

# Using Python to measure the expected wait in a queue with two waiting rooms

Michalis Panayides

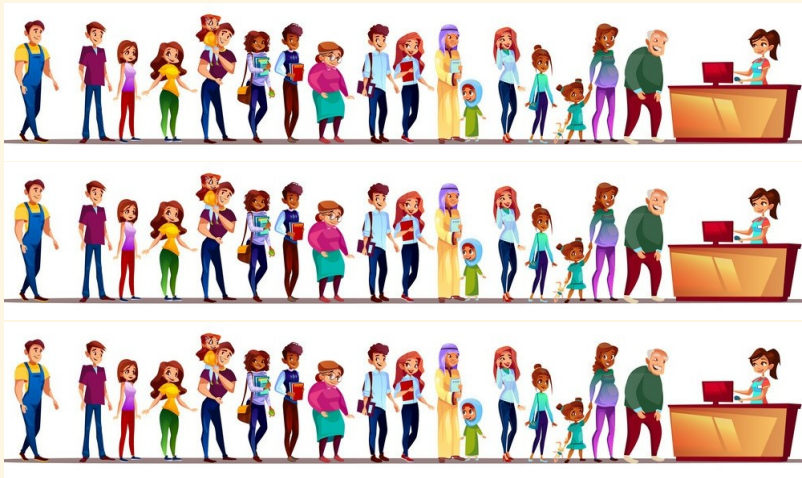
June 16, 2021

About me

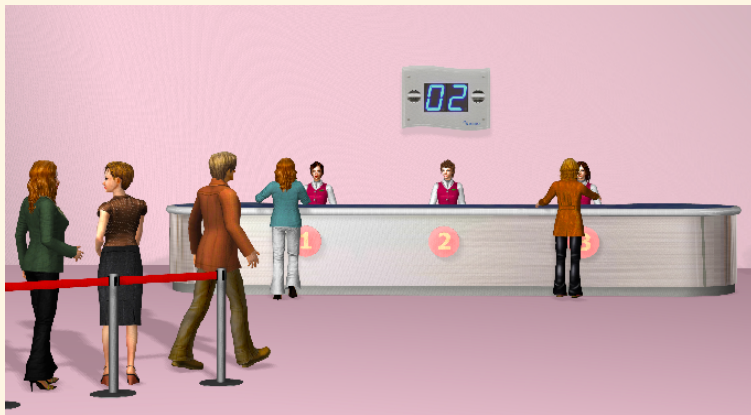


THIS.

# Queues



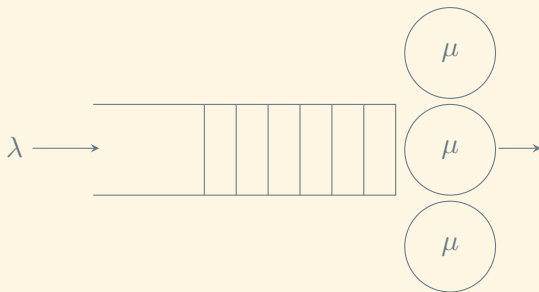
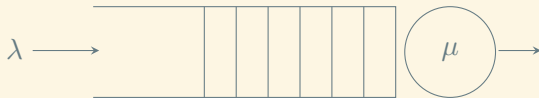
# Queues



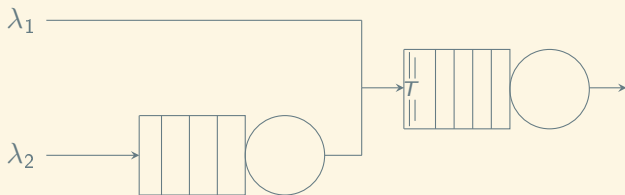
# Queues



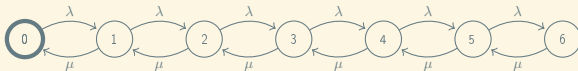
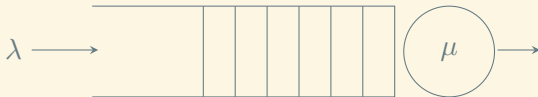
# Discrete Event Simulation



