

Matrices

Shirley Chu

De La Salle University

March 13, 2020

Matrices

Definition

A *matrix* is a rectangular array of numbers.

A matrix with m rows and n columns is called an $m \times n$ matrix.

Matrices

Definition

A *matrix* is a rectangular array of numbers.

A matrix with m rows and n columns is called an $m \times n$ matrix.

Note: $n \times n$ matrix is a *square matrix*

Convention: Boldface uppercase letters are used to represent matrices.

Matrix Operations

Addition Let \mathbf{A} and \mathbf{B} be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} =$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} & & \\ & & \end{bmatrix}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & & \end{bmatrix}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & & \\ & & \end{bmatrix}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & & \end{bmatrix}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & & \end{bmatrix}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & \end{bmatrix}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & \end{bmatrix}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Matrix Operations

Addition Let \mathbf{A} and \mathbf{B} be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let \mathbf{A} be an $m \times k$ matrix and \mathbf{B} be a $k \times n$ matrix.

Matrix Operations

Addition Let \mathbf{A} and \mathbf{B} be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let \mathbf{A} be an $m \times k$ matrix and \mathbf{B} be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where}$$

Matrix Operations

Addition Let \mathbf{A} and \mathbf{B} be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let \mathbf{A} be an $m \times k$ matrix and \mathbf{B} be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} =$$

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} =$$

2×3 matrix 3×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} =$$

2×3 matrix 3×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} =$$

2×3 matrix 3×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} =$$

2×3 matrix 3×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} =$$

2×3 matrix 3×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) & \dots \\ \dots & \dots \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) & \dots \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & \\ & \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & \\ & \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) \\ \dots & \dots \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) + 2(6) \\ \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) + 2(6) + 3(8) \\ \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) + 2(6) + 3(8) \\ \dots & \dots \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) + 2(6) + 3(8) \\ 4(2) & \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) + 2(6) + 3(8) \\ 4(2) + 5(3) & \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) + 2(6) + 3(8) \\ 4(2) + 5(3) + 6(4) & \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) + 2(6) + 3(8) \\ 4(2) + 5(3) + 6(4) & \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) + 2(6) + 3(8) \\ 4(2) + 5(3) + 6(4) & 4(4) \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) + 2(6) + 3(8) \\ 4(2) + 5(3) + 6(4) & 4(4) + 5(6) \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) + 2(6) + 3(8) \\ 4(2) + 5(3) + 6(4) & 4(4) + 5(6) + 6(8) \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 1(2) + 2(3) + 3(4) & 1(4) + 2(6) + 3(8) \\ 4(2) + 5(3) + 6(4) & 4(4) + 5(6) + 6(8) \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let \mathbf{A} and \mathbf{B} be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let \mathbf{A} be an $m \times k$ matrix and \mathbf{B} be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 1(4) + 2(6) + 3(8) \\ 4(2) + 5(3) + 6(4) & 4(4) + 5(6) + 6(8) \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let \mathbf{A} and \mathbf{B} be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let \mathbf{A} be an $m \times k$ matrix and \mathbf{B} be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 1(4) + 2(6) + 3(8) \\ 4(2) + 5(3) + 6(4) & 4(4) + 5(6) + 6(8) \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let \mathbf{A} and \mathbf{B} be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let \mathbf{A} be an $m \times k$ matrix and \mathbf{B} be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 40 \\ 4(2) + 5(3) + 6(4) & 4(4) + 5(6) + 6(8) \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 40 \\ 4(2) + 5(3) + 6(4) & 4(4) + 5(6) + 6(8) \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 40 \\ 47 & 4(4) + 5(6) + 6(8) \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 40 \\ 47 & 4(4) + 5(6) + 6(8) \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Matrix Operations

Addition Let **A** and **B** be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let **A** be an $m \times k$ matrix and **B** be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^k a_{ix} b_{xj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 40 \\ 47 & 94 \end{bmatrix}$$

2×3 matrix 3×2 matrix 2×2 matrix

Exercise 1

Given that $r = 3$ and $s = -2$, and the following matrices:

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 4 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 4 & 1 & 2 \\ 6 & -1 & 5 \\ 1 & 3 & 2 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 2 & 4 \\ 6 & -1 \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} 4 & -6 \\ 1 & 3 \\ 2 & -1 \end{bmatrix}$$

Compute the following (if possible):

1 $\mathbf{A} + \mathbf{D}$

2 $r(s\mathbf{C})$

3 $\mathbf{BA} + \mathbf{D}$

4 $r(\mathbf{A} + \mathbf{D})$

5 \mathbf{DC}

6 \mathbf{CA}

7 $\mathbf{A} + s\mathbf{D}$

8 \mathbf{BA}

Exercise 2

Find w , x , y , and z :

$$\begin{bmatrix} x + y & 2x - 3y \\ z - w & z + 2w \end{bmatrix} = \begin{bmatrix} 4 & -7 \\ -6 & 6 \end{bmatrix}$$

References



[Rosen, 2007] Kenneth Rosen.

Discrete Mathematics and Its Applications 7th edition, 2007