

Annual Review

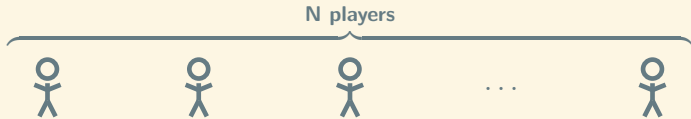
Michalis Panayides

2020-06-10

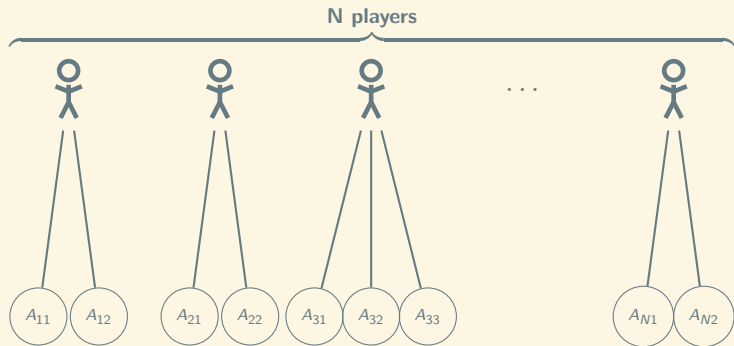
Game Theory - Syllabus

- ▶ Normal Form Games
- ▶ Mixed-Strategy Nash Equilibrium
- ▶ Alternate Solution Concepts
- ▶ Extensive-Form Games
- ▶ Repeated Games (TBC)
- ▶ Bayesian Games (TBC)
- ▶ Coalitional Games (TBC)

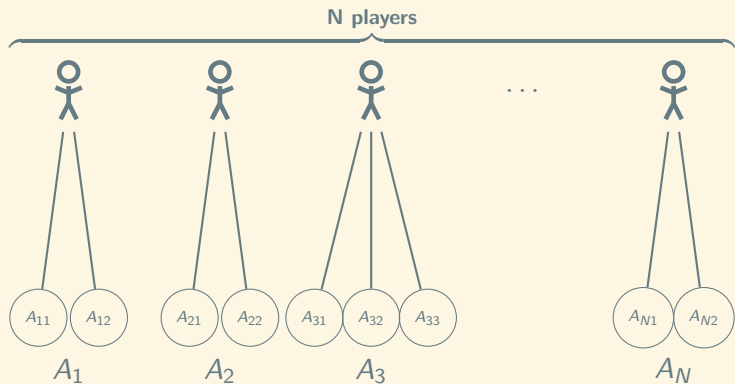
Normal Form Games



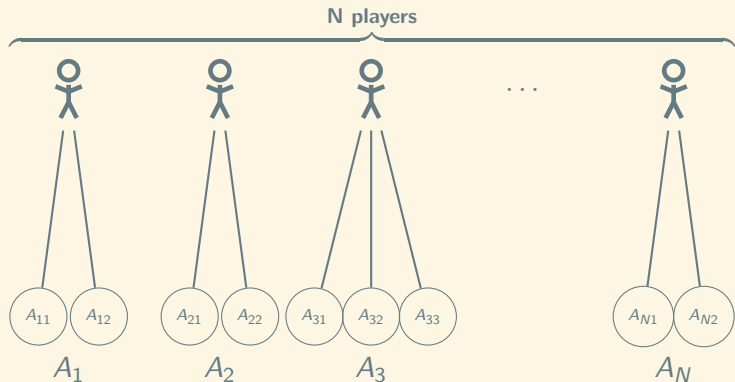
Normal Form Games



Normal Form Games









Normal Form Games



$$u_i = A_1 \times A_2 \times A_3 \times \cdots \times A_N \rightarrow \mathbb{R}$$

Rock-Paper-Scissors

			
	$(0, 0)$	$(1, -1)$	$(-1, 1)$
	$(-1, 1)$	$(0, 0)$	$(1, -1)$
	$(1, -1)$	$(-1, 1)$	$(0, 0)$

