RULES OF INFERENCE

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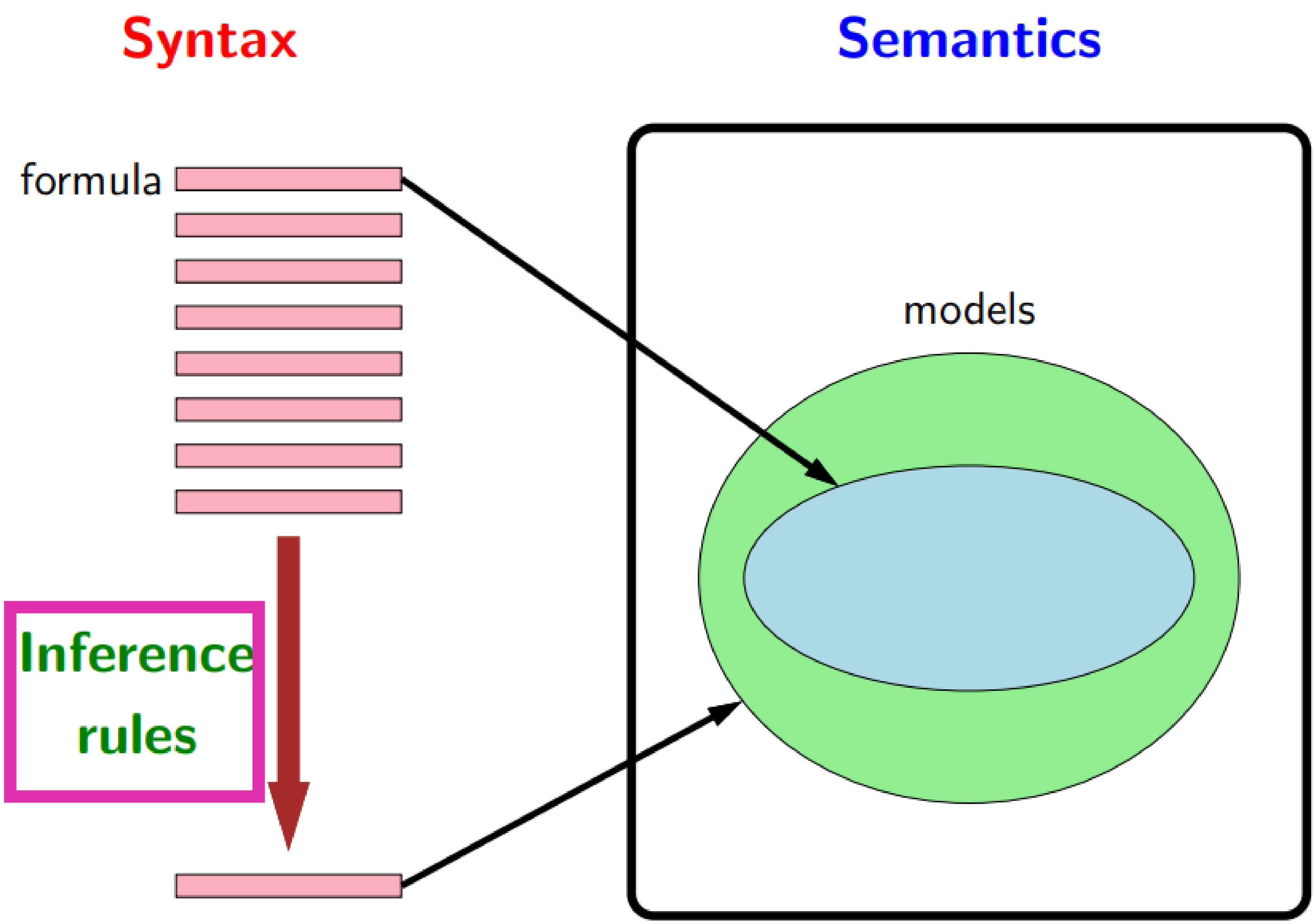
Intelligent Logic-Based Systems

- A smart logic-based system **should be able to make inferences** based on the known facts in the knowledge base.
- **Example:**

KB

Rain
Rain → Traffic

Based on these facts, the knowledge base should be able to derive that **there is traffic**, even without being explicitly told!


Syntax
Semantics
formula
models
Inference rules

Inference Rule

- **Example of making an inference:**

KB

Rain
Rain → Traffic

$$\frac{\text{Rain, Rain} \rightarrow \text{Traffic}}{\text{Traffic}} \qquad \frac{\text{(premises)}}{\text{(conclusion)}}$$

- Inference rules operate directly on **syntax**, not on semantics!

