Solution 1:

a) multiclass classification (plate digits) (supervised learning)
b) binary classification (supervised)
c) outlier detection ((un)supervised)
d) frequent pattern mining (unsupervised)
e) classification (supervised) / clustering (unsupervised)
f) classification (supervised)
g) clustering / assocation rules (unsupervised)
h) not a machine learning task
i) not a machine learning task

Solution 2:

a) We use the least squares-estimator introduced in the lecture: $\hat{\beta} = (X^T X)^{-1} X^T y$ with

$$X = \begin{bmatrix} 1 & x_{1,1} & x_{1,2} & ... & x_{1,m} \\ 2 & x_{2,1} & x_{2,2} & ... & x_{2,m} \\ \vdots & \vdots & \vdots & ... & \vdots \\ n & x_{n,1} & x_{n,2} & ... & x_{n,m} \end{bmatrix}$$

$$x = \begin{bmatrix} 0.56 \\ 0.22 \\ 1.7 \\ 0.63 \\ 0.36 \\ 1.2 \end{bmatrix}, X = \begin{bmatrix} 1 & 0.56 \\ 1 & 0.22 \\ 1 & 1.7 \\ 1 & 0.63 \\ 1 & 0.36 \\ 1 & 1.2 \end{bmatrix} \text{ and } y = \begin{bmatrix} 160 \\ 150 \\ 175 \\ 185 \\ 165 \\ 170 \end{bmatrix}$$

Then

$\hat{\beta} = (X^TX)^{-1}X^Ty$

$$= \left(\begin{bmatrix} 1 & 1 & 1 & ... & 1 \\ x_{1,1} & x_{2,1} & x_{3,1} & ... & x_{n,1} \\ \vdots & \vdots & \vdots & ... & \vdots \\ x_{1,m} & x_{2,m} & x_{3,m} & ... & x_{n,m} \end{bmatrix} \begin{bmatrix} 1 & x_{1,1} & x_{1,2} & ... & x_{1,m} \\ 1 & x_{2,1} & x_{2,2} & ... & x_{2,m} \\ \vdots & \vdots & \vdots & ... & \vdots \\ 1 & x_{n,1} & x_{n,2} & ... & x_{n,m} \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 1 & ...1 & \\ x_{1,1} & x_{2,1} & x_{3,1} & ... & x_{n,1} \\ \vdots & \vdots & \vdots & ... & \vdots \\ x_{1,m} & x_{2,m} & x_{3,m} & ... & x_{n,m} \end{bmatrix} \begin{bmatrix} 160 \\ 150 \\ 175 \\ 185 \\ 165 \\ 170 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0.56 & 0.22 & 1.7 & 0.63 & 0.36 & 1.2 \end{bmatrix} \begin{bmatrix} 1 & 0.56 \\ 1 & 0.22 \\ 1 & 1.7 \\ 1 & 0.63 \\ 1 & 0.36 \\ 1 & 1.2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0.56 & 0.22 & 1.7 & 0.63 & 0.36 & 1.2 \end{bmatrix} \begin{bmatrix} 160 \\ 150 \\ 175 \\ 185 \\ 165 \\ 170 \end{bmatrix}$$

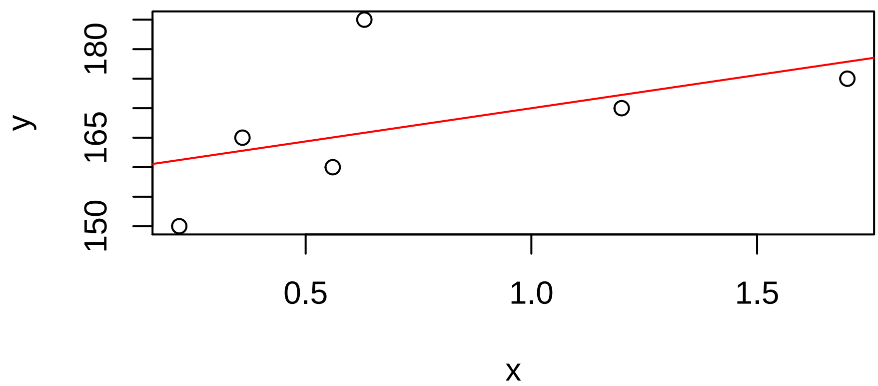
$$= \begin{bmatrix} 6 & 4.67 \\ 4.67 & 5.2185 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0.56 & 0.22 & 1.7 & 0.63 & 0.36 & 1.2 \end{bmatrix} \begin{bmatrix} 160 \\ 150 \\ 175 \\ 185 \\ 165 \\ 170 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5491944 & -0.4914703 \\ -0.4914703 & 0.6314394 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0.56 & 0.22 & 1.7 & 0.63 & 0.36 & 1.2 \end{bmatrix} \begin{bmatrix} 160 \\ 150 \\ 175 \\ 185 \\ 165 \\ 170 \end{bmatrix}$$

