CSARCH Lecture Series: Number System Base Conversion

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Overview

Reflect on the following question:

- What is the octal equivalent of 0x829.A?

Overview

- This sub-module introduces how to perform base conversion between number systems
- The objectives are as follows:
  - ✓Describe the process of converting from decimal to other number system
  - ✓Describe the process of converting special cases involving binary, octal and hexadecimal
  - ✓Describe the process of converting from one number system to another number system

Base conversion from decimal to other number system

- How to convert a decimal number to the target number system:
- For numbers on the left side of the radix point (i.e., whole number), it is done by successively “reverse” dividing the whole decimal number by the target base.
- For numbers on the right side of the radix point (i.e., fraction number), it is done by successively multiplying the fraction decimal number by the target base.

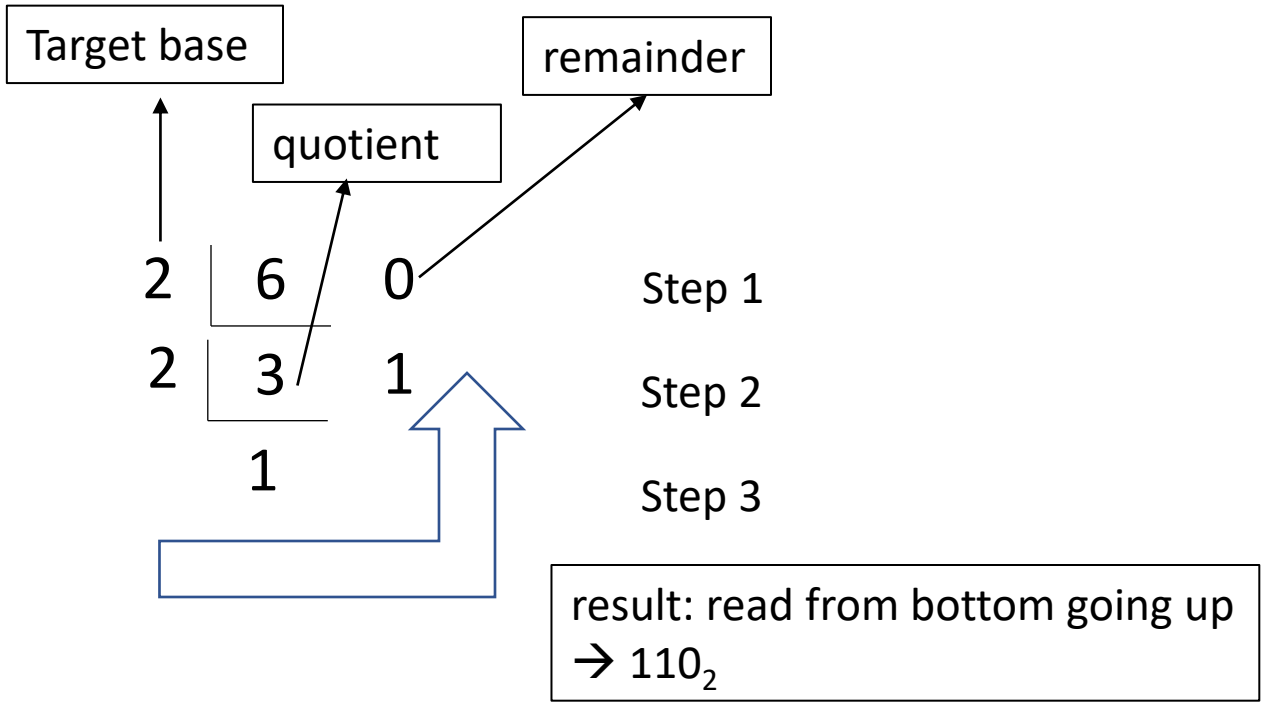
Base conversion from decimal to other number system

Example: Convert 6 to binary

1.) (Step 1) For whole number, “reverse divide” 6 with the target base (e.g., 2). Write the quotient below and remainder on the side

2.) (step 2) Repeat while the quotient is greater than the target base

3.) (Step 3) Read the answer bottom up



Base conversion from decimal to other number system

Convert 0.625 to binary

1.) (Step 1) for the fractional part, multiply it with the target base (e.g., 2). Copy (to the "answer") the digit on the left side of the radix point (0.1) then zero it out (0.250). Continue while the fractional part is not zero.

2.) (Step 2) After multiplying, copy the digit on the left side of the radix point even it is ZERO (0.10).

