

A 3-player game theoretic model of a choice
between two queueing systems with strategic
managerial decision making

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EURO 2021 Athens

About me



About me

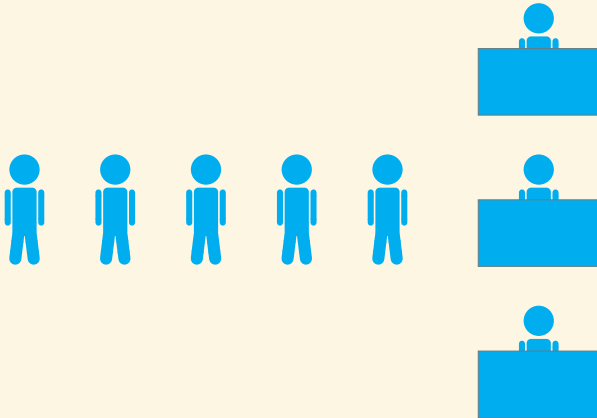


THIS.

Queues - Examples



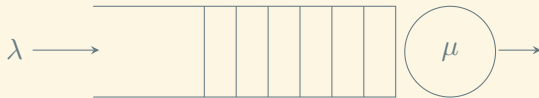
Queues - Examples



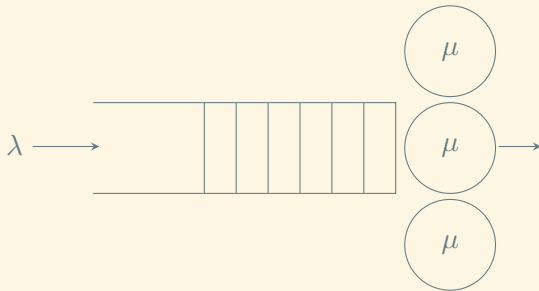
Queues - Examples



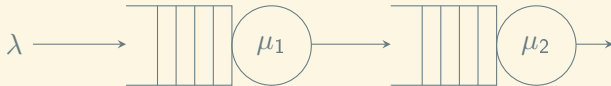
Discrete Event Simulation - M|M|1



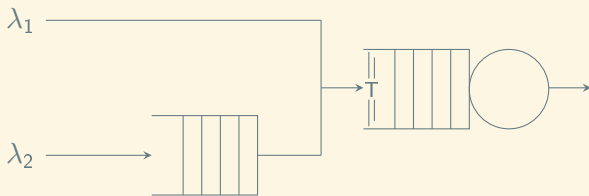
Discrete Event Simulation - M|M|3



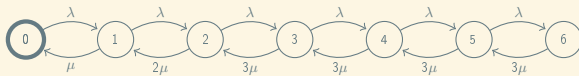
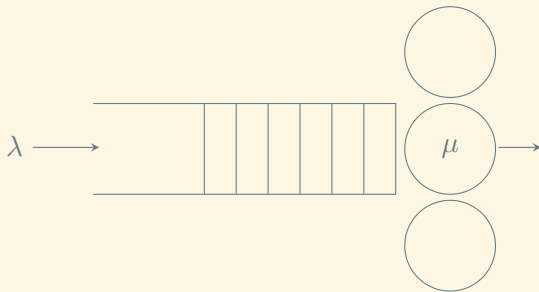
Discrete Event Simulation - Network of queues



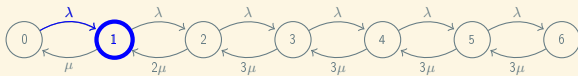
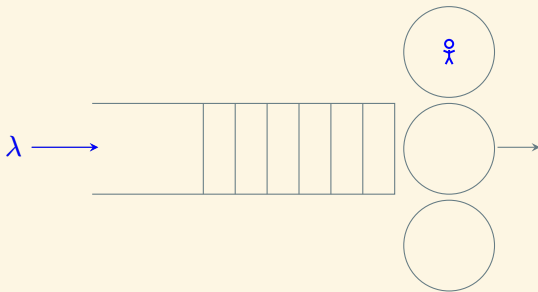
Discrete Event Simulation - Custom network



