



Figure 1: Simulation results of  $\text{rank}(\mathbf{S})$ , where  $\mathbf{S} = \sum_{i=1}^n \mathbf{G}_i^T \mathbf{G}_i$ , with  $\mathbf{G}_i$  being SRHT or  $\mathbf{G}_i$  being the subsampling matrix. We repeat the simulation and compute the rank of  $\mathbf{S}$  1000 times with different  $n$  and  $d$  values. Among all results, it is not hard to observe  $\text{rank}(\mathbf{S})$  is full, i.e.,  $= nk$ , when  $\mathbf{G}_i$  is SRHT, all the time; while  $\text{rank}(\mathbf{S}) < nk$ , when  $\mathbf{G}_i$  is the subsampling matrix, all the time. Since the higher rank  $\mathbf{S}$  is, the lower the MSE is. This is an evidence showing the superior performance of Rand-Proj-Spatial with  $\mathbf{G}_i$  being SRHT compared to the baseline Rand- $k$ -Spatial with  $\mathbf{G}_i$  being subsampling matrix.