UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion 9
Spring 2017

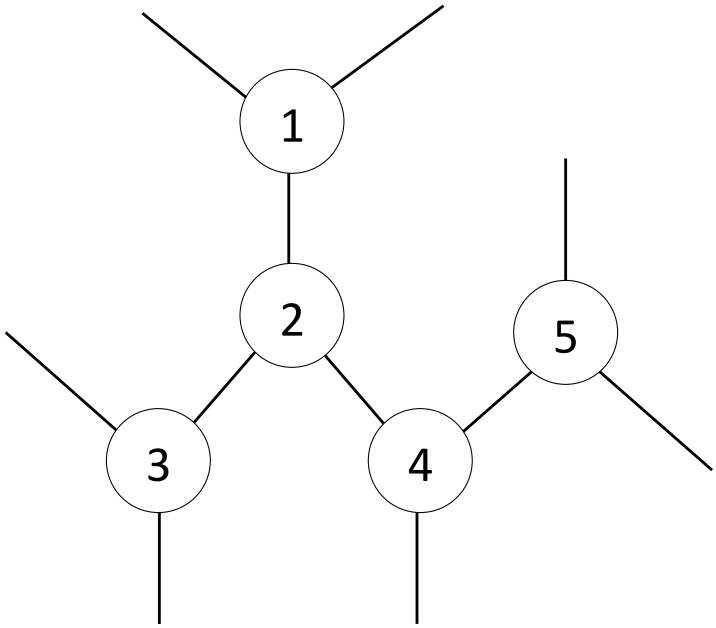
Date: Wednesday, April 5, 2017

Problem 1. (Infection source detection) Consider a graph where each node represent each person and edges represent connectivity between them. At time 1, the source of the rumor u^* appears. At time 2, the source chooses one of its neighbors, and infects the chosen neighbor. Similarly, in the following time slots, one of the uninfected nodes that are neighboring the nodes that are already infected in the previous time slots is chosen uniformly at random, and get infected. Right after time n, you observe the infected network with n infected nodes, and you want to detect the source of the infection.

(a) Consider an infinitely long linear-network: node i is connected with node $(i-1)$ and node $(i+1)$ for all $i \in \mathbb{Z}$.

At time 11, 11 nodes, $\{-5, -4, \ldots, 4, 5\}$, are infected. Find the MLE of the source of the infection.

(b) Consider the following infection graph: at time 5, the following 5 nodes are infected. Find the MLE of the source of the infection.



(c) Consider the same graph. Given that node 4 has twice higher probability of being the source than the others, find the MAP estimate of the source of the infection.

(d) Consider an infinitely large 2D grid: node (i, j) is connected with node $(i + u, j + v)$ for all $(u, v) \in \{(\pm 1, 0), (0, \pm 1)\}$.

At time 4, 4 nodes $\{(0, 0), (1, 0), (0, 1), (-1, 0)\}$ are infected. Find the MLE of the source of the infection. Which node is the second most likely source?

Problem 2. (Fall 2008, MT2) Given $X \in \{0, 1\}$, the random variable Y is exponentially distributed with rate $3X + 1$.

(a) Assume $P(X = 1) = p$ and $P(X = 0) = 1 - p$. Find the MAP estimate of X given Y.

(b) Find the MLE of X given Y.

(c) Solve the hypothesis testing problem of X given Y with a probability of false alarm at most 0.1. That is, find $\hat{X}$ as a function of Y that maximizes $P[\hat{X} = 1|X = 1]$ subject to $P[\hat{X} = 1|X = 0] \leq 0.1$.

(d) For what value of p does one have the same solution for (a) and (c)?