Functions

Pau Rivera

S11/S12: TH 1500-1730

S13/S14: WF 900-1130

1 whole sheet of yellow paper (front)

A. Draw graphs of each of these functions from Z to Z.

1. $f(x) = \lfloor x + \frac{1}{2} \rfloor$
2. $f(x) = \left\lceil \frac{x}{3} \right\rceil$

Determine whether each function is one-to-one, onto, or one-to-one correspondence. For each that is not satisfied by the function, give one **counterexample**.

Functions	One-to-one	Onto	One-to-one Correspondence
1. $f(x) = \lfloor x + \frac{1}{2} \rfloor$			
2. $f(x) = \left\lceil \frac{x}{3} \right\rceil$			

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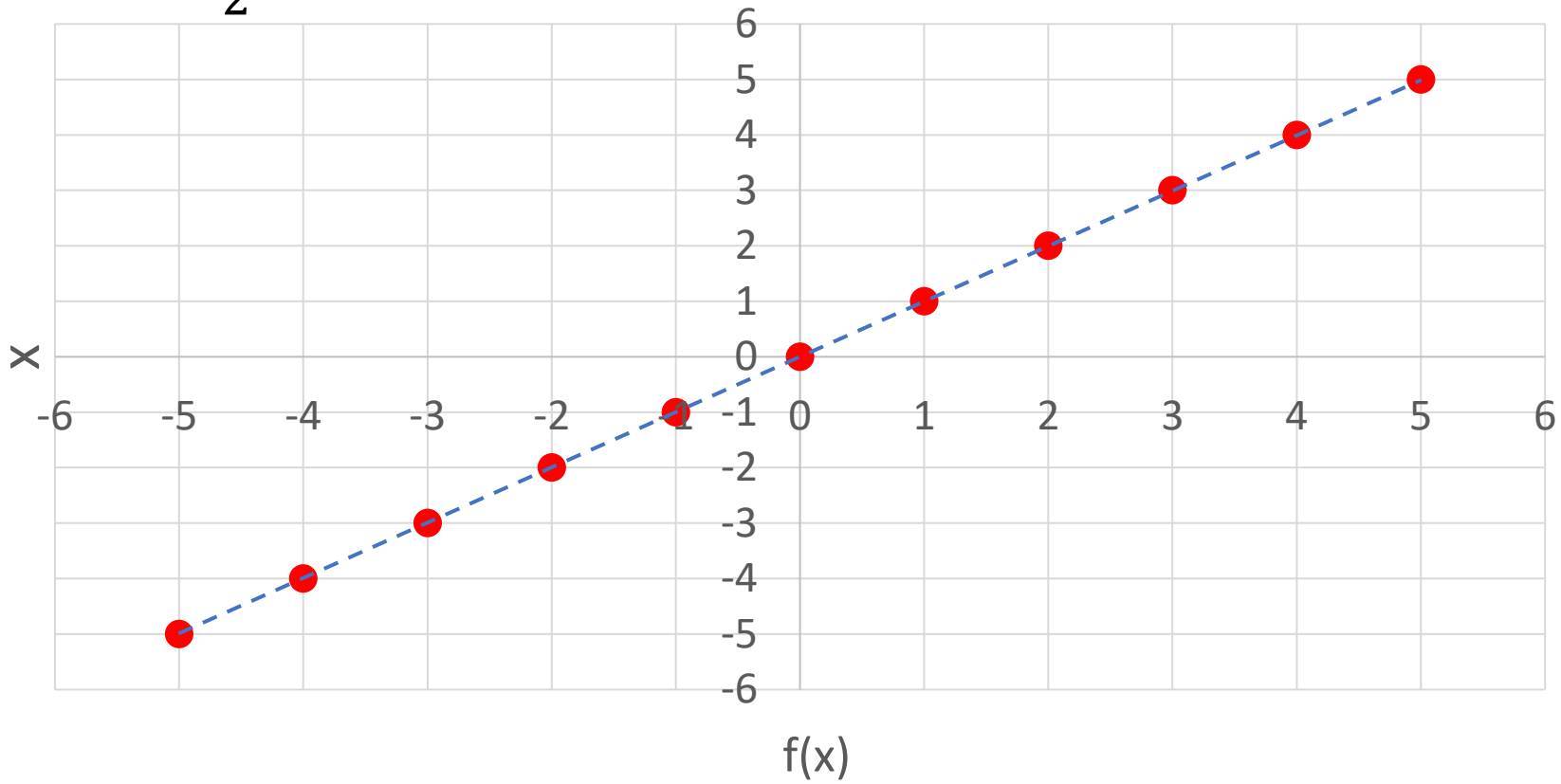
A. Draw graphs of each of these functions from Z to Z.