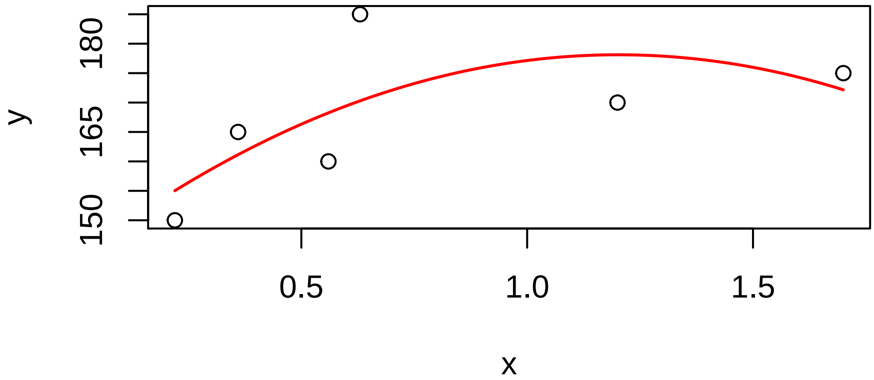
$$= \begin{bmatrix} 0.2739710 & 0.4410709 & -0.2863051 & 0.23956809 & 0.3722651 & -0.04056998 \\ -0.1378643 & -0.3525536 & 0.5819766 & -0.09366351 & -0.2641521 & 0.26625693 \end{bmatrix} \begin{bmatrix} 160 \\ 150 \\ 175 \\ 185 \\ 165 \\ 170 \end{bmatrix}$$

$$= \begin{bmatrix} 158.73954 \\ 11.25541 \end{bmatrix}$$

Hence the linear model $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x = 158.73954 + 11.25541x$



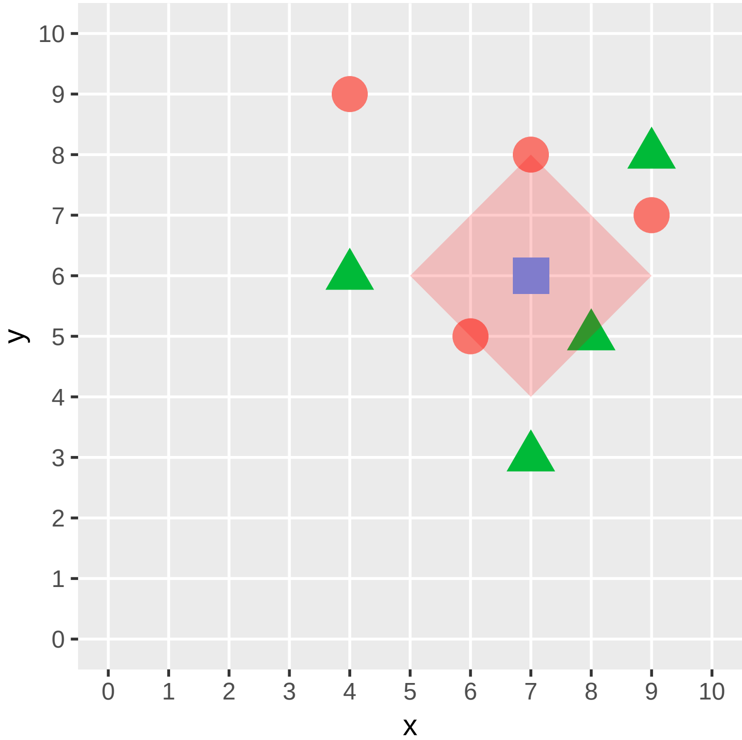
b) Here $X = \begin{bmatrix} 1 & 0.56 & 0.3136 \\ 1 & 0.22 & 0.0484 \\ 1 & 1.7 & 2.89 \\ 1 & 0.63 & 0.3969 \\ 1 & 0.36 & 0.1296 \\ 1 & 1.2 & 1.44 \end{bmatrix}$ and $\hat{\beta} = \begin{bmatrix} 143.51682 \\ 57.59155 \\ -23.96347 \end{bmatrix}$



