UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECTRICAL ENGINEERING 126: PROBABILITY AND RANDOM PROCESSES

Discussion 2
Spring 2017

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1. **Poisson Properties**

   (a) Suppose X and Y are independent Poisson random variables with mean λ and μ respectively. Prove that $X+Y$ has the Poisson distribution with mean $\lambda+\mu$. (This is known as **Poisson merging**.)

   (b) Suppose that X has the Poisson distribution with mean λ. View X as the number of arrivals of a process. Independently, for each arrival, mark the arrival as 0 with probability p and 1 with probability $1-p$. Let Y be the number of 0 arrivals and Z be the number of 1 arrivals. Prove that Y and Z are independent Poisson random variables with means λp and $\lambda(1-p)$ respectively. (This is known as **Poisson splitting**.)

2. **Sampling Without Replacement**

   Suppose you have N items, G of which are good and B of which are bad ($B+G=N$). You start to draw items without replacement, and suppose that the first good item appears on draw X. Compute the mean and variance of X.

3. **Clustering Coefficient**

   This problem will explore an important probabilistic concept of clustering that is widely used in machine learning applications today. Consider n students. For each pair of students, say student i and student j, they are friends with probability p, independently of other pairs. We assume that friendship is mutual. We can see that the friendship among the n students can be represented by an undirected graph G. Let $N(i)$ be the number of friends of student i and $T(i)$ be the number of triangles attached to student i. We define the **clustering coefficient** $C(i)$ for student i as follows:

$$C(i) = \frac{T(i)}{\binom{N(i)}{2}}.$$

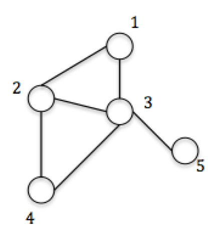

Figure 1: Friendship and clustering coefficient.

Clustering coefficient is not defined for the students who have no friends. An example is shown in Figure 1. Student 3 has 4 friends (1, 2, 4, 5) and there are two triangles attached to student 3, i.e., triangle 1-2-3 and triangle 2-3-4. Therefore $C(3) = 2/\binom{4}{2} = 1/3$.

Find $E[C(i) \mid N(i) \geq 2]$.

4. **Packet Routing**

Consider a system with n inputs and n outputs. At each input, a packet appears independently with probability p. If a packet appears, it is destined for one of the n outputs uniformly randomly, independently of the other packets.

(a) Let X denote the number of packets destined for the first input. What is the distribution of X?

(b) What is the probability of a collision, that is, more than one packet heading to the same output?