Using Python to measure the expected wait in a queue with two waiting rooms

Michalis Panayides

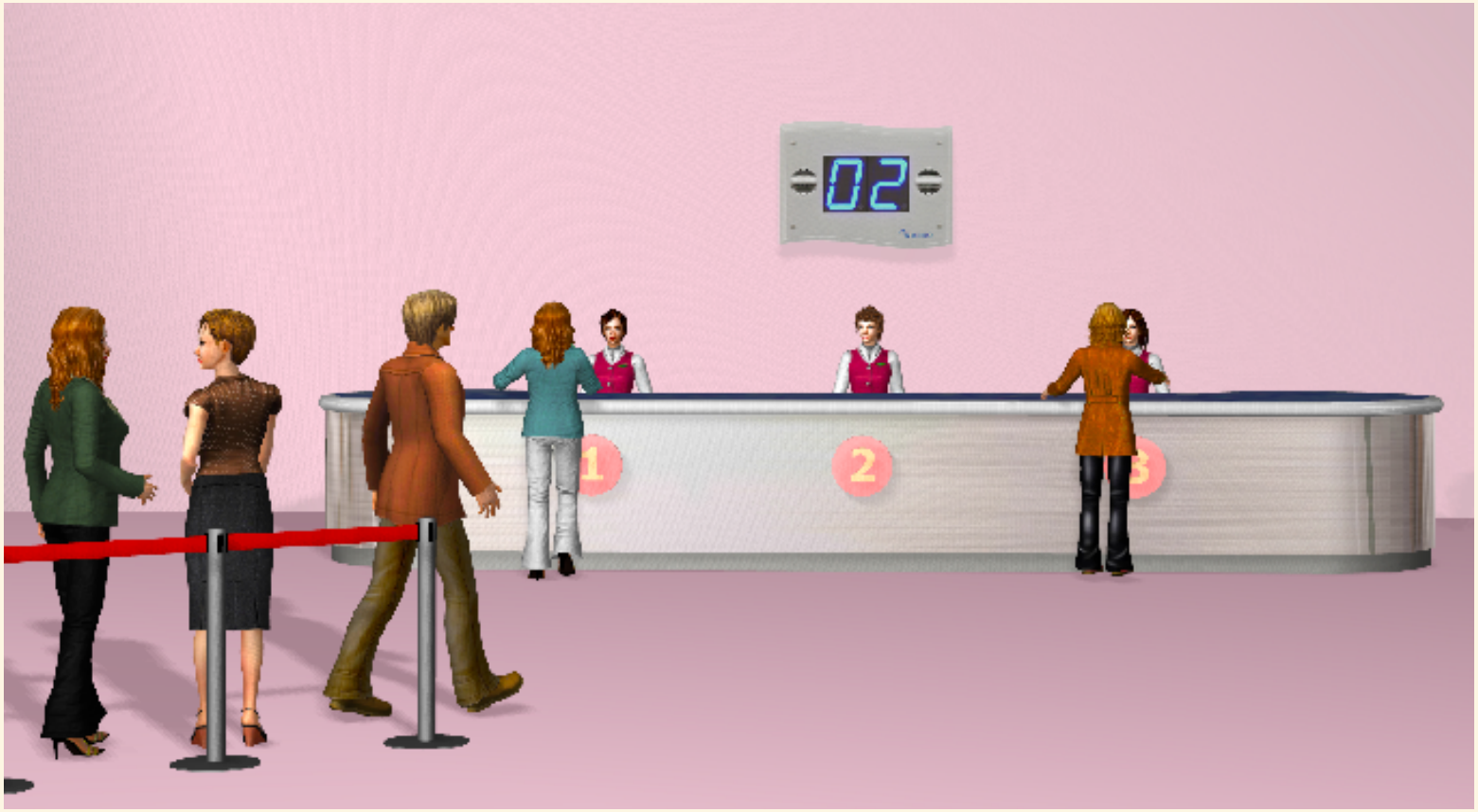
June 16, 2021

About me

THIS.

