

NUMBER SYSTEM (ARITHMETIC)



BINARY ARITHMETIC

Binary Addition & Subtraction

- Addition and subtraction of binary numbers
 - Addition – *rule of carrying*
 - $0 + 0 = 0$ (or 0_{10})
 - $0 + 1 = 1$ (or 1_{10})
 - $1 + 0 = 1$ (or 1_{10})
 - $1 + 1 = 10$ (or 2_{10})
 - Subtraction – *rule of borrowing*
 - $0 - 0 = 0$
 - $0 - 1 = -1$
 - $1 - 0 = 1$
 - $1 - 1 = 0$



BINARY ARITHMETIC

Binary Addition

$$11010_2 + 1100_2 = \mathbf{100110_2}$$

$$\begin{array}{r} \textcolor{blue}{1} \textcolor{blue}{1} \longleftarrow \textit{carry} \\ 1\ 1\ 0\ 1\ 0 \\ +\ 1\ 1\ 0\ 0 \\ \hline \mathbf{1\ 0\ 0\ 1\ 1\ 0} \end{array}$$

$$10110_2 + 1011_2 = ? \mathbf{100001_2}$$

$$11101_2 + 1110 + 1010_2 = ?$$
$$\mathbf{110101_2}$$



BINARY ARITHMETIC

Binary Subtraction

$$10011_2 - 1001_2 = \mathbf{1010_2}$$

$$\begin{array}{r} \overset{1}{\cancel{1}} \leftarrow \text{borrow} \\ \cancel{1} \\ \cancel{1} \cancel{0} 0 1 1 \\ - \cancel{1} 0 0 1 \\ \hline \mathbf{1\ 0\ 1\ 0} \end{array}$$

$$11001_2 - 1110_2 = ?$$

$$\mathbf{1011_2}$$



BINARY MULTIPLICATION

Multiplication Rules:

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

Example: Binary Multiplication

				1	1	0	0
				1	0	1	0
				<hr/>			
				0	0	0	0
			1	1	0	0	
		0	0	0	0		
	1	1	0	0			
	<hr/>						
	1	1	1	1	0	0	0



BINARY DIVISION

Division Rules:

$0 \div 0 = \text{undefined}$

$0 \div 1 = 0$

$1 \div 0 = \text{undefined}$

$1 \div 1 = 1$

Example: Binary Division

$$\begin{array}{r} 10 \overline{) 111} \quad \text{r. } 1 \\ \underline{10} \\ 11 \\ \underline{10} \\ 10 \\ \underline{10} \\ 0 \end{array}$$



OCTAL ARITHMETIC

- Octal Addition
- Octal Subtraction
- Octal Multiplication
- Octal Division



OCTAL ADDITION

Example: Octal Addition

$$\begin{array}{r} \\ +) \\ \hline \end{array}$$

$$\begin{array}{r} \\ + \\ \hline \end{array}$$

If the sum is greater than or equal 8, **divide it by 8**, **bring down the remainder** and **carry the quotient**.



OCTAL SUBTRACTION

Example: Octal Subtraction

	0	10	13
	1	3	5
-)		4	6
		6	7

If the minuend is less than the subtrahend, it will **borrow 8** from the other number



OCTAL MULTIPLICATION

Example: Octal Multiplication

	2	3
X)	1	5
	10	15
2	3	
2	13	15
1	1	
3	6	7



OCTAL DIVISION

Octal Division Example:

$$422 / 7 = 47 \text{ r. } 1$$

Refer to the octal multiplication:

$$0 \times 7 = 0$$

$$1 \times 7 = 7$$

$$2 \times 7 = 16$$

$$3 \times 7 = 25$$

$$4 \times 7 = 34$$

$$5 \times 7 = 43$$

$$6 \times 7 = 52$$

$$7 \times 7 = 61$$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	10	12	14	16
3	0	3	6	11	14	17	22	25
4	0	4	10	14	20	24	30	34
5	0	5	12	17	24	31	36	43
6	0	6	14	22	30	36	44	52
7	0	7	16	25	34	43	52	61



OCTAL DIVISION

Octal Division Example:

$$513 / 6 = 67 \text{ r. } 1$$

Refer to the octal multiplication:

$$0 \times 6 = 0$$

$$1 \times 6 = 6$$

$$2 \times 6 = 14$$

$$3 \times 6 = 22$$

$$4 \times 6 = 30$$

$$5 \times 6 = 36$$

$$6 \times 6 = 44$$

$$7 \times 6 = 52$$

	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	10	12	14	16
3	0	3	6	11	14	17	22	25
4	0	4	10	14	20	24	30	34
5	0	5	12	17	24	31	36	43
6	0	6	14	22	30	36	44	52
7	0	7	16	25	34	43	52	61