Nash Equilibrium

	C	D
C	(3, 3)	(0, 5)
D	(5, 0)	(1, 1)

Nash Equilibrium

$$\begin{array}{c} \text{C} \longrightarrow \\ \text{D} \longrightarrow \end{array} \left[\begin{array}{cc} (3, -) & (0, -) \\ (\underline{5}, -) & (\underline{1}, -) \end{array} \right]$$

Nash Equilibrium

C	D
\downarrow	\downarrow
$\begin{bmatrix} (-, 3) \\ (-, 0) \end{bmatrix}$	$\begin{bmatrix} (-, \underline{5}) \\ (-, \underline{1}) \end{bmatrix}$

Nash Equilibrium

		D
		↓
D	→	$\begin{bmatrix} (3, 3) & (0, \underline{5}) \\ (\underline{5}, 0) & (\underline{1}, \underline{1}) \end{bmatrix}$

Pareto Optimality

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

$$\overbrace{(3, 3), (0, 5), (5, 0), (1, 1)}$$

Pareto Optimality

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

$$\overbrace{(3, 3), (0, 5), (5, 0), (1, 1)}$$

$$(\underline{3}, \underline{3}) > (1, 1)$$

Computing the Nash Equilibria

- ▶ Lemke-Howson Algorithm
- ▶ Support Enumeration
- ▶ Iterative removal of strictly dominated strategies

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	L	C	R
U	(3, 0)	(2, 1)	(0, 0)
M	(1, 1)	(1, 1)	(5, 0)
D	(0, 1)	(4, 2)	(0, 1)

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	L	C	R
U	$(3, 0)$	$(2, \underline{1})$	$(0, 0)$
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Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	C
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D	$(4, 2)$

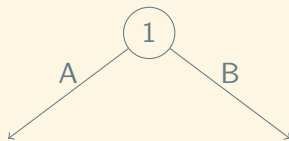
Iterative Removal of Strictly Dominated Strategies

$P1 \backslash P2$	C
U	$(2, 1)$
D	$(\underline{4}, 2)$

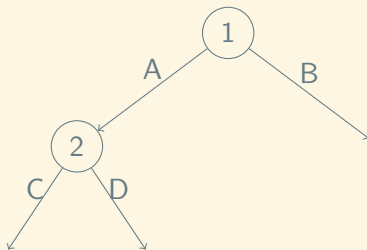
Iterative Removal of Strictly Dominated Strategies

$$\begin{array}{cc} P1 \setminus P2 & C \\ D & (4, 2) \end{array}$$

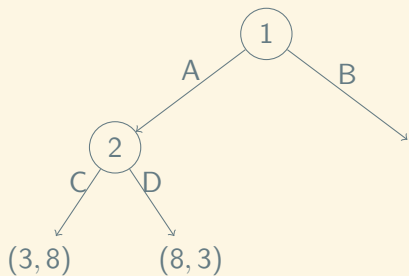
Perfect Information Extensive Form Games



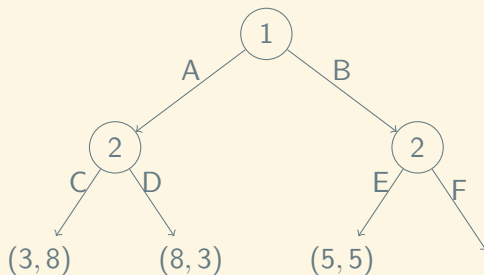
Perfect Information Extensive Form Games



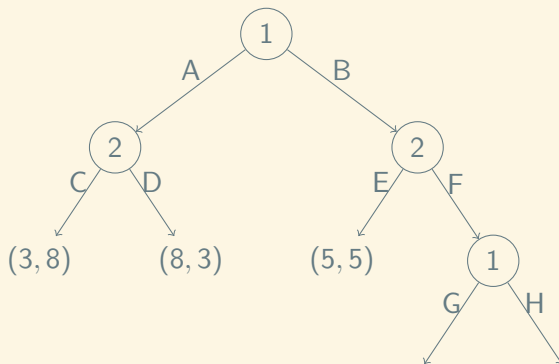
Perfect Information Extensive Form Games



