Boosted HP Filter

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Illustartion of bHP, by Iris Shi

```r
library(bHP)
library(magrittr)
```

Introduction

This vignette introduces the HP filter, the boosted HP filter, and the usage of the R package `bHP`. The Hodrick-Prescott filter (HP filter; Hodrick and Prescott (1997)) is one of the fundamental statistical tools in macroeconomic data analysis. Thanks to its simplicity, it has been widely used in empirical macroeconomics studies. As an operational algorithm, its pros and cons have been debated over decades, and academic interest has been renewed in recent years to investigate its properties and extensions. While Hamilton (2018) argues against the usage of the HP filter, Phillips and Shi (2019) propose a machine learning enhancement of the original HP filter, called the boosted HP filter (bHP), and establish consistency when it is applied to a large class of trended time series in macroeconomic applications.

HP filter

Given a time series $(x_t)_{t=1}^n$ the HP method decomposes it into two additive components: a trend component f_t and a cyclical component (residual) c_t. The trend is estimated as

$$(\hat{f}_t^{\mathrm{HP}}) = \arg\min_{(f_t)} \left\{ \sum_{t=1}^{n} (x_t - f_t)^2 + \lambda \sum_{t=2}^{n} (\Delta^2 f_t)^2 \right\},$$

where $\Delta f_t = f_t - f_{t-1}$, and $\Delta^2 f_t = \Delta f_t - \Delta f_{t-1} = f_t - 2f_{t-1} + f_{t-2}$, and $\lambda \geq 0$ is a tuning parameter that controls the level of the penalty. The corresponding cycle is

$$(\hat{c}_t^{\mathrm{HP}}) = (x_t - \hat{f}_t^{\mathrm{HP}}).$$

The optimization problem admits a closed form solution. The estimated trend can be written as

$$\widehat{f}^{\mathrm{HP}} = Sx, \tag{1}$$

where S is a deterministic $n \times n$ matrix and $x = (x_1, ..., x_n)'$ is the sample data. The estimated trend is

$$\widehat{c}^{\mathrm{HP}} = (I_n - S)\, x,$$

where I_n is the $n \times n$ identity matrix. The explicit form of S can be found in Phillips and Shi (2019).

The choice of the tuning parameter is crucial for the behavior of the HP filter. In practice, Hodrick and Prescott (1997) recommend $\lambda = 1600$ for quarterly data, and this number and its sampling frequency adjusted version (Ravn and Uhlig 2002) are widely adopted. However, recent research (Phillips and Jin 2015) (Hamilton 2018) find the "gold standard" is too rigid for the length of time series that often used in macroeconomic studies.

Boosted HP filter

The intuition of bHP is that, if the cyclical component $\widehat{c}_t^{\mathrm{HP}}$ still exhibits trending behavior after HP filtering, we continue to apply the HP filter to $\widehat{c}^{\mathrm{HP}}$ to remove the leftover trend residual. After a second fitting, the cyclical component can be written as

$$\widehat{c}^{(2)} = (I_n - S)\,\widehat{c}^{\mathrm{HP}} = (I_n - S)^2\, x,$$

where the superscript "(2)" indicates that the HP filter is fitted twice. The corresponding trend component becomes

$$\widehat{f}^{(2)} = x - \widehat{c}^{(2)} = \left(I_n - (I_n - S)^2\right) x.$$

If $\widehat{c}^{(2)}$ continues to exhibit trend behavior, the filtering process may be continued for a third or further time. After m repeated applications of the filter, the cyclical and trend component are

$$\begin{aligned} \widehat{c}^{(m)} &= (I_n - S)\,\widehat{c}^{(m-1)} = (I_n - S)^m\, x \\ \widehat{f}^{(m)} &= x - \widehat{c}^{(m)}. \end{aligned}$$

The number of iterations m is an additional tuning parameter in bHP. In practice, it is recommended that we choose λ according to the convention, say $\lambda = 1600$ for quarterly data, and then monitor a stopping criterion as the iteration proceeds. Phillips and Shi (2019) suggest using either the ADF test or the Bayesian Information Criterion (BIC) to terminate the iteration.

This package `bHP` automates bHP. The main function is `BoostedHP`. The user chooses the two tuning parameters `lambda` for λ and `stopping` for the stopping criterion. In particular, three options are available for `stopping`:

- `"BIC"` for the BIC stopping criterion
- `"adf"` for the ADF stopping criterion (default p-value 5%)
- `"nonstop"` keeps iteration until it reaches `Max_iter` (default is 100 iterations).

The basic usage with the default options is as follows:

```
BoostedHP(x, lambda = 1600, iter= TRUE, stopping = "BIC", Max_Iter = 100)
```

The above line produces an object of the `bHP` class. We can extract the trend by `$trend`, the cycle by `$cycle`. The sequence of trend for each iteration is stored in `$trend_hist`, and `$iter_num` reports the number of iterations. The original HP filter can also be implemented by setting `iter = FALSE` along with a `lambda`.

