

A 3-player game theoretic model of a choice  
between two queueing systems with strategic  
managerial decision making

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# About me



About me

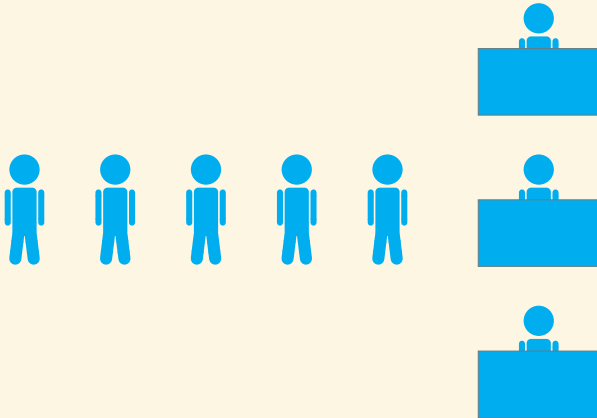


THIS.

# Queues - Examples



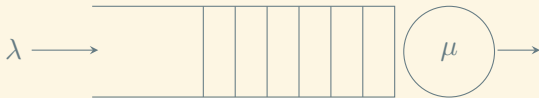
# Queues - Examples



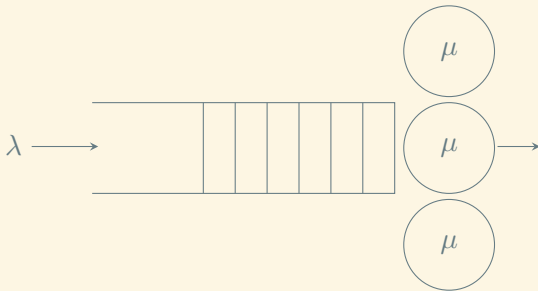
# Queues - Examples



## Queues - Discrete Event Simulation

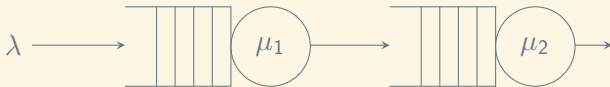


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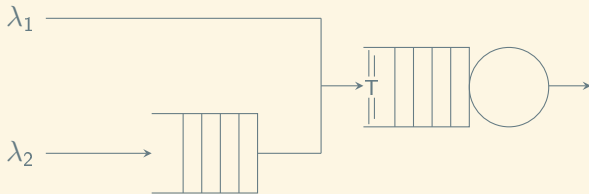




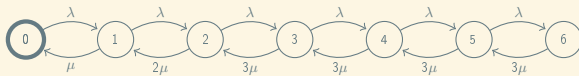
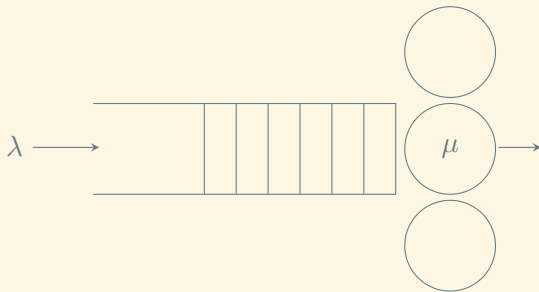
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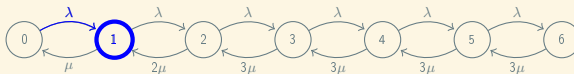
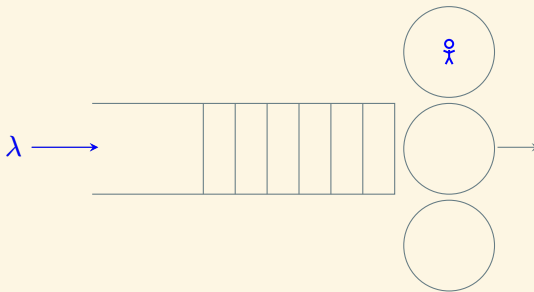
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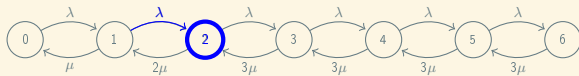
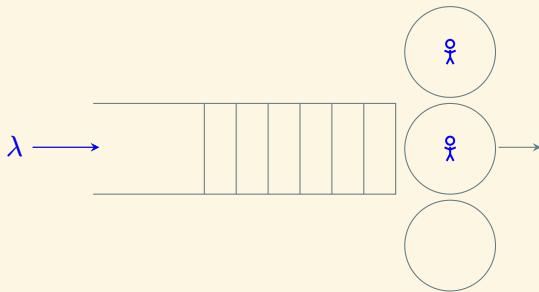
# Queues - Markov Chain



