Computing for Mathematics: Week 1

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- `vknight.org/cfm`

(Gauss, 1777-1855)

$$\sum_{i=0}^{100} i$$

Cryptography:

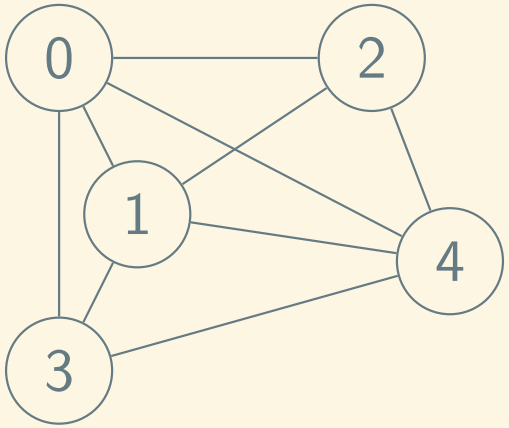
https://www.youtube.com/watch?v=_i-TcU0zLE0

Programming and Mathematics

There are various areas in which computers are of major importance to Mathematicians:

- Computer assisted proofs;
- Implementation of mathematics;
- Computer generated proofs;
- Everyday mathematics.

Computer assisted proofs



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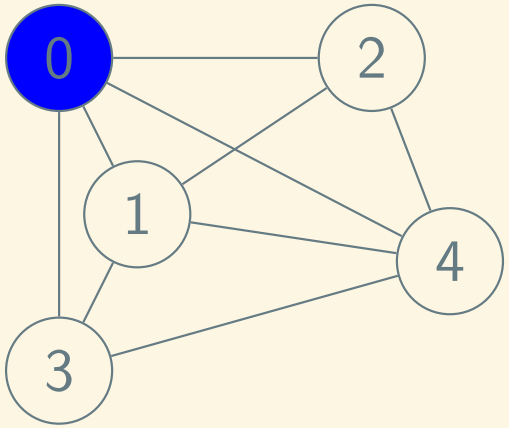

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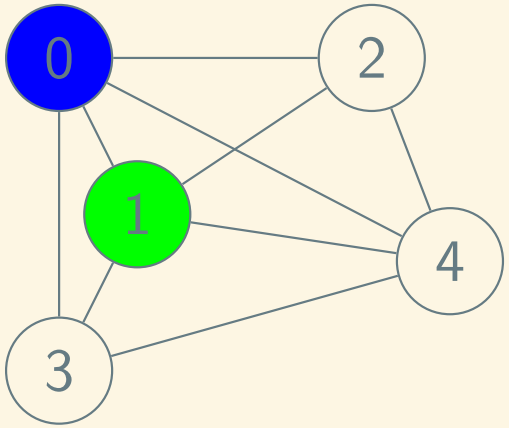

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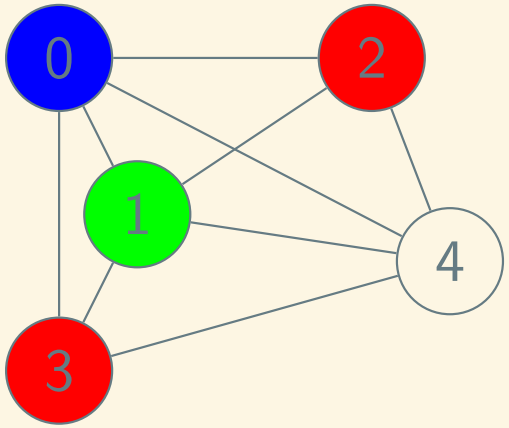