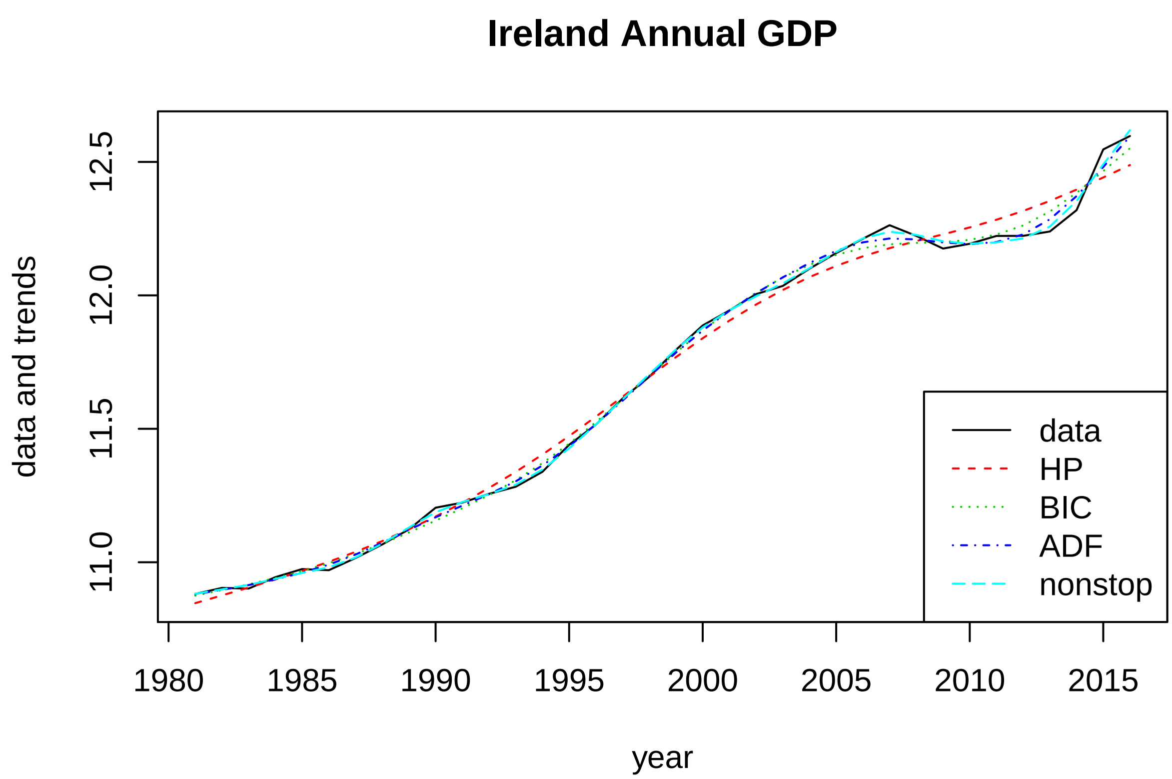
Examples

One of the applications in Phillips and Shi (2019) is concerning the international comparison of the Okun's law. We use Ireland's annual GDP here for illustration.

```r
lam <- 100 # tuning parameter for the annual data
data(IRE) # load the data 'IRE'

bx_HP <- BoostedHP(IRE, lambda = lam, iter= FALSE)$trend
bx_BIC <- BoostedHP(IRE, lambda = lam, stopping = "BIC")$trend
bx_ADF <- BoostedHP(IRE, lambda = lam, stopping = "adf")$trend
bx_nonstop <- BoostedHP(IRE, lambda = lam, iter= TRUE,
                        stopping = "nonstop") %>% predict( )
# use the generic method `predict` is an alternative way to get the trend

matplot( y = cbind(IRE, bx_HP, bx_BIC, bx_ADF, bx_nonstop),
         type = "l", x = 1981:2016, ylab = "data and trends",
         xlab = "year", main = "Ireland Annual GDP")
legend("bottomright", legend = c("data","HP", "BIC", "ADF", "nonstop"),
       col = 1:5, lty = 1:5)
```



The trend and cycle can also be extracted by the generic methods `predict` and `residuals`, respectively.

```r
bx <- BoostedHP(IRE, lambda = lam, stopping = "BIC")
IRE_cycle <- residuals(bx)
#> Retrun the trend component of BIC criterion.
```

```
#> Number of iterations: 5
```

Version

This is our first R package released on `github`, labeled with Version 1.0. The main function `BoostedHP` and associated methods `predict`, `residuals` and `BIC` are complete and well documented. The package also contains experimental generic methods `print`, `plot` and `summary`, which are still preliminary.

References

Hamilton, James D. 2018. "Why You Should Never Use the Hodrick-Prescott Filter." *Review of Economics and Statistics* 100 (5): 831–43.

Hodrick, Robert J, and Edward C Prescott. 1997. "Postwar Us Business Cycles: An Empirical Investigation." *Journal of Money, Credit, and Banking*, 1–16.

Phillips, Peter C B, and Sainan Jin. 2015. "Business Cycles, Trend Elimination, and the Hp Filter." Yale University.

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Ravn, Morten O, and Harald Uhlig. 2002. "On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations." *Review of Economics and Statistics* 84 (2): 371–76.