Unit 3 Project

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Question 1

Find an exponential function with points (0, 4) and (5, 972)

Solution

1) Rewrite the equation into two exponent equation

$4 = a \cdot b^0$ and $972 = 4 \cdot b^5$

2) Solve for a, since any number raised to the zero power is equal to 1, we left with 4 = a·1, which a will be 4

$4 = a \cdot b^0$

$4 = a \cdot 1$

$4 = a$

3) Plug 4 into b for the next equation and solve for b

$972 = 4 \cdot b^5$

$972= 4 \cdot b^5$

$243= b^5$

$\sqrt[5]{(243)} = b$

$b = 3$

Solution cont.

4) Plug a and b into the equation and check for correction

$4 = 4 \cdot 3^0$

$4 = 4 \cdot 1$

$4 = 4$

$972 = 4 \cdot 3^5$

$972 = 4 \cdot 243$

$243 = 972$

5) If both equations are correct, then you have solved the problem

Answer:

$f(x) = 4 \cdot 3^x$

Significant

This question demonstrate the basic of exponent and will prepare the student for the sections ahead

Question 2

The population of Las Vegas is 260,050 in 2000 and it is increasing at a rate of 5% each year, what will the population of Las Vegas be in 2020?

Equation

This is the equation that will be used to solve the problem.

$$A = P(1 + r)^t$$

A = final product, in this case is the population at 2020

P = principal amount, in this case is the population at 2000

r = rate of change, the rate that is increasing or decreasing, in this case is 5%

t = time, in this case is the year between 2000 and 2020

Growth factor: $b = (1 + r)$
The positive rate of change

Decay factor $b = (1 - r)$
The negative rate of change

In this case, we are using the Growth factor

Solution cont.

1) Rewrite the problem into an exponential equation. (1 + .05) is the growth factor, and it means that the population will increase 5% each year. The power is 20 because 2020 is 20 years after 2000.

$$A = 260{,}050(1 + .05)^{20}$$

2) Solve the equation

$$A = 260{,}050(1 + .05)^{20}$$

$$A = 260{,}050(1.05)^{20}$$

$$A = 260{,}050(2.653297705)$$

$$A = 689{,}990$$

Answer:

The population for Las Vegas in 2020 will be 689,990 people

Significant

This question introduce growth factor and decay factor, which gives a better understanding of rate when doing logistic function.

Question 3

Rewrite $\text{Log}_5 3125 = 5$ into exponential form

Logarithm vs. Exponential

$$\log_b Y = X$$

$$b^x = Y$$

b = the base of the exponent
x = the power of exponent
Y = the result of the exponent

Since logarithm is the inverse of exponent, they both share the same element of an exponent

Solution cont.

Because logarithm and exponent share the same element, we can rewrite it into exponent by plugging in the element

$\text{Log}_5 3125 = 5$

$5^5 = 3125$

Significant

This question demonstrate the understanding of the difference between exponent and logarithm

Question 4

Solve the following equation

$$\log_3 X^3 = \log_3 27$$

Solution

Since both part of the equal sign had the same base we can ignore the the log and focus on the X value

$$\log_3 X^3 = \log_3 27$$

same

Now we left with $X^3 = 27$, which we can solve the equation by getting the cubic root of 27.

$$X^3 = 27$$

$$X = \sqrt[3]{(27)}$$

$$X = 3$$

Answer:

X = 3

Significant

This question demonstrate the understanding of two logarithm with the same base

Question 5

Graph the following function

$$f(x) = 3^{2x}$$

Solution

To graph an exponential function, you must identify the following things:

- Domain and range
- Asymptotes
- Intercepts
- End behavior

Solution cont.

The domain is all real number because an exponent can have negative power.

Domain: $(-\infty, \infty)$

The range is 0 to positive infinity because you can’t get 0 and negative number when taking a number to a power

Range: $(0, \infty)$

Solution cont.

Asymptote is the line that the graph can’t touch.

Since this exponential graph don’t have a vertical shift, it will not touch or be less than 0, so it had a horizontal asymptote at

y = 0.

Horizontal Asymptote:

Y = 0

The x value is all real number which mean that it had no vertical asymptote

Vertical Asymptote:

None

Solution cont.

