UNIFORM COST SEARCH

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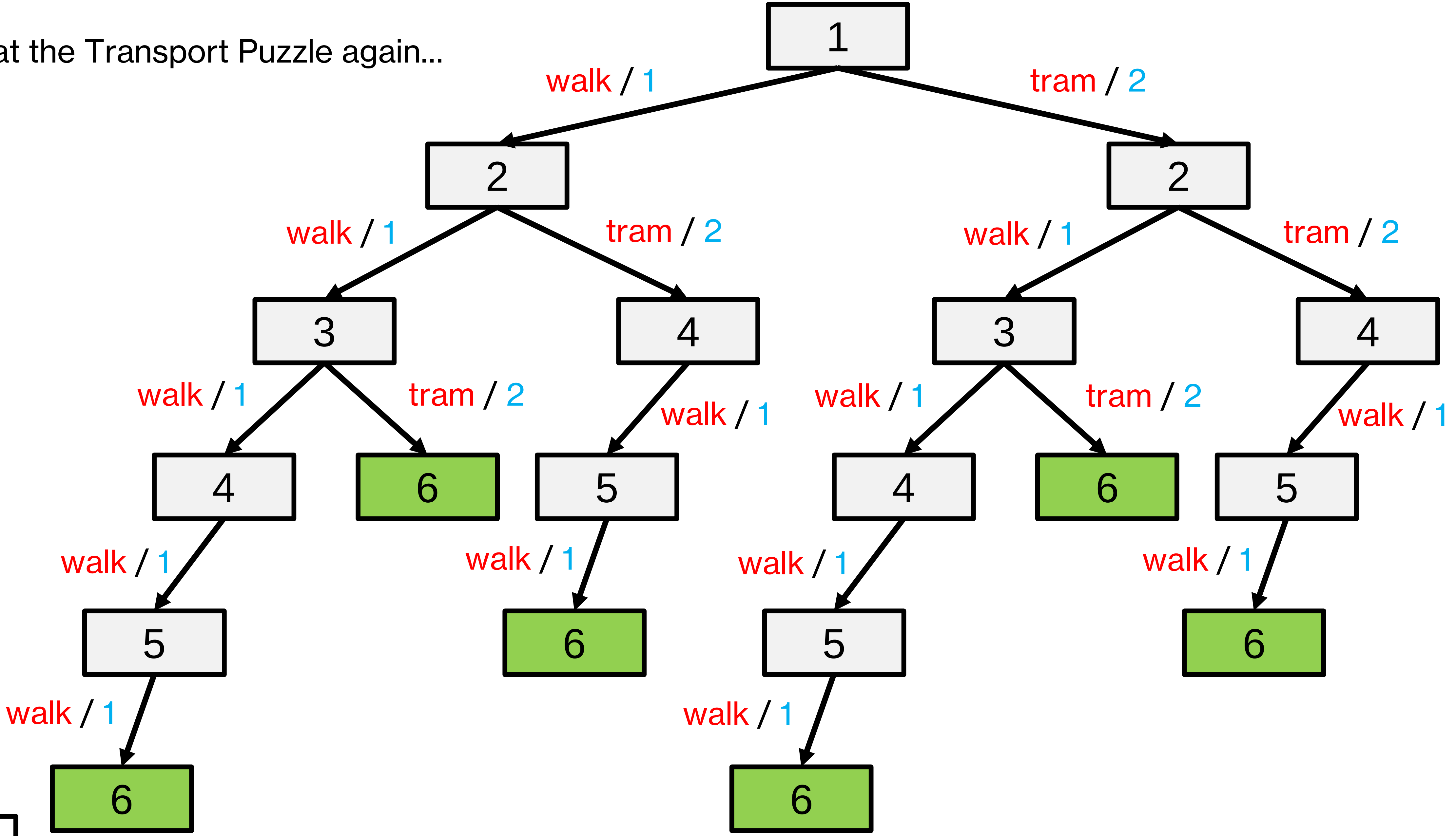
College of Computer Studies

Let's look at the Transport Puzzle again...

$n = 6$



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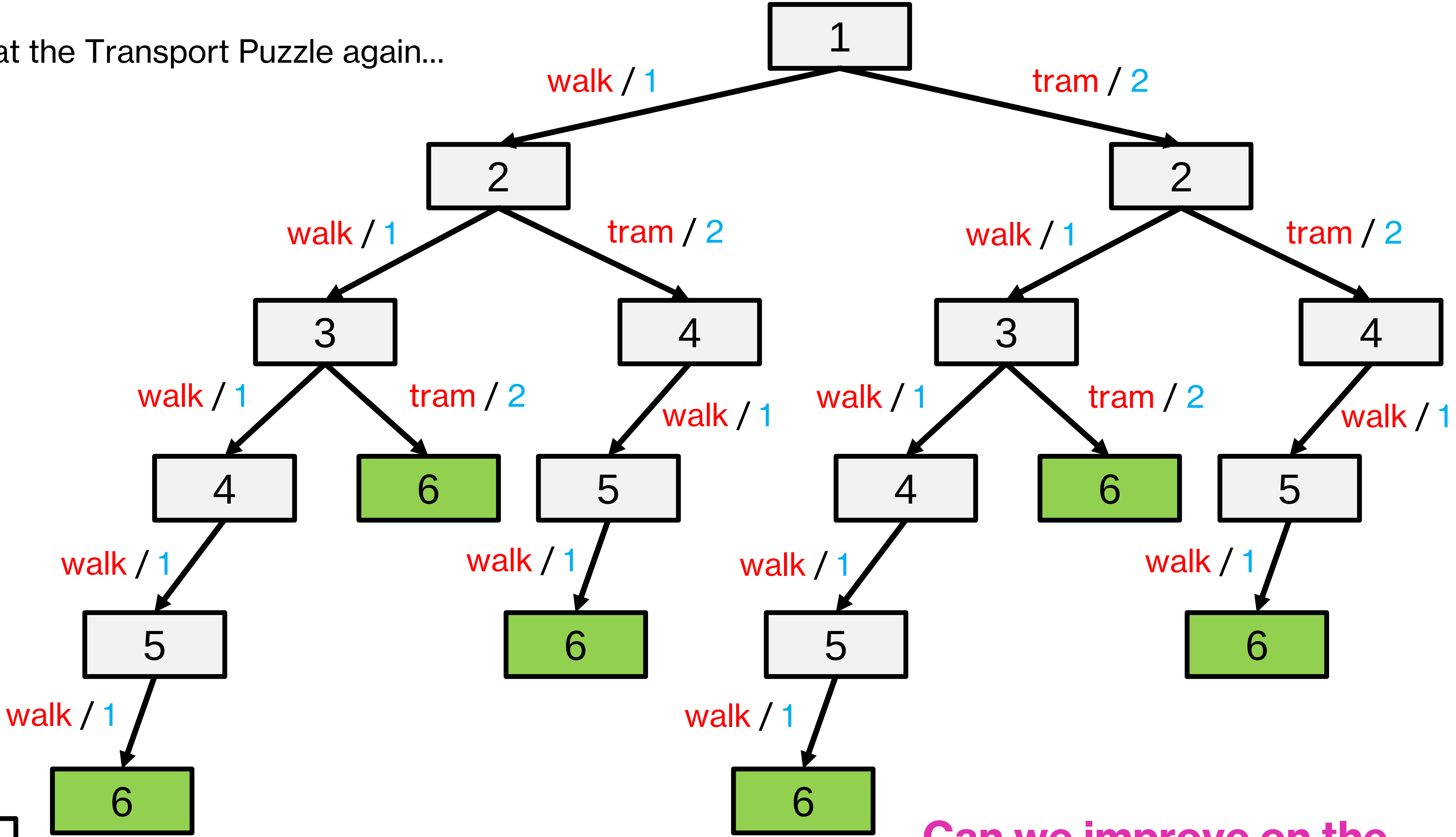

Action
Cost

Can we improve on the backtracking solution?

Let's look at the Transport Puzzle again…

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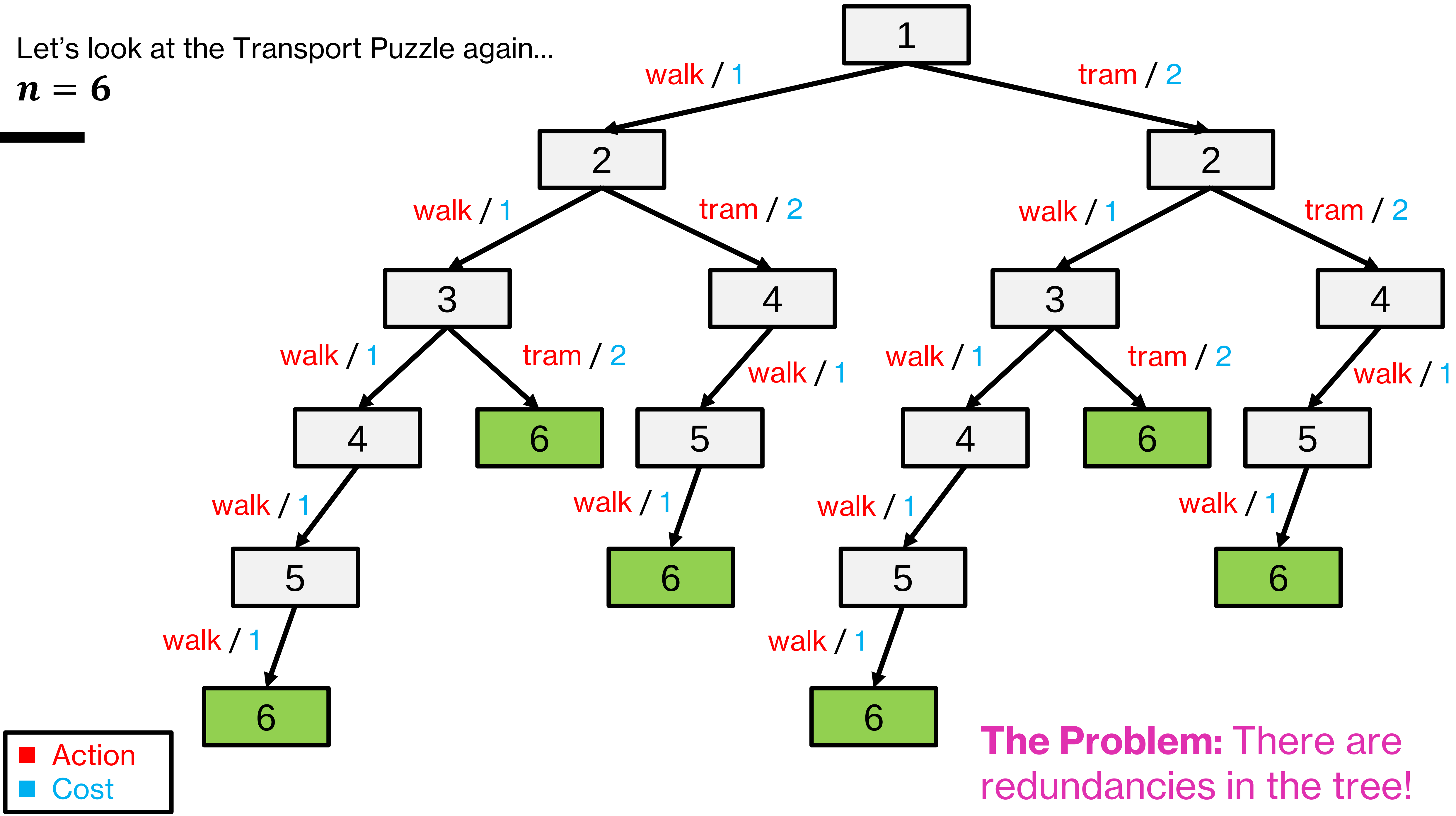


The Problem: There are redundancies in the tree!

Dynamic Programming

- Programming paradigm in which **solutions to subproblems are stored so they can easily be retrieved later**.
- Goal: reduce the number of states that must be explored (to reduce the execution time)

$$MinCost(s) = \begin{cases} 0 & IsEnd(s) \\ \min\limits_{a \in Actions} Cost(s,a) + MinCost(Succ(s,a)) & otherwise \end{cases}$$

Dynamic Programming in the Transport Puzzle ($n = 6$)

MinCost cache

1	2	3	4	5	6

Dynamic Programming in the Transport Puzzle ($n = 6$)

MinCost cache

1	2	3	4	5	6
					0

- If you are already at the goal, then the minimum cost from the current state to the goal is 0 (base case).

Dynamic Programming in the Transport Puzzle ($n = 6$)

MinCost cache

1	2	3	4	5	6
				1	0

- MinCost(5) = 1 + MinCost(6)
  = 1 + 0
  = 1

From block 5, no choice but to walk.

Dynamic Programming in the Transport Puzzle ($n = 6$)

MinCost cache

1	2	3	4	5	6
			2	1	0

- MinCost(4) = 1 + MinCost(5)

  = 1 + 1

  = 2

From block 4, no choice but to walk.

