SECTION-A

1. If $f(x)=\begin{cases} x^3\sin\left(\frac{1}{x}\right) & , \quad x\neq 0 \\ 0 & , \quad x=0 \end{cases}$, then

(1) $f''(0)=1$ (2) $f''\left(\frac{2}{\pi}\right)=\frac{24-\pi^2}{2\pi}$

(3) $f''\left(\frac{2}{\pi}\right)=\frac{12-\pi^2}{2\pi}$ (4) $f''(0)=0$

Ans. (2)

Sol. $f'(x) = 3x^2\sin\left(\frac{1}{x}\right)-x\cos\left(\frac{1}{x}\right)$

$$f''(x)=6x\sin\left(\frac{1}{x}\right)-3\cos\left(\frac{1}{x}\right)-\cos\left(\frac{1}{x}\right)-\frac{\sin\left(\frac{1}{x}\right)}{x}$$

$$f''\left(\frac{2}{\pi}\right)=\frac{12}{\pi}-\frac{\pi}{2}=\frac{24-\pi^2}{2\pi}$$

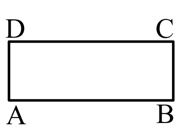
2. If $A(3,1,-1)$, $B\left(\frac{5}{3},\frac{7}{3},\frac{1}{3}\right)$, $C(2,2,1)$ and $D\left(\frac{10}{3},\frac{2}{3},\frac{-1}{3}\right)$ are the vertices of a quadrilateral ABCD, then its area is

(1) $\frac{4\sqrt{2}}{3}$ (2) $\frac{5\sqrt{2}}{3}$

(3) $2\sqrt{2}$ (4) $\frac{2\sqrt{2}}{3}$

Ans. (1)

Sol.



Area $=\frac{1}{2}\left|\overline{BD}\times\overline{AC}\right|$

$\overline{BD}=\frac{5}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{2}{3}\hat{k}$

$\overline{AC}=\hat{i}-\hat{j}-2\hat{k}$

3. $\int_0^{\pi/4}\frac{\cos^2x\sin^2x}{\left(\cos^3x+\sin^3x\right)^2}dx$ is equal to

(1) 1/12 (2) 1/9

(3) 1/6 (4) 1/3

Ans. (3)

Sol. Divide Nr & Dr by cosx

$$\int_0^{\pi/4}\frac{\tan^2x\sec^2xdx}{\left(1+\tan^3x\right)^2}dx$$

Let $1+\tan^3x=t$

$\tan^2x\sec^2x\,dx=\frac{dt}{3}$

$$\frac{1}{3}\int_1^2\frac{dt}{t^2}=\frac{1}{6}$$

4. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On respectively, it was found that an observation by mistake was taken 8 instead of 12. The correct standard deviation is

(1) $\sqrt{3.86}$ (2) 1.8

(3) $\sqrt{3.96}$ (4) 1.94

Ans. (3)

Sol. Mean $(\bar{x})=10$

$\Rightarrow\frac{\Sigma x_i}{20}=10$

$\Sigma x_i=10\times20=200$

If 8 is replaced by 12, then $\Sigma x_i=200-8+12=204$

$\therefore$ Correct mean $(\overline{x}) = \frac{\Sigma x_i}{20}$

$= \frac{204}{20} = 10.2$

$\because$ Standard deviation = 2

$\therefore$ Variance = $(S.D.)^2 = 2^2 = 4$

$\Rightarrow \frac{\Sigma x_i^2}{20} - \left(\frac{\Sigma x_i}{20}\right)^2 = 4$

$\Rightarrow \frac{\Sigma x_i^2}{20} - (10)^2 = 4$

$\Rightarrow \frac{\Sigma x_i^2}{20} = 104$

$\Rightarrow \Sigma x_i^2 = 2080$

Now, replaced '8' observations by '12'

Then, $\Sigma x_i^2 = 2080 - 8^2 + 12^2 = 2160$

$\therefore$ Variance of removing observations

$\Rightarrow \frac{\Sigma x_i^2}{20} - \left(\frac{\Sigma x_i}{20}\right)^2$

$\Rightarrow \frac{2160}{20} - (10.2)^2$

$\Rightarrow 108 - 104.04$

$\Rightarrow 3.96$

Correct standard deviation

$= \sqrt{3.96}$

5. The function $f(x) = \frac{x^2 + 2x - 15}{x^2 - 4x + 9}$, $x \in R$ is

(1) both one-one and onto.

(2) onto but not one-one.

(3) neither one-one nor onto.

(4) one-one but not onto.

NTA Ans. (3)

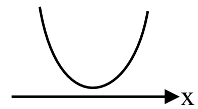
Ans. Bonus

Sol. $f(x) = \frac{(x+5)(x-3)}{x^2 - 4x + 9}$

Let $g(x) = x^2 - 4x + 9$

$D < 0$

$g(x) > 0$ for $x \in R$



$\therefore \left[\begin{array}{l} f(-5) = 0 \\ f(3) = 0 \end{array}\right.$

So, f(x) is many-one.

again,

$yx^2 - 4xy + 9y = x^2 + 2x - 15$

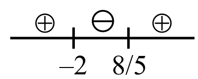
$x^2(y - 1) - 2x(2y + 1) + (9y + 15) = 0$

for $\forall x \in R \Rightarrow D \geq 0$

$D = 4(2y + 1)^2 - 4(y - 1)(9y + 15) \geq 0$

$5y^2 + 2y + 16 \leq 0$

$(5y - 8)(y + 2) \leq 0$



$\boxed{y \in \left[-2, \frac{8}{5}\right]}$ range

Note : If function is defined from $f : R \to R$ then only correct answer is option (3)

$\Rightarrow$ Bonus

6. Let $A = \{n \in [100, 700] \cap N$: n is neither a multiple of 3 nor a multiple of 4$\}$. Then the number of elements in A is

(1) 300 (2) 280

(3) 310 (4) 290

Ans. (1)

Sol. $n(3) \Rightarrow$ multiple of 3

102, 105, 108, ... , 699

$T_n = 699 = 102 + (n - 1)(3)$

$n = 200$

$n(3) = 200$

$\because n(4) \Rightarrow$ multiple of 4

100, 104, 108, ..., 700

$T_n = 700 = 100 + (n-1)(4)$

$n = 151$

$n(4) = 151$

$n(3\cap 4) \Rightarrow$ multiple of 3 & 4 both

108, 120, 132, ..., 696

$T_n = 696 = 108 + (n-1)(12)$

$n = 50$

$n(3 \cap 4) = 50$

$n(3 \cup 4) = n(3) + n(4) - n(3 \cap 4)$

$= 200 + 151 - 50$

$= 301$

$n\left(\overline{3\cup 4}\right)$ = Total – $n(3 \cup 4)$ = neither a multiple of 3 nor a multiple of 4

$= 601 - 301 = 300$

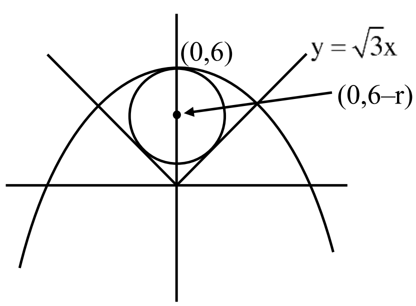
7. Let C be the circle of minimum area touching the parabola $y = 6 - x^2$ and the lines $y=\sqrt{3}|x|$. Then, which one of the following points lies on the circle C ?

(1) (2, 4) (2) (1, 2)

(3) (2, 2) (4) (1, 1)

Ans. (1)

Sol.



Equation of circle

$x^2 + (y - (6 - r))^2 = r^2$

touches $\sqrt{3}x - y = 0$

$p = r$

$$\frac{|0-(6-r)|}{2} = r$$

$|r - 6| = 2r$

$r = 2$

$\therefore$ Circle $x^2 + (y - 4)^2 = 4$

(2, 4) Satisfies this equation

8. For $\alpha, \beta \in R$ and a natural number n, let

$$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$. Then $2A_{10} - A_8$ is

(1) $4\alpha + 2\beta$ (2) $2\alpha + 4\beta$

(3) 2n (4) 0

Ans. (1)

Sol. $$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$2A_{10} - A_8 = \begin{vmatrix} 20 & 1 & \frac{n^2}{2}+\alpha \\ 40 & 2 & n^2-\beta \\ 56 & 3 & \frac{n(3n-1)}{2} \end{vmatrix} - \begin{vmatrix} 8 & 1 & \frac{n^2}{2}+\alpha \\ 16 & 2 & n^2-\beta \\ 22 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 12 & 1 & \frac{n^2}{2}+\alpha \\ 24 & 2 & n^2-\beta \\ 34 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & 1 & \frac{n^2}{2}+\alpha \\ 0 & 2 & n^2-\beta \\ -2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$\Rightarrow -2\left((n^2-\beta)-(n^2+2\alpha)\right)$

$\Rightarrow -2(-\beta-2\alpha) \Rightarrow 4\alpha+2\beta$

9. The shortest distance between the lines

$\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ is

(1) $6\sqrt{3}$ (2) $4\sqrt{3}$

(3) $5\sqrt{3}$ (4) $8\sqrt{3}$

Ans. (2)

Sol. $\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$ & $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$

$$\text{S.D} = \frac{\left|(\bar{a}_2.\bar{a}_1).(\bar{b}_1.\bar{b}_2)\right|}{\left|\bar{b}_1\times\bar{b}_2\right|}$$

$a_1 = 3, -15, 9$ $b_1 = 2, -7, 5$

$a_2 = -1, 1, 9$ $b_2 = 2, 1, -3$

$a_2 - a_1 = -4, 16, 0$

$$\bar{b}_1\times\bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix} = \hat{i}(16)-\hat{j}(-16)+\hat{k}(16)$$

$16(\hat{i}+\hat{j}+\hat{k})$

$\left|\bar{b}_1\times\bar{b}_2\right| = 16\sqrt{3}$

$\therefore\ (\bar{a}_2-\bar{a}_1).(\bar{b}_1-\bar{b}_2) = 16[-4+16] = (16)(12)$

$$\text{S.D.} = \frac{(16)(12)}{16\sqrt{3}} = 4\sqrt{3}$$

10. A company has two plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B. 80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then 126p is

(1) 54 (2) 64

(3) 66 (4) 56

Ans. (1)

Sol.

	A	B
Manufactured	60%	40%
Standard quality	80%	90%

P(Manufactured at B / found standard quality) = ?

