Computational Finance

AMSTERDAM BUSINESS SCHOOL

Business

Preliminaries

General Information

- My name is Simon Broda. You can find me at REC E4.27 (by appointment). Email: s.a.broda@uva.nl (mailto:s.a.broda@uva.nl)
- Format of this course: 12 lectures of 2h each (2 per week), plus computer labs.
- Final grade based on a group assignment (groups of two; 40%), an indiviual assignment (35%), and a final exam (closed book, 2h; 25%).
- Additional exercises will be made available but not graded.

Material

- These lecture slides. Available on Blackboard (https://blackboard.uva.nl), Github (https://github.com/s-broda/ComputationalFinance), and Microsoft Azure (https://notebooks.azure.com/s-broda/libraries/ComputationalFinance).
- Books:
  - Yves Hilpisch. Python for Finance: Analyze Big Financial Data. O'Reilly, 2014. ISBN 978-1-4919-4528-5 (603 pages, c. EUR 31). Code is available on Github (https://github.com/yhilpisch/py4fi).
  - John C. Hull. Options, Futures and Other Derivatives. 8th Edition (or later), Prentice Hall, 2012. ISBN 978-0273759072 (847 pages, c. EUR 58).
- Further reading:
  - Python documentation (https://docs.python.org/2/index.html)
  - Yves Hilpisch. Derivatives Analytics with Python. Wiley, 2015. ISBN 978-1-119-03799-6 (374 pages, c. EUR 72). Code is available on Github (https://github.com/yhilpisch/dawp).
  - Python for Data Analysis. 2nd Edition, O'Reilly, 2017. ISBN 978-1-4919-5766-0 (544 pages, c. EUR 34). Code is available on Github (https://github.com/wesm/pydata-book).

Outline and Reading List

Week	Topic	Read: Hilpisch (2014)	Read: Hull (2012)
1	Introduction to Python	Chs. 2, 4 (pp. 79-95)	
2	Dealing with Data	Chs. 4 (pp. 95-108), 6, App. C	
3	Risk Measures; Plotting	Chs. 5, 10 (pp. 398-301), 11 (pp. 307-322)	Chs. 21.1-21.2, 22.1-22.2
4	Binomial Trees	Ch. 8 (pp. 218-223)	Chs. 12, 14, 19, 20.1-20.5
5	Monte Carlo Methods	Ch. 10 (pp. 265-287, 290-294)	Chs. 13, 16.3, 18, 20.6, 25.8-25.12
6	Variance Reduction	Ch. 10 (pp. 287-290)	Ch. 20.7

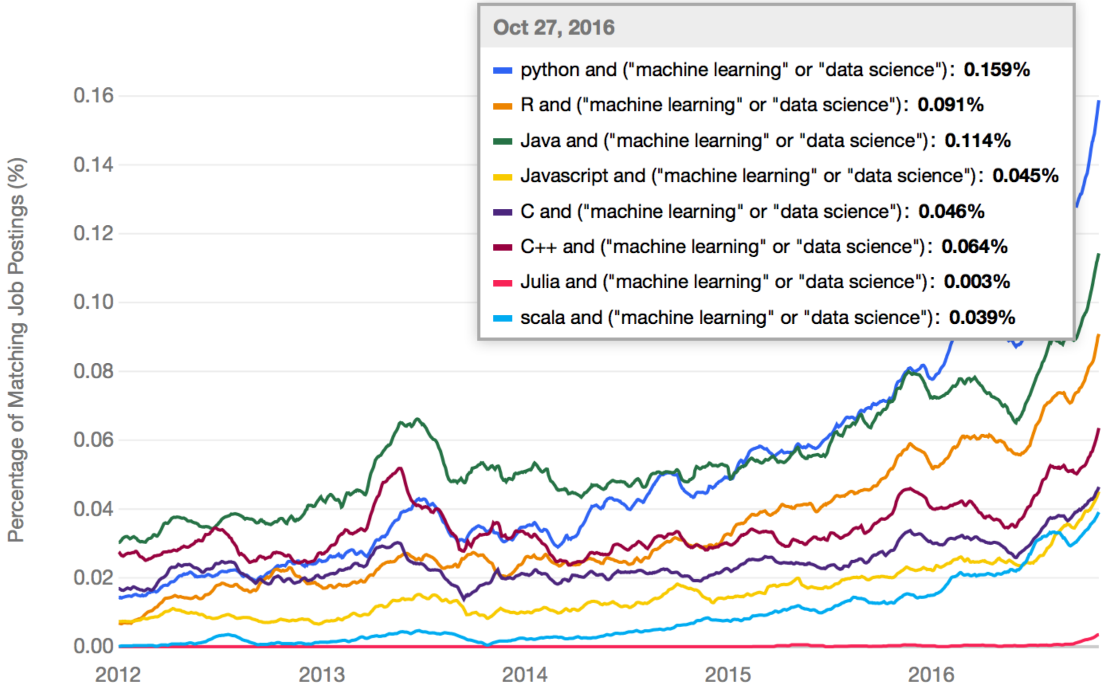
Note: Chapter numbers for Hull refer to the 8th edition. Increment by 1 for the 9th and 10th editions.