Inference Algorithm

ALGORITHM

Input: set of inference rules

Repeat until no changes to KB:

  Choose set of formulas $f_1, f_2, \dots, f_k \in KB$.

  If matching rule $\frac{f_1, f_2, \dots, f_k}{g}$ exists:

    Add g to KB.

Any rule can be fired only once.

- KB **derives / proves** f ($KB \vdash f$) if and only if f gets added to KB

Modus Ponens Inference Rule

For any propositional symbols $p_1, p_2, \ldots, p_k$ and q:

$$\boxed{\frac{p_1, p_2, p_3, \ldots, p_k, (p_1 \wedge p_2 \wedge \ldots \wedge p_k) \rightarrow q}{q}}$$

Modus Ponens Inference Rule: Example

KB

Rain
Snow
Rain → Traffic
Traffic ∧ Slippery → Late
Snow → Slippery

$$\frac{p_1, p_2, p_3, \ldots, p_k, (p_1 \land p_2 \land \ldots \land p_k) \to q}{q}$$

Modus Ponens Inference Rule: Example

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Snow
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$$\frac{\text{Rain},\ \text{Rain} \rightarrow \text{Traffic}}{\text{Traffic}}$$

$$\frac{p_1, p_2, p_3, \ldots, p_k, (p_1 \land p_2 \land \ldots \land p_k) \rightarrow q}{q}$$

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$$\frac{p_1, p_2, p_3, \ldots, p_k,\ (p_1 \wedge p_2 \wedge \ldots \wedge p_k) \rightarrow q}{q}$$

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$$\frac{\text{Snow},\ \text{Snow} \rightarrow \text{Slippery}}{\text{Slippery}}$$

$$\frac{p_1, p_2, p_3, \ldots, p_k, (p_1 \land p_2 \land \ldots \land p_k) \rightarrow q}{q}$$

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$$\frac{\text{Snow},\ \text{Snow} \rightarrow \text{Slippery}}{\text{Slippery}}$$

$$\frac{p_1, p_2, p_3, \dots, p_k, (p_1 \land p_2 \land \dots \land p_k) \rightarrow q}{q}$$

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$$\frac{p_1, p_2, p_3, \ldots, p_k, (p_1 \land p_2 \land \ldots \land p_k) \rightarrow q}{q}$$

Modus Ponens Inference Rule: Example

KB

Rain
Snow
Rain → Traffic
Traffic ∧ Slippery → Late
Snow → Slippery
Traffic
Slippery

The knowledge base changed, so we need to scan the knowledge base again.

$$\frac{p_1, p_2, p_3, \ldots, p_k, (p_1 \land p_2 \land \ldots \land p_k) \rightarrow q}{q}$$

Modus Ponens Inference Rule: Example

KB

Rain
Snow
Rain → Traffic
Traffic ∧ Slippery → Late
Snow → Slippery
Traffic
Slippery

$$\frac{\text{Traffic, Slippery, Traffic} \land \text{Slippery} \rightarrow \text{Late}}{\text{Late}}$$

$$\frac{p_1, p_2, p_3, \dots, p_k, (p_1 \land p_2 \land \dots \land p_k) \rightarrow q}{q}$$

Modus Ponens Inference Rule: Example

KB

Rain
Snow
Rain → Traffic
Traffic ∧ Slippery → Late
Snow → Slippery
Traffic
Slippery
Late

$$\frac{\text{Traffic, Slippery, Traffic} \land \text{Slippery} \rightarrow \text{Late}}{\text{Late}}$$

$$\frac{p_1, p_2, p_3, \ldots, p_k, (p_1 \land p_2 \land \ldots \land p_k) \rightarrow q}{q}$$

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$$\frac{p_1, p_2, p_3, \ldots, p_k, (p_1 \land p_2 \land \ldots \land p_k) \to q}{q}$$

Modus Ponens Inference Rule: Example

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The knowledge base changed, so we need to scan the knowledge base again.

$$\frac{p_1, p_2, p_3, \ldots, p_k, (p_1 \land p_2 \land \ldots \land p_k) \rightarrow q}{q}$$

Modus Ponens Inference Rule: Example

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Rain
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Rain → Traffic
Traffic ∧ Slippery → Late
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Traffic
Slippery
Late

There are no more sets of formulas that match the inference rule!

