Mathematical Induction

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Introduction

Mathematical induction

- proof technique
- proves that the formula $P(n)$ is true for the specified set of numbers.

Steps

Mathematical induction has **two** steps:

- **Basis Step**

- **Inductive Step**

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Mathematical induction has **two** steps:

- **Basis Step**

  Prove that $P(n)$ is true for the smallest/largest possible value of n.

- **Inductive Step**

  1. Assume that the given $P(n)$ is true.
  2. Prove that it is also true for the next value of n, i.e. $n + 1$

Proof Template

by Mathematical Induction:
Basis Step: Show $P(\underline{\quad\quad})$

solution here

Inductive Step:
Assume $P(n)$: ______________________________

Show $P(n+1)$: ______________________________

solution here

Example 1

Prove that $1 + 2 + 3 + \cdots + n = \frac{n^2+n}{2}$ for all $n \in \mathbb{Z}^+$

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Note that:

- $P(1)$ means sum of the first odd integer

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- $P(n)$ means sum of the first n odd integers
  thus, $P(n): 1 + 3 + 5 + \cdots + (2n-1) = n^2$

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Basis Step: Show $P(1)$

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$$1 = 1 \qquad \checkmark$$

Inductive Step:

Assume $P(n)$: $1 + 3 + 5 + \cdots + (2n - 1) = n^2$

Show $P(n+1)$: $1 + 3 + 5 + \cdots + (2n - 1) + (2n + 1) = (n + 1)^2$

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Show that the sum of the first n positive odd integers is n^2.

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Exercises

Use Mathematical Induction to prove the following.

1. Show that $1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}$, for all $n > 1$.
2. Show that for positive integer n,

   $1 + 3 + 6 + \cdots + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2}{6}$
3. Prove that $3 + 3^2 + 3^3 + \cdots + 3^{n+1} = \frac{3^{n+2}-3}{2}$, for $n \in \mathbb{Z}^+$