# Differential Equations

## Question1

**Let x = x(t) and y = y(t) be solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, a, b ∈ R. Given that x(0) = 2; y(0) = 1 and 3y(1) = 2x(1), the value of t, for which x(t) = y(t), is :**
**[27-Jan-2024 Shift 1]**

**Options:**

A. $\log_{\frac{2}{3}} 2$

B. $\log_4 3$

C. $\log_3 4$

D. $\log_{\frac{4}{3}} 2$

**Answer: D**

### Solution:

$$\frac{dx}{dt} + ax = 0$$

$$\frac{dx}{x} = -a\,\mathrm{dt}$$

$$\int \frac{dx}{x} = -a\int \mathrm{dt}$$

$$\ln|x| = -at + c$$

$$att = 0, x = 2$$

$$\ln 2 = 0 + c$$

$$\ln x = -at + \ln 2$$

$$\frac{x}{2} = e^{-at}$$

$$x = 2e^{-at} \text{ ..............(I)}$$

$$\frac{dy}{dt} + by = 0$$

$$\frac{dy}{y} = -b\,dt$$

$$\ln|y| = -bt + \lambda$$

$t = 0,\ y = 1$

$0 = 0 + \lambda$

$y = e^{-bt}$ .................(II)

According to question

$3y(1) = 2x(1)$

$3e^{-b} = 2(2e^{-a})$

$$e^{a-b} = \frac{4}{3}$$

For $x(t) = y(t)$

$\Rightarrow 2e^{-at} = e^{-bt}$

$2 = e^{(a-b)t}$

$$2 = \left(\frac{4}{3}\right)^t$$

$$\log_{\frac{4}{3}} 2 = t$$

---

# Question2

**If the solution of the differential equation $(2x + 3y - 2)\,dx + (4x + 6y - 7)\,dy = 0$, $y(0) = 3$, is $\alpha x + \beta y + 3\log_e 2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to____**
**[27-Jan-2024 Shift 1]**

**Answer: 29**

## Solution:

$2x + 3y - 2 = t$

$4x + 6y - 4 = 2t$

$$2 + 3\frac{dy}{dx} = \frac{dt}{dx}$$

$4x + 6y - 7 = 2t - 3$

$$\frac{dy}{dx} = \frac{-(2x + 3y - 2)}{4x + 6y - 7}$$

$$\frac{dt}{dx} = \frac{-3t + 4t - 6}{2t - 3} = \frac{t - 6}{2t - 3}$$

$\int \frac{2t-3}{t-6}\,dt = \int dx$

$\int \left( \frac{2t-12}{t-6} + \frac{9}{t-6} \right) \cdot dt = x$

$2t + 9\ln(t-6) = x + c$

$2(2x+3y-2) + 9\ln(2x+3y-8) = x + c$

$x = 0, y = 3$

$c = 14$

$4x + 6y - 4 + 9\ln(2x+3y-8) = x + 14$

$x + 2y + 3\ln(2x+3y-8) = 6$

$\alpha = 1, \beta = 2, \gamma = 8$

$\alpha + 2\beta + 3\gamma = 1 + 4 + 24 = 29$

----------------------------------------------------------------------------------------------------

# Question3

**If y = y(x) is the solution curve of the differential equation $(x^2 - 4)dy - (y^2 - 3y)dx = 0$,
$x > 2$, $y(4) = \frac{3}{2}$ and the slope of the curve is never zero, then the value of y(10) equals :
[27-Jan-2024 Shift 2]**

**Options:**

A. $\frac{3}{1 + (8)^{1/4}}$

B. $\frac{3}{1 + 2\sqrt{2}}$

C. $\frac{3}{1 - 2\sqrt{2}}$

D. $\frac{3}{1 - (8)^{1/4}}$

**Answer: A**

## Solution:

$(x^2-4)dy-(y^2-3y)\,dx=0$

$\Rightarrow \int \frac{dy}{y^2-3y}=\int \frac{dx}{x^2-4}$

$\Rightarrow \frac{1}{3}\int \frac{y-(y-3)}{y(y-3)}dy=\int \frac{dx}{x^2-4}$

$\Rightarrow \frac{1}{3}(\ln|y-3|-\ln|y|)=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

$\Rightarrow \frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

At $x=4, y=\frac{3}{2}$

$\therefore C=\frac{1}{4}\ln 3$

$\therefore \frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+\frac{1}{4}\ln(3)$

At $x=10$

$\frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{2}{3}\right|+\frac{1}{4}\ln(3)$

$\ln\left|\frac{y-3}{y}\right|=\ln 2^{3/4}, \forall x>2, \frac{dy}{dx}<0$

as $y(4)=\frac{3}{2}\Rightarrow y\in(0,3)$

$-y+3=8^{1/4}\cdot y$

$y=\frac{3}{1+8^{1/4}}$

-----------------------------------------------------------------------------------------------

# Question4

**If the solution curve, of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passing through the point (2, 1) is**
**$\tan^{-1}\left(\frac{y-1}{x-1}\right)-\frac{1}{\beta}\log_e\left(\alpha+\left(\frac{y-1}{x-1}\right)^2\right)=\left|\log_e x-1\right|$**
**then $5\beta+\alpha$ is equal to__**
**[27-Jan-2024 Shift 2]**

**Answer: 11**

**Solution:**

$$\frac{dy}{dx} = \frac{x+y-2}{x-y}$$

$x = X + h,\ y = Y + k$

$$\frac{dY}{dX} = \frac{X+Y}{X-Y}$$

$$\left.\begin{array}{r} h+k-2=0 \\ h-k=0 \end{array}\right\} h = k = 1$$

$Y = vX$

$$v + \frac{dv}{dX} = \frac{1+v}{1-v} \Rightarrow X - \frac{dv}{dX} = \frac{1+v^2}{1-v}$$

$$\frac{1-v}{1+v^2}dv = \frac{dX}{X}$$

$$\tan^{-1}v - \frac{1}{2}\ln(1+v^2) = \ln|X| + C$$

As curve is passing through (2, 1)

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\left(1 + \left(\frac{y-1}{x-1}\right)^2\right) = \ln|x-1|$$

$\therefore \alpha = 1$ and $\beta = 2$

$\Rightarrow 5\beta + \alpha = 11$

----------------------------------------------------------------------------------------------------

# Question5

## A function $y = f(x)$ satisfies $f(x)\sin 2x + \sin x - (1 + \cos^2 x)f'(x) = 0$ with condition $f(0) = 0$. Then $f\left(\frac{\pi}{2}\right)$ is equal to [29-Jan-2024 Shift 1]

**Options:**

A. 1

B. 0

C. -1

D. 2

**Answer: A**

## Solution:

**Solution:**

$$\frac{dy}{dx}-\left(\frac{\sin 2x}{1+\cos^2 x}\right)y=\sin x$$

$$\text{I.F.} = 1+\cos^2 x$$

$$y\cdot(1+\cos^2 x)=\int(\sin x)\,dx$$

$$=-\cos x+C$$

$$x=0,\ C=1$$

$$y\left(\frac{\pi}{2}\right)=1$$

---

# Question6

**If the solution curve y = y(x) of the differential equation $(1+y^2)(1+\log_e x)\,dx+x\,dy=0$, $x>0$ passes through the point (1, 1) and $y(e)=\dfrac{\alpha-\tan\left(\frac{3}{2}\right)}{\beta+\tan\left(\frac{3}{2}\right)}$, then $\alpha+2\beta$ is**

**[29-Jan-2024 Shift 1]**

**Answer: 3**

## Solution:

$$\int\left(\frac{1}{x}+\frac{\ln x}{x}\right)dx+\int\frac{dy}{1+y^2}=0$$

$$\ln x+\frac{(\ln x)^2}{2}+\tan^{-1}y=C$$

Put $x=y=1$

$$\therefore C=\frac{\pi}{4}$$

$$\Rightarrow \ln x+\frac{(\ln x)^2}{2}+\tan^{-1}y=\frac{\pi}{4}$$

Put $x=e$

$$\Rightarrow y=\tan\left(\frac{\pi}{4}-\frac{3}{2}\right)=\frac{1-\tan\frac{3}{2}}{1+\tan\frac{3}{2}}$$

$$\therefore \alpha=1,\beta=1$$

$$\Rightarrow \alpha+2\beta=3$$

---

# Question7

**If $\sin\left(\frac{y}{x}\right) = \log_e|x| + \frac{\alpha}{2}$ is the solution of the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ and $y = \frac{\pi}{3}$, then $\alpha^2$ is equal to**
**[29-Jan-2024 Shift 2]**

**Options:**

A. 3

B. 12

C. 4

D. 9

**Answer: A**

## Solution:

**Solution:**

Differential equation :-

$$x\cos\frac{y}{x}\frac{dy}{dx} = y\cos\frac{y}{x} + x$$

$$\cos\frac{y}{x}\left[x\frac{dy}{dx} - y\right] = x$$

Divide both sides by $x^2$

$$\cos\frac{y}{x}\left(\frac{x\frac{dy}{dx} - y}{x^2}\right) = \frac{1}{x}$$

Let $\frac{y}{x} = t$

$$\cos t\left(\frac{dt}{dx}\right) = \frac{1}{x}$$

$$\cos tt = \frac{1}{x}dx$$

Integrating both sides

$$\sin t = \ln|x| + c$$

$$\sin\frac{y}{x} = \ln|x| + c$$

Using $y(1) = \frac{\pi}{3}$, we get $c = \frac{\sqrt{3}}{2}$

So, $\alpha = \sqrt{3} \Rightarrow \alpha^2 = 3$

----------------------------------------------------------------------------------------------------

