

## Exercise 1

思路: 动态构建树; Hill Climbing 方法的本质是添加排序的深搜; Best-First Search 方法的本质是添加了排序的广搜。

```
public class Treesearch
{
    public static void main(String[] args)
    {
        Node T1 = new Node();
        initializecube(T1);
        Node T2 = new Node();
        initializecube(T2);
        Node T3 = new Node();
        initializecube(T3);
        Node T4 = new Node();
        initializecube(T4);
        System.out.println("Output:");
        System.out.println("可达");
        long starttime = System.currentTimeMillis();
        breadthfirst(T1);
        long endtime = System.currentTimeMillis();
        long exacttime = endtime - starttime;
        System.out.println("Breadth-First: " + exacttime + "ms");
        starttime = System.currentTimeMillis();
        depthfirst(T2, 0);
        endtime = System.currentTimeMillis();
        exacttime = endtime - starttime;
        System.out.println("Depth-First: " + exacttime + "ms");
        starttime = System.currentTimeMillis();
        hillclimbing(T3);
        endtime = System.currentTimeMillis();
        exacttime = endtime - starttime;
        System.out.println("Hill Climbing: " + exacttime + "ms");
        starttime = System.currentTimeMillis();
        bestfirstsearch(T4);
        endtime = System.currentTimeMillis();
        exacttime = endtime - starttime;
        System.out.println("Best-First Search: " + exacttime + "ms");
    }
}
```

//先广搜索。

```
public static boolean breadthfirst(Node T)
```

```

{
    Queue<Node> q = new LinkedList<>();
    q.offer(T);
    while(q.peek() != null)
    {
        Node t = q.poll();
        t.visited = true;
        creatsons(t);
        if(t.cost == 0)
            return true;
        for(int i = 0 ; i < t.sons.size() ; i++)
        {
            if(t.sons.get(i).visited == false)
            {
                q.offer(t.sons.get(i));
            }
        }
    }
    return false;
}

```

//先深搜索。

```

public static boolean depthfirst(Node T , int counter)
{
    counter++;
    if(counter > 12)
        return false;
    if(T.cost == 0)
        return true;
    creatsons(T);
    for(int i = 0 ; i < T.sons.size() ; i++)
    {
        if(depthfirst(T.sons.get(i),counter))
            return true;
    }
    return false;
}

```

```

public static boolean hillclimbing(Node T)
{
    if(T.cost == 0)
        return true;
}

```

```

    creatsons(T);
    //添加排序，使每一次都优先遍历 cost 最小的节点
    for(int i = 0 ; i < T.sons.size() ; i++)
    {
        for(int j = i ; j < T.sons.size() ; j++)
        {
            if(T.sons.get(i).cost > T.sons.get(j).cost)
            {
                Node temp = T.sons.get(i);
                T.sons.set(i, T.sons.get(j));
                T.sons.set(j, temp);
            }
        }
    }
    for(int i = 0 ; i < T.sons.size() ; i++)
    {
        if(hillclimbing(T.sons.get(i)))
            return true;
    }
    return false;
}

```

```

public static boolean bestfirstsearch(Node T)
{
    myQueue<Node> q = new myQueue<>();
    q.offer(T);
    while(!q.isEmpty())
    {
        Node t = q.poll();
        t.visited = true;
        creatsons(t);
        if(t.cost == 0)
            return true;
        for(int i = 0 ; i < t.sons.size() ; i++)
        {
            if(t.sons.get(i).visited == false)
            {
                q.offer(t.sons.get(i));
            }
        }
        //对队列进行排序，以保证每次取出的是 cost 最小的。
        sort(q);
    }
    return false;
}

