Całki nieoznaczone

Zad 1. Oblicz całkę:

(a) $\int(5x^2-6x+3-\frac{2}{x}+\frac{5}{x^2})dx$; **(b)** $\int(x^2-x+1)(x^2+x+1)dx$; **(c)** $\int\frac{(x^2-1)^3}{x}dx$;
(d) $\int\frac{2x^2\sqrt[3]{x}-5x+3x^2e^x-4}{x^2}dx$; **(e)** $\int\frac{x\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt[3]{x}}dx$; **(f)** $\int(3+2\sqrt[4]{x})^3dx$;
(g) $\int\frac{1-x^2}{x\sqrt{x}}dx$; **(h)** $\int\frac{x^2dx}{1+x^2}$;

Zad 2. Oblicz całkę (przez podstawienie):

(a) $\int(x^2+4)^5xdx$; **(b)** $\int\sin 7xdx$; **(c)** $\int\frac{xdx}{1+x^2}$; **(d)** $\int\frac{x^3dx}{1+x^2}$; **(e)** $\int\frac{xdx}{(x^2+3)^6}$;
(f) $\int\sqrt{3x+1}dx$; **(g)** $\int\sqrt{3x+1}xdx$; **(h)** $\int\frac{xdx}{\sqrt[3]{2x^2-1}}$; **(i)** $\int x\sqrt{1+x^2}dx$; **(j)** $\int(5-3x)^{10}dx$;
(k) $\int\frac{dx}{\sqrt{1-4x^2}}$; **(l)** $\int x^2\sqrt[5]{5x^3+1}dx$; **(m)** $\int\frac{\ln x}{x}dx$; **(n)** $\int\frac{e^x}{e^{2x}+1}dx$; **(o)** $\int\frac{5\sin x}{3-2\cos x}dx$;
(p) $\int e^x\sin e^xdx$; **(q)** $\int e^x\sqrt{e^x}dx$; **(r)** $\int\frac{e^{\sqrt{x}}}{\sqrt{x}}dx$; **(s)** $\int\frac{\cos\sqrt{x}}{\sqrt{x}}dx$; **(t)** $\int\frac{dx}{x\ln x}$;
(u) $\int\frac{\arcsin^3 x}{\sqrt{1-x^2}}dx$; **(v)** $\int\sin^3 xdx$; **(w)** $\int\frac{dx}{\sqrt{4x-x^2}}$; **(x)** $\int x^2e^{x^3}dx$; **(y)** $\int\frac{\operatorname{arc\,tg}^2 x}{1+x^2}dx$;
(z) $\int\frac{dx}{3+4x^2}$;

Zad 3. Oblicz całkę (przez podstawienie):

(a) $\int\frac{dx}{\sqrt{-9x^2+18x-5}}$; **(b)** $\int\frac{\cos x}{\sqrt{\sin x}}dx$; **(c)** $\int\operatorname{tg}xdx$; **(d)** $\int\operatorname{tg}^2xdx$; **(e)** $\int\sqrt{\sin x}\cos xdx$;
(f) $\int\sin^5x\cos xdx$; **(g)** $\int\cos^3xdx$; **(h)** $\int\frac{1+\sqrt{\operatorname{ctg}x}}{\sin^2x}dx$; **(i)** $\int\frac{\operatorname{arc\,tg}(\ln x)}{x(1+\ln^2x)}dx$; **(j)** $\int\frac{\sqrt{x}+\ln x}{x}dx$;
(k) $\int\frac{e^{\frac{1}{x}}}{x^2}dx$; **(l)** $\int\frac{2x+1}{x^2+x+1}dx$; **(m)** $\int x\sqrt{x+1}dx$; **(n)** $\int\frac{x\,dx}{\sqrt{x+1}}$; **(o)** $\int\frac{e^x\,dx}{e^x-1}$;
(p) $\int\frac{\sqrt{x+1}}{x}dx$; **(q)** $\int 2x\cos(x^2+1)dx$; **(r)** $\int\frac{\cos 2x}{\cos^2x\sin^2x}dx$; **(s)** $\int\frac{dx}{\sqrt{4-x^2}}$; **(t)** $\int\frac{dx}{\sin^2(5x+1)}$;
(u) $\int\frac{\arcsin^2x}{\sqrt{1-x^2}}dx$; **(v)** $\int\frac{\sin 3x}{3+\cos 3x}dx$; **(w)** $\int\frac{dx}{2+\sqrt{x}}$; **(x☠)** $\int\sqrt{1-\cos x}\,dx$