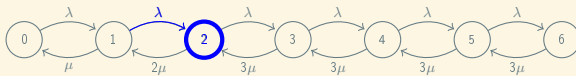
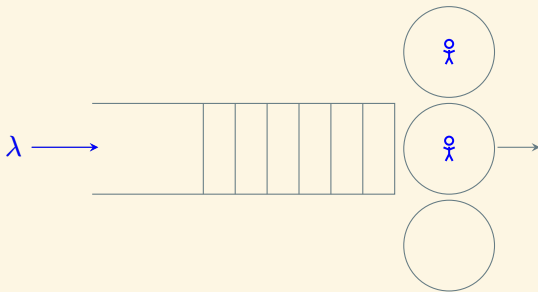
Markov Chain - M|M|3



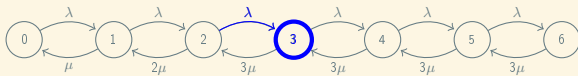
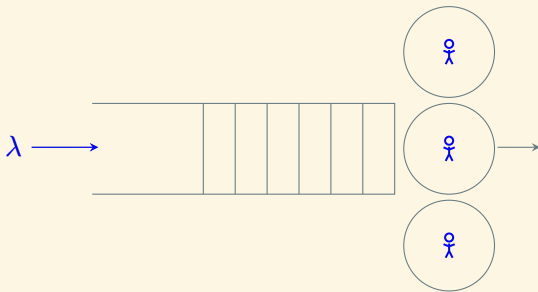
Markov Chain - M|M|3



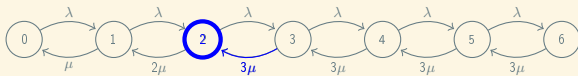
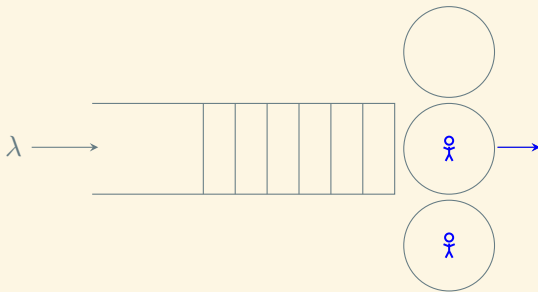
Markov Chain - M|M|3



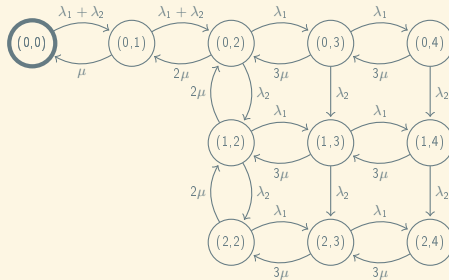
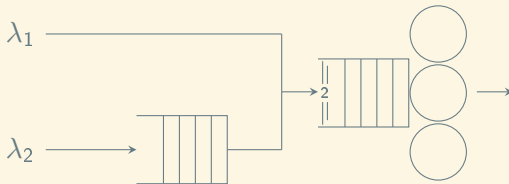
Markov Chain - M|M|3



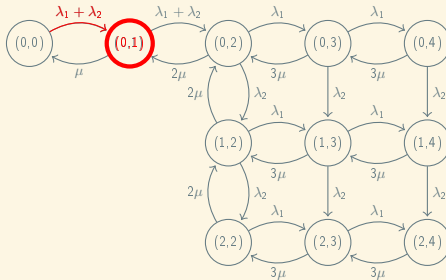
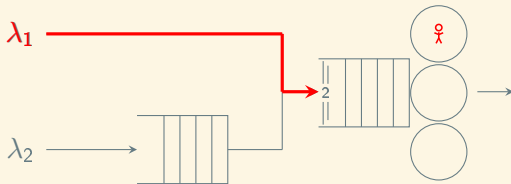
Markov Chain - M|M|3



