

# Annual Review

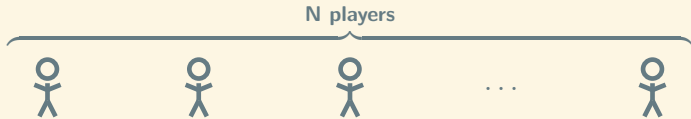
Michalis Panayides

2020-06-10

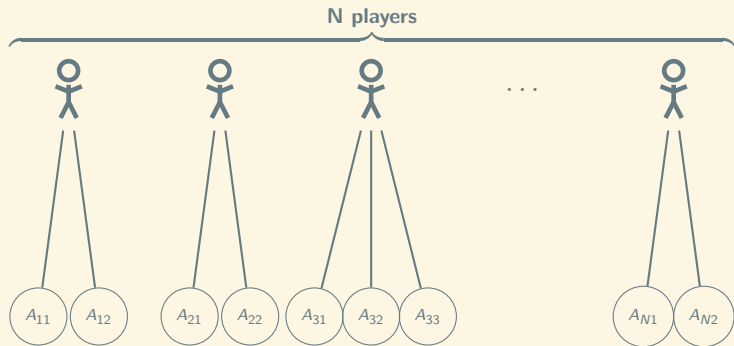
# Game Theory - Syllabus

- ▶ Normal Form Games
- ▶ Mixed-Strategy Nash Equilibrium
- ▶ Alternate Solution Concepts
- ▶ Extensive-Form Games
- ▶ Repeated Games (TBC)
- ▶ Bayesian Games (TBC)
- ▶ Coalitional Games (TBC)

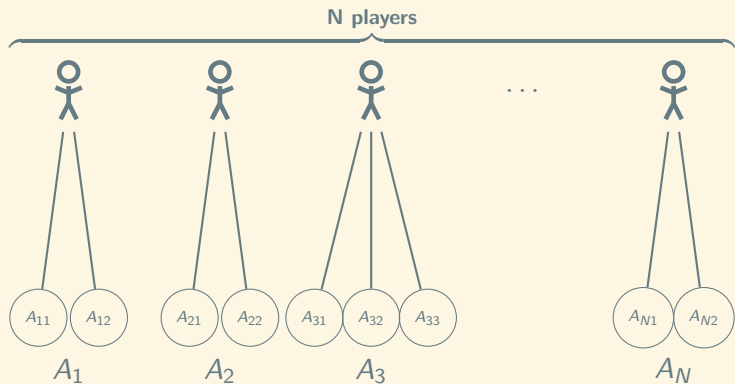
# Normal Form Games



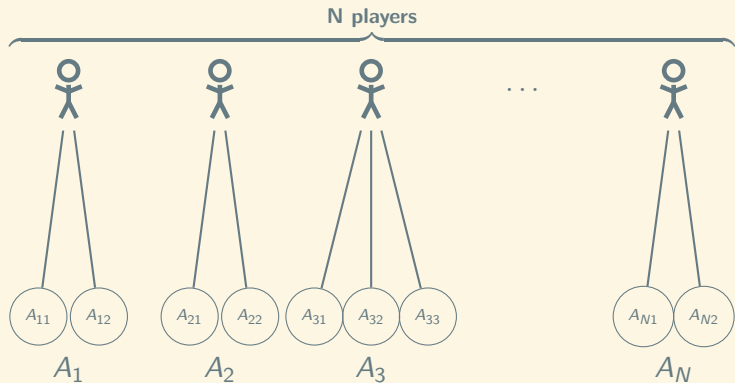
# Normal Form Games



# Normal Form Games









# Normal Form Games



$$u_i = A_1 \times A_2 \times A_3 \times \cdots \times A_N$$

# Rock-Paper-Scissors

|   |  |  |  |
|---|---|---|---|
|  | $(0, 0)$  | $(1, -1)$   | $(-1, 1)$   |
|  | $(-1, 1)$   | $(0, 0)$  | $(1, -1)$   |
|  | $(1, -1)$   | $(-1, 1)$   | $(0, 0)$  |

# Nash Equilibrium

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$



# Nash Equilibrium

$$\begin{array}{c} \text{C} \longrightarrow \\ \text{D} \longrightarrow \end{array} \begin{bmatrix} (3, -) & (0, -) \\ (\underline{5}, -) & (\underline{1}, -) \end{bmatrix}$$