1. $f(x) = \lfloor x + \frac{1}{2} \rfloor$



Functions	One-to-one	Onto	One-to-one Correspondence
$1. f(x) = \lfloor x + \frac{1}{2} \rfloor$	Yes	Yes	Yes

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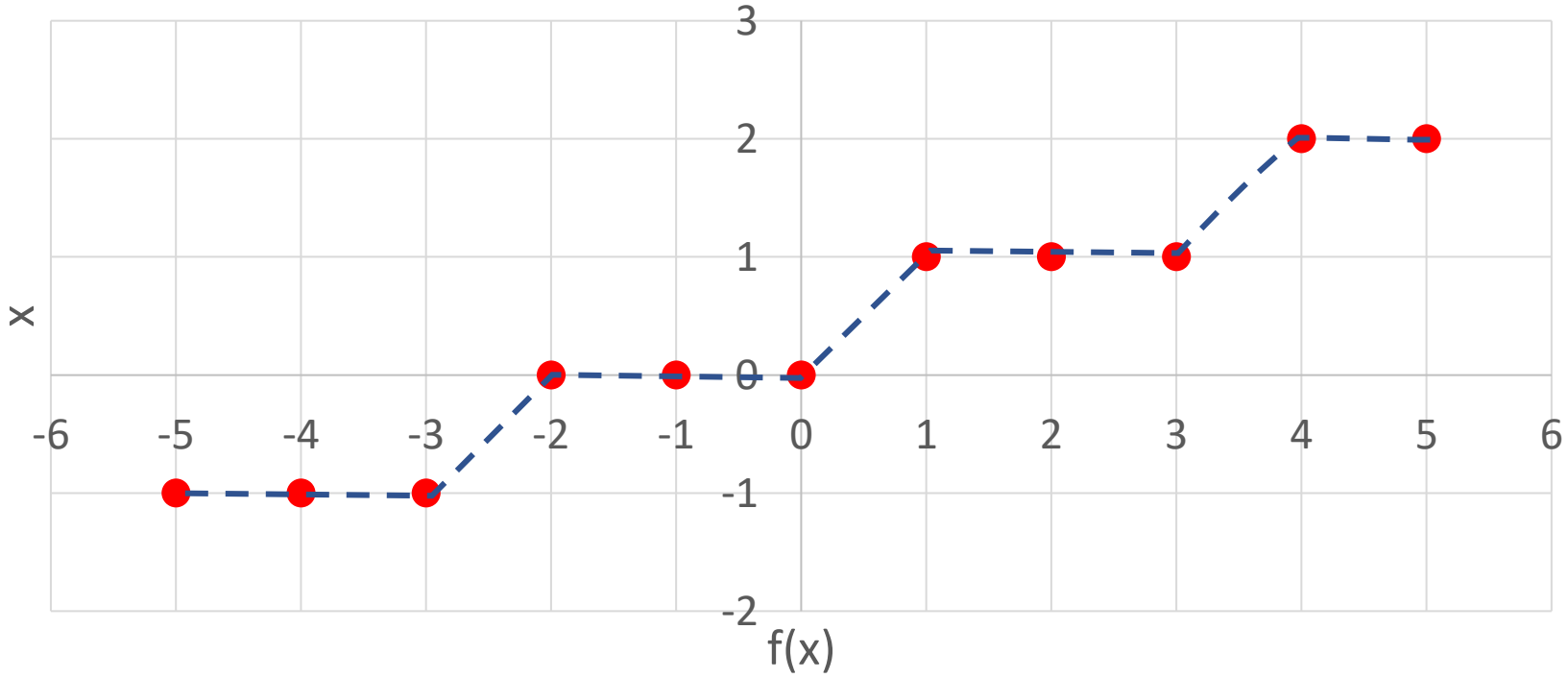
A. Draw graphs of each of these functions from Z to Z.

2. $f(x) = \left\lceil \frac{x}{3} \right\rceil$



Functions	One-to-one	Onto	One-to-one Correspondence
$2. f(x) = \left\lceil \frac{x}{3} \right\rceil$	No, f(-5) = f(-4)	Yes	No, not one-to-one

Relations

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S11/S12: TH 1500-1730

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B. The following are relations on $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$. Determine whether each relation is reflexive, symmetric, antisymmetric or transitive. For each property that is not satisfied by the relation, give one **counterexample**.