Dynamic Programming in the Transport Puzzle ($n = 6$)

MinCost cache

1	2	3	4	5	6
		2	2	1	0

- MinCost(3) = min(1 + MinCost(4), 2 + MinCost(6))
  = min(1 + 2, 2 + 0)
  = min(3, 2) = 2

From block 3, it's better to take the tram

Dynamic Programming in the Transport Puzzle ($n = 6$)

MinCost cache

1	2	3	4	5	6
	3	2	2	1	0

- MinCost(2) = min(1 + MinCost(3), 2 + MinCost(4))
  = min(1 + 2, 2 + 2)
  = min(3, 4) = 3

From block 2, it's better to walk

Dynamic Programming in the Transport Puzzle ($n = 6$)

MinCost cache

1	2	3	4	5	6
4	3	2	2	1	0

- MinCost(1) = min(1 + MinCost(2), 2 + MinCost(2))

  = min(1 + 3, 2 + 3)

  = min(4, 5) = 4

From block 1, it's better to walk

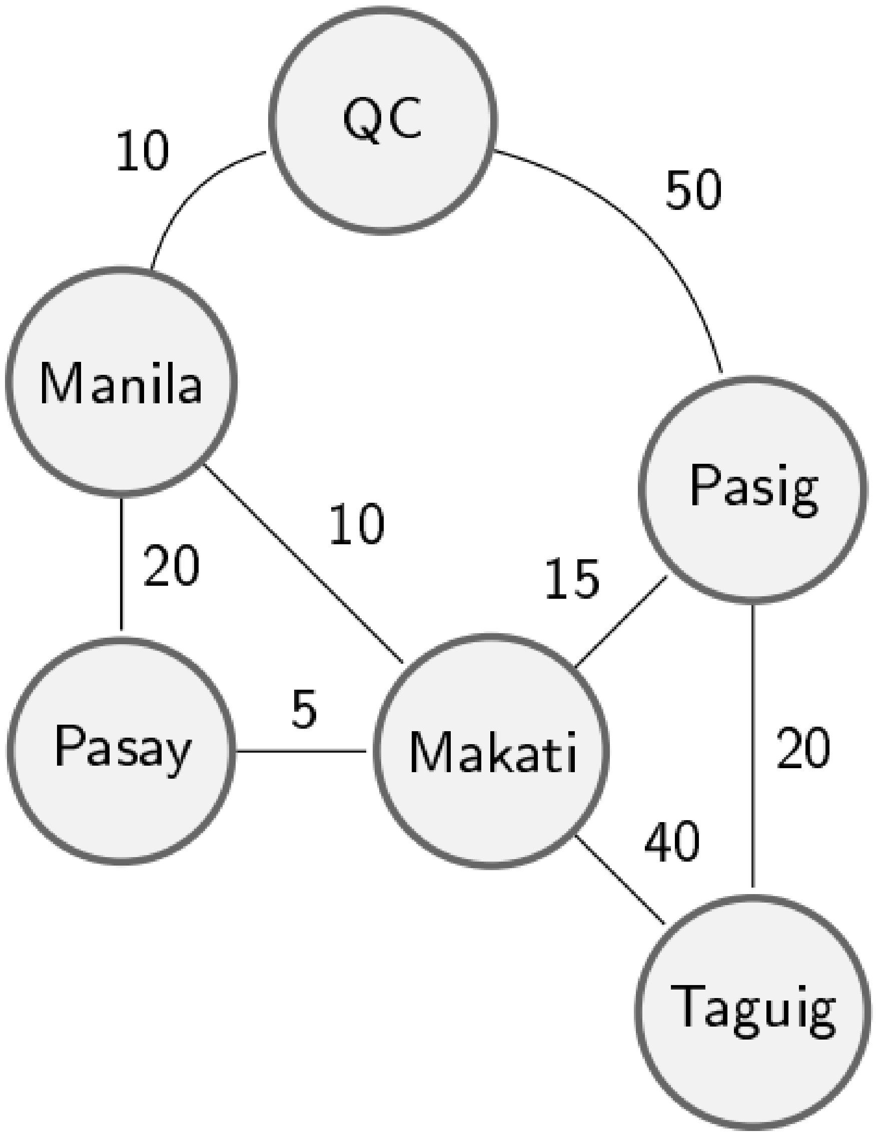
Performance Gains

Total number of states to explore (Transport Puzzle)

n	Backtracking	Dynamic Programming
6	19	6
10	59	10
50	9827	50
100	205657	100
1000	7389571	1000

Routing Problem

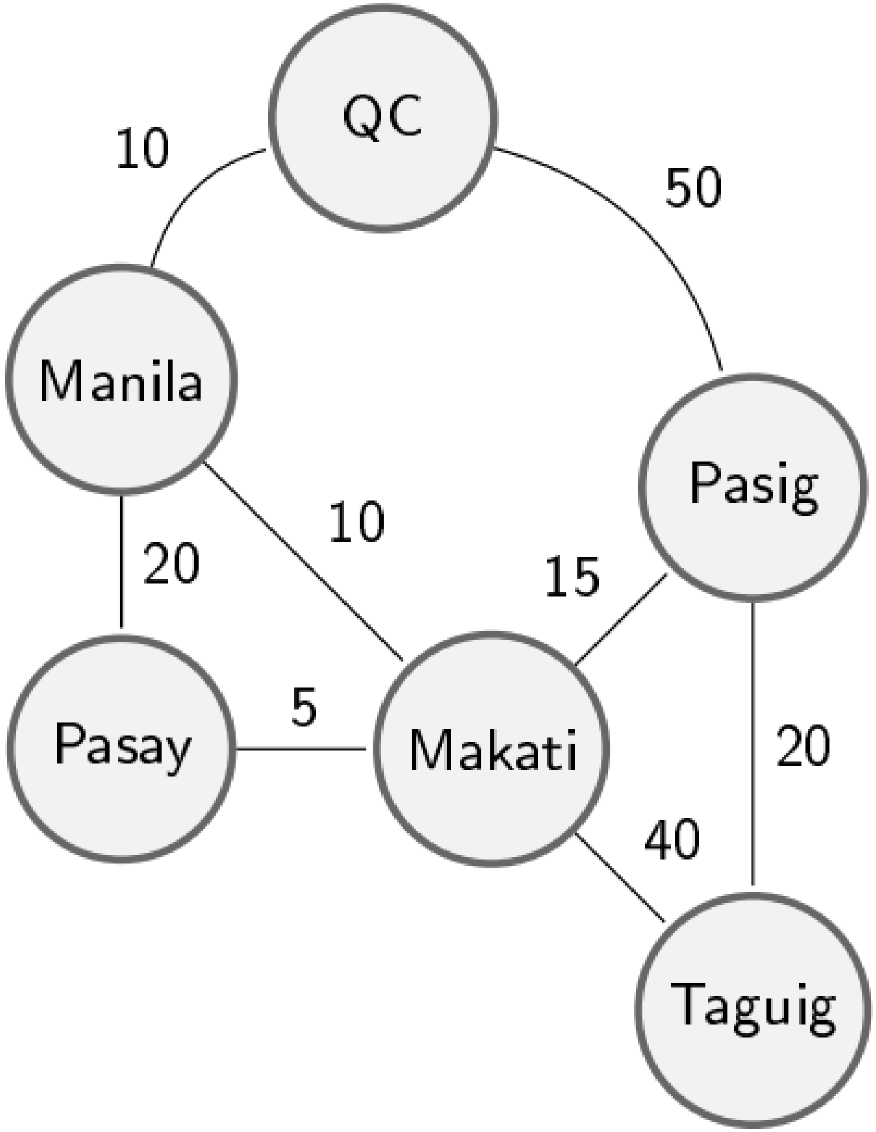
- Start at QC
- Want to travel to Taguig
- Want to take the path with the least traffic
- The weights of the edges in the graph represent the amount of traffic between the cities
- **Can dynamic programming handle this problem?**



Routing Problem

- Dynamic programming cannot handle this because there are cycles in the state transitions!

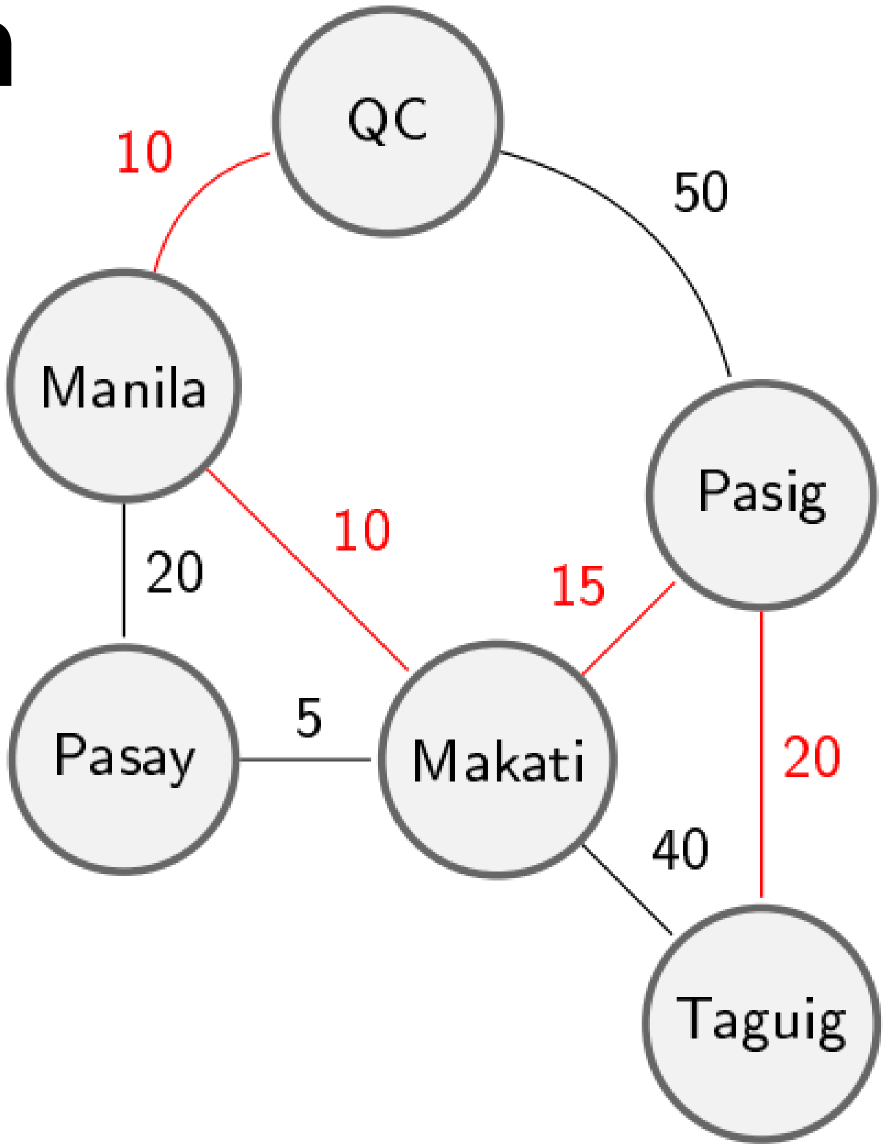
- *(from Pasay you can go to Makati, but from Makati you can also go to Pasay, and either one may come first)*



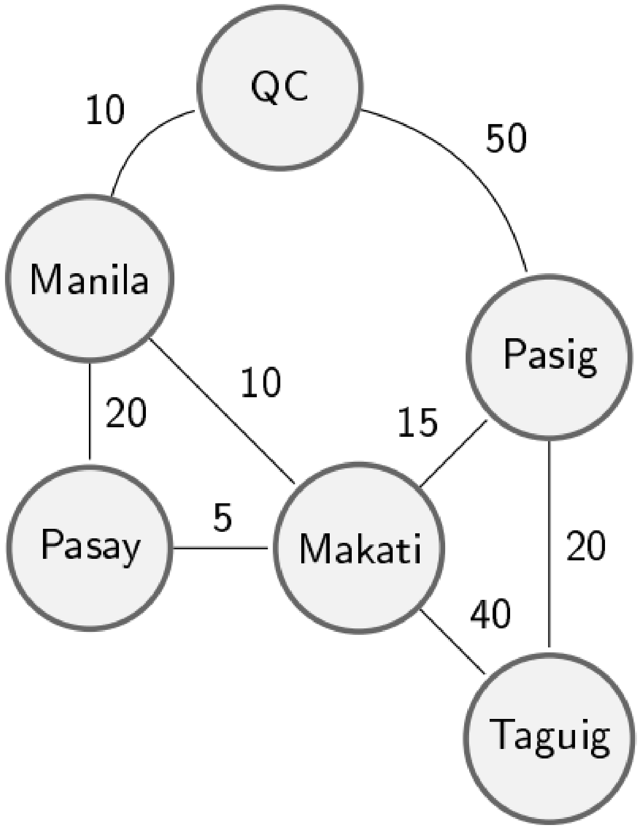
Uniform Cost Search

- **Observation:** Prefixes of the optimal path are also optimal.

- Key Idea: maintain a frontier list, and **always explore the state with the lowest cost from the start state**.



