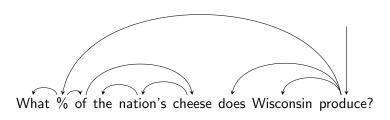
# CS 4650/7650 Dependency parsing<sup>1</sup>

Jacob Eisenstein

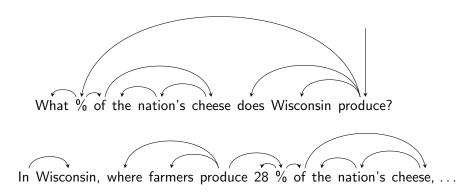
October 2, 2014

Dependency parsing is used in many real-world applications, like question answering (Cui et al, 2005):

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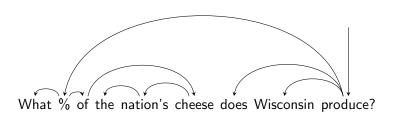


Dependency parsing is used in many real-world applications, like question answering (Cui et al, 2005):



Question answering works by searching for statements which match well against the query.

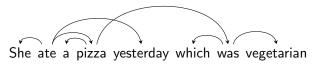
- ▶ In the surface form of the question, *produce* and % are six words apart.
- ▶ But in the dependency parse, they're adjacent.



### Projectivity

In projective dependency parsing, there are no crossing edges.

Crossing edges are rare in English:



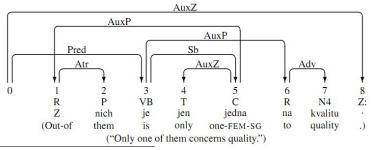
#### Projectivity

In **projective** dependency parsing, there are no crossing edges.

Crossing edges are rare in English:

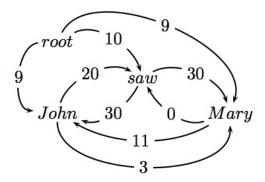


▶ They are more common in other languages, like Czech:<sup>2</sup>

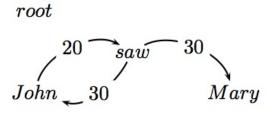




 $\triangleright x = \text{root John saw Mary}$ 

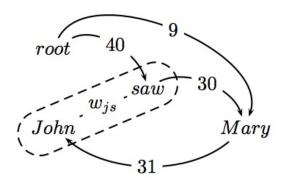


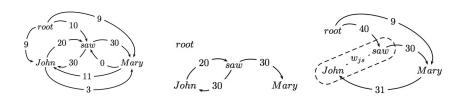
► Find highest scoring incoming arc for each vertex



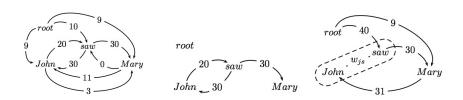
▶ If this is a tree, then we have found MST!!

- ▶ If not a tree, identify cycle and contract
- ▶ Recalculate arc weights into and out-of cycle





- Outgoing arc weights
  - Equal to the max of outgoing arc over all vertexes in cycle
  - $\,\blacktriangleright\,$  e.g., John  $\to$  Mary is 3 and saw  $\to$  Mary is 30

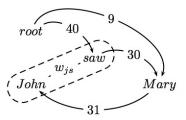


#### ► Incoming arc weights

- Equal to the weight of best spanning tree that includes head of incoming arc, and all nodes in cycle
- ▶ root  $\rightarrow$  saw  $\rightarrow$  John is 40 (\*\*)
- ▶ root  $\rightarrow$  John  $\rightarrow$  saw is 29

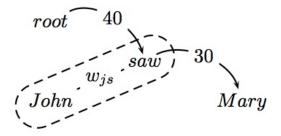
#### Theorem

The weight of the MST of this contracted graph is equal to the weight of the MST for the original graph



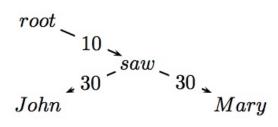
▶ Therefore, recursively call algorithm on new graph

▶ This is a tree and the MST for the contracted graph!!



▶ Go back up recursive call and reconstruct final graph

► This is the MST!!



