Liczby zespolone - zadania

Zad 1. Oblicz:

(a) i^2; **(b)** i^3; **(c)** i^4; **(d)** i^5; **(e)** i^{22}; **(f)** i^{89}; **(g)** i^{2007}; **(h)** i^{-1}; **(i)** i^{-2}; **(j)** i^{-3};
(k) i^{-4}; **(l)** i^{-129}; **(m)** i^{-75}; **(n)** i^{-2008};

Zad 2. Wykonaj działania; wynik zapisz w postaci algebraicznej:

(a) $\left(2+\frac{1}{4}i\right)(5+i)$; **(b)** $\left(\frac{1}{2}+\frac{\sqrt{2}}{2}i\right)\left(\frac{1}{2}-\frac{\sqrt{2}}{2}i\right)$; **(c)** $\left(\frac{1}{4}+i\right)^2$; **(d)** $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^3$; **(e)** $\left(\frac{1}{2}+\frac{2}{3}i\right)^4$; **(f)** $\frac{4+i}{1-2i}$;
(g) $\frac{2+i}{2i-5}$; **(h)** $\frac{(3+2i)^2}{4i-3}$; **(i)** $\frac{(5-i)(3-i)}{(4+i)(i-2)}$; **(j)** $\frac{(2+3i)(1+i)}{(1-i)(2+i)}$;

Zad 3. Znaleźć $x, y \in \mathbb{R}$ spełniające równanie:

(a) $x(2+3i)+y(4-5i)=6-2i$; **(b)** $\frac{1+yi}{x-2i}=3i-1$; **(c)** $(2+yi)(x-3i)=7-i$;
(d) $x(2+3i)+y(5-2i)=-8+7i$; **(e)** $\frac{x}{2-3i}+\frac{y}{3+2i}=1$; **(f)** $x(4-3i)^2+y(1+i)^2=7-12i$;

Zad 4. Oblicz pierwiastek kwadratowy z liczby:

(a) $z=i$; **(b)** $z=-8i$; **(c)** $z=-1+i$; **(d)** $z=3+4i$; **(e)** $z=-16+30i$; **(f)** $z=\frac{1}{2}+\frac{\sqrt{3}}{2}i$;
(g) $z=\frac{(14-2i)(i+3)}{4-2i}$;

Zad 5. Rozwiązać w zbiorze liczb zespolonych równanie:

(a) $z^2-z+1=0$; **(b)** $z^2+4z+5=0$; **(c)** $(i-3)z=5+i-z$; **(d)** $\frac{1-3i}{3z+2i}=\frac{2i-3}{5-2iz}$;
(e) $z^2-4z+13=0$; **(f)** $\frac{2+i}{z-1+4i}=\frac{1-i}{2z+1}$; **(g)** $z^3-6iz^2-12z+8i=0$; **(h)** $z^4+3z^2-4=0$;
(i) $4z^3-4z^2+z-1=0$; **(j)** $z^4+81=0$; **(k)** $z^6-1=0$;
(l) $z^4-(18+4i)z^2+77-36i=0$; **(m)** $z^4-10z^2-20z-16=0$; **(n)** $z^3-4z^2+6z-4=0$;
(o) $z^5-3z^4+2z^3-6z^2+z-3=0$; **(p)** $(3+i)z^2+(1-i)z-6i=0$; **(q)** $(z-i)^4=(iz+3)^4$;
(r) $(1-i)z^2-(6-2i)z+11-3i=0$; **(s)** $x^4+6ix^3-9x^2+4ix-12=0$; **(t)** $z^6=(1-3i)^{12}$;
(u) $(1+i)z^2-(2+2i)z-(1-3i)=0$; **(v)** $(z+1)^6+z^6=0$;

Zad 6. Rozwiązać w zbiorze liczb zespolonych równanie:

(a) $z^2+3\overline{z}=0$; **(b)** $2z+(1+i)\overline{z}=1-3i$; **(c)** $(z+2)^2=(\overline{z}+2)^2$; **(d)** $\overline{z+i}-z+i=0$;

Zad 7. Obliczyć:

