Exercise 1:

Summarize the logistic regression model (shortly!). Show that the decision boundaries, which are defined as the area where the estimated probability for class 1 is exactly a specific threshold $a \in [0, 1]$, is always a hyperplane between both classes. What happens if you are choosing $a = 0.5$?

Exercise 2:

Choose some of the classifiers already introduced in the lecture and visualize their decision boundaries for relevant hyperparameters. Use `mlbench::mlbench.spirals` to generate data and use `plotLearnerPrediction` for visualization.

Exercise 3:

a) What is the relationship between softmax $\pi_i(x) = \frac{e^{\theta_i^T x}}{\sum_k e^{\theta_k^T x}}$ and the logistic function $\pi(x) = \frac{1}{1+e^{\theta^T x}}$ for $k = 2$ (binary classification)?

b) Derive the negative log likelihood of softmax regression. Suppose there are n instances and p features.

c) Explain how the predictions of softmax regression (multiclass classification) looks like (probabilities and classes) and define the parameter space.

Exercise 4:

You are given the following table with the target variable `Banana`:

ID	Color	Form	Origin	Banana ?
1	yellow	oblong	imported	yes
2	yellow	round	domestic	no
3	yellow	oblong	imported	no
4	brown	oblong	imported	yes
5	brown	round	domestic	no
6	green	round	imported	yes
7	green	oblong	domestic	no
8	red	round	imported	no

a) We want to use a naive Bayes classifier to predict whether a new fruit is a Banana or not. Calculate the posterior probability $\pi(x)$ for a new observation (yellow, round, imported). How would you classify the object?

b) Assume you have an addtional feature "Length", which measures the length in cm. Describe in 1-2 sentences how you would handle this numeric feature with Naive Bayes.