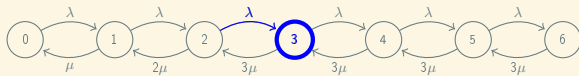
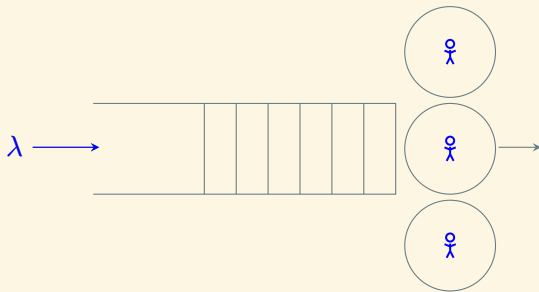
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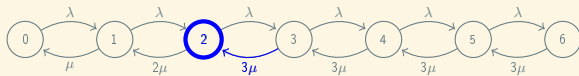
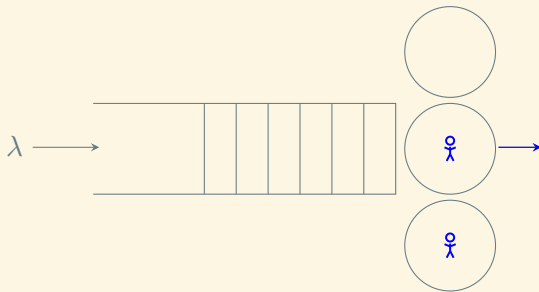
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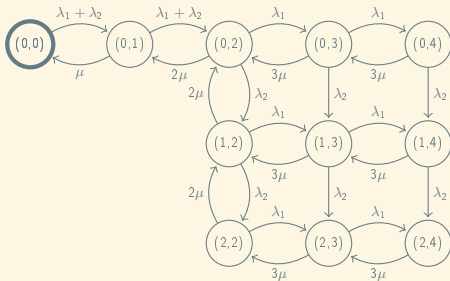
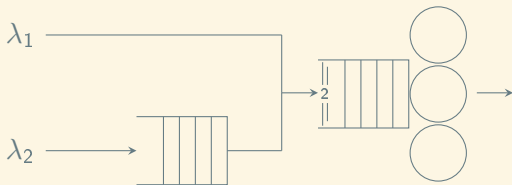
# Queues - Markov Chain



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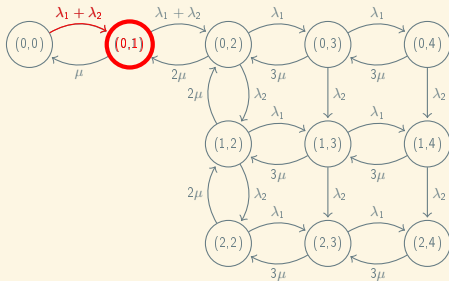
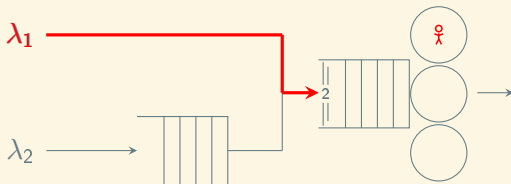


# Analytical formulation - Custom Queue

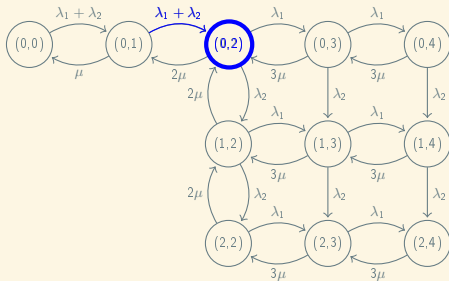
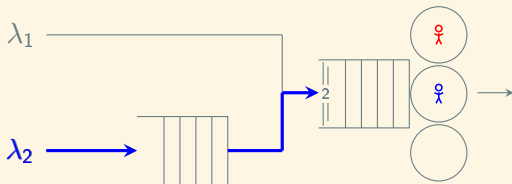




