Digital Signature

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Outline

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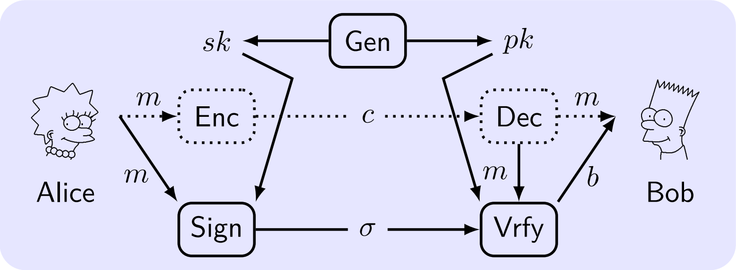
Digital Signatures – An Overview

- **Digital signature scheme** is a mathematical scheme for demonstrating the authenticity/integrity of a digital message
- allow a **signer** S to "**sign**" a message with its own sk, anyone who knows S's pk can **verify** the authenticity/integrity
- (Comparing to MAC) digital signature is:
  - publicly verifiable
  - transferable
  - non-repudiation
  - but slow
- Q: What are the differences between digital signatures and handwritten signatures?
- Digital signature is NOT the "inverse" of public-key encryption

The Syntax of Digital Signature Scheme



- signature σ, a bit b means valid if $b = 1$; invalid if $b = 0$.
- **Key-generation** algorithm $(pk, sk) \leftarrow \mathsf{Gen}(1^n), |pk|, |sk| \geq n$.
- **Signing** algorithm $\sigma \leftarrow \mathsf{Sign}_{sk}(m)$.
- **Verification** algorithm $b := \mathsf{Vrfy}_{pk}(m, \sigma)$.
- **Basic correctness requirement**: $\mathsf{Vrfy}_{pk}(m, \mathsf{Sign}_{sk}(m)) = 1$.

Defining Signature Security

The signature experiment $\mathsf{Sigforge}_{\mathcal{A},\Pi}(n)$:

1. $(pk, sk) \leftarrow \mathsf{Gen}(1^n)$.
2. $\mathcal{A}$ is given input 1^n and oracle access to $\mathsf{Sign}_{sk}(\cdot)$, and outputs (m, σ). $\mathcal{Q}$ is the set of queries to its oracle.
3. $\mathsf{Sigforge}_{\mathcal{A},\Pi}(n) = 1 \iff \mathsf{Vrfy}_{pk}(m, \sigma) = 1 \wedge m \notin \mathcal{Q}$.

Definition 1

A signature scheme Π is **existentially unforgeable under an adaptive CMA** if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that:

$$\Pr[\mathsf{Sigforge}_{\mathcal{A},\Pi}(n) = 1] \leq \mathsf{negl}(n).$$

Q: What's the difference on the ability of adversary between MAC and digital signature? What if an adversary is not limited to PPT?

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Insecurity of "Textbook RSA"

Construction 2

- Gen*: on input* 1^n *run* GenRSA(1^n) *to obtain* N, e, d*.*
  $pk = \langle N, e \rangle$ *and* $sk = \langle N, d \rangle$*.*
- Sign*: on input* sk *and* $m \in \mathbb{Z}_N^*$, $\sigma := [m^d \bmod N]$*.*
- Vrfy*: on input* pk *and* $m \in \mathbb{Z}_N^*$, $m \stackrel{?}{=} [\sigma^e \bmod N]$*.*

- **A no-message attack**: choose an arbitrary $\sigma \in \mathbb{Z}_N^*$ and compute $m := [\sigma^e \bmod N]$. Output the forgery (m, σ).

$pk = \langle 15, 3 \rangle, \ \sigma = 2, \ m =? \ m^d =?$

- **Forging a signature on an arbitrary message**:
  To forge a signature on m, choose a random m_1, set $m_2 := [m/m_1 \bmod N]$, obtain signatures σ_1, σ_2 on m_1, m_2.
  Q: $\sigma := [\_\_\_\_ \bmod N]$ is a valid signature on m.

