

# Annual Review

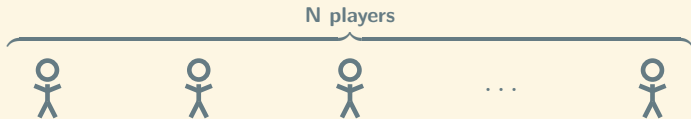
Michalis Panayides

2020-06-10

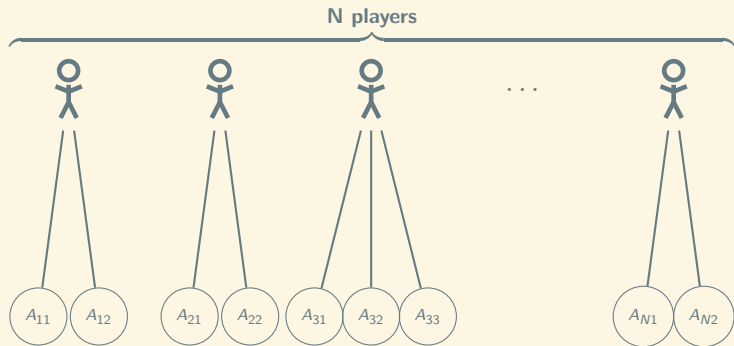
# Game Theory - Syllabus

- ▶ Normal Form Games
- ▶ Mixed-Strategy Nash Equilibrium
- ▶ Alternate Solution Concepts
- ▶ Extensive-Form Games
- ▶ Repeated Games (TBC)
- ▶ Bayesian Games (TBC)
- ▶ Coalitional Games (TBC)

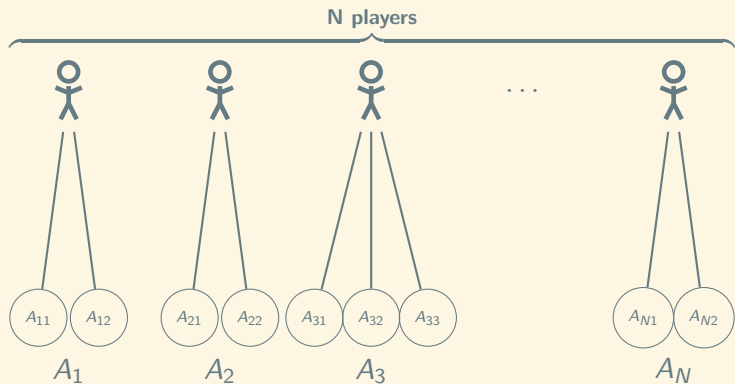
# Normal Form Games



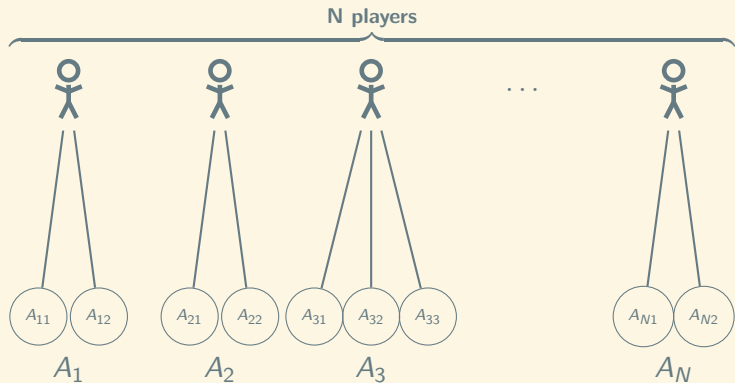
# Normal Form Games



# Normal Form Games



# Normal Form Games



$$u_i = A_1 \times A_2 \times A_3 \times \cdots \times A_N$$

# Rock-Paper-Scissors



$$\begin{bmatrix} (0, 0) & (1, -1) & (-1, 1) \\ (-1, 1) & (0, 0) & (1, -1) \\ (1, -1) & (-1, 1) & (0, 0) \end{bmatrix}$$