# Nash Equilibrium

| C  | D  |
|--|--|
| $\downarrow$                                     | $\downarrow$   |
| $\begin{bmatrix} (-, 3) \\ (-, 0) \end{bmatrix}$ | $\begin{bmatrix} (-, \underline{5}) \\ (-, \underline{1}) \end{bmatrix}$ |

# Nash Equilibrium

|   |   |  |
|---|---|--|
|   |   | D  |
|   |   | ↓  |
| D | → | $\begin{bmatrix} (3, 3) & (0, \underline{5}) \\ (\underline{5}, 0) & (\underline{1}, \underline{1}) \end{bmatrix}$ |

# Pareto Optimality

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

$$\overbrace{(3, 3), (0, 5), (5, 0), (1, 1)}$$

# Pareto Optimality

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

$$\overbrace{(3, 3), (0, 5), (5, 0), (1, 1)}$$

$$(\underline{3}, \underline{3}) > (1, 1)$$

# Computing the Nash Equilibria

- ▶ Lemke-Howson Algorithm
- ▶ Support Enumeration
- ▶ Iterative removal of strictly dominated strategies

## Iterative Removal of Strictly Dominated Strategies

| $P1 \setminus P2$ | $L$    | $C$    | $R$    |
|-------------------|--------|--------|--------|
| $U$               | (3, 0) | (2, 1) | (0, 0) |
| $M$               | (1, 1) | (1, 1) | (5, 0) |
| $D$               | (0, 1) | (4, 2) | (0, 1) |

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# Iterative Removal of Strictly Dominated Strategies

| $P1 \setminus P2$ | $C$      |
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| $D$               | $(4, 2)$ |

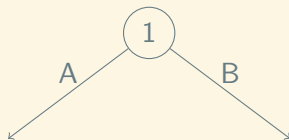
# Iterative Removal of Strictly Dominated Strategies

| $P1 \backslash P2$ | $C$                  |
|--------------------|----------------------|
| $U$                | $(2, 1)$             |
| $D$                | $(\underline{4}, 2)$ |

# Iterative Removal of Strictly Dominated Strategies

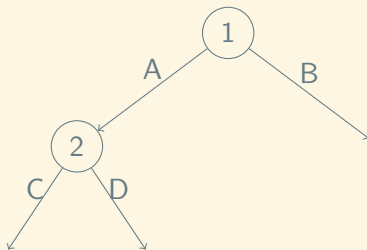
$$\begin{array}{cc} P1 \backslash P2 & C \\ D & (4, 2) \end{array}$$

# Perfect Information Extensive Form Games

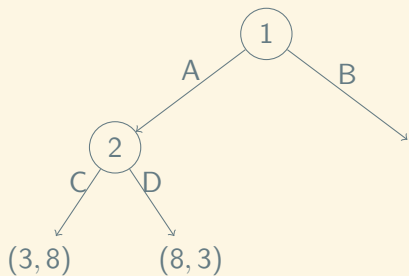




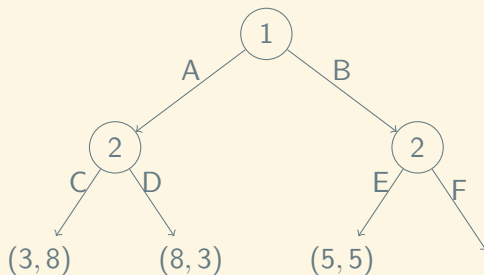
# Perfect Information Extensive Form Games



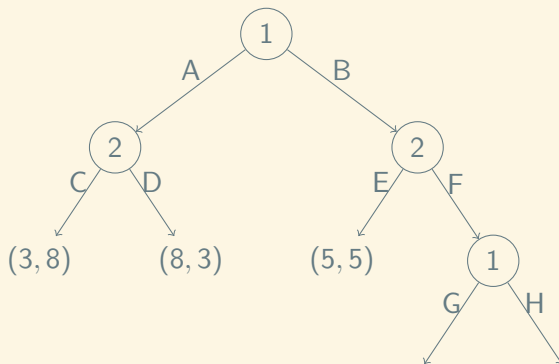
# Perfect Information Extensive Form Games



