Annual Review

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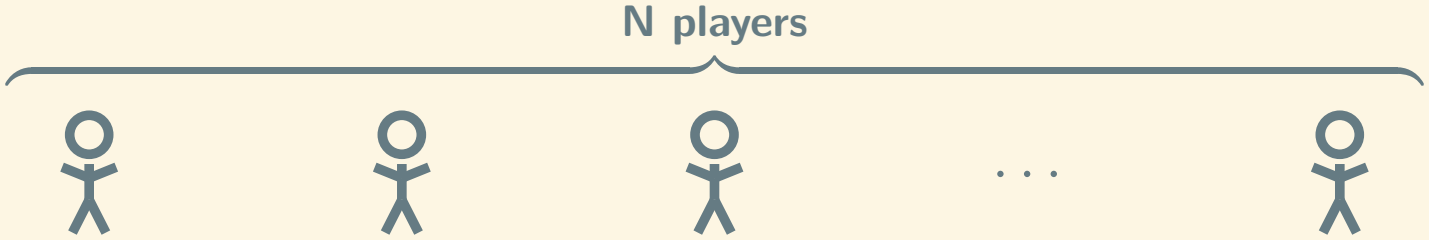
2020-06-10

Game Theory - Syllabus

- Normal Form Games
- Mixed-Strategy Nash Equilibrium
- Alternate Solution Concepts
- Extensive-Form Games
- Repeated Games (TBC)
- Bayesian Games (TBC)
- Coalitional Games (TBC)

Normal Form Games



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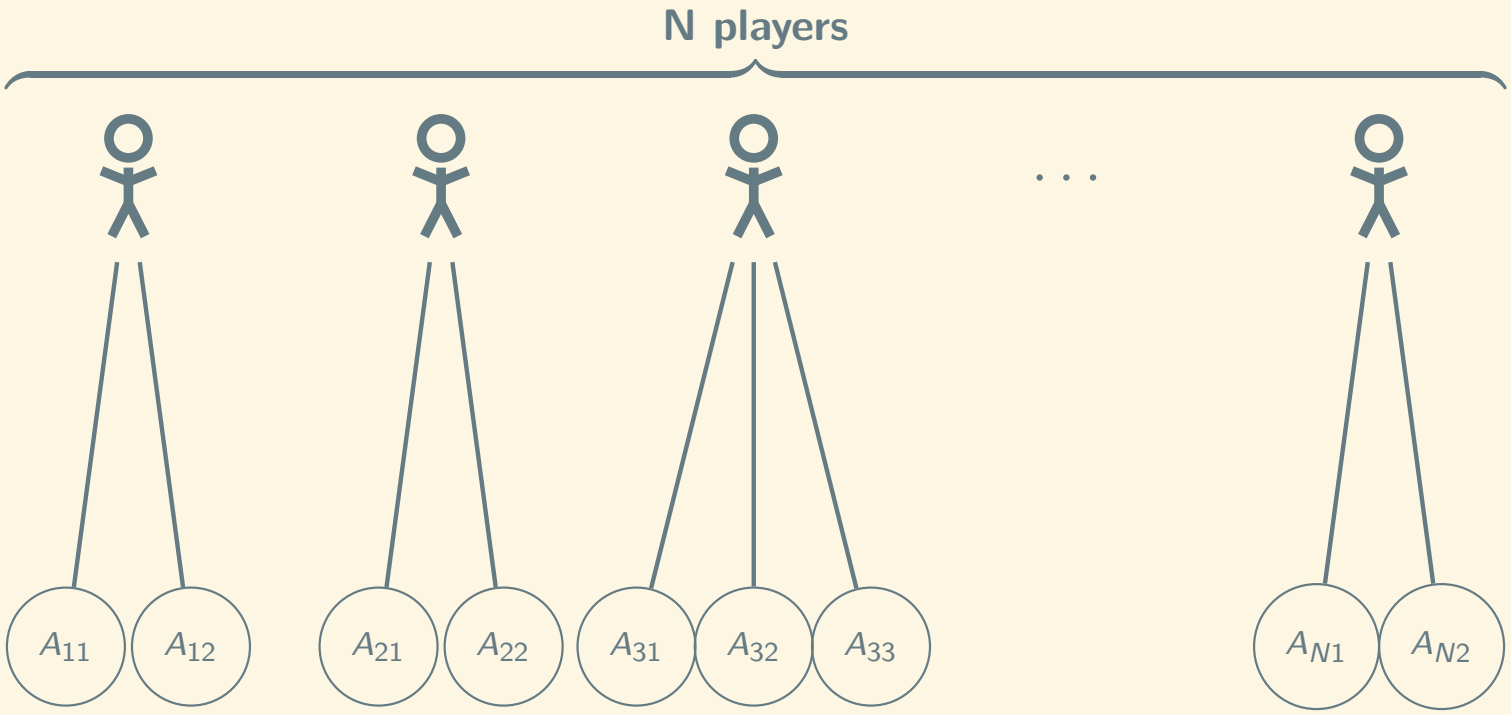

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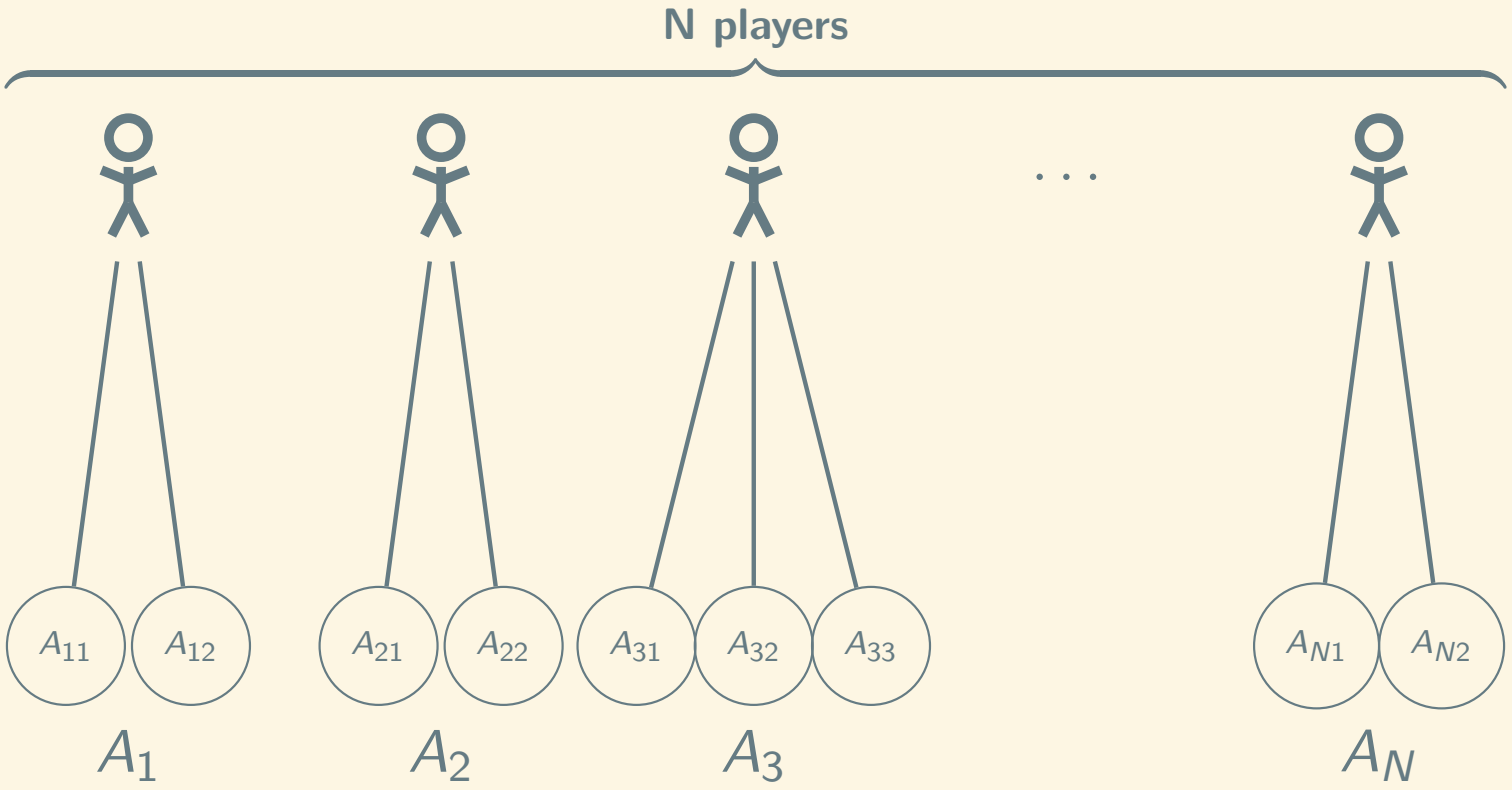
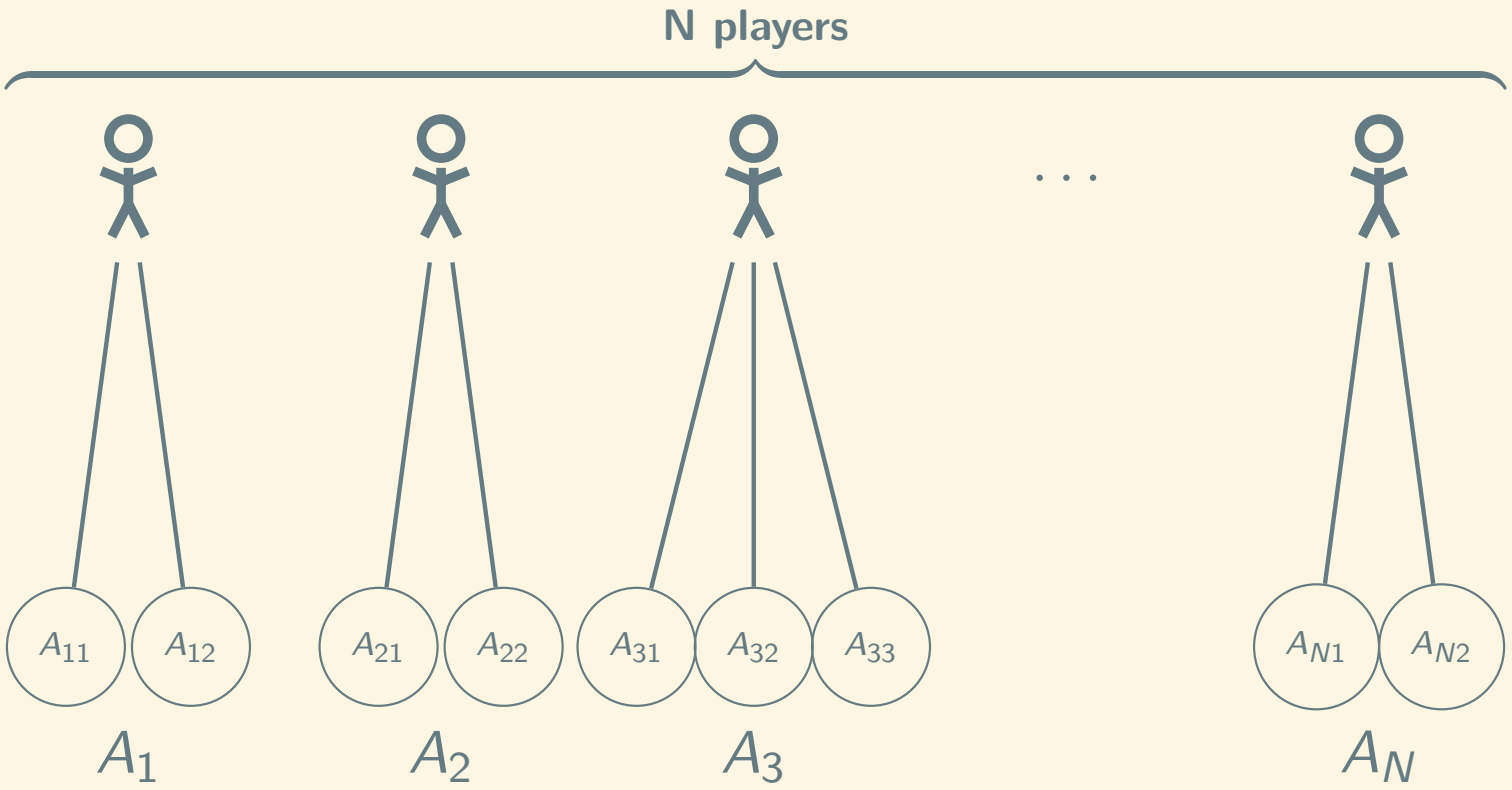
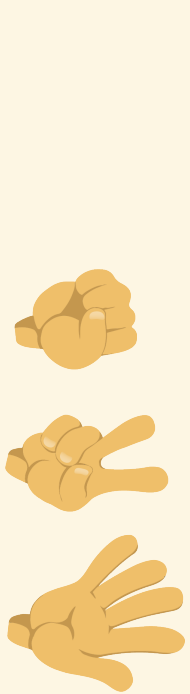


