

DAOCROSS AND DAOSWAP

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1. INTRODUCTION

DaoCross and DaoSwap are smart contracts¹ that facilitate on-chain ERC-20² token exchange. DaoSwap provides a mechanism for efficient relative price discovery, and DaoCross enables parties to engage in zero market impact transactions with price improvement.

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¹<https://ethereum.org/en/developers/docs/smart-contracts/>

²<https://ethereum.org/en/developers/docs/standards/tokens/erc-20/>

DaoSwap performs price discovery by matching supply and demand at regular time intervals. Supply and demand is expressed through a collection of buy and sell limit orders, and the clearing price is determined by a Walrasian-style auction ([12]).

DaoCross is a decentralized liquidity pool that crosses buy and sell orders with respect to an external reference price, i.e., a price oracle. This allows traders to interact directly with each other and share price improvement with respect to exchanges, e.g., by trading at bid-ask midpoint. Additionally, order intentions are not publicly disclosed prior to a cross, which enables block or other high-volume traders to execute quickly with minimal market impact.

DaoSwap and DaoCross differ in how the token exchange rate is determined, but both share several common advantages:

- (1) Low cost: they are a DeFi primitive that allows trading tokens peer-to-peer without incurring spread costs
- (2) Capital efficiency: participants provide liquidity only when they wish to trade, and only in the amount they wish to trade
- (3) No impermanent loss: liquidity providers are not exposed to the risk of impermanent loss (a form of unrealized loss that becomes realized upon withdrawing liquidity), unlike the case with other decentralized exchange protocols based on automated market makers
- (4) No intermediary custody: exchanged tokens always remain under the control of their respective owners
- (5) No rent extraction from high-frequency traders: speed alone does not confer an advantage to traders, as the discrete timing prevents predatory tactics such as front-running, limit order scalping, and spoofing, which are known issues seen with continuous limit order books
- (6) Ease-of-use: the interaction between buyers and sellers occurs through smart contracts, with a website front-end available

2. MATCHING LIQUIDITY

Suppose there are two parties, Alice an ETH holder and Bob a wBTC holder (wrapped bitcoin, an ERC-20 compliant coin that tracks bitcoin), who wish to swap tokens at a competitive rate. Suppose also that there are many additional ETH and wBTC holders who may be interested in participating in a swap. How should the liquidity be pooled? At first glance, it appears that there should be a single pool of liquidity. The wrinkle manifests in determining how to express a desire to trade. A simple expression of a desire to trade is a limit order representing a commitment to trade up to q in quantity of a token at a token exchange rate up to p . But what if some parties express quantity in terms ETH and some in terms of wBTC, and some parties express limit prices in terms of ETH per wBTC, while others express limit prices in terms of wBTC per ETH? Under these conditions, setting a single clearing price and determining fill prioritization rules raises several questions. We will consider and address these in the sequel, but to begin, we will discuss a more constrained setting.

2.1. Breaking the symmetry. To simplify, we follow the practice of FX (foreign exchange) futures markets, which break the symmetry by specifying two non-interchangeable roles:

- (1) a *base* currency (for quantity)
- (2) a *quote* currency (for price)

The roles are expressed by writing *base currency/quote currency*, e.g., “EUR/USD”.

A limit order then consists of a quantity expressed in the units of the base currency and a price expressed in terms of the quote currency (per unit or suitable multiple of base currency). See, for example, [6] (e.g., footnotes on page 25) for usage of this terminology. See [7] for how this works in practice for Euro FX contracts, where contract units are denominated in Euros and price is quoted in U.S. dollars and cents per Euro increment.

By breaking the symmetry, one may run a standard limit order book, hold a classical Walrasian-style auction, and handle size quantization effects by using one of a variety of priority rules based on quantity, price, and time. See [4] for discussion around priority rule variations and their effects on market quality outcomes.

An effect of breaking the symmetry in this way is that either liquidity for a pair of currencies may be expressed in only one form, or liquidity for a pair of currencies is split across two separate contracts (with roles of base and quote token reversed), each with its own limit order book. Using our example, this would mean treating wBTC/ETH and ETH/wBTC as separate token pairs, even though the union of the underlying parties and sources of liquidity are the same.

2.2. Basket case. It seems plausible that by retaining the symmetry in the two-token case and instead treating a pair of tokens as a single source of liquidity, one may contrive a more efficient exchange mechanism.