#### Chu-Liu-Edmonds Code

▶ Reminder:  $w_{ii} = \arg \max_k w_{ii}^k$ 

```
Chu-Liu-Edmonds(G_x, w)
       Let M = \{(i^*, j) : j \in V_x, i^* = \arg \max_{i'} w_{ii} \}
       Let G_M = (V_x, M)
       If G_M has no cycles, then it is an MST: return G_M
 3
       Otherwise, find a cycle C in G_M
 5.
       Let \langle G_C, c, ma \rangle = \text{contract}(G, C, w)
       Let G = \text{Chu-Liu-Edmonds}(G_C, w)
 6.
       Find vertex i \in C such that (i', c) \in G and ma(i', c) = i
 7.
 8.
       Find arc (i'', i) \in C
       Find all arc (c, i''') \in G
 9.
       G = G \cup \{(ma(c, i'''), i''')\}_{\forall (c, i''') \in G} \cup C \cup \{(i', i)\} - \{(i'', i)\}
10.
11.
       Remove all vertices and arcs in G containing c
12
       return G
```

## Projective dependency parsing: The Eisner Algorithm

- ► ROOT She gave him the ball
- ▶ ROOT (She  $\leftarrow$  gave) (gave  $\rightarrow$  him) (the  $\leftarrow$  ball)
- ▶ [ ROOT ] [She  $\leftarrow$  gave] [gave  $\rightarrow$  him] [the  $\leftarrow$  ball]

ROOT she gave him the ball

## Projective dependency parsing: The Eisner Algorithm

- ► ROOT She gave him the ball
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ROOT she gave him the ball

 $\bullet \ \ (\ [\ \mathsf{ROOT}\ ] \to [\mathsf{She} \leftarrow \mathsf{gave}]) \ ([\mathsf{gave} \to \mathsf{him}] \to [\mathsf{the} \leftarrow \mathsf{ball}])$ 

ROOT she gave him the ball

ROOT she gave him the ball

## Projective dependency parsing: The Eisner Algorithm

- ► ROOT She gave him the ball
- ▶ ROOT (She  $\leftarrow$  gave) (gave  $\rightarrow$  him) (the  $\leftarrow$  ball)
- $\blacktriangleright \ [ \ \mathsf{ROOT} \ ] \ [\mathsf{She} \leftarrow \mathsf{gave}] \ [\mathsf{gave} \rightarrow \mathsf{him}] \ [\mathsf{the} \leftarrow \mathsf{ball}]$

 $\blacktriangleright \ \ (\ [\ \mathsf{ROOT}\ ] \to [\mathsf{She} \leftarrow \mathsf{gave}]) \ ([\mathsf{gave} \to \mathsf{him}] \to [\mathsf{the} \leftarrow \mathsf{ball}])$ 

- $\blacktriangleright \ \ (\ [\ \mathsf{ROOT}\ ] \to [\mathsf{She} \leftarrow \mathsf{gave}])\ [[\mathsf{gave} \to \mathsf{him}] \to [\mathsf{the} \leftarrow \mathsf{ball}]]$
- $\blacktriangleright \ \ [(\ [\ \mathsf{ROOT}\ ] \to [\mathsf{She} \leftarrow \mathsf{gave}]) \to [[\mathsf{gave} \to \mathsf{him}] \to [\mathsf{the} \leftarrow \mathsf{ball}]]]$



|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| _             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

|              | incomp  | lete |         |                                | incomplete    |         |         |  |
|--------------|---------|------|---------|--------------------------------|---------------|---------|---------|--|
| $\leftarrow$ | plastic | cup  | holders | $\overset{-}{\longrightarrow}$ | plastic       | cup     | holders |  |
| ROOT         |         |      |         | ROOT                           |               |         |         |  |
| plastic      |         |      |         | plastic                        |               |         |         |  |
| cup          |         |      |         | cup                            |               |         |         |  |
|              | compl   | ete  |         |                                | compl         | ete     | _       |  |
| $\leftarrow$ | plastic | cup  | holders | $\rightarrow$                  | plastic       | cup     | holders |  |
| ROOT         |         |      |         | ROOT                           |               |         |         |  |
| plastic      |         |      |         | plastic                        |               |         |         |  |
| cup          |         |      |         | cup                            |               | _       |         |  |
| ·            |         |      |         | ·                              | 4 11 1 4 11 1 | 4 = 7 4 | = = *)4 |  |

|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| _             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