Normal Form Games



$$u_i = A_1 \times A_2 \times A_3 \times \cdots \times A_N \to \mathbb{R}$$

Rock-Paper-Scissors



$$\begin{bmatrix} (0,0) & (1,-1) & (-1,1) \\ (-1,1) & (0,0) & (1,-1) \\ (1,-1) & (-1,1) & (0,0) \end{bmatrix}$$

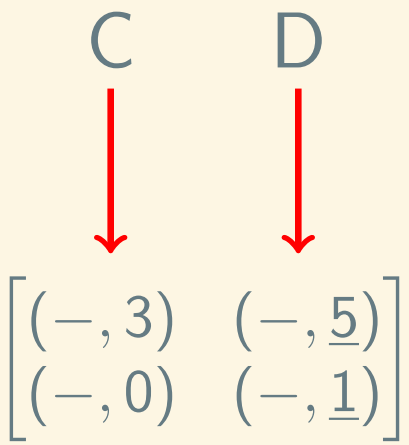
Nash Equilibrium

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{bmatrix} (3,3) & (0,5) \\ (5,0) & (1,1) \end{bmatrix} \end{array}$$

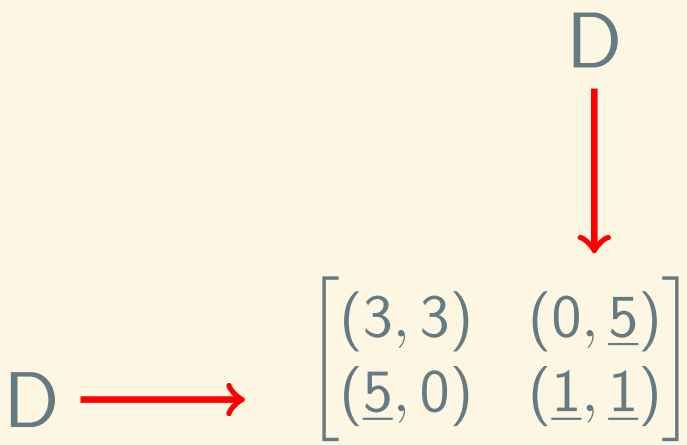
Nash Equilibrium

$$\begin{array}{c} \mathrm{C} \longrightarrow \\ \mathrm{D} \longrightarrow \end{array} \begin{bmatrix} (3,-) & (0,-) \\ (\underline{5},-) & (\underline{1},-) \end{bmatrix}$$

Nash Equilibrium



Nash Equilibrium



Pareto Optimality

$$\begin{bmatrix} (3,3) & (0,5) \\ (5,0) & (1,1) \end{bmatrix}$$

$$\overbrace{(3,3),\ (0,5),\ (5,0),\ (1,1)}$$

Pareto Optimality

$$\begin{bmatrix} (3,3) & (0,5) \\ (5,0) & (1,1) \end{bmatrix}$$

$$\overbrace{(3,3),\ (0,5),\ (5,0),\ (1,1)}$$

$$(\underline{3},\underline{3}) > (1,1)$$

Computing the Nash Equilibria

- Lemke-Howson Algorithm
- Support Enumeration
- Iterative removal of strictly dominated strategies

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	L	C	R
U	$(3,0)$	$(2,1)$	$(0,0)$
M	$(1,1)$	$(1,1)$	$(5,0)$
D	$(0,1)$	$(4,2)$	$(0,1)$

Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	L	C	R
U	$(3, 0)$	$(2, \underline{1})$	$(0, 0)$
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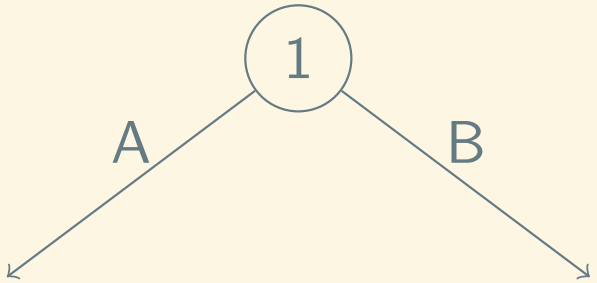
Perfect Information Extensive Form Games



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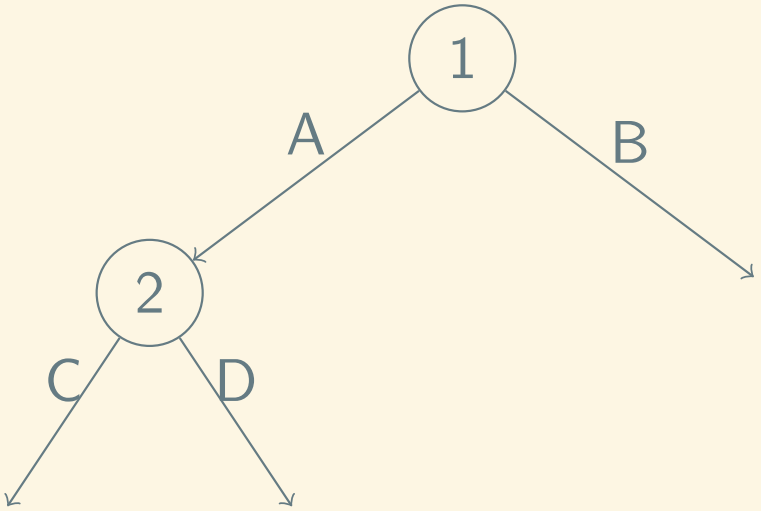

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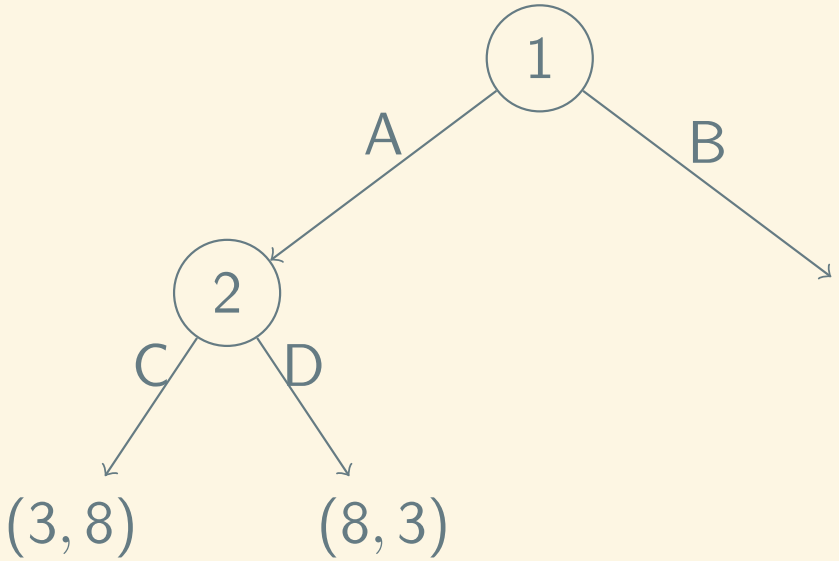

