Propositions Exercises

Exercise 1. Proposition or not. Truth value?

1) Read and follow instructions.
Not proposition

2) I answered the assignment.
Proposition
T or F

3) $2+3=5$
Proposition
T

4) Let x be 5, $x+y=10$.
Not Proposition

5) DLSU-CCS is in Quezon City.
Proposition
F

Exercise 2

Let f and s be the propositions:

f It is below freezing.

s It is snowing.

Write these propositions using f and s and logical connectives.

1. It is below freezing and snowing. $\boxed{f \wedge s}$
2. It is below freezing, but not snowing. $\boxed{f \wedge \neg s}$
3. It is either below freezing, or snowing (or both). $\boxed{f \vee s}$
4. It is neither below freezing, nor snowing. $\boxed{\neg(f \vee s) \equiv \neg f \wedge \neg s}$
5. That it is below freezing is necessary and sufficient for it to be snowing. $\boxed{f \longleftrightarrow s}$

Implication, Converse, Inverse, Contrapositive

State the converse, contrapositive, and inverse of each of the given statements.

1. If it snows tonight, I will stay home.
2. I go to the beach whenever it is a sunny summer day.
3. A positive integer is prime only if it has no divisors other than 1 and itself.

Implication, Converse, Inverse, Contrapositive

1. If it snows tonight, I will stay home.

CONVERSE:

If I will stay home, it snows tonight.

INVERSE:

If it does not snow tonight, I will not stay home.

CONTRAPOSITIVE:

If I will not stay home, it did not snow tonight.

Implication, Converse, Inverse, Contrapositive

2. I go to the beach whenever it is a sunny summer day.

CONVERSE:

It is a sunny summer day whenever I go to the beach

If ... then ... format:

If I go to the beach, then it is a sunny summer day.

INVERSE

I do not go to the beach whenever it is not a sunny summer day.

If... then... format:

If it is not a sunny summer day, then I do not go to the beach.

CONTRAPOSITIVE:

It is not a sunny summer day whenever I do not go to the beach.

If... then... format:

If I do not go to the beach, then it is not a sunny summer day.

Implication, Converse, Inverse, Contrapositive

3. A positive integer is prime only if it has no divisors other than 1 and itself.

CONVERSE:

A positive integer has no divisors other than 1 and itself only if it is prime.

If... then... format

If a positive integer has no divisors other than 1 and itself, then it is prime.

INVERSE:

A positive integer is not prime only if it has divisors other than 1 and itself.

If... then... format

If a positive integer is not prime, then it has divisors other than 1 and itself.

CONTRAPOSITIVE:

A positive integer has divisors other than 1 and itself only if it is not prime.

If... then... format

If a positive integer has divisors others than 1 and itself, then it is not prime.

Exercise 3

Use Logical Equivalence Rules to simplify the following. Construct the truth tables for each of the compound propositions and its simplified expression.

1. $\neg b \land (a \to b) \land a$
2. $(a \to b) \leftrightarrow (\neg a \lor b)$
3. $((p \to q) \to r) \land (\neg q \leftrightarrow (p \land \neg q))$
4. $\neg a \land (b \oplus c) \land (\neg b \lor c)$

Exercise 3

1. $\neg b \land (a \to b) \land a$

STATEMENT/S	REASON/S
1) $\neg b \land (a \to b) \land a$	Given
2) $(a \land \neg b) \land (a \to b)$	Commutative, Associative
3) $\neg(\neg a \lor b) \land (a \to b)$	De Morgan's
4) $\neg(a \to b) \land (a \to b)$	Material Implication
5) $\boxed{F}$	Negation

a	b	$\neg b$	$a \to b$	$\neg b \land (a \to b)$	$\neg b \land (a \to b) \land a$
T	T	F	T	F	F
T	F	T	F	F	F
F	T	F	T	F	F
F	F	T	T	F	F