Zad 4. Oblicz całkę (przez części):

(a) $\int x\cos xdx$; **(b)** $\int x\ln xdx$; **(c)** $\int\ln xdx$; **(d)** $\int x^2\sin xdx$; **(e)** $\int\frac{x}{\cos^2x}dx$; **(f)** $\int\arcsin xdx$;
(g) $\int\arccos xdx$; **(h)** $\int\operatorname{arc\,tg}xdx$; **(i)** $\int xe^{-3x}dx$; **(j)** $\int e^x\sin xdx$; **(k)** $\int\cos^2xdx$; **(l)** $\int 6x\ln x^2dx$;
(m) $\int e^{-2x}\cos(3x+2)dx$; **(n)** $\int\ln\sqrt{x}dx$; **(o)** $\int x\sin^2xdx$; **(p)** $\int e^{\sqrt{x}}dx$; **(q)** $\int\cos\sqrt{x}dx$;
(r) $\int\ln x^2\,dx$; **(s)** $\int\sin x^2\,dx$; **(t)** $\int\frac{x}{\cos^2x}\,dx$;

Zad 5. Oblicz całkę (wymierna):

(a) $\int\frac{dx}{x^2+2x+8}$; **(b)** $\int\frac{2}{x^2+6x+18}dx$; **(c)** $\int\frac{5-4x}{x^2-4x+10}dx$; **(d)** $\int\frac{x^2}{x^2+2x+5}dx$; **(e)** $\int\frac{x^2+4}{x^2+3}dx$;
(f) $\int\frac{x^2+3x+2}{x^2+x+1}dx$; **(g)** $\int\frac{dx}{x(x-1)^2}$; **(h)** $\int\frac{x}{(x-1)^2(x^2+1)}dx$; **(i)** $\int\frac{x^2}{x^3-1}dx$; **(j)** $\int\frac{dx}{x(x^2+4)}$;
(k) $\int\frac{x\,dx}{(x-1)(x+2)(x+3)}$; **(l)** $\int\frac{dx}{x^3-4x}$; **(m)** $\int\frac{2x^4+5x^2-2}{2x^3-x-1}dx$; **(n)** $\int\frac{x}{1-x^4}dx$; **(o)** $\int\frac{dx}{(x-2)^2(x+3)^3}$;
(p) $\int\frac{dx}{x^8+x^6}$; **(q)** $\int\frac{x^2+5x+7}{x+3}dx$; **(r)** $\int\frac{x}{x^2-7x+13}dx$; **(s)** $\int\frac{x}{2x^2+3}dx$; **(t)** $\int\frac{dx}{x^3+2x^2+x}$;
(u) $\int\frac{dx}{x^3-5x^2+7x-3}$;

Zad 6. Oblicz całkę (wymierna):

(a) $\int\frac{x}{x^3+1}dx$; **(b)** $\int\frac{x}{(x-1)(x+1)^2}dx$; **(c)** $\int\frac{x^2-5x+9}{x^2+5x+6}dx$; **(d)** $\int\frac{x+1}{(x^2+4x+5)^2}dx$; **(e)** $\int\frac{-x^3+2x}{(x^2+4)(x^2+1)^2}dx$;
(f) $\int\frac{2x^4-10x^3+21x^2-20x+5}{x^3-3x+2}dx$; **(g)** $\int\frac{e^x}{(e^x+2)(e^x-1)}dx$; **(h)** $\int\frac{e^x}{e^{2x}-4}dx$; **(i)** $\int\frac{dx}{e^x-1}$;
(j) $\int\frac{\cos x}{\sin x(\sin x-1)^2}dx$; **(k)** $\int\frac{3x^3-16x^2+31x-31}{x^4-7x^3+21x^2-32x+20}dx$;

Zad 7. Oblicz całkę (trygonometryczna):

(a) $\int\sin x\cos(3x)dx$; **(b)** $\int\sin(3x)\cos(2x)dx$; **(c)** $\int\sin(2x)\sin(5x)dx$; **(d)** $\int\cos(7x)\sin(-2x)dx$;
(e) $\int\cos(2x)\cos(3x)dx$;

