CS 4650/7650
Examples of Viterbi, Forward, and Backward algorithms

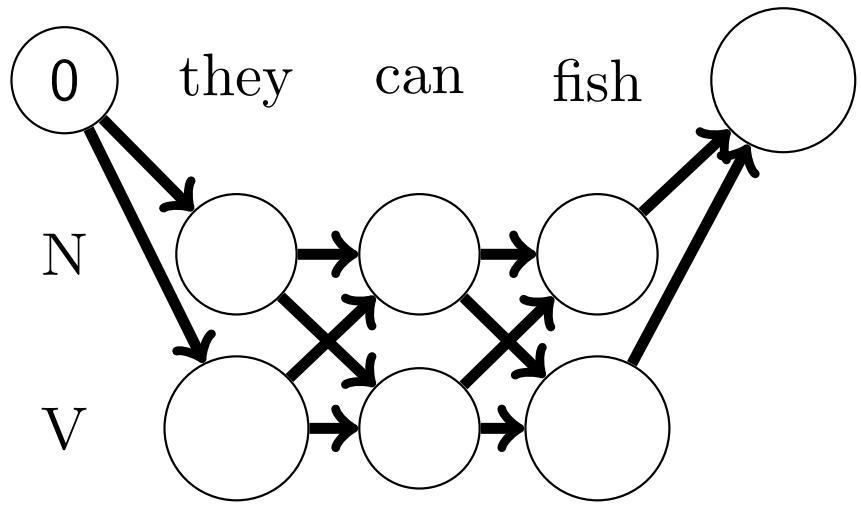
Jacob Eisenstein

September 19, 2013

Viterbi example

Transitions			
	N	V	END
N	-3	-1	-2
V	-1	-3	-2
START	-1	-2	

Emissions			
	they	can	fish
N	-2	-3	-3
V	-10	-1	-3



Viterbi example

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Emissions	they	can	fish
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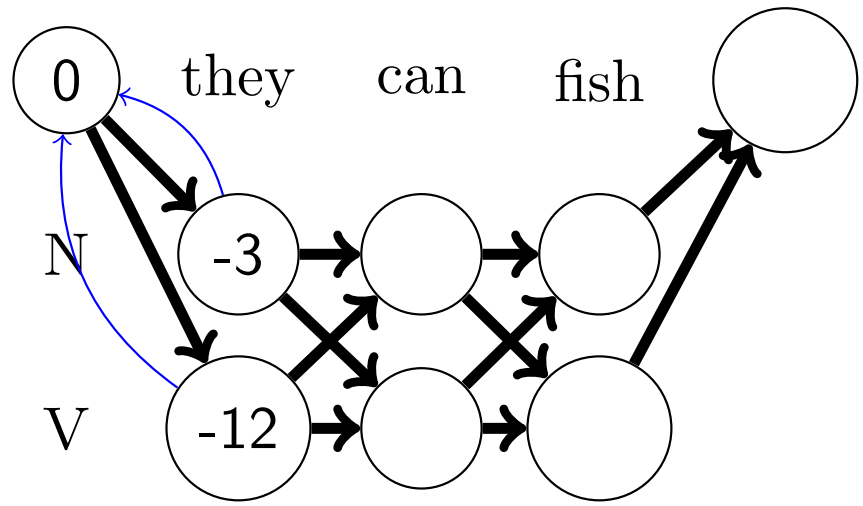

Viterbi example

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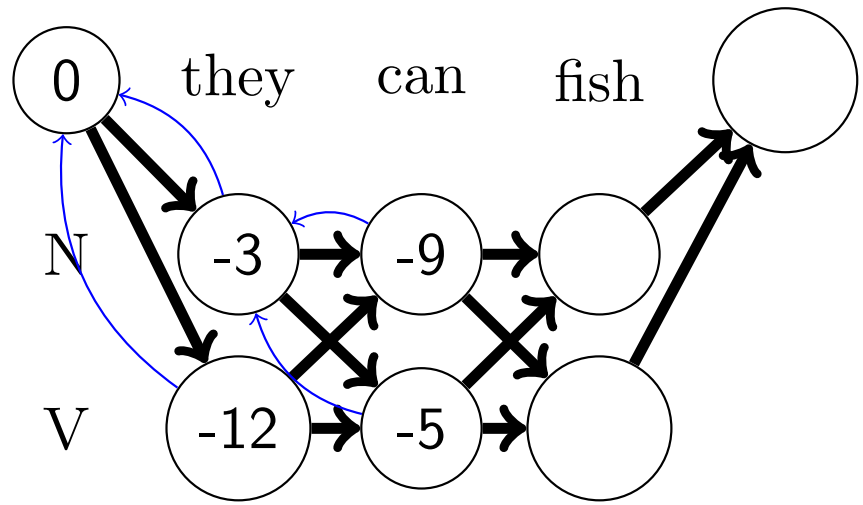

Viterbi example

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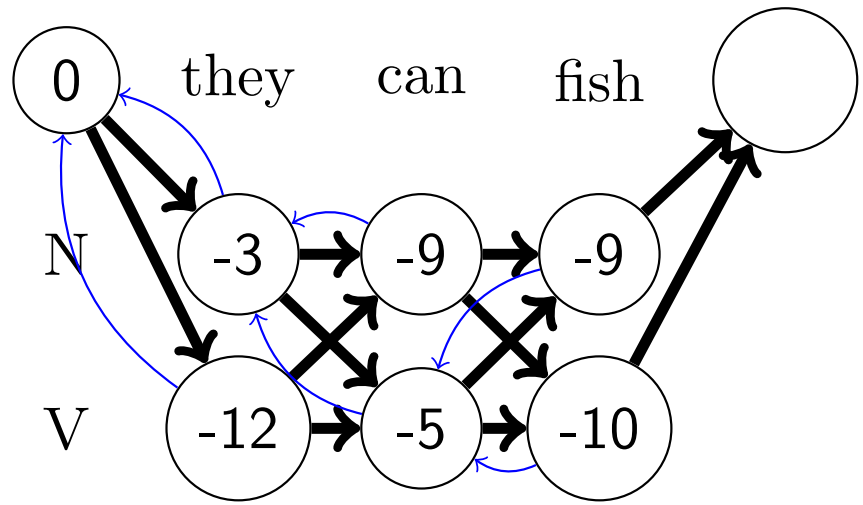

