UC Berkeley
Department of Electrical Engineering and Computer Sciences
EE126: PROBABILITY AND RANDOM PROCESSES
Discussion 12
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Problem 1. (EE126 Fall 2007, PS9) Bob has gone hiking, and is lost in the forest. In order to try and find a road, he decides on the following distance/coin-flip strategy. At time instants $t = 1, 2, 3, \ldots$, he chooses a distance uniformly at random between t and $t + 1$. Independently of the chosen random distance, he then flips a fair coin; if it comes up heads, he moves the chosen random distance to the right (positive on the real line), and otherwise for a tails toss, he moves the chosen random distance to the left (negative on the real line). Both the random distance and the coin flip are independent random variables for different time instants. Assume that he starts at the origin at time instant $t = 0$.

(a) Let Y_s be Bobs position after repeating his distance/coin-flip strategy for a fixed number of s time instants. Compute its expected value and variance as a function of s.

Now suppose that Bob repeats his distance/coin-flip strategy for a random number S of time rounds, after which he stops. Assume that $S \sim \text{Geo}(p)$ has a geometric distribution with parameter p, and let $X \in \mathbb{R}$ be his final position. For any question below, you may feel free to express your answer (if appropriate) in terms of the moments $\mu_i = E[S^i]$, $i = 1, 2, 3, \ldots$.

(b) Suppose that you observe that $S = s$. What is the minimum mean squared error (MMSE) estimator of X given this information?

(c) What is the expected value and variance of his position X?

(d) Now suppose that you observe that Bob finishes at position $X = x$. Given this information, what is the linear least squares estimator (LLSE) of the number of time rounds S that he repeated his distance/coin-flip strategy?

Answer 1. (a) $E[Y_s] = \sum_{t=1}^{s} E[X_t] = 0$.
$\text{Var}(X_t) = E[X_t^2] = \int_{-\infty}^{\infty} x_t^2 f_{X_t}(x_t) dx_t = 2 \int_t^{t+1} x_t^2 \frac{1}{2} dx_t = \frac{(t+1)^3}{3} - \frac{t^3}{3}$.
$\text{Var}(Y_s) = \sum_{t=1}^{s} \text{Var}(X_t) = \frac{(s+1)^3 - 1}{3}$.

(b) $\text{MMSE}[X|S = s] = E[X|S = s] = E[Y_s] = 0$.

(c) $E[X] = E[E[X|S]] = E[0] = 0$.
$\text{Var}(X) = E[\text{Var}(X|S)] + \text{Var}(E[X|S]) = E[\text{Var}(Y_S)] = E[\frac{(S+1)^3-1}{3}] = \mu_1 + \mu_2 + \frac{\mu_3}{3}$.

(d) $\text{Cov}(X, S) = E[XS] - E[X]E[S] = E[XS] = E[E[XS|S]] = E[SE[X|S]] = E[S \cdot 0] = 0$. Thus, $\text{LLSE}[X|S] = \mu_1$.