RESOLUTION REFUTATION, FORWARD & BACKWARD CHAINING

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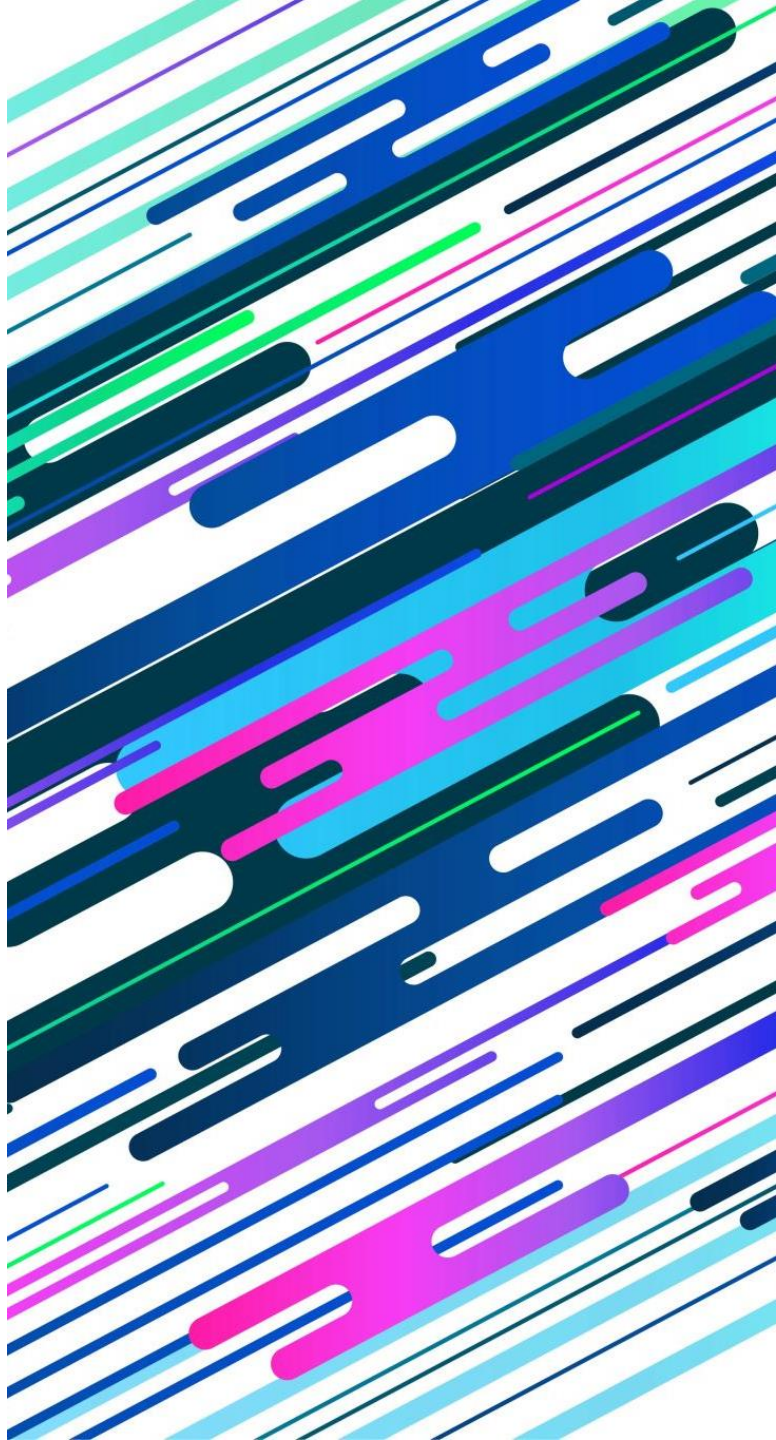

Inference: Resolution Refutation

- Using resolution inference **to prove the truthfulness of a formula** against a knowledge base
- **Approach:** Proof by contradiction

1. Remove the implications
   - $p \rightarrow q \equiv \neg p \vee q$
   - $p \wedge q \rightarrow r \equiv \neg p \vee \neg q \vee r$
   - $p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$
   - $p \wedge q \equiv p, q$ (separate p and q into individual clause)
2. Push negations inside
   - $\neg(p \vee q) \equiv \neg p \wedge \neg q$
   - $\neg(p \wedge q) \equiv \neg p \vee \neg q$
3. Remove double negations
   - $\neg\neg p \equiv p$
4. Distribute $\vee$ over $\wedge$
   - $p \vee (q \vee r) \equiv (p \vee q) \wedge (p \vee r)$
5. Remove all quantifiers

- **Resolution Refutation Steps:**
  - Convert to predicate logic form
  - Convert to Conjunctive Normal Form (CNF)
  - Negate the goal
  - Apply resolution inference algorithm
  - If empty clause appears, then the formula is **proven**.

