

# Całki nieoznaczone

**Zad 1.** Oblicz całkę:

$$\begin{array}{lll} \text{(a)} \int (5x^2 - 6x + 3 - \frac{2}{x} + \frac{5}{x^2}) dx; & \text{(b)} \int (x^2 - x + 1)(x^2 + x + 1) dx; & \text{(c)} \int \frac{(x^2-1)^3}{x} dx; \\ \text{(d)} \int \frac{2x^2 \sqrt[3]{x-5x+3x^2} e^x - 4}{x^2} dx; & \text{(e)} \int \frac{x \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt[3]{x}} dx; & \text{(f)} \int (3 + 2\sqrt[4]{x})^3 dx; \\ \text{(g)} \int \frac{1-x^2}{x\sqrt{x}} dx; & \text{(h)} \int \frac{x^2 dx}{1+x^2}; \end{array}$$

**Zad 2.** Oblicz całkę (przez podstawienie):

$$\begin{array}{lllll} \text{(a)} \int (x^2 + 4)^5 x dx; & \text{(b)} \int \sin 7x dx; & \text{(c)} \int \frac{x dx}{1+x^2}; & \text{(d)} \int \frac{x^3 dx}{1+x^2}; & \text{(e)} \int \frac{x dx}{(x^2+3)^6}; \\ \text{(f)} \int \sqrt{3x+1} dx; \text{ red} & \text{(g)} \int \sqrt{3x+1} x dx; & \text{(h)} \int \frac{x dx}{\sqrt[3]{2x^2-1}}; & \text{(i)} \int x \sqrt{1+x^2} dx; & \text{(j)} \int (5-3x)^{10} dx; \\ \text{(k)} \int \frac{dx}{\sqrt{1-4x^2}}; & \text{(l)} \int x^2 \sqrt[5]{5x^3+1} dx; & \text{(m)} \int \frac{\ln x}{x} dx; & \text{(n)} \int \frac{e^x}{e^{2x}+1} dx; & \text{(o)} \int \frac{5 \sin x}{3-2 \cos x} dx; \\ \text{(p)} \int e^x \sin e^x dx; & \text{(q)} \int e^x \sqrt{e^x} dx; & \text{(r)} \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx; & \text{(s)} \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx; & \text{(t)} \int \frac{dx}{x \ln x}; \\ \text{(u)} \int \frac{\arcsin^3 x}{\sqrt{1-x^2}} dx; & \text{(v)} \int \sin^3 x dx; & \text{(w)} \int \frac{dx}{\sqrt{4x-x^2}}; & \text{(x)} \int x^2 e^{x^3} dx; & \text{(y)} \int \frac{\arcsin^2 x}{1+x^2} dx; \\ \text{(z)} \int \frac{dx}{3+4x^2}; \end{array}$$

**Zad 3.** Oblicz całkę (przez podstawienie):

$$\begin{array}{lllll} \text{(a)} \int \frac{dx}{\sqrt{-9x^2+18x-5}}; & \text{(b)} \int \frac{\cos x}{\sqrt{\sin x}} dx; & \text{(c)} \int \operatorname{tg} x dx; & \text{(d)} \int \operatorname{tg}^2 x dx; & \text{(e)} \int \sqrt{\sin x} \cos x dx; \\ \text{(f)} \int \sin^5 x \cos x dx; & \text{(g)} \int \cos^3 x dx; & \text{(h)} \int \frac{1+\sqrt{\operatorname{ctg} x}}{\sin^2 x} dx; & \text{(i)} \int \frac{\arcsin(\ln x)}{x(1+\ln^2 x)} dx; & \text{(j)} \int \frac{\sqrt{x+\ln x}}{x} dx; \\ \text{(k)} \int \frac{e^{\frac{1}{x}}}{x^2} dx; & \text{(l)} \int \frac{2x+1}{x^2+x+1} dx; & \text{(m)} \int x \sqrt{x+1} dx; & \text{(n)} \int \frac{x dx}{\sqrt{x+1}}; & \text{(o)} \int \frac{e^x dx}{e^x-1}; \\ \text{(p)} \int \frac{\sqrt{x+1}}{x} dx; & \text{(q)} \int 2x \cos(x^2+1) dx; & \text{(r)} \int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx; & \text{(s)} \int \frac{dx}{\sqrt{4-x^2}}; & \text{(t)} \int \frac{dx}{\sin^2(5x+1)}; \\ \text{(u)} \int \frac{\arcsin^2 x}{\sqrt{1-x^2}} dx; & \text{(v)} \int \frac{\sin 3x}{3+\cos 3x} dx; & \text{(w)} \int \frac{dx}{2+\sqrt{x}}; & \text{(x)} \int \sqrt{1-\cos x} dx \end{array}$$

