Trigonometric Functions

Question1

Let the set of all a ∈ R such that the equation cos2 x + a sin x = 2a − 7 has a solution be [p, q] and

$r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ} + \tan 81^\circ$, then pqr is equal to

[27-Jan-2024 Shift 1]

Answer: 48

Solution:

$\cos 2x + a \cdot \sin x = 2a - 7$

$a(\sin x - 2) = 2(\sin x - 2)(\sin x + 2)$

$\sin x = 2,\ a = 2(\sin x + 2)$

$\Rightarrow a \in [2, 6]$

$p = 2 \quad q = 6$

$r = \tan 9^\circ + \cot 9^\circ - \tan 27 - \cot 27$

$r = \frac{1}{\sin 9 \cdot \cos 9} - \frac{1}{\sin 27 \cdot \cos 27}$

$= 2\left[\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right]$

$r = 4$

$p \cdot q \cdot r = 2 \times 6 \times 4 = 48$

Question2

If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in the interval [0, nπ/2], for the least value of n ∈ N then $\sum_{k=1}^{n} \frac{k}{2^k}$ is equal to :

[27-Jan-2024 Shift 2]

Options:

A.

$\frac{1}{2^{15}}(2^{14}-14)$

B.

$\frac{1}{2^{14}}(2^{15}-15)$

C.

$1-\frac{15}{2^{13}}$

D.

$\frac{1}{2^{13}}(2^{14}-15)$

Answer: D

Solution:

$2\tan^2\theta-5\,sec\theta-1=0$

$\Rightarrow 2\,sec^2\theta-5\,sec\theta-3=0$

$\Rightarrow (2\,sec\theta+1)(\,sec\theta-3)=0$

$\Rightarrow\ sec\theta=-\frac{1}{2}, 3$

$\Rightarrow \cos\theta=-2,\ \frac{1}{3}$

$\Rightarrow \cos\theta=\ \frac{1}{3}$

For 7 solutions $\mathbf{n}=13$

So, $\sum_{k=1}^{13}\frac{\mathbf{k}}{2^{\mathbf{k}}}=S$ (say)

$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+\ldots+\frac{13}{2^{13}}$

$\frac{1}{2}S=\frac{1}{2^2}+\frac{1}{2^3}+\ldots.+\frac{12}{2^{13}}+\frac{13}{2^{14}}$

$\Rightarrow \frac{S}{2}=\frac{1}{2}\cdot\frac{1-\frac{1}{2^{13}}}{1-\frac{1}{2}}-\frac{13}{2^{14}}\Rightarrow S=2\cdot\left(\frac{2^{13}-1}{2^{13}}\right)-\frac{13}{2^{13}}$

Question3

If α, $-\frac{\pi}{2}<\alpha<\frac{\pi}{2}$ is the solution of 4cosθ + 5 sinθ = 1, then the value of tanα is

[29-Jan-2024 Shift 1]

Options:

A.

$$\frac{10-\sqrt{10}}{6}$$

B.

$$\frac{10-\sqrt{10}}{12}$$

C.

$$\frac{\sqrt{10}-10}{12}$$

D.

$$\frac{\sqrt{10}-10}{6}$$

Answer: C

Solution:

$4+5\tan\theta = sec\theta$

Squaring : $24\tan^2\theta+40\tan\theta+15=0$

$\tan\theta = \dfrac{-10\pm\sqrt{10}}{12}$

and $\tan\theta = -\left(\dfrac{10+\sqrt{10}}{12}\right)$ is Rejected.

(3) is correct.

Question4

The sum of the solutions x ∈ ℝ of the equation $\dfrac{3\cos 2x+\cos^3 2x}{\cos^6 x-sin^6 x}=x^3-x^2+6$ is

[29-Jan-2024 Shift 2]

Options:

A.

0

B.

1

C.

-1

D.

3

Answer: C

Solution:

$$\frac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin^6 x}=x^3-x^2+6$$

$$\Rightarrow \frac{\cos 2x(3+\cos^2 2x)}{\cos 2x(1-\sin^2 x\cos^2 x)}=x^3-x^2+6$$

$$\Rightarrow \frac{4(3+\cos^2 2x)}{(4-\sin^2 2x)}=x^3-x^2+6$$

$$\Rightarrow \frac{4(3+\cos^2 2x)}{(3+\cos^2 2x)}=x^3-x^2+6$$

$$x^3-x^2+2=0\Rightarrow (x+1)(x^2-2x+2)=0$$

so, sum of real solutions $=-1$

Question5

If $2\sin^3 x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $[0, n\pi/2]$, $n \in N$, then the roots of the equation

$x^2 + nx + (n - 3) = 0$ belong to :

[30-Jan-2024 Shift 1]

Options:

A.

$(0, \infty)$

B.

$(-\infty, 0)$

C.

$\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$

D.

Z

Answer: B

Solution:

$2sin^3x + 2sin\,x \cdot \cos^2x + 4sin\,x - 4 = 0$

$2sin^3x + 2sin\,x \cdot (1 - sin^2x) + 4sin\,x - 4 = 0$

$6sin\,x - 4 = 0$

$sin\,x = \frac{2}{3}$

$n = 5$ (in the given interval)

$x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{17}}{2}$

$n = 5$ (in the given interval)

$x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{17}}{2}$

Required interval $(-\infty, 0)$

Question6

The number of solutions of the equation $4\sin^2x - 4\cos^3x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is :

[1-Feb-2024 Shift 2]

Options:

A.

1

B.

3

C.

2

D.

0

Answer: D

Solution:

$4sin^2x - 4\cos^3x + 9 - 4\cos x = 0; x \in [-2\pi, 2\pi]$

$4 - 4\cos^2x - 4\cos^3x + 9 - 4\cos x = 0$

$4\cos^3x + 4\cos^2x + 4\cos x - 13 = 0$

$4\cos^3x + 4\cos^2x + 4\cos x = 13$

L.H.S. ≤ 12 can't be equal to 13.

Question7

For α, β ∈ (0, π/2), let 3sin (α + β) = 2sin (α − β) and a real number k be such that tanα = k tan β. Then the value of k is equal to :

[30-Jan-2024 Shift 2]

Options:

A.

$-\frac{2}{3}$

B.

-5

C.

2/3

D.

5

Answer: B

Solution:

$3sin\ \alpha\cos\beta + 3sin\ \beta\cos\alpha$

$= 2sin\ \alpha\cos\beta - 2sin\ \beta\cos\alpha$

$5sin\ \beta\cos\alpha = -sin\ \alpha\cos\beta$

$\tan\beta = -\frac{1}{5}\tan\alpha$

$\tan\alpha = -5\tan\beta$

Question8

If $\tan A = \frac{1}{\sqrt{x(x^2+x+1)}}, \tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and

$\tan C = (x^{-3}+x^{-2}+x^{-1})^{\frac{1}{2}}, 0 < A, B, C < \frac{\pi}{2}$, then $A+B$ is equal to :

[1-Feb-2024 Shift 1]

Options:

A.

C

B.

π − C

C.

2π − C

D.

π/2 −C

Answer: A

Solution:

Finding $\tan(A+B)$ we get

$\Rightarrow \tan(A+B) =$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{\sqrt{x(x^2+x+1)}} + \frac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1 - \frac{1}{x^2+x+1}}$$

$$\Rightarrow \tan(A+B) = \frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{(x^2+x)(\sqrt{x})}$$

$$\frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{(x^2+x)(\sqrt{x})}$$

$$\tan(A+B) = \frac{\sqrt{x^2+x+1}}{x\sqrt{x}} = \tan C$$

$$A + B = C$$

Question9

If m and n respectively are the numbers of positive and negative value of θ in the interval [−π, π] that satisfy the equation $\cos 2\theta \cos \frac{\theta}{2} = \cos 3\theta \cos \frac{9\theta}{2}$, then mn is equal to ________.
[25-Jan-2023 Shift 2]

Answer: 25

Solution:

$$\cos 2\theta \cdot \cos \frac{\theta}{2} = \cos 3\theta \cdot \cos \frac{9\theta}{2}$$
$$\Rightarrow 2\cos 2\theta \cdot \cos \frac{\theta}{2} = 2\cos \frac{9\theta}{2} \cdot \cos 3\theta$$

$\Rightarrow \cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} = \cos\frac{15\theta}{2} + \cos\frac{3\theta}{2}$

$\Rightarrow \cos\frac{15\theta}{2} = \cos\frac{5\theta}{2}$

$\Rightarrow \frac{15\theta}{2} = 2k\pi \pm \frac{5\theta}{2}$

$5\theta = 2k\pi$ or $10\theta = 2k\pi$

$\theta = \frac{2k\pi}{5}$

$\therefore \theta = \left\{ -\pi, \frac{-4\pi}{5}, \frac{-3\pi}{5}, \frac{-2\pi}{5}, \frac{-\pi}{5}, 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi \right\}$

$\therefore m \cdot n = 25$

Question10

Let $S = \{\theta \in [0, 2\pi) : \tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0\}$.

Then $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to

[24-Jan-2023 Shift 2]

Answer: 2

Solution:

Solution:

$\tan(\pi\cos\theta) + \tan(\pi\sin\theta) = 0$

$\tan(\pi\cos\theta) = -\tan(\pi\sin\theta)$

$\tan(\pi\cos\theta) = \tan(-\pi\sin\theta)$

$\pi\cos\theta = n\pi - \pi\sin\theta$

$\sin\theta + \cos\theta = n$ where $n \in I$

possible values are $n = 0$, 1 and -1 because

$-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$

Now it gives $\theta \in \left\{ 0, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{2}, \pi \right\}$

So $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right) = 2(0) + 4\left(\frac{1}{2}\right) = 2$

Question11

Let $f(\theta) = 3\left(\sin^4\left(\frac{3\pi}{2} - \theta\right) + \sin^4(3\pi + \theta)\right) - 2(1 - \sin^2 2\theta)$ and

$S = \left\{ \theta \in [0, \pi] : f'(\theta) = -\frac{\sqrt{3}}{2} \right\}$. If $4\beta = \sum_{\theta \in S} \theta$, then $f(\beta)$ is equal to

[29-Jan-2023 Shift 1]

Options:

A. $\frac{11}{8}$

B. $\frac{5}{4}$

C. $\frac{9}{8}$

D. $\frac{3}{2}$

Answer: B

Solution:

Solution:

$f(\theta)=3\left(\sin^4\left(\frac{3\pi}{2}-\theta\right)+\sin^4(3x+\theta)\right)-2(1-\sin^2 2\theta)$

$S=\left\{\theta\in[0,\pi]:f'(\theta)=-\frac{\sqrt{3}}{2}\right\}$

$\Rightarrow f(\theta)=3(\cos^4\theta+\sin^4\theta)-2\cos^2 2\theta$

$\Rightarrow f(\theta)=3\left(1-\frac{1}{2}\sin^2 2\theta\right)-2\cos^2 2\theta$

$\Rightarrow f(\theta)=3-\frac{3}{2}\sin^2 2\theta-2\cos^2\theta$

$=\frac{3}{2}-\frac{1}{2}\cos^2 2\theta=\frac{3}{2}-\frac{1}{2}\left(\frac{1+\cos 4\theta}{2}\right)$

$f(\theta)=\frac{5}{4}-\frac{\cos 4\theta}{4}$

$f'(\theta)=\sin 4\theta$

$\Rightarrow f'(\theta)=\sin 4\theta=-\frac{\sqrt{3}}{2}$

$\Rightarrow 4\theta=n\pi+(-1)^n\frac{\pi}{3}$

$\Rightarrow \theta=\frac{n\pi}{4}+(-1)^n\frac{\pi}{12}$

$\Rightarrow \theta=\frac{\pi}{12},\left(\frac{\pi}{4}-\frac{\pi}{12}\right),\left(\frac{\pi}{2}+\frac{\pi}{12}\right),\left(\frac{3\pi}{4}-\frac{\pi}{12}\right)$

$\Rightarrow 4\beta=\frac{\pi}{4}+\frac{\pi}{2}+\frac{3\pi}{4}=\frac{3\pi}{2}$

$\Rightarrow \beta=\frac{3\pi}{8}\Rightarrow f(\beta)=\frac{5}{4}-\frac{\cos\frac{3\pi}{2}}{4}=\frac{5}{4}$

Question12

The set of all values of λ for which the equation
$\cos^2 2x-2\sin^4 x-2\cos^2 x=\lambda$
[29-Jan-2023 Shift 2]

Options:

A. $[-2,-1]$

B. $\left[-2,-\frac{3}{2}\right]$

C. $\left[-1,-\frac{1}{2}\right]$

D. $\left[-\frac{3}{2},-1\right]$

Answer: D

Solution:

Solution:

$\lambda = \cos^2 2x - 2\sin^4 x - 2\cos^2 x$

convert all in to $\cos x$.

$\lambda = (2\cos^2 x - 1)^2 - 2(1 - \cos^2 x)^2 - 2\cos^2 x$

$= 4\cos^4 x - 4\cos^2 x + 1 - 2(1 - 2\cos^2 x + \cos^4 x) -$
$2\cos^2 x$

$= 2\cos^4 x - 2\cos^2 x + 1 - 2$

$= 2\cos^4 x - 2\cos^2 x - 1$

$= 2\left[\cos^4 x - \cos^2 x - \frac{1}{2}\right]$

$= 2\left[\left(\cos^2 x - \frac{1}{2}\right)^2 - \frac{3}{4}\right]$

$\lambda_{max} = 2\left[\frac{1}{4} - \frac{3}{4}\right] = 2 \times \left(-\frac{2}{4}\right) = -1$ (max Value)

$\lambda_{min} = 2\left[0 - \frac{3}{4}\right] = -\frac{3}{2}$ (MinimumValue)

So, Range $= \left[-\frac{3}{2}, -1\right]$

Question13

If $\tan 15^\circ + \frac{1}{\tan 75^\circ} + \frac{1}{\tan 105^\circ} + \tan 195^\circ = 2a$ then the value of $\left(a + \frac{1}{a}\right)$ is : [30-Jan-2023 Shift 1]

Options:

A. 4

B. $4 - 2\sqrt{3}$

C. 2

D. $5 - \frac{3}{2}\sqrt{3}$

Answer: A

Solution:

Solution:

Option (1)

$\tan 15^\circ = 2 - \sqrt{3}$

$\frac{1}{\tan 75^\circ} = \cot 75^\circ = 2 - \sqrt{3}$

$\frac{1}{\tan 105^\circ} = \cot(105^\circ) = -\cot 75^\circ = \sqrt{3} - 2$

$\tan 195^\circ = \tan 15^\circ = 2 - \sqrt{3}$

$\therefore\ 2(2 - \sqrt{3}) = 2a \Rightarrow a = 2 - \sqrt{3}$

$\Rightarrow\ a + \frac{1}{a} = 4$

Question14

If the solution of the equation

$\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1, x \in \left(0, \frac{\pi}{2}\right)$, is $\sin^{-1}\left(\frac{\alpha+\sqrt{\beta}}{2}\right)$, where α, β are integers, then $\alpha + \beta$ is equal to:
[30-Jan-2023 Shift 1]

Options:

A. 3

B. 5

C. 6

D. 4

Answer: D

Solution:

Solution:

$\log_{\cos x} \cot x + 4\log_{\sin x} \tan x = 1$

$\Rightarrow \frac{\ln \cos x - \ln \sin x}{\ln \cos x} + 4\frac{\ln \sin x - \ln \cos x}{\ln \sin x} = 1$

$\Rightarrow (\ln \sin x)^2 - 4(\ln \sin x)(\ln \cos x) + 4(\ln \cos x)^2 = 1$

$\Rightarrow \ln \sin x = 2\ln \cos x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0 \Rightarrow \sin x = \frac{-1+\sqrt{5}}{2}$

$\therefore \alpha + \beta = 4$

Correct option (4)