# Question8

**Let $y = y(x)$ be the solution of the differential equation**

**$(1-x^2)dy = \left[xy + (x^3+2)\sqrt{3(1-x^2)}\right]dx\ -1 < x < 1$, $y(0) = 0$. If $y\left(\frac{1}{2}\right) = \frac{m}{n}$, m and n are coprime numbers, then m + n is equal to**
**[30-Jan-2024 Shift 1]**

**Answer: 97**

## Solution:

**Solution:**

$$\frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{(x^3+2)\sqrt{3(1-x^2)}}{1-x^2}$$

$$\text{IF} = e^{-\int \frac{x}{1-x^2}dx} = e^{+\frac{1}{2}\ln(1-x^2)} = \sqrt{1-x^2}$$

$$y\sqrt{1-x^2} = \sqrt{3}\int(x^3+2)\,dx$$

$$y\sqrt{1-x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right) + c$$

$$\Rightarrow y(0) = 0 \quad \therefore c = 0$$

$$y\left(\frac{1}{2}\right) = \frac{65}{32} = \frac{m}{n}$$

$$m + n = 97$$

---------------------------------------------------------------------------------------------------

# Question9

**Let y = y(x) be the solution of the differential equation $\sec x\,dy + \{2(1-x)\tan x + x(2-x)\}\,dx = 0$ such that $y(0) = 2$. Then $y(2)$ is equal to :**
**[30-Jan-2024 Shift 1]**

**Options:**

A. 2

B. $2\{1 - \sin(2)\}$

C. $2\{\sin(2) + 1\}$

D. 1

**Answer: A**

## Solution:

**Solution:**

$$\frac{dy}{dx} = 2(x-1)\sin x + (x^2-2x)\cos x$$

Now both side integrate

$$y(x) = \int 2(x-1)\sin x\,dx + [(x^2-2x)(\sin x) - \int (2x-2)\sin x\,dx]$$

$$y(x) = (x^2-2x)\sin x + \lambda$$

$$y(0) = 0 + \lambda \Rightarrow 2 = \lambda$$

$$y(x) = (x^2-2x)\sin x + 2$$

$$y(2) = 2$$

---------------------------------------------------------------------------------------------------

# Question10

## The solution curve of the differential equation $y\frac{dx}{dy} = x(\log_e x - \log_e y + 1),\ x > 0,\ y > 0$ passing through the point (e, 1) is [31-Jan-2024 Shift 1]

**Options:**

A. $\left|\log_e \frac{y}{x}\right| = x$

B. $\left|\log_e \frac{y}{x}\right| = y^2$

C. $\left|\log_e \frac{x}{y}\right| = y$

D. $2\left|\log_e \frac{x}{y}\right| = y + 1$

**Answer: C**

## Solution:

$$\frac{dx}{dy} = \frac{x}{y}\left(\ln\left(\frac{x}{y}\right) + 1\right)$$

Let $\frac{x}{y} = t \Rightarrow x = ty$

$$\frac{dx}{dy} = t + y\frac{dt}{dy}$$

$$t + y\frac{dt}{dy} = t(\ln(t) + 1)$$

$$y\frac{dt}{dy} = t\ln(t) \Rightarrow \frac{dt}{t\ln(t)} = \frac{dy}{y}$$

$$\Rightarrow \int \frac{dt}{t\cdot\ln(t)} = \int \frac{dy}{y}$$

$$\Rightarrow \int \frac{dp}{p} = \int \frac{dy}{y}$$

let $\ln t = p$

$\frac{1}{t}\,dt = dp$

$\Rightarrow \ln p = \ln y + c$

$\ln(\ln t) = \ln y + c$

$\ln\left(\ln\left(\frac{x}{y}\right)\right) = \ln y + c$

at $x = e, y = 1$

$\ln\left(\ln\left(\frac{e}{1}\right)\right) = \ln(1) + c \Rightarrow c = 0$

$\ln\left|\ln\left(\frac{x}{y}\right)\right| = \ln y$

$\left|\ln\left(\frac{x}{y}\right)\right| = e^{\ln y}$

$\left|\ln\left(\frac{x}{y}\right)\right| = y$

----------------------------------------------------------------------------------------------------

# Question11

**Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = \frac{(\tan x) + y}{\sin x(\sec x - \sin x \tan x)}$, $x \in \left(0, \frac{\pi}{2}\right)$ satisfying the condition $y\left(\frac{\pi}{4}\right) = 2$. Then, $y\left(\frac{\pi}{3}\right)$ is**

**[31-Jan-2024 Shift 1]**

**Options:**

A. $\sqrt{3}(2 + \log_e\sqrt{3})$

B. $\frac{\sqrt{3}}{2}(2 + \log_e 3)$

C. $\sqrt{3}(1 + 2\log_e 3)$

D. $\sqrt{3}(2 + \log_e 3)$

**Answer: A**

**Solution:**

$$\frac{dy}{dx} = \frac{\sin x + y\cos x}{\sin x \cdot \cos x\left(\frac{1}{\cos x} - \sin x \cdot \frac{\sin x}{\cos x}\right)}$$

$$= \frac{\sin x + y\cos x}{\sin x(1-\sin^2 x)}$$

$$\frac{dy}{dx} = \sec^2 x + y \cdot 2(\operatorname{cosec} 2x)$$

$$\frac{dy}{dx} - 2\operatorname{cosec}(2x) \cdot y = \sec^2 x$$

$$\frac{dy}{dx} + p \cdot y = Q$$

$$\text{I.F.} = e^{\int \mathrm{pdx}} = e^{\int -2\operatorname{cosec}(2x)\,dx}$$

$$\text{Let } 2x = t$$

$$2\frac{dx}{dt} = 1$$

$$dx = \frac{dt}{2}$$

$$= e^{-\int \operatorname{cosec}(t)\,dt}$$

$$= e^{-\ln\left|\tan\frac{t}{2}\right|}$$

$$= e^{-\ln|\tan x|} = \frac{1}{|\tan x|}$$

$$y(\mathrm{IF}) = \int Q(\mathrm{IF})\,dx + c$$

$$\Rightarrow y\frac{1}{|\tan x|} = \int \sec^2 x \cdot \frac{1}{|\tan x|} + c$$

$$y \cdot \frac{1}{|\tan x|} = \int \frac{dt}{|t|} + c \quad \text{for } \tan x = t$$

$$y \cdot \frac{1}{|\tan x|} = \ln|t| + c$$

$$y = |\tan x|(\ln|\tan x| + c)$$

$$\text{Put } x = \frac{\pi}{4}, y = 2$$

$$2 = \ln 1 + c \Rightarrow c = 2$$

$$y = |\tan x|(\ln|\tan x| + 2)$$

$$y\left(\frac{\pi}{3}\right) = \sqrt{3}(\ln\sqrt{3} + 2)$$

---------------------------------------------------------------------------------------------------

# Question12

**The temperature T(t) of a body at time t = 0 is 160° F and it decreases continuously as per the differential equation $\frac{dT}{dt} = -K(T-80)$, where K is positive constant. If T(15) = 120°F, then T(45) is equal to**

**[31-Jan-2024 Shift 2]**

**Options:**

A. 85˚F

B. 95˚F

C. 90˚F

D. 80˚F

**Answer: C**

## Solution:

**Solution:**

$$\frac{dT}{dt} = -k(T-80)$$

$$\int_{160}^{T} \frac{dT}{(T-80)} = \int_{0}^{t} -Kdt$$

$$[\ln|T-80|]_{160}^{T} = -kt$$

$$\ln|T-80| - \ln 80 = -kt$$

$$\ln\left|\frac{T-80}{80}\right| = -kt$$

$$T = 80 + 80e^{-kt}$$

$$120 = 80 + 80e^{-k \cdot 15}$$

$$\frac{40}{80} = e^{-k15} = \frac{1}{2}$$

$$\therefore T(45) = 80 + 80e^{-k \cdot 45}$$

$$= 80 + 80(e^{-k.15})^{3}$$

$$= 80 + 80 \times \frac{1}{8}$$

$$= 90$$

------------------------------------------------------------------------------------------------

# Question13

**Let y = y(x) be the solution of the differential equation $\sec^2 x\,dx + (e^{2y}\tan^2 x + \tan x)dy = 0$**

**$0 < x < \frac{\pi}{2}$, $y\left(\frac{\pi}{4}\right) = 0$. If $y\left(\frac{\pi}{6}\right) = \alpha$**

**Then $e^{8\alpha}$ is equal to**

**[31-Jan-2024 Shift 2]**

**Answer: 9**

## Solution:

$$sec^2 x \frac{dx}{dy} + e^{2y}\tan^2 x + \tan x = 0$$

$$\left( \text{Put } \tan x = t \Rightarrow sec^2 x \frac{dx}{dy} = \frac{dt}{dy} \right)$$

$$\frac{dt}{dy} + e^{2y} \times t^2 + t = 0$$

$$\frac{dt}{dy} + t = -t^2 \cdot e^{2y}$$

$$\frac{1}{t^2}\frac{dt}{dy} + \frac{1}{t} = -e^{2y}$$

$$\left( \text{Put } \frac{1}{t} = u \ \frac{-1}{t^2}\frac{dt}{dy} = \frac{du}{dy} \right)$$

$$\frac{-du}{dy} + u = -e^{2y}$$

$$\frac{du}{dy} - u = e^{2y}$$

$$\text{I.F. } = e^{-\int dy} = e^{-y}$$

$$ue^{-y} = \int e^{-y} \times e^{2y} dy$$

$$\frac{1}{\tan x} \times e^{-y} = e^{y} + c$$

$$x = \frac{\pi}{4}, y = 0, c = 0$$

$$x = \frac{\pi}{6}, y = \alpha$$

$$\sqrt{3}e^{-\alpha} = e^{\alpha} + 0$$

$$e^{2\alpha} = \sqrt{3}$$

$$e^{8\alpha} = 9$$

----------------------------------------------------------------------------------------------------

