A 3-player game theoretic model of a choice between two queueing systems with strategic managerial decision making

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About me

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Queues - Examples

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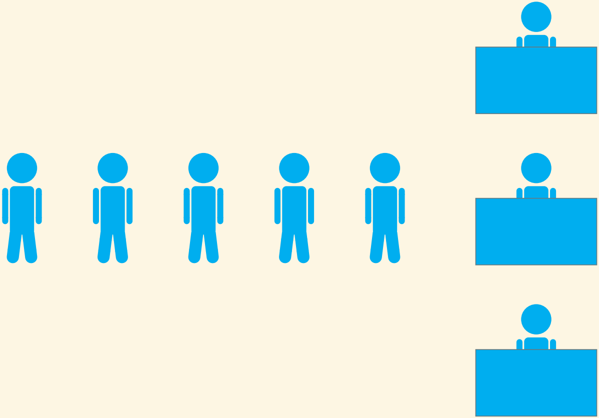
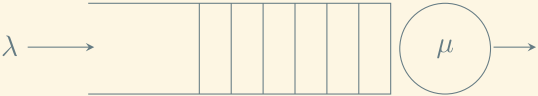
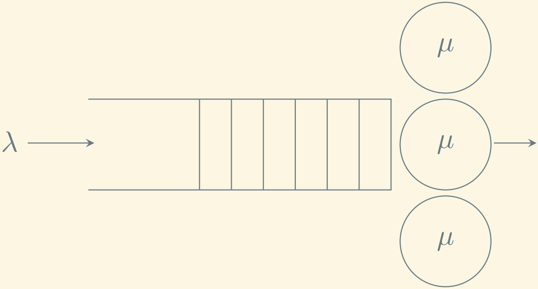


Queues - Examples

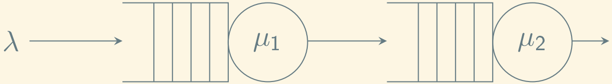
Discrete Event Simulation - M|M|1



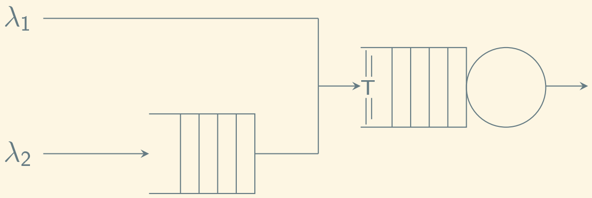
Discrete Event Simulation - M|M|3



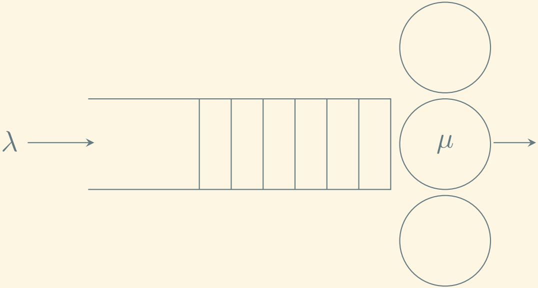
Discrete Event Simulation - Network of queues