Zad 8. Oblicz całkę (trygonometryczna):

(a) $\int \frac{dx}{\sin x}$; **(b)** $\int \frac{dx}{5+4\cos x}$; **(c)** $\int \frac{dx}{\sin x \cos^3 x}$; **(d)** $\int \frac{\sin x \cos x}{1+\sin^4 x}dx$; **(e)** $\int \frac{\sin^3 x}{3+\sin^2 x}dx$;
(f) $\int \frac{dx}{(\sin^2 x+3\cos^2 x)^2}$; **(g)** $\int \frac{dx}{\sin x+\cos x}$; **(h)** $\int \frac{dx}{3\sin x+4\cos x+5}$; **(i)** $\int \cos^4 x dx$; **(j)** $\int \frac{dx}{\sin x \cos^2 x}$;
(k) $\int \frac{dx}{\cos x}$; **(l)** $\int \frac{dx}{\cos^2 x}$; **(m)** $\int \frac{dx}{1+\sin x+\cos x}$; **(n)** $\int \frac{2\sin x+3\cos x}{\sin^2 x \cos x+2\cos^3 x}dx$; **(o)** $\int \frac{dx}{\sin^2 x \cos x}$;
(p) $\int \sin^5 x dx$; **(q)** $\int \cos^7 x dx$; **(r)** $\int \frac{\cos x}{\sin^8 x}dx$; **(s)** $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}dx$; **(t)** $\int \frac{dx}{\cos^3 x}$;
(u) $\int \frac{\cos x+\sin x}{(\sin x-\cos x)^2}dx$; **(v)** $\int \frac{1+\mathrm{tg}\, x}{\sin(2x)}dx$; **(w)** $\int \frac{7\sin x \cos x}{3+\cos^2 x}dx$;

Zad 9. Oblicz całkę (niewymierna):

(a) $\int \frac{x+\sqrt[3]{x}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx$; **(b)** $\int \frac{dx}{\sqrt[3]{(1+x)^2}+\sqrt{1+x}}$; **(c)** $\int \frac{dx}{\sqrt[3]{3x-4}}$; **(d)** $\int x\sqrt{2+3x}dx$; **(e)** $\int \frac{\sqrt{x+1}}{x}dx$;
(f) $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$; **(g)** $\int \frac{dx}{\sqrt{x^2+2x-3}}$; **(h)** $\int \frac{dx}{\sqrt{-3x^2+2x+1}}$; **(i)** $\int \frac{dx}{\sqrt{x^2+4x+5}}$; **(j)** $\int \frac{dx}{\sqrt{-9x^2+18x-5}}$;
(k) $\int \frac{dx}{\sqrt{-2x^2+4x+6}}$; **(l)** $\int \frac{dx}{x+\sqrt{x^2-x+1}}$; **(m)** $\int x^2\sqrt{9-x^2}dx$; **(n)** $\int x^3\sqrt{1+x^2}dx$; **(o)** $\int \frac{x^3}{\sqrt{25+x^2}}dx$;
(p) $\int \frac{dx}{x^2\sqrt{4+x^2}}$; **(q)** $\int \frac{\sqrt{x^2-1}}{x}dx$; **(r)** $\int \sqrt{x^2+25}dx$; **(s)** $\int \sqrt{1-4x^2}dx$; **(t)** $\int \sqrt{36-x^2}dx$;
(u) $\int \frac{dx}{(1+x^2)\sqrt{1+x^2}}$; **(v)** $\int \sqrt{x^2-36}dx$; **(w)** $\int \frac{x^2}{\sqrt{1-3x^2}}dx$; **(x)** $\int \frac{\sqrt{9-x^2}}{x}dx$; **(y)** $\int \frac{x^2}{\sqrt{x^2-1}}dx$;

Całki nieoznaczone - odpowiedzi

Zad 1.