HEXADECIMAL ARITHMETIC

- Addition and subtraction of hexadecimal numbers

- **Addition**

- Performed starting at the lowest (first from the right) digit
 - A carry to the upper digit is performed when the result is higher than 16

- **Subtraction**

- Performed starting at the lowest (first from the right) digit
 - A borrow from the upper digit is performed when the result is negative



HEXADECIMAL ADDITION

$$\begin{array}{r}
 \boxed{1} \boxed{1} \leftarrow \text{Carry} \\
 \text{A } 8 \text{ D} \\
 + \text{B } 1 \text{ 7} \\
 \hline
 1 \text{ 5 A } 4
 \end{array}$$

$$\left[\begin{array}{r}
 10 \quad 8 \quad 13 \\
 + 11 \quad 1 \quad 7 \\
 \hline
 21 \quad 9 \quad 20
 \end{array} \right]$$

- First column from right

$D + 7 = (\text{In the decimal system: } 13 + 7 = 20) = 16 (\text{carried } 1) + 4$

The sum of the first column is 4 and 1 is carried to the second column.

- Second column from right

$1 + 8 + 1 = (\text{In the decimal system: } 10) = A$

Carried from the first column

- Third column from right

$A + B = (\text{In the decimal system: } 10 + 11 = 21) = 16 (\text{carried } 1) + 5$

The sum of the third column is 5 and 1 is carried to the fourth column.

- The result is $(15A4)_{16}$.



HEXADECIMAL SUBTRACTION

$$\begin{array}{r}
 \heartsuit \leftarrow \text{Borrow} \\
 6 \text{ D } 3 \\
 - 1 \text{ 7 } 4 \\
 \hline
 5 \text{ 5 } \text{F}
 \end{array}$$

$$\left[\begin{array}{r}
 \heartsuit \text{ 16} \\
 6 \quad 13 \quad 3 \\
 - 1 \quad 7 \quad 4 \\
 \hline
 5 \quad 5 \quad 15
 \end{array} \right]$$

- First column from right

Since $3 - 4 = -1$, a borrow is performed from D in the second digit (D becomes C).
 $16 \text{ (borrowed 1)} + 3 - 4 = \text{F}$ (In the decimal system: $19 - 4 = 15$)

- Second column from right

$\text{C} - 7 = 5$ (In the decimal system: $12 - 7 = 5$)

- Third column

$6 - 1 = 5$

- The result is $(55\text{F})_{16}$.



HEXADECIMAL MULTIPLICATION

Example: Hexadecimal Multiplication

$$\begin{array}{r} \text{X)} \quad \quad \quad 2 \quad 3 \\ \hline \quad \quad \quad 3 \quad 5 \\ \hline \quad \quad 10 \quad 15 \\ \quad 6 \quad 9 \\ \hline \quad 6 \quad 19 \quad 15 \\ \quad 1 \\ \hline 7 \quad 3 \quad F \end{array}$$



HEXADECIMAL DIVISION

Hexadecimal Division Example:

$$67D4 / A = A62 \text{ r. } 0$$

x	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
1	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	2	4	6	8	A	C	E	10	12	14	16	18	1A	1C	1E	20
3	3	6	9	C	F	12	15	18	1B	1E	21	24	27	2A	2D	30
4	4	8	C	10	14	18	1C	20	24	28	2C	30	34	38	3C	40
5	5	A	F	14	19	1E	23	28	2D	32	37	3C	41	46	4B	50
6	6	C	12	18	1E	24	2A	30	36	3C	42	48	4E	54	5A	60
7	7	E	15	1C	23	2A	31	38	3F	46	4D	54	5B	62	69	70
8	8	10	18	20	28	30	38	40	48	50	58	60	68	70	78	80
9	9	12	1B	24	2D	36	3F	48	51	5A	63	6C	75	7E	87	90
A	A	14	1E	28	32	3C	46	50	5A	64	6E	78	82	8C	96	A0
B	B	16	21	2C	37	42	4D	58	63	6E	79	84	8F	9A	A5	B0
C	C	18	24	30	3C	48	54	60	6C	78	84	90	9C	A8	B4	C0
D	D	1A	27	34	41	4E	5B	68	75	82	8F	9C	A9	B6	C3	D0
E	E	1C	2A	38	46	54	62	70	7E	8C	9A	A8	B6	C4	D2	E0
F	F	1E	2D	3C	4B	5A	69	78	87	96	A5	B4	C3	D2	E1	F0
10	10	20	30	40	50	60	70	80	90	A0	B0	C0	D0	E0	F0	100