```

```
}
```

//动态生成树的方法。

```
public static void creatsons(Node t)
{
    int x = 0, y = 0;
    for(int i = 1 ; i <= 3 ; i++ )
    {
        for(int j = 1 ; j <= 3 ; j++)
        {
            if(t.magiccube[i][j] == -1)
            {
                x = i;
                y = j;
            }
        }
    }
    if(x == 2 && y == 2)
    {
        Node t1 = new Node();
        Node t2 = new Node();
        Node t3 = new Node();
        Node t4 = new Node();
        move(t,t1,x,y,"up");
        move(t,t2,x,y,"down");
        move(t,t3,x,y,"right");
        move(t,t4,x,y,"left");
        if(isLegal(t, t1))
        {
            t1.father = t;
            t1.cost = calcostfun(t1.magiccube);
            t.sons.add(t1);
        }
        if(isLegal(t, t2))
        {
            t2.father = t;
            t2.cost = calcostfun(t2.magiccube);
            t.sons.add(t2);
        }
        if(isLegal(t, t3))
        {
            t3.father = t;
            t3.cost = calcostfun(t3.magiccube);
```

```

        t.sons.add(t3);
    }
    if(isLegal(t, t4))
    {
        t4.father = t;
        t4.cost = calcostfun(t4.magiccube);
        t.sons.add(t4);
    }
}
else if(x == 1 && y == 1)
{
    Node t2 = new Node();
    Node t3 = new Node();
    move(t,t2,x,y,"down");
    move(t,t3,x,y,"right");
    if(isLegal(t, t2))
    {
        t2.father = t;
        t2.cost = calcostfun(t2.magiccube);
        t.sons.add(t2);
    }
    if(isLegal(t, t3))
    {
        t3.father = t;
        t3.cost = calcostfun(t3.magiccube);
        t.sons.add(t3);
    }
}
else if(x == 1 && y == 2)
{
    Node t2 = new Node();
    Node t3 = new Node();
    Node t4 = new Node();
    move(t,t2,x,y,"down");
    move(t,t3,x,y,"right");
    move(t,t4,x,y,"left");
    if(isLegal(t, t2))
    {
        t2.father = t;
        t2.cost = calcostfun(t2.magiccube);
        t.sons.add(t2);
    }
    if(isLegal(t, t3))
    {

```

```

        t3.father = t;
        t3.cost = calcostfun(t3.magiccube);
        t.sons.add(t3);
    }
    if(isLegal(t, t4))
    {
        t4.father = t;
        t4.cost = calcostfun(t4.magiccube);
        t.sons.add(t4);
    }
}
else if(x == 1 && y == 3)
{
    Node t2 = new Node();
    Node t4 = new Node();
    move(t,t2,x,y,"down");
    move(t,t4,x,y,"left");
    if(isLegal(t, t2))
    {
        t2.father = t;
        t2.cost = calcostfun(t2.magiccube);
        t.sons.add(t2);
    }
    if(isLegal(t, t4))
    {
        t4.father = t;
        t4.cost = calcostfun(t4.magiccube);
        t.sons.add(t4);
    }
}
else if(x == 2 && y == 1)
{
    Node t1 = new Node();
    Node t2 = new Node();
    Node t3 = new Node();
    move(t,t1,x,y,"up");
    move(t,t2,x,y,"down");
    move(t,t3,x,y,"right");
    if(isLegal(t, t1))
    {
        t1.father = t;
        t1.cost = calcostfun(t1.magiccube);
        t.sons.add(t1);
    }
}

```

```

    if(isLegal(t, t2))
    {
        t2.father = t;
        t2.cost = calcostfun(t2.magiccube);
        t.sons.add(t2);
    }
    if(isLegal(t, t3))
    {
        t3.father = t;
        t3.cost = calcostfun(t3.magiccube);
        t.sons.add(t3);
    }
}
else if(x == 2 && y == 3)
{
    Node t1 = new Node();
    Node t2 = new Node();
    Node t4 = new Node();
    move(t,t1,x,y,"up");
    move(t,t2,x,y,"down");
    move(t,t4,x,y,"left");
    if(isLegal(t, t1))
    {
        t1.father = t;
        t1.cost = calcostfun(t1.magiccube);
        t.sons.add(t1);
    }
    if(isLegal(t, t2))
    {
        t2.father = t;
        t2.cost = calcostfun(t2.magiccube);
        t.sons.add(t2);
    }
    if(isLegal(t, t4))
    {
        t4.father = t;
        t4.cost = calcostfun(t4.magiccube);
        t.sons.add(t4);
    }
}
else if(x == 3 && y == 1)
{
    Node t1 = new Node();
    Node t3 = new Node();

