UC Berkeley
Department of Electrical Engineering and Computer Sciences

Electrical Engineering 126: Probability and Random Processes

**Discussion 9**
Spring 2017

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**1. Illegal U-Turns**

Each morning, as you pull out of your driveway, you would like to make a U-turn rather than drive around the block. Unfortunately, U-turns are illegal and police cars drive by according to a Poisson process with rate $\lambda$. You decide to make a U-turn once you see that the road has been clear of police cars for $\tau$ units of time. Let $N$ be the number of police cars you see before you make a U-turn.

(a) Find $E[N]$.

(b) Find the conditional expectation of the time elapsed between police cars $n-1$ and $n$, given that $N \geq n$.

(c) Find the expected time that you wait until you make a U-turn.

**2. Two-Server System**

A company has two servers (the second server is a backup in case the first server fails, so only one server is ever used at a time). When a server is running, the time until it breaks down is exponentially distributed with rate $\mu$. When a server is broken, it is taken to the repair shop. The repair shop can only fix one server at a time, and its repair time is exponentially distributed with rate $\lambda$. Find the long-run probability that no servers are operational.

**3. $M/M/2$ Queue**

A queue has Poisson arrivals with rate $\lambda$. It has two servers that work in parallel. Where there are at least two customers in the queue, two are being served. When there is only one customer, only one server is active. The service times are i.i.d. exponential random variables with rate $\mu$.

(a) Argue that the queue length is a Markov chain.

(b) Draw the state transition diagram.

(c) Find the minimum value of $\mu$ so that the queue is positive recurrent and solve the balance equations.