# Question14

**Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$, $y(0) = 1$.**

**Then, $\left( \frac{1}{\sqrt{2}} + y\left( \frac{1}{\sqrt{2}} \right) \right)^2$ equals :**

**[1-Feb-2024 Shift 1]**

**Options:**

A. $\frac{4}{4+\sqrt{e}}$

B. $\frac{3}{3-\sqrt{e}}$

C. $\frac{2}{1+\sqrt{e}}$

D. $\frac{1}{2-\sqrt{e}}$

**Answer: D**

## Solution:

$$\frac{dy}{dx}=2x(x+y)^3-x(x+y)-1$$

$$x+y=t$$

$$\frac{dt}{dx}-1=2xt^3-xt-1$$

$$\frac{dt}{2t^3-t}=x\,dx$$

$$\frac{tdt}{2t^4-t^2}=xdx$$

Let $t^2=z$

$$\int\frac{dz}{2(2z^2-z)}=\int x\,dx$$

$$\int\frac{dz}{4z\left(z-\frac{1}{2}\right)}=\int x\,dx$$

$$\ln\left|\frac{z-\frac{1}{2}}{z}\right|=x^2+k$$

$$z=\frac{1}{2-\sqrt{e}}$$

---

# Question15

**If x = x(t) is the solution of the differential equation $(t+1)\,dx=(2x+(t+1)^4)\,dt$, $x(0)=2$, then, x(1) equals ________ [1-Feb-2024 Shift 1]**

**Answer: 14**

## Solution:

$(t+1)\,dx = (2x+(t+1)^4)\,dt$

$$\frac{dx}{dt} = \frac{2x+(t+1)^4}{t+1}$$

$$\frac{dx}{dt} - \frac{2x}{t+1} = (t+1)^3$$

$$I \cdot F = e^{-\int \frac{2}{t+1}dt} = e^{-2\ln(t+1)} = \frac{1}{(t+1)^2}$$

$$\frac{x}{(t+1)^2} = \int \frac{1}{(t+1)^2}(t+1)^3\,dt + c$$

$$\frac{x}{(t+1)^2} = \frac{(t+1)^2}{2} + c$$

$$\Rightarrow c = \frac{3}{2}$$

$$x = \frac{(t+1)^4}{2} + \frac{3}{2}(t+1)^2$$

put, $t = 1$

$x = 2^3 + 6 = 14$

-----------------------------------------------------------------------------------------------

# Question16

**Let α be a non-zero real number. Suppose $f : R \longrightarrow R$ is a differentiable function such that $f(0) = 2$ and $\lim\limits_{x \to -\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(-\log_e 2)$ is equal to**
**[1-Feb-2024 Shift 2]**

**Options:**

A. 3

B. 5

C. 9

D. 7

**Answer: A**

## Solution:

**Solution:**

$f(0) = 2,\ \lim\limits_{x \to -\infty} f(x) = 1$

$f'(x) - \alpha \cdot f(x) = 3$

$I \cdot F = e^{-\alpha x}$

$y(e^{-\alpha x}) = \int 3 \cdot e^{-\alpha x}\,dx$

$f(x) \cdot (e^{-\alpha x}) = \dfrac{3e^{-\alpha x}}{-\alpha} + c$

$x = 0 \Rightarrow 2 = \dfrac{-3}{\alpha} + c \Rightarrow \dfrac{3}{\alpha} = c - 2$

$f(x) = \dfrac{-3}{\alpha} + c \cdot e^{\alpha x}$

$x \to -\infty \Rightarrow 1 = \frac{-3}{\alpha} + c(0)$

$\alpha = -3 \therefore c = 1$

$f(-\ln 2) = \frac{-3}{\alpha} + c \cdot e^{\alpha x}$

$= 1 + e^{3 \ln 2} = 9$

(But α should be greater than 0 for finite value of c)

------------------------------------------------------------------------------------------------

# Question17

**If $\frac{dx}{dy} = \frac{1 + x - y^2}{y}$, x(1) = 1, then 5x(2) is equal to :**
**[1-Feb-2024 Shift 2]**

**Answer: 5**

## Solution:

**Solution:**

$\frac{dx}{dy} - \frac{x}{y} = \frac{1 - y^2}{y}$

Integrating factor $= e^{\int -\frac{1}{y}} = \frac{1}{y}$

$x \cdot \frac{1}{y} = \int \frac{1 - y^2}{y^2} dy$

$\frac{x}{y} = \frac{-1}{y} - y + c$

$x = -1 - y^2 + cy$

$x(1) = 1$

$1 = -1 - 1 + c \Rightarrow c = 3$

$x = -1 - y^2 + 3y$

$5x(2) = 5(-1 - 4 + 6)$

$= 5$

------------------------------------------------------------------------------------------------

# Question18

**Let y = y(x) be the solution of the differential equation**
**$x^3 dy + (xy - 1) dx = 0$, $x > 0$, $y\left(\frac{1}{2}\right) = 3 - e$. Then y(1) is equal to**
**[24-Jan-2023 Shift 1]**

**Options:**

A. 1

B. e

C. $2-e$

D. 3

**Answer: A**

## Solution:

**Solution:**

$$\frac{dy}{dx}=\frac{1-xy}{x^3}=\frac{1}{x^3}-\frac{y}{x^2}$$

$$\frac{dy}{dx}+\frac{y}{x^2}=\frac{1}{x^3}$$

$$\text{If } = e^{\int\frac{1}{x^2}dx}=e^{-\frac{1}{x}}$$

$$y\cdot e^{-\frac{1}{x}}=\int e^{-\frac{1}{x}}\cdot\frac{1}{x^3}dx\left(\text{ put } -\frac{1}{x}=t\right)$$

$$y\cdot e^{-\frac{1}{x}}=-\int e^t\cdot t\,dt$$

$$y=\frac{1}{x}+1+Ce^{\frac{1}{x}}$$

Where C is constant

Put $x=\frac{1}{2}$

$$3-e=2+1+Ce^2$$

$$C=-\frac{1}{e}$$

$$y(1)=1$$

-----------------------------------------------------------------------------------------------

# Question19

**Let y = y(x) be the solution of the differential equation left$(x^2-3y^2$ right$)\,dx+3xyd\,y=0$, $y(1)=1$. Then $6y^2(e)$ is equal to [24-Jan-2023 Shift 2]**

**Options:**

A. $3e^2$

B. $e^2$

C. $2e^2$

D. $\frac{3e^2}{2}$

**Answer: C**

## Solution:

**Solution:**

$$(x^2-3y^2)\,dx+3xyd\,y=0$$

$$\frac{dy}{dx}=\frac{3y^2-x^2}{3xy}\Rightarrow\frac{dy}{dx}=\frac{y}{x}-\frac{1}{3}\frac{x}{y}$$

Put $y=vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$
$$(1) \Rightarrow v + x\frac{dv}{dx} = v - \frac{1}{3}\frac{1}{v}$$
$$\Rightarrow vd\,v = \frac{-1}{3x}$$
Integrating both side
$$\frac{v^2}{2} = \frac{-1}{3}\ln x + c$$
$$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + c$$
$y(1) = 1$
$$\Rightarrow \frac{1}{2} = c$$
$$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3}\ln x + \frac{1}{2}$$
$$\Rightarrow y^2 = -\frac{2}{3}x^2\ln x + x^2$$
$$y^2(e) = -\frac{2}{3}e^2 + e^2 = \frac{e^2}{3}$$
$$\Rightarrow 6y^2(e) = 2e^2$$

---------------------------------------------------------------------------------------------------

# Question20

## Let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx} = \frac{y}{x}(1 + xy^2(1 + \log_e x))$, $x > 0$, $y(1) = 3$. Then $\frac{y^2(x)}{9}$ is equal to : [25-Jan-2023 Shift 1]

**Options:**

A. $\frac{x^2}{5 - 2x^3(2 + \log_e x^3)}$

B. $\frac{x^2}{2x^3(2 + \log_e x^3) - 3}$

C. $\frac{x^2}{3x^3(1 + \log_e x^2) - 2}$

D. $\frac{x^2}{7 - 3x^3(2 + \log_e x^2)}$

**Answer: A**

## Solution:

**Solution:**

$$\frac{dy}{dx} - \frac{y}{x} = y^3(1 + \log_e x)$$
$$\frac{1}{y^3}\frac{dy}{dx} - \frac{1}{xyy^2} = 1 + \log_e x$$
$$\text{Let } -\frac{1}{y^2} = t \Rightarrow \frac{2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$$
$$\therefore \frac{dt}{dx} + \frac{2t}{x} = 2(1 + \log_e x)$$
$$\text{I.F.} = e^{\int \frac{2}{x}dx} = x^2$$
$$\frac{-x^2}{y^2} = \frac{2}{3}\left((1 + \log_e x)x^3 - \frac{x^3}{3}\right) + C$$
$y(1) = 3$

$$\frac{y^2}{9} = \frac{x^2}{5 - 2x^3(2 + \log_e x^3)}$$
$$x\,dy = y\,dx + xy^3(1 + \log_e x)\,dx$$
$$\frac{x\,dy - y\,dx}{y^3} = x(1 + \log_e x)\,dx$$
$$-\frac{x}{y} d\left(\frac{x}{y}\right) = x^2(1 + \log_e x)\,dx$$
$$-\left(\frac{x}{y}\right)^2 = 2\int x^2(1 + \log_e x)\,dx$$