(a) $|4+3i|$; **(b)** $|\sqrt{3}-2i|$; **(c)** $|14+i|$; **(d)** $|(2i+3)(1-i)|$; **(e)** $\left|\frac{\frac{1}{2}-\frac{\sqrt{3}}{2}i}{\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i}\right|$;
(f) $\left|\frac{1+i}{2-3i}\right|$; **(g)** $\arg(5+5i)$; **(h)** $\arg(-3+3\sqrt{3}i)$; **(i)** $\arg(8\sqrt{3}-8i)$; **(j)** $\arg(2-5i)$
(k) $\arg\left(\frac{\frac{1}{2}-\frac{\sqrt{3}}{2}i}{\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i}\right)$; **(l)** $\arg\left(\left(\frac{-\sqrt{3}}{2}-\frac{1}{2}i\right)\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)\right)$; **(m)** $\overline{(2i-1)\left(\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)}$; **(n)** $\overline{1+3i}$;
(o) $\overline{i-1}$; **(p)** $\overline{\left(\frac{1+i}{3-2i}\right)}$;

Zad 8. Udowodnić że dla dowolnych $z_1, z_2 \in \mathbb{C}$ zachodzi:

(a) $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$; **(b)** $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$; **(c)** $\overline{z_1 \cdot z_2} = \overline{z_1} \cdot \overline{z_2}$; **(d)** $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$; **(e)** $z\overline{z} = |z|^2$;
(f) $\arg(\overline{z}) = 2\pi - \arg(z)$; **(g)** $\arg\left(\frac{1}{z}\right) = 2\pi - \arg(z)$; **(h)** $|z_1+z_2|^2+|z_1-z_2|^2 = 2\left(|z_1|^2+|z_2|^2\right)$;

Zad 9. Zapisać w postaci algebraicznej liczby:

(a) $3(\cos\pi + i\sin\pi)$; **(b)** $2^3\left(\cos(\frac{\pi}{3})+i\sin(\frac{\pi}{3})\right)$; **(c)** $2\left(\cos(\frac{7}{6}\pi)+i\sin(\frac{7}{6}\pi)\right)$; **(d)** $\cos(\frac{3}{4}\pi)+i\sin(\frac{3}{4}\pi)$;
(e) $\cos(-\frac{\pi}{4})+i\sin(-\frac{\pi}{4})$; **(f)** $\cos(7\frac{2}{3}\pi)+i\sin(7\frac{2}{3}\pi)$; **(g)** $\cos(3\frac{5}{6}\pi)+i\sin(3\frac{5}{6}\pi)$;

Zad 10. Zapisać w postaci trygonometrycznej liczby:

(a) i; **(b)** $1-i$; **(c)** $1+i\sqrt{3}$; **(d)** $-1-\frac{i}{\sqrt{3}}$; **(e)** $\sqrt{3}-i$; **(f)** $-\frac{1}{2}+\frac{\sqrt{3}}{2}i$; **(g)** $9\sqrt{3}-9i$;
(h) $-7\sqrt{2}-7\sqrt{2}i$; **(i)** $7-4i$; **(j)** $-3+2i$; **(k)** $-5-3i$; **(l)** $2+3i$; **(m)** $0.34-1.23i$
(n) $\sin(\alpha)+i\cos(\alpha)$; **(o)** $-\cos(\alpha)+i\sin(\alpha)$; **(p)** $1+i\,\mathrm{tg}(\alpha)$;

Uwaga. W ostatnich podpunktach przyjmujemy $\alpha\in(0,\frac{\pi}{2})$.