Question15

The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ is _______ :
[6-Apr-2023 shift 2]

Answer: 4

Solution:

Solution:

$(\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)$

$\frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin 27^\circ \cos 27^\circ}$

$\frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$

$\frac{2(4)}{\sqrt{5}-1} - \frac{2(4)}{(\sqrt{5}+1)}$

$\frac{8(\sqrt{5}+1)}{4} - \frac{8(\sqrt{5}-1)}{4}$

$2[(\sqrt{5}+1) - (\sqrt{5}-1)]$

$= 4$

Question16

The value of $36(4\cos^2 9^\circ - 1)(4\cos^2 27^\circ - 1)(4\cos^2 81^\circ - 1)(4\cos^2 243^\circ - 1)$ is
[8-Apr-2023 shift 2]

Options:

A. 27

B. 54

C. 18

D. 36

Answer: D

Solution:

Solution:
$4\cos^2\theta - 1 = 4(1 - \sin^2\theta) - 1 = 3 - 4\sin^2\theta = \frac{\sin 3\theta}{\sin\theta}$
so given expression can be written as
$$36\times\frac{\sin 27^\circ}{\sin 9^\circ}\times\frac{\sin 81^\circ}{\sin 27^\circ}\times\frac{\sin 243^\circ}{\sin 81^\circ}\times\frac{\sin 729^\circ}{\sin 243^\circ}$$
$$36\times\frac{\sin 729^\circ}{\sin 9^\circ} = 36$$

Question17

$96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ is equal to :
[10-Apr-2023 shift 1]

Options:

A. 4

B. 2

C. 3

D. 1

Answer: C

Solution:

Solution:
$$96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{2^2\pi}{33}\cos\frac{2^3\pi}{33}\cos\frac{2^4\pi}{33}$$
$$\because \cos A\cos 2A\cos 2^2A\ldots\cos 2^{n-1}A = \frac{\sin(2^nA)}{2^n\sin A}$$
Here $A = \frac{\pi}{33}$, $n = 5$

$$= \frac{96\sin\left(2^5\frac{\pi}{33}\right)}{2^5\sin\left(\frac{\pi}{33}\right)}$$

$$= \frac{96\sin\left(\frac{32\pi}{33}\right)}{32\sin\left(\frac{\pi}{33}\right)}$$

$$= \frac{3\sin\left(\pi - \frac{\pi}{33}\right)}{\sin\left(\frac{\pi}{33}\right)} = 3$$

Question18

Let S = $\left\{ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) : 9^{1-\tan^2 x} + 9^{\tan^2 x} = 10 \right\}$ and b = $\sum_{x \in S\tan}^{2}\left(\frac{x}{3}\right)$, then $\frac{1}{6}(\beta - 14)^2$ is equal to

[10-Apr-2023 shift 2]

Options:

A. 16

B. 32

C. 8

D. 64

Answer: B

Solution:

Solution:

Let $9^{\tan^2 x} = P$

$\frac{9}{P} + P = 10$

$P^2 - 10P + 9 = 0$

$(P - 9)(P - 1) = 0$

$P = 1, 9$

$9^{\tan^2 x} = 1, 9^{\tan^2 x} = 9$

$\tan^2 x = 0, \tan^2 x = 1$

$x = 0, \pm\frac{\pi}{4} \quad \therefore x \in \left(-\frac{\pi}{2}, \frac{p}{2}\right)$

$\beta = \tan^2(0) + \tan^2\left(+\frac{\pi}{12}\right) + \tan^2\left(-\frac{\pi}{12}\right)$

$= 0 + 2(\tan 15^\circ)^2$

$2(2 - \sqrt{3})^2$

$2(7 - 4\sqrt{3})$

Than $\frac{1}{6}(14 - 8\sqrt{3} - 14)^2 = 32$

Question19

The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is :

[25-Jul-2022-Shift-1]

Options:

A. 4

B. 6

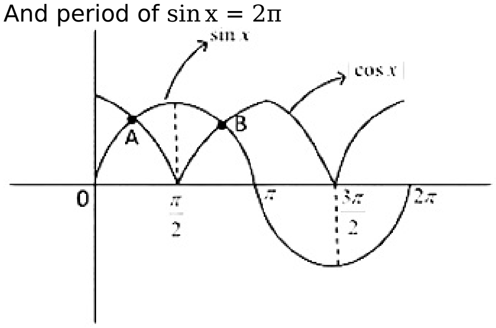
C. 8

D. 12

Answer: C

Solution:

Solution:
Period of $|\cos x| = \pi$
And period of $\sin x = 2\pi$



Graph of $\sin x$ and $|\cos x|$ cuts each other at two points A and B in $[0, 2\pi]$
So, in $[-4\pi, 4\pi]$, total 4 similar graph will be present and graph of $\sin x$ and $|\cos x|$ will cut $4 \times 2 = 8$ times.
$\therefore$ Total possible solutions $= 8$

Question20

Let $S = \left\{ \theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16 \right\}$. Then $n(s) + \sum_{\theta \in S} \left(\sec\left(\frac{\pi}{4} + 2\theta \right) \operatorname{cosec}\left(\frac{\pi}{4} + 2\theta \right) \right)$ is equal to:
[26-Jul-2022-Shift-1]

Options:

A. 0

B. -2

C. -4

D. 12

Answer: C

Solution:

Solution:
$S = \left\{ \theta \in [0, 2\pi] : 8^{2\sin^2\theta} + 8^{2\cos^2\theta} = 16 \right\}$
Now apply AM $\geq$ GM for $8^{2\sin^2\theta}$, $8^{2\cos^2\theta}$

$\frac{8^{2\sin^2\theta}+8^{2\cos^2\theta}}{2} \geq \left(8^{2\sin^2\theta+2\cos^2\theta}\right)\frac{1}{2}$

$8 \geq 8$

$\Rightarrow 8^{2\sin^2\theta} = 8^{2\cos^2\theta}$

or $\sin^2\theta = \cos^2\theta$

$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$n(S) + \sum_{\theta \in S} \sec\left(\frac{\pi}{4}+2\theta\right) \operatorname{cosec}\left(\frac{\pi}{4}+2\theta\right)$

$4 + \sum_{\theta \in S} \frac{2}{2\sin\left(\frac{\pi}{4}+2\theta\right)\cos\left(\frac{\pi}{4}+2\theta\right)}$

$= 4 + \sum_{\theta \in S} \frac{2}{\sin\left(\frac{\pi}{2}+4\theta\right)} = 4 + 2\sum_{\theta \in S} \operatorname{cosec}\left(\frac{\pi}{2}+4\theta\right)$

$= 4 + 2\left[\operatorname{cosec}\left(\frac{\pi}{2}+\pi\right) \operatorname{cosec}\left(\frac{\pi}{2}+3\pi\right) + \operatorname{cosec}\left(\frac{\pi}{2}+5\pi\right) + \operatorname{cosec}\left(\frac{\pi}{2}+7\pi\right)\right]$

$= 4 + 2\left[-\operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2} - \operatorname{cosec}\frac{\pi}{2}\right]$

$= 4 - 2(4)$

$= 4 - 8$

$= -4$

Question21

If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to ________.

[26-Jul-2022-Shift-2]

Answer: 3

Solution:

Equation (1) $2\sin^2\theta = 1 - 2\sin^2\theta$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \pm\frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

Equation (2) $2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow 2\sin^2\theta - 4\sin\theta + \sin\theta - 2 = 0$

$\Rightarrow (\sin\theta - 2)(2\sin\theta + 1) = 0$

$\Rightarrow \sin\theta = \frac{-1}{2}$

$\Rightarrow \theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\therefore$ Common solutions $= \frac{7\pi}{6}; \frac{11\pi}{6}$

Sum of solutions $= \frac{7\pi + 11\pi}{6} = \frac{18\pi}{6} = 3\pi$

$\therefore k = 3$

Question22

Let $S = \{\theta \in (0, 2\pi) : 7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2\}$. Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$, $\theta \in S$, is ______.
[29-Jul-2022-Shift-1]

Answer: 16

Solution:

Solution:
$7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$
$\Rightarrow 4\left(\frac{1+\cos 2\theta}{2}\right) + 3\cos 2\theta - 2\cos^2 2\theta = 2$
$\Rightarrow 2 + 5\cos^2\theta - 2\cos^2 2\theta = 2$
$\Rightarrow \cos 2\theta = 0$ or $\frac{5}{2}$(rejected)
$\Rightarrow \cos 2\theta = 0 = \frac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \tan^2\theta = 1$
$\therefore$ Sum of roots $= 2(\tan^2\theta + \cot^2\theta) = 2 \times 2 = 4$
But as $\tan\theta = \pm 1$ for $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ in the interval $(0, 2\pi)$
$\therefore$ Four equations will be formed
Hence sum of roots of all the equations $= 4 \times 4 = 16$.

Question23

$2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to :
[25-Jul-2022-Shift-2]

Options:

A. $\frac{3}{16}$

B. $\frac{1}{16}$

C. $\frac{1}{32}$

D. $\frac{9}{32}$

Answer: B

Solution:

Solution:
$2\sin\frac{\pi}{22}\sin\frac{3\pi}{22}\sin\frac{5\pi}{22}\sin\frac{7\pi}{22}\sin\frac{9\pi}{22}$

$= 2\sin\left(\frac{11\pi - 10\pi}{22}\right)\sin\left(\frac{11\pi - 8\pi}{22}\right)\sin\left(\frac{11\pi - 6\pi}{22}\right)\sin\left(\frac{11\pi - 4\pi}{22}\right)\sin\left(\frac{11\pi - 2\pi}{22}\right)$

$= 2\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}\cos\frac{4\pi}{11}\cos\frac{5\pi}{11}$

$= \frac{2\sin\frac{32\pi}{11}}{2^5\sin\frac{\pi}{11}}$

$= \frac{1}{16}$

Question24

Let $S = \left\{\theta \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9}\sec\left(\theta + (m-1)\frac{\pi}{6}\right)\sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}$. Then
[27-Jul-2022-Shift-2]

Options:

A. $S = \left\{\frac{\pi}{12}\right\}$

B. $S = \left\{\frac{2\pi}{3}\right\}$

C. $\sum_{\theta \in S}\theta = \frac{\pi}{2}$

D. $\sum_{\theta \in S}\theta = \frac{3\pi}{4}$

Answer: C

Solution:

Solution:

$s = \left\{0 \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9}\sec\left(\theta + (m-1)\frac{\pi}{6}\right)\sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}$.

$\sum_{m=1}^{9}\frac{1}{\cos\left(\theta + (m-1)\frac{\pi}{6}\right)}\cos\left(\theta + m\frac{\pi}{6}\right)$

$\frac{1}{\sin\left(\frac{\pi}{6}\right)}\sum_{m=1}^{9}\frac{\sin\left[\left(\theta + \frac{m\pi}{6}\right) - \left(\theta + (m-1)\frac{\pi}{6}\right)\right]}{\cos\left(\theta + (m-1)\frac{\pi}{6}\right)\cos\left(\theta + \frac{\pi}{6}\right)}$

$= 2\sum_{m=1}^{9}\left[\tan\left(\theta + \frac{m\pi}{6}\right) - \tan\left(\theta + (m-1)\frac{\pi}{6}\right)\right]$

Now,

$m = 1 \quad 2\left[\tan\left(\theta + \frac{\pi}{6}\right) - \tan(\theta)\right]$

$m = 2 \quad 2\left[\tan\left(\theta + \frac{2\pi}{6}\right) - \tan\left(\theta + \frac{\pi}{6}\right)\right]$

.
.
.

$m = 9 \quad 2\left[\tan\left(\theta + \frac{9\pi}{6}\right) - \tan\left(\theta + 8\frac{\pi}{6}\right)\right]$

$\therefore = 2\left[\tan\left(\theta + \frac{3\pi}{2}\right) - \tan\theta\right] = \frac{-8}{\sqrt{3}}$

$= -2[\cot\theta + \tan\theta] = \frac{-8}{\sqrt{3}}$

$= -\frac{2 \times 2}{2\sin\theta\cos\theta} = \frac{-8}{\sqrt{3}}$

$= \frac{1}{\sin 2\theta} = \frac{2}{\sqrt{3}}$

$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$

$2\theta = \frac{\pi}{3}$

$2\theta = \frac{2\pi}{3}$

$\theta = \frac{\pi}{6}$

$\theta = \frac{\pi}{3}$

$\sum\theta_i = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$

Question25

Let $S = \left[-\pi, \frac{\pi}{2}\right) - \left\{-\frac{\pi}{2}, -\frac{\pi}{4}, -\frac{3\pi}{4}, \frac{\pi}{4}\right\}$. Then the number of elements in the set $|A = \{\theta \in S : \tan\theta(1 + \sqrt{5}\tan(2\theta)) = \sqrt{5} - \tan(2\theta)\}$ is ______.
[28-Jul-2022-Shift-2]

Answer: 5

Solution:

Solution:
Let $\tan\alpha = \sqrt{5}$
$\therefore \tan\theta = \frac{\tan\alpha - \tan 2\theta}{1 + \tan\alpha\tan 2\theta}$
$\therefore \tan\theta = \tan(\alpha - 2\theta)$
$\alpha - 2\theta = n\pi + \theta$
$\Rightarrow 3\theta = \alpha - n\pi$
$\Rightarrow \theta = \frac{\alpha}{3} - \frac{n\pi}{3}$; $n \in Z$
If $\theta \in [-\pi, \pi / 2]$ then $n = 0, 1, 2, 3, 4$ are acceptable
$\therefore$5 solutions.