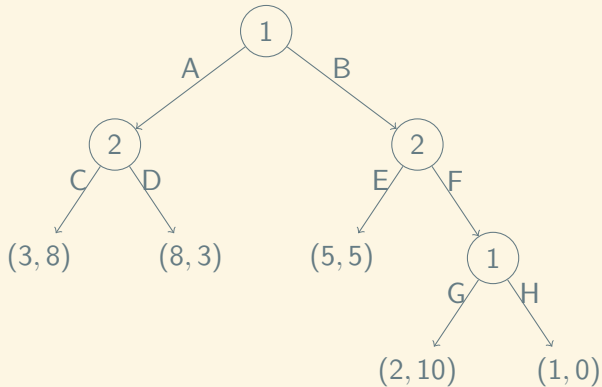
Perfect Information Extensive Form Games



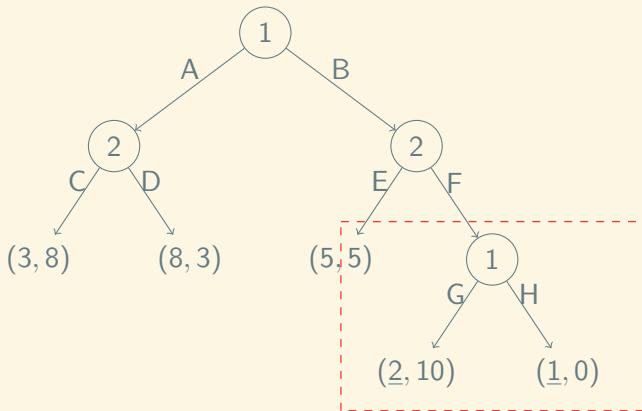
Perfect Information Extensive Form Games



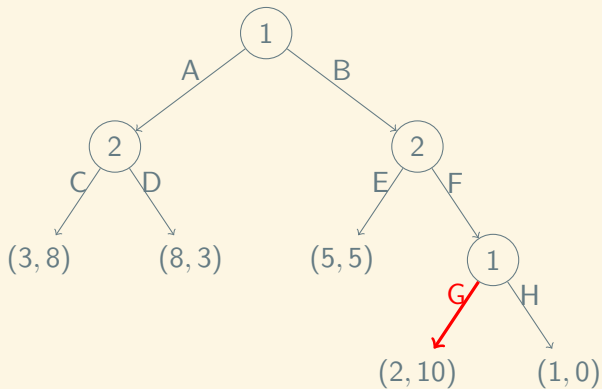
Perfect Information Extensive Form Games



