# A 3-player game theoretic model of a choice between two queueing systems with strategic managerial decision making

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# About me

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# Queues - Examples

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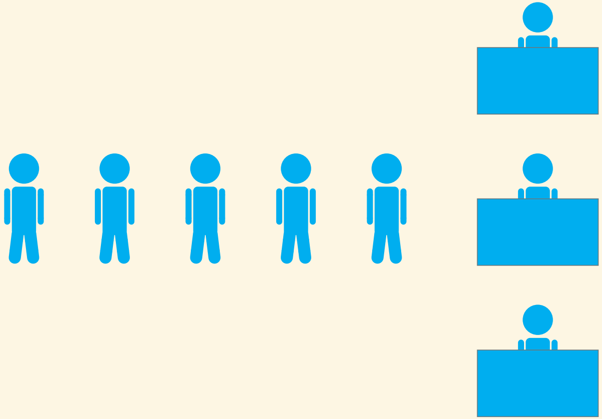
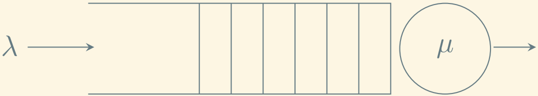
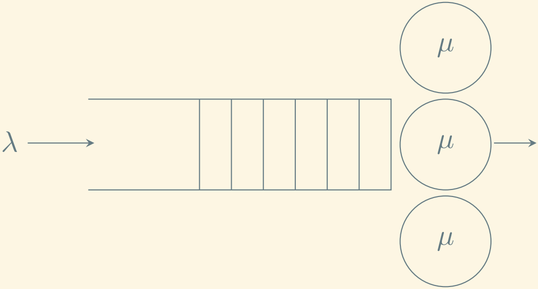


# Queues - Examples

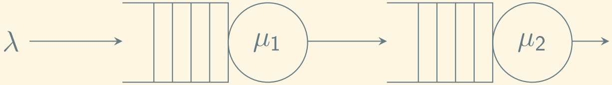
# Discrete Event Simulation - M|M|1



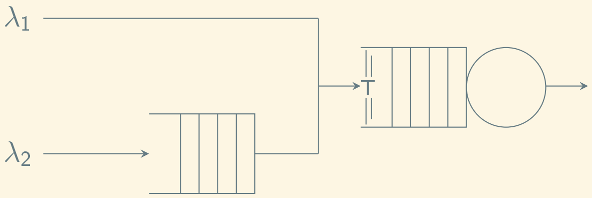
# Discrete Event Simulation - M|M|3



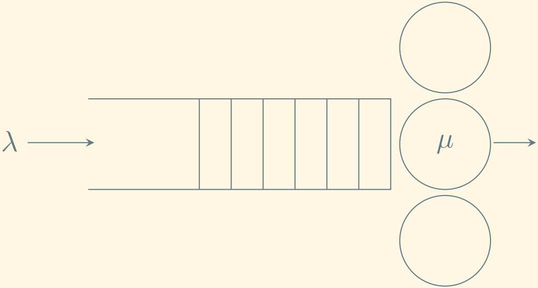
# Discrete Event Simulation - Network of queues

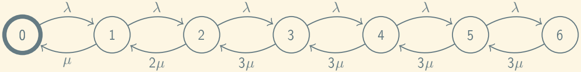


# Discrete Event Simulation - Custom network



# Markov Chain - M|M|3



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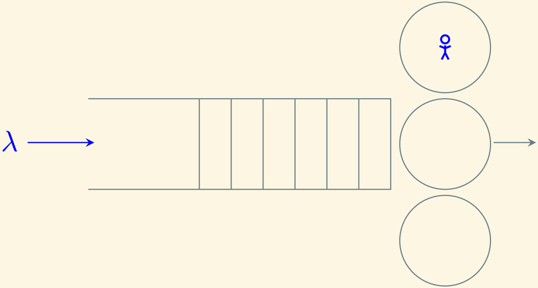