3.) (Step 3) After multiplying, the fractional part is now 0.000. Process end and the result is 0.101_2

4.) If it is non-terminating (fractional part never reaches 0), we can stop after 4 fractional places

```
  0.625
x   2
---------
0.1   1.250    Step 1
  0.250
x   2
---------
0.10  0.500    Step 2
  0.500
x   2
---------
0.101 1.000    Step 3
  0.000
```

Try

Convert 314 to hex

314 = ___$_{16}$

Try

Convert 0.625 to hexadecimal

0.625 = ____$_{16}$

For discussion ---

- NOTE that using reverse division can be a long process especially if the target base is small.
- Try converting 2052 to binary, how many reverse divisions do you have to perform?
- Are there other faster methods to convert decimal to other number system?

Base conversion between binary and octal

- Binary needs at least 3 digits to represent 1 octal digit.
- Even though, 1 binary digit is enough to represent octal digit 0, but you need 3 bits to represent octal digit 7.

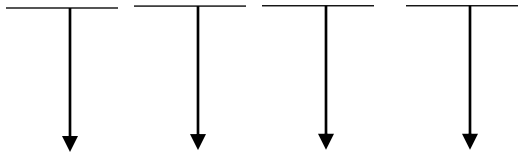
Octal	
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Base conversion between binary and octal

- Binary to octal

→ starts from binary point, group every 3 binary digits and convert to its equivalent octal digit

$011\ 101\ 001.111_2 = \underline{351.7}_8$



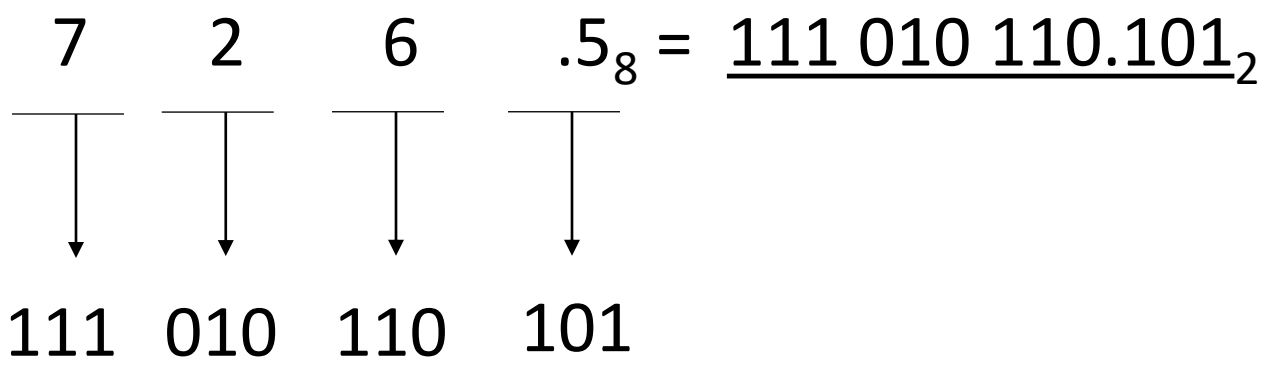
3 5 1 7

Octal	
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Base conversion between binary and octal

- Octal to binary

→ Convert each octal digit to 3 binary digits



Octal	
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Base conversion between binary and hexadecimal

- Binary needs at least 4 digits to represent 1 hex digit.
- Even though, 1 binary digit is enough to represent hex digit 0, but you need 4 bits to represent hex digit F.

Hexadecimal			
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Base conversion between binary and hexadecimal

- Binary to hexadecimal

→ starts from binary point, group every 4 binary digits and convert to its equivalent hexadecimal digit

