UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion 7

Date: Wednesday, March 8, 2017

Problem 1. (Final Sp06) Consider a particle moving according to the following Markov Chain:

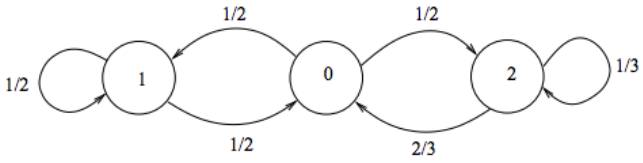

Figure 1: Markov Chain for Problem 1

(a.) Classify the states in the Markov Chain. Is it periodic?

(b.) In the long run, what fraction of time does it spend in state 1?

(c.) Suppose X_0 is selected according to the steady state distribution. Conditioned on $X_2 = 2$, what is the probability that $X_0 = 0$?

(d.) Suppose $X_0 = 0$, find the expected amount of time until the particle has visited all the states.

(e.) Suppose there are two particles moving according to the Markov Chain. One particle starts in state 1 and the other starts in state 2. What is the expected amount of time before at least one of the particles is at state 0?

Problem 2. (Final Fa14) An online dating website tries to match couples. Let X_n be the number of members of this site at time slot n. We want to analyze the discrete-time process $\{X_n, n \geq 0\}$. At each time slot, exactly one of the following events happens: (i) Two persons are happily matched and leave the website forever with probability p, (ii) A single frustrated person leaves the system individually with probability q , and (iii) a new member joins the system with probability $r = 1-p-q$. If there is only one member in the system, that member leaves with probability $p + q = 1 - r$. Suppose that $r - q - 2p > 0$: is the Markov Chain $\{X_n, n \geq 0\}$ positive recurrent, null recurrent, or transient? Prove your answer.