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**Exam Date:** March 12, 2025
**Academic Year and Term:** Term 2, A.Y. 2024 - 2025
**Total Points:** 50
**REMINDERS. READ BEFORE YOU START ANSWERING!** Daniel Gavrie Y. Clemente

1. This is an open notes exam. Only non-digital notes are allowed.
2. This exam is worth 70 points, where 50 points will be credited. Any excess score will be recorded as is under your Exam 2 grade component.
3. Show your solution as detailed as possible because partial credits will be given when appropriate. Clearly indicate the final answer for each item (you may box the final answer to indicate it when applicable).
4. When a question asks for a deterministic machine, any nondeterministic answer will receive **ZERO** points.
5. For state diagrams and transition tables, failure to indicate the start state will automatically result in a score of **ZERO** for that item. For items with a state limit, exceeding the state limit will also result in a score of **ZERO**.
6. After finishing the exam, insert the questionnaire inside the test booklet and submit both to the assigned proctor.
7. Cheating in any form is punishable with a grade of 0.0 for the course and a disciplinary offense. This includes but is not limited to passing notes during the exam or communicating with other people.

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1. Given the following three languages:

   - $L_1 = \{\omega \in \{0,1\}^* \mid \omega = (00^*1)^*\}$
   - $L_2 = \{\omega \in \{0,1\}^* \mid \omega = (01^*0 \cup 1)^*\}$
   - $L_3 = \{\omega \in \{0,1\}^* \mid \omega = 1^*0^*\}$

   represented by the following DFA's:

| | 0 | 1 |
|---|---|---|
| $\rightarrow A^*$ | B | C |
| $B$ | B | A |
| $C$ | C | C |

Table 1: DFA for $L_1$

| | 0 | 1 |
|---|---|---|
| $\rightarrow D^*$ | E | D |
| $E$ | D | E |

Table 2: DFA for $L_2$

| | 0 | 1 |
|---|---|---|
| $\rightarrow F^*$ | G | F |
| $G^*$ | G | H |
| $H$ | H | H |

Table 3: DFA for $L_3$

   Provide the transition table for the DFA that recognizes $\overline{(L_1 \cup L_2) - (L_3)^R}$. (10 points)

2. Using the pumping lemma for regular languages, prove that $L = \{\omega \in \{1\}^* \mid \omega = 1^n \wedge n$ is a prime number$\}$ is NOT regular. (10 points)

3. Juan Tamad created a function that separates words into morphemes (which are the smaller meaningful units of a word). Morphemes can be classified into root morphemes (morphemes at the core of the word) and affixes (morphemes attached to root morphemes). For example, in the English word unspeakable, un and able are affixes, while speak is the root morpheme. Juan Tamad's friend Tina Masipag used this function and created grammar $G$ for Japanese words.

   In this grammar, a root morpheme is followed by an affix, and both the affix and the root morpheme serve as terminal symbols. Their other friend Naka Zero, a researcher of Japanese and Spanish descent, created a function Backwards($X$) that swaps the position of root morphemes and affixes. The production rules for Backwards($G$) after the swap are defined below, where $\Sigma$ is a non- terminal symbol, $R_1$ and $R_2$ are root morphemes, and $A_1$ and $A_2$ are affixes:

   Backwards$(G) = (N, T, P, \Sigma)$
   $N = \{A_1, R_1, A_2, R_2\}$
   $T = \{$mu, no, yo, moriko, ru, tabe, ne, de$\}$
   $P$ :
   $\Sigma \rightarrow A_1R_1 \mid A_2R_2$
   $A_1 \rightarrow$ mu
   $R_1 \rightarrow$ no | yo | moriko
   $A_2 \rightarrow$ ru

$R_2 \rightarrow$ tabe | ne | de

For the items below, write TRUE if the string can be derived from the original grammar ($G$) that Tina Masipag created. Otherwise write FALSE. (2 points each)

(a) rutabe FALSE
(b) muno FALSE
(c) rune FALSE
(d) mude FALSE
(e) taberu TRUE

4. Given the language $L = \{\alpha 2^* \alpha^R \mid \alpha \in \{0,1\}^*\}$.

   (a) Design a Pushdown Automaton (PDA), $M$, that recognizes language $L$. You may use $\{e, f\}$ as stack symbols (in addition to the initial stack symbol $Z$ or #). Provide the **state diagram**. Follow the constraint $|Q| \leq 4$. (10 points)

   (b) Consider an accepter $J$ that includes the PDA from the previous sub-item and the Moore machine $Y$ below. The PDA accepts as input the output of Moore machine $Y$. The start state of Moore machine $Y$ is also the start state of accepter $J$. The final state of the PDA is also the final state of accepter $J$.

| | 0 | 1 |
|---|---|---|
| $\rightarrow A:0$ | A | B |
| $B:1$ | C | A |
| $C:2$ | B | C |

Table 4: Transition Table for Moore Machine $Y$

To formally state the behavior of this *cascade machine*:
Let $Y : \{0,1\}^* \mapsto \{0,1,2\}^*$ be a function that is the transduction of a string $\omega$ using Moore machine $Y$. This means that $Y(\omega) = \phi$ if and only if when $\omega$ is the input of Moore Machine $Y$, the output of the Moore machine is $\phi$.
$\phi$ is then fed as input to the accepter you built in the previous item $M$. The cascading operation of these two machines forms the accepter $J$. The language recognized by $J$ is:

$$L(J) = \{\omega \in \{0,1\} \mid Y(\omega) \in L(M)\}$$

or alternatively:

$$L(J) = \{\omega \in \{0,1\} \mid M \text{ accepts } Y(\omega)\}$$

Determine if these inputs will be accepted by machine $J$. Write TRUE if it is accepted; otherwise write FALSE. (2 points each)

i. 10101 TRUE
ii. 11111 FALSE
iii. 01010 FALSE
iv. 00000 TRUE
v. 10001 FALSE

5. Prove that the language $L = \{\omega \in \{a,b,c\}^* \mid a^i b^j c^k, i,j,k \geq 0 \wedge i \geq j+k\}$ is context-free by:

   (a) Creating a Push-Down Automaton (PDA) that recognizes $L$ (Provide the **state diagram**. Follow the constraint $|Q| \leq 4$) (10 points), and

   (b) Designing the Context-Free Grammar (CFG) that generates $L$ (Follow the constraints $|N| \leq 3 \wedge |P| \leq 10$). (10 points)

- Accept: *aac* , *aaabbc* , *aaaaabccc* , *aaaaaabbcc*
- Reject: *abc* , *bbc* , *abba*, *aaaabbbccccc*

**- - END OF EXAM - -**