Introduction to Python

Why Python?

- General purpose programming language, unlike, e.g., Matlab®.
- High-level language with a simple syntax, interactive (*REPL*: read-eval-print loop). Hence ideal for rapid development.
- Vast array of libraries available, including for scientific computing and finance.
- Native Python is usually slower than compiled languages like C++. Alleviated by highly optimized libraries, e.g. NumPy for calculations with arrays.
- Free and open source software. Cross-platform.
- Python skills are a marketable asset: most popular language for data science.

Job Postings on Indeed.com



[Source (https://www.ibm.com/developerworks/community/blogs/jfp/entry/What_Language_Is_Best_For_Machine_Learning_And_Data_Science?lang=en)](https://www.ibm.com/developerworks/community/blogs/jfp/entry/What_Language_Is_Best_For_Machine_Learning_And_Data_Science?lang=en)

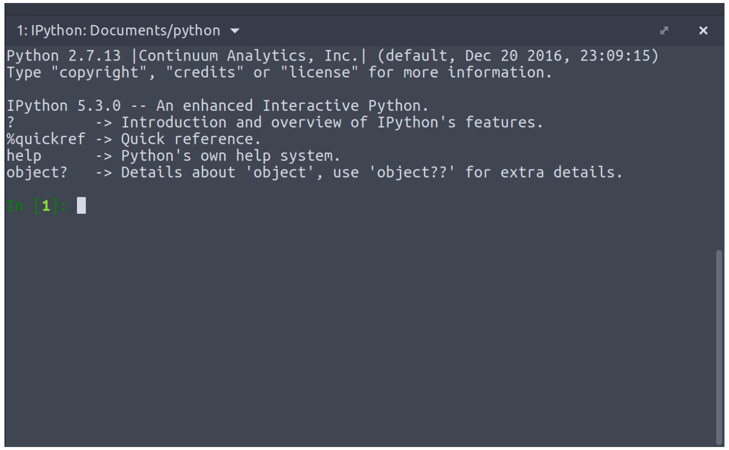
Obtaining Python

- Anaconda is a Python distribution, developed by Continuum Analytics, and specifically designed for scientific computing.
- Comes with its own package manager (conda). Many important packages (the *SciPy stack*) are pre-installed.
- Two versions: Python 2.7 and 3.6. Like the book, we will be using Python 2.7, which is still the industry standard. Most of our code should run on both with minimal adjustments.
- Obtain it from here (https://www.anaconda.com/download/). I recommend adding it to your PATH upon installation.
- Install the RISE plugin:

In [1]:
```
#uncomment the next line to install. Note: "!" executes shell commands.
#!conda install -y -c damianavila82 rise
```

