Sequences

Exercise 1 Determine the next 3 terms and the explicit formula of the following sequences.

n: 0 1 2 3 4 5 6

1) $1, 3, 5, 7, \underbrace{9, 11, 13}$

2 2 2 ✓ common diff. $\longrightarrow a_n = a + dn$

$\{a_n\}$, where $a_n = 1 + 2n$ where n starts at 0.

n = 0 1 2 3

2) $2, 4, 8, 16, 32, 64, 128$

2 2 2 ✓ common ratio $\longrightarrow a_n = a \cdot r^n$

$\{a_n\}$, where $a_n = 2 \cdot 2^n$ where n starts at 0.

$= 2^{n+1}$ " 0.

$= 2^n$ " 1.

n = 0 1 2 3

3) $2, -4, 8, -16, 32, -64, 128$

-2 -2 -2 ✓ common ratio $\rightarrow a_n = a \cdot r^n$

$\{a_n\}$, where $a_n = 2 \cdot (-2)^n$ where n starts at 0.

$n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

5) 1, 3, 9, 27, 81, 243, 729

$\quad$ 3 $\quad$ 3 $\quad$ 3 $\quad$ common ratio $\rightarrow a_n = a \cdot r^n$

$\{a_n\}$, where $a_n = 1 \cdot 3^n$, n starts at 0.

$n = 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$

4) 2, 3, 7, 25, 121, 721, 5041, 40321

$\quad$ 1 $\quad$ 4 $\quad$ 18 $\quad$ 96 $\rightarrow$ factorial $\rightarrow n!$

x common diff.

$3/2 \quad 7/3 \quad 25/7 \quad \ldots$ x common ratio

$-1 \quad 1 \quad 2 \quad 6 \quad 24 \quad 120$ ✓

$\quad 1 \quad 1 \cdot 2 \quad 1 \cdot 2 \cdot 3 \quad 1 \cdot 2 \cdot 3 \cdot 4 \quad 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$ ✓

$\{a_n\}$, where $a_n = n! + 1$, n starts at 1

$1 \quad 2 \quad 6 \quad 24 \quad 120$

$\quad 2 \quad 3 \quad 4 \quad 5$ ✓

Exercise 2 Determine the recurrence relation of each of the following sequences.

1) $\overset{+2}{1}, \overset{+2}{3}, \overset{+2}{5}, 7$

☒ $1+2=3$

$1*3=3$

$$\{a_n\} = \begin{cases} a_0 = 1 \\ a_n = a_{n-1} + 2 \end{cases}$$

2) $\overset{*2}{2}, \overset{*2}{4}, \overset{*2}{8}, 16$

☒ $2+2=4$

☑ $2*2=4$

☐ $2^2 = 4$

$$\{a_n\} = \begin{cases} a_0 = 2 \\ a_n = a_{n-1} * 2 \\ \quad = 2a_{n-1} \end{cases}$$

5) $\overset{*3}{1}, \overset{*3}{3}, \overset{*3}{9}, 27$

☒ $1+2=3$

☑ $1*3=3$

$$\{a_n\} = \begin{cases} a_0 = 1 \\ a_n = a_{n-1} * 3 \\ \quad = 3a_{n-1} \end{cases}$$

3) $0, 1, 1, 2, 3, 5, 8$

☒ $0+1=1$ ☑ $0+1=1$

☒ $x^0 = 1$ ☐ $1^0 = 1$

$\overset{n-2}{0}$ $\overset{n-1}{1}$ $\overset{n}{1}$ 2 3 5 8

Fibonacci

$$\{a_n\} = \begin{cases} a_0 = 0 \\ a_1 = 1 \\ a_n = a_{n-1} + a_{n-2} \end{cases}$$

4) $\overset{*1}{1}, \overset{*2}{1}, \overset{*3}{2}, \overset{*4}{6}, 24$

☒ $1+0...=1$

☑ $1*1...=1$

☐ $1^1 = 1$

n: $\overset{0}{1}, \overset{1}{\boxed{1}}, \overset{2}{\boxed{2}}, \overset{3}{6}, \overset{4}{24}$

$a_1 = a_{1-1} * 1$

$= a_0 * 1$

$= 1 * 1$

$$\{a_n\} = \begin{cases} a_0 = 1 \\ a_n = a_{n-1} * n \end{cases}$$

Factorial

$$\{a_n\} = \begin{cases} a_{-1} = 1 ✓ \\ a_0 = 1 ✓ \\ a_n = (a_{n-1} + a_{n-2}) * n ✓ \end{cases}$$

Exercise 3. Determine the next term & the formula for a_n

1) 5 , 11, 29, 83

-2 3 9 27 81 $+2$

3 3 3 ratio

$3^n, n \geq 1$ –
$3^{n+1}, n \geq 0$

n	$3^n+2, n \geq 1$
1	$3^1+2 = 5$
2	$3^2+2 = 11$
3	$3^3+2 = 29$
4	$3^4+2 = 83$

5, 11, 29, 83

6 18 54 diff.