$0111\ 0101\ 1001.1011_2 = \underline{759.B}_{16}$

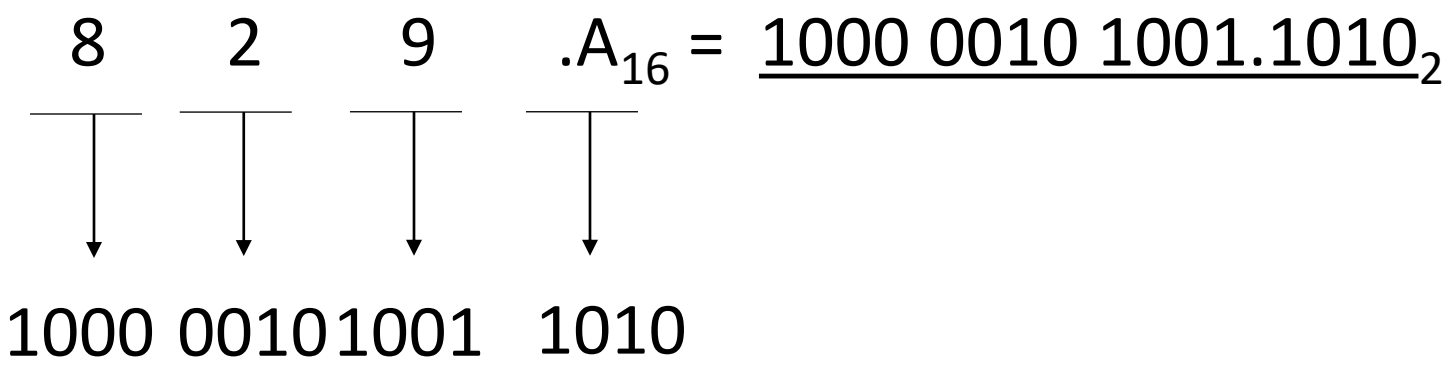
7 5 9 B

Hexadecimal			
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Base conversion between binary and hexadecimal

- Hexadecimal to binary

→ Convert each hex digit to 4 binary digits



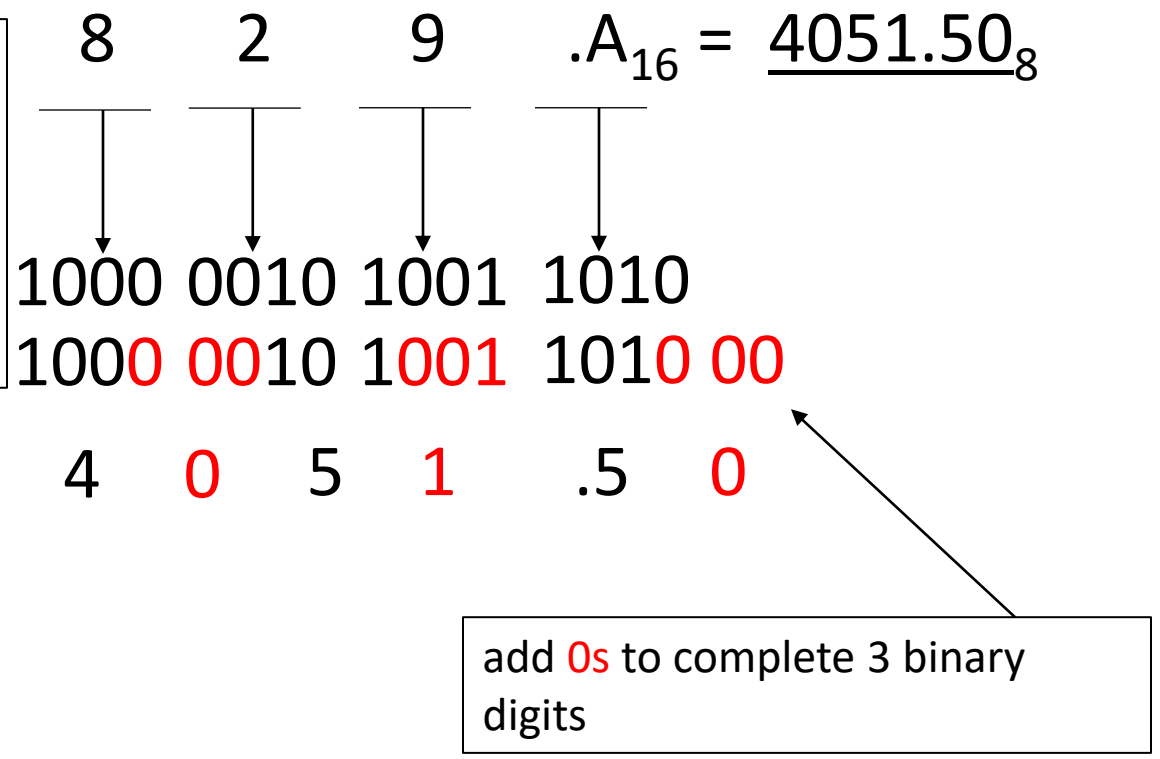
Hexadecimal			
0	0000	8	1000
1	0001	9	1001
2	0010	A	1010
3	0011	B	1011
4	0100	C	1100
5	0101	D	1101
6	0110	E	1110
7	0111	F	1111