# Nash Equilibrium



$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$



# Nash Equilibrium

$$\begin{array}{l} \text{C} \longrightarrow \\ \text{D} \longrightarrow \end{array} \begin{bmatrix} (3, -) & (0, -) \\ (5, -) & (1, -) \end{bmatrix}$$

# Nash Equilibrium

C	D
	
$\begin{bmatrix} (-, 3) \\ (-, 0) \end{bmatrix}$	$\begin{bmatrix} (-, 5) \\ (-, 1) \end{bmatrix}$

# Nash Equilibrium

	D	
D	$(3, 3)$	$(0, 5)$
	$(5, 0)$	$(1, 1)$

# Pareto Optimality

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

$$\overbrace{(3, 3), (0, 5), (5, 0), (1, 1)}$$

# Pareto Optimality

$$\begin{bmatrix} (3, 3) & (0, 5) \\ (5, 0) & (1, 1) \end{bmatrix}$$

$$\overbrace{(3, 3), (0, 5), (5, 0), (1, 1)}$$

$$(3, 3) > (1, 1)$$

# Computing the Nash Equilibria

- ▶ Lemke-Howson Algorithm
- ▶ Support Enumeration
- ▶ Iterative removal of strictly dominated strategies

# Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	$L$	$C$	$R$
$U$	(3, 0)	(2, 1)	(0, 0)
$M$	(1, 1)	(1, 1)	(5, 0)
$D$	(0, 1)	(4, 2)	(0, 1)

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$P1 \setminus P2$	$C$
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$D$	$(4, 2)$

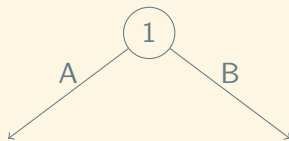
# Iterative Removal of Strictly Dominated Strategies

$P1 \setminus P2$	$C$
$U$	$(2, 1)$
$D$	$(4, 2)$

# Iterative Removal of Strictly Dominated Strategies

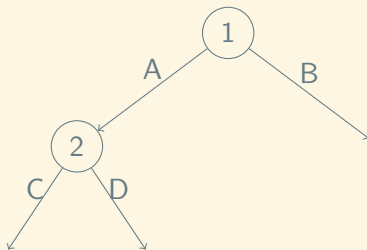
$$\begin{array}{cc} P_1 \backslash P_2 & C \\ D & (4, 2) \end{array}$$

# Perfect Information Extensive Form Games

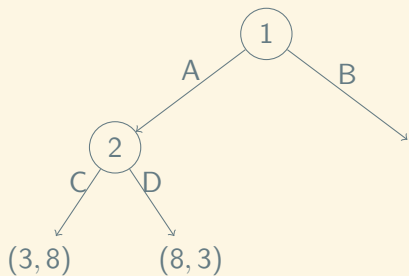




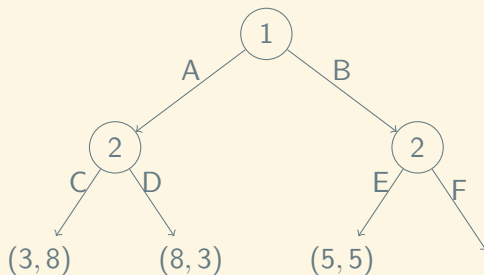
# Perfect Information Extensive Form Games



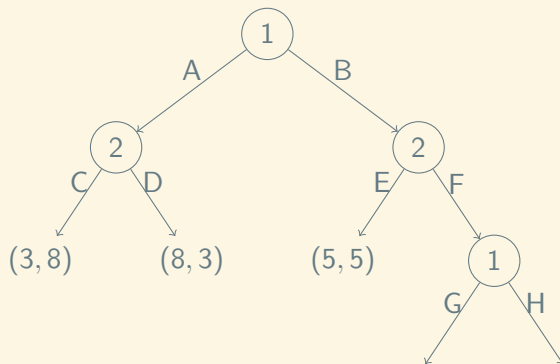
# Perfect Information Extensive Form Games