This minor expansion, however, suggests widening even further the universe of eligible swaps. For example, if there are three tokens available for exchange, one could contemplate many-for-one, one-for-many, or many-for-many token swaps, all to be carried out in an atomic fashion, and perhaps with complex constraints. A toy case along these lines one may consider is as follows:

Problem 1 (Triangular liquidity). Suppose there are the following three parties:

- Party A, who holds wBTC and wants ETH
- Party B, who holds ETH and wants DAI
- Party C, who holds DAI and wants wBTC

How should an auction be structured to “best” facilitate exchange?

While there may be other use cases and even demand for such auctions, unfortunately, the problem in general is NP-complete (e.g., [13]), which is a significant limiting factor in terms of feasibility and auction expense.

3. PROTOCOL INTRODUCTION

In the previous section, we discussed the notions of base currency and quote currency from foreign exchange. Here we extend them to tokens.

3.1. Base/quote tokens. A token (ordered) pair consists of a *base token* and a *quote token*. The shorthand notation for expressing the pair is *base token/quote token* (e.g., “ wBTC/ETH ”), and as such indicates the respective roles of the tokens.

By convention, *quantity* to exchange is expressed in terms of the base token, and *price* is a token exchange rate expressed in terms of the quote token per unit of base token (or multiple thereof).

Example 1 (base/quote tokens). By way of example, in the pair wBTC/ETH , wBTC is the base token and ETH is the quote token. Interest to trade is expressed in terms of orders with wBTC as base token and ETH as quote token, as follows:

- A “buy” order represents a commitment to purchase the base token wBTC and pay with the quote token ETH
- A “sell” order represents a commitment to sell the base token wBTC and receive the quote token ETH .
- Quantity q is in terms of the base token (“buy/sell a certain amount of wBTC ”)
- Limit price is in terms of the quote token (“buy/sell wBTC with a limit price of p ETH per wBTC ”)

A party who places a “buy” order must have the quote token (ETH) in custody, i.e., in its wallet. A party who places a “sell” order must have the base token (wBTC) in custody.

3.2. Order attributes and notation. We introduce the following tuple notation for a general limit order (valid for both DaoCross and DaoSwap).

Definition 1 (Limit order). A DaoCross or DaoSwap limit order is represented by a tuple of the form

```
(timestamp,
  action,
  quantity,
  base_token,
  limit_price,
  quote_token,
  deposit_address)
```

The quantities are arranged to make the order simple to read in natural language: “At timestamp `timestamp` create an order to `action` a `quantity` of the token `base_token` for a limit price of `limit_price` with respect to the token `quote_token` and deposit the resulting tokens at `deposit_address`.”

We have previously discussed the roles of `action`, `quantity`, `base_token`, `limit_price`, and `quote_token`. The roles of `timestamp` include determining eligibility in a swap or cross and can extend to influencing order fill priority. The `deposit_address` attribute mirrors standard functionality available in traditional finance, e.g., in purchasing T-Bills on TreasuryDirect³. This feature facilitates account management through the use of multiple wallets. For example, a miner may have a supply of `wBTC` collected and held in one wallet which it would like to systematically diversify into `ETH` and perhaps other tokens. In specifying a deposit address, it may use a separate wallet to collect incoming `ETH`.

For brevity, in the sequel we may:

- omit `timestamp` and `deposit_address` when not relevant to the discussion
- omit the (infinite) `limit_price` for market orders

Example 2 (Limit order notation). The order

```
(1678660406, buy, 3.2, ETH, 4.0, wBTC, 0xdeadcd0de)
```

corresponds to the natural language description: “At timestamp `Mon Mar 13 2023 02:33:25 GMT+0000`, the user commits to buy up to 3.2 units of `ETH` in exchange for `wBTC` up to a `limit_price` of 4.0 `wBTC` per `ETH` with proceeds deposited at `0xdeadcd0de`”.

Next we introduce some examples of market order and limit order behavior when a clearing rate has been set. In particular, consider a swap for the tokens `ETH`, `wBTC` and assume that the exchange rate between `ETH` and `wBTC` is fixed at 0.2 (i.e., 0.2 `wBTC` can be exchanged for 1 `ETH` and vice versa).