Zad 11. Obliczyć:

(a) $\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)^{12}$; **(b)** $(1+i)^4$; **(c)** $\left(\sqrt{2}-i\sqrt{2}\right)^7$; **(d)** $\left(\frac{-1+i\sqrt{3}}{12}\right)^{12}$; **(e)** $\frac{(1+i)^9}{(1-i)^7}$; **(f)** $\frac{(1-i)^5-1}{(1+i)^5+1}$;
(g) $\left(\frac{1+i\sqrt{3}}{1-i}\right)^{20}$; **(h)** $\left(\frac{1-i\sqrt{3}}{2}\right)^{2007}$; **(i)** $\frac{(1+i\sqrt{3})^{15}}{(1+i)^{10}}$; **(j)** $\frac{(1-i\sqrt{3})^6}{(1+i\sqrt{3})^4}+(1+i)(3-i)$; **(k)** $\frac{(1-i)^6}{(1+i)^4}i^{74}$;

Zad 12. Obliczyć:

(a) $\sqrt[3]{i}$; **(b)** $\sqrt[6]{-1}$; **(c)** $\sqrt[5]{1}$; **(d)** $\sqrt[3]{27i}$; **(e)** $\sqrt[3]{(1+i)^3}$; **(f)** $\sqrt[3]{-8}$; **(g)** $\sqrt{8i-15}$; **(h)** $\sqrt[3]{2-2i}$;
(i) $\sqrt[4]{z}$ dla $z=\frac{(1-i)^6(\sqrt{3}+i)}{(1+i)^4}i^{74}$; **(j)** $\sqrt[4]{\left(\sqrt{3}-i\right)^{12}}$; **(k)** $\sqrt{\left(\frac{7}{16}+\frac{i}{16}\right)(1-i)^7}$; **(l)** $\sqrt[5]{-4+4i}$;
(m) $\sqrt[4]{-\frac{1}{32}(1+\sqrt{3}i)(1-\sqrt{3}i)^8}$;

Zad 13. Znając jeden z pierwiastków wyznaczyć wszystkie pozostałe pierwiastki:

(a) $\sqrt{-2i}$; $z_1=-1+i$; **(b)** $\sqrt[4]{-8-8i\sqrt{3}}$; $z_1=-\sqrt{3}+i$; **(c)** $\sqrt[6]{-1}$; $z_1=i$;

Zad 14. Korzystając ze wzoru Moivre'a wyrazić za pomocą $\sin x$ oraz $\cos x$ funkcje:

(a) $\sin(3x)$ oraz $\cos(3x)$; **(b)** $\sin(4x)$ oraz $\cos(4x)$; **(c)** $\sin(5x)$ oraz $\cos(5x)$;

Zad 15. Narysować na płaszczyźnie zespolonej obszary określone warunkami:

(a) $|z-1+i|=1$; **(b)** $2<|z-1|\leq 4$; **(c)** $\frac{|z|}{|z-1|}=2$; **(d)** $|z|<2\wedge\arg(z)\in\langle 0,\pi)$; **(e)** $|z|^2=2|z|$;
(f) $z\overline{z}+z+\overline{z}=0$; **(g)** $\left|\frac{z+2}{z-2}\right|>\sqrt{3}$; **(h)** $\frac{4}{|z|}\geq|\overline{z}|\wedge\arg(z)\in\langle-\frac{\pi}{6},\frac{\pi}{3}\rangle$; **(i)** $\overline{z-i}=z-1$;
(j) $1\leq\left|\frac{z-2}{2z-1}\right|\leq 2$; **(k)** $\mathrm{Re}(z^2)>\mathrm{Re}(\frac{1}{z^2})$; **(l)** $\arg(z-i\,z)=\frac{3\pi}{4}$; **(m)** $\arg(z^3)<\frac{\pi}{2}$;

Zad 16. Zamienić postać wykładniczą na algebraiczną:

(a) $e^{\pi i}$; **(b)** $e^{1+\frac{\pi}{2}i}$; **(c)** $e^{2\pi i}$; **(d)** e^{i}; **(e)** e^{-2i}; **(f)** $e^{1-\frac{3}{4}\pi i}$ **(g)** $e^{2+\frac{2}{3}\pi i}$ **(h)** $e^{1-\frac{7}{6}\pi i}$

Zad 17. Zamienić postać algebraiczną na wykładniczą:

(a) -1; **(b)** $1+i$; **(c)** $-i$; **(d)** $1-\sqrt{3}i$; **(e)** $-2+7i$; **(f)** $3-5i$;

Zad 18. Wykonać działania. Wynik zapisać w postaci wykładniczej.