A : Found S.Q

B : Manufacture B

C : Manufacture A

$P(E_1) = \frac{40}{100}$

$P(E_2) = \frac{60}{100}$

$P(A/E_1) = \frac{90}{100}$

$P(A/E_2) = \frac{80}{100}$

$$\because P(E_1/A) = \frac{P(A/E_1)\,P(E_1)}{P(A/E_1)\,P(E_1)+P(A/E_2)\,P(E_2)} = \frac{3}{7}$$

$\therefore$ 126 P = 54

11. Let, α, β be the distinct roots of the equation

$x^2-(t^2-5t+6)x+1=0,\ t\in R$ and $a_n = \alpha^n+\beta^n$.

Then the minimum value of $\frac{a_{2023}+a_{2025}}{a_{2024}}$ is

(1) 1/4 (2) –1/2

(3) –1/4 (4) 1/2

Ans. (3)

Sol. by newton's theorem

$a_{n+2} - (t^2 - 5t + 6)a_{n+1} + a_n = 0$

$\therefore\ a_{2025} + a_{2023} = (t^2 - 5t + 6)\ a_{2024}$

$$\therefore\ \frac{a_{2025}+a_{2023}}{a_{2024}} = t^2-5t+6$$

$$\because\ t^2-5t+6 = \left(t-\frac{5}{2}\right)^2-\frac{1}{4}$$

$$\therefore\ \text{minimum value} = -\frac{1}{4}$$

12. Let the relations R_1 and R_2 on the set $X = \{1, 2, 3, ..., 20\}$ be given by $R_1 = \{(x, y) : 2x - 3y = 2\}$ and $R_2 = \{(x, y) : -5x + 4y = 0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then M + N equals

(1) 8 (2) 16

(3) 12 (4) 10

Ans. (4)

Sol. $x = \{1, 2, 3, ...20\}$

$R_1 = \{(x, y) : 2x - 3y = 2\}$

$R_2 = \{(x, y) : -5x + 4y = 0\}$

$R_1 = \{(4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)\}$

$R_2 = \{(4, 5), (8, 10), (12, 15), (16, 20)\}$

in R_1 6 element needed

in R_2 4 element needed

So, total 6+4 = 10 element

13. Let a variable line of slope m > 0 passing through the point (4, –9) intersect the coordinate axes at the points A and B. the minimum value of the sum of the distances of A and B from the origin is

(1) 25 (2) 30

(3) 15 (4) 10

Ans. (1)

Sol. equation of line is

$y + 9 = m(x - 4)$

$\therefore\ A = \left(\frac{9+4m}{m}, 0\right)$

$B = (0, -9 - 4m)$

$\therefore\ OA + OB = \frac{9+4m}{m} + 9 + 4m$

$\because\ m > 0$

$= 13 + \frac{9}{m} + 4m$

$\because\ \frac{4m + \frac{9}{m}}{2} \geq \sqrt{36} \Rightarrow 4m + \frac{9}{m} \geq 12$

$\therefore\ OA + OB \geq 25$

14. The interval in which the function $f(x) = x^x$, $x > 0$, is strictly increasing is

(1) $\left(0, \frac{1}{e}\right]$ (2) $\left[\frac{1}{e^2}, 1\right)$

(3) $(0, \infty)$ (4) $\left[\frac{1}{e}, \infty\right)$

Ans. (4)

Sol. $f(x) = x^x$; $x > 0$

$\ell ny = x\ell nx$

$\frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \ell nx$

$\frac{dy}{dx} = x^x(1 + \ell nx)$

for strictly increasing

$\frac{dy}{dx} \geq 0 \quad \Rightarrow x^x(1 + \ell nx) \geq 0$

$\Rightarrow \ell nx \geq -1$

$x \geq e^{-1}$

$x \geq \frac{1}{e}$

$x \in \left[\frac{1}{e}, \infty\right)$

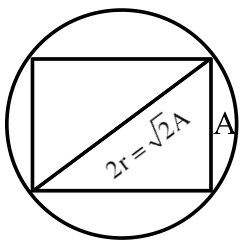
15. A circle in inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n, respectively, then m + n^2 is equal to

(1) 396 (2) 408

(3) 312 (4) 414

Ans. (2)

Sol. $\because r = \frac{\Delta}{s} = \frac{\sqrt{3}a^2}{4.\frac{3a}{2}} = \frac{a}{2\sqrt{3}} = \frac{12}{2\sqrt{3}} = 2\sqrt{3}$



$\therefore$ **A =** $r\sqrt{2} = 2\sqrt{6}$

Area = m = A^2 = 24

Perimeter = n = 4A = $8\sqrt{6}$

$\therefore$ m + n^2 = 24 + 384

= 408

16. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is

(1) 24 (2) 56

(3) 16 (4) 48

Ans. (3)

Sol. $\because$ no. of triangles having no side common with a n sided polygon = $\frac{{}^{n}C_1 \cdot {}^{n-4}C_2}{3}$

$= \frac{{}^{8}C_1 \cdot {}^{4}C_2}{3} = 16$

17. Let y = y(x) be the solution of the differential equation $\left(1+x^2\right)\frac{dy}{dx} + y = e^{\tan^{-1}x}$, y(1) = 0. Then y(0) is

(1) $\frac{1}{4}\left(e^{\pi/2}-1\right)$ (2) $\frac{1}{2}\left(1-e^{\pi/2}\right)$

(3) $\frac{1}{4}\left(1-e^{\pi/2}\right)$ (4) $\frac{1}{2}\left(e^{\pi/2}-1\right)$

Ans. (2)

Sol. $\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$

I.F. = $e^{\int\frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$

$y.e^{\tan^{-1}x} = \int\left(\frac{e^{\tan^{-1}x}}{1+x^2}\right)e^{\tan^{-1}x}.dx$

Let $\tan^{-1}x = z$ $\quad\therefore \frac{dx}{1+x^2} = dz$

$\therefore y.e^{z} = \int e^{2z}dz = \frac{e^{2z}}{2} + C$

$y.e^{\tan^{-1}x} = \frac{e^{2\tan^{-1}x}}{2} + C$

$\Rightarrow y = \frac{e^{\tan^{-1}x}}{2} + \frac{C}{e^{\tan^{-1}x}}$

$\because y(1) = 0 \Rightarrow 0 = \frac{e^{\pi/4}}{2} + \frac{C}{e^{\pi/4}} \Rightarrow C = \frac{-e^{\pi/2}}{2}$

$\therefore y = \frac{e^{\tan^{-1}x}}{2} - \frac{e^{\pi/2}}{2e^{\tan^{-1}x}}$

$\boxed{\therefore y(0) = \frac{1-e^{\pi/2}}{2}}$

18. Let y = y(x) be the solution of the differential equation $\left(2x\log_e x\right)\frac{dy}{dx}+2y=\frac{3}{x}\log_e x$, x > 0 and $y(e^{-1}) = 0$. Then, y(e) is equal to

(1) $-\frac{3}{2e}$ (2) $-\frac{2}{3e}$

(3) $-\frac{3}{e}$ (4) $-\frac{2}{e}$

Ans. (3)

Sol. $\frac{dy}{dx}+\frac{y}{x\,\ell n\,x}=\frac{3}{2x^2}$

$\therefore$ I.F. = $e^{\int\frac{1}{x\,\ell n\,x}dx}=e^{\ell n(\ell n(x))}=\ell nx$

$\therefore\ y\ell nx=\int\frac{3\ell n\,x}{2x^2}dx$

$=\frac{3\ell n\,x}{2}\int x^{-2}dx-\int\left(\frac{3}{2x}.\int x^{-2}\,dx\right)dx$

$=\frac{3\ell n\,x}{2}\left(-\frac{1}{x}\right)-\int\frac{3}{2x}\left(-\frac{1}{x}\right)dx$

$y.\ \ell nx=\frac{-3\ell nx}{2x}-\frac{3}{2x}+C$

$\because\ y(e^{-1})=0$

$\therefore\ 0\,(-1)=\frac{3e}{2}-\frac{3e}{2}+C\quad\Rightarrow C=0$

$\therefore\ y=\frac{-3\ell nx}{2x}-\frac{3}{2x}$

$\boxed{\therefore y(e)=\frac{-3}{2e}-\frac{3}{2e}=\frac{-3}{e}}$

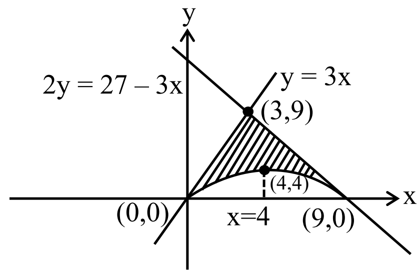
19. Let the area of the region enclosed by the curves y = 3x, 2y = 27 – 3x and $y=3x-x\sqrt{x}$ be A. Then 10 A is equal to

(1) 184 (2) 154

(3) 172 (4) 162

Ans. (4)

Sol. y = 3x, 2y = 27 – 3x & $y = 3x - x\sqrt{x}$



$A=\int_0^3 3x-\left(3x-x\sqrt{x}\right)dx+\int_3^9\left(\frac{27-3x}{2}-\left(3x-x\sqrt{x}\right)\right)dx$

$A=\int_0^3 x^{3/2}dx+\int_3^9\frac{27}{2}-\frac{9x}{2}+x^{3/2}dx$

$A=\left[\frac{2x^{5/2}}{5}\right]_0^3+\frac{27}{2}[x]_3^9-\frac{9}{2}\left[\frac{x^2}{2}\right]_3^9+\left[\frac{2x^{5/2}}{5}\right]_3^9$

$A=\frac{2}{5}\left(3^{5/2}\right)+\frac{27}{2}(6)-\frac{9}{4}(72)+\frac{2}{5}\left(9^{5/2}-3^{5/2}\right)$

$A=\frac{2}{5}\left(3^{5/2}\right)+81-162+\frac{2}{5}\times3^5-\frac{2}{5}\times3^{5/2}$

$A=\frac{486}{5}-81=\frac{81}{5}$

10A = 162

Ans. = 4

20. Let $f:(-\infty,\infty)-\{0\}\to R$ be a differentiable function such that $f'(1)=\lim_{a\to\infty}a^2f\left(\frac{1}{a}\right)$.

