UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion 4
Spring 2017

Date: Wednesday, February 8, 2017

Problem 1. (a.) We proceed by convolution:

$$\begin{aligned}
f_Y(y) &= \int_{-\infty}^{\infty} f_{X_1}(x_1) f_{X_2}(y - x_1) dx_1 \\
&= \int_0^y f_{X_1}(x_1) f_{X_2}(y - x_1) dx_1 \\
&= \int_0^y \lambda e^{-\lambda x_1} \lambda e^{-\lambda(y - x_1)} dx_1 \\
&= \lambda^2 y e^{-\lambda y}
\end{aligned}$$

(b.) Let $Y = \min(X_1, X_2)$. We proceed by finding the CCDF of Y:

$$\begin{aligned}
P(Y \geq y) &= P(\min(X_1, X_2) \geq Y) \\
&= P(X_1 \geq Y, X_2 \geq Y) \\
&= P(X_1 \geq Y) P(X_2 \geq Y) \\
&= e^{-2\lambda y}
\end{aligned}$$

and thus, Y is exponential with rate 2λ.

(c.) We find the CDF $P(Y < y)$:

$$\begin{aligned}
P(Y < y) &= P(X^2 < y) \\
&= P(X < \sqrt{y}) \\
&= \sqrt{y}
\end{aligned}$$

Taking the derivative gives $f_Y(y) = \frac{1}{2\sqrt{y}}$. For a more general approach, please see p. 258 in the book of Walrand.

Problem 2. (a) We need the total area to be 1 for this to be a valid PDF, so $2A + 3A + A = 1$, and $A = \frac{1}{6}$.

(b) We are calculating $cov(X, Y) = E[XY] - E[X]E[Y]$. We consider the events E_1, E_2, E_3 as shown in the diagram below, and note that the joint pdf is uniform conditioned on the rectangle it is in. This implies that the marginals are independent. Intuitively, one may see this by conditioning on one of the rectangles, and then conditioning on a given x, the value of y is equally likely to be any of the points on the line. The same holds true if we pick a y. Thus, we have:

$$\begin{aligned}
E[XY] &= E[XY|E_1]P(E_1) + E[XY|E_2]P(E_2) + E[XY|E_3]P(E_3) \\
&= -\frac{3}{2} \cdot \frac{1}{2} \cdot \frac{2}{6} - \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{3}{6} + \frac{1}{2} \cdot \frac{5}{2} \cdot \frac{1}{6} \\
&= -\frac{5}{12}
\end{aligned}$$

In the same manner, we see that $E[X] = -\frac{2}{3}$, $E[Y] = \frac{4}{3}$, so $cov(X,Y) = \frac{17}{36}$.

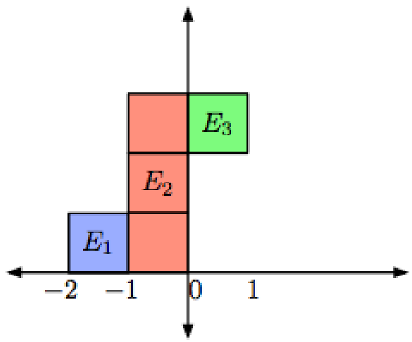

Figure 1: Joint pdf of X and Y with events E_1, E_2, E_3.