CSARCH Lecture Series: Binary Integer Multiplication

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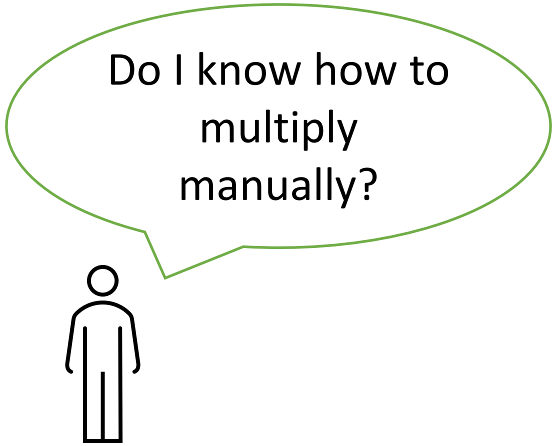
Overview

Reflect on the following question:

- Are there other ways to perform integer binary multiplication besides the usual pencil-and-paper method?

```
     0100
x    0101
--------
```



Overview

- This sub-module introduces the concepts of integer binary multiplication
- The objectives are as follows:
  - ✓Describe the process of performing integer binary multiplication using pencil-and-paper method
  - ✓Describe the process of performing integer binary multiplication using Booth's algorithm
  - ✓Describe the process of performing integer binary multiplication using extended Booth's algorithm

ALU Multiplication

- Some methods in performing integer binary multiplication:
  - Pencil-and-paper method
  - Booth’s algorithm
  - Extended Booth’s algorithm

Pencil-and-Paper Method

Steps:

1.) represent both multiplicand (m) and multiplier (n) using 2’s complement format

2.) the first intermediate product has a length of $m+n$ bits. Sign extend as needed

3.) As with pencil-and-paper multiplication, succeeding intermediate products are shifted right

4.) if the multiplier is negative, add the 2’s complement of the multiplicand, sign-extended and shifted to sign-bit position

Pencil-and-Paper Method

```
      0100
 x    0101        step 1
 --------
 00000100         step 2
 0000000
 000100           step 3
 00000
 --------
 00010100
```

step 4 not needed

Example: +4 * +5

Steps:
1.) represent both multiplicand (m) and multiplier (n) using 2’s complement format
2.) the first intermediate product has a length of *m*+*n* bits. Sign extend as needed
3.) As with pencil-and-paper multiplication, succeeding intermediate products are shifted right
4.) if the multiplier is negative, add the 2’s complement of the multiplicand, sign-extended and shifted to sign-bit position

Pencil-and-Paper Method

```
       1100
 x     1011        step 1
 --------
 11111100          step 2
 1111100
 000000            step 3
 11100
 0100              step 4
 --------
 00010100
```

Example: -4 * -5

Steps:

1.) represent both multiplicand (m) and multiplier (n) using 2's complement format

2.) the first intermediate product has a length of *m*+*n* bits. Sign extend as needed

3.) As with pencil-and-paper multiplication, succeeding intermediate products are shifted right

4.) if the multiplier is negative, add the 2's complement of the multiplicand, sign-extended and shifted to sign-bit position

Pencil-and-Paper Method

```
      0100
 x    1011      step 1
 --------
 00000100   <-  step 2
 0000100
 000000         step 3
 00100
 1100       <-  step 4
 --------
 11101100
```

Example: +4 * -5

Steps:

1.) represent both multiplicand (m) and multiplier (n) using 2's complement format

2.) the first intermediate product has a length of *m+n* bits. Sign extend as needed

3.) As with pencil-and-paper multiplication, succeeding intermediate products are shifted right

4.) if the multiplier is negative, add the 2's complement of the multiplicand, sign-extended and shifted to sign-bit position

Pencil-and-Paper Method

```
    1100
x   0101        step 1
--------
11111100   <--  step 2
0000000
111100          step 3
00000
--------
11101100
```

step 4 not needed

Example: -4 * +5

Steps:

1.) represent both multiplicand (m) and multiplier (n) using 2's complement format

2.) the first intermediate product has a length of *m+n* bits. Sign extend as needed

3.) As with pencil-and-paper multiplication, succeeding intermediate products are shifted right

4.) if the multiplier is negative, add the 2's complement of the multiplicand, sign-extended and shifted to sign-bit position

Try

Try: +13*-6 (using Pencil-and-paper)

```
    01101
x   11010
---------
```

Try

Try: +13*-6 (using Pencil-and-paper)

```
     01101
x    11010
  --------
```

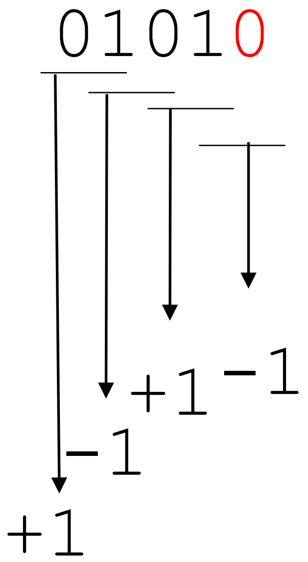
```
           01101
        x  11010
        --------
0000000000          step 2
000001101
00000000
0001101             step 3
001101
10011               step 4
------------
1110110010
```

step 1

Booth's Algorithm

- Treats positive and negative multipliers uniformly.
- Rewrites multiplier in terms of sums and differences.
- Convert code according to next bit at right
- 0 to 1 $\Rightarrow$ +1
- 1 to 0 $\Rightarrow$ –1
- Otherwise, 0
- Right of lsb is "nothing", *i.e.*, equal to 0

Booth’s Algorithm



Steps in obtaining Booth’s equivalent of a binary number

1. append 0 at the LSb side
2. pair 2 bits starting at LSb
3. 00 → 0; 01→ +1; 10→-1;11→0