(a) $e^{-3+5i}\cdot e^{2-3i}$; **(b)** $\frac{e^{-2-i}}{e^{5+3i}}$; **(c)** $e^{1+i}+e^{2+3i}$; **(d)** $e^{-2+3i}+e^{7-3i}$; **(e)** $e^{-3i-5}-e^{2-2i}$;

Liczby zespolone - odpowiedzi

Zad 1.

(a) -1; **(b)** $-i$; **(c)** 1; **(d)** i; **(e)** -1; **(f)** i; **(g)** $-i$; **(h)** $-i$; **(i)** -1; **(j)** i; **(k)** 1; **(l)** $-i$; **(m)** i; **(n)** 1;

Zad 2.

(a) $\frac{39}{4}+\frac{13}{4}i$; **(b)** $\frac{3}{4}$; **(c)** $\frac{i}{2}-\frac{15}{16}$; **(d)** -1; **(e)** $-\frac{7}{27}i-\frac{527}{1296}$; **(f)** $\frac{2}{5}+\frac{9}{5}i$; **(g)** $-\frac{8}{29}-\frac{9}{29}i$; **(h)** $\frac{33}{25}-\frac{56}{25}i$; **(i)** $-\frac{142}{85}+\frac{44}{85}i$; **(j)** $-\frac{4}{5}+\frac{7}{5}i$;

Zad 3.

(a) $[x=1, y=1]$; **(b)** $[x=5, y=17]$; **(c)** brak rozwiązań w $\mathbb{R}$; **(d)** $[x=1, y=-2]$; **(e)** $[x=2, y=3]$; **(f)** $[x=1, y=6]$;

Zad 4.

(a) $\left[\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i, -\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right]$; **(b)** $[-2+2i, 2-2i]$; **(c)** ; **(d)** ; **(e)** ; **(f)** ;

Zad 5.

(a) $z_1=\frac{1}{2}-\frac{\sqrt{3}i}{2}, z_2=\frac{1}{2}+\frac{\sqrt{3}i}{2}$; **(b)** $z_1=-2-i, z_2=-2+i$; **(c)** $z=-\frac{7}{5}i-\frac{9}{5}$;
(d) $z=-\frac{45}{73}i-\frac{99}{73}$; **(e)** $z_1=2-3i, z_2=3i+2$; **(f)** $z=\frac{1}{2}i+\frac{5}{6}$;
(g) $z_1=z_2=z_3=2i$; **(h)** $z_1=-2i, z_2=2i, z_3=-1, z_4=1$; **(i)** $z_1=-\frac{i}{2}, z_2=\frac{i}{2}, z=1$;
(j) $z_1=\frac{3\sqrt{2}}{2}i-\frac{3\sqrt{2}}{2},\ z_2=-\frac{3\sqrt{2}}{2}i-\frac{3\sqrt{2}}{2},\ z_3=\frac{3\sqrt{2}}{2}-\frac{3\sqrt{2}}{2}i,\ z_4=\frac{3\sqrt{2}}{2}i+\frac{3\sqrt{2}}{2}$;
(k) $z_1=\frac{\sqrt{3}i+1}{2},\ z_2=\frac{\sqrt{3}i-1}{2},\ z_3=-1,\ z_4=-\frac{\sqrt{3}i+1}{2},\ z_5=-\frac{\sqrt{3}i-1}{2},\ z_6=1$;
(l) $z_1=-i-4,\ z_2=i+4,\ z_3=i-2,\ z_4=2-i$;
(m) $z_1=4,\ z_2=-2,\ z_3=-i-1,\ z_4=i-1$;
(n) $z_1=1-i, z_2=i+1, z_3=2$; **(o)** $z_1=3, z_2=-i, z_3=i$; **(p)** $z_1=-\frac{3}{5}i-\frac{6}{5},\ z_2=i+1$;
(q) ; **(r)** $z_1=-2+i, z_2=-2-3i$; **(s)** $z_1=-2i, z_2=i, z_3=-3i$;
(t) ; **(u)** $z_1=i,\ z_2=2-i$; **(v)** ;

Zad 6.