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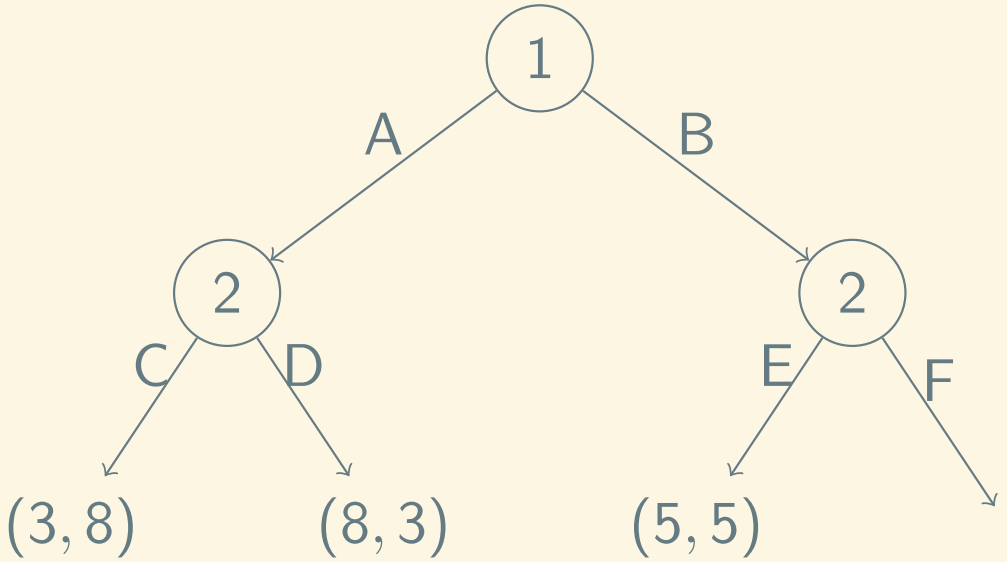

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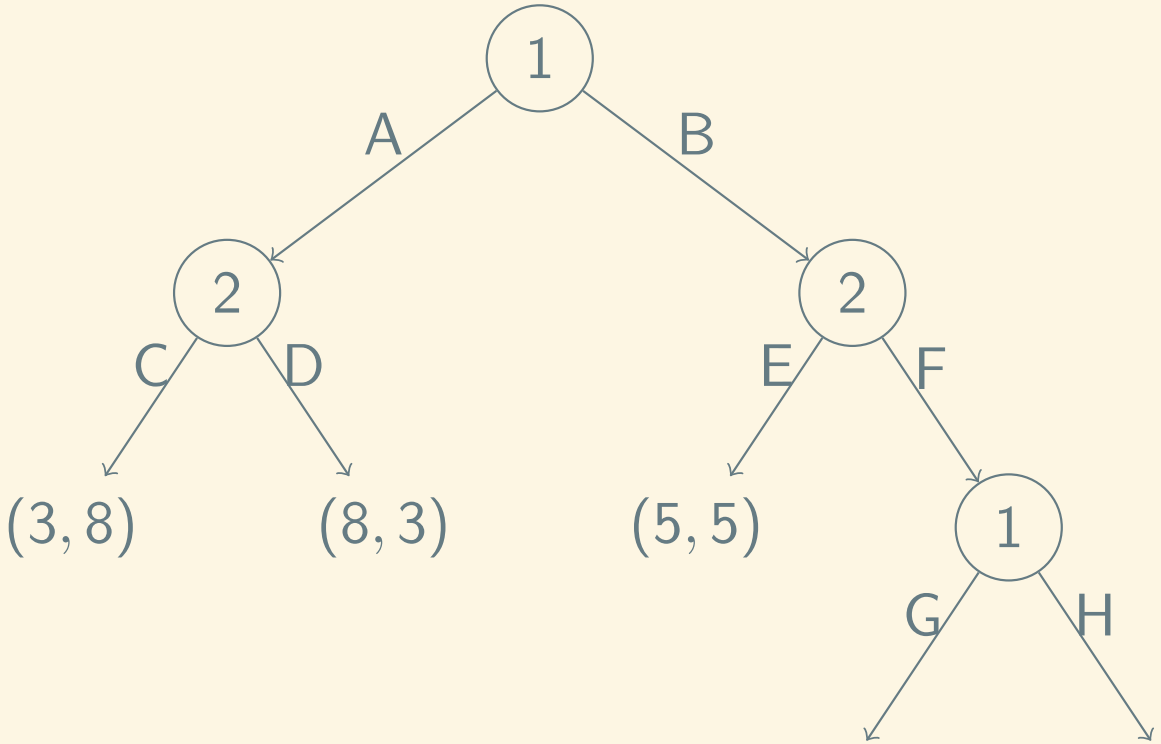

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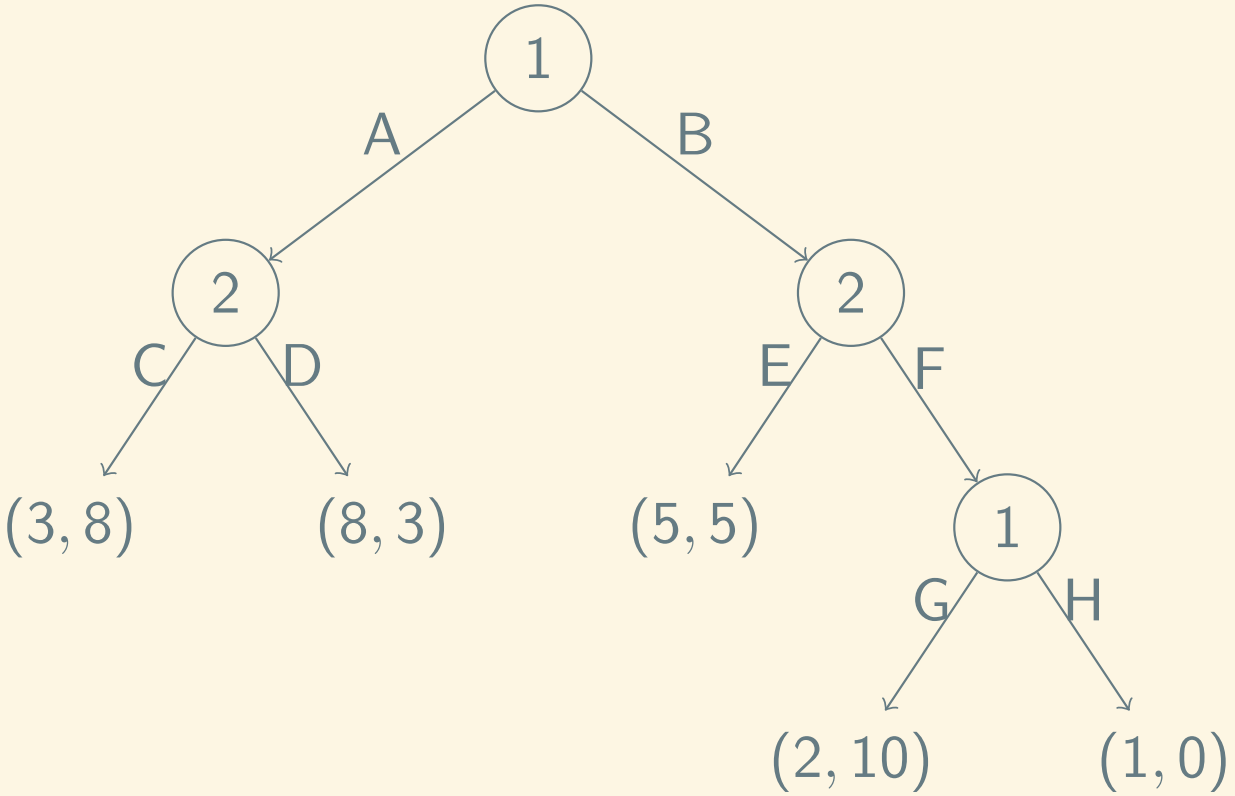
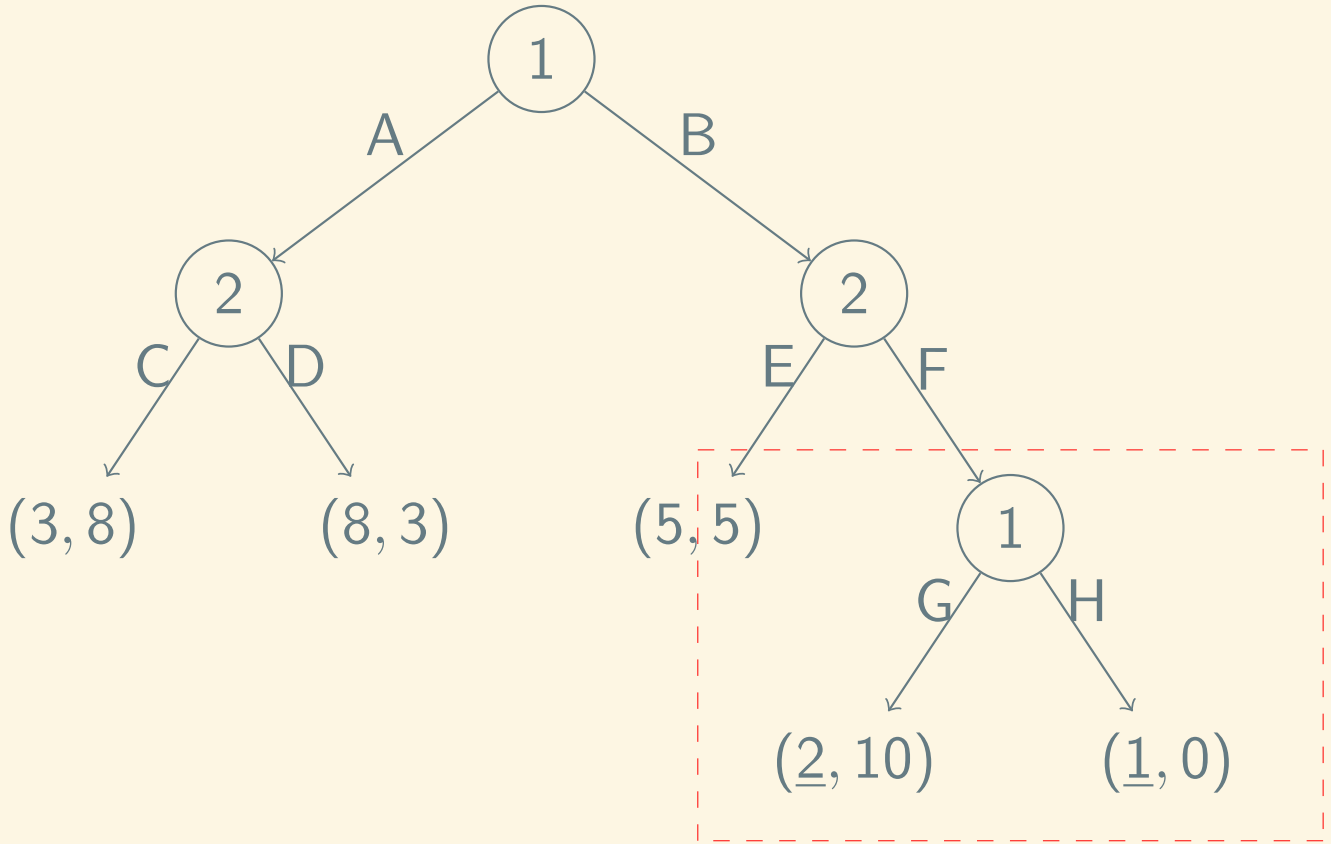


