反向传播算法 (DNN)、

$$a^l = \sigma(z^l) = \sigma(W^l a^{l-1} + b^l)$$

$$J(W, b, x, y) = \frac{1}{2}\|a^L - y\|_2^2$$

损失对第l层未激活输出的梯度为δ^l

$$\delta^L = \frac{\partial J}{\partial z^L} = (a^L - y) \odot \sigma'(z^L)$$

$$\therefore \frac{\partial J}{\partial W} = \frac{\partial J}{\partial z^l}\frac{\partial z^l}{\partial W} = [(a^L - y) \odot \sigma'(z^L)](a^{L-1})^T$$

$$\frac{\partial J}{\partial b^l} = \frac{\partial J}{\partial z^l}\frac{\partial z^l}{\partial b^l} = (a^L - y) \odot \sigma'(z^L)$$

$$\delta^l = \frac{\partial J}{\partial z^l} = \left(\frac{\partial z^{l+1}}{\partial z^{l}}\right)^T \frac{\partial J}{\partial z^{l+1}}$$

$$\because z^{l+1} = W^{l+1} a^l + b^{l+1}$$

$$= W^{l+1}\sigma(z^l) + b^{l+1}$$

$$\delta^l = \left(\frac{\partial z^{l+1}}{\partial z^l}\right)^T \frac{\partial J}{\partial z^{l+1}}$$

$$= [W^{l+1}\mathrm{diag}(\sigma'(z^l))]^T \delta^{l+1}$$

$$= \mathrm{diag}(\sigma'(z^l))(W^{l+1})^T \delta^{l+1}$$

$$= (W^{l+1})^T \delta^{l+1} \odot \sigma'(z^l)$$

$$\frac{\partial J}{\partial W^l} = \frac{\partial J}{\partial z^l}\frac{\partial z^l}{\partial W^l} = \delta^l (a^{l-1})^T$$

$$\frac{\partial J}{\partial b^l} = \delta^l$$

使用交叉熵损失函数

$$\delta^L = \frac{\partial J}{\partial z^L} = -y\frac{1}{a^L}(a^L)(1-a^L) + (1-y)\frac{1}{1-a^L}(a^L)(1-a^L)$$

$$= a^L - y$$

没有了$\sigma'(z)$

使用softmax激活，对数似然损失

$$J = -\sum_k y_k \ln a_k \qquad y_k = 0 \text{或} 1$$

对于某个样本i $y_i = 1$ $j \neq i$ 则 $y_j = 0$

所以，$J = -\ln a_i^L$，真实i类，对应的第j个
连接w_{ij}^L

$$\frac{\partial J}{\partial w_{ij}^L} = \frac{\partial J}{\partial a_i^L} \cdot \frac{\partial a_i^L}{\partial z_i^L} \cdot \frac{\partial z_i^L}{\partial w_{ij}^L}$$

$$= -\frac{1}{a_i^L} \cdot \frac{(e^{z_i^L})\sum_{j=1}^{n_L} e^{z_j^L} - e^{z_i^L}e^{z_i^L}}{\left(\sum_{j=1}^{n_L} e^{z_j^L}\right)^2} a_j^{L-1}$$

$$= -\frac{1}{a_i^L} \cdot a_i^L(1 - a_i^L)a_j^{L-1}$$

$$= (a_i^L - 1)a_j^{L-1}$$

$$\frac{\partial J}{\partial b_i^L} = a_i^L - 1$$