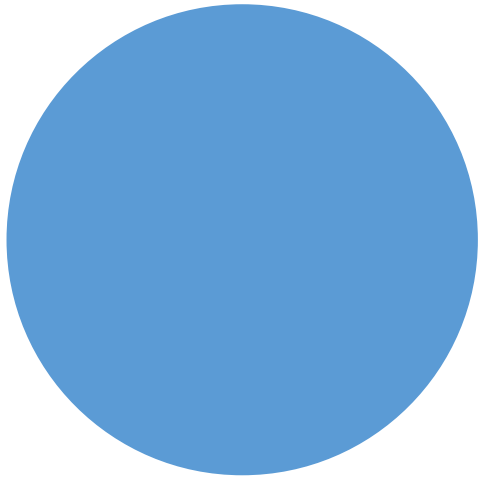
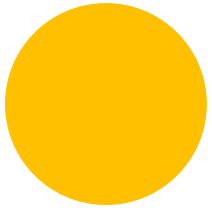

# CSARCH Lecture Series: Integer operation and overflow


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# Overview

Reflect on the following questions:

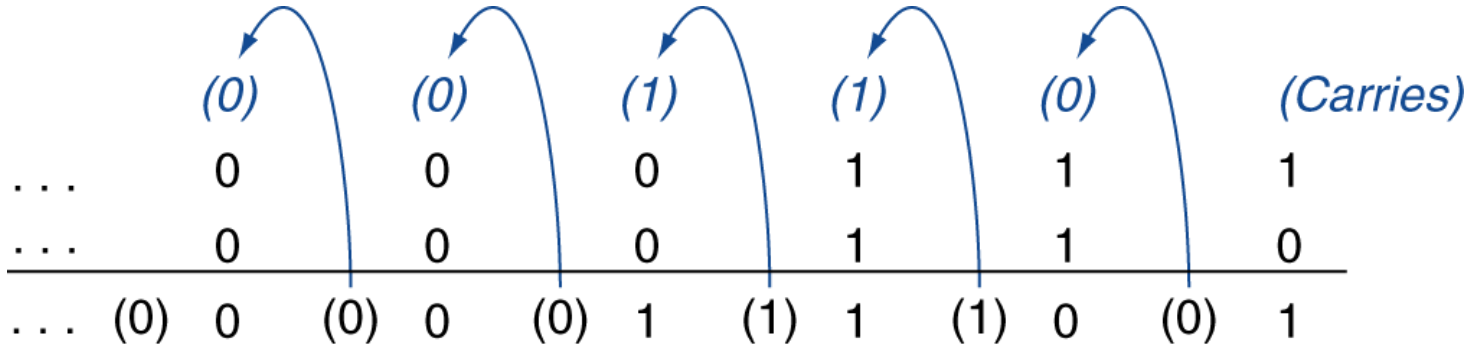
- How do you perform addition and subtraction for binary integer ?
- How do we know if the result is correct (i.e., not overflow)?

$$10111111_2 + 11000001_2 = ?$$

# Overview

- This sub-module discusses how to perform addition and subtraction for binary integer and determine if the result of the operation is an overflow.
- The objectives are as follows:
  - ✓Describe the process of performing integer addition and subtraction
  - ✓Explain if overflow occurs after performing addition and subtraction

# Addition of Integer




```
   111   (carry)
   1111
+) 1111
------
1 1110
(carry out)
```

2= $10_2$ → 0 carry 1

3= $11_2$ → 1 carry 1

- In order to add multiple-bit numbers, we used a method analogous to that used for manual computation with decimal numbers ➔ Add bit pairs starting from the LSb of the bit vectors, propagating carries toward the MSb

# Overflow and Addition of Unsigned Integer

```
   111   (carry)
   1111
+) 1111
------
1  1110
(carry out)
```

- 4-bit binary addition of unsigned integer:

  15 + 15 = ? [$1111_2 + 1111_2 = ?$]

  Is the result an overflow?

- 15 + 15 = 30 [$1111_2 + 1111_2 = 1110_2$]
- The result is $1110_2$ or 14 but the correct answer is 30
- The result is therefore an overflow.
- There is an overflow because the result needs an additional bit to store the result (i.e., there is a carry-out)
- For unsigned integer, overflow occurs if there is a carry out from the MSb

# Overflow and Addition of Unsigned Integer

```
   111   (carry)
    0111
+)  0111
------
    1110
```

- 4-bit binary addition of unsigned integer:

  $7 + 7 = ?$ $[0111_2 + 0111_2 = ?]$

  Is the result an overflow?

- $7 + 7 = 14$ $[0111_2 + 0111_2 = 1110_2]$
- The result is $1110_2$ or 14 and the correct answer is 14
- The result is not an overflow (i.e., there is no carry out)

# Overflow and Addition of Unsigned Integer

- Unsigned integer: overflow occurs if there is a carry-out

# Overflow and subtraction of unsigned integer

Subtraction of two *n*-digit unsigned numbers *M*-*N* in base *r* can be done as follows:

- Add the minuend *M* to the r's complement of the subtrahend *N*
- If *M* >=*N*, the sum will produce and end carry which can be discarded. What is left is the result of *M*-*N*
- If *M*<*N*, the sum does not produce an end carry and the result is the *r*'s complement of (*M*-*N*). To "view" the answer in a familiar format, take the *r*'s complement of the sum and place a negative sign in front. (Note: since its unsigned, negative should not exist!).

