

Annual Review

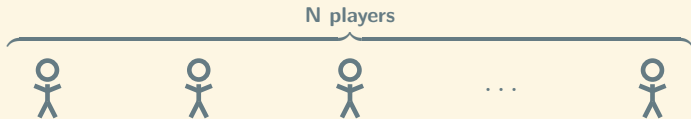
Michalis Panayides

2020-06-10

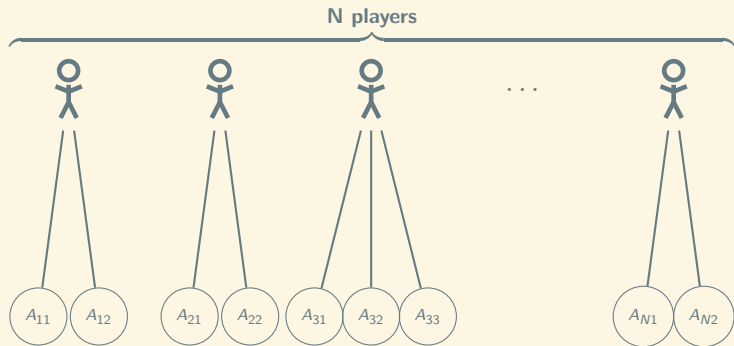
Game Theory - Syllabus

- ▶ Normal Form Games
- ▶ Mixed-Strategy Nash Equilibrium
- ▶ Alternate Solution Concepts
- ▶ Extensive-Form Games
- ▶ Repeated Games (TBC)
- ▶ Bayesian Games (TBC)
- ▶ Coalitional Games (TBC)

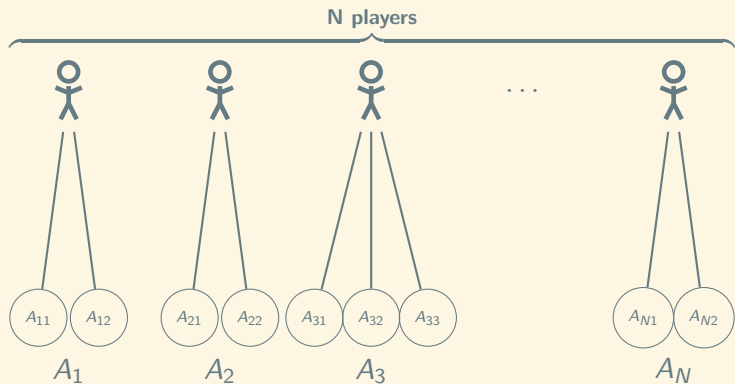
Normal Form Games



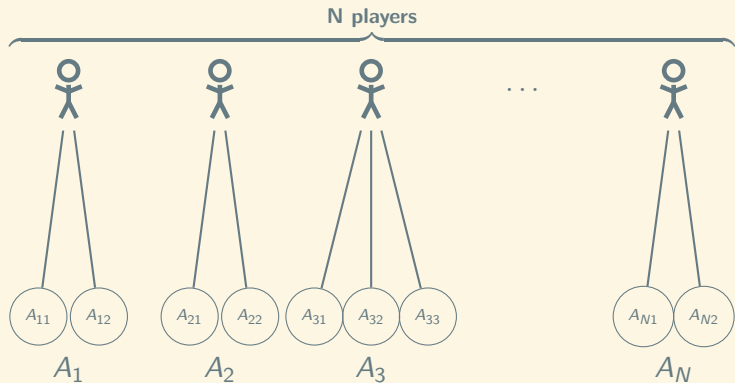
Normal Form Games



Normal Form Games









Normal Form Games



$$u_i = A_1 \times A_2 \times A_3 \times \cdots \times A_N \rightarrow \mathbb{R}$$

Rock-Paper-Scissors

			
	$(0, 0)$	$(1, -1)$	$(-1, 1)$
	$(-1, 1)$	$(0, 0)$	$(1, -1)$
	$(1, -1)$	$(-1, 1)$	$(0, 0)$