Hashed RSA

- Gen: a hash function $H : \{0,1\}^* \to \mathbb{Z}_N^*$ is part of public key.
- Sign: $\sigma := [H(m)^d \bmod N]$.
- Vrfy: $\sigma^e \stackrel{?}{=} H(m) \bmod N$.

If H is not efficiently invertible, then the no-message attack and forging a signature on an arbitrary message is difficult.

Insecurity

There is NO known function H for which hashed RSA signatures are secure.

RSA-FDH Signature Scheme: Random Oracle as a **Full Domain Hash (FDH)** whose image size $=$ the RSA modulus $N-1$.

The "Hash-and-Sign" Paradigm

Construction 3

$\Pi = (\mathsf{Gen}_S, \mathsf{Sign}, \mathsf{Vrfy})$, $\Pi_H = (\mathsf{Gen}_H, H)$. *A signature scheme* Π'*:*

- Gen'*: on input* 1^n *run* $\mathsf{Gen}_S(1^n)$ *to obtain* (pk, sk)*, and run* $\mathsf{Gen}_H(1^n)$ *to obtain* s*. The public key is* $pk' = \langle pk, s \rangle$ *and the private key is* $sk' = \langle sk, s \rangle$*.*
- Sign'*: on input* sk' *and* $m \in \{0,1\}^*$*,* $\sigma \leftarrow \mathsf{Sign}_{sk}(H^s(m))$*.*
- Vrfy'*: on input* pk'*,* $m \in \{0,1\}^*$ *and* σ*, output 1* $\iff$ $\mathsf{Vrfy}_{pk}(H^s(m), \sigma) = 1$*.*

Theorem 4

If Π *is existentially unforgeable under an adaptive CMA and* Π_H *is collision resistant, then Construction is existentially unforgeable under an adaptive CMA.*

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Identification Schemes

An identification scheme $\Pi = (\mathsf{Gen}, \mathcal{P}_1, \mathcal{P}_2, \mathcal{V})$ is a 3-round protocol between the prover and the verifier. The attacker can do eavesdropping and has an access to an oracle Trans_{sk} to learn (I, r, s) by executing the protocol as a verifier.

Prover (sk) | Verifier (pk)

$(I, \mathsf{st}) \leftarrow \mathcal{P}_1(sk) \xrightarrow{\quad I \quad}$

$\xleftarrow{\quad r \quad} \quad r \leftarrow \Omega_{pk}$

$s := \mathcal{P}_2(sk, \mathsf{st}, r) \xrightarrow{\quad s \quad}$

$\mathcal{V}(pk, r, s) \stackrel{?}{=} I$

Identification Schemes: Definition

The identification experiment $\mathsf{Ident}_{\mathcal{A},\Pi}(n)$:

1. $(pk, sk) \leftarrow \mathsf{Gen}(1^n)$.
2. $\mathcal{A}$ is given input 1^n and oracle access to $\mathsf{Trans}_{sk}(\cdot)$, and outputs a message I.
3. A uniform challenge r is chosen and given to $\mathcal{A}$, and $\mathcal{A}$ outpus s. ($\mathcal{A}$ may continue to query the oracle.)
4. $\mathsf{Ident}_{\mathcal{A},\Pi}(n) = 1 \iff \mathcal{V}(pk, r, s) \stackrel{?}{=} I$.

Definition 5

An identification scheme $\Pi = (\mathsf{Gen}, \mathcal{P}_1, \mathcal{P}_2, \mathcal{V})$ is **secure** if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that:

$$\Pr[\mathsf{Ident}_{\mathcal{A},\Pi}(n) = 1] \leq \mathsf{negl}(n).$$

The Schnorr Identification Scheme

Prover (x)

Verifier $(\mathbb{G}, q, g, y)$

$$k \leftarrow \mathbb{Z}_q;\ I := g^k \xrightarrow{\quad I \quad}$$

$$\xleftarrow{\quad r \quad} \quad r \leftarrow \mathbb{Z}_q$$

$$s := [rx + k \mod q] \xrightarrow{\quad s \quad}$$

$$g^s \cdot y^{-r} \stackrel{?}{=} I$$

Theorem 6

If the discrete-log problem is hard, then the Schnorr identification scheme is secure.

