Q(s, a) & \text{以概率 } 1 - \epsilon \\ \text{随机从 } A \text{ 中选取动作} & \text{以概率 } \epsilon \end{cases}$$



# Q-learning算法

- 蒙特卡洛强化学习算法需要采样一个完整的轨迹来更新值函数，效率较低，此外该算法没有充分利用强化学习任务的序贯决策结构。
- Q-learning算法结合了动态规划与蒙特卡洛方法的思想，使得学习更加高效。

# Q-learning算法

假设对于状态动作对 $(s, a)$ 基于 $t$ 次采样估算出其值函数为：

$$Q_t^\pi(s, a) = \frac{1}{t} \sum_{i=1}^t r_i$$

在进行 $t+1$ 次采样后，依据增量更新得到：

$$Q_{t+1}^\pi(s, a) = Q_t^\pi(s, a) + \frac{1}{t+1} (r_{t+1} - Q_t^\pi(s, a))$$

然后，将 $\frac{1}{t+1}$ 替换成系数 $\alpha$ （步长），得到：

$$Q_{t+1}^\pi(s, a) = Q_t^\pi(s, a) + \alpha(r_{t+1} - Q_t^\pi(s, a))$$

# Q-learning算法

以 $\gamma$ 折扣累计奖赏为例：

$$\mathbf{r}_{t+1} = \mathbf{R}_s^a + \gamma \mathbf{Q}_t^\pi(\mathbf{s}', \mathbf{a}')$$

则值函数的更新方式如下：

$$\mathbf{Q}_{t+1}^\pi(\mathbf{s}, \mathbf{a}) = \mathbf{Q}_t^\pi(\mathbf{s}, \mathbf{a}) + \alpha(\mathbf{R}_s^a + \gamma \mathbf{Q}_t^\pi(\mathbf{s}', \mathbf{a}') - \mathbf{Q}_t^\pi(\mathbf{s}, \mathbf{a}))$$

# Q-learning算法流程

---

**输入：**环境 $E$ ；动作空间 $A$ ；起始状态 $s_0$ ；奖励折扣 $\gamma$ ；更新步长 $\alpha$ ；

**过程：**

1 :  $Q(s, a) = 0, \pi(s, a) = 1/|A|$ ;

2 :  $s = s_0$ ;

3 : for  $t = 1, 2, \dots$  do

4 :    $r, s' =$  在 $E$ 中执行动作 $\pi^e(s)$ 产生的奖赏和转移的状态;

5 :    $a' = \pi(s')$ ;

6 :    $Q(s, a) = Q(s, a) + \alpha(r + \gamma Q(s', a') - Q(s, a))$ ;

7 :    $\pi(s) = \operatorname{argmax}_{a''} Q(s, a'')$ ;

8 :    $s = s', a = a'$ ;

9 : end for

**输出：**策略 $\pi$

---

请参考周志华老师《机器学习》一书



# 深度强化学习

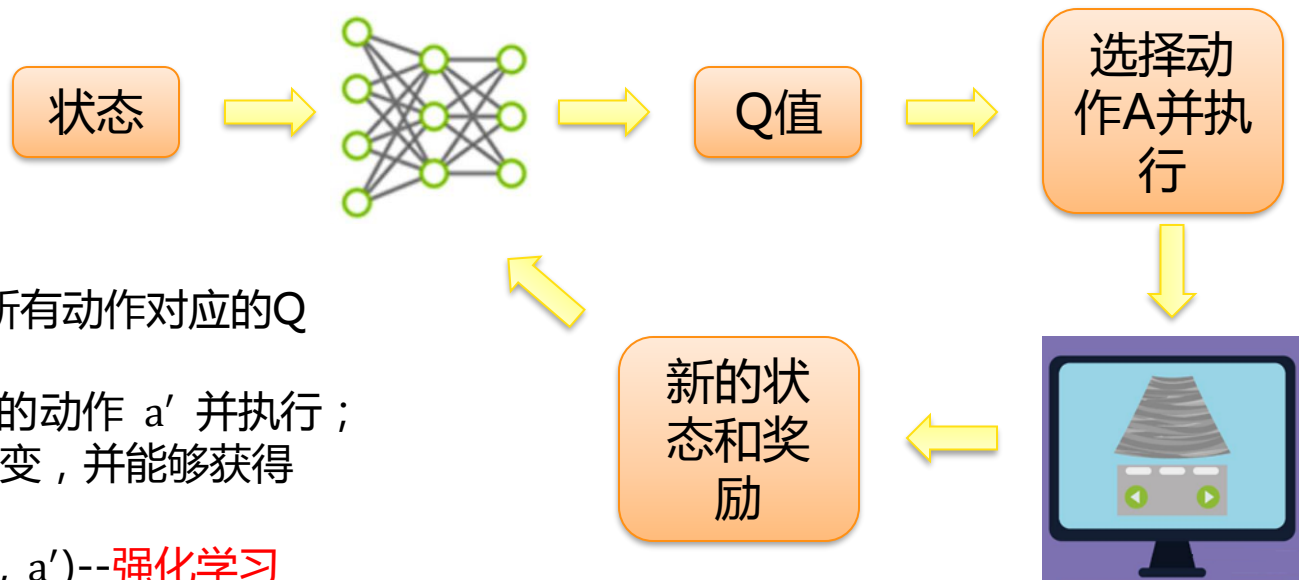
# 深度强化学习 ( DRL )