**Zad 4.** Oblicz całkę (przez części):

$$\begin{array}{lllll} \text{(a)} \int x \cos x dx; & \text{(b)} \int x \ln x dx; & \text{(c)} \int \ln x dx; & \text{(d)} \int x^2 \sin x dx; & \text{(e)} \int \frac{x}{\cos^2 x} dx; & \text{(f)} \int \arcsin x dx; \\ \text{(g)} \int \arcsin x dx; & \text{(h)} \int \arcsin \operatorname{tg} x dx; & \text{(i)} \int x e^{-3x} dx; & \text{(j)} \int e^x \sin x dx; & \text{(k)} \int \cos^2 x dx; & \text{(l)} \int 6x \ln x^2 dx; \\ \text{(m)} \int e^{-2x} \cos(3x+2) dx; & \text{(n)} \int \ln \sqrt{x} dx; & \text{(o)} \int x \sin^2 x dx; & \text{(p)} \int e^{\sqrt{x}} dx; & \text{(q)} \int \cos \sqrt{x} dx; \\ \text{(r)} \int \ln x^2 dx; & \text{(s)} \int \sin x^2 dx; & \text{(t)} \int \frac{x}{\cos^2 x} dx; \end{array}$$

**Zad 5.** Oblicz całkę (wymierna):

$$\begin{array}{lllll} \text{(a)} \int \frac{dx}{x^2+2x+8}; & \text{(b)} \int \frac{2}{x^2+6x+18} dx; & \text{(c)} \int \frac{5-4x}{x^2-4x+10} dx; & \text{(d)} \int \frac{x^2}{x^2+2x+5} dx; & \text{(e)} \int \frac{x^2+4}{x^2+3} dx; \\ \text{(f)} \int \frac{x^2+3x+2}{x^2+x+1} dx; & \text{(g)} \int \frac{dx}{x(x-1)^2}; & \text{(h)} \int \frac{x}{(x-1)^2(x^2+1)} dx; & \text{(i)} \int \frac{x^2}{x^3-1} dx; & \text{(j)} \int \frac{dx}{x(x^2+4)}; \\ \text{(k)} \int \frac{x dx}{(x-1)(x+2)(x+3)}; & \text{(l)} \int \frac{dx}{x^3-4x}; & \text{(m)} \int \frac{2x^4+5x^2-2}{2x^3-x-1} dx; & \text{(n)} \int \frac{x}{1-x^4} dx; & \text{(o)} \int \frac{dx}{(x-2)^2(x+3)^3}; \\ \text{(p)} \int \frac{dx}{x^8+x^6}; & \text{(q)} \int \frac{x^2+5x+7}{x+3} dx; & \text{(r)} \int \frac{x}{x^2-7x+13} dx; & \text{(s)} \int \frac{x}{2x^2+3} dx; & \text{(t)} \int \frac{dx}{x^3+2x^2+x}; \\ \text{(u)} \int \frac{dx}{x^3-5x^2+7x-3}; \end{array}$$

