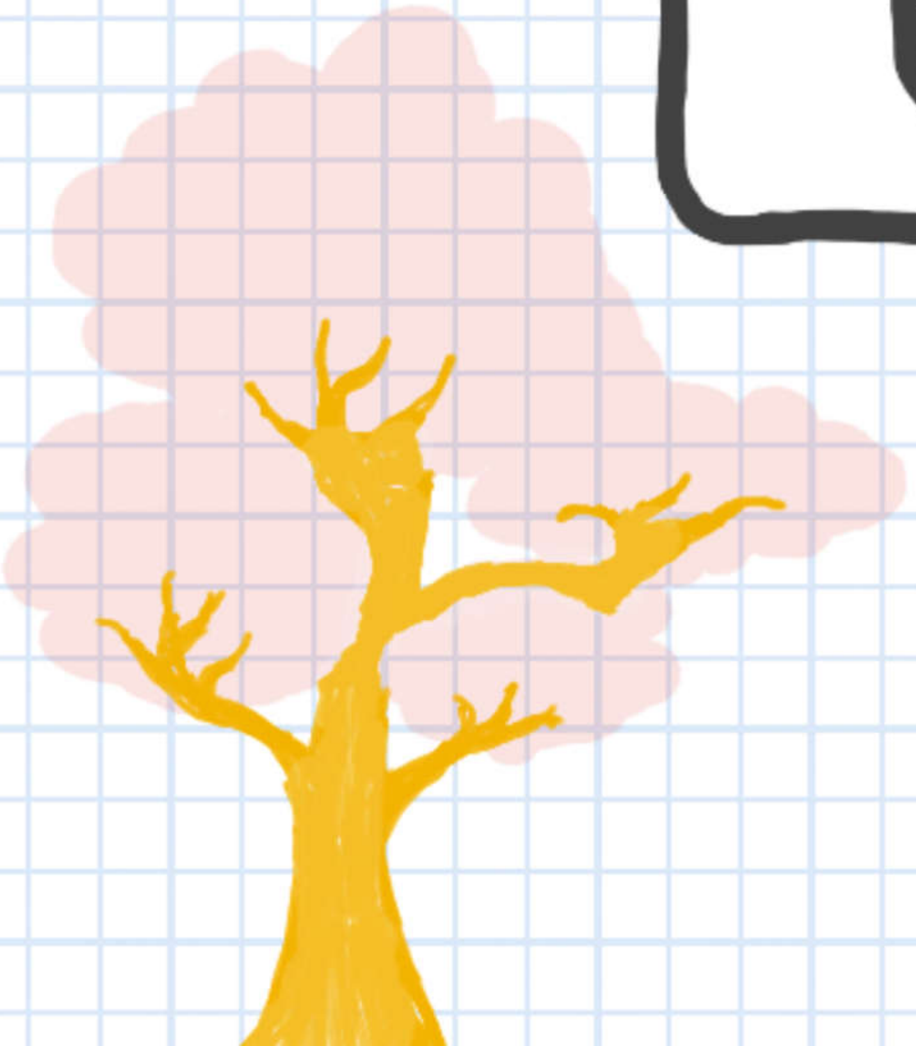
Logical Equivalences



Exercise 1. Derive using Logical Equivalence Rules.

1) Absorption laws:

$p \wedge (p \vee q) \equiv p$

Logical Expressions	Rules Applied
$p \wedge (p \vee q)$	Left-hand side given
$(p \vee F) \wedge (p \vee q)$	Identity
$p \vee (F \wedge q)$	Distributive
$p \vee F$	Domination
p	Identity ☑ $\therefore p \wedge (p \vee q) \equiv p$

p	Right-hand side given
$p \vee F$	Identity
$p \vee (F \wedge q)$	Domination
$(p \vee F) \wedge (p \vee q)$	Distributive
$p \wedge (p \vee q)$	Identity ☑ $\therefore p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

$p \vee (p \wedge q)$	Left-hand sign given
$(p \wedge T) \vee (p \wedge q)$	Identity
$p \wedge (T \vee q)$	Distributive
$p \wedge T$	Domination
p	Identity ☑ $\therefore p \vee (p \wedge q) \equiv p$