Question26

Let $S = \left\{\theta \in [-\pi, \pi] - \left\{\pm\frac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}$.
If $T = \sum_{\theta \in S}\cos 2\theta$, then $T + n(S)$ is equal to:
[24-Jun-2022-Shift-1]

Options:

A. $7 + \sqrt{3}$

B. 9

C. $8 + \sqrt{3}$

D. 10

Answer: B

Question27

The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\operatorname{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$ holds, is____
[25-Jun-2022-Shift-1]

Answer: 4

Solution:

$$\frac{14}{\sin^2 x} - 2\sin^2 x = 21 - 4(1 - \sin^2 x)$$

Let $\sin^2 x = t$

$\Rightarrow 14 - 2t^2 = 21t - 4t + 4t^2$

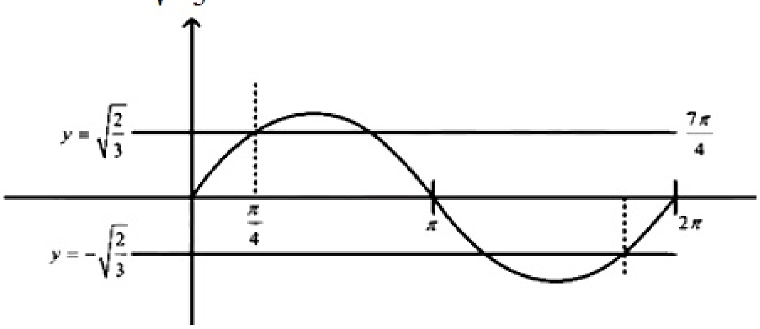
$\Rightarrow 6t^2 + 17t - 14 = 0$

$\Rightarrow 6t^2 + 21t - 4t - 14 = 0$

$\Rightarrow 3t(2t + 7) - 2(2t + 7) = 0$

$\Rightarrow \sin 2x = \frac{2}{3}$ or $-\frac{7}{3}$ (rejected)

$\Rightarrow \sin x = \pm\sqrt{\frac{2}{3}}$



Question28

The number of elements in the set
$S = \{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\}$ is
[29-Jun-2022-Shift-1]

Answer: 32

Solution:

Solution:

$3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$

$3\cos^2 2\theta + 6\cos 2\theta - 5(1 + \cos 2\theta) + 5 = 0$

$3\cos^2 2\theta + \cos 2\theta = 0$

$\text{Cos } 2\theta = 0 \text{ OR } \cos 2\theta = -1/3$

$\theta \in [-4\pi, 4\pi]$

$2\theta = (2n+1)\cdot\frac{\pi}{2}$

$\therefore \theta = \pm\pi/4. \pm 3\pi/4 \ldots\ldots \pm 15\pi/4$

Similarly $\cos 2\theta = -1/3$ gives 16 solution

Question29

The number of solutions of the equation $2\theta - \cos^2\theta + \sqrt{2} = 0$ in R is equal to [29-Jun-2022-Shift-1]

Answer: 1

Solution:

Solution:

Given,

$2\theta - \cos^2\theta + \sqrt{2} = 0$

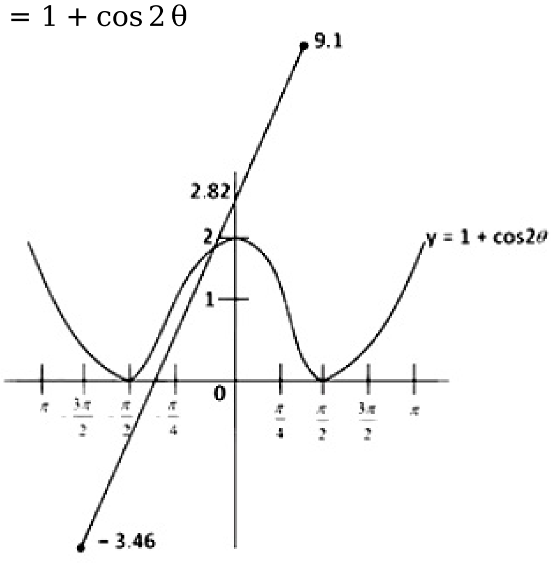
$\Rightarrow 2\theta + \sqrt{2} = \cos^2\theta$

$\Rightarrow 2\theta + \sqrt{2} = \dfrac{1 + \cos 2\theta}{2}$

$\Rightarrow 4\theta + 2\sqrt{2} = 1 + \cos 2\theta = y$ (Assume)

$\therefore y = 4\theta + 2\sqrt{2}$ and

$y = 1 + \cos 2\theta$



For $y = 1 + \cos 2\theta$

when $\theta = 0$, $y = 1 + 1 = 2$

when $\theta = \dfrac{\pi}{4}$, $y = 1 + \cos\dfrac{\pi}{2} = 1$

$\theta = \frac{\pi}{2}$, $y = 1 + \cos\pi = 1 - 1 = 0$

For $y = 4\theta + 2\sqrt{2}$
when $\theta = 0$, $y = 2\sqrt{2}$
when $\theta = \frac{\pi}{2}$, $y = 2\pi + 2\sqrt{2}$

$= 2(\pi + \sqrt{2})$
$= 2(3.14 + 1.41)$
$= 2(4.55)$
$= 9.1$

when $\theta = -\frac{\pi}{2}$, $y = -2\pi + 2\sqrt{2}$

$= 2(-\pi + \sqrt{2})$
$= 2(-3.14 + 1.41)$
$= -3.46$

∴ Two graph cut's at only one point so one solution possible.

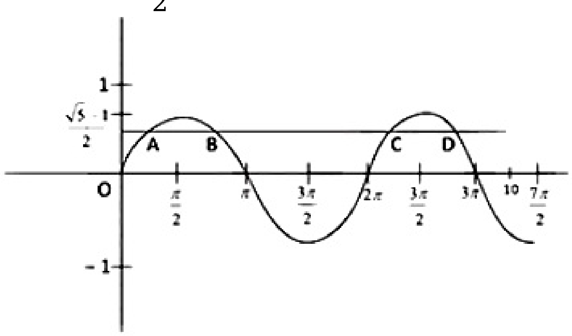
Question30

The number of solutions of the equation $\sin x = \cos^2 x$ in the interval (0, 10) is____
[29-Jun-2022-Shift-2]

Answer: 4

Solution:

Solution:
$\sin x = \cos^2 x$, $x \in (0, 10)$
$\Rightarrow \sin x = 1 - \sin^2 x$
$\Rightarrow \sin^2 x + \sin x - 1 = 0$
$\therefore \sin x = \frac{-1 \pm \sqrt{1+4}}{2}$
$\Rightarrow \sin x = \frac{-1 \pm \sqrt{5}}{2}$
We know $\sin \in (-1, 1)$
$\therefore \frac{-1 - \sqrt{5}}{2}$ can't be a value of $\sin x$
$\therefore \sin x = \frac{\sqrt{5} - 1}{2}$



$3\pi = 3 \times 3.14 = 9.42 < 10$
$\frac{7\pi}{2} = \frac{7}{2} \times 3.14 = 10.99 > 10$
$\therefore 10$ will be in between 3π and $\frac{7\pi}{2}$.
There are 4 intersection at A, B, C and D between $\sin x$ graph and $y = \frac{\sqrt{5} - 1}{2}$ graph.
∴ possible solution $= 4$

Question31

The number of solutions of the equation
$\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{4}\cos^2 2x, x \in [-3\pi, 3\pi]$ is :
[24-Jun-2022-Shift-2]

Options:

A. 8

B. 5

C. 6

D. 7

Answer: D

Solution:

Solution:

$\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right) = \frac{1}{4}\cos^2 2x, x \in [-3\pi, 3\pi]$

$\Rightarrow \cos 2x + \cos\frac{2\pi}{3} = \frac{1}{2}\cos^2 2x$

$\Rightarrow \cos^2 2x - 2\cos 2x - 1 = 0$

$\Rightarrow \cos 2x = 1$

$\therefore x = -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi$

$\therefore$ Number of solutions $= 7$

Question32

The value of $2\sin(12^\circ) - \sin(72^\circ)$ is :
[25-Jun-2022-Shift-2]

Options:

A. $\frac{\sqrt{5}(1-\sqrt{3})}{4}$

B. $\frac{1-\sqrt{5}}{8}$

C. $\frac{\sqrt{3}(1-\sqrt{5})}{2}$

D. $\frac{\sqrt{3}(1-\sqrt{5})}{4}$

Answer: D

Solution:

Solution:

$2\sin 12^\circ - \sin 72^\circ$

$= \sin 12^\circ + (-2\cos 42^\circ \cdot \sin 30^\circ)$

$= \sin 12^\circ - \cos 42^\circ$

$= \sin 12^\circ - \sin 48^\circ$

$= 2\sin 18^\circ \cdot \cos 30^\circ$

$= -2\left(\dfrac{\sqrt{5}-1}{4}\right) \cdot \dfrac{\sqrt{3}}{2}$

$= \dfrac{\sqrt{3}(1-\sqrt{5})}{4}$

Question33

If $\sin^2(10^\circ)\sin(20^\circ)\sin(40^\circ)\sin(50^\circ)\sin(70^\circ) = \alpha - \frac{1}{16}\sin(10^\circ)$, then $16+\alpha^{-1}$ is equal to____
[26-Jun-2022-Shift-1]

Answer: 80

Solution:

Solution:

$(\sin 10^\circ \cdot \sin 50^\circ \cdot \sin 70^\circ) \cdot (\sin 10^\circ \cdot \sin 20^\circ \cdot \sin 40^\circ)$

$= \left(\dfrac{1}{4}\sin 30^\circ\right) \cdot \left[\dfrac{1}{2}\sin 10^\circ(\cos 20^\circ - \cos 60^\circ)\right]$

$= \dfrac{1}{16}\left[\sin 10^\circ\left(\cos 20^\circ - \dfrac{1}{2}\right)\right]$

$= \dfrac{1}{32}[2\sin 10^\circ \cdot \cos 20^\circ - \sin 10^\circ]$

$= \dfrac{1}{32}[\sin 30^\circ - \sin 10^\circ - \sin 10^\circ]$

$= \dfrac{1}{64} - \dfrac{1}{64}\sin 10^\circ$

Clearly, $\alpha = \dfrac{1}{64}$

Hence $16 + \alpha^{-1} = 80$

Question34

$16\sin(20^\circ)\sin(40^\circ)\sin(80^\circ)$ is equal to :
[26-Jun-2022-Shift-2]

Options:

A. $\sqrt{3}$

B. $2\sqrt{3}$

C. 3

D. $4\sqrt{3}$

Answer: B

Solution:

Solution:

$$16 \sin 20^{\circ} \cdot \sin 40^{\circ} \cdot \sin 80^{\circ}$$

$$= 4 \sin 60^{\circ} \{\because 4 \sin \theta \cdot \sin(60^{\circ} - \theta) \cdot \sin(60^{\circ} + \theta) = \sin 3\theta\}$$

$$= 2\sqrt{3}$$

Question35

The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is equal to: [27-Jun-2022-Shift-1]

Options:

A. -1

B. $-\frac{1}{2}$

C. $-\frac{1}{3}$

D. $-\frac{1}{4}$

Answer: B

Solution:

Solution:

$$\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = \frac{\sin 3\left(\frac{\pi}{7}\right)}{\sin\frac{\pi}{7}} \cos\frac{\left(\frac{2\pi}{7} + \frac{6\pi}{7}\right)}{2}$$

$$= \frac{\sin\left(\frac{3\pi}{7}\right) \cdot \cos\left(\frac{4\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)}$$

$$= \frac{2\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}}{2\sin\frac{\pi}{7}}$$

$$= \frac{\sin\left(\frac{3\pi}{7}\right)}{2\sin\frac{\pi}{7}} = \frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}} = \frac{-1}{2}$$

Question36

$\alpha = \sin 36^{\circ}$ is a root of which of the following equation? [27-Jun-2022-Shift-2]

Options:

A. $16x^4 - 10x^2 - 5 = 0$

B. $16x^4 + 20x^2 - 5 = 0$

C. $16x^4 - 20x^2 + 5 = 0$

D. $16x^4 - 10x^2 + 5 = 0$

Answer: C

Solution:

Solution:

$\alpha = \sin 36^{\circ} = x$ (say)

$\therefore x = \frac{\sqrt{10 - 2\sqrt{5}}}{4}$

$\Rightarrow 16x^2 = 10 - 2\sqrt{5}$

$\Rightarrow (8x^2 - 5)^2 = 5$

$\Rightarrow 16x^4 - 80x^2 + 20 = 0$

$\therefore 4x^4 - 20x^2 + 5 = 0$

Question37

If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are : [28-Jun-2022-Shift-2]

Options:

A. $-\frac{1}{7}$ and IV^{th} quadrant

B. 7 and I^{st} quadrant

C. -7 and IV^{th} quadrant

D. $\frac{1}{7}$ and I^{st} quadrant

Answer: A

Solution:

Solution:
$\because \cot\alpha = 1, \quad \alpha \in \left(\pi, \frac{3\pi}{2}\right)$
then $\tan\alpha = 1$
and $\sec\beta = -\frac{5}{3}, \quad \beta \in \left(\frac{\pi}{2}, \pi\right)$
then $\tan\beta = -\frac{4}{3}$
$\therefore \tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$
$= \frac{1 - \frac{4}{3}}{1 + \frac{4}{3}}$
$= -\frac{1}{7}$
$\alpha + \beta \in \left(\frac{3\pi}{2}, 2\pi\right)$ i.e. fourth quadrant

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Question38

$\operatorname{cosec} 18°$ is a root of the equation [31 Aug 2021 Shift 1]

Options:

A. $x^2 + 2x - 4 = 0$

B. $4x^2 + 2x - 1 = 0$

C. $x^2 - 2x + 4 = 0$

D. $x^2 - 2x - 4 = 0$

Answer: D

Solution:

Solution:
$\operatorname{cosec} 18° = \frac{1}{\sin 18°} = \frac{4}{\sqrt{5} - 1} = \sqrt{5} + 1$
If $x = \sqrt{5} + 1$, then
$(x - 1)^2 = 5$
$\Rightarrow x^2 - 2x - 4 = 0$

Question39

The value of $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$ is [26 Aug 2021 Shift 2]

Options:

A. $\frac{1}{4\sqrt{2}}$

B. $\frac{1}{4}$

C. $\frac{1}{8}$

D. $\frac{1}{8\sqrt{2}}$

Answer: C

Solution:

Solution:

$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$= 2\sin\frac{\pi}{8}\sin\frac{2\pi}{8}\sin\frac{3\pi}{8}\sin\left(\pi-\frac{3\pi}{8}\right)\sin\left(\pi-\frac{2\pi}{8}\right)\sin\left(\pi-\frac{\pi}{8}\right)$

$= 2\sin\frac{\pi}{8}\sin\frac{2\pi}{8}\sin\frac{3\pi}{8}\sin\frac{3\pi}{8}\sin\frac{2\pi}{8}\sin\frac{\pi}{8}$

$= 2\sin^2\left(\frac{\pi}{8}\right)\sin^2\left(\frac{2\pi}{8}\right)\sin^2\left(\frac{3\pi}{8}\right)2\sin^2\frac{\pi}{8}\left(\frac{1}{\sqrt{2}}\right)^2\sin^2\left(\frac{\pi}{2}-\frac{\pi}{8}\right)$

$= 2\sin^2\frac{\pi}{8}\times\frac{1}{2}\times\cos^2\frac{\pi}{8} = \sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right)$

$= \frac{1}{4}\left(2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right)^2\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right) = \frac{1}{4}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{8}$

Question40

If $\sin\theta + \cos\theta = \frac{1}{2}$, then16 $(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to: [27 Jul 2021 Shift 1]

Options:

A. 23

B. -27

C. -23

D. 27

Answer: C

Solution:

Solution:

$\sin\theta + \cos\theta = \frac{1}{2}$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{1}{4}$

$\sin 2\theta = -\frac{3}{4}$

Now :

$\cos 4\theta = 1 - 2\sin^2 2\theta$

$= 1 - 2\left(-\frac{3}{4}\right)^2$

$= 1 - 2\times\frac{9}{16} = -\frac{1}{8}$

$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$

$= (3 - 4\sin^2 2\theta) \cdot \sin 2\theta$

$= \left[3 - 4\left(\frac{9}{16}\right)\right] \cdot \left(-\frac{3}{4}\right)$

$\Rightarrow \left[\frac{3}{4}\right] \times \left(-\frac{3}{4}\right) = -\frac{9}{16}$

$16[\sin 2\theta + \cos 4\theta + \sin 6\theta]$

$16\left(-\frac{3}{4} - \frac{1}{8} - \frac{9}{16}\right) = -23$

Question41

The value of $\cot \frac{\pi}{24}$ is:
[25 Jul 2021 Shift 2]

Options:

A. $\sqrt{2} + \sqrt{3} + 2 - \sqrt{6}$

B. $\sqrt{2} + \sqrt{3} + 2 + \sqrt{6}$

C. $\sqrt{2} - \sqrt{3} - 2 + \sqrt{6}$

D. $3\sqrt{2} - \sqrt{3} - \sqrt{6}$

Answer: B

Solution:

Solution:

$\cot\theta = \frac{1 + \cos 2\theta}{\sin 2\theta} = \frac{1 + \left(\frac{\sqrt{3} + 1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right)}$

$\theta = \frac{\pi}{24}$

$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \frac{1 + \left(\frac{\sqrt{3} + 1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right)}$

$= \frac{(2\sqrt{2} + \sqrt{3} + 1)}{(\sqrt{3} - 1)} \times \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)}$

$= \frac{2\sqrt{6} + 2\sqrt{2} + 3 + \sqrt{3} + \sqrt{3} + 1}{2}$

$= \sqrt{6} + \sqrt{2} + \sqrt{3} + 2$

Question42

If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to
[18 Mar 2021 Shift 2]

Options:

A. 350

B. 500

C. 400

D. 250

Answer: D

Solution:

Solution:
Given, $15\sin^4\alpha + 10\cos^4\alpha = 6$
$\Rightarrow\ 15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^2\alpha + \cos^2\alpha)^2$
$\Rightarrow\ 15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^4\alpha + \cos^4\alpha + 2\sin^2\alpha\cos^2\alpha)$
$\Rightarrow 9\sin^4\alpha + 4\cos^4\alpha - 12\sin^2\alpha\cos^2\alpha = 0$
$\Rightarrow\ (3\sin^2\alpha - 2\cos^2\alpha)^2 = 0$
$\Rightarrow\ 3\sin^2\alpha - 2\cos^2\alpha = 0$
$\Rightarrow 3\sin^2\alpha = 2\cos^2\alpha$
$\Rightarrow \tan^2\alpha = \frac{2}{3}$
$\therefore\ \cot^2\alpha = 3 / 2$
Now,
$27\sec^6\alpha + 8\operatorname{cosec}^6\alpha = 27(\sec^2\alpha)^3 + 8(\operatorname{cosec}^2\alpha)^3$
$= 27(1 + \tan^2\alpha)^3 + 8(+\cot^2\alpha)^3$
$= 27\left(1 + \frac{2}{3}\right)^3 + 8\left(1 + \frac{3}{2}\right)^3 = 250$

Question43

If for $x \in \left(0,\ \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10}n - 1)$, $n > 0$, then the value of n is equal to [16 Mar 2021 Shift 1]

Options:

A. 20

B. 12

C. 9

D. 16

Answer: B

Solution:

Solution:
$\log_{10}\sin x + \log_{10}\cos x = -1,\ x \in (0, \pi/2)$
$\log_{10}(\sin x\cos x) = -1$
$\Rightarrow\ \sin x\cos x = 10^{-1} = 1/10$
$\log_{10}(\sin x + \cos x) = 1/2(\log_{10}n - 1),\ n > 0$
$2\log_{10}(\sin x + \cos x) = (\log_{10}n - \log_{10}10)$
$\Rightarrow\ \log_{10}(\sin x + \cos x)^2 = \log_{10}(n/10)$
$\Rightarrow\ (\sin x + \cos x)^2 = n/10$

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x \cos x = \frac{n}{10}$
$\Rightarrow$ 1 + 2(1/10) = n/10 $\Rightarrow$ 12/10 = n/10
$\therefore$ n = 12

Question44

If $0 < a, b < 1$ and $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$, then the value of $(a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \ldots$ is [26 Feb 2021 Shift 2]

Options:

A. $\log_e 2$

B. $e^2 - 1$

C. e

D. $\log_e\left(\frac{e}{2}\right)$

Answer: A

Solution:

Solution:
Given, $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$
$\Rightarrow \tan^{-1}\left[\frac{a+b}{1-ab}\right] = \frac{\pi}{4}$
$\Rightarrow \frac{a+b}{1-ab} = \tan\frac{\pi}{4} = 1$
$\Rightarrow a + b = 1 - ab$
$\Rightarrow (1+a)(1+b) = 2$ . . . (i)
Now, $(a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \ldots$
$= \left[a - \frac{a^2}{2} + \frac{a^3}{3} - \frac{a^4}{4} + \ldots\right] + \left[b - \frac{b^2}{2} + \frac{b^3}{3} - \frac{b^4}{4} + \ldots\right]$
$= \log_e(1+a) + \log_e(1+b)$ [Using expansion of $\log_e(1+x)$]
$= \log_e(1+a)(1+b)$ [$\because \log a + \log b = \log ab$]
$= \log_e 2$ [use Eq. (i)]

Question45

All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in [25 Feb 2021 Shift 1]

Options:

A. $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

B. $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

C. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

D. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup$

Answer: D

Solution:

Solution:

$\sin 2\theta + \tan 2\theta > 0$

$\Rightarrow \sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0$

$\Rightarrow \sin 2\theta\left(1 + \frac{1}{\cos 2\theta}\right) > 0$

$\Rightarrow \sin 2\theta\left(\frac{\cos 2\theta + 1}{\cos 2\theta}\right) > 0$

$\Rightarrow \tan 2\theta(2\cos^2\theta) > 0$

$\cos 2\theta \neq 0$

$\Rightarrow 1 - 2\sin^2\theta \neq 0$

$\sin\theta \neq \pm\frac{1}{\sqrt{2}}$

Now, $\tan 2\theta(1 + \cos 2\theta) > 0$

$\Rightarrow \tan 2\theta > 0$ as $1 + \cos 2\theta > 0$

$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$

$\therefore \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

Since, $\sin\theta \neq \pm\frac{1}{\sqrt{2}}$]

Question46

If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to
[25 Feb 2021 Shift 2]

Options:

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{1-\sqrt{3}}{2}$

D. $\frac{1+\sqrt{3}}{2}$

Answer: D

Solution:

Solution:

Given, $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - \left[2\cos^2\left(\frac{x+y}{2}\right) - 1\right] = \frac{3}{2}$ [Use formula,

$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$

$\cos 2x = 2\cos^2 x - 1$]

$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 2\cos^2\left(\frac{x+y}{2}\right)$

$= \frac{3}{2} - 1 = \frac{1}{2}$

$\Rightarrow 4\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 4\cos^2\left(\frac{x+y}{2}\right)$

$= \frac{1}{2} \times 2 = 1 = \cos^2\frac{(x-y)}{2} + \sin^2\frac{(x-y)}{2}$

$\Rightarrow \left[\cos\left(\frac{x-y}{2}\right) - 2\cos\left(\frac{x+y}{2}\right)\right]^2 + \sin^2\left(\frac{x-y}{2}\right) = 0$

$\Rightarrow \sin\left(\frac{x-y}{2}\right) = 0$ and $\cos\left(\frac{x-y}{2}\right) - 2\cos\left(\frac{x+y}{2}\right) = 0$

$\Rightarrow x = y$ and $\cos 0 - 2\cos x = 0$

Gives, $\cos x = \frac{1}{2} = \cos y$

$\therefore \sin x = \sqrt{1 - \cos^2 x} = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$

$\therefore \sin x + \cos y = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1 + \sqrt{3}}{2}$

Question47

If n is the number of solutions of the equation $2\cos x\left(4\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) - 1\right) = 1, x \in [0, \pi]$ and S is the sum of all these solutions, then the ordered pair (n, S) is
[1 Sep 2021 Shift 2]

Options:

A. $\left(3, \frac{13\pi}{9}\right)$

B. $\left(2, \frac{2\pi}{3}\right)$

C. $\left(2, \frac{8\pi}{9}\right)$

D. $\left(3, \frac{5\pi}{3}\right)$

Answer: A

Solution:

Solution:

$2\cos x\left(4\sin\left(\frac{\pi}{4} + x\right)\sin\left(\frac{\pi}{4} - x\right) - 1\right)$

$\Rightarrow 2\cos x\left(2\cos(2x) - 2\cos\left(\frac{\pi}{2}\right) - 1\right) = 1$

$\Rightarrow 2\cos x(4\cos^2 x - 3) = 1$

$\Rightarrow \cos 3x = \frac{1}{2}$

$\Rightarrow 3x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

$\Rightarrow x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

Number of solutions = n = 3

Sum of solutions = S = $\frac{13\pi}{9}$

Question48

The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81, 0 \le x \le \frac{\pi}{4}$ is
[31 Aug 2021 Shift 2]

Options:

A. 3

B. 1

C. 0

D. 2

Answer: B

Solution:

Solution:

$32^{\tan^2 x} + 32^{\sec^2 x} = 81$

$\Rightarrow 32^{\tan^2 x} + 32^{1 + \tan^2 x} = 81$

$\Rightarrow 33 \times 32^{\tan^2 x} = 81$

$\Rightarrow 32^{\tan^2 x} = \frac{27}{11}$

$\Rightarrow \tan^2 x = \ln_{32}\left(\frac{27}{11}\right)$

$\tan x = \sqrt{\ln_{32}\left(\frac{27}{11}\right)} \in (0, 1)$

$\Rightarrow$ One solution in $\left[0, \frac{\pi}{4}\right]$

Question49

Section B : Numerical Type Questions
Let S be the sum of all solutions (in radians) of the equation $\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$. Then, $\frac{8S}{\pi}$ is equal to
[27 Aug 2021 Shift 2]

Answer: 56

Solution:

Solution:
$\sin^4\theta + \cos^4\theta - \sin\theta\cos\theta = 0$ in $[0, 4\pi]$
$\Rightarrow 1 - 2\sin^2\theta\cos^2\theta - \sin\theta\cos\theta = 0$
$\Rightarrow 2 - \sin^2 2\theta - \sin 2\theta = 0$
$\Rightarrow \sin^2 2\theta + \sin 2\theta - 2 = 0$
$\Rightarrow (\sin 2\theta + 2)(\sin 2\theta - 1) = 0$
$\Rightarrow \sin 2\theta = 1,\ 2\theta \in [0, 8\pi]$
$\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$
Sum of solutions $S = \frac{28\pi}{4}$
Then, $\frac{8S}{\pi} = \frac{8}{\pi} \times \frac{28\pi}{4} = 56$

Question50

The sum of solutions of the equation $\frac{\cos x}{1 + \sin x}$ $= |\tan 2x|, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is [26 Aug 2021 Shift 1]

Options:

A. $-\frac{11\pi}{30}$

B. $\frac{\pi}{10}$

C. $-\frac{7\pi}{30}$

D. $-\frac{\pi}{15}$

Answer: A

Solution:

Solution:
We have, $\frac{\cos x}{1} = |\tan 2x|$

$$\Rightarrow \frac{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\cos\frac{x}{2}\sin\frac{x}{2}} = |\tan 2x|$$

$$\Rightarrow \frac{\left[\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right]\left[\cos\frac{x}{2} + \sin\frac{x}{2}\right]}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2} = |\tan 2x|$$

$$\Rightarrow \frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}} = |\tan 2x|$$

$\Rightarrow \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) = |\tan 2x|$
$\Rightarrow \tan^2\left(\frac{\pi}{4} - \frac{x}{2}\right) = \tan^2 2x$

$\Rightarrow 2x = n\pi \pm \left(\frac{\pi}{4} - \frac{x}{2}\right)$
$\Rightarrow 2x = n\pi + \frac{\pi}{4} - \frac{x}{2}$
or$2x = n\pi - \left(\frac{\pi}{4} - \frac{x}{2}\right)$
$\Rightarrow \frac{5x}{2} = \left(n + \frac{1}{4}\right)\pi$
or $\frac{3x}{2} = \left(n - \frac{1}{4}\right)\pi$
$\Rightarrow \frac{-\pi}{2} < \frac{2}{5}\left(n + \frac{1}{4}\right)\pi < \frac{\pi}{2}$
or $\frac{-\pi}{2} < \frac{2}{3}\left(n - \frac{1}{4}\right)\pi < \frac{\pi}{2}$
$\Rightarrow -\frac{5}{4} < n + \frac{1}{4} < \frac{5}{4}$
or $\frac{-3}{4} < n - \frac{1}{4} < \frac{3}{4}$
$\Rightarrow \frac{-6}{4} < n < 1$
or $\frac{-1}{2} < n < 1$
$\Rightarrow n = -1, 0$
or n = 0
When $n = -1$, $x = \left(\frac{-3}{10}\right)\pi$
or when $n = 0$, $x = -\frac{\pi}{6}$
$n = 0$, $x = \left(\frac{1}{10}\right)\pi$
$\therefore$ Required sum $= \left(\frac{-3}{10}\right)\pi + \left(\frac{1}{10}\right)\pi + \left(\frac{-1}{6}\right)\pi = \left(\frac{-11}{30}\right)\pi$

Question51

The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to [22 Jul 2021 Shift 2]

Options:

A. 11

B. 7

C. 5

D. 9

Answer: C

Solution:

Solution:
$\sin^7 x \le \sin^2 x \le \lambda 1$ ....(1)
and $\cos^7 x \le \cos^2 x \le 1$ ....(2)
also $\sin^2 x + \cos^2 x = 1$
$\Rightarrow$ equality must hold for (1) & (2)
$\Rightarrow \sin^7 x = \sin^2 x$ & $\cos^7 = \cos^2 x$
$\Rightarrow \sin x = 0$ & $\cos x = 1$
or
$\cos x = 0$ & $\sin x = 1$
$\Rightarrow x = 0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$

$\Rightarrow$5 solutions

Question52

Let $\alpha = \max_{x \in R}\{8^{2\sin 3x} . 4^{4\cos 3x}\}$ andbeta $= \min_{x \in R}\{8^{2\sin 3x} . 4^{4\cos 3x}\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{1/5}$ and $\beta^{1/5}$,then the value of $c - b$ is equal to :
[27 Jul 2021 Shift 2]

Options:

A. 42

B. 47

C. 43

D. 50

Answer: A

Solution:

Solution:
$\alpha = \max\{8^{2\sin 3x} . 4^{4\cos 3x}\}$
$= \max\{2^{6\sin 3x} . 2^{8\cos 3x}\}$
$= \max\{2^{6\sin 3x + 8\cos 3x}\}$
and $\beta = \min\{8^{2\sin 3x} . 4^{4\cos 3x}\} = \min\{2^{6\sin 3x + 8\cos 3x}\}$
Now range of $6\sin 3x + 8\cos 3x$
$= \left[-\sqrt{6^2 + 8^2}, +\sqrt{6^2 + 8^2}\right] = [-10, 10]$
$\alpha = 2^{10}$ & $\beta = 2^{-10}$
So, $\alpha^{1/5} = 2^2 = 4$
$\Rightarrow \beta^{1/5} = 2^{-2} = 1/4$
quadratic $8x^2 + bx + c = 0$, $c - b =$
$8 \times$ [(product of roots] + (sum of roots)
$= 8 \times \left[4 \times \frac{1}{4} + 4 + \frac{1}{4}\right] = 8 \times \left[\frac{21}{4}\right] = 42$

Question53

The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is
[18 Mar 2021 Shift 1]

Answer: 1

Solution:

Given, $|\cot x| = \cot x + \frac{1}{\sin x}$....(i)
If $\cot x > 0$, then $|\cot x| = \cot x$
From Eq. (i), $\cot x = \cot x + \frac{1}{\sin x}$
$\Rightarrow \frac{1}{\sin x} = 0$ (not possible)
If $\cot x < 0$, then $|\cot x| = -\cot x$
From Eq. (ii), $-\cot x = \cot x + \frac{1}{\sin x}$
$\Rightarrow 2\cot x + \frac{1}{\sin x} = 0 \Rightarrow 2\cos x = -1$
$\Rightarrow x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$
Here, $x = \frac{4\pi}{3}$ rejected because $\frac{4\pi}{3} \in$ third quadrant and in third quadrant $\cot x$ is positive. Since, we considered $\cot x < 0$. $\therefore x = 2\pi/3$ is the only one solution.

Question54

The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is
[17 Mar 2021 Shift 2]

Options:

A. 3

B. 4

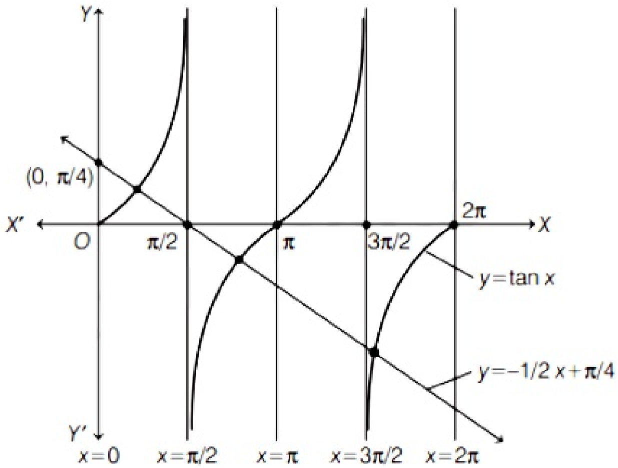
C. 2

D. 5

Answer: A

Solution:

Solution:
Given, $x + 2\tan x = \frac{\pi}{2}$
$\Rightarrow 2\tan x = \frac{\pi}{2} - x \Rightarrow \tan x = \frac{\pi}{4} - \frac{x}{2}$
$\Rightarrow \tan x = \left(-\frac{1}{2}\right)x + \frac{\pi}{4}$...(i)
Approach In this type of problem solving, graphical approach is best because we have to find only number of solutions, not the solution (i.e. not the value(s) of x).
Concept To find the number of solution(s) for Eq. (i), first of all, let $y = \tan x$... (ii) and $y = \left(\frac{-1}{2}\right)x + \frac{\pi}{4}$... (iii) and then draw the graph of Eqs. (ii) and (iii).
Now, total number of solution(s) = Total number of point(s) of intersection of the graph (ii) and (iii).