**Zad 6.** Oblicz całkę (wymierna):

$$\begin{array}{lllll} \text{(a)} \int \frac{x}{x^3+1} dx; & \text{(b)} \int \frac{x}{(x-1)(x+1)^2} dx; & \text{(c)} \int \frac{x^2-5x+9}{x^2+5x+6} dx; & \text{(d)} \int \frac{x+1}{(x^2+4x+5)^2} dx; & \text{(e)} \int \frac{-x^3+2x}{(x^2+4)(x^2+1)^2} dx; \\ \text{(f)} \int \frac{2x^4-10x^3+21x^2-20x+5}{x^3-3x+2} dx; & \text{(g)} \int \frac{e^x}{(e^x+2)(e^x-1)} dx; & \text{(h)} \int \frac{e^x}{e^{2x}-4} dx; & \text{(i)} \int \frac{dx}{e^x-1}; \\ \text{(j)} \int \frac{\cos x}{\sin x(\sin x-1)^2} dx; & \text{(k)} \int \frac{3x^3-16x^2+31x-31}{x^4-7x^3+21x^2-32x+20} dx; \end{array}$$

**Zad 7.** Oblicz całkę (trygonometryczna):

$$\begin{array}{llll} \text{(a)} \int \sin x \cos(3x) dx; & \text{(b)} \int \sin(3x) \cos(2x) dx; & \text{(c)} \int \sin(2x) \sin(5x) dx; & \text{(d)} \int \cos(7x) \sin(-2x) dx; \\ \text{(e)} \int \cos(2x) \cos(3x) dx; \end{array}$$

**Zad 8.** Oblicz całkę (trygonometryczna):

$$\begin{array}{lllll}
 \text{(a)} \int \frac{dx}{\sin x}; & \text{(b)} \int \frac{dx}{5+4\cos x}; & \text{(c)} \int \frac{dx}{\sin x \cos^3 x}; & \text{(d)} \int \frac{\sin x \cos x}{1+\sin^4 x} dx; & \text{(e)} \int \frac{\sin^3 x}{3+\sin^2 x} dx; \\
 \text{(f)} \int \frac{dx}{(\sin^2 x + 3\cos^2 x)^2}; & \text{(g)} \int \frac{dx}{\sin x + \cos x}; & \text{(h)} \int \frac{dx}{3\sin x + 4\cos x + 5}; & \text{(i)} \int \cos^4 x dx; & \text{(j)} \int \frac{dx}{\sin x \cos^2 x}; \\
 \text{(k)} \int \frac{dx}{\cos x}; & \text{(l)} \int \frac{dx}{\cos^2 x}; & \text{(m)} \int \frac{dx}{1+\sin x + \cos x}; & \text{(n)} \int \frac{2\sin x + 3\cos x}{\sin^2 x \cos x + 2\cos^3 x} dx; & \text{(o)} \int \frac{dx}{\sin^2 x \cos x}; \\
 \text{(p)} \int \sin^5 x dx; & \text{(q)} \int \cos^7 x dx; & \text{(r)} \int \frac{\cos x}{\sin^8 x} dx; & \text{(s)} \int \frac{\cos x}{\sqrt[3]{\sin^2 x}} dx; & \text{(t)} \int \frac{dx}{\cos^3 x}; \\
 \text{(u)} \int \frac{\cos x + \sin x}{(\sin x - \cos x)^2} dx; & \text{(v)} \int \frac{1+\operatorname{tg} x}{\sin(2x)} dx; & \text{(w)} \int \frac{7\sin x \cos x}{3+\cos^2 x} dx;
 \end{array}$$