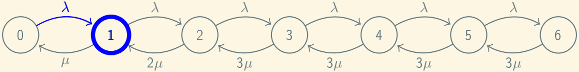

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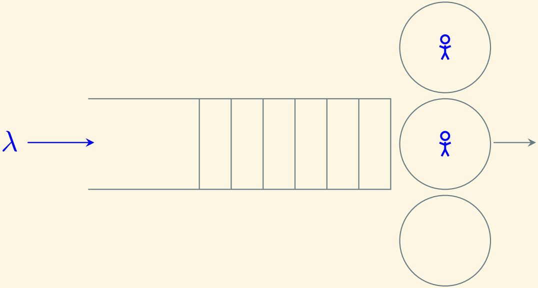


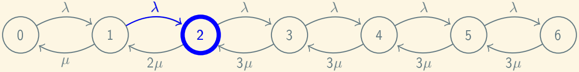

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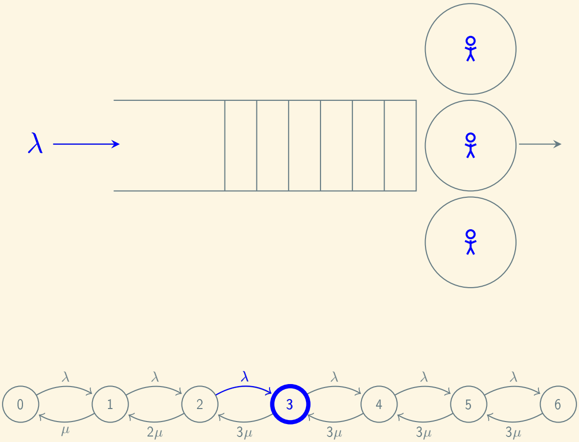

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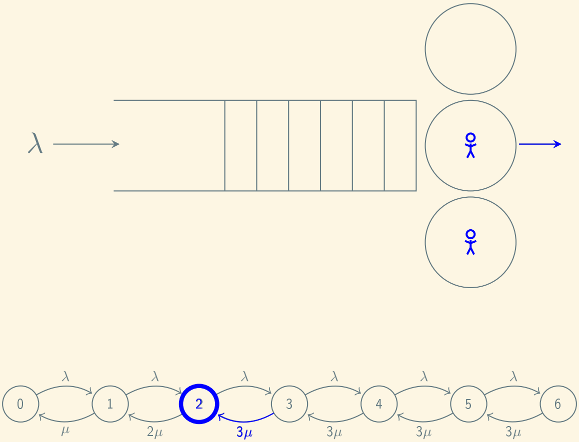
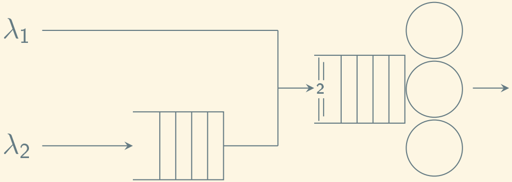
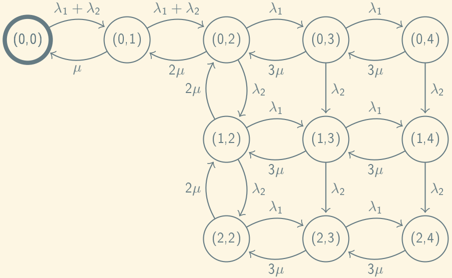


# Markov Chain - Custom network



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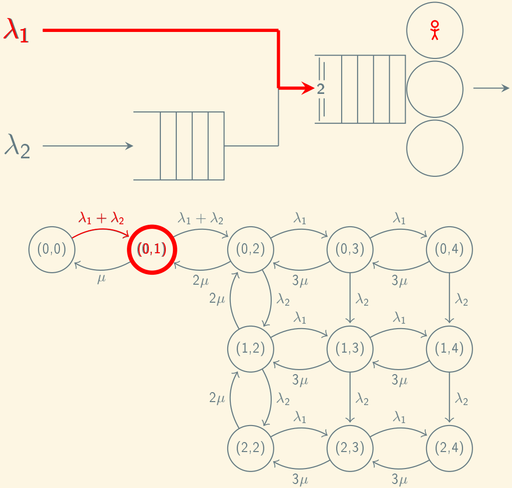

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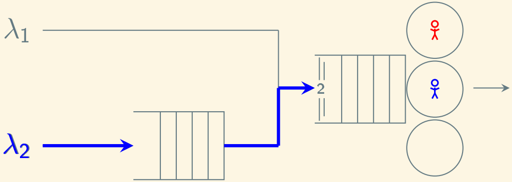