(a) $z_1=-3,\ z_2=0,\ z_3=\frac{3}{2}+\frac{3\sqrt{3}}{2}i,\ z_4=\frac{3}{2}-\frac{3\sqrt{3}}{2}i$; **(b)** $z=2-5i$; **(c)** $z_1=k,\ z_2=-2+ki \quad k\in\mathbb{R}$; **(d)** $z=k, \quad k\in\mathbb{R}$;

Zad 7.

(a) 5; **(b)** $\sqrt{7}$; **(c)** $\sqrt{197}$; **(d)** $\sqrt{26}$; **(e)** 1; **(f)** $\sqrt{26}$; **(g)** $\frac{1}{4}\pi$; **(h)** $\frac{2}{3}\pi$; **(i)** $-\frac{1}{6}\pi$;
(j) $-\arctan\left(\frac{5}{2}\right)\approx -1.19$; **(k)** $\arctan\left(\sqrt{3}-2\right)\approx -0.262$; **(l)** $\pi-\arctan\left(2-\sqrt{3}\right)\approx 2.88$;
(m) $\left(-\frac{\sqrt{3}}{2}-1\right)i+\sqrt{3}-\frac{1}{2}$; **(n)** $1-3i$; **(o)** $-i-1$;
(p) $\frac{1}{13}-\frac{5}{13}i$;

Zad 8.

(a) Niech $z_1=x_1+iy_1$ oraz niech $z_2=x_2+y_2i$. Wtedy

$$\begin{aligned}
|z_1\cdot z_2| &= |(x_1+y_1i)\cdot(x_2+y_2i)| = |(x_1y_2+x_2y_1)\,i-y_1y_2+x_1x_2| \\
&= \sqrt{(x_1x_2-y_1y_2)^2+(x_1y_2+x_2y_1)^2} = \sqrt{{y_1}^2{y_2}^2+{x_1}^2{y_2}^2+{x_2}^2{y_1}^2+{x_1}^2{x_2}^2} \\
&= \sqrt{{y_1}^2+{x_1}^2}\cdot\sqrt{{y_2}^2+{x_2}^2} \\
&= |z_1|\cdot|z_2|\,.
\end{aligned}$$

Zad 9.

(a) -3; **(b)** $4+4\sqrt{3}i$; **(c)** $-\sqrt{3}-i$; **(d)** $-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$; **(e)** $\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i$; **(f)** $\frac{1}{2}-\frac{\sqrt{3}}{2}i$; **(g)** $\frac{\sqrt{3}}{2}-\frac{1}{2}i$;

Zad 10.

(a) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$; **(b)** $\sqrt{2}\left(\cos\left(\frac{7}{4}\pi\right) + i\sin\left(\frac{7}{4}\pi\right)\right)$; **(c)** $2\left(\cos\left(\frac{1}{3}\pi\right) + i\sin\left(\frac{1}{3}\pi\right)\right)$;
(d) $\frac{2\sqrt{3}}{3}\left(\cos\left(\frac{7}{6}\pi\right) + i\sin\left(\frac{7}{6}\pi\right)\right)$; **(e)** $\sqrt{2}\left(\cos\left(\frac{11}{6}\pi\right) + i\sin\left(\frac{11}{6}\pi\right)\right)$; **(f)** $\cos\left(\frac{4}{3}\pi\right) + i\sin\left(\frac{4}{3}\pi\right)$;
(g) $18\left(\cos\left(\frac{11}{6}\pi\right) + i\sin\left(\frac{11}{6}\pi\right)\right)$; **(h)** $14\left(\cos\left(\frac{5\pi}{4}\right) + i\sin\left(\frac{5\pi}{4}\right)\right)$; **(i)** $\approx 8,06\,(\cos(5,76) + i\sin(5,76))$;
(j) $\approx 3,61\,(\cos(2,55) + i\sin(2,55))$; **(k)** $\approx 5,83\,(\cos(3,68) + i\sin(3,68))$; **(l)** $\cos\left(\frac{\pi}{2} - \alpha\right) + i\sin\left(\frac{\pi}{2} - \alpha\right)$;
(m) $\cos(\pi - \alpha) + i\sin(\pi - \alpha)$; **(n)** $\frac{1}{\cos\alpha}(\cos\alpha + i\sin\alpha)$;

Zad 11.

