

Annual Review

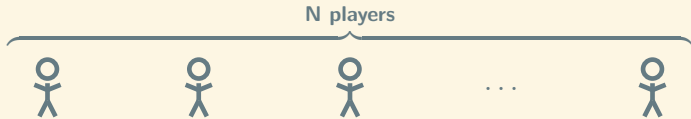
Michalis Panayides

2020-06-10

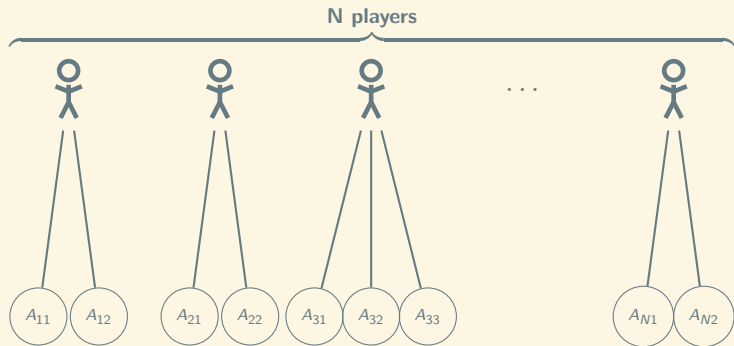
Game Theory - Syllabus

- ▶ Normal Form Games
- ▶ Mixed-Strategy Nash Equilibrium
- ▶ Alternate Solution Concepts
- ▶ Extensive-Form Games
- ▶ Repeated Games (TBC)
- ▶ Bayesian Games (TBC)
- ▶ Coalitional Games (TBC)

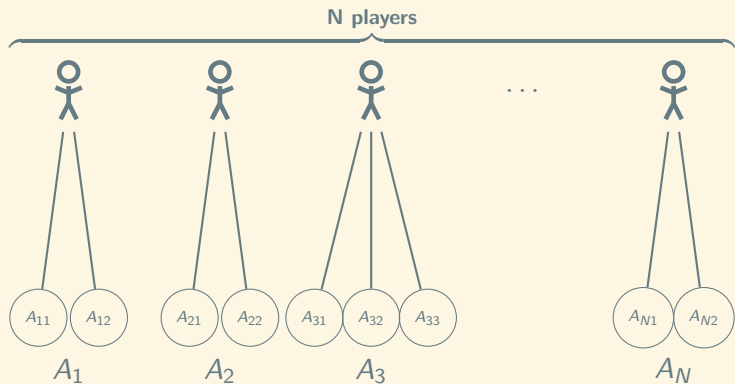
Normal Form Games



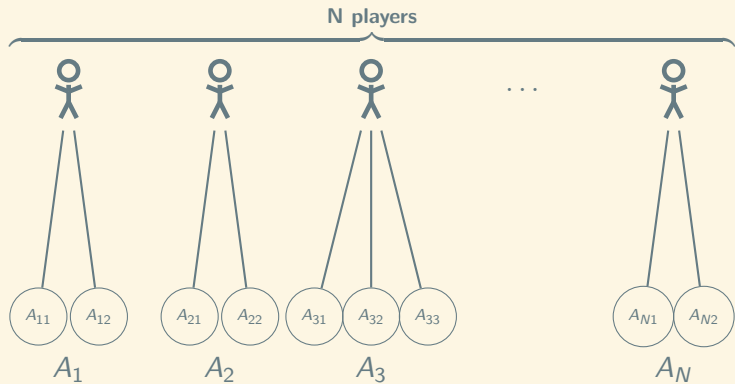
Normal Form Games



Normal Form Games









Normal Form Games



$$u_i = A_1 \times A_2 \times A_3 \times \cdots \times A_N$$

Rock-Paper-Scissors

			
	$(0, 0)$	$(1, -1)$	$(-1, 1)$
	$(-1, 1)$	$(0, 0)$	$(1, -1)$
	$(1, -1)$	$(-1, 1)$	$(0, 0)$

Nash Equilibrium

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

Nash Equilibrium

$$\begin{array}{l} \text{C} \longrightarrow \\ \text{D} \longrightarrow \end{array} \left[\begin{array}{cc} (3, -) & (0, -) \\ (\underline{5}, -) & (\underline{1}, -) \end{array} \right]$$

Nash Equilibrium

C	D
\downarrow	\downarrow
$\begin{bmatrix} (-, 3) \\ (-, 0) \end{bmatrix}$	$\begin{bmatrix} (-, \underline{5}) \\ (-, \underline{1}) \end{bmatrix}$

Nash Equilibrium

A 2x2 normal form game matrix. The horizontal axis is labeled 'D' with a red arrow pointing to the matrix. The vertical axis is labeled 'D' with a red arrow pointing to the matrix. The matrix contains four payoff pairs: (3, 3) at the top-left, (0, 5) at the top-right, (5, 0) at the bottom-left, and (1, 1) at the bottom-right. The underlined values 5, 1, and 1 indicate the best responses for each player, identifying the Nash Equilibria at (5, 0) and (0, 5).