$y = -\frac{1}{2}x + \frac{\pi}{4}$ intersects $y = \tan x$ at three distinct points in $[0, 2\pi]$.
∴ Total number of solutions $= 3$

Question55

The number of roots of the equation, $(81)^{\sin^2 x} + (81)^{\cos^2 x} = 30$ in the interval $[0, \pi]$ is equal to
[16 Mar 2021 Shift 1]

Options:

A. 3

B. 4

C. 8

D. 2

Answer: B

Solution:

Solution:
Given, $81^{\sin^2 x} + 81^{\cos^2 x} = 30$
$\Rightarrow 81^{\sin^2 x} + 81^{(1 - \sin^2 x)} = 30$
$\Rightarrow 81^{\sin^2 x} + \frac{81}{81^{\sin^2 x}} = 30$
Let $81^{\sin^2 x} = y$.
$\therefore y + \frac{81}{y} = 30$
$\Rightarrow y^2 - 30y + 81 = 0$
$\Rightarrow (y - 27)(y - 3) = 0$
$\Rightarrow y = 3$ or $y = 27$
$81^{\sin^2 x} = 3$ or $81^{\sin^2 x} = 27$
$3^{4\sin^2 x} = 3$ or $3^{4\sin^2 x} = 3^3$
$\Rightarrow 4\sin^2 x = 1$ or $4\sin^2 x = 3$
$\Rightarrow \sin^2 x = 1/4$ or $\sin^2 x = 3/4$
$\Rightarrow \sin^2 x = \sin^2(\pi/6)$ or $\sin^2 x = \sin^2(\pi/3)$
$\Rightarrow x = n\pi \pm \pi/6$ or $x = n\pi \pm \pi/3$
From $[0, \pi]$,
$x = \pi/6, 5\pi/6$ or $x = \pi/3, 2\pi/3$
Hence, the total number of solutions $= 4$

Question56

If $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is ....
[26 Feb 2021 Shift 1]

Answer: 1

Solution:

Solution:
Given, $\sqrt{3}\cos^2 x = (\sqrt{3} - 1)\cos x + 1$, $x \in [0, \pi / 2]$
Let $\cos x = t$, then
$\sqrt{3}t^2 = (\sqrt{3} - 1)t + 1$
$\Rightarrow \sqrt{3}t^2 - \sqrt{3}t + t - 1 = 0$
$\Rightarrow (\sqrt{3}t^2 - \sqrt{3}t) + (t - 1) = 0$
$\Rightarrow \sqrt{3}t(t - 1) + 1(t - 1) = 0$
$\Rightarrow (t - 1)(\sqrt{3}t + 1) = 0$
This gives $t = 1$ and $t = \frac{-1}{\sqrt{3}}$
Put, $t = \cos x$, then
$\cos x = 1$ and $\cos x = \frac{-1}{\sqrt{3}}$
$\cos x = -1 / \sqrt{3}$ is rejected as $x \in [0, \pi / 2]$
$\cos x = -1 / \sqrt{3}$ is rejected as $x \in [0, \pi$
$\therefore \cos x = 1$
$\text{Since, } x \in \left[0, \frac{\pi}{2}\right], \text{ then } \cos x = \cos 0$
This gives $x = 0$ is only solution.
Therefore, number of solution when $x \in [0, \pi / 2]$ is 1 .

Question57

The number of integral values of k for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is ....
[26 Feb 2021 Shift 1]

Answer: 11

Solution:

Solution:
Given, $3\sin x + 4\cos x = k + 1 \ldots$ (i)
Multiply and divide LHS of Eq. (i) by $\sqrt{3^2 + 4^2} = 5$

i.e. $5\left(\frac{3}{5}\sin x + \frac{4}{5}\cos x\right) = k + 1$
$\Rightarrow 5(\cos\alpha\sin x + \sin\alpha\cos x) = k + 1$
[Let $\cos\alpha = 3/5$ then $\sin\alpha = \sqrt{1-(3/5)^2} = \frac{4}{5}$]
$\Rightarrow 5\sin(x+\alpha) = k + 1$ [Use $\sin(a+b) = \sin a\cos b + \cos a\sin b$]
$\Rightarrow \sin(x+\alpha) = \frac{k+1}{5}$
Let $x + \alpha = \theta$
Then, $\sin\theta = \frac{k+1}{5}$
$\because \; -1 \le \sin\theta \le 1$
$\Rightarrow \; -1 \le \frac{k+1}{5} \le 1$
$\Rightarrow \; -5 \le k + 1 \le 5$
$\Rightarrow \; -6 \le k \le 4$
$\therefore$ Possible integral values of k are $-6, -5, -4, -3, -2, -1, 0, 1, 2, 3$ and 4. i.e.
Total 11 integral values of k are possible for which Eq. (i) has solution.

Question58

The value of $\cos^3\left(\frac{\pi}{8}\right)\cdot\cos\left(\frac{3\pi}{8}\right) + \sin^3\left(\frac{\pi}{8}\right)\cdot\sin\left(\frac{3\pi}{8}\right)$ is [Jan. 9, 2020 (I)]

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. $\frac{1}{2}$

D. $\frac{1}{4}$

Answer: B

Solution:

Solution:
$\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right] + \sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right]$
$= 4\cos^6\frac{\pi}{8} - 4\sin^6\frac{\pi}{8} - 3\cos^4\frac{\pi}{8} + 3\sin^4\frac{\pi}{8}$
$= 4\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\right]$
$\left[\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)\right]$;
-3$\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right)\right]$;
$= \cos\frac{\pi}{4}\left[4\left(1 - \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right) - 3\right]$
$= \frac{1}{\sqrt{2}}\left[1 - \frac{1}{2}\right] = \frac{1}{2\sqrt{2}}$].

Question59

If L = $\sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and M = $\cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then :

[Sep. 05, 2020 (II)]

Options:

A. $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

B. $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

C. $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$

D. $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

Answer: D

Solution:

Solution:

$L + M = 1 - 2\sin^2\frac{\pi}{8} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ ....(i)

and $L - M = -\cos\frac{\pi}{8}$ ....(ii)

From equations (i) and (ii), $L = \frac{1}{2}\left(\frac{1}{\sqrt{2}} - \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\frac{\pi}{8}$ and $M = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + \cos\frac{\pi}{8}\right) = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

Question60

If $\frac{\sqrt{2}\sin\alpha}{\sqrt{1+\cos 2\alpha}} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos 2\beta}{2}} = \frac{1}{\sqrt{10}}$ $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, then $\tan(\alpha + 2\beta)$ is equal to _______

[Jan. 8, 2020 (II)]

Answer: 1

Solution:

Solution:

$\frac{\sqrt{2}\sin\alpha}{\sqrt{2}\cos\alpha} = \frac{1}{7}$ and $\sqrt{\frac{1-\cos^2\beta}{2}} = \frac{1}{10}$

$\Rightarrow \frac{\sqrt{2}\sin\beta}{\sqrt{2}} = \frac{1}{\sqrt{10}}$

$\therefore\ \tan\alpha = \frac{1}{7}$ and $\sin\beta = \frac{1}{\sqrt{10}}$

$\tan\beta = \frac{1}{3}$

$\therefore \tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2\cdot\frac{1}{3}}{1-\frac{1}{9}} = \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{3}{4}$

$\tan(\alpha + 2\beta) = \frac{\tan\alpha + \tan 2\beta}{1 - \tan\alpha\tan 2\beta}$

$= \frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}\cdot\frac{3}{4}} = \frac{\frac{4+21}{28}}{\frac{25}{28}} = 1$

Question61

The set of all possible values of θ in the interval $(0, \pi)$ for which the points (1,2) and $(\sin\theta, \cos\theta)$ lie on the same side of the line $x + y = 1$ is: [Sep. 02, 2020 (II)]

Options:

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

C. $\left(0, \frac{3\pi}{4}\right)$

D. $\left(0, \frac{\pi}{4}\right)$

Answer: A

Solution:

Solution:
Let $f(x, y) = x + y - 1$
Given (1,2) and $(\sin\theta, \cos\theta)$ are lies on same side.
$\therefore f(1, 2) \cdot f(\sin\theta, \cos\theta) > 0$
$\Rightarrow 2[\sin\theta + \cos\theta - 1] > 0$
$\Rightarrow \sin\theta + \cos\theta| > 1 \Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) > \frac{1}{\sqrt{2}}$
$\Rightarrow \theta + \frac{\pi}{4} \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \Rightarrow \theta \in \left(0, \frac{\pi}{2}\right)$

Question62

If the equation $\cos^4\theta + \sin^4\theta + \lambda = 0$ has real solutions for θ, then λ lies in the interval : [Sep. 02, 2020 (II)]

Options:

A. $\left(-\frac{5}{4}, -1\right)$

B. $\left[-1,-\frac{1}{2}\right]$

C. $\left(-\frac{1}{2},-\frac{1}{4}\right]$

D. $\left[-\frac{3}{2},-\frac{5}{4}\right]$

Answer: B

Solution:

Solution:
Given equation is $\cos^4(\theta)+\sin^4(\theta)+\lambda=0$
$\Rightarrow \lambda=-[\cos^4(\theta)+\sin^4(\theta)]$
$\Rightarrow\lambda=-[(\cos^2(\theta)+\sin^2(\theta))^2-2\cos^2(\theta)\sin^2(\theta)]$
$[\because a^2+b^2=(a+b)^2-2ab]$
$\Rightarrow\lambda=-(1)^2+\frac{1}{2}(2\cos(\theta)\sin(\theta))^2$
$[\because\cos^2(\theta)+\sin^2(\theta)=1]$
$\Rightarrow\lambda=\frac{1}{2}\sin^2(2\theta)-1$
We know that $-1\le\sin(x)\le1$
Therefore $0\le\sin^2(x)\le1$
So $\frac{1}{2}\sin^2(2\theta)\in\left[0,\frac{1}{2}\right]$
for $\frac{1}{2}\sin^2(2\theta)=0\Rightarrow\lambda=0-1=-1$
for $\frac{1}{2}\sin^2(2\theta)=\frac{1}{2}\Rightarrow\lambda=\frac{1}{2}-1=\frac{-1}{2}$
$\therefore\lambda\in\left[-1,-\frac{1}{2}\right]$
Hence, Option(B) i.e. $\left[-1,\frac{-1}{2}\right]$ is correct

----

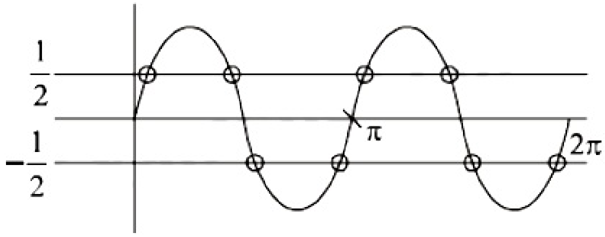
Question63

The number of distinct solutions of the equation,
$\log_{1/2}\sin x|=2-\log_{1/2}|\cos x|$ in the interval $[0,2\pi]$, is ______
[Jan. 9, 2020 (I)]

Answer: 8

Solution:

Solution:
$\log_{1/2}|\sin x|=2-\log_{1/2}|\cos x$
$\Rightarrow\log_{1/2}|\sin x\cos x|=2$
$\Rightarrow |\sin x\cos x\Big|=\frac{1}{4}$
$\Rightarrow \sin 2x=\pm\frac{1}{2}$



Hence, total number of solutions = 8.

Question64

For any $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ the expression $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$ equals:
[Jan. 9, 2019 (I)]

Options:

A. $13 - 4\cos^2\theta + 6\sin^2\theta\cos^2\theta$

B. $13 - 4\cos^6\theta$

C. $13 - 4\cos^2\theta + 6\cos^4\theta$

D. $13 - 4\cos^4\theta + 2\sin^2\theta\cos^2\theta$

Answer: B

Solution:

Solution:

$3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4\sin^6\theta$
$= 3(1 - 2\sin\theta\cos\theta)^2 + 6(1 + 2\sin\theta\cos\theta) + 4\sin^6\theta$
$= 3(1 + 4\sin^2\theta\cos^2\theta - 4\sin\theta\cos\theta) + 6$
$- 12\sin\theta\cos\theta + 4\sin^6\theta$
$= 9 + 12\sin^2\theta\cos^2\theta + 4\sin^6\theta$
$= 9 + 12\cos^2\theta(1 - \cos^2\theta) + 4(1 - \cos^2\theta)^3$
$= 9 + 12\cos^2\theta - 12\cos^4\theta + 4(1 - \cos^6\theta - 3\cos^2\theta + 3\cos^4\theta)$
$= 9 + 4 - 4\cos^6\theta$
$= 13 - 4\cos^6\theta$

Question65

The angle of elevation of the top of a vertical tower standing on a horizontal plane is observed to be 45° from a point A on the plane. Let B be the point 30m vertically above the point A. If the angle of elevation of the top of the tower from B be 30°, then the distance (inm) of the foot of the tower from the point A is:
[April 12, 2019 (II)]

Options:

A. $15(3 + \sqrt{3})$

B. $15(5-\sqrt{3})$

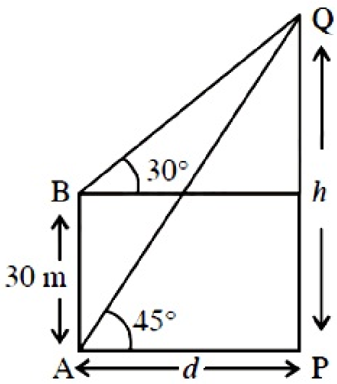
C. $15(3-\sqrt{3})$

D. $15(1+\sqrt{3})$

Answer: A

Solution:

Solution:
Let the height of the tower be h and distance of the foot of the tower from the point A is d . By the diagram,



$\tan 45^{\circ} = \frac{\mathrm{h}}{\mathrm{d}} = 1$

$\mathrm{h} = \mathrm{d}$ ......(i)

$\tan 30^{\circ} = \frac{\mathrm{h}-30}{\mathrm{d}}$

$\sqrt{3}(\mathrm{h}-30) = \mathrm{d}$ ....(ii)

Put the value of h
from (i) to (ii),

$\sqrt{3}\mathrm{d} = \mathrm{d} + 30\sqrt{3}$

$\mathrm{d} = \frac{30\sqrt{3}}{\sqrt{3}-1} = 15\sqrt{3}(\sqrt{3}+1) = 15(3+\sqrt{3})$

Question66

The value of $\cos^2 10^{\circ} - \cos 10^{\circ} \cos 50^{\circ} + \cos^2 50^{\circ}$ is [April 9, 2019 (II)]

Options:

A. $\frac{3}{4}+\cos 20^{\circ}$

B. .3/4

C. $\frac{3}{2}(1+\cos 20^{\circ})$

D. $\frac{3}{2}$

Answer: B

Solution:

$\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$
$= \left(\frac{1+\cos 20^\circ}{2}\right) + \left(\frac{1+\cos 100^\circ}{2}\right) - \frac{1}{2}(2\cos 10^\circ \cos 50^\circ)$
$= 1 + \frac{1}{2}(\cos 20^\circ + \cos 100^\circ) - \frac{1}{2}[\cos 60^\circ + \cos 40^\circ]$
$= \left(1 - \frac{1}{4}\right) + \frac{1}{2}[\cos 20^\circ + \cos 100^\circ - \cos 40^\circ]$
$= \frac{3}{4} + \frac{1}{2}[2\cos 60^\circ \times \cos 40^\circ - \cos 40^\circ]$
$= \frac{3}{4}$

Question67

Two poles standing on a horizontal ground are of heights 5m and 10m respectively. The line joining their tops makes an angle of 15° with the ground. Then the distance (in m) between the poles, is: [April. 09, 2019 (II)]

Options:

A. $5(2+\sqrt{3})$

B. $5(\sqrt{3}+1)$

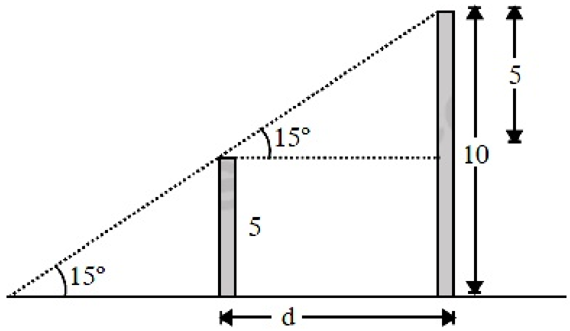
C. $\frac{5}{2}(2+\sqrt{3})$

D. $10(\sqrt{3}-1)$

Answer: A

Solution:

Solution:



By the diagram.

