

Elementary Model Theory in Rocq

Current Progress Report

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1 Introduction

The study of Mathematical “models” and their capabilities, interpretations, and theories.

Models?

- Formulas
- Languages
- Sentences
- What are they composed of?
- How do we write things down?

Idea: Given a language, a sentence, and a theory, what does it mean?
Is the theory valid? Are there some things which cannot be proven in this language?

- Implement Elementary Model Theory in Rocq
 - Gödel's First Incompleteness Theorem
 - Łoś's Theorem (ultraproducts)
 - Löwenheim–Skolem Theorem
 - Concerning completeness/compactness
 - Stability

Proofs requires non-trivial definitions, theorems and lemmas.

From what he have seen, it is very difficult to find Model Theory libraries and or implementations in Rocq.

- Why?

- Define primary concepts and syntax.
- State lemmas and work upwards towards desired theorem.
- Restrictions on `Stdlib`

2 Our Progress

Definitions so Far:

- First-order language: \mathcal{L}
 - Logical symbols of \mathcal{L} : $\text{Var}, \doteq, \wedge, \vee, \neg, \rightarrow, \leftrightarrow, \forall, \exists$
 - Signature of \mathcal{L} : $\text{Const}(\mathcal{L}), \text{Rel}(\mathcal{L}), \text{Fun}(\mathcal{L})$
- Terms of \mathcal{L} or \mathcal{L} -terms
- Formulas of \mathcal{L} or \mathcal{L} -formulas

Sections:

- Languages
- Semantics

- Translating definitions into Rocq:
 - Enforcing properties?
- Weak definitions and or induction schema
- Proving simple things are hard
- Proving in Rocq requires deep knowledge of proof.
 - Learning Model Theory
- Getting stuck

3 What's Next

Fallbacks:

- Focus on most important results:
 - Gödel's Results
 - Completeness/Compactness

- Rocq's "Limitations"
- No Sandboxing
- Acknowledging Previous Efforts