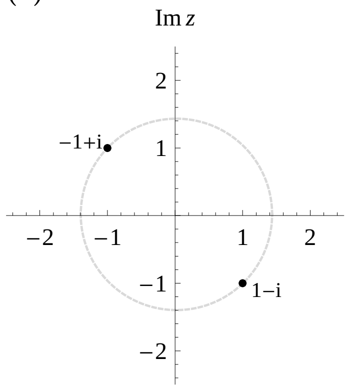
(a) -1; **(b)** -4; **(c)** $64\sqrt{2}i + 64\sqrt{2}$; **(d)** $\frac{1}{2^{12}3^{12}}$; **(e)** 2; **(f)** $-\frac{32}{25}i - \frac{1}{25}$; **(g)** -2^15i; **(h)** -1;
(i) $2^{10}i$; **(j)** $\left(2\sqrt{3}+2\right)i + 2$; **(k)** $2i$;

Zad 12.

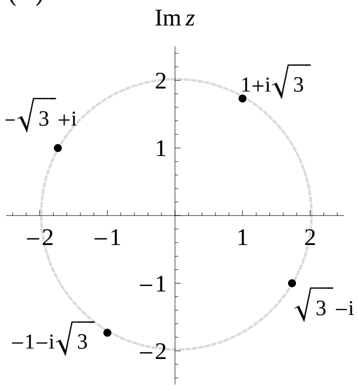
(a) $z_1 = \frac{i}{2} - \frac{\sqrt{3}}{2}, z_2 = -i, z_3 = \frac{i}{2} + \frac{\sqrt{3}}{2}$;
(b) $z_1 = i, z_2 = \frac{i}{2} - \frac{\sqrt{3}}{2}, z_3 = -\frac{i}{2} - \frac{\sqrt{3}}{2}, z_4 = -i, z_5 = \frac{\sqrt{3}}{2} - \frac{i}{2}, z_6 = \frac{i}{2} + \frac{\sqrt{3}}{2}$;
(c) $z_1 = i\sin\left(\frac{2\pi}{5}\right) + \cos\left(\frac{2\pi}{5}\right), z_2 = i\sin\left(\frac{4\pi}{5}\right) + \cos\left(\frac{4\pi}{5}\right), z_3 = \cos\left(\frac{4\pi}{5}\right) - i\sin\left(\frac{4\pi}{5}\right), z_4 = \cos\left(\frac{2\pi}{5}\right) - i\sin\left(\frac{2\pi}{5}\right), z_5 = 1$;
(d) $z_1 = \frac{3i-3\sqrt{3}}{2}, z_2 = -3i, z_3 = \frac{3i+3\sqrt{3}}{2}$;
(e) $z_1 = \frac{\left(\sqrt{3}-1\right)i-\sqrt{3}-1}{2}, z_2 = -\frac{\left(\sqrt{3}+1\right)i-\sqrt{3}+1}{2}, z_3 = i+1$;
(f) $z_1 = 1 - \sqrt{3}i, z_2 = \sqrt{3}i + 1, z_3 = -2$;
(g) $z_1 = -4i - 1, z_2 = 4i + 1$;
(h) $z_1 = -\frac{\left(\sqrt{3}-1\right)i-\sqrt{3}-1}{2}, z_2 = \frac{\left(\sqrt{3}+1\right)i-\sqrt{3}+1}{2}, z_3 = -i - 1$;
(i) $z_1 = \frac{\sqrt{6}i-\sqrt{2}}{2}, z_2 = -\frac{\sqrt{2}i+\sqrt{6}}{2}, z_3 = -\frac{\sqrt{6}i-\sqrt{2}}{2}, z_4 = \frac{\sqrt{2}i+\sqrt{6}}{2}$;
(j) $z_1 = 8i, z_2 = -8, z_3 = -8i, z_4 = 8$;

Zad 13.