	D	
D	$\begin{bmatrix} (3, 3) & (0, \underline{5}) \\ (\underline{5}, 0) & (\underline{1}, \underline{1}) \end{bmatrix}$	

Pareto Optimality

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

$$\overbrace{(3, 3), (0, 5), (5, 0), (1, 1)}$$

Pareto Optimality

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

$$\overbrace{(3, 3), (0, 5), (5, 0), (1, 1)}$$

$$(\underline{3}, \underline{3}) > (1, 1)$$

Computing the Nash Equilibria

- ▶ Lemke-Howson Algorithm
- ▶ Support Enumeration
- ▶ Iterative removal of strictly dominated strategies

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	L	C	R
U	(3, 0)	(2, 1)	(0, 0)
M	(1, 1)	(1, 1)	(5, 0)
D	(0, 1)	(4, 2)	(0, 1)

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	L	C	R
U	(3, 0)	(2, 1)	(0, 0)
M	(1, 1)	(1, 1)	(5, 0)
D	(0, 1)	(4, 2)	(0, 1)

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	L	C
U	(3, 0)	(2, 1)
M	(1, 1)	(1, 1)
D	(0, 1)	(4, 2)

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	L	C
U	(3, 0)	(2, 1)
M	(1, 1)	(1, 1)
D	(0, 1)	(4, 2)

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	L	C
U	$(3, 0)$	$(2, 1)$
D	$(0, 1)$	$(4, 2)$

Iterative Removal of Strictly Dominated Strategies

$P1 \backslash P2$	L	C
U	$(3, 0)$	$(2, 1)$
D	$(0, 1)$	$(4, 2)$

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	C
U	$(2, 1)$
D	$(4, 2)$

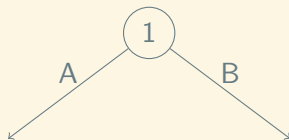
Iterative Removal of Strictly Dominated Strategies

$P1 \backslash P2$	C
U	$(2, 1)$
D	$(4, 2)$

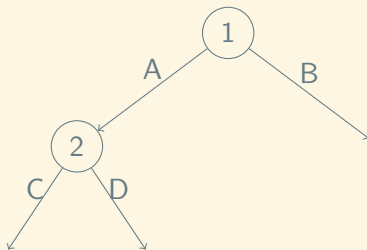
Iterative Removal of Strictly Dominated Strategies

$$\begin{array}{cc} P1 \setminus P2 & C \\ D & (4, 2) \end{array}$$

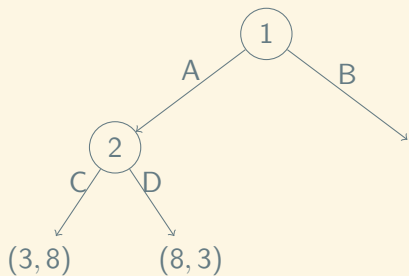
Perfect Information Extensive Form Games



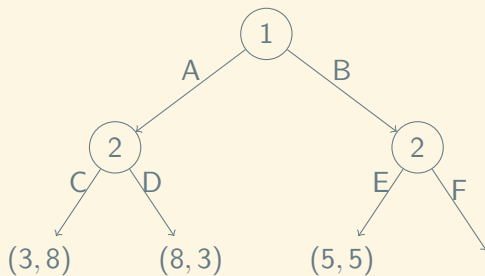
Perfect Information Extensive Form Games



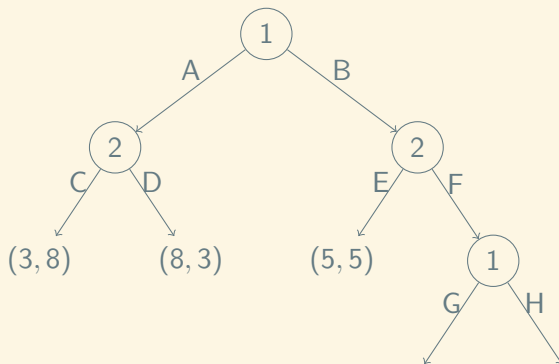
Perfect Information Extensive Form Games