(a) $-2\ln|x|+\frac{5x^3}{3}-3x^2+3x-\frac{5}{x}+C$; **(b)** $\frac{x^5}{5}+\frac{x^3}{3}+x+C$; **(c)** $-\ln|x|+\frac{x^6}{6}-\frac{3x^4}{4}+\frac{3x^2}{2}+C$;

(d) $3e^x+\frac{3x^{\frac{4}{3}}}{2}+\frac{4}{x}-5\ln|x|+C$; **(e)** $\frac{x^2}{2}+\frac{12x^{\frac{11}{12}}}{11}+C$; **(f)** $\frac{32x^{\frac{7}{4}}}{7}+24x^{\frac{3}{2}}+\frac{216x^{\frac{5}{4}}}{5}+27x+C$;

(g) $-\frac{2x^{\frac{3}{2}}}{3}-\frac{2}{\sqrt{x}}+C$; **(h)** ;

Zad 2.

(a) $\frac{\left(x^2+4\right)^6}{12}+C$; **(b)** $-\frac{1}{7}\cos 7x+C$; **(c)** $\frac{1}{2}\ln(1+x^2)+C$; **(d)** $\frac{x^2}{2}-\frac{1}{2}\ln\left(x^2+1\right)+C$;

(e) $-\frac{1}{10(x^2+3)^5}+C$; **(f)** $\frac{2(3x+1)^{\frac{3}{2}}}{9}+C$; **(g)** $\frac{2}{135}(3x+1)^{\frac{3}{2}}(9x-2)+C$; **(h)** $\frac{3\left(2x^2-1\right)^{\frac{2}{3}}}{8}+C$;

(i) $\frac{\left(x^2+1\right)^{\frac{3}{2}}}{3}+C$; **(j)** $-\frac{(5-3x)^{11}}{33}+C$; **(k)** $\frac{\arcsin(2x)}{2}+C$; **(l)** $\frac{\left(5x^3+1\right)^{\frac{4}{3}}}{20}+C$;

(m) $\frac{1}{2}\ln^2 x+C$; **(n)** $\operatorname{arc\,tg} e^x+C$; **(o)** $\frac{5\ln(|2\cos(x)-3|)}{2}+C$; **(p)** $-\cos e^x+C$;

(q) $\frac{2}{3}e^{\frac{3}{2}x}+C$; **(r)** $2e^{\sqrt{x}}+C$; **(s)** $2\sin\sqrt{x}+C$; **(t)** $\ln|\ln x|+C$;

(u) $\frac{1}{4}\arcsin^4 x+C$; **(v)** $\frac{\cos(x)^3}{3}-\cos(x)+C$; **(w)** $-\arcsin\left(\frac{4-2x}{4}\right)+C$; **(x)** $\frac{e^{x^3}}{3}+C$;

(y) $\frac{\operatorname{arc\,tg}^3 x}{3}+C$; **(z)** $\frac{\operatorname{arc\,tg}\left(\frac{2x}{\sqrt{3}}\right)}{2\sqrt{3}}+C$;

Zad 3.

(a) $\frac{\arcsin\left(\frac{3x-3}{2}\right)}{3}+C$; **(b)** $2\sqrt{\sin(x)}+C$; **(c)** $-\ln|\cos(x)|+C$; **(d)** $\tan x-x+C$;

(e) $\frac{2}{3}\sin^{\frac{3}{2}} x+C$; **(f)** $\frac{1}{6}\sin^6 x+C$; **(g)** $\sin x-\frac{1}{3}\sin^3 x+C$; **(h)** $-\frac{1}{\operatorname{tg}(x)}-\frac{2}{3\operatorname{tg}(x)^{\frac{3}{2}}}+\frac{1}{3}+C$;

(i) $\frac{\operatorname{arc\,tg}(\ln(x))^2}{2}+C$; **(j)** $2\left(\frac{\ln(x)^2}{4}+\sqrt{x}\right)+C$; **(k)** $-e^{\frac{1}{x}}+C$; **(l)** $\ln|x^2+x+1|+C$;

(m) $\frac{2(x+1)^{\frac{5}{2}}}{5}-\frac{2(x+1)^{\frac{3}{2}}}{3}+C$; **(n)** $\frac{2(x+1)^{\frac{3}{2}}}{3}-2\sqrt{x+1}+C$; **(o)** $\ln|e^x-1|+C$;

(p) $\ln\left(\left|\sqrt{x+1}-1\right|\right)-\ln\left(\sqrt{x+1}+1\right)+2\sqrt{x+1}+C$; **(q)** $\sin(x^2+1)+C$; **(r)** $-\operatorname{tg} x-\operatorname{ctg} x+C$;