---

# Question21

**Let y = y(t) be a solution of the differential equation $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$ Where, $\alpha > 0$, $\beta > 0$ and $\gamma > 0$. Then $\lim\limits_{t \to \infty} y(t)$**

**[25-Jan-2023 Shift 2]**

**Options:**

A. is 0

B. does not exist

C. is 1

D. is - 1

**Answer: A**

## Solution:

**Solution:**

$$\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$$
$$\text{I.F.} = e^{\int \alpha\,dt} = e^{\alpha t}$$
$$\text{Solution} \Rightarrow y \cdot e^{\alpha t} = \int \gamma e^{-\beta T} \cdot e^{\alpha t}\,dt$$
$$\Rightarrow ye^{\alpha t} = \gamma\frac{e^{(\alpha - \beta)t}}{(\alpha - \beta)} + c$$
$$\Rightarrow y = \frac{\gamma}{e^{\beta t}(\alpha - \beta)} + \frac{c}{e^{\alpha t}}$$
So, $\lim\limits_{t \to \infty} y(t) = \frac{\gamma}{\infty} + \frac{c}{\infty} = 0$

---

# Question22

**Let y = f(x) be the solution of the differential equation $y(x + 1)\,dx - x^2 dy = 0$, $y(1) = e$. Then $\lim\limits_{x \to 0^+} f(x)$ is equal to**

**[29-Jan-2023 Shift 1]**

**Options:**

A. 0

B. $\frac{1}{e}$

C. $e^2$

D. $\frac{1}{e^2}$

**Answer: A**

## Solution:

**Solution:**

$\frac{x+1}{x^2}dx = \frac{dy}{y}$

$\ln x - \frac{1}{x} = \ln y + c$

$(1, e)$

$c = -2$

$\ln x - \frac{1}{x} = \ln y - 2$

$y = e^{\ln x} - \frac{1}{x} + 2$

$\lim_{x \to 0^+} e^{\ln x - 1} - \frac{1}{x} + 2$

$= e^{-\infty}$

$= 0$

-----------------------------------------------------------------------------------------------

# Question23

**Let y = y(x) be the solution of the differential equation $x\log_e x \frac{dy}{dx} + y = x^2\log_e x$, (x > 1). If y(2) = 2, then y(e) is equal to [29-Jan-2023 Shift 2]**

**Options:**

A. $\frac{4+e^2}{4}$

B. $\frac{1+e^2}{4}$

C. $\frac{2+e^2}{2}$

D. $\frac{1+e^2}{2}$

**Answer: A**

## Solution:

**Solution:**

$x\log_e x \frac{dy}{dx} + y = x^2\log_e x, (x > 1)$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x\ln x} = x$

Linear differential equation

I.F. $= e^{\int \frac{1}{x\ln x}dx} = \left|\ln x\right|$

∴ Solution of differential equation

$y\,|\ln x| = \int x\,|\ln x|\,dx$

$= |\ln x| \left| \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \right.$

$\Rightarrow y\,|\ln x| = |\ln x| \left| \left( \frac{x^2}{2} \right) - \frac{x^2}{4} + c \right.$

For constant

$y(2) = 2 \Rightarrow c = 1$

So, $y(x) = \frac{x^2}{2} - \frac{x^2}{4\,|\ln x|} + \frac{1}{|\ln x|}$

Hence, $y(e) = \frac{e^2}{2} - \frac{e^2}{4} + 1 = 1 + \frac{e^2}{4}$

---

# Question24

## Let the solution curve y = y(x) of the differential equation $\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}} y = 2x$ $\exp \frac{x^3 - \tan^{-1}x^3}{\sqrt{(1+x)^6}}$ pass through the origin.

## Then y(1) is equal to :

## [30-Jan-2023 Shift 1]

**Options:**

A. $\exp\left( \frac{4-\pi}{4\sqrt{2}} \right)$

B. $\exp\left( \frac{\pi-4}{4\sqrt{2}} \right)$

C. $\exp\left( \frac{1-\pi}{4\sqrt{2}} \right)$

D. $\exp\left( \frac{4+\pi}{4\sqrt{2}} \right)$

**Answer: A**

## Solution:

$$\frac{dy}{dx} + \left( \frac{-3x^5 \tan^{-1}x^3}{(1+x^6)^{3/2}} \right) y = 2e^{\left\{ \frac{x - \tan x}{\sqrt{1+x^6}} \right\}}$$

$$\text{I.F.} = e^{\int \frac{-3x^5 \tan^{-1}x^3}{(1+x^6)^{3/2}} dx}$$

$$= e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}}$$

Solution of differential equation

$$y \cdot e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}} = \int 2xe^{\left( \frac{x^3 - \tan^{-1}x^3}{\sqrt{1+x^6}} \right)} \cdot e^{\left( \frac{\tan^{-1}(x^3) - x^3}{\sqrt{1+x^6}} \right)} dx$$

$$= \int 2x\,dx + c$$

$$y \cdot e^{\frac{\tan^{-1}x^3 - x^3}{\sqrt{1+x^6}}} = x^2 + c$$

Also it passes through origin

$c = 0$

$$y(1) \cdot e^{\frac{\tan^{-1}(1) - 1}{\sqrt{2}}} = 1$$

$$y(1) \cdot e^{\frac{\frac{\pi}{4} - 1}{\sqrt{2}}} = 1$$

$$y(1) \cdot e^{\frac{\pi - 4}{4\sqrt{2}}} = 1$$

$$y(1) = \frac{1}{e^{\frac{\pi - 4}{4\sqrt{2}}}} = e^{\frac{4 - \pi}{4\sqrt{2}}}$$

---------------------------------------------------------------------------------------------------

# Question25

## The solution of the differential equation $\frac{dy}{dx} = -\left(\frac{x^2 + 3y^2}{3x^2 + y^2}\right)$, y(1) = 0 is [30-Jan-2023 Shift 2]

**Options:**

A. $\log_e |x + y| - \frac{xy}{(x + y)^2} = 0$

B. $\log_e |x + y| + \frac{xy}{(x + y)^2} = 0$

C. $\log_e |x + y| + \frac{2xy}{(x + y)^2} = 0$

D. $\log_e |x + y| - \frac{2xy}{(x + y)^2} = 0$

**Answer: C**

## Solution:

**Solution:**

Put $y = vx$

$$v + x\frac{dv}{dx} = -\left(\frac{1 + 3v^2}{3 + v^2}\right)$$

$$x\frac{dv}{dx} = -\frac{(v + 1)^3}{3 + v^2}$$

$$\frac{(3 + v^2)dv}{(v + 1)^3} + \frac{dx}{x} = 0$$

$$\int \frac{4dv}{(v + 1)^3} + \int \frac{dv}{v + 1} - \int \frac{2dv}{(v + 1)^2} + \int \frac{dx}{x} = 0$$

$$\frac{-2}{(v + 1)^2} + \ln(v + 1) + \frac{2}{v + 1} + \ln x = c$$

$$\frac{-2x^2}{(x + y)^2} + \ln\left(\frac{x + y}{x}\right) + \frac{2x}{x + y} + \ln x = c$$

$$\frac{2xy}{(x + y)^2} + \ln(x + y) = c$$

$\therefore c = 0$, as $x = 1, y = 0$

$$\therefore \frac{2xy}{(x + y)^2} + \ln(x + y) = 0$$

# Question26

**Let $y = y(x)$ be the solution of the differential equation $(3y^2 - 5x^2)y\,dx + 2x(x^2 - y^2)d\,y = 0$ such that $y(1) = 1$. then $|(y(2))^3 - 12y(2)|$ is equal to:**
**[31-Jan-2023 Shift 2]**

**Options:**

A. $32\sqrt{2}$

B. 64

C. $16\sqrt{2}$

D. 32

**Answer: A**

## Solution:

**Solution:**

$(3y^2 - 5x^2)y \cdot dx + 2x(x^2 - y^2)d\,y = 0$

$\Rightarrow \frac{d\,y}{d\,x} = \frac{y(5x^2 - 3y^2)}{2x(x^2 - y^2)}$

Put $y = mx$

$\Rightarrow m + x \cdot \frac{dm}{dx} = \frac{m(5 - 3m^2)}{2(1 - m^2)}$

$x \cdot \frac{d\,m}{dx} = \frac{(5 - 3m^2)m - 2m(1 - m^2)}{2(1 - m^2)}$

$\Rightarrow \frac{dx}{x} = \frac{2(m^2 - 1)}{m(m^2 - 3)}\,dm$

$\Rightarrow \frac{dx}{x} = \left( \frac{2}{m} - \frac{\frac{4}{3}}{m} + \frac{\frac{4m}{3}}{m^2 - 3} \right) dm$

$\Rightarrow \int \frac{dx}{x} = \int \frac{\left(\frac{2}{3}\right)}{m} + \int \frac{2}{3}\left( \frac{2m}{m^2 - 3} \right) dm$

$\Rightarrow \ln|x| = \frac{2}{3}\ln|m| + \frac{2}{3}\ln|m^2 - 3| + C$

Or, $\ln|x| = \frac{2}{3}\ln\left|\frac{y}{x}\right| + \frac{2}{3}\ln\left|\left(\frac{y}{x}\right)^2 - 3\right| + C$

Put $(x = 1, y = 1)$ : we get $c = -\frac{2}{3}\ln(2)$

$\Rightarrow \ln|x| = \frac{2}{3}\ln\left|\frac{y}{x}\right| + \frac{2}{3}\ln\left|\left(\frac{y}{x}\right)^2 - 3\right| - \frac{2}{3}\ln(2)$