Viterbi example

Transitions	N	V	END
N	-3	-1	-2
V	-1	-3	-2
START	-1	-2	

Emissions	they	can	fish
N	-2	-3	-3
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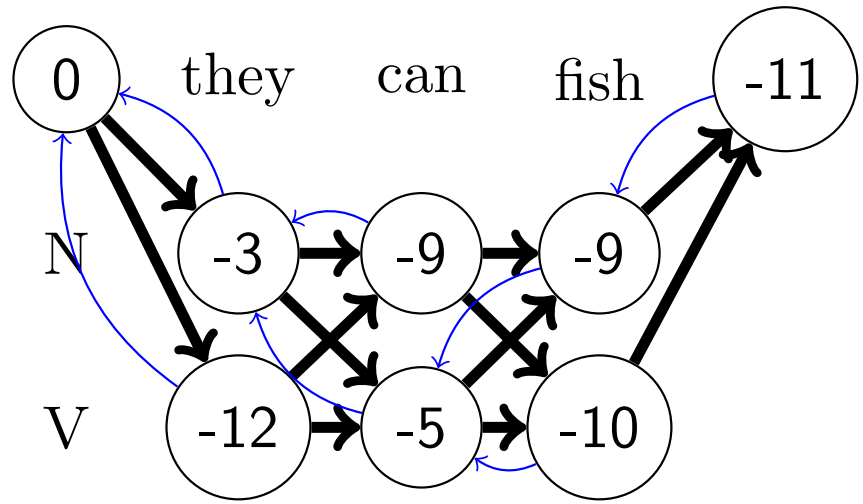
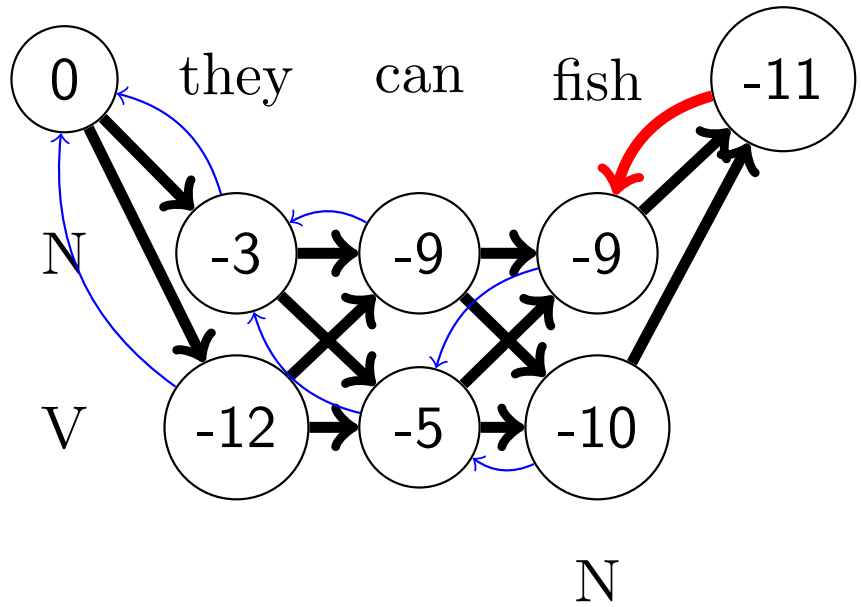


Viterbi example

Transitions	N	V	END
N	-3	-1	-2
V	-1	-3	-2
START	-1	-2	

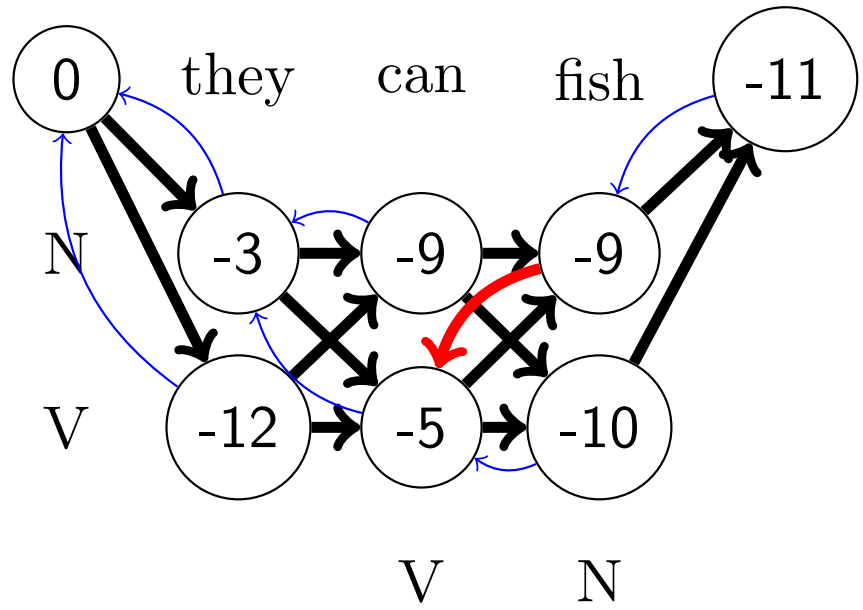
Emissions	they	can	fish
N	-2	-3	-3
V	-10	-1	-3



Viterbi example

Transitions	N	V	END
N	-3	-1	-2
V	-1	-3	-2
START	-1	-2	

Emissions	they	can	fish
N	-2	-3	-3
V	-10	-1	-3



Viterbi example

Transitions	N	V	END
N	-3	-1	-2
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Emissions	they	can	fish
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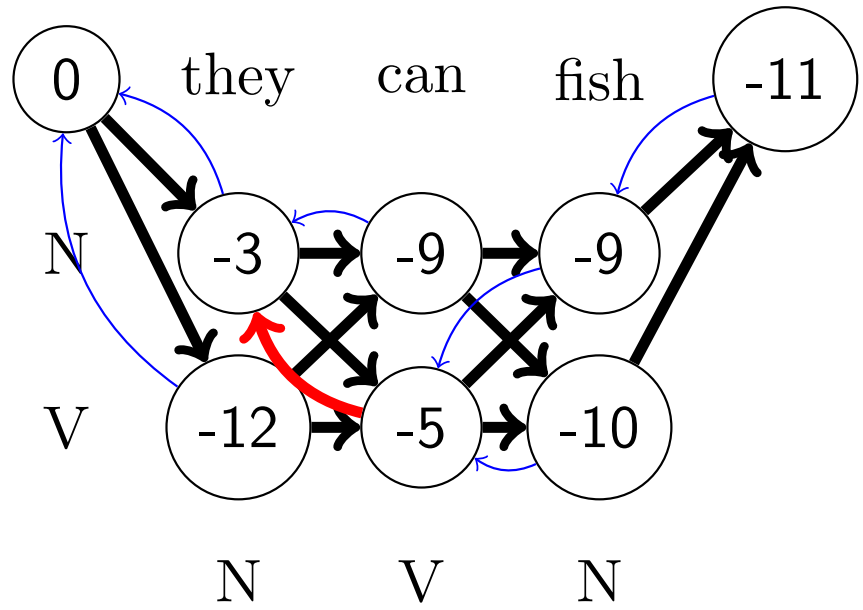