## Custom Queue with two waiting spaces



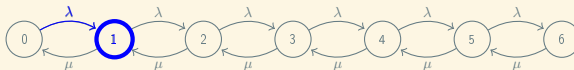
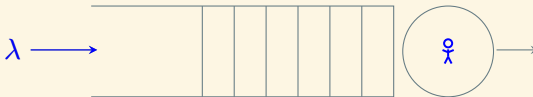
# Analytical formulation - M|M|1 queue



$$Q = \begin{matrix} & \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{matrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix} & \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & \mu & -\mu - \lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & \mu & -\mu - \lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & \mu & -\mu - \lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & \mu & -\mu \end{pmatrix} \end{matrix}$$

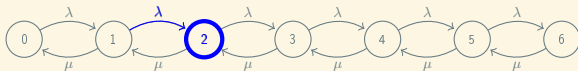
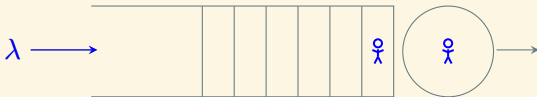


# Analytical formulation - M|M|1 queue



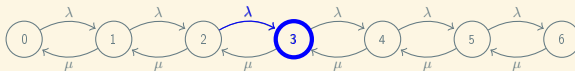
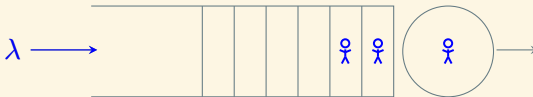
$$Q = \begin{matrix} & \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{matrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix} & \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & \mu & -\mu - \lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & \mu & -\mu - \lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & \mu & -\mu - \lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & \mu & -\mu \end{pmatrix} \end{matrix}$$

# Analytical formulation - M|M|1 queue



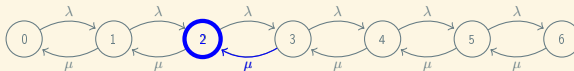
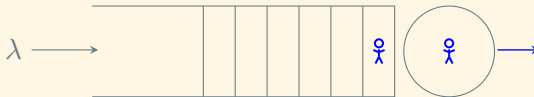
$$Q = \begin{matrix} & \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{matrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix} & \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & \mu & -\mu - \lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & \mu & -\mu - \lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & \mu & -\mu - \lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & \mu & -\mu \end{pmatrix} \end{matrix}$$

# Analytical formulation - M|M|1 queue



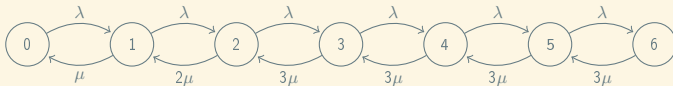
$$Q = \begin{matrix} & \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{matrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix} & \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & \mu & -\mu - \lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & \mu & -\mu - \lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & \mu & -\mu - \lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & \mu & -\mu \end{pmatrix} \end{matrix}$$

# Analytical formulation - M|M|1 queue



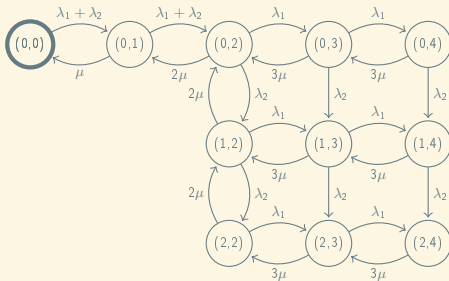
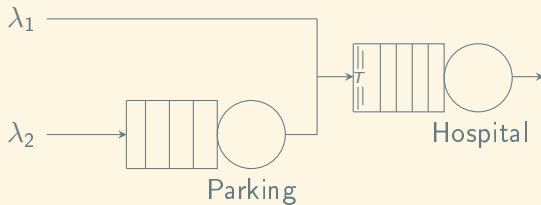
$$Q = \begin{matrix} & \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{matrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix} & \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & \mu & -\mu - \lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & \mu & -\mu - \lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & \mu & -\mu - \lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & \mu & -\mu \end{pmatrix} \end{matrix}$$