(s) $\arcsin\left(\frac{x}{2}\right)+C$; **(t)** $-\frac{1}{5}\operatorname{ctg}(5x+1)+C$; **(u)** $\frac{\arcsin^3(x)}{3}+C$; **(v)** $-\frac{\ln(|\cos(3x)+3|)}{3}+C$;

(w) $2\left(\sqrt{x}+2\right)-4\ln\left(\sqrt{x}+2\right)+C$; **(x)** $-2\sqrt{1-\cos(x)}\operatorname{ctg}\frac{x}{2}+C$;

Zad 4.

(a) $x\sin x+\cos x+C$; **(b)** $\frac{x^2\ln x}{2}-\frac{x^2}{4}+C$; **(c)** $x\ln x-x+C$; **(d)** $2x\sin x+\left(2-x^2\right)\cos x+C$;

(e) $\frac{\ln\left(4\cos x^2\right)}{2}+x\operatorname{tg} x+C$; **(f)** $x\arcsin x+\sqrt{1-x^2}+C$; **(g)** $x\arccos x-\sqrt{1-x^2}+C$;

(h) $x\operatorname{arc\,tg} x-\frac{\ln\left(x^2+1\right)}{2}+C$; **(i)** $-\frac{(3x+1)e^{-3x}}{9}+C$; **(j)** $\frac{e^x(\sin x-\cos x)}{2}+C$; **(k)** $\frac{\sin(2x)+2x}{4}+C$;

(l) $6x^2\ln x-3x^2+C$; **(m)** $\frac{e^{-2x}(-2\sin(3x+2)-3\cos(3x+2))}{13}+C$; **(n)** $x\ln\sqrt{x}-\frac{1}{2}x+C$;

(o) $-\frac{2x\sin(2x)+\cos(2x)-2x^2}{8}+C$; **(p)** $2\left(\sqrt{x}-1\right)e^{\sqrt{x}}+C$; **(q)** $2\left(\sin\left(\sqrt{x}\right)\sqrt{x}+\cos\left(\sqrt{x}\right)\right)+C$;

Zad 5.

(a) $\frac{1}{\sqrt{7}}\operatorname{arc\,tg}\left(\frac{x+1}{\sqrt{7}}\right)+C$; **(b)** $\frac{2}{3}\operatorname{arc\,tg}\left(\frac{x+3}{3}\right)+C$; **(c)** $-2\ln\left(\left|x^2-4x+20\right|\right)-\frac{3}{4}\operatorname{arc\,tg}\left(\frac{x-2}{4}\right)+C$;

(d) $-\ln\left(\left|x^2+2x+5\right|\right)-\frac{3}{2}\operatorname{arc\,tg}\left(\frac{x+1}{2}\right)+x+C$; **(e)** $x+\frac{\sqrt{3}}{3}\operatorname{arc\,tg}\left(\frac{\sqrt{3}}{3}x\right)+C$;

(f) $x+\ln(x^2+x+1)+C$; **(g)** $\ln|x|-\ln|x-1|-\frac{1}{x-1}+C$; **(h)** $-\frac{1}{2}\operatorname{arc\,tg} x-\frac{1}{2x-2}+C$;

(i) $\frac{1}{3}\ln|x^3-1|+C$; **(j)** $\frac{\ln|x|}{4}-\frac{\ln\left(x^2+4\right)}{8}+C$; **(k)** $-\frac{3\ln|x+3|}{4}+\frac{2\ln|x+2|}{3}+\frac{\ln|x-1|}{12}+C$;

(l) $\frac{\ln|x+2|}{8}-\frac{\ln|x|}{4}+\frac{\ln|x-2|}{8}+C$;

(m) $\ln\left|2x^2+2x+1\right|+\ln|x-1|+\operatorname{arc\,tg}(2x+1)+\frac{x^2}{2}+C$; **(n)** $\frac{\ln\left(x^2+1\right)}{4}-\frac{\ln\left|x^2-1\right|}{4}+C$;

(o) $\frac{3\ln|x+3|}{625}-\frac{3\ln|x-2|}{625}-\frac{6x^2+21x-16}{250x^3+1000x^2-750x-4500}+C$; **(p)** $-\operatorname{arc\,tg} x-\frac{15x^4-5x^2+3}{15x^5}+C$;