Backwards Induction



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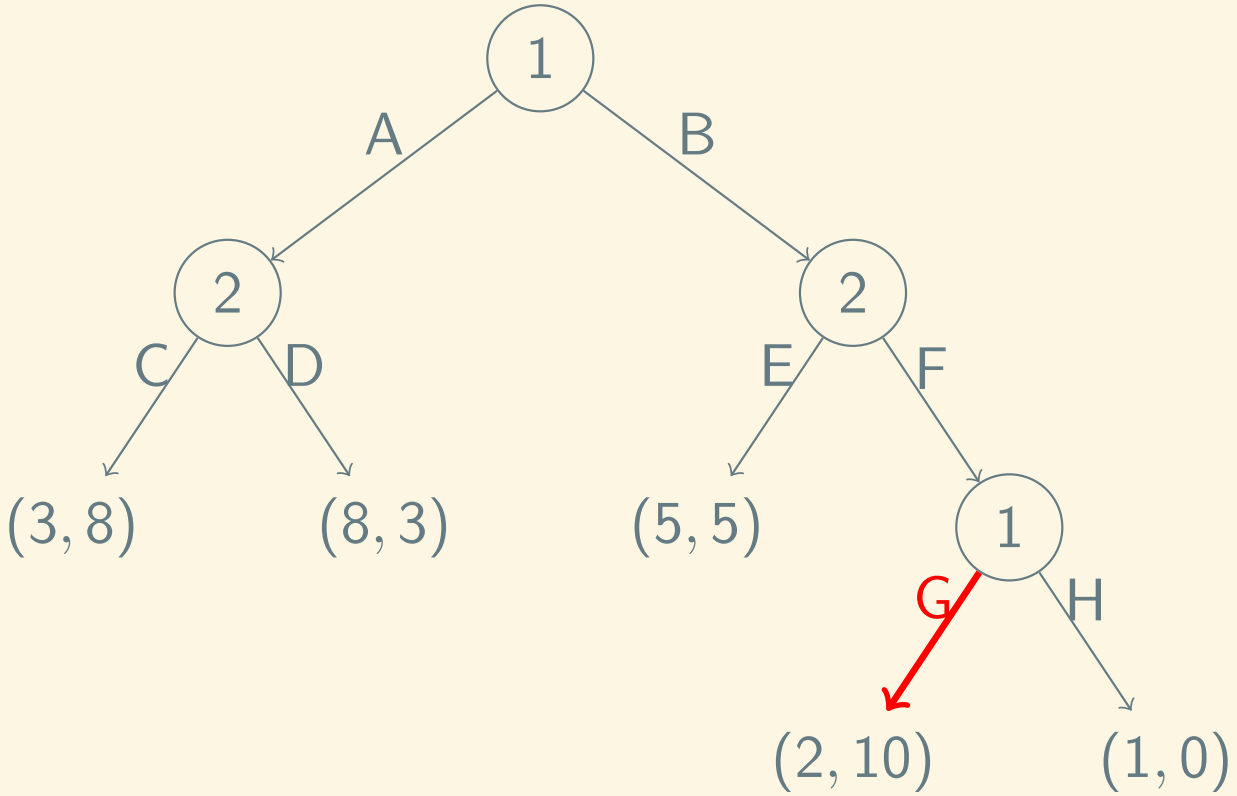

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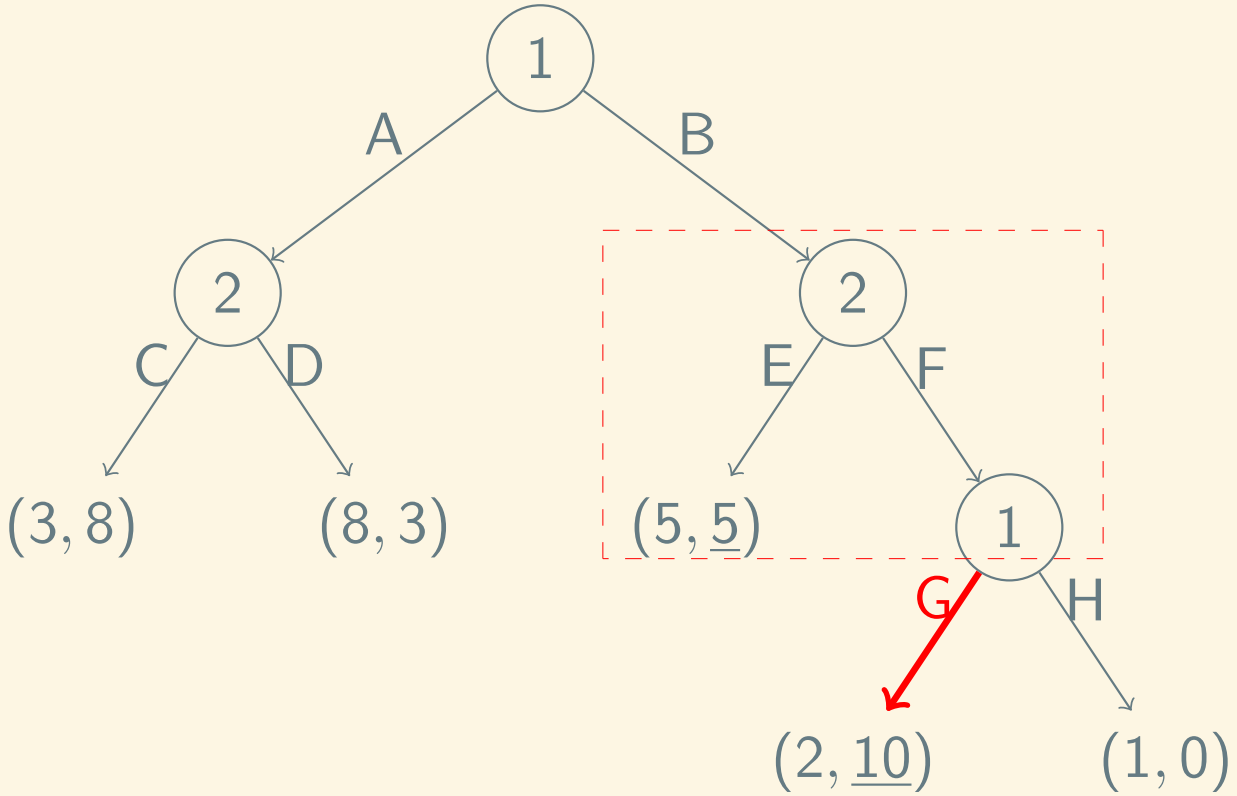