- 传统强化学习：真实环境中的状态数目过多，求解困难。
- 深度强化学习：将深度学习和强化学习结合在一起，通过深度神经网络直接学习环境（或观察）与状态动作值函数 $Q(s,a)$ 之间的映射关系，简化问题的求解。

# Deep Q Network ( DQN )

- Deep Q Network ( DQN ) : 是将神经网络(neural network) 和Q-learning结合, 利用神经网络近似模拟函数 $Q(s,a)$ , 输入是问题的状态 ( e.g., 图形 ), 输出是每个动作a对应的Q值, 然后依据Q值大小选择对应状态执行的动作, 以完成控制。
- 神经网络的参数: 应用监督学习完成

# DQN学习过程



## 学习流程：

1. 状态 $s$ 输入，获得所有动作对应的Q值 $Q(s, a)$ ；
2. 选择对应Q值最大的动作  $a'$  并执行；
3. 执行后环境发生改变，并能够获得环境的奖励 $r$ ；
4. 利用奖励 $r$ 更新 $Q(s, a')$ --**强化学习**  
利用新的 $Q(s, a')$ 更新网络参数—**监督学习**



# DQN算法流程

初始化D：用于存放采集的  
( $S_t, a_t, r_t, S_{t+1}$ ) 状态  
转移过程，用于网络参数的  
训练

## Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory  $\mathcal{D}$  to capacity  $N$

Initialize action-value function  $Q$  with random weights

**for** episode = 1,  $M$  **do**

    Initialize sequence  $s_1 = \{x_1\}$  and preprocessed sequenced  $\phi_1 = \phi(s_1)$

**for**  $t = 1, T$  **do**

        With probability  $\epsilon$  select a random action  $a_t$

        otherwise select  $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

        Execute action  $a_t$  in emulator and observe reward  $r_t$  and image  $x_{t+1}$

        Set  $s_{t+1} = s_t, a_t, x_{t+1}$  and preprocess  $\phi_{t+1} = \phi(s_{t+1})$

        Store transition  $(\phi_t, a_t, r_t, \phi_{t+1})$  in  $\mathcal{D}$

        Sample random minibatch of transitions  $(\phi_j, a_j, r_j, \phi_{j+1})$  from  $\mathcal{D}$

        Set  $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

        Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equation 3

**end for**

**end for**

进行多次采样

一次完整的采样

# DQN算法流程

## Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory  $\mathcal{D}$  to capacity  $N$

Initialize action-value function  $Q$  with random weights

随机初始化神经网络的参数

**for** episode = 1,  $M$  **do**

    Initialize sequence  $s_1 = \{x_1\}$  and preprocessed sequenced  $\phi_1 = \phi(s_1)$

**for**  $t = 1, T$  **do**

        With probability  $\epsilon$  select a random action  $a_t$

        otherwise select  $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

        Execute action  $a_t$  in emulator and observe reward  $r_t$  and image  $x_{t+1}$

        Set  $s_{t+1} = s_t, a_t, x_{t+1}$  and preprocess  $\phi_{t+1} = \phi(s_{t+1})$

        Store transition  $(\phi_t, a_t, r_t, \phi_{t+1})$  in  $\mathcal{D}$

        Sample random minibatch of transitions  $(\phi_j, a_j, r_j, \phi_{j+1})$  from  $\mathcal{D}$