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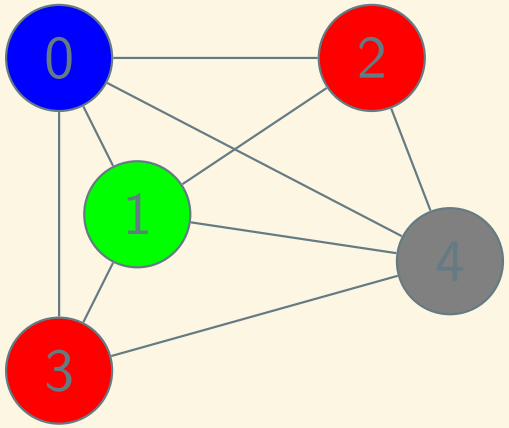
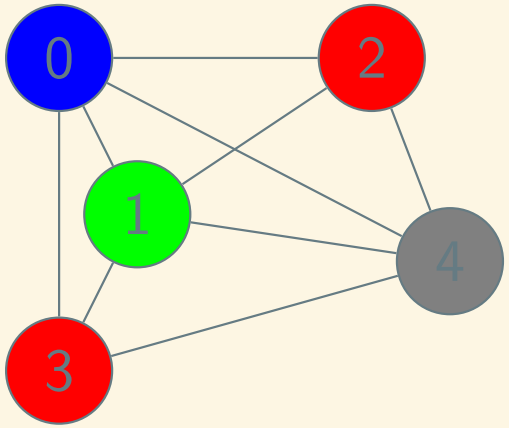


- '4 colour theorem': **Any map can be coloured using 4 colours.**

Computer assisted proofs



- '4 colour theorem': **Any map can be coloured using 4 colours.**
- Proved in 1976 by Kenneth Appel and Wolfgang Haken:

  Used computers to check 1936 particular cases.

Computer assisted proofs

How to pack 3 dimensional spheres?

- In 1611 Kepler conjectured the best possible way.
- Proof in 1998 by Hales which involved a computer to minimize a function of 150 variables (100,000 times).
- **Also** involved a 100 page paper for the 'non computer assisted aspects'.

Computer assisted proofs

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- **Also** involved a 100 page paper for the 'non computer assisted aspects'.
- Referees are 99% sure.

Implementation of mathematics

Here at Cardiff Dr Leanne Smith studied the best way to locate ambulances in Wales. This took in to account:

- Queues;
- Survival probabilities of patients;
- Time of the day...

Once the mathematics was done a computer program was built to be able to demonstrate to the Welsh Ambulance Trust.

Computer generated proofs

Timothy Gowers

Computer generated proofs

Timothy Gowers

Theorem: Let X and Y be sets, let $f : X \to Y$ be an injection and let A and B be subsetsof X. Then $f(A) \cap f(B) \subset f(A \cap B)$.

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Proof: Take $x \in f(A) \cap f(B)$. So there is some $y \in A$ and $z \in B$ such that $f(y) = f(z) = x$. As f is injective, y and z are equal. So $y \in A \cap B$. So $x = f(y) \in f(A \cap B)$.

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The above is an example of a computer generated proof. **You do not need to know any of this!**

Everyday mathematics

Everyday mathematicians might need to calculate an integral for a bigger project. This is some code to calculate an integral:

```
sympy.integrate(x ** 3, x)
```

which returns:

$$\frac{x^4}{4}$$

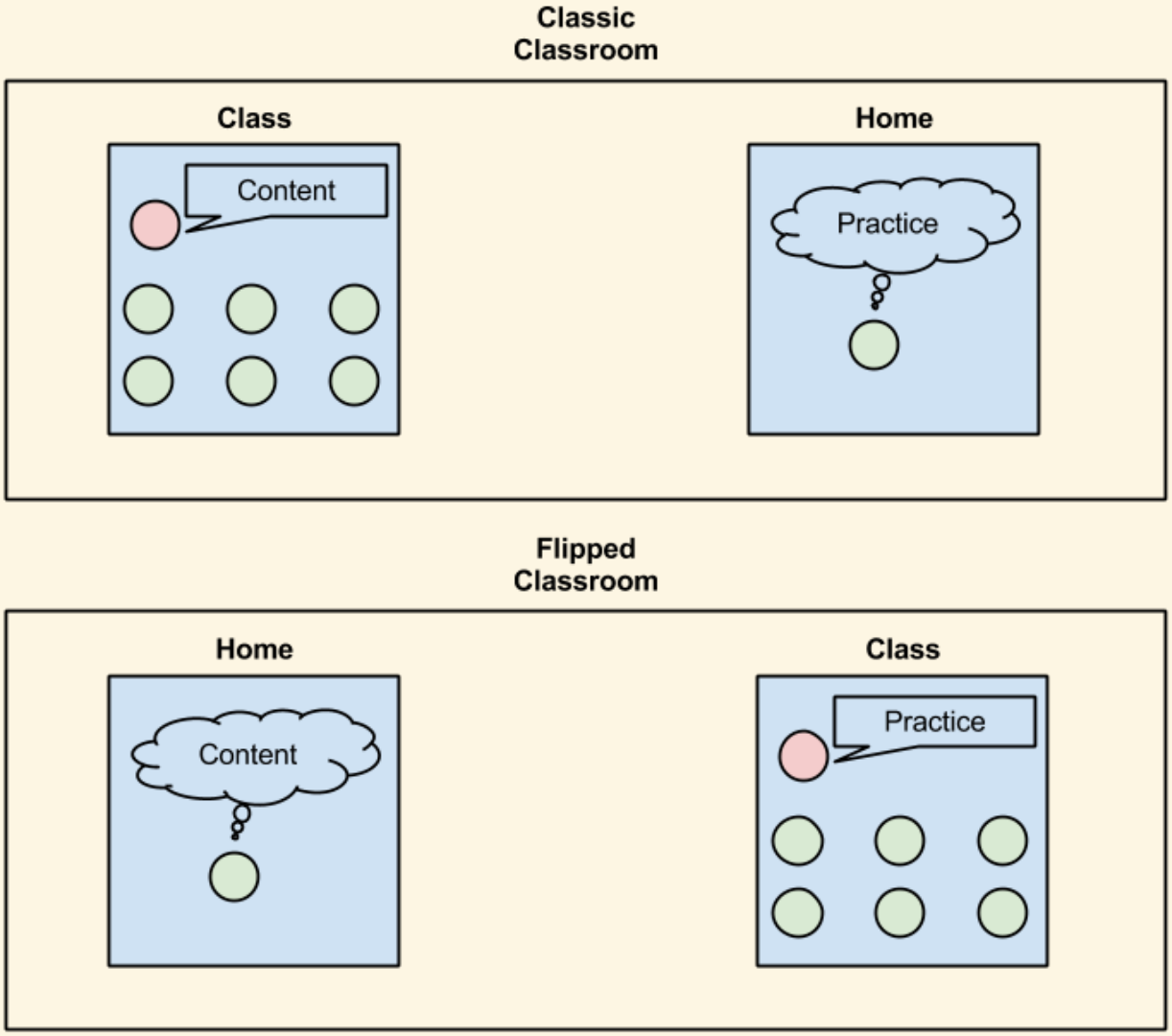
What we will learn

- Python: general purpose programming (Weeks 1-5).
- LaTeX: a package for writing mathematics (Week 6).
- Python: mathematical programming (Weeks 1-5).