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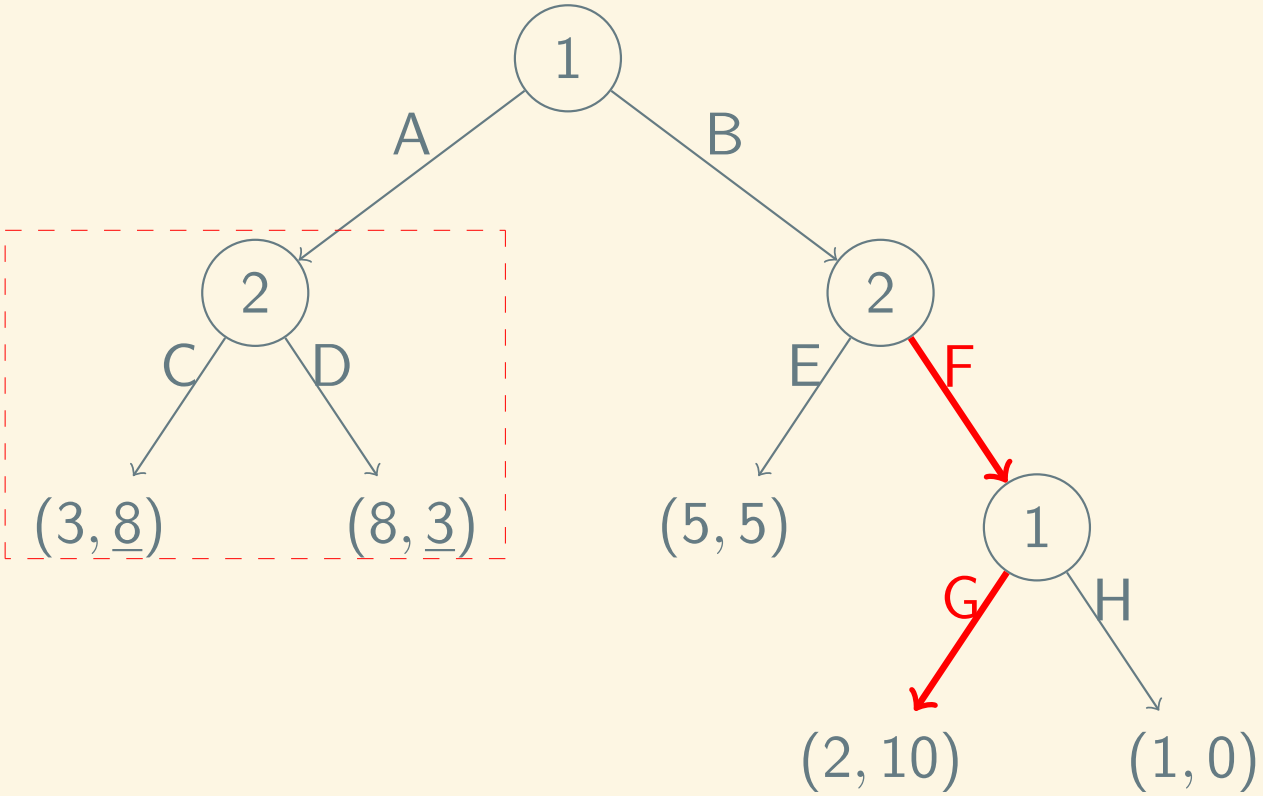

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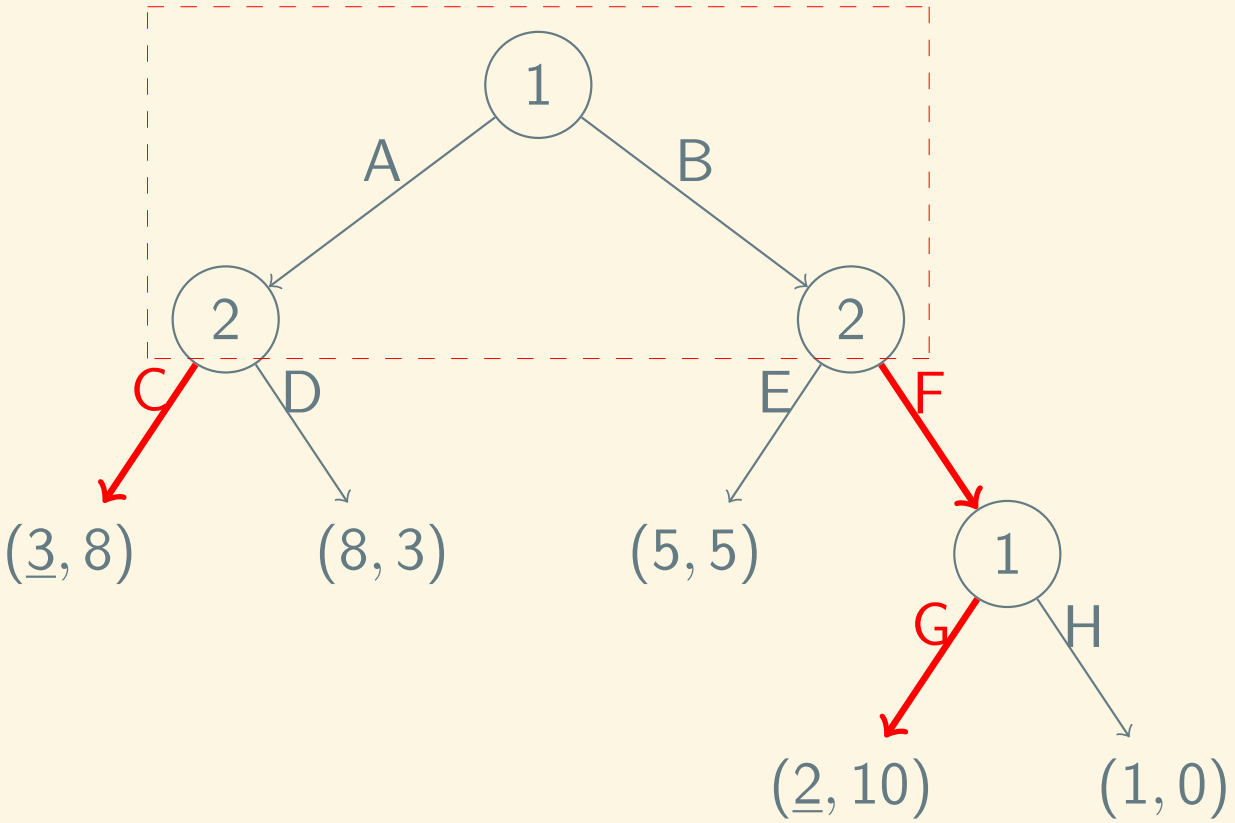

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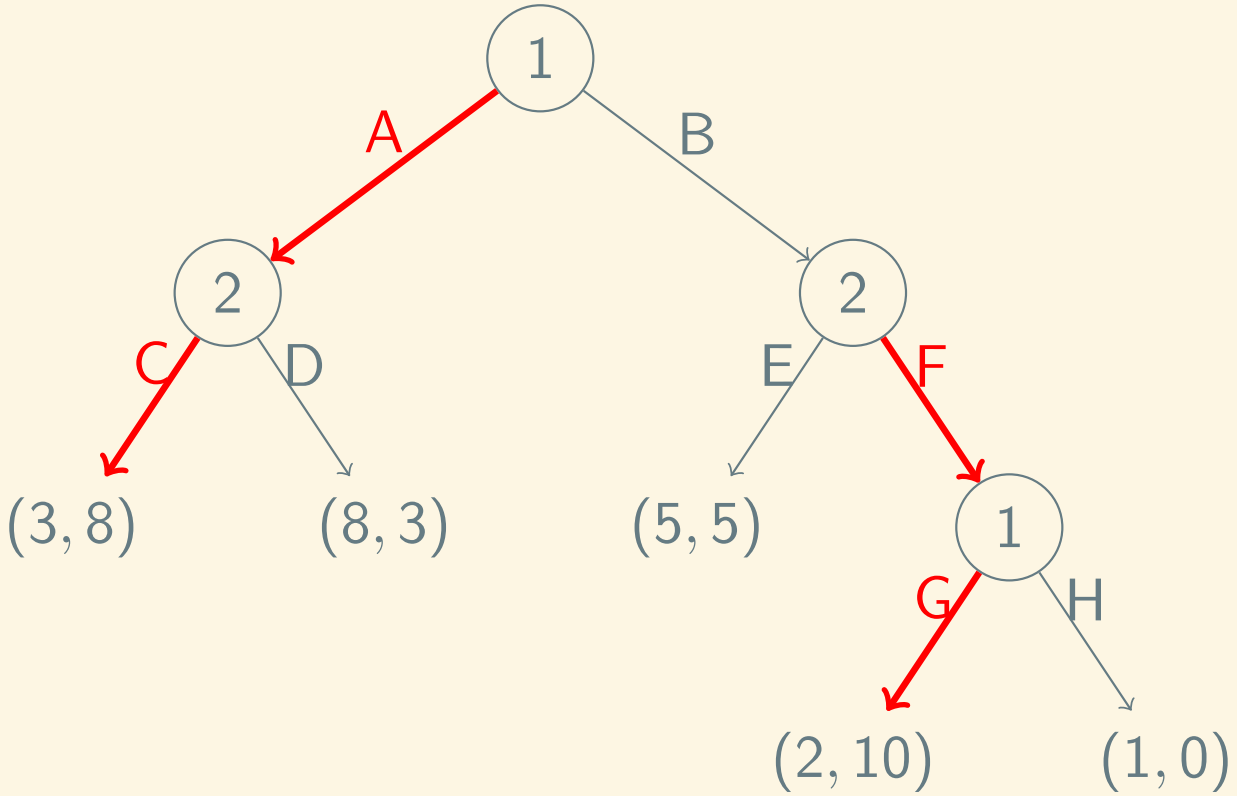