Solution 3:

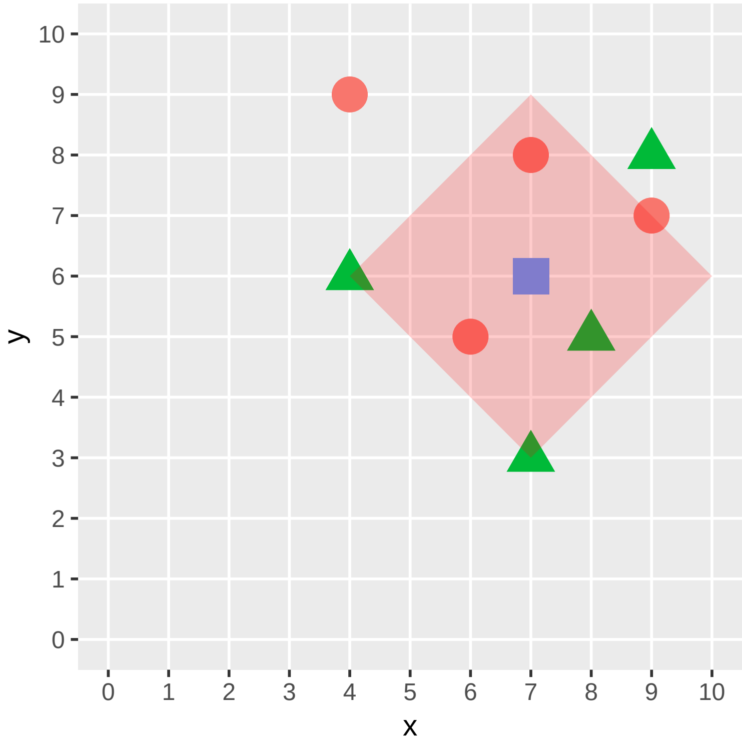
a) $k = 3$

2 circles and 1 triangle, so our point is also a circle



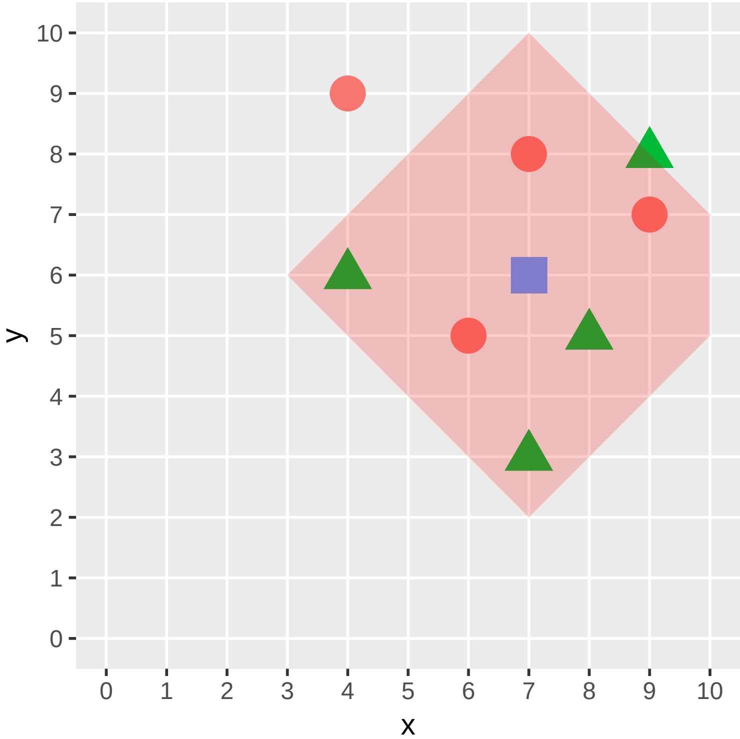
b) $k = 5$

3 circles and 3 triangles, we have to specify beforehand what to do in case of a tie



c) $k = 7$

3 circles and 4 triangles, so our point is also a triangle



Solution 4:

a) Learning consists of *representation*, *evaluation* and *optimization*.
A learner in mlr3 can be thought of as the implementation of these components, since

