Computing for Mathematics: Week 1

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- `vknight.org/cfm`

(Gauss, 1777-1855)

$$\sum_{i=0}^{100} i$$

Cryptography:

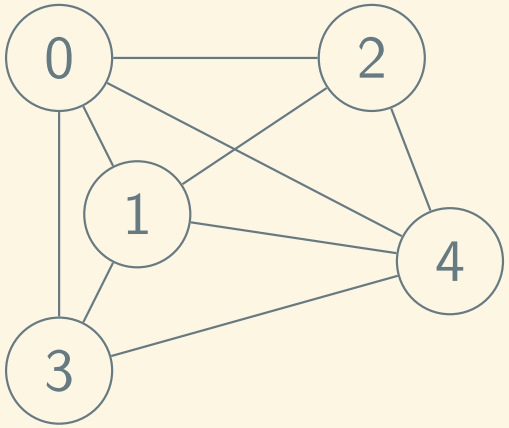
https://www.youtube.com/watch?v=_i-TcU0zLE0

Programming and Mathematics

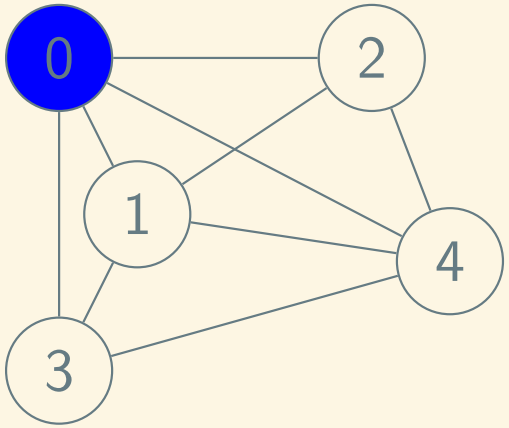
There are various areas in which computers are of major importance to Mathematicians:

- Computer assisted proofs;
- Implementation of mathematics;
- Computer generated proofs;
- Everyday mathematics.

Computer assisted proofs



Computer assisted proofs



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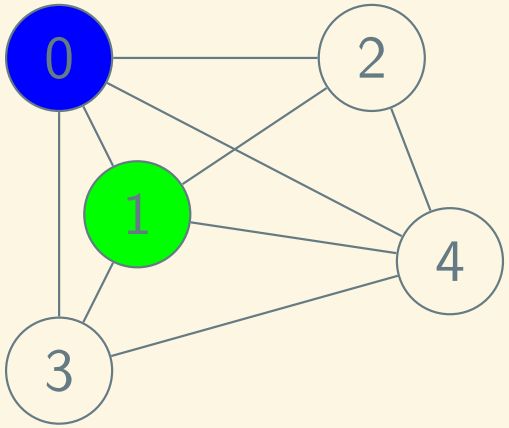
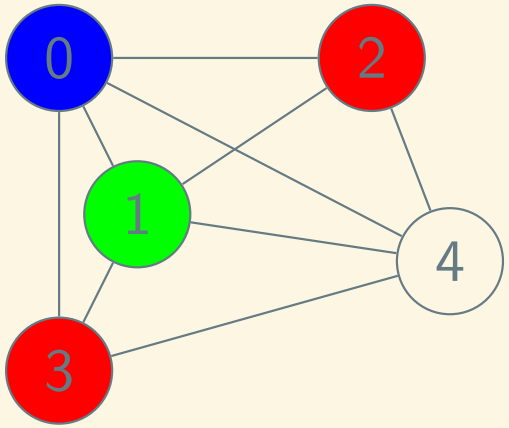
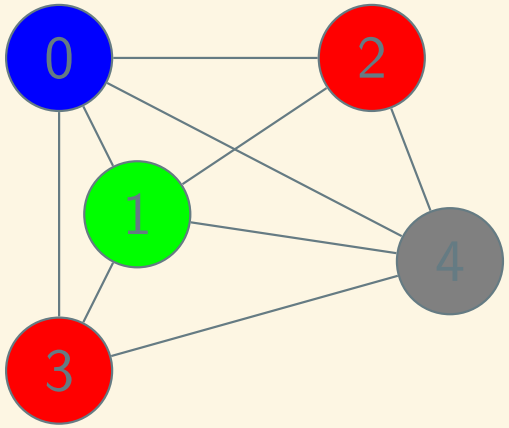