Nash Equilibrium

	C	D
C	(3, 3)	(0, 5)
D	(5, 0)	(1, 1)

Nash Equilibrium

$$\begin{array}{l} \text{C} \longrightarrow \\ \text{D} \longrightarrow \end{array} \begin{bmatrix} (3, -) & (0, -) \\ (\underline{5}, -) & (\underline{1}, -) \end{bmatrix}$$

Nash Equilibrium

C	D
	
$\begin{bmatrix} (-, 3) \\ (-, 0) \end{bmatrix}$	$\begin{bmatrix} (-, \underline{5}) \\ (-, \underline{1}) \end{bmatrix}$

Nash Equilibrium

A 2x2 normal form game matrix. The row player's strategies are D and D (both labeled 'D'). The column player's strategies are D and D (both labeled 'D'). The payoffs are (row, column): (3, 3), (0, 5), (5, 0), and (1, 1). Red arrows point to the bottom-left and bottom-right cells, indicating Nash Equilibria.

	D	D
D	(3, 3)	(0, <u>5</u>)
D	(<u>5</u> , 0)	(<u>1</u> , <u>1</u>)

Pareto Optimality

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

$$\overbrace{(3, 3), (0, 5), (5, 0), (1, 1)}$$

Pareto Optimality

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

$$\overbrace{(3, 3), (0, 5), (5, 0), (1, 1)}$$

$$(\underline{3}, \underline{3}) > (1, 1)$$

Computing the Nash Equilibria

- ▶ Lemke-Howson Algorithm
- ▶ Support Enumeration
- ▶ Iterative removal of strictly dominated strategies

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	L	C	R
U	(3, 0)	(2, 1)	(0, 0)
M	(1, 1)	(1, 1)	(5, 0)
D	(0, 1)	(4, 2)	(0, 1)

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	L	C	R
U	$(3, 0)$	$(2, \underline{1})$	$(0, 0)$
M	$(1, 1)$	$(1, \underline{1})$	$(5, 0)$
D	$(0, 1)$	$(4, \underline{2})$	$(0, 1)$

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Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	C
U	$(2, 1)$
D	$(4, 2)$

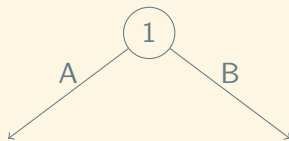
Iterative Removal of Strictly Dominated Strategies

$P1 \backslash P2$	C
U	$(2, 1)$
D	$(\underline{4}, 2)$

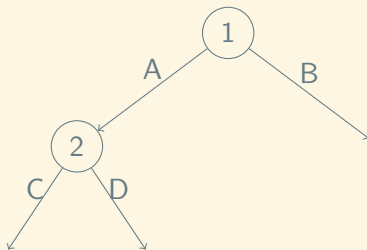
Iterative Removal of Strictly Dominated Strategies

$$\begin{array}{cc} P_1 \backslash P_2 & C \\ D & (4, 2) \end{array}$$

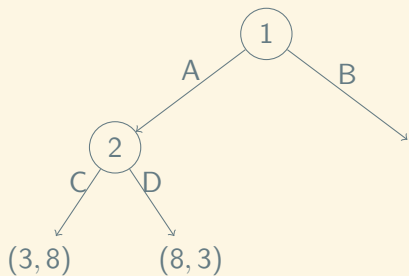
Perfect Information Extensive Form Games



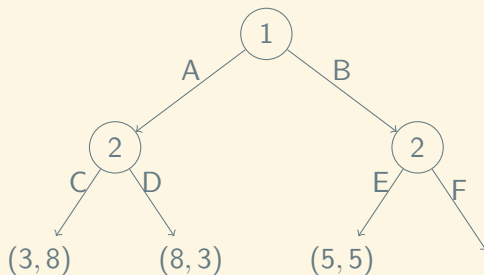
Perfect Information Extensive Form Games