**Zad 9.** Oblicz całkę (niewymierna):

$$\begin{array}{lllll}
 \text{(a)} \int \frac{x + \sqrt[3]{x} + \sqrt[6]{x}}{x(1 + \sqrt[3]{x})} dx; & \text{(b)} \int \frac{dx}{\sqrt[3]{(1+x)^2} + \sqrt{1+x}}; & \text{(c)} \int \frac{dx}{\sqrt[3]{3x-4}}; & \text{(d)} \int x\sqrt{2+3x} dx; & \text{(e)} \int \frac{\sqrt{x+1}}{x} dx; \\
 \text{(f)} \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}; & \text{(g)} \int \frac{dx}{\sqrt{x^2+2x-3}}; & \text{(h)} \int \frac{dx}{\sqrt{-3x^2+2x+1}}; & \text{(i)} \int \frac{dx}{\sqrt{x^2+4x+5}}; & \text{(j)} \int \frac{dx}{\sqrt{-9x^2+18x-5}}; \\
 \text{(k)} \int \frac{dx}{\sqrt{-2x^2+4x+6}}; & \text{(l)} \int \frac{dx}{x + \sqrt{x^2-x+1}}; & \text{(m)} \int x^2\sqrt{9-x^2} dx; & \text{(n)} \int x^3\sqrt{1+x^2} dx; & \text{(o)} \int \frac{x^3}{\sqrt{25+x^2}} dx; \\
 \text{(p)} \int \frac{dx}{x^2\sqrt{4+x^2}}; & \text{(q)} \int \frac{\sqrt{x^2-1}}{x} dx; & \text{(r)} \int \sqrt{x^2+25} dx; & \text{(s)} \int \sqrt{1-4x^2} dx; & \text{(t)} \int \sqrt{36-x^2} dx; \\
 \text{(u)} \int \frac{dx}{(1+x^2)\sqrt{1+x^2}}; & \text{(v)} \int \sqrt{x^2-36} dx; & \text{(w)} \int \frac{x^2}{\sqrt{1-3x^2}} dx; & \text{(x)} \int \frac{\sqrt{9-x^2}}{x} dx; & \text{(y)} \int \frac{x^2}{\sqrt{x^2-1}} dx;
 \end{array}$$

## Całki nieoznaczone - odpowiedzi

### Zad 1.

- (a)  $-2 \ln |x| + \frac{5x^3}{3} - 3x^2 + 3x - \frac{5}{x} + C$ ; (b)  $\frac{x^5}{5} + \frac{x^3}{3} + x + C$ ; (c)  $-\ln |x| + \frac{x^6}{6} - \frac{3x^4}{4} + \frac{3x^2}{2} + C$ ;  
 (d)  $3e^x + \frac{3x^{\frac{4}{3}}}{2} + \frac{4}{x} - 5 \ln |x| + C$ ; (e)  $\frac{x^2}{2} + \frac{12x^{\frac{11}{2}}}{11} + C$ ; (f)  $\frac{32x^{\frac{7}{4}}}{7} + 24x^{\frac{3}{2}} + \frac{216x^{\frac{5}{4}}}{5} + 27x + C$ ;  
 (g)  $-\frac{2x^{\frac{3}{2}}}{3} - \frac{2}{\sqrt{x}} + C$ ; (h) ;

### Zad 2.

- (a)  $\frac{(x^2+4)^6}{12} + C$ ; (b)  $-\frac{1}{7} \cos 7x + C$ ; (c)  $\frac{1}{2} \ln(1+x^2) + C$ ; (d)  $\frac{x^2}{2} - \frac{1}{2} \ln(x^2+1) + C$ ;  
 (e)  $-\frac{1}{10(x^2+3)^5} + C$ ; (f)  $\frac{2(3x+1)^{\frac{3}{2}}}{9} + C$ ; (g)  $\frac{2}{135} (3x+1)^{\frac{3}{2}} (9x-2) + C$ ; (h)  $\frac{3(2x^2-1)^{\frac{3}{2}}}{8} + C$ ;  
 (i)  $\frac{(x^2+1)^{\frac{3}{2}}}{3} + C$ ; (j)  $-\frac{(5-3x)^{11}}{33} + C$ ; (k)  $\frac{\arcsin(2x)}{2} + C$ ; (l)  $\frac{(5x^3+1)^{\frac{4}{3}}}{20} + C$ ;  
 (m)  $\frac{1}{2} \ln^2 x + C$ ; (n)  $\arctg e^x + C$ ; (o)  $\frac{5 \ln(|2 \cos(x)-3|)}{2} + C$ ; (p)  $-\cos e^x + C$ ;  
 (q)  $\frac{2}{3} e^{\frac{3}{2}x} + C$ ; (r)  $2e^{\sqrt{x}} + C$ ; (s)  $2 \sin \sqrt{x} + C$ ; (t)  $\ln |\ln x| + C$ ;  
 (u)  $\frac{1}{4} \arcsin^4 x + C$ ; (v)  $\frac{\cos(x)^3}{3} - \cos(x) + C$ ; (w)  $-\arcsin\left(\frac{4-2x}{4}\right) + C$ ; (x)  $\frac{e^{x^3}}{3} + C$ ;  
 (y)  $\frac{\arctg^3 x}{3} + C$ ; (z)  $\frac{\arctg\left(\frac{2x}{\sqrt{3}}\right)}{2\sqrt{3}} + C$ ;

