Mathematical Induction

Pau Rivera

S11/S12: MW 1100-1230

S11/S12: MW 1630-1730

Principle of Mathematical Induction

To prove that $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function, we complete two steps:

BASIS STEP: We verify that $P(1)$ is true.

INDUCTIVE STEP: We show that the conditional statement $P(k) \rightarrow P(k+1)$ is true for **all positive integers *k***.

Example: Prove that $1+2+3+\cdots+n=\frac{n(n+1)}{2}$

for all positive integers n

BASIS STEP: Show $\boxed{\text{P(1)}}$ ← $\boldsymbol{1, because\ 1\ is\ the\ first\ positive\ integer}$

$$\frac{1(1+1)}{2}=1\ \checkmark$$

INDUCTIVE STEP:

Assume P(k): $\frac{k(k+1)}{2}$

Show P(k+1): $\frac{(k+1)(k+2)}{2}$

$$1+2+3+\cdots+k+(k+1)=\boxed{\frac{(k+1)(k+2)}{2}} \leftarrow \boldsymbol{P(k+1)}$$

$\boldsymbol{P(k)}$: summation from 1 to k →

$$\boxed{\left(\frac{k(k+1)}{2}\right)}+\boxed{(k+1)}=$$

← term after k

$$\frac{k(k+1)+2(k+1)}{2}=$$

$$\frac{(k+1)(k+2)}{2}=\frac{(k+1)(k+2)}{2}\ \checkmark$$

Example: Prove that $1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$ for all nonnegative integers *n*

BASIS STEP: Show $\boxed{\text{P(0)}}$

$\mathbf{0},$ ***because 0 is the first nonnegative integer***

$2^{0+1} - 1 = 1$ ✓

INDUCTIVE STEP:

Assume P(k): $2^{k+1} - 1$

Show P(k+1): $2^{k+2} - 1$

$1 + 2 + 4 \ldots + 2^k + 2^{k+1} = \boxed{2^{k+2} - 1}$

$\boldsymbol{P(k+1)}$

$\boldsymbol{P(k)}$: summation from 1 to 2^k

$\boxed{(2^{k+1} - 1)} + \boxed{2^{k+1}} =$

term after 2^k

$2(2^{k+1}) - 1 =$

$2^{k+2} - 1 = 2^{k+2} - 1$ ✓

Tips

- For Basis Step, prove $P(initial\ value)$
  - Substitute $initial\ value\ of\ n$
- For Inductive Step, assuming P(k) is true, show that P(k+1) is also true
  1. Since you assume that $P(k)$ is true, start from $P(k)$
  2. Substitute $k + 1$ to the formula for getting the next term in the series.
     - Ex: $1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$
       the formula to get the next term is 2^n
       substitute $k + 1$ to n, 2^{k+1}
  3. Add the result of number 2 to $P(k)$
     - Ex: If $P(k)$: $2^{k+1} - 1$
       $(2^{k+1} - 1) + 2^{k+1}$
  4. Simplify the result of number 3 to make it look exactly like $P(k + 1)$.

Exercise - Formulas

1. Provide an explicit formula to get the sum of the first n positive odd integers.
2. Prove that your explicit formula in number 1 is correct using Mathematical Induction.
3. Prove the explicit formula of the summation of j^3 from 1 to n using Mathematical Induction.

Questions? ☺