Queues

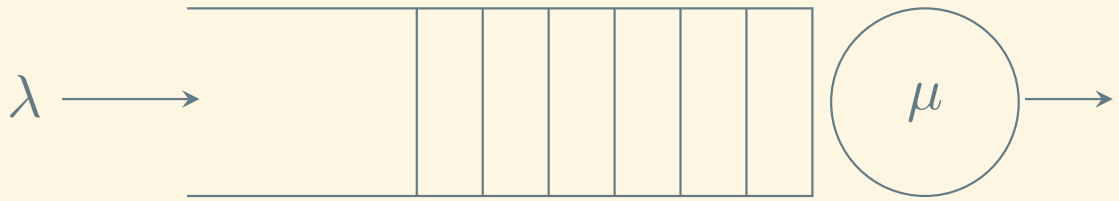
Queues



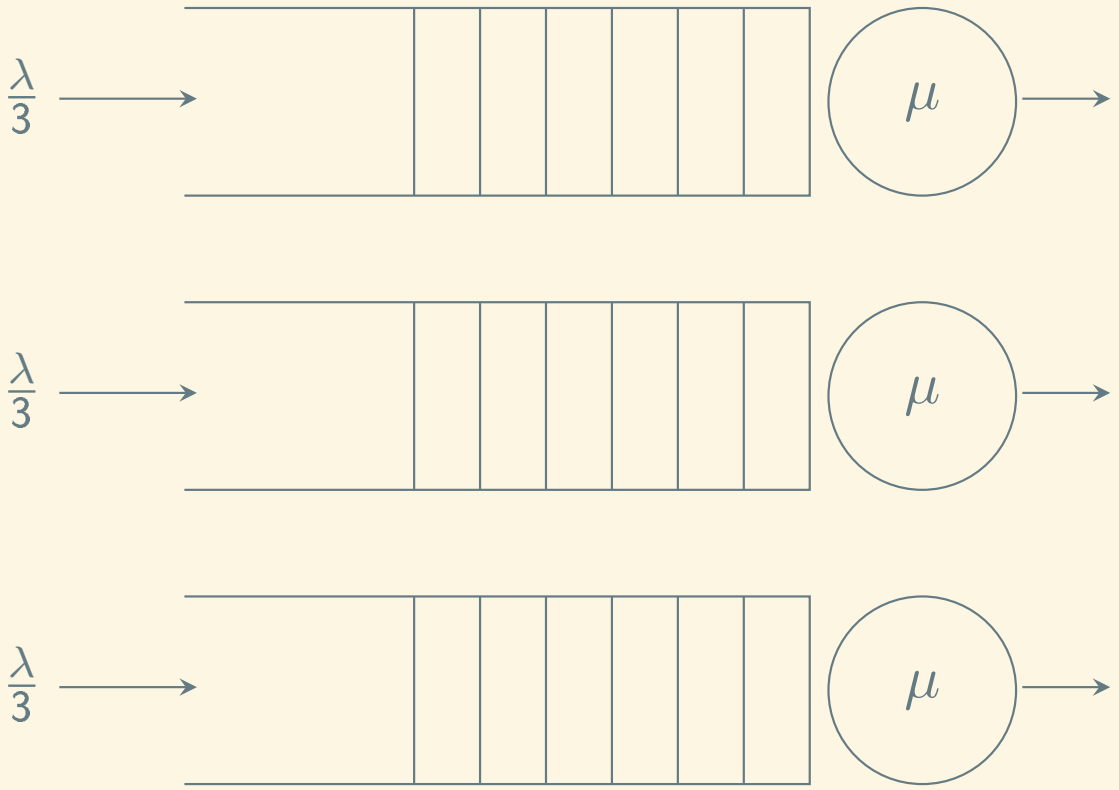
Queues


Queen Elizabeth Hospital Birmingham
EMERGENCY AMBULANCE

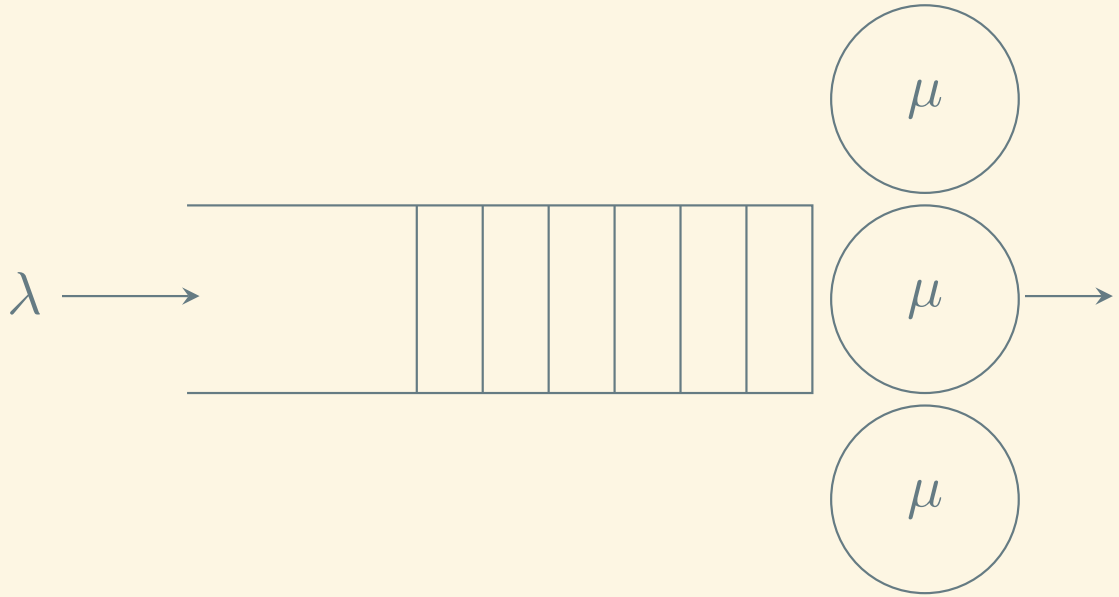
Discrete Event Simulation



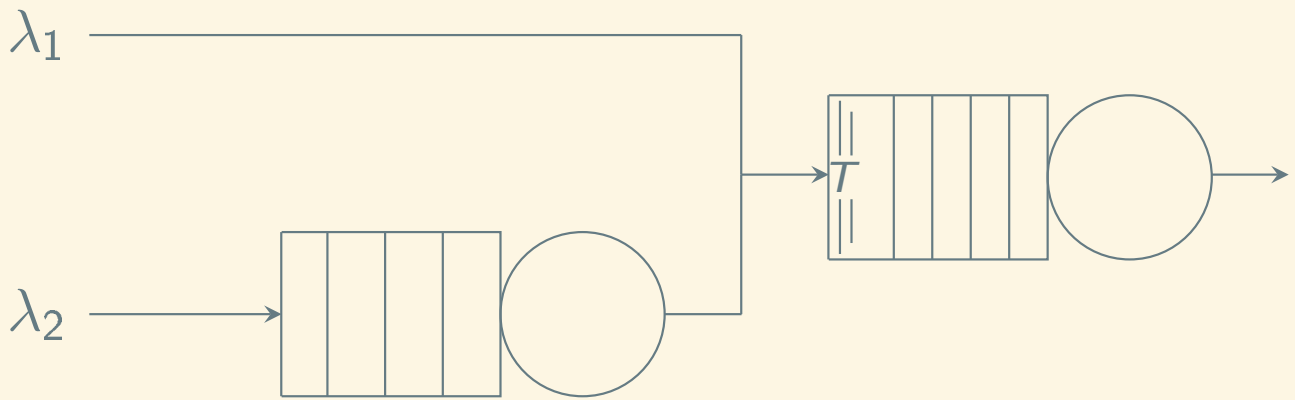
Discrete Event Simulation



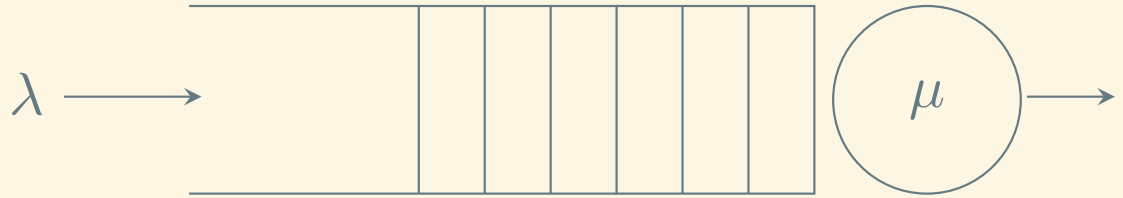
Discrete Event Simulation

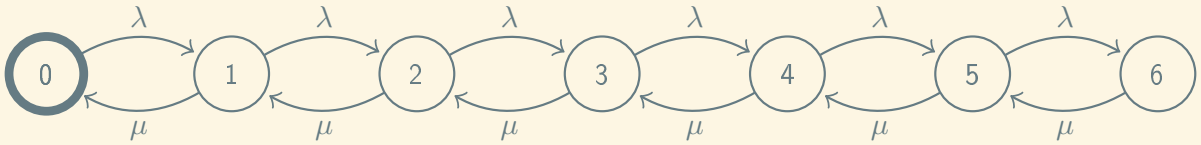


Custom Queue with two waiting spaces



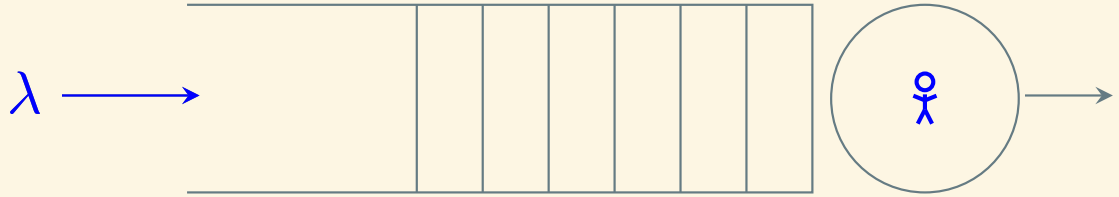
Analytical formulation - M|M|1 queue

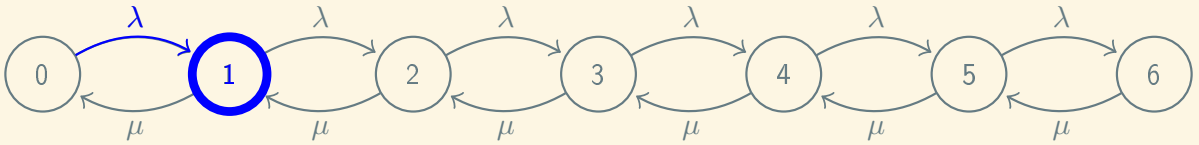




$$
Q = \begin{array}{c} \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu-\lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & -\mu-\lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & \mu & -\mu-\lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & \mu & -\mu-\lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & \mu & -\mu-\lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & \mu & -\mu \end{pmatrix} \end{array} \begin{matrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix}
$$

Analytical formulation - M|M|1 queue





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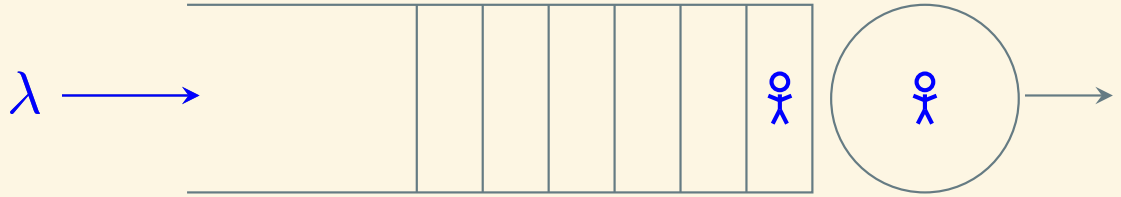
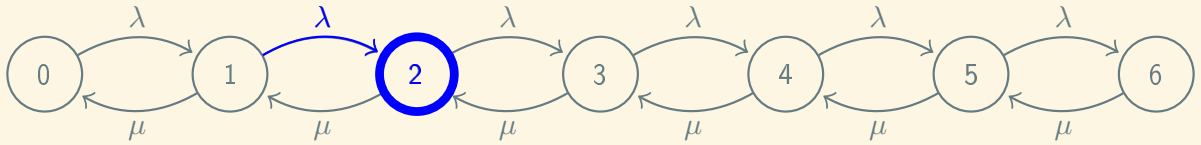
Analytical formulation - M|M|1 queue





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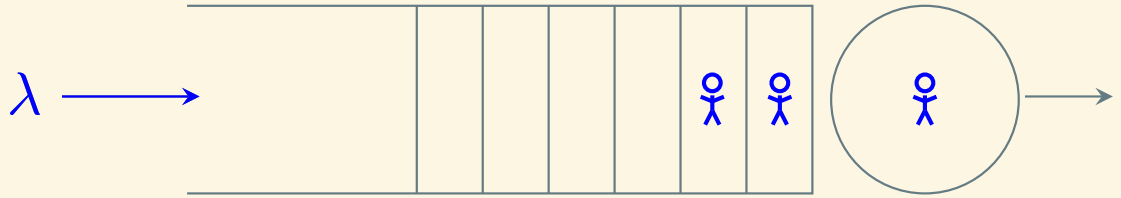
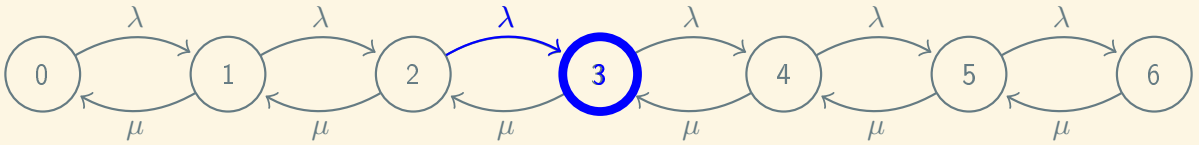
$$