### Zad 3.

- (a)  $\frac{\arcsin\left(\frac{3x-3}{2}\right)}{3} + C$ ; (b)  $2\sqrt{\sin(x)} + C$ ; (c)  $-\ln |\cos(x)| + C$ ; (d)  $\tan x - x + C$ ;  
 (e)  $\frac{2}{3} \sin^{\frac{3}{2}} x + C$ ; (f)  $\frac{1}{6} \sin^6 x + C$ ; (g)  $\sin x - \frac{1}{3} \sin^3 x + C$ ; (h)  $-\frac{1}{\operatorname{tg}(x)} - \frac{2}{3 \operatorname{tg}(x)^{\frac{3}{2}}} + \frac{1}{3} + C$ ;  
 (i)  $\frac{\arctg(\ln(x))^2}{2} + C$ ; (j)  $2\left(\frac{\ln(x)^2}{4} + \sqrt{x}\right) + C$ ; (k)  $-e^{\frac{1}{x}} + C$ ; (l)  $\ln |x^2 + x + 1| + C$ ;  
 (m)  $\frac{2(x+1)^{\frac{5}{2}}}{5} - \frac{2(x+1)^{\frac{3}{2}}}{3} + C$ ; (n)  $\frac{2(x+1)^{\frac{3}{2}}}{3} - 2\sqrt{x+1} + C$ ; (o)  $\ln |e^x - 1| + C$ ;  
 (p)  $\ln(|\sqrt{x+1}-1|) - \ln(\sqrt{x+1}+1) + 2\sqrt{x+1} + C$ ; (q)  $\sin(x^2+1) + C$ ; (r)  $-\operatorname{tg} x - \operatorname{ctg} x + C$ ;  
 (s)  $\arcsin\left(\frac{x}{2}\right) + C$ ; (t)  $-\frac{1}{5} \operatorname{ctg}(5x+1) + C$ ; (u)  $\frac{\arcsin^3(x)}{3} + C$ ; (v)  $-\frac{\ln(|\cos(3x)+3|)}{3} + C$ ;  
 (w)  $2(\sqrt{x}+2) - 4 \ln(\sqrt{x}+2) + C$ ; (x)  $-2\sqrt{1-\cos(x)} \operatorname{ctg} \frac{x}{2} + C$ ;

### Zad 4.

- (a)  $x \sin x + \cos x + C$ ; (b)  $\frac{x^2 \ln x}{2} - \frac{x^2}{4} + C$ ; (c)  $x \ln x - x + C$ ; (d)  $2x \sin x + (2-x^2) \cos x + C$ ;  
 (e)  $\frac{\ln(4 \cos x^2)}{2} + x \operatorname{tg} x + C$ ; (f)  $x \arcsin x + \sqrt{1-x^2} + C$ ; (g)  $x \arccos x - \sqrt{1-x^2} + C$ ;  
 (h)  $x \arctg x - \frac{\ln(x^2+1)}{2} + C$ ; (i)  $-\frac{(3x+1)e^{-3x}}{9} + C$ ; (j)  $\frac{e^x(\sin x - \cos x)}{2} + C$ ; (k)  $\frac{\sin(2x)+2x}{4} + C$ ;  
 (l)  $6x^2 \ln x - 3x^2 + C$ ; (m)  $\frac{e^{-2x}(-2 \sin(3x+2)-3 \cos(3x+2))}{13} + C$ ; (n)  $x \ln \sqrt{x} - \frac{1}{2}x + C$ ;  
 (o)  $-\frac{2x \sin(2x) + \cos(2x) - 2x^2}{8} + C$ ; (p)  $2(\sqrt{x}-1)e^{\sqrt{x}} + C$ ; (q)  $2(\sin(\sqrt{x})\sqrt{x} + \cos(\sqrt{x})) + C$ ;

