实验报告一

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| 题目（摘要）  前 言：（目的和意义） |
| 数学原理 |
| 程序设计流程 |
| 实验结果、结论与讨论 问题一 实验结果：   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | 1/(1+x^2) | | | | | | | n\x | 0.75 | 1.75 | 2.75 | 3.75 | 4.75 | | 5 | 0.528974 | 0.373325 | 0.153733 | -0.02595 | -0.01574 | | 10 | 0.67899 | 0.19058 | 0.215592 | -0.23146 | 1.923631 | | 20 | 0.636755 | 0.238446 | 0.08066 | -0.44705 | -39.9524 | | inf | 0.64 | 0.246154 | 0.116788 | 0.06639 | 0.04244 |  |  |  |  |  |  | | --- | --- | --- | --- | --- | | exp(x) | | | | | | n/x | -0.95 | -0.05 | 0.05 | 0.95 | | 5 | 0.386798 | 0.951248 | 1.05129 | 2.585785 | | 10 | 0.386741 | 0.951229 | 1.051271 | 2.58571 | | 20 | 0.386741 | 0.951229 | 1.051271 | 2.58571 | | inf | 0.386741 | 0.951229 | 1.051271 | 2.58571 |   不难看出，对于1/(1+x^2)而言，由于Runge现象，在x较大时，高次的插值函数反而效果不佳，但是对于exp(x)而言，直到x=20，在所给定的范围内，n越大效果越好。  可以将目标插值区间等分为若干个小区间，等分为若干个小区间，在每一个小区间上分别作低次插值来避免Runge现象。 问题二  |  |  |  |  |  | | --- | --- | --- | --- | --- | | 1/(1+x^2) | | | | | | n/x | -0.95 | -0.05 | 0.05 | 0.95 | | 5 | 0.517147289 | 0.992790671 | 0.992790671 | 0.517147289 | | 10 | 0.526407983 | 0.997506857 | 0.997506857 | 0.526407983 | | 20 | 0.525620366 | 0.997506234 | 0.997506234 | 0.525620366 | | inf | 0.525624179 | 0.997506234 | 0.997506234 | 0.525624179 |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | exp(x) | | | | | | | n/x | 0.75 | 1.75 | 2.75 | 3.75 | 4.75 | | 5 | 2.37395687 | 4.871634881 | 15.00806092 | 45.86225678 | 119.6210071 | | 10 | 2.117135895 | 5.754367326 | 15.64324806 | 42.51843098 | 115.6073601 | | 20 | 2.117000017 | 5.754602676 | 15.64263188 | 42.521082 | 115.5842845 | | inf | 2.117000017 | 5.754602676 | 15.64263188 | 42.521082 | 115.5842845 |   比较发现，对于exp(x)而言，差异不显著，然而从绝对误差上看，小区间的效果略好。对于1/(1+x^2)而言，小区间的结果显著的好。  原因是小区间上，点的分布更密集，对f(x)逼近效果更好。 问题三  |  |  |  |  |  | | --- | --- | --- | --- | --- | | 1/(1+x^2) | | | | | | n/x | -0.95 | -0.05 | 0.05 | 0.95 | | 5 | 0.523881 | 0.987881 | 0.987881 | 0.523881 | | 10 | 0.525682 | 0.997509 | 0.997509 | 0.525682 | | 20 | 0.525624 | 0.997506 | 0.997506 | 0.525624 | | inf | 0.525624 | 0.997506 | 0.997506 | 0.525624 | | exp(x) | | | | | | n/x | -0.95 | -0.05 | 0.05 | 0.95 | | 5 | 0.386754 | 0.951272 | 1.051314 | 2.585727 | | 10 | 0.386741 | 0.951229 | 1.051271 | 2.58571 | | 20 | 0.386741 | 0.951229 | 1.051271 | 2.58571 | | inf | 0.386741 | 0.951229 | 1.051271 | 2.58571 |   在更改插值节点之后，插值对1/(1+x^2)和exp(x)的效果都有所提升。不同的插值节点限制了插值函数的导数，这是Hermite插值的思想。 问题四  |  |  |  |  |  | | --- | --- | --- | --- | --- | | 实验次数/x | 5 | 50 | 115 | 185 | | 1 | 2.266667 | -20.2333 | -171.9 | -492.733 | | 2 | 3.115751 | 7.071795 | 10.16703 | 10.03883 | | 3 | 4.439112 | 7.284961 | 10.72276 | 13.53567 | | 4 | 5.497172 | 7.800128 | 10.80049 | 13.60062 | | 真实值 | 2.236068 | 7.071068 | 10.72381 | 13.60147 |   观察可知，相比于外推，内插更加可靠。插值问题的外推是指插值节点都位于待计算点的两侧，内推则指插值节点分布于待计算点两侧。 |