# Analytical formulation - M|M|3 queue

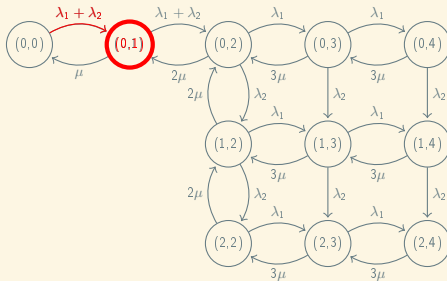
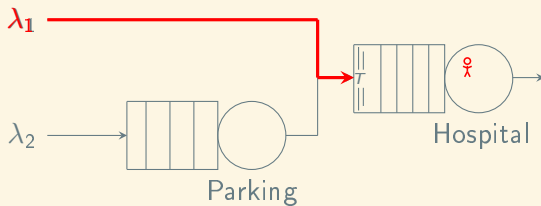


$$Q = \begin{matrix} & \begin{matrix} (0) & (1) & (2) & (3) & (4) & (5) & (6) \end{matrix} \\ \begin{matrix} (0) \\ (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix} & \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 \\ \mu & -\mu - \lambda & \lambda & 0 & 0 & 0 & 0 \\ 0 & 2\mu & -2\mu - \lambda & \lambda & 0 & 0 & 0 \\ 0 & 0 & 3\mu & -3\mu - \lambda & \lambda & 0 & 0 \\ 0 & 0 & 0 & 3\mu & -3\mu - \lambda & \lambda & 0 \\ 0 & 0 & 0 & 0 & 3\mu & -3\mu - \lambda & \lambda \\ 0 & 0 & 0 & 0 & 0 & 3\mu & -3\mu \end{pmatrix} \end{matrix}$$

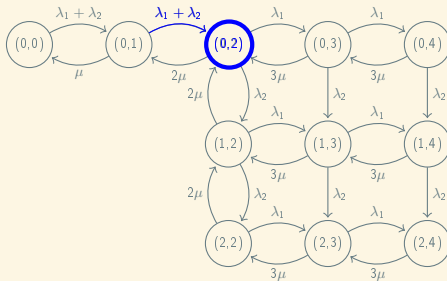
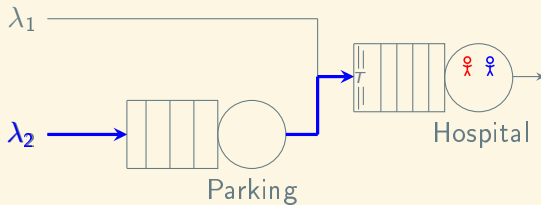
# Analytical formulation - Custom Queue



# Analytical formulation - Custom Queue

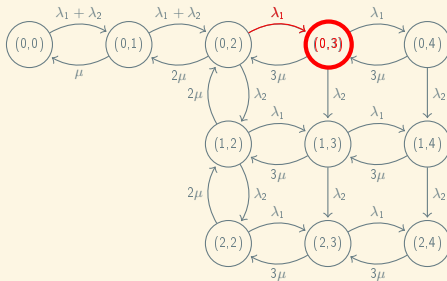
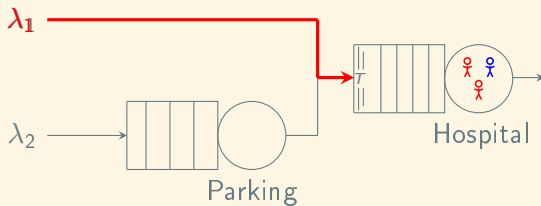


# Analytical formulation - Custom Queue

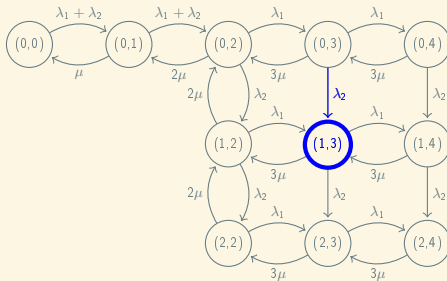
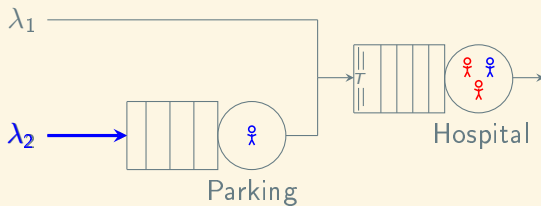




