# STALGCM Exam #2 Reviewer

**Prepared By:**
Ryan Austin Fernandez (ryan.fernandez@dlsu.edu.ph)

---

1. Given the following three languages:

   - $L_1 = \{\omega \in \{0,1\}^* \mid \omega = (1^*0)^*\}$
   - $L_2 = \{\omega \in \{0,1\}^* \mid \omega = 0^*1(0 \cup 10^*1)^*\}$
   - $L_3 = \{\omega \in \{0,1\}^* \mid \omega = 0(00)^*\}$

   represented by the following DFA's:

| | 0 | 1 |
|---|---|---|
| $\rightarrow A^*$ | A | B |
| $B$ | A | B |

Table 1: DFA for $L_1$

| | 0 | 1 |
|---|---|---|
| $\rightarrow C$ | C | D |
| $D^*$ | D | C |

Table 2: DFA for $L_2$

| | 0 | 1 |
|---|---|---|
| $\rightarrow E$ | F | G |
| $F^*$ | E | G |
| $G$ | G | G |

Table 3: DFA for $L_3$

   Provide the transition table for the $\lambda -$ NFA that recognizes $(\overline{(L_1^R \cup L_3) - L_2})^*$. (10 points)

2. Using the pumping lemma for regular languages, prove that $L = \{\alpha\alpha \mid \alpha \in \{0,1\}^*\}$ is NOT regular. (10 points)

3. Given the grammar $G$ given by:
   $G = (N, T, P, \Sigma)$
   $N = \{A, B, C, D\}$
   $T = \{0, 1\}$
   $P$ :
   $\Sigma \rightarrow A$
   $A \rightarrow 0B11 \mid 0C1$
   $B \rightarrow 0A11 \mid 0D1 \mid 01$
   $C \rightarrow 0A1 \mid 0D11 \mid 011$
   $D \rightarrow 0C11 \mid 0B1$

   For the items below, write TRUE if the string is generated by $G$ and FALSE otherwise (2 points each)

   (a) $000011111$
   (b) $00111$
   (c) $\lambda$
   (d) $00001111$
   (e) $01$

4. Given the language $L = \{\omega \in \{a,b,c\}^* \mid \omega = a^n b^n c b^m a^m \wedge n, m \geq 0\}$, design a Pushdown Automaton (PDA), $M$, that recognizes language $L$. (Provide the state diagram. Follow the constraint $|Q| \leq 5$.) (10 points)

5. Prove that the language $L = \{\omega \in \{0,1\}^* \mid 0^n 1^m \wedge n \neq m\}$ is context-free by:

   (a) Creating a Push-Down Automaton (PDA) that recognizes $L$ (Provide the state diagram. Follow the constraint $|Q| \leq 4$) (10 points), and
   (b) Designing the Context-Free Grammar (CFG) that generates $L$ (Follow the constraints $|N| \leq 5 \wedge |P| \leq 12$). (10 points)

   - Accept: $001, 00111, 0001111, 00011$
   - Reject: $01, 0011, 000111, \lambda$

**- - END OF REVIEWER - -**

# STALGCM Exam #2 Reviewer Answer Key

**Prepared By:**
Ryan Austin Fernandez (ryan.fernandez@dlsu.edu.ph)

---

1. Given the following three languages:

   Answered in 13 minutes, 49 seconds

| | 0 | 1 | $\lambda$ |
|---|---|---|---|
| $\rightarrow q_0^*$ | | | ACE |
| $ACE$ | BCF | HDG | |
| $ACF$ | BCE | HDG | $q_0$ |
| $ACG$ | BCG | HDG | |
| $ADE$ | BDF | HCG | |
| $ADF$ | BDE | HCG | $q_0$ |
| $ADG$ | BDG | HCG | |
| $BCE$ | BCF | BDG | |
| $BCF$ | BCE | BDG | $q_0$ |
| $BCG$ | BCG | BDG | |
| $BDE$ | BDF | BCG | |
| $BDF$ | BDE | BCG | $q_0$ |
| $BDG$ | BDG | BCG | |
| $HCE$ | HCF | HDG | $q_0$ |
| $HCF$ | HCE | HDG | $q_0$ |
| $HCG$ | HCG | HDG | $q_0$ |
| $HDE$ | HDF | HCG | |
| $HDF$ | HDE | HCG | $q_0$ |
| $HDG$ | HDG | HCG | |

Table 4: Transition Table for the $\lambda - NFA$ for $(\overline{(L_1^R \cup L_3) - L_2})^*$

2. Answered in 4 minutes, 41 seconds

   For any positive integer $p$, we can generate a string $0^p1^p0^p1^p \in L$.