Analytical formulation - M|M|1 queue





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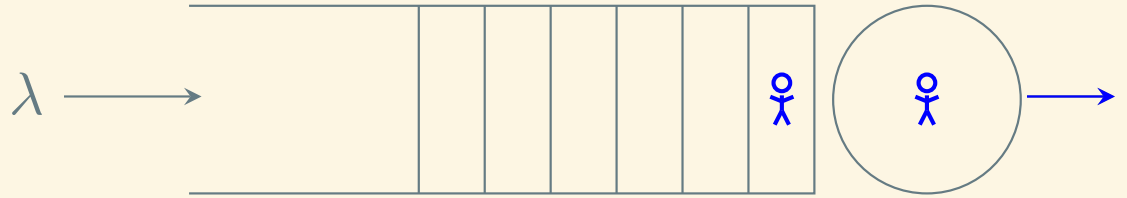
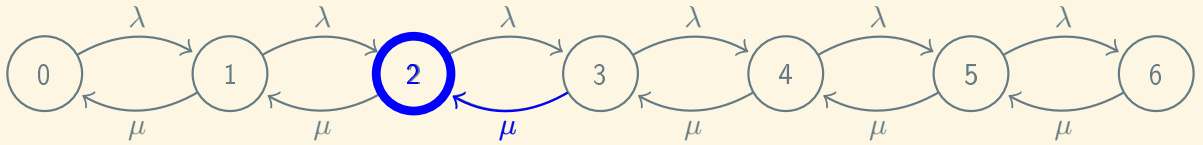
Q = \begin{array}{c} \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu-\lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & -\mu-\lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & \mu & -\mu-\lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & \mu & -\mu-\lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & \mu & -\mu-\lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & \mu & -\mu \end{pmatrix} \end{array} \begin{matrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix}
$$

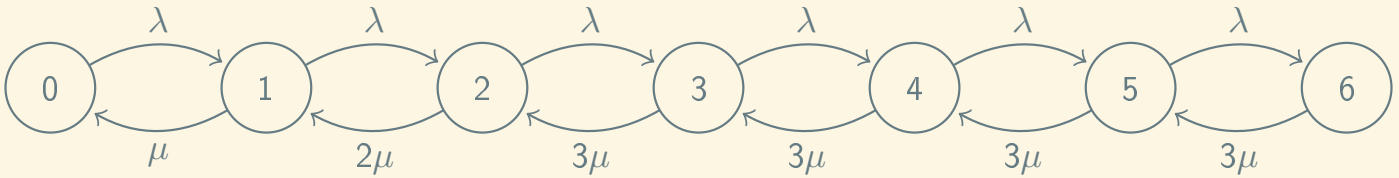
Analytical formulation - M|M|1 queue





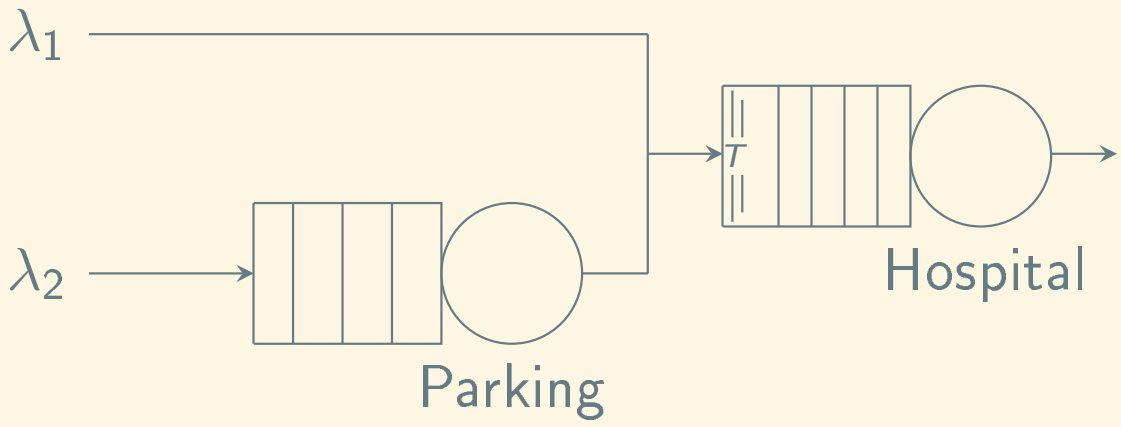
$$
Q = \begin{array}{c} \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu-\lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & -\mu-\lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & \mu & -\mu-\lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & \mu & -\mu-\lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & \mu & -\mu-\lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & \mu & -\mu \end{pmatrix} \end{array} \begin{matrix} \\ (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix}
$$