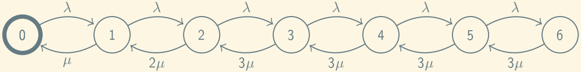


Discrete Event Simulation - Custom network

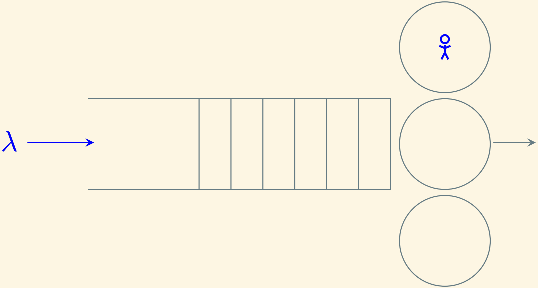


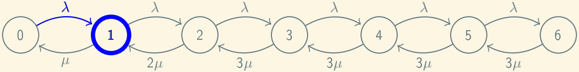
Markov Chain - M|M|3



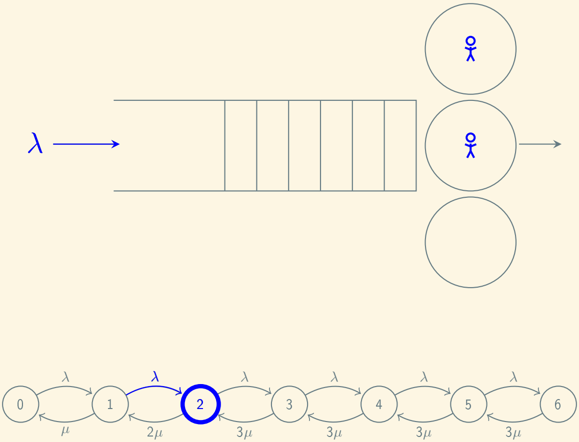


Markov Chain - M|M|3

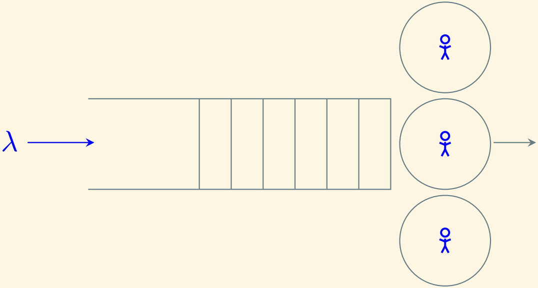


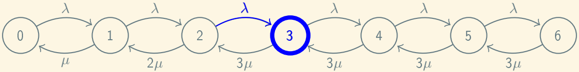


Markov Chain - M|M|3

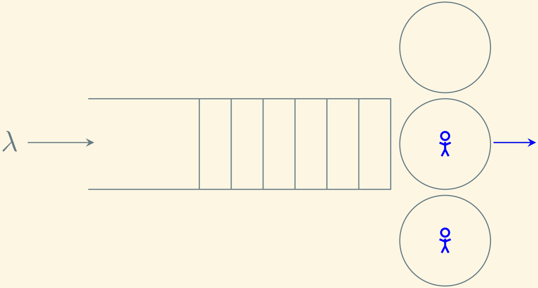


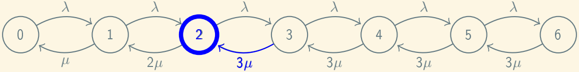
Markov Chain - M|M|3



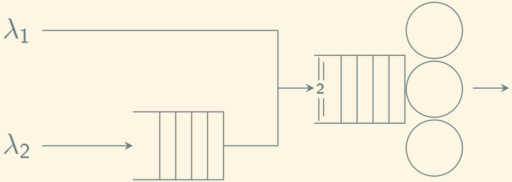


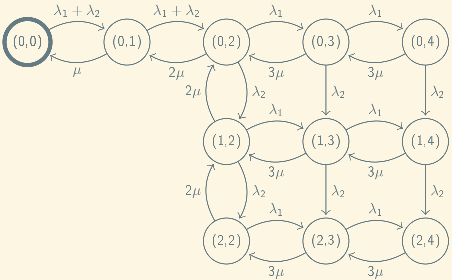
Markov Chain - M|M|3



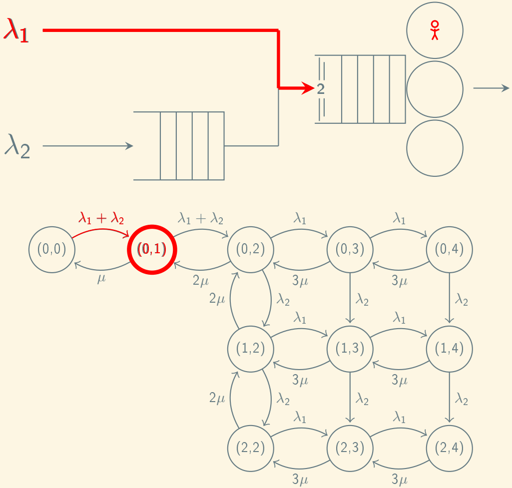


Markov Chain - Custom network





Markov Chain - Custom network



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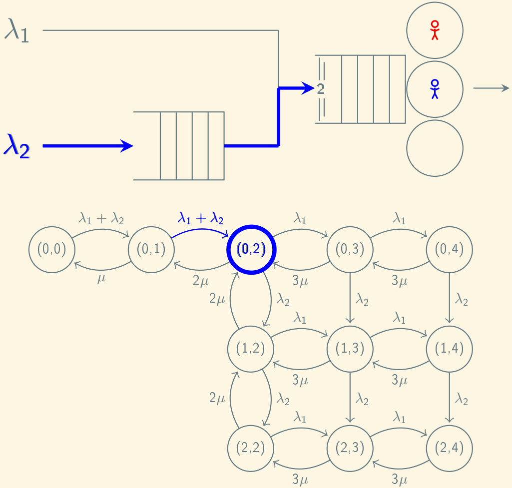