Relations	Reflexive	Symmetric	Antisymmetric	Transitive
1. $A = \{(x, y) \mid x \text{ is divisible by } y\}$	Yes	No $(4, 2) \in A \wedge (2, 4) \notin A$	Yes	Yes
2. $B = \{(x, y) \mid 2x + y \leq 15\}$	No $(12, 12) \notin B$	No $(2, 8) \in B \wedge (8, 2) \notin B$	No $(2, 3) \in B \wedge (3, 2) \in B$	No $(6, 2) \in B \wedge (2, 6) \in B \wedge (6, 6) \notin B$

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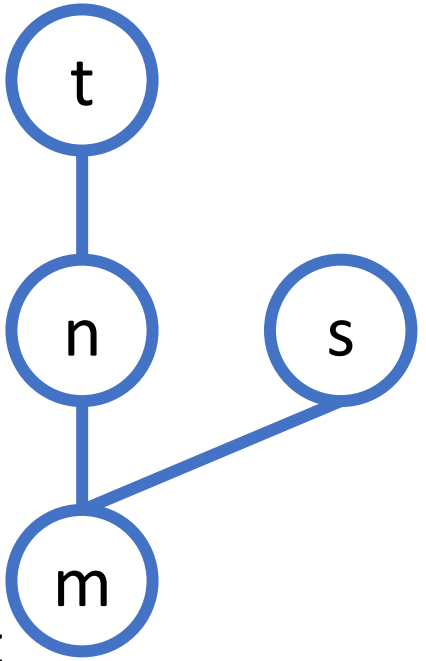
From the relations given previously, choose one relation that is reflexive, antisymmetric, and transitive. Draw the diagram for the relation you chose by following the steps below.

1. List the elements of the relation.
2. Cross out all the reflexive pairs (a, a) in the list.
3. Remove all transitive pairs in the list (i.e. if pairs (a, b) and (b, c) exists, cross out the transitive pair (a, c)).
4. Rewrite the elements that are not crossed out.
5. For every pair (a, b) in final list, draw a line that from point a to point b, where point a is below point b. For example, if your final list contains {(m, n), (n, t), (m, s)}, your drawing will look like the figure at the right.



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From the relations given previously, choose one relation that is reflexive, antisymmetric, and transitive. Draw the diagram for the relation you chose by following the steps below.

$A = \{(x, y) | x \ is \ divisible \ by \ y\}$ A is a relation on $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$

1. List the elements of the relation.

$$A = \{(2,2), (3,3), (4,2), (4,4), (5,5), (6,2), (6,3), (6,6), (7,7), (8,2), (8,4), (8,8), (9,3), (9,9), (10,2), (10,5), (10,10), (12,2), (12,3), (12,4), (12,6), (12,12)\}$$

2. Cross out all the reflexive pairs (a, a) in the list.

$$A = \{\cancel{(2,2)}, \cancel{(3,3)}, (4,2), \cancel{(4,4)}, \cancel{(5,5)}, (6,2), (6,3), \cancel{(6,6)}, \cancel{(7,7)}, (8,2), (8,4), \cancel{(8,8)}, (9,3), \cancel{(9,9)}, (10,2), (10,5), \cancel{(10,10)}, (12,2), (12,3), (12,4), (12,6), \cancel{(12,12)}\}$$

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$$A = \{\cancel{(2,2)}, \cancel{(3,3)}, (4,2), \cancel{(4,4)}, \cancel{(5,5)}, (6,2), (6,3), \cancel{(6,6)}, \cancel{(7,7)},$$
$$(8,2), (8,4), \cancel{(8,8)}, (9,3), \cancel{(9,9)}, (10,2), (10,5), \cancel{(10,10)}, (12,2),$$
$$(12,3), (12,4), (12,6), \cancel{(12,12)}\}$$

3. Remove all transitive pairs in the list (i.e. if pairs (a, b) and (b, c) exists, cross out the transitive pair (a, c)).

$$A = \{\cancel{(2,2)}, \cancel{(3,3)}, (4,2), \cancel{(4,4)}, \cancel{(5,5)}, (6,2), (6,3), \cancel{(6,6)}, \cancel{(7,7)},$$
$$\cancel{(8,2)}, (8,4), \cancel{(8,8)}, (9,3), \cancel{(9,9)}, (10,2), (10,5), \cancel{(10,10)}, \cancel{(12,2)},$$
$$\cancel{(12,3)}, (12,4), (12,6), \cancel{(12,12)}\}$$

4. Rewrite the elements that are not crossed out.

$$A = \{(4,2), (6,2), (6,3), (8,4), (9,3), (10,2), (10,5), (12,4), (12,6)\}$$

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$$A = \{(4,2),(6,2),(6,3),(8,4),(9,3),(10,2),(10,5),(12,4),(12,6)\}$$

5. For every pair (a, b) in final list, draw a line that from point a to point b, where point a is below point b. For example, if your final list contains {(m, n), (n, t), (m, s)}, your drawing will look like the figure at the right.