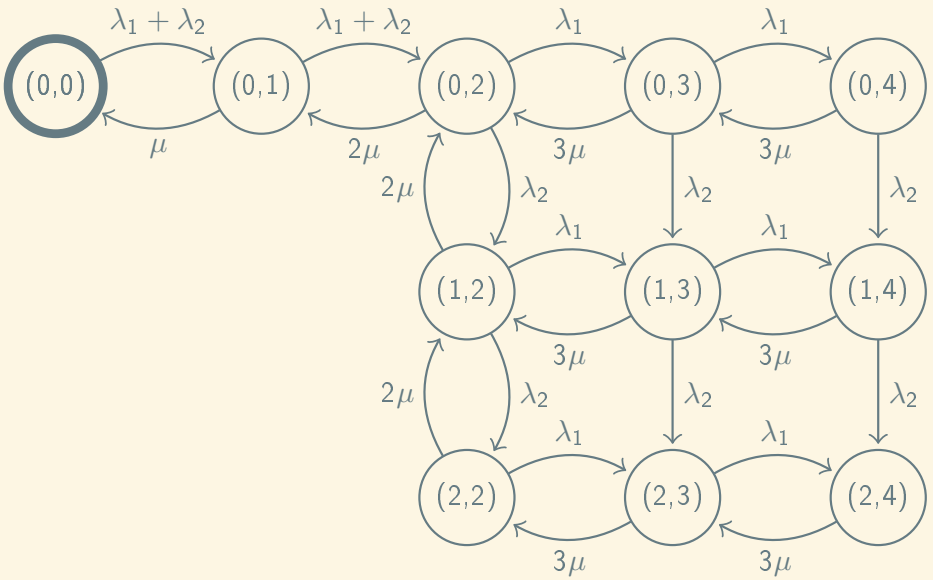
Analytical formulation - M|M|3 queue



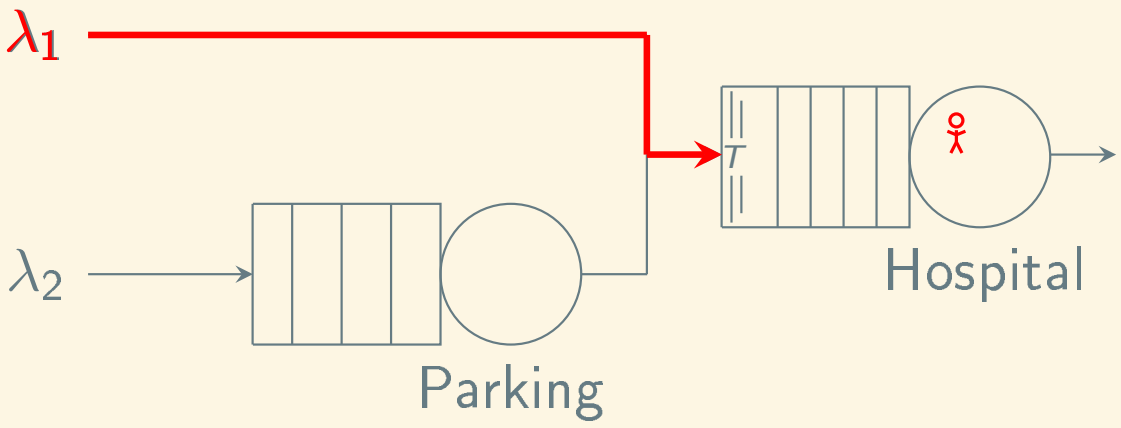
$$Q = \begin{array}{c} \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu-\lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & 2\mu & -2\mu-\lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & 3\mu & -3\mu-\lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & 3\mu & -3\mu-\lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & 3\mu & -3\mu-\lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & 3\mu & -3\mu \end{pmatrix} \end{array} \begin{matrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix}$$