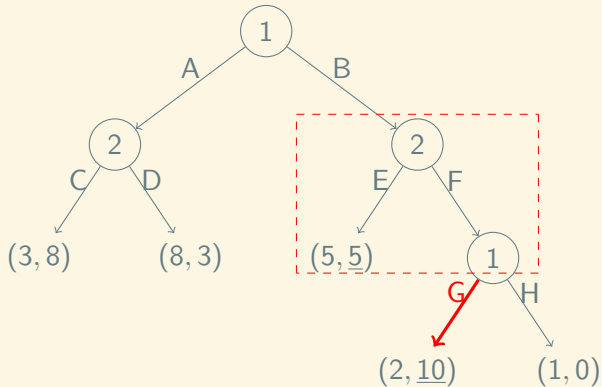
Backwards Induction



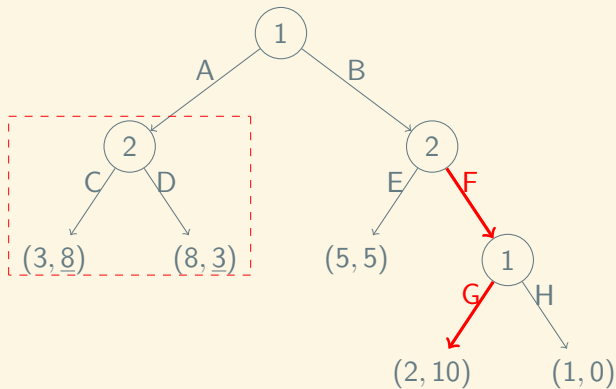
Backwards Induction



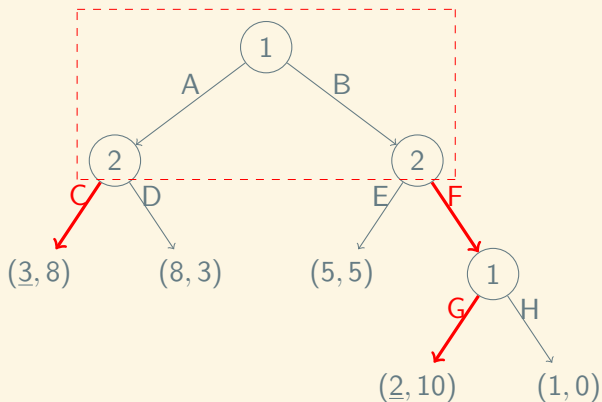
Backwards Induction



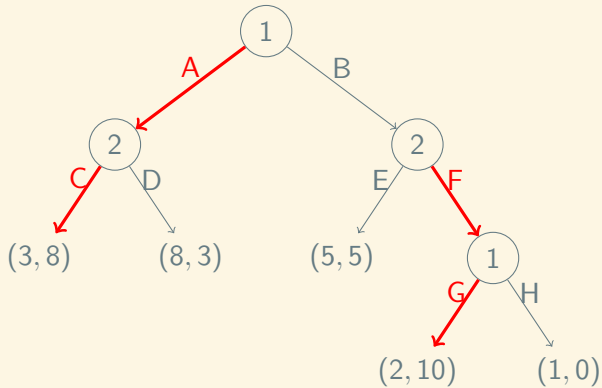
Backwards Induction



Backwards Induction



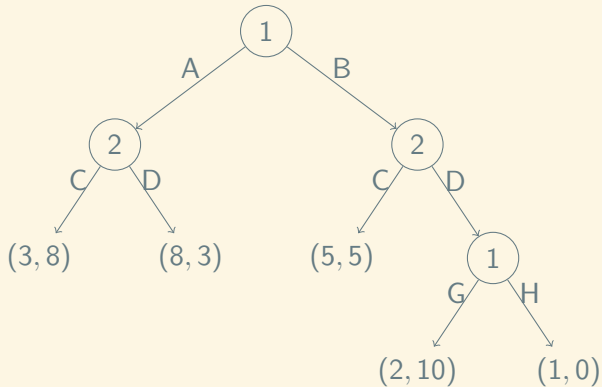
Backwards Induction



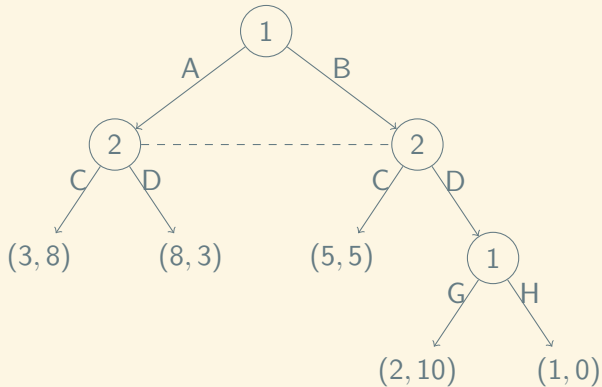
Imperfect Information Extensive-form Games