```

```

    move(t,t1,x,y,"up");
    move(t,t3,x,y,"right");
    if(isLegal(t, t1))
    {
        t1.father = t;
        t1.cost = calcostfun(t1.magiccube);
        t.sons.add(t1);
    }
    if(isLegal(t, t3))
    {
        t3.father = t;
        t3.cost = calcostfun(t3.magiccube);
        t.sons.add(t3);
    }
}
else if(x == 3 && y == 2)
{
    Node t1 = new Node();
    Node t3 = new Node();
    Node t4 = new Node();
    move(t,t1,x,y,"up");
    move(t,t3,x,y,"right");
    move(t,t4,x,y,"left");
    if(isLegal(t, t1))
    {
        t1.father = t;
        t1.cost = calcostfun(t1.magiccube);
        t.sons.add(t1);
    }
    if(isLegal(t, t3))
    {
        t3.father = t;
        t3.cost = calcostfun(t3.magiccube);
        t.sons.add(t3);
    }
    if(isLegal(t, t4))
    {
        t4.father = t;
        t4.cost = calcostfun(t4.magiccube);
        t.sons.add(t4);
    }
}
else if(x == 3 && y == 3)
{

```



```

        Node t1 = new Node();
        Node t4 = new Node();
        move(t,t1,x,y,"up");
        move(t,t4,x,y,"left");
        if(isLegal(t, t1))
        {
            t1.father = t;
            t1.cost = calcostfun(t1.magiccube);
            t.sons.add(t1);
        }
        if(isLegal(t, t4))
        {
            t4.father = t;
            t4.cost = calcostfun(t4.magiccube);
            t.sons.add(t4);
        }
    }
}

```

//初始化根节点。

```

public static void initializecube(Node t)
{
    t.magiccube[1][1] = 2;
    t.magiccube[1][2] = 3;
    t.magiccube[1][3] = -1;
    t.magiccube[2][1] = 1;
    t.magiccube[2][2] = 8;
    t.magiccube[2][3] = 5;
    t.magiccube[3][1] = 7;
    t.magiccube[3][2] = 4;
    t.magiccube[3][3] = 6;
    t.cost = calcostfun(t.magiccube);
}

```

//移动魔方块的方法。

```

public static void move(Node t1 , Node t2 , int x , int y , String choice )
{
    for(int i = 1 ; i <= 3 ; i++ )
    {
        for(int j = 1 ; j <= 3 ; j++)
        {
            t2.magiccube[i][j] = t1.magiccube[i][j];
        }
    }
}

```

```

        if(choice.equals("up"))
        {
            int temp = t2.magiccube[x][y];
            t2.magiccube[x][y] = t2.magiccube[x-1][y];
            t2.magiccube[x-1][y] = temp;
        }
        if(choice.equals("down"))
        {
            int temp = t2.magiccube[x][y];
            t2.magiccube[x][y] = t2.magiccube[x+1][y];
            t2.magiccube[x+1][y] = temp;
        }
        if(choice.equals("right"))
        {
            int temp = t2.magiccube[x][y];
            t2.magiccube[x][y] = t2.magiccube[x][y+1];
            t2.magiccube[x][y+1] = temp;
        }
        if(choice.equals("left"))
        {
            int temp = t2.magiccube[x][y];
            t2.magiccube[x][y] = t2.magiccube[x][y-1];
            t2.magiccube[x][y-1] = temp;
        }
    }
}

```

//判断路径中是否已经经过 t1 时的状态（动态生成树的时候要用到）

```

public static boolean islegal(Node t , Node t1)
{
    int flag1 = 0 , flag2 = 0;
    Node visitor = t;
    while(visitor.father != null)
    {
        visitor = visitor.father;
        for(int i = 1 ; i <= 3 ; i++)
        {
            for(int j = 1 ; j <= 3 ; j++)
            {
                if(visitor.magiccube[i][j] != t1.magiccube[i][j])
                    flag1 = 1;
            }
        }
    }
    if(flag1 == 0)