Proof of the Schnorr Identification Scheme

Idea: If the attacker can let $g^s \cdot y^{-r} = I$, then the attacker can compute x.

Proof.

Reduce $\mathcal{A}'$ inverting y to $\mathcal{A}$ attacking the Schnorr scheme:

1. $\mathcal{A}'$ as a verifier, answering all queries, runs $\mathcal{A}$ as a prover.
2. When $\mathcal{A}$ outputs I, $\mathcal{A}'$ choose $r_1 \in \mathbb{Z}_q$ and give it to $\mathcal{A}$, who responds with s_1.
3. Run $\mathcal{A}$ a second time, send $r_2 \in \mathbb{Z}_q$ to $\mathcal{A}$ who responds with s_2.
4. If $g^{s_1} \cdot h^{-r_1} = I$ and $g^{s_2} \cdot h^{-r_2} = I$ and $r_1 \neq r_2$ then output $x = [(s_1 - s_2) \cdot (r_1 - r_2)^{-1} \mod q]$. Else, output nothing.

$\square$

The Fiat-Shamir Transform

The Fiat-Shamir transform constructs a (non-interactive) signature scheme by letting the signer run the protocol by itself.

Construction 7

Let $\Pi = (\mathsf{Gen}_{\mathsf{id}}, \mathcal{P}_1, \mathcal{P}_2, \mathcal{V})$ *be an identification scheme.*

- Gen*:* $(pk, sk) \leftarrow \mathsf{Gen}_{\mathsf{id}}$*. A function* $H : \{0,1\}^* \to \Omega_{pk}$ *(a set of challenges).*
- Sign*: On input* sk *and* $m \in \{0,1\}^*$*, do*
  1. *Compute* $(I, \mathsf{st}) \leftarrow \mathcal{P}_1(sk)$
  2. *Compute* $r := H(I, m)$
  3. *Compute* $s := \mathcal{P}_2(sk, \mathsf{st}, r)$

  Outpus the signature r, s*.*
- Vrfy*:* $I := \mathcal{V}(pk, r, s)$*. Output* $1 \iff H(I, m) \stackrel{?}{=} r$*.*

Theorem 8

If Π *is a secure identification scheme and* H *is a random oracle, then the Fiat-Shamir transform results a secure signature scheme.*

The Schnorr Signature Scheme

Construction 9

- Gen*: $(\mathcal{G}, q, g) \leftarrow \mathcal{G}(1^n)$. Choose $x \in \mathbb{Z}_q$ and set $y := g^x$. The private key is x and the public key is $(\mathcal{G}, q, g, y)$. A function $H : \{0,1\}^* \rightarrow \mathbb{Z}_q$.*
- Sign*: On input x and $m \in \{0,1\}^*$, do*
  1. *Compute $I := g^k$, where a uniform $k \in \mathbb{Z}_q$*
  2. *Compute $r := H(I, m)$*
  3. *Compute $s := [rx + k \mod q]$*

  Outpus the signature (r, s).
- Vrfy*: Compute $I := g^s \cdot y^{-r}$ and output $1 \iff H(I, m) \stackrel{?}{=} r$.*

NIST published DSS (Digital Signature Standard) which uses Digital Signature Algorithm (DSA, a variant of ElGamal signature scheme), Elliptic Curve Digital Signature Algorithm (ECDSA), and RSA Signature Algorithm.

Construction 10

- Gen: $(\mathbb{G}, q, g) \leftarrow \mathcal{G}$. *Two hash functions* $H, F : \{0,1\}^* \to \mathbb{Z}_q$.
  $x \leftarrow \mathbb{Z}_q$ *and* $y := g^x$.
  $pk = \langle \mathbb{G}, q, g, y, H, F \rangle$. $sk = \langle \mathbb{G}, q, g, x, H, F \rangle$.
- Sign: $k \leftarrow \mathbb{Z}_q^*$ *and* $r := F(g^k)$, $s := (H(m) + xr) \cdot k^{-1}$.
  Output (r, s).
- Vrfy: *Output* $1 \iff r \stackrel{?}{=} F(g^{H(m) \cdot s^{-1}} y^{r \cdot s^{-1}})$.

Q: Is the verification correct?