$$\frac{p_1, p_2, p_3, \dots, p_k, (p_1 \land p_2 \land \dots \land p_k) \rightarrow q}{q}$$

Modus Ponens Inference Rule: Example

KB

Rain
Snow
Rain → Traffic
Traffic ∧ Slippery → Late
Snow → Slippery
Traffic
Slippery
Late

Converged!

$$\frac{p_1, p_2, p_3, \dots, p_k, (p_1 \land p_2 \land \dots \land p_k) \to q}{q}$$

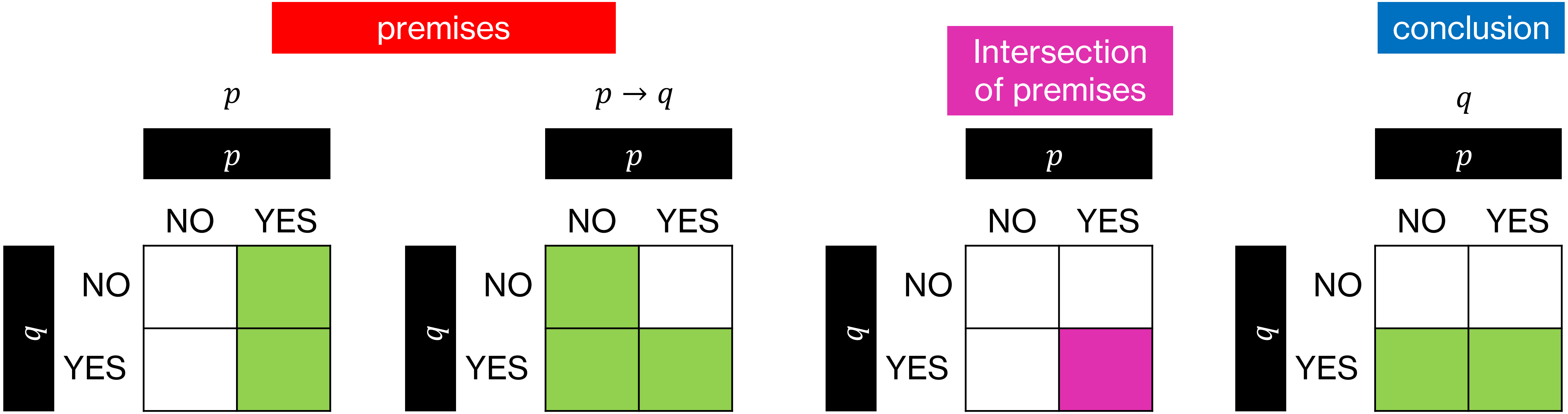
Inference Rule Properties

- **Soundness:** every formula that gets added is **entailed by the KB**
  - "**correctness**" of the derived formulas.

- **Completeness:** the inference rule **can derive every formula in the given form** that is entailed by the KB

Checking for Soundness

- Check for one rule at a time and verify whether **each rule always holds**.
- **Modus ponens is sound** because of the rule that it uses.



Modus Ponens is Incomplete

- Illustrative examples:

KB

Rain
Rain $\vee$ Snow $\rightarrow$ Traffic

Cannot derive anything!
“Traffic” was not derived

KB

Rain
Rain $\rightarrow$ Traffic $\wedge$ Wet

Cannot derive anything!
“Traffic” was not derived
“Wet” was not derived

Addressing Incompleteness

- 2 general strategies
  - **Use more powerful inference rule/s**
  - **Restrict the format of the formulas in the KB**

Literals and Clauses

- **Clause:** is a disjunction of literals.
- **Literals:** a single proposition or its negation

- ***<literal>*** V ***<literal>*** V ***<literal>*** V ... V ***<literal>***

- Note: A single literal is also a clause!

Horn Clause

- A horn clause is a propositional logic formula in a specific form:

A disjunction of literals with **at one most** positive (un-negated) literal

Horn Clause Examples

- Examples of Horn Clauses:
  - $\neg A \lor \neg B \lor \neg C \lor D$
  - $\neg X \lor \neg Y \lor \neg Z$
  - Q

- Examples of non-Horn Clauses:
  - $\neg A \lor \neg B \lor C \lor D$
  - $Q \lor P$

Horn Clause Examples

- **Key Idea:** horn clauses can be re-written as implications!