Base conversion between hexadecimal and octal

- Hexadecimal to octal

Hexadecimal → binary → octal

*expand 4 binary digits → group by 3 binary digits

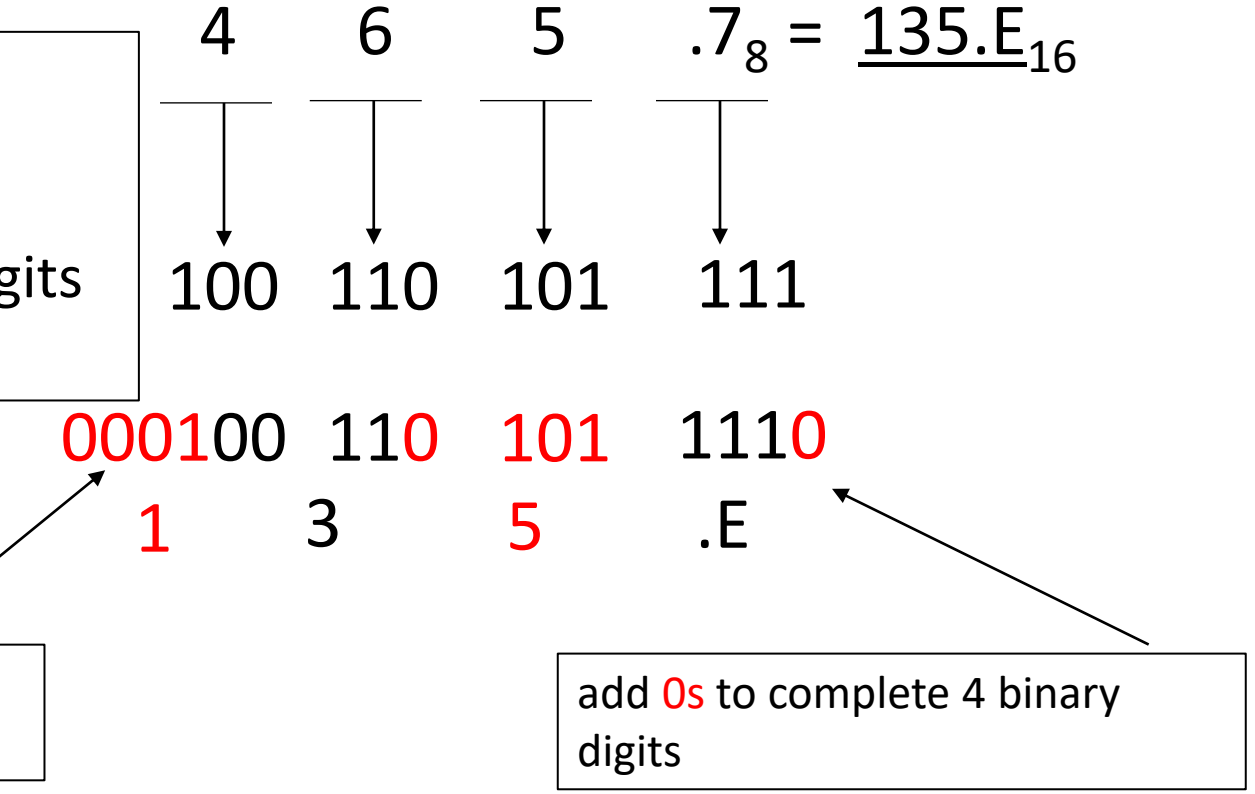


Base conversion between octal and hexadecimal

- Octal to hexadecimal

Octal → binary → hexadecimal

*expand 3 binary digits → group by 4 binary digits



Try

- Convert $CAF.E_{16}$ to octal

- $CAF.E_{16}$ = _____$_8$

Base conversion between numeral systems

- *x* numeral system to *y* numeral system (example: 32.2_5 to base-7)
- *x* numeral system → decimal → *y* numeral system
- $32.2_5 = 17.4_{10} = 23.2541_7$

32.2_5

$2*5^{-1} = 0.4$

$2*5^{0} = 2$

$3*5^{1} = 15$

summation: 17.4

7 | 17 3

2

```
    0.4
  x   7
--------
0.2 2.8

    0.8
  x   7
--------
0.25 5.6

    0.6
  x   7
--------
0.254 4.2

    0.2
  x   7
--------
0.2541 1.4
```

we stop at 4 fractional places

To recall…

- What we have learned:
  - ✓Describe the process of converting from decimal to other number system
  - ✓Describe the process of converting special cases involving binary, octal and hexadecimal
  - ✓Describe the process of converting from one number system to another number system