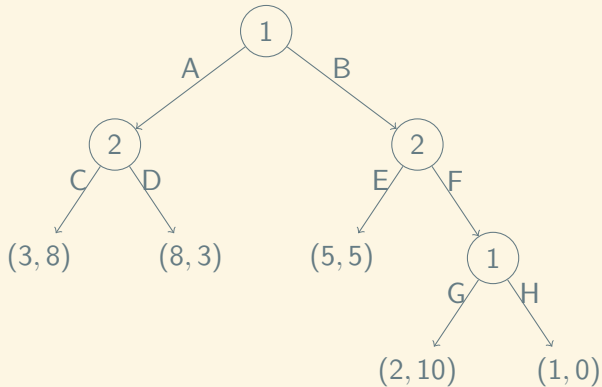
# Perfect Information Extensive Form Games



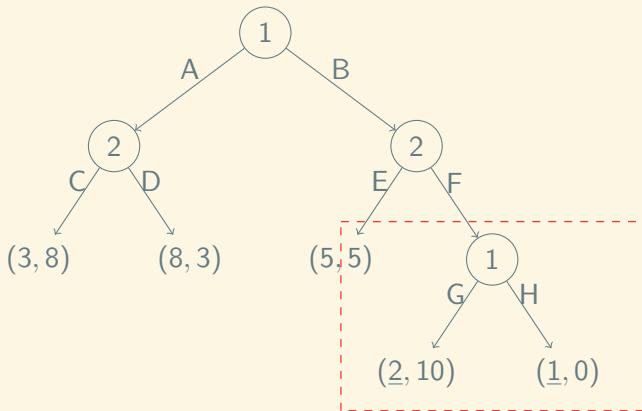
# Perfect Information Extensive Form Games



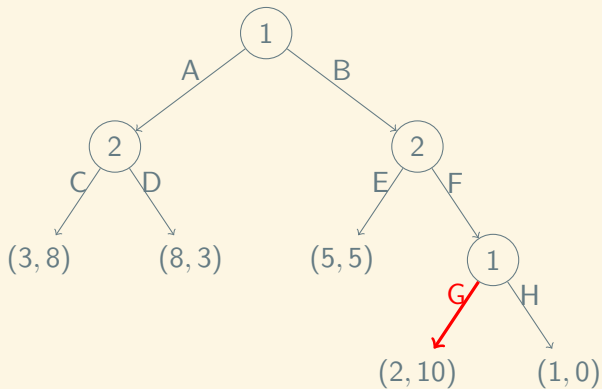
# Perfect Information Extensive Form Games