Resolution Refutation: Example

- All dogs are animals. Fido is a dog. All animals will die. **Prove that Fido will die**.

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$\forall X$ Dog(X) $\rightarrow$ Animal(X)
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$\forall Y$ Animal(Y) $\rightarrow$ Die(Y)
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Convert to CNF

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$\neg \text{Die}(\text{Fido})$ → Negate goal

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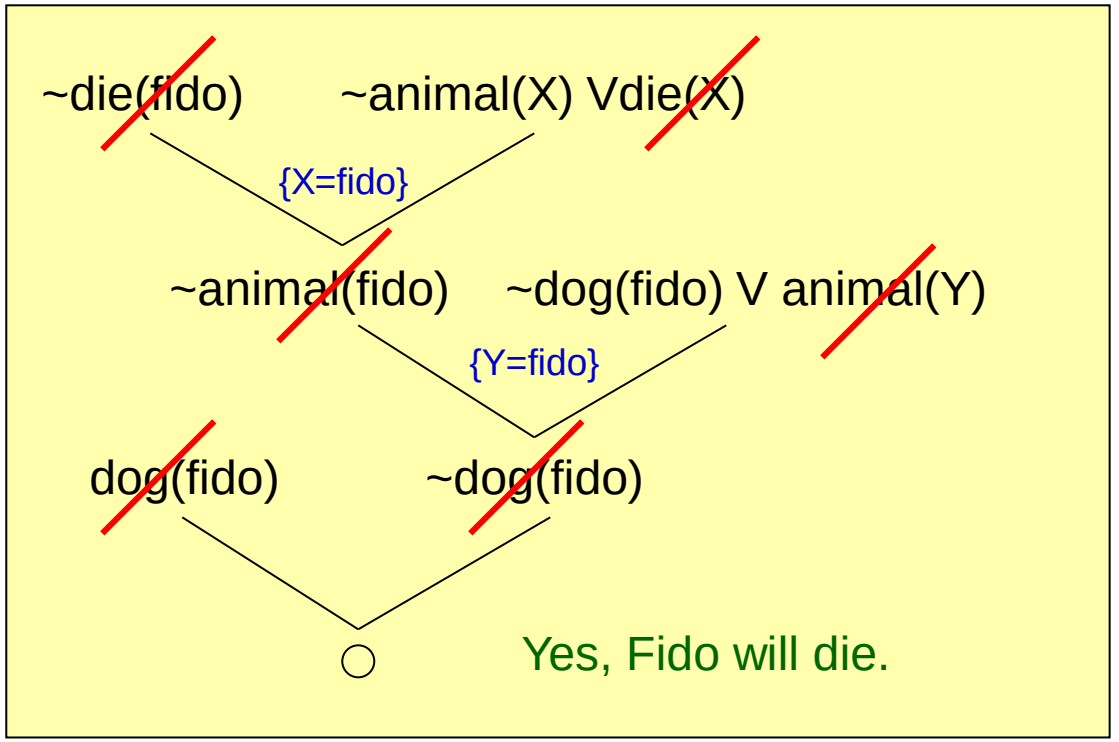
empty clause – contradiction!

Therefore, Fido will **die**!

Alternative Visualization

- All dogs are animals. Fido is a dog. All animals will die. **Prove that Fido will die**.



Resolution Refutation: Try this!

Exciting Life

Everyone who are not poor and are smart they are happy. Everyone who read are not stupid. John can read and is wealthy. Everyone who are happy have exciting lives. Can someone be found with an exciting life?

Forward and Backward Chaining

- Reasoning strategies used in automated reasoning for rule-based systems.
  - **Forward-Chaining:** Start with the facts and keep on deriving new formulas.
  - **Backward-Chaining:** Start with a goal, and then go backwards until the goal is proven / disproven.

