A cool proof [1]:

$$V_0(0) = \frac{1}{1+R}\left[q_0^1(0)V_1(1) + q_0^0(0)V_1(0)\right],$$

of the theorem:

$$\begin{aligned} V_0(0) = \frac{M}{(1+R)^N}\sum_{j=0}^{2^N-1} e^{\beta-1}\sum_{k=0}^{N-1}\alpha_k\left(\lfloor j/2^{N-k}\rfloor\right) \\ \Phi\left(Su^{r_N(j)}d^{N-r_N(j)}\right)\prod_{k=1}^{N} q_{k-1}^{\mathrm{mod}(\lfloor j/2^{N-k}\rfloor,2)}\left(\lfloor j/2^{N-(k-1)}\rfloor\right) = \\ \frac{1}{(1+R)^N}\sum_{j=0}^{2^N-1} V_N(j)\prod_{k=1}^{N} q_{k-1}^{\mathrm{mod}(\lfloor j/2^{N-k}\rfloor,2)}\left(\lfloor j/2^{N-(k-1)}\rfloor\right). \end{aligned}$$

[1] Gerardo Hernández-del-Valle, Explicit formulae for the valuation of European options with price impacts, 2025. https://www.sciencedirect.com/science/article/pii/S2405918824000187