CCA-Secure and Authentication Encryption

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Cryptography, Autumn, 2020

Outline

1 Authenticated Encryptions

2 Deterministic Encryptions

3 Key Derivation Function

Content

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Recall Security Against CCA

The CCA indistinguishability experiment $PrivK_{\mathcal{A},\Pi}^{\mathsf{cca}}(n)$:

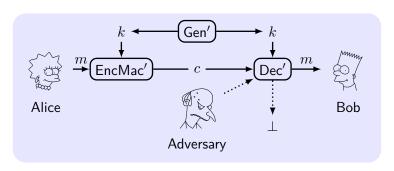
- 2 \mathcal{A} is given input 1^n and oracle access $\mathcal{A}^{\mathsf{Enc}_k(\cdot)}$ and $\mathcal{A}^{\mathsf{Dec}_k(\cdot)}$, outputs m_0, m_1 of the same length.
- **3** a random bit $b \leftarrow \{0,1\}$ is chosen. Then $c \leftarrow \operatorname{Enc}_k(m_b)$ is given to \mathcal{A} .
- 4 \mathcal{A} continues to have oracle access **except for** c, outputs b'.
- **5** If b' = b, \mathcal{A} succeeded PrivK $_{\mathcal{A},\Pi}^{\mathsf{cca}} = 1$, otherwise 0.

Definition 1

 Π has indistinguishable encryptions under a CCA (CCA-secure) if \forall PPT \mathcal{A} , \exists negl such that

$$\Pr\left[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi}(n) = 1\right] \leq \frac{1}{2} + \mathsf{negl}(n).$$

Message Transmission Scheme



- **Key-generation** algorithm outputs $k \leftarrow \text{Gen}'(1^n)$. $k = (k_1, k_2)$. $k_1 \leftarrow \text{Gen}_E(1^n)$, $k_2 \leftarrow \text{Gen}_M(1^n)$.
- Message transmission algorithm is derived from $\operatorname{Enc}_{k_1}(\cdot)$ and $\operatorname{Mac}_{k_2}(\cdot)$, outputs $c \leftarrow \operatorname{EncMac'}_{k_1,k_2}(m)$.
- **Decryption** algorithm is derived from $\mathsf{Dec}_{k_1}(\cdot)$ and $\mathsf{Vrfy}_{k_2}(\cdot)$, outputs $m \leftarrow \mathsf{Dec}'_{k_1,k_2}(c)$ or \bot .
- **Correctness requirement**: $Dec'_{k_1,k_2}(EncMac'_{k_1,k_2}(m)) = m$.

Defining Secure Message Transmission

The secure message transmission experiment $Auth_{A,\Pi'}(n)$:

- $1 k = (k_1, k_2) \leftarrow \text{Gen}'(1^n).$
- 2 \mathcal{A} is given input 1^n and oracle access to $\operatorname{EncMac'}_k$, and outputs $c \leftarrow \operatorname{EncMac'}_k(m)$.
- $3 \ m := \mathsf{Dec}_k'(c). \ \mathsf{Auth}_{\mathcal{A},\Pi'}(n) = 1 \iff m \neq \bot \land \ m \notin \mathcal{Q}.$

Definition 2

 Π' achieves authenticated communication if \forall PPT $\mathcal{A},$ \exists negl such that

$$\Pr[\mathsf{Auth}_{\mathcal{A},\Pi'}(n) = 1] \le \mathsf{negl}(n).$$

Definition 3

 Π' is **secure Authenticated Encryption (A.E.)** if it is both CCA-secure and also achieves authenticated communication.