2) Material Equivalence Laws

$p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$

First: $p \leftrightarrow q \equiv (p \to q) \land (q \to p)$ (by definition)

$(\neg p \lor q) \land (\neg q \lor p)$

- Flip operators
- Reposition variables

1

LOGICAL EXPRESSIONS	RULES APPLIED
$(p \land q) \lor (\neg p \land \neg q)$	Right-hand side given
$((p \land q) \lor \neg p) \land ((p \land q) \lor \neg q)$	Distributive
$((\neg p \lor p) \land (\neg p \lor q)) \land ((\neg q \lor p) \land (\neg q \lor q))$	Distributive, Commutative
$(T \land (\neg p \lor q)) \land ((\neg q \lor p) \land T)$	Negation
$(\neg p \lor q) \land (\neg q \lor p)$	Identity (x2)
$(p \to q) \land (q \to p)$	Material Implication
$p \leftrightarrow q$	First Material Equivalence ☑
	$\therefore p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$

Negation Law

$\neg p \lor p \equiv T$

$\neg q \lor q \equiv T$

$\neg p \land p \equiv F$

2

$p \leftrightarrow q$	Left-hand side given
$(p \to q) \land (q \to p)$	First Material Equivalence
$(\neg p \lor q) \land (\neg q \lor p)$	Material Implication
$(\neg p \land \neg q) \lor (\neg p \land p) \lor (q \land \neg q) \lor (q \land p)$	Distributive (x2)
$(\neg p \land \neg q) \lor F \lor F \lor (q \land p)$	Negation
$(\neg p \land \neg q) \lor (q \land p)$	Identity (x2), Associative
$(p \land q) \lor (\neg p \land \neg q)$	Commutative (x2) ☑
	$\therefore p \leftrightarrow q \equiv (p \land q) \lor (\neg p \land \neg q)$

Exercise 2. Simplify and identify whether tautology, contradiction, or contingency.

1) $\neg p \to (p \to q)$

LOGICAL EXPRESSIONS	RULES APPLIED
1) $\neg p \to (p \to q)$	Given
2) $p \vee (\neg p \vee q)$	Material Implication (x2), Involution $\neg(\neg p) \vee (\neg p \vee q)$
3) $T \vee q$	Negation, Associative
4) T	Domination

$\therefore \neg p \to (p \to q)$ is a $\boxed{\text{Tautology.}}$

2) $(p \vee q) \to (p \wedge q)$

LOGICAL EXPRESSIONS	RULES APPLIED
1) $(p \vee q) \to (p \wedge q)$	Given
2) $\neg(p \vee q) \vee (p \wedge q)$	Material Implication
3) $(\neg p \wedge \neg q) \vee (p \wedge q)$	De Morgan's
4) $p \leftrightarrow q$	Material Equivalence, Commutative

$\therefore (p \vee q) \to (p \wedge q)$ is a $\boxed{\text{Contingency.}}$

3) $p \leftrightarrow \neg p$ (by Evaluation, F)

LOGICAL EXPRESSIONS	RULES APPLIED
1) $p \leftrightarrow \neg p$	Given
2) $(p \wedge \neg p) \vee (\neg p \wedge p)$	Material Equivalence, Involution $(p \wedge \neg p) \vee (\neg p \wedge \neg(\neg p))$
3) $F \vee F$	Negation (x2)
4) F	Idempotent

$\therefore p \leftrightarrow \neg p$ is a $\boxed{\text{Contradiction.}}$

Exercise 3. Determine if logically Equivalent by using RLE. verify answers using Truth Table.

1) $p \to q \overset{?}{\equiv} \neg q \to \neg p$

LOGICAL EXPRESSIONS	RULES APPLIED
A	
1) $p \to q$	Left-hand side given
2) $\neg q \to \neg p$	Contrapositive
$\therefore p \to q \equiv \neg q \to \neg p$	
B	
1) $p \to q$	Left-hand side given
2) $\neg p \vee q$	Material Implication
3) $q \vee \neg p$	Commutative
4) $\neg q \to \neg p$	Material Implication
$\therefore p \to q \equiv \neg q \to \neg p$	
C	
1) $\neg q \to \neg p$	Right-hand side given
2) $q \vee \neg p$	Material Implication
3) $\neg p \vee q$	Commutative
4) $p \to q$	Material Implication
$\therefore p \to q \equiv \neg q \to \neg p$	

p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

same!