Example 3 (Market order notation and clearing). The following market orders omit `timestamp`, `limit_price`, and `deposit_address` in favor of emphasizing the amounts of tokens exchanged:

- An order (`buy, 1.0, ETH, wBTC`) means buying 1 `ETH` in exchange for 0.2 `wBTC`
- An order (`sell, 1.0, ETH, wBTC`) means selling 1 `ETH`, receiving the corresponding amount of 0.2 `wBTC`
- An order (`buy, 1.0, wBTC, ETH`) means buying 1 `wBTC`, paying with 5 `ETH`
- An order (`sell, 1.0, wBTC, ETH`) means exchanging 1 `wBTC` in return for 5 `ETH`

Next we consider the behavior of limit orders under the same prevailing exchange rate (and we assume that there is sufficient supply of tokens to fully fill the orders).

³<https://www.treasurydirect.gov>

Example 4 (Executable buy limit order). A limit order

$$(\text{buy}, 1.0, \text{ETH}, 0.5, \text{wBTC})$$

means “buy up to 1 ETH in exchange for wBTC at a rate up to 0.5 wBTC per ETH.” In this case, since the price of one ETH is equal to 0.2 wBTC, the order can be executed at the prevailing market rate.

Example 5 (Non-executable buy limit order). On the other hand, a limit order

$$(\text{buy}, 1.0, \text{ETH}, 0.1, \text{wBTC})$$

requires that the rate of wBTC per ETH be lower than the current market rate, and so the limit price prevents a token swap from being carried out.

Example 6 (Non-executable sell limit order). A limit order

$$(\text{sell}, 1.0, \text{ETH}, 0.5, \text{wBTC})$$

means “sell up to 1 unit of ETH in exchange for wBTC at a rate down to 0.5 wBTC per ETH.” Since the current rate of wBTC per ETH is 0.2, which is below the limit price of 0.5, the order cannot be executed.

3.3. Limit order as a set of inequalities. Any limit order as defined above can be translated into inequalities involving quantity of exchanged tokens and token exchange rates. Multiple limit orders can be converted into a system of inequalities, which collectively constrain potential exchange outcomes.

In both DaoCross and DaoSwap, a swap between a given base/quote token pair occurs at a single exchange rate, i.e., the clearing price is the same for all executed orders. On the other hand, quantity exchanged is specific to each order and each order’s constraint on quantity exchanged cannot be violated. In the same way each order can be executed with a non-null quantity only when the single exchange rate is compatible with the limit price.

In the sequel we use an asterisk to denote clearing quantity and exchange rate:

- $q_{base}^*(o_i)$ denotes quantity of the base token exchanged in a swap from order o_i
- $p_{quote_per_base}^*$ denotes exchange rate between the quote and base tokens for all the swaps

Note that:

$$p_{quote_per_base} = \frac{1}{p_{base_per_quote}}$$

If prices of base and quote tokens are expressed in terms of a common currency (e.g., USD) then it holds that:

$$p_{quote_per_base} = \frac{p_{quote}}{p_{base}}$$

3.3.1. Extracting attributes from orders. Given an order $o_i = (\text{buy}, q, A, lb, B)$ we indicate with

- $action(o_i)$ the desired action (e.g., buy)
- $q_{base}(o_i)$ the maximum desired quantity q
- $lb(o_i)$ the limit price lb
- $base(o_i)$ the base token A
- $quote(o_i)$ the quote token B

3.3.2. Inequalities for buy order. An order of the form $o_i = (\text{buy}, q, A, lb, B)$ means “buy q units of token A in exchange for token B with a limit price up to lb B per unit of A” and it corresponds to the following constraint on realized quantity exchanged and clearing exchange rate:

$$(p_{B_per_A})^* \leq lb(o_i) \wedge (0 \leq q_A^*(o_i) \leq q(o_i)) \vee (q_A^*(o_i) = 0)$$

The constraint on the quantity exchanged is conditioned on the corresponding price satisfying the desired limit price constraint, otherwise the swap cannot be carried out and the exchanged quantity is 0.