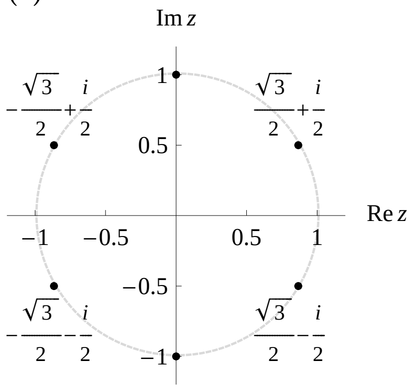
(a)



(b)



(c)

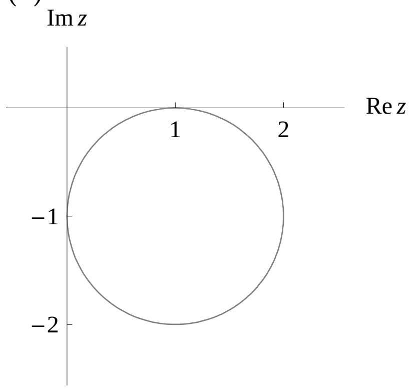


Zad 14.

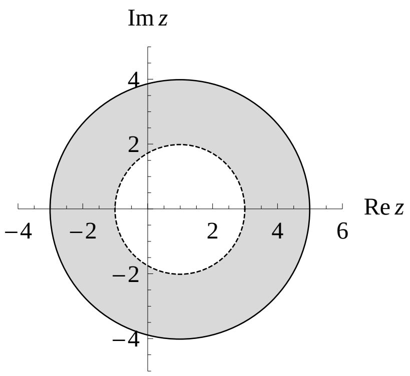
(a) $\sin(3x) = 3\cos^2 x\sin x - \sin^3(x),\ \cos(3x) = \cos^3 x - 3\cos x\sin^2 x$;
(b) $\sin(4x) = 4\cos^3 x\sin x - 4\cos x\sin^3 x,\ \cos(4x) = \cos^4 x - 6\sin^2 x\cos^2 x + \sin^4 x$;
(c) $\sin(5x) = \sin^5 x - 10\cos^2 x\sin^3 x + 5\cos^4 x\sin x,\ \cos(5x) = \cos^5 x - 10\sin^2 x\cos^3 x + 5\sin^4 x\cos x$;

Zad 15.

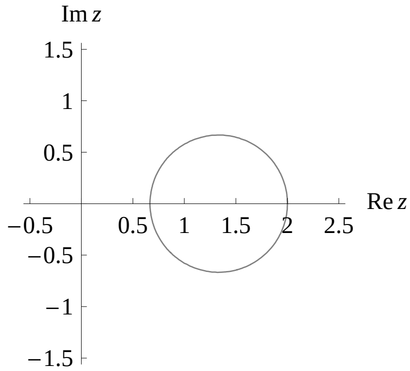
(a)



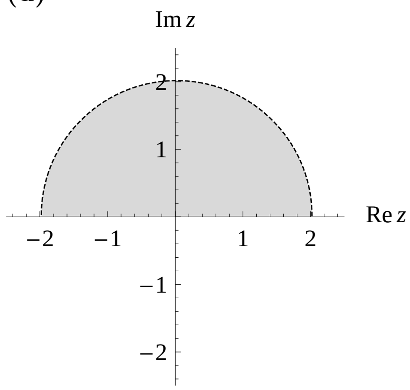
(b)



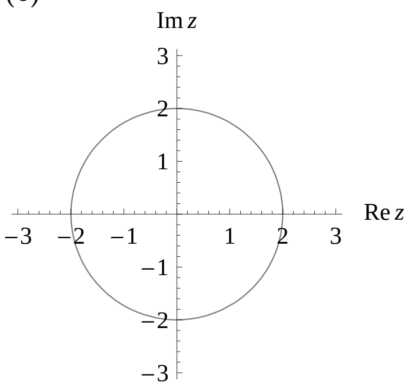
(c)



