UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: PROBABILITY AND RANDOM PROCESSES

Problem Set 11
Spring 2017

Issued: Thursday, April 20, 2017 **Due:** 8am Thursday, April 27, 2017

Problem 1. Let $(X, Y, Z)^T \sim N(\mu, \Sigma)$, and

$$\mu = [0, 0, 0]^T,$$

and

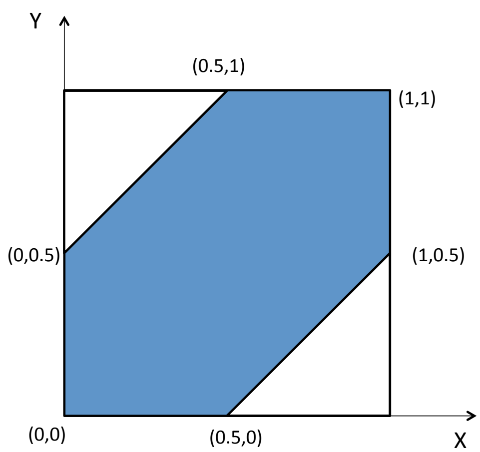
$$\Sigma = \begin{bmatrix} 5 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 2 \end{bmatrix}.$$

Find $E[X|Y, Z]$.

Problem 2. Suppose that $X = Y + 2Z + Y^2$, where Y and Z are independent random variables distributed uniformly between -1 and 1. Find the quadratic least squares estimate $Q[X|Y]$.

Problem 3. (a.) Let N be a geometric random variable with parameter $1 - p$, and X_i be i.i.d. exponential random variables with parameter λ. Let $T = X_1 + \cdots + X_N$. Compute the LLSE and MMSE of N given T.

(b.) Consider the following figure:



Suppose that (X, Y) are uniformly distributed on the shaded region. Find the linear least squares estimate of X given Y ($L[X|Y]$).

Problem 4. Consider the state space equations:

$$x_n = \frac{1}{\sqrt{2}} x_{n-1} + v_n$$
$$y_n = 2x_n + w_n$$

Where $v_n, w_n \sim \mathcal{N}(0,1)$ are independent sources of noise. Additionally, one has the initial conditions: $n = 0, \hat{x}_{0|0} = 0, E[x_0^2] = 2$.

(a.) Find $\sigma^2_{1|0}, k_1, \sigma^2_{1|1}$ as well as $\sigma^2_{2|1}, k_2, \sigma^2_{2|2}$ and $\sigma^2_{3|2}, k_3, \sigma^2_{3|3}$.

(b.) Now suppose that as $n \to \infty$, the error in the estimator converges to a constant. Find $\lim_{n\to\infty} \sigma^2_{n|n}$ as well as k_n and $\sigma^2_{n|n-1}$. Give an interpretation.

Problem 5. Consider an observation $Y = \sum_{i=1}^{n} W_i X_i + Z$ where $Z \sim \mathcal{N}(0,1)$ is independent measurement noise and all the X_i are zero-mean. In addition, W_i are i.i.d. and independent of all other parameters with pdf $f_W(w) = \frac{\lambda}{2} e^{-\lambda|w|}$.

(a.) Find $\text{MAP}[W_1, W_2, \ldots, W_n | X_1, X_2, \ldots, X_n, Y]$

(b.) Suppose that you are trying to reconstruct Y based on $X_1, X_2, \ldots, X_n$, but it costs \$100 per X_i that you use in your reconstruction. Given a budget of \$100, what is $\hat{Y}$, your reconstruction of Y (here you measure how good $\hat{Y}$ is using squared error)? Assume also that you are subject to a constant bound on the weights w_i: $\sum_{i=1}^{n} |w_i| \le C$.

(c.) What is $\hat{Y}$ given a budget of \$200? You are subject to the same upper bound on the weights.

Problem 6. Assume that the Markov chain $\{X_n, n \ge 0\}$ with states 0 and 1, and initial distribution $\pi_0(0) = \pi_0(1) = 0.5$ and $P(x, x') = 0.3$ for $x \ne x'$ and $P(x,x) = 0.7$ $(x, x' \in \{0, 1\})$. Assume also that X_n is observed through a BSC with error probability 0.1. The observations are denoted by Y_n. Suppose the observations are $(Y_0, \ldots, Y_4) = (0, 0, 1, 1, 1)$. Use the Viterbi algorithm to find the most likely sequence of the states $(X_0, \ldots, X_4)$.