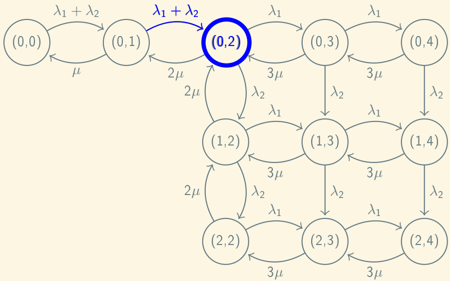

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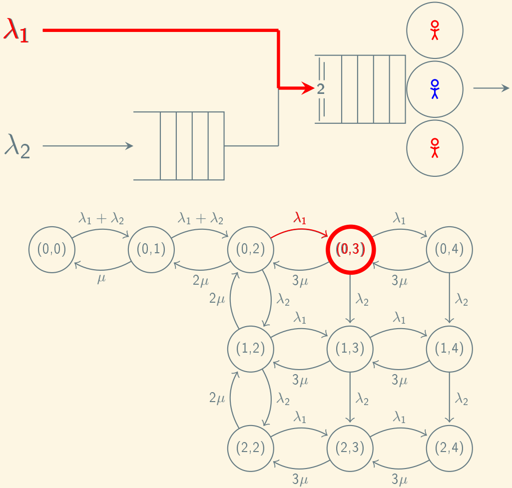

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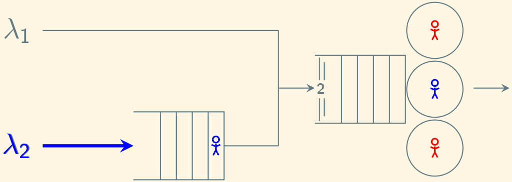


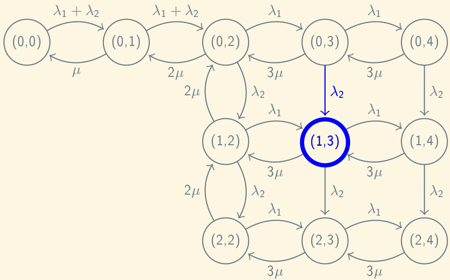

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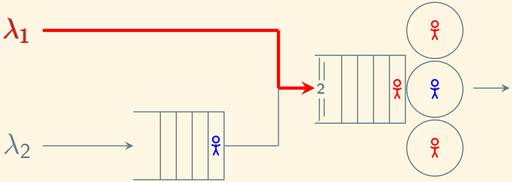


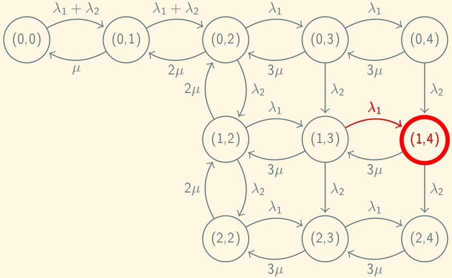

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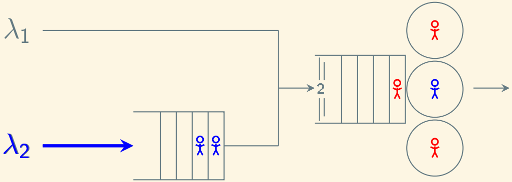


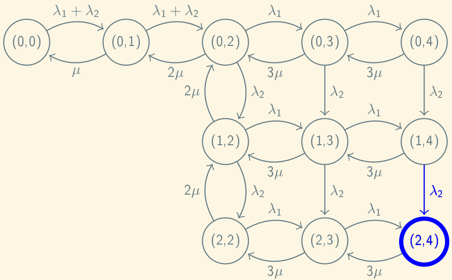

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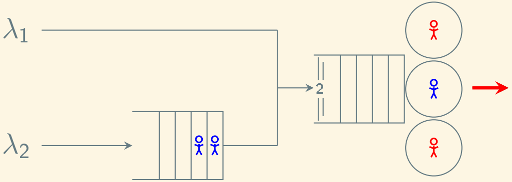