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START	-1	-2	

Emissions			
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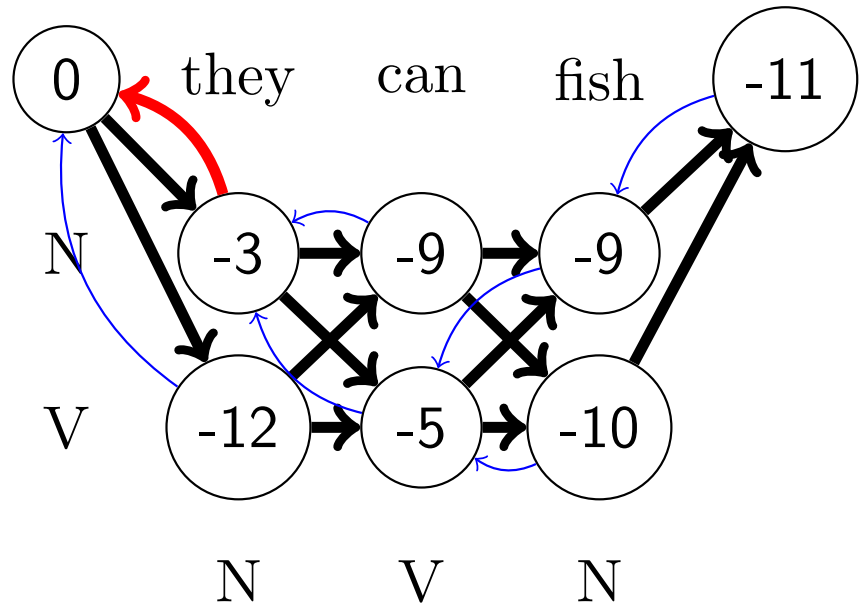
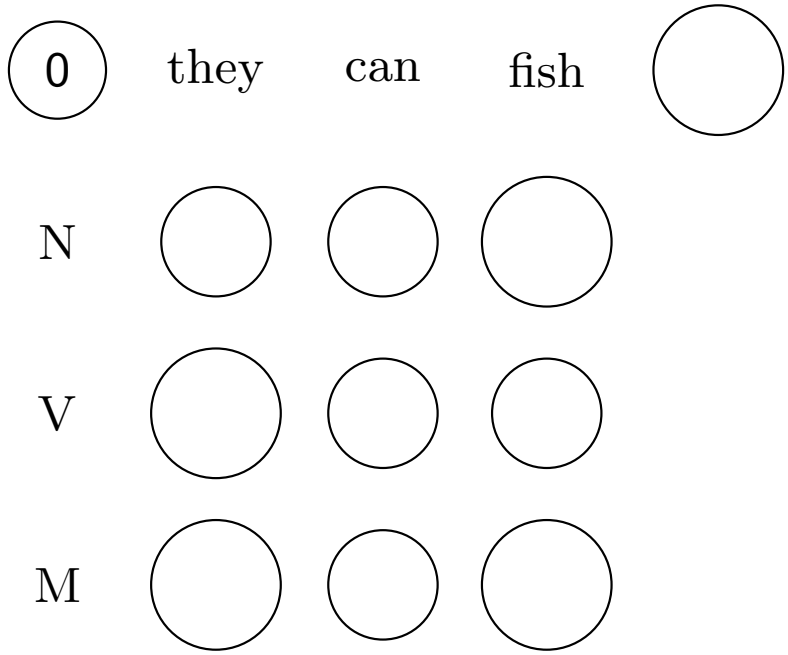


Now let's add a tag M for modal verbs.

Viterbi example

Transition				
	N	V	M	END
N	-3	-1	-2	-2
V	-1	-2	-5	-2
M	-10	0	-3	-10
Start	-1	-2	-3	

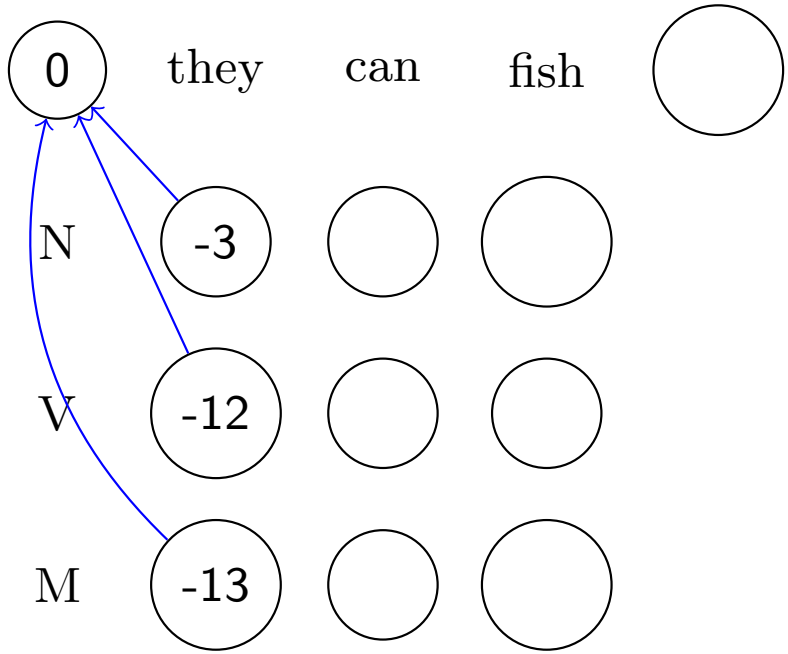
Emission			
	they	can	fish
N	-2	-3	-3
V	-10	-2	-3
M	-10	-1	-10



Viterbi example

Transition				
	N	V	M	END
N	-3	-1	-2	-2
V	-1	-2	-5	-2
M	-10	0	-3	-10
Start	-1	-2	-3	