Computer assisted proofs

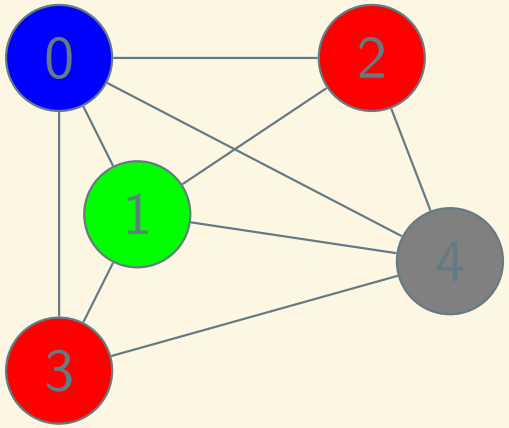


Computer assisted proofs



- '4 colour theorem': **Any map can be coloured using 4 colours.**

Computer assisted proofs



- '4 colour theorem': **Any map can be coloured using 4 colours.**
- Proved in 1976 by Kenneth Appel and Wolfgang Haken:

  Used computers to check 1936 particular cases.

Computer assisted proofs

How to pack 3 dimensional spheres?

- In 1611 Kepler conjectured the best possible way.
- Proof in 1998 by Hales which involved a computer to minimize a function of 150 variables (100,000 times).
- **Also** involved a 100 page paper for the 'non computer assisted aspects'.

Computer assisted proofs

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- **Also** involved a 100 page paper for the 'non computer assisted aspects'.
- Referees are 99% sure.

Implementation of mathematics

Here at Cardiff Dr Leanne Smith studied the best way to locate ambulances in Wales. This took in to account:

- Queues;
- Survival probabilities of patients;
- Time of the day...

Once the mathematics was done a computer program was built to be able to demonstrate to the Welsh Ambulance Trust.

Computer generated proofs

Timothy Gowers

Computer generated proofs

Timothy Gowers

Theorem: Let X and Y be sets, let $f : X \to Y$ be an injection and let A and B be subsetsof X. Then $f(A) \cap f(B) \subset f(A \cap B)$.

Computer generated proofs

Timothy Gowers

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Proof: Take $x \in f(A) \cap f(B)$. So there is some $y \in A$ and $z \in B$ such that $f(y) = f(z) = x$. As f is injective, y and z are equal. So $y \in A \cap B$. So $x = f(y) \in f(A \cap B)$.

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The above is an example of a computer generated proof. **You do not need to know any of this!**

Everyday mathematics

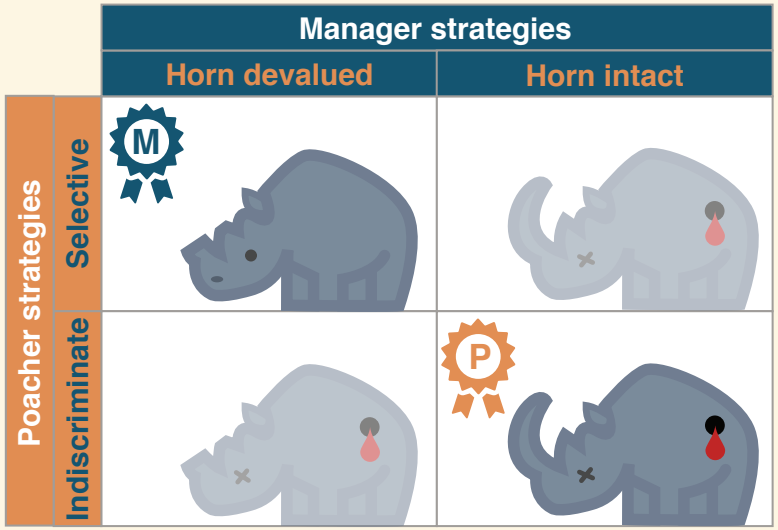
Everyday mathematicians might need to calculate an integral for a bigger project. This is some code to calculate an integral:

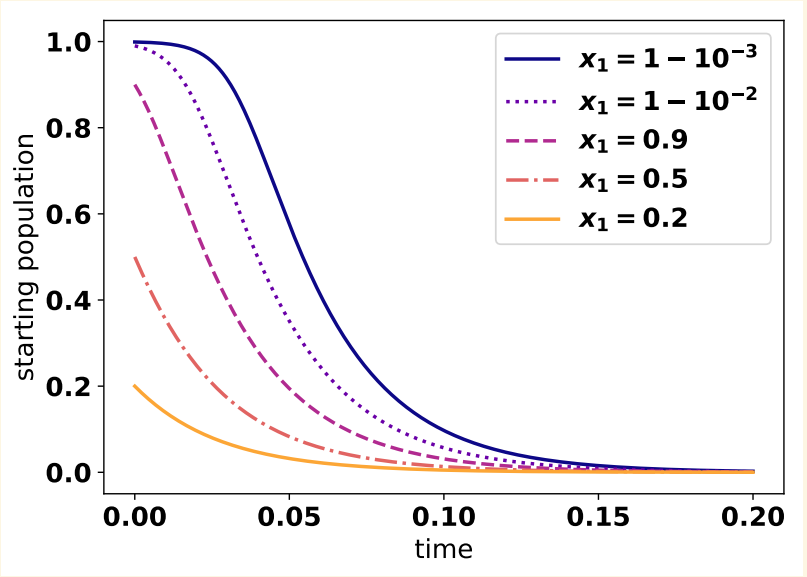
```
sympy.integrate(x ** 3, x)
```

which returns:

$$\frac{x^4}{4}$$

Rhino Poaching behaviour





$$-H\theta_r r\theta(r,s^*)^{-\alpha} + Fr(1-rs^*)^{\gamma}(1-r)^{\beta} = 0.$$

$$\frac{F}{H} = \frac{\theta_r}{(1-rs)^{\gamma}} + \frac{1}{(1-rs)^{\gamma}(1-r)^{\beta} r(1-s)}$$

