CSARCH Lecture Series: Single-precision floating-point format for decimal (decimal-32)

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Overview

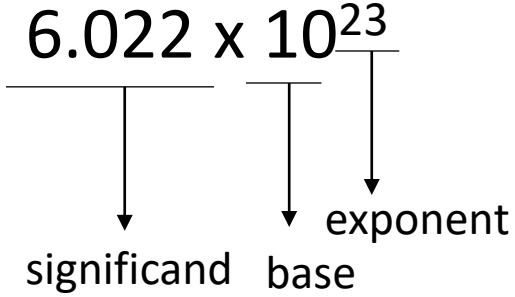
- This sub-module introduces the IEEE-754 decimal-32 floating-point format
- The objective is as follows:
  - ✓Describe the process of representing decimal-32 floating-point data using IEEE-754 standard

Floating Point

- Scientists and engineers use scientific notation where a number is expressed as

$$+/- S \times 10^{\pm E}$$

- Where S is the significand (also known as mantissa), E is the exponent and 10 is the base.



Floating Point

- Floating point standard for floating-point numbers in computer is the IEEE-754 (Institute of Electrical and Electronics Engineers Standard 754). Originally 1985, revised 2008, current version 2019
- Decimal floating point is introduced in 2008.
- The representation is to be used in applications that need to emulate decimal rounding exactly (i.e., financial and tax computations)

IEEE-754 decimal floating-point

- Decimal32 precision
- Decimal64 precision
- Decimal128 precision

Length of field

Format	Decimal32	Decimal64	Decimal128
Format length	32	64	128
Sign bit	1	1	1
Combination bit	5	5	5
Exponent continuation bit	6	8	12
coefficient continuation bit	20	50	110
Total coefficient in digits	7	16	34
E_{max} (denormalized/normalized)	96/90	384/369	6144/6111
E_{min} (denornamlized/normalized)	-95/-101	-383/-398	-6143/-6176
Bias	101	398	6176
E_{limit}	191	767	12287

IEEE-754 decimal-32 floating-point format

Sign	Combination field	Exponent continuation	Coefficient continuation
1	5	6	20

normalized to this format before representation:

$ddddddd \times 10^{e}$

- IEEE-754 decimal32 floating-point format is 32-bit in width
- The 32-bit is partition as 1 bit for sign bit; 5 bits for combination field, 6 bits for exponent continuation and 20 bits for coefficient continuation
  - Significand in decimal
  - Base 10
  - sign bit: 0→ positive; 1→ negative
  - e’ = e+101
  - significand normalized to 7 whole decimal digits

Combination field

Combination Field	Type	Exp MSbs	coefficient MSD
a b c d e	Finite	a b	0 c d e
1 1 c d e	Finite	c d	1 0 0 e
1 1 1 1 0	Infinity	- -	- - - -
1 1 1 1 1	NaN	- -	- - - -

- 5-bit combination field is composed of:
  - two most significant bits of the exponent representation (valid bits: 00, 01 and 10 only)
  - 1 or 3 bits of the most significant digit of the significand

Exponent continuation field

- Exponent representation is e+101
  - two most significant bits of the exponent representation (valid bits: 00, 01 and 10 only) in the combination field
  - The rest of the 6 bits in the exponent continuation field
  - Largest exponent (normalized) value that can be represented is 90
  - smallest exponent (normalized) value that can be represented is -101

Coefficient continuation field

- 7 whole decimal digits
- most significant digit is stored in the combination field
- remaining 6 digits are represented as densely-packed BCD and stored in the coefficient continuation field

Redundant encoding

- 7.5
  - 75×10^{-1}
  - 750×10^{-2}
  - 7500×10^{-3}
  - 75000×10^{-4}
  - 750000×10^{-5}
  - 7500000×10^{-6}

All are the representation of 7.5 (they are all called “cohorts”)

Example

- 7123456×10^{20}

Combination Field	Type	Exp MSbs	coefficient MSD
a b c d e	Finite	a b	0 c d e
1 1 c d e	Finite	c d	1 0 0 e
1 1 1 1 0	Infinity	- -	- - - -
1 1 1 1 1	NaN	- -	- - - -

Significand in decimal?	yes
Base-10?	yes
Normalized?	Yes, 7 whole digits
	MSD = 7 (0111)
Sign bit	0 (+)
e' = e+101	20+101 = 121 (01 111001)

Sign	Combination field	Exponent continuation	Coefficient continuation
0	01 111	111001	0010100011 1001010110

Example

- -8765432×10^{-20}

Combination Field	Type	Exp MSbs	coefficient MSD
a b c d e	Finite	a b	0 c d e
1 1 c d e	Finite	c d	1 0 0 e
1 1 1 1 0	Infinity	- -	- - - -
1 1 1 1 1	NaN	- -	- - - -

Significand in decimal?	yes
Base-10?	yes
Normalized?	Yes, 7 whole digits
	MSD = 8 (1000)
Sign bit	1 (-)
e’ = e+101	-20+101 = 81 (01 010001)

Sign	Combination field	Exponent continuation	Coefficient continuation
1	11 010	010001	1111100101 1000110010

Try

- -1.234567×10^{15}

Combination Field	Type	Exp MSbs	coefficient MSD
a b c d e	Finite	a b	0 c d e
1 1 c d e	Finite	c d	1 0 0 e
1 1 1 1 0	Infinity	- -	- - - -
1 1 1 1 1	NaN	- -	- - - -

Significand in decimal?	
Base-10?	
Normalized?	
Sign bit	
e’ = e+101	

Sign	Combination field	Exponent continuation	Coefficient continuation

Try

- -1.234567×10^{15}

Combination Field	Type	Exp MSbs	coefficient MSD
a b c d e	Finite	a b	0 c d e
1 1 c d e	Finite	c d	1 0 0 e
1 1 1 1 0	Infinity	- -	- - - -
1 1 1 1 1	NaN	- -	- - - -

Significand in decimal?	yes
Base-10?	yes
Normalized?	No, -1234567×10^9
	MSD = 1 (0001)
Sign bit	1 (-)
e' = e+101	9+101 = 110 (01 101110)

Sign	Combination field	Exponent continuation	Coefficient continuation
1	01 001	101110	0100110100 1011100111

To recall …

- What have we learned:
  - ✓Describe the process of representing decimal-32 floating-point data using IEEE-754 standard