Then $\lim_{a\to\infty}\frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right)+a^2-2\log_e a$ is equal to

(1) $\frac{3}{2}+\frac{\pi}{4}$ (2) $\frac{3}{8}+\frac{\pi}{4}$

(3) $\frac{5}{2}+\frac{\pi}{8}$ (4) $\frac{3}{4}+\frac{\pi}{8}$

Ans. (3)

Sol. $f:(-\infty,\infty)-\{0\}\to R$

$$f'(1)=\lim_{a\to\infty}a^2f\left(\frac{1}{a}\right)$$

$$\lim_{a\to\infty}\frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right)+a^2-2\ell n(a)$$

$$\lim_{a\to\infty}a^2\left(\frac{\left(1+\frac{1}{a}\right)}{2}\tan^{-1}\left(\frac{1}{a}\right)+1-\frac{2}{a^2}\ell n(a)\right)$$

$$f(x)=\frac{1}{2}(1+x)\tan^{-1}(x)+1-2x^2\ell n(x)$$

$$f'(x)=\frac{1}{2}\left(\frac{1+x}{1+x^2}+\tan^{-1}(x)+4x\ell n(x)\right)+2x$$

$$f'(1)=\frac{1}{2}\left(1+\frac{\pi}{4}\right)+2$$

$$f'(1)=\frac{5}{2}+\frac{\pi}{8}$$

Ans. (3)

SECTION-B

21. Let $\alpha\beta\gamma=45$; $\alpha,\beta,\gamma\in R$. If $x(\alpha,1,2)+y(1,\beta,2)+z(2,3,\gamma)=(0,0,0)$ for some $x,y,z\in R$, $xyz\neq 0$, then $6\alpha+4\beta+\gamma$ is equal to_______

Ans. (55)

Sol. $\alpha\beta\gamma=45$, $\alpha\beta\gamma\in R$

$x(\alpha,1,2)+y(1,\beta,2)+z(2,3,\gamma)=(0,0,0)$

$x,y,z\in R$, $xyz\neq 0$

$\alpha x+y+2z=0$

$x+\beta y+3z=0$

$2x+2y+\gamma z=0$

$xyz\neq 0\Rightarrow$ non-trivial

$$\begin{vmatrix}\alpha & 1 & 2\\ 1 & \beta & 3\\ 2 & 2 & \gamma\end{vmatrix}=0$$

$\Rightarrow \alpha(\beta\gamma-6)-1(\gamma-6)+2(2-2\beta)=0$

$\Rightarrow \alpha\beta\gamma-6\alpha-\gamma+6+4-4\beta=0$

$\Rightarrow 6\alpha+4\beta+\gamma=55$

22. Let a conic C pass through the point (4, –2) and P(x, y), $x\geq 3$, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and (3, –5). If the focal distance of the point (7, 1) on C is d, then 12d equals_______.

Ans. (75)

Sol. P(x, y) & $x\geq 3$

Slope of line at P(x, y) will be $\frac{dy}{dx}=\frac{1}{2}\left(\frac{y+5}{x-3}\right)$

$$\Rightarrow 2\frac{dy}{(y+5)}=\frac{1}{(x-3)}dx$$

$\Rightarrow 2\ell n(y+5)=\ell n(x-3)+C$

Passes through (4, –2)

$\Rightarrow 2\ell n(3)=\ell n(1)+C$

$\Rightarrow C=2\ell n(3)$

$\Rightarrow 2\ell n(y+5)=\ell n(x-3)+2\ell n(3)$

$$\Rightarrow 2\left(\ell n\left(\frac{y+5}{3}\right)\right)=\ell n(x-3)$$

$$\Rightarrow \left(\frac{y+5}{3}\right)^2=(x-3)$$

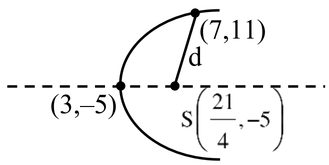
$\Rightarrow (y+5)^2=9(x-3)$

↓

Parabola

$4a=9$

$$a=\frac{9}{4}$$



$$d=\sqrt{\left(\frac{7}{4}\right)^2+6^2}$$

$$d=\frac{\sqrt{625}}{4}$$

$$d=\frac{25}{4}$$

$12d = 75$

23. Let $r_k=\dfrac{\int_0^1\left(1-x^7\right)^k dx}{\int_0^1\left(1-x^7\right)^{k+1} dx}, k\in N$. Then the value of $\sum_{k=1}^{10}\dfrac{1}{7\left(r_k-1\right)}$ is equal to _______.

Ans. (65)

Sol. $I_K=\int 1.(1-x^7)^K dx$

$$I_K=(1-x^7)^K x\Big|_0^1+7K\int_0^1(1-x^7)^{K-1}x^6.xdx$$

$$I_K=-7K\int_0^1(1-x^7)^{K-1}((1-x^7)-1)dx$$

$I_K = -7K\ I_K + 7K\ I_{K-1}$

$$\Rightarrow \frac{I_K}{I_{K+1}}=\frac{7K+8}{7K+7}$$

$$r_K=\frac{7K+8}{7K+7}$$

$$r_K-1=\frac{1}{7(K+1)}$$

$$\Rightarrow 7(r_K-1)=\frac{1}{K+1}$$

$$\sum_{K=1}^{10}(K+1)=11(6)-1=65$$

24. Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and

$$\left(4+x_1^2\right)\left(4+x_2^2\right)\left(4+x_3^2\right)\left(4+x_4^2\right)=\frac{125}{16}m.$$

Then the value of m is ________.

Ans. (221)

Sol. $4x^4 + 8x^3 - 17x^2 - 12x + 9$

$= 4(x - x_1)(x - x_2)(x - x_3)(x - x_4)$

Put $x = 2i$ & $-2i$

$64 - 64i + 68 - 24i + 9 = (2i - x_1)(2i - x_2)(2i - x_3)(2i - x_4)$

$= 141 - 88i$ ......(1)

$64 + 64i + 68 + 24i + 9 = 4(-2i - x_1)(-2i - x_2)(-2i - x_3)(-2i - x_4)$

$= 141 + 88i$ ....(2)

$$\frac{125}{16}m=\frac{141^2+88^2}{16}$$

$m = 221$

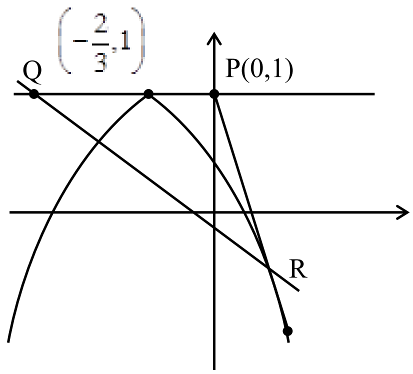
25. Let L_1, L_2 be the lines passing through the point P(0, 1) and touching the parabola $9x^2 + 12x + 18y - 14 = 0$. Let Q and R be the points on the lines L_1 and L_2 such that the ΔPQR is an isosceles triangle with base QR. If the slopes of the lines QR are m_1 and m_2. then $16\left(m_1^2+m_2^2\right)$ is equal to _______.

Ans. (68)

Sol. $9x^2 + 12x + 4 = -18(y - 1)$

$(3x + 2)^2 = -18(y - 1)$

$$\left(x+\frac{2}{3}\right)^2=-2(y-1)$$



(0, 1)

y = mx + 1

$$\left(x+\frac{2}{3}\right)^2=-2(y-1)$$

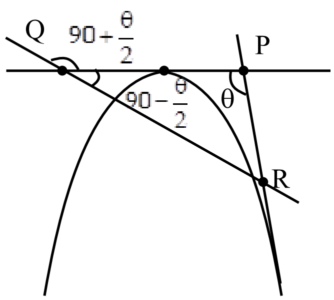
$(3x + 2)^2 = –18mx$

$9x^2 + (12 + 18m)x + 4 = 0$

$4(6 + 9m)^2 = 4(36)$

6 + 9m = 6, – 6

$m = 0, \frac{-4}{3}$



$$\tan\theta=-\frac{4}{3}$$

$$\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}=\frac{-4}{3}$$

$$\left(\tan\frac{\theta}{2}-2\right)\left(2\tan\frac{\theta}{2}+1\right)=0$$

$$\tan\frac{\theta}{2}=2,\frac{-1}{2}$$

$$m_{QR}=\tan\left(90+\frac{\theta}{2}\right)$$

$$=-\cot\frac{\theta}{2}$$

$$m_1=\frac{-1}{2} \qquad m_2=\frac{-1}{-1/2}=2$$

$$16\left(m_1^2+m_2^2\right)=16\left(\frac{1}{4}+4\right)$$

= 4 + 64 = 68

26. If the second, third and fourth terms in the expansion of $(x + y)^n$ are 135, 30 and $\frac{10}{3}$, respectively, then $6\left(n^3+x^2+y\right)$ is equal to _____.

Ans. (806)

Sol. $^nC_1x^{n-1}y=135$ ....(i)

$^nC_2x^{n-2}y^2=30$ ....(ii)

$^nC_3x^{n-3}y^3=\frac{10}{3}$ ....(iii)

By $\frac{(i)}{(ii)}$

$$\frac{^nC_1}{^nC_2}\frac{x}{y}=\frac{9}{2} \quad .....(iv)$$

By $\frac{(ii)}{(iii)}$

$$\frac{^nC_2}{^nC_3}\frac{x}{y}=9 \quad .....(v)$$

By $\frac{(iv)}{(v)}$

$$\frac{^nC_1\,{}^nC_3}{^nC_2\,{}^nC_2}=\frac{1}{2}$$

$$\frac{2n^2(n-1)(n-2)}{6}=\frac{n(n-1)}{2}\frac{n(n-1)}{2}$$

4n – 8 = 3n – 3

$\Rightarrow \boxed{n=5}$

put in (v)

$\frac{x}{y} = 9$

x = 9y

put in (i)

$^5C_1 x^4 \left(\frac{x}{9}\right) = 135$

$x^5 = 27 \times 9$

$\Rightarrow x = 3, \quad y = \frac{1}{3}$

$6\left(n^3 + x^2 + y\right)$

$= 6\left(125 + 9 + \frac{1}{3}\right)$

= 806

27. Let the first term of a series be $T_1 = 6$ and its r^{th} term $T_r = 3\,T_{r-1} + 6^r$, r = 2, 3, ..., n. If the sum of the first n terms of this series is $\frac{1}{5}\left(n^2 - 12n + 39\right)$ $\left(4.6^n - 5.3^n + 1\right)$. Then n is equal to ______.

