Workforce behaviours in healthcare systems

Michalis Panayides

March 14, 2022

Contents

1 Introduction

2 Literature review

2.1 EMS/ED OR models

2.2 Game theory in Healthcare

2.3 Game theory and Queueing theory

2.4 Behavioural modelling

3 Queueing theoretic model

One of the main outcomes of this research is the creation of a queueing model that consists of two waiting zones for two types of individuals.

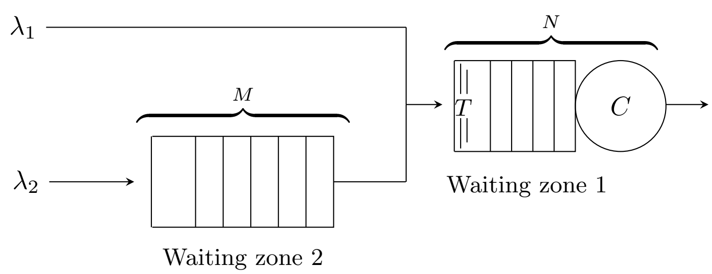

Figure 1: A diagrammatic representation of the queueing model. The threshold T only applies to type 2 individuals. If the number of individuals in the hospital is T, only individuals of type 1 are accepted (at a rate λ_1) and individuals of type 2 (arriving at a rate λ_2) are blocked in the parking space.

The model consists of two types of individuals; type 1 and type 2. Type 1 individuals arrive instantly at waiting zone 1 and wait to receive their service. Type 2 individuals arrive at waiting zone 2 and wait there until they are allowed to move to waiting zone 1. They are allowed to proceed only when the number of individuals in waiting zone 1 **and** in service is less than a predetermined threshold T. When the number of individuals is equal to or exceeds this threshold, all type 2 individuals that arrive will stay *blocked* in waiting zone 2 until the number of people in waiting zone 1 falls below T. This is shown diagrammatically in Figure 1. The parameters of the described queueing model are:

- λ_i: The arrival rate of type i individuals where $i \in \{1, 2\}$
- μ: The service rate for individuals receiving service after waiting zone 1
- C: The number of servers

- T: The threshold at which individuals of the second type are blocked

3.1 Markov chain

Under the assumption that all rates (arrival and service) are Markovian the queueing system corresponds to a Markov chain [1]. The states of the Markov chain are denoted by (u, v) where:

- u is the number of individuals blocked
- v is the number of individuals either in waiting zone 1 or in the service centre

We denote the state space of the Markov chain as $S = S(T)$ which can be written as the disjoint union (1).

$$\begin{aligned} S(T) =& S_1(T) \cup S_2(T) \text{ where:} \\ S_1(T) =& \left\{(0, v) \in \mathbb{N}_0^2 \mid v < T\right\} \\ S_2(T) =& \{(u, v) \in \mathbb{N}_0^2 \mid v \geq T\} \end{aligned} \tag{1}$$

3.1.1 Generator matrix

The generator matrix Q of the Markov chain consists of the rates between the numerous states of the model. Every entry $Q_{ij} = Q_{(u_i,v_i),(u_j,v_j)}$ represents the rate from state $i = (u_i, v_i)$ to state $j = (u_j, v_j)$ for all $(u_i, v_i), (u_j, v_j) \in S$. The entries of Q can be calculated using the state-mapping function described in (2):

$$Q_{ij} = \begin{cases} \Lambda, & \textbf{if } (u_i, v_i) - (u_j, v_j) = (0, -1) \textbf{ and } v_i < \text{t} \\ \lambda_1, & \textbf{if } (u_i, v_i) - (u_j, v_j) = (0, -1) \textbf{ and } v_i \geq \text{t} \\ \lambda_2, & \textbf{if } (u_i, v_i) - (u_j, v_j) = (-1, 0) \\ v_i\mu, & \textbf{if } (u_i, v_i) - (u_j, v_j) = (0, 1) \textbf{ and } v_i \leq C \textbf{ or} \\ & \quad (u_i, v_i) - (u_j, v_j) = (1, 0) \textbf{ and } v_i = T \leq C \\ C\mu, & \textbf{if } (u_i, v_i) - (u_j, v_j) = (0, 1) \textbf{ and } v_i > C \textbf{ or} \\ & \quad (u_i, v_i) - (u_j, v_j) = (1, 0) \textbf{ and } v_i = T > C \\ -\sum_{j=1}^{|Q|} Q_{ij} & \textbf{if } i = j \\ 0, & \textbf{otherwise} \end{cases} \tag{2}$$

Note that Λ here denotes the overall arrival rate in the model by both types of individuals (i.e. $\Lambda = \lambda_1 + \lambda_2$). A visualisation of how the transition rates relate to the states of the model can be seen in the general Markov chain model shown in Figure 2.