Uniform Cost Search Example

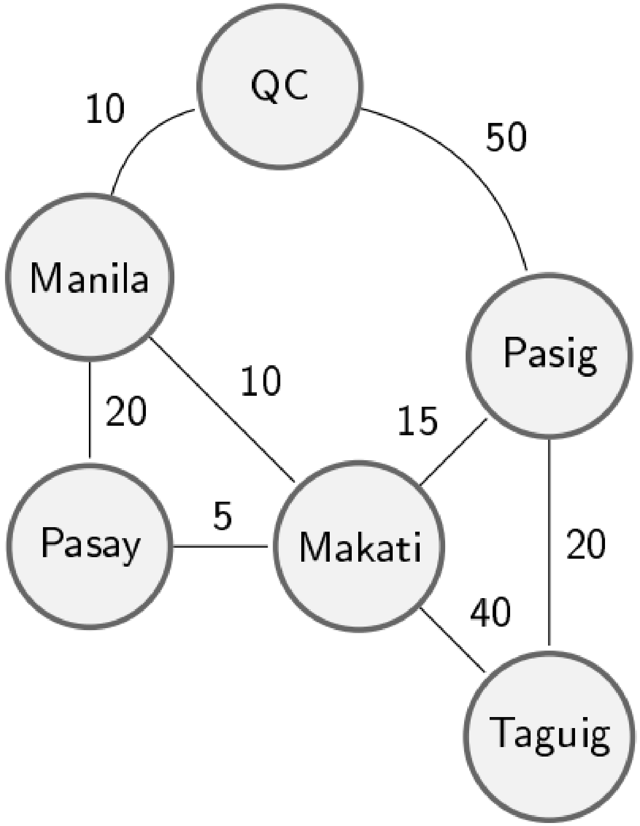


FRONTIER

QC (0)

EXPLORED

Uniform Cost Search Example



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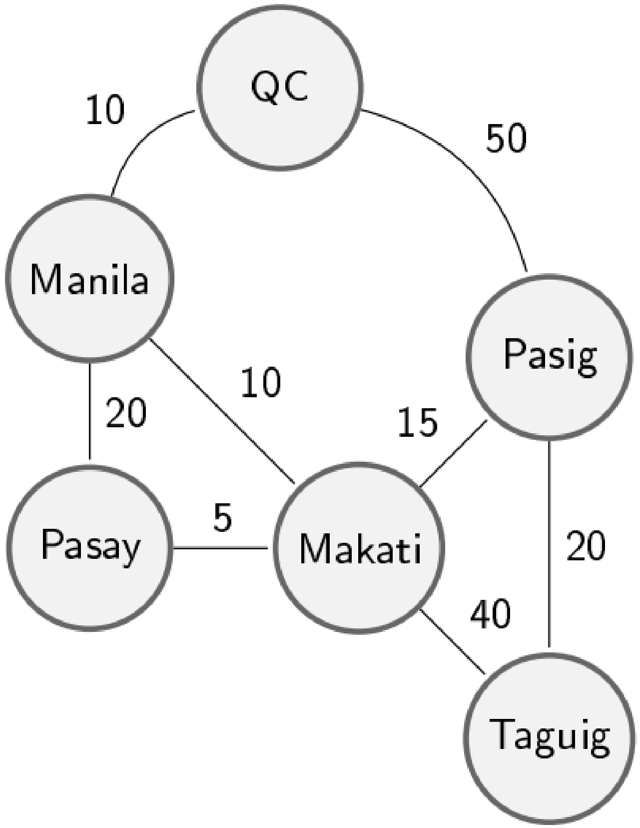


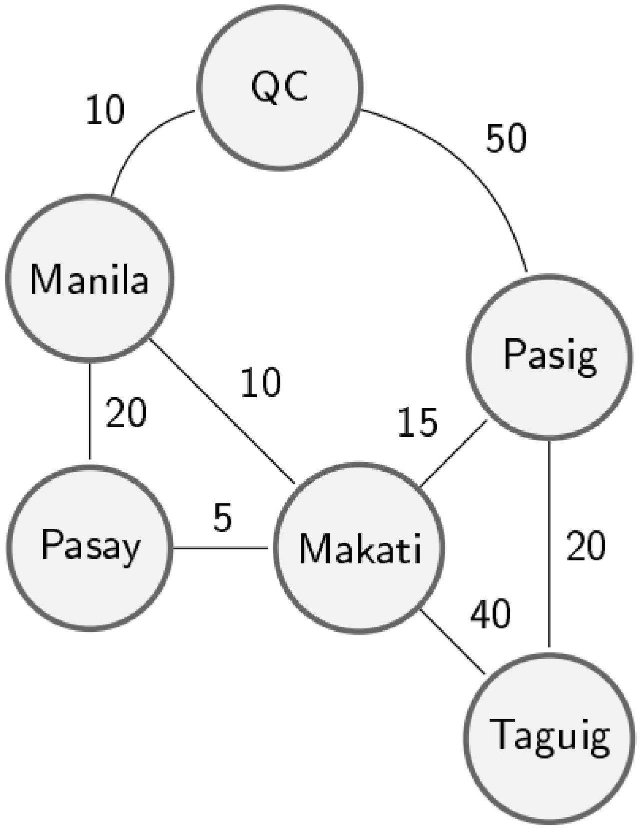
FRONTIER

Manila (10 from QC)
Pasig (50 from QC)

EXPLORED

QC (0)

Uniform Cost Search Example



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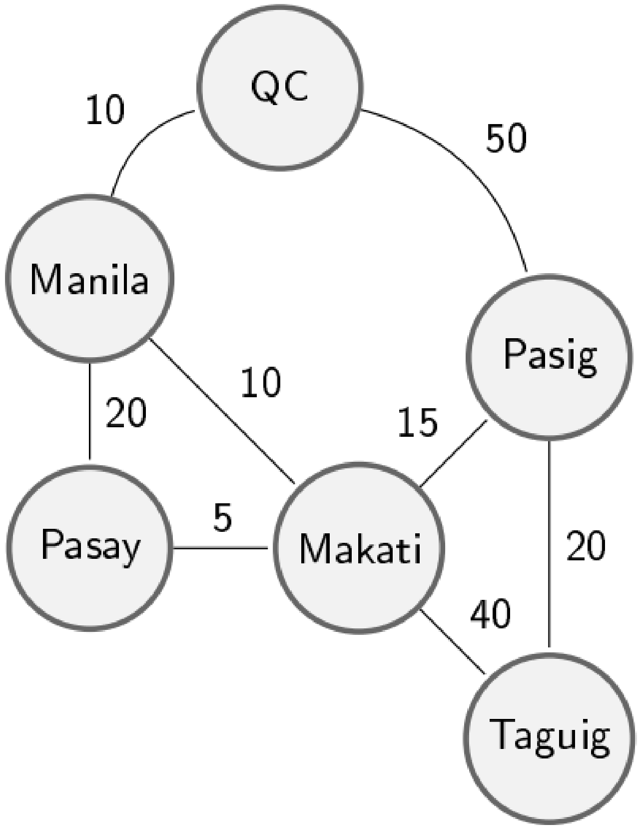


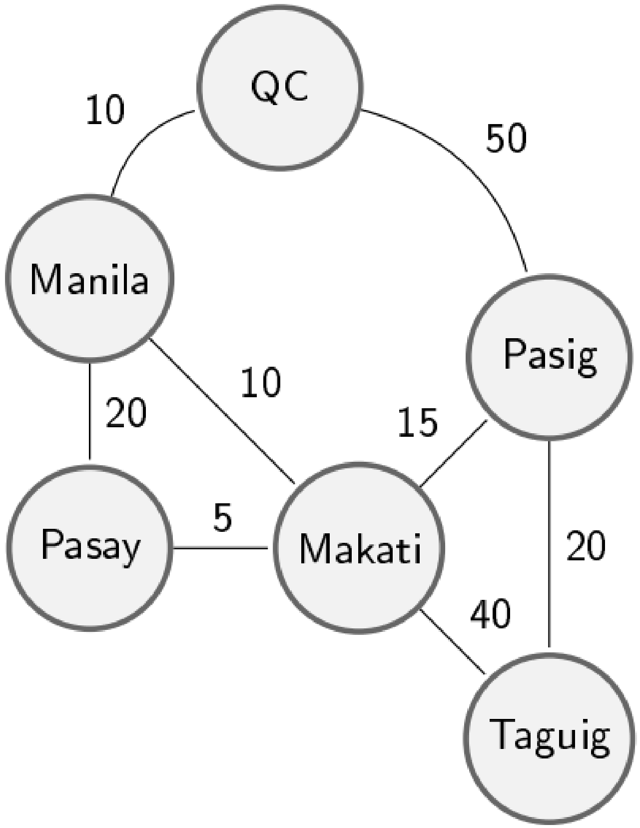
FRONTIER

Pasig (50 from QC)
Pasay (30 from Manila)
Makati (20 from Manila)

EXPLORED

QC (0)
Manila (10 from QC)

Uniform Cost Search Example



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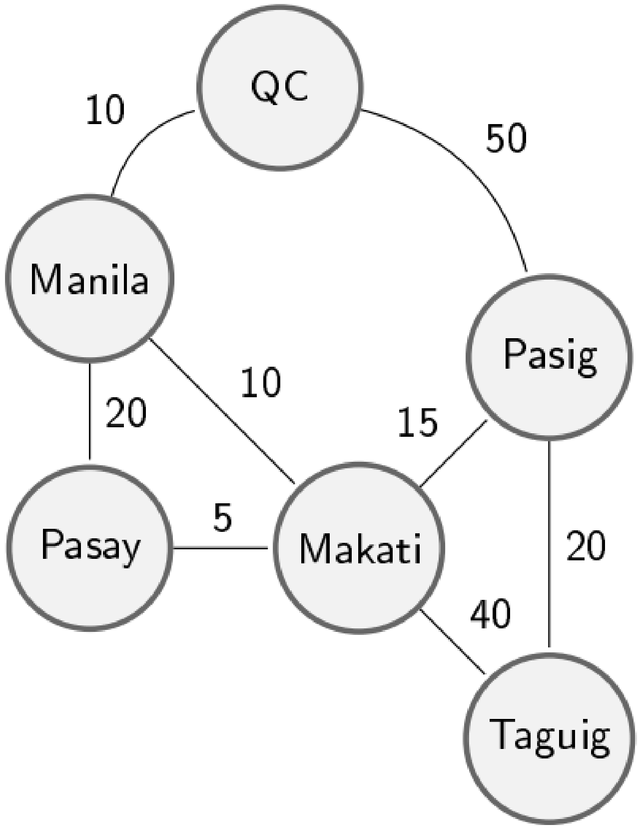


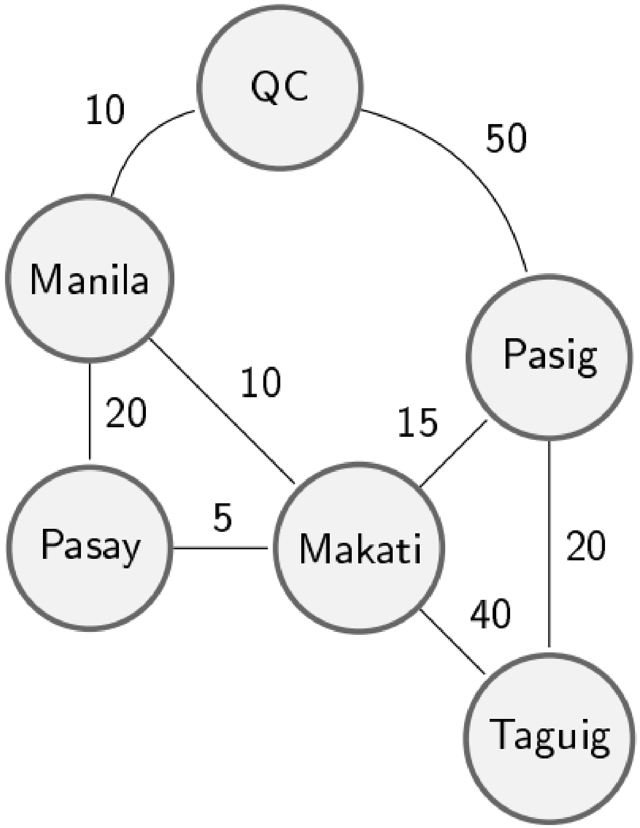
FRONTIER

Pasay (25 from Makati)
Pasig (35 from Makati)
Taguig (60 from Makati)

EXPLORED

QC (0)
Manila (10 from QC)
Makati (20 from Manila)

Uniform Cost Search Example



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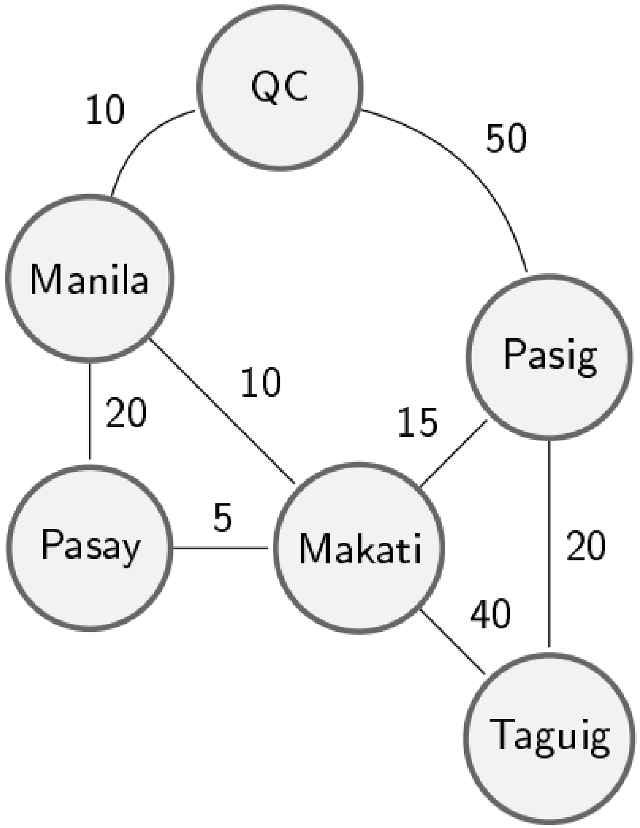

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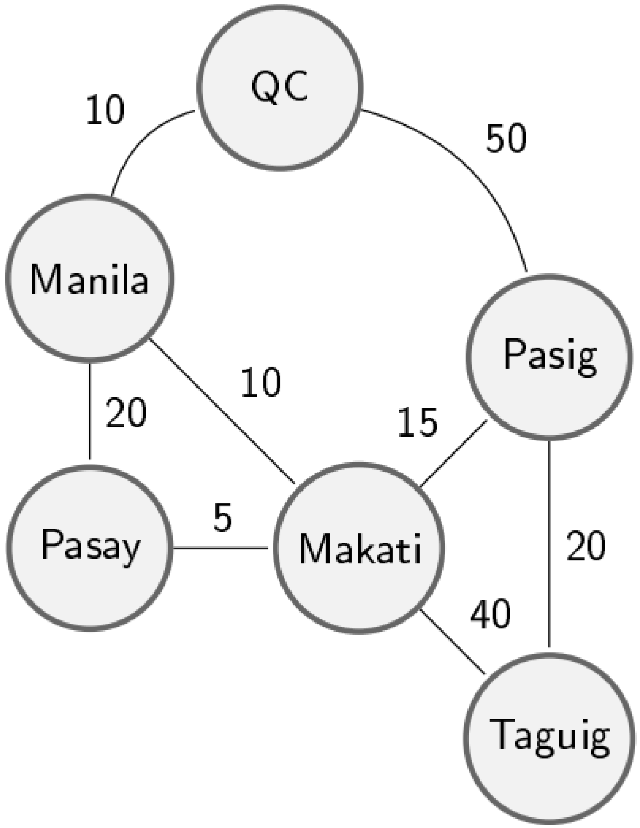