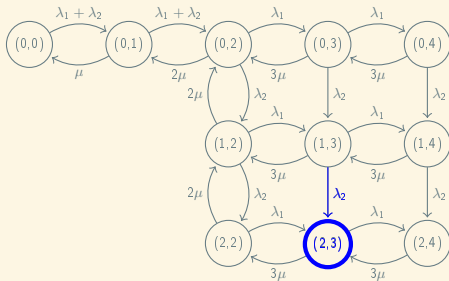
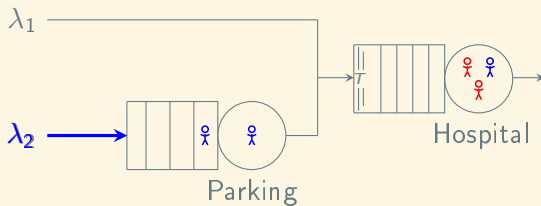
# Analytical formulation - Custom Queue



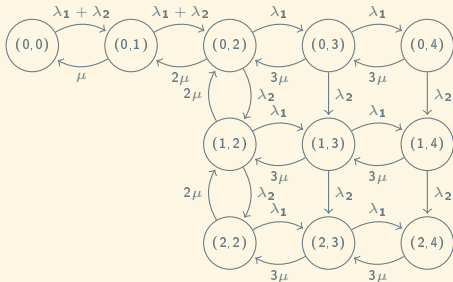
# Analytical formulation - Custom Queue



## Analytical formulation - Custom Queue

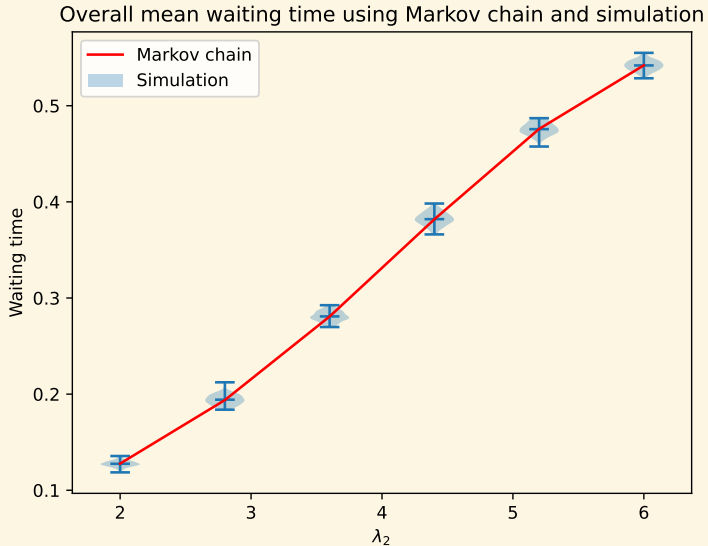


# Analytical formulation - Custom Queue



$$Q = \begin{matrix} & \begin{matrix} (0, 0) & (0, 1) & (0, 2) & (2, 3) & (2, 4) \end{matrix} \\ \begin{pmatrix} -\lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & 0 & \dots & 0 & 0 \\ \mu & -\mu - \lambda_1 - \lambda_2 & \lambda_1 + \lambda_2 & \dots & 0 & 0 \\ 0 & 2\mu & -2\mu - \lambda_1 - \lambda_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\lambda_1 - 3\mu & \lambda_1 \\ 0 & 0 & 0 & \dots & 3\mu & -3\mu \end{pmatrix} & \begin{matrix} (0, 0) \\ (0, 1) \\ (0, 2) \\ \\ (2, 3) \\ (2, 4) \end{matrix} \end{matrix}$$

# Performance Measures - Waiting time



# Performance Measures - Blocking time