This exponential graph doesn't have a x-intercept because the x-axis is the asymptote.

To find the y-intercept, you will have to take the number to the 0 power, and any number take to the 0 power will be 1, which mean it will have a y-intercept at (0,1)

X-intercept:

None

Y-intercept:

(0,1)

Solution cont.

The end behavior of a graph will help you determine the end behavior of the graph.

This means that as x heads to negative infinity, y will become closer to 0

As $x \rightarrow -\infty$,

$y \rightarrow 0$

This means that as x heads to negative infinity, y will heads to positive infinity

As $x \rightarrow \infty$,

$y \rightarrow \infty$

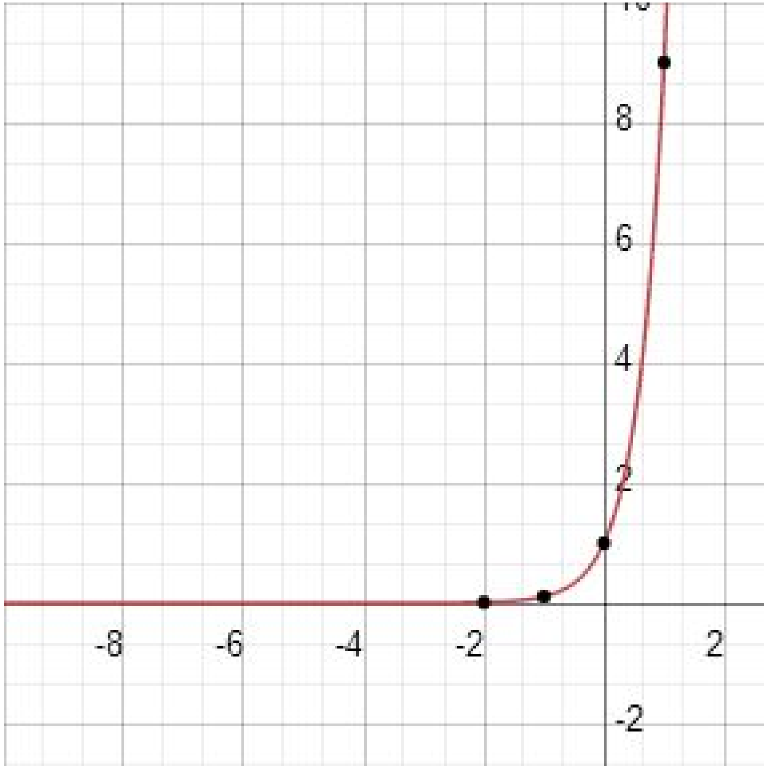
Solution cont.

- Domain: $(-\infty,\infty)$
- Range: $(0, \infty)$
- Horizontal Asymptote:

  $Y = 0$
- Y-intercept: (0,1)
- As $x \to -\infty$,

  $y \to 0$
- As $x \to \infty$,

  $y \to \infty$



Significant

This question demonstrate the understanding of the effect of the exponent equation compare to the parent function

Question 6

Solve the following equation

$$\log 3^{x} = \log 300$$

In order to solve this equation, you have to use the properties of logarithm.

Use the Power rule to get the x value to become a term. Then divide both side by log 3 to isolate the variable. Finally perform the division

$\log 3^x = \log 300$

$x \log 3 = \log 300$

$x = \log 300 / \log 3$

$x = 5.19180654$

Answer:

$x = 5.19180654$

Significant

This question demonstrate the understanding of the properties of logarithms which will help student have a better understanding of logarithms and logic

Question 7

Write a logistic function from the following information

Initial growth is 4,

limit of growth is 200, and

pass through (2,40)

Logistic Function

$$f(x) = \frac{c}{1 + ab^x}$$

c = the limit of growth

(the factor that stop the function from infinity)

a = the initial growth

(the growth factor of the function)

b = base number

Solution

1) Plug in the information into the function, because the function pass through the point (2, 40), it fill out the x and the y value of the function

$$40 = \frac{200}{1 + 4b^2}$$

2) multiply both side by $1 + 4b^2$ to get rid of the division

$$40 + 160b^2 = 200$$

3) solve for b

$$40 + 160b^2 = 200$$
$$160b^2 = 160$$
$$b^2 = 1$$
$$b = 1$$

Solution cont.