$\tan 15^\circ = \frac{5}{d} \Rightarrow d = \frac{5}{\tan 15^\circ} = \frac{5(\sqrt{3}+1)}{\sqrt{3}-1}$
$= \frac{5(4+2\sqrt{3})}{2} = 5(2+\sqrt{3})$

Question68

The value of $\sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ$ is: [April. 09, 2019 (II)]

Options:

A. $\frac{1}{16}$

B. $\frac{1}{32}$

C. $\frac{1}{18}$

D. $\frac{1}{36}$

Answer: A

Solution:

Solution:

$\because \sin(60^\circ + A) \cdot \sin(60^\circ - A) \sin A = \frac{1}{4} \sin 3A$

$\therefore \sin 10^\circ \sin 50^\circ \sin 70^\circ = \sin 10^\circ \sin(60^\circ - 10^\circ)$

$\sin(60^\circ + 10^\circ) = \frac{1}{4} \sin 30^\circ$

$\Rightarrow \sin 10^\circ \sin 30^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{4} \sin^2 30 = \frac{1}{16}$

Question69

If $\cos(\alpha + \beta) = \frac{3}{5}$, $\sin(\alpha - \beta) = \frac{5}{13}$ and $0 < \alpha, \beta < \frac{\pi}{4}$, then $\tan(2\alpha)$ is equal to :
[April 8, 2019 (I)]

Options:

A. $\frac{63}{52}$

B. $\frac{63}{16}$

C. $\frac{21}{16}$

D. $\frac{33}{52}$

Answer: B

Solution:

Solution:

$\because \alpha + \beta$ and $\alpha - \beta$ both are acute angles.

$\cos(\alpha + \beta) = \frac{3}{5}$, then $\sin(\alpha + \beta) = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$

$\tan(\alpha + \beta) = \frac{4}{3}$

And $\sin(\alpha - \beta) = \frac{5}{13}$, then

$\cos(\alpha-\beta) = \sqrt{1-\left(\frac{5}{13}\right)^2} = \frac{12}{13}$

$\Rightarrow \tan(\alpha-\beta) = \frac{5}{12}$

Now, $\tan 2\alpha = \tan((\alpha+\beta)+(\alpha-\beta))$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\cdot\tan(\alpha-\beta)} = \frac{\frac{4}{3}+\frac{5}{12}}{1-\frac{4}{3}\cdot\frac{5}{12}} = \frac{63}{16}$$

Question70

If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to : [Jan. 12, 2019 (II)]

Options:

A. 0

B. -1

C. $\sqrt{2}$

D. $-\sqrt{2}$

Answer: D

Solution:

Solution:
∵ The given equation is
$\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cdot\cos\beta$, $\alpha, \beta \in [0, \pi]$
Then, by A.M., G.M. ineqality;
A.M. ≥ G.M.

$$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq (\sin^4\alpha\cdot 4\cos^4\beta\cdot 1\cdot 1)^{\frac{1}{4}}$$

$\sin^4\alpha + 4\cos^4\beta + 1 + 1 \geq 4\sqrt{2}\sin\alpha\cdot|\cos\beta|$
Inequality still holds when $\cos\beta < 0$ but L.H.S. is positive than $\cos\beta > 0$, then
$L.H.S. = R.H.S$

$\therefore \sin^4\alpha = 1$ and $\cos^4\beta = \frac{1}{4}$

$\Rightarrow \alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{4}$

$\therefore \cos(\alpha+\beta) - \cos(\alpha-\beta)$

$= \cos\left(\frac{\pi}{2}+\beta\right) - \cos\left(\frac{\pi}{2}-\beta\right)$

$= -\sin\beta - \sin\beta = -2\sin\frac{\pi}{4} = -\sqrt{2}$

Question71

Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ for k = 1, 2, 3, ... Then for all $x \in R$, the value of $f_4(x) - f_6(x)$ is equal to : [Jan. 11, 2019 (I)]

Options:

A. $\frac{1}{12}$

B. $\frac{1}{4}$

C. $\frac{-1}{12}$

D. $\frac{5}{12}$

Answer: A

Solution:

Solution:

$f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$

$f_4(x) = \frac{1}{4}[\sin^4 x + \cos^4 x]$

$= \frac{1}{4}\left[(\sin^2 x + \cos^2 x)^2 - \frac{(\sin 2x)^2}{2}\right]$

$= \frac{1}{4}\left[1 - \frac{(\sin 2x)^2}{2}\right]$

$f_6(x) = \frac{1}{6}[\sin^6 x + \cos^6 x]$

$= \frac{1}{6}\left[\left(\sin^2 x + (\cos^2 x) - \frac{3}{4}(\sin^2 x)^2\right]\right.$

$= \frac{1}{6}\left[1 - \frac{3}{4}(\sin 2x)^2\right]$

Now $f_4(x) - f_{(6)}(x) = \frac{1}{4} - \frac{1}{6} - \frac{(\sin 2x)^2}{8} + \frac{1}{8}(\sin 2x)^2$

$= \frac{1}{12}$

Question72

The value of $\cos\frac{\pi}{2^2} \cdot \cos\frac{\pi}{2^3} \cdot \ldots \cdot \cos\frac{\pi}{2^{10}} \cdot \sin\frac{\pi}{2^{10}}$ [Jan. 10, 2019 (II)]

Options:

A. $\frac{1}{512}$

B. $\frac{1}{1024}$

C. $\frac{1}{256}$

D. $\frac{1}{2}$

Answer: A

Solution:

Solution:

$A = \cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\ldots\cos\frac{\pi}{2^{10}}\cdot\sin\frac{\pi}{2^{10}}$

$= \frac{1}{2}\left(\cos\frac{\pi}{2^2}\cdot\cos\frac{\pi}{2^3}\ldots\cos\frac{\pi}{2^9}\sin\frac{\pi}{2^9}\right)$

$= \frac{1}{2^8}\left(\cos\frac{\pi}{2^2}\cdot\sin\frac{\pi}{2^2}\right) = \frac{1}{2^9}\sin\frac{\pi}{2}$

$= \frac{1}{512}$

Question73

The number of solutions of the equation $1+\sin^4 x = \cos^2 3x,\ x \in \left[-\frac{5\pi}{2},\ \frac{5\pi}{2}\right]$ is : [April 12, 2019 (I)]

Options:

A. 3

B. 5

C. 7

D. 4

Answer: C

Solution:

Solution:

Consider equation, $1+\sin^4 x = \cos^2 3x$

L.H.S. $= 1+\sin^4 x$ and $R\cdot H\ .S. = \cos^2 3x$

$\because L.H\ .S. \ge 1$ and $R.H\ .S. \le 1 \Rightarrow L.H\ .S. = R.H\ .S. = 1$

$\sin^4 x = 0$, and $\cos^2 3x = 1$

$\Rightarrow\ \sin x = 0$ and $(4\cos^2 x - 3)^2\cos^2 x = 1$

$\Rightarrow\ \sin x = 0$ and $\cos^2 x = 1 \Rightarrow x = 0, \pm\pi, \pm 2\pi$

Hence, total number of solutions is 5.

Question74

Let S be the set of all $\alpha \in R$ such that the equation, $\cos 2x + \alpha\sin x = 2\alpha - 7$ has a solution. Then S is equal to [April 12, 2019 (II)]

Options:

A. R

B. [1,4]

C. [3,7]

D. [2,6]

Answer: D

Solution:

Solution:
Given equation is, $\cos 2x + \alpha \sin x = 2\alpha - 7$ $1 - 2\sin^2 x + \alpha \sin x = 2\alpha - 7$
$2\sin^2 x - \alpha \sin x + (2\alpha - 8) = 0$
$\Rightarrow \sin x = \dfrac{\alpha \pm \sqrt{\alpha^2 - 8(2\alpha + 8)}}{4}$
$\Rightarrow \sin x = \dfrac{\alpha \pm (\alpha - 8)}{4} \Rightarrow \sin x = \dfrac{\alpha - 4}{4}$
$[\sin x = 2(\text{ rejected })]$
$\because$ equation has solution, then $\dfrac{\alpha - 4}{4} \in [-1, 1]$
$\Rightarrow \alpha \in [2, 6]$

Question75

If [x] denotes the greatest integer ≤x, then the system of linear equations
$[\sin\theta]x + [-\cos\theta]y = 0$
$[\cot\theta]x + y = 0$
[April 12, 2019 (II)]

Options:

A. have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and has a unique solution if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$.

B. has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$.

C. has a unique solution if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$ and have infinitely many solutions if $\theta \in \left(\pi, \frac{7\pi}{6}\right)$

D. have infinitely many solutions if $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

Answer: A

Solution:

Solution:
According to the question, there are two cases.
Case 1 : $\theta \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right)$
In this interval, $[\sin\theta] = 0$, $[-\cos\theta] = 0$ and $[\cot\theta] = -1$ Then the system of equations will be ;
$0 \cdot x + 0 \cdot y = 0$ and $-x + y = 0$
Which have infinitely many solutions.
Case 2 : $\theta \in \left(\pi, \frac{7\pi}{6}\right)$
In this interval, $[\sin\theta] = -1$ and $[-\cos\theta] = 0$
Then the system of equations will be ;
$-x + 0 \cdot y = 0$ and $[\cot\theta]x + y = 0$
Clearly, $x = 0$ and $y = 0$ which has unique solution.

Question76

Let $S = \{\theta \in [-2\pi, 2\pi] : 2\cos^2\theta + 3\sin\theta = 0\}$. Then the sum of the elements of S is:
[April 9, 2019 (I)]

Options:

A. $\frac{13\pi}{6}$

B. $\frac{5\pi}{3}$

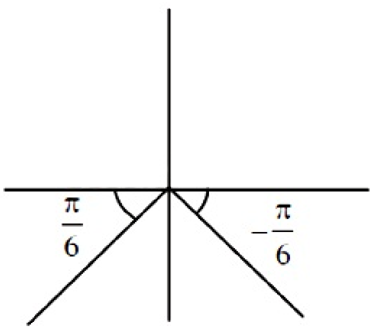
C. 2π

D. π

Answer: C

Solution:

Solution:
$2\cos^2\theta + 3\sin\theta = 0$
$(2\sin\theta + 1)(\sin\theta - 2) = 0$
$\Rightarrow \sin\theta = -\frac{1}{2}$ or $\sin\theta = 2 \rightarrow$ Not possibe



The required sum of all solutions in $[-2\pi, 2\pi]$ is
$= \left(\pi + \frac{\pi}{6}\right) + \left(2\pi - \frac{\pi}{6}\right) + \left(-\frac{\pi}{6}\right) + \left(-\pi + \frac{\pi}{6}\right) = 2\pi$

Question77

If $0 \le x < \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$, is:
[Jan. 09, 2019 (II)]

Options:

A. 3

B. 1

C. 4

D. 2

Answer: D

Solution:

Solution:
$\sin x - \sin 2x + \sin 3x = 0$
$\Rightarrow\ \sin x - 2\sin x \cdot \cos x + 3\sin x - 4\sin^3 x = 0$
$\Rightarrow\ 4\sin x - 4\sin^3 x - 2\sin x \cdot \cos x = 0$
$\Rightarrow\ 2\sin x(1 - \sin^2 x) - \sin x \cdot \cos x = 0$
$\Rightarrow\ 2\sin x \cdot \cos^2 x - \sin x \cdot \cos x = 0$
$\Rightarrow\ \sin x \cdot \cos x(2\cos x - 1) = 0$
$\therefore\ \sin x = 0, \cos x = 0, \cos x = \frac{1}{2}$
$\therefore\ x = 0, \frac{\pi}{3}\ \ \because x \in \left[0, \frac{\pi}{2}\right)$

Question78

The number of solutions of $\sin 3x = \cos 2x$, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is
[Online April 15, 2018]

Options:

A. 3

B. 4

C. 2

D. 1

Answer: D

Solution:

Solution:
$\sin 3x = \cos 2x$
$\Rightarrow 3\sin x - 4\sin^3 x = 1 - 2\sin^2 x$
$\Rightarrow 4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$
$\Rightarrow \sin x = 1, \frac{-2 \pm 2\sqrt{5}}{8}$
In the interval $\left(\frac{\pi}{2}, \pi\right)$, $\sin x = \frac{-2 + 2\sqrt{5}}{8}$
So, there is only one solution.