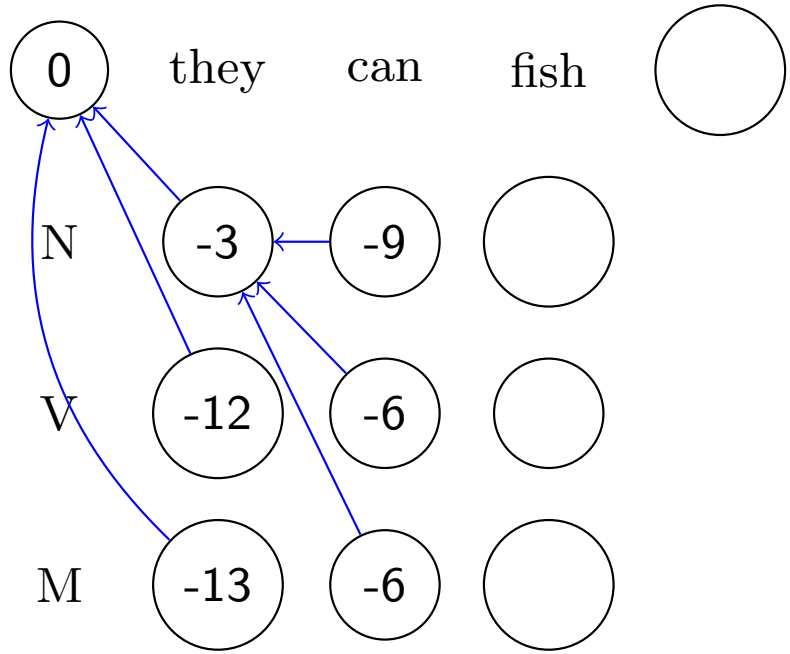
Emission			
	they	can	fish
N	-2	-3	-3
V	-10	-2	-3
M	-10	-1	-10



Viterbi example

Transition				
	N	V	M	END
N	-3	-1	-2	-2
V	-1	-2	-5	-2
M	-10	0	-3	-10
Start	-1	-2	-3	

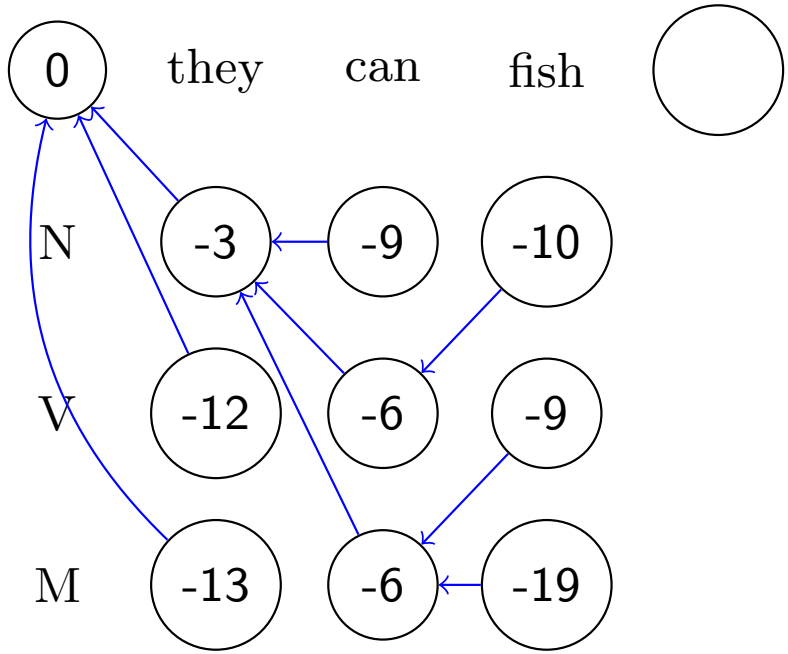
Emission			
	they	can	fish
N	-2	-3	-3
V	-10	-2	-3
M	-10	-1	-10



Viterbi example

Transition				
	N	V	M	END
N	-3	-1	-2	-2
V	-1	-2	-5	-2
M	-10	0	-3	-10
Start	-1	-2	-3	

Emission			
	they	can	fish
N	-2	-3	-3
V	-10	-2	-3
M	-10	-1	-10



Viterbi example

Transition				
	N	V	M	END
N	-3	-1	-2	-2
V	-1	-2	-5	-2
M	-10	0	-3	-10
Start	-1	-2	-3	

Emission			
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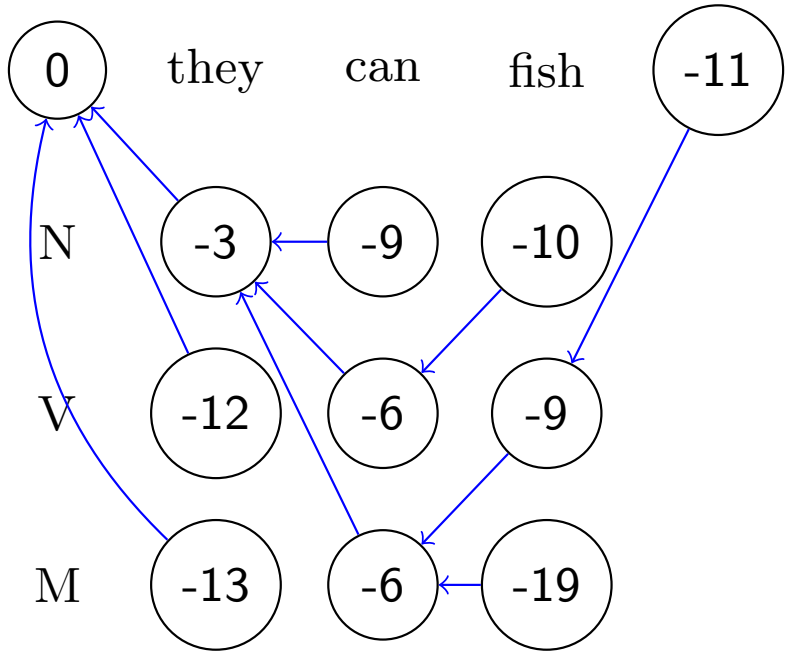

Viterbi example

Transition				
	N	V	M	END
N	-3	-1	-2	-2
V	-1	-2	-5	-2
M	-10	0	-3	-10
Start	-1	-2	-3	