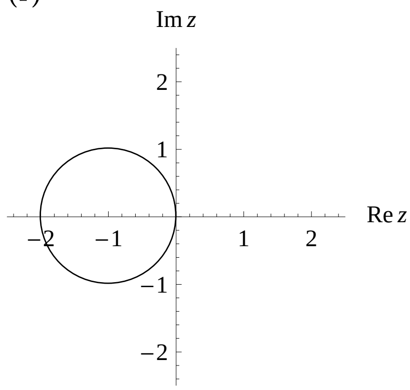
(d)



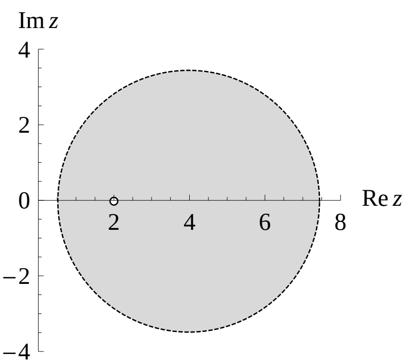
(e)



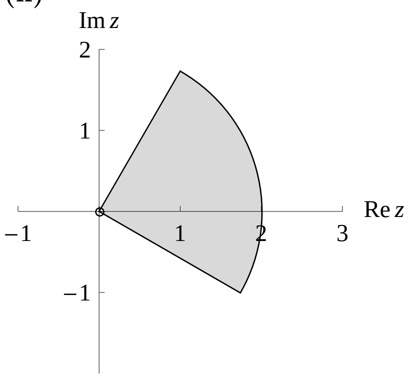
(f)



(g)

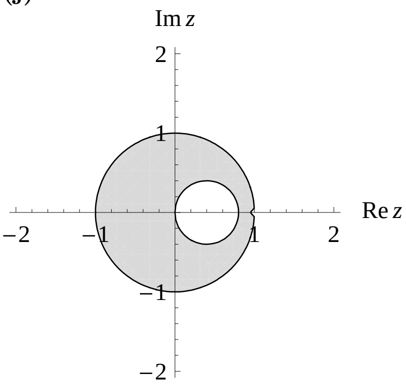


(h)

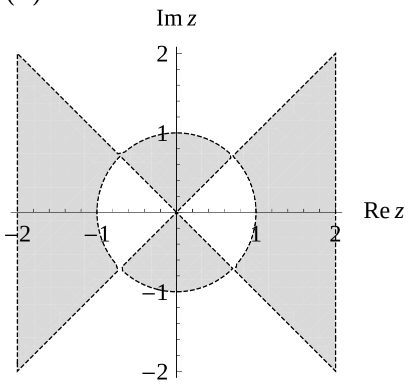


(i) $z \in \emptyset$

(j)



(k)



(l)

Zad 16.

(a) -1; **(b)** ei; **(c)** 1; **(d)** $\cos(1) + i\sin(1) \approx 0.54 + 0.84\,i$; **(e)** $\cos(2) - i\sin(2) \approx 0.42 - 0.91\,i$;
(f) $e\left(-\frac{\sqrt{2}}{2}i - \frac{\sqrt{2}}{2}\right)$; **(g)** $e^2\left(\frac{\sqrt{3}}{2}i - \frac{1}{2}\right)$; **(h)** $e^2\left(-\frac{i}{2} - \frac{\sqrt{3}}{2}\right)$.

Zad 17.

(a) $e^{\pi i}$; **(b)** $\sqrt{2}e^{\frac{\pi}{4}i}$; **(c)** $e^{-\frac{\pi}{2}i}$; **(d)** $2\,e^{\frac{\pi}{3}i}$; **(e)** $\sqrt{53}\,e^{i\left(\pi-\arctan\left(\frac{7}{2}\right)\right)} \approx 7,28\,e^{1,85\,i}$;
(f) $\sqrt{34}\,e^{-i\arctan\left(\frac{5}{3}\right)} \approx 5.83\,e^{-1.03\,i}$;

Zad 18.

(a) e^{-1+2i}; **(b)** e^{-7-4i}; **(c)** $\approx 6,73\,e^{2,62\,i}$; **(d)** $\approx 1096,76\,e^{-3i}$; **(e)** ;