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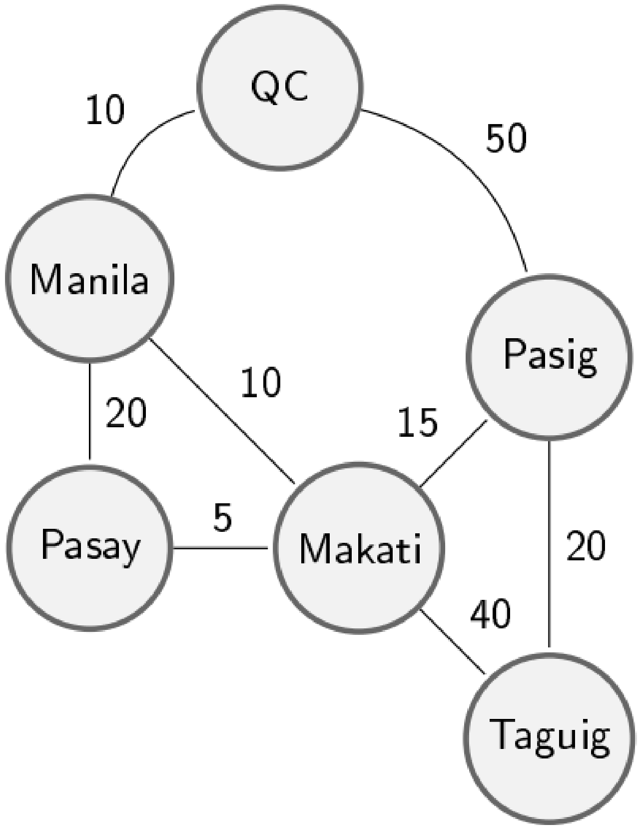

FRONTIER

Taguig (55 from Pasig)

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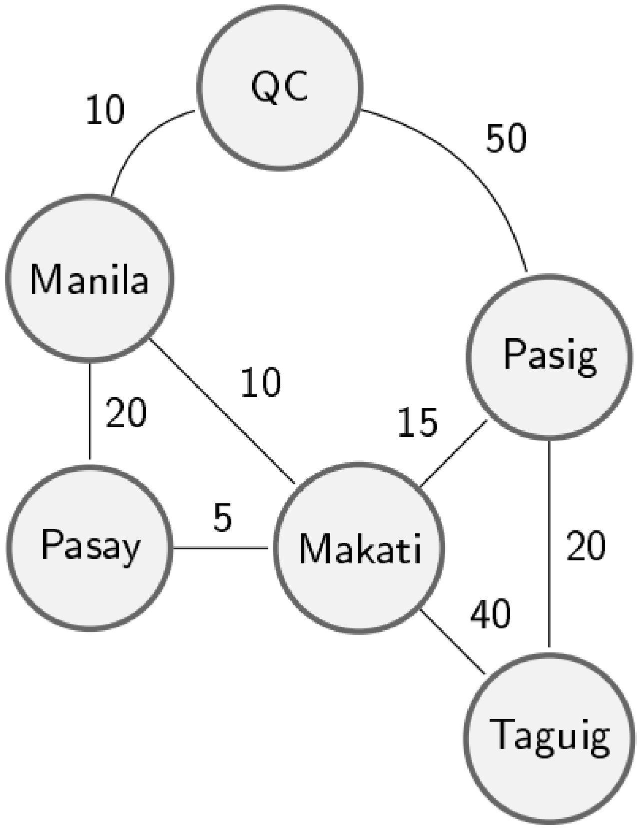

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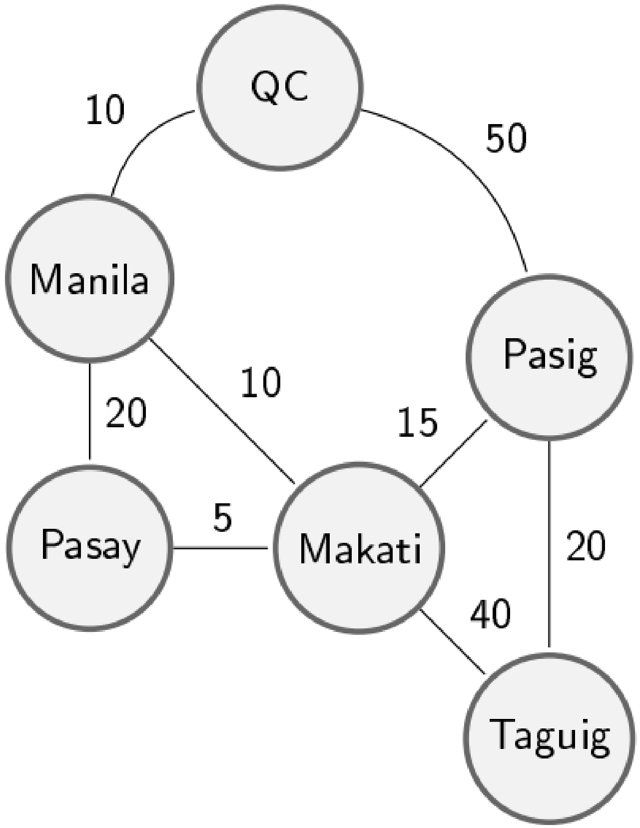

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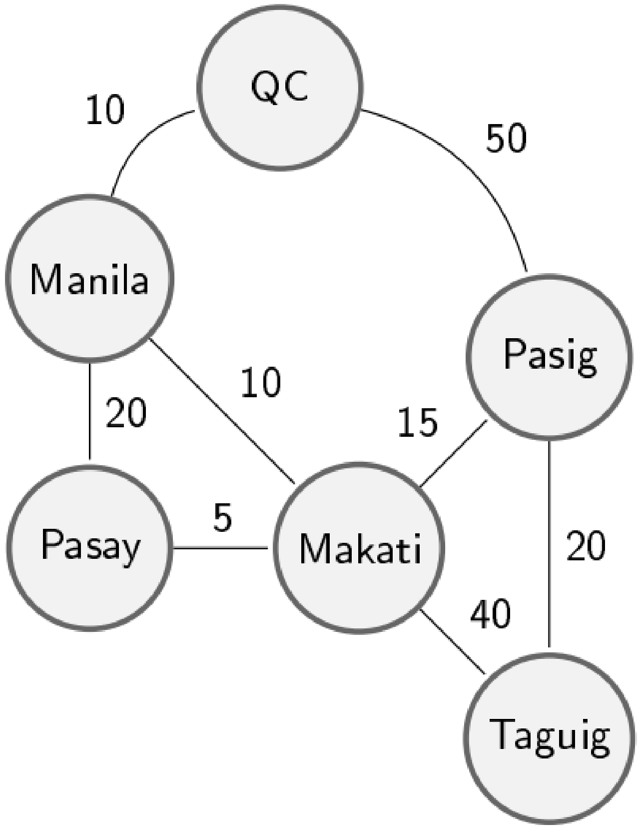

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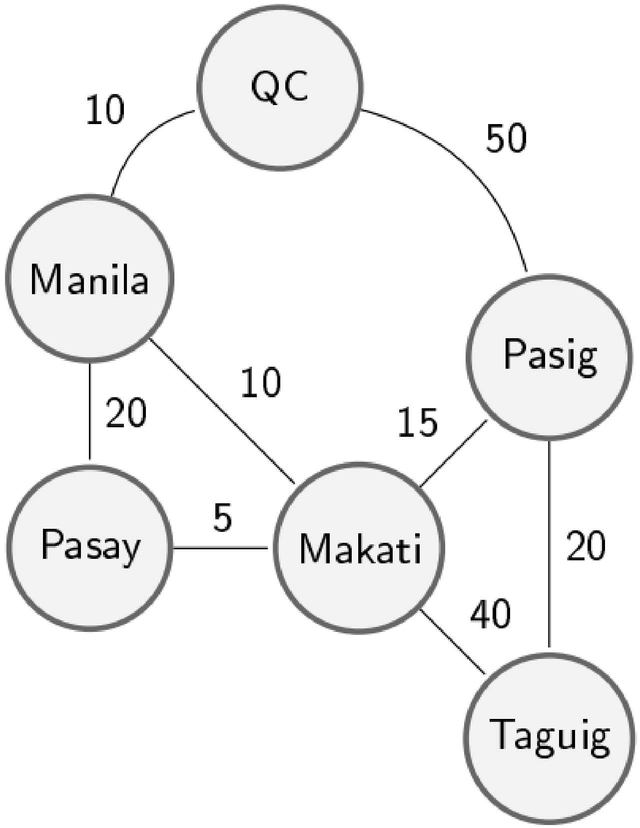

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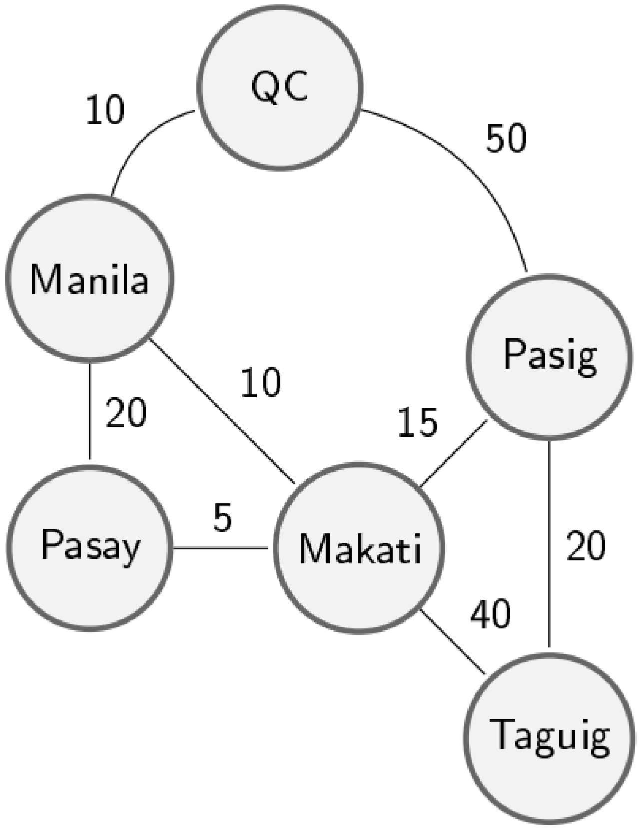

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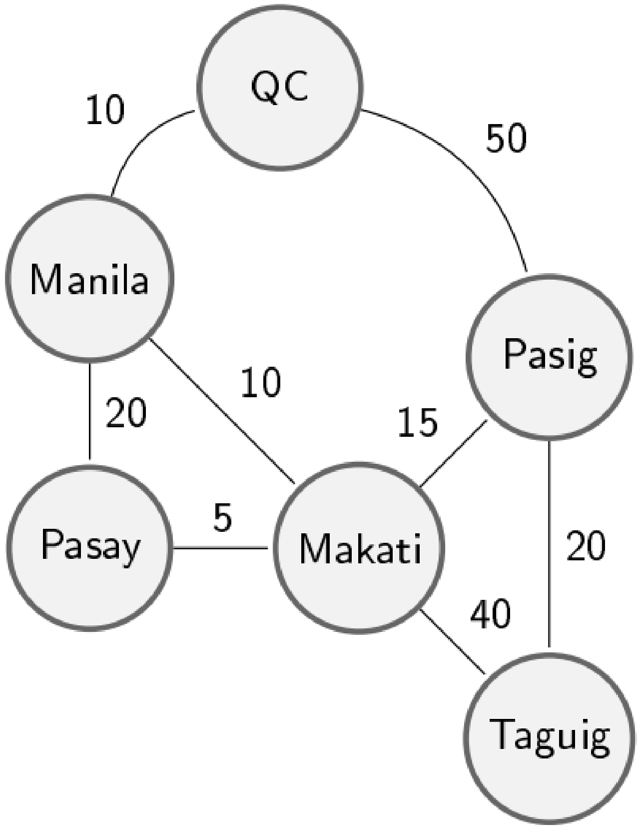

