CCDSTRU HW # 3 Date: Mar 16, 2020

Name (SURNAME, First Names): ______________________ Section: _____

A . Relations

1 . Let $\boldsymbol{R}$ be a relation on $\mathbb{Z}^+$, where $\boldsymbol{R = \{(x, y) \mid xy \geq 1\}}$ Determine if $\boldsymbol{R}$ is reflexive, symmetric, antisymmetric, and transitive. Give one counterexample for each property not satisfied.

reflexive: ______________________

symmetric: ______________________

antisymmetric: ______________________

transitive: ______________________

2 . Let $\boldsymbol{R}$ be a relation on $\mathbb{Z}$, where $\boldsymbol{R = \{(x, y) \mid x = y + 1 \vee x = y - 1\}}$ Determine if $\boldsymbol{R}$ is reflexive, symmetric, antisymmetric, and transitive. Give one counterexample for each property not satisfied.

reflexive: ______________________

symmetric: ______________________

antisymmetric: ______________________

transitive: ______________________

B . Functions

1 . Let $\boldsymbol{f} : \mathbb{R} \to \mathbb{R}$, and $\boldsymbol{f(x) = x^2 - 6x}$. Determine if the function is injective, surjective, and bijective. Provide a counterexample, if not.

injective: ______________________

surjective: ______________________

bijective: ______________________

2 . Let $\boldsymbol{f} : \mathbb{R} \to \mathbb{R}$, and $\boldsymbol{f(x) = 4x - 1}$. Determine if the function is injective, surjective, and bijective. Provide a counterexample, if not.

injective: ______________________

surjective: ______________________

bijective: ______________________

C . **Sequences**

1 . Determine the formula for $\boldsymbol{a_n}$, where $\boldsymbol{n > 0}$

a.) $\mathbf{8, 20, 32, 44, \ldots}$ $\quad \boldsymbol{a_n} =$ ____

b.) $\mathbf{7, \frac{14}{3}, \frac{28}{9}, \frac{56}{27}, \ldots}$ $\quad \boldsymbol{a_n} =$ ____

2 . Answer the following.

a.) Given an arithmetic sequence, where $\boldsymbol{a_1 = 100}$, and $\boldsymbol{d = -5}$ $\quad \boldsymbol{a_{10}} =$____________________

b.) Given a geometric sequence, where $\boldsymbol{a_1 = 3}$, and $\boldsymbol{a_3 = \frac{4}{3}}$. $\quad \boldsymbol{a_5} =$____________________

D . **Sums.** Show the first few steps (as indicated below) and the final answer in evaluating the given summation. Final answers must be in its simplest whole or rational number, or expression.

$$\sum_{j=8}^{n-1} 10 \cdot 3^j = \underline{\hspace{10cm}} \textit{ first step}$$

$$= \underline{\hspace{10cm}} \mathbf{2^{nd}} \textit{ step}$$

$$= \underline{\hspace{10cm}} \mathbf{3^{rd}} \textit{ step}$$

$$\vdots$$

$$= \underline{\hspace{10cm}} \textit{ final answer}$$

$$\sum_{r=0}^{n+1} \sum_{s=0}^{2n} \sum_{t=0}^{n} rst = \underline{\hspace{10cm}} \textit{ first step}$$

$$= \underline{\hspace{10cm}} \mathbf{2^{nd}} \textit{ step}$$

$$= \underline{\hspace{10cm}} \mathbf{3^{rd}} \textit{ step}$$

$$= \underline{\hspace{10cm}} \mathbf{4^{th}} \textit{ step}$$

$$\vdots$$

$$= \underline{\hspace{10cm}} \textit{ final answer}$$

E . Use Mathematical Induction to prove the following.

1 . Prove that

$$\mathbf{\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{\boldsymbol{n}(\boldsymbol{n}+1)}=\frac{\boldsymbol{n}}{\boldsymbol{n}+1}}$$

2 . Prove that for positive integer $\boldsymbol{n > 1}$,

$$\mathbf{1^2 + 3^2 + 5^2 + \cdots + (2\boldsymbol{n} - 1)^2 = \frac{4\boldsymbol{n}^3 - \boldsymbol{n}}{3}}$$