(1)即证$\exists c_1, c_2 > 0, n_0 \quad s.t. \forall n > n_0, c_1 f(n) \le f(n) + o(f(n)) \le c_2 f(n)$

由定义知$\exists n_1, s.t. \forall n > n_1, \forall c > 0, 0 \le o(f(n)) \le c f(n)$

取$c = c_0 > 0$，即$\forall n > n_1, 0 \le o(f(n)) \le c_0 f(n)$

于是可以取$c_1 = 1, c_2 = c + 1, n_0 = n_1$，有$\forall n > n_0, c_1 f(n) \le f(n) + o(f(n)) \le c_2 f(n)$，得证

(2)即证$\exists c_4 > 0, n_2, s.t. \forall n > n_2, 0 \le \theta(f(n)) + O(g(n)) \le c_4 max \{f(n), g(n)\}$

由定义，$\exists c_1, c_2 > 0, n_0 \quad s.t. \forall n > n_0, c_1 f(n) \le \theta(f(n)) \le c_2 f(n)$，以及$\forall c_3 > 0, n_1 \quad s.t. \forall n > n_1, 0 \le O(g(n)) \le c_3 g(n)$

进而有$\exists n_3 = max \{n, n_0\} \, s.t. \forall n > n_3, c_1 f(n) \le \theta(f(n)) + O(g(n)) \le c_2 f(n) + c_3 g(n)$

取$n_0 = n_2, c_3 = 1, c_4 = c_2 + 1$即可

(3)由定义，$\exists c_1, c_2 > 0, n_0 \quad s.t. \forall n > n_0, c_1 f(n) \le \theta(f(n)) \le c_2 f(n)$，以及$\forall c_3 > 0, n_1 \quad s.t. \forall n > n_1, 0 \le O(g(n)) \le c_3 g(n)$

取$c_3 = \frac{c_1}{2}$，于是有$0 \le O(g(n)) \le c_3 g(n) = \frac{c_1}{2} g(n) < c_1 f(n) \le \theta(f(n)) \le c_2 f(n)$，得证

(4)$\lim\limits_{x\to\infty} \frac{\log_k n}{n} = \lim\limits_{x\to\infty} \frac{1}{nlnk} = 0$，得证

(5)由Strling公式知$n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$，于是$\log n! \sim \frac{1}{2} \log 2\pi n + n(\log n - \log e) = \theta(n \log n)$