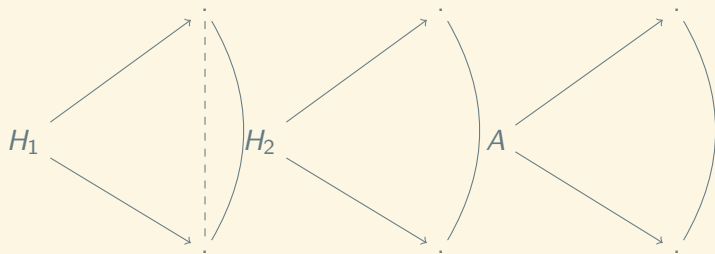
Imperfect Information Extensive-form Games



Imperfect Information Extensive-form Games



PhD - Motivation



PhD - Motivation

NHS Wales: A&E record low performance as ambulance service struggles

By Owain Clarke
BBC Wales health correspondent

19 December 2019

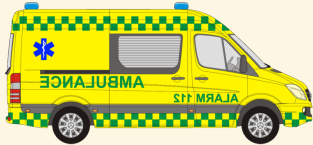
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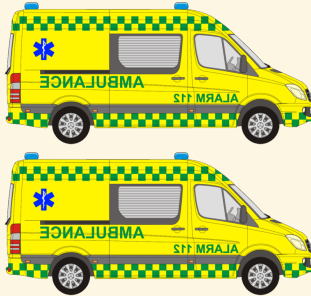
Ambulances outside Morriston Hospital in Swansea this week



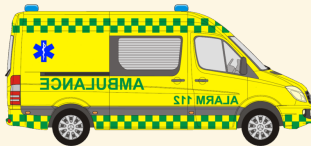
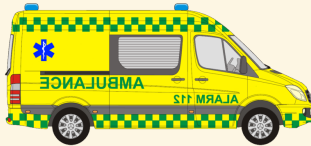
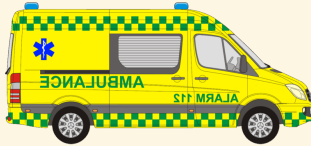
PhD - Motivation



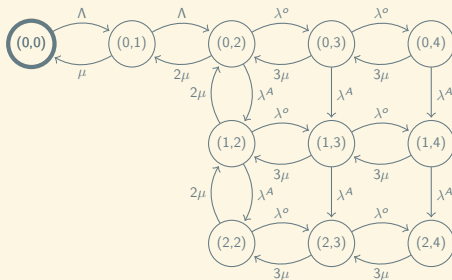
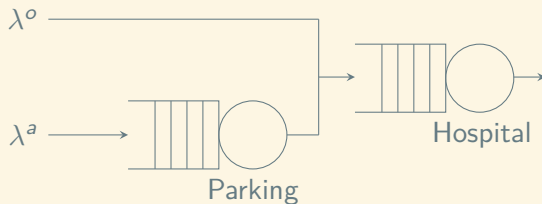
PhD - Motivation



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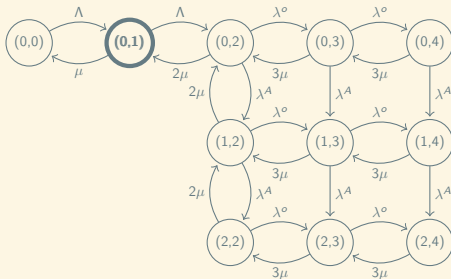
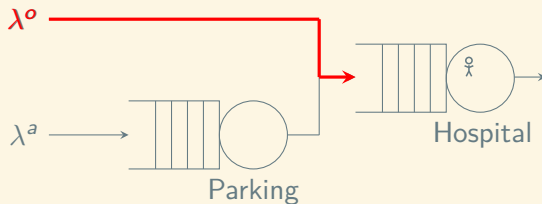