### Zad 5.

- (a)  $\frac{1}{\sqrt{7}} \arctg\left(\frac{x+1}{\sqrt{7}}\right) + C$ ; (b)  $\frac{2}{3} \arctg\left(\frac{x+3}{3}\right) + C$ ; (c)  $-2 \ln(|x^2-4x+20|) - \frac{3}{4} \arctg\left(\frac{x-2}{4}\right) + C$ ;  
 (d)  $-\ln(|x^2+2x+5|) - \frac{3}{2} \arctg\left(\frac{x+1}{2}\right) + x + C$ ; (e)  $x + \frac{\sqrt{3}}{3} \arctg\left(\frac{\sqrt{3}x}{3}\right) + C$ ;  
 (f)  $x + \ln(x^2+x+1) + C$ ; (g)  $\ln|x| - \ln|x-1| - \frac{1}{x-1} + C$ ; (h)  $-\frac{1}{2} \arctg x - \frac{1}{2x-2} + C$ ;  
 (i)  $\frac{1}{3} \ln|x^3-1| + C$ ; (j)  $\frac{\ln|x|}{4} - \frac{\ln(x^2+4)}{8} + C$ ; (k)  $-\frac{3 \ln|x+3|}{4} + \frac{2 \ln|x+2|}{3} + \frac{\ln|x-1|}{12} + C$ ;  
 (l)  $\frac{\ln|x+2|}{8} - \frac{\ln|x|}{4} + \frac{\ln|x-2|}{8} + C$ ;  
 (m)  $\ln|2x^2+2x+1| + \ln|x-1| + \arctg(2x+1) + \frac{x^2}{2} + C$ ; (n)  $\frac{\ln(x^2+1)}{4} - \frac{\ln|x^2-1|}{4} + C$ ;  
 (o)  $\frac{3 \ln|x+3|}{625} - \frac{3 \ln|x-2|}{625} - \frac{6x^2+21x-16}{250x^3+1000x^2-750x-4500} + C$ ; (p)  $-\arctg x - \frac{15x^4-5x^2+3}{15x^5} + C$ ;

$$\begin{aligned}
 \text{(q)} \quad & \ln|x+3| + \frac{x^2+4x}{2} + C; & \text{(r)} \quad & \frac{\ln|x^2-7x+13|}{2} + \frac{7 \arctg\left(\frac{2x-7}{\sqrt{3}}\right)}{\sqrt{3}} + C; \\
 \text{(s)} \quad & \frac{1}{4} \ln(2x^2+3) + C; & \text{(t)} \quad & -\ln|x+1| + \ln|x| + \frac{1}{x+1} + C; & \text{(u)} \quad & -\frac{\ln|x-1|}{4} + \frac{\ln|x-3|}{4} + \frac{1}{2x-2} + C;
 \end{aligned}$$