# Overflow and subtraction of unsigned integer

Using 10's complement, perform 72532-3250

```
M  =   72532
N* = + 96750
       -----
      169282
(ignore carry)= 69282
```

*10's complement; 5 digits (i.e., 03250) since minuend is 5 digits

Using 10's complement, perform 3250 - 72532

```
M  =   03250
N = +  27468
       -----
       30718
(Overflow)
```

# Overflow and subtraction of unsigned integer

Using 2's complement, perform $1010100_2 - 1000011_2$

```
 M   =    1010100
 N* = + 0111101
        -------
       10010001
(ignore carry)= 0010001
```

*2's complement

Using 2's complement, perform $1000011_2 - 1010100_2$

```
M   =    1000011
N   = + 0101100
        -------
        1101111
no carry
(overflow)
```

# Try

Using 10’s complement, perform

(1)15-14

(2)14-15

and determine if the result is an overflow

Using 2’s complement, perform

(3)$1111_2$-$1110_2$

(4)$1110_2$-$1111_2$

and determine if the result is an overflow

# Try

Using 10's complement, perform

(1)15-14

(2)14-15

(1)

M = 15

N = + 86

--

101

(ignore carry)= 01

→ not overflow

(2)

M = 14

N = + 85

--

99

(no carry)

→Overflow

# Try

Using 2's complement, perform

(3) $1111_2$-$1110_2$

(4) $1110_2$-$1111_2$

(3)

M = 1111

N = + 0010

---

10001

(ignore carry)= 0001

→ not overflow

(4)

M = 1110

N = + 0001

---

1111

(no carry)

→ overflow

# Overflow and Addition of Signed Integer

```
  111   (carry)
  0111
+)0111
------
  1110
```

- 4-bit binary addition of signed integer:

  $7 + 7 = ?\ [0111_2 + 0111_2 = ?]$

  Is the result an overflow?

- $7 + 7 = 14\ [0111_2 + 0111_2 = 1110_2]$
- The result is $1110_2$ or -2 but the correct answer should be 14
- The result is therefore an overflow.
- There is an overflow because adding two positive integers and the result is a negative integer

# Overflow and Addition of Signed Integer

- 4-bit binary addition of signed integer:

  $3 + 4 = ?$ $[0011_2 + 0100_2 = ?]$

  Is the result an overflow?

```
  0011
+)0100
------
  0111
```

- $3 + 4 = 7$ $[0011_2 + 0100_2 = 0111_2]$
- The result is $0111_2$ or 7 and the correct answer is 7
- The result is not an overflow (adding 2 positive integers and the result is a positive integer)

# Overflow and Addition of Signed Integer

- 4-bit binary addition of signed integer:

  -7 + -6 = ? [$1001_2 + 1010_2 = ?$]

  Is the result an overflow?

```
   1001
+) 1010
------
1  0011
(carry out)
```

- -7 + -6 = -13 [$1001_2 + 1010_2 = 0011_2$]
- The result is $0011_2$ or 3 but the correct answer should be -13
- The result is therefore an overflow.
- There is an overflow because adding two negative numbers results in a positive result

# Overflow and Addition of Signed Integer

```
   1   (carry)
   1101
+) 1100
------
   1001
```

- 4-bit binary addition of signed integer:

  -3 + -4 = ? [$1101_2 + 1100_2$ = ?]

  Is the result an overflow?

- -3 + -4 = -7 [$1101_2 + 1100_2 = 1001_2$]
- The result is $1001_2$ or -7 and the correct answer is -7
- The result is not an overflow (adding 2 negative integers and the result is a negative integer)

# Overflow and Addition of Signed Integer

```
   111   (carry)
   1101
+) 0111
------
1  0100
(carry out)
```

- 4-bit binary addition of signed integer:

  -3 + 7 = ? [$1101_2 + 0111_2 = ?$]

  Is the result an overflow?

- -3 + 7 = +4 [$1101_2 + 0111_2 = 0100_2$]
- The result is $0100_2$ or +4 and the correct answer is +4
- The result is not an overflow (adding a positive with a negative integer is similar to performing subtraction)
- Subtraction operation never cause an overflow result. Why?

# Overflow and Addition of Signed Integer

- Adding positive integer with negative integer and vice-versa→ no overflow
- Adding two positive integers
  - Overflow if the result is negative
- Adding two negative integers
  - Overflow if the result is positive

# Try

$10111111_2 + 11000001_2 = ?$

| Is it an overflow (8-bit integer)? | Yes/No |
|---|---|
| View the result as unsigned integer | |
| View the result as signed integer | |

# Try

$10111111_2 + 11000001_2 = ?$

| Is it an overflow (8-bit integer)? | Yes/No |
|---|---|
| View the result as unsigned integer | Yes |
| View the result as signed integer | No |

```
  10111111
+)11000001
----------
1 10000000
```

```
Unsigned:
  191
+)193
-------
  384
```

```
Signed:
  -65
+)-63
-------
 -128
```

# Subtraction of Signed Integer

- Subtraction can be viewed as adding the 2’s complement of the second operand
  - A – B = A+ (-B)
  - 2’s complement is  used to change the sign of the second operand

# Subtraction of Signed Integer

```
  1       (carry)
  0110
+)1101
------
1 0011
(carry out)
```

- 4-bit binary subtraction using 2's complement addition

  $6 - (+3) = 6 + (-3) = [0110_2 + 1101_2 = ?]$

  Is the result an overflow?

- $6 - (+3) = 6 + (-3) = [0110_2 + 1101_2 = 0011]$
- The result is $0011_2$ or +3 and the correct answer is 3
- The result is not an overflow since "adding" positive with negative integer will never cause an overflow

# To recall …

- What have we learned:
  - ✓Describe the process of performing integer addition and subtraction
  - ✓Explain if overflow occurs after performing addition and subtraction