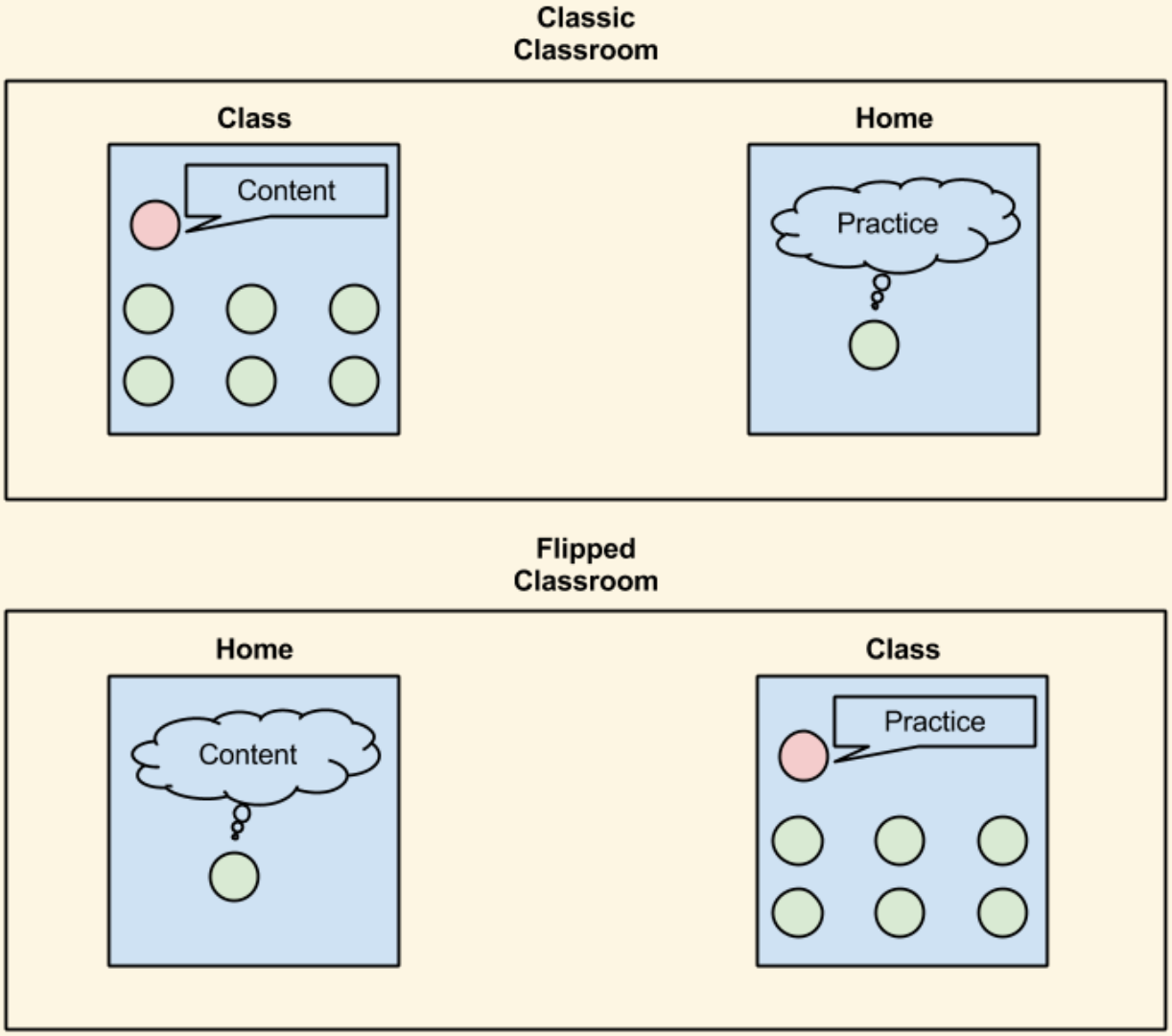
What we will learn

- Python: general purpose programming (Weeks 1-5).
- LaTeX: a package for writing mathematics (Week 6).
- Python: mathematical programming (Weeks 1-5).