IPython Shell

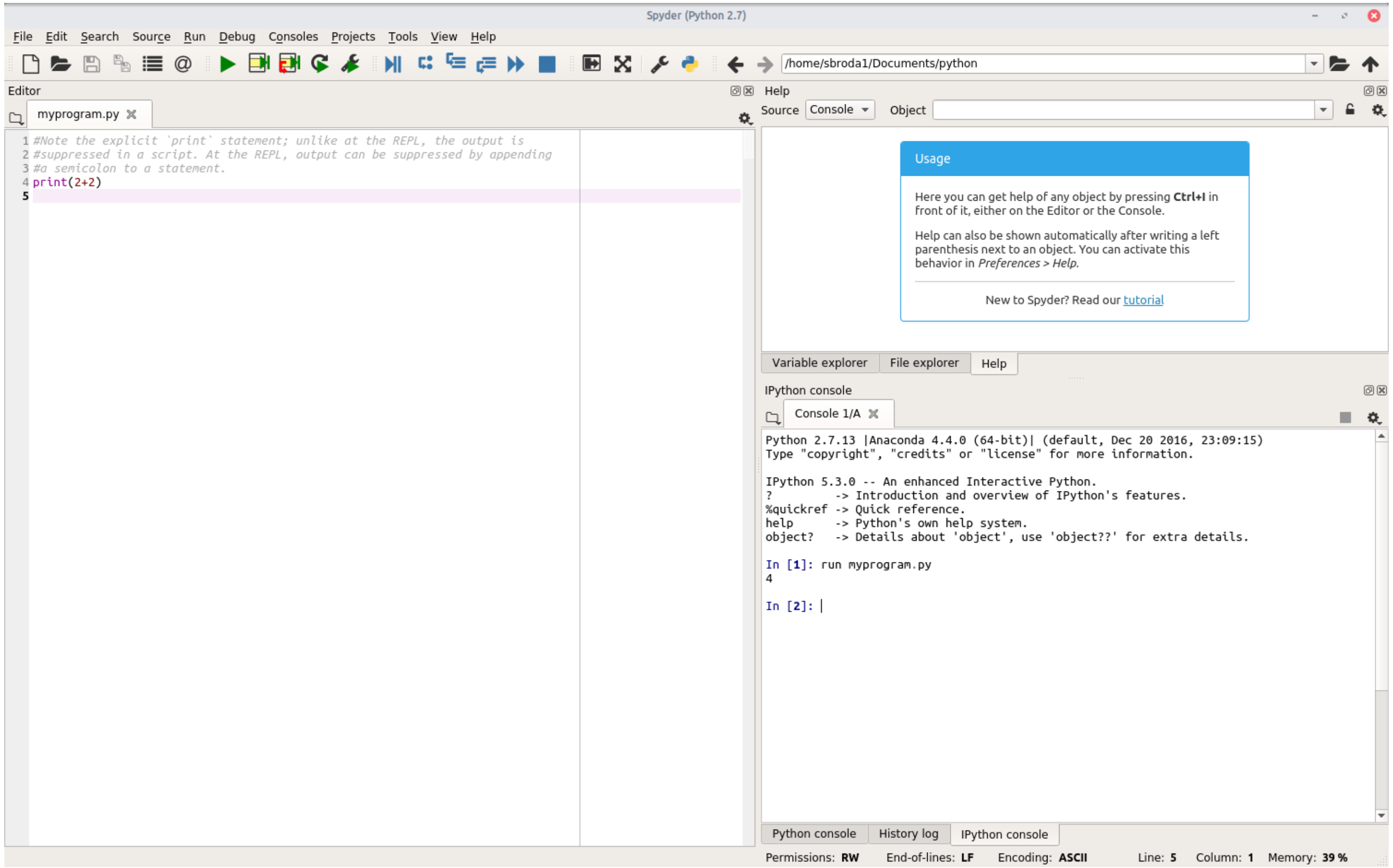
- Python features a *read-eval-print loop* (REPL) which allows you to interact with it.
- The most bare-bones way to interact with it is via the *IPython shell*:



- For now you can treat it like a fancy calculater. Try entering `2+2`. Use `quit()` or `exit()` to quit, `help()` for Python's interactive help.