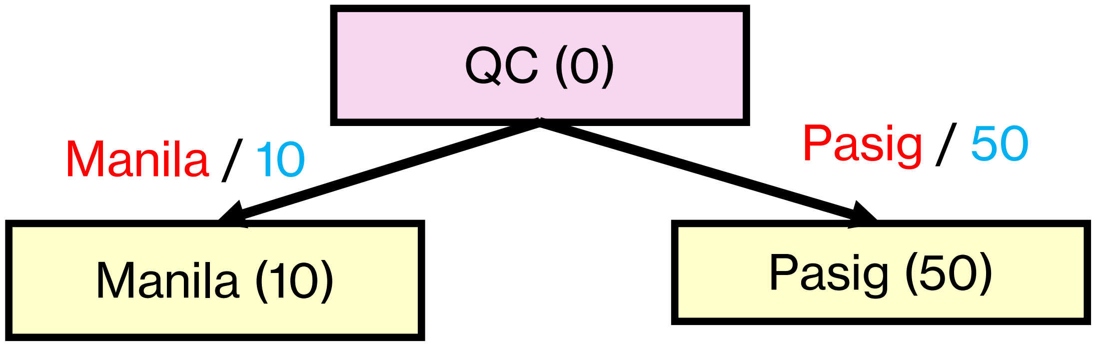

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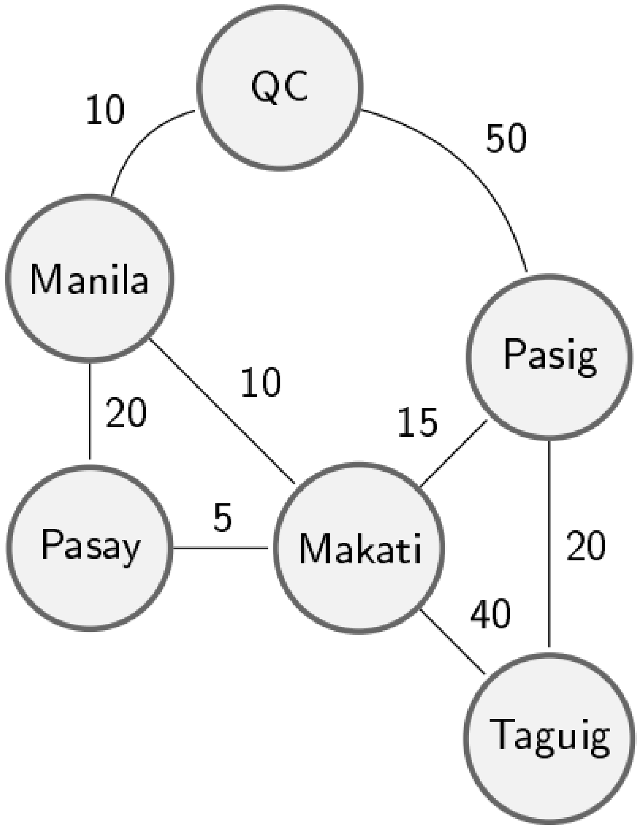

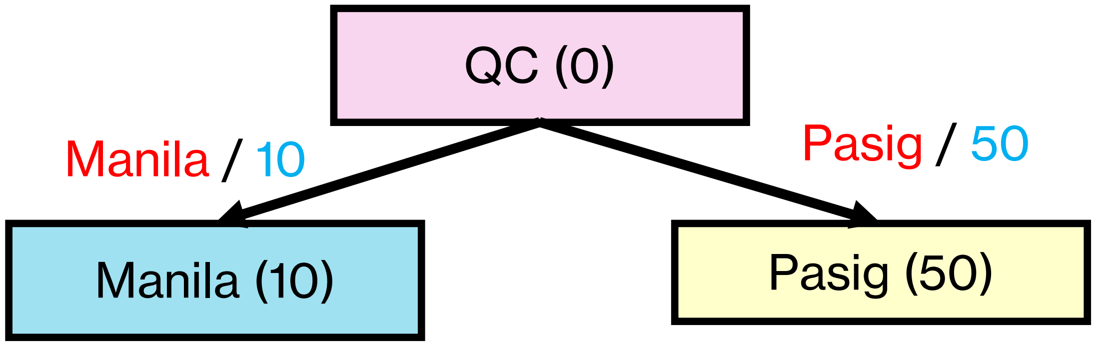

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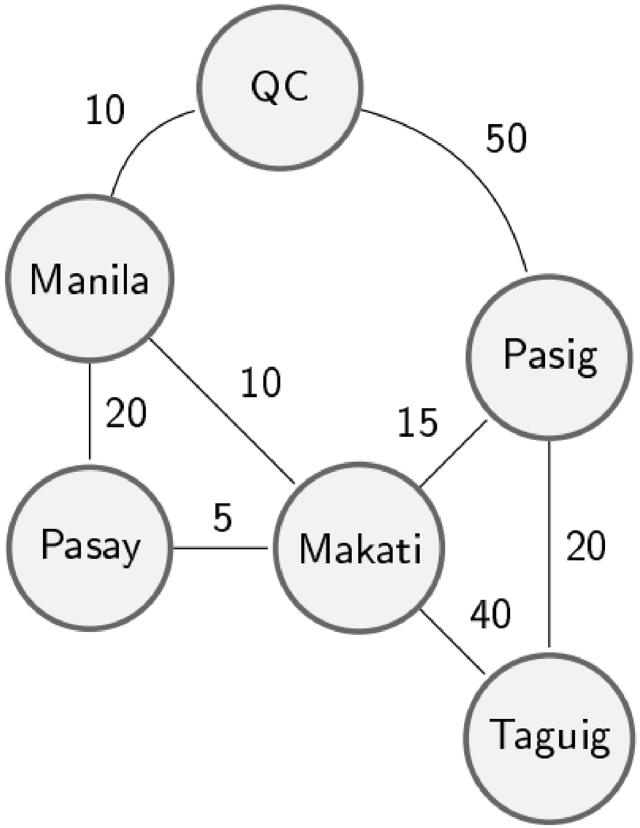

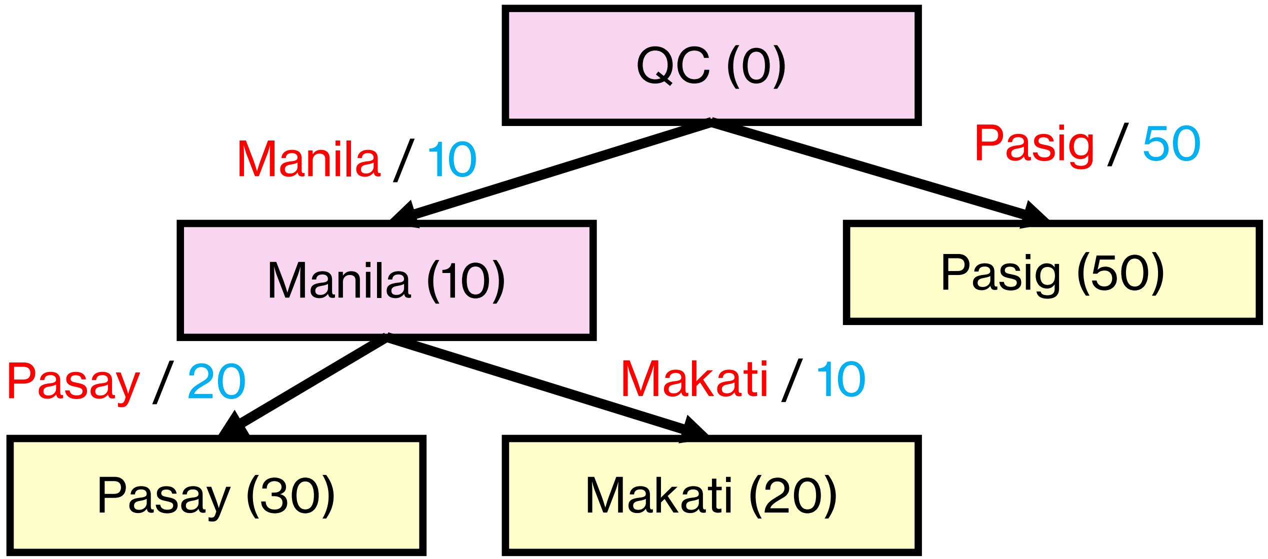

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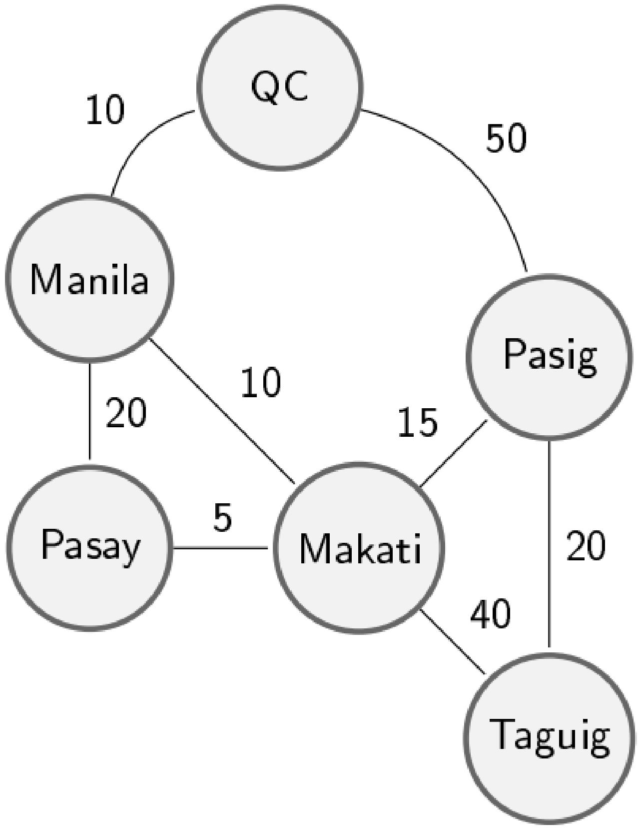

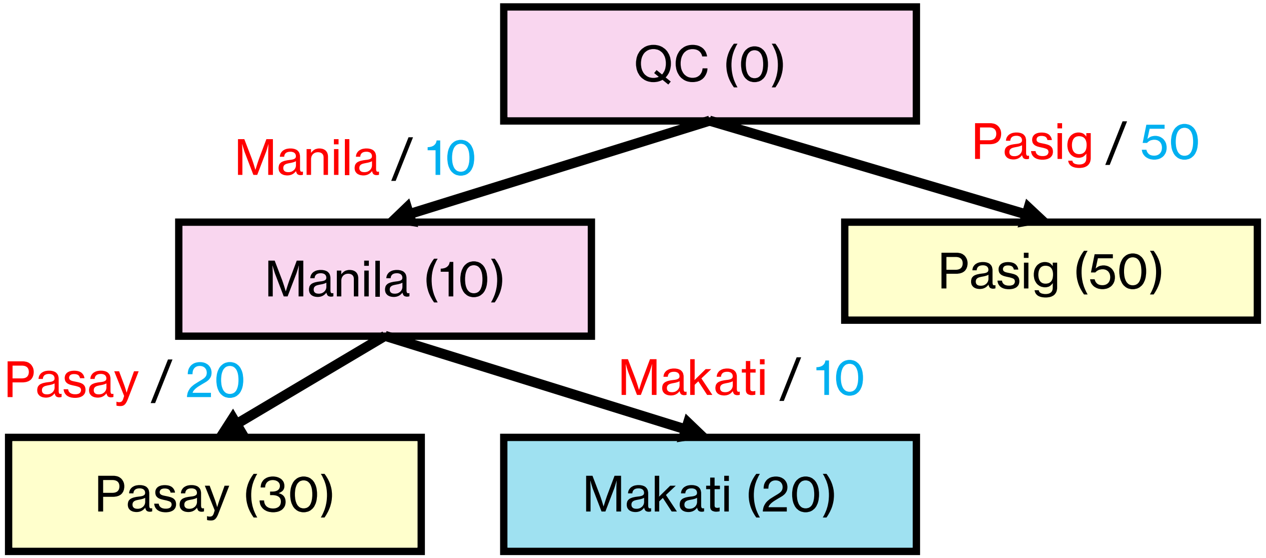

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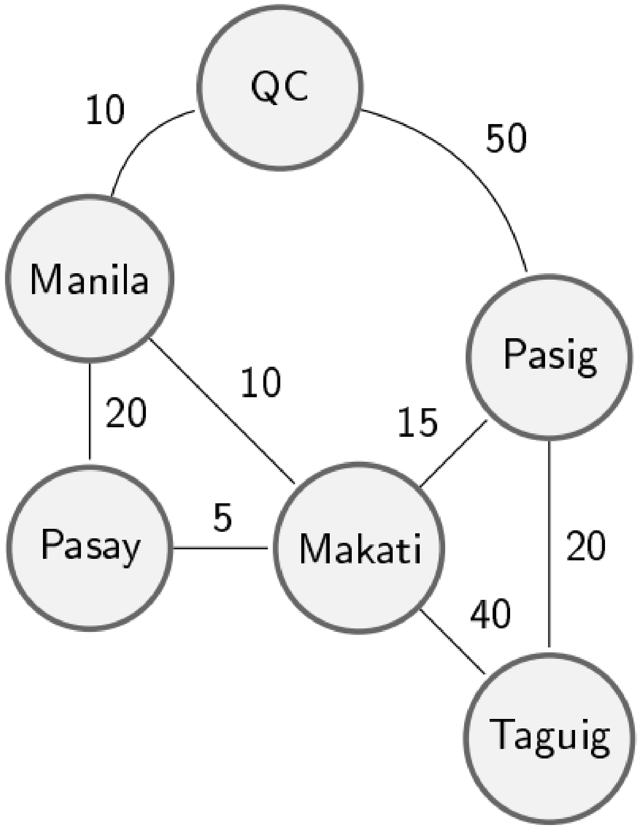