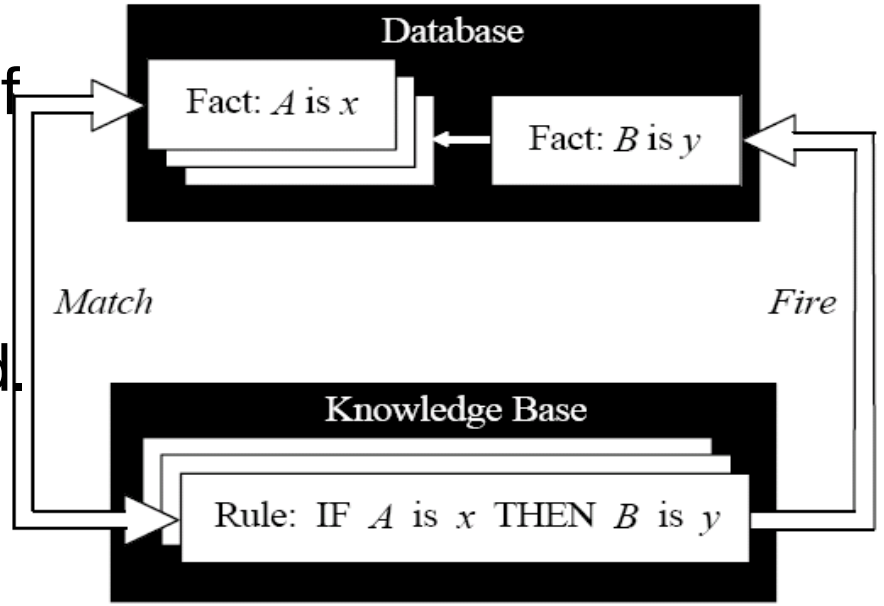
Forward and Backward Chaining

- **Forward Chaining:** data-driven
- **Backward Chaining:** goal-driven

Forward and Backward Chaining:

Inferencing method

- Domain knowledge is represented by a set of IF-THEN rules and data is represented by a set of facts.
- The inference engine compares each rule stored in the knowledge base with facts contained in the database.
- When the antecedent part of the rule matches a fact, the rule is FIRED and its consequent part is executed.
- The fired rule may change the set of facts by adding a new fact to the database.



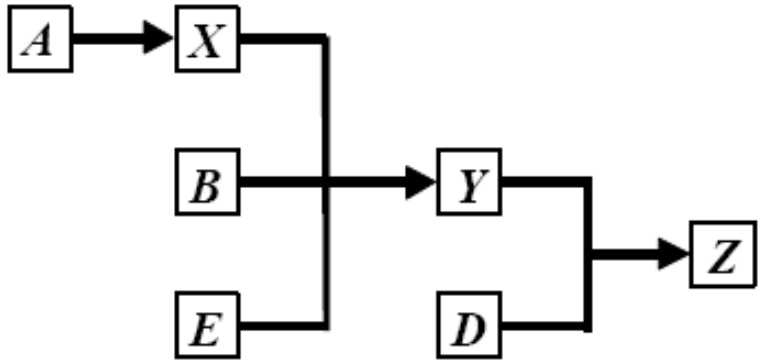
Forward and Backward Chaining:
Inferencing method

- The matching of the rule antecedent parts to the facts in DB produces an inference chain:

Rule 1: IF *Y* is true
AND *D* is true
THEN *Z* is true

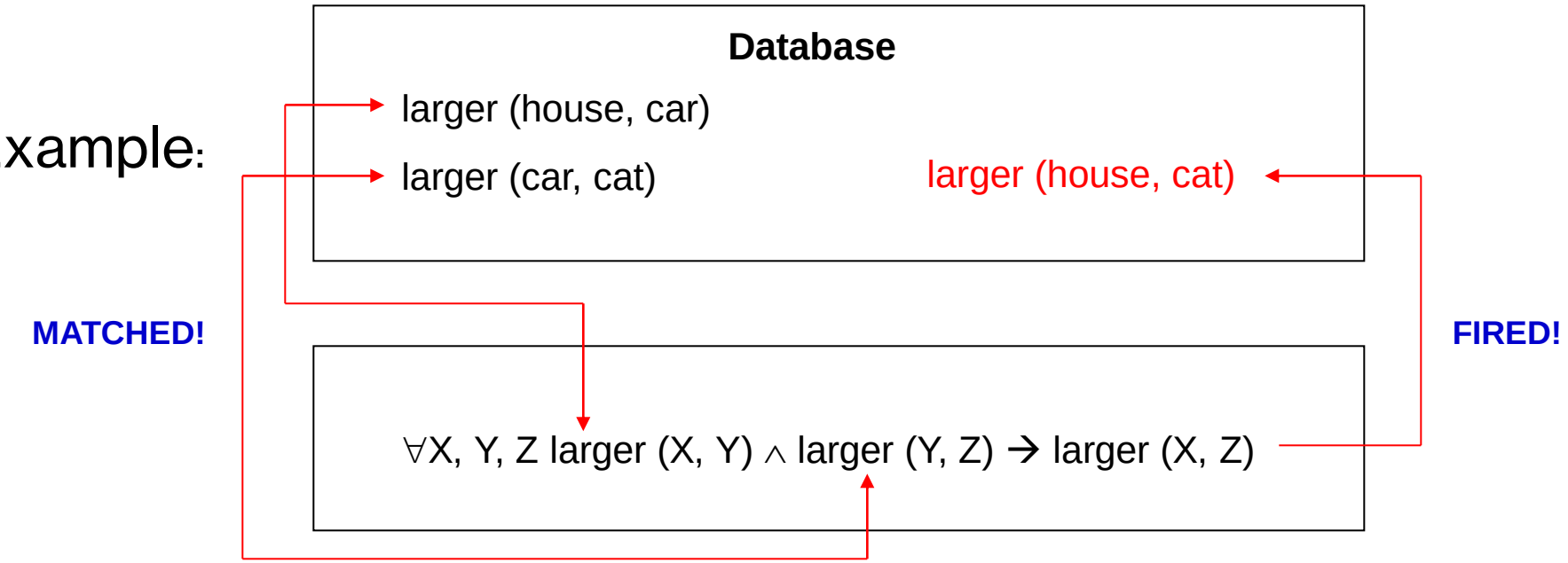
Rule 2: IF *X* is true
AND *B* is true
AND *E* is true
THEN *Y* is true

Rule 3: IF *A* is true
THEN *X* is true



Forward and Backward Chaining:
Inferencing method

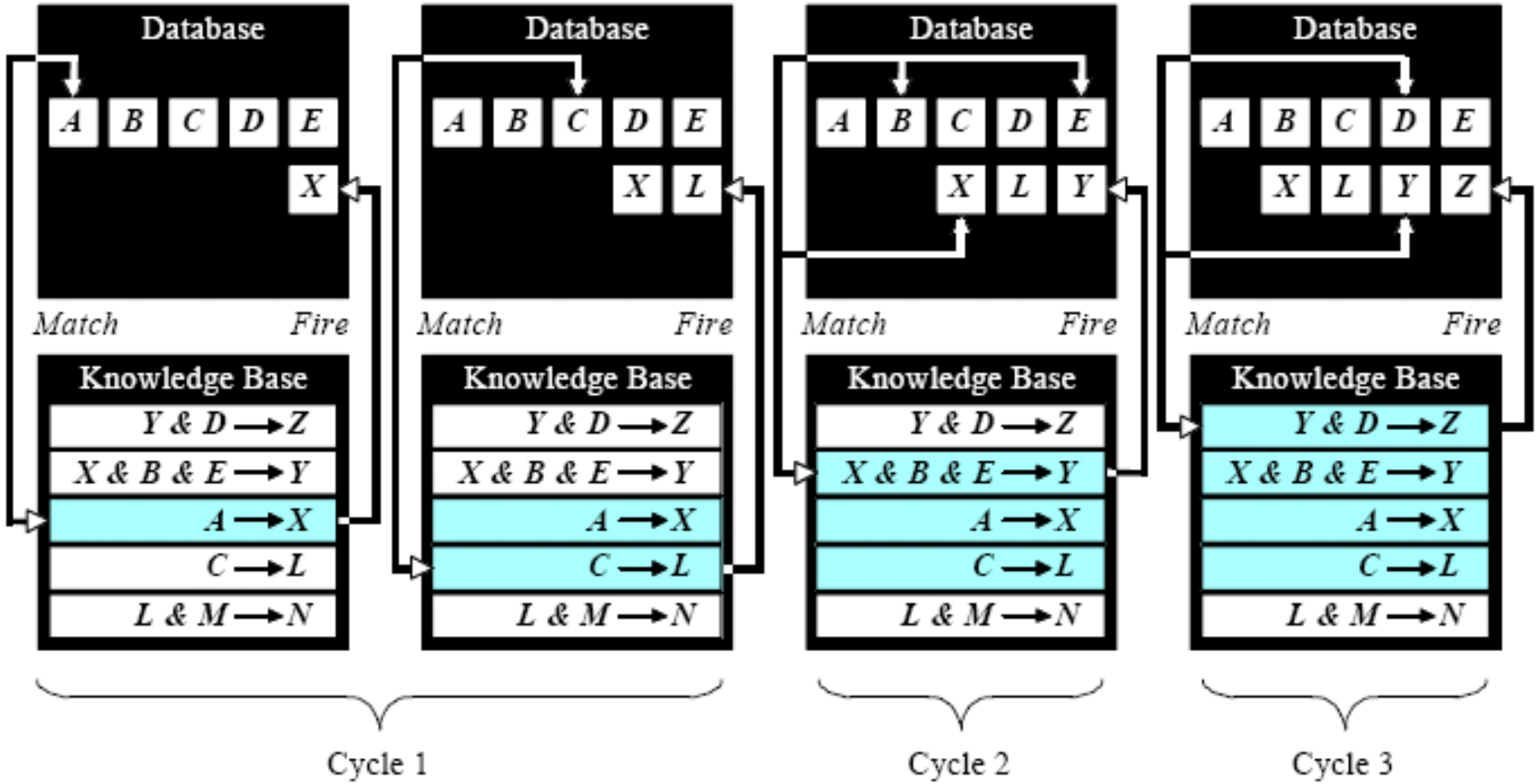
- Example:



Forward Chaining:
Inferencing method

- Forward Chaining:
  - Also known as the **data-driven reasoning**.
  - The reasoning starts from the known data and proceeds forward with that data.
  - Any rule can be executed only once.
  - When fired, the rule adds a new fact to the database.
  - The match-fire cycle stops when no further rules can be fired.

Forward Chaining:
Inferencing method



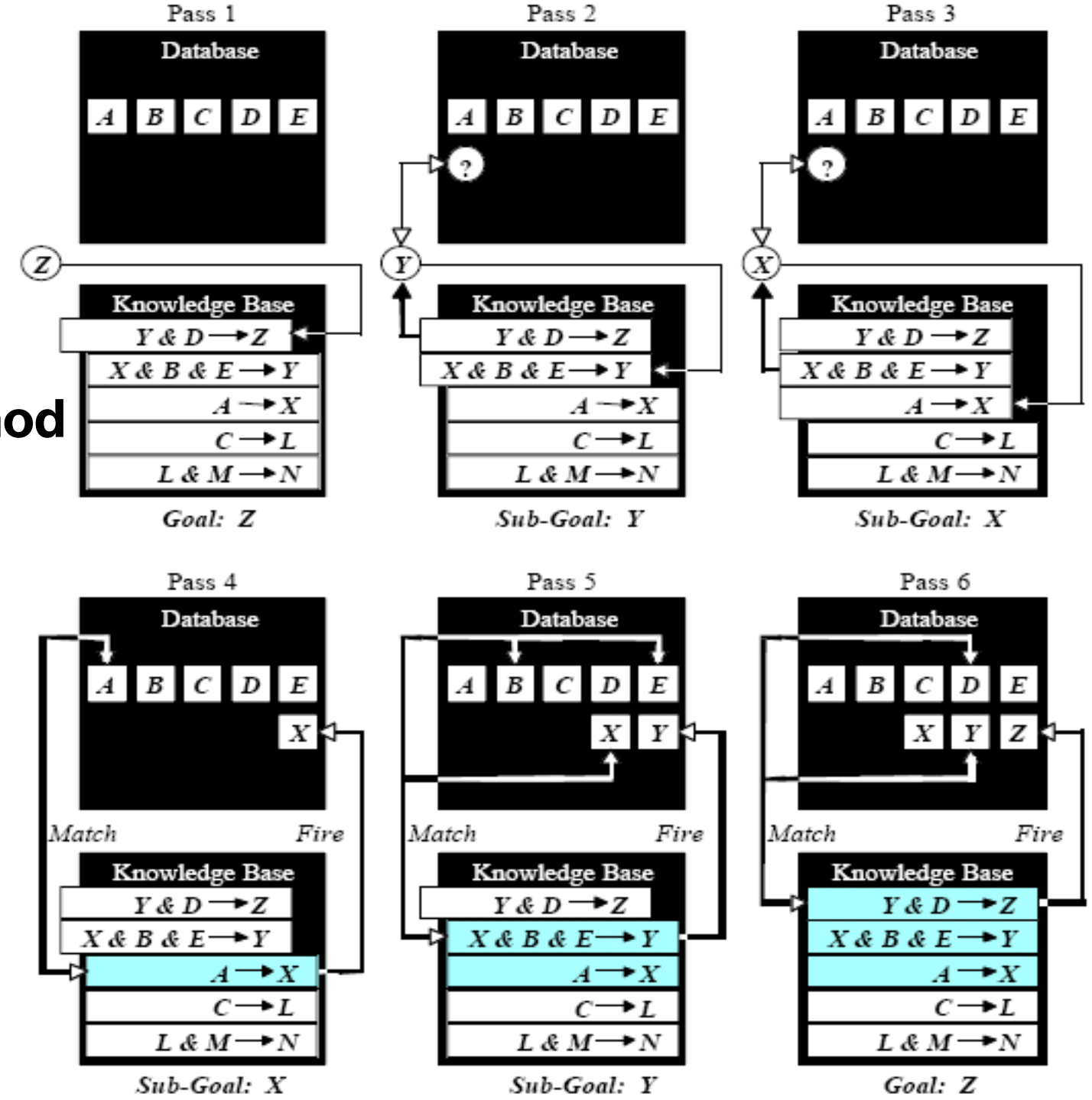
Forward Chaining:
Inferencing method

You are given a set of rules. Assume that an investor has USD 10000 and she is 25 years old. Using forward chaining inference technique, advice whether she should get a pension plan or not.
RULE 1: IF a person has USD 10000 to invest AND has a degree THEN the person should invest in real estate
RULE 2: IF a person's annual income >= USD 50000 AND has a degree THEN the person should invest in stocks
RULE 3: IF a person is younger than 30 AND is investing in real estate THEN the person should also invest in retirement planning
RULE 4: IF a person is younger than 30 AND older than 22 THEN the person has a degree
RULE 5: IF a person wants to invest in retirement planning THEN the retirement planning is the pension plan

Backward Chaining:
Inferencing method

- Backward Chaining:
  - Also known as the **goal-driven reasoning**.
  - ES has the goal and the inference engine attempts to find the evidence to prove it.
  - First, the knowledge base is searched to find rules that might have the desired solution.
    - Such rules must have the goal in their consequent parts.
    - If such a rule is found and its antecedent part matches data in the database, then the rule is fired and the goal is proved.

Backward Chaining:
Inferencing method



Backward Chaining:

Example Using backward chaining inferencing, show that the patient has a strep throat

Observed facts:

- Patient has a temperature of 102
- Patient has been sick for 2 months
- Patient has a sore throat

- *Rule 1*

  If the patient has a sore throat
  and we suspect a bacterial infection
  then we believe the patient has a strep throat
- *Rule 2*

  If the patient's temperature is > 100
  Then the patient has a fever
- *Rule 3*

  If the patient has been sick over a month
  and the patient has a fever
  Then we suspect a bacterial infection

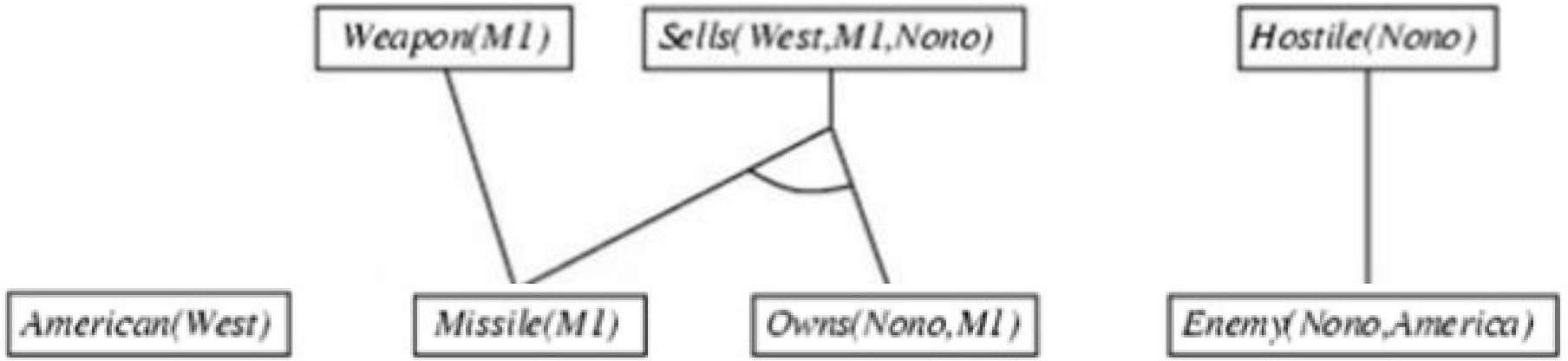
Example

- The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.
- **Query:** “Is Col. West a criminal?”