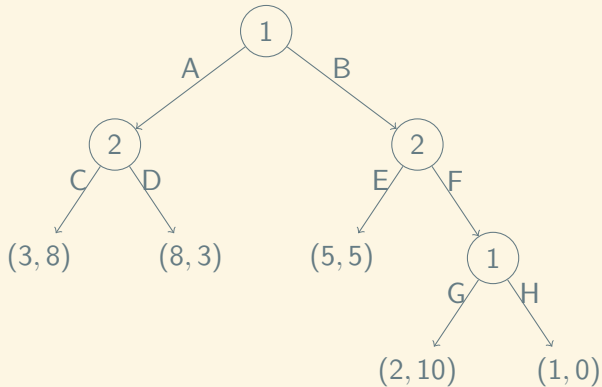
Perfect Information Extensive Form Games



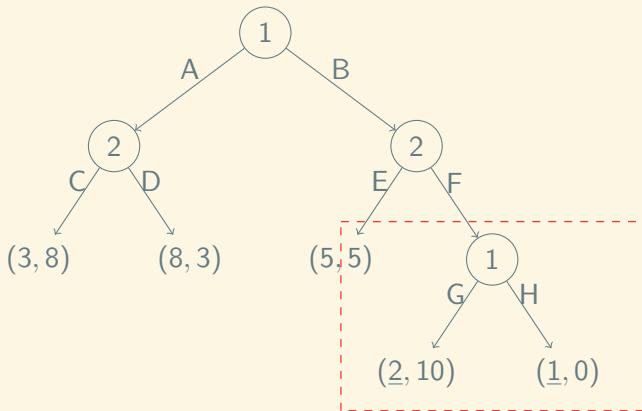
Perfect Information Extensive Form Games



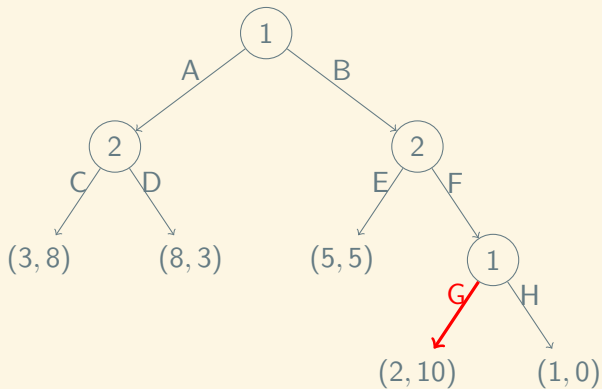
Perfect Information Extensive Form Games