|              | incomp    | lete |         |               | incomplete |     |         |  |
|--------------|-----------|------|---------|---------------|------------|-----|---------|--|
| $\leftarrow$ | plastic   | cup  | holders | $\rightarrow$ | plastic    | cup | holders |  |
| ROOT         | $-\infty$ |      |         | ROOT          |            |     |         |  |
| plastic      |           | 2    |         | plastic       |            |     |         |  |
| cup          |           |      | 4       | cup           |            |     |         |  |
|              | compl     | ete  |         |               | compl      | ete |         |  |
| $\leftarrow$ | plastic   | cup  | holders | $\rightarrow$ | plastic    | cup | holders |  |
| ROOT         |           |      |         | ROOT          |            |     |         |  |
| plastic      |           |      |         | plastic       |            |     |         |  |
| cup          |           |      |         | cup           |            |     |         |  |

|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| _             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

|              | incomp    | lete |         | incomplete    |         |     |         |
|--------------|-----------|------|---------|---------------|---------|-----|---------|
| $\leftarrow$ | plastic   | cup  | holders | $\rightarrow$ | plastic | cup | holders |
| ROOT         | $-\infty$ |      |         | ROOT          | 1       |     |         |
| plastic      |           | 2    |         | plastic       |         | -1  |         |
| cup          |           |      | 4       | cup           |         |     | -1      |
|              | compl     | ete  |         |               | compl   | ete |         |
| $\leftarrow$ | plastic   | cup  | holders | $\rightarrow$ | plastic | cup | holders |
| ROOT         |           |      |         | ROOT          |         |     |         |
| plastic      |           |      |         | plastic       |         |     |         |
| cup          |           |      |         | cup           |         |     |         |

|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| -             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

| cup holders |
|-------------|
|             |
|             |
| -1          |
| -1          |
| lete        |
| cup holders |
|             |
|             |
|             |
| le          |

|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| _             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

| incomp    | lete   |  |   | incomp  | lete  |   |
|-----------|--|--|---|---|---|---|
| plastic   | cup  | holders  | $\rightarrow$   | plastic   | cup   | holders   |
| $-\infty$ |  |  | ROOT  | 1   |   |   |
|           | 2  |  | plastic   |   | -1  |   |
|           |  | 4  | cup   |   |   | -1  |
| compl     | ete  |  |   | compl   | ete   |   |
| plastic   | cup  | holders  | $\rightarrow$   | plastic   | cup   | holders   |
| $-\infty$ |  |  | ROOT  | 1   |   |   |
|           | 2  |  | plastic   |   | -1  |   |
|           |  | 4  | cup   |   |   | -1  |
|           | $\begin{array}{c} \text{plastic} \\ -\infty \end{array}$ | $ \begin{array}{c c} -\infty & 2 \\ \hline  & complete \\ \hline  & plastic & cup \\  & -\infty &  \end{array} $ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ |

incomplete

|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| _             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

|              | incomp    | lete      |         |               | incomp      | piete   |          |  |
|--------------|-----------|-----------|---------|---------------|-------------|---------|----------|--|
| $\leftarrow$ | plastic   | cup       | holders | $\rightarrow$ | plastic     | cup     | holders  |  |
| ROOT         | $-\infty$ | $-\infty$ |         | ROOT          | 1           |         |          |  |
| plastic      |           | 2         | 4       | plastic       |             | -1      |          |  |
| cup          |           |           | 4       | cup           |             |         | -1       |  |
|              | compl     | ete       |         |               | complete    |         |          |  |
| $\leftarrow$ | plastic   | cup       | holders | $\rightarrow$ | plastic     | cup     | holders  |  |
| ROOT         | $-\infty$ |           |         | ROOT          | 1           |         |          |  |
| plastic      |           | 2         |         | plastic       |             | -1      |          |  |
| cup          |           |           | 4       | cup           |             |         | -1       |  |
|              |           |           |         |               | 4 D > 4 D > | 4 = 7 4 | <u> </u> |  |

incomplete

|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| _             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

|              | incomp    | lete      |              |               | incomp  | nete |         |
|--------------|-----------|-----------|--------------|---------------|---------|------|---------|
| $\leftarrow$ | plastic   | cup       | holders      | $\rightarrow$ | plastic | cup  | holders |
| ROOT         | $-\infty$ | $-\infty$ |              | ROOT          | 1       | 3    |         |
| plastic      |           | 2         | 4            | plastic       |         | -1   | 3       |
| cup          |           |           | 4            | cup           |         |      | -1      |
|              | compl     | ete       | ete complete |               |         |      |         |
| $\leftarrow$ | plastic   | cup       | holders      | $\rightarrow$ | plastic | cup  | holders |
| ROOT         | $-\infty$ |           |              | ROOT          | 1       |      |         |
| plastic      |           | 2         |              | plastic       |         | -1   |         |
| cup          |           |           | 4            | cup           |         |      | -1      |
|              |           |           |              |               |         |      |         |

