

Quantifiers Exercises

Exercise 1

Determine the truth value of each of the following statements if the domain consists of:

- 1 $\forall x(x^2 \geq 0)$
- 2 $\exists x(x^3 = -1)$
- 3 $\forall x((-x)^2 = x^2)$
- 4 $\exists n(n^2 = 2)$
- 5 $\forall x(x^2 + 2 \geq 1)$
- 6 $\forall x(x^2 \neq x)$
- 7 $\forall x(2x > x)$
- 8 $\exists n(n = -n)$
- 9 $\exists n(2n = 3n)$
- 10 $\forall x(x > 0 \vee x < 0)$

\mathbb{Z}

T

T

T

F

T

F, $x = 0$

F, $x \geq 0$

T

T

F, $x = 0$

\mathbb{R}

T

T

T

T

T

F, $x = 0$

F, $x \geq 0$

T

T

F, $x = 0$

Exercise 2

Determine the truth value of each of the following statements if the domain consists of:



1 $\forall x \exists y (x^2 < y)$

2 $\exists y \forall x (xy = x)$

3 $\exists x \exists y (x^2 + y^2 = 5)$

4 $\forall x \exists y (x + y = 0)$



\mathbb{Z}

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\mathbb{R}

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Exercise 3

Translate the paragraph to logical expressions.

- 1 Randy works hard. If Randy works hard, then he is a dull boy. If Randy is a dull boy, he will get the job. Randy did not get the job.
- 2 No man is an island. Manhattan is an island. Manhattan is not a man.
- 3 All men are mortal. Socrates is a man. Socrates is mortal.
- 4 Someone in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. There is a person in this class who cares about ocean pollution.
- 5 Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Zeke, a student in this class, can use a word processing program.

Exercise 3

Translate the paragraph to logical expressions.

- 1 Randy works hard. If Randy works hard, then he is a dull boy. If Randy is a dull boy, he will get the job. Randy did not get the job.

Let $W(x)$ - x works hard.
 $D(x)$ - x is a dull boy.
 $G(x)$ - x gets the job.

The domain of x is the set of persons.

Translation/s:

- 1) $W(\text{Randy})$
- 2) $W(\text{Randy}) \rightarrow D(\text{Randy})$
- 3) $D(\text{Randy}) \rightarrow G(\text{Randy})$
- 4) $\neg G(\text{Randy})$

Let w - Randy works hard.
 d - Randy is a dull boy.
 g - Randy gets the job.

Translation/s:

- 1) w
- 2) $w \rightarrow d$
- 3) $d \rightarrow g$
- 4) $\neg g$

Exercise 3

Translate the paragraph to logical expressions.

2 No man is an island. Manhattan is an island. Manhattan is not a man.

Handwritten notes:
- Above "No man is an island": $\neg \exists x (M(x) \wedge I(x))$
- Above "Manhattan is an island": $I(\text{Manhattan})$
- Above "Manhattan is not a man": $\neg M(\text{Manhattan})$
- A note "x? no predicate." with an arrow pointing to the variable 'x' in the first sentence.

Let $I(x)$ - x is an island.
 $M(x)$ - x is a man.

The domain of x is the set of All.
OR
 $x \in U$

Translation is:

- 1) $\neg \exists x (M(x) \wedge I(x))$
- 2) $I(\text{Manhattan})$
- 3) $\neg M(\text{Manhattan})$

Exercise 3

Translate the paragraph to logical expressions.

3 $\forall x (A(x) \rightarrow O(x))$
All men are mortal. $A(\text{Socrates})$ Socrates is a man. $O(\text{Socrates})$ Socrates is mortal.
predicate

Let $O(x)$ - x is a mortal.
 $A(x)$ - x is a man.

The domain of x is a set of AI.
OR
 $x \in U$

Translation/s:

- 1) $\forall x (A(x) \rightarrow O(x))$
- 2) $A(\text{Socrates})$
- 3) $O(\text{Socrates})$

Exercise 3

Translate the paragraph to logical expressions.

- 4 Someone in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. There is a person in this class who cares about ocean pollution.

Let: $C(x)$ - x is in this class.
 $W(x)$ - x enjoys whale watching.
 $O(x)$ - x cares about ocean pollution.

The domain of x is set of persons.

Translation/s:

- 1) $\exists x (C(x) \wedge W(x))$
- 2) $\forall x (W(x) \rightarrow O(x))$
- 3) $\exists x (C(x) \wedge O(x))$

Exercise 3

Translate the paragraph to logical expressions.

- 5 Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Zeke, a student in this class, can use a word processing program.

Let: $C(x)$ - x is a student in this class.

$P(x)$ - x owns a personal computer.

$W(x)$ - x can use a word processing program.

The domain of x is set of students.

Translations:

1) $\forall x(C(x) \rightarrow P(x))$

2) $\forall x(P(x) \rightarrow W(x))$

3) $C(\text{Zeke})$

4) $W(\text{Zeke})$

] OR $C(\text{Zeke}) \wedge W(\text{Zeke})$