Emission			
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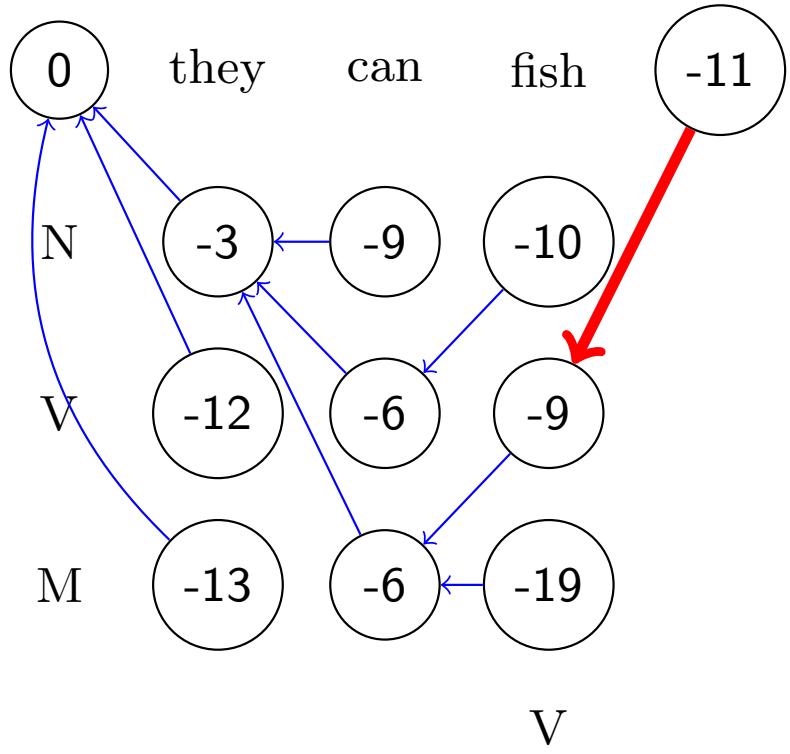

Viterbi example

Transition				
	N	V	M	END
N	-3	-1	-2	-2
V	-1	-2	-5	-2
M	-10	0	-3	-10
Start	-1	-2	-3	

Emission			
	they	can	fish
N	-2	-3	-3
V	-10	-2	-3
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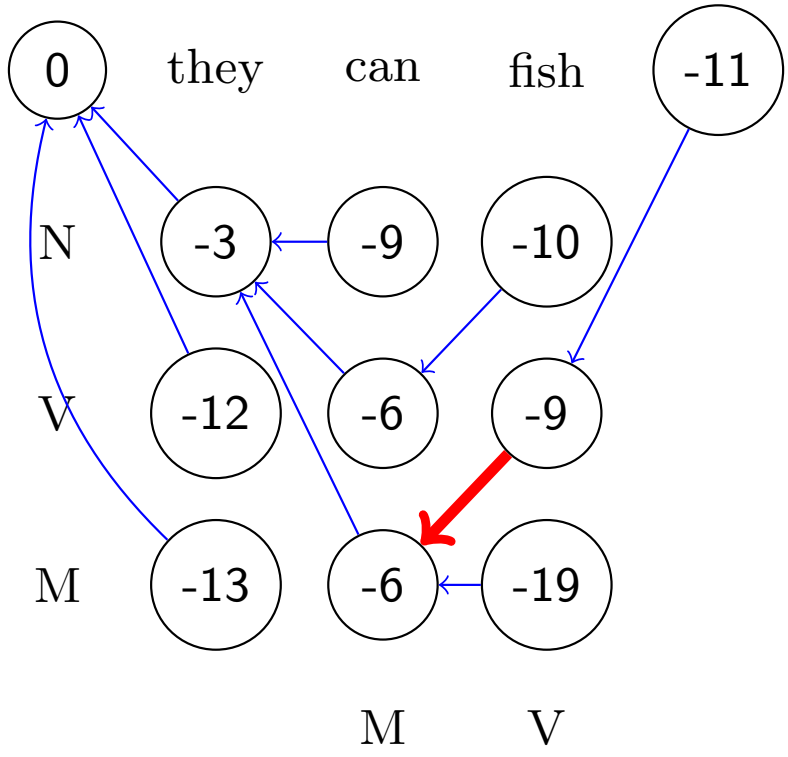

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Transition				
	N	V	M	END
N	-3	-1	-2	-2
V	-1	-2	-5	-2
M	-10	0	-3	-10
Start	-1	-2	-3	

Emission			
	they	can	fish
N	-2	-3	-3
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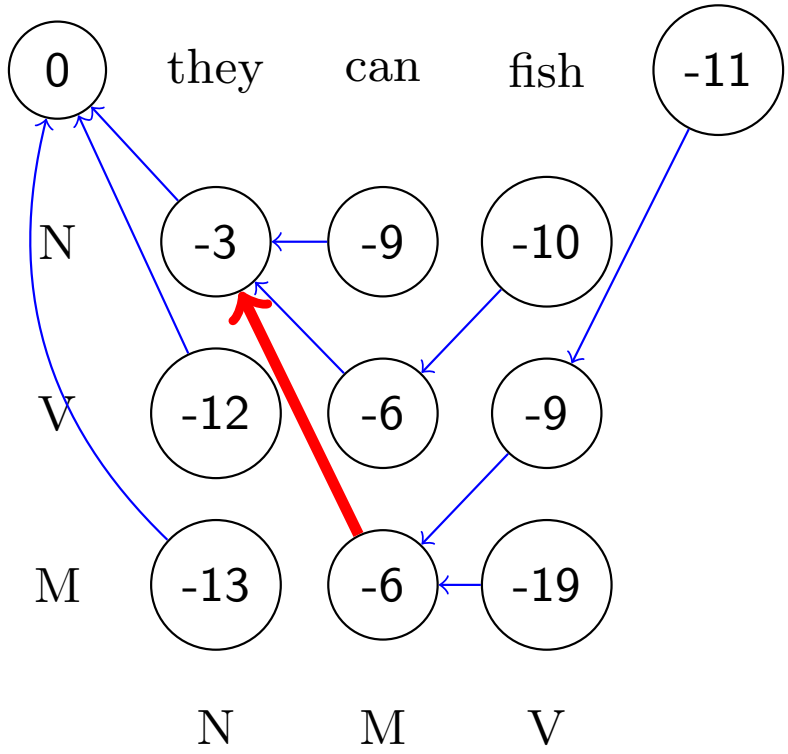