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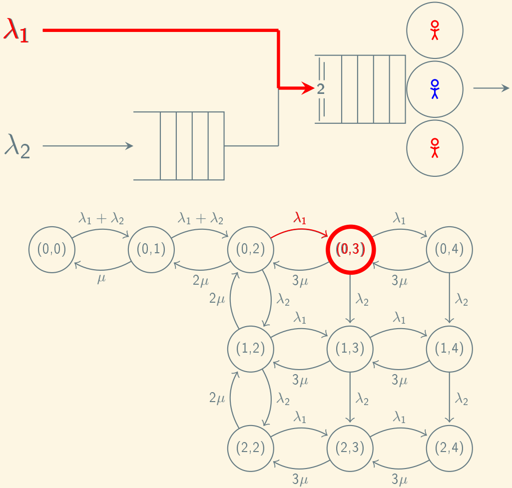

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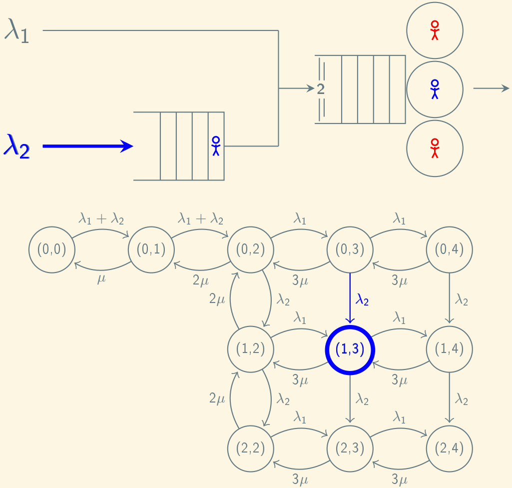

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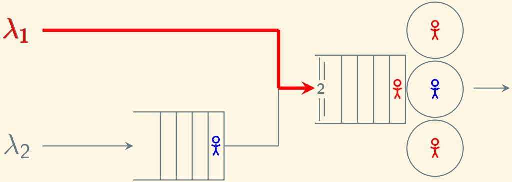
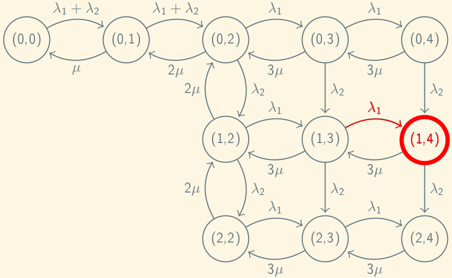
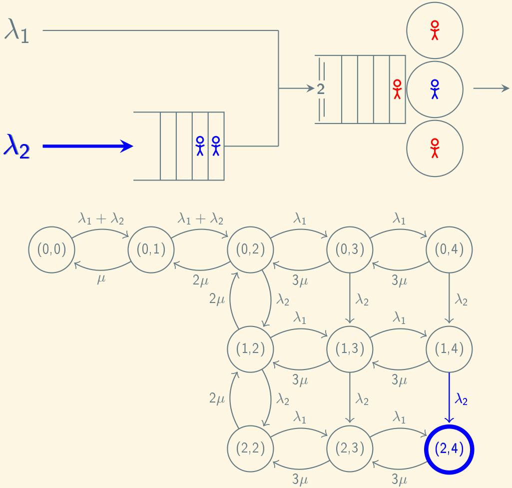
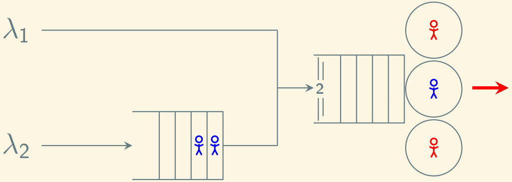