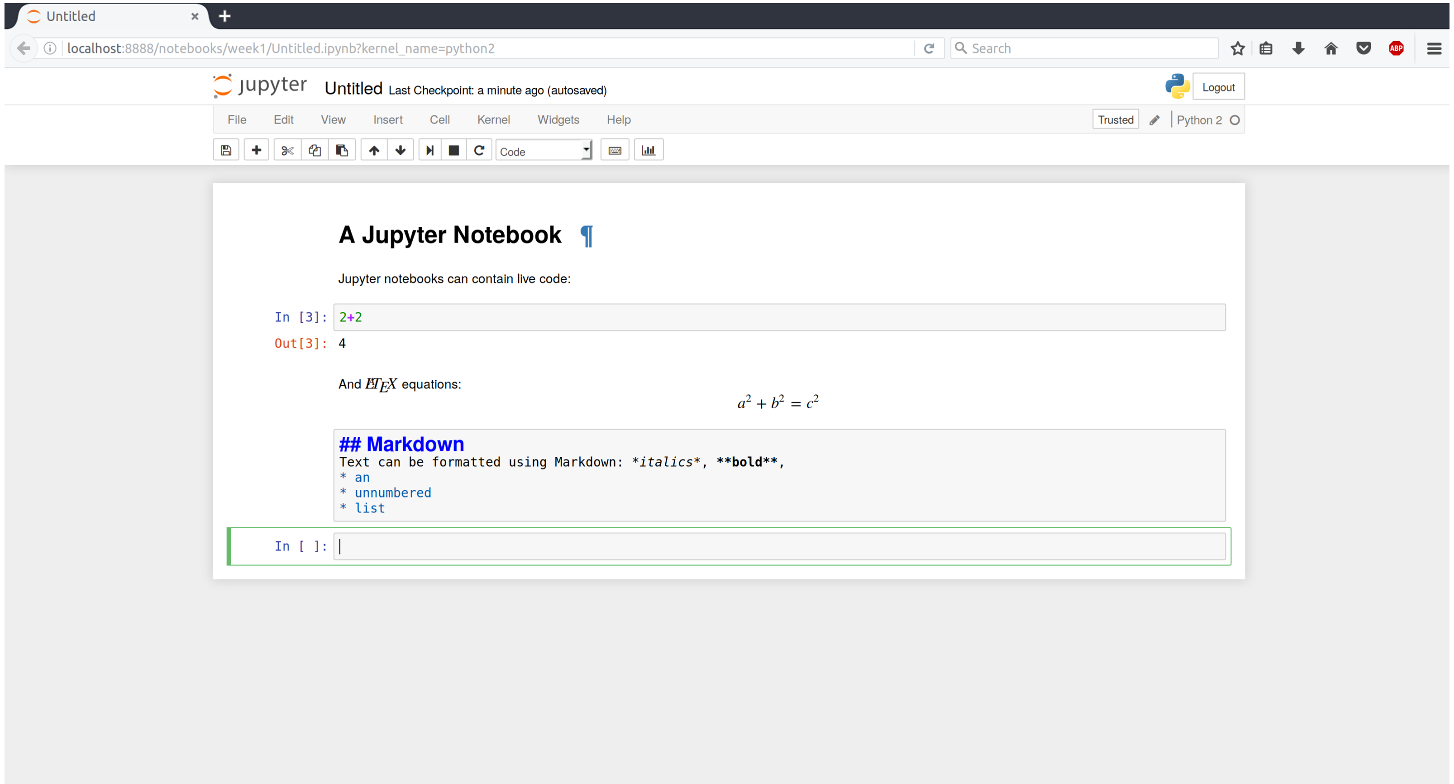
Writing Python Programs

- Apart from using it interactively, we can also write Python programs so we can rerun the code later.
- A Python program (called a *script* or a *module*) is just a text file, typically with the file extension .py.
- Contains Python commands and comments (introduced by the # character)
- To execute a program, do `run filename.py` in IPython (you may need to navigate to the right directory by using the `cd` command).
- While it's possible to code Python using just the REPL and a text editor, many people prefer to use an *integrated development environment* (IDE).
- Anaconda comes with an IDE called *Spyder* (Scientific PYthon Development EnviRonment), which integrates an editor, an IPython shell, and other useful tools.


Spyder (Python 2.7)
File Edit Search Source Run Debug Consoles Projects Tools View Help
/home/sbroda1/Documents/python
Editor
myprogram.py
1 #Note the explicit `print` statement; unlike at the REPL, the output is
2 #suppressed in a script. At the REPL, output can be suppressed by appending
3 #a semicolon to a statement.
4 print(2+2)
5
Help
Source Console Object
Usage
Here you can get help of any object by pressing Ctrl+I in front of it, either on the Editor or the Console.
Help can also be shown automatically after writing a left parenthesis next to an object. You can activate this behavior in Preferences > Help.
New to Spyder? Read our tutorial
Variable explorer File explorer Help
IPython console
Console 1/A
Python 2.7.13 |Anaconda 4.4.0 (64-bit)| (default, Dec 20 2016, 23:09:15)
Type "copyright", "credits" or "license" for more information.
IPython 5.3.0 -- An enhanced Interactive Python.
? -> Introduction and overview of IPython's features.
%quickref -> Quick reference.
help -> Python's own help system.
object? -> Details about 'object', use 'object??' for extra details.
In [1]: run myprogram.py
4
In [2]:
Python console History log IPython console
Permissions: RW End-of-lines: LF Encoding: ASCII Line: 5 Column: 1 Memory: 39 %

Jupyter Notebooks

- Another option is the *Jupyter notebook* (JUlia PYThon (e) R, formerly known as IPython notebook).
- Web app that allows you to create documents (*.ipynb) that contain text (formatted in Markdown (https://daringfireball.net/projects/markdown)), live code, and equations (formatted in $\LaTeX$).
- In fact these very slides are based on Jupyter notebooks. You can find them on my Github page (https://github.com/s-broda/ComputationalFinance).
- The slides are also available on my Microsoft Azure page (https://notebooks.azure.com/s-broda/libraries/ComputationalFinance), where you can also see them as a slide show and/or run them (after cloning them; requires a free Microsoft account).