3.3.3. Example of inequalities for buy order. An order of the form $o_1 = (\text{buy}, 2, A, 3, B)$ means “buy 2 units of token A in exchange for token B with a limit price up to 3 B per unit of A” and it corresponds to the following constraint on realized quantity exchanged and clearing exchange rate:

$$(p_{B_per_A}) \leq 3 \wedge (0 \leq q_A^*(o_1) \leq 2) \vee (q_A^*(o_1) = 0)$$

3.3.4. Inequalities for sell order. An order of the form $o_i = (\text{sell}, q, A, lb, B)$ means “sell q units of token A paying in token B with a limit price of at least to lb B per unit of A” and it corresponds to the following constraint on realized quantity exchanged and clearing exchange rate:

$$(p_{B_per_A})^* \geq lb(o_i) \wedge (0 \leq q_A^*(o_i) \leq q(o_i)) \vee (q_A^*(o_i) = 0)$$

3.3.5. Example of inequalities for sell order. In the same way, an order like $o_2 = (\text{sell}, 3, A, 2, B)$ means “sell 3 units of token A paying in token B with a limit price for one unit of A of at least 2 B” which corresponds to

$$(p_{B_per_A}^*(o_2) \geq 2) \wedge (0 \leq q_A^*(o_2) \leq 3) \vee (q_A^*(o_2) = 0)$$

3.3.6. Example of inequalities for buy market order. A market order (i.e., without a limit price) has no constraint on exchange rate and thus is reduced to

$$0 \leq q_A^*(o_1) \leq q(o_i)$$

3.3.7. Order equivalence. Let

$$o_1 = (\text{buy}, q, A, p_{B_per_A}, B).$$

Suppose that $p_{B_per_A}^*$ is fixed and known. Then the order

$$o_2 = (\text{sell}, q', B, 1/p_{B_per_A}, A)$$

may be handled in order crossing in the same way that o_1 may be provided that

$$q' = q \cdot p_{B_per_A}^*$$

4. REFERENCE CLEARING PRICES AND PRIORITIZATION

DaoCross relies on a *price oracle* for determining the effective token exchange rate in a cross. The price oracle may come from a lit exchange, centralized or decentralized, or even on-chain automated market makers such as Uniswap ([2, §2.2]).

The case of using an automated market maker as a price oracle is relatively straightforward, provided there is an automated market maker trading the target currency pair with sufficient liquidity.

To use an exchange as a price reference, we must consider some additional steps. First, exchanges typically use a dollar stablecoin (e.g., USDC, USDT) as the quote currency and all other currencies as base currencies. Our example of BTC/ETH would be considered a cross pair in the world of traditional finance. Because most cross pairs are not traded directly on exchanges, we must derive a clearing price from pairs involving stablecoins.

4.1. Lit exchange bid-ask midpoint reference price. Let bid_{BTC} , ask_{BTC} be stablecoin-denominated top-of-book bid-ask prices for wBTC (e.g., expressed in USDC), and bid_{ETH} , ask_{ETH} be the analogous prices for ETH . Then, a commitment to buy one wBTC and pay ETH would clear at a midpoint price of

$$\text{wBTC/ETH}_{\text{midpoint}} = \frac{\text{bid}_{\text{BTC}} + \text{ask}_{\text{BTC}}}{\text{bid}_{\text{ETH}} + \text{ask}_{\text{ETH}}}$$

expressed in BTC per ETH . Note that if the dollar price of BTC is significantly more than the dollar price of ETH , then this price implies paying many multiples of ETH for BTC (as is the case at the time of this writing).

In general, DaoCross may use the following reference price for a base/quote token pair, where the bids and asks come from a lit exchange and are expressed in terms of stablecoin prices:

$$\frac{\text{bid}_{\text{quote token}} + \text{ask}_{\text{quote token}}}{\text{bid}_{\text{base token}} + \text{ask}_{\text{base token}}} \quad (1)$$

When using either an on-chain price oracle or an exchange as a price oracle, it is possible to perform a time or volume weighted average of prices so as to avoid extreme price variations and to mitigate the risk and effects of any market manipulation attempts.

4.2. Order crossing. At regular time intervals, order submissions are cut off and reference prices are determined. An element of randomness is used to determine order cutoff times and reference price cutoff times in order to mitigate manipulative behaviors.

An order is eligible for matching if its timestamp is within the cutoff window and if the external reference price does not exceed its limit price.

Except on occasions where eligible buy/sell volume is perfectly matched, not all orders can be fully crossed. There are also discretization effects to consider arising from discrete order sizes and a discrete price grid [5][§3.2.1]. See [4] for a discussion of the trade-offs surrounding different prioritization rule choices. See [11] for the prioritization rules used by one of Morgan Stanley’s equity liquidity pools.