Viterbi example

Transition				
	N	V	M	END
N	-3	-1	-2	-2
V	-1	-2	-5	-2
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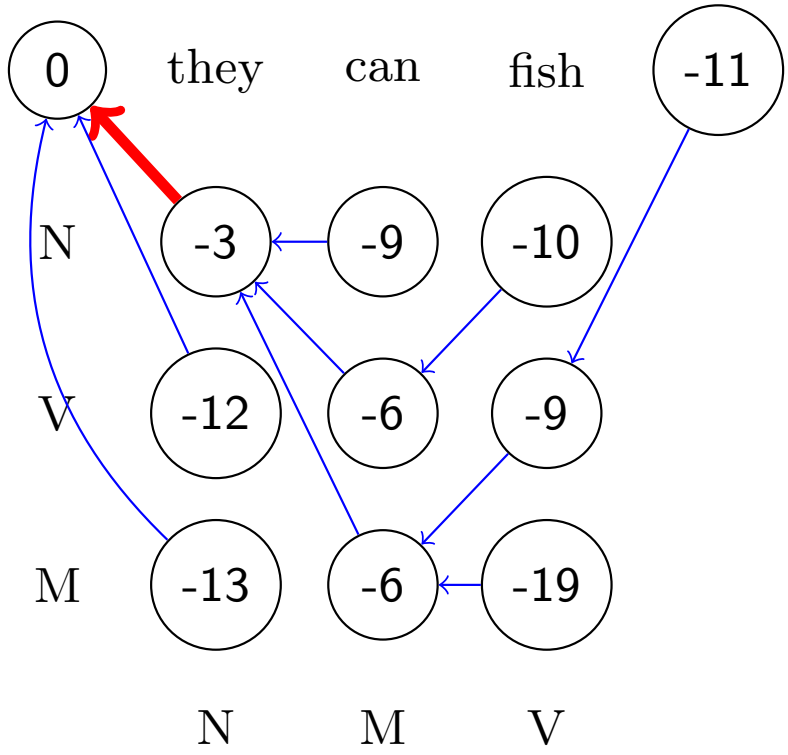