https://www.continuum.io/downloads

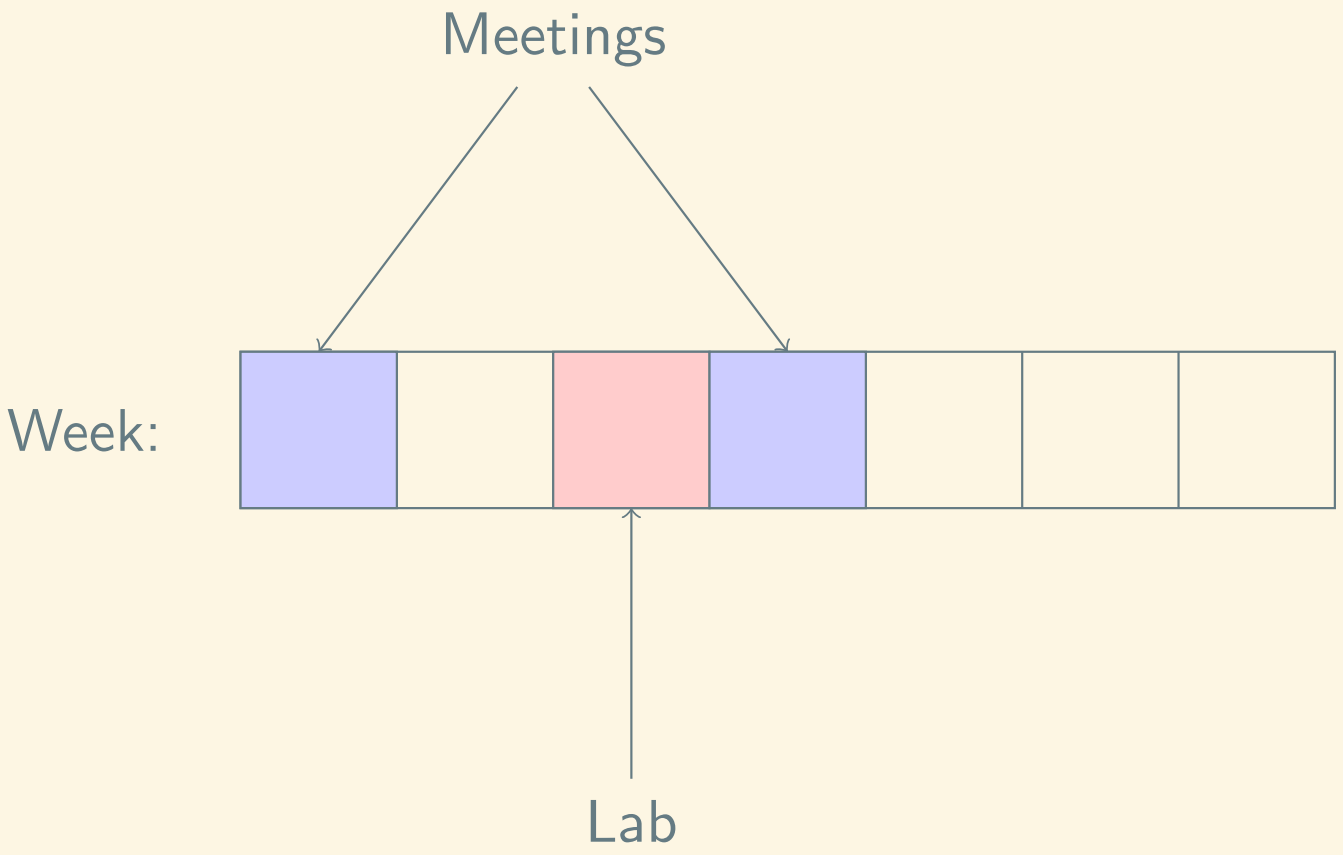
Flipped classrooms

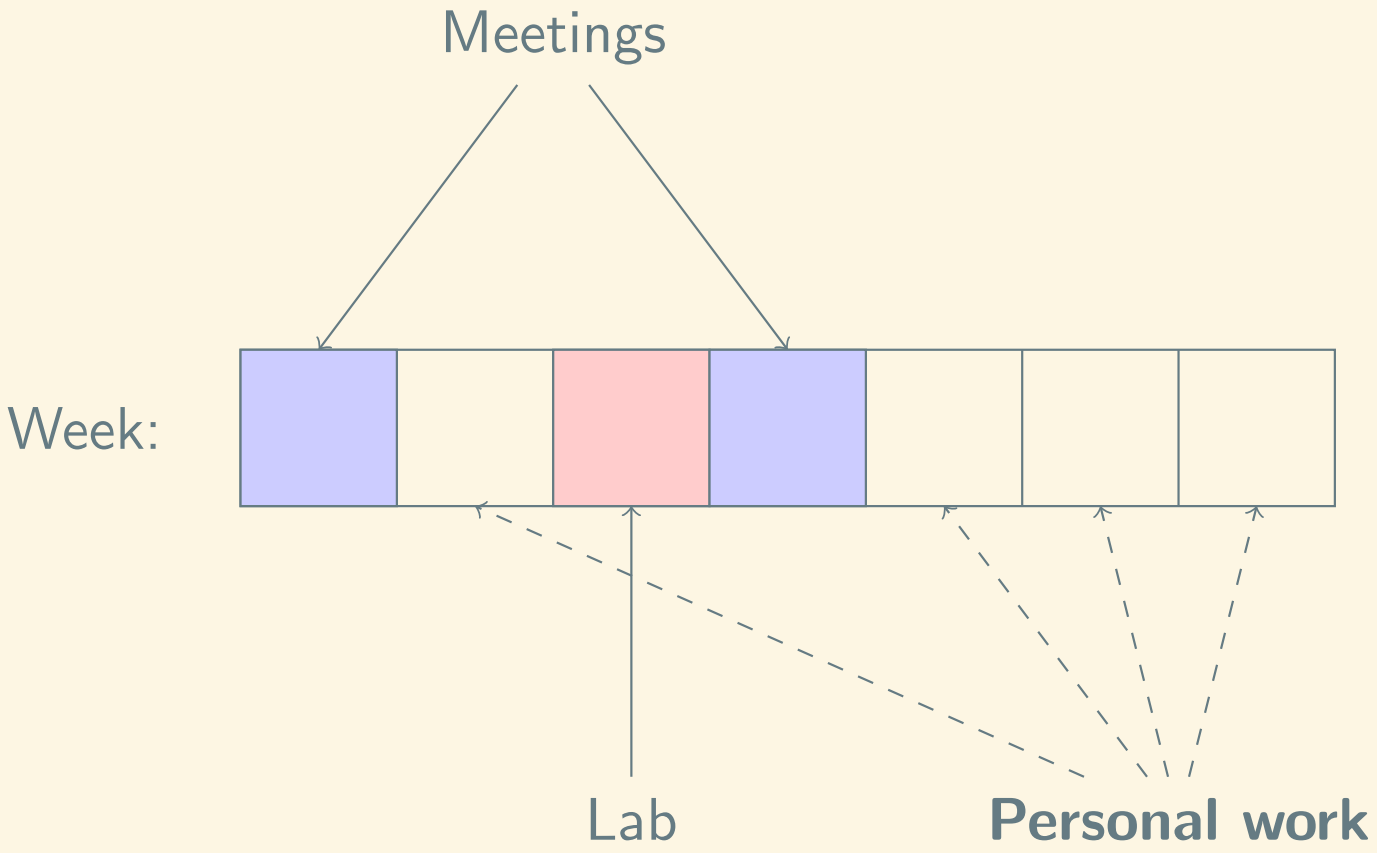
Flipped classrooms



Lab and Class meetings

- Every week you have 1 class meeting to look ahead.
- Every week you have 1 lab sheet: you should aim to work on your lab sheets before the lab session.
- Every week you have 1 class meeting to look back and address difficulties.


Meetings
Week:
Lab


Meetings
Week:
Lab
Personal work

Resources

http://vknight.org/cfm/

Some Feedback

Vince

"Vince very approachable"

(50%)

"You are intimidating and I would personally rather approach a tutor for help - no offence. Where is your accent from?"

(20%)

The class meeting

"The lecture is useful to go over what we struggle with."

(60%)

"Would be better to discuss the upcoming lab sheets in lectures instead of the one we just did."

(4%)

"Some aspects should be taught first in lectures."

(4%)

Labs

"Some (not all) [tutors] just give us the answers and don't explain it clear enough **AND***."*

(3%)

"Sometimes asking if you're watched videos when you have is a bit demoralising, makes it hard to ask for help."

(3%)

"Would like to know about all assessment from the start, class test was only recently revealed and don't know much about the remaining 45%"

- Individual Coursework: Week 11 - 70%
- Group Coursework: Spring semester - 30%

“Do you have snapchat?”

Getting help

- Gitter (chat) room: `vknight.org/cfm`.
- Message boards: `vknight.org/cfm`.
- email: knightva@cf.ac.uk.
- Office hours (M1.30): Thursday 1300 - 1500.
- @drvinceknight (and fb)

http://www.pydiff.wales
2017.pyconuk.org