X: t时刻 Y: t+1 时刻

$$Z = \begin{bmatrix} X \\ Y \end{bmatrix} \qquad Var(Z) = E(Z) - E^2(Z) = \begin{bmatrix} Var(X) & Cov(X,Y) \\ Cov(Y,X) & Var(Y) \end{bmatrix}$$

$$= \begin{bmatrix} C_0 & B \\ B^t & C \end{bmatrix}$$

discrete: $M_{ji} = P(j|i) = \dfrac{P_{ij}}{\sum_j P_{ij}}$

continuous: $P(Y|X) = \dfrac{P(X,Y;\mu,\Sigma)}{\int_Y P(X,Y;\mu,\Sigma)\,dY}$

$$\Rightarrow Y|X \sim G(Y;\mu_0,\Sigma_0) \qquad (1)$$

where $\mu_0 = E(Y) + A(X - E(X))$

$A = B^t C_0^{-1}$

$\Sigma_0 = C_1 - A C_0 A^t$

$$Y = E(Y) + A(X - E(X)) + n \qquad (2)$$

where n satisfies:

$E(n) = 0$; $Var(n) = \Sigma_0$

$E(n \cdot X^T) = 0$

Special case: ~~sat~~ sationary process

$E(X_i) = Const\ 1$ $\quad C_i = Const\ 2.$ $\quad i = 1, 2, 3 \ldots$

$= Con\ 1$

rewrite (2):

$$Y = AX + n$$

$$Var(Z) = \begin{pmatrix} C & B \\ B^t & C \end{pmatrix}$$

factorization.

$X: \ X^A, X^B \qquad Y: \ Y^A, Y^B$

$$Z_{fac} = \begin{bmatrix} X^A \\ X^B \\ Y^A \\ Y^B \end{bmatrix}; \quad Var(Z_{fac}) = \begin{bmatrix} Var(X^A) & \cdots & \cdots \\ Cov(X^A, X^B) & \cdots & \cdots \\ \vdots & & \\ Cov(Y^B, X^A) & & Var(Y^B) \end{bmatrix}$$

$$Var(Z_{fac}) = \text{perm}(Var(Z))$$

$$M^A(Y^A, X^A) = G(X^A; \hat{A}_A X^A, \hat{\Sigma}_A)$$

$$M^B(Y^B, X^B) = G(X^B; \hat{A}_B X^B, \hat{\Sigma}_B)$$

where $\hat{A}_A = cov(X^A_A, Y^A) Var^{-1}(A)$

$$\hat{\Sigma}_A = Var(A) - cov(X^A, Y^A) Var^{-1}(A) cov(Y^A, X^A)$$

$$\hat{Y} = \begin{pmatrix} Y^A \\ Y^B \end{pmatrix} \doteq \begin{bmatrix} \hat{A}_A & 0 \\ 0 & \hat{A}_B \end{bmatrix} \begin{bmatrix} X^A \\ X^B \end{bmatrix} + \hat{n}$$

where $\hat{n}$ satisfies $\begin{cases} Var(\hat{n}_A) = \Sigma_A \\ E(\hat{n}_A) = 0 \end{cases}$