$\Rightarrow \left(\frac{y}{x}\right)\left[\left(\frac{y}{x}\right)^2 - 3\right] = 2 . (x^{3/2})$

Put $x = 2$ to get $y(2)$

$\Rightarrow y(y^2 - 12) = 4 \times 2 \times 2 \times 2\sqrt{2}$

$\Rightarrow y^3 - 12y = 32\sqrt{2}$

$\Rightarrow |y^3(2) - 12y(2)| = 32\sqrt{2}$

----------------------------------------------------------------------------------------------------

# Question27

**If $y = y(x)$ is the solution curve of the differential equation $\frac{dy}{dx} + y\tan x = x\sec x$, $0 \le x \le \frac{\pi}{3}$, $y(0) = 1$, then $y\left(\frac{\pi}{6}\right)$ is equal to**
**[1-Feb-2023 Shift 1]**

**Options:**

A. $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

B. $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

C. $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$

D. $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

**Answer: A**

## Solution:

**Solution:**
Here I.F. = $\sec x$
Then solution of D.E :
$y(\sec x) = x\tan x - \ln(\sec x) + c$
Given $y(0) = 1 \Rightarrow c = 1$
$\therefore y(\sec x) = x\tan x - \ln(\sec x) + 1$
At $x = \frac{\pi}{6}$, $y = \frac{\pi}{12} + \frac{\sqrt{3}}{2}\ln\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}$

---------------------------------------------------------------------------------------------------

# Question28

**Let $\alpha x = \exp(x^{\beta}y^{\gamma})$ be the solution of the differential equation $2x^2y\,dy - (1 - xy^2)\,dx = 0$, $x > 0$, $y(2) = \sqrt{\log_e 2}$. Then $\alpha + \beta - \gamma$ equals :**
**[1-Feb-2023 Shift 2]**

**Options:**

A. 1

B. $-1$

C. 0

D. 3

**Answer: A**

## Solution:

**Solution:**
$\alpha x = e^{x^{\beta} \cdot y^{\gamma}}$

$2x^2y\frac{dy}{dx} = 1 - x \cdot y^2 \quad y^2 = t$

$x^2\frac{dt}{dx} = 1 - xt$

$\frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^2} \quad \text{I.F.} = e^{tnx} = x$

$t(x) = \int \frac{1}{x^2} \cdot x\,dx$

$y^2 \cdot x = \ell nx + C$

$\therefore 2 . \ln 2 = \ln 2 + C$

$\therefore C = \ell n2$

Hence, $xy^2 = \ell n2x$

$\therefore 2x = e^{x \cdot y^2}$

Hence $\alpha = 2, \beta = 1, \gamma = 2$

---------------------------------------------------------------------------------------------------

# Question29

**Let y = y(x) be a solution of the differential $(x\cos x)dy + (xy\sin x + y\cos x - 1)dx = 0$, $0 < x < \frac{\pi}{2}$. If $\frac{\pi}{3}y\left(\frac{\pi}{3}\right) = \sqrt{3}$, then $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + 2y'\left(\frac{\pi}{6}\right)\right|$ is equal to ______.**

**[6-Apr-2023 shift 1]**

**Answer: 2**

## Solution:

**Solution:**

$(x\cos x)dy + (xy\sin x + y\cos x - 1)dx = 0, 0 < x < \frac{\pi}{2}$

$\frac{dy}{dx} + \left(\frac{x\sin x + \cos x}{x\cos x}\right)y = \frac{1}{x\cos x}$

$\text{I F} = x\sec x$

$y \cdot x\sec x = \int \frac{x\sec x}{x\cos x}dx = \tan x + c$

Since $y\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{\pi}$ Hence $c = \sqrt{3}$

Hence $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + y'\left(\frac{\pi}{6}\right)\right| = |-2| = 2$

---------------------------------------------------------------------------------------------------

# Question30

**If the solution curve f(x, y) = 0 of the differential equation $(1 + \log_e x)\frac{dx}{dy} - x\log_e x = e^y$, $x > 0$, passes through the points (1, 0) and (α, 2), then $\alpha^\alpha$ is equal to :**

**[6-Apr-2023 shift 2]**

**Options:**

A. $e^{\sqrt{2c^2}}$

B. $e^{c^2}$

C. $e^{2e^{\sqrt{2}}}$

D. $e^{2c^2}$

**Answer: D**

## Solution:

**Solution:**

$(1+\ell\ln x)\frac{dx}{dy} - x\ell\ln x = e^y$

Let $x\ell nx = t$

$(1+\ln x)\frac{dx}{dy} = \frac{dt}{dy}$

$\frac{dt}{dy} - t = e^y \quad P = -1, Q = e^y$

$I \cdot F = e^{\int -dy} = e^{-y}$

Solution -

$(t)(e^{-y}) = \int (e^{-y})(e^y)dy$

$t(e^{-y}) = y + c$

$(x\ell\ln x)e^{-y} = y + c$ pass $(1, 0) \Rightarrow c = 0$

pass $(\alpha, 2)$

$\alpha^\alpha = e^{2e^2}$

Ans. Option 4

-----------------------------------------------------------------------------------------------

# Question31

**Let the solution curve $x = x(y)$, $0 < y < \frac{\pi}{2}$, of the differential equation $(\log_e(\cos y))^2 \cos y\, dx - (1 + 3x\log_e.(\cos y))\sin y\, dy = 0$ satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_c 2}$. If $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_c n}$, where m and n are co-prime, then mn is equal to**
**[8-Apr-2023 shift 2]**

**Answer: 12**

## Solution:

$\text{Cos}\, y \ln^2 \cos y\, dx = (1 + 3x \ln \cos y)\sin y\, dy$

$\frac{dx}{dy} = \tan y\left(\frac{3x}{\ln\cos y} + \frac{1}{\ln^2\cos y}\right)$

$\frac{dx}{dy} - \left(\frac{3\tan y}{\ln\cos y}\right)x = \frac{\tan y}{\ln^2\cos y}$

$\text{If} = e^{-\frac{3}{\ln\cos y\cos y}dy}$

$\ln \cos y = t$

$\frac{1}{\cos y} - \sin y d y = d t$

If $= e^{\frac{3}{t} dt} = e^{3 \ln t} = t^3 = \ln^3 \cos y$

solution is $x \cdot \ln^3 \cos y = \int \frac{\sin y}{\cos y} \cdot \ln \cos y \, d\, y + C.$

$x \ln^3 \cos y = \frac{-\ln^2 \cos y}{2} + C$

$x\left(\frac{\pi}{3}\right) = \frac{1}{2 \ln 2}$ so $\frac{1}{2 \ln^2} \times \ln^3\left(\frac{1}{2}\right) = -\frac{\ln^3\left(\frac{1}{2}\right)}{2} + C$

$C = 0$ x $\ln^3 \frac{\sqrt{3}}{2} = -\frac{1}{2} \ln^2 \frac{\sqrt{3}}{2} + 0$

$y = \frac{\pi}{6}$

$x = -\frac{1}{2 \ln\left(\frac{\sqrt{3}}{2}\right)}$

$x = \frac{1}{\ln \frac{4}{3}} = \frac{1}{\ln 4 - \ln 3}$

$mn = 12$

---

# Question32

## Let f be a differentiable function such that $x^2 f(x) - x = 4 \int_0^x t f(t)\, dt$, $f(1) = \frac{2}{3}$.. Then 18f(3) is equal to : [10-Apr-2023 shift 1]

**Options:**

A. 180

B. 150

C. 210

D. 160

**Answer: D**

## Solution:

**Solution:**

$x^2 f(x) - x = 4 \int_0^x t f(t)\, dt$

Differentiate w.r.t. x

$x^2 f'(x) + 2x f(x) - 1 = 4x f(x)$

Let $y = f(x)$

$\Rightarrow x^2 \frac{d y}{d x} - 2xy - 1 = 0$

$\frac{d y}{d x} - \frac{2}{x} y = \frac{1}{x^2}$

I.F. $= e^{\int \frac{-2}{x} dx} = \frac{1}{x^2}$

Its solution is

$\frac{y}{x^2} = \int \frac{1}{x^4} dx + C$

$\frac{y}{x^2} = \frac{-1}{3x^3} + C$

$\because f(1) = \frac{2}{3} \Rightarrow y(1) = \frac{2}{3}$
$\Rightarrow \frac{2}{3} = -\frac{1}{3} + C$
$\Rightarrow C = 1$
$\because y = -\frac{1}{3x} + x^2$
$f(x) = -\frac{1}{3x} + x^2$
$f(3) = -\frac{1}{9} + 9 = \frac{80}{9} \Rightarrow 18f(3) = 160$

-----------------------------------------------------------------------------------------------

# Question33

## The slope of tangent at any point (x, y) on a curve y = y(x) is $\frac{x^2+y^2}{2xy}$, x > 0. If y(2) = 0, then a value of y(8) is: [10-Apr-2023 shift 1]

**Options:**

A. $4\sqrt{3}$

B. $-4\sqrt{2}$

C. $-2\sqrt{3}$

D. $2\sqrt{3}$

**Answer: A**

## Solution:

**Solution:**
$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$
$y = vx$
$y(2) = 0$
$y(8) = ?$
$\frac{dv}{dx} = v + x\frac{dv}{dx}$
$v + \frac{xdv}{dx} = \frac{x^2+v^2x^2}{2vx^2}$
$x \cdot \frac{dv}{dx} = \left(\frac{v^2+1}{2v} - v\right)$
$\frac{2vdv}{(1-v^2)} = \frac{dx}{x}$
$-\ln(1-v^2) = \ln x + C$
$\ln x + \ln(1-v^2) = C$
$\ln\left[x\left(1-\frac{y^2}{x^2}\right)\right] = C$
$\ln\left[\left(\frac{x^2-y^2}{x}\right)\right] = C$
$x^2 - y^2 = cx$
$y(2) = 0$ at $x = 2, y = 0$
$4 = 2C \Rightarrow C = 2$
$x^2 - y^2 = 2x$
Hence, at $x = 8$
$64 - y^2 = 16$
$y = \sqrt{48} = 4\sqrt{3}$
$y(8) = 4\sqrt{3}$