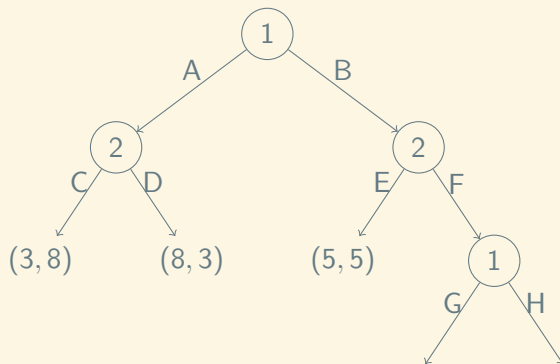
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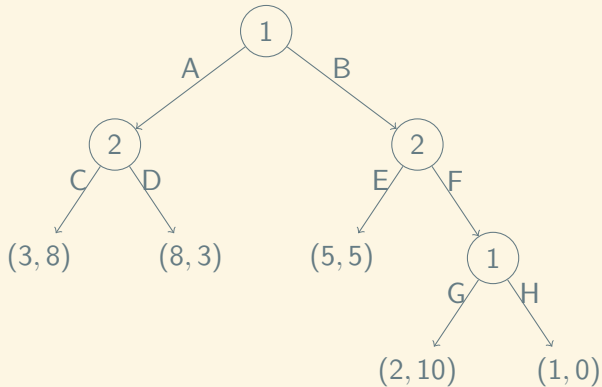
Perfect Information Extensive Form Games



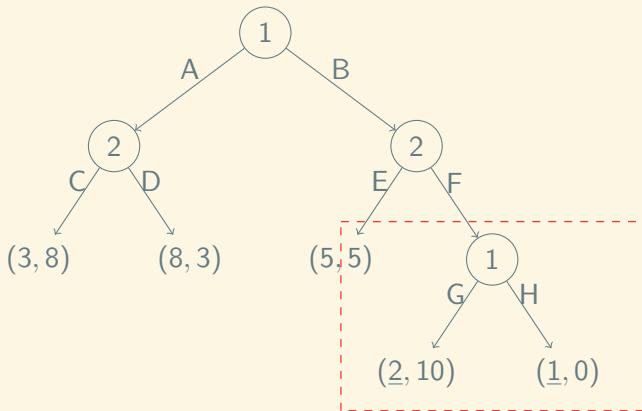
Perfect Information Extensive Form Games



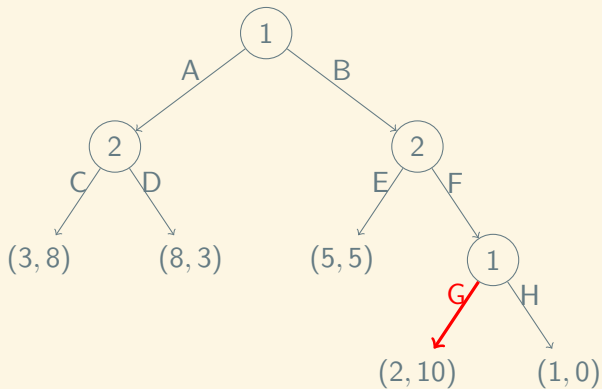
Perfect Information Extensive Form Games