Question79

If sum of all the solutions of the equation
$8\cos x \cdot \left(\cos\left(\frac{\pi}{6} + x\right) \cdot \cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2}\right) - 1$ in $[0, \pi]$ is $k\pi$ then k is equal to :
[2018]

Options:

A. $\frac{13}{9}$

B. $\frac{8}{9}$

C. $\frac{20}{9}$

D. $\frac{2}{3}$

Answer: A

Solution:

Solution:

$\because 8\cos x\left(\cos^2\frac{\pi}{6}-\sin^2 x-\frac{1}{2}\right)=1$

$\Rightarrow\ 8\cos x\left(\frac{3}{4}-\frac{1}{2}-\sin^2 x\right)=1$

$\Rightarrow\ 8\cos x\left(\frac{1}{4}-(1-\cos^2 x)\right)=1$

$\Rightarrow 8\cos x\left(\frac{1}{4}-1+\cos^2 x\right)=1$

$\Rightarrow\ 8\cos x\left(\cos^2 x-\frac{3}{4}\right)=1$

$\Rightarrow 8\left(\frac{4\cos^3 x-3\cos x}{4}\right)=1$

$\Rightarrow\ 2(4\cos^3 x-3\cos x)=1$

$\Rightarrow\ 2\cos 3x=1\Rightarrow\cos 3x=\frac{1}{2}$

$\therefore\ 3x=2n\pi\pm\frac{\pi}{3},\ n\in 1$

$\Rightarrow\ x=\frac{2n\pi}{3}\pm\frac{\pi}{9}$

In $x\in[0,\pi]: x=\frac{\pi}{9},\ \frac{2\pi}{3}+\frac{\pi}{9},\ \frac{2\pi}{3}-\frac{\pi}{9}$, only

Sum of all the solutions of the equation

$=\left(\frac{1}{9}+\frac{2}{3}+\frac{1}{9}+\frac{2}{3}-\frac{1}{9}\right)\pi=\frac{13}{9}\pi$

Question80

If $5(\tan^2 x-\cos^2 x)=2\cos 2x+9$, then the value of $\cos 4x$ is [2017]

Options:

A. $-\frac{7}{9}$

B. $-\frac{3}{5}$

C. $\frac{1}{3}$

D. $\frac{2}{9}$

Answer: A

Solution:

Solution:
We have
$5\tan^2 x - 5\cos^2 x = 2(2\cos^2 x - 1) + 9$
$\Rightarrow 5\tan^2 x - 5\cos^2 x = 4\cos^2 x - 2 + 9$
$\Rightarrow 5\tan^2 x = 9\cos^2 x + 7$
$\Rightarrow 5(\sec^2 x - 1) = 9\cos^2 x + 7$
Let $\cos^2 x = t$
$\Rightarrow \frac{5}{t} - 9t - 12 = 0$
$\Rightarrow 9t^2 + 12t - 5 = 0$
$\Rightarrow 9t^2 + 15t - 3t - 5 = 0$
$\Rightarrow (3t - 1)(3t + 5) = 0$
$\Rightarrow t = \frac{1}{3}$ as $t \neq -\frac{5}{3}$
$\cos 2x = 2\cos^2 x - 1 = 2\left(\frac{1}{3}\right) - 1 = -\frac{1}{3}$
$\cos 4x = 2\cos^2 2x - 1 = 2\left(-\frac{1}{3}\right)^2 - 1 = -\frac{7}{9}$

Question81

If m and M are the minimum and the maximum values of $4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x$, $x \in R$, then $M - m$ is equal to : [Online April 9, 2016]

Options:

A. $\frac{9}{4}$

B. $\frac{15}{4}$

C. $\frac{7}{4}$

D. $\frac{1}{4}$

Answer: B

Solution:

Solution:
$4 + \frac{1}{2}\sin^2 2x - 2\cos^4 x$
$4 + 2(1 - \cos^2 x)\cos^2 x - 2\cos^4 x$
$-4\left\{\cos^4 x - \frac{\cos^2 x}{2} - 1 + \frac{1}{16} - \frac{1}{16}\right\}$
$-4\left\{\left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16}\right\}$
$0 \leq \cos^2 x \leq 1$
$-\frac{1}{4} \leq \cos^2 x - \frac{1}{4} \leq \frac{3}{4}$
$0 \leq \left(\cos^2 x - \frac{1}{4}\right)^2 \leq \frac{9}{16}$

$$-\frac{17}{16} \le \left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16} \le \frac{9}{16} - \frac{17}{16}$$
$$\frac{17}{4} \ge -4\left\{\left(\cos^2 x - \frac{1}{4}\right)^2 - \frac{17}{16}\right\} \ge \frac{1}{2}$$
$$M = \frac{17}{4}$$
$$m = \frac{1}{2}$$
$$M - m = \frac{17}{4} - \frac{2}{4} = \frac{15}{4}$$

Question82

If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$ is:
[2016]

Options:

A. 7

B. 9

C. 3

D. 5

Answer: A

Solution:

Solution:
$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$
$\Rightarrow 2\cos 2x \cos x + 2\cos 3x \cos x = 0$
$\Rightarrow 2\cos x\left(2\cos\frac{5x}{2}\cos\frac{x}{2}\right) = 0$
$\cos x = 0, \cos\frac{5x}{2} = 0, \cos\frac{x}{2} = 0$
$x = \pi, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{9\pi}{5}$

Question83

The number of $x \in [0, 2\pi]$ for which $\left|\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x}\right| = 1$ is
[Online April 9, 2016]

Options:

A. 2

B. 6

C. 4

D. 8

Answer: D

Solution:

Solution:

$\left| \sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x} \right| = 1$

$\sqrt{2\sin^4 x + 18\cos^2 x} - \sqrt{2\cos^4 x + 18\sin^2 x} = \pm 1$

$\sqrt{2\sin^4 x + 18\cos^2 x} = \pm 1 + \sqrt{2\cos^4 x + 18\sin^2 x}$

by squaring both the sides we will get 8 solutions

Question84

If $\cos\alpha + \cos\beta = \frac{3}{2}$ and $\sin\alpha + \sin\beta \frac{1}{2}$ and θ is the the arithmetic mean of α and β, then $\sin 2\theta + \cos 2\theta$ is equal to : [Online April 11, 2015]

Options:

A. $\frac{3}{5}$

B. $\frac{7}{5}$

C. $\frac{4}{5}$

D. $\frac{8}{5}$

Answer: B

Solution:

Solution:

Let $\cos\alpha + \cos\beta = \frac{3}{2}$

$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \frac{3}{2}$ ....(i)

and $\sin\alpha + \sin\beta = \frac{1}{2}$

$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \frac{1}{2}$ ....(ii)

On dividing (ii) by (i),

we get $\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{1}{3}$

Given : $\theta = \frac{\alpha+\beta}{2} \Rightarrow 2\theta = \alpha + \beta$

Consider $\sin 2\theta + \cos 2\theta = \sin(\alpha+\beta) + \cos(\alpha+\beta) = \frac{\frac{2}{3}}{1+\frac{1}{9}} + \frac{1-\frac{1}{9}}{1+\frac{1}{9}} = \frac{6}{10} + \frac{8}{10} = \frac{7}{5}$

Question85

Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ where $x \in R$ and $k \geq 1$ Then $f_4(x) - f_6(x)$ equals
[2014]

Options:

A. $\frac{1}{4}$

B. $\frac{1}{12}$

C. $\frac{1}{6}$

D. $\frac{1}{3}$

Answer: B

Solution:

Solution:
(b) Let $f_k(x) = \frac{1}{k}(\sin^k x + \cos^k x)$ Consider $f_4(x) - f_6(x) = \frac{1}{4}(\sin^4 x + \cos^4 x)$
$- \frac{1}{6}(\sin^6 x + \cos^6 x)$
$= \frac{1}{4}[1 - 2\sin^2 x\cos^2 x] - \frac{1}{6}[1 - 3\sin^2 x\cos^2 x]$
$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

Question86

If $2\cos\theta + \sin\theta = 1 \left(\theta \neq \frac{\pi}{2}\right)$ then $7\cos\theta + 6\sin\theta$ is equal to:
[Online April 11, 2014]

Options:

A. $\frac{1}{2}$

B. 2

C. $\frac{11}{2}$

D. $\frac{46}{5}$

Answer: D

Solution:

Solution:
Given $2\cos\theta + \sin\theta = 1$
Squaring both sides,
we get

$(2\cos\theta + \sin\theta)^2 = 1^2$
$\Rightarrow 4\cos^2\theta + \sin^2\theta + 4\sin\theta\cos\theta = 1$
$\Rightarrow 3\cos^2\theta + (\cos^2\theta + \sin^2\theta) + 4\sin\theta\cos\theta = 1$
$\Rightarrow 3\cos^2\theta + \text{not}\, x + 4\sin\theta\cos\theta = \text{not}$
$\Rightarrow 3\cos^2\theta + 4\sin\theta\cos\theta = 0$
$\Rightarrow \cos\theta(3\cos\theta + 4\sin\theta) = 0$
$\Rightarrow 3\cos\theta + 4\sin\theta = 0 \Rightarrow 3\cos\theta = -4\sin\theta$
$\Rightarrow \frac{-3}{4} = \tan\theta = \sqrt{\sec^2\theta - 1} = \frac{-3}{4}$
$\left(\because \tan\theta = \sqrt{\sec^2\theta - 1}\right)$
$\Rightarrow \sec^2\theta - 1 = \left(\frac{-3}{4}\right)^2 = \frac{9}{16}$
$\Rightarrow \sec^2\theta = \frac{9}{16} + 1 = \frac{25}{16} \Rightarrow \sec\theta = \frac{5}{4}$
or $\cos\theta = \frac{4}{5}$ ......(1)
Now, $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta + \left(\frac{4}{5}\right)^2 = 1$
$\sin^2\theta + \frac{4}{5} = 1 \Rightarrow \sin^2\theta = 1 - \frac{16}{25} = \frac{9}{25}$
$\sin\theta = \pm\frac{3}{5}$ ... (2)
Taking $\left(\sin\theta = +\frac{3}{5}\right)$ because $\left(\sin\theta = -\frac{3}{5}\right)$ cannot satisfy the given equation.
Therefore; $7\cos\theta + 6\sin\theta$
$= 7\times\frac{4}{5} + 6\times\frac{3}{5} = \frac{28}{5} + \frac{18}{5} = \frac{46}{5}$

Question87

If $\operatorname{cosec}\theta = \frac{p+q}{p-q}$ $(p \neq q \neq 0)$, then $\left|\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right|$ is equal to: [Online April 9, 2014]

Options:

A. $\sqrt{\frac{p}{q}}$

B. $\sqrt{\frac{q}{p}}$

C. $\sqrt{pq}$

D. pq

Answer: B

Solution:

Solution:
$\operatorname{cosec}\theta = \frac{p+q}{p-q}$, $\sin\theta = \frac{p-q}{p+q}$
$\cos\theta = \pm\sqrt{1 - \sin^2\theta} = \sqrt{1 - \left(\frac{p-q}{p+q}\right)^2} = \frac{2\sqrt{pq}}{(p+q)}$
$\left|\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right| = \left|\frac{\cot\frac{\pi}{4}\cot\frac{\theta}{2} - 1}{\cot\frac{\pi}{4} + \cot\frac{\theta}{2}}\right| = \left|\frac{\cot\frac{\theta}{2} - 1}{\cot\frac{\theta}{2} + 1}\right|$

$$\left| \frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}} \right|$$

On rationalizing denominator, we get

$$\left| \left(\frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}} \right) \left(\frac{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + \sin\frac{\theta}{2}} \right) \right|$$

$$= \left| \frac{\cos\theta}{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} \right|$$

$$= \left| \frac{\cos\theta}{1 + \sin\theta} \right| = \left| \frac{2\sqrt{pq} / (p+q)}{1 + \frac{(p-q)}{p+q}} \right| = \frac{\sqrt{pq}}{p} = \sqrt{\frac{q}{p}}$$

Question88

The number of values of α in [0, 2π] for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$, is:
[Online April 9, 2014]

Options:

A. 6

B. 4

C. 3

D. 1

Answer: C

Solution:

Solution:

$2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha - 2 = 0$

$\Rightarrow 2\sin^2\alpha(\sin\alpha - 1) - 5\sin\alpha(\sin\alpha - 1)$

$+ 2(\sin\alpha - 1) = 0$

$\Rightarrow (\sin\alpha - 1)(2\sin^2\alpha - 5\sin\alpha + 2) = 0$

$\Rightarrow \sin\alpha - 1 = 0$ or $2\sin^2\alpha - 5\sin\alpha + 2 = 0$

$\sin\alpha = 1$ or $\sin\alpha = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4}$

$\alpha = \frac{\pi}{2}$ or $\sin\alpha = \frac{1}{2}, 2$

Now, $\sin\alpha \neq 2$

for, $\sin\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}, \frac{2\pi}{3}$

There are three values of α between [0, 2π]

Question89

The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as:
[2013]

Options:

A. $\sin A \cos A + 1$

B. $\sec A \operatorname{cosec} A + 1$

C. $\tan A + \cot A$

D. $\sec A + \operatorname{cosec} A$

Answer: B

Solution:

Solution:
Given expression can be written as

$$\frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

$$\left(\begin{array}{c} \because \ \tan A = \frac{\sin A}{\cos A} \text{ and} \\ \cot A = \frac{\cos A}{\sin A} \end{array} \right)$$

$$= \frac{1}{\sin A - \cos A} \left\{ \frac{\sin^3 A - \cos^3 A}{\cos A \sin A} \right\}$$

$\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= \frac{\sin^2 A + \sin A \cos A + \cos^2 A}{\sin A \cos A}$$

=1+sec A cosec A

Question90

Let A = {θ : sin(θ) = tan(θ)} and B = {θ : cos(θ) = 1} be two sets. Then : [Online April 25, 2013]

Options:

A. $A = B$

B. $A \subseteq B$

C. $B \subseteq A$

D. $A \subset B$ and $B - A \neq \varphi$

Answer: B

Solution:

Solution:
Let $A = \{\theta : \sin \theta = \tan \theta\}$
and $B = \{\theta : \cos \theta = 1\}$
Now, $A = \left\{ \theta : \sin \theta = \frac{\sin \theta}{\cos \theta} \right\}$
$= \{\theta : \sin \theta(\cos \theta - 1) = 0\}$
$= \{\theta = 0, \pi, 2\pi, 3\pi, \ldots..\}$
For B : $\cos \theta = 1 \Rightarrow \theta = \pi, 2\pi, 4\pi, \ldots\ldots$
This shows that A is not contained in B. i.e. $A \text{not} \subset B$. but $B \subset A$

Question91

The number of solutions of the equation $\sin 2x - 2\cos x + 4\sin x = 4$ in the interval $[0, 5\pi]$ is:
[Online April 23, 2013]

Options:

A. 3

B. 5

C. 4

D. 6

Answer: A

Solution:

Solution:
$\sin 2x - 2\cos x + 4\sin x = 4$
$\Rightarrow 2\sin x \cdot \cos x - 2\cos x + 4\sin x - 4 = 0$
$\Rightarrow (\sin x - 1)(\cos x - 2) = 0$
$\because \cos x - 2 \neq 0, \quad \therefore \sin x = 1$
$\therefore x = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$

Question92

Statement-1: The number of common solutions of the trigonometric equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is two.
Statement- 2 : The number of solutions of the equation, $2\cos^2\theta - 3\sin\theta = 0$ in the interval $[0, \pi]$ is two.
[Online April 22, 2013]

Options:

A. Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1 .

B. Statement-1 is true; Statement- 2 is true; Statement-2 is not a correct explanation for statement-1.

C. Statement-1 is false; Statement- 2 is true.

D. Statement- 1 is true; Statement- 2 is false.

Answer: B

Solution:

Solution:
$2\sin^2\theta - \cos 2\theta = 0$
$\Rightarrow 2\sin^2\theta - (1 - 2\sin^2\theta) = 0$
$\Rightarrow 2\sin^2\theta - 1 + 2\sin^2\theta = 0$
$\Rightarrow 4\sin^2\theta = 1 \Rightarrow \sin\theta = \pm\frac{1}{2}$
$\therefore \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \theta \in [0, 2\pi]$
$\therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$
Now $2\cos^2\theta - 3\sin\theta = 0$
$\Rightarrow 2(1 - \sin^2\theta) - 3\sin\theta = 0$
$\Rightarrow -2\sin^2\theta - 3\sin\theta + 2 = 0$
$\Rightarrow -2\sin^2\theta - 4\sin\theta + \sin\theta + 2 = 0$
$\Rightarrow 2\sin^2\theta - \sin\theta + 4\sin\theta - 2 = 0$
$\Rightarrow \sin\theta(2\sin\theta - 1) + 2(2\sin\theta - 1) = 0$
Now $2\cos^2\theta - 3\sin\theta = 0$
$\Rightarrow 2(1 - \sin^2\theta) - 3\sin\theta = 0$
$\Rightarrow -2\sin^2\theta - 3\sin\theta + 2 = 0$
$\Rightarrow -2\sin^2\theta - 4\sin\theta + \sin\theta + 2 = 0$
$\Rightarrow 2\sin^2\theta - \sin\theta + 4\sin\theta - 2 = 0$
$\Rightarrow \sin\theta(2\sin\theta - 1) + 2(2\sin\theta - 1) = 0$
$\Rightarrow \sin\theta = \frac{1}{2}, -2$
But $\sin\theta = -2$, is not possible $\therefore \sin\theta = \frac{1}{2}, -2 \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}$
Hence, there are two common solution, there each of the statement- 1 and 2 are true but statement- 2 is not a correct explanation for statement-1.