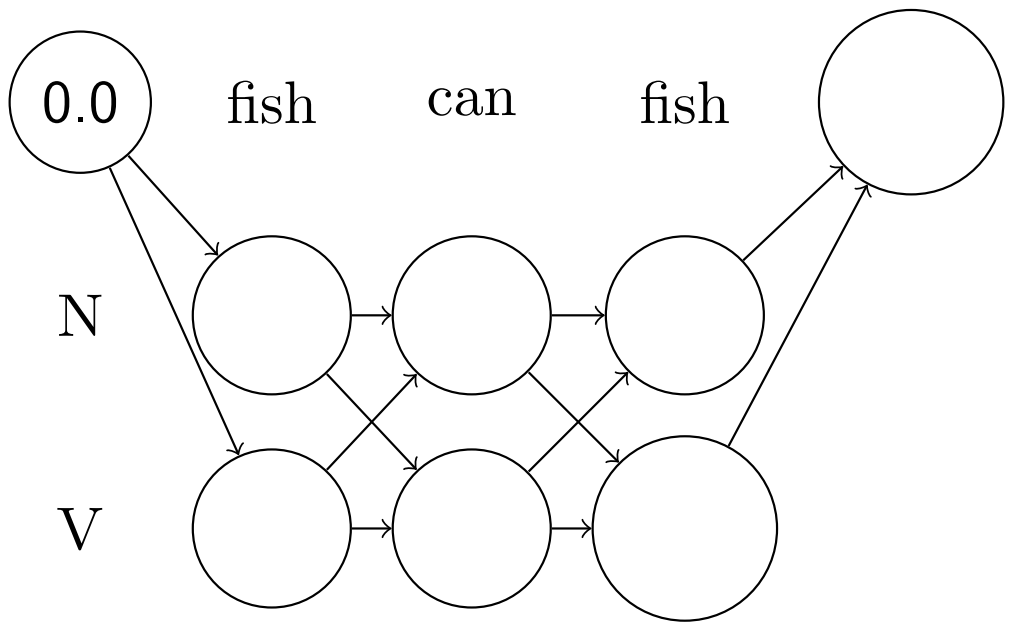
Forward example

Transition weights

	N	V
N	-3	-1
V	-1	-2

Emission weights

	fish	can	fish
N	-3	-3	-3
V	-3	-1	-3



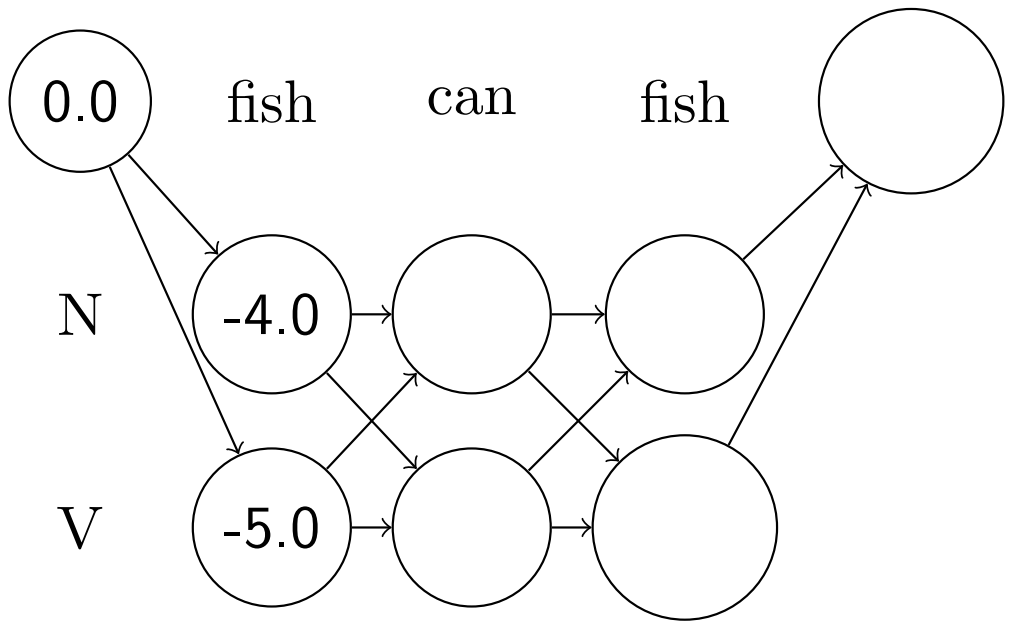
Forward example

Transition weights

	N	V
N	-3	-1
V	-1	-2

Emission weights

	fish	can	fish
N	-3	-3	-3
V	-3	-1	-3



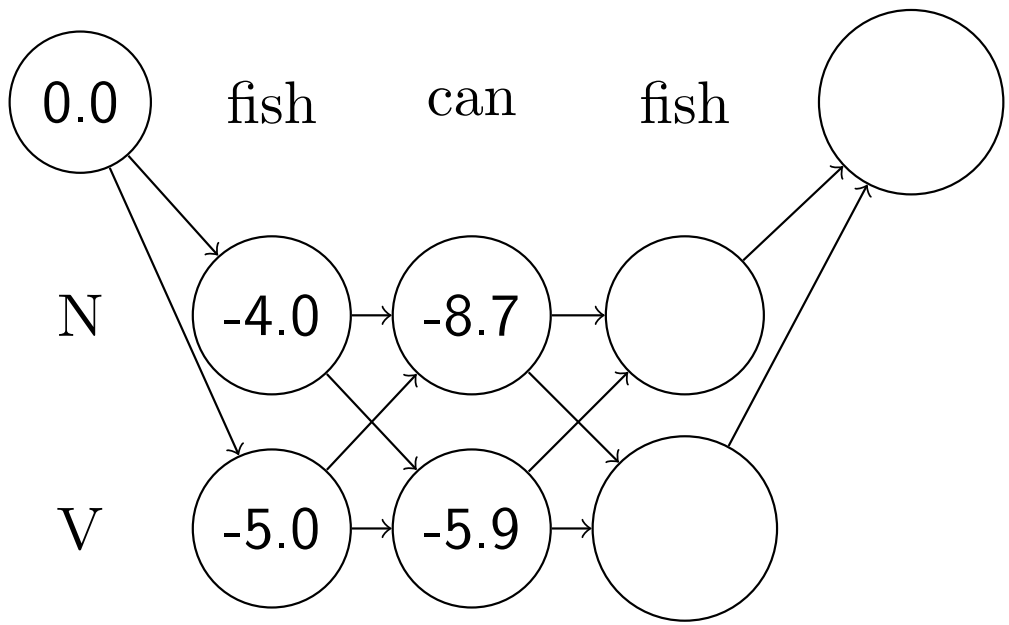
Forward example

Transition weights

	N	V
N	-3	-1
V	-1	-2

Emission weights

	fish	can	fish
N	-3	-3	-3
V	-3	-1	-3



Forward example

Transition weights

	N	V
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Emission weights

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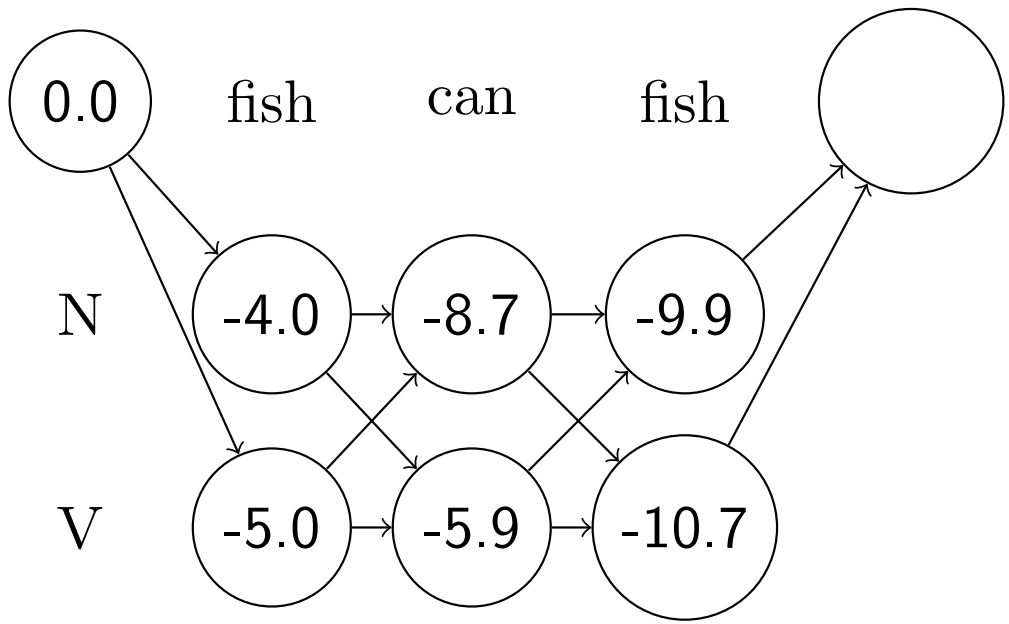

Forward example

Transition weights

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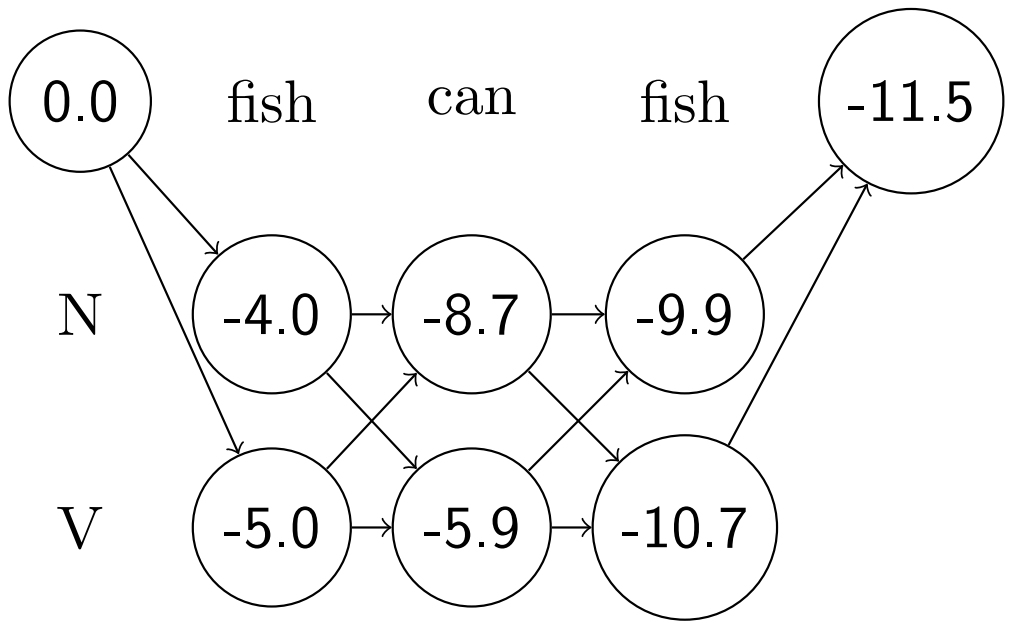

Example: backward algorithm

Transition weights
(log-probabilities)

	N	V	END
N	-2	-1	-2
V	-2	-3	-2
START	-1	-2	

Emission weights
(log-probabilities)

	they	can	fish
N	-2	-3	-3
V	-6	-1	-2

Example: backward algorithm

Transition weights
(log-probabilities)