0*m = 0

1*m = m

-1*M = 2’s complement(m)

Booth's Algorithm

```
0100
0101      step 1

01010
+1-1+1-1  step 2
```

```
        0100
  x  +1-1+1-1
  -----------
    11111100
    0000100
    111100      step 3
    00100
  -----------
    00010100
```

Example: +4 * +5

Steps:

1.) represent both multiplicand (m) and multiplier (n) using 2's complement format

2.) convert multiplier to its Booth's equivalent

- append 0 at the LSb side
- pair 2 bits starting at LSb
- 00 → 0; 01→ +1; 10→-1;11→0

3.) Multiply using pencil-and-paper method ignoring extra steps if multiplier is negative

Booth's Algorithm

```
0100 ┐
1011 ┘ step 1

10110 ┐
-1+1 0-1 ┘ step 2
```

```
          0100
x    -1+10-1
-----------
      11111100
      0000000
      000100
      11100
-----------
      11101100
```

step 3

Example: +4 * -5

Steps:

1.) represent both multiplicand (m) and multiplier (n) using 2's complement format

2.) convert multiplier to its Booth's equivalent

- append 0 at the LSb side
- pair 2 bits starting at LSb
- 00 → 0; 01→ +1; 10→-1;11→0

3.) Multiply using pencil-and-paper method ignoring extra steps if multiplier is negative

Try

Try: +13*-6 (using Booth's algorithm)

```
    01101
x   11010
-------
```

Try

Try: +13*-6 (using Booth's algorithm)

```
  01101
x 11010
-------
```

```
 01101
 11010
```
step 1

```
110100
0-1+1-10
```
step 2

```
    01101
x 0-1+1-10
----------
```

step 3

```
0000000000
111110011
00001101
1110011
000000
----------
1110110010
```

Extended Booth's Algorithm

- Also known as fast multiplication or bit-pair recording
- Bit-Pair Recording – reduces to half the number of summands The number of summands is reduced by pairing multiplier bits

Bit-pair recording:

0 0 0 => 0

0 0 1 => +1

0 1 0 => +1

0 1 1 => +2

1 0 0 => -2

1 0 1 => -1

1 1 0 => -1

1 1 1 => 0

Extended Booth's Algorithm

01010

+1

+1

Steps in obtaining Booth's equivalent of a binary number

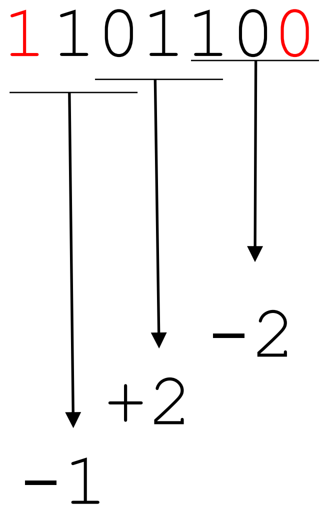
1. append 0 at the LSb side
2. if odd number of bits, sign-extend
3. bit-pair starting at LSb

0*m = 0
1*m = m
-1*m = 2's complement(m)
+2*m = m0
-2*m = [2's complement(m)]0

Bit-pair recording:

0 0 0 => 0
0 0 1 => +1
0 1 0 => +1
0 1 1 => +2
1 0 0 => -2
1 0 1 => -1
1 1 0 => -1
1 1 1 => 0

Extended Booth’s Algorithm



Steps in obtaining Extended Booth’s equivalent of a binary number

1. append 0 at the LSb side
2. if odd number of bits, sign-extend
3. bit-pair starting at LSb

0*m = 0

1*m = m

-1*m = 2’s complement(m)

+2*m = m0

-2*m = [2’s complement(m)]0

Bit-pair recording:

0 0 0 => 0

0 0 1 => +1

0 1 0 => +1

0 1 1 => +2

1 0 0 => -2

1 0 1 => -1

1 1 0 => -1

1 1 1 => 0

Extended Booth's Algorithm

```
 0100
 0101      step 1

01010
+1+1       step 2
```

```
       0100
x     +1+1
----------
   00000100
   000100        step 3
 ---------
   00010100
```

Example: +4 * +5

Steps:
1.) represent both multiplicand (m) and multiplier (n) using 2's complement format
2.) convert multiplier to Extended Booth's equivalent
3.) Multiply using pencil-and-paper method ignoring extra steps if multiplier is negative. Since each bit-pair is equivalent to 2 bits, skip two after the initial intermediate product

Extended Booth's Algorithm

```
0100
1011     step 1

10110
-1-1     step 2
```

```
        0100
x       -1-1
  ----------
    11111100
    111100
    --------
    11101100
```

step 3

Example: +4 * -5

Steps:
1.) represent both multiplicand (m) and multiplier (n) using 2's complement format
2.) convert multiplier to Extended Booth's equivalent
3.) Multiply using pencil-and-paper method ignoring extra steps if multiplier is negative. Since each bit-pair is equivalent to 2 bits, skip two after the initial intermediate product

Try

Try: +13*-6 (using Extended Booth's algorithm)

```
    01101
x   11010
 -------
```

Try

Try: +13*-6 (using Extended Booth's algorithm)

```
   01101
x  11010
--------
```

```
 01101  } step 1
 11010

1110100  } step 2
0-1-2
```

```
       01101
x      0-1-2
------------
        1111100110  } step 3
        11110011
        000000
------------
        1110110010
```

To recall …

- What have we learned:
  - ✓Describe the process of performing integer binary multiplication using pencil-and-paper method
  - ✓Describe the process of performing integer binary multiplication using Booth's algorithm
  - ✓Describe the process of performing integer binary multiplication using extended Booth's algorithm