4.2.1. Priority rules. DaoCross prioritizes fills according to:

- Volume (higher volume comes first in priority)
- Price (higher limit price breaks volume ties)
- Timestamp (earlier timestamp breaks ties in volume and price)

If, in the unlikely scenario that all three of these parameters perfectly agree, a certain priority is not guaranteed.

4.2.2. Matching algorithm. The mechanism for matching eligible buy and sell orders consists of two priority queues, one for eligible buy orders and one for eligible sell orders. Priority is determined according to the volume/price/timestamp priority rules introduced above. Top-of-queue orders are compared, and the lesser of the two volumes is fully filled. Once an order is fully filled at the established reference price. Once an order is fully filled, it is removed from the priority queue. The procedure continues until one of the two priority queues is empty.

4.2.3. Computational complexity. Let n denote the sum (or max) of the number of eligible buy and sell orders. Priority queue construction occurs in $O(n)$ time, order removal costs $O(n \log n)$, and order remove occurs $O(n)$ times. So, the computational time complexity of the task is $O(n \log n)$. Memory requirements are $O(n)$.

4.2.4. DaoSwap variant. The mechanics for DaoSwap are similar to those above, with the primary difference being that an auction is used to determine a single clearing price. Variations in prioritization rules also possible.

5. FORMULATION OF THE GENERAL DAO\$WAP PROBLEM

The general problem of determining the token equilibrium price and the allocation among the swap participants can be formulated in terms of the inequalities on quantities and prices corresponding to the orders and on the need that the total quantity exchanged of each token be preserved across the swaps, i.e., the quantity bought for each token is equal to the quantity sold.

5.1. Limit order inequalities. Let $\{o_i\}_{i=1}^n$ be a set of limit orders, each of the form

$$o_i = (a_i, q_i, \text{base_token}_i, p_i, \text{quote_token}_i)$$

where the quote token is implicit in the quantity q_i and the quote and base tokens are implicit in the limit price p_i . To encode directionality in the limit price inequality, we introduce

$$\mathcal{I}(a) := \begin{cases} +1 & \text{for } a = \text{buy} \\ -1 & \text{for } a = \text{sell} \end{cases}$$

For each i , define

$$c_i := \left(p_{\text{quote_token}_i \text{ per } \text{base_token}_i}^* \leq \mathcal{I}(a_i) \cdot p_i \right)$$

where $p_{\text{quote_token}_i \text{ per } \text{base_token}_i}^*$ is the unique clearing price for the indicated base token/quote token ordered pair. TODO(Paul): revive the “to token” notation. Then, each limit order o_i may be expressed as

$$c_i \wedge (q_i^* \leq q_i) \vee (q_i^* = 0)$$

5.2. No arbitrage. Let T denote the intersection of the union of all base tokens and the union of all quote tokens:

$$T := (\cup_i \text{base_token}_i) \cap (\cup_i \text{quote_token}_i)$$

The set of tokens T is the set of tokens eligible for swapping. We require the following *no arbitrage* constraints be satisfied for all tokens $u, v, w \in T$ involved in a swap:

$$p_{u \text{ per } v}^* \cdot p_{v \text{ per } w}^* = p_{u \text{ per } w}^*$$

Note that, in the case $w = u$, this enforces a unique exchange rate between a pair of tokens (changing the roles of the base and quote tokens inverts the exchange rate).

5.3. Conservation of exchanged value. For each order i , let \tilde{q}_i^* denote the quantity of quote_token_i exchanged. Then we must have

$$\tilde{q}_i^* = q_i^* \cdot p_{\text{quote_token}_i \text{ per } \text{base_token}_i}^*$$

for each i . In effect, this enforces the market clearing price for each order.

5.4. Conservation of exchanged tokens. Assume by convention that a buy order removes base tokens from the system and provides quote tokens to the system, whereas a sell order does the opposite.