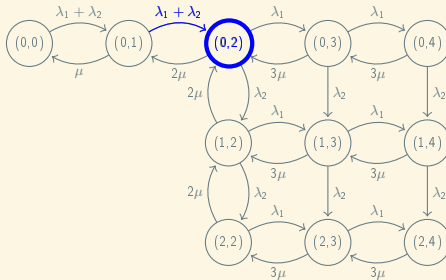
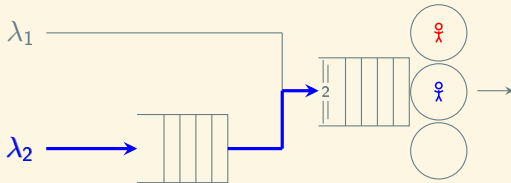
Markov Chain - Custom network



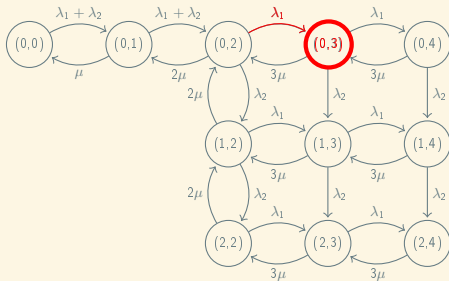
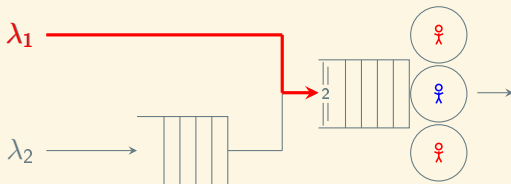
Markov Chain - Custom network



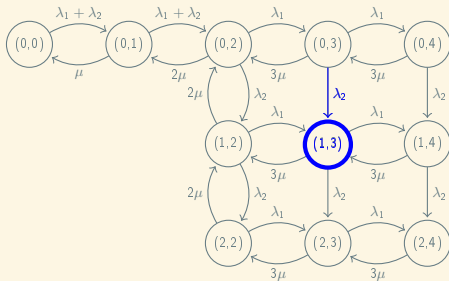
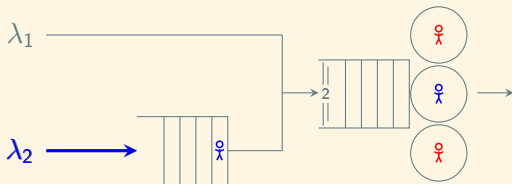
Markov Chain - Custom network



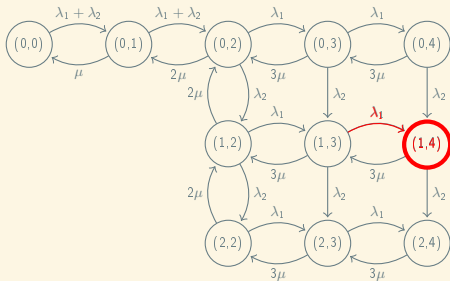
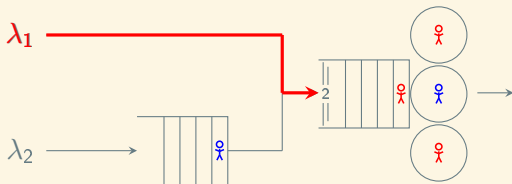
Markov Chain - Custom network



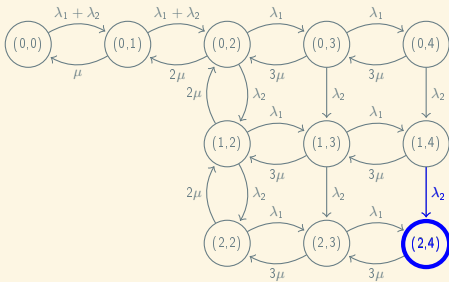
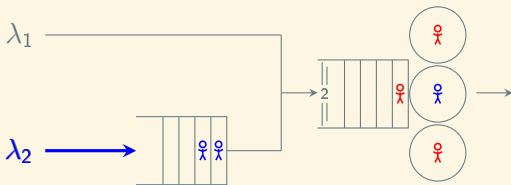
Markov Chain - Custom network



