INTRODUCTION TO NUMBER SYSTEMS

CCICOMP

OUTLINE

- Number Systems
- Base Conversions

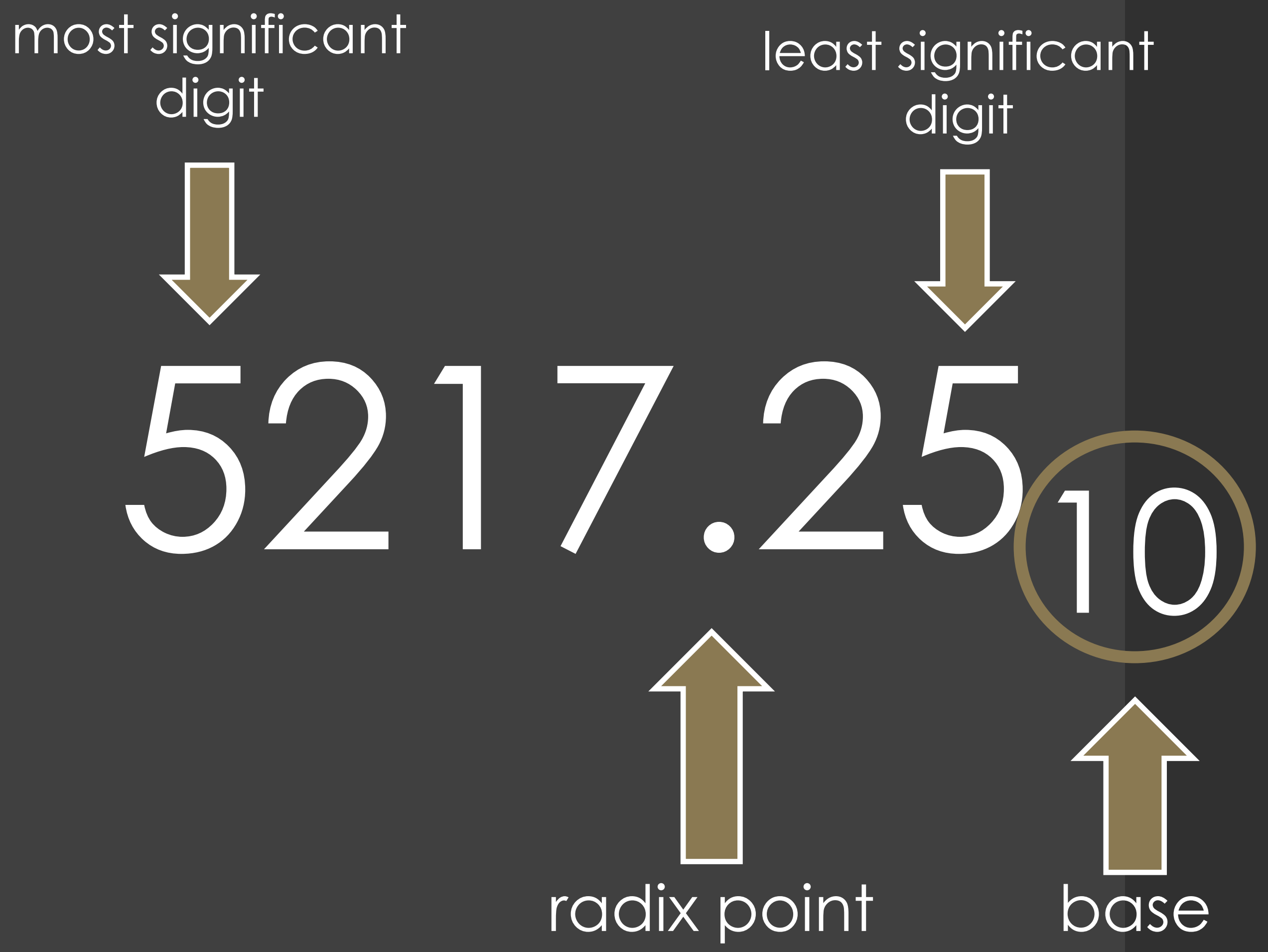
RECALL:

- Computers process data into information
- Computers work exclusively with numbers. All data is represented in binary form
- Binary represents data in combinations of 1’s and 0’s
- But, what exactly is ‘binary’??

‘Binary’ is a **number system**

WHAT IS A NUMBER SYSTEM?

- A way of representing numerical values
- Has its own arithmetic method
- Any arbitrary numerical value can be represented by a fixed set of symbols consisting of ***r*** elements (also known as **radix** or **base**).
- The largest-valued symbol always has a magnitude of one less than the radix.
- Each variable that denotes a digit is assigned a weight dependent on its position relative to a radix point.