Hospital Formulation



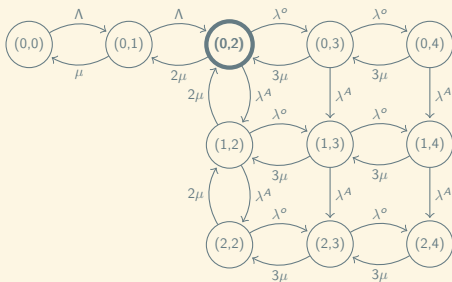
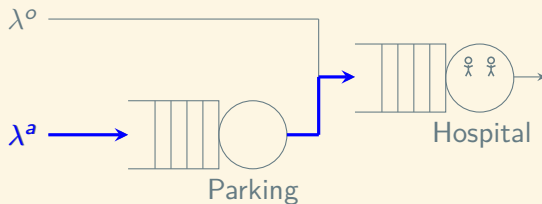
$$C=3, T=2, N=4, M=2$$

Hospital Formulation



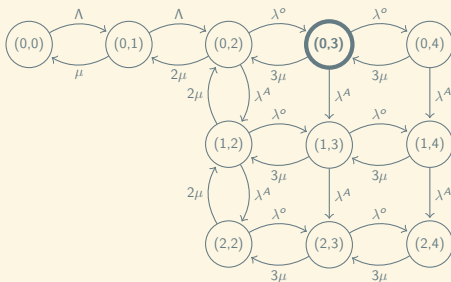
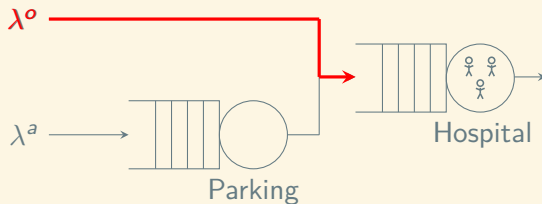
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Hospital Formulation



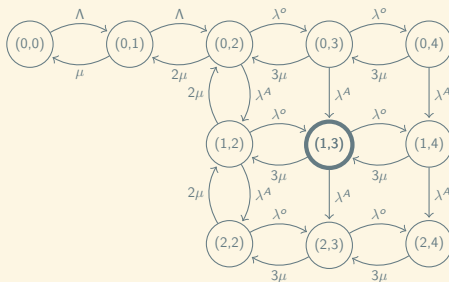
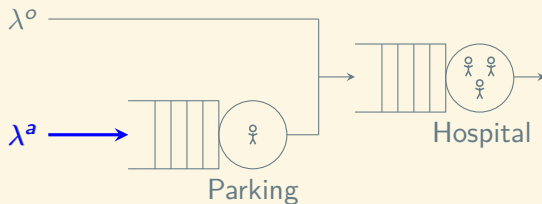
$$C=3, T=2, N=4, M=2$$

Hospital Formulation



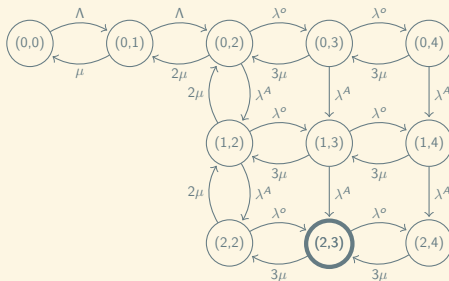
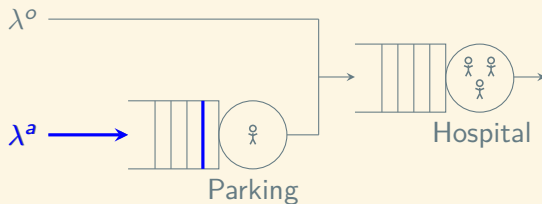
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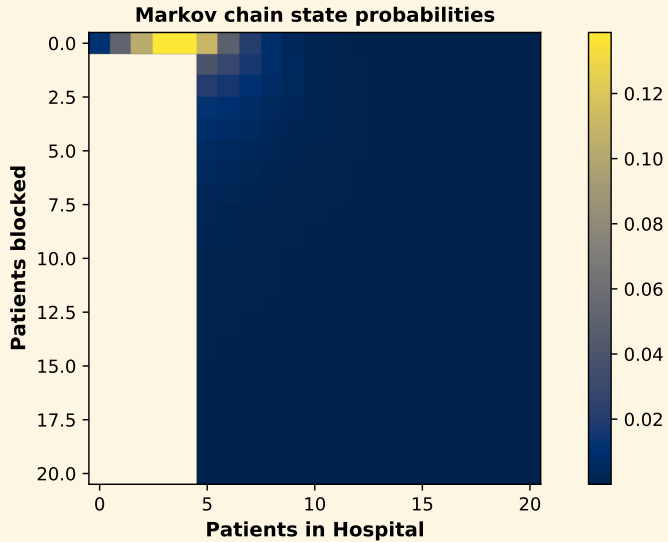
$$C=3, T=2, N=4, M=2$$