  - $\neg A \lor \neg B \lor \neg C \lor D$
    - Can be expressed as: $(\boldsymbol{A} \land \boldsymbol{B} \land \boldsymbol{C}) \rightarrow \boldsymbol{D}$
  - $\neg X \lor \neg Y \lor \neg Z$
    - Can be expressed as: $(\boldsymbol{X} \land \boldsymbol{Y} \land \boldsymbol{Z}) \rightarrow \mathbf{0}$
  - Q
    - Can be expressed as: $1 \rightarrow Q$

Modus Ponens Inference Rule

For any propositional symbols $p_1, p_2, \dots, p_k$ and q:

$$\boxed{\frac{p_1, p_2, p_3, \dots, p_k, (p_1 \wedge p_2 \wedge \dots \wedge p_k) \rightarrow q}{q}}$$

Modus Ponens Inference Rule

For any propositional symbols $p_1, p_2, \ldots, p_k$ and q:

$$\boxed{\frac{p_1, p_2, p_3, \ldots, p_k, (p_1 \land p_2 \land \ldots \land p_k) \to q}{q}}$$

Note that these are all horn clauses.

Modus Ponens Inference Rule

- Modus Ponens is **complete** if we restrict the KB to only contain horn clauses!

KB

Rain

$\boxed{\text{Rain} \rightarrow \text{Traffic} \land \text{Wet}}$

This is not a horn clause!

Cannot derive anything!
“Traffic” was not derived
“Wet” was not derived

Modus Ponens Inference Rule

- Modus Ponens is **complete** if we restrict the KB to only contain horn clauses!

KB

Rain

Rain → Traffic ∧ Wet

Cannot derive anything!
"Traffic" was not derived
"Wet" was not derived

Rewrite it into 2 horn clauses!

Rain → (Traffic ∧ Wet)
¬Rain ∨ (Traffic ∧ Wet)
(¬Rain ∨ Traffic) ∧ (¬Rain ∨ Wet)
Rain → Traffic ∧ **Rain → Wet**

Modus Ponens Inference Rule

- Modus Ponens is **complete** if we restrict the KB to only contain horn clauses!

KB

Rain
Rain → Traffic
Rain → Wet

Rewrite it into 2 horn clauses!

Rain → Traffic ∧ Wet
¬Rain ∨ Traffic ∧ Wet
(¬Rain ∨ Traffic) ∧ (¬Rain ∨ Wet)
Rain → Traffic ∧ **Rain → Wet**

Resolution Inference Rule

- Robinson (1965)

$$\frac{f_1 \vee \ldots \vee f_n \vee p, \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

Resolution Inference Rule Example

$$\frac{\text{Rain} \vee \text{Snow}, \neg\text{Snow} \vee \text{Traffic}}{\text{Rain} \vee \text{Traffic}}$$

- If it’s snowing, then there must be traffic.
- If it’s not snowing, then there must be rain.
- Therefore, at least one of Rain and Traffic must be true.

Resolution Inference Rule Example

$$\frac{\neg Hot \vee Lazy,\ Hot}{Lazy}$$

- Hot must be true (because it is a fact).
- If it's hot, then Lazy must be true.
- Therefore, Lazy should be true.

Conjunctive Normal Form (CNF)

- Resolution inference rule requires all formulas to be expressed in **Conjunctive Normal Form (CNF)**.
- Unlike Horn Clauses, **any propositional logic formula can be converted into CNF**.
  - Implication: the expressivity of the language is not diminished.

Conjunctive Normal Form (CNF)

- CNF is a **conjunction of clauses**.
  - ***<clause>*** ∧ ***<clause>*** ∧ ***<clause>*** ∧ ... ∧ ***<clause>***

- **Clause:** is a **disjunction of literals**.
  - ***<literal>*** ∨ ***<literal>*** ∨ ***<literal>*** ∨ ... ∨ ***<literal>***
- **Literals:** a single proposition or its negation

- Example of CNF form:
  - (***<literal>*** ∨ ***<literal>***) ∧ (***<literal>*** ∨ ***<literal>*** ∨ ***<literal>***)

General Steps to Convert to CNF

1. Remove the implications

   $p \rightarrow q \equiv \neg p \vee q$

   $p \wedge q \rightarrow r \equiv \neg p \vee \neg q \vee r$

   $p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$

   $p \wedge q \equiv p , q$ (separate p and q into individual clause)