Untitled
localhost:8888/notebooks/week1/Untitled.ipynb?kernel_name=python2
Search
jupyter
Untitled Last Checkpoint: a minute ago (autosaved)
Logout
File
Edit
View
Insert
Cell
Kernel
Widgets
Help
Trusted
Python 2
Code
A Jupyter Notebook ¶
Jupyter notebooks can contain live code:
In [3]: 2+2
Out[3]: 4
And LaTeX equations:
a² + b² = c²
Markdown
Text can be formatted using Markdown: *italics*, **bold**,
* an
* unnumbered
* list
In [ ]:

- A notebook consists of cells, each of which is either designated as Markdown (for text and equations), or as code.
- You should take a moment to familiarize yourself with the keyboad shortcuts. E.g., enter enters edit mode, esc enters command mode, ctrl-enter evaluates a cell, shift-enter evaluates a cell and selects the one below.
- Useful references:
  - Jupyter documentation (http://jupyter-notebook.readthedocs.io/en/latest/index.html)
  - Markdown cheat sheet (https://github.com/adam-p/markdown-here/wiki/Markdown-Cheatsheet)
  - Latex math cheat sheet (https://en.wikibooks.org/wiki/LaTeX/Mathematics)

Python Basics

Variables

- A variable is a named memory location. It is assigned using "=" (technically, "=" binds the name on the LHS to the result of the expression on the RHS).

In [2]:
```
a=2
a=a+1 #bind the value a to the result of the expression a+1
print(a) #show the result
```

```
3
```

In [3]:
```
a+=1 #shorthand for a=a+1
print(a)
```

```
4
```

- Variable names can be made up from letters, numbers and the underscore. They may not start with a number. Python is case-sensitive: A is not the same as `a`.

Built-in Types

Attributes and Methods

- Any Python object has a *type*.
- One can use the `type` function to show the type of an object:

In [4]:
```
type(a) #Functions take one or more inputs (in parens) and return an output.
```

Out[4]:
```
int
```

- Objects can have attributes and methods associated with them:

In [5]:
```
a.real #an attribute (internal variable stored inside an object)
```

Out[5]:
```
4
```

In [6]:
```
a.bit_length() #a method (function that operates on objects of a particular type)
```

Out[6]:
```
3
```

Numeric Types

- Computers distinguish between integers and floating point numbers.
- Python integers can be arbitrary large (will use as many bits as necessary)
- Python floats are between $\pm 1.8 \cdot 10^{308}$, but are stored with just 64 bit precision.
- Hence floating point arithmetic is not exact:

In [7]:
```
a=1.0;type(a) #Note that variables can change type: a was an int before
```

Out[7]:
```
float
```

In [8]:
```
a-0.9
```

Out[8]:
```
0.09999999999999998
```

Arithmetic

- The basic arithmetic operations are `+`, `-`, `*`, `/`, and `**` for exponentiation:

In [9]:
```
2*(3-1)**2
```

Out[9]:
```
8
```

- If any of the operands is `float`, then Python will convert the others to float too:

In [10]:
```
2*(3-1.0)**2
```

Out[10]:
```
8.0
```

- Note that `/` performs floor division in Python 2.7 (not 3.6) when both arguments are `ints`:

In [11]:
```
c=3
c/2
```
Out[11]:
```
1
```

- Need to convert one argument to float to get the usual division:

In [12]:
```
c/2.0
```
Out[12]:
```
1.5
```

In [13]:
```
float(c)/2
```
Out[13]:
```
1.5
```

Booleans

- A `bool` can take one of two values: `True` or `False`.
- They are returned by *relational operators*: <, <=, >, >=, == (equality), `!=` (inequality)
- Can be combined using the *logical operators* `and`, `or`, and `not`.

In [14]:
```
1<=2<4
```
Out[14]:
```
True
```

In [15]:
```
1<2 and 2<1
```
Out[15]:
```
False
```

In [16]:
```
not(1<2)
```
Out[16]:
```
False
```

Sequence Types: Containers with Integer Indexing

Strings

- Strings hold text. Constructed using either single or double quotes.

In [17]:
```
s1="Python"; s2=' is easy'; s1+s2 #Concatenation
```

Out[17]:
```
'Python is easy'
```

- Strings can be indexed into:

In [18]:
```
s1[0] #Note zero-based indexing
```

Out[18]:
```
'P'
```

In [19]:
```
s1[-1] #Negative indexes count from the right:
```

Out[19]:
```
'n'
```

- Can also pick out several elements ("*slicing*"). Works for all *sequence types* (lists, NumPy arrays, ...)