|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| _             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

| incomplete        |           |           |              | incomp          | lete    |     |         |
|-------------------|-----------|-----------|--------------|-----------------|---------|-----|---------|
| $\leftarrow$      | plastic   | cup       | holders      | $\rightarrow$   | plastic | cup | holders |
| ROOT              | $-\infty$ | $-\infty$ |              | ROOT            | 1       | 3   |         |
| plastic           |           | 2         | 4            | plastic         |         | -1  | 3       |
| cup               |           |           | 4            | cup             |         |     | -1      |
|                   | compl     | 0±0       |              | complete        |         |     |         |
|                   | compi     | ete       |              |                 | compi   | ete |         |
| $\leftarrow$      | plastic   | cup       | holders      | ${\rightarrow}$ | plastic | cup | holders |
| $\leftarrow$ ROOT |           |           | holders      | o ROOT          |         |     | holders |
|                   | plastic   | cup       | holders<br>6 | ightarrowROOT   |         |     | holders |
| ROOT              | plastic   | $-\infty$ |              |                 |         | cup | holders |

|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| _             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

| incomplete   |                  |            | incomplete   |                        |                  |            |         |
|--------------|------------------|------------|--------------|------------------------|------------------|------------|---------|
| $\leftarrow$ | plastic          | cup        | holders      | $\rightarrow$          | plastic          | cup        | holders |
| ROOT         | $-\infty$        | $-\infty$  |              | ROOT                   | 1                | 3          |         |
| plastic      |                  | 2          | 4            | plastic                |                  | -1         | 3       |
| cup          |                  |            | 4            | cup                    |                  |            | -1      |
|              |                  |            |              | complete               |                  |            |         |
|              | compl            | ete        |              |                        | compl            | ete        |         |
| <del></del>  | compl<br>plastic | ete<br>cup | holders      | ${\longrightarrow}$    | compl<br>plastic | ete<br>cup | holders |
| ←<br>ROOT    |                  |            | holders      | o ROOT                 | <u> </u>         |            | holders |
| `            | plastic          | cup        | holders<br>6 | ightarrow ROOT plastic | <u> </u>         | cup        | holders |
| ROOT         | plastic          | $-\infty$  |              |                        | <u> </u>         | cup<br>3   |         |

incomplete

|         | ROOT           | plastic   | cup   | holders   |
|---------|----------------|---|---|---|
| ROOT    |                | 1   | 1   | 1   |
| plastic | $-\infty$      |   | -1  | -1  |
| cup     | $-\infty$      | 2   |   | -1  |
| holders | $-\infty$      | 0   | 4   |   |
|         | plastic<br>cup | $\begin{array}{ccc} ROOT & & \\ plastic & -\infty & \\ cup & -\infty & \end{array}$ | $\begin{array}{ccc} ROOT & & 1 \\ plastic & -\infty & \\ cup & -\infty & 2 \end{array}$ | $\begin{array}{c cccc} ROOT & & 1 & 1 \\ plastic & -\infty & & -1 \\ cup & -\infty & 2 & \end{array}$ |

|              | mcomp     | ricte     |           |               | mcomp   | rictc   |         |
|--------------|-----------|-----------|-----------|---------------|---------|---------|---------|
| $\leftarrow$ | plastic   | cup       | holders   | $\rightarrow$ | plastic | cup     | holders |
| ROOT         | $-\infty$ | $-\infty$ | $-\infty$ | ROOT          | 1       | 3       |         |
| plastic      |           | 2         | 4         | plastic       |         | -1      | 3       |
| cup          |           |           | 4         | cup           |         |         | -1      |
|              | complete  |           |           |               | compl   | ete     |         |
| $\leftarrow$ | plastic   | cup       | holders   | $\rightarrow$ | plastic | cup     | holders |
| ROOT         | $-\infty$ | $-\infty$ |           | ROOT          | 1       | 3       |         |
| plastic      |           | 2         | 6         | plastic       |         | -1      | 3       |
| cup          |           |           | 4         | cup           |         |         | -1      |
|              |           |           |           |               |         | 4 = 1 4 | = > = 9 |