        Set  $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

        Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equation 3

**end for**

**end for**

进行多次采样

一次完整的采样

# DQN算法流程

## Algorithm 1 Deep Q-learning with Experience Replay

Initialize replay memory  $\mathcal{D}$  to capacity  $N$

Initialize action-value function  $Q$  with random weights

**for** episode = 1,  $M$  **do**

    Initialize sequence  $s_1 = \{x_1\}$  and preprocessed sequenced  $\phi_1 = \phi(s_1)$

**for**  $t = 1, T$  **do**

        With probability  $\epsilon$  select a random action  $a_t$

        otherwise select  $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

        Execute action  $a_t$  in emulator and observe reward  $r_t$  and image  $x_{t+1}$

        Set  $s_{t+1} = s_t, a_t, x_{t+1}$  and preprocess  $\phi_{t+1} = \phi(s_{t+1})$

        Store transition  $(\phi_t, a_t, r_t, \phi_{t+1})$  in  $\mathcal{D}$

        Sample random minibatch of transitions  $(\phi_j, a_j, r_j, \phi_{j+1})$  from  $\mathcal{D}$

        Set  $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

        Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equation 3

**end for**

**end for**

获取环境的初始状态（ $x$ 是采集的图像，使用图像作为agent的状态；预处理过程是说，使用4张图像代表当前状态，这里可以先忽略掉）

进行多次采样

一次完整的采样

# DQN算法流程

---

**Algorithm 1** Deep Q-learning with Experience Replay

---

Initialize replay memory  $\mathcal{D}$  to capacity  $N$

Initialize action-value function  $Q$  with random weights

**for** episode = 1,  $M$  **do**

    Initialize sequence  $s_1 = \{x_1\}$  and preprocessed sequence  $\phi_1 = \phi(s_1)$

**for**  $t = 1, T$  **do**

        With probability  $\epsilon$  select a random action  $a_t$

        otherwise select  $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

        Execute action  $a_t$  in emulator and observe reward  $r_t$

        Set  $s_{t+1} = s_t, a_t, x_{t+1}$  and preprocess  $\phi_{t+1} = \phi(s_{t+1})$

        Store transition  $(\phi_t, a_t, r_t, \phi_{t+1})$  in  $\mathcal{D}$

        Sample random minibatch of transitions  $(\phi_j, a_j, r_j, \phi_{j+1})$  from  $\mathcal{D}$

        Set  $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

        Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equation 3

**end for**

**end for**

---

进行多次采样

一次完整的采样

**$\epsilon$ 贪心策略**：使用 $\epsilon$ 概率随机选取动作或 $1 - \epsilon$ 的概率根据神经网络的输出选择动作

# DQN算法流程

---

**Algorithm 1** Deep Q-learning with Experience Replay

---

Initialize replay memory  $\mathcal{D}$  to capacity  $N$

Initialize action-value function  $Q$  with random weights

**for** episode = 1,  $M$  **do**

    Initialize sequence  $s_1 = \{x_1\}$  and preprocessed sequenced  $\phi_1 = \phi(s_1)$

**for**  $t = 1, T$  **do**

        With probability  $\epsilon$  select a random action  $a_t$

        otherwise select  $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

        Execute action  $a_t$  in emulator and observe reward  $r_t$  and image  $x_{t+1}$

        Set  $s_{t+1} = s_t, a_t, x_{t+1}$  and preprocess  $\phi_{t+1} = \phi(s_{t+1})$

        Store transition  $(\phi_t, a_t, r_t, \phi_{t+1})$  in  $\mathcal{D}$

        Sample random minibatch of transitions  $(\phi_j, a_j, r_j, \phi_{j+1})$  from  $\mathcal{D}$

        Set  $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

        Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equation 3

**end for**

**end for**

---

在模拟器中执行选定的动作，获得奖励 $r_t$ 和一个观察 $x_{t+1}$

进行多次采样

一次完整的采样

# DQN算法流程

---

**Algorithm 1** Deep Q-learning with Experience Replay

---

Initialize replay memory  $\mathcal{D}$  to capacity  $N$

Initialize action-value function  $Q$  with random weights

**for** episode = 1,  $M$  **do**

    Initialize sequence  $s_1 = \{x_1\}$  and preprocessed sequenced  $\phi_1 = \phi(s_1)$

**for**  $t = 1, T$  **do**

        With probability  $\epsilon$  select a random action  $a_t$

        otherwise select  $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

        Execute action  $a_t$  in emulator and observe reward  $r_t$  and image  $x_{t+1}$

