Introduction to Propositional Logic

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Summary

Summary

p	q
T	T
T	F
F	T
F	F

Not

$\neg p$
F
T

And

$p \wedge q$
T
F
F
F

Or

$p \vee q$
T
T
T
F

Xor

$p \oplus q$
F
T
T
F

Implies

$p \rightarrow q$
T
F
T
T

Iff

$p \leftrightarrow q$
T
F
F
T

Introduction

Definition

Compound propositions whose truth values are the same for all possible cases are said to be **logically equivalent**.
We write $p \equiv q$ or $p \Leftrightarrow q$ when p is logically equivalent to q.

Introduction

Definition

A compound proposition whose truth value is always true is called a **tautology** or a **fact**.

Definition

A compound proposition whose truth value is always false is called a **contradiction** or a **fallacy**.

Definition

A compound proposition whose truth value is not a tautology nor a contradiction is called a **contingency**.

Logical Equivalence Rules

Identity Laws

$$p \wedge T \equiv p$$

$$p \vee F \equiv p$$

Domination Laws

$$p \wedge F \equiv F$$

$$p \vee T \equiv T$$

Negation Laws

$$p \wedge \neg p \equiv F$$

$$p \vee \neg p \equiv T$$

Idempotent Laws

$$p \wedge p \equiv p$$

$$p \vee p \equiv p$$

Involution Laws

$$\neg(\neg p) \equiv p$$

De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Logical Equivalence Rules

Commutative Laws

$$p \wedge q \equiv q \wedge p$$

$$p \vee q \equiv q \vee p$$

Associative Laws

$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

Distributive Laws

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

Absorption Laws

$$p \wedge (p \vee q) \equiv p$$

$$p \vee (p \wedge q) \equiv p$$

Material Implication Laws

$$p \rightarrow q \equiv \neg p \vee q$$

Material Equivalence Laws

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$$

Exercise 1

Derive the following rules from the Logical Equivalence Rules.

1. Absorption Laws
2. Material Equivalence Laws

Exercise 2

Simplify the following propositions. Identify whether the proposition is a tautology, contradiction or a contingency.

1. $\neg p \rightarrow (p \rightarrow q)$
2. $(p \vee q) \rightarrow (p \wedge q)$
3. $p \leftrightarrow \neg p$

Exercise 3

Determine whether the following pairs of propositions are logically equivalent or not using Rules of Logical Equivalences. Verify your answers using the truth table.

1. $p \to q$ and $\neg q \to \neg p$
2. $(p \land q) \to r$ and $(p \to r) \land (q \to r)$
3. $\neg(p \leftrightarrow q)$ and $\neg p \leftrightarrow q$

Reference

Discrete Mathematics and Its Applications, 7^{th} edition by Kenneth Rosen