|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| _             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

|                | incomp    | lete       |              |                                   | incomp           | lete       |         |  |  |
|----------------|-----------|------------|--------------|-----------------------------------|------------------|------------|---------|--|--|
| $\leftarrow$   | plastic   | cup        | holders      | $\rightarrow$                     | plastic          | cup        | holders |  |  |
| ROOT           | $-\infty$ | $-\infty$  | $-\infty$    | ROOT                              | 1                | 3          | 7       |  |  |
| plastic        |           | 2          | 4            | plastic                           |                  | -1         | 3       |  |  |
| cup            |           |            | 4            | cup                               |                  |            | -1      |  |  |
|                | complete  |            |              | ·                                 | complete         |            |         |  |  |
|                | compl     | ete        |              |                                   | compl            | ete        |         |  |  |
| ${\leftarrow}$ | plastic   | ete<br>cup | holders      | ${\longrightarrow}$               | compl<br>plastic | ete<br>cup | holders |  |  |
| ←<br>ROOT      |           |            | holders      | $\overset{\longrightarrow}{ROOT}$ | <u> </u>         |            | holders |  |  |
| `              | plastic   | cup        | holders<br>6 | ightarrow ROOT                    | <u> </u>         | cup        | holders |  |  |
| ROOT           | plastic   | cup        |              |                                   | <u> </u>         | cup<br>3   |         |  |  |

incomplete

|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| _             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

|              | шсоппр    | ricte     |           |               | шсошь   | nete    |             |
|--------------|-----------|-----------|-----------|---------------|---------|---------|-------------|
| $\leftarrow$ | plastic   | cup       | holders   | $\rightarrow$ | plastic | cup     | holders     |
| ROOT         | $-\infty$ | $-\infty$ | $-\infty$ | ROOT          | 1       | 3       | 7           |
| plastic      |           | 2         | 4         | plastic       |         | -1      | 3           |
| cup          |           |           | 4         | cup           |         |         | -1          |
|              | complete  |           |           |               | compl   | ete     | <del></del> |
| $\leftarrow$ | plastic   | cup       | holders   | $\rightarrow$ | plastic | cup     | holders     |
| ROOT         | $-\infty$ | $-\infty$ | $-\infty$ | ROOT          | 1       | 3       |             |
| plastic      |           | 2         | 6         | plastic       |         | -1      | 3           |
| cup          |           |           | 4         | cup           |         |         | -1          |
|              |           |           |           |               |         | 4 = 1 4 | = P = ♥ Q   |

|               |         | ROOT      | plastic | cup | holders |
|---------------|---------|-----------|---------|-----|---------|
| _             | ROOT    |           | 1       | 1   | 1       |
| Edge weights: | plastic | $-\infty$ |         | -1  | -1      |
|               | cup     | $-\infty$ | 2       |     | -1      |
|               | holders | $-\infty$ | 0       | 4   |         |

| incomplete   |                  |            | incomplete |                      |                  |            |                   |  |
|--------------|------------------|------------|------------|----------------------|------------------|------------|-------------------|--|
| $\leftarrow$ | plastic          | cup        | holders    | $\rightarrow$        | plastic          | cup        | holders           |  |
| ROOT         | $-\infty$        | $-\infty$  | $-\infty$  | ROOT                 | 1                | 3          | 7                 |  |
| plastic      |                  | 2          | 4          | plastic              |                  | -1         | 3                 |  |
| cup          |                  |            | 4          | cup                  |                  |            | -1                |  |
| complete     |                  |            |            | complete             |                  |            |                   |  |
|              | compl            | ete        |            |                      | compl            | ete        |                   |  |
| $\leftarrow$ | compl<br>plastic | ete<br>cup | holders    | ${\longrightarrow}$  | compl<br>plastic | ete<br>cup | holders           |  |
| ←<br>ROOT    |                  |            | $-\infty$  | $\overset{-}{}$ ROOT |                  |            | holders<br>7      |  |
| _ `          | plastic          | cup        |            | ·                    |                  | cup        | holders<br>7<br>3 |  |
| ROOT         | plastic          | cup        | $-\infty$  | ROOT                 |                  | cup<br>3   | 7                 |  |