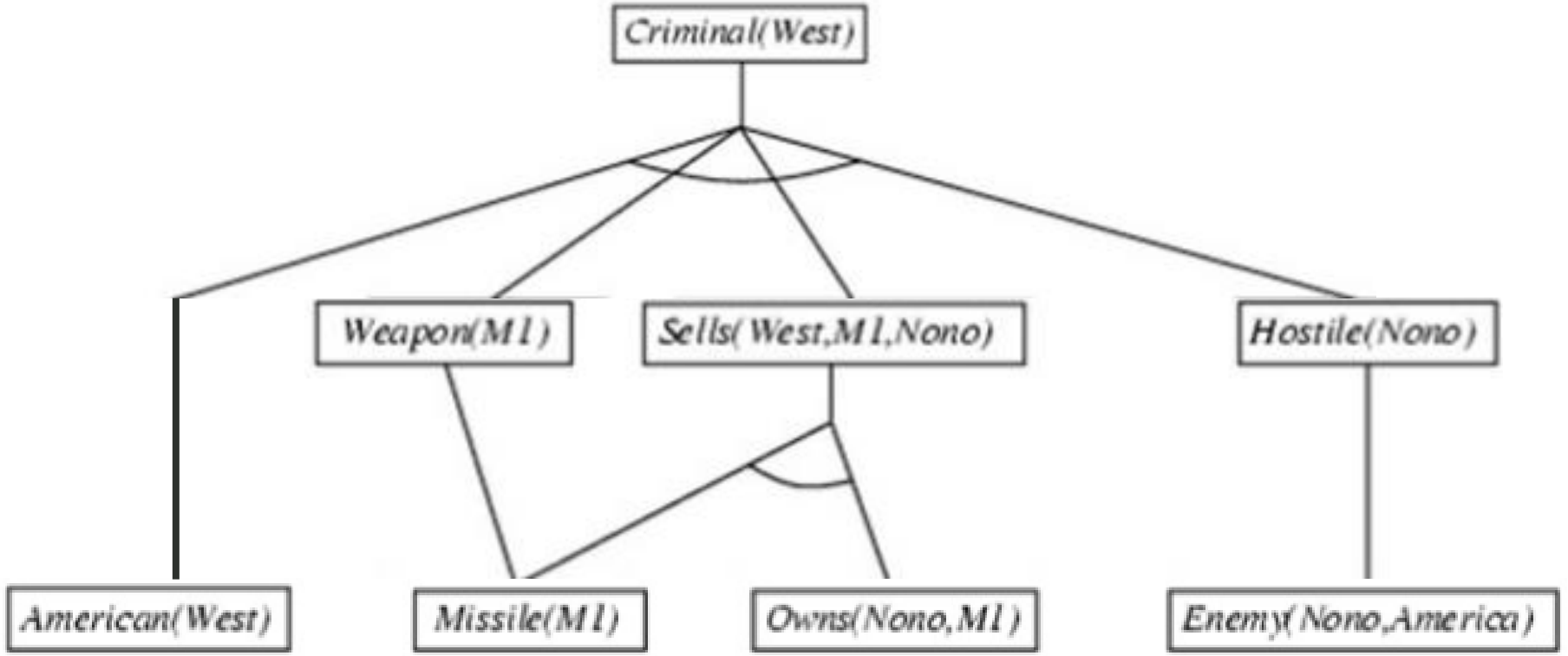
Forward Chaining

Forward Chaining

Forward Chaining



Forward Chaining



Backward Chaining

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Forward or Backward Chaining?

- If an expert first needs to gather some information and then tries to infer from it whatever can be inferred, choose the forward chaining inference engine.
  - Example: DENDRAL
- However, if your expert begins with a hypothetical solution and then attempts to find facts to prove it choose the backward chaining inference engine.
  - Example: MYCIN
- There are many ES shells use a combination of both techniques. However, the common inference method is backward chaining.

Acknowledgments

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  - Norshuhani Zamin, PhD