TaskB1

zerofrom

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## B.1

### (1) Value of a

Considering chat follows the probability density function:

The probability density function must follows:

Since when , it follows that:

Replacing expression (1) into expression (3) yields:

Assuming that , it can be obtained that:

Assuming that , then , therefore the upper and lower limits of integration become , substituting expression :

Using the exponential integration formula:

Thus:

The solution is:

### In conclusion, the value of a is .

#### (2) i) Population Mean :

From the definition of the mean, there is:

Replacing and :

Assuming , then , and :

Integral formula for exponential functions:

Replacing expression (t) into expression (3), there is:

### In conclusion, the population mean .

#### ii) Standard Deviation :

The standard deviation formula:

From the derivation of , it follows that:

The formula for is:

Substituting :

Assuming . then , and :

Integrals formula for exponential functions:

Replacing formulas (6) into expression (5), there is:

Replacing expression (2) and (7) into expression (1):

### In conclusion, the standard deviation of is .

#### (3) i) Cumulative distribution function (CDF): Considering the definition of CDF, it follows that:

If , then , thus there is:

If , then , thus there is:

Assuming that , then . and , the upper and Cower limit of integration becomes .

### In conclusion, the cumulative distribution function for the random variable is

#### ii) Quartile Function (QF)

Considering the CDF of random variable :

Quartile Function satisfies:

Replacing (1) into cl), it follows that:

Thus there is:

Taking the natural logarithm of both sides:

Thus

### In conclusion, the Quartile Function of random variable is

#### (4) Maxmum Likehood estimat :

The definition of likehood function is:

Taking the natural logarithm of both sides:

Derivativing with respect to :

Assuming that the derivative is zero, it follows that:

Calculating the value of is:

#### In conclusion, the maxmum likehood estimate for is

### (5) Given the sample, compute and display the maximum likelihood estimate λMLE of the parameter λ.

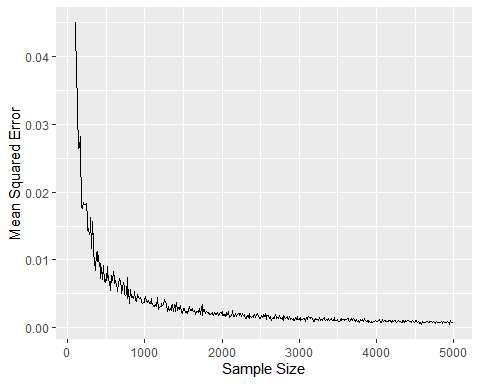
## [1] "The maximum likelihood estimate lambda\_MLE is: 0.019884260798572"

### (6) Compute the Bootstrap confidence interval

## [1] "The Bootstrap confidence interval is: [ 0.0191115894121709 , 0.0206792527343937 ]\n"

### (7)) Conduct a simulation study to explore the behaviour of the maximum likelihood estimator λMLE for λ on simulated data X1, · · · , Xn

## SampleSize MSE  
## 1 100 0.04507438  
## 2 110 0.03664986  
## 3 120 0.03907616  
## 4 130 0.03197187  
## 5 140 0.02650689



## B2

### (1) The formula for the probability mass function

The range of possible values of the discrete ranom variable is:

Event 1: Draw 2 blue balls

Event 2: Draw 1 red ball and 1 blue ball

Event 3: Draw 2 red balls

### In conclustion, the formula for the probability mass function pros:

### (2) The expression of the expectation

According to the expectation formula for random variables:

Replacing into (1) , there is:

### In conclusion, the expectation of is:

### (3)The expression of the variance Var(X)

According to the formula of :

Replacing in to , there is:

According to the formula of :

Replacing and into (3), it follows that:

### In conclusion, the expression of the variance Var(X) is:

### (4)Write a function called compute\_expectation\_X that takes a and b as inputs and outputs the expectation E(X). Write a function called compute\_variance\_X that takes a and b as input and outputs the variance Var(X).

### (5) The expression of the expectation of the random variable X

According to the linear nature of the mean of a random variable:

are i.i.d., thus , thus:

### In conclusion, the expression of the expectation of the random variable is:

### (6) The expression of the variance of the random variable X

Considering the effect of a linear transformation of a random variable on the variance :

$$
\begin{equation}
\operatorname{Var}(a X+b)=a^{2} \operatorname{Var}(X) \tag{1}\\
\end{equation}
$$

thus:

### In conclusion, the expression of the variance of the random variable is

### (7) Create a function called sample\_Xs

### (8) Calculate E(X),Var(X),E({X}),Var({X})

## [1] "Expectation E(X): -0.5 \n"

## [1] "Expectation of samples E(bar{X}): -0.501 \n"

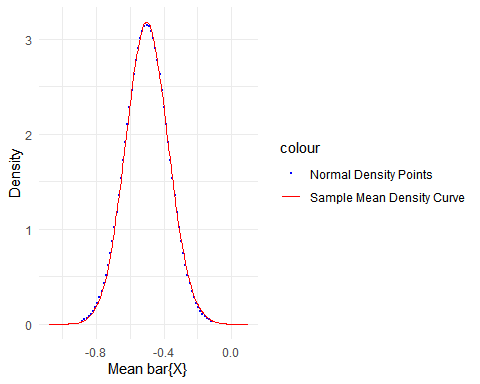
## [1] "Variance Var(X): 1.60714285714286 \n"

## [1] "Variance of samples Var(bar{X}): 1.60309503095031 \n"

### (9) Compute the corresponding sample mean X based on X1, · · · , Xn

### (10) Create a scatter plot of the points

## Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.  
## ℹ Please use `linewidth` instead.  
## This warning is displayed once every 8 hours.  
## Call `lifecycle::last\_lifecycle\_warnings()` to see where this warning was  
## generated.



### (11) Describe the relationship between the density of X and the function fµ,σ displayed in your plot. Try to explain the reason.

1. The blue points in the figure depicts the density distribution of the mean of the discrete random variable X over the interval [µ - 3σ, µ + 3σ], satisfying the normal distribution.
2. The red curve in the figure is the kernel density estimate obtained by representing the kernel density of the sample mean X within simulation study with 50000 trials.
3. It can be seen that the two are very close to each other, proving that the sampling distribution of the sample means tends to the standard normal distribution when the sample size is large enough.