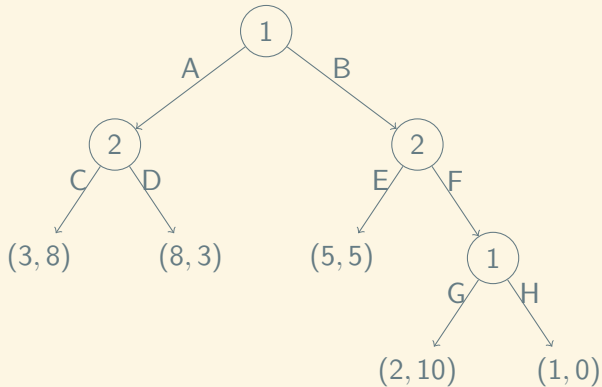
# Perfect Information Extensive Form Games



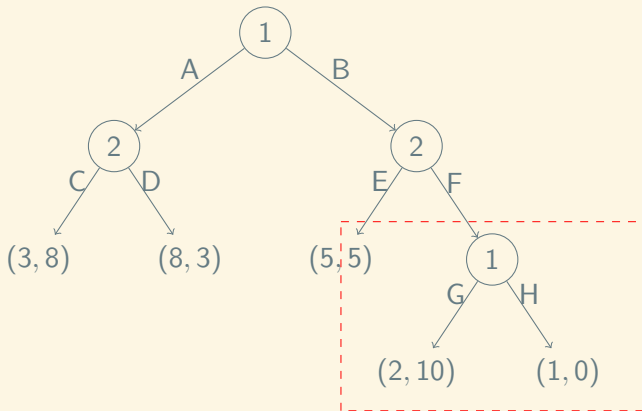
# Perfect Information Extensive Form Games



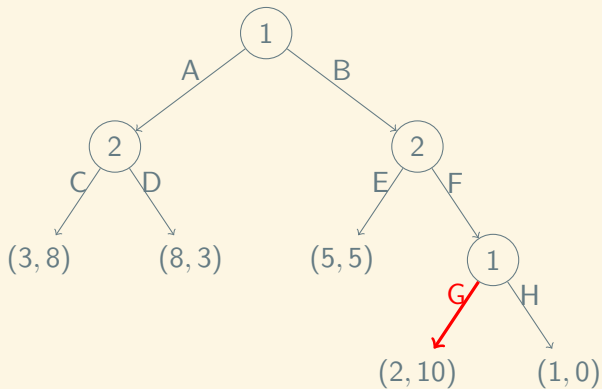
# Perfect Information Extensive Form Games



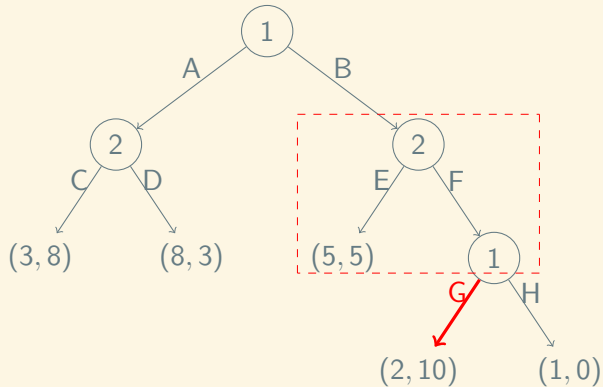
# Backwards Induction



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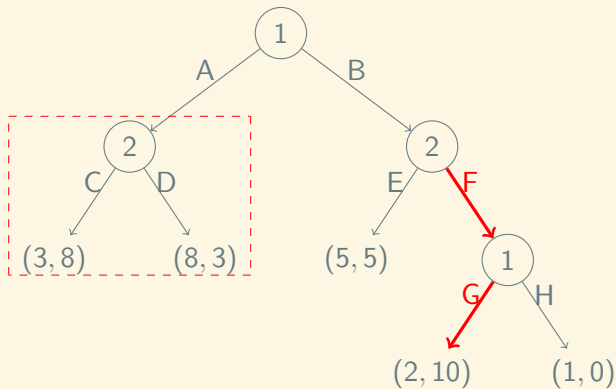