DIFFERENT NUMBER SYSTEMS

Radix/Base	Number system	Radix/Base	Number System
1	Unary	11	Undecimal
2	Binary	12	Duodecimal
3	Ternary	13	Tridecimal
4	Quatenary	14	Tetradecimal
5	Quinary	15	Pentadecimal
6	Senary	16	Hexadecimal
7	Septenary	18	Octodecimal
8	Octal	20	Vigesimal
9	Nonary	24	Tetravigesimal
10	Decimal	25	Pentavigesimal

DECIMAL NUMBER SYSTEM

- **Radix/Base:** 10
- **Symbols:** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- **Written as:** 25, 25d or 25_{10}
- Each digit is weighted by powers of 10

Example:

5217.25

$5 \times 10^{-2} = 5 / 10 / 10$

$2 \times 10^{-1} = 2 / 10$

$7 \times 10^{0} = 7$

$1 \times 10^{1} = 1 \times 10$

$2 \times 10^{2} = 2 \times 10 \times 10$

$5 \times 10^{3} = 5 \times 10 \times 10 \times 10$

BINARY NUMBER SYSTEM

- **Radix/Base:** 2
- **Symbols:** 0, 1
- **Written as:** 1011b or 1011_2
- Each digit is weighted by powers of 2

$$1011_2$$

$1 \times 2^0 = 1$

$1 \times 2^1 = 1 \times 2$

$0 \times 2^2 = 0 \times 2 \times 2$

$1 \times 2^3 = 1 \times 2 \times 2 \times 2$

OCTAL NUMBER SYSTEM

- **Radix/Base:** 8
- **Symbols:** 0, 1, 2, 3, 4, 5, 6, 7
- **Written as:** 327o or 327_8
- Each digit is weighted by powers of 8

$$327_8$$

$7 \times 8^0 = 7$

$2 \times 8^1 = 2 \times 8$

$3 \times 8^2 = 3 \times 8 \times 8$

HEXADECIMAL NUMBER SYSTEM

- **Radix/Base:** 16
- **Symbols:** 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A (10), B (11), C (12), D (13), E (14), F (15)
- **Written as:** 5C8h or $5C8_{16}$
- Each digit is weighted by powers of 16

$5C8_{16}$

8 $\times 16^0 = 16$

12 $\times 16^1 = 12 \times 16$

5 $\times 16^2 = 5 \times 16 \times 16$

BASE CONVERSION

- Binary ↔ Decimal
- Binary ↔ Octal
- Binary ↔ Hexadecimal
- Decimal ↔ Octal
- Decimal ↔ Hexadecimal

BINARY → DECIMAL CONVERSION

- Each digit appearing to the left of the binary point represents a value of either zero or one times an increasing power of two.

EXAMPLE 1:

$101_2 = ?_{10}$

1	0	1
$1\text{x}2^2$	$0\text{x}2^1$	$1\text{x}2^0$

$= 1\text{x}2^2 + 0\text{x}2^1 + 1\text{x}2^0$

$= 4 + 0 + 1$

$= \mathbf{5_{10}}$

BINARY → DECIMAL CONVERSION

- Each digit appearing to the left of the binary point represents a value of either zero or one times an increasing power of two.

EXAMPLE 2:

$101010_2 = ?_{10}$

1	0	1	0	1	0
$1x2^5$	$0x2^4$	$1x2^3$	$0x2^2$	$1x2^1$	$0x2^0$

$=1x2^5 + 0x2^4 + 1x2^3 + 0x2^2 + 1x2^1 + 0x2^0$

$=32 + 0 + 8 + 0 + 2 + 0$

$=\mathbf{42_{10}}$

DECIMAL → BINARY CONVERSION

- Converting a decimal number to a binary number is done by successively dividing the decimal number by 2 until a 0 is obtained
- Answer consists of the remainders read bottom up

EXAMPLE 1:

$6_{10} = ?_2$

		R
2		0
2		1
2		1
	0	

= $\mathbf{110_2}$

EXAMPLE 2:

$23_{10} = ?_2$

		R
2		1
2		1
2		1
2		0
2		1
	0	

= $\mathbf{10111_2}$

BINARY → OCTAL CONVERSION

- One octal digit is equivalent to three binary digits.
- Group the bits by three starting from the least significant bit.
- If the grouping does not have enough to form 3 bits then append 0 to the most significant bit.