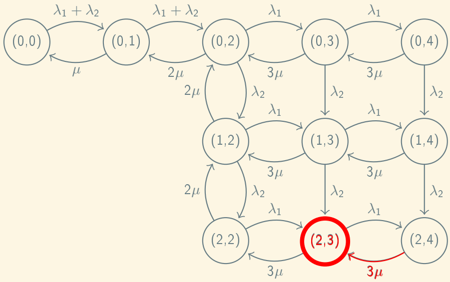

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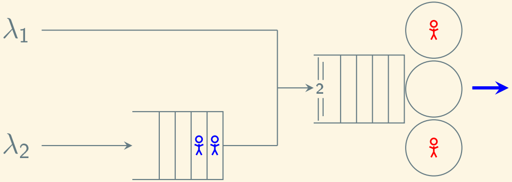


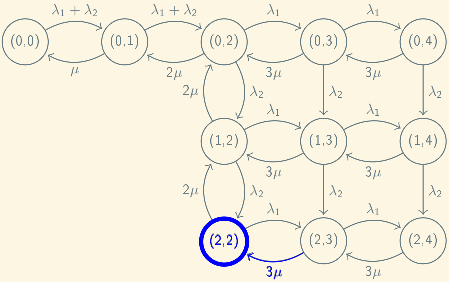

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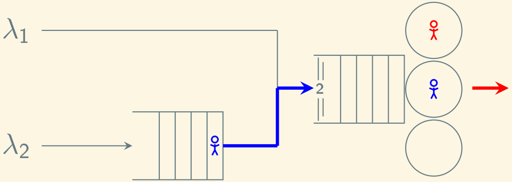
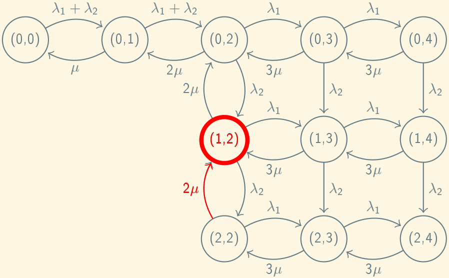
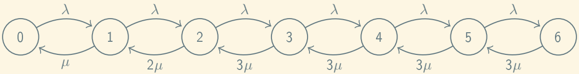


# Steady state probabilities - M|M|3 queue




$$
\begin{array}{cc}
 & \begin{array}{ccccccc} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{array} \\
Q = & \begin{pmatrix}
-\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\
\mu & -\mu-\lambda & \lambda & 0 & 0 & 0 & 0 \\
0 & 2\mu & -2\mu-\lambda & \lambda & 0 & 0 & 0 \\
0 & 0 & 3\mu & -3\mu-\lambda & \lambda & 0 & 0 \\
0 & 0 & 0 & 3\mu & -3\mu-\lambda & \lambda & 0 \\
0 & 0 & 0 & 0 & 3\mu & -3\mu-\lambda & \lambda \\
0 & 0 & 0 & 0 & 0 & 3\mu & -3\mu
\end{pmatrix}
\begin{array}{c} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{array}
\end{array}
$$

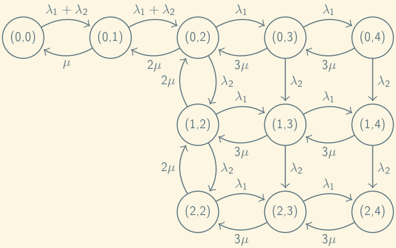
$$\frac{d\pi}{dt} = \pi Q = 0$$

$$\sum \pi_i = 1$$

$$\pi = \begin{pmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \\ \pi_6 \end{pmatrix}$$