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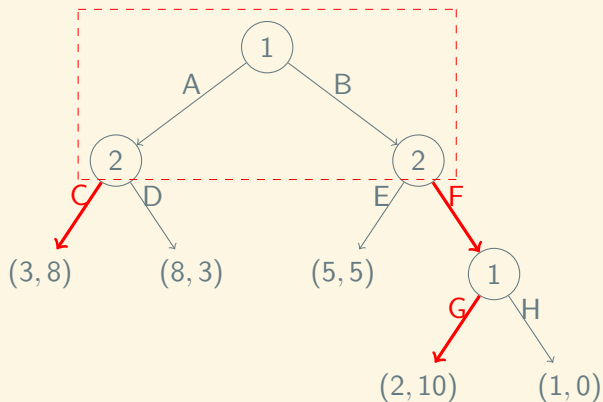




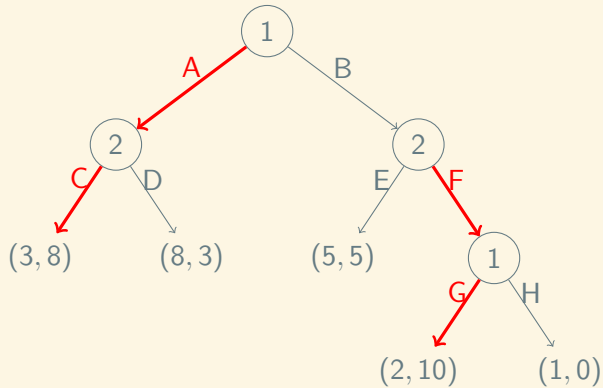
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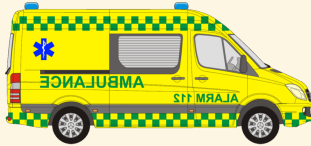
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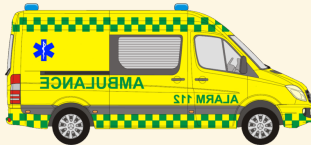
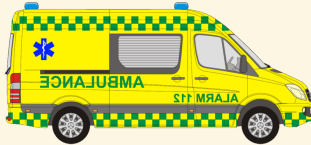
# Imperfect Information Extensive-form Games



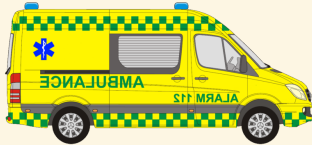
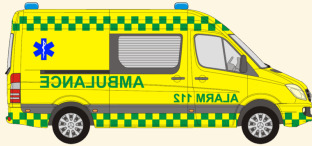
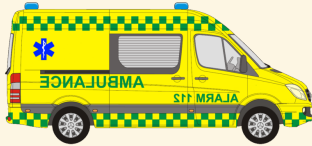
# PhD - Motivation