   Let:

   $x = 0^\alpha$
   $y = 0^(p - \alpha)$
   $z = 1^p0^p1^p$, where $\alpha < p$

   In this case, $xy^*z$ will generate the language $L' = \{\omega \in \{0,1\}^* \mid \omega = 0^{kp-(k-1)\alpha}1^p0^p1^p, k \geq 0\}$. Since $kp - (k-1)\alpha \neq p$ for all $k \neq 1$, only the string in $L'$ where $k = 1$ is a member of the original language $L$.

   Therefore, $L$ is not regular.

3. Answered in 47 seconds

   (a) TRUE
   (b) TRUE
   (c) FALSE
   (d) FALSE
   (e) TRUE

4. Answered in 6 minutes

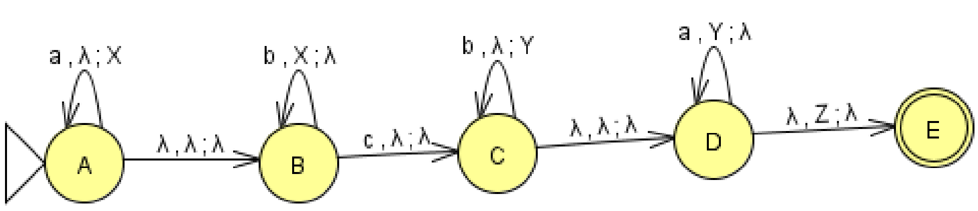


Figure 1: State Diagram for the PDA for #4a

   (a)

5. (a) Answered in 3 minutes, 43 seconds

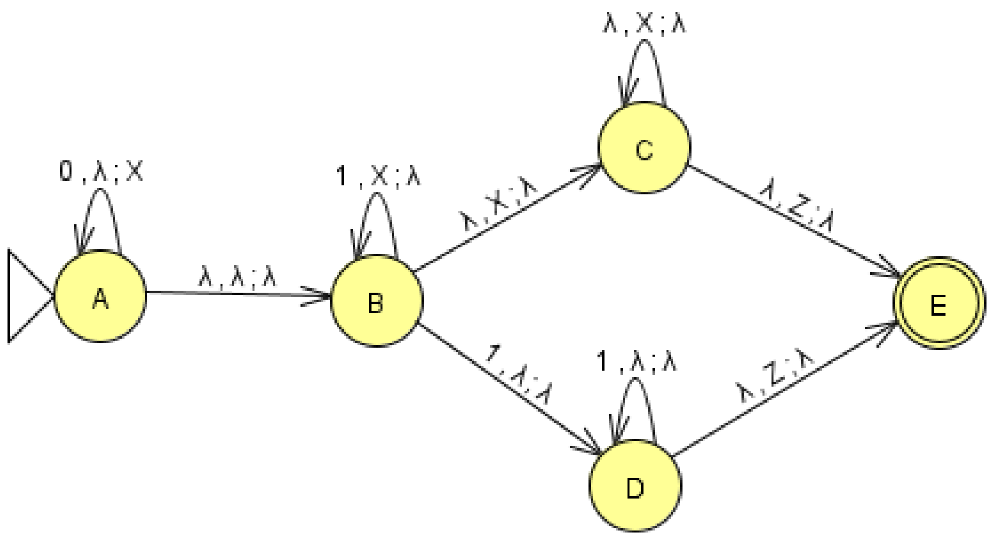


Figure 2: State Diagram for #5a

(b) Answered in 2 minutes, 35 seconds

$G = (N, T, P, \Sigma)$
$N = \{A, B, C, D, E\}$
$T = \{0, 1\}$
$P:$
$\Sigma \to A \mid B$
$A \to CD \mid C$
$B \to DE \mid E$
$C \to 0C \mid 0$
$D \to 0D1 \mid 01$
$E \to 1E \mid 1$

- - **END OF REVIEWER ANSWER KEY** - -