(q) $\ln|x+3| + \frac{x^2+4x}{2} + C$; **(r)** $\frac{\ln|x^2-7x+13|}{2} + \frac{7\operatorname{arc\,tg}\left(\frac{2x-7}{\sqrt{3}}\right)}{\sqrt{3}} + C$;

(s) $\frac{1}{4}\ln(2x^2+3) + C$; **(t)** $-\ln|x+1| + \ln|x| + \frac{1}{x+1} + C$; **(u)** $-\frac{\ln|x-1|}{4} + \frac{\ln|x-3|}{4} + \frac{1}{2x-2} + C$;

Zad 6.

(a) $\frac{\ln|x^2-x+1|}{6} - \frac{\ln|x+1|}{3} + \frac{\operatorname{arc\,tg}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}} + C$; **(b)** $-\frac{\ln|x+1|}{4} + \frac{\ln|x-1|}{4} - \frac{1}{2x+2} + C$;

(c) $-33\ln|x+3| + 23\ln|x+2| + x + C$; **(d)** $-\frac{\operatorname{arc\,tg}(x+2)}{2} - \frac{x-3}{2(x^2+4x+5)} + C$;

(e) $\frac{\ln\left(x^2+4\right)}{3} - \frac{\ln\left(x^2+1\right)}{3} - \frac{1}{2x^2+2} + C$; **(f)** $x^2 - 10x + \frac{2}{3(x-1)} + \frac{2}{9}\ln|x-1| + \frac{241}{9}\ln|x+2| + C$

(g) $\frac{1}{3}\ln|e^x-1| - \frac{1}{3}\ln|e^x+2| + C$; **(h)** $\frac{1}{4}\ln|e^x-2| - \frac{1}{4}\ln|e^x+2| + C$;

(i) $\ln|e^x-1| - x + C$; **(j)** $\ln|\sin x| - \ln|\sin x - 1| - \frac{1}{\sin x - 1} + C$;

Zad 7.

(a) $\frac{\cos(2x)}{4} - \frac{\cos(4x)}{8} + C$; **(b)** $-\frac{\cos(5x)+5\cos x}{10} + C$; **(c)** $\frac{\sin(3x)}{6} - \frac{\sin(7x)}{14} + C$; **(d)** $\frac{\cos(9x)}{18} - \frac{\cos(5x)}{10} + C$;

(e) $\frac{\sin(5x)}{10} + \frac{\sin(x)}{2} + C$;

Zad 8.

(a) $\ln\left|\operatorname{tg}\frac{x}{2}\right| + C$; **(b)** $\frac{2}{3}\operatorname{arc\,tg}\left(\frac{1}{3}\tan\left(\frac{x}{2}\right)\right) + C$;

(c) $\frac{1}{2\cos^2 x} + \ln\left(\frac{\sin x}{\cos x}\right) + C$; **(d)** $\frac{1}{2}\operatorname{arc\,tg}\left(\sin^2 x\right) + C$;

(e) $-\cos x - \frac{3}{4}\ln(2-\cos x) + \frac{3}{4}\ln(2+\cos x) + C$; **(f)** $\frac{2\operatorname{arc\,tg}\left(\frac{\tan x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\sin(2x)}{6(\cos(2x)+2)} + C$;

(g) $\frac{\sqrt{2}}{2}\ln\left|\frac{\operatorname{tg}\frac{x}{2}+(1-\sqrt{2})}{\operatorname{tg}\frac{x}{2}-(1+\sqrt{2})}\right| + C$; **(h)** $\frac{2}{3+9\cot\frac{x}{2}} + C$;

(i) $\frac{1}{32}(12x + 8\sin(2x) + \sin(4x)) + C$; **(j)** $\ln\left|\tan\left(\frac{x}{2}\right)\right| + \frac{1}{\cos x} + C$;

(k) $\ln\left|\frac{1+\operatorname{tg}\frac{x}{2}}{1-\operatorname{tg}\frac{x}{2}}\right| + C$; **(l)** $\operatorname{tg} x + C$;

(m) $\ln\left|\operatorname{tg}\frac{x}{2} + 1\right| + C$;