**Zad 6.**

$$\begin{aligned}
 \text{(a)} \quad & \frac{\ln|x^2-x+1|}{6} - \frac{\ln|x+1|}{3} + \frac{\arctg\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}} + C; & \text{(b)} \quad & -\frac{\ln|x+1|}{4} + \frac{\ln|x-1|}{4} - \frac{1}{2x+2} + C; \\
 \text{(c)} \quad & -33 \ln|x+3| + 23 \ln|x+2| + x + C; & \text{(d)} \quad & -\frac{\arctg(x+2)}{2} - \frac{x-3}{2(x^2+4x+5)} + C; \\
 \text{(e)} \quad & \frac{\ln(x^2+4)}{3} - \frac{\ln(x^2+1)}{3} - \frac{1}{2x^2+2} + C; & \text{(f)} \quad & x^2 - 10x + \frac{2}{3(x-1)} + \frac{2}{9} \ln|x-1| + \frac{241}{9} \ln|x+2| + C \\
 \text{(g)} \quad & \frac{1}{3} \ln|e^x-1| - \frac{1}{3} \ln|e^x+2| + C; & \text{(h)} \quad & \frac{1}{4} \ln|e^x-2| - \frac{1}{4} \ln|e^x+2| + C; \\
 \text{(i)} \quad & \ln|e^x-1| - x + C; & \text{(j)} \quad & \ln|\sin x| - \ln|\sin x-1| - \frac{1}{\sin x-1} + C;
 \end{aligned}$$

**Zad 7.**

$$\begin{aligned}
 \text{(a)} \quad & \frac{\cos(2x)}{4} - \frac{\cos(4x)}{8} + C; & \text{(b)} \quad & -\frac{\cos(5x)+5 \cos x}{10} + C; & \text{(c)} \quad & \frac{\sin(3x)}{6} - \frac{\sin(7x)}{14} + C; & \text{(d)} \quad & \frac{\cos(9x)}{18} - \frac{\cos(5x)}{10} + C; \\
 \text{(e)} \quad & \frac{\sin(5x)}{10} + \frac{\sin(x)}{2} + C;
 \end{aligned}$$

**Zad 8.**

$$\begin{aligned}
 \text{(a)} \quad & \ln\left|\operatorname{tg} \frac{x}{2}\right| + C; & \text{(b)} \quad & \frac{2}{3} \arctg\left(\frac{1}{3} \tan\left(\frac{x}{2}\right)\right) + C; \\
 \text{(c)} \quad & \frac{1}{2 \cos^2 x} + \ln\left(\frac{\sin x}{\cos x}\right) + C; & \text{(d)} \quad & \frac{1}{2} \arctg(\sin^2 x) + C; \\
 \text{(e)} \quad & -\cos x - \frac{3}{4} \ln(2 - \cos x) + \frac{3}{4} \ln(2 + \cos x) + C; & \text{(f)} \quad & \frac{2 \arctg\left(\frac{\tan x}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\sin(2x)}{6(\cos(2x)+2)} + C; \\
 \text{(g)} \quad & \frac{\sqrt{2}}{2} \ln\left|\frac{\operatorname{tg} \frac{x}{2} + (1-\sqrt{2})}{\operatorname{tg} \frac{x}{2} - (1+\sqrt{2})}\right| + C; & \text{(h)} \quad & \frac{2}{3+9 \cot \frac{x}{2}} + C; \\
 \text{(i)} \quad & \frac{1}{32} (12x + 8 \sin(2x) + \sin(4x)) + C; & \text{(j)} \quad & \ln\left|\tan\left(\frac{x}{2}\right)\right| + \frac{1}{\cos x} + C; \\
 \text{(k)} \quad & \ln\left|\frac{1+\operatorname{tg} \frac{x}{2}}{1-\operatorname{tg} \frac{x}{2}}\right| + C; & \text{(l)} \quad & \operatorname{tg} x + C; \\
 \text{(m)} \quad & \ln\left|\operatorname{tg} \frac{x}{2} + 1\right| + C; \\
 \text{(n)} \quad & -\frac{2}{3} \ln|1 + \cos x| + \frac{4}{3} \ln|\cos^2 x - \cos x + 1| - 8 \arctg\left(\frac{2 \cos x - 1}{3}\right) + 12 \arctg\left(\frac{1}{2} \operatorname{tg} x\right) + C; \\
 \text{(o)} \quad & \ln\left|\frac{1+\operatorname{tg} \frac{x}{2}}{1-\operatorname{tg} \frac{x}{2}}\right| - \frac{1}{\sin x} + C; & \text{(p)} \quad & -\frac{5 \cos x}{8} + \frac{5}{48} \cos(3x) - \frac{1}{80} \cos(5x) + C; \\
 \text{(q)} \quad & \frac{35 \sin x}{64} + \frac{7}{64} \sin(3x) + \frac{7}{320} \sin(5x) + \frac{1}{448} \sin(7x) + C; & \text{(r)} \quad & -\frac{1}{7 \sin^7 x} + C; \\
 \text{(s)} \quad & 3 \sqrt[3]{\sin x} + C; & \text{(t)} \quad & ; \\
 \text{(u)} \quad & \frac{1}{\cos x - \sin x} + C; & \text{(v)} \quad & \frac{1}{2} \operatorname{tg} x + \frac{1}{2} \ln|\operatorname{tg} x| + C; \\
 \text{(w)} \quad & ;
 \end{aligned}$$