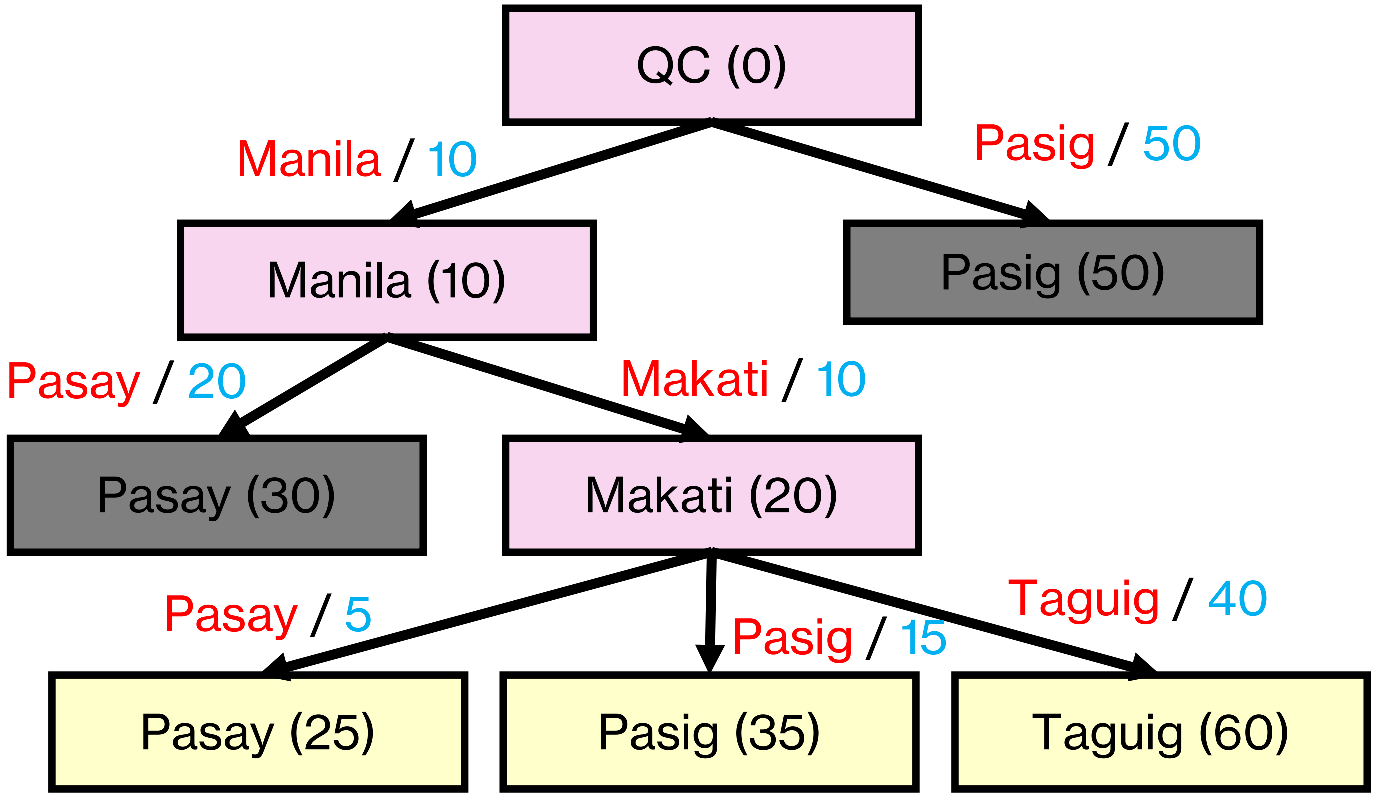

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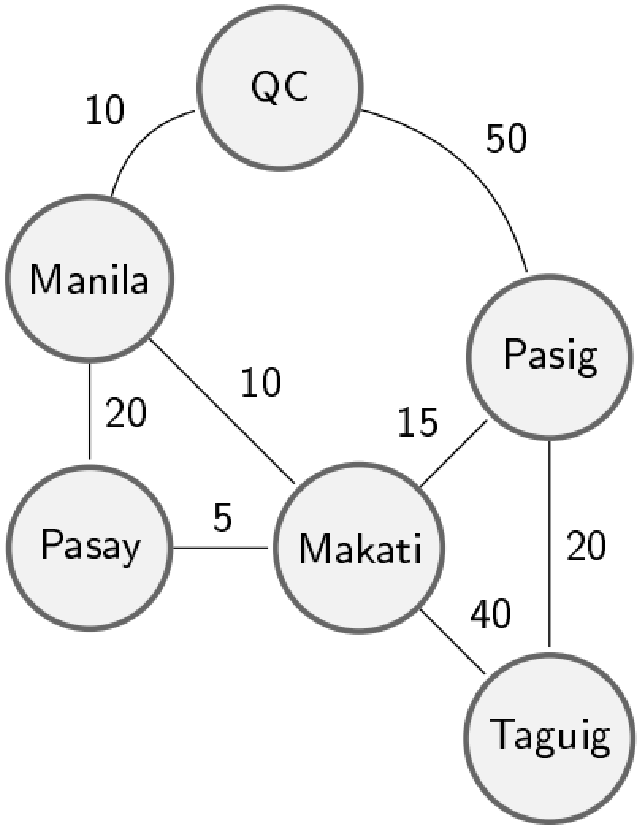

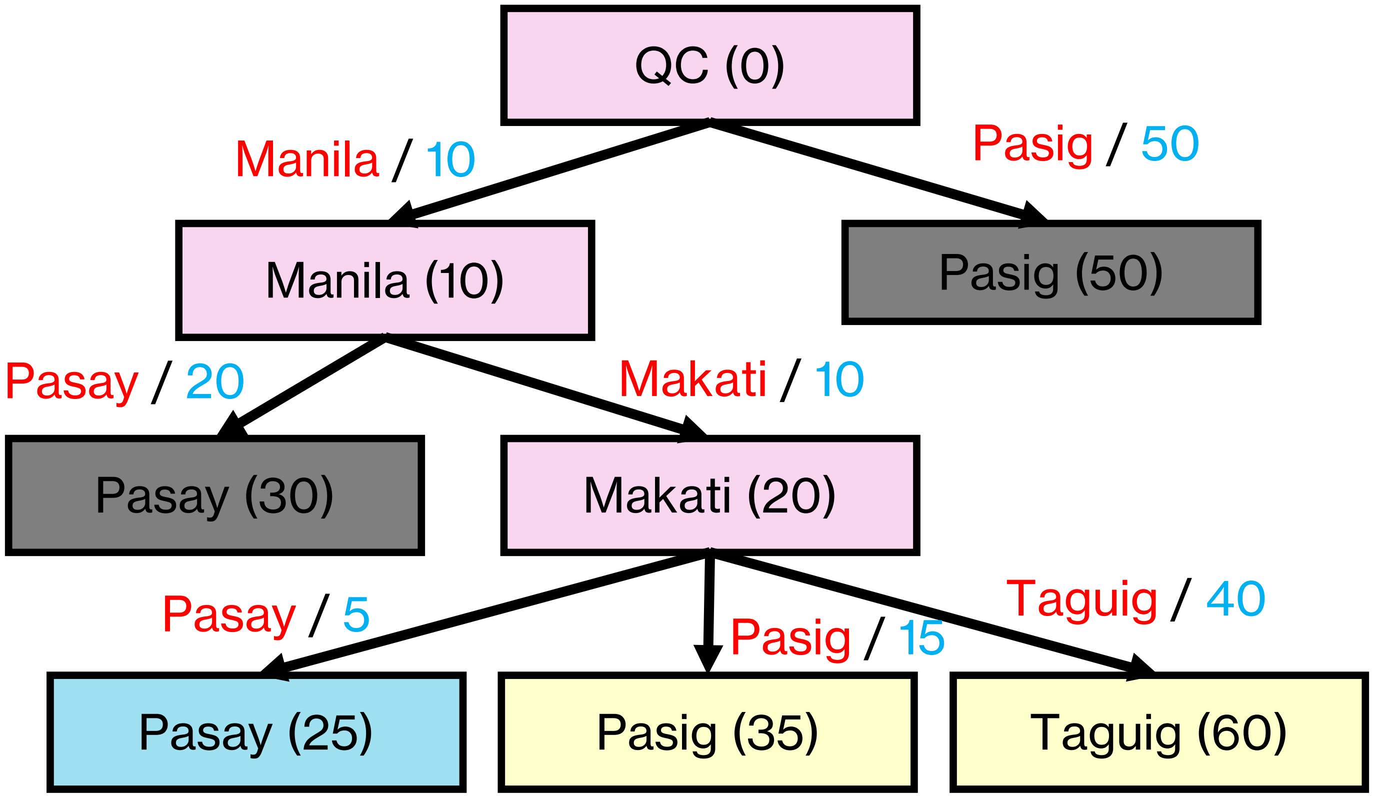

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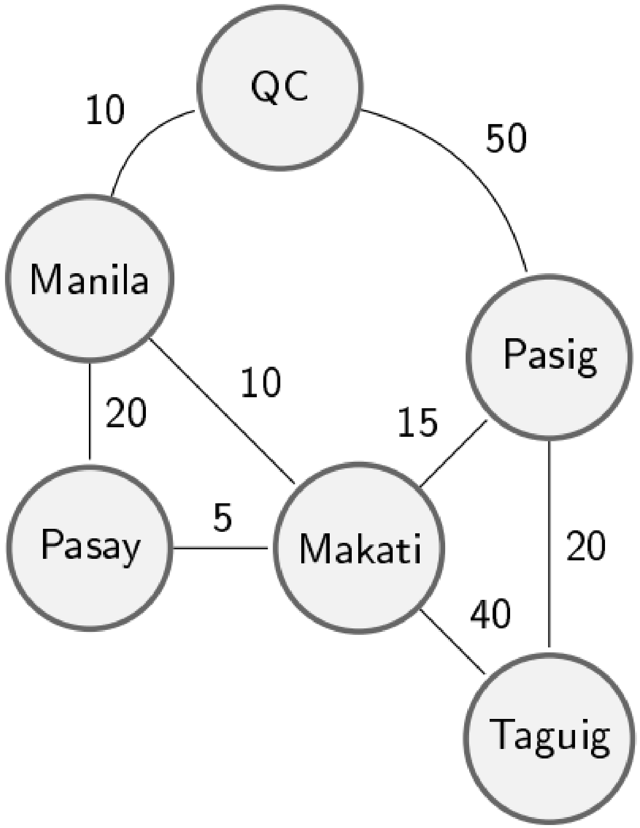

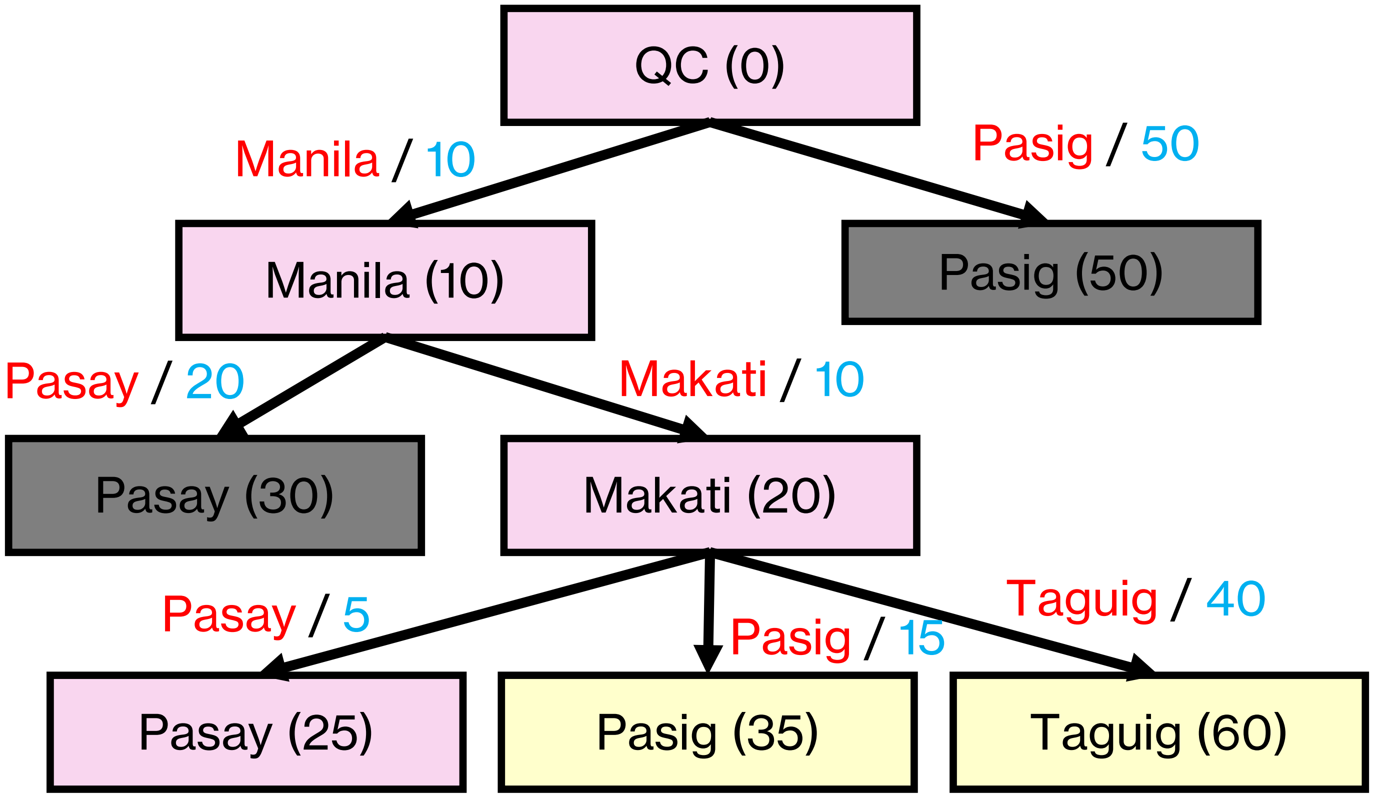

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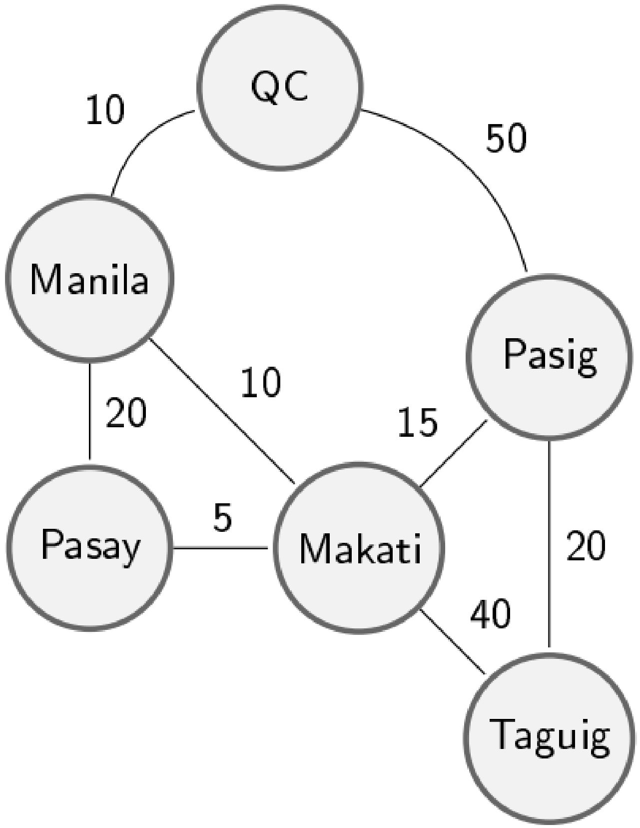

