Exercises

CCDSTRU

Logical Equivalences

Pau Rivera

S11/S12: TH 1500-1730

S13/S14: WF 900-1130

Use Logical Equivalence Rules to simplify the following.

1. $\neg b \land (a \rightarrow b\ ) \land a$
2. $(a \rightarrow b) \leftrightarrow (\neg a \lor b)$

Determine whether each pair of propositions are logically equivalent or not. Use Logical Equivalence Rules.

1. $\neg\big(p \vee (\neg p \wedge q)\big)\ and\ \neg p \wedge \neg q$
2. $\neg(p \leftrightarrow q)\ and\ p \leftrightarrow \neg q$

Rules of Inference

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S11/S12: TH 1500-1730

S13/S14: WF 900-1130

In 1 whole sheet of yellow paper, determine if the following arguments are valid:

1. By using rules of inference, determine whether the argument below is valid. Clearly state your reasons for each step.

$$\begin{array}{l} p \to (q \to r) \\ p \lor s \\ \underline{\neg s \land \neg r} \\ \therefore \neg q \end{array}$$

2. Bees like red flowers, or my hat is red and bees like hats. However, my hat is not red, or bees don’t like hats but they like red flowers. Therefore, bees like red flowers.
3. No animals, except giraffes, are 15 feet or higher. There are no animals in this zoo that belong to anyone but me. I have no animals less than 15 feet high. Therefore, all animals in this zoo are giraffes.

In 1 whole sheet of yellow paper, determine if the following arguments are valid:

4. If Dominic goes to the racetrack, then Helen will be mad. If Ralph plays cards all night, then Carmela will be mad. If either Helen or Carmela gets mad, then Veronica (their attorney) will be notified. Veronica has not heard from either of these two clients. Therefore, Dominic didn't make it to racetracks and Ralph didn't play cards all night.
5. A student in this class has not read the book. Everyone in this class passed the first exam. Therefore, someone who passed the first exam has not read the book.
6. Carrots are vegetables and peaches are fruit. There are carrots and peaches in this garden. So, there are vegetables and fruits in this garden.

Sets word problem

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Let $A = \{1, 2, 3, 4, 5,6\}$, $B = \{2, 4, 6\}$, $C = \{1, 2, 3\}$, and $D = \{7, 8, 9\}$. If the universe of discourse is $U = \{1, 2, 3, \ldots, 10\}$, find:

1. $A \cup B$
2. $A \cap B$
3. $B \cap C$
4. $A \cap D$
5. $\overline{B \cup C}$
6. $A - B$
7. $(D \cap \bar{C}\ ) \cup \overline{A \cap B}$
8. $\emptyset \cup C$
9. $\emptyset \cap C$

Functions

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1 whole sheet of yellow paper (front)

A. Draw graphs of each of these functions from Z to Z.

1. $f(x) = \lfloor x + \frac{1}{2} \rfloor$
2. $f(x) = \left\lceil \frac{x}{3} \right\rceil$

Determine whether each function is one-to-one, onto, or one-to-one correspondence. For each that is not satisfied by the function, give one **counterexample**.

Functions	One-to-one	Onto	One-to-one Correspondence
1. $f(x) = \lfloor x + \frac{1}{2} \rfloor$			
2. $f(x) = \left\lceil \frac{x}{3} \right\rceil$			

Relations

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1 whole sheet of yellow paper (back)

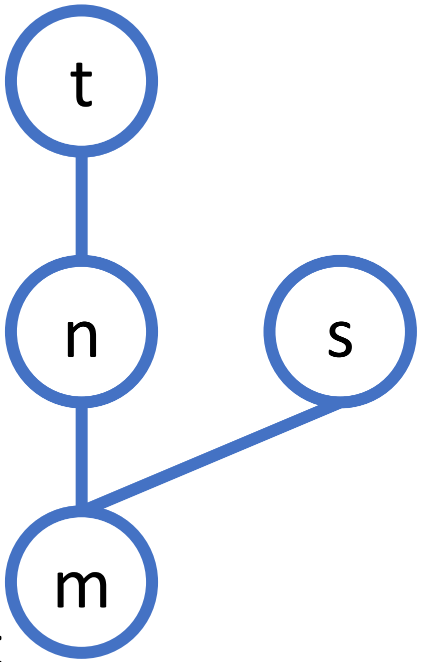
B. The following are relations on $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$. Determine whether each relation is reflexive, symmetric, antisymmetric or transitive. For each property that is not satisfied by the relation, give one **counterexample**.

Relations	Reflexive	Symmetric	Antisymmetric	Transitive
1. $A = \{(x, y) \mid x\ is\ divisible\ by\ y\}$				
2. $B = \{(x, y) \mid 2x + y \leq 15\}$				

1 whole sheet of yellow paper (back) – cont'd.

From the relations given previously, choose one relation that is reflexive, antisymmetric, and transitive. Draw the diagram for the relation you chose by following the steps below.

1. List the elements of the relation.
2. Cross out all the reflexive pairs (a, a) in the list.
3. Remove all transitive pairs in the list (i.e. if pairs (a, b) and (b, c) exists, cross out the transitive pair (a, c)).
4. Rewrite the elements that are not crossed out.
5. For every pair (a, b) in final list, draw a line that from point a to point b, where point a is below point b. For example, if your final list contains {(m, n), (n, t), (m, s)}, your drawing will look like the figure at the right.