```

```

        flag2 = 1;
        flag1 = 0;
    }
    if(flag2 == 0)
        return true;
    else
        return false;
}

```

//计算代价函数（在错误位置上的数字个数）

```

public static int calcostfun(int[][] magiccube)
{
    int counter = 0;
    if(magiccube[1][1] != 1)
        counter++;
    if(magiccube[1][2] != 2)
        counter++;
    if(magiccube[1][3] != 3)
        counter++;
    if(magiccube[2][1] != 8)
        counter++;
    if(magiccube[2][3] != 4)
        counter++;
    if(magiccube[3][1] != 7)
        counter++;
    if(magiccube[3][2] != 6)
        counter++;
    if(magiccube[3][3] != 5)
        counter++;
    return counter;
}

```

//对队列进行排序（为实现最佳优先方法）

```

public static void sort(myQueue<Node> q)
{
    int i , j;
    for(i = 0 ; i < q.queue.size() ; i++)
    {
        for(j = i ; j < q.queue.size() ; j++)
        {
            if(q.queue.get(i).cost > q.queue.get(j).cost)
            {
                Node temp = q.queue.get(i);

```

```

        q.queue.set(i, q.queue.get(j));
        q.queue.set(j, temp);
    }
}
}
}
}
}

```

```

class myQueue<E>
{
    public List<E> queue = new ArrayList<>();

    public boolean isempty()
    {
        if(queue.size() == 0)
            return true;
        else
            return false;
    }

    public void offer(E obj)
    {
        queue.add(obj);
    }

    public E poll()
    {
        E obj = queue.get(0);
        queue.remove(obj);
        return obj;
    }
}

```

```

class Node
{
    boolean visited = false;
    int cost;
    int[][] magiccube = new int[4][4];
    Node father = null;
    List<Node> sons = new ArrayList<>();
}

```

}

## Exercise 2

### Exercise 2.

1. 数据结构含义: 定义数组  $L[8]$ , 则  $L[i] = j$  表示棋盘第  $i$  列的皇后在第  $j$  行

伪代码:  $\text{dfs}(\text{row}, L)$  // row 表示从第 row 列开始深搜

```
if (row == 9) // 表示8个皇后已摆完
    then 得到结果;
else
    for (i = 1 To 8)
    {
        L[row] = i;
        if (L[row] 位置皇后摆放与前面不冲突) // O(n)
            then dfs(row+1, queen);
    }
}
```

### 2. 算法复杂度分析:

空间复杂度: 用数组  $L$ , 故复杂度为  $O(n)$

时间复杂度: 每次 dfs 内循环为  $O(n)$ , 进行  $n$  次 dfs, 故  $T(n) = O(n^2)$

### 改进方法:

定义代价函数  $h(\text{row})$  为产生冲突的皇后的个数

使用爬山策略进行分支界限搜索 (前面的深搜其实利用了此思想)

### Exercise 3

#### Exercise 3

1. 要用到的数据结构:  $visit1$  数组: 用于判断某点是否作为起始点过  
 $visit2$  数组: 在深度优先搜索中是否遍历过该点  
 $G'$ : 由  $G$  生成的子图

伪代码:

```
findlongest()
{
    for (G' 上任意一点)
    {
        if (visit1[p] != true) // p 还未访问过
            num[p] = dfs(p); // 记录点的数目
    }
    return max{num}; // 返回点数目最大值
}
```

dfs(p)