In [20]:
```
s1[0:2] #Elements 0 and 1; left endpoint is included, right endpoint excluded.
```

Out[20]:
```
'Py'
```

In [21]:
```
s1[0:6:2] #start:stop:step
```

Out[21]:
```
'Pto'
```

In [22]:
```
s1[::-1] #start and stop can be ommitted; default to 0 and len(str)
```

Out[22]:
```
'nohtyP'
```

- Strings are *immutable*:

In [23]:
```
#Wrapping this in a try block so the error doesn't break `Run all` in Jupyter
try:
    s1[0]="C" #This errors
except TypeError as e:
    print(e)
```

```
'str' object does not support item assignment
```

- Python has many useful methods for strings:

In [24]:
```
print(', '.join(filter(lambda m: callable(getattr(s1, m)) and not m.startswith("_")
, dir(s1))))
```

```
capitalize, center, count, decode, encode, endswith, expandtabs, find, format, i
ndex, isalnum, isalpha, isdigit, islower, isspace, istitle, isupper, join, ljust
, lower, lstrip, partition, replace, rfind, rindex, rjust, rpartition, rsplit, r
strip, split, splitlines, startswith, strip, swapcase, title, translate, upper,
zfill
```

In [25]:
```
help(s1.upper)
```

```
Help on built-in function upper:

upper(...)
    S.upper() -> string

    Return a copy of the string S converted to uppercase.
```

In [26]:
```
(s1+s2).replace('easy','hard').upper()
```

Out[26]:
```
'PYTHON IS HARD'
```

Lists

- Lists are indexable collections of arbitrary (though usually homogenous) things:

In [27]:
```
list1=[1, 2., 'hi']; print(list1)
```

```
[1, 2.0, 'hi']
```

- The function `len` returns the length of a list (or any other sequence):

In [28]:
```
len(list1)
```

Out[28]:
```
3
```

- Like strings, they support indexing, but unlike strings, they are *mutable*:

In [29]:
```
list1[2]=42; print(list1)
```

```
[1, 2.0, 42]
```

- Note the following:

In [30]:
```python
list2=list1 #bind the name list2 to the object list1. This does not create a copy:
list2[0]=13
print(list1) #list2 and list1 are the _same_ object!
```

```
[13, 2.0, 42]
```

In [31]:
```python
list3=list1[:] #This creates a copy
list3==list1 #Tests if all elements are equal
```

Out[31]:
```
True
```

In [32]:
```python
list3 is list1 #Tests if list3 and list1 refer to the same object
```

Out[32]:
```
False
```

In [33]:
```python
list2 is list1
```

Out[33]:
```
True
```

- Lists of integers can be constructed using the `range` function:

In [34]:
```
range(1,11,2) #start,stop,[,step]
```

Out[34]:
```
[1, 3, 5, 7, 9]
```

In [35]:
```
range(5) #start and step can be ommited
```

Out[35]:
```
[0, 1, 2, 3, 4]
```

- *List comprehensions* allow creating lists programmatically:

In [36]:
```
[x**2 for x in range(1,10) if x>3 and x<7]
```

Out[36]:
```
[16, 25, 36]
```

- The `for` and `if` statements will be discussed in more detail later.

- Methods for lists:

In [37]:
```
print(', '.join(filter(lambda m: callable(getattr(list1, m)) and not m.startswith("
_"), dir(list1))))
```

```
append, count, extend, index, insert, pop, remove, reverse, sort
```

In [38]:
```
list1.append(13); #append 13 to the list `l1`
print(list1)
```

```
[13, 2.0, 42, 13]
```

In [39]:
```
list1.remove(13) #remove first occurence of 13 from l1
print(list1)
```

```
[2.0, 42, 13]
```

- Note: Table 4-2 in the book incorrectly states that `remove[i]` removes the element at index `i`. For that, use

In [40]:
```
del(list1[0]); print(list1)
```

```
[42, 13]
```

- `del` can also be used to delete variables (technically, unbind the variable name)

xranges

- an xrange is similar to a list created with `range`, but they are more memory efficient because the list elements are created on demand (*lazily*).

In [41]:
```
xrange(1,10,2)[3]
```

Out[41]:
```
7
```

Tuples

- Tuples are immutable sequences. They are created with round brackets:

In [42]:
```
(1, 2., 'hi')
```

Out[42]:
```
(1, 2.0, 'hi')
```

Other built-in datatypes

- Other built-in datatypes include `sets` (unordered collections) and `dicts` (collections of key-value pairs). See Hilpisch (2014), pp. 92-94.