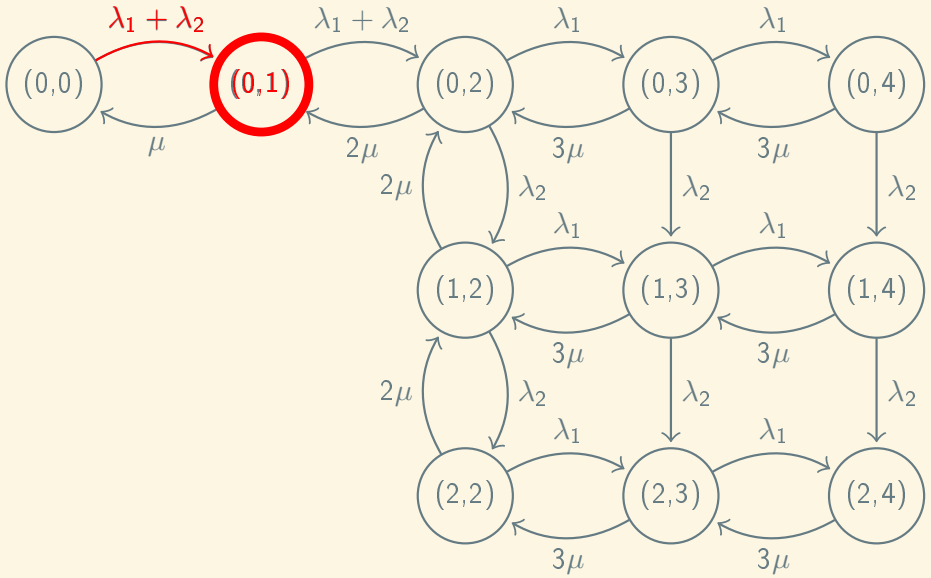
Analytical formulation - Custom Queue



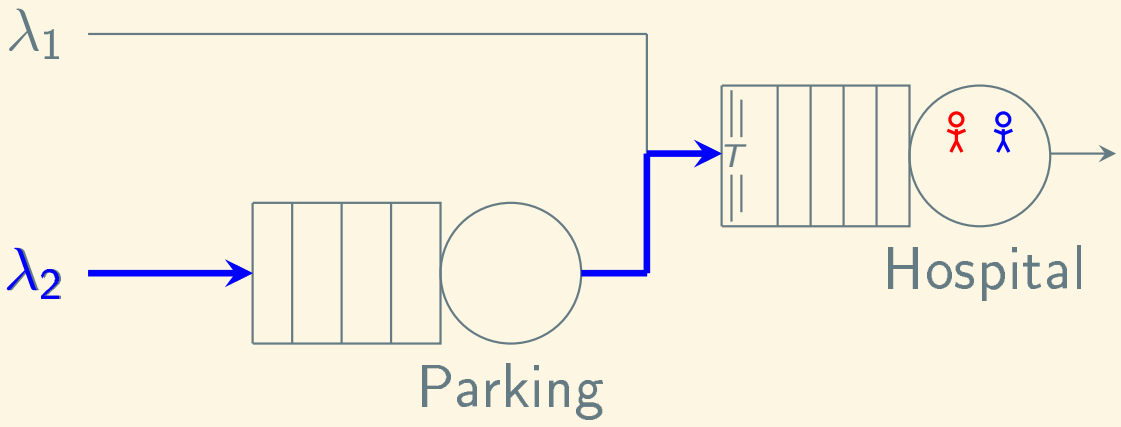


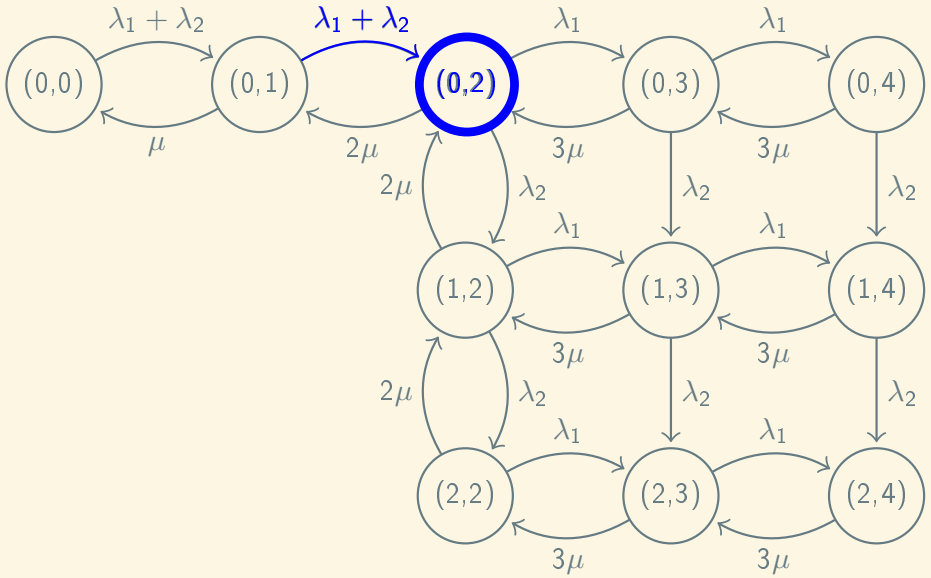
Analytical formulation - Custom Queue



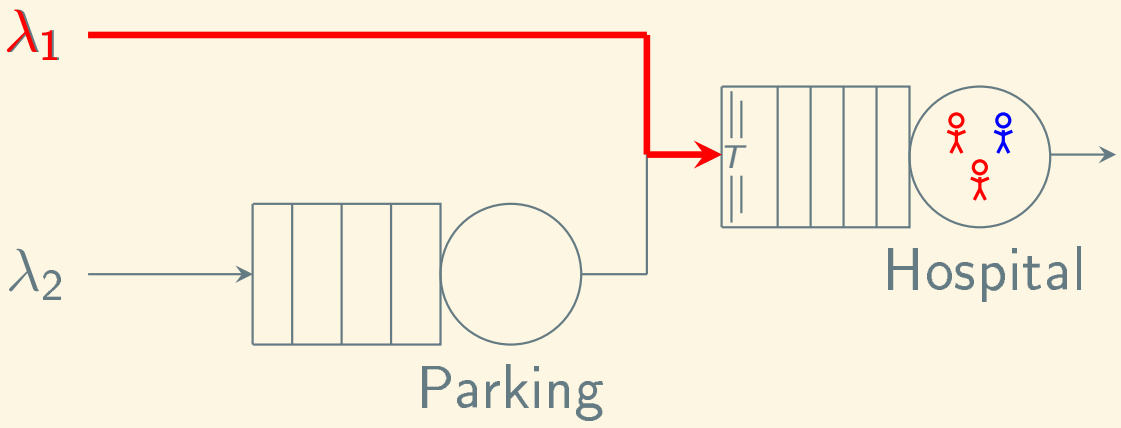


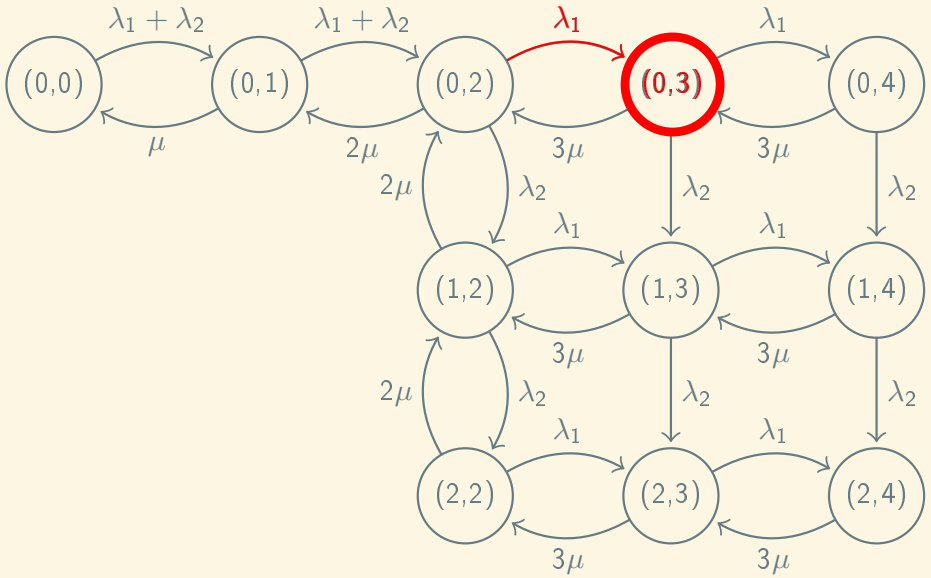
Analytical formulation - Custom Queue



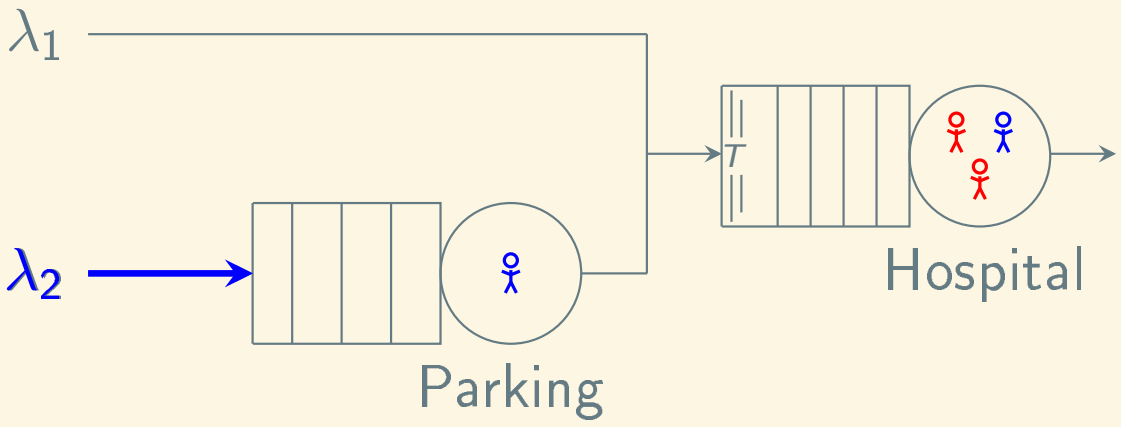


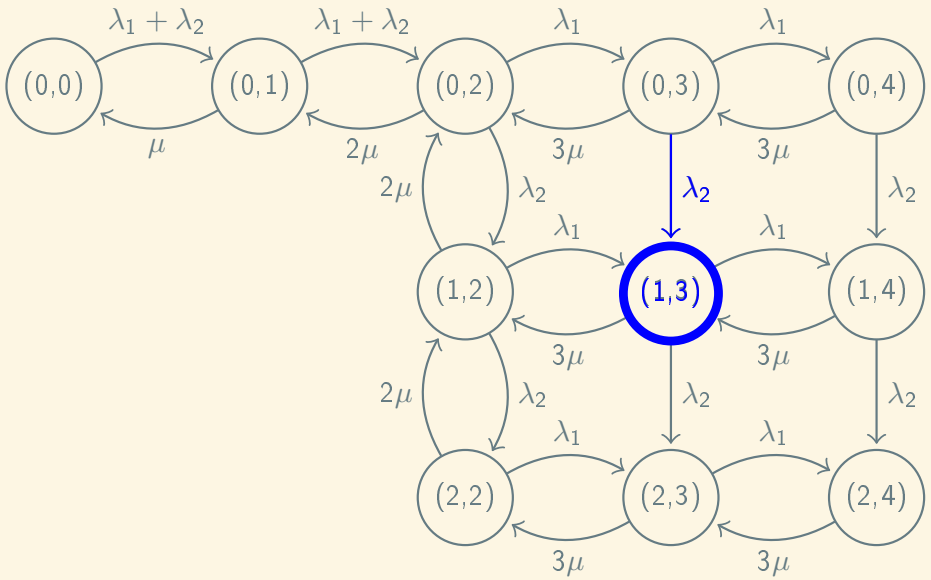
Analytical formulation - Custom Queue