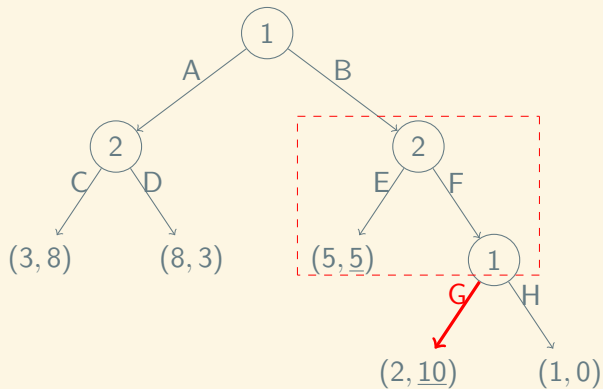
Backwards Induction



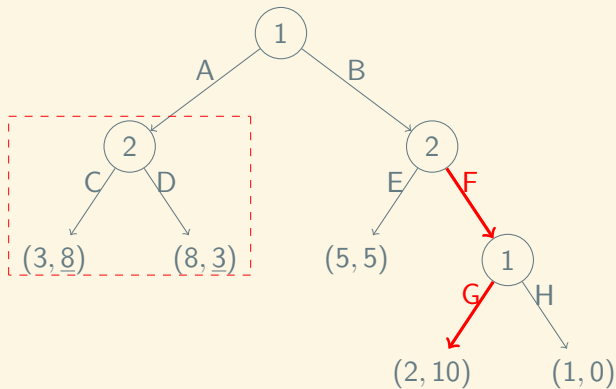
Backwards Induction



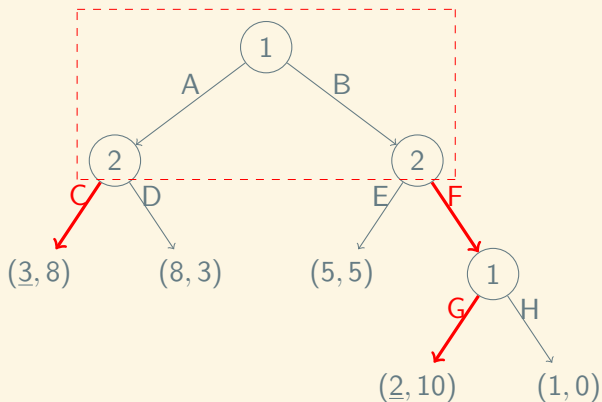
Backwards Induction



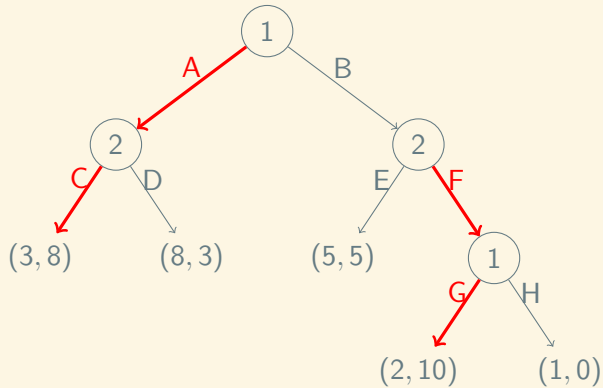
Backwards Induction



Backwards Induction



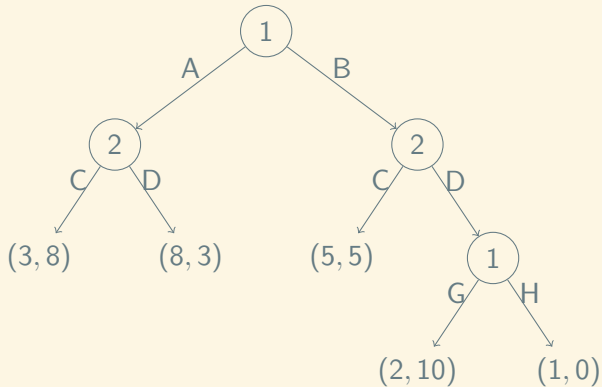
Backwards Induction



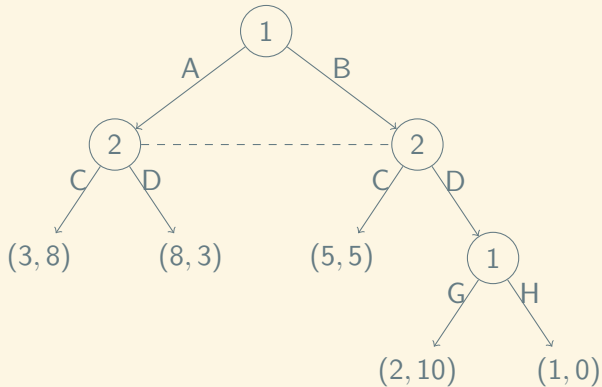
Imperfect Information Extensive-form Games



