Introduction to Machine Learning

Classification: Naive Bayes

compstat-lmu.github.io/lecture_i2ml

NAIVE BAYES CLASSIFIER

NB is a generative multiclass technique. Remember: We use Bayes' theorem and only need $p(\mathbf{x}|y = k)$ to compute the posterior as:

$$\pi_k(\mathbf{x}) = \mathbb{P}(y = k \mid \mathbf{x}) = \frac{\mathbb{P}(\mathbf{x}|y = k)\mathbb{P}(y = k)}{\mathbb{P}(\mathbf{x})} = \frac{p(\mathbf{x}|y = k)\pi_k}{\sum_{j=1}^{g} p(\mathbf{x}|y = j)\pi_j}$$

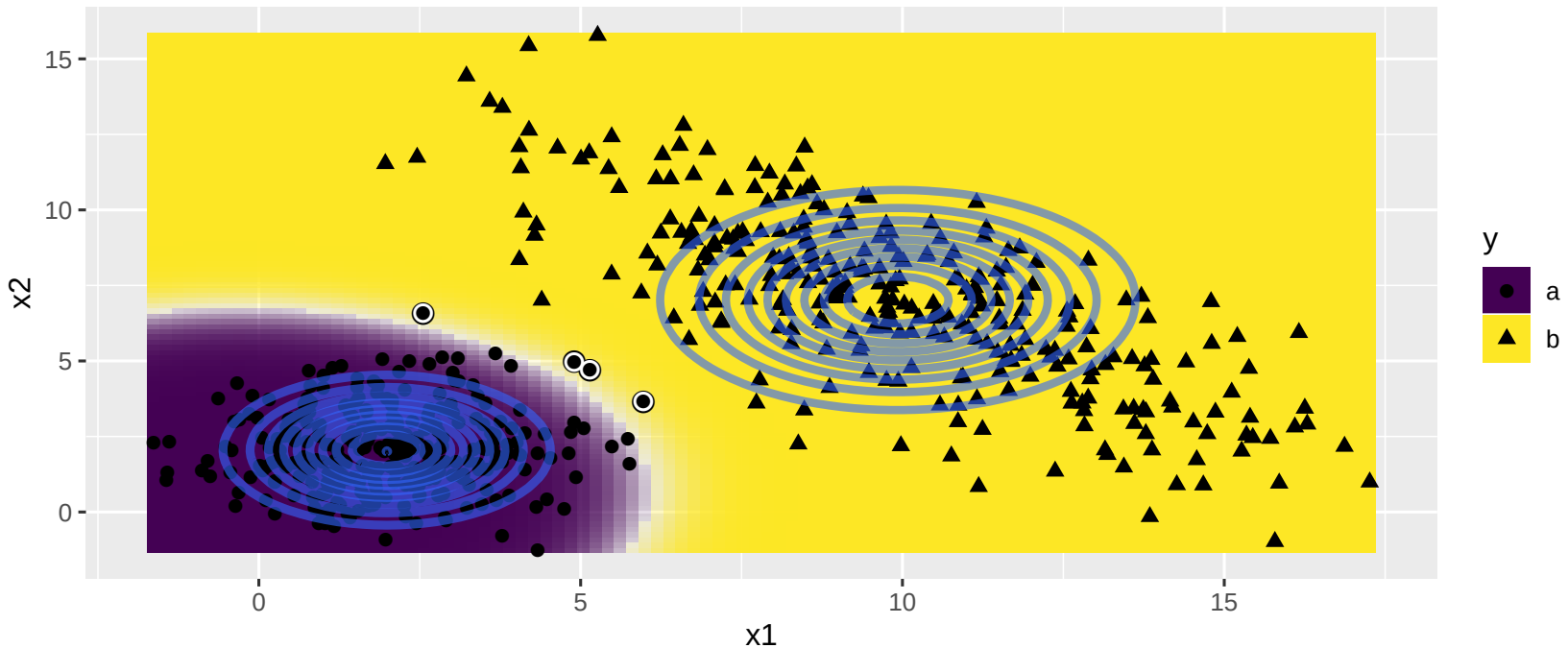
NB is based on a simple **conditional independence assumption**: the features are conditionally independent given class y.

$$p(\mathbf{x}|y = k) = p((x_1, x_2, ..., x_p)|y = k) = \prod_{j=1}^{p} p(x_j|y = k).$$

So we only need to specify and estimate the univariate distribution $p(x_j|y = k)$, which is considerably simpler as this is univariate.

NB: NUMERICAL FEATURES

We use a univariate Gaussian for $p(x_j, | y = k)$, and estimate (μ_j, σ_j^2) in the standard manner. Because $p(\mathbf{x}|y = k) = \prod_{j=1}^{p} p(x_j|y = k)$, The joint conditional density is Gaussian, per class. With diagonal, but non-isotropic covariance structure, and potentially different per class. Hence, NB is a (specific) QDA model, with quadratic decision boundary.



NB: CATEGORICAL FEATURES

We use a categorical distribution for $p(x_j|y = k)$ and estimate the probabilities p_{kjm} that in class in k our j-th feature has value m, $x_j = m$, simply by counting the frequencies.

$$p(x_j|y = k) = \prod_m p_{kjm}^{[x_j=m]}$$

Because of the simple conditional independence structure, it is also very easy to deal with mixed numerical / categorical feature spaces.

LAPLACE SMOOTHING

If a given class and feature value never occur together in the training data, then the frequency-based probability estimate will be zero.

This is problematic because it will wipe out all information in the other probabilities when they are multiplied.

A simple numerical correction is to set these zero probabilities to a small value to regularize against this case.

NAIVE BAYES: APPLICATION AS SPAM FILTER

- In the late 90s, Naive Bayes became popular for email spam-filter programs
- Word counts were used as features to detect spam mails (e.g. "Viagra" often occurs in spam mail)
- Independence assumption implies: Occurrence of two words in mail is not correlated
- Seems naive ("Viagra" more likely to occur in context with "Buy now" than "flower"), but leads to less required parameters and therefore better generalization, of works well in practice.