# Steady state probabilities - Custom network

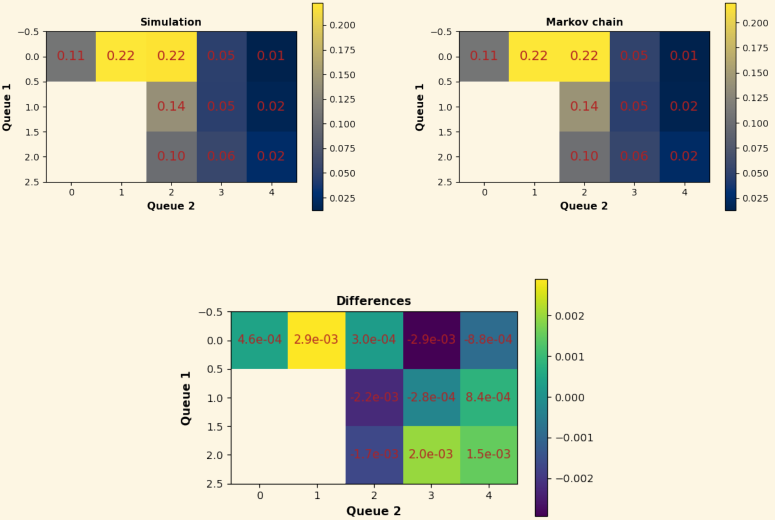
$$Q = \begin{array}{c} \begin{array}{cccccc} (0,0) & (0,1) & (0,2) & & (2,3) & (2,4) \end{array} \\ \begin{pmatrix} -\lambda_1-\lambda_2 & \lambda_1+\lambda_2 & 0 & \dots & 0 & 0 \\ \mu & -\mu-\lambda_1-\lambda_2 & \lambda_1+\lambda_2 & \dots & 0 & 0 \\ 0 & 2\mu & -2\mu-\lambda_1-\lambda_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda_1-3\mu & \lambda_1 \\ 0 & 0 & 0 & \dots & 3\mu & -3\mu \end{pmatrix} \end{array} \begin{array}{c} \\ (0,0) \\ (0,1) \\ (0,2) \\ \\ (2,3) \\ (2,4) \end{array}$$




$$\frac{d\pi}{dt} = \pi Q = 0, \qquad \sum \pi_{(u,v)} = 1$$

$$\pi = \begin{bmatrix} \pi_{(0,0)} \\ \pi_{(0,1)} \\ \pi_{(0,2)} \\ \vdots \\ \pi_{(2,3)} \\ \pi_{(2,4)} \end{bmatrix}$$

# Steady state probabilities - Comparison



# Performance Measures - Number of individuals

$$(u_i, v_i) \longrightarrow \pi_{(u_i, v_i)}$$

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$$(u_i, v_i) \longrightarrow \pi_{(u_i, v_i)}$$

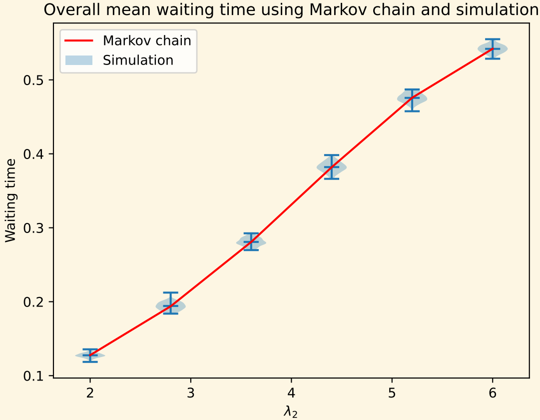
$$L = \sum_{i=1}^{|\pi|} \pi_i (u_i + v_i)$$

$$L_1 = \sum_{i=1}^{|\pi|} \pi_i u_i$$

$$L_2 = \sum_{i=1}^{|\pi|} \pi_i v_i$$

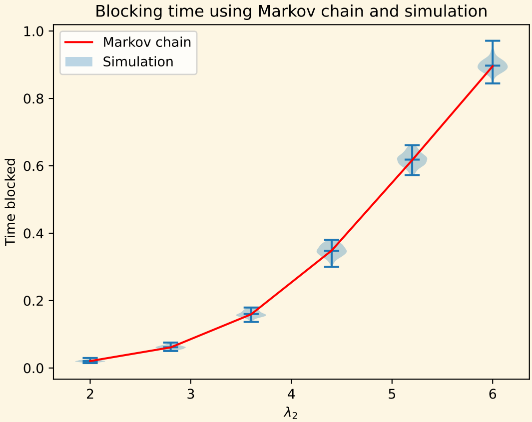
# Performance Measures - Waiting time

$$W = \frac{\lambda_1 P_{L'_1}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} W^{(1)} + \frac{\lambda_2 P_{L'_2}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} W^{(2)} \tag{1}$$