Option (1)

---------------------------------------------------------------------------------------------------

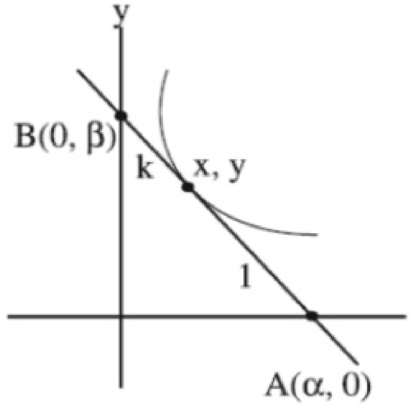
# Question34

**Let the tangent at any point P on a curve passing through the points (1, 1) and $\left(\frac{1}{10}, 100\right)$, intersect positive x axis and y-axis at the points A and B respectively. If PA : PB = 1 : k and y = y(x) is the solution of the differential equation $e^{\frac{dy}{dx}} = kx + \frac{k}{2}$, y(0) = k, then 4y then $4y(1) - 5\log e^3$ is equal to ________.**
**[10-Apr-2023 shift 2]**

**Answer: 5**

## Solution:

$Y - y = \frac{dy}{dx}(X - x)$
$Y = 0$
$X = \frac{-y\,dx}{dy} + x$




$\frac{k\alpha + 0}{k + 1} = x,\ \alpha = \frac{k + 1}{k}x$
$\frac{k + 1}{k}x = -y\frac{dx}{dy} + x$
$x + \frac{x}{k} = -y\frac{dx}{dy} + x$
$x\frac{dy}{dx} + ky = 0$
$\frac{dy}{dx} + \frac{k}{x}y = 0$
$y \cdot x^k = C$
$C = 1$
$100 \cdot \left(\frac{1}{10}\right)^k = 1$
$K = 2$
$\frac{dy}{dx} = \ln(2x + 1)$
$y = \frac{(2x + 1)}{2}(\ln(2x + 1) - 1) + c$
$2 = \frac{1}{2}(0 - 1) + C$

$C = 2 + \frac{1}{2} = \frac{5}{2}$

$y(1) = \frac{3}{2}(\ell \ln 3 - 1) + \frac{5}{2}$

$= \frac{3}{2}\ln 3 + 1$

$4y(1) = 6\ln 3 + 4$

$4y(1) - 5\ln 3 = 4 + \ln 3$

---------------------------------------------------------------------------------------------------

# Question35

## Let $y = y(x)$ be a solution curve of the differential equation. $(1 - x^2y^2)\,dx = y\,dx + x\,d\,y$. If the line $x = 1$ intersects the curve $y = y(x)$ at $y = 2$ and the line $x = 2$ intersects the curve $y = y(x)$ at $y = \alpha$, then a value of $\alpha$ is : [11-Apr-2023 shift 1]

**Options:**

A. $\frac{1 + 3e^2}{2(3e^2 - 1)}$

B. $\frac{1 - 3e^2}{2(3e^2 + 1)}$

C. $\frac{3e^2}{2(3e^2 - 1)}$

D. $\frac{3e^2}{2(3e^2 + 1)}$

**Answer: A**

## Solution:

**Solution:**

$(1 - x^2y^2)\,dx = y\,dx + x\,d\,y,\ y(1) = 2$

$y(2) = \infty$

$d\,x = \frac{d\,(xy)}{1 - (xy)^2}$

$\int dx = \int \frac{d\,(xy)}{1 - (xy)^2}$

$x = \frac{1}{2}\ln\left|\frac{1 + xy}{1 - xy}\right| + C$

Put $x = 1$ and $y = 2$:

$1 = \frac{1}{2}\ln\left|\frac{1 + 2}{1 - 2}\right| + C$

$C = 1 - \frac{1}{2}\ln 3$

Now put $x = 2$ :

$2 = \frac{1}{2}\ln\left|\frac{1 + 2\alpha}{1 - 2\alpha}\right| + 1 - \frac{1}{2}\ln 3$

$1 + \frac{1}{2}\ln 3 = \frac{1}{2}\left|\frac{1 + 2\alpha}{1 - 2\alpha}\right|$

$2 + \ln 3 = \left|\frac{1 + 2\alpha}{1 - 2\alpha}\right|$

$\left|\frac{1 + 2\alpha}{1 - 2\alpha}\right| = 3e^2$

$\frac{1 + 2\alpha}{1 - 2\alpha} = 3e^2, -3e^2$

$\frac{1+2\alpha}{1-2\alpha} = 3e^2 \Rightarrow \alpha = \frac{3e^2-1}{2(3e^2+1)}$

And $\frac{1+2\alpha}{1-2\alpha} = -3e^2 \Rightarrow \alpha = \frac{3e^2+1}{2(3e^2-1)}$

------------------------------------------------------------------------------------------------

# Question36

## Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} + \frac{5}{x(x^5+1)}y = \frac{(x^5+1)^2}{x^7}$, x > 0. If y(1) = 2, then y(2) is equal to [11-Apr-2023 shift 2]

**Options:**

A. $\frac{693}{128}$

B. $\frac{637}{128}$

C. $\frac{697}{128}$

D. $\frac{679}{128}$

**Answer: A**

## Solution:

**Solution:**

$\text{I.F} = \; = e^{\int \frac{5\,dx}{x(x^5+1)}} = e^{e}\frac{5x^{-6}-6x}{(x^{-5}+1)}$

Put, $1 + x^{-5} = t \Rightarrow -5x^{-6}\,dx = dt$

$\Rightarrow e^{\int \frac{-dt}{1}} = \frac{1}{t} = \frac{x^5}{1+x^5}$

$y \cdot \frac{x^5}{1+x^5} = \int \frac{x^5}{(1+x^5)} \times \frac{(1+x^5)^2}{x^7}\,dx$

$= \int x^3\,dx + \int x^{-2}\,dx$

$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + c$

Given than: $x = 1 \Rightarrow y = 2$

$2 \cdot \frac{1}{2} = \frac{1}{4} - 1 + c$

$c = \frac{7}{4}$

$y \cdot \frac{x^5}{1+x^5} = \frac{x^4}{4} - \frac{1}{x} + \frac{7}{4}$

Now put, $x = 2$

$y \cdot \left(\frac{32}{33}\right) = \frac{21}{4}$

$y = \frac{693}{128}$

------------------------------------------------------------------------------------------------

# Question37

**Let $y = y(x)$, $y > 0$, be a solution curve of the differential equation $(1 + x^2)dy = y(x - y)dx$. If $y(0) = 1$ and $y(2\sqrt{2}) = \beta$, then**
**[12-Apr-2023 shift 1]**

**Options:**

A. $e^{3\beta^{-1}} = e(5 + \sqrt{2})$

B. $e^{3\beta^{-1}} = e(3 + 2\sqrt{2})$

C. $e^{\beta^{-1}} = e^{-2}(3 + 2\sqrt{2})$

D. $e^{\beta^{-1}} = e^{-2}(5 + \sqrt{2})$

**Answer: B**

## Solution:

**Solution:**

$(1 + x^2)dy = y(x - y)dx$

$Y(0) = 1 \cdot y(2\sqrt{2}) = \beta$

$\frac{dy}{dx} = \frac{yx - y^2}{1 + x^2}$

$\frac{dy}{dx} + y\left(\frac{-x}{1 + x^2}\right) = \left(\frac{-1}{1 + x^2}\right)y^2$

$\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y}\left(\frac{-x}{1 + x^2}\right) = \frac{-1}{1 + x^2}$

Put $\frac{1}{y} = t$ then $\frac{-1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} + t\frac{x}{1 + x^2} = \frac{1}{1 + x^2}$

If $= e^{\int \frac{x}{1 + x^2}dx} = e^{\frac{1}{2}\ln(1 + x^2)}\sqrt{1 + x^2}$

$t\sqrt{1 + x^2} = \int \frac{1}{\sqrt{1 + x^2}}dx$

$\frac{\sqrt{1 + x^2}}{y} = \ln\left(x + \sqrt{x^2 + 1}\right) + c$

$y(0) = 1 \Rightarrow c = 1$

$\Rightarrow \sqrt{1 + x^2} = y\ln\left(e\left(x + \sqrt{x^2 + 1}\right)\right)$

$\beta = \frac{3}{\ln(e(3 + 2\sqrt{2}))} \Rightarrow \frac{3}{\beta} = \ln(e(3 + 2\sqrt{2}))$

$e^{\frac{3}{\beta}} = e(3 + 2\sqrt{2})$

----------------------------------------------------------------------------------------------------

# Question38

**Let $y = y_1(x)$ and $y = y_2(x)$ be the solution curves of the differential equation $\frac{dy}{dx} = y + 7$ with initial conditions $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then the curves $y = y_1(x)$ and $y = y_2(x)$ intersect at**
**[13-Apr-2023 shift 1]**

**Options:**

A. no point

B. infinite number of points

C. one point

D. two points

**Answer: A**

## Solution:

**Solution:**

$\frac{dy}{dx} = y + 7 \Rightarrow \frac{dy}{dx} - y = 7$

I.F. $= e^{-x}$

$ye^{-x} = \int 7e^{-x}dx$

$\Rightarrow ye^{-x} = -7e^{-x} + c$

$\Rightarrow y = -7 + ce^{x}$

$-7 + 7e^{x} = -7 + 8e^{x} \Rightarrow e^{x} = 0$

No solution

------------------------------------------------------------------------------------------------

# Question39

**If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + \frac{4x}{(x^2-1)}y = \frac{x+2}{(x^2-1)^{\frac{5}{2}}}$, $x > 1$ such that $y(2) = \frac{2}{9}\log_e(2+\sqrt{3})$ and $y(\sqrt{2}) = \alpha\log_e(\sqrt{\alpha}+\beta) + \beta - \sqrt{\gamma}$, $\alpha, \beta, \gamma, \in N$, then $\alpha\beta\gamma$ is equal to _______ [13-Apr-2023 shift 2]**

**Answer: 6**

## Solution:

given differential equation $\frac{dy}{dx} + \frac{4x}{(x^2-1)}y = \frac{x+2}{(x^2-1)^{5/2}}$ is linear D.E.