Ans. (6)

Sol. $T_r = 3T_{r-1} + 6^r$, r = 2, 3, 4, … n

$T_2 = 3.T_1 + 6^2$

$T_2 = 3.6 + 6^2 \qquad \ldots(1)$

$T_3 = 3T_2 + 6^3$

$T_3 = 3T_2 + 6^3$

$T_3 = 3(3.6 + 6^2) + 6^3$

$T_3 = 3^2.6 + 3.6^2 + 6^3 \qquad \ldots(2)$

$T_r = 3^{r-1}.6 + 3^{r-2}.6^2 + \ldots + 6^r$

$T_r = 3^{r-1} \cdot 6\left[1 + \frac{6}{3} + \left(\frac{6}{3}\right)^2 + \ldots + \left(\frac{6}{3}\right)^{r-1}\right]$

$T_r = 3^{r-1}.6(1 + 2 + 2^2 + \ldots + 2^{r-1})$

$T_r = 6 \cdot 3^{r-1} 1.\frac{(1-2^r)}{(-1)}$

$T_r = 6.3^{r-1}.(2^r - 1)$

$T_r = \frac{6 \cdot 3^r}{3}.(2^r - 1)$

$T_r = 2.(6^r - 3^r)$

$S_n = 2\Sigma\left(6^r - 3^r\right)$

$S_n = 2.\left[\frac{6.(6^n - 1)}{5} - \frac{3.(3^n - 1)}{2}\right]$

$S_n = 2\left[\frac{12(6^n - 1) - 15(3^n - 1)}{10}\right]$

$S_n = \frac{3}{5}\left[4.6^4 - 5.3^n + 1\right]$

$\therefore n^2 - 12n + 39 = 3$

$n^2 - 12n + 36 = 0$

n = 6

28. For n ∈ N, if $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{1} n = \frac{\pi}{4}$, then n is equal to ________.

Ans. (47)

Sol. $\cot^{-1}3 + \cot^{-1}4 + \cot^{-1}5 + \cot^{1} n = \frac{\pi}{4}$

$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{n} = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{46}{48}\right) + \tan^{-1}\frac{1}{n} = \frac{\pi}{4}$

$\tan^{-1}\left(\frac{23}{24}\right) + \tan^{-1}\frac{1}{n} = \frac{\pi}{4}$

$\tan^{-1}\frac{1}{n} = \tan^{-1}1 - \tan^{-1}\frac{23}{24}$

$\tan^{-1}\frac{1}{n} = \tan^{-1}\left(\frac{1 - \frac{23}{24}}{1 + \frac{23}{24}}\right)$

$\tan^{-1}\frac{1}{n} = \tan^{-1}\left(\frac{\frac{1}{24}}{\frac{47}{24}}\right)$

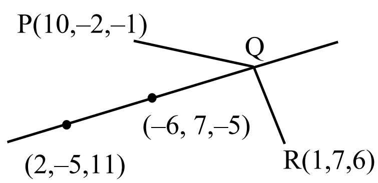
$\tan^{-1}\frac{1}{n} = \tan^{-1}\frac{1}{47}$

n = 47

29. Let P be the point (10, –2, –1) and Q be the foot of the perpendicular drawn from the point R(1, 7, 6) on the line passing through the points (2, –5, 11) and (–6, 7, –5). Then the length of the line segment PQ is equal to ________.

Ans. (13)

Sol.



Line : $\frac{x+6}{-8}=\frac{y-7}{12}=\frac{z+5}{-16}$

$\frac{x+6}{2}=\frac{y-7}{-3}=\frac{z+5}{4}=\lambda$

Q($2\lambda - 6, 7 - 3\lambda, 4\lambda - 5$)

$\overline{QR}\left(2\lambda-7,-3\lambda,4\lambda-11\right)$

$\overline{QR}$ · dr's of line = 0

$4\lambda - 14 + 9\lambda + 16\lambda - 44 = 0$

$29\lambda = 58 \Rightarrow \lambda = 2$

Q(–2, 1, 3)

$PQ=\sqrt{144+9+16}=\sqrt{169}=13$

30. Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}$, $\vec{b}=3\hat{i}+4\hat{j}-5\hat{k}$, and a vector $\vec{c}$ be such that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times\vec{c}=\hat{i}+8\hat{j}+13\hat{k}$. If $\vec{a}\cdot\vec{c}=13$, then $(24-\vec{b}\cdot\vec{c})$ is equal to ______.

Ans. (46)

Sol. $\vec{a}\times\vec{b}+\vec{a}\times\vec{c}+\vec{b}\times\vec{c}=(1,8,13)$

$\vec{a}\times\left(\vec{a}\times\vec{b}\right)+\vec{a}\times\left(\vec{a}\times\vec{c}\right)+\vec{a}\times\left(\vec{b}\times\vec{c}\right)$

$=\vec{a}\times\left(\hat{i}+8\hat{j}+13\hat{k}\right)$

$\left(\vec{a}\cdot\vec{b}\right)\vec{a}-a^2\vec{b}+\left(\vec{a}\cdot\vec{c}\right)\vec{a}-a^2\vec{c}+\left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{c}=\vec{a}\times\left(\hat{i}+8\hat{j}+13\hat{k}\right)$

$\Rightarrow -26\vec{a}-29\vec{b}+13\vec{a}-29\vec{c}+13\vec{b}+26\vec{c}=\vec{a}\times\left(\hat{i}+8\hat{j}+13\hat{k}\right)$

$\Rightarrow \quad -13\vec{a}-16\vec{b}-3\vec{c}=\vec{a}\times\left(\hat{i}+8\hat{j}+13\hat{k}\right)$

$\Rightarrow \quad -13\vec{a}\cdot\vec{b}-16b^2-3\vec{b}\cdot\vec{c}=\left\{\vec{a}\times\left(\hat{i}+8\hat{j}+13\hat{k}\right)\right\}\cdot\vec{b}$

$\Rightarrow \quad (-13)(-26)-16(50)-3\vec{b}\cdot\vec{c}=\begin{vmatrix}2 & -3 & 4\\ 1 & 8 & 13\\ 3 & 4 & -5\end{vmatrix}$

$\Rightarrow \quad -462-3\vec{b}\cdot\vec{c}=-396$

$\Rightarrow \quad \vec{b}\cdot\vec{c}=-22$

Hence $24-\vec{b}\cdot\vec{c}=46$

SECTION-A

31. To find the spring constant (k) of a spring experimentally, a student commits 2% positive error in the measurement of time and 1% negative error in measurement of mass. The percentage error in determining value of k is :

(1) 3% (2) 1%

(3) 4% (4) 5%

Ans. (4)

Sol. $T = 2\pi\sqrt{\frac{m}{k}}$

$T^2 \propto \frac{m}{k}$

$\frac{2\Delta T}{T}\% = \frac{\Delta m}{m}\% - \frac{\Delta k}{k}\%$

$\frac{\Delta k}{k}\% = \frac{\Delta m}{m}\% - \frac{2\Delta T}{T}\%$

$\frac{\Delta k}{k}\% = (-1)\% - 2(2)\% = |-5\%| = 5\%$

32. A bullet of mass 50 g is fired with a speed 100 m/s on a plywood and emerges with 40 m/s. The percentage loss of kinetic energy is :

(1) 32% (2) 44%

(3) 16% (4) 84%

Ans. (4)

Sol. $K_i = \frac{1}{2}m(100)^2$

$K_f = \frac{1}{2}m(40)^2$

$\%loss = \frac{|K_f - K_i|}{K_i} \times 100$

$= \frac{\left|\frac{1}{2}m(40)^2 - \frac{1}{2}m(100)^2\right|}{\frac{1}{2}m(100)^2} \times 100$

$= \frac{|1600 - 100 \times 100|}{100} = 84\%$

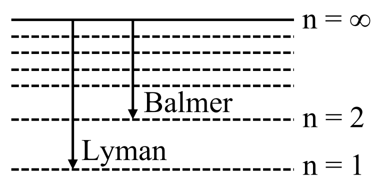
33. The ratio of the shortest wavelength of Balmer series to the shortest wavelength of Lyman series for hydrogen atom is :

(1) 4 : 1 (2) 1 : 2

(3) 1 : 4 (4) 2 : 1

Ans. (1)

Sol



$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\frac{\frac{1}{\lambda_L} = Rz^2\left(\frac{1}{1^2}\right)}{\frac{1}{\lambda_B} = Rz^2\left(\frac{1}{2^2}\right)}$

$\frac{\lambda_B}{\lambda_L} = 4:1$

34. To project a body of mass m from earth's surface to infinity, the required kinetic energy is (assume, the radius of earth is R_E, g = acceleration due to gravity on the surface of earth) :

(1) $2mgR_E$ (2) mgR_E

(3) $\frac{1}{2}mgR_E$ (4) $4mgR_E$

Ans. (2)

Sol. $\frac{1}{2}mv_e^2 = \frac{GMm}{R_E}$

$g = \frac{GM}{R_E^2}$

$K = mgR_E$

35. Electromagnetic waves travel in a medium with speed of 1.5×10^8 ms^{-1}. The relative permeability of the medium is 2.0. The relative permittivity will be :

(1) 5 (2) 1

(3) 4 (4) 2

Ans. **(4)**

Sol. $$\frac{\varepsilon_m \times \mu_m}{\varepsilon_0 \times \mu_0} = \frac{\frac{1}{v^2}}{\frac{1}{c^2}}$$

$$\varepsilon_r \times \mu_r = \frac{c^2}{v^2}$$

$$\varepsilon_r \times 2 = \frac{(3\times10^8)^2}{(1.5\times10^8)^2}$$

$$\varepsilon_r \times 2 = 4$$

$$\varepsilon_r = 2$$

36. Which of the following phenomena does not explain by wave nature of light.

(A) reflection (B) diffraction

(C) photoelectric effect (D) interference

(E) polarization

Choose the **most appropriate** answer from the options given below :

(1) E only (2) C only

(3) B, D only (4) A, C only

Ans. **(2)**

Sol. (Theory)

Photoelectric effect prove particle nature of light.

37. While measuring diameter of wire using screw gauge the following readings were noted. Main scale reading is 1 mm and circular scale reading is equal to 42 divisions. Pitch of screw gauge is 1 mm and it has 100 divisions on circular scale. The diameter of the wire is $\frac{x}{50}$ mm . The value of x is :

(1) 142 (2) 71

(3) 42 (4) 21

Ans. **(2)**

Sol. MSR = 1mm, CSR = 42, pitch = 1 mm

$$LC = \frac{\text{pitch}}{\text{No. of CSD}} = \left(\frac{1}{100}\right) = 0.01\text{mm}$$

Diameter = MSR + LC × CSD

Diameter = 1 + (0.01) × 42 mm

Diameter = 1.42 mm = $\frac{x}{50}$

$\therefore$ x = 71

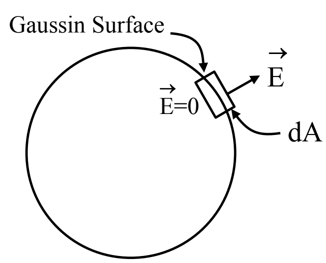
38. σ is the uniform surface charge density of a thin spherical shell of radius R. The electric field at any point on the surface of the spherical shell is :

(1) $\sigma/\epsilon_0 R$ (2) $\sigma/2\epsilon_0$

(3) σ/ϵ_0 (4) $\sigma/4\epsilon_0$

Ans. **(3)**

Sol.