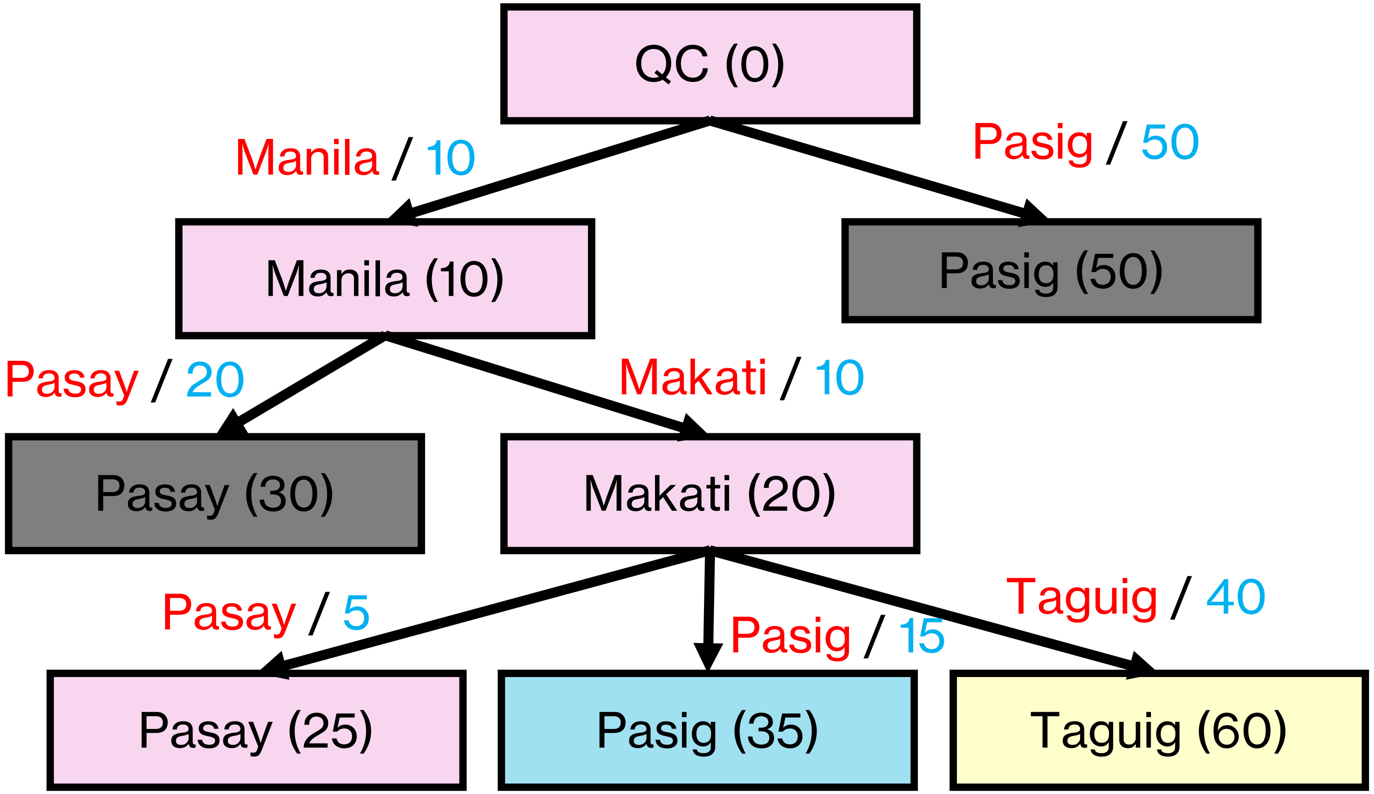

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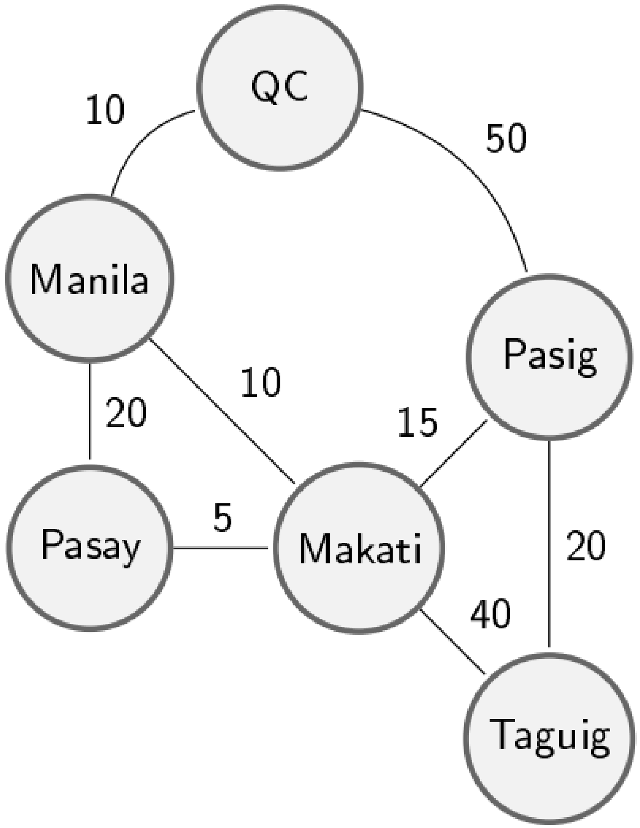

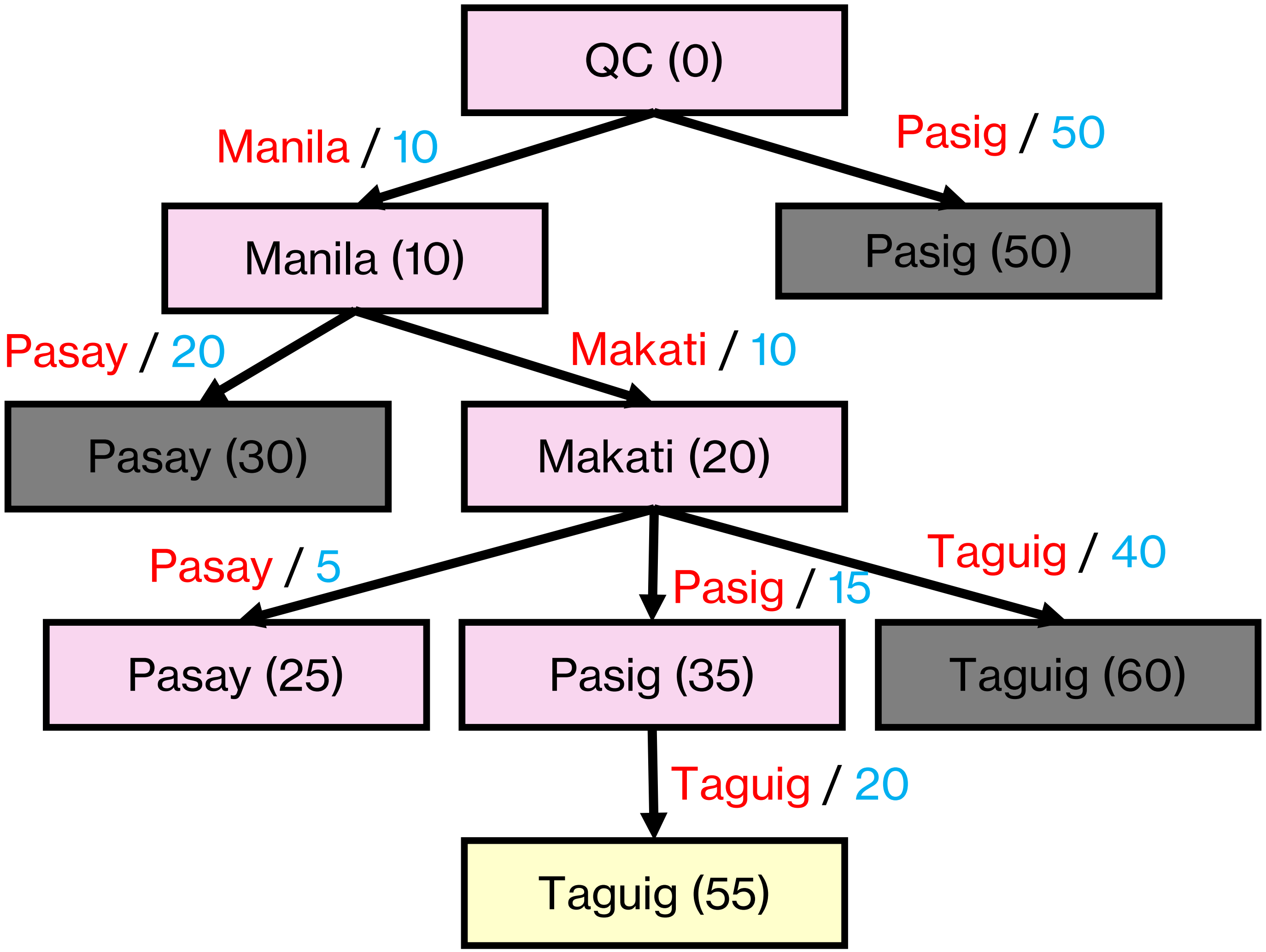

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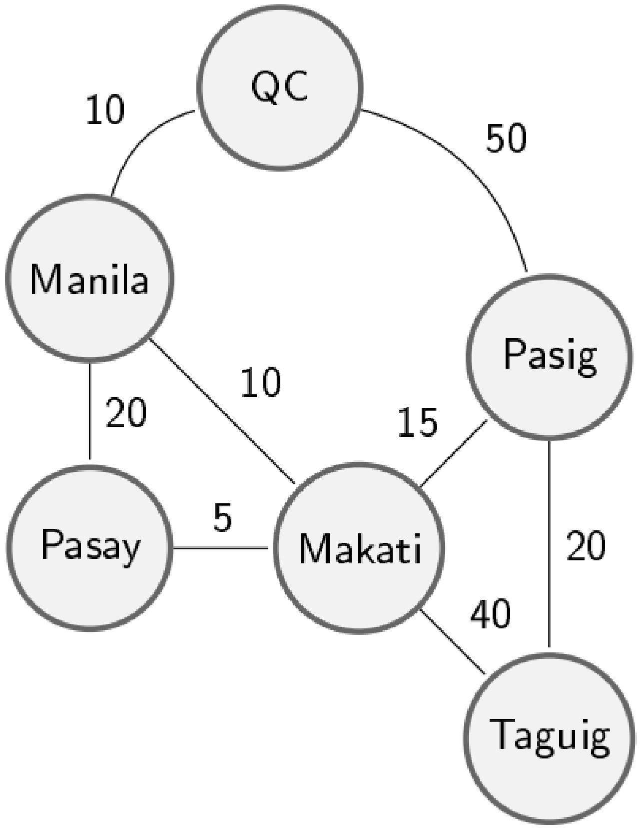