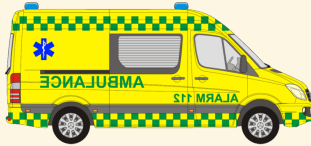
Imperfect Information Extensive-form Games



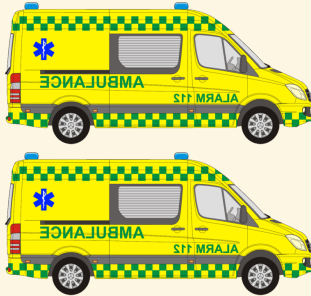
Imperfect Information Extensive-form Games



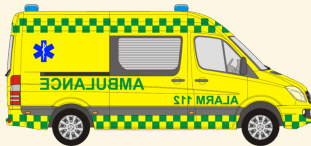
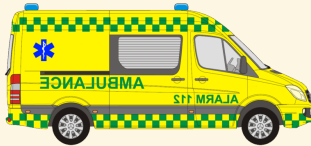
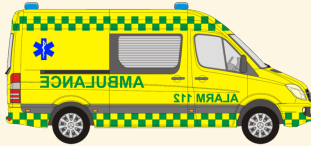
PhD - Motivation



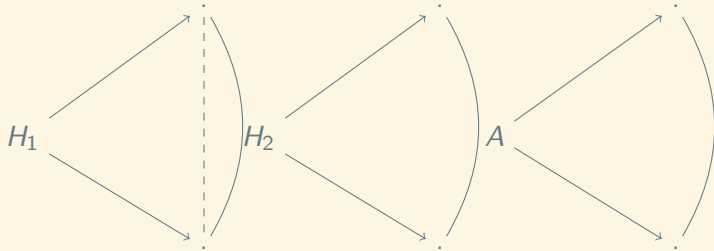
PhD - Motivation



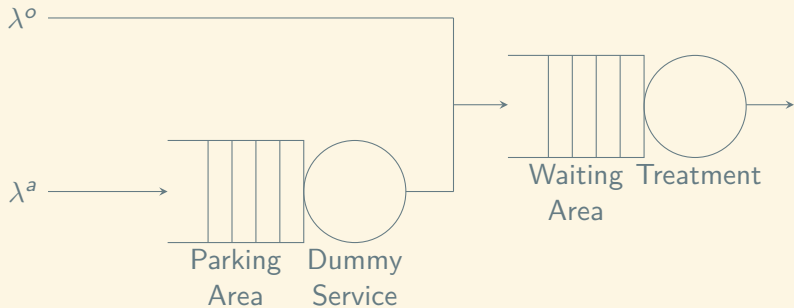
PhD - Motivation