**Zad 9.**

$$\begin{aligned}
 \text{(a)} \quad & \ln x + 6 \arctg\left(x^{\frac{1}{6}}\right) + C; & \text{(b)} \quad & -6 \ln\left((x+1)^{\frac{1}{6}} + 1\right) + 2\sqrt{x+1} - 3(x+1)^{\frac{1}{3}} + 6(x+1)^{\frac{1}{6}} + C; \\
 \text{(c)} \quad & \frac{(3x-4)^{\frac{2}{3}}}{2} + C; & \text{(d)} \quad & \frac{4(3x+2)^{\frac{3}{2}}}{9} + C; \\
 \text{(e)} \quad & -\ln(\sqrt{x+1}+1) + \ln|\sqrt{x+1}-1| + 2\sqrt{x+1} + C; \\
 \text{(f)} \quad & 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6 \ln\left(x^{\frac{1}{6}} + 1\right) + C; & \text{(g)} \quad & \ln(2x+2+2\sqrt{x^2+2x-3}) + C; \\
 \text{(h)} \quad & ; & \text{(i)} \quad & ; \\
 \text{(j)} \quad & \frac{1}{3} \arcsin\left(\frac{3(x-1)}{2}\right) + C; & \text{(k)} \quad & \frac{\arcsin\left(\frac{x-1}{2}\right)}{\sqrt{2}} + C; \\
 \text{(l)} \quad & ; & \text{(m)} \quad & \frac{81 \arcsin\left(\frac{x}{3}\right)}{8} - \frac{x(9-x^2)^{\frac{3}{2}}}{4} + \frac{9x\sqrt{9-x^2}}{8} + C; \\
 \text{(n)} \quad & \frac{\sqrt{x^2+1}(3x^4+x^2-2)}{15} + C; & \text{(o)} \quad & \frac{(x^2-50)\sqrt{x^2+25}}{3} + C; \\
 \text{(p)} \quad & -\frac{\sqrt{x^2+4}}{4x} + C; & \text{(q)} \quad & \arcsin\left(\frac{1}{|x|}\right) + \sqrt{x^2-1} + C; \\
 \text{(r)} \quad & \frac{1}{2} x \sqrt{x^2+25} + \frac{25}{2} \ln(x + \sqrt{x^2+25}) + C; & \text{(s)} \quad & \frac{\arcsin(2x)}{4} + \frac{x\sqrt{1-4x^2}}{2} + C;
 \end{aligned}$$

$$(t) \quad 18 \arcsin\left(\frac{x}{6}\right) + \frac{x\sqrt{36-x^2}}{2} + C$$

$$(u) \quad \frac{x}{\sqrt{x^2+1}} + C$$

$$(v) \quad \frac{x\sqrt{x^2-36}}{2} - 18 \ln(2\sqrt{x^2-36} + 2x) + C;$$

$$(w) \quad \frac{\arcsin\left(\frac{3x}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{x\sqrt{1-3x^2}}{6} + C;$$

$$(x) \quad \sqrt{9-x^2} - 3 \ln\left(\frac{6\sqrt{9-x^2}}{|x|} + \frac{18}{|x|}\right) + C;$$

$$(y) \quad \frac{\ln(2\sqrt{x^2-1}+2x)}{2} + \frac{x\sqrt{x^2-1}}{2} + C;$$