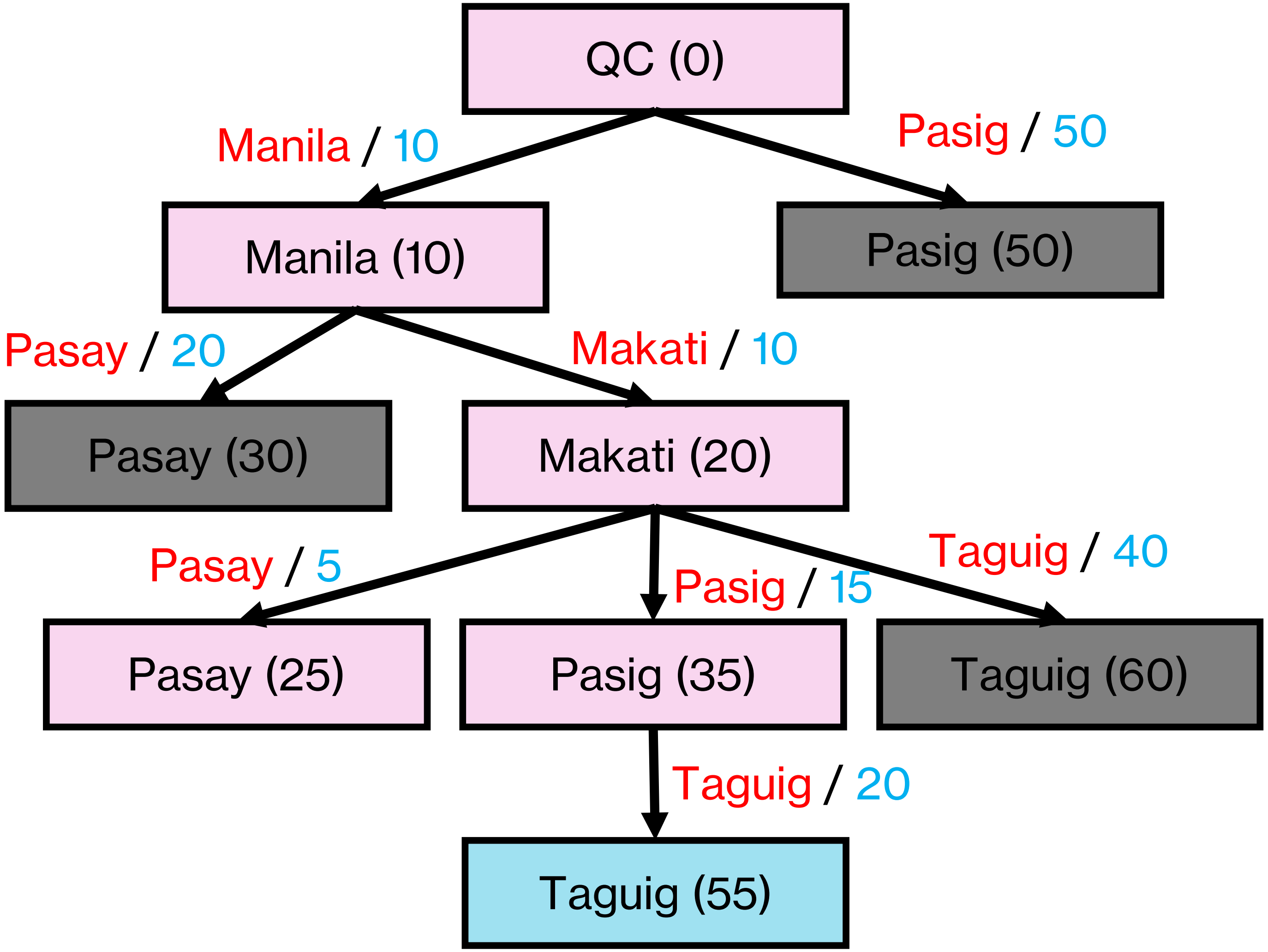

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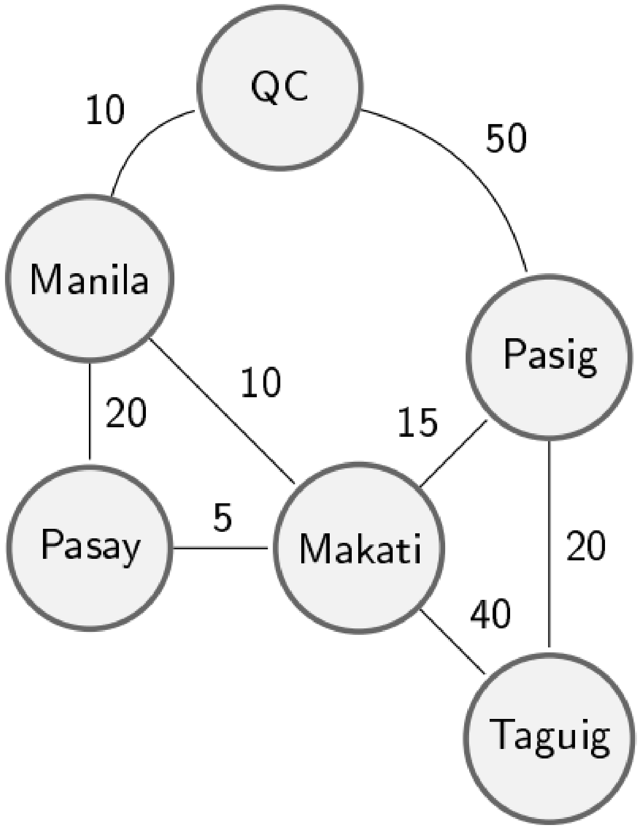
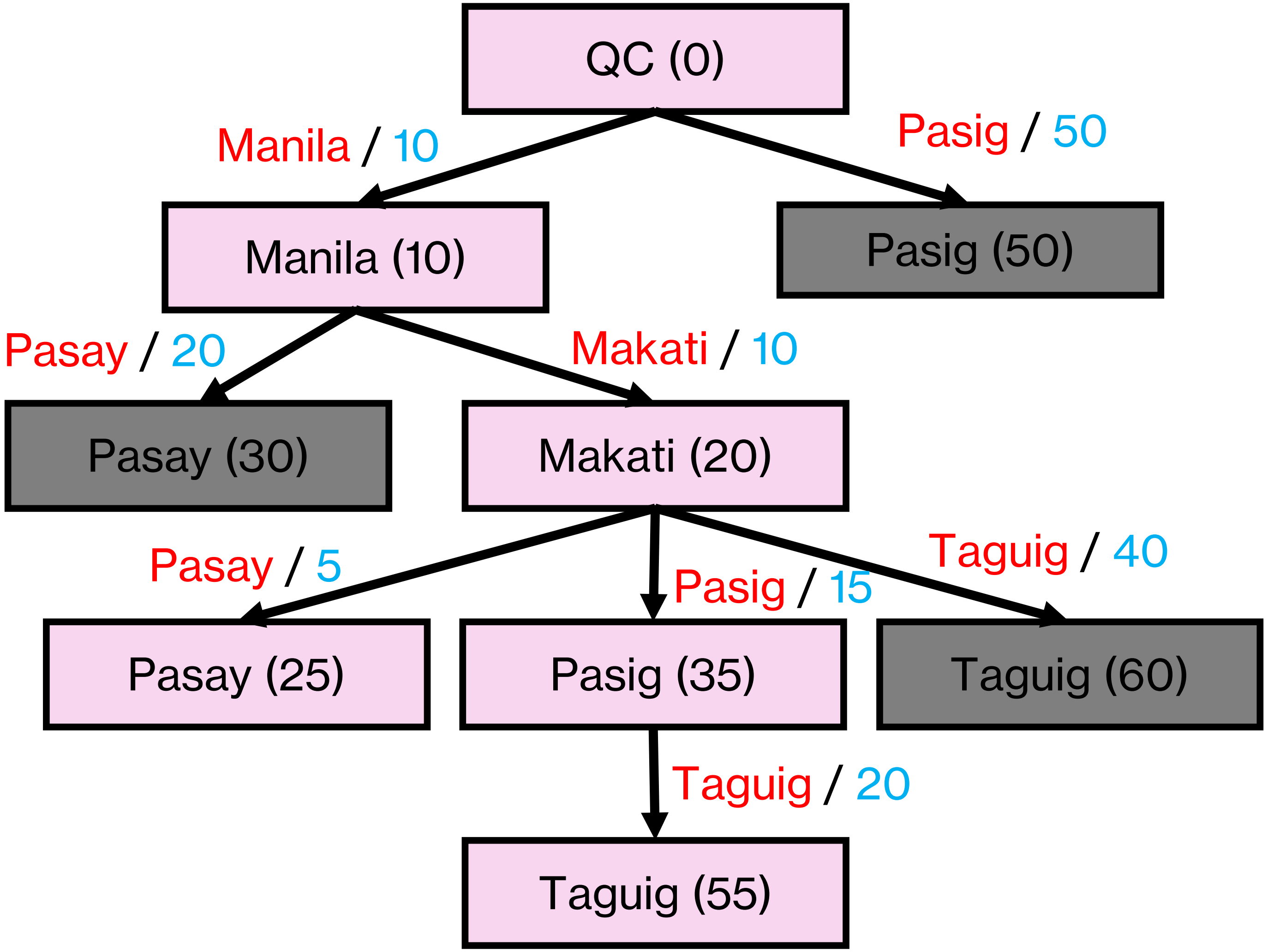
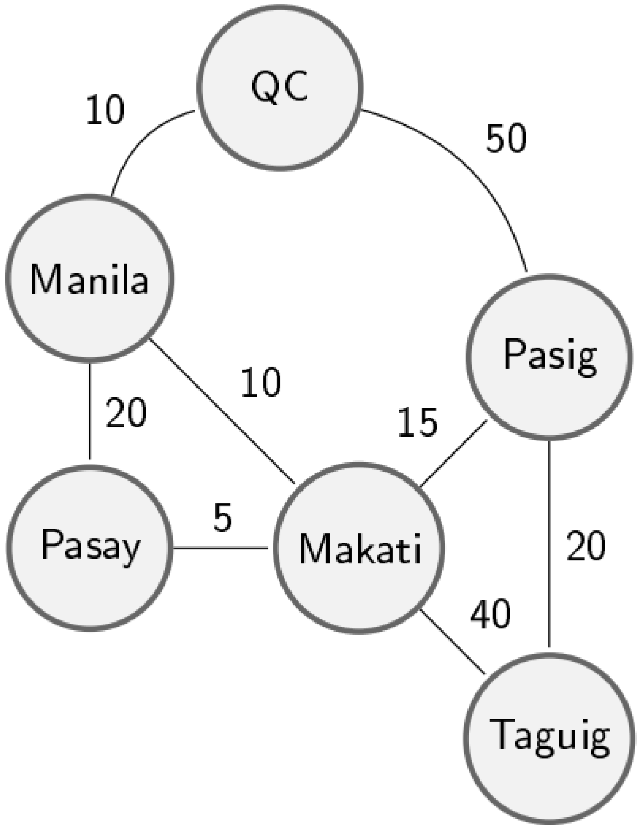
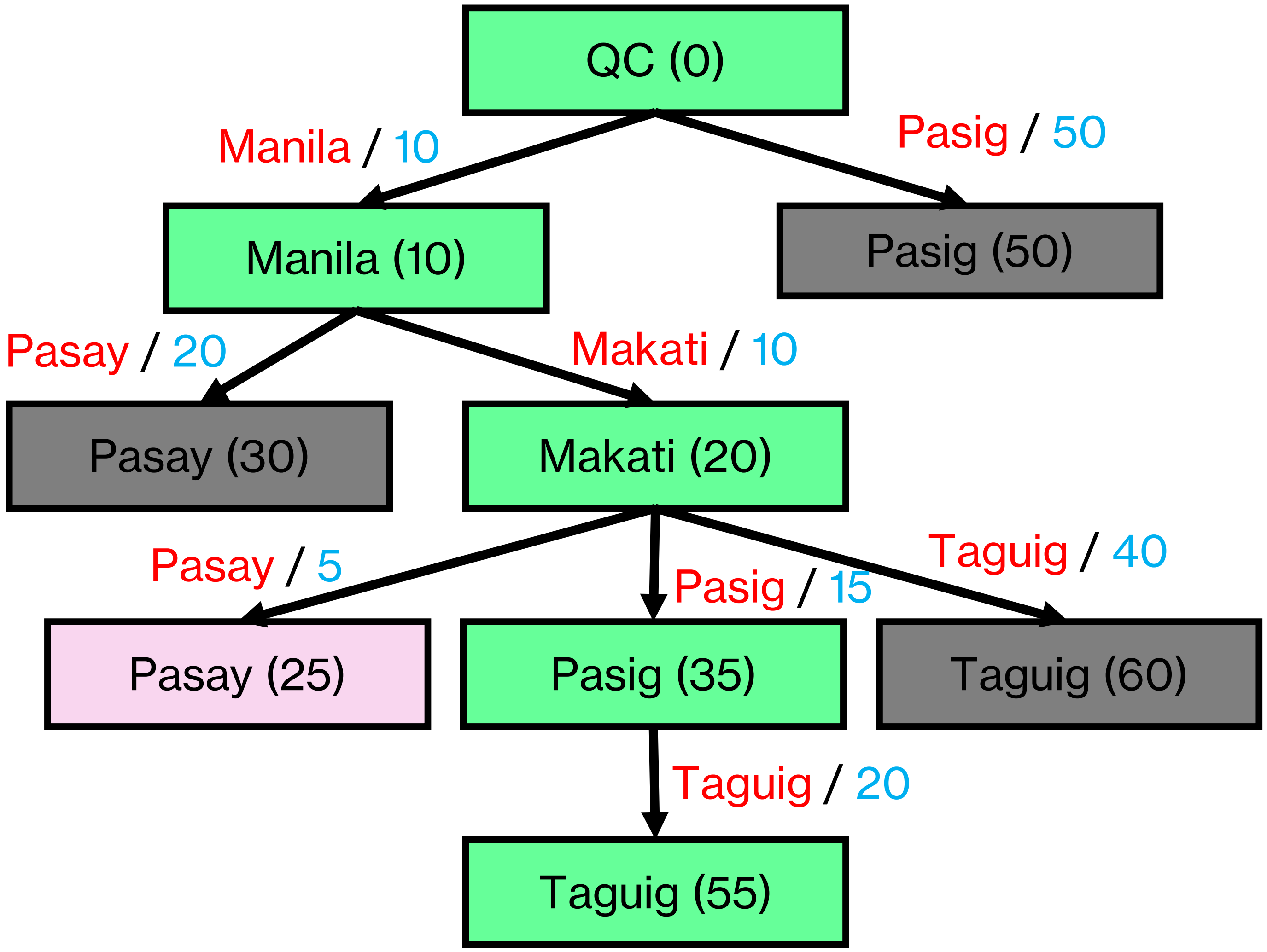




Solution found!

Uniform Cost Search Example





Solution found!

Uniform Cost Search

ALGORITHM

Add s_{start} to FRONTIER with priority 0.
While FRONTIER is not empty do
    Remove s with the lowest priority p from FRONTIER
    if IsEnd(s) then
        return solution
    Add s to EXPLORED
    for each $a \in Actions(s)$ do
        Get successor $s' \leftarrow Succ(s, a)$
        if s' not yet in EXPLORED
            Update s' in FRONTIER with priority $p + Cost(s, a)$

Characteristics of Uniform Cost Search

- Cannot handle negative costs.
- If state space is finite, it is **complete**.
- It is **optimal**.

- Time and space complexity: $\boldsymbol{O(n \log n)}$, where n is the number of states that are closer to the start state than the goal state.

DP vs UCS

- N total states, n of which are closer to start state than the goal.

Algorithm	Cycles?	Action Costs	Time / Space Complexity
Dynamic Programming	no	any	$O(N)$
Uniform Cost Search	yes	≥ 0	$O(n \log n)$

Acknowledgments

- Stanford University CS221 Autumn 2021 course. Available online at: https://stanford-cs221.github.io/autumn2021
- Previous CSINTSY slides by the following instructors:
  - Raymund Sison, PhD
  - Judith Azcarraga, PhD
  - Merlin Suarez, PhD
  - Joanna Pauline Rivera

Readings

- https://www.youtube.com/watch?v=dRMvK76xQJI
- https://www.educba.com/uniform-cost-search/
- https://www.youtube.com/watch?v=9iE9Mj4m8jk