2. Push negations inside

   $\neg(p \vee q) \equiv \neg p \wedge \neg q$

   $\neg(p \wedge q) \equiv \neg p \vee \neg q$

3. Remove double negations

   $\neg\neg p \equiv p$

4. Distribute ∨ over ∧

   $p \vee (q \vee r) \equiv (p \vee q) \wedge (p \vee r)$

Example Conversion to CNF

$A \vee \neg B \rightarrow \neg C \vee D$ Initial formula

Example Conversion to CNF

$A \vee \neg B \rightarrow \neg C \vee D$	Initial formula
$\neg(A \vee \neg B) \vee (\neg C \vee D)$	Remove implication

Example Conversion to CNF

$A \vee \neg B \rightarrow \neg C \vee D$	Initial formula
$\neg(A \vee \neg B) \vee (\neg C \vee D)$	Remove implication
$(\neg A \wedge \neg\neg B) \vee (\neg C \vee D)$	Push negation inside

Example Conversion to CNF

$A \vee \neg B \rightarrow \neg C \vee D$	Initial formula
$\neg(A \vee \neg B) \vee (\neg C \vee D)$	Remove implication
$(\neg A \wedge \neg\neg B) \vee (\neg C \vee D)$	Push negation inside
$(\neg A \wedge B) \vee (\neg C \vee D)$	Remove double negation

Example Conversion to CNF

$A \vee \neg B \rightarrow \neg C \vee D$	Initial formula
$\neg(A \vee \neg B) \vee (\neg C \vee D)$	Remove implication
$(\neg A \wedge \neg\neg B) \vee (\neg C \vee D)$	Push negation inside
$(\neg A \wedge B) \vee (\neg C \vee D)$	Remove double negation
$(\neg \boldsymbol{A} \vee \neg \boldsymbol{C} \vee \boldsymbol{D}) \wedge (\boldsymbol{B} \vee \neg \boldsymbol{C} \vee \boldsymbol{D})$	Distribute $\vee$ over $\wedge$

Each clause is to be added to the KB separately.

Resolution Inference Rule: Example

KB

Rain
Snow
Rain → Traffic
Traffic ∧ Slippery → Late
Snow → Slippery

$$\frac{f_1 \vee \ldots \vee f_n \vee p, \quad \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

Resolution Inference Rule: Example

KB

Rain
Snow
Rain → Traffic
Traffic ∧ Slippery → Late
Snow → Slippery

Convert to CNF

$$\frac{f_1 \lor \ldots \lor f_n \lor p, \quad \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

Resolution Inference Rule: Example

KB

Rain
Snow
¬Rain ∨ Traffic
Traffic ∧ Slippery → Late
Snow → Slippery

$$\frac{f_1 \vee \ldots \vee f_n \vee p, \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

Resolution Inference Rule: Example

KB

Rain
Snow
¬Rain ∨ Traffic
$\boxed{\text{Traffic} \land \text{Slippery} \rightarrow \text{Late}}$ → Convert to CNF
Snow → Slippery

$$\frac{f_1 \lor \ldots \lor f_n \lor p, \quad \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

Resolution Inference Rule: Example

KB

Rain
Snow
¬Rain ∨ Traffic
¬Traffic ∨ ¬Slippery ∨ Late
Snow → Slippery

$$\frac{f_1 \lor \ldots \lor f_n \lor p, \quad \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

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Convert to CNF

$$\frac{f_1 \lor \ldots \lor f_n \lor p, \quad \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

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¬ Snow ∨ Slippery

$$\frac{f_1 \vee \ldots \vee f_n \vee p,\ \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

Resolution Inference Rule: Example

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Rain
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¬Traffic ∨ ¬Slippery ∨ Late
¬ Snow ∨ Slippery

$$\frac{\text{Rain},\ \neg\text{Rain} \lor \text{Traffic}}{\text{Traffic}}$$

$$\frac{f_1 \lor \ldots \lor f_n \lor p,\ \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

Resolution Inference Rule: Example

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Traffic

$$\frac{\text{Rain},\ \neg\text{Rain} \lor \text{Traffic}}{\text{Traffic}}$$

$$\frac{f_1 \lor \dots \lor f_n \lor p,\ \neg p \lor g_1 \lor \dots \lor g_n}{f_1 \lor \dots \lor f_n \lor g_1 \lor \dots \lor g_n}$$

Resolution Inference Rule: Example

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Traffic

$$\frac{f_1 \vee \ldots \vee f_n \vee p, \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