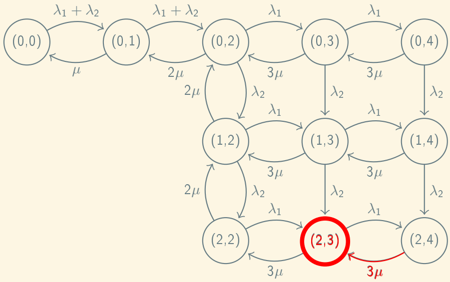


Markov Chain - Custom network

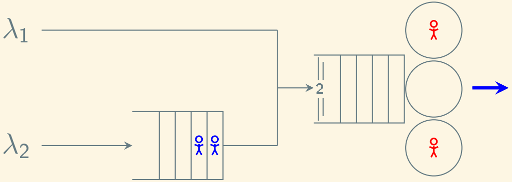


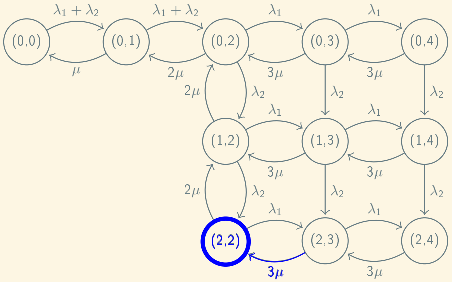
Markov Chain - Custom network



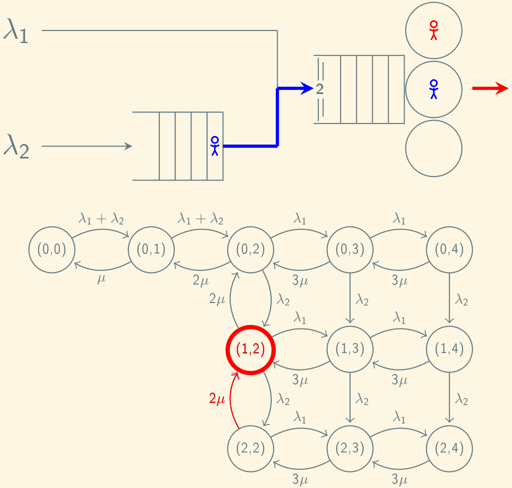


Markov Chain - Custom network

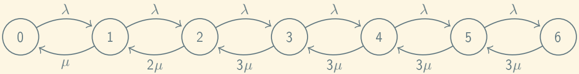




Markov Chain - Custom network



Steady state probabilities - M|M|3 queue



$$
\begin{array}{c c}
\begin{array}{ccccccc}
(0) & (1) & (2) & (3) & (4) & (5) & (6)
\end{array} & \\
Q = \begin{pmatrix}
-\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\
\mu & -\mu-\lambda & \lambda & 0 & 0 & 0 & 0 \\
0 & 2\mu & -2\mu-\lambda & \lambda & 0 & 0 & 0 \\
0 & 0 & 3\mu & -3\mu-\lambda & \lambda & 0 & 0 \\
0 & 0 & 0 & 3\mu & -3\mu-\lambda & \lambda & 0 \\
0 & 0 & 0 & 0 & 3\mu & -3\mu-\lambda & \lambda \\
0 & 0 & 0 & 0 & 0 & 3\mu & -3\mu
\end{pmatrix} &
\begin{array}{c}
(0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6)
\end{array}
\end{array}
$$

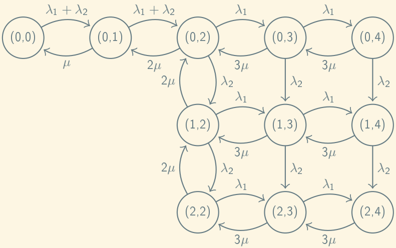
$$\frac{d\pi}{dt} = \pi Q = 0$$

$$\sum \pi_i = 1$$

$$\pi = \begin{pmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \\ \pi_6 \end{pmatrix}$$