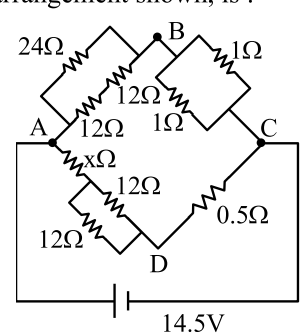


By Gauss law $\int \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$

$$EdA = \frac{\sigma \times dA}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

39. The value of unknown resistance (x) for which the potential difference between B and D will be zero in the arrangement shown, is :

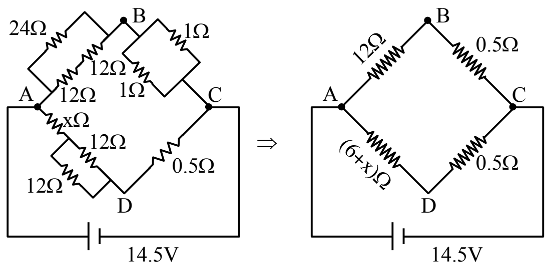


(1) 3 Ω (2) 9 Ω

(3) 6 Ω (4) 42 Ω

Ans. **(3)**

Sol.



In case of balanced Wheatstone Bridge

$$\frac{V_{AB}}{V_{AD}} = \frac{V_{BC}}{V_{CD}} \Rightarrow \frac{12}{6+x} = \frac{0.5}{0.5}$$

$x = 6\,\Omega$

40. The specific heat at constant pressure of a real gas obeying $PV^2 = RT$ equation is :

(1) $C_V + R$ (2) $\frac{R}{3} + C_V$

(3) R (4) $C_V + \frac{R}{2V}$

Ans. (4)

Sol. $dQ = du + dW$

$CdT = C_V dT + PdV$ …..(1)

$\therefore\ PV^2 = RT$

$P = \text{constant}$

$P(2VdV) = RdT$

$PdV = \frac{RdT}{2V}$

Put in equation (1)

$$C = C_V + \frac{R}{2V}$$

41. Match **List I** with **List II**

	LIST I		**LIST II**
A.	Torque	I.	$[M^1L^1T^{-2}A^{-2}]$
B.	Magnetic field	II.	$[L^2A^1]$
C.	Magnetic moment	III.	$[M^1T^{-2}A^{-1}]$
D.	Permeability of free space	IV.	$[M^1L^2T^{-2}]$

Choose the **correct** answer from the options given below :

(1) A-I, B-III, C-II, D-IV

(2) A-IV, B-III, C-II, D-I

(3) A-III, B-I, C-II, D-IV

(4) A-IV, B-II, C-III, D-I

Ans. (2)

Sol. $[\vec{\tau}] = [\vec{r} \times \vec{F}] = [ML^2T^{-2}]$

$[F] = [qVB]$

$$\Rightarrow\ B = \left(\frac{F}{qV}\right) = \left[\frac{MLT^{-2}}{ATLT^{-1}}\right] = [MA^{-1}T^{-2}]$$

$[M] = [I \times A] = [AL^2]$

$$B = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

$$\Rightarrow\ [\mu] = \left[\frac{Br^2}{Idl}\right] = \left[\frac{MT^{-2}A^{-1} \times L^2}{AL}\right]$$

$$= [MLT^{-2}A^{-2}]$$

42. Given below are two statements :

Statement I : In an LCR series circuit, current is maximum at resonance.

Statement II : Current in a purely resistive circuit can never be less than that in a series LCR circuit when connected to same voltage source.

In the light of the above statements, choose the ***correct*** from the options given below :

(1) Statement I is true but Statement II is false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Ans. (3)

Sol. **Statement-I**

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$ at resonance $X_L = X_C$

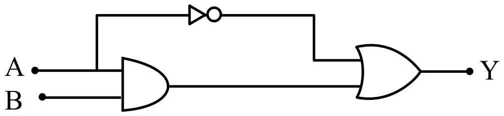
Thus, $I_m = \frac{V_m}{R}$

$\because$ Impendence is minimum therefore I is maximum at resonance.

Statement-II

$I = \left(\frac{V}{R}\right)$ in purely resistive circuit.

43. The correct truth table for the following logic circuit is :



Options :

(1)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	1

(3)

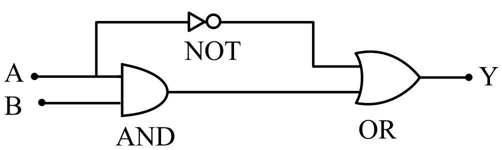
A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(4)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Ans. (2)

Sol.



44. A sample contains mixture of helium and oxygen gas. The ratio of root mean square speed of helium and oxygen in the sample, is :

(1) $\frac{1}{32}$ (2) $\frac{2\sqrt{2}}{1}$

(3) $\frac{1}{4}$ (4) $\frac{1}{2\sqrt{2}}$

Ans. (2)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M_w}}$

$$\Rightarrow \frac{V_{O_2}}{V_{He}} = \sqrt{\frac{M_{w,He}}{M_{w,O_2}}}$$

$$= \sqrt{\frac{4}{32}} = \frac{1}{2\sqrt{2}}$$

$$\frac{V_{He}}{V_{O_2}} = \frac{2\sqrt{2}}{1}$$

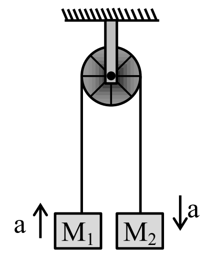
45. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (where $m_2 > m_1$). If the acceleration of the system is $\frac{g}{\sqrt{2}}$, then the ratio of the masses $\frac{m_1}{m_2}$ is :

(1) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (2) $\frac{1+\sqrt{5}}{\sqrt{5}-1}$

(3) $\frac{1+\sqrt{5}}{\sqrt{2}-1}$ (4) $\frac{\sqrt{3}+1}{\sqrt{2}-1}$

Ans. (1)

Sol. $a = \left(\frac{M_2 - M_1}{M_1 + M_2}\right)g$



$$\frac{g}{\sqrt{2}} = \left(\frac{M_2 - M_1}{M_1 + M_2}\right)g$$

$$(M_1 + M_2) = \sqrt{2}M_2 - \sqrt{2}M_1$$

$$\frac{M_1}{M_2} = \left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)$$

46. Four particles A, B, C, D of mass $\frac{m}{2}$, m, 2m, 4m, have same momentum, respectively. The particle with maximum kinetic energy is :

(1) D (2) C

(3) A (4) B

Ans. (3)

Sol. $KE = \frac{p^2}{2m}$

Same momentum, so less mass means more KE.

So $\frac{m}{2}$ will have max. KE.

47. A train starting from rest first accelerates uniformly up to a speed of 80 km/h for time t, then it moves with a constant speed for time 3t. The average speed of the train for this duration of journey will be (in km/h) :

(1) 80 (2) 70

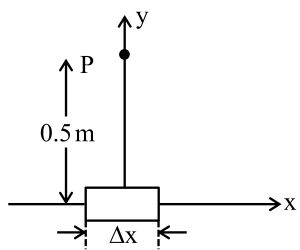
(3) 30 (4) 40

Ans. (2)

Sol. Average speed = $\dfrac{\text{total distance}}{\text{time taken}}$

$$= \frac{\frac{80\times t}{2} + 80\times 3t}{4t} = 70 \text{ km/hr.}$$

48. An element $\Delta l = \Delta x\hat{i}$ is placed at the origin and carries a large current I = 10A. The magnetic field on the y-axis at a distance of 0.5 m from the elements Δx of 1 cm length is :

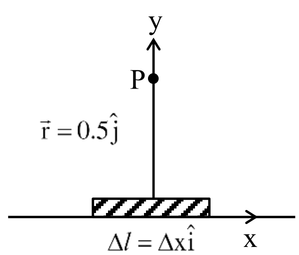


(1) 4×10^{-8}T (2) 8×10^{-8}T

(3) 12×10^{-8}T (4) 10×10^{-8}T

Ans. **(1)**

Sol.



$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi}\frac{(\overrightarrow{dl}\times\vec{r})}{r^3} \text{ (Tesla)}$$

$$= \frac{10^{-7}\times10\times\left(\frac{1}{2}\times\frac{1}{100}\right)(+\hat{k})}{\left(\frac{1}{2}\right)^3} = 4\times10^{-8}\,T\,(+\hat{k})$$

49. A small ball of mass m and density ρ is dropped in a viscous liquid of density ρ_0. After sometime, the ball falls with constant velocity. The viscous force on the ball is :

(1) $mg\left(\frac{\rho_0}{\rho}-1\right)$ (2) $mg\left(1+\frac{\rho}{\rho_0}\right)$

(3) $mg(1-\rho\rho_0)$ (4) $mg\left(1-\frac{\rho_0}{\rho}\right)$

Ans. **(4)**

Sol. $mg - F_B - F_v = ma$

a = 0 for constant velocity

$mg - F_B = F_v$

$$F_v = mg - v\,\rho_0\,g = mg - \frac{m}{\rho}\rho_0 g = mg\left(1-\frac{\rho_0}{\rho}\right)$$

50. In photoelectric experiment energy of 2.48 eV irradiates a photo sensitive material. The stopping potential was measured to be 0.5 V. Work function of the photo sensitive material is :

(1) 0.5 eV (2) 1.68 eV

(3) 2.48 eV (4) 1.98 eV

Ans. **(4)**

Sol. $eV_s = h\nu - \phi$

$0.5\text{ V} = 2.48 - \phi$

work function $(\phi) = 2.48\text{ V} - 0.5\text{ V} = 1.98\text{ V}$

SECTION-B

51. If the radius of earth is reduced to three-fourth of its present value without change in its mass then value of duration of the day of earth will be ______ hours 30 minutes.