Imperfect Information Extensive-form Games

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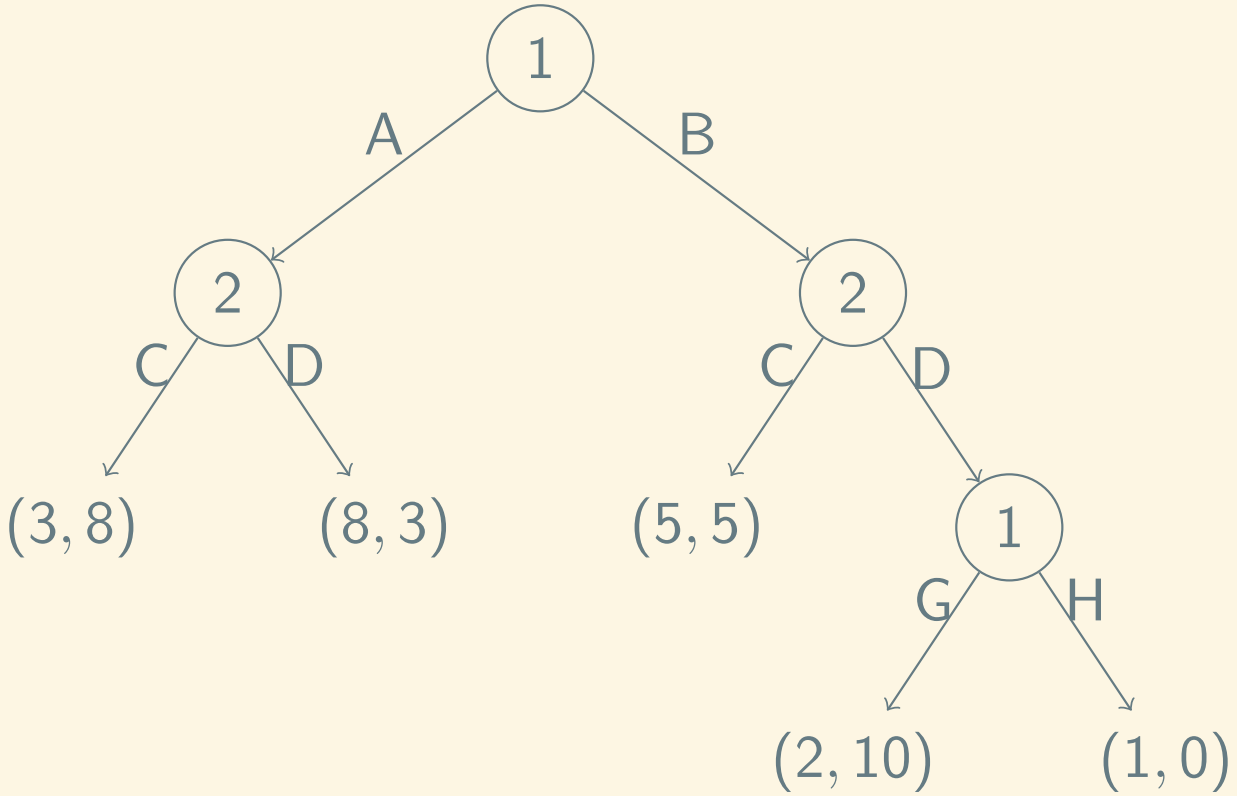
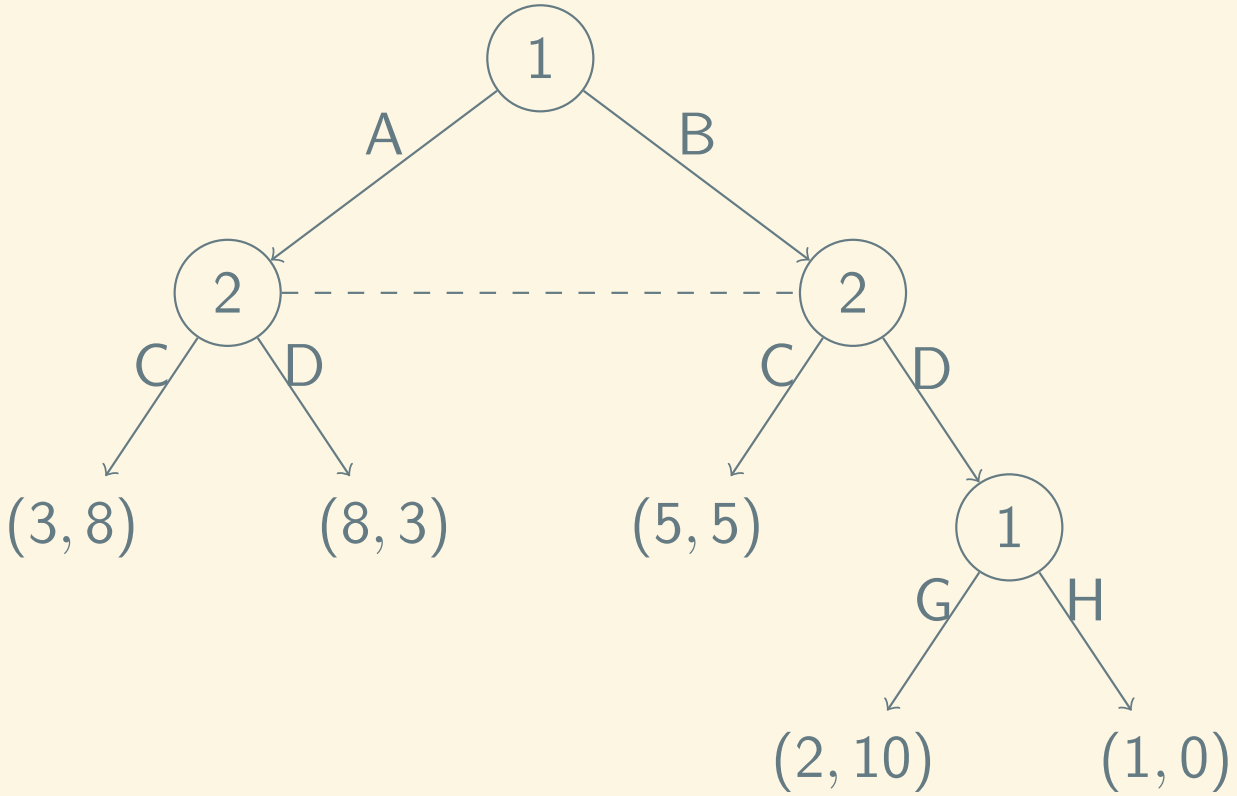
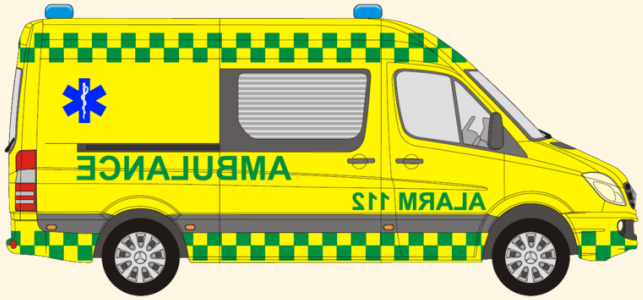


Imperfect Information Extensive-form Games



PhD - Motivation



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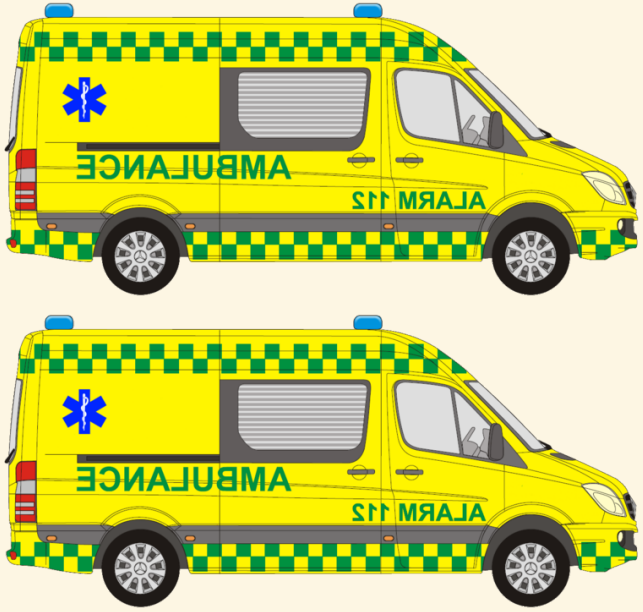