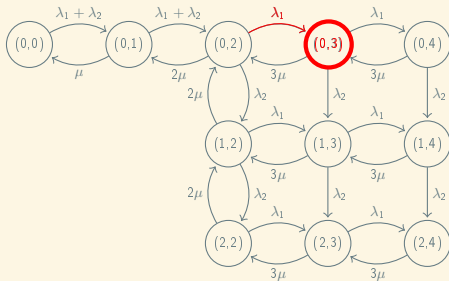
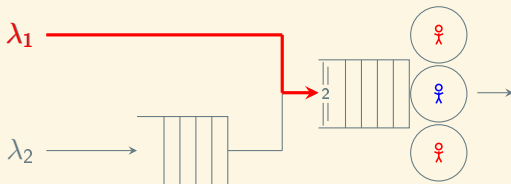
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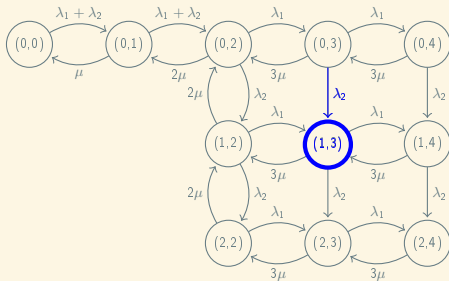
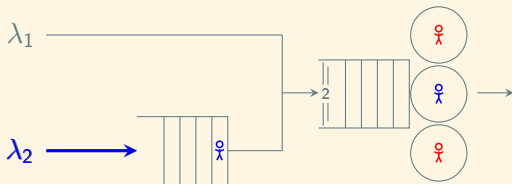
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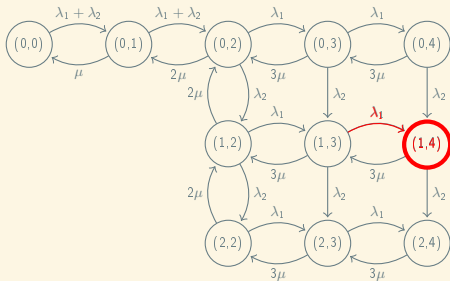
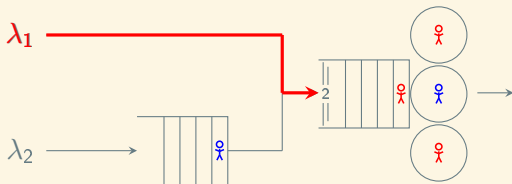
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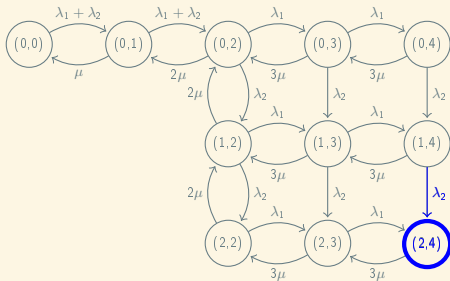
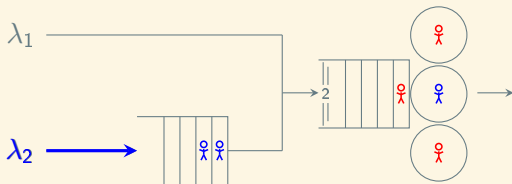
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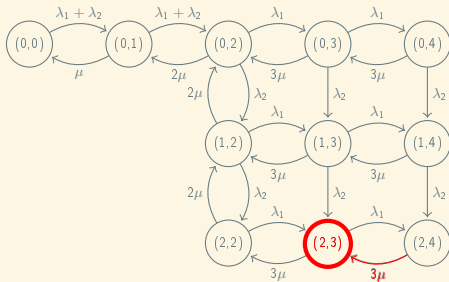
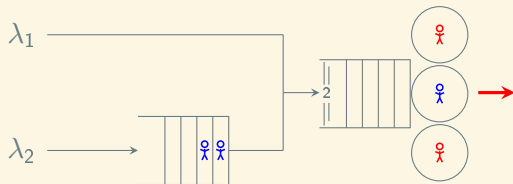
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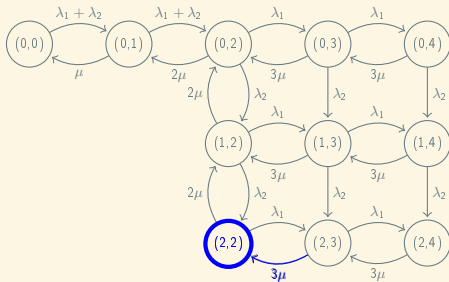
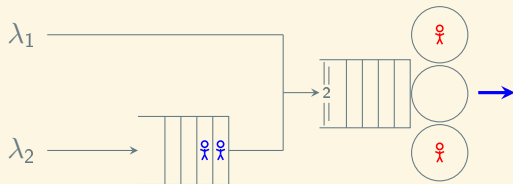
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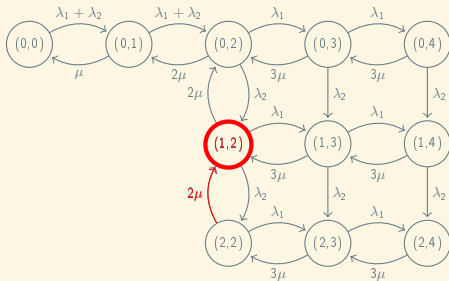
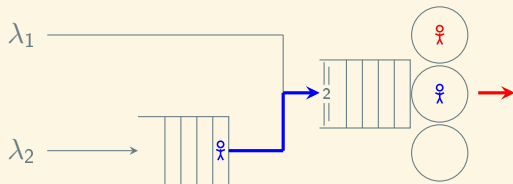


