Introduction to Machine Learning

Evaluation: Simple Measures for Classification

compstat-lmu.github.io/lecture_i2ml

LABELS VS PROBABILITIES

In classification we predict:

1. Class labels $\rightarrow \hat{h}(\mathbf{x}) = \hat{y}$
2. Class probabilites $\rightarrow \hat{\pi}_k(\mathbf{x})$

$\rightarrow$ We evaluate based on those

LABELS: MCE

The misclassification error rate (MCE) counts the number of incorrect predictions and presents them as a rate:

$$MCE = \frac{1}{n} \sum_{i=1}^{n} [y^{(i)} \neq \hat{y}^{(i)}] \in [0; 1]$$

Accuracy is defined in a similar fashion for correct classifications:

$$ACC = \frac{1}{n} \sum_{i=1}^{n} [y^{(i)} = \hat{y}^{(i)}] \in [0; 1]$$

- If the data set is small this can be brittle
- The MCE says nothing about how good/skewed predicted probabilities are
- Errors on all classes are weighed equally (often inappropriate)

LABELS: CONFUSION MATRIX

True classes in columns.
Predicted classes in rows.

```
           setosa versicolor virginica -err.- -n-
setosa         50          0         0      0  50
versicolor      0         46         4      4  50
virginica       0          4        46      4  50
-err.-          0          4         4      8  NA
-n-            50         50        50     NA 150
```

We can see class sizes (predicted and true) and where errors occur.

LABELS: CONFUSION MATRIX

In binary classification

		True Class y	
		$+$	$-$
Pred.	$+$	True Positive (TP)	False Positive (FP)
$\hat{y}$	$-$	False Negative (FN)	True Negative (TN)

LABELS: COSTS

We can also assign different costs to different errors via a cost matrix.

$$Costs = \frac{1}{n} \sum_{i=1}^{n} C[y^{(i)}, \hat{y}^{(i)}]$$

Example:
Predict if person has a ticket (yes / no).
Should train conductor check ticket of a person?

Costs:

Ticket checking:	3 EUR
Fee for fare-dodging:	40 EUR

http://www.oslobilder.no/OMU/OB.%C3%9864/2902

LABELS: COSTS

Predict if person has a ticket (yes / no).

```
Cost matrix C
     predicted
true   no yes
  no  -37  40
  yes   3   0
```

```
Confusion matrix
     predicted
true  no yes
  no   7   0
  yes 93   0
```

```
Confusion matrix * C
     predicted
true    no yes
  no  -259   0
  yes  279   0
```

Costs:

Ticket checking:	3 EUR
Fee for fare-dodging:	40 EUR

Our model says that we should not trust anyone and check the tickets of all passengers.

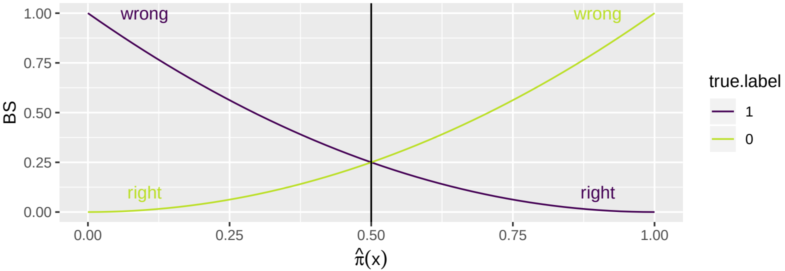
$$
\begin{aligned}
Costs &= \frac{1}{n}\sum_{i=1}^{n} C[y^{(i)}, \hat{y}^{(i)}] \\
&= \frac{1}{100}\left(-37 \cdot 7 + 40 \cdot 0 + 3 \cdot 93 + 0 \cdot 0\right) \\
&= \frac{20}{100} = 0.2
\end{aligned}
$$

PROBABILITIES: BRIER SCORE

Measures squared distances of probabilities from the true class labels:

$$BS1 = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{\pi}(\mathbf{x}^{(i)}) - y^{(i)}\right)^2$$

- Fancy name for MSE on probabilities
- Usual definition for binary case, $y^{(i)}$ must be coded as 0 and 1.



PROBABILITIES: BRIER SCORE

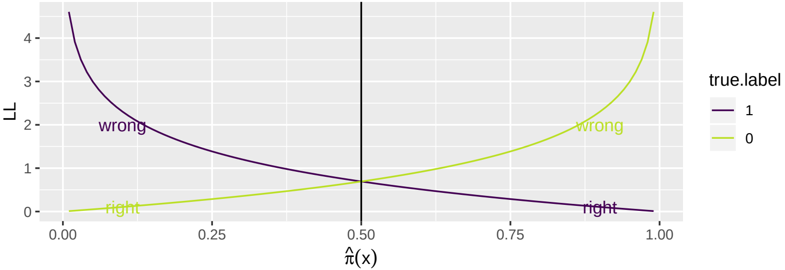
$$BS2 = \frac{1}{n}\sum_{i=1}^{n}\sum_{k=1}^{g}\left(\hat{\pi}_k(\mathbf{x}^{(i)}) - o_k^{(i)}\right)^2$$

- Original by Brier, works also for multiple classes
- $o_k^{(i)} = [y^{(i)} = k]$ is a 0-1-one-hot coding for labels
- For the binary case, BS2 is twice as large as BS1, because in BS2 we sum the squared difference for each observation regarding class 0 **and** class 1, not only the true class.

PROBABILITIES: LOG-LOSS

Logistic regression loss function, a.k.a. Bernoulli or binomial loss, $y^{(i)}$ coded as 0 and 1.

$$LL = \frac{1}{n} \sum_{i=1}^{n} \left(-y^{(i)} \log(\hat{\pi}(\mathbf{x}^{(i)})) - (1 - y^{(i)}) \log(1 - \hat{\pi}(\mathbf{x}^{(i)})) \right)$$



- Optimal value is 0, "confidently wrong" is penalized heavily

PROBABILITIES: LOG-LOSS

- Multiclass version: $LL = -\frac{1}{n}\sum\limits_{i=1}^{n} \sum_{k=1}^{g} o_k^{(i)} \log(\hat{\pi}_k(\mathbf{x}^{(i)}))$