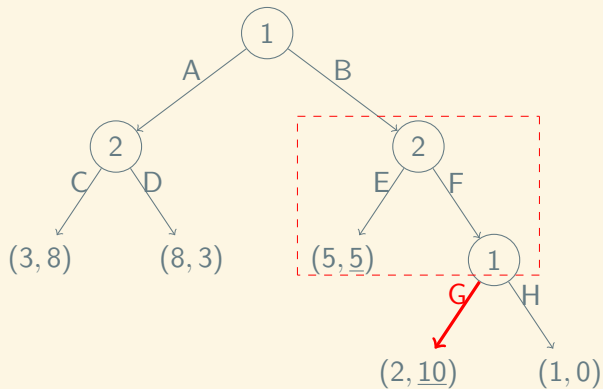
# Backwards Induction



# Backwards Induction

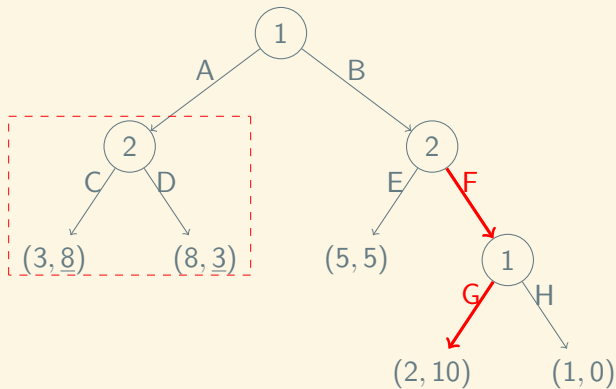


# Backwards Induction

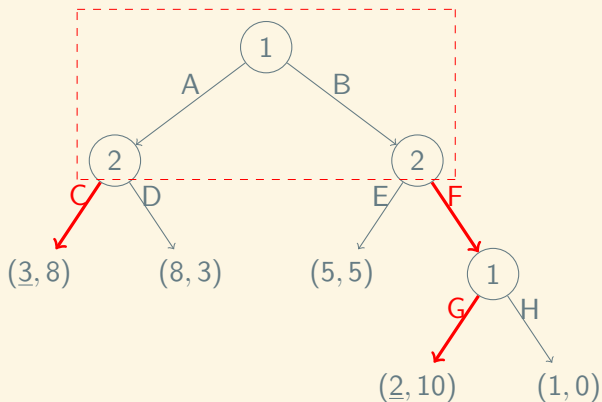




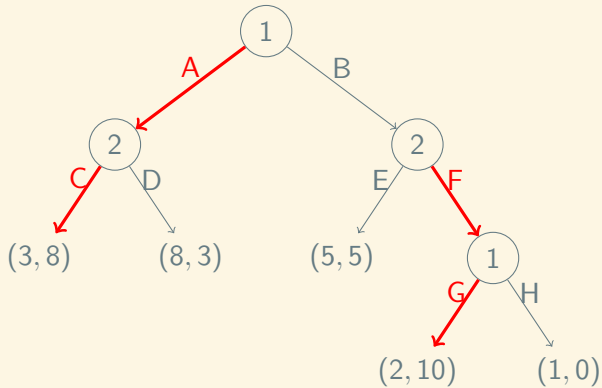
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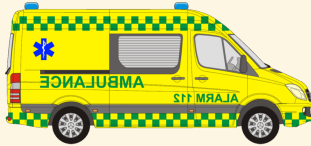
# Backwards Induction



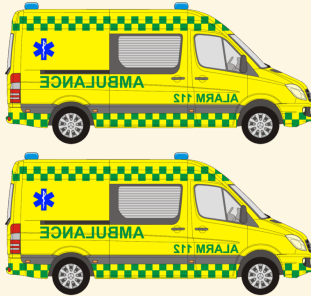
# Imperfect Information Extensive-form Games



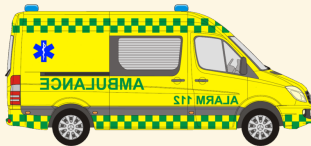
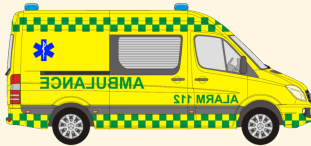
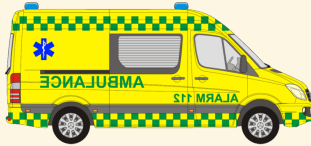
# PhD - Motivation