Does CCA-security imply A.E.? (homework)

Questions

Suppose (E,D) provides A.E. Which of the following systems provide A.E.?

$$lacksquare E_k'(m) = (E_k(m), E_k(m)) \text{ and } D_k'(c_1, c_2) = D_k(c_1)$$

Combining Encryption and Authentication



■ Encrypt-and-authenticate (e.g., SSH):

$$c \leftarrow \mathsf{Enc}_{k_1}(m), \ t \leftarrow \mathsf{Mac}_{k_2}(m).$$

Authenticate-then-encrypt (e.g, SSL):

$$t \leftarrow \mathsf{Mac}_{k_2}(m), \ c \leftarrow \mathsf{Enc}_{k_1}(m||t).$$

■ Encrypt-then-authenticate (e.g, IPsec):

$$c \leftarrow \mathsf{Enc}_{k_1}(m), \ t \leftarrow \mathsf{Mac}_{k_2}(c).$$

Analyzing Security of Combinations

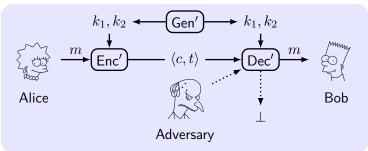
All-or-nothing: Reject any combination for which there exists even a single counterexample is insecure.

- **Encrypt-and-authenticate**: $Mac'_k(m) = (m, Mac_k(m))$.
- Authenticate-then-encrypt:
 - Trans : $0 \rightarrow 00$; $1 \rightarrow 10/01$; Enc' uses CTR mode; c = Enc'(Trans(m||Mac(m))).
 - Flip the first two bits of c and verify whether the ciphertext is valid. $10/01 \rightarrow 01/10 \rightarrow 1$, $00 \rightarrow 11 \rightarrow \bot$.
 - If valid, the first bit of message is 1; otherwise 0.
 - For any MAC, this is not CCA-secure.
- Encrypt-then-authenticate:

Decryption: If $Vrfy(\cdot) = 1$, then $Dec(\cdot)$; otherwise output \bot .

Constructing Authenticated Encryption Schemes

Idea: Make decryption oracle useless. AE(/CCA-secure) = CPA-then-MAC.



Construction 4

 $\Pi_E = (\mathsf{Gen}_E, \mathsf{Enc}, \mathsf{Dec}), \ \Pi_M = (\mathsf{Gen}_M, \mathsf{Mac}, \mathsf{Vrfy}). \ \Pi'$:

- $\operatorname{\mathsf{Gen}}'(1^n)$: $k_1 \leftarrow \operatorname{\mathsf{Gen}}_E(1^n)$ and $k_2 \leftarrow \operatorname{\mathsf{Gen}}_M(1^n)$
- $\operatorname{Enc}'_{k_1,k_2}(m)$: $c \leftarrow \operatorname{Enc}_{k_1}(m)$, $t \leftarrow \operatorname{Mac}_{k_2}(c)$ and output $\langle c, t \rangle$
- $\operatorname{Dec}'_{k_1,k_2}(\langle c,t\rangle) = \operatorname{Dec}_{k_1}(c)$ if $\operatorname{Vrfy}_{k_2}(c,t) \stackrel{?}{=} 1$; otherwise \bot

Proof of AE(/CCA-Secure) Encryption Schemes

Theorem 5

If Π_E is a CPA-secure private-key encryption scheme and Π_M is a secure MAC with unique tags, then Construction Π' is AE(/CCA-secure).

Proof.

VQ: ${\cal A}$ submits a "new" query to oracle ${\sf Dec}'$ and ${\sf Vrfy}=1.$

$$\Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1] \leq \Pr[\mathsf{VQ}] + \Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1 \wedge \overline{\mathsf{VQ}}]$$

We need to prove the following claims.

- $\mathbf{1}$ $\Pr[VQ]$ is negligible.
- $Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1 \wedge \overline{\mathsf{VQ}}] \leq \frac{1}{2} + \mathsf{negl}(n).$

Proof of "Pr[VQ] is negligible"

Idea: Reduce A_M (attacking Π_M with an oracle $\mathsf{Mac}_{k_2}(\cdot)$) to A.

Proof.