Security of DSS/DSA

Insecurity

Security of DSS relies on the hardness of discrete log problem. But NO proof of security for DSS based on discrete log assumption.

The entropy, secrecy and uniqueness of k is critical.

- Case I: If k is predictable, then x leaks, since $s := [(H(m) + xr) \cdot k^{-1} \bmod q]$, and only x is known;
- Case II: If the same k is ever used to generate two different signatures under the same x, then both k and x leaks.
  Q: how?
  This attack has been used to learn the private key of the Sony PlayStation (PS3) in 2010.

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One-Time Signature (OTS)

One-Time Signature (OTS): Under a weaker attack scenario, sign only one message with one secret.

The OTS experiment $\mathsf{Sigforge}^{\text{1-time}}_{\mathcal{A},\Pi}(n)$:

1. $(pk, sk) \leftarrow \mathsf{Gen}(1^n)$.
2. $\mathcal{A}$ is given input 1^n and a single query m' to $\mathsf{Sign}_{sk}(\cdot)$, and outputs (m, σ), $m \neq m'$.
3. $\mathsf{Sigforge}^{\text{1-time}}_{\mathcal{A},\Pi}(n) = 1 \iff \mathsf{Vrfy}_{pk}(m, \sigma) = 1$.

Definition 11

A signature scheme Π is **existentially unforgeable under a single-message attack** if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that:

$$\Pr[\mathsf{Sigforge}^{\text{1-time}}_{\mathcal{A},\Pi}(n) = 1] \leq \mathsf{negl}(n).$$

Lamport's OTS (1979)

Idea: OTS from OWF; one mapping per bit.

Construction 12

f is a one-way function.

- Gen*: on input 1^n, for $i \in \{1, \ldots, \ell\}$:*
  1. *choose random $x_{i,0}, x_{i,1} \leftarrow \{0,1\}^n$.*
  2. *compute $y_{i,0} := f(x_{i,0})$ and $y_{i,1} := f(x_{i,1})$.*

$$pk = \begin{pmatrix} y_{1,0} & y_{2,0} & \cdots & y_{\ell,0} \\ y_{1,1} & y_{2,1} & \cdots & y_{\ell,1} \end{pmatrix} \quad sk = \begin{pmatrix} x_{1,0} & x_{2,0} & \cdots & x_{\ell,0} \\ x_{1,1} & x_{2,1} & \cdots & x_{\ell,1} \end{pmatrix}.$$

- Sign*: $m = m_1 \cdots m_\ell$, output $\sigma = (x_{1,m_1}, \ldots, x_{\ell,m_\ell})$.*
- Vrfy*: $\sigma = (x_1, \ldots, x_\ell)$, output $1 \iff f(x_i) = y_{i,m_i}$, for all i.*

Theorem 13

If f is OWF, Π is OTS for messages of length polynomial ℓ.

Example of Lamport's OTS

Signing $m = 011$

$$sk = \begin{pmatrix} x_{1,0} & x_{2,0} & x_{3,0} \\ x_{1,1} & x_{2,1} & x_{3,1} \end{pmatrix} \implies \sigma = \underline{\qquad}$$

$\sigma = (x_1, x_2, x_3)$:

$$pk = \begin{pmatrix} y_{1,0} & y_{2,0} & y_{3,0} \\ y_{1,1} & y_{2,1} & y_{3,1} \end{pmatrix} \implies \begin{array}{l} f(x_1) \stackrel{?}{=} \underline{\qquad} \\ f(x_2) \stackrel{?}{=} \underline{\qquad} \\ f(x_3) \stackrel{?}{=} \underline{\qquad} \end{array}$$

Proof of Lamport's OTS Security

Idea: If $m \neq m'$, then $\exists i^*, m_{i*} = b^* \neq m'_{i*}$. So to forge a signature on m can invert a single y_{i^*,b^*} at least.

Proof.

