

Name (SURNAME, First Names): _____

Section: _____

A . Relations

1 . Let $A = \{2, 3, 4, 5, 6, 7, 8\}$, R is a relation on A , and xRy iff $x - y = 3n$, where $n \in \mathbb{Z}$

a.) $R =$ _____

b.) Determine if R is reflexive, symmetric, antisymmetric, and transitive. Give one counterexample for each property not satisfied.

reflexive: _____

symmetric: _____

antisymmetric: _____

transitive: _____

2 . Let $A = \{0, 1, 2, 3\}$, R is a relation on A , and $R = \{(0, 0), (0, 1), (0, 3), (1, 1), (1, 0), (2, 3), (3, 3)\}$. Determine if R is reflexive, symmetric, antisymmetric, and transitive. Give one counterexample for each property not satisfied.

reflexive: _____

symmetric: _____

antisymmetric: _____

transitive: _____

B . Functions

1 . Let $f : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$, and $f(x) = 2x \bmod 5$. Determine if the function is injective, surjective, and bijective. Provide a counterexample, if not.

injective: _____

surjective: _____

bijective: _____

2 . Let $f : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+$, and $f(x) = 3x \bmod 3$. Determine if the function is injective, surjective, and bijective. Provide a counterexample, if not.

injective: _____

surjective: _____

bijective: _____

C . Sequences

1 . Determine the formula for a_n , where $n > 0$

a.) $4, 16, 36, 64, 100, \dots$ $a_n =$ _____

b.) $-2, 6, -18, 54, -162, \dots$ $a_n =$ _____

2 . Find the 8th term.

a.) $a_n = -3 \cdot 2^{n-1}, n \geq 1$ $a_8 =$ _____

b.) $a_n = a_{n-1} + 2^{n-1}, a_0 = \frac{1}{2}$ $a_8 =$ _____

c.) Given an arithmetic sequence, where $a_3 = -38$, and $a_{20} = -191$ $a_8 =$ _____

d.) Given a geometric sequence, where $a_4 = 108$, and $a_9 = 26244$. $a_8 =$ _____

D . **Sums.** Show the first few steps (as indicated below) and the final answer in evaluating the given summation. Final answers must be in its simplest whole or rational number, or expression.

$$\sum_{j=5}^{n+1} (2j+1)^3 =$$

first step

$$=$$

2nd *step*

\vdots

$$=$$

final answer

$$\sum_{k=0}^{n+1} \sum_{j=9}^{2k} \sum_{x=11}^{2k} k =$$

first step

$$=$$

2nd *step*

$$=$$

3rd *step*

$$=$$

4th *step*

\vdots

$$=$$

final answer