Exercises

Pau Rivera

De La Salle University

S11/S12: MW 1100-1230
S11/S12: WF 1630-1730

Exercise 1

Determine if each statement is a proposition. If so, also determine its truth value.

1. Read and follow instructions.
2. I answered the assignment.
3. $2 + 3 = 5$
4. Let x be 5, $x + y = 10$.
5. DLSU-CCS is in Quezon City.

Exercise 2

Let f and s be the propositions:

f It is below freezing.

s It is snowing.

Write these propositions using f and s and logical connectives.

1. It is below freezing and snowing.
2. It is below freezing, but not snowing.
3. It is either below freezing, or snowing (or both).
4. It is neither below freezing, nor snowing.
5. That it is below freezing is necessary and sufficient for it to be snowing.

Implication, Converse, Inverse, Contrapositive

State the converse, inverse, and contrapositive of each of the given statements.

1. If it snows tonight, I will stay home.
2. I go to the beach whenever it is a sunny summer day.
3. A positive integer is prime only if it has no divisors other than 1 and itself.

Exercise 3

Use Logical Equivalence Rules to simplify the following. Construct the truth tables for each of the compound propositions and its simplified expression.

1. $\neg b \wedge (a \rightarrow b) \wedge a$
2. $(a \rightarrow b) \leftrightarrow (\neg a \vee b)$
3. $((p \rightarrow q) \rightarrow r) \wedge (\neg q \leftrightarrow (p \wedge \neg q))$
4. $\neg a \wedge (b \oplus c) \wedge (\neg b \vee c)$

Exercise 4

Determine whether each pair of propositions are logically equivalent or not. Use Logical Equivalence Rules.

1. $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$
2. $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$
3. $p \leftrightarrow (q \wedge r)$ and $(p \leftrightarrow q) \wedge (p \leftrightarrow r)$
4. $\neg(p \leftrightarrow q)$ and $p \oplus q$

Definition

Consistent System Specification

A system specification is said to be *consistent* when it does not contain conflicting requirements that could derive to a contradiction.

In an inconsistent system specification, it is not possible to develop a system that will satisfy all the requirements.

Example

Derive whether the given specifications are consistent.

Specs 1

The diagnostic message is stored in a buffer or it is retransmitted. The diagnostic message is not stored in the buffer. If the diagnostic message is stored in the buffer then it is retransmitted.

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Translate: Definition of propositional variables:

- b The diagnostic message is stored in the buffer.
- r The diagnostic message is retransmitted.

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Translate: Definition of propositional variables:

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Translation: $b \vee r$ $\neg b$

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Translation: $b \vee r$ $\neg b$ $b \rightarrow r$

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Create a new propositions using the conjunction connective.

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$$(b \vee r) \wedge \neg b \wedge (b \rightarrow r)$$

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$$(b \vee r) \wedge \neg b \wedge (b \rightarrow r)$$

Simplify the compound proposition formed.

Simplifying $(b \vee r) \wedge \neg b \wedge (b \rightarrow r)$

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$$(b \vee r) \wedge \neg b \wedge (b \rightarrow r)$$
$$\equiv$$

Simplifying $(b \vee r) \wedge \neg b \wedge (b \rightarrow r)$

$$
\begin{aligned}
&(b \vee r) \wedge \neg b \wedge (b \rightarrow r) \\
&\quad \equiv (b \vee r) \wedge \neg b \wedge (\neg b \vee r) \qquad \text{mat'l implication} \\
&\quad \equiv
\end{aligned}
$$

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$\equiv (b \vee r) \wedge \neg b \wedge (\neg b \vee r)$ mat'l implication

$\equiv$

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&\quad \equiv (b \vee r) \wedge \neg b \wedge (\neg b \vee r) && \text{mat'l implication} \\
&\quad \equiv (b \vee r) \wedge \neg b && \text{absorption} \\
&\quad \equiv
\end{aligned}
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&\quad \equiv (\neg b \wedge b) \vee (\neg b \wedge r) && \text{distributivity} \\
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&\quad \equiv (\neg b \wedge b) \vee (\neg b \wedge r) && \text{distributivity} \\
&\quad \equiv \mathrm{F} \vee (\neg b \wedge r) && \text{negation law} \\
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\end{aligned}
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$$
\begin{aligned}
&(b \vee r) \wedge \neg b \wedge (b \rightarrow r) && \\
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\end{aligned}
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Interpretation:

- The specification is **consistent**.

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Interpretation:

- The specification is **consistent**.
- The diagnostic message is not stored in the buffer and it is retransmitted.

Simplifying $(b \vee r) \wedge \neg b \wedge (b \rightarrow r)$

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\begin{array}{lll}
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Derive whether the given specifications are consistent.

Diagnostice Message - modified

The diagnostic message is stored in a buffer or it is retransmitted. The diagnostic message is not stored in the buffer. If the diagnostic message is stored in the buffer then it is retransmitted. The diagnostic message is not retransmitted.

Exercise 2

Derive whether the given specifications are consistent.

Router

The router can send packets to the edge system only if it supports the new address space. For the router to support the new address space it it necessary that the latest software release be installed. The router can send packets to the edge system if the latest software release is installed. The router does not support the new address space.

Exercise 3

Derive whether the given specifications are consistent.

File System

If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer. New messages will not be sent to the message buffer.

Exercise 4

Derive whether the given specifications are consistent.

Specs 1

The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode.

Exercise 5

Five friends have access to the chat room. Who are chatting?

Chat Room

Either Kevin or Heather, or both, are chatting. Either Randy or Jay, but not both, are chatting. If Abby is chatting, so is Randy. Jay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin.

References

 [Rosen, 2012] Kenneth Rosen.
Discrete Mathematics and Its Applications 7^{th} edition, 2012