Reduce $\mathcal{I}$ inverting y to $\mathcal{A}$ attacking Π:

1. Construct pk: Choose $i^* \leftarrow \{1, \ldots, \ell\}$ and $b^* \leftarrow \{0, 1\}$, set $y_{i^*,b^*} := y$. For $i \neq i^*$, $y_{i,b} := f(x_{i,b})$.
2. $\mathcal{A}$ queries m': If $m'_{i_*} = b^*$, stop. Otherwise, return $\sigma = (x_{1,m'_1}, \ldots, x_{\ell,m'_\ell})$.
3. When $\mathcal{A}$ outputs (m, σ), $\sigma = (x_1, \ldots, x_\ell)$, if $\mathcal{A}$ output a forgery at (i^*, b^*): $\mathsf{Vrfy}_{pk}(m, \sigma) = 1$ and $m_{i^*} = b^* \neq m'_{i^*}$, then output x_{i^*,b^*}.

$$\Pr[\mathcal{I} \text{ succeeds}] \geq \frac{1}{2\ell} \Pr[\mathcal{A} \text{ succeeds}]$$

□

Stateful Signature Scheme

Idea: OTS by signing with **“new” key** derived from **“old” state**.

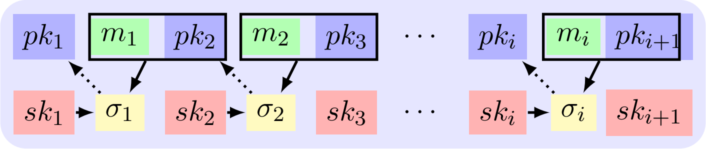
Definition 14 (Stateful signature scheme)

- **Key-generation** algorithm $(pk, sk, s_0) \leftarrow \mathsf{Gen}(1^n)$. s_0 is **initial state**.
- **Signing** algorithm $(\sigma, s_i) \leftarrow \mathsf{Sign}_{sk,s_{i-1}}(m)$.
- **Verification** algorithm $b := \mathsf{Vrfy}_{pk}(m, \sigma)$.

A simple stateful signature scheme for OTS:
Generate (pk_i, sk_i) independently, set $pk := (pk_1, \ldots, pk_\ell)$ and $sk := (sk_1, \ldots, sk_\ell)$.
Start from the state 1, sign the s-th message with sk_s, verify with pk_s, and update the state to $s + 1$.
Weakness: the upper bound ℓ must be fixed in advance.

"Chain-Based" Signatures

Idea: generate keys "on-the-fly" and sign the key chain.



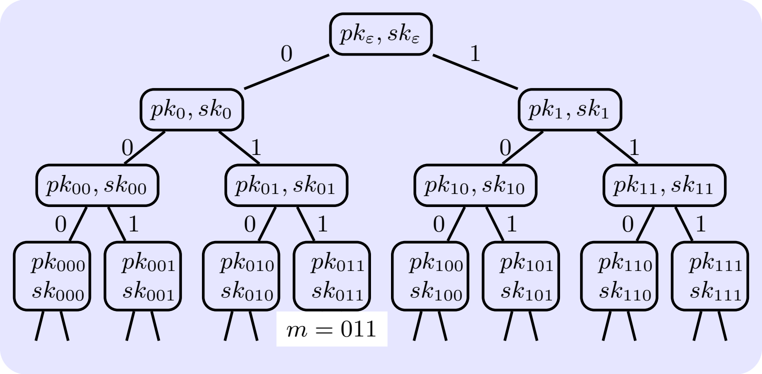
Use a single public key pk_1, sign each m_i and pk_{i+1} with sk_i:

$$\sigma_i \leftarrow \mathsf{Sign}_{sk_i}(m_i \| pk_{i+1}),$$

output $\langle pk_{i+1}, \sigma_i \rangle$, and verify σ_i with pk_i.
The signature is $(pk_{i+1}, \sigma_i, \{m_j, pk_{j+1}, \sigma_j\}_{j=1}^{i-1})$.
Weakness: stateful, not efficient, revealing all previous messages.

"Tree-Based" Signatures

Idea: generate a chain of keys for each message and sign the chain.



- root is ε (empty string), leaf is a message m, and internal nodes (pk_w, sk_w), where w is the prefix of m.
- each node pk_w "certifies" its children $pk_{w0} \| pk_{w1}$ or w.

A Stateless Solution

Idea: use deterministic randomness to emulate the state of tree.