Sums and Sequences

In a 1 whole sheet of yellow paper

A. List the first 10 terms of each of these sequences. Provide a formula or recurrence relation that generates the terms of the sequence.

1. the sequence obtained by starting with 10 and obtaining each term by subtracting 3 from the previous term
2. the sequence whose *n*th term is the sum of the first *n* positive integers
3. the sequence whose first two terms are 1 and 5, and each succeeding term is the sum of the two previous terms

Sequence	Explicit Formula	Recursive Formula
1. 10, 7, 4, 1, -2, -5, -8, -11, -14, -17		
2. 1, 3, 6, 10, 15, 21, 28, 36, 45, 55		
3. 1, 5, 6, 11, 17, 28, 45, 73, 118, 191		

B. Answer the following:

1. Suppose that the number of bacteria in a colony triples every hour.
   a. Set up a recurrence relation for the number of bacteria after *n* hours have elapsed.
   b. If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?

2. Assume that the population of the world in 2010 was 6.9 billion and is growing at the rate of 1.1% a year.
   a. Set up a recurrence relation for the population of the world *n* years after 2010.
   b. Find an explicit formula for the population of the world *n* years after 2010.

Summations

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S11/S12: MW 1100-1230
S11/S12: MW 1630-1530

D. Find the value of each of these sums. Show your solution.

1.

$$\sum_{j=4}^{8} (-3)^j$$

2.

$$\sum_{j=0}^{8} (2^{j+1} - 2^j)$$

3.

$$\sum_{i=1}^{3} \sum_{j=1}^{3} \frac{i^3}{4}$$

Mathematical Induction

Pau Rivera

S11/S12: TH 1500-1730

S13/S14: WF 900-1130

A. 1 whole sheet of yellow paper (front)

1. If $P(2) = 1 \cdot 2 + 2 \cdot 3 = \frac{2(3)(4)}{3}$ and $P(5) = 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + 5 \cdot 6 = \frac{5(6)(7)}{3}$,
   a. What is $P(n)$?
   b. Prove that $P(n)$ is true for all positive integers n using mathematical induction.
2. Given that
$$P(n) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$
   a. What is $P(n)$?
   b. Prove that your formula is correct using mathematical induction.

B. 1 whole sheet of yellow paper (back)

1. Using mathematical induction, prove that
$$1^2 + 4^2 + 7^2 + \cdots + (3n-2)^2 = \frac{n(6n^2 - 3n - 1)}{2}$$
for all positive integers *n*.

2. Using mathematical induction, prove that whenever *n* is a positive integer,
$$\frac{1}{2!} + \frac{2}{3!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}$$

Matrices

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S11/S12: TH 1500-1730

S13/S14: WF 900-1130

A. 1 whole sheet of yellow paper

$$A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & 0 & 4 & 6 \\ 1 & 1 & 3 & 7 \end{bmatrix}$$

Given matrix A, answer the following questions:

1. What size is A?
2. What is the third column of A?
3. What is the second row of A?
4. What is the element of A in (3, 2)?

B. 1 whole sheet of yellow paper

$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad \mathrm{B} = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$ $\quad \mathrm{C} = \begin{bmatrix} 4 & 5 \\ 6 & 6 \end{bmatrix}$ $\quad \mathrm{D} = \begin{bmatrix} -1 & 0 \\ -2 & 0 \end{bmatrix}$ $\quad \mathrm{E} = \begin{bmatrix} 14 \\ 18 \end{bmatrix}$

Considering the matrices A, B, C, and D, which of the following statements are true? Write True or False.

1. A + B = C
2. B + C = D
3. B – C = D
4. AC = E
5. CA = E

C. 1 whole sheet of yellow paper

s= 2 $\quad A = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \mathrm{B} = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix} \quad \mathrm{C} = \begin{bmatrix} 4 & 5 \\ 6 & 6 \end{bmatrix}$

1. Considering the scalar value *s*, and the matrices *A*, *B*, and *C*, solve for: $(\boldsymbol{sB} - \boldsymbol{C}^{\boldsymbol{t}})\boldsymbol{A}$.

$$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} A = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$$

2. Given the equation above, find matrix A.

Number Theory

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S11/S12: TH 1500-1730

S13/S14: WF 900-1130

A. 1 whole sheet of yellow paper

Determine whether the each of the statements is True or False.

1. 17 divides 1001.
2. 103 is congruent to 8 modulo 19.
3. 1919 and 38 are congruent modulo 19.
4. 143 is a prime number.
5. 25, 34, 49, and 64 are pairwise relatively prime.

B. 1 whole sheet of yellow paper

1. What is the quotient and remainder when 2002 is divided by 87?
2. What is 101 mod 13?
3. What time does a 12-hour clock read 80 hours after it reads 11:00?
4. Given $a \equiv 11 \ (mod\ 19)$ and a is an integer, what is c with $0 \leq c \leq 18$ such that $c \equiv 13a \ (mod\ 19)$?
5. Which positive integers less than 15 are relatively prime to 15?

C. 1 whole sheet of yellow paper

1. Show that if *a*, *b*, *c*, and *d* are integers, where $a \neq 0$ and $b \neq 0$, such that $a|c$ and $b|d$, then $ab|cd$.
2. Using prime factorization, find $gcd(1000, 625)$.
3. Using prime factorization, find $lcm(1000, 625)$.
4. Use the Euclidean algorithm to find $gcd(1529, 14\,038)$.