Matrices

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Matrices

Definition

A *matrix* is a rectangular array of numbers.

A matrix with m rows and n columns is called an $m \times n$ matrix.

Note: $n \times n$ matrix is a *square matrix*

Convention: Boldface uppercase letters are used to represent matrices.

Matrix Operations

Addition Let $\mathbf{A}$ and $\mathbf{B}$ be $m \times n$ matrices.

$$\mathbf{A} + \mathbf{B} = [a_{ij} + b_{ij}]$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 5 & 8 & 11 \end{bmatrix}$$

Multiplication Let $\mathbf{A}$ be an $m \times k$ matrix and $\mathbf{B}$ be a $k \times n$ matrix.

$$\mathbf{AB} = [c_{ij}], \text{ where } c_{ij} = \sum_{x=1}^{k} a_{ik} b_{kj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{bmatrix} = \begin{bmatrix} 20 & 40 \\ 47 & 94 \end{bmatrix}$$

Exercise 1

Given that $r = 3$ and $s = -2$, and the following matrices:

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \\ 3 & 4 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 4 & 1 & 2 \\ 6 & -1 & 5 \\ 1 & 3 & 2 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 2 & 4 \\ 6 & -1 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} 4 & -6 \\ 1 & 3 \\ 2 & -1 \end{bmatrix}$$

Compute the following (if possible):

1. $\mathbf{A} + \mathbf{D}$
2. $r(s\mathbf{C})$
3. $\mathbf{BA} + \mathbf{D}$
4. $r(\mathbf{A} + \mathbf{D})$
5. $\mathbf{DC}$
6. $\mathbf{CA}$
7. $\mathbf{A} + s\mathbf{D}$
8. $\mathbf{BA}$

Exercise 2

Find w, x, y, and z:

$$\begin{bmatrix} x+y & 2x-3y \\ z-w & z+2w \end{bmatrix} = \begin{bmatrix} 4 & -7 \\ -6 & 6 \end{bmatrix}$$

References

 [Rosen, 2007] Kenneth Rosen.
Discrete Mathematics and Its Applications 7^{th} edition, 2007