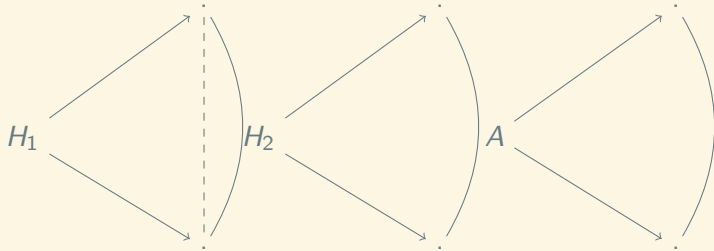
# PhD - Motivation



# PhD - Motivation

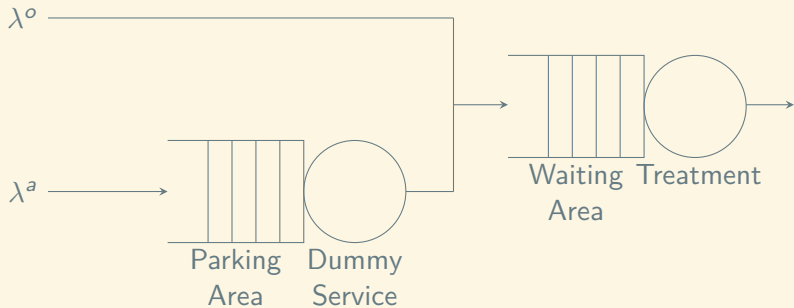


# Ambulance - Hospital Interface





# Hospital Formulation



# Hospital - Markov Chain

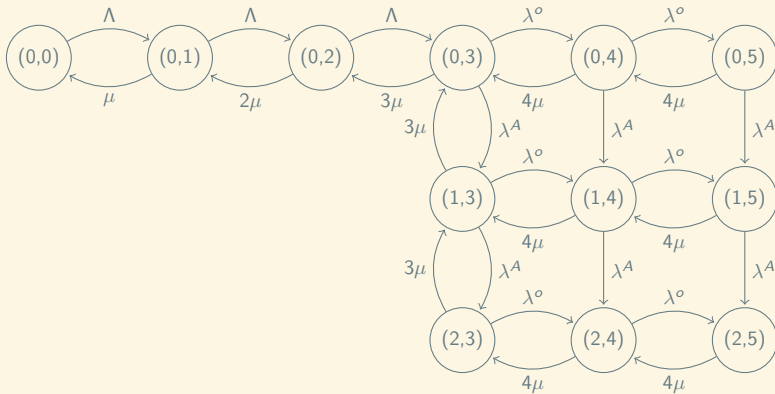
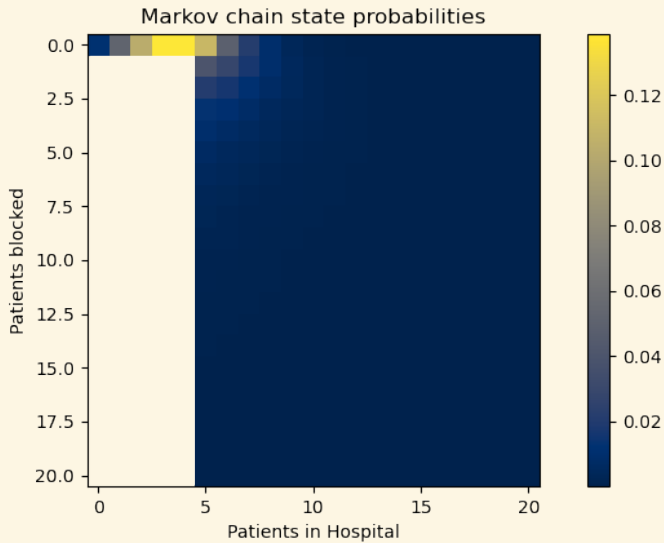
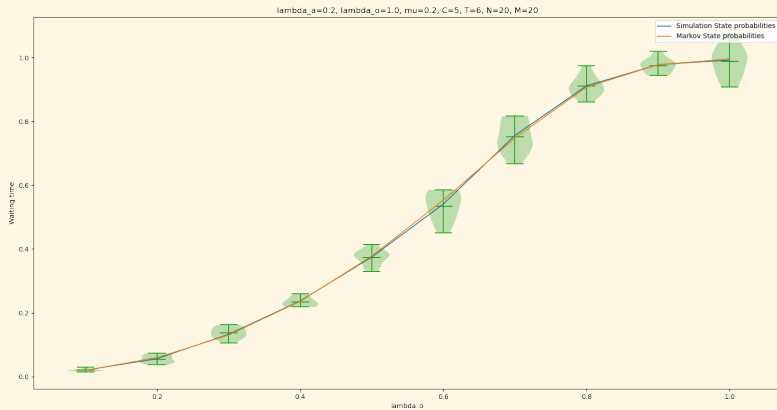


Figure:  $C=4$ ,  $T=3$ ,  $N=5$ ,  $M=2$

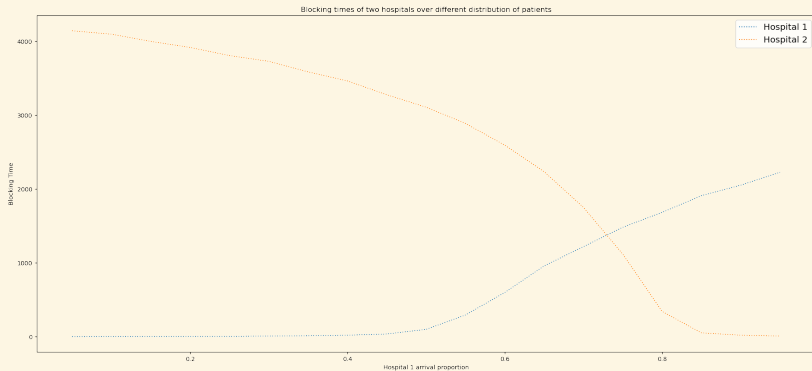
## State Probabilities



# Waiting Times



# Optimal patient distribution



# Future Plans

- ▶ Performance Measures
- ▶ Game theoretic interface