https://www.continuum.io/downloads

Flipped classrooms

Flipped classrooms



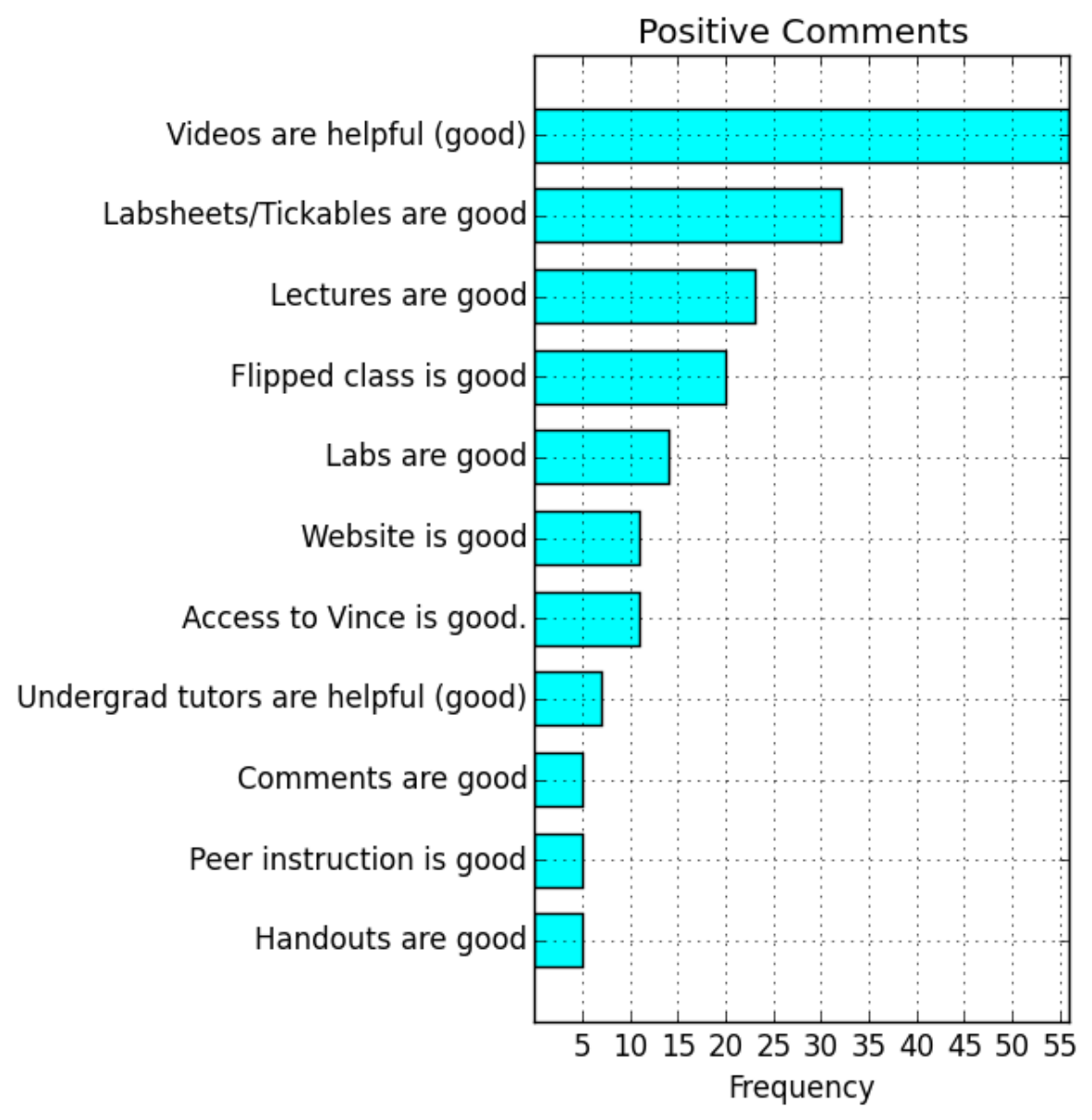
Labs and ‘Tickables’

- Every week you have 1 class meeting to look ahead.
- Every week you have 1 lab sheet: you should aim to work on your lab sheets before the lab session.
- Every week you have 1 class meeting to look back and address difficulties.