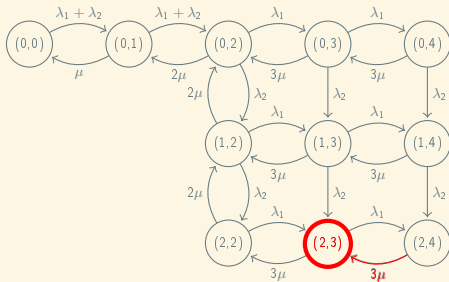
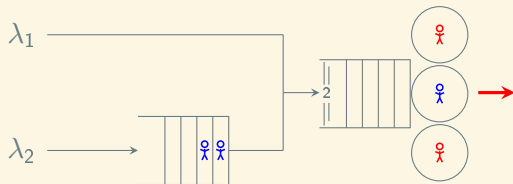
Markov Chain - Custom network



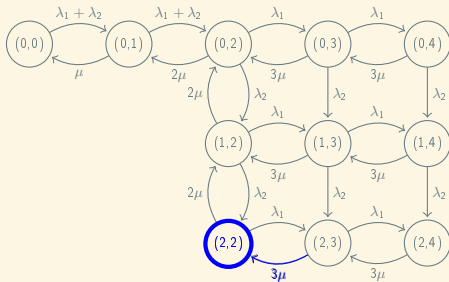
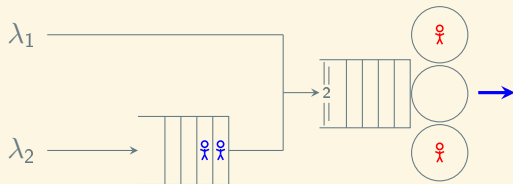
Markov Chain - Custom network



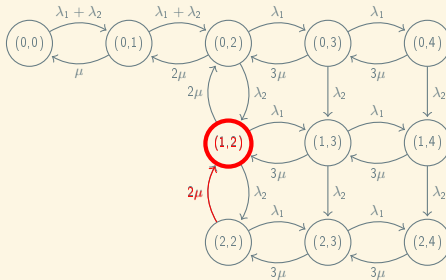
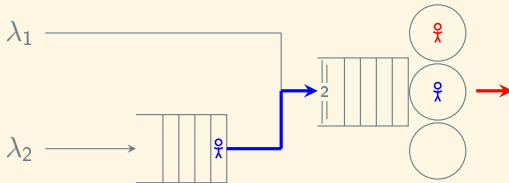
Markov Chain - Custom network



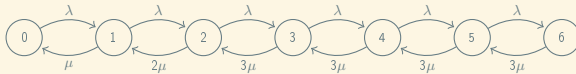
Markov Chain - Custom network



Markov Chain - Custom network



Steady state probabilities - M|M|3 queue



$$Q = \begin{matrix} & \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{matrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix} & \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & 2\mu & -2\mu - \lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & 3\mu & -3\mu - \lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & 3\mu & -3\mu - \lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & 3\mu & -3\mu - \lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & 3\mu & -3\mu \end{pmatrix} \end{matrix}$$

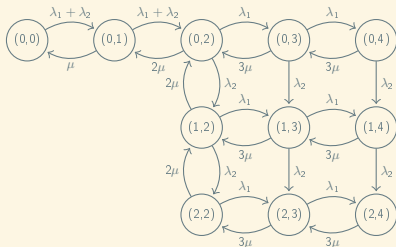
$$\frac{d\pi}{dt} = \pi Q = 0$$

$$\sum \pi_i = 1$$

$$\pi = \begin{pmatrix} \pi_0 \\ \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \\ \pi_6 \end{pmatrix}$$