Steady state probabilities - Custom network

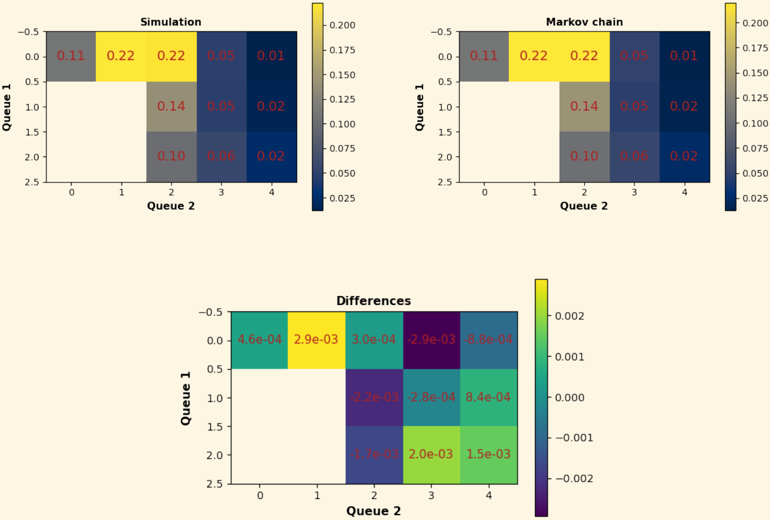
$$
Q = \begin{array}{c c}
\begin{array}{cccccc} (0,0) & (0,1) & (0,2) & & (2,3) & (2,4) \end{array} & \\
\begin{pmatrix}
-\lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & 0 & \dots & 0 & 0 \\
\mu & -\mu - \lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & \dots & 0 & 0 \\
0 & 2\mu & -2\mu - \lambda_1 - \lambda_2 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \dots & -\lambda_1 - 3\mu & \lambda_1 \\
0 & 0 & 0 & \dots & 3\mu & -3\mu
\end{pmatrix} &
\begin{array}{c} (0,0) \\ (0,1) \\ (0,2) \\ \\ (2,3) \\ (2,4) \end{array}
\end{array}
$$



$$\frac{d\pi}{dt} = \pi Q = 0, \qquad \sum \pi_{(u,v)} = 1$$

$$\pi = \begin{bmatrix} \pi_{(0,0)} \\ \pi_{(0,1)} \\ \pi_{(0,2)} \\ \vdots \\ \pi_{(2,3)} \\ \pi_{(2,4)} \end{bmatrix}$$

Steady state probabilities - Comparison



Performance Measures - Number of individuals

$$(u_i, v_i) \longrightarrow \pi_{(u_i,v_i)}$$

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$$(u_i, v_i) \longrightarrow \pi_{(u_i, v_i)}$$

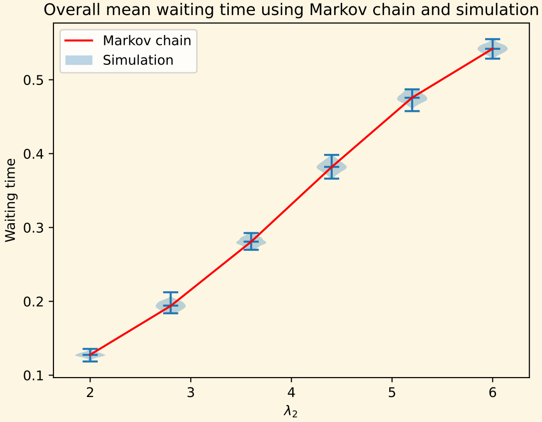
$$L = \sum_{i=1}^{|\pi|} \pi_i (u_i + v_i)$$

$$L_1 = \sum_{i=1}^{|\pi|} \pi_i u_i$$

$$L_2 = \sum_{i=1}^{|\pi|} \pi_i v_i$$

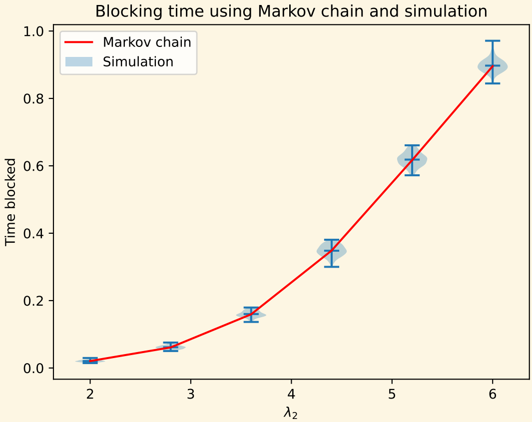
Performance Measures - Waiting time

$$W = \frac{\lambda_1 P_{L_1'}}{\lambda_2 P_{L_2'} + \lambda_1 P_{L_1'}} W^{(1)} + \frac{\lambda_2 P_{L_2'}}{\lambda_2 P_{L_2'} + \lambda_1 P_{L_1'}} W^{(2)} \tag{1}$$



Performance Measures - Blocking time

$$B = \frac{\sum_{(u,v)\in S_A^{(2)}} \pi_{(u,v)} \; b(u,v)}{\sum_{(u,v)\in S_A^{(2)}} \pi_{(u,v)}} \tag{2}$$



Performance Measures - Proportion within time

$$P(X < t) = \frac{\lambda_1 P_{L'_1}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} P(X^{(1)} < t) + \frac{\lambda_2 P_{L'_2}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} P(X^{(2)} < t) \tag{3}$$