EXAMPLE 1:

$1101_2 = ?_8$

0	0	1	1	0	1
$0x2^2 + 0x2^1 + 1x2^0$			$1x2^2 + 0x2^1 + 1x2^0$		

$=0x2^2 + 0x2^1 + 1x2^0 \mid 1x2^2 + 0x2^1 + 1x2^0$

$=0 \quad + 0 \quad + 1 \quad \mid 4 \quad + 0 \quad + 1$

$=\mathbf{15_8}$

BINARY → OCTAL CONVERSION

- One octal digit is equivalent to three binary digits.
- Group the bits by three starting from the least significant bit (rightmost).
- If the grouping does not have enough to form 3 bits then append 0 to the most significant bit.

EXAMPLE 2:

$1101010_2 = ?_8$

0	0	1	1	0	1	0	1	0
$0x2^2 + 0x2^1 + 1x2^0$			$1x2^2 + 0x2^1 + 1x2^0$			$0x2^2 + 1x2^1 + 0x2^0$		

$=0x2^2 + 0x2^1 + 1x2^0 \mid 1x2^2 + 0x2^1 + 1x2^0 \mid 0x2^2 + 1x2^1 + 0x2^0$

$=0 \quad + 0 \quad + 1 \quad \mid 4 \quad + 0 \quad + 1 \quad \mid 0 \quad + 2 \quad + 0$

$=\mathbf{152_8}$

OCTAL → BINARY CONVERSION

- Since one octal digit is equivalent to three binary digits, just convert the individual octal digit into three binary digits.
- Pad with 0s if the binary equivalent of a hex digit is less than 3 digits.

EXAMPLE 1:

$57_8 = ?_2$

5			7		
1	0	1	1	1	1

$= \mathbf{101111_2}$

EXAMPLE 2:

$326_8 = ?_2$

3			2			6		
0	1	1	0	1	0	1	1	0

$= \mathbf{11010110_2}$

BINARY → HEXADECIMAL CONVERSION

- One hexadecimal digit is equivalent to four binary digits.
- Group the bits by four starting from the least significant bit.
- If the grouping does not have enough to form 4 bits then append 0 to the most significant bit.

EXAMPLE 1:

$1101_2 = ?_{16}$

1	1	0	1
$1x2^3$ +	$1x2^2$ +	$0x2^1$ +	$1x2^0$

$=1x2^3 + 1x2^2 + 0x2^1 + 1x2^0$

$=8 \quad +4 \quad + 0 \quad + 1$

$=\mathbf{D_{16}}$ *RECALL: $13_{10} = D_{16}$

BINARY → HEXADECIMAL CONVERSION

- One hexadecimal digit is equivalent to four binary digits.
- Group the bits by four starting from the least significant bit (rightmost).
- If the grouping does not have enough to form 4 bits then append 0 to the most significant bit.

EXAMPLE 2:

$1101010_2 = ?_{16}$

0	1	1	0	1	0	1	0
$0x2^3$	$+1x2^2$	$+ 1x2^1$	$+0x2^0$	$1x2^3$	$+0x2^2$	$+ 1x2^1$	$+ 0x2^0$

$= 0x2^3 + 1x2^2 + 1x2^1 + 0x2^0 \mid 1x2^3 + 0x2^2 + 1x2^1 + 0x2^0$

$= 0 \quad +4 \quad +2 \quad +0 \quad \mid 8 \quad +0 \quad +2 \quad +0$

$= \mathbf{6A_{16}}$

HEXADECIMAL → BINARY CONVERSION

- Since one hexadecimal digit is equivalent to four binary digits, just convert the individual hexadecimal digit into four binary digits
- Pad with 0s if the binary equivalent of a hex digit is less than 4 digits.

EXAMPLE 1:

$BAE_{16} = ?_2$

B				A				E			
1	0	1	1	1	0	1	0	1	1	1	0

= **101110101110_2**

EXAMPLE 2:

$91D_{16} = ?_2$

9				1				D			
1	0	0	1	0	0	0	1	1	1	0	1

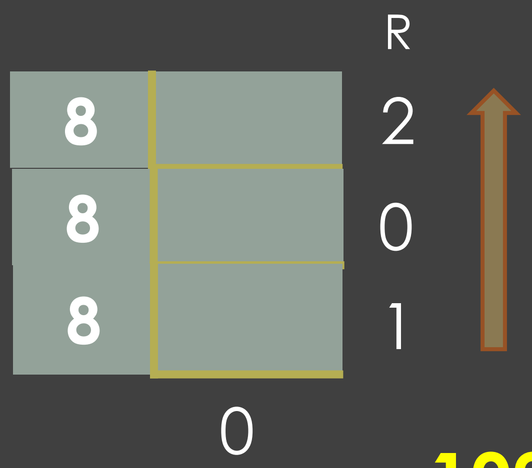
= **100100011101_2**

DECIMAL → OCTAL CONVERSION

- Converting a decimal number to an octal number is done by successively dividing the decimal number by 8
- Answer consists of the remainders read bottom up