Ans. **(13)**

Sol. By conservation of angular momentum

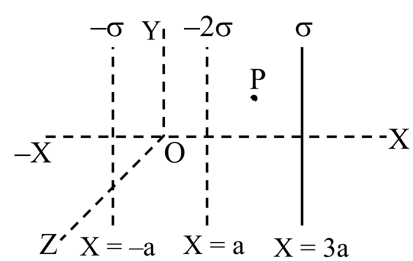
$I_1\omega_1 = I_2\omega_2$

$$\left(\frac{2}{5}MR^2\right)\frac{2\pi}{T_1} = \frac{2}{5}M\left(\frac{3}{4}R\right)^2\frac{2\pi}{T_2}$$

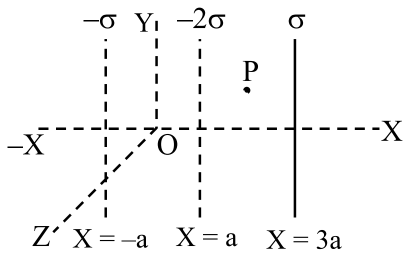
$$\frac{1}{T_1} = \frac{9}{16T_2}$$

$$\frac{1}{T_2} = \frac{9}{16}\times T_1 = \frac{9}{16}\times24\text{hr} = \frac{27}{2}\text{hr} = 13\text{ hr }30\text{ mins.}$$

52. Three infinitely long charged thin sheets are placed as shown in figure. The magnitude of electric field at the point P is $\frac{x\sigma}{\epsilon_0}$. The value of x is _________ (all quantities are measured in SI units).



Ans. **(2)**

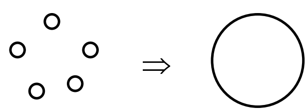
Sol. 

$$\vec{E}_p = \left(\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right)(-\hat{i})$$

$$= -\frac{2\sigma}{\varepsilon_0}\hat{i}$$

53. A big drop is formed by coalescing 1000 small droplets of water. The ratio of surface energy of 1000 droplets to that of energy of big drop is $\frac{10}{x}$. The value of x is _________.

Ans. **(1)**

Sol. 

1000 drops $\Rightarrow$ Big drop

$$1000\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$10r = R$$

$$R = 10r$$

$$\frac{\text{S.E. of 1000 drops}}{\text{S.E. of Big drop}} = \frac{1000(4\pi r^2)T}{4\pi R^2 T}$$

$$= \frac{1000 \times r^2}{(10r)^2} = 10 = \frac{10}{x}$$

$\therefore$ x = 1

54. When a dc voltage of 100V is applied to an inductor, a dc current of 5A flows through it. When an ac voltage of 200V peak value is connected to inductor, its inductive reactance is found to be $20\sqrt{3}\,\Omega$. The power dissipated in the circuit is _________W.

Ans. **(250)**

Sol. For DC voltage

$$R = \frac{V}{I} = \frac{100}{5} = 20\,\Omega$$

for AC voltage

$$X_L = 20\sqrt{3}\,\Omega$$

$$R = 20\,\Omega$$

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{3 \times 400 + 400} = 40\,\Omega$$

Power = $i_{rms}^2 R$

$$= \left(\frac{V_{rms}}{Z}\right)^2 \times R = \left(\frac{\frac{200}{\sqrt{2}}}{40}\right)^2 \times 20 = 250\,\text{W}$$

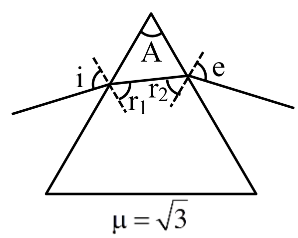
55. The refractive index of prism is $\mu = \sqrt{3}$ and the ratio of the angle of minimum deviation to the angle of prism is one. The value of angle of prism is _________°.

Ans. **(60)**

Sol. For δ_{min}

i = e

$$r_1 = r_2 = \frac{A}{2}$$



$$\frac{\delta_{min}}{A} = 1$$

$$\frac{2i - A}{A} = 1$$

$$2i = 2A$$

$$i = A$$

Snell's law

$$1 \times \sin i = \mu \sin r$$

$$\sin i = \mu \sin\left(\frac{A}{2}\right)$$

$$\sin A = \mu \sin\left(\frac{A}{2}\right)$$

$$2\sin\frac{A}{2}\cos\frac{A}{2} = \sqrt{3}\sin\left(\frac{A}{2}\right)$$

$$\cos\left(\frac{A}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\therefore \frac{A}{2} = 30°$$

$$\therefore A = 60°$$

56. A wire of resistance R and radius r is stretched till its radius became r/2. If new resistance of the stretched wire is x R, then value of x is _________.

Ans. **(16)**

Sol. We know $R = \frac{\rho l}{A}$, $R \propto \frac{l}{r^2}$

As we starch the wire, its length will increase but its radius will decrease keeping the volume constant

$$V_i = V_f$$

$$\pi r^2 l = \pi \frac{r^2}{4} l_f$$

$$l_f = 4l$$

$$\frac{R_{new}}{R_{old}} = \left(\frac{4l}{\frac{r^2}{4}}\right)\frac{r^2}{l} = 16$$

$$R_{new} = 16R$$

$\therefore$ x = 16

57. Radius of a certain orbit of hydrogen atom is 8.48 Å. If energy of electron in this orbit is E/x, then x = _________.

(Given a_0 = 0.529Å, E = energy of electron in ground state)

Ans. **(16)**

Sol. We know

$$r = 0.529\frac{n^2}{Z} \Rightarrow 8.48 = 0.529\frac{n^2}{1}$$

$n^2 = 16 \Rightarrow n = 4$

We know

$$E \propto \frac{1}{n^2}$$

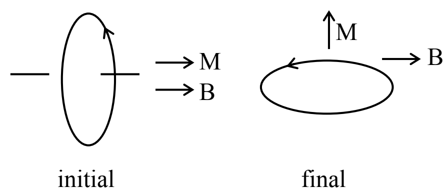
$$E_{n^{th}} = \frac{E}{16}$$

x = 16

58. A circular coil having 200 turns, 2.5×10^{-4} m^2 area and carrying 100 μA current is placed in a uniform magnetic field of 1 T. Initially the magnetic dipole moment $(\vec{M})$ was directed along $\vec{B}$. Amount of work, required to rotate the coil through 90° from its initial orientation such that $\vec{M}$ becomes perpendicular to $\vec{B}$, is _________ μJ.

Ans. **(5)**

Sol.



We know

$W_{ext} = \Delta U + \Delta KE \qquad \left(P.E. = -\vec{M}\cdot\vec{B}\right)$

$= -\vec{M}\cdot\vec{B}_f + \vec{M}\cdot\vec{B}_i + 0$

$= -MB \cos 90 + MB \cos 0$

$= MB$

$= NIAB$

$$= 200 \times 100 \times 10^{-6} \times \frac{5}{2} \times 10^{-4} \times 1 = 5\mu J$$

59. A particle is doing simple harmonic motion of amplitude 0.06 m and time period 3.14 s. The maximum velocity of the particle is _______ cm/s.

Ans. **(12)**

Sol. We know

$v_{max} = \omega A$ at mean position

$$= \frac{2\pi}{T}A = \frac{2\pi}{\pi} \times 0.06 = 0.12 \text{ m/sec}$$

v_{max} = 12 cm/sec

60. For three vectors $\vec{A} = (-x\hat{i} - 6\hat{j} - 2\hat{k})$, $\vec{B} = (-\hat{i} + 4\hat{j} + 3\hat{k})$ and $\vec{C} = (-8\hat{i} - \hat{j} + 3\hat{k})$, if $\vec{A}\cdot(\vec{B}\times\vec{C}) = 0$, them value of x is _________.

Ans. **(4)**

Sol. $$\vec{B}\times\vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = 15\hat{i} - 21\hat{j} + 33\hat{k}$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = (-x\hat{i} - 6\hat{j} - 2\hat{k})\cdot(15\hat{i} - 21\hat{j} + 33\hat{k})$$

0 = –15x + 126 – 66

15x = 60

x = 4

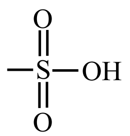
SECTION-A

61. Functional group present in sulphonic acid is :

(1) SO_4H (2) SO_3H

(3) $-\underset{\underset{O}{\|}}{S}-OH$ (4) $-SO_2$

Ans. (2)

Sol. 

Group present in sulphonic acids

62. Match **List I** with **List II** :

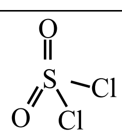
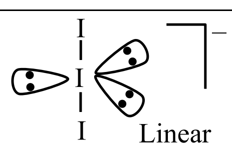
List I (Molecule / Species)		**List II (Property / Shape)**	
A.	SO_2Cl_2	I.	Paramagnetic
B.	NO	II.	Diamagnetic
C.	NO_2^-	III.	Tetrahedral
D.	I_3^-	IV.	Linear

Choose the **correct** answer from the options given below :

(1) A-IV, B-I, C-III, D-II
(2) A-III, B-I, C-II, D-IV
(3) A-II, B-III, C-I, D-IV
(4) A-III, B-IV, C-II, D-I

Ans. (2)

Sol.

(A)	SO_2Cl_2	sp^3	 Tetrahedral
(B)	NO		Paramagnetic
(C)	NO_2^-		Diamagnetic
(D)	I_3^-	sp^3d	 Linear

63. Given below are two statements :

Statement I : Picric acid is 2, 4, 6-trinitrotoluene.

Statement II : Phenol-2, 4-disulphuric acid is treated with conc. HNO_3 to get picric acid.

In the light of the above statement, choose the **most appropriate** answer from the options given below :

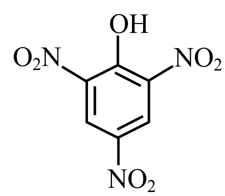
(1) Statement I is incorrect but Statement II is correct.

(2) Both Statement I and Statement II are incorrect.

(3) Statement I is correct but Statement II is incorrect.

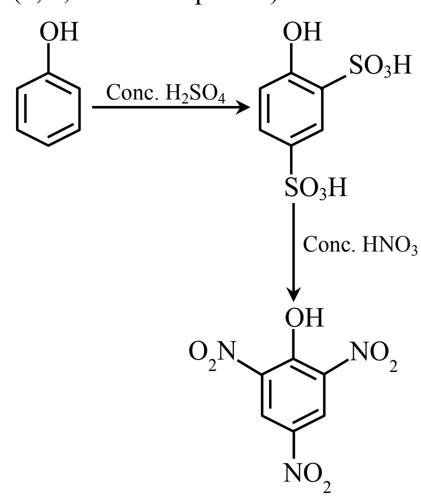
(4) Both Statement I and Statement II are correct.

Ans. (1)

Sol. 