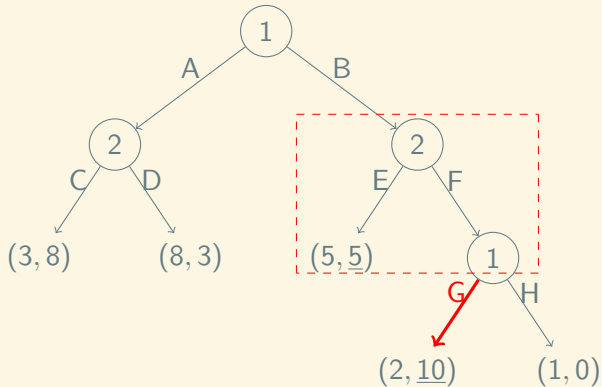
Backwards Induction



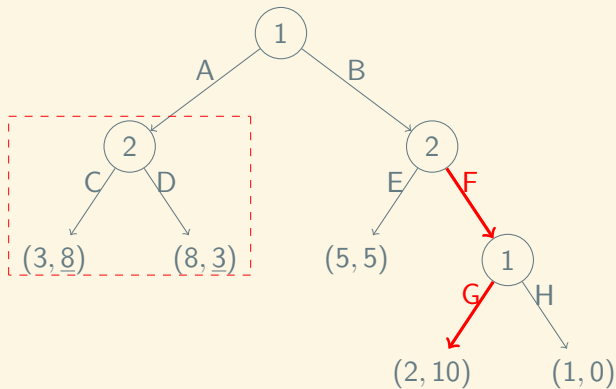
Backwards Induction



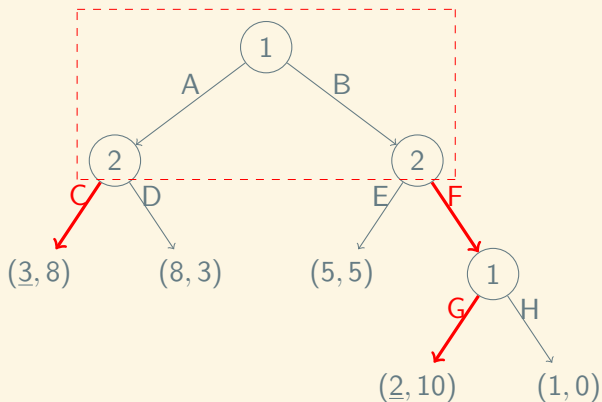
Backwards Induction



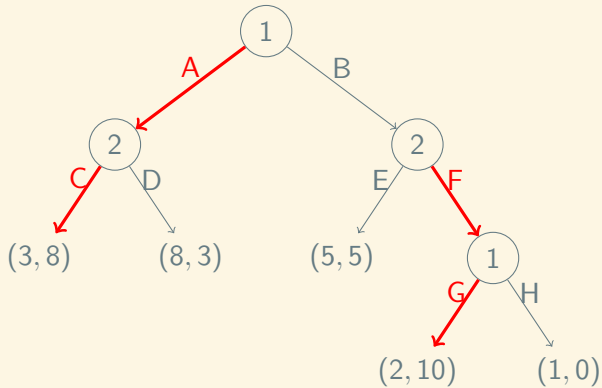
Backwards Induction



Backwards Induction



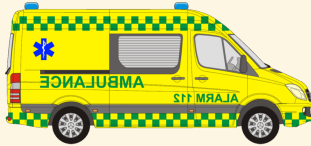
Backwards Induction



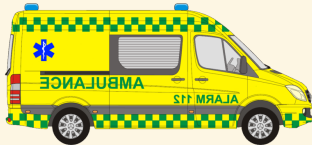
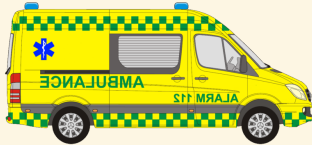
Imperfect Information Extensive-form Games



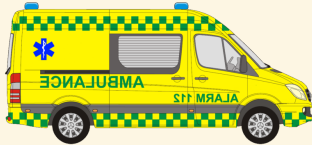
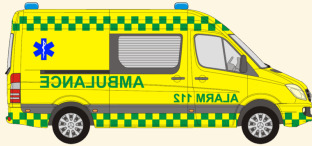
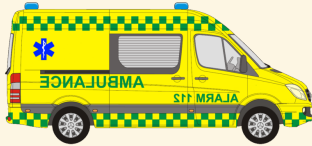
PhD - Motivation