Use PRF F and two keys k, k' (secrets) to generate pk_w, sk_w:

1. compute $r_w := F_k(w)$.
2. compute $(pk_w, sk_w) :=$ Gen$(1^n; r_w)$, using r_w as random coins.

k' is used to generate r'_w that is used to compute σ_w.

Lemma 15

If OWF exist, then $\exists$ OTS (for messages of arbitrary length).

Theorem 16

If OWF exists, then $\exists$ (stateless) secure signature scheme.

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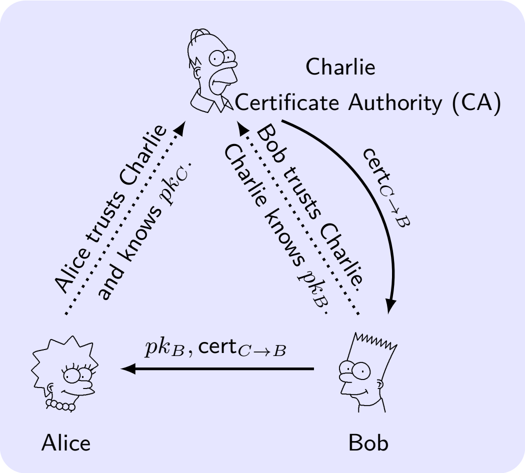
Certificates



Certificates $\mathsf{cert}_{C\to B} \stackrel{\text{def}}{=} \mathsf{Sign}_{sk_C}(\text{‘Bob's key is } pk_B\text{'}).$

How Alice learn CA's key? How CA learn Bob's key?

Public-Key Infrastructure (PKI)

- **A single CA**: is trusted by everybody.
  - Strength: simple
  - Weakness: single-point-of-failure
- **Multiple CAs**: are trusted by everybody.
  - Strength: robust
  - Weakness: cannikin law
- **Delegation and certificate chains**: The trust is transitive.
  - Strength: ease the burden on the root CA.
  - Weakness: difficult for management, cannikin law.
- **"Web of trust"**: No central points of trust, e.g., PGP.
  - Strength: robust, work at "grass-roots" level.
  - Weakness: difficult to manage/give a guarantee on trust.

Invalidating Certificates

- **Expiration**: include an *expiry date* in the certificate.

$$\mathsf{cert}_{C\to B} \stackrel{\mathsf{def}}{=} \mathsf{Sign}_{sk_C}(\text{‘bob's key is } pk_B\text{'}, \text{ date}).$$

- **Revocation**: explicitly revoke the certificate.

$$\mathsf{cert}_{C\to B} \stackrel{\mathsf{def}}{=} \mathsf{Sign}_{sk_C}(\text{‘bob's key is } pk_B\text{'}, \ \#\#\#).$$

"###" represents the serial number of this certificate.
Cumulated Revocation: CA generates *certificate revocation list* (CRL) containing the serial numbers of all revoked certificates, signs CRL with the current date.

Signcryption

Signcryption: which scheme is secure?

In a group, each has two pairs of keys: (ek, dk) for enc, and (vk, sk) for sig. And all public keys are distributed to everyone. How a sender S and a receiver R should do to secure privacy and authenticity?

- Enc-then-Auth I: send $\left\langle S, c \leftarrow \mathsf{Enc}_{ek_R}(m), \mathsf{Sign}_{sk_S}(c) \right\rangle$
- Auth-then-Enc I: $\sigma \leftarrow \mathsf{Sign}_{sk_S}(m)$, send $\langle S, \mathsf{Enc}_{ek_R}(m \| \sigma) \rangle$
- Auth-then-Enc II: $\sigma \leftarrow \mathsf{Sign}_{sk_S}(m \| R)$, send $\langle S, \mathsf{Enc}_{ek_R}(S \| m \| \sigma) \rangle$
- Any other method?

Summary

- Textbook RSA, Hashed RSA, Hash-and-Sign
- Identification, Fiat-Shamir Transform, Schnorr Signature, DSS/DSA
- Lamport's OTS, Stateful/Chain-based/Tree-based/Stateless Signature
- Certificates, PKI, CA, Web-of-trust, Invalidation