	N	V	END
N	-2	-1	-2
V	-2	-3	-2
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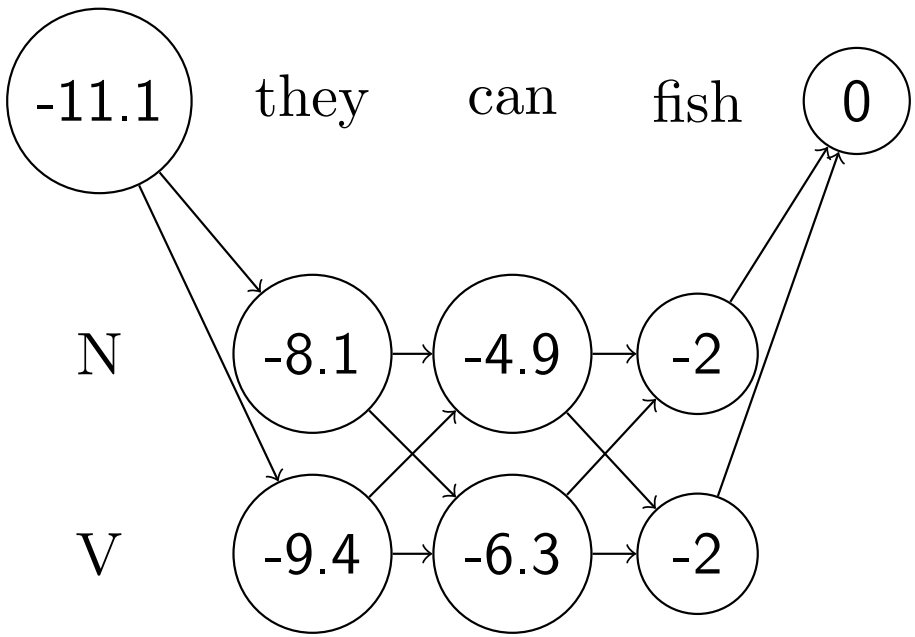
Emission weights
(log-probabilities)

	they	can	fish
N	-2	-3	-3
V	-6	-1	-2



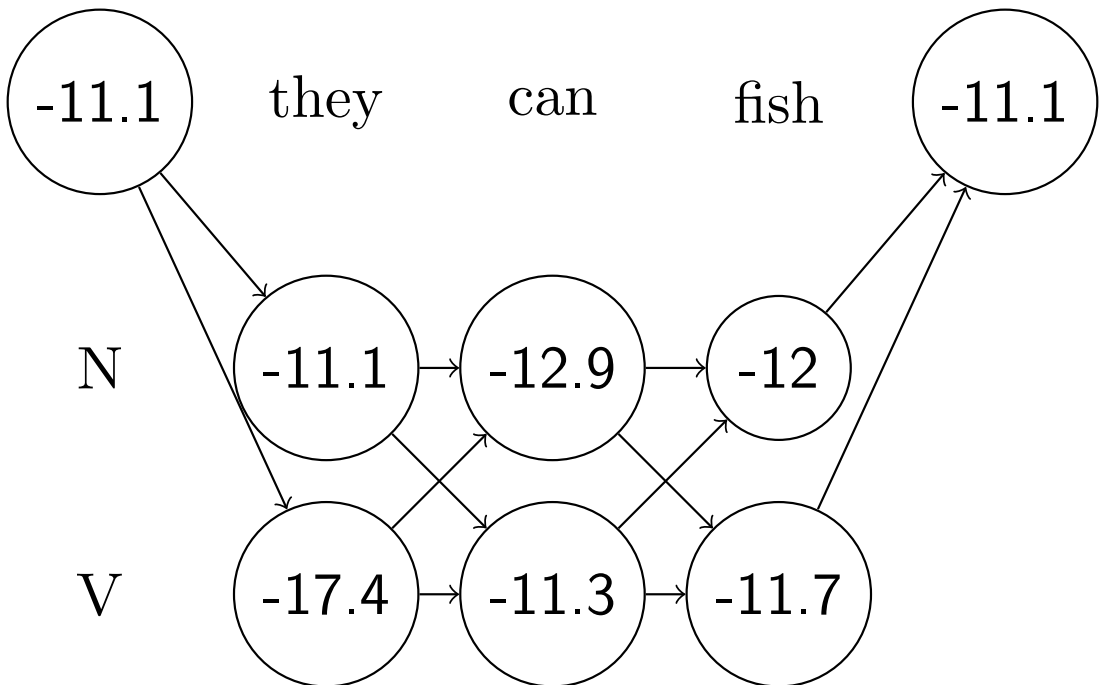
Example: marginal probabilities

Transition weights
(log-probabilities)

	N	V	END
N	-2	-1	-2
V	-2	-3	-2
START	-1	-2	

Emission weights
(log-probabilities)

	they	can	fish
N	-2	-3	-3
V	-6	-1	-2



Example: marginal probabilities

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	they	can	fish
N	-2	-3	-3
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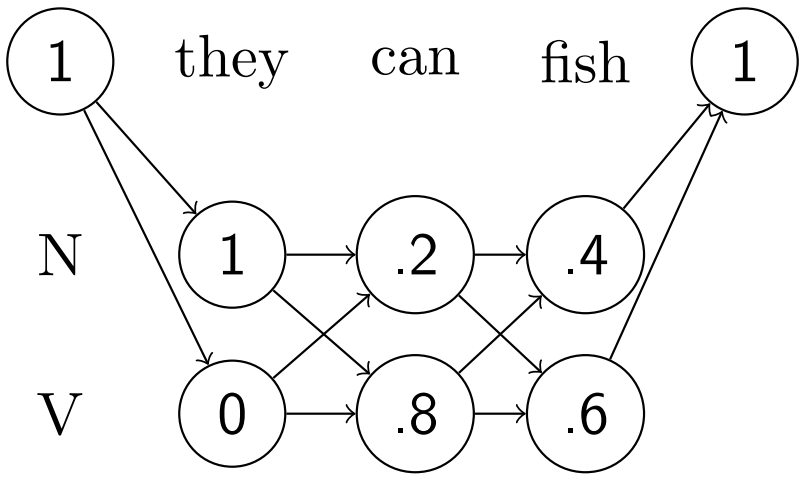

Example: marginal probabilities

Transition weights (log-probabilities)

	N	V	END
N	-2	-1	-2
V	-2	-3	-2
START	-1	-2	

Emission weights (log-probabilities)

	they	can	fish
N	-2	-3	-3
V	-6	-1	-2

$$\log P(\textit{they}/N\ \textit{can}/V\ \textit{fish}/V) = -1-2-1-1-3-2-2 = -12$$

$$\log P(\textit{they}/N\ \textit{can}/V\ \textit{fish}/N) = -1-2-1-1-2-3-2 = -12$$

$$\log P(\textit{they}/V\ \ldots) \approx -\infty$$

$$\log \sum_y \exp P(y_1, y_2 = V, y_3, x_{1:3}) = \log(e^{-12} + e^{-12} + e^{-\infty} + e^{-\infty})$$

$$= -11.3 = f_2(V) + b_2(V)$$