EXAMPLE 1:

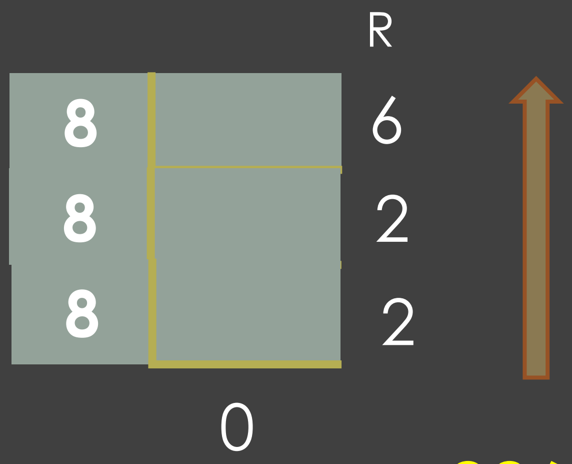
$66_{10} = ?_8$



$= 102_8$

EXAMPLE 2:

$150_{10} = ?_8$



$= 226_8$

OCTAL → DECIMAL CONVERSION

- Each digit appearing to the left of the radix point represents a value between zero to seven (inclusive) times an increasing power of eight.

EXAMPLE 1:

$27_8 = ?_{10}$

2	7
$2x8^1$	$7x8^0$

$$= 2x8^1 + 7x8^0$$
$$= 16 + 7$$
$$= \mathbf{23_{10}}$$

EXAMPLE 2:

$175_8 = ?_{10}$

1	7	5
$1x8^2$	$7x8^1$	$5x8^0$

$$= 1x8^2 + 7x8^1 + 5x8^0$$
$$= 64 + 56 + 5$$
$$= \mathbf{125_{10}}$$

DECIMAL → HEXADECIMAL CONVERSION

- Converting a decimal number to an octal number is done by successively dividing the decimal number by 16
- Answer consists of the remainders read bottom up

EXAMPLE 1:

$286_{10} = ?_{16}$

		R
16		14 (E)
16		1
16		1
	0	

$= \mathbf{11E_{16}}$

EXAMPLE 2:

$351_{10} = ?_{16}$

		R
16		15 (F)
16		5
16		1
	0	

$= \mathbf{15F_{16}}$

HEXADECIMAL → DECIMAL CONVERSION

- Each digit appearing to the left of the radix point represents a value between zero to 15 (inclusive) times an increasing power of 16.

EXAMPLE 1:

$2A_{16} = ?_{10}$

2	A
2×16^1	10×16^0

$= 2 \times 16^1 + 10 \times 16^0$

$= 32 + 10$

$= \mathbf{42_{10}}$

EXAMPLE 2:

$305_{16} = ?_{10}$

3	0	5
3×16^2	0×16^1	5×16^0

$= 3 \times 16^2 + 0 \times 16^1 + 5 \times 16^0$

$= 768 + 0 + 5$

$= \mathbf{773_{10}}$

SUMMARY

- A number system is a method of representing numerical values wherein the base/radix determines the number of symbols used to represent digits and each digit is weighted based on its position relative to the radix point

- Although binary (base 2) is the native language of computers, humans commonly also use decimal (base 10), octal (base 8) and hexadecimal (base 16) to represent values for computer data

SUMMARY

From	To	
Binary	Decimal	Multiply each binary digit by increasing powers of 2 then add
Octal	Decimal	Multiply each octal digit by increasing powers of 8 then add
Hexadecimal	Decimal	Multiply each hex digit by increasing powers of 16 then add
Decimal	Binary	Continuous division by 2, remainders will be binary digits
Decimal	Octal	Continuous division by 8, remainders will be octal digits
Decimal	Hexadecimal	Continuous division by 16, remainders will be hex digits
Binary	Octal	Group by 3 digits from the right, convert to corresponding octal digit
Binary	Hexadecimal	Group by 4 digits from the right, convert to corresponding hex digit
Octal	Binary	Convert each octal digit to corresponding 3-digit binary value
Hexadecimal	Binary	Convert each hex digit to corresponding 4-digit binary value

Short Quiz: Do the following conversion (2 pts each)

Binary	Decimal	Octal	Hexadecimal
10001011			
	58		
		26	
			D9