Steady state probabilities - Custom network

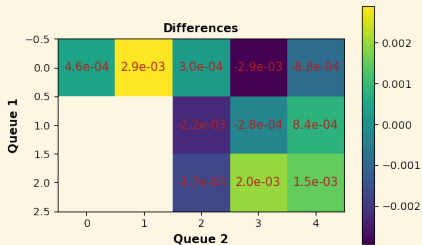
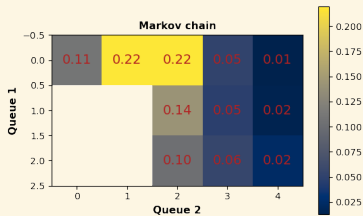
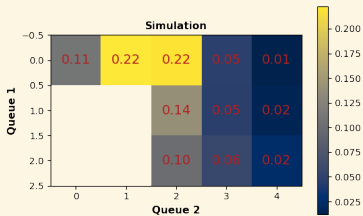
$$Q = \begin{matrix} & \begin{matrix} (0,0) & (0,1) & (0,2) & \dots & (2,3) & (2,4) \end{matrix} \\ \begin{matrix} (0,0) \\ (0,1) \\ (0,2) \\ \vdots \\ (2,3) \\ (2,4) \end{matrix} & \begin{pmatrix} -\lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & 0 & \dots & 0 & 0 \\ \mu & -\mu - \lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & \dots & 0 & 0 \\ 0 & 2\mu & -2\mu - \lambda_1 - \lambda_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda_1 - 3\mu & \lambda_1 \\ 0 & 0 & 0 & \dots & 3\mu & -3\mu \end{pmatrix} \end{matrix}$$



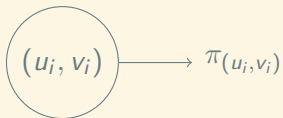
$$\frac{d\pi}{dt} = \pi Q = 0, \quad \sum \pi_{(u,v)} = 1$$

$$\pi = \begin{bmatrix} \pi(0,0) \\ \pi(0,1) \\ \pi(0,2) \\ \vdots \\ \pi(2,3) \\ \pi(2,4) \end{bmatrix}$$

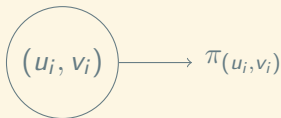
Steady state probabilities - Comparison



Performance Measures - Number of individuals



Performance Measures - Number of individuals



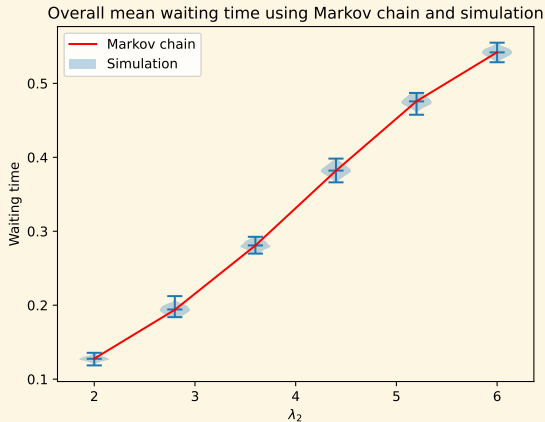
$$L = \sum_{i=1}^{|\pi|} \pi_i (u_i + v_i)$$

$$L_1 = \sum_{i=1}^{|\pi|} \pi_i u_i$$

$$L_2 = \sum_{i=1}^{|\pi|} \pi_i v_i$$

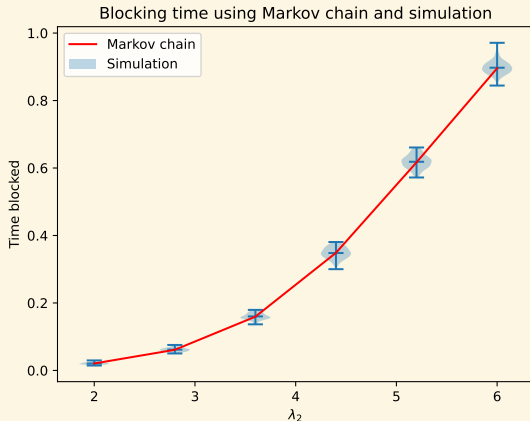
Performance Measures - Waiting time

$$W = \frac{\lambda_1 P_{L'_1}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} W^{(1)} + \frac{\lambda_2 P_{L'_2}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} W^{(2)} \quad (1)$$