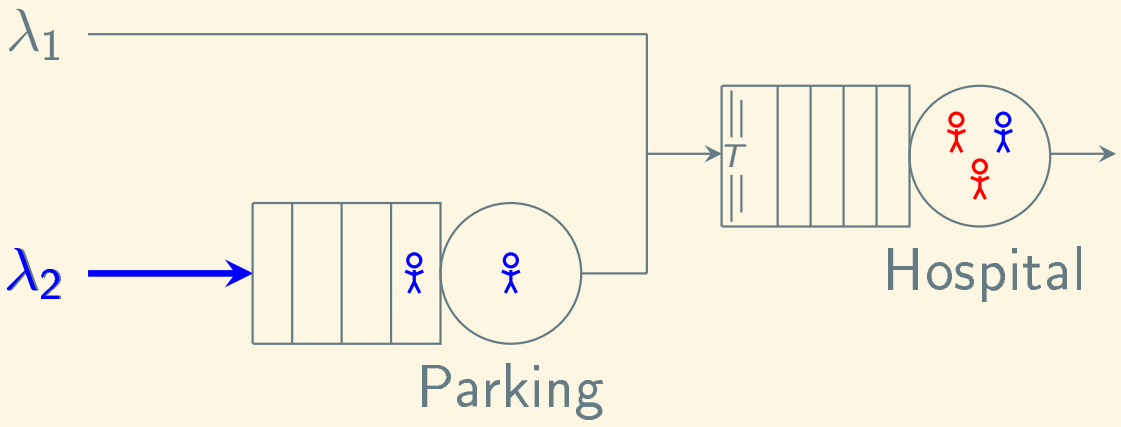


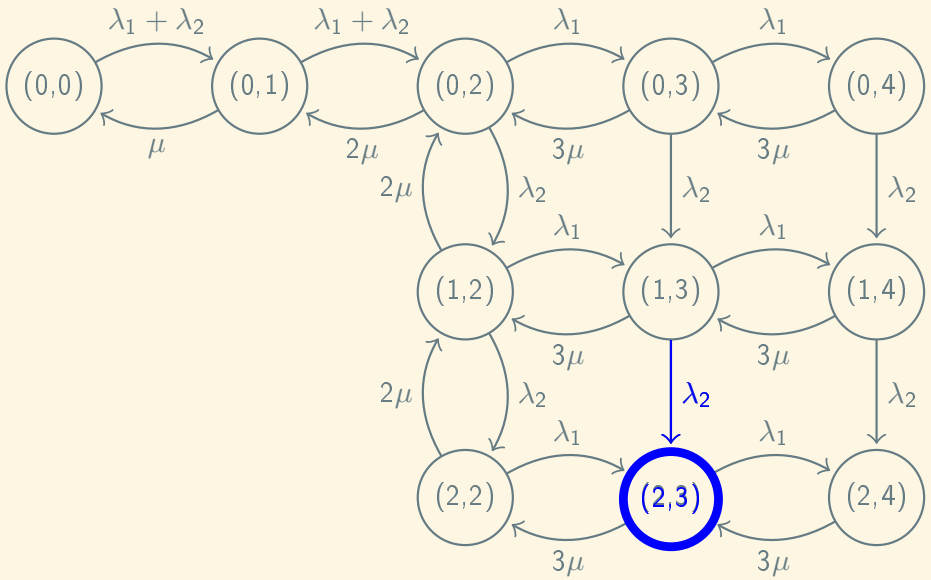
Analytical formulation - Custom Queue



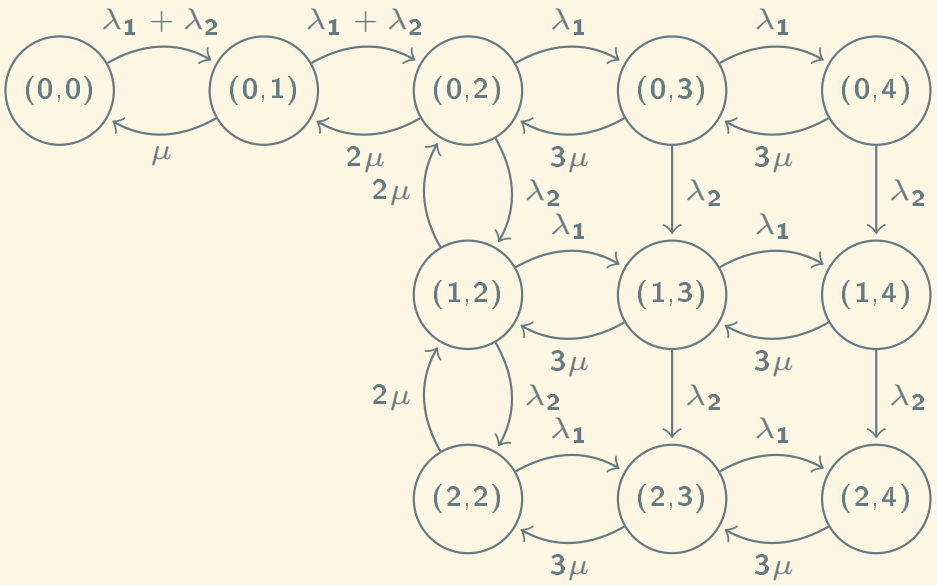


Analytical formulation - Custom Queue



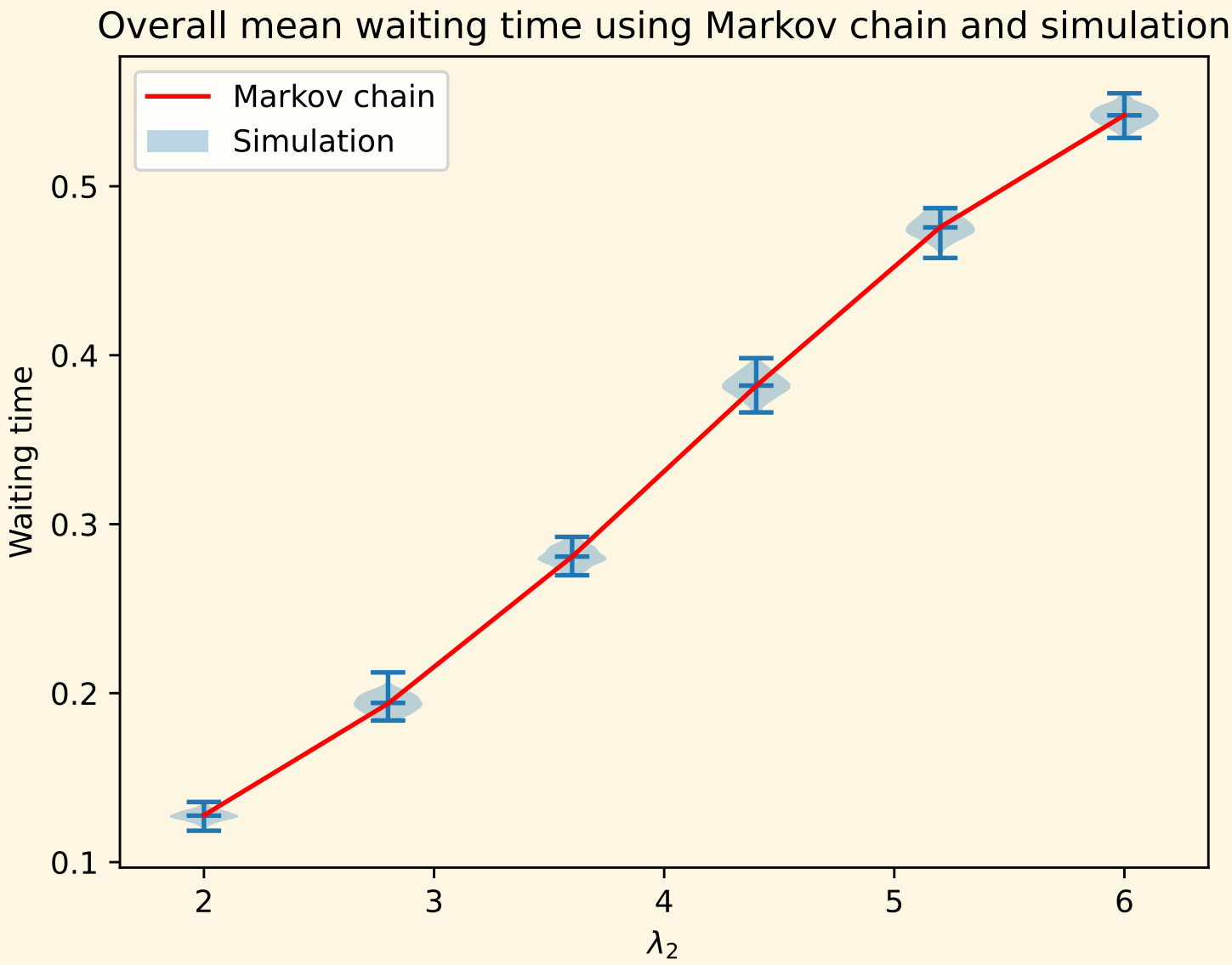


Analytical formulation - Custom Queue



$$
Q = \begin{array}{c}
\begin{array}{ccccccc} (0,0) & (0,1) & (0,2) & & (2,3) & (2,4) & \end{array} \\
\begin{pmatrix}
-\lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & 0 & \dots & 0 & 0 \\
\mu & -\mu - \lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & \dots & 0 & 0 \\
0 & 2\mu & -2\mu - \lambda_1 - \lambda_2 & \dots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \dots & -\lambda_1 - 3\mu & \lambda_1 \\
0 & 0 & 0 & \dots & 3\mu & -3\mu
\end{pmatrix}
\begin{array}{c} (0,0) \\ (0,1) \\ (0,2) \\ \\ (2,3) \\ (2,4) \end{array}
\end{array}
$$

Performance Measures - Waiting time



Performance Measures - Blocking time