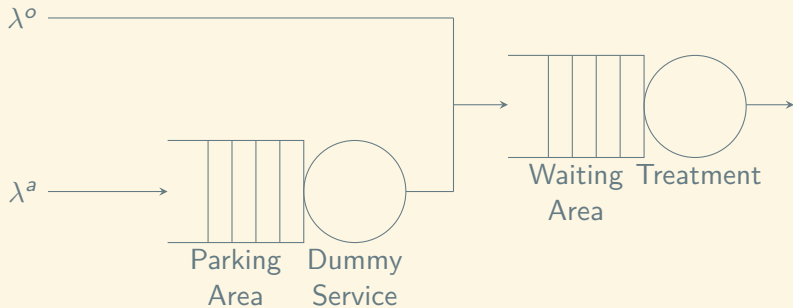
# PhD - Motivation



# PhD - Motivation

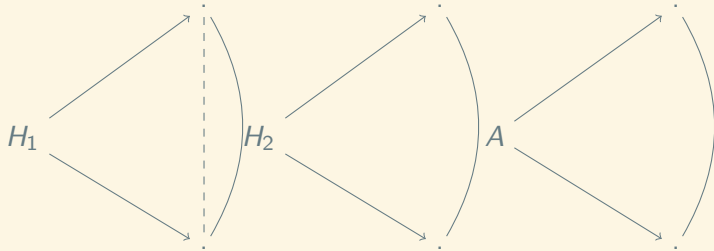


# Hospital Formulation





# Ambulance - Hospital Interface



# Hospital - Markov Chain

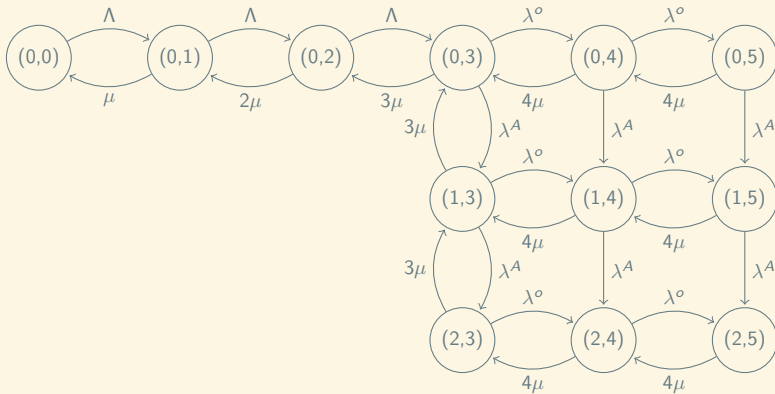
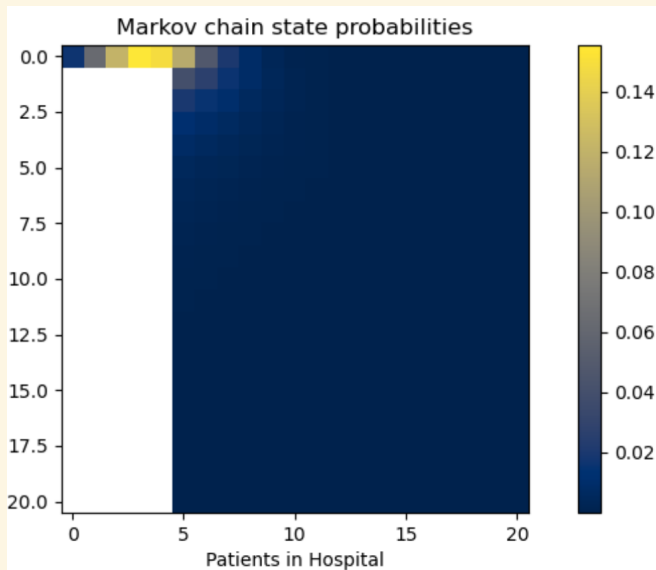
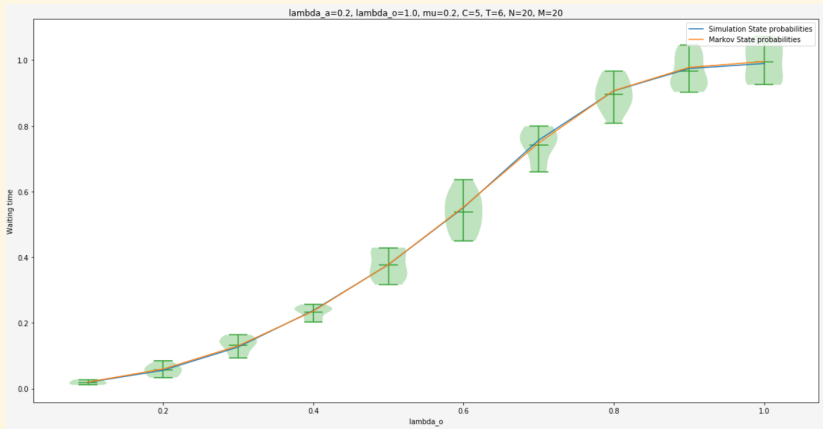


Figure:  $C=4$ ,  $T=3$ ,  $N=5$ ,  $M=2$

## State Probabilities



# Waiting Times



# Optimal patient distribution

