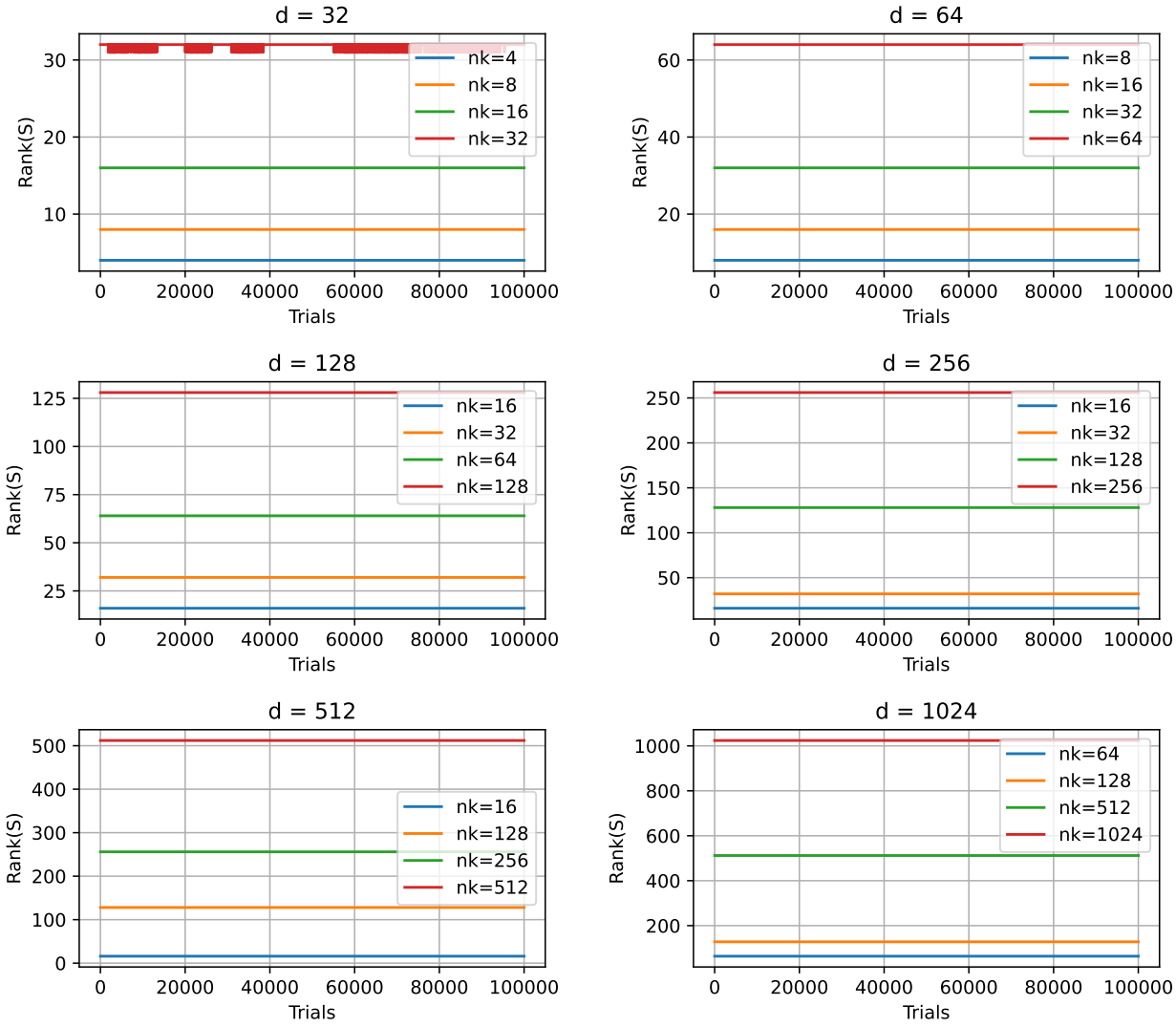

Figure 1: Simulation results of rank($\boldsymbol{S}$), where $\boldsymbol{S} = \sum_{i=1}^{n} \boldsymbol{G}_i^T \boldsymbol{G}_i$, with $\boldsymbol{G}_i$ being SRHT. With $d \in \{32, 64, 128, \ldots, 1024\}$ and 4 different nk values such that $nk \leq d$ for each d, we compute rank($\mathbf{S}$) for 10^5 trials for each pairs of (nk, d) values and plot the results for all trials. When $d = 32$ and $nk = 32$ in the first plot, rank($\boldsymbol{S}$) $= 31$ in 2100 trials, and rank($\boldsymbol{S}$) $= nk = 32$ in all the rest of the trials. For all other (nk, d) pairs, $\boldsymbol{S}$ always has rank nk in the 10^5 trials. This verifies that $\delta = \Pr[\text{rank}(\boldsymbol{S}) < nk] \approx 0$.