UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 4
Spring 2017

Issued: Thursday, February 16, 2017 **Due:** 8:00am Thursday, February 23, 2017

Problem 1. Midterm 01.

Problem 2. Consider a population of N individuals. At the end of each year, each individual, independently of others, leaves behind ξ offspring. Assume $E[\xi] = \mu$ and $\text{Var}(\xi) = \sigma^2$. Let X_n denote the size of the population at the end of the n^{th} year. Compute $E[X_n]$ and $\text{Var}(X_n)$.

Hint : You may need to consider $\mu = 1$ and $\mu \neq 1$ cases separately while computing the variance.

Problem 3. Assume that you have a random variable U which is uniformly distributed on $(0, 1)$. Using U, you want to simulate an exponential random variable T with rate λ, i.e. $f_T(x) = \lambda e^{-\lambda x}\mathbf{1}\{x \geq 0\}$. Find a strictly increasing function $h : (0, 1) \to (0, \infty)$ such that $T \sim h(U)$.

Problem 4. A bin contains balls numbered $1, 2, \ldots, n$. You select m balls at random without replacement and order them: $X_{(1)} < X_{(2)} < \cdots < X_{(m)}$. For this question, you may assume $a, b \in \mathbb{N}$.

(a) Find $P(X_{(1)} = a)$ for $1 \leq a \leq n - m + 1$.

(b) Find $P(X_{(2)} = b)$ for $2 \leq b \leq n - m + 2$.

(c) Assume $m = 10$, $n = 100$.

  (i) Find $P(X_{(1)} = 3, X_{(2)} = 7, X_{(3)} = 15 | X_{(5)} = 20)$.
  (ii) Find $P(X_{(1)} = 8, X_{(2)} = 11, X_{(3)} = 15 | X_{(5)} = 20)$.

Problem 5. Consider a random variable Z with transform:

$$M_Z(s) = \frac{a - 3s}{s^2 - 6s + 8}$$

(a) Find the numerical value for the parameter a.

(b) Find $P(Z \geq 0.5)$.

(c) Find $E[Z]$ by using the probability distribution of Z.

(d) Find $E[Z]$ by using the transform of Z and without explicitly using the probability distribution of Z.

(e) Find $\mathrm{Var}(Z)$ by using the probability distribution of Z.

(f) Find $\mathrm{Var}(Z)$ by using the transform of Z and without explicity using the probability distribution of Z.

Problem 6. Suppose $E[X] = 0, \mathrm{Var}(X) = \sigma^2 < \infty$ and $\alpha > 0$. Prove that

$$P(X \geq \alpha) \leq \frac{\sigma^2}{\alpha^2 + \sigma^2}.$$