Question93

The value of $\cos 255^\circ + \sin 195^\circ$ is [Online May 26, 2012]

Options:

A. $\frac{\sqrt{3} - 1}{2\sqrt{2}}$

B. $\frac{\sqrt{3} - 1}{\sqrt{2}}$

C. $-\left(\frac{\sqrt{3} - 1}{\sqrt{2}}\right)$

D. $\frac{\sqrt{3} + 1}{\sqrt{2}}$

Answer: C

Solution:

Solution:
Consider $\cos 255^\circ + \sin 195^\circ$
$= \cos(270^\circ - 15^\circ) + \sin(180^\circ + 15^\circ)$
$= -\sin 15^\circ - \sin 15^\circ$
$= -2\sin 15^\circ = -2\left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right) = -\left(\frac{\sqrt{3} - 1}{\sqrt{2}}\right)$

Question94

Let $f(x) = \sin x$, $g(x) = x$
Statement 1: $f(x) \leq g(x)$ for x in $(0, \infty)$
Statement 2 : $f(x) \leq 1$ for x in $(0, \infty)$ but $g(x) \to \infty$ as $x \to \infty$.
[Online May 7, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1
.

C. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1 .

D. Statement 1 is false, Statement 2 is true.

Answer: C

Solution:

Solution:
Let$f(x) = \sin x$ and $g(x) = x$
Statement-1: $f(x) \leq g(x)\ \forall x \in (0, \infty)$
i.e., $\sin x \leq x\ \forall x \in (0, \infty)$
which is true
Statement- $2 : f(x) \leq 1\ \forall x \in (0, \infty)$
i.e., $\sin x \leq 1\ \forall x \in (0, \infty)$
It is true and
$g(x) = x \to \infty$ as $x \to \infty$ also true.

Question95

The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has:
[2012]

Options:

A. infinite number of real roots

B. no real roots

C. exactly one real root

D. exactly four real roots

Answer: B

Solution:

Solution:
Given equation is $e^{\sin x} - e^{-\sin x} - 4 = 0$
Put $e^{\sin x} = t$ in the given equation, we get
$t^2 - 4t - 1 = 0$

$\Rightarrow \ t = \frac{4 \pm \sqrt{16+4}}{2} = \frac{4 \pm \sqrt{20}}{2}$

$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

$\Rightarrow \ e^{\sin x} = 2 \pm \sqrt{5} (\because t = e^{\sin x})$

$\Rightarrow e^{\sin x} = 2 - \sqrt{5}$ and $e^{\sin x} = 2 + \sqrt{5}$

$\Rightarrow \ e^{\sin x} = 2 - \sqrt{5} < 0$

and $\sin x = \ln(2 + \sqrt{5}) > 1$

So, rejected.

Hence, given equation has no solution.

$\therefore$ The equation has no real roots.

Question96

If $A = \sin^2 x + \cos^4 x$, then for all real x : [2011]

Options:

A. $\frac{13}{16} \leq A \leq 1$

B. $1 \leq A \leq 2$

C. $\frac{3}{4} \leq A \leq \frac{13}{16}$

D. $\frac{3}{4} \leq A \leq 1$

Answer: D

Solution:

Solution:

$A = \sin^2 x + \cos^4 x$

$= \sin^2 x + \cos^2 x (1 - \sin^2 x)$

$= \sin^2 x + \cos^2 x - \frac{1}{4}(2 \sin x \cdot \cos x)^2$

$= 1 - \frac{1}{4}\sin^2(2x)$

$\because -1 \leq \sin 2x \leq 1$

$\Rightarrow 0 \leq \sin^2(2x) \leq 1$

$\Rightarrow 0 \geq -\frac{1}{4}\sin^2(2x) \geq -\frac{1}{4}$

$\Rightarrow 1 \geq 1 - \frac{1}{4}\sin^2(2x) \geq 1 - \frac{1}{4}$

$\Rightarrow 1 \geq A \geq \frac{3}{4}$

Question97

The possible values of $\theta \in (0, \pi)$ such that $\sin(\theta) + \sin(4\theta) + \sin(7\theta) = 0$ are [2011RS]

Options:

A. $\frac{\pi}{4}, \frac{5\pi}{12}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

B. $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{35\pi}{36}$

C. $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{8\pi}{9}$

D. $\frac{2\pi}{9}, \frac{\pi}{4}, \frac{4\pi}{9}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{8\pi}{9}$

Answer: D

Solution:

Solution:

$\sin 4\theta + 2\sin 4\theta \cos 3\theta = 0$

$\sin 4\theta(1 + 2\cos 3\theta) = 0$

$\sin 4\theta = 0$

or $\cos 3\theta = -\frac{1}{2}$

$4\theta = n\pi;\ n \in I$

or $3\theta = 2n\pi \pm \frac{2\pi}{3}, n \in I$

$\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ or $\theta = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}$ $[\because \theta, \in (0, \pi)]$

Question98

Let $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$ where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$ [2010]

Options:

A. $\frac{56}{33}$

B. $\frac{19}{12}$

C. $\frac{20}{7}$

D. $\frac{25}{16}$

Answer: A

Solution:

Solution:

$\cos(\alpha + \beta) = \frac{4}{5} \Rightarrow \tan(\alpha + \beta) = \frac{3}{4}$

$\sin(\alpha - \beta) = \frac{5}{13} \Rightarrow \tan(\alpha - \beta) = \frac{5}{12}$

$\tan 2\alpha = \tan[(\alpha + \beta) + (\alpha - \beta)$

$$= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta)\tan(\alpha - \beta)} = \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{56}{33}$$

Question99

Let A and B denote the statements
A : $\cos\alpha + \cos\beta + \cos\gamma = 0$
B : $\sin\alpha + \sin\beta + \sin\gamma = 0$
I f $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$, then :
[2009]

Options:

A. A is false and B is true

B. both A and B are true

C. both A and B are false

D. A is true and B is false

Answer: B

Solution:

Solution:
Given that
$\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -\frac{3}{2}$
$\Rightarrow\ 2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)] + 3 = 0$
$\Rightarrow\ 2[\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)]$
$+\sin^2\alpha + \cos^2\alpha + \sin^2\beta + \cos^2\beta$
$+\sin^2\gamma + \cos^2\alpha = 0$
$\Rightarrow\ [\sin^2\alpha + \sin^2\beta + \sin^2\gamma + 2\sin\alpha\sin\beta$
$+2\sin\beta\sin\gamma + 2\sin\gamma\sin\alpha] + [\cos^2\alpha + \cos^2\beta$
$+\cos^2\gamma + 2\cos\alpha\cos\beta + 2\cos\beta\cos\gamma$
$+2\cos\gamma\cos\alpha] = 0$
$[\because \cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B]$
$\Rightarrow\ [\sin\alpha + \sin\beta + \sin\gamma]^2 + (\cos\alpha + \cos\beta + \cos\gamma)^2 = 0$
$\Rightarrow\ \sin\alpha + \sin\beta + \sin\gamma = 0$ and $\cos\alpha + \cos\beta + \cos\gamma = 0$
$\therefore$ A and B both are true.

Question100

If p and q are positive real numbers such that $p^2 + q^2 = 1$ then the maximum value of $(p + q)$ is
[2007]

Options:

A. $\frac{1}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\sqrt{2}$

D. 2.

Answer: C

Solution:

Solution:
Given that $p^2 + q^2 = 1$
$\therefore p = \cos\theta$ and $q = \sin\theta$ satisfy the given equation
Then $p + q = \cos\theta + \sin\theta$
We know that
$-\sqrt{a^2 + b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2 + b^2}$
$\therefore -\sqrt{2} \le \cos\theta + \sin\theta \le \sqrt{2}$
Hence max. value of $p + q$ is $\sqrt{2}$

Question101

A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length x. The maximum area enclosed by the park is [2006]

Options:

A. $\frac{3}{2}x^2$

B. $\sqrt{\frac{x^3}{8}}$

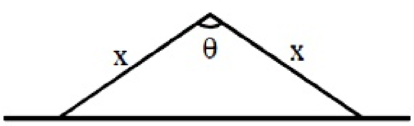
C. $\frac{1}{2}x^2$

D. πx^2

Answer: C

Solution:

Solution:
Area $= \frac{1}{2}x^2\sin\theta$



Maximum value of $\sin\theta$ is 1 at $\theta = \frac{\pi}{2}$

$A_{max} = \frac{1}{2}x^2$

Question102

If $0 < x < \pi$ and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is [2006]

Options:

A. $\frac{(1-\sqrt{7})}{4}$

B. $\frac{(4-\sqrt{7})}{3}$

C. $-\frac{(4+\sqrt{7})}{3}$

D. $\frac{(1+\sqrt{7})}{4}$

Answer: C

Solution:

Solution:

$\cos x + \sin x = \frac{1}{2} \Rightarrow 1 + \sin 2x = \frac{1}{4}$

$\Rightarrow \sin 2x = -\frac{3}{4}$

$\therefore \pi < 2x < 2\pi$

$\Rightarrow \frac{\pi}{2} < x \leq \pi$ ....(i)

$\frac{2\tan x}{1+\tan^2 x} = -\frac{3}{4}$

$\Rightarrow 3\tan^2 x + 8\tan x + 3 = 0$

$\therefore \tan x = \frac{-8 \pm \sqrt{64-36}}{6} = -\frac{-4 \pm \sqrt{7}}{3}$

for $\frac{\pi}{2} < x < \pi$, $\tan x < 0$

$\therefore \tan x = \frac{-4-\sqrt{7}}{3}$

Question103

The number of values of x in the interval [0, 3π] satisfying the equation $2\sin^2 x + 5\sin x - 3 = 0$ is [2006]

Options:

A. 4

B. 6

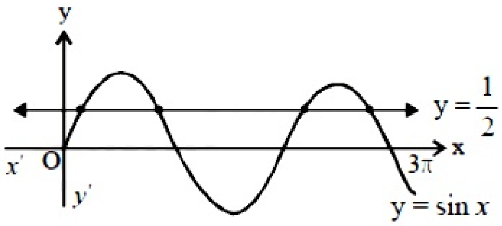
C. 1

D. 2

Answer: A

Solution:

Solution:



$2\sin^2 x + 5\sin x - 3 = 0$
$\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$
$\Rightarrow \sin x = \frac{1}{2}$ and $\sin x \neq -3$
∴ In [0, 3π], x has 4 values.

Question104

If $\mathbf{u = \sqrt{a^2\cos^2\theta + b^2\sin^2\theta} + \sqrt{a^2\sin^2\theta + b^2\cos^2\theta}}$
then the difference between the maximum and minimum values of $\mathbf{u^2}$ **is given by**
[2004]

Options:

A. $(a - b)^2$

B. $2\sqrt{a^2 + b^2}$

C. $(a + b)^2$

D. $2(a^2 + b^2)$

Answer: A

Solution:

Solution:
$u^2 = a^2 + b^2 + 2\sqrt{\left(a^4 + b^4\right)\cos^2\theta\sin^2\theta; + a^2b^2(\cos^4\theta + \sin^4\theta)}$. ....(1)
Now, $(a^4 + b^4)\cos^2\theta\sin^2\theta + a^2b^2(\cos^4\theta + \sin^4\theta)$
$= (a^4 + b^4)\cos^2\theta\sin^2\theta + a^2b^2(1 - 2\cos^2\theta\sin^2\theta)$
$= (a^4 + b^4 - 2a^2b^2)\cos^2\theta\sin^2\theta + a^2b^2$
$= (a^2 - b^2)^2 \cdot \frac{\sin^2 2\theta}{4} + a^2b^2$ ....(2)
$\because 0 \leq \sin^2 2\theta \leq 1$
$\Rightarrow 0 \leq (a^2 - b^2)^2 \frac{\sin^2 2\theta}{4} \leq \frac{(a^2 - b^2)^2}{4}$
$\Rightarrow a^2b^2 \leq (a^2 - b^2)^2 \frac{\sin^2 2\theta}{4} + a^2b^2$
$\leq (a^2 - b^2)^2 \cdot \frac{1}{4} + a^2b^2$ ....(3)
From (1)
$a^2 + b^2 + 2\sqrt{a^2b^2} \leq u^2 \leq a^2 + b^2 + \frac{2}{2}\sqrt{(a^2 + b^2)^2}$
$(a + b)^2 \leq u^2 \leq 2(a^2 + b^2)$
∴ Max. value − Min. value
$= 2(a^2 + b^2) - (a + b^2) = (a - b)^2$

Question105

Let α, β be such that $\pi < \alpha - \beta < 3\pi$.
If $\sin\alpha + \sin\beta = -\frac{21}{65}$ and $\cos\alpha + \cos\beta = -\frac{27}{65}$, then the value of $\cos\frac{\alpha-\beta}{2}$
[2004]

Options:

A. $\frac{-6}{65}$

B. $\frac{3}{\sqrt{130}}$

C. $\frac{6}{65}$

D. $-\frac{3}{\sqrt{130}}$

Answer: D

Solution:

Solution:
$\pi < \alpha - \beta < 3\pi$
$\Rightarrow \frac{\pi}{2} < \frac{\alpha-\beta}{2} < \frac{3\pi}{2} \Rightarrow \cos\frac{\alpha-\beta}{2} < 0$ ......(1)
$\sin\alpha + \sin\beta = -\frac{21}{65}$
$\Rightarrow 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{21}{65}$ ....(2)
$\cos\alpha + \cos\beta = -\frac{27}{65}$
$\Rightarrow 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = -\frac{27}{65}$ ......(3)
Squaring and adding (2) and (3) , we get
$4\cos^2\frac{\alpha-\beta}{2} = \frac{(21)^2+(27)^2}{(65)^2} = \frac{1170}{65\times 65}$
$\therefore \cos^2\frac{\alpha-\beta}{2} = \frac{9}{130} \Rightarrow \cos\frac{\alpha-\beta}{2} = -\frac{3}{\sqrt{130}}$ [from(1)]

Question106

The function $f(x) = \log\left(x + \sqrt{x^2+1}\right)$, is
[2003]

Options:

A. neither an even nor an odd function

B. an even function

C. an odd function

D. a periodic function.

Answer: C

Solution:

Given $f(x) = \log\left(x + \sqrt{x^2 + 1}\right)$

$f(-x) = \log\left\{-x + \sqrt{x^2 + 1}\right\} = \log\left\{\dfrac{x^2 - x^2 + 1}{x + \sqrt{x^2 + 1}}\right\}$

$= -\log\left(x + \sqrt{x^2 + 1}\right) = -f(x)$

$\Rightarrow f(x)$ is an odd function.

Question107

The period of $\sin^2\theta$ is [2002]

Options:

A. π^2

B. π

C. 2π

D. $\pi / 2$

Answer: B

Solution:

We know that $\sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$;

Since period of $\cos 2\theta = \dfrac{2\pi}{2} = \pi$

Hence period of $\sin^2\theta$ is also π.

Question108

Which one is not periodic? [2002]

Options:

A. $|\sin 3x| + \sin^2 x$

B. $\cos\sqrt{x} + \cos^2 x$

C. $\cos 4x + \tan^2 x$

D. $\cos 2x + \sin x$

Answer: B

Solution:

we know that $\cos\sqrt{x}$ is non periodic
$\therefore \cos\sqrt{x} + \cos^2 x$ can not be periodic.

Question109

The number of solution of $\tan x + \sec x = 2\cos x$ in $[0, 2\pi)$ is [2002]

Options:

A. 2

B. 3

C. 0

D. 1

Answer: B

Solution:

$\because \tan x + \sec x = 2\cos x$
$\Rightarrow \sin x + 1 = 2\cos^2 x$
$\Rightarrow \sin x + 1 = 2(1 - \sin^2 x)$
$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$
$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$
$\Rightarrow \sin x = \dfrac{1}{2}, -1.$
$\Rightarrow x = 30^\circ, 150^\circ, 270^\circ$
Number of solution $= 3$
