UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Problem Set 8
Spring 2017

Issued: Thursday, March 23, 2017 **Due:** 8:00AM, Thursday, April 6, 2017

Problem 1. Midterm 2.

Problem 2. The random variable X is exponentially distributed with mean 1. Given X, the random variable Y is exponentially distributed with rate X (with mean $1/X$).

(a) Find $MLE[X|Y]$;

(b) Find $MAP[X|Y]$.

Problem 3. The stochastic block model (SBM) is a random graph $G(n, p, q)$ consisting of two communities of size $\frac{n}{2}$ each such that the probability an edge exists between two nodes of the same community is p and the probability an edge exists between two nodes in different communities is q, where $p > q$. The goal of the problem is to exactly determine the two communities given only the graph. Show that the MAP-decision rule is equivalent to finding the min-bisection of the graph (ie the split of G into two groups of size $\frac{n}{2}$ that has the minimum edge weight across the partition).

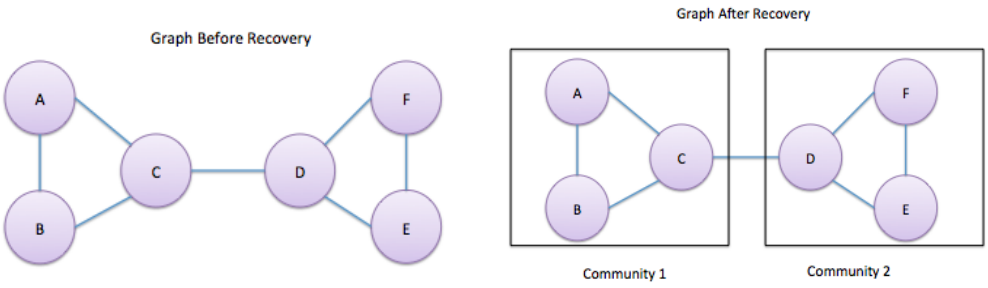

Figure 1: An example of a graph before and after recovery

Problem 4. You are testing a digital link that corresponds to a BSC with some error probability $\epsilon \in [0, 0.5)$.

(a) Assume you observe the input and the output of the link. How do you find the MLE of ϵ?

(b) You are told that the inputs are i.i.d. bits that are equal to 1 with probability 0.6 and to 0 with probability 0.4. You observe n outputs. How do you calculate the MLE of ϵ.

(c) The situation is as in the previous case, but you are told that ϵ has pdf $4 - 8x$ on $[0, 0.5)$. How do you calculate the MAP of ϵ given n outputs.

Problem 5. You are trying to detect whether voltage V_1 or voltage V_2 was sent over a channel with independent Gaussian noise $Z \sim N(V_3, \sigma^2)$. Assume that both voltages are equally likely to be sent.

(a) Derive the MAP detector for this channel.

(b) Using the Gaussian Q-function, determine the average error probability for the MAP detector.

(c) Suppose that the average transmit energy is $\frac{V_1^2+V_2^2}{2}$ and that the average transmit energy is constrained such that it cannot be more than E. What voltage levels V_1, V_2 should you choose to meet this energy constraint but still minimize the average error probability?