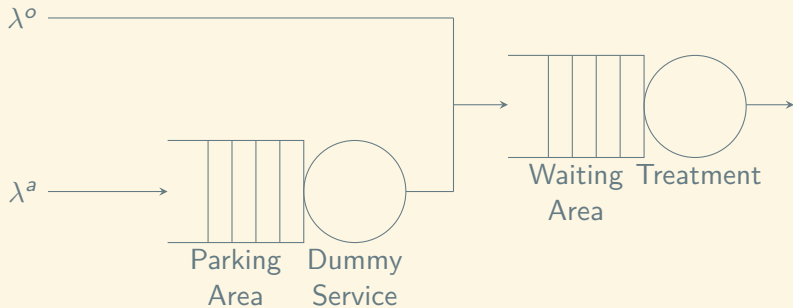
PhD - Motivation



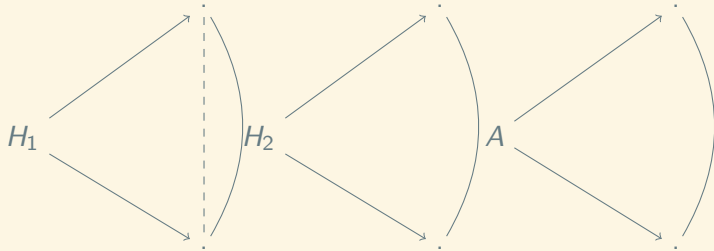
PhD - Motivation



Hospital Formulation



Ambulance - Hospital Interface



Hospital - Markov Chain

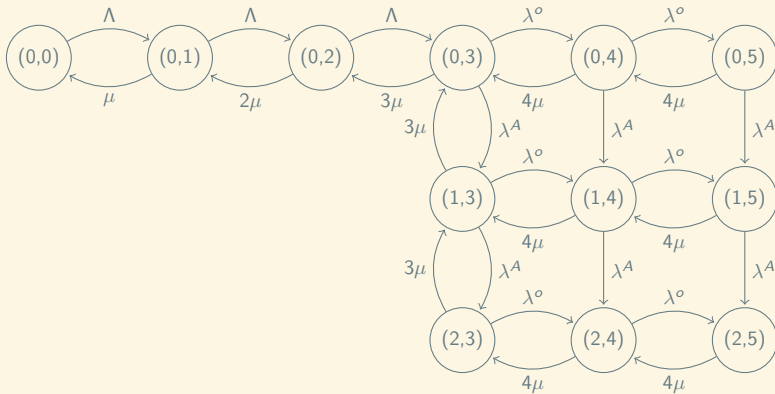
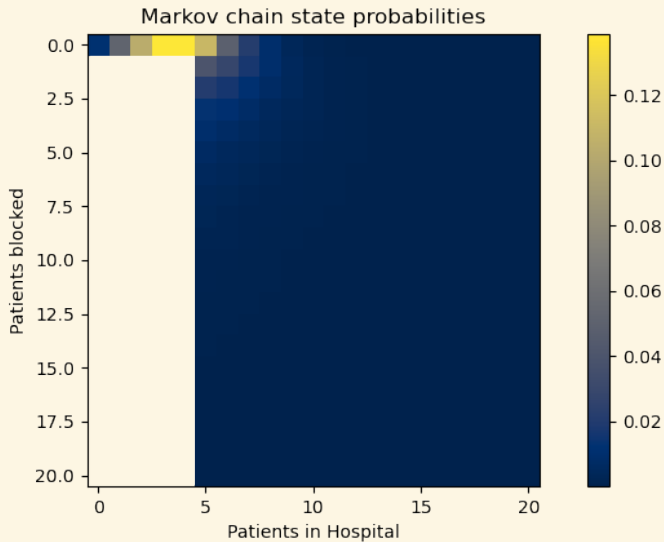
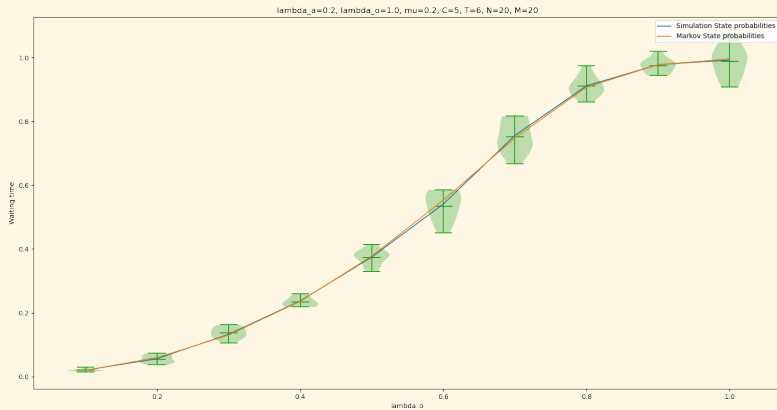


Figure: $C=4$, $T=3$, $N=5$, $M=2$

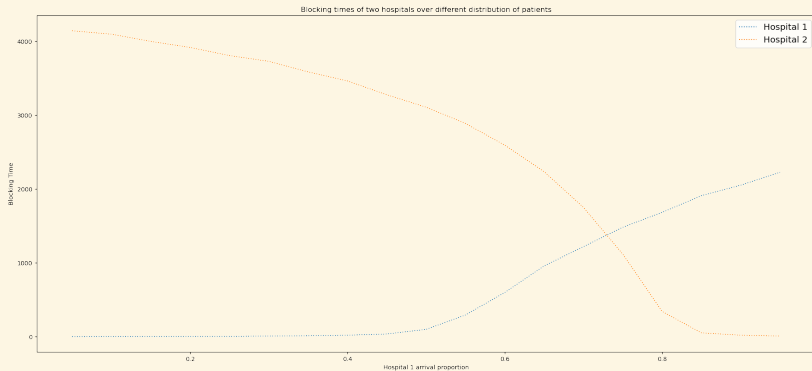
State Probabilities



Waiting Times



Optimal patient distribution



Future Plans

- ▶ Performance Measures
- ▶ Game theoretic interface