Annual Review

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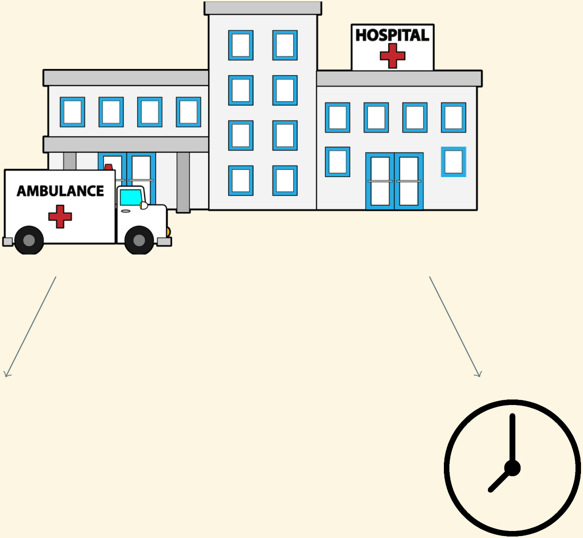
2021-06-11

Motivation

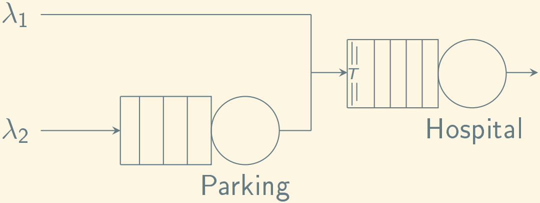
Queen Elizabeth Hospital Birmingham
EMERGENCY AMBULANCE

Emergency Ambulance
LONDON AMBULANCE
LX64 DZK

Motivation


HOSPITAL
AMBULANCE

Hospital queueing model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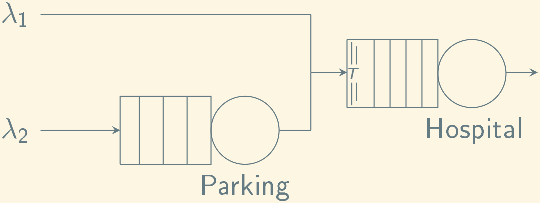
Hospital queueing model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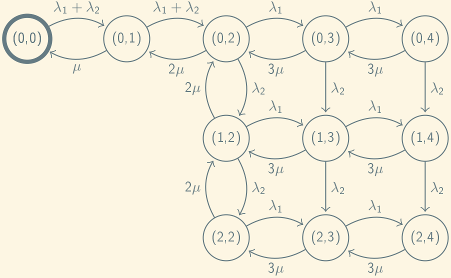
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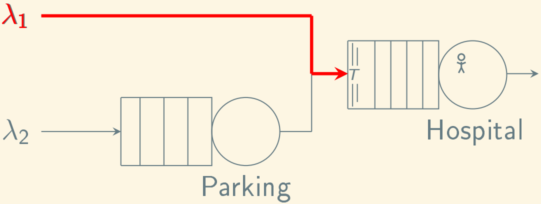
Hospital queueing model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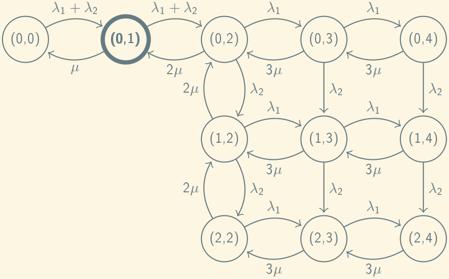
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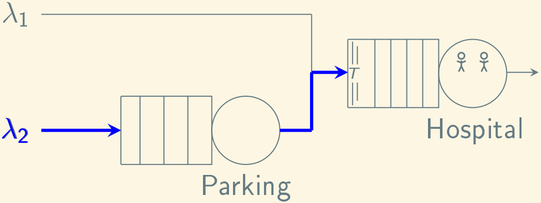
Hospital queueing model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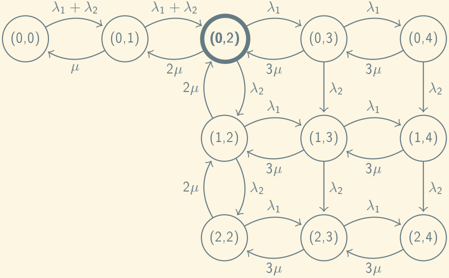
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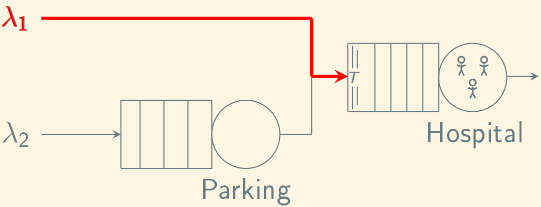
Hospital queueing model



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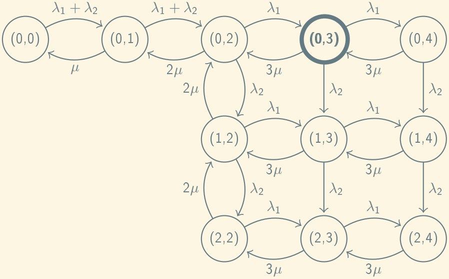