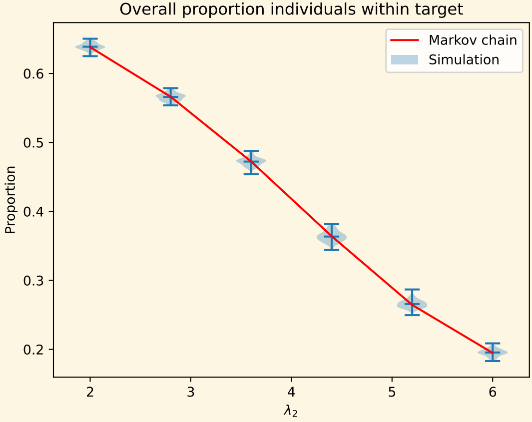
# Performance Measures - Blocking time

$$B = \frac{\sum_{(u,v) \in S_A^{(2)}} \pi_{(u,v)} \; b(u,v)}{\sum_{(u,v) \in S_A^{(2)}} \pi_{(u,v)}} \tag{2}$$



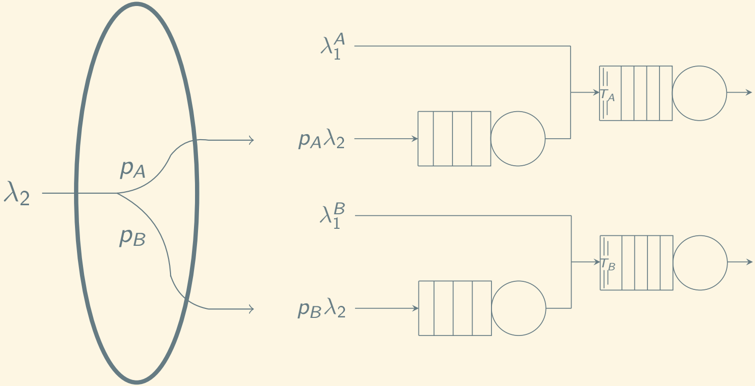
# Performance Measures - Proportion within time

$$P(X < t) = \frac{\lambda_1 P_{L'_1}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} P(X^{(1)} < t) + \frac{\lambda_2 P_{L'_2}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} P(X^{(2)} < t) \tag{3}$$

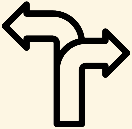


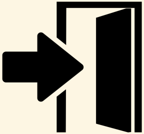
# Game - Definition

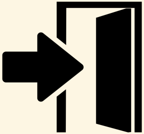
# Game - Players



# Game - Strategies







$p_A, p_B \in [0, 1]$

$T_A \in [1, N_A]$

$T_B \in [1, N_B]$

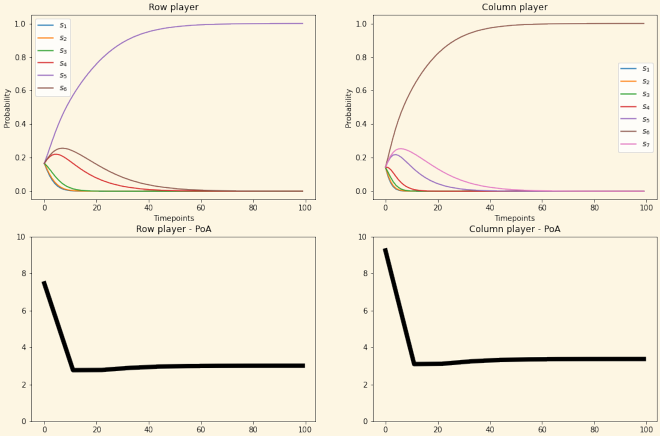
$p_A + p_B = 1$

# Game - Formulation

$$A = \begin{pmatrix} U^A_{1,1} & U^A_{1,2} & \cdots & U^A_{1,N_B} \\ U^A_{2,1} & U^A_{2,2} & \cdots & U^A_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ U^A_{N_A,1} & U^A_{N_A,2} & \cdots & U^A_{N_A,N_B} \end{pmatrix}, \quad B = \begin{pmatrix} U^B_{1,1} & U^B_{1,2} & \cdots & U^B_{1,N_B} \\ U^B_{2,1} & U^B_{2,2} & \cdots & U^B_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ U^B_{N_A,1} & U^B_{N_A,2} & \cdots & U^B_{N_A,N_B} \end{pmatrix}$$

$$R = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,N_B} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N_A,1} & p_{N_A,2} & \cdots & p_{N_A,N_B} \end{pmatrix}$$

# Learning algorithms - Asymmetric replicator dynamics



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