Resolution Inference Rule: Example

KB

Rain
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¬Rain ∨ Traffic
¬Traffic ∨ ¬Slippery ∨ Late
¬ Snow ∨ Slippery
Traffic

$$\frac{\text{Snow},\ \neg\,\text{Snow} \lor \text{Slippery}}{\text{Slippery}}$$

$$\frac{f_1 \lor \ldots \lor f_n \lor p,\ \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

Resolution Inference Rule: Example

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Slippery

$$\frac{f_1 \vee \ldots \vee f_n \vee p, \quad \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

Resolution Inference Rule: Example

KB

Rain
Snow
¬Rain ∨ Traffic
¬Traffic ∨ ¬Slippery ∨ Late
¬ Snow ∨ Slippery
Traffic
Slippery

The knowledge base changed, so we need to scan the knowledge base again.

$$\frac{f_1 \vee \ldots \vee f_n \vee p, \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

Resolution Inference Rule: Example

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Rain
Snow
¬Rain ∨ Traffic
¬Traffic ∨ ¬Slippery ∨ Late
¬ Snow ∨ Slippery
Traffic
Slippery

$$\frac{\text{Traffic},\ \neg\text{Traffic} \lor \neg\text{Slippery} \lor \text{Late}}{\neg\text{Slippery} \lor \text{Late}}$$

$$\frac{f_1 \lor \ldots \lor f_n \lor p,\ \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

Resolution Inference Rule: Example

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Rain
Snow
¬Rain ∨ Traffic
¬Traffic ∨ ¬Slippery ∨ Late
¬ Snow ∨ Slippery
Traffic
Slippery
¬Slippery ∨ Late

$$\frac{f_1 \vee \ldots \vee f_n \vee p, \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

Resolution Inference Rule: Example

KB

Rain
Snow
¬Rain ∨ Traffic
¬Traffic ∨ ¬Slippery ∨ Late
¬ Snow ∨ Slippery
Traffic
Slippery
¬Slippery ∨ Late

$$\frac{\text{Slippery},\ \neg\text{Traffic} \lor \neg\text{Slippery} \lor \text{Late}}{\neg\text{Traffic} \lor \text{Late}}$$

$$\frac{f_1 \lor \ldots \lor f_n \lor p,\ \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

Resolution Inference Rule: Example

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Traffic
Slippery
¬Slippery ∨ Late
¬Traffic ∨ Late

$$\frac{f_1 \lor \ldots \lor f_n \lor p, \quad \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

Resolution Inference Rule: Example

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Traffic
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¬Slippery ∨ Late
¬Traffic ∨ Late

The knowledge base changed, so we need to scan the knowledge base again.

$$\frac{f_1 \vee \ldots \vee f_n \vee p, \ \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

Resolution Inference Rule: Example

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Slippery
¬Slippery ∨ Late
¬Traffic ∨ Late

$$\frac{\text{Traffic},\ \neg\text{Traffic} \vee \text{Late}}{\text{Late}}$$

$$\frac{f_1 \vee \ldots \vee f_n \vee p,\ \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

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¬Slippery ∨ Late
¬Traffic ∨ Late
Late

$$\frac{f_1 \lor \ldots \lor f_n \lor p, \quad \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

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Traffic
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¬Traffic ∨ Late
Late

$$\frac{\text{Slippery}, \neg\text{Slippery} \lor \text{Late}}{\text{Late}}$$

$$\frac{f_1 \lor \ldots \lor f_n \lor p, \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

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$$\frac{f_1 \vee \ldots \vee f_n \vee p, \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

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¬ Snow ∨ Slippery
Traffic
Slippery
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¬Traffic ∨ Late
Late

The knowledge base changed, so we need to scan the knowledge base again.

$$\frac{f_1 \lor \ldots \lor f_n \lor p, \neg p \lor g_1 \lor \ldots \lor g_n}{f_1 \lor \ldots \lor f_n \lor g_1 \lor \ldots \lor g_n}$$

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Slippery
¬Slippery ∨ Late
¬Traffic ∨ Late
Late

Converged!

$$\frac{f_1 \vee \ldots \vee f_n \vee p, \neg p \vee g_1 \vee \ldots \vee g_n}{f_1 \vee \ldots \vee f_n \vee g_1 \vee \ldots \vee g_n}$$

Resolution Inference Rule

- Resolution Inference rule is **sound** and **complete**.
- It requires formulas to be converted into CNF first.
- Any propositional logic formula can be converted into CNF.

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- Stanford University CS221 Autumn 2021 course. Available online at: https://stanford-cs221.github.io/autumn2021
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