2) $(p \land q) \to r \stackrel{?}{\equiv} (p \to r) \land (q \to r)$

A

LOGICAL EXPRESSIONS	RULES APPLIED
1) $(p \to r) \land (q \to r)$	Right-hand side given
2) $(\neg p \lor r) \land (\neg q \lor r)$	Material Implication (x2)
3) $(\neg p \land \neg q) \lor r$	Distributive, Commutative
4) $\neg(p \lor q) \lor r$	De Morgan's
5) $(p \lor q) \to r$	Material Implication

$\therefore (p \land q) \to r \not\equiv (p \to r) \land (q \to r)$

B

1) $(p \land q) \to r$	Right-hand side given
2) $\neg(p \land q) \lor r$	Material Implication
3) $\neg p \lor \neg q \lor r$	De Morgan's
4) $\neg p \lor r \lor \neg q \lor r$	Idempotent, Commutative
5) $(p \to r) \lor (q \to r)$	Material Implication

$\therefore (p \land q) \to r \not\equiv (p \to r) \land (q \to r)$

different

p	q	r	$p \land q$	$(p \land q) \to r$	$p \to r$	$q \to r$	$(p \to r) \land (q \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	F	F	F	T	F	T	F
F	T	T	F	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

3) $\neg(p \leftrightarrow q) \overset{?}{\equiv} \neg p \leftrightarrow q$

A

LOGICAL EXPRESSIONS	RULES APPLIED
1) $\neg(p \leftrightarrow q)$	Left-hand side given
2) $\neg((p \wedge q) \vee (\neg p \wedge \neg q))$	Material Equivalence
3) $(\neg p \vee \neg q) \wedge (p \vee q)$	De Morgan's (x2), Involution
4) $(\neg q \vee \neg p) \wedge (p \vee q)$	Commutative
5) $(q \rightarrow \neg p) \wedge (\neg p \rightarrow q)$	Material Implication, Involution
6) $(\neg p \rightarrow q) \wedge (q \rightarrow \neg p)$	Commutative
7) $\neg p \leftrightarrow q$	Material Equivalence
$\therefore \neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$	

B

1) $\neg p \leftrightarrow q$	Right-hand side given
2) $(\neg p \wedge q) \vee (p \wedge \neg q)$	Material Equivalence, Involution
3) $(\neg p \vee p) \wedge (q \vee p) \wedge (\neg p \vee \neg q) \wedge (q \vee \neg q)$	Distributive (x2) $\rightarrow$ FOIL
4) $T \wedge (q \vee p) \wedge (\neg p \vee \neg q) \wedge T$	Negation (x2)
5) $(q \vee p) \wedge (\neg p \vee \neg q)$	Identity
6) $\neg((\neg q \wedge \neg p) \vee (p \wedge q))$	De Morgan's (x2)
7) $\neg(p \leftrightarrow q)$	Material Equivalence
$\therefore \neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q$	

C

1) $\neg p \leftrightarrow q$	Right-hand side given
2) $(\neg p \wedge q) \vee (p \wedge \neg q)$	Material Equivalence
3) $\neg((\neg q \vee p) \wedge (\neg p \vee q))$	De Morgan's (x2), Commutative
4) $\neg((p \rightarrow q) \wedge (q \rightarrow p))$	Material Implication, Commutative
5) $\neg(p \leftrightarrow q)$	Material Equivalence
$\therefore \neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q)$	

3) $\neg(p \leftrightarrow q) \stackrel{?}{\equiv} \neg p \leftrightarrow q$

p	q	$p \leftrightarrow q$	$\neg(p \leftrightarrow q)$	$\neg p$	$\neg p \leftrightarrow q$
T		T	F	F	F
|		F	T	F	T
		F	T	T	T
	F	T	F	T	F

same!