Ambulance - Hospital Interface



Hospital Formulation



Hospital - Markov Chain

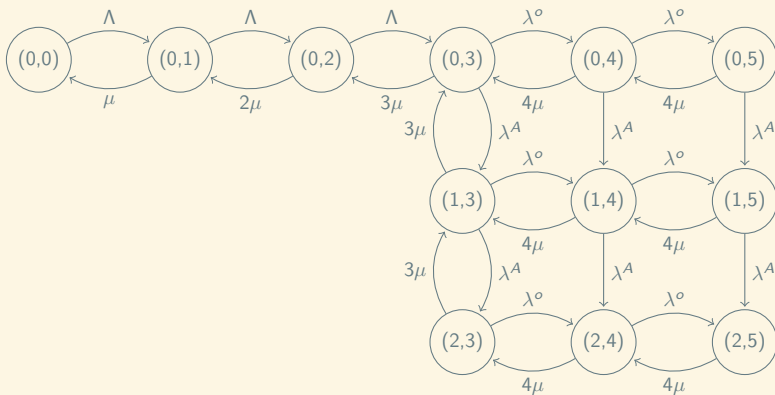
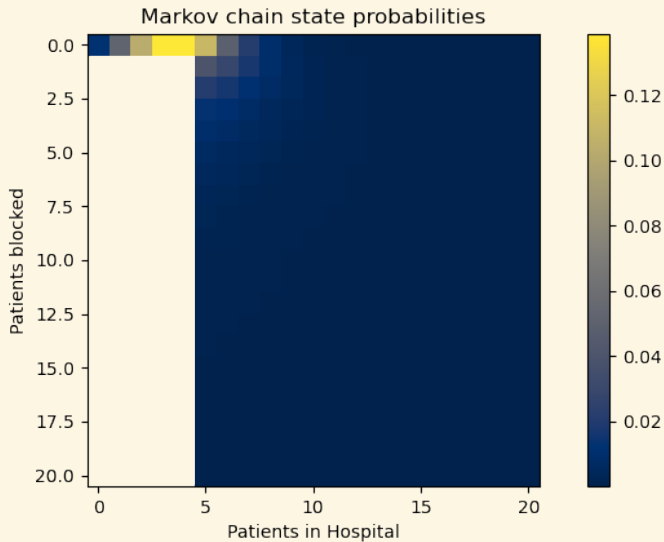
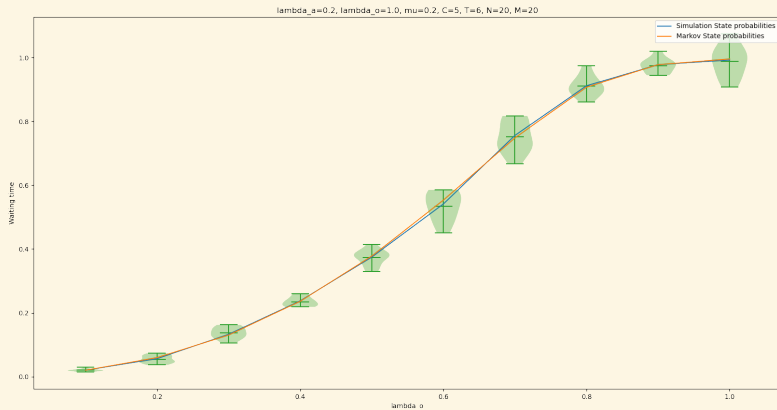


Figure: $C=4$, $T=3$, $N=5$, $M=2$

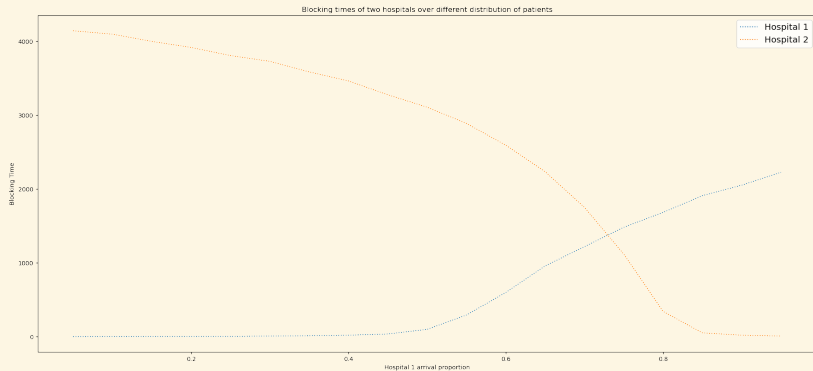
State Probabilities



Waiting Times



Optimal patient distribution



Future Plans

- ▶ Performance Measures
- ▶ Game theoretic interface