Control Flow

- Control flow refers to the order in which commands are executed within a program.
- Often we would like to alter the linear way in which commands are executed. Examples:
  1. *Conditional branch*: Code that is only evaluated if some condition is true.
  2. *Loop*: Code that is evaluated more than once.

Conditional Branch: The `if-else` statement

In [43]:
```
x=3#uncomment the next line for interactive use
#x=int(raw_input("Enter a number between 0 and 9: ")) #`raw_input` returns a string
. `int` converts to integer.
if x<0:
    print("You entered a negative number.")
elif x>9:
    print("You entered a number greater than 9.")
else:
    print("Thank you. You entered %s." %x) #String interpolation.
```

```
Thank you. You entered 3.
```

- Notes:
  1. Code blocks are introduced by colons and *have* to be indented.
  2. The `if` block is executed only if the first condition is true
  3. The optional `elif` (short for 'else if') block is executed only if the first condition is false and the second one is true. There could be more than one.
  4. The optional `else` block is executed when none of the others is.

`While` loops

- Similar to `if`, but jumps back to the `while` statement after the `while` block has finished.
- The `else` block is executed when the condition becomes false (not if the loop is exited through a `break` statement; see next).

In [44]:
```
x=1#set this to -1 to run
while x<0 or x>9:
    x=int(raw_input("Enter a number between 0 and 9: "))
    if x<0:
        print("You entered a negative number.")
    elif x>9:
        print("You entered a number greater than 9.")
else:
    print("Thank you. You entered %s." %x)
```

```
Thank you. You entered 1.
```

- Alternative implementation:

In [45]:
```
while False: #Change to True to run
    x=int(raw_input("Enter a number between 0 and 9: "))
    if x<0:
        print("You entered a negative number.")
        continue #skip remainder of loop body, go back to `while`
    if x>9:
        print("You entered a number greater than 9.")
        continue
    print("Thank you. You entered %s." %x)
    break #exit innermost enclosing loop
```

`For` Loops

- A `for` loop iterates over the elements of a sequence (e.g., a list):

In [46]:
```
for letter in "Python":
    print(letter)
```

```
P
y
t
h
o
n
```

- `letter` is called the loop variable. Every time the loop body is executed, it will in turn assume the value of each element of the sequence.

- `For` loops are typically used to execute a block of code a pre-specified number of times; `range` and `xrange` are often used in that case:

In [47]:
```
squares=[]
for i in xrange(10):
    squares.append(i**2)
print(squares)
```

```
[0, 1, 4, 9, 16, 25, 36, 49, 64, 81]
```

- Question: What does the following compute?

In [48]:
```
n=7
f=1
for i in xrange(n):
    f*=i+1
```

Modules

- Python's functionality is organized in *Modules*.
- Some of these are part of Python's *standard library* (e.g., `math`). Others are part of *packages*, many of which come preinstalled with Anaconda (e.g., `numpy`).
- Modules need to be imported in order to make them available:

In [49]:
```
import math
math.factorial(7)
```

Out[49]:
```
5040
```

- Can use *tab completion* to discover functions defined by `math`: after importing, enter `math.` and press the `Tab` key. Alternatively, use dir(math):

In [50]:
```
print(', '.join(filter(lambda m: not m.startswith("_"), dir(math)))) #just so the o
utput fits on the slide
```

```
acos, acosh, asin, asinh, atan, atan2, atanh, ceil, copysign, cos, cosh, degrees
, e, erf, erfc, exp, expm1, fabs, factorial, floor, fmod, frexp, fsum, gamma, hy
pot, isinf, isnan, ldexp, lgamma, log, log10, log1p, modf, pi, pow, radians, sin
, sinh, sqrt, tan, tanh, trunc
```

- Note that importing the module does not bring the functions into the *global namespace*: they need to be called as `module.function()`
- A function can be imported into the global namespace like so:

In [51]:
```
from math import factorial
factorial(7)
```

Out[51]:
```
5040
```

- It is possible to import all functions from the module into the global namespace using `from math import *`, but this is frowned upon; it pollutes the namespace, which may lead to name collisions.
- Packages can contain several modules. They are imported the same way:

In [52]:
```
import numpy
numpy.random.rand()
```

Out[52]:
```
0.5496655339625427
```

- Optionally, you can specify a shorthand name for the imported package/module:

In [53]:
```
import numpy as np
np.sqrt(2.0)  #Note that this is not the same function as math.sqrt
```

Out[53]:
```
1.4142135623730951
```

- Conventions have evolved for the shorthands of some packages (e.g., np for numpy). Following them improves code readability.
- For the same reason, it is good practice to put your `import` statements at the beginning of your document (which I didn't do here).