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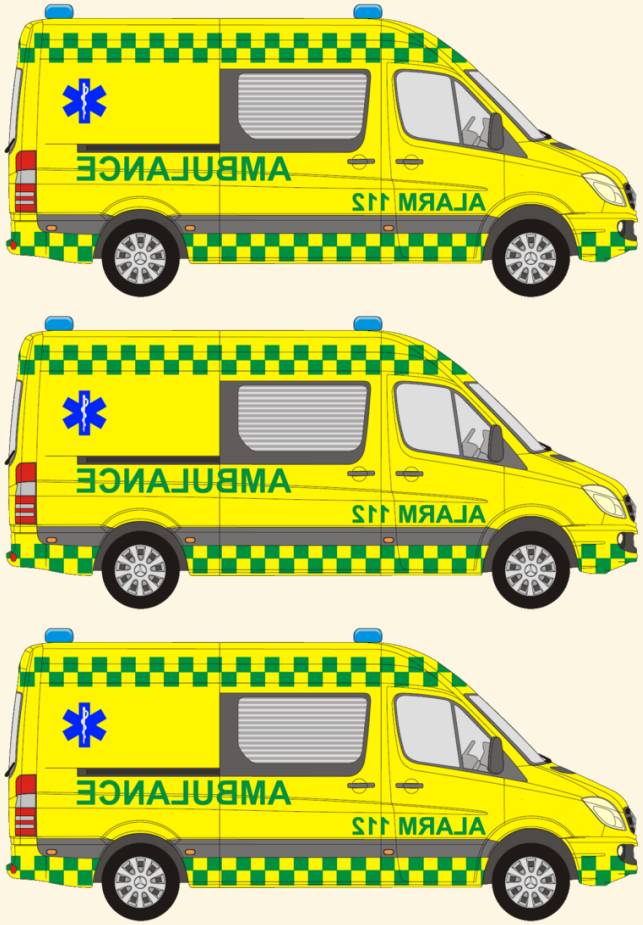
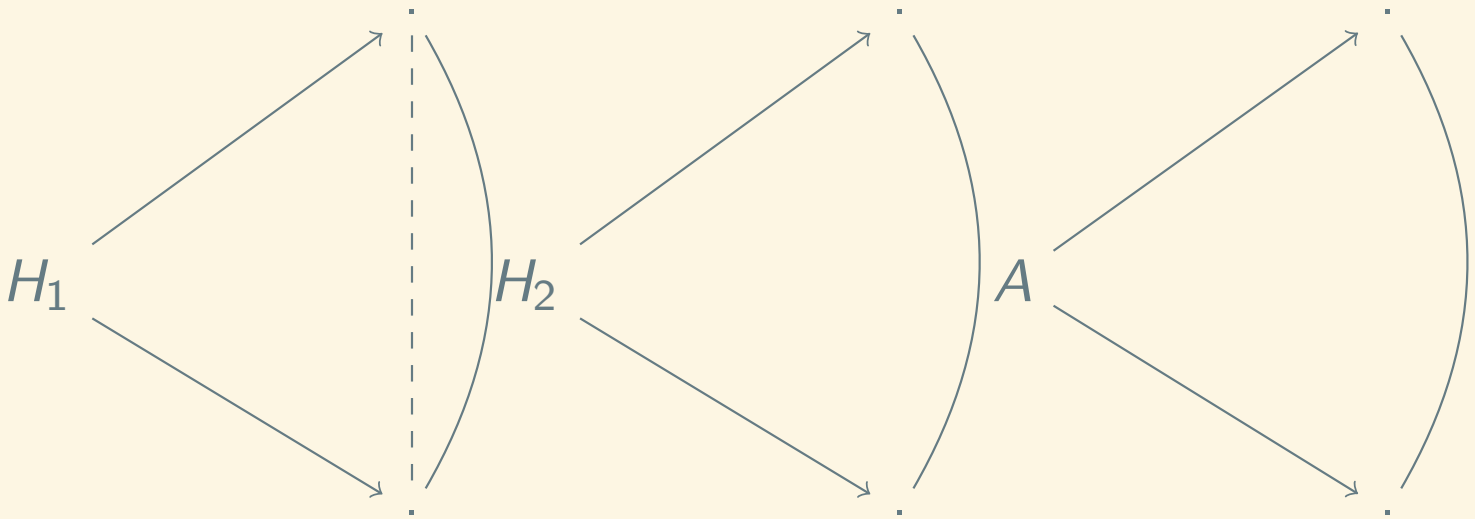
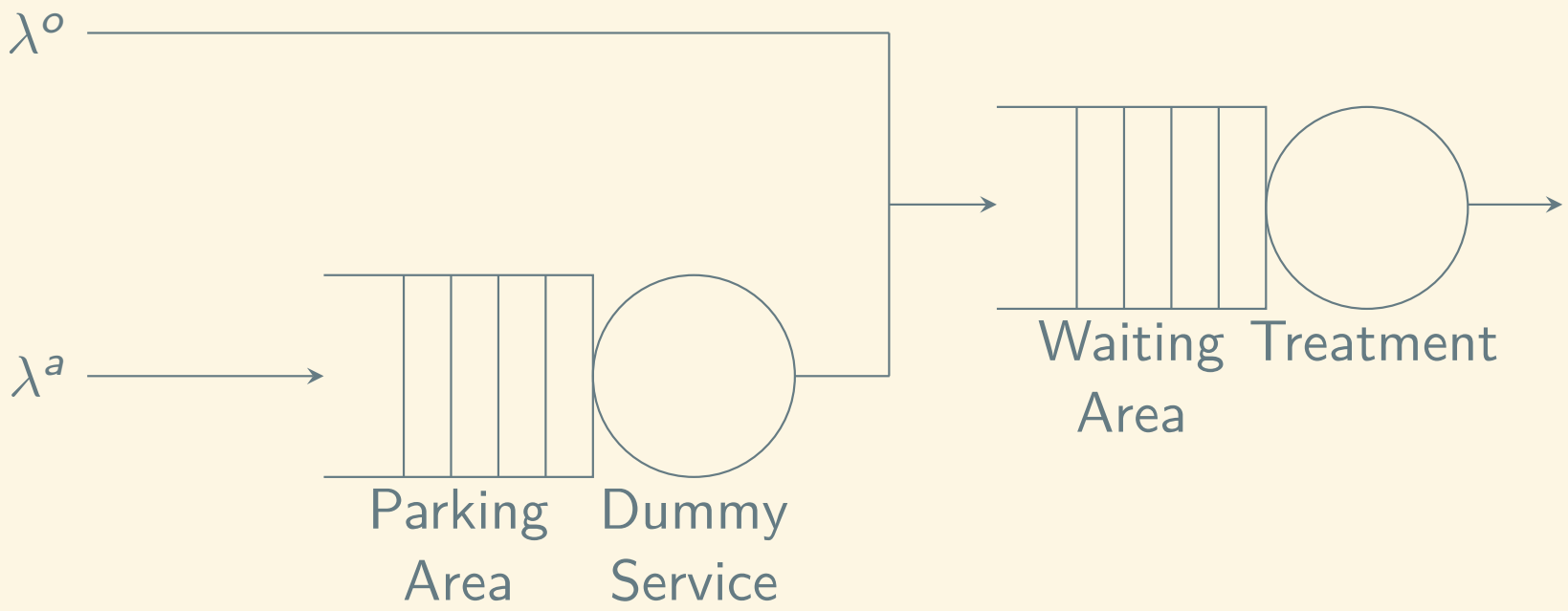