Exercise 3

2. $(a \to b) \leftrightarrow (\neg a \lor b)$

STATEMENT/S	REASON/S
1) $(a \to b) \leftrightarrow (\neg a \lor b)$	Given
2) $(a \to b) \leftrightarrow (a \to b)$	Material Implication
3) $\boxed{T}$	Biconditional Definition

a	b	$a \to b$	$\neg a$	$\neg a \lor b$	$(a \to b) \leftrightarrow (\neg a \lor b)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Exercise 3

3 $((p \to q) \to r) \land (\neg q \leftrightarrow (p \land \neg q))$

STATEMENT/S	REASON/S
1) $((p \to q) \to r) \land (\neg q \leftrightarrow (p \land \neg q))$	Given
2) $(\neg(\neg p \lor q) \lor r) \land (\neg q \leftrightarrow (p \land \neg q))$	Material Implication (x2)
3) $((p \land \neg q) \lor r) \land (\neg q \leftrightarrow (p \land \neg q))$	De Morgan's
4) $((p \land \neg q) \lor r) \land ((\neg q \land (p \land \neg q)) \lor (q \land \neg(p \land \neg q)))$	Material Equivalence
5) $((p \land \neg q) \lor r) \land ((p \land \neg q) \lor (q \land \neg(p \land \neg q)))$	Idempotent, Commutative & Associative
6) $((p \land \neg q) \lor r) \land ((p \land \neg q) \lor (q \land (\neg p \lor q)))$	De Morgan's
7) $((p \land \neg q) \lor r) \land ((p \land \neg q) \lor q)$	Absorption
8) $\boxed{(p \land \neg q) \lor (r \land q)}$	Distributive

same!

p	q	r	a ≡ $p \to q$	b ≡ $a \to r$	$\neg q$	c ≡ $p \land \neg q$	d ≡ $\neg q \leftrightarrow c$	$b \land d$	e ≡ $r \land q$	$c \lor e$
T	T	T	T	T	F	F	T	T	T	T
T	T	F	T	F	F	F	T	F	F	F
T	F	T	F	T	T	T	T	T	F	T
T	F	F	F	T	T	T	T	T	F	T
F	T	T	T	T	F	F	T	T	T	T
F	T	F	T	F	F	F	T	F	F	F
F	F	T	T	T	T	F	F	F	F	F
F	F	F	T	F	T	F	F	F	F	F

Exercise 3

4 $\neg a \wedge (b \oplus c) \wedge (\neg b \vee c)$

STATEMENT/S	REASON/S
1) $\neg a \wedge (b \oplus c) \wedge (\neg b \vee c)$	Given
2) $\neg a \wedge ((b \vee c) \wedge (\neg b \vee \neg c)) \wedge (\neg b \vee c)$	XOR(Negation of Material Equivalence law #2)
3) $\neg a \wedge (b \vee c) \wedge (\neg b \vee (\neg c \wedge c))$	Distributive
4) $\neg a \wedge (b \vee c) \wedge (\neg b \vee F)$	Negation
5) $\neg a \wedge (b \vee c) \wedge \neg b$	Identity
6) $\neg a \wedge ((b \wedge \neg b) \vee (c \wedge \neg b))$	Distributive
7) $\neg a \wedge (F \vee (c \wedge \neg b))$	Negation
8) $\boxed{\neg a \wedge c \wedge \neg b}$	Identity

same!

a	b	c	$\neg a$	$p \equiv b \oplus c$	$\neg b$	$r \equiv \neg b \vee c$	$q \equiv \neg a \wedge p$	$q \wedge r$	$s \equiv \neg a \wedge c$	$s \wedge \neg b$
T	T	T	F	F	F	T	F	F	F	F
T	T	F	F	T	F	F	F	F	F	F
T	F	T	F	T	T	T	F	F	F	F
T	F	F	F	F	T	T	F	F	F	F
F	T	T	T	F	F	T	F	F	T	F
F	T	F	T	T	F.	F	T	F	F	F
F	F	T	T	T	T	T	T	T	T	T
F	F	F	T	F	T	T	F	F	F	F

Exercise 4

Determine whether each pair of propositions are logically equivalent or not. Use Logical Equivalence Rules.