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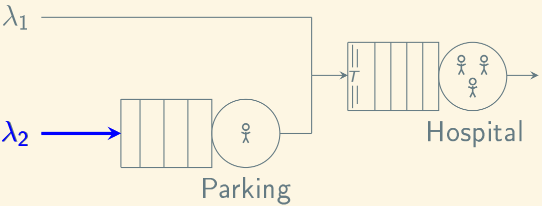
Hospital queueing model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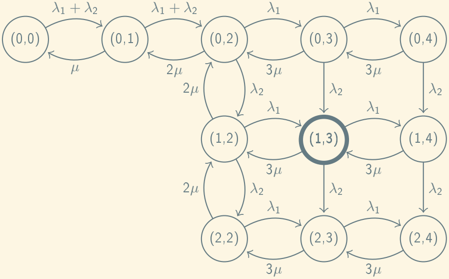
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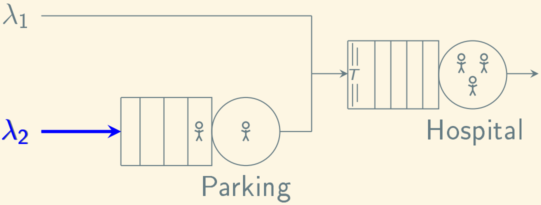
Hospital queueing model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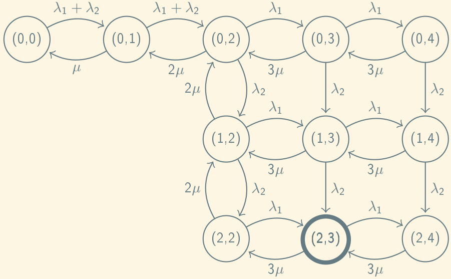
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Performance Measures - Steady state vector

$$Q = \begin{array}{c} \\ \end{array}\begin{array}{ccccccc} (0,0) & (0,1) & (0,2) & & (2,3) & (2,4) & \\ \end{array}$$

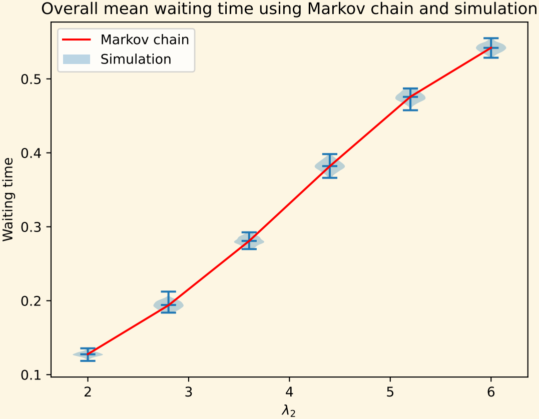
$$Q = \left(\begin{array}{cccccc|c} -\lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & 0 & \dots & 0 & 0 & (0,0) \\ \mu & -\mu - \lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & \dots & 0 & 0 & (0,1) \\ 0 & 2\mu & -2\mu - \lambda_1 - \lambda_2 & \dots & 0 & 0 & (0,2) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \\ 0 & 0 & 0 & \dots & -\lambda_1 - 3\mu & \lambda_1 & (2,3) \\ 0 & 0 & 0 & \dots & 3\mu & -3\mu & (2,4) \end{array}\right)$$

$$\pi Q = 0$$

$$\pi = \left[\pi_{(0,0)}, \pi_{(0,1)}, \pi_{(0,2)}, \dots, \pi_{(2,3)}, \pi_{(2,4)}\right]$$

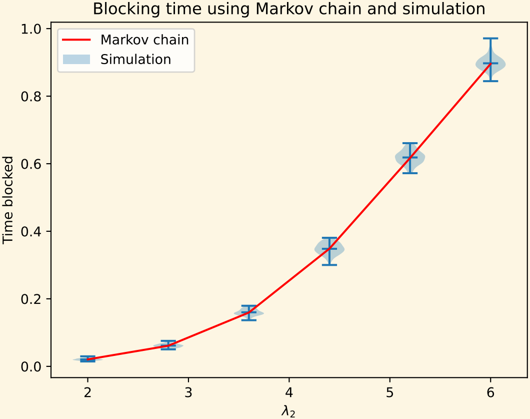
Performance Measures - Waiting time

$$W = \frac{\lambda_1 P_{L'_1}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} W^{(1)} + \frac{\lambda_2 P_{L'_2}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} W^{(2)} \tag{1}$$

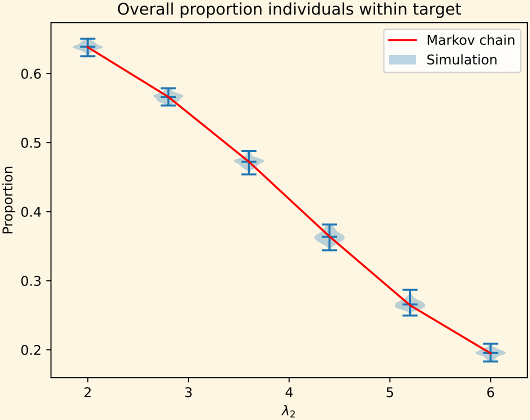


Performance Measures - Blocking time

$$B = \frac{\sum_{(u,v) \in S_A^{(2)}} \pi_{(u,v)} \; b(u,v)}{\sum_{(u,v) \in S_A^{(2)}} \pi_{(u,v)}} \tag{2}$$



Performance Measures - Proportion within time

$$P(X < t) = \frac{\lambda_1 P_{L_1'}}{\lambda_2 P_{L_2'} + \lambda_1 P_{L_1'}} P(X^{(1)} < t) + \frac{\lambda_2 P_{L_2'}}{\lambda_2 P_{L_2'} + \lambda_1 P_{L_1'}} P(X^{(2)} < t) \tag{3}$$