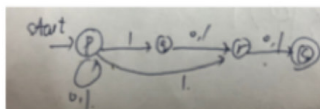


哈尔滨工业大学2019年《形式语言与自动机》期末试题

1. Design a DFA for the language $L = \{w \in \{0,1\}^* \mid w \text{ contains both } 01 \text{ and } 10 \text{ as substrings}\}$.
2. Design a NFA within four states for the language $\{a\}^* \cup \{ab\}^*$.
3. Design regular expressions for language over $\Sigma = \{0,1\}$.
 - (1). All strings contain the substring 001.
 - (2). All strings except the string 001.
4. Prove that $L = \{0^m 1^n \mid m/n \text{ is an integer}\}$ is not regular with pumping lemma.
5. Convert the following NFA into DFA with subset construction.



6. Give a context-free grammar for $L = \{a^i b^j c^{i+j} \mid i, j \geq 0\}$
7. Let L be the language generated by the grammar G below

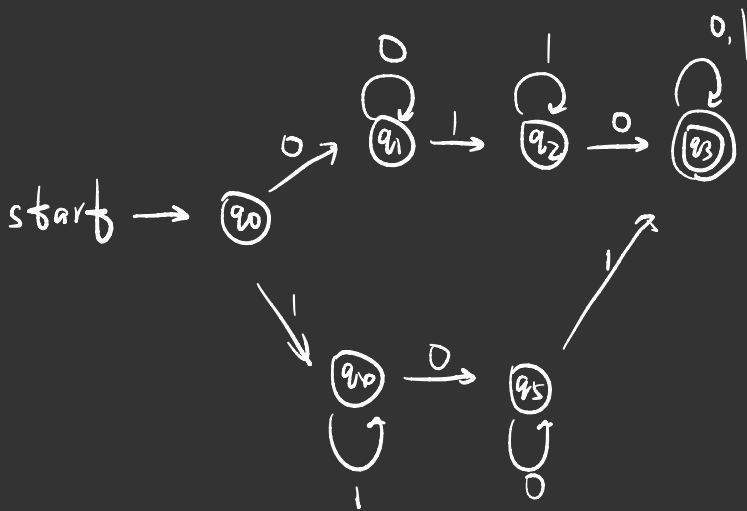
$S \rightarrow AB \mid BBB$
 $A \rightarrow Bb \mid \epsilon$
 $B \rightarrow aB \mid A$

 - (1). 消除空产生式
 - (2). 消除单元产生式
 - (3). 转换到CNF
8. Design a PDA for $L = \{w \in \{a,b\}^* \mid w \text{ has more } a's \text{ than } b's\}$
9. Prove : for every context free language L , the language $L' = \{0^{|w|} \mid w \in L\}$ is also context free.
10. Design a Turing Machine that computes the following function $f: 0^n \rightarrow \text{Binary}(n)$
 Where integer $n \geq 1$ and $\text{binary}(n)$ is the binary representation of n .
 For example: $f(0^3) = 11$ $f(0^5) = 101$.

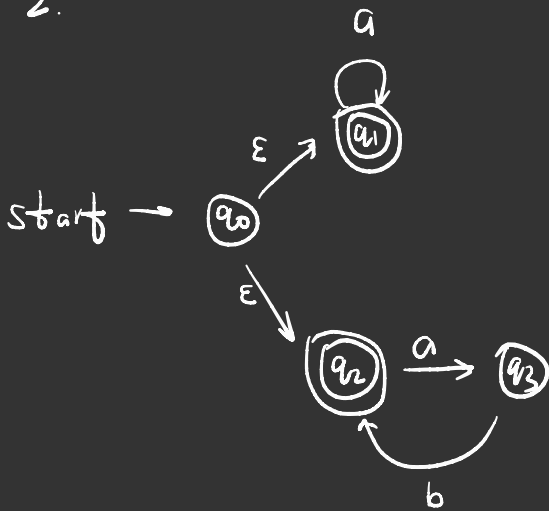
1.

0000|110

10--1



2.



3.

$$(1) (0+1)^* 001 (0+1)^*$$

$$(2) (0+1)^4 (0+1)^*$$

$$+ (0+1)' + (0+1)^2 + 000 + 010$$

$$+ 011 + 100 + 101 + 110 + 111$$

4. ① 假设 L 是正则语言

② 取 $w = 0^{2N} 1^{2N}$, $2N/2N=1$, 故 $w \in L$

③ $|w| = 4N \geq N$,

④ 故 $\exists x, y$ 子, 使 $w = xy$

$w = xy$ 且满足 $|y| > 0$, $|xy| \leq N$

$\forall i \in \mathbb{N}$,

xy^i 子 L

⑤ 由于 $|xy| \leq N$, 不妨设 $x = 0^a$, $y = 0^b$,

有 $a \geq 0$, $b > 0$, $a+b \leq N$

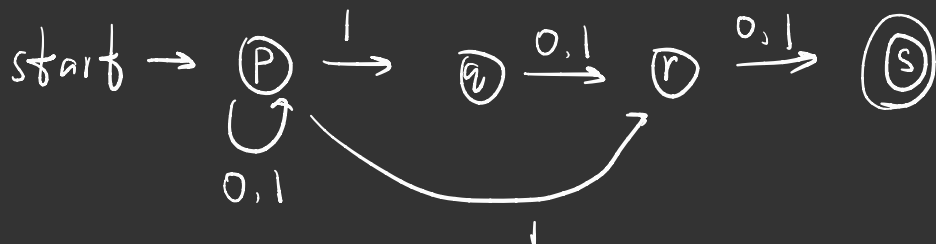
⑥ xy^2 子 $0^{2N+b} 0^{2N}$

$2N < 2N+b \leq 3N$

即 $2N < 2N+b < 4N$, 故 $(2N+b)/2N$
不是整数

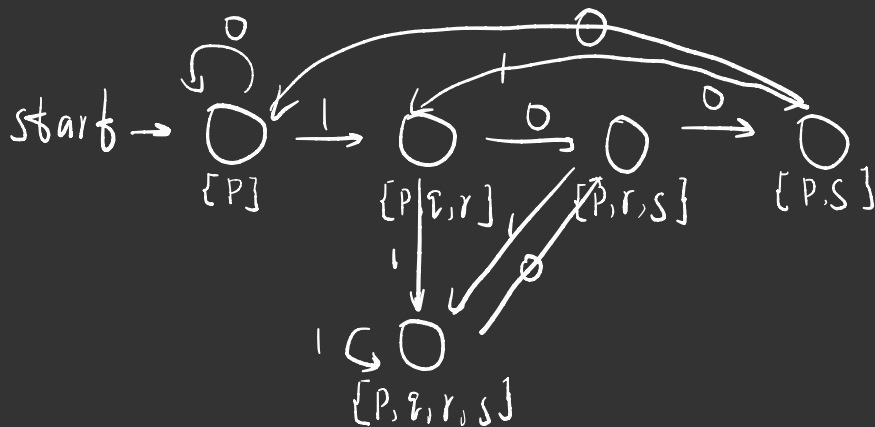
⑦ 不满足泵引理, 故 L 不是正则语言

5.



	0	1
→ P	{P}	{P, q, r}
q	{r}	{r}
r	{s}	{s}
* S	∅	∅

	0	1
→ {P}	{P}	{P, q, r}
{P, q, r}	{P, r, s}	{P, q, r, s}
* {P, r, s}	{P, s}	{P, q, r, s}
* {P, q, r, s}	{P, r, s}	{P, q, r, s}
* {P, s}	{P}	{P, q, r}



$$6. \quad S \rightarrow a S c \mid D$$

$$D \rightarrow b D c \mid \varepsilon$$

$$7. \quad S \rightarrow AB \mid BBB$$

$$A \rightarrow Bb \mid \varepsilon$$

$$B \rightarrow aB \mid A$$

(1) 消空

$$S \rightarrow AB \mid A \mid B \mid BBB \mid BB$$

$$A \rightarrow Bb \mid b$$

$$B \rightarrow aB \mid a \mid A$$

(2) 消单元

$$S \rightarrow AB \mid Bb \mid b \mid aB \mid a \mid BBB \mid BB$$

$$A \rightarrow Bb \mid b$$

$$B \rightarrow aB \mid a \mid Bb \mid b$$

(2) 消单元

$$S \rightarrow AB \mid Bb \mid b \mid aB \mid a \mid BBB \mid BB$$

$$A \rightarrow Bb \mid b$$

$$B \rightarrow aB \mid a \mid Bb \mid b$$

(3) $S \rightarrow AB \mid BCb \mid b \mid CaB \mid a \mid DbB \mid BB$

$$Ca \rightarrow a$$

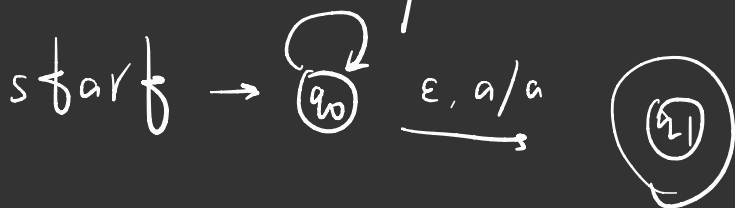
$$Cb \rightarrow b$$

$$Db \rightarrow BB$$

$$A \rightarrow BCb \mid b$$

$$B \rightarrow CaB \mid a \mid BCb \mid b$$

8.

$$\begin{array}{l}
 b, a / \varepsilon \\
 b, b / bb \\
 a, b / \varepsilon \\
 a, a / aa \\
 b, \bar{z}_0 / b\bar{z}_0 \\
 a, \bar{z}_0 / a\bar{z}_0
 \end{array}$$


9. 有同志

$$h(a) = 0$$

$$\text{则 } h(w) = 0^{|w|}$$

$$\text{故 } h(L) = \{0^{|w|} \mid w \in L\} = L'$$

由于 CFL 对同志封闭.

L 是 CFL, 故 L' 也是 CFL