Functions

Defining Functions

- User defined functions are declared using the `def` keyword:

In [54]:
```
def mypower(x, y): #zero or more arguments, here two
    """Compute x^y."""
    return x**y
mypower(2, 3) #positional arguments
```

Out[54]:
```
8
```

- The *docstring* is shown by the help function:

In [55]:
```
help(mypower)
```

```
Help on function mypower in module __main__:

mypower(x, y)
    Compute x^y.
```

Several outputs

- Functions can have more than one output argument:

In [56]:
```python
def plusminus(a, b):
    return a+b, a-b
c, d = plusminus(1, 2); c, d
```

Out[56]:
```
(3, -1)
```

Keyword arguments

- Instead of *positional arguments*, can also pass in *keyword arguments*:

In [57]:
```
mypower(y=2, x=3)
```

Out[57]:

9

- Functions can specify *default arguments*:

In [58]:
```
def mypower(x, y=2): #default arguments have to appear at the end
    """Compute x^y.
    """
    return x**y
mypower(3)
```

Out[58]:

9

In [59]:
```
mypower(3, 3)
```

Out[59]:

27

Variable Scope

- Variables defined in functions are local (not visible in the calling scope):

In [60]:
```
def f():
    z=1
f()
```

In [61]:
```
try:
    print(z) #x is local to function f
except NameError as e:
    print(e)
```

```
name 'z' is not defined
```

Calling Convention

- Python uses a *calling convention* known as *call by object reference*.
- This means that modifications to its arguments made by a function are visible to the caller (i.e., outside the function):

In [62]:

```
x=[1] #Recall that lists are mutable
def f(y):
    y[0]=2 #Note no return statement; equivalent to `return None`
f(x);print(x) #Note that x has been modified in calling scope
```

```
[2]
```

Nested Functions

- Functions can be defined inside other functions. They will only be visible to the enclosing function.
- Nested functions can see variables defined in the enclosing function.

In [63]:

```python
def mypower(x, y):
    def helper(): #no need to pass in x and y
        return x**y #because the nested function can see them
    a=helper()
    return a
mypower(2, 3)
```

Out[63]:

```
8
```

Advanced Material on Functions

Splatting and Slurping

- Splatting: passing the elements of a sequence into a function as positional arguments, one by one.

In [64]:

```
def mypower(x, y):
    return x**y
args=[2, 3] #a list or a tuple
mypower(*args) #splat (unpack) positional arguments into function
```

Out[64]:

```
8
```

- We can splat keyword arguments too, but we need to use a `dict` (key-value store):

In [65]:

```
kwargs={'y': 3, 'x': 2} # a dict
mypower(**kwargs) #splat keyword arguments
```

Out[65]:

```
8
```

- Slurping allows us to create *vararg* functions: functions that can be called with any number of positional and/or keyword arguments.

In [66]:
```
def myfunc(*myargs, **mykwargs):
    for (i, a) in enumerate(myargs): print("The %sth positional argument was %s." % (i, a))
    for a in mykwargs: print("Got keyword argument %s=%s." %(a,mykwargs[a]))
myfunc(0, 1, x=2, y=3)
```

```
The 0th positional argument was 0.
The 1th positional argument was 1.
Got keyword argument y=3.
Got keyword argument x=2.
```

- The asterisk means "collect all (remaining) positional arguments into a tuple".
- The double asterisk means "collect all (remaining) keyword arguments into a dict".

Closures

- Functions are *first class objects* in Python
- This implies that functions can return other functions.
- Such functions are called *closures*, because they close about (capture) the local variables of the enclosing function.

In [67]:

```
def makemultiplier(factor):
    """Return a function that multiplies its argument by `factor`."""
    def multiplier(x):
        return x*factor
    return multiplier
timesfive=makemultiplier(5)
type(timesfive)
```

Out[67]:

```
function
```

In [68]:

```
timesfive(3)
```

Out[68]:

```
15
```

Anonymous Functions

- Anonymous functions (or *lambdas*) are functions without a name (duh...) and whose function body is a single expression.
- They are often useful for functions that are needed only once (e.g., to return from a function, or to pass to a function)
- E.g., the previous example could be written

In [69]:

```
def makemultiplier(factor):
    """Return a function that multiplies its argument by `factor`."""
    multiplier = lambda x: x*factor
    return multiplier
timesfive=makemultiplier(5)
timesfive(3)
```

Out[69]: 15