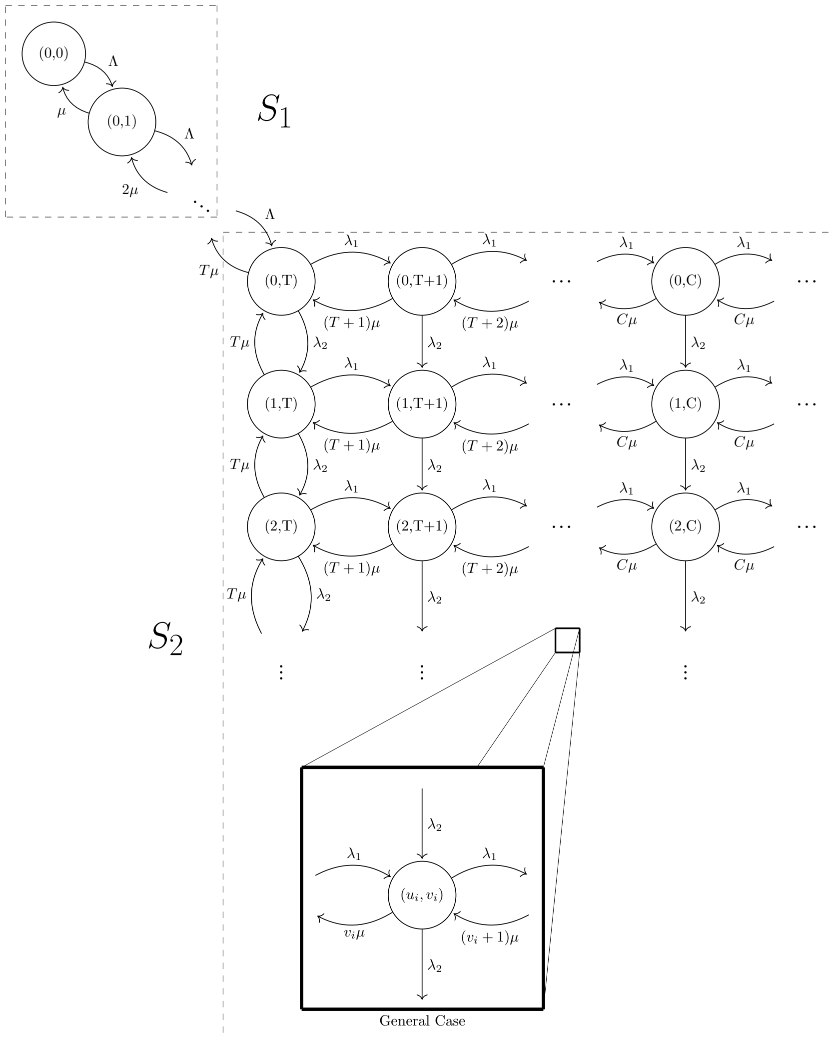

Figure 2: General case of the Markov chain model

In order to consider this model numerically an adjustment needs to be made. The problem defined above assumes no upper boundary to the number of individuals that can wait for service or for the ones that are blocked in the buffer centre. Therefore, a different state space $\tilde{S}$ is constructed where $\tilde{S} \subseteq S$ and there is a maximum allowed number of individuals N that can be in the system and a maximum allowed number of individuals M that can be blocked in the buffer centre:

$$\tilde{S} = \{(u, v) \in S \,|u \leq M, v \leq N\} \tag{3}$$

3.1.2 Steady state

The generator matrix Q defined in (2) can be used to get the probability vector π. The vector π is commonly used to study stochastic systems and it's main purpose is to keep track of the probability of being at any given state of the system. π_i is the steady state probability of being in state $(u_i, v_i) \in \tilde{S}$ which is the i^{th} state of $\tilde{S}$ for some ordering of $\tilde{S}$. The term *steady state* refers to the instance of the vector π where the probabilities of being at any state become stable over time. Thus, by considering the steady state vector π the relationship between it and Q is given by:

$$\frac{d\pi}{dt} = \pi Q = 0$$

3.1.3 Graph theoretic approach to steady state

3.2 Performance measures

Using vector π there are numerous performance measures of the model that can be calculated. The following equations utilise π to get performance measures on the average number of people at certain sets of state:

- Average number of people in the system:

$$L = \sum_{i=1}^{|\pi|} \pi_i (u_i + v_i)$$

- Average number of people in the service centre:

$$L_H = \sum_{i=1}^{|\pi|} \pi_i v_i$$

- Average number of people in waiting zone 2:

$$L_A = \sum_{i=1}^{|\pi|} \pi_i u_i$$

Consequently, there are some additional performance measures of interest that are not as straightforward to calculate. Such performance measures are the mean waiting time in the system (for both type 1 and type 2 individuals), the mean time blocked in waiting zone 2 (only valid for type 2 individuals) and the proportion of individuals that wait in waiting zone 1 within a predefined time target (for both types).

3.2.1 Waiting time

3.2.2 Blocking time

3.2.3 Proportion of individuals within target

4 Game theoretic model

4.1 Formulation

5 Methodology

5.1 Backwards Induction

5.2 Nash equilibrium

5.3 Learning algorithms

6 EMS-ED application

6.1 Formulation

6.2 Numerical results

7 Agent-based extension

7.1 State-dependent model

7.2 Server-dependent model

7.3 Reinforcement learning

8 Results

9 Conclusion

References

[1] John G Kemeny and J Laurie Snell. *Markov chains*, volume 6. Springer-Verlag, New York, 1976.