3 3 ratio $3^n, n \geq 1$

n	$3^n, n \geq 1$		goal
1	$3^1 = 3$	$\rightarrow +2$	5
2	$3^2 = 9$	$\rightarrow +2$	11
3	$3^3 = 27$	$\rightarrow +2$	29
4	$3^4 = 81$	$\rightarrow +2$	83

$$\boxed{a_n = 3^n + 2, \text{ where } n \geq 1}$$

$$a_5 = 3^5 + 2 = 243 + 2 = \boxed{245}$$

2) 1, 9, 17, 25, 33

$\underset{8}{\vee}\ \underset{8}{\vee}\ \underset{8}{\vee}\ \underset{8}{\vee}$ — common differences

Arithmetic Progression = $a+dn$, $n \geq 0$

n	$1+8n$
0	$1+8(0) = 1$
1	$1+8(1) = 9$
2	$1+8(2) = 17$
3	$1+8(3) = 25$
4	$1+8(4) = 33$

$$\boxed{a_n = 1+8n,\ n \geq 0}$$ closed-form formula

$$a_5 = 1+8(5) = 1+40 = \boxed{41}$$

3) -1, 1, 9, 45, 237

2 8 36 192 ÷ 2

1 4 18 96 -

1	2	3	4	5
1	2	6	24	120

1 4 18 96 -

n	Goal	n! * 2 - 3	
1	-1	1 * 2 = 2 - 3	= -1
2	1	2 * 2 = 4 - 3	= 1
3	9	6 * 2 = 12 - 3	= 9
4	45	24 * 2 = 48 - 3	= 45
5	237	120 * 2 = 240 - 3	= 237

$a_1 = -1$

$a_n = a_{n-1} * n + 3(n-1)$

$= (a_{n-1} + 3)n - 3$

$-1 * 1 + 2 = 1$
$1 * 2 + 7 = 9$
$9 * 3 + 18 = 45$
$45 * 4 + 57 = 237$

$-1 * \boxed{2} + 3\boxed{(1)} = 1$
$1 * \boxed{3} + 3\boxed{(2)} = 9$
$9 * \boxed{4} + 3\boxed{(3)} = 45$
$45 * \boxed{5} + 3\boxed{(4)} = 237$

$a_n = 2n! - 3, \; n \geq 1$

$a_0 = -1$

$= a_{n-1} * n$

$a_n = 2n(n-1)! - 3, \; n \geq 1$

n

5 $\quad 5 \cdot \boxed{4 \cdot 3 \cdot 2 \cdot 1}$

$2n! - 3$

$1 \cdot 2 \quad 2 \cdot 5 \quad 3 \cdot 8 \quad 4 \cdot 11$ ✗

4) 2, 10, 24, 44, . . .

8 14 20 — Growth is big at first, then slow

6 6

4 7 10 ÷ 2

3 3 ⟷ 3 (n ?)

SOLUTION 2:

2+0	2	0
2+8	10	8 = 2+6
2+8+14	24	14 = 2+12
2+8+14+20	44	20 = 2+18

Similar to growth of summation

	0	1	2	3	4
	0	0+1	0+1+2	0+1+2+3	0+1+2+3+4
	0	1	3	6	10
x3	0	3	9	18	30

2^n Growth is small

0	1	2	3	4	5
1	2	4	8	16	32

✗

$n!$ Growth is small at first, then too big later.

1	2	3	4	5
1	2	6	24	120

✗

3^n A bit similar w/ $n!$

0	1	2	3	4	5
1	3	9	27	81	243

✗

n^2 Nearer

	0	1	2	3	4	5
	0	1	4	9	16	25
x3	0	3	12	27	48	

✓

n	3n	$3n^2$	− n	Goal
1	3(1)=3	3(1)=	3−1=	2 ✓
2	3(2)=6	3(4)=	12−2=	10 ✓
3	3(3)=9	3(9)=	27−3=	24 ✓
4	3(4)=12	3(16)=	48−4=	44 ✓

$$\boxed{a_n = 3n^2 - n \ , \ n \geq 1}$$