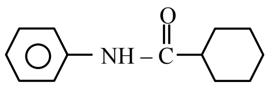
picric acid

(2, 4, 6 – trinitrophenol)

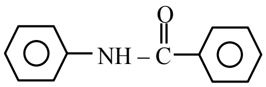


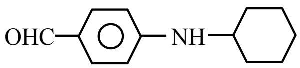
Picric acid

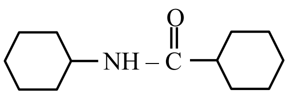
64. Which of the following is metamer of the given compound (X) ?

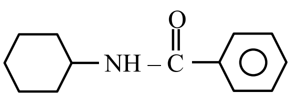


(X)

(1) 

(2) 

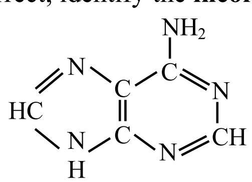
(3) 

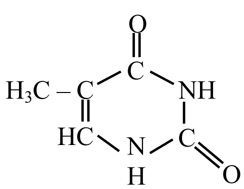
(4) 

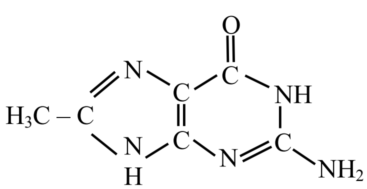
Ans. (4)

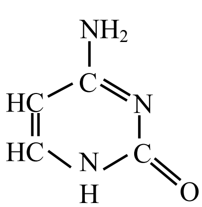
Sol. Metamer $\Rightarrow$ Isomer having same molecular formula, same functional group but different alkyl/aryl groups on either side of functional group.

65. DNA molecule contains 4 bases whoes structure are shown below. One of the structure is not correct, identify the **incorrect** base structure.

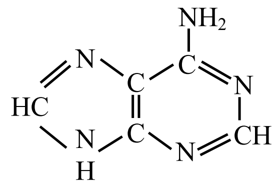
(1) 

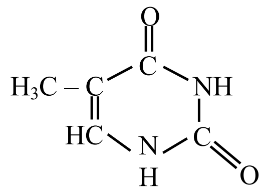
(2) 

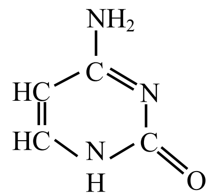
(3) 

(4) 

Ans. (3)

Sol.  $\Rightarrow$ Adenine

 $\Rightarrow$ Thymine

 $\Rightarrow$ Cytosine

Are bases of DNA molecule. As DNA contain four bases, which are adenine, guanine, cytosine and thymine.

66. Match **List I** with **List II** :

LIST I (Hybridization)		**LIST II (Orientation in Space)**	
A.	sp^3	I.	Trigonal bipyramidal
B.	dsp^2	II.	Octahedral
C.	sp^3d	III.	Tetrahedral
D.	sp^3d^2	IV.	Square planar

Choose the **correct** answer from the options given below :

(1) A-III, B-I, C-IV, D-II
(2) A-II, B-I, C-IV, D-III
(3) A-IV, B-III, C-I, D-II
(4) A-III, B-IV, C-I, D-II

Ans. (4)

Sol. $sp^3 \rightarrow$ Tetrahedral
$dsp^2 \rightarrow$ Square planar
$sp^3d \rightarrow$ Trigonal Bipyramidal
$sp^3d^2 \rightarrow$ Octahedral

67. Given below are two statements :

Statement I : Gallium is used in the manufacturing of thermometers.

Statement II : A thermometer containing gallium is useful for measuring the freezing point (256 K) of brine solution.

In the light of the above statement, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are false.

(2) Statement I is false but Statement II is true.

(3) Both Statement I and Statement II are true.

(4) Statement I is true but Statement II is false.

Ans. **(4)**

Sol. **Statement - I** $\Rightarrow$ Correct

Statement - II $\Rightarrow$ False

Ga is used to measure high temperature

68. Which of the following statements are correct ?

A. Glycerol is purified by vacuum distillation because it decomposes at its normal boiling point.

B. Aniline can be purified by steam distillation as aniline is miscible in water.

C. Ethanol can be separated from ethanol water mixture by azeotropic distillation because it forms azeotrope.

D. An organic compound is pure, if mixed M.P. is remained same.

Choose the **most appropriate** answer from the options given below :

(1) A, B, C only

(2) A, C, D only

(3) B, C, D only

(4) A, B, D only

Ans. **(2)**

Sol. Option (B) is incorrect because aniline is immisible in water.

69. Match **List I** with **List II** :

LIST I (Compound / Species)		LIST II (Shape / Geometry)	
A.	SF_4	I.	Tetrahedral
B.	BrF_3	II.	Pyramidal
C.	BrO_3^-	III.	See saw
D.	NH_4^+	IV.	Bent T-shape

Choose the **correct** answer from the options given below :

(1) A-II, B-III, C-I, D-IV

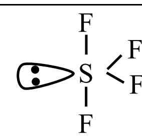
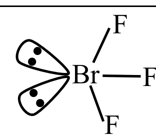
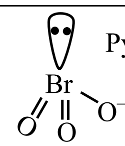
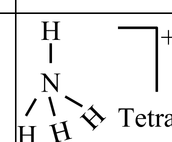
(2) A-III, B-IV, C-II, D-I

(3) A-II, B-IV, C-III, D-I

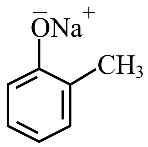
(4) A-III, B-II, C-IV, D-I

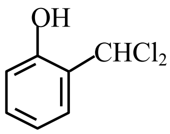
Ans. **(2)**

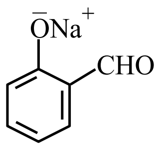
Sol.

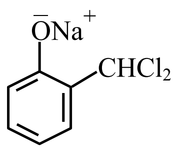
(A)	SF_4	sp^3d hybridisation	
(B)	BrF_3	sp^3d hybridisation	 Bent T-Shape
(C)	BrO_3^-	sp^3 hybridisation	 Pyramidal
(D)	NH_4^+	sp^3 hybridisation	 Tetrahedral

70. In Reimer - Tiemann reaction, phenol is converted into salicylaldehyde through an intermediate. The structure of intermediate is ______.

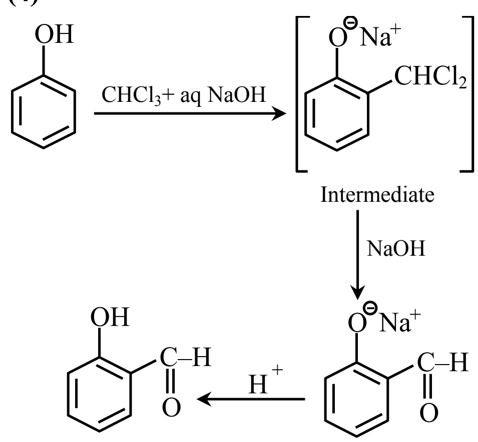
(1) 

(2) 

(3) 

(4) 

Ans. **(4)**

Sol. 

71. Which of the following material is not a semiconductor.

(1) Germanium

(2) Graphite

(3) Silicon

(4) Copper oxide

Ans. **(2)**

Sol. Graphite is conductor

72. Consider the following complexes.

$[CoCl(NH_3)_5]^{2+}$, (A) $[Co(CN)_6]^{3-}$, (B)

$[Co(NH_3)_5(H_2O)]^{3+}$, (C) $[Cu(H_2O)_4]^{2+}$ (D)

The correct order of A, B, C and D in terms of *wavenumber* of light absorbed is :

(1) C < D < A < B

(2) D < A < C < B

(3) A < C < B < D

(4) B < C < A < D

Ans. **(2)**

Sol. As ligand field increases, light of more energy is absorbed

Energy $\propto$ wave number

$(\bar{\upsilon})$

73. Match **List I** with **List II** :

LIST I **(Precipitating reagent and conditions)**		**LIST II** **(Cation)**	
A.	$NH_4Cl + NH_4OH$	I.	Mn^{2+}
B.	$NH_4OH + Na_2CO_3$	II.	Pb^{2+}
C.	$NH_4OH + NH_4Cl + H_2S$ gas	III.	Al^{3+}
D.	dilute HCl	IV.	Sr^{2+}

Choose the **correct** answer from the options given below :

(1) A-IV, B-III, C-II, D-I

(2) A-IV, B-III, C-I, D-II

(3) A-III, B-IV, C-I, D-II

(4) A-III, B-IV, C-II, D-I

Ans. **(3)**

Sol. Theory based question

74. The electron affinity value are negative for :

A. $Be \rightarrow Be^-$

B. $N \rightarrow N^-$

C. $O \rightarrow O^{2-}$

D. $Na \rightarrow Na^-$

E. $Al \rightarrow Al^-$

Choose the most appropriate answer from the options given below :

(1) D and E only (2) A, B, D and E only

(3) A and D only (4) A, B and C only

Allen Ans. (4)

NTA Ans. (1)

Sol. (A) $Be + e^- \rightarrow Be^-$, E.A = –ive

(B) $N + e^- \rightarrow N^-$ E.A = –ive

(C) $O + e^- \rightarrow O^-$

$O^- + e^- \rightarrow O^{-2}$ E.A = –ive

(D) $Na + e^- \rightarrow Na^-$ E.A = +ive

(E) $A\ell + e^- \rightarrow A\ell^-$ E.A = +ive

Ans. A,B and C only

75. The number of element from the following that do not belong to lanthanoids is :

Eu, Cm, Er, Tb, Yb and Lu

(1) 3 (2) 4

(3) 1 (4) 5

Ans. (3)

Sol. Cm is Actinide

76. The density of 'x' M solution ('x' molar) of NaOH is 1.12 g mL^{-1}. while in molality, the concentration of the solution is 3 m (3 molal). Then x is

(Given : Molar mass of NaOH is 40 g/mol)

(1) 3.5 (2) 3.0

(3) 3.8 (4) 2.8

Ans. (2)

Sol. $\text{Molality} = \dfrac{1000 \times M}{1000 \times d - M \times (Mw)_{solute}}$

$3 = \dfrac{1000 \times x}{1000 \times 1.12 - (x \times 40)}$

$x = 3$

77. Which among the following aldehydes is most reactive towards nucleophilic addition reactions?

(1) $H-\overset{\overset{\displaystyle O}{\|}}{C}-H$ (2) $C_2H_5-\overset{\overset{\displaystyle O}{\|}}{C}-H$

(3) $CH_3-\overset{\overset{\displaystyle O}{\|}}{C}-H$ (4) $C_3H_7-\overset{\overset{\displaystyle O}{\|}}{C}-H$

Ans. (1)

Sol. $H-\overset{\overset{\displaystyle O}{\|}}{C}-H$ has low steric hindrance at carbonyl carbon and high partial positive charge at carbonyl carbon.