(n) $-\frac{2}{3}\ln|1+\cos x| + \frac{4}{3}\ln|\cos^2 x - \cos x + 1| - 8\operatorname{arc\,tg}\left(\frac{2\cos x - 1}{3}\right) + 12\operatorname{arc\,tg}\left(\frac{1}{2}\operatorname{tg} x\right) + C$;

(o) $\ln\left|\frac{1+\operatorname{tg}\frac{x}{2}}{1-\operatorname{tg}\frac{x}{2}}\right| - \frac{1}{\sin x} + C$; **(p)** $-\frac{5\cos x}{8} + \frac{5}{48}\cos(3x) - \frac{1}{80}\cos(5x) + C$;

(q) $\frac{35\sin x}{64} + \frac{7}{64}\sin(3x) + \frac{7}{320}\sin(5x) + \frac{1}{448}\sin(7x) + C$; **(r)** $-\frac{1}{7\sin^7 x} + C$;

(s) $3\sqrt[3]{\sin x} + C$; **(t)** ;

(u) $\frac{1}{\cos x - \sin x} + C$; **(v)** $\frac{1}{2}\operatorname{tg} x + \frac{1}{2}\ln|\operatorname{tg} x| + C$;

(w) ;

Zad 9.

(a) $\ln x + 6\operatorname{arc\,tg}\left(x^{\frac{1}{6}}\right) + C$; **(b)** $-6\ln\left((x+1)^{\frac{1}{6}} + 1\right) + 2\sqrt{x+1} - 3(x+1)^{\frac{1}{3}} + 6(x+1)^{\frac{1}{6}} + C$;

(c) $\frac{(3x-4)^{\frac{2}{3}}}{2} + C$; **(d)** $\frac{4(3x+2)^{\frac{3}{2}}}{9} + C$;

(e) $-\ln\left(\sqrt{x+1}+1\right) + \ln\left|\sqrt{x+1}-1\right| + 2\sqrt{x+1} + C$;

(f) $2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\ln\left(x^{\frac{1}{6}} + 1\right) + C$; **(g)** $\ln\left(2x + 2 + 2\sqrt{x^2+2x-3}\right) + C$;

(h) ; **(i)** ;

(j) $\frac{1}{3}\operatorname{arc\,sin}\left(\frac{3(x-1)}{2}\right) + C$; **(k)** $\frac{\operatorname{arc\,sin}\left(\frac{x-1}{2}\right)}{\sqrt{2}} + C$;

(l) ; **(m)** $\frac{81\operatorname{arc\,sin}\left(\frac{x}{3}\right)}{8} - \frac{x\left(9-x^2\right)^{\frac{3}{2}}}{4} + \frac{9x\sqrt{9-x^2}}{8} + C$;

(n) $\frac{\sqrt{x^2+1}\left(3x^4+x^2-2\right)}{15} + C$; **(o)** $\frac{\left(x^2-50\right)\sqrt{x^2+25}}{3} + C$;

(p) $-\frac{\sqrt{x^2+4}}{4x} + C$; **(q)** $\operatorname{arc\,sin}\left(\frac{1}{|x|}\right) + \sqrt{x^2-1} + C$;

(r) $\frac{1}{2}x\sqrt{x^2+25} + \frac{25}{2}\ln\left(x + \sqrt{x^2+25}\right) + C$; **(s)** $\frac{\operatorname{arc\,sin}(2x)}{4} + \frac{x\sqrt{1-4x^2}}{2} + C$;

(t) $18 \arcsin\left(\frac{x}{6}\right) + \frac{x\sqrt{36-x^2}}{2} + C$

(u) $\frac{x}{\sqrt{x^2+1}} + C$

(v) $\frac{x\sqrt{x^2-36}}{2} - 18\ln\left(2\sqrt{x^2-36}+2x\right) + C;$

(w) $\frac{\arcsin\left(\frac{3x}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{x\sqrt{1-3x^2}}{6} + C;$

(x) $\sqrt{9-x^2} - 3\ln\left(\frac{6\sqrt{9-x^2}}{|x|} + \frac{18}{|x|}\right) + C;$

(y) $\frac{\ln\left(2\sqrt{x^2-1}+2x\right)}{2} + \frac{x\sqrt{x^2-1}}{2} + C;$