- a representation of the associated model learnt from the data by using the implemented optimization is stored in such a learner object,
- its performance measures can be accessed afterwards.

b)
```
library(mlr3)
library(mlr3learners)

# show all available learners
mlr_learners$keys()

##  [1] "classif.debug"        "classif.featureless" "classif.glmnet"
##  [4] "classif.kknn"         "classif.lda"         "classif.log_reg"
##  [7] "classif.multinom"     "classif.naive_bayes" "classif.qda"
## [10] "classif.ranger"       "classif.rpart"       "classif.svm"
## [13] "classif.xgboost"      "regr.featureless"    "regr.glmnet"
## [16] "regr.kknn"            "regr.km"             "regr.lm"
## [19] "regr.ranger"          "regr.rpart"          "regr.svm"
## [22] "regr.xgboost"

# see settings for a specific learner, e.g., for a regression tree
rpart_learner <- lrn("regr.rpart")
print(rpart_learner)
```

```
## <LearnerRegrRpart:regr.rpart>
## * Model: -
## * Parameters: xval=0
## * Packages: rpart
## * Predict Type: response
## * Feature types: logical, integer, numeric, factor, ordered
## * Properties: importance, missings, selected_features, weights
```

Solution 5:
See R code **lm_knn_l_1a.R**

Solution 6*:

Firstly note that for $n = 1$ the median $y_{\text{med}} = y^{(1)}$ obviously minimizes the empirical risk $\mathcal{R}_{\text{emp}}$ associated to the L1 loss L. Hence let $n > 1$ in the following:
Since for $a, b \in \mathbb{R}, S_{a,b} : \mathbb{R} \to \mathbb{R}_0^+, c \mapsto |a - c| + |b - c|$ it holds that

$$S_{a,b}(c) = \begin{cases} |a - b|, & \text{for } c \in [a, b] \\ |a - b| + 2 \cdot \min\{|a - c|, |b - c|\}, & \text{otherwise} \end{cases}$$

$c^* \in [a, b]$ minimizes $S_{a,b}$.
With this $\mathcal{R}_{\text{emp}}$ can be expressed for a constant $c \in \mathbb{R}$, s.t.

$$n \cdot \mathcal{R}_{\text{emp}}(c) = \sum_{i=1}^{n} L(y^{(i)}, c) = \sum_{i=1}^{n} |y^{(i)} - c| = \begin{cases} \sum_{i=1}^{\overbrace{n/2}^{=:i_{\max}}} \overbrace{S_{y^{(i)}, y^{(n+1-i)}}(c)}^{=:S_i(c)}, & \text{for } n \text{ is even} \\ (\sum_{i=1}^{\overbrace{(n-1)/2}^{=:i_{\max}}} \underbrace{S_{y^{(i)}, y^{(n+1-i)}}(c)}_{=:S_i(c)}) + \underbrace{|y^{(n+1)/2} - c|}_{=:S_0(c)}, & \text{for } n \text{ is odd} \end{cases}.$$

Now we define for $i \in \{1, \dots, i_{\max}\}$ $\mathcal{I}_i := [y^{(i)}, y^{(n+1-i)}]$.
From construction it follows that for $j \in \{1, \dots, i_{\max} - 1\}$

$$\mathcal{I}_{j+1} \subseteq \mathcal{I}_j \Rightarrow \forall i \in \{1, \dots, i_{\max}\} : \mathcal{I}_{i_{\max}} \subseteq \mathcal{I}_i.$$

From this it follows, that

- for "n is even": $c^* \in \mathcal{I}_{i_{\max}} = [y^{(n/2)}, y^{(n/2+1)}]$ minimizes S_i for all $i \in \{1, \dots, i_{\max}\}$ $\Rightarrow \mathcal{R}_{\text{emp}}$ reaches its global minimum at c^*,
- for "n is odd": $c^* = y^{(n+1)/2} \in \mathcal{I}_{i_{\max}}$ minimizes S_i for all $i \in \{0, 1, \dots, i_{\max}\}$ $\Rightarrow \mathcal{R}_{\text{emp}}$ reaches its global minimum at c^*.

Since the median fulfills these conditions, we can conclude that it minimizes the L1 loss.