4) After getting all the information, plug it in back to the equation and check for correction.

$$40 = \frac{200}{1 + 4(1)^2}$$

$$40 = \frac{200}{1 + 4(1)}$$

$$40 = \frac{200}{5}$$

$$40 = 40$$

Answer:

$$f(x) = \frac{200}{1 + 4(1)^x}$$

Significant

This question demonstrate the understanding of the properties of logarithms which will help student have a better understanding of logarithms and logic

Question 8

Calculate the balance if $4500 is invested for 17 years at 3% compounded monthly

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = balance amount

P = principal amount

r = annual interest rate

n = numbers of times compounded in a year

t = times in year

Solution

1) Plug in the information into the function

$$A = 4500\left(1 + \frac{.03}{12}\right)^{12(17)}$$

2) Solve for A

$$A = 4500(1 + .0025)^{204}$$
$$A = 4500(1.0025)^{204}$$
$$A = 4500(1.6642316751)$$
$$A = 7489.04$$

Answer:

The balance will be \$7489.04 after 17 years

Significant

This question is similar to the denominator of a logistic function. This question will give a better understanding when it comes to logistic function

Question 9

A single cell amoeba doubles every 4 days how long to get a population of 10,000?

Solution

Based on the question, we can find out that it is exponential. The power is x/4 because x is the number of days, and divide that by 4 will result in doubled every 4 days.

To solve this equation, first have to take the log of both side and use the power properties to isolate the variable. Then solve for x.

$10{,}000 = 2^{x/4}$

$\log 10{,}000 = \log 2^{x/4}$

$\log 10{,}000 = (x/4) \log 2$

$\dfrac{\log 10{,}000}{\log 2} = x/4$

$13.28771238 = x/4$

$53.2 = x$

Answer:

It will take 53 days to reach 10,000 cells

Significant

This question is relating back to the properties of logarithm and it demonstrate the use of logarithm in real life situation.

Question 10

The number P of students infected with flu at SWCTA t days after exposure is modeled by

$$P(t) = \frac{300}{1 + e^{4 - t}}$$

Question 10

a) What was the initial number of students infected with the flu?
b) How many students were infected after 3 days?
c) When will 100 students be infected?
d) What would be the maximum number of students infected?

Solution a

To find the number of initial students that get infected, you will have to set the t to 0. This will tell us the number of students that get infected at day 0 which is the starting day.

Answer:

At first, there are 6students infected.

$$P(t) = \frac{300}{1 + e^{4-0}}$$

$$P(t) = \frac{300}{1 + e^{4}}$$

$$P(t) = \frac{300}{1 + 54.59815}$$

$$P(t) = \frac{300}{55.9815}$$

$$P(t) = 5.5$$

Solution b

To find the number of infected students, you will plug 3 in for t and solve for the equation

$$P(t) = \frac{300}{1 + e^{4-3}}$$

$$P(t) = \frac{300}{1 + e^{1}}$$

$$P(t) = \frac{300}{1 + 2.71828183}$$

$$P(t) = \frac{300}{3.71828183}$$

$$P(t) = 80.6$$

Answer:

At day 3, there will be 81 students with the flu.

Solution c

To find out the numbers of day it takes to infect 100 students, first have to set P(t) to 100 and solve for t. Next will have to get the ln of both side to cancel out the e. Since ln e is equal to 1, it left with 4 - t equal to ln 2. Finally, solve for t

Answer:

It will take 3 days to get 100 students infected

$$100 = \frac{300}{1 + e^{4-t}}$$

$$100 + 100e^{4-t} = 300$$

$$100e^{4-t} = 200$$

$$e^{4-t} = 2$$

$$\ln e^{4-t} = \ln 2$$

$$(4 - t) \ln e = \ln 2$$

$$4 - t = .6931471806$$

$$-t = -3.306852819$$

$$t = 3.3$$

Solution d

The maximum students that can get infected by the flu is 300, because 300 is the growth limit and it can never be 300 or go beyond 300

Answer:

The maximum students that can get infected is 300 students.

Significant

This question is relating back to logistic function and exponential function, it demonstrate the difference between exponential and logistic that can help students understand this unit better. It also demonstrates the use of logistic function in a real life situation.