1. $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$
2. $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$
3. $p \leftrightarrow (q \wedge r)$ and $(p \leftrightarrow q) \wedge (p \leftrightarrow r)$
4. $\neg(p \leftrightarrow q)$ and $p \oplus q$

Exercise 4

1. $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$

STATEMENT/S	REASON/S
1) $\neg(p \vee (\neg p \wedge q))$	Given
2) $\neg p \wedge (p \vee \neg q)$	De Morgan's
3) $\neg p \wedge p \vee \neg p \wedge \neg q$	Distributive
4) $F \vee \neg p \wedge \neg q$	Negation
5) $\neg p \wedge \neg q$	Identity

$\boxed{\therefore \neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q}$

Exercise 4

2. $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$

STATEMENT/S	REASON/S
1) $\neg((p \wedge q) \vee (\neg p \wedge \neg q))$	Given
2) $(\neg p \vee \neg q) \wedge (p \vee q)$	De Morgan's
3) $(p \to \neg q) \wedge (\neg q \to p)$	Material Implication
4) $p \leftrightarrow \neg q$	Material Equivalence

$$\boxed{\therefore \neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q}$$

Exercise 4

3. $p \leftrightarrow (q \wedge r)$ and $(p \leftrightarrow q) \wedge (p \leftrightarrow r)$

STATEMENTS	REASONS
1) $p \leftrightarrow (q \wedge r)$	Left side given
2) $(p \wedge (q \wedge r)) \vee (\neg p \wedge \neg(q \wedge r))$	Material Equivalence
3) $(p \wedge (q \wedge r)) \vee (\neg p \wedge (\neg q \vee \neg r))$	De Morgan's
4) $(p \leftrightarrow q) \wedge (p \leftrightarrow r)$	
5) $(p \rightarrow q) \wedge (q \rightarrow p) \wedge (p \rightarrow r) \wedge (r \rightarrow p)$	Right side given
6) $(\neg p \vee q) \wedge (\neg q \vee p) \wedge (\neg p \vee r) \wedge (\neg r \vee p)$	Material Implication (x2)
7) $(\neg p \vee (q \wedge r)) \wedge (p \vee (\neg q \wedge \neg r))$	Distributive
8) $(\neg p \wedge p) \vee (\neg p \wedge (\neg q \wedge \neg r)) \vee ((q \wedge r) \wedge p) \vee ((q \wedge r) \wedge (\neg q \wedge \neg r))$	Distributive
9) $F \vee (\neg p \wedge (\neg q \wedge \neg r)) \vee (p \wedge (q \wedge r)) \vee F \wedge F$	Negation, Commutative
10) $p \wedge (q \wedge r)) \vee (\neg p \wedge (\neg q \wedge \neg r))$	Identity, Commutative

$$\boxed{p \leftrightarrow (q \wedge r) \not\equiv (p \leftrightarrow q) \wedge (p \leftrightarrow r)}$$

Exercise 4

4. $\neg(p \leftrightarrow q)$ and $p \oplus q$

STATEMENT/S	REASON/S
1) $p \oplus q$	Right side given
2) $(p \vee q) \wedge (\neg p \vee \neg q)$	XOR definition
3) $\neg(\neg(p \vee q) \vee \neg(\neg p \vee \neg q))$	De Morgan's (x2)
4) $\neg((\neg p \wedge \neg q) \vee (p \wedge q))$	De Morgan's
5) $\neg(p \leftrightarrow q)$	Material Equivalence

$\boxed{\therefore\ \neg(p \leftrightarrow q) \equiv p \oplus q}$