Cheat sheet

1. Discrete Random Variables

1) Geometric with parameter $p \in [0,1]$:
   $P(X = n) = (1-p)^{n-1}p,\ n \geq 1$
   $E[X] = 1/p$, $\text{var}(X) = (1-p)p^{-2}$
   $M_X(s) = \frac{pe^s}{1-(1-p)e^s}$

2) Binomial with parameters N and p:
   $P(X = n) = \binom{N}{n} p^n (1-p)^{N-n},\ n = 0, \ldots, N,$ where $\binom{N}{n} = \frac{N!}{(N-n)!n!}$
   $E[X] = Np$, $\text{var}(X) = Np(1-p)$
   $M_X(s) = (1-p+pe^s)^n$

3) Poission with parameter λ:
   $P(X = n) = \frac{\lambda^n}{n!}e^{-\lambda},\ n \geq 0$
   $E[X] = \lambda$, $\text{var}(X) = \lambda$
   $M_X(s) = e^{\lambda(e^s-1)}$

2. Continuous Random Variables

1) Uniformly distributed in $[a,b]$, for some $a < b$:
   $f_X(x) = \frac{1}{b-a}\mathbf{1}\{a \leq x \leq b\}$
   $E[X] = \frac{a+b}{2}$, $\text{var}(X) = \frac{(b-a)^2}{12}$
   $M_X(s) = \frac{1}{b-a}\frac{e^{sb}-e^{sa}}{s}$

2) Exponentially distributed with rate $\lambda > 0$:
   $f_X(x) = \lambda e^{-\lambda x}\mathbf{1}\{x \geq 0\}$
   $E[X] = \lambda^{-1}$, $\text{var}(X) = \lambda^{-2}$
   $M_X(s) = \frac{\lambda}{\lambda-s}$

3) Gaussian, or normal, with mean μ and variance σ^2:
   $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\{-\frac{(x-\mu)^2}{2\sigma^2}\}$
   $E[X] = \mu$, $\text{var}(X) = \sigma^2$
   $M_X(s) = \exp(\frac{\sigma^2 s^2}{2} + \mu s)$

4) Erlang distribution, i.e., sum of k i.i.d. exponential random variables with rate λ:

$f_X(x) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!} \mathbf{1}\{x \geq 0\}$
$E[X] = \frac{k}{\lambda}$, $\text{var}(X) = \frac{k}{\lambda^2}$
$M_X(s) = \left(\frac{\lambda}{\lambda - s}\right)^k$

3. Estimation

1) LLSE: Let X and Y be random variables. Then,
$L[X|Y] = E(X) + \frac{\text{cov}(X,Y)}{\text{var}(Y)}(Y - E[Y])$.

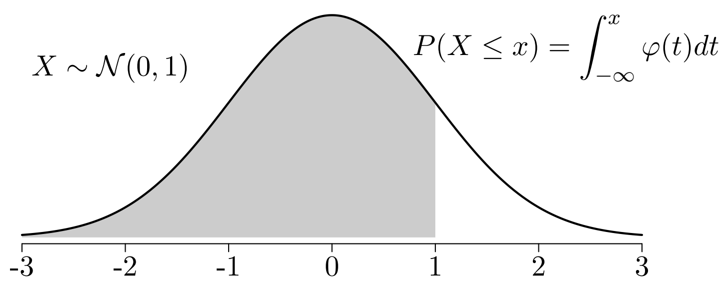
2) Scalar Kalman Filter:

$$\begin{aligned}
\hat{x}_{n|n} &= \hat{x}_{n|n-1} + k_n \tilde{y}_n \\
\tilde{y}_n &= y_n - a\hat{x}_{n-1|n-1} \\
k_n &= \frac{c\sigma^2_{n|n-1}}{c^2\sigma^2_{n|n-1} + \sigma^2_w} \\
\sigma^2_{n|n-1} &= a^2\sigma^2_{n-1|n-1} + \sigma^2_v \\
\sigma^2_{n|n} &= \sigma^2_{n|n-1}(1 - k_n c)
\end{aligned}$$

3) Vector Kalman Filter:

$$\begin{aligned}
\hat{X}_n &= \hat{X}_{n|n-1} + K_n(\tilde{Y}_n) \\
\tilde{Y}_n &= Y_n - C\hat{X}_{n|n-1} \\
K_n &= \Sigma_{n|n-1}C^T[C\Sigma_{n|n-1}C^T + \Sigma_W]^{-1} \\
\Sigma_{n|n-1} &= A\Sigma_{n-1|n-1}A^T + \Sigma_V \\
\Sigma_{n|n} &= \Sigma_{n|n-1}(I - K_n C)
\end{aligned}$$

3. Normal Distribution Table



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990