I.F. $= e^{\int \frac{4x}{x^2-1}dx} = e_e 2\ln(x^2-1) = e_e \ln(x^2-1)^2 = (x^2-1)^2$

$y(x^2-1)^2 = \int \frac{x+2}{(x^2-1)^{5/2}}(x^2-1)^2 dx$

$= \int \frac{x}{\sqrt{x^2-1}}dx + \int \frac{2\,dx}{\sqrt{x^2-1}}$

$= \sqrt{x^2-1} + 2\ln\left[x + \sqrt{x^2-1}\right] + C$

put $y(2) = \frac{2}{9}\ln(2+\sqrt{3})$

$\frac{2}{9}\ln(2+\sqrt{3})(9) = \sqrt{3} + 2\ln[2+\sqrt{3}] + C$

$= C = -\sqrt{3}$

put $x = \sqrt{2}$

$y = 1 + 2\ln[\sqrt{2}+1] - \sqrt{3}$

$\alpha = 2, \beta = 1 = \gamma = 3$

$\alpha\beta\gamma = 2(1)(3) = 6$

# Question40

**Let $x = x(y)$ be the solution of the differential equation $2(y+2)\log_e(y+2)\,dx + (x+4-2\log_e(y+2))\,dy = 0,\ y > -1$ with $x(e^4-2) = 1$. Then $x(e^9-2)$ is equal to**
**[15-Apr-2023 shift 1]**

**Options:**

A. $\frac{4}{9}$

B. $\frac{32}{9}$

C. $\frac{10}{3}$

D. 3

**Answer: B**

## Solution:

**Solution:**

$$2(y+2)\ln(y+2)\,dx + (x+4-2\ln(y+2))\,dy = 0$$

$$2\ln(y+2) + (x+4-2\ln(y+2))\frac{1}{y+2}\cdot\frac{dy}{dx} = 0$$

let, $\ln(y+2) = t$

$$\frac{1}{y+2}\cdot\frac{dy}{dx} = \frac{dt}{dx}$$

$$2t + (x+4-2t)\cdot\frac{dt}{dx} = 0$$

$$(x+4-2t)\frac{dt}{dx} = -2t$$

$$\frac{dx}{dt} = \frac{2t-4-x}{2t}$$

$$\frac{dx}{dt} + \frac{x}{2t} = \frac{2t-4}{2t}$$

$$x\cdot t^{1/2} = \int \frac{2t-4}{2t}\cdot t^{1/2}\cdot dt$$

$$x\cdot t^{1/2} = \int\left(t^{1/2} - \frac{2}{t^{1/2}}\right)\cdot dt$$

$$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2\cdot\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$x\cdot t^{\frac{1}{2}} = \frac{2t^{\frac{3}{2}}}{3} - 4t^{\frac{1}{2}} + C$$

$$x = \frac{2}{3}\cdot t - 4 + C\cdot t^{\frac{-1}{2}}$$

$$x = \frac{2}{3}\ln(y+2) - 4 + C\cdot(\ln(y+2))^{\frac{-1}{2}}$$

Put $y = e^4 - 2,\ x = 1$

$$1 = \frac{2}{3}\times 4 - 4 + C\times\frac{1}{2}$$

$$\frac{C}{2} = 5 - \frac{8}{3} = \frac{7}{3}$$

$$\Rightarrow C = \frac{14}{3}$$

$x = \frac{2}{3} \times 9 - 4 + \frac{14}{3} \times \frac{1}{3}$

$= 2 + \frac{14}{9}$

$= 3$

-------------------------------------------------------------------------------------------------------

# Question41

**The slope of normal at any point (x, y), x > 0, y > 0 on the curve y = y(x) is given by $\frac{x^2}{xy - x^2y^2 - 1}$. If the curve passes through the point (1, 1), then e. y(e) is equal to**
**[24-Jun-2022-Shift-2]**

**Options:**

A. $\frac{1 - \tan(1)}{1 + \tan(1)}$

B. $\tan(1)$

C. 1

D. $\frac{1 + \tan(1)}{1 - \tan(1)}$

**Answer: D**

## Solution:

**Solution:**

$\because -\frac{dx}{dy} = \frac{x^2}{xy - x^2y^2 - 1}$

$\therefore \frac{dy}{dx} = \frac{x^2y^2 - xy + 1}{x^2}$

Let $xy = v \Rightarrow y + x\frac{dy}{dx} = \frac{dv}{dx}$

$\therefore \frac{dv}{dx} - y = \frac{(v^2 - v + 1)y}{v}$

$\therefore \frac{dv}{dx} = \frac{v^2 + 1}{x}$

$\because y(1) = 1 \Rightarrow \tan^{-1}(xy) = \ln x + \tan^{-1}(1)$

Put $x = e$ and $y = y(e)$ we get

$\tan^{-1}(e \cdot y(e)) = 1 + \tan^{-1} 1$

$\tan^{-1}(e \cdot y(e)) - \tan^{-1} 1 = 1$

$\therefore e(y(e)) = \frac{1 + \tan(1)}{1 - \tan(1)}$

-------------------------------------------------------------------------------------------------------

# Question42

**Let y = y(x) be the solution of the differential equation $(x + 1)y' - y = e^{3x}(x + 1)^2$, with $y(0) = \frac{1}{3}$. Then, the point $x = -\frac{4}{3}$ for the**

curve y = y(x) is :
[25-Jun-2022-Shift-1]

**Options:**

A. not a critical point

B. a point of local minima

C. a point of local maxima

D. a point of inflection

**Answer: B**

## Solution:

**Solution:**

$(x+1)\frac{dy}{dx}-y=e^{3x}(x+1)^2$

$\frac{dy}{dx}-\frac{y}{x+1}=e^{3x}(x+1)$

If $e^{-\int\frac{1}{x+1}x}=e^{-\log(x+1)}=\frac{1}{x+1}$

$\therefore y\left(\frac{1}{x+1}\right)=\int\frac{e^{3x}(x+1)}{x+1}dx$

$\frac{y}{x+1}=\int e^{3x}dx$

$\frac{y}{x+1}=\frac{e^{3x}}{3}+c$

$\because y(0)=\frac{1}{3}$

$\frac{1}{3}=\frac{1}{3}+c$

$\therefore c=0$

So : $y=\frac{e^{3x}}{3}(x+1)$

$y'=e^{3x}(x+1)+\frac{e^{3x}}{3}=e^{3x}\left(x+\frac{4}{3}\right)$

$y''=3e^{3x}\left(x+\frac{4}{3}\right)+e^{3x}=e^{3x}(3x+5)$

$y'=0$ at $x=\frac{-4}{3}$ & $y''=e^{-4}(1)>0$ at $x=\frac{-4}{3}$

$\Rightarrow x=\frac{-4}{3}$ is point of local minima

---------------------------------------------------------------------------------------------------

# Question43

**If the solution curve y = y(x) of the differential equation $y^2dx+(x^2-xy+y^2)dy=0$, which passes through the point (1, 1) and intersects the line $y=\sqrt{3}x$ at the point $(\alpha, \sqrt{3}\alpha)$, then value of $\log_e(\sqrt{3}\alpha)$ is equal to :**
**[25-Jun-2022-Shift-1]**

**Options:**

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{12}$

D. $\frac{\pi}{6}$

**Answer: C**

## Solution:

**Solution:**

$$\frac{dy}{dx} = \frac{y^2}{xy - x^2 - y^2}$$

Put $y = vx$ we get

$$v + x\frac{dv}{dx} = \frac{v^2}{v - 1 - v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v^2 - v^2 + v + v^3}{v - 1 - v^2}$$

$$\Rightarrow \int \frac{v - 1 - v^2}{v(1 + v^2)} dv = \int \frac{dx}{x}$$

$$\tan^{-1}\left(\frac{y}{x}\right) - \ln\left(\frac{y}{x}\right) = \ln x + c$$

As it passes through $(1, 1)$

$$c = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) \ln\left(\frac{y}{x}\right) = \ln x + \frac{\pi}{4}$$

Put $y = \sqrt{3}x$ we get

$$\Rightarrow \frac{\pi}{3} - \ln\sqrt{3} = \ln x + \frac{\pi}{4}$$

$$\Rightarrow \ln x = \frac{\pi}{12} - \ln\sqrt{3} = \ln\alpha$$

$$\therefore \ln(\sqrt{3}\alpha) = \ln\sqrt{3} + \ln\alpha$$

$$= \ln\sqrt{3} + \frac{\pi}{12} - \ln\sqrt{3} = \frac{\pi}{12}$$

----------------------------------------------------------------------------------------------------