State Probabilities

$$\pi Q = 0$$

$$\sum_i \pi_i = 1$$

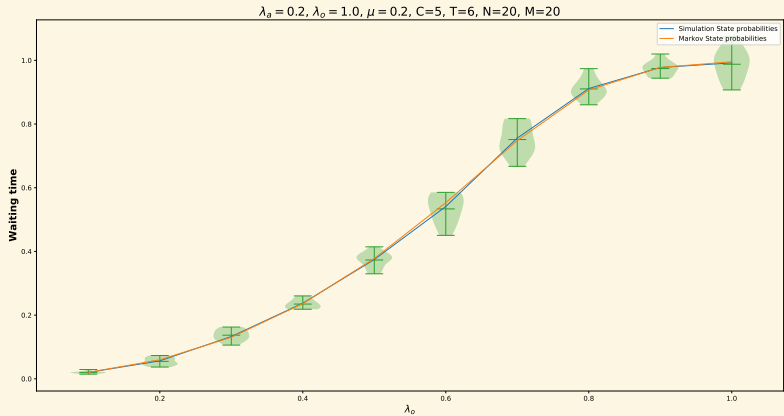
State Probabilities



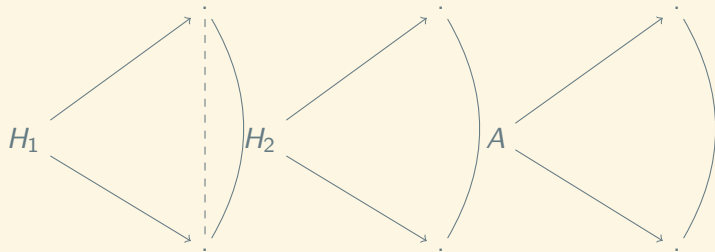
Waiting Times

$$W = \frac{\sum_{(u,v) \in S_A} w(u,v) \pi(u,v)}{\sum_{(u,v) \in S_A} \pi(u,v)}$$

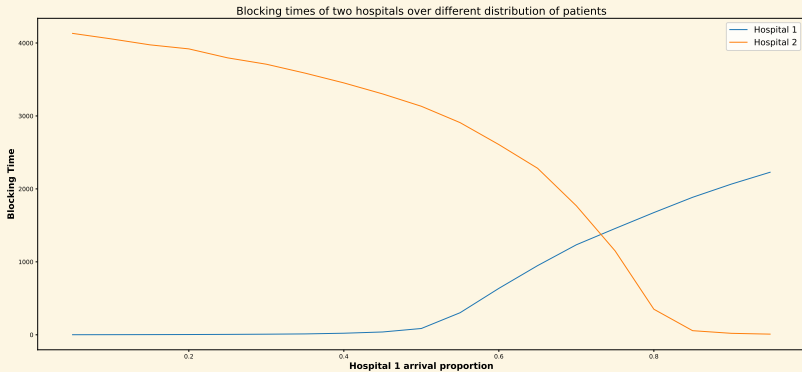
Waiting Times



Future Plans



Future Plans



Future Plans

- ▶ Harper, P., Knight, V. and Panayides, M., 2020. *On a queueing model with two waiting rooms.*
- ▶ Harper, P., Knight, V. and Panayides, M., 2020. *A game theoretic model of the ED-EMS interface.*