Ambulance - Hospital Interface



Hospital Formulation



Hospital - Markov Chain

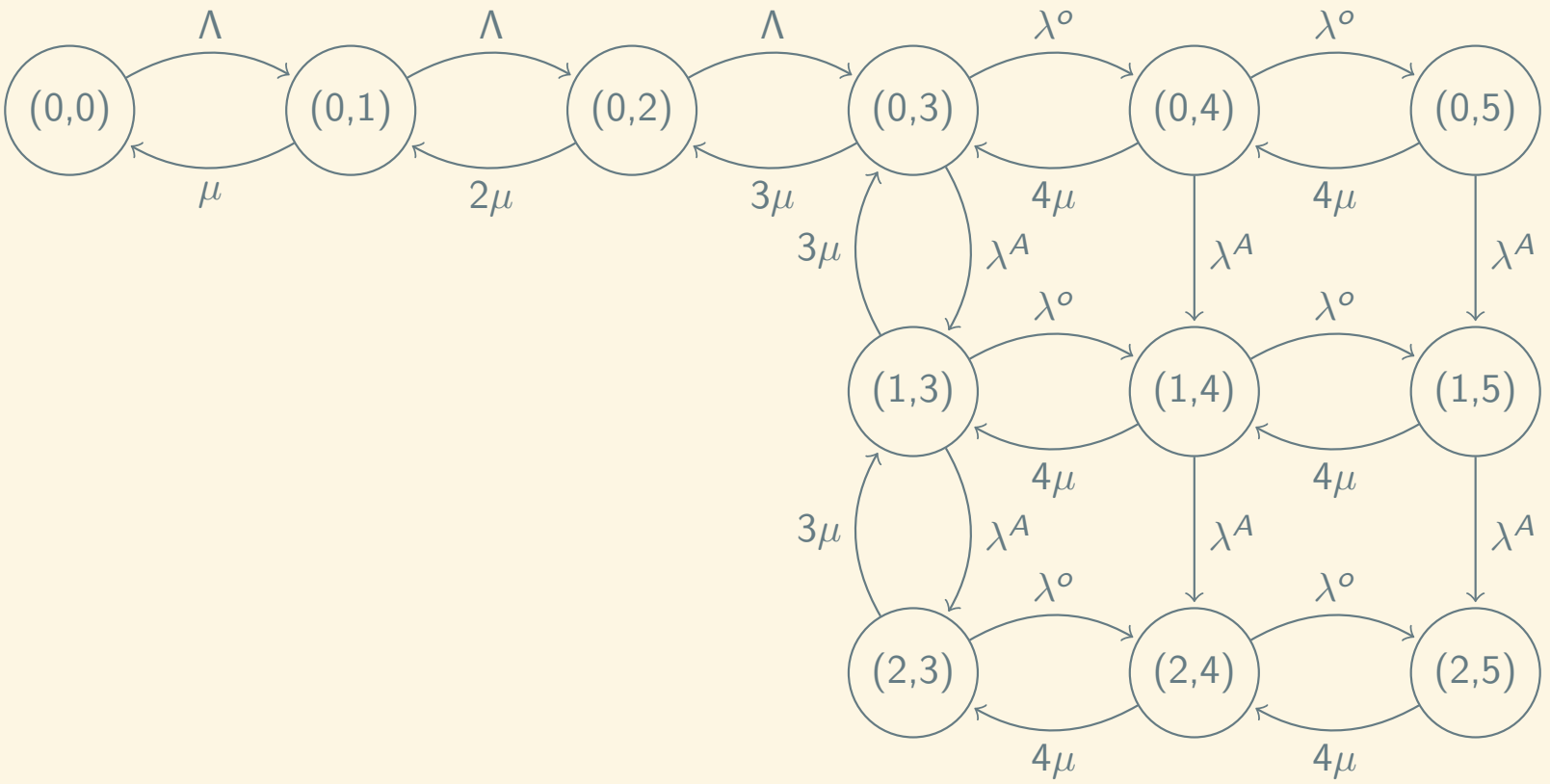
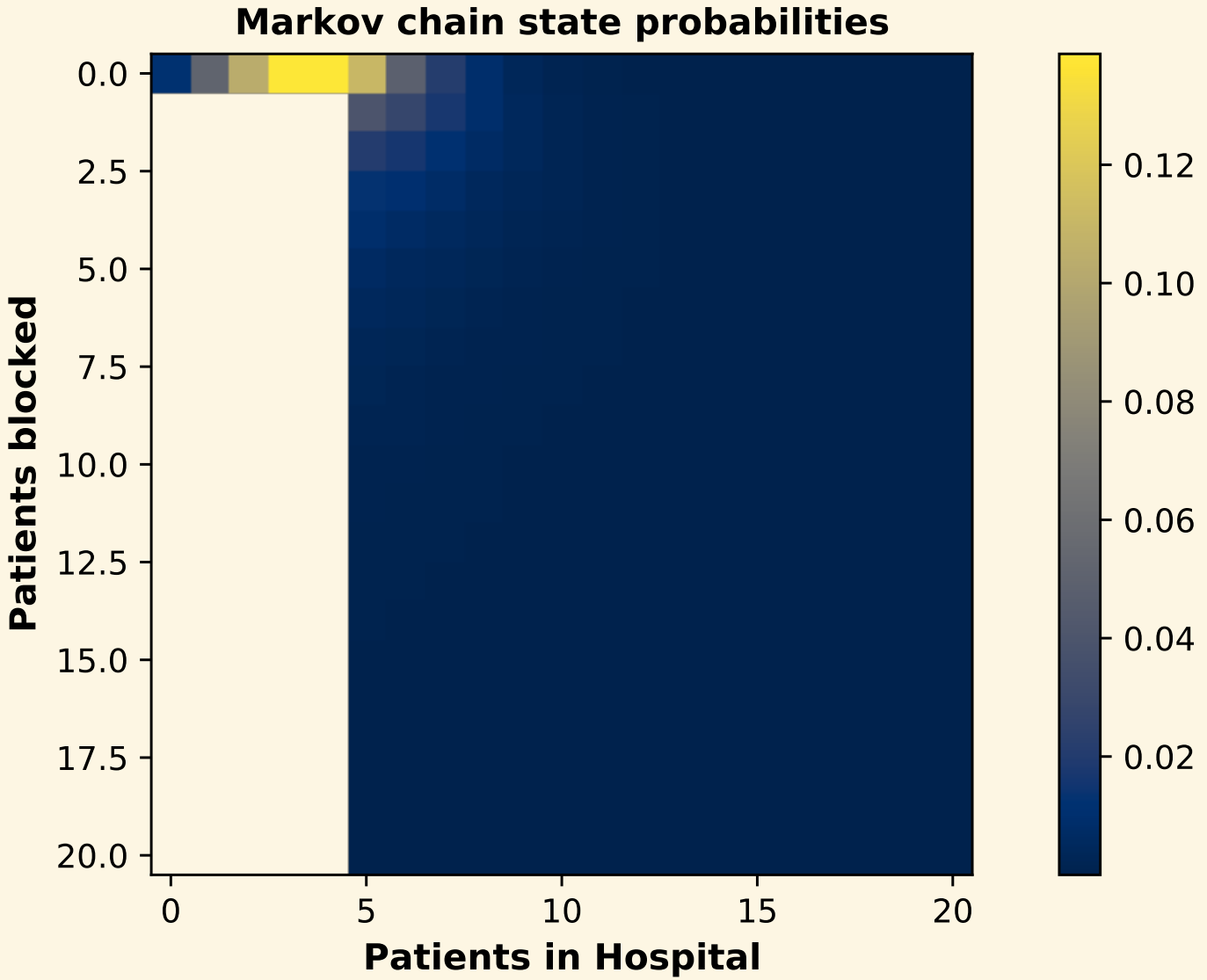
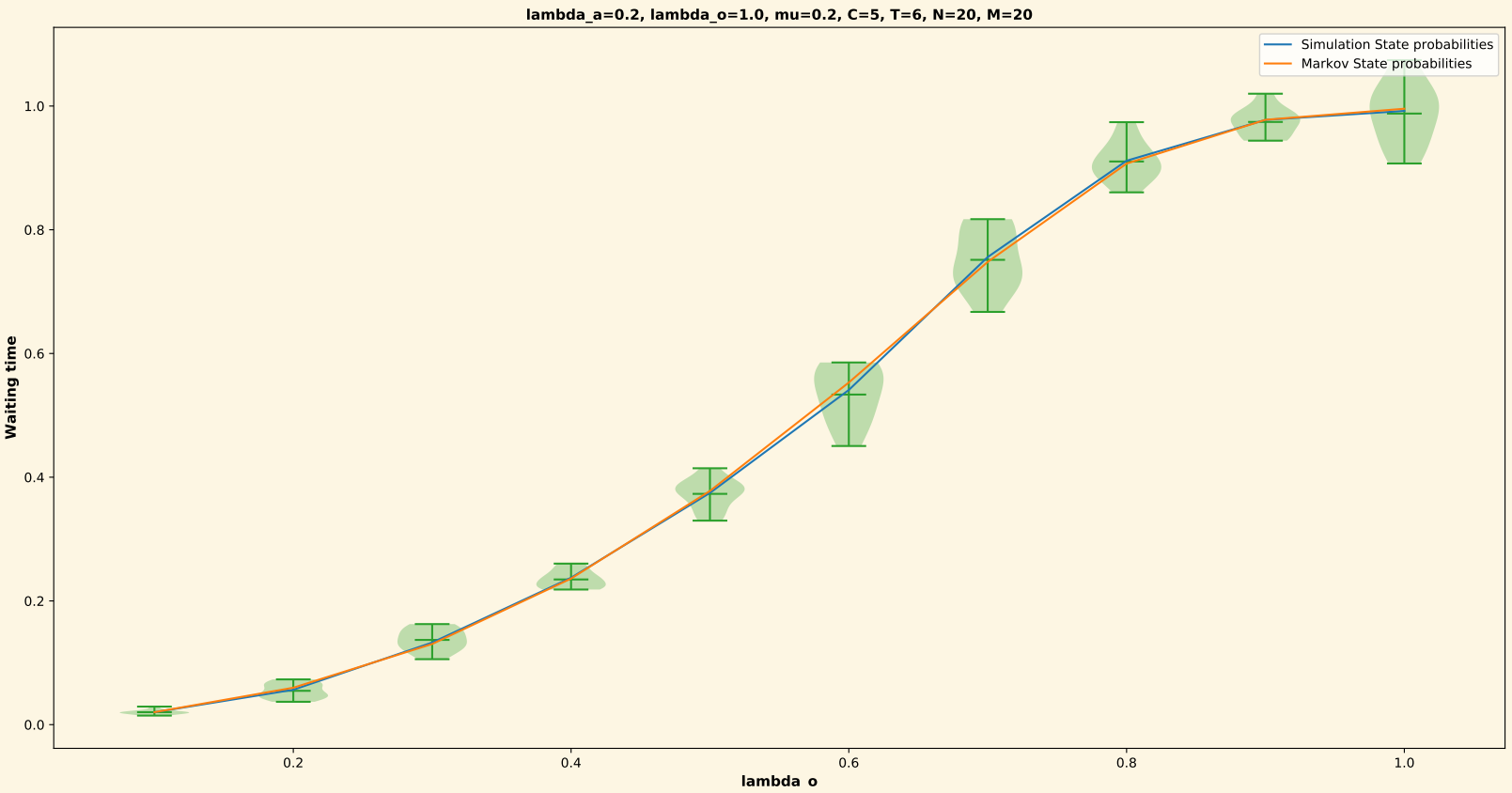


Figure: C=4, T=3, N=5, M=2

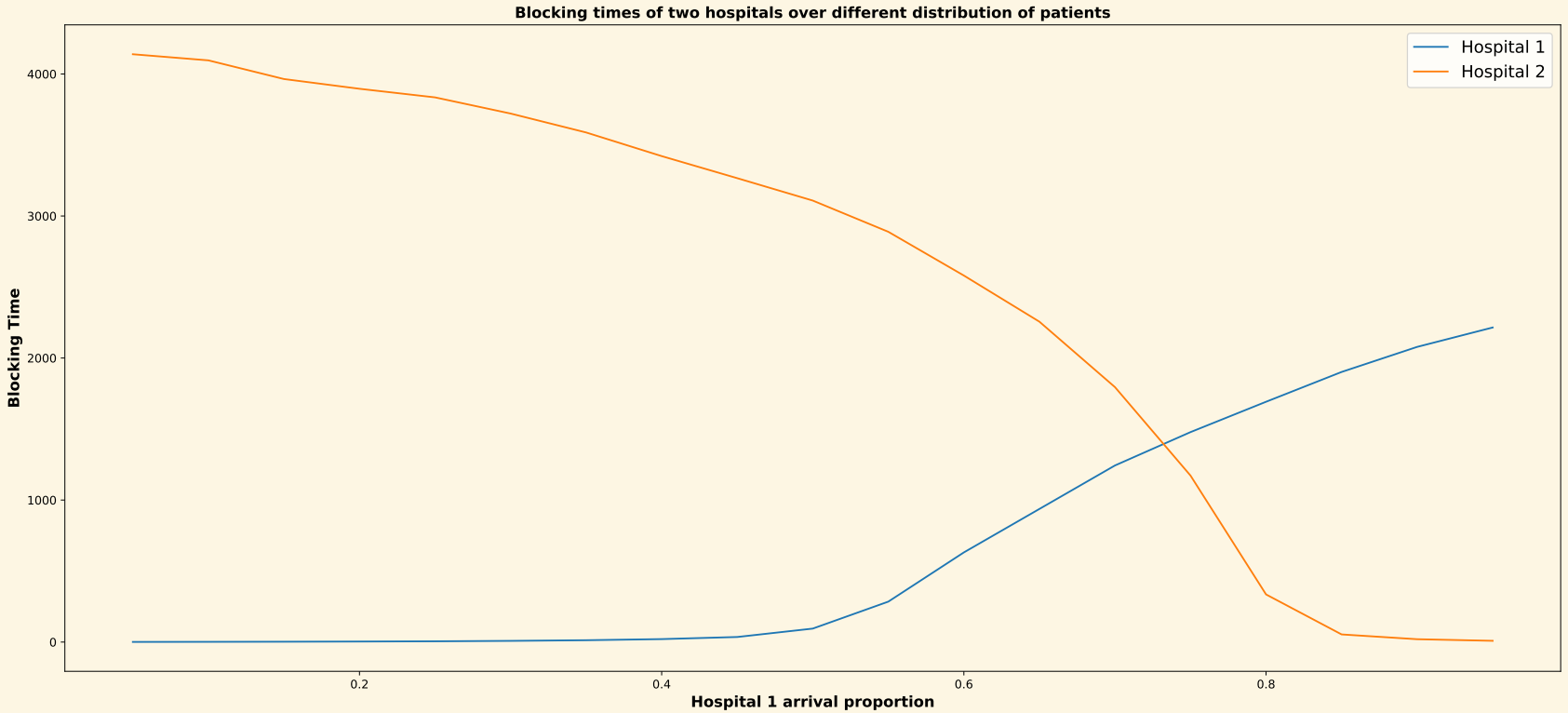
State Probabilities



Waiting Times



Optimal patient distribution



Future Plans

- Performance Measures
- Game theoretic interface