# Question44

**If y = y(x) is the solution of the differential equation $2x^2\frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(e) = \frac{e}{3}$, then y(1) is equal to [25-Jun-2022-Shift-2]**

**Options:**

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{3}{2}$

D. 3

**Answer: B**

## Solution:

**Solution:**

$2x^2\frac{dy}{dx} - 2xy + 3y^2 = 0$

$\Rightarrow 2x(xdy - ydx) + 3y^3dx = 0$

$\Rightarrow 2\left(\frac{xdy - ydx}{y^2}\right) + 3\frac{dx}{x} = 0$

$\Rightarrow -\frac{2x}{y} + 3\ln x = C$

$\because y(e) = \frac{e}{3} \Rightarrow -6 + 3 = C \Rightarrow C = -3$

Now, at $x = 1$, $-\frac{2}{y} + 0 = -3$

$y = \frac{2}{3}$

-----------------------------------------------------------------------------------------------------

# Question45

## Let the solution curve $y = y(x)$ of the differential equation $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$ pass through the origin. Then $y(2)$ is equal to____ [26-Jun-2022-Shift-1]

**Answer: 12**

## Solution:

$(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$

$\Rightarrow \frac{dy}{dx} = \left(\frac{6x}{x^2+4}\right)y + 2x$

$\Rightarrow \frac{dy}{dx} - \left(\frac{6x}{x^2+4}\right)y = 2x$

I. F. $= e^{-3\ln(x^2+4)} = \frac{1}{(x^2+4)^3}$

So $\frac{y}{(x^2+4)^3} = \int \frac{2x}{(x^2+4)^3}dx + c$

$\Rightarrow y = -\frac{1}{2}(x^2+4) + c(x^2+4)^3$

When $x = 0$, $y = 0$ gives $c = \frac{1}{32}$,

So, for $x = 2$, $y = 12$

## Question46

**Let S = (0, 2π) − $\left\{ \frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4} \right\}$. Let y = y(x), x ∈ S, be the solution curve of the differential equation $\frac{dy}{dx} = \frac{1}{1 + \sin 2x}$, $y\left(\frac{\pi}{4}\right) = \frac{1}{2}$. If the sum of abscissas of all the points of intersection of the curve y = y(x) with the curve $y = \sqrt{2}\sin x$ is $\frac{k\pi}{12}$, then k is equal to____**
**[26-Jun-2022-Shift-1]**

**Answer: 42**

## Solution:

$\frac{dy}{dx} = \frac{1}{1+\sin 2x}$

$\Rightarrow dy = \frac{\sec^2 x dx}{(1+\tan x)^2}$

$\Rightarrow y = -\frac{1}{1+\tan x} + c$

When $x = \frac{\pi}{4}, y = \frac{1}{2}$ gives c = 1

So $y = \frac{\tan x}{1+\tan x} \Rightarrow y = \frac{\sin x}{\sin x + \cos x}$

Now, $y = \sqrt{2}\sin x \Rightarrow \sin x = 0$

or $\sin x + \cos x = \frac{1}{\sqrt{2}}$

$\sin x = 0$ gives $x = \pi$ only.

and $\sin x + \cos x = \frac{1}{\sqrt{2}} \Rightarrow \sin\left(x + \frac{\pi}{4}\right) = \frac{1}{2}$

So $x + \frac{\pi}{4} = \frac{5\pi}{6}$ or $\frac{13\pi}{6} \Rightarrow x = \frac{7\pi}{12}$ or $\frac{23\pi}{12}$

Sum of all solutions $= \pi + \frac{7\pi}{12} + \frac{23\pi}{12} = \frac{42\pi}{12}$

Hence, $k = 42$.

## Question47

**If the solution of the differential equation $\frac{dy}{dx} + e^x(x^2 - 2)y = (x^2 - 2x)(x^2 - 2)e^{2x}$ satisfies y(0) = 0, then the value of y(2) is__**
**[26-Jun-2022-Shift-2]**

**Options:**

A. −1

B. 1

C. 0

D. e

**Answer: C**

## Solution:

$\because \frac{dy}{dx} + e^x(x^2-2)y = (x^2-2x)(x^2-2)e^{2x}$

Here, $I.F. = e^{\int e^x(x^2-2)dx}$

$= e^{(x^2-2x)e^x}$

$\therefore$ Solution of the differential equation is

$y \cdot e^{(x^2-2x)e^x} = \int (x^2-2x)(x^2-2)e^{2x} \cdot e^{(x^2-2x)e^x} dx$

$= \int (x^2-2x)e^x \cdot (x^2-2)e^x \cdot e^{(x^2-2x)e^x} dx$

Let $(x^2-2x)e^x = t$

$\therefore (x^2-2)e^x dx = dt$

$y \cdot e^{(x^2-2x)e^x} = \int t \cdot e^t dt$

$y \cdot e^{(x^2-2x)e^x} = (x^2-2x-1)e^{(x^2-2x)e^x} + c$

$\therefore y(0) = 0$

$\therefore c = 1$

$\therefore y = (x^2-2x-1) + e^{(2x-x^2)e^x}$

$\therefore y(2) = -1 + 1 = 0$

# Question48

**Let $\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$, where a, b, c are constants, represent a circle passing through the point (2, 5). Then the shortest distance of the point (11, 6) from this circle is :**
**[27-Jun-2022-Shift-1]**

**Options:**

A. 10

B. 8

C. 7

D. 5

**Answer: B**

## Solution:

$\frac{dy}{dx} = \frac{ax - by + a}{bx + cy + a}$
$= bxdy + cydy + ady = axdx - bydx + adx$
$= cydy + ady - axdx - adx + b(xdy + ydx) = 0$
$= c\int ydy + a\int xdx - a\int dx + b\int d(xy) = 0$
$= \frac{cy^2}{2} + ay - \frac{ax^2}{2} - ax + bxy = k$
$= ax^2 - cy^2 + 2ax - 2ay - 2bxy = k$
Above equation is circle
$\Rightarrow a = -c$ and $b = 0$
$ax^2 + ay^2 + 2ax - 2ay = k$
$\Rightarrow x^2 + y^2 + 2x - 2y = \lambda \; \left[\lambda = \frac{k}{a}\right]$
Passes through (2, 5)
$4 + 25 + 4 - 10 = \lambda \Rightarrow \lambda = 23$

Circle $\equiv x^2 + y^2 + 2x - 2y - 23 = 0$
Centre $(-1, 1) r = \sqrt{(-1)^2 + 1^2 + 23} = 5$
Shortest distance of $(11, 6) = \sqrt{12^2 + 5^2} - 5$
$= 13 - 5$
$= 8$

------------------------------------------------------------------------------------------------

# Question49

## If $\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0$, x, y > 0, y(1) = 1, then y(2) is equal to : [27-Jun-2022-Shift-1]

**Options:**

A. $2 + \log_2 3$

B. $2 + \log_3 2$

C. $2 - \log_3 2$

D. $2 - \log_2 3$

**Answer: D**

## Solution:

**Solution:**
$\frac{dy}{dx} + \frac{2^{x-y}(2^y - 1)}{2^x - 1} = 0, x, y > 0, y(1) = 1$
$\frac{dy}{dx} = -\frac{2^x(2^y - 1)}{2^y(2^z - 1)}$
$\int \frac{2^y}{2^y - 1} dy = -\int \frac{2^x}{2^x - 1} dx$
$= \frac{\log_e(2^y - 1)}{\log_{e^2}} = -\frac{\log_e(2^x - 1)}{\log_e 2} + \frac{\log_e c}{\log_e 2}$
$= |(2^y - 1)(2^x - 1)| = c$
$\because y(1) = 1$
$\therefore c = 1$
$= |(2^y - 1)(2^x - 1)| = 1$
For $x = 2$
$|(2^y - 1)3| = 1$
$2^y - 1 = \frac{1}{3} \Rightarrow 2y = \frac{4}{3}$
Taking log to base 2.
$\therefore y = 2 - \log_2 3$

------------------------------------------------------------------------------------------------

# Question50

## If the solution curve of the differential equation $((\tan^{-1}y) - x)dy = (1 + y^2)dx$ passes through the point (1, 0), then the abscissa of the point on the curve whose ordinate is tan(1), is [27-Jun-2022-Shift-2]

**Options:**

A. 2e

B. $\frac{2}{e}$

C. 2

D. $\frac{1}{e}$

**Answer: B**

## Solution:

**Solution:**

$((\tan^{-1}y) - x)dy = (1 + y^2)dx$

$\frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{\tan^{-1}y}{1 + y^2}$

$\text{I. F.} = e^{\int \frac{1}{1 + y^2}dy} = e^{\tan^{-1}y}$

$\therefore$ Solution

$x.\ e^{\tan^{-1}y} = \int \frac{e^{\tan^{-1}y}\tan^{-1}y}{1 + y^2}dy$

Let $e^{\tan^{-1}y} = t$

$\frac{e^{\tan - 1}y}{1 + y^2} = dt$

$= xe^{\tan^{-1}y} = \int \ln t\, dt = t \ln t - t + c$

$\therefore = xe^{\tan^{-1}y} = e^{\tan^{-1}y}\tan^{-1}y - e^{\tan^{-1}y} + c \ldots..\ (i)$

$\because$ It passes through $(1, 0) \Rightarrow c = 2$

Now put $y = \tan 1$, then

$ex = e - e + 2$

$$

$\Rightarrow x = \frac{2}{e}$

-----------------------------------------------------------------------------------------------

# Question51

**Let y = y