78. At –20 °C and 1 atm pressure, a cylinder is filled with equal number of H_2. I_2 and HI molecules for the reaction

$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$, the K_P for the process is $x \times 10^{-1}$. x = _________.

[Given : R = 0.082 L atm K^{-1} mol^{-1}]

(1) 2 (2) 1

(3) 10 (4) 0.01

Ans. (3)

Sol. $\Delta n_g = 0$ $K_p = \dfrac{(n_{HI})^2}{n_{H_2} n_{I_2}} \left(\dfrac{P_T}{n_T}\right)^{\Delta n_g}$

$n_{HI} = n_{H_2} = n_{I_2}$ so $K_P = 1$

$1 = x \times 10^{-1}$ $x = 10$

79. Match **List I** with **List II** :

	LIST I (Compound)		LIST II (Uses)
A.	Iodoform	I.	Fire extinguisher
B.	Carbon tetrachloride	II.	Insecticide
C.	CFC	III.	Antiseptic
D.	DDT	IV.	Refrigerants

Choose the **correct** answer from the options given below :

(1) A-I, B-II, C-III, D-IV

(2) A-III, B-II, C-IV, D-I

(3) A-III, B-I, C-IV, D-II

(4) A-II, B-IV, C-I, D-III

Ans. (3)

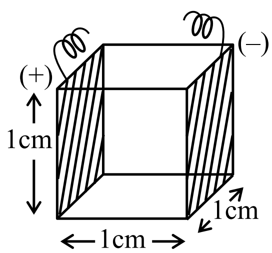
Sol. Iodoform – Antiseptic

CCl_4 – Fire extinguisher

CFC – Refrigerants

DDT – Insecticide

80. A conductivity cell with two electrodes (dark side) are half filled with infinitely dilute aqueous solution of a weak electrolyte. If volume is doubled by adding more water at constant temperature, the molar conductivity of the cell will -



(1) increase sharply

(2) remain same or can not be measured accurately

(3) decrease sharply

(4) depend upon type of electrolyte

Ans. (2)

Sol. Solution is already infinitely dilute, hence no change in molar conductivity upon addition of water

SECTION-B

81. Consider the dissociation of the weak acid HX as given below

$HX(aq) \rightleftharpoons H^+(aq) + X^-(aq)$, $Ka = 1.2 \times 10^{-5}$

[K_a : dissociation constant]

The osmotic pressure of 0.03 M aqueous solution of HX at 300 K is _____ $\times 10^{-2}$ bar (nearest integer).

[Given : $R = 0.083$ L bar Mol^{-1} K^{-1}]

Ans. (76)

Sol. $HX \rightleftharpoons H^+ + X^-$ $K_a = 1.2 \times 10^{-5}$

0.03M

$0.03 - x$ $\quad x \quad x$

$K_a = 1.2 \times 10^{-5} = \dfrac{x^2}{0.03 - x}$

$0.03 - x \approx 0.03$ (K_a is very small)

$\dfrac{x^2}{0.03} = 1.2 \times 10^{-5}$

$x = 6 \times 10^{-4}$

Final solution : $0.03 - x + x + x$

$= 0.03 + x = 0.03 + 6 \times 10^{-4}$

$\Pi = (0.03 + (6 \times 10^{-4})) \times 0.083 \times 300$

$= 76.19 \times 10^{-2} \approx 76 \times 10^{-2}$

82. The difference in the 'spin-only' magnetic moment values of $KMnO_4$ and the manganese product formed during titration of $KMnO_4$ against oxalic acid in acidic medium is _____ BM. (nearest integer)

Ans. (6)

Sol. Spin only magnetic moment of Mn in $KMnO_4 = 0$

Spin only value of manganese product fromed during titration of $KMnO_4$ aganist oxalic acid in acidic medium is = 6

Ans. 6

83. Time required for 99.9% completion of a first order reaction is _____ time the time required for completion of 90% reaction.(nearest integer).

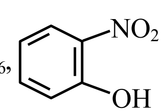
Ans. (3)

Sol. $K = \dfrac{1}{t_{99.9\%}} \ell n\left(\dfrac{100}{0.1}\right) = \dfrac{1}{t_{90\%}} \ell n\left(\dfrac{100}{10}\right)$

$t_{99.9\%} = t_{90\%} \dfrac{\ell n(10^3)}{\ell n 10}$

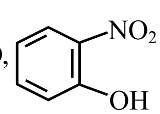
$t_{99.9\%} = t_{90\%} \times 3$

84. Number of molecules from the following which can exhibit hydrogen bonding is ______. (nearest integer)

CH_3OH, H_2O, C_2H_6, C_6H_6, 

HF, NH_3

Ans. (5)

Sol. CH_3OH, H_2O, 

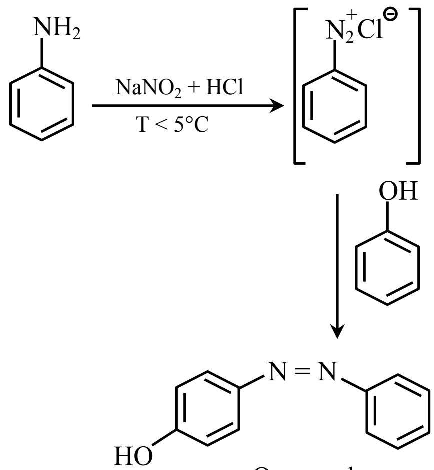
HF, NH_3

Can show H–bonding.

85. 9.3 g of pure aniline upon diazotisation followed by coupling with phenol gives an orange dye. The mass of orange dye produced (assume 100% yield/ conversion) is ________g. (nearest integer)

Ans. (20)

Sol.



Orange dye

Reaction suggests that 1 mole of aniline give 1 mole of orange dye.

so $(\text{mol})_{\text{aniline}} = (\text{mole})_{\text{orange dye}}$

$$\frac{9.3\text{g}}{93\text{g mol}^{-1}} = \frac{\text{mass of orange dye}}{199\text{g mol}^{-1}}$$

mass of orange dye = 19.9 g $\simeq$ 20 g

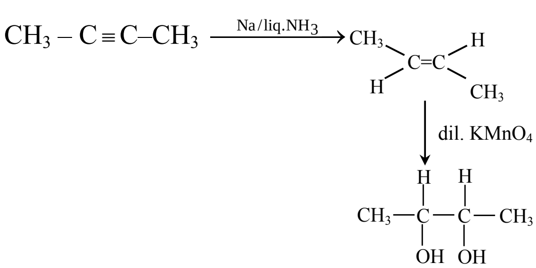
86. The major product of the following reaction is P.

$$CH_3C \equiv C - CH_3 \xrightarrow[\text{(ii)dil.KMnO}_4\ 273\text{K}]{\text{(i)Na/liq.NH}_3} \text{'P'}$$

Number of oxygen atoms present in product 'P' is _______ (nearest integer).

Ans. (2)

Sol.



(Product **P**)

87. Frequency of the de-Broglie wave of election in Bohr's first orbit of hydrogen atom is ____ $\times 10^{13}$ Hz (nearest integer).

[Given : R_H (Rydberg constant) = 2.18×10^{-18} J. h (Plank's constant) = 6.6×10^{-34} J.s.]

Allen Ans. (661)

NTA Ans. (658)

Sol. $\lambda = \dfrac{h}{mv}$

$$\lambda = \frac{hv}{mv^2}$$

$$\frac{mv^2}{h} = \frac{v}{\lambda} = \nu \text{ (frequency)}$$

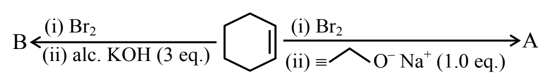
Given $\dfrac{1}{2}mv^2 = 2.18 \times 10^{-18}$ J

$h = 6.6 \times 10^{-34}$

$$\nu = \frac{4.36 \times 10^{-18}}{6.6 \times 10^{-34}} = 660.60 \times 10^{13} \text{ Hz}$$

$\approx 661 \times 10^{13}$ Hz

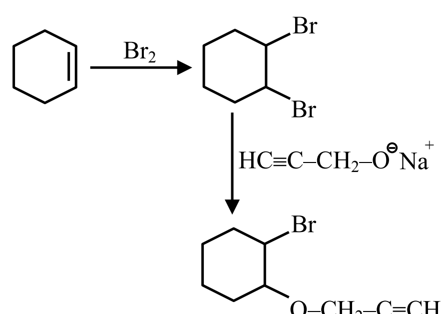
88. The major products from the following reaction sequence are product A and product B.



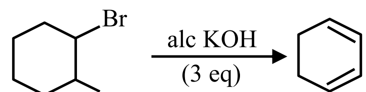
The total sum of π electrons in product A and product B are ______ (nearest integer)

Ans. (8)

Sol.



(A)



(B)

89. Among CrO, Cr_2O_3 and CrO_3, the sum of spin-only magnetic moment values of basic and amphoteric oxides is ______ 10^{-2} BM (nearest integer).

(Given atomic number of Cr is 24)

Ans. **(877)**

Sol. CrO Basic oxide

Cr_2O_3 Amphoteric oxide

In CrO, Cr exist as Cr^{+2} and have μ only = 4.90

In Cr_2O_3, Cr exist as Cr^{+3} and have μ only = 3.87

Sum of spin only magnetic moment

= 4.90 + 3.87 = 8.77

$\mu_{only} = 877 \times 10^{-2}$

Ans. 877

90. An ideal gas, $\overline{C}_V = \frac{5}{2}R$, is expanded adiabatically against a constant pressure of 1 atm untill it doubles in volume. If the initial temperature and pressure is 298 K and 5 atm, respectively then the final temperature is ______ K (nearest integer).

[$\overline{C}_V$ is the molar heat capacity at constant volume]

Ans. **(274)**

Sol. $\Delta U = q + w \ (q = 0)$

$nC_V\Delta T = -P_{ext}(V_2 - V_1)$

$V_2 = 2V_1$

$$\frac{nRT_2}{P_2} = \frac{2nRT_1}{P_1}$$

$P_1 = 5, \ T_1 = 298$

$$P_2 = \frac{5T_2}{2 \times 298}$$

$$n\frac{5}{2}R(T_2 - T_1) = -1\left(\frac{nRT_2}{P_1} - \frac{nRT_1}{P_1}\right)$$

Put $T_1 = 298$

and $P_2 = \dfrac{5T_2}{2 \times 298}$

Solve and we get $T_2 = 274.16$ K

$T_2 \approx 274$ K