- lacksquare \mathcal{A}_M chooses $i \leftarrow \{1, \dots, q(n)\}$ u.a.r.
- \blacksquare Run \mathcal{A} with the encryption/decryption oracles.
- If the ith decryption oracle query from $\mathcal A$ uses a "new" c, output (c,t) and stop.
- Macforge_{A_M,Π_M}(n) = 1 only if VQ occurs.
- A_M correctly guesses i with probability 1/q(n).

$$\Pr[\mathsf{Macforge}_{\mathcal{A}_M,\Pi_M}(n) = 1] \ge \Pr[\mathsf{VQ}]/q(n).$$

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Proof of " $\Pr[\operatorname{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1 \land \overline{\mathsf{VQ}}] \leq \frac{1}{2} + \mathsf{negl}(n)$ "

Idea: Reduce A_E (attacking Π_E with an oracle $\operatorname{Enc}_{k_1}(\cdot)$) to A.

Proof.

- lacksquare Run ${\cal A}$ with the encryption/decryption oracles.
- Run PrivK $_{\mathcal{A}_{E},\Pi_{E}}^{\mathsf{cpa}}$ as PrivK $_{\mathcal{A},\Pi'}^{\mathsf{cca}}$.
- lacksquare \mathcal{A}_E outputs the same b' that is output by \mathcal{A}_{\cdot}
- $\Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A}_E,\Pi_E}(n) = 1 \land \overline{\mathsf{VQ}}] = \Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1 \land \overline{\mathsf{VQ}}]$ unless VQ occurs.

$$\Pr[\mathsf{PrivK}^{\mathsf{cpa}}_{\mathcal{A}_E,\Pi_E}(n) = 1] \geq \Pr[\mathsf{PrivK}^{\mathsf{cca}}_{\mathcal{A},\Pi'}(n) = 1 \wedge \overline{\mathsf{VQ}}].$$

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Authenticated Encryption Theory and Practice

Theorem 6

 Π_E is CPA-secure and Π_M is a secure MAC with unique tages, Π' deriving from encrypt-then-authenticate approach is secure.

GCM(Galois/Counter Mode): CTR encryption then Galois MAC. (RFC4106/4543/5647/5288 on IPsec/SSH/TLS) **EAX**: CTR encryption then CMAC (Cipher-based MAC).

Proposition 7

Authenticate-then-encrypt approach is secure if Π_E is rand-CTR mode or rand-CBC mode.

CCM (Counter with CBC-MAC): CBC-MAC then CTR encryption. (802.11i, RFC3610)

OCB (Offset Codebook Mode): integrating MAC into encryption. (two times fast as CCM, EAX)

All support AEAD (A.E. with associated data): part of message is in clear, and all is authenticated

Remarks on Secure Message Transmission

- Authentication may leak the message.
- Secure message transmission implies CCA-security. The opposite direction is not necessarily true.
- Different security goals should always use different keys.
 - lacksquare otherwise, the message may be leaked if $\mathsf{Mac}_k(c) = \mathsf{Dec}_k(c)$.
- Implementation may destroy the security proved by theory.
 - Attack with padding oracle (in TLS 1.0):
 Dec return two types of error: padding error, MAC error.
 Adv. learns last bytes if no padding error with guessed bytes.
 - Attack non-atomic dec. (in SSH Binary Packet Protocol): Dec (1)decrypt length field; (2)read packets as specified by the length; (3)check MAC.
 - **Adv.** (1)send c; (2)send l packets until "MAC error" occurs; (3)learn l = Dec(c).

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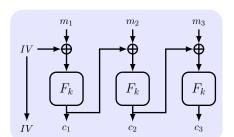
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Deterministic CPA Security

- **Application**: encrypted database index, disk encryption
- **Deterministic encryption**: the same message is encrypted to the same ciphertext under the same key.

Q: would it be CPA-secure?