For each token $u \in T$, define the indicator function $\mathcal{T} : T \rightarrow \{0, 1\}$ via

$$\mathcal{T}_u(v) = \begin{cases} 1 & \text{if } v = u \\ 0 & \text{if } v \neq u \end{cases}$$

Then, for each $u \in T$, the following conservation law must hold:

$$\sum_i \mathcal{T}_u(\text{base_token}_i) \cdot q_i^* = - \sum_i \mathcal{T}_u(\text{quote_token}_i) \cdot \tilde{q}_i^*$$

Note that this translates into one equality per token participating in the swap. Note also that this is a global constraint on the swap rather than local to each order; in particular, it expresses the notion that each filled order must have, across the collection of orders, suitable counterparties.

5.5. Combining the constraints. We now collect the constraints that must be satisfied:

$$\begin{cases} c_i \wedge (q_i^* \leq q_i) \vee (q_i^* = 0) & \forall i \in \{1, \dots, n\} \\ \tilde{q}_i^* = q_i^* \cdot p_{\text{quote_token}_i\text{-per_base_token}_i}^* & \forall i \in \{1, \dots, n\} \\ p_{u\text{-per-}v}^* \cdot p_{v\text{-per-}w}^* = p_{u\text{-per-}w}^* & \forall u, v, w \in T \\ \sum_{j=1}^n \mathcal{T}_u(\text{base_token}_j) \cdot q_j^* = - \sum_{j=1}^n \mathcal{T}_u(\text{quote_token}_j) \cdot \tilde{q}_j^* & \forall u \in T \end{cases} \quad (2)$$

5.6. Solution of the general DaoSwap problem. The general solution of the DaoSwap problem must satisfy a set of non-linear, combinatorial constraints, whose solution is NP-complete.

It can still be solved in an effective way with solvers like ...

TODO(gp): explain better, cite

5.7. Reformulating the DaoCross problem. In the DaoCross set-up, the price of the tokens involved in the swap is determined by an external oracle. This allows simplifying the general problem by applying the equivalence principle between orders and removing the non-linearity from the set of inequalities above.

In fact, the actual prices p^* compared to the limit price verifies or not the boolean condition c_i , which then allows specifying the quantity constraints:

$$\begin{cases} \text{cond}_i := \text{to_ineq}(a(o_i))(p_{\text{base}}(o_i) \leq lp_{\text{base}}(o_i)p_{\text{quote}}(o_i)) \\ 0 \leq q_{\text{base}}(o_i) \leq q_{\text{base}} \text{ if } \text{cond}_i \\ q_{\text{base}}(o_i) = 0 \text{ if } \neg \text{cond}_i \end{cases} \quad (3)$$

The problem then becomes a set of linear inequalities that can be solved with various efficient methods (e.g., simplex-method).

TODO(gp): check

6. ALTERNATIVE SOLUTIONS FOR TOKEN SWAPPING

TODO(gp): Some of the text should be in the abstract / intro.

DaoSwap is a new approach to token swapping that brings together advantages of efficiency from centralized limit-order book approaches and transparency and self-custody of decentralized approaches.

Exchanges are ubiquitous in financial systems and create value by bringing together different types of participants (e.g., investors, traders, brokers, arbitrageurs, market makers).

Typically an exchange (or swap) requires two components:

- Trade matching: participants find a counterparty agreeing on quantity and price of the assets to swap (this includes price discovery)
- Trade settlement: the assets are actually swapped after finding matching counterparties

In decentralized finance a token swap can be accomplished in different ways, e.g., using

- a centralized exchange (e.g., Coinbase, Binance, OkX)
- a decentralized exchange (e.g., Uniswap, SushiSwap)

Centralized exchanges are usually organized around different versions of a central limit order book (CLOB)

Decentralized exchanges can be organized as

- off-chain order books (e.g., 1inch)
- on-chain order books (e.g., SwapSwap, KyberSwap)

- automatic market makers (AMM) (e.g., Uniswap)

Any off-chain solution incurs into custodial and censorship issues, that makes that solution untenable from the goals of decentralized and self-determination.

Off-chain order books are an hybrid version of CEX and DEX where the matching

On-chain order book matching has the advantages of being custodial, since users are completely in charge of their funds

Running an exchange is extremely profitable since the business model of transaction fees.

