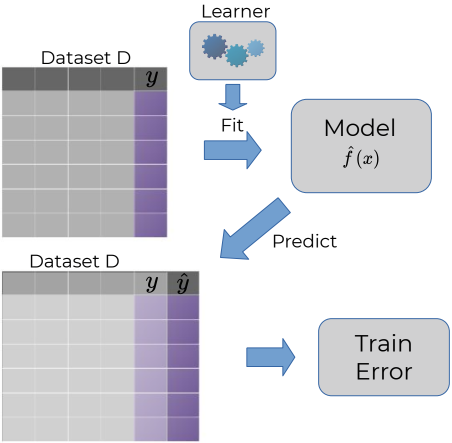
Introduction to Machine Learning

Evaluation: Training Error

compstat-lmu.github.io/lecture_i2ml

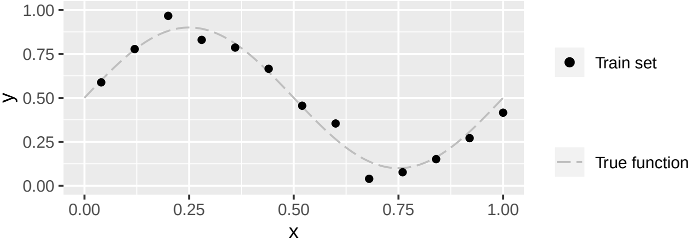
TRAINING ERROR

(also: apparent error / resubstitution error)



EXAMPLE: POLYNOMIAL REGRESSION

Sample data from sinusoidal function $0.5 + 0.4 \cdot \sin(2\pi x) + \epsilon$ with measurement error ϵ.

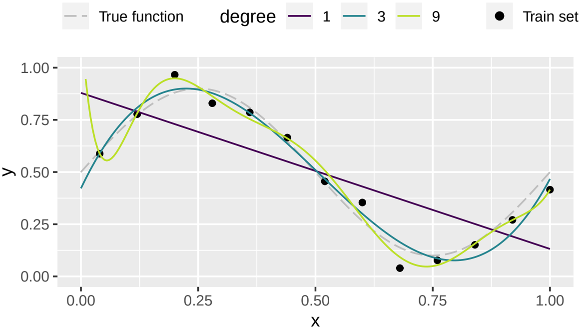


Assume data generating process unknown.
Try to approximate with a dth-degree polynomial:

$$f(\mathbf{x} \mid \boldsymbol{\theta}) = \theta_0 + \theta_1 x + \cdots + \theta_d x^d = \sum_{j=0}^{d} \theta_j x^j.$$

EXAMPLE: POLYNOMIAL REGRESSION

Models of different *complexity*, i.e., of different orders of the polynomial are fitted. How should we choose *d*?



- d=1: MSE = 0.036: Clear underfitting
- d=3: MSE = 0.003: Pretty OK?
- d=9: MSE = 0.001: Clear overfitting

Simply using the training error seems to be a bad idea.

TRAINING ERROR PROBLEMS

- Unreliable and overly optimistic estimator of future performance.
  E.g. training error of 1-NN is always zero, as each observation is its own NN during test time.
- Goodness-of-fit measures like (classical) R^2, likelihood, AIC, BIC, deviance are all based on the training error.
- For models of restricted capacity, and given enough data, the training error may provide reliable information.
  E.g. LM with $p = 5$ features, 10^6 training points.
  But: impossible to determine when training error becomes unreliable.