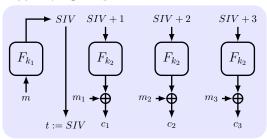
- **Deterministic CPA Security**: CPA-secure if *never encrypt same message twice* using same key. The pair $\langle k, m \rangle$ is unique.
- Deterministic Authenticated Encryption (DAE)
- **Common Mistake**: CBC/CTR with **fixed** *IV*.



Adversary can query (m_{q1},m_{q2}) and get (c_{q1},c_{q2}) ; then output PT: $IV\oplus c_{q1}\oplus m_{q2}$ and expect CT: c_{q2} .

Synthetic IV (SIV) for Det. Encryption

- SIV (fixed IV for the same $\langle k, m \rangle$): PRF F, CPA-secure Π : (Enc_k(r, m), Dec_k(r, s)) $(k_1, k_2) \leftarrow$ Gen; $SIV \leftarrow F_{k_1}(m)$ $c = \langle SIV, \text{Enc}_{k_2}(SIV, m) \rangle$.
- DAE for free with SIV-CTR:

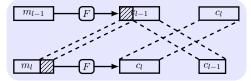


Wide Block PRP for Det. Encryption

- Wide block PRP: PRP with longer block length (e.g. a sector on disk) from PRP with short block length (e.g. AES).
- **PRP-based DAE**: If F is PRP, then F is also det. CPA-secure. $\operatorname{Enc}_k(m||0^\ell)$. In Dec, if $\neq 0^\ell$, output \perp .
- Narrow block may leak info. as some blocks are the same.
- **Standards**: CBC-mask-CBC (CMC) and ECB-mask-ECB (EME) in IEEE P1619.2.
- **Cost**: 2x slower than SIV due to two-pass encryption.

Tweakable Encryption

- **Encryption without expansion**: $\mathcal{M} = \mathcal{C}$ implies det. encryption without integrity (e.g., disk encryption).
- **Tweak**: like *IV*, different tweak for different block.
- Trivial solution: $k_t = F_k(t), t = 1, \dots, \ell$.
- Tweakable block ciphers: many PRPs from one key $\mathcal{K} \times \mathcal{T} \times \mathcal{X} \to \mathcal{X}$, \mathcal{T} is the set of tweaks.
- XTS: XEX(Xor-Encrypt-Xor)-based tweaked-codebook mode with ciphertext stealing. (XTS-AES, NIST SP 800-38E)
- **XEX**: To encrypt block j in sector I, $c = F_k(m \oplus x) \oplus x$, where $x = F_k(I) \otimes 2^j$ in Galois field, (I, j) is tweak.
- Ciphertext stealing (CTS): no padding, no expansion.



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Key Derivation Function (KDF)

Key Derivation Function (KDF) generates many keys from a secret source key sk.

For uniformly random sk: F is PRF, ctx is a unique string identifying application,

$$\mathsf{KDF}(sk,ctx,l) = \langle F_{sk}(ctx||0), F_{sk}(ctx||1) \cdots, F_{sk}(ctx||l) \rangle.$$

For not-uniform sk: extract-then-expand paradigm.

extract: HKDF $k \leftarrow \mathsf{HMAC}(salt, sk)$. salt is a random number.

expand: as the above.

Password-Based KDF (PBKDF)

Key stretching increases the time of testing key (with slow hash function).

Key strengthening increases the length/randomness of key (with salt).

PKCS#5 (PBKDF1): $H^{(c)}(pwd||salt)$, iterate hash function c times.

Attacker: either try the enhanced key (larger key space), or else try the initial key (longer time per key).

IV, Nonce, Counter and Salt

IV an input to a cryptographic primitive, providing randomness.
 nonce a number used only once to sign a communication.
 counter a sequence number used as nonce or IV.
 tweak an input used only once for each block in a cipher.
 salt consists of random bits, creating the input to a function.

Summary

- CCA-secure, Authenticated Encryption, Enc-then-Auth(/MAC).
- SIV, Wide Block Cipher, Tweakable Encryption, CTS.
- PBKDF, salt.