TODO(gp): Add colors: in green what's good, yellow, red

TODO(gp): Reorg, maybe from best to worst? DaoSwap, LOB, DaoCross, AMM

	<i>LOB</i>	<i>AMM</i>	<i>DaoSwap</i>	<i>DaoCross</i>
Custodial risk	Yes	No	No	No
Transparency	Low	High	High	Medium
Censorship resistant	No	Yes	Yes	No
Intrinsic efficiency	Medium	Low	High	Medium
Support for limit order	Yes	No	Yes	Yes
Risk of predatory behaviors	High	High	Low	Medium
Impermanent loss	No	Yes	No	No
Multi-coin exchange support	Low	Medium	High	High

6.1. DaoSwap.

- Custodial risk: Low
 - Private keys and funds are always under the control of the users
- Transparency: High
 - Work is done on-chain and the off-chain computations are independently verifiable
- Censorship resistance: Yes
 - The application runs on-chain and even if the web front-end is attacked or disabled. The off-chain computation can be made robust using similar approaches to distributed oracles, or even ported on-chain if gas prices are not an issue (e.g., using Layer2 solutions)
- Intrinsic efficiency: High
 - DaoSwap relies on periodic auctions and it has been argued in previous literature that for small enough intervals (e.g., seconds) periodic auctions result in the same quality of continuous matching without allowing predatory behaviors
 - TODO(gp): Add a reference to Cramton paper
- Support for limit order: Yes
 - Limit orders are supported natively
- Risk of predatory behaviors: Low
 - TODO(gp):
- Impermanent loss: No
 - TODO(gp):
- Support for multi-coin swap: Yes
 - TODO(gp): DaoSwap/Cross pools all the liquidity in a single optimization problem

6.2. DaoCross.

- Custodial risk: Low
 - Same as DaoSwap
- Transparency: Medium
 - DaoCross relies on price discovery carried out on a lit exchange

- Censorship resistance: No
 - The robustness of DaoCross is the same as the lit exchange that it relies on. See comments on the corresponding topic for LOB
- Intrinsic efficiency: Medium
 - The efficiency in matching trades is the same as the limit-order book. See comments on the corresponding topic for LOB
- Support for limit order: Yes
 - Same as DaoSwap
- Risk of predatory behaviors:
 - TODO(gp)
- Impermanent loss: No
 - TODO(gp)
- Support for multi-coin swap:
 - TODO(gp): DaoSwap/Cross pools all the liquidity in a single optimization problem

6.3. Limit Order Book (LOB).

- Custodial risk: High
 - Exchanges own user private keys and funds, creating issues with fraud (e.g., FTX) and hacking (Mt. Gox)
- Transparency: Low
 - Many centralized exchanges can easily exaggerate volume (e.g., through wash trading) to attract ICOs and liquidity
 - There is no guarantee on the
- Censorship resistance: No
 - Users need to link their bank account to CEX and transfer funds, which can take 2-3 days and be subjected to Know-Your-Customer (KYC) and AML (Anti money laundering policies)
 - Governments can censor, interfere with, or even sever the connection between traditional and decentralized finance (see recent interventions of SEC against Coinbase and Binance, and FDIC rescue of Silvergate Bank, Signature Bank)
- Intrinsic efficiency: Medium
 - Limit order books are considered efficient in matching continuously trades, although recently researchers have criticized their time-continuous-time nature as source of latency arbitrage (TODO(gp): Add ref to Compton, IEX, Flash Boys)
 - The same token can be traded on different exchanges causing liquidity and price discovery to be fragmented with detriment on market quality
- Support for limit order: Yes
 - Limit order books naturally support limit orders from users
 - Risk of predatory behaviors: High
 - LOB operators often welcome and incentivize high-frequency traders as way to increase trading revenues and liquidity, at expense of predatory tactics (e.g., front-running, latency arbitrage, spoofing, sniping) which arm
- Impermanent loss: No
 - LOB
- Support for multi-coin swap: No

6.4. Automatic Market Makers (AMM).

- Custodial risk: Low
 - Private keys and the

- Transparency:
- Censorship risk:
- Intrinsic efficiency: Low
 - Arbitrageurs are needed to keep liquidity pool and prices in alignment with other centers for price discovery
- Support for limit order: No
 - Uniswap V3 doesn't support limit orders directly but only in terms of
- Risk of predatory behaviors: High
 - TODO(gp):
- Impermanent loss: Yes
 - Uniswap is affected by impermanent loss because liquidity providers and liquidity takers are in general different users operating at different time scales
- Support for multi-coin swap: Medium
 - Uniswap requires multiple swaps and fees for arbitrary tokens

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