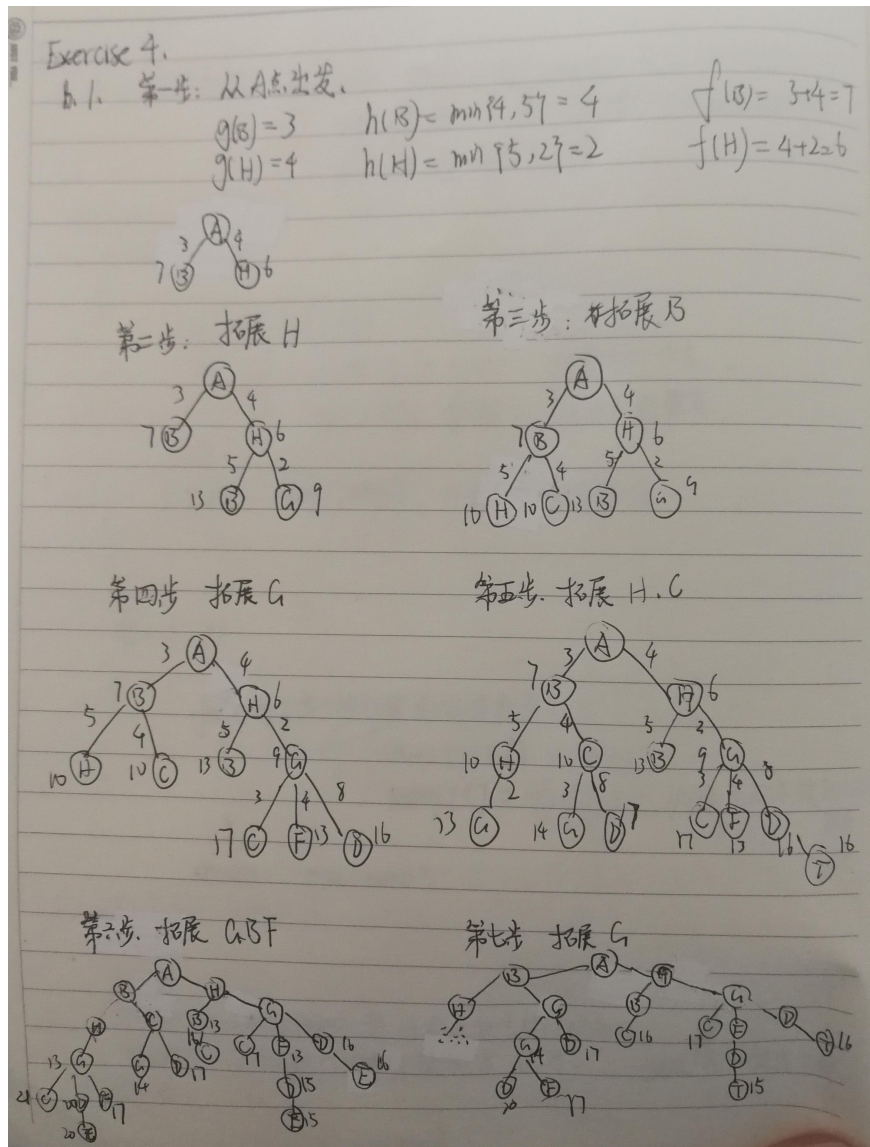
```
{
    for (与 p 不相邻的所有点 u)
    {
        if (visit2[p][u] != true)
            num[u] = dfs(u); // 记录 u 点未搜索过的数目
    }
    return max{num};
}
```

2. 可定义代价函数  $h(num)$  为: 当前已包含的点的数目。

使用爬山策略进行分支界限搜索 (上界)。可剪枝: 减少搜索次数, 提升效率。



## Exercise 4



第八步 扩展 E

故最优解为 A-H-G-F-D-T

2. 用到的数据结构 heap: 以  $f$  排序的小根堆, way: 存储路径

findminway()

{ for (与 A 相邻的点 p)

{  $g(p) = d(A, p);$

way(p).add(A, p);

$h(p) = \min \{d(p, \text{非 way 上的点})\};$

$f(p) = g(p) + h(p);$

heap.add(p, (f(p), way(p))); // 向堆中添加包含  $f(p)$  与 way(p) 的数据结构;

{ while (heap.top != E) // 堆顶非终点 E.

{ k = heap.top;

for (与 k 相邻的点 l)

{  $g(l) = \text{way}(k).length + d(k, l);$

way(l).add(way(k) + k, l);

$h(l) = \min \{d(l, \text{非 way}(l) \text{ 上的点})\};$

$f(l) = g(l) + h(l);$

heap.add(l, (f(l), way(l)));

}

}

return heap.top.way; // 得到最短路径

}