Resources

http://vknight.org/cfm/

Feedback from last year



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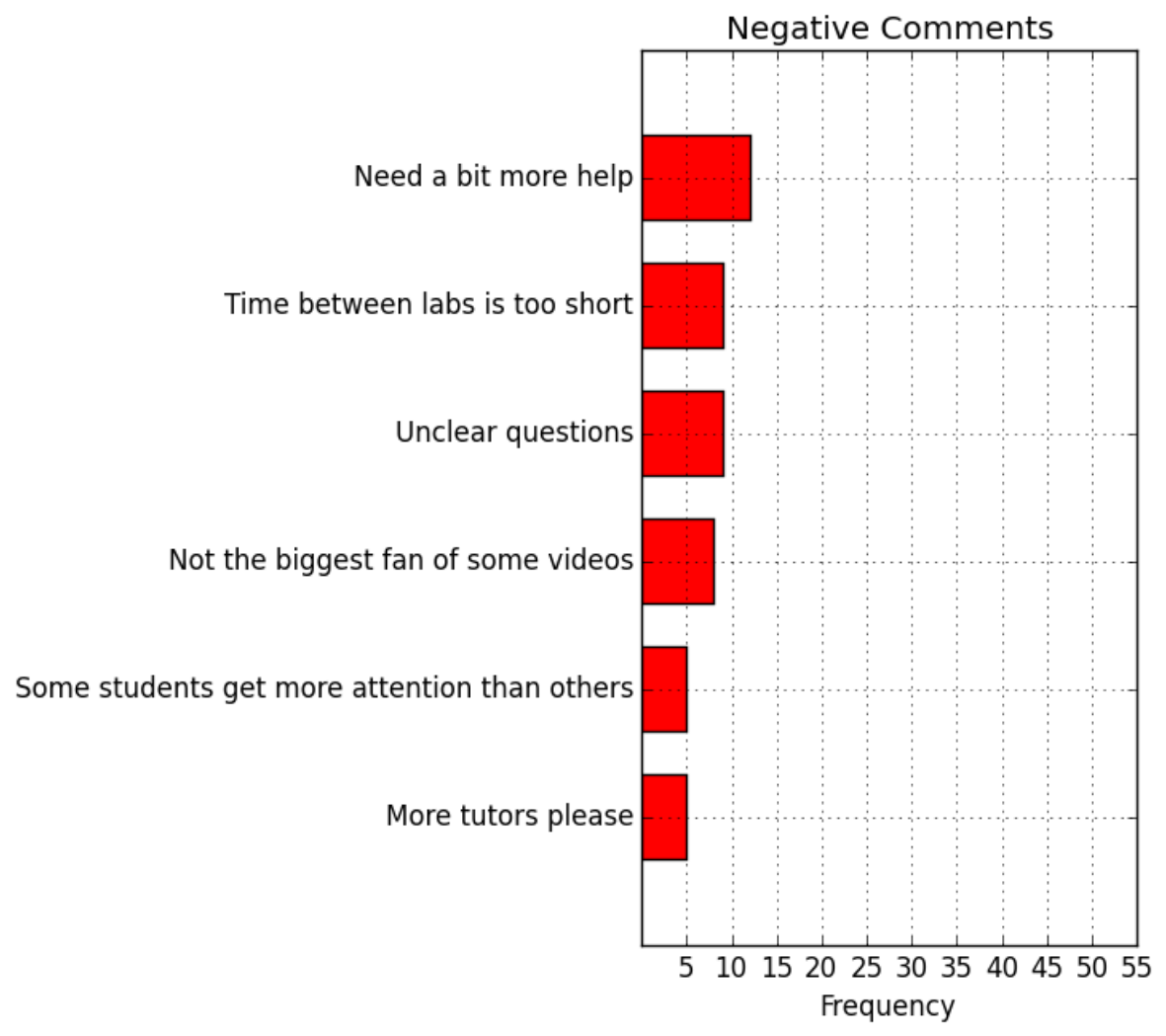

"No matter how difficult the lab sheets are, it always feels like there is sufficient support, either from Vince or from various online resources."

"Would like to know about all assessment from the start, class test was only recently revealed and don't know much about the remaining 45%"

- Individual Coursework: Week 11 - 70%
- Group Coursework: Spring semester - 30%

“Do you have snapchat?”

Getting help

- Gitter (chat) room: `vknight.org/cfm`.
- Message boards: `vknight.org/cfm`.
- email: knightva@cf.ac.uk.
- Office hours (M1.30): Thursday 1400 - 1600.
- @drvinceknight (and fb)

http://cardiffmathematicscodeclub.github.io/

@cardiffmathcode

http://www.pydiff.wales
@pydiff