

主管
领导
审核
签字

1. [10 points] Design a DFA for $L = \{w \in \{0,1\}^* \mid w \text{ has exactly three 0s.}\}$

2. [10 points] Design an NFA for the language:

$$L = \{w \in \{a,b,c\}^* \mid w \text{ starts with } ac \text{ and ends with } cb.\}$$

3. [10 points] Design regular expressions for languages over $\Sigma = \{a,b\}$.

(1) All strings that do not end with aba .

(2) $L = \{w \mid w \text{ has no more than 5 } a\text{'s.}\}$

4. [10 points] Prove that the language $L = \{w \in \{a,b\}^* \mid w = w^R\}$ is not regular with pumping lemma.

5. [10 points] Consider the following ε -NFA.

	ε	a	b	c
$\rightarrow p$	$\{q, r\}$	\emptyset	$\{q\}$	$\{r\}$
q	\emptyset	$\{p\}$	$\{r\}$	$\{p, q\}$
$*r$	\emptyset	\emptyset	\emptyset	\emptyset

(1) Compute the ε -closure of each state.

(2) Give all the strings of length three or less accepted by the automaton.

(3) Convert the automaton to a DFA by subset construction. (diagram of transition function)

6. [10 points] Give a CFG for $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j + k\}$.

7. [10 points] Find a grammar equivalent to

$$S \rightarrow AB \mid CA$$

$$A \rightarrow a$$

$$B \rightarrow BC \mid AB$$

$$C \rightarrow aB \mid b$$

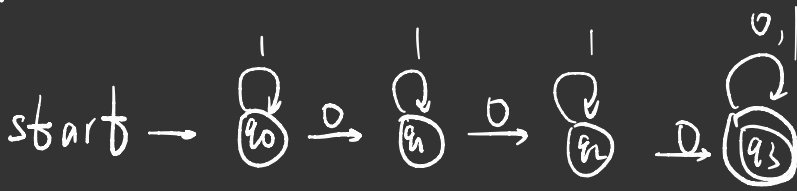
with no useless symbols.

8. [10 points] Design a PDA for $L_{eq} = \{w \in \{0,1\}^* \mid w \text{ contains the same number of 0's and 1's}\}$.

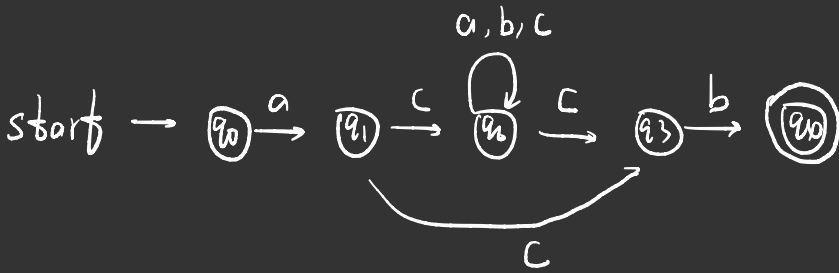
9. [10 points] Prove or disprove: if L_1 is CFL and $L_1 \cup L_2$ is also CFL, then L_2 must be CFL.

10. [10 points] Design Turing machine for the language $\{0^{2n}1^n \mid n \geq 0\}$.

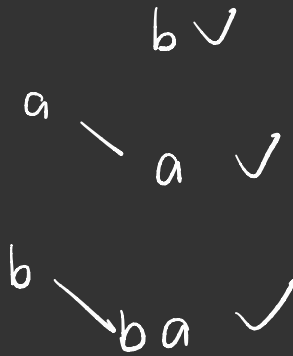
1.



2.



3.



$\epsilon + a + ba$

1) $(0+1)^* (b+aa+bbba)^*$

2)

$$b^*(a+\epsilon) b^* (a+\epsilon) b^* (a+\epsilon) b^*$$

$$(a+\epsilon) b^* (a+\epsilon) b^*$$

$$4. \quad a^N b^N b^N a^N$$

① 证明 L 是正则语言

② 取 $w = a^N b^N b^N a^N$, $|w| = 4N \geq N$
 $w \in L$.

③ 故 $\exists x, y, z$ $w = xyz$. 且

$$|y| > 0 \quad |xy| \leq N \quad xy^iz \in L$$

④ $x = a^i \quad y = a^j$

有 $j > 0 \quad i+j \leq N \quad i \geq 0$

⑤ xy^iz

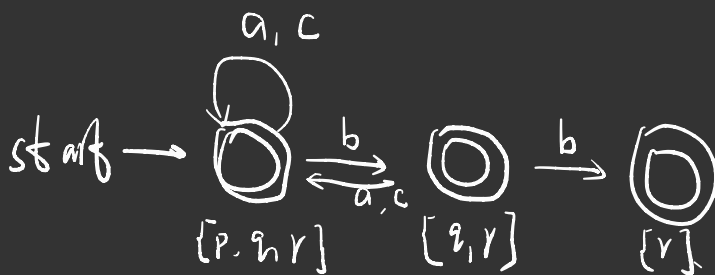
$$= a^{N+j} b^N b^N a^N$$

由于 $N+j > N$. 故 $xy^iz \notin L$.

⑥ L 不是正则语言

	a	b	c	ε close
→ P	\emptyset	$\{q\}$	$\{r\}$	$\{P, q, r\}$
q	$\{P\}$	$\{r\}$	$\{P, q\}$	$\{c\}$
* r	\emptyset	\emptyset	\emptyset	$\{r\}$

	a	b	c
* $\{P, q, r\}$	$\{P, q, r\}$	$\{q, r\}$	$\{P, q, r\}$
* $\{q, r\}$	$\{P, q, r\}$	$\{r\}$	$\{P, q, r\}$
* $\{r\}$	\emptyset	\emptyset	\emptyset



$\epsilon, a, b, c, aa^*, ab^*, ac^*$

$ba^*, bb, bc^*, ca^*, cb^*, cc^*$

$$6. S \rightarrow aSc \mid D$$

$$D \rightarrow aDb \mid \epsilon$$

$$7. S \rightarrow \cancel{AB} \mid CA$$

$$A \rightarrow a$$

$$\cancel{B \rightarrow \cancel{BC} \mid \cancel{AB}}$$

$$C \rightarrow \cancel{aB} \mid b.$$

(1) ✓

(2) ✓

$$(3) S \rightarrow CA$$

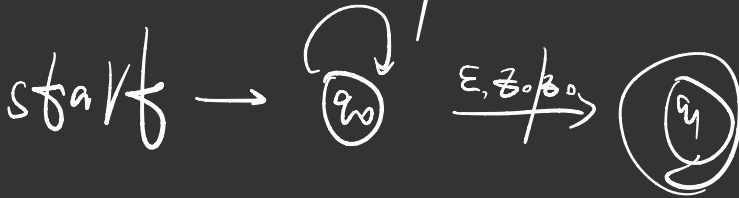
$$A \rightarrow a$$

$$C \rightarrow b$$

$$S \rightarrow ba$$

8.

$1, 1 / 1$
 $1, 0 / \epsilon$
 $1, z_0 / z_0$
 $0, 1 / \epsilon$
 $0, 0 / 00$
 $0, z_0 / 0z_0$



\uparrow

$L_1 : CFL$

$a^* b^* c^*$

$L_2 : \text{not } CFL$

$a^n b^n c^n$

$L_1 \cup L_2 : CFL$

$a^* b^* c^*$

Ex 1

B