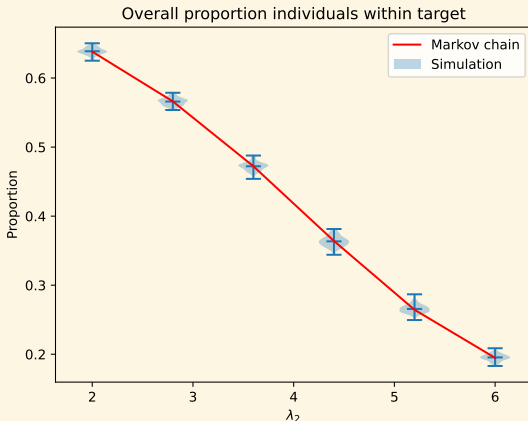
Performance Measures - Blocking time

$$B = \frac{\sum_{(u,v) \in S_A^{(2)}} \pi(u,v) b(u,v)}{\sum_{(u,v) \in S_A^{(2)}} \pi(u,v)} \quad (2)$$



Performance Measures - Proportion within time

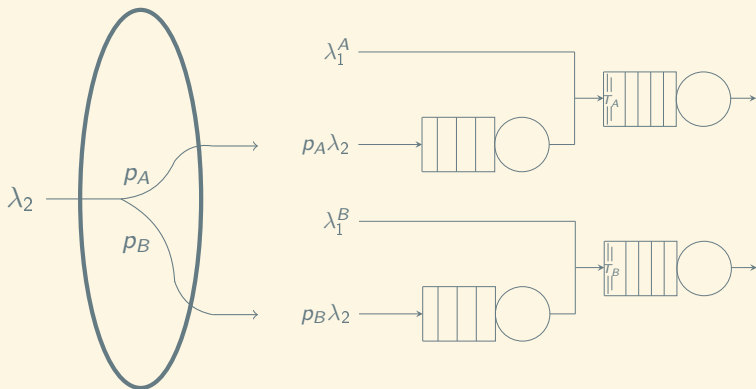
$$P(X < t) = \frac{\lambda_1 P_{L'_1}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} P(X^{(1)} < t) + \frac{\lambda_2 P_{L'_2}}{\lambda_2 P_{L'_2} + \lambda_1 P_{L'_1}} P(X^{(2)} < t) \quad (3)$$



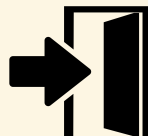
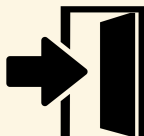
Game - Definition



Game - Players



Game - Strategies



$$p_A, p_B \in [0, 1]$$

$$T_A \in [1, N_A]$$

$$T_B \in [1, N_B]$$

$$p_A + p_B = 1$$

Game - Formulation

$$A = \begin{pmatrix} U_{1,1}^A & U_{1,2}^A & \cdots & U_{1,N_B}^A \\ U_{2,1}^A & U_{2,2}^A & \cdots & U_{2,N_B}^A \\ \vdots & \vdots & \ddots & \vdots \\ U_{N_A,1}^A & U_{N_A,2}^A & \cdots & U_{N_A,N_B}^A \end{pmatrix}, \quad B = \begin{pmatrix} U_{1,1}^B & U_{1,2}^B & \cdots & U_{1,N_B}^B \\ U_{2,1}^B & U_{2,2}^B & \cdots & U_{2,N_B}^B \\ \vdots & \vdots & \ddots & \vdots \\ U_{N_A,1}^B & U_{N_A,2}^B & \cdots & U_{N_A,N_B}^B \end{pmatrix}$$

$$R = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,N_B} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,N_B} \\ \vdots & \vdots & \ddots & \vdots \\ p_{N_A,1} & p_{N_A,2} & \cdots & p_{N_A,N_B} \end{pmatrix}$$