        Set  $s_{t+1} = s_t, a_t, x_{t+1}$  and preprocess  $\phi_{t+1} = \phi(s_{t+1})$

        Store transition  $(\phi_t, a_t, r_t, \phi_{t+1})$  in  $\mathcal{D}$

        Sample random minibatch of transitions  $(\phi_j, a_j, r_j, \phi_{j+1})$  from  $\mathcal{D}$

        Set  $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

        Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equation 3

**end for**

**end for**

---

设置  $S_{t+1}$  , 并将状态转移过程  $(S_t, a_t, r_t, S_{t+1})$  存放在  $\mathcal{D}$  中

进行多次采样

一次完整的采样

# DQN算法流程

---

**Algorithm 1** Deep Q-learning with Experience Replay

---

Initialize replay memory  $\mathcal{D}$  to capacity  $N$

Initialize action-value function  $Q$  with random weights

**for** episode = 1,  $M$  **do**

    Initialize sequence  $s_1 = \{x_1\}$  and preprocessed sequenced  $\phi_1 = \phi(s_1)$

**for**  $t = 1, T$  **do**

        With probability  $\epsilon$  select a random action  $a_t$

        otherwise select  $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

        Execute action  $a_t$  in emulator and observe reward  $r_t$  and image  $x_{t+1}$

        Set  $s_{t+1} = s_t, a_t, x_{t+1}$  and preprocess  $\phi_{t+1} = \phi(s_{t+1})$

        Store transition  $(\phi_t, a_t, r_t, \phi_{t+1})$  in  $\mathcal{D}$

        Sample random minibatch of transitions  $(\phi_i, a_i, r_i, \phi_{i+1})$  from  $\mathcal{D}$

        Set  $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

        Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equation 3

**end for**

**end for**

---

进行多次采样

一次完整的采样

从D中进行随机采样，  
获得一部分状态转移过程历史信息

# DQN算法流程

---

**Algorithm 1** Deep Q-learning with Experience Replay

---

Initialize replay memory  $\mathcal{D}$  to capacity  $N$

Initialize action-value function  $Q$  with random weights

**for** episode = 1,  $M$  **do**

    Initialize sequence  $s_1 = \{x_1\}$  and preprocessed sequenced  $\phi_1 = \phi(s_1)$

**for**  $t = 1, T$  **do**

        With probability  $\epsilon$  select a random action  $a_t$

        otherwise select  $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

        Execute action  $a_t$  in emulator and observe reward  $r_t$  and image  $x_{t+1}$

        Set  $s_{t+1} = s_t, a_t, x_{t+1}$  and preprocess  $\phi_{t+1} = \phi(s_{t+1})$

        Store transition  $(\phi_t, a_t, r_t, \phi_{t+1})$  in  $\mathcal{D}$

        Sample random minibatch of transitions  $(\phi_j, a_j, r_j, \phi_{j+1})$  from  $\mathcal{D}$

        Set  $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

        Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equation 12.1

**end for**

**end for**

---

进行多次采样

一次完整的采样

使用Q-learning方法更新状态值函数的值 (终止与非终止状态的更新不同)



# DQN算法流程

---

**Algorithm 1** Deep Q-learning with Experience Replay

---

Initialize replay memory  $\mathcal{D}$  to capacity  $N$

Initialize action-value function  $Q$  with random weights

**for** episode = 1,  $M$  **do**

    Initialize sequence  $s_1 = \{x_1\}$  and preprocessed sequenced  $\phi_1 = \phi(s_1)$

**for**  $t = 1, T$  **do**

        With probability  $\epsilon$  select a random action  $a_t$

        otherwise select  $a_t = \max_a Q^*(\phi(s_t), a; \theta)$

        Execute action  $a_t$  in emulator and observe reward  $r_t$  and image  $x_{t+1}$

        Set  $s_{t+1} = s_t, a_t, x_{t+1}$  and preprocess  $\phi_{t+1} = \phi(s_{t+1})$

        Store transition  $(\phi_t, a_t, r_t, \phi_{t+1})$  in  $\mathcal{D}$

        Sample random minibatch of transitions  $(\phi_j, a_j, r_j, \phi_{j+1})$  from  $\mathcal{D}$

        Set  $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$

        Perform a gradient descent step on  $(y_j - Q(\phi_j, a_j; \theta))^2$  according to equ

**end for**

**end for**

---

进行多次采样

一次完整的采样

使用监督学习方法更新网络的参数