# Analytical formulation - Custom Queue

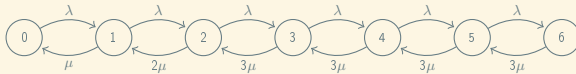




# Analytical formulation - Custom Queue



# Steady state probabilities - M|M|3 queue



$$Q = \begin{matrix} & \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{matrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix} & \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & 2\mu & -2\mu - \lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & 3\mu & -3\mu - \lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & 3\mu & -3\mu - \lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & 3\mu & -3\mu - \lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & 3\mu & -3\mu \end{pmatrix} \end{matrix}$$

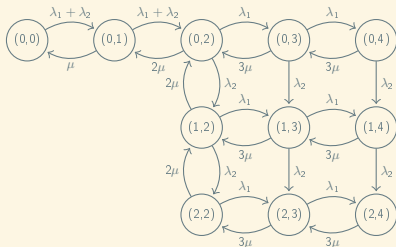
$$\frac{d\pi}{dt} = \pi Q = 0$$

$$\sum \pi_i = 1$$

$$\pi = \begin{pmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \\ \pi_6 \end{pmatrix}$$

# Steady state probabilities - Custom queue

$$Q = \begin{matrix} & \begin{matrix} (0,0) & (0,1) & (0,2) & \dots & (2,3) & (2,4) \end{matrix} \\ \begin{matrix} (0,0) \\ (0,1) \\ (0,2) \\ \vdots \\ (2,3) \\ (2,4) \end{matrix} & \begin{pmatrix} -\lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & 0 & \dots & 0 & 0 \\ \mu & -\mu - \lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & \dots & 0 & 0 \\ 0 & 2\mu & -2\mu - \lambda_1 - \lambda_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda_1 - 3\mu & \lambda_1 \\ 0 & 0 & 0 & \dots & 3\mu & -3\mu \end{pmatrix} \end{matrix}$$



$$\frac{d\pi}{dt} = \pi Q = 0, \quad \sum \pi_{(u,v)} = 1$$

$$\pi = \begin{bmatrix} \pi(0,0) \\ \pi(0,1) \\ \pi(0,2) \\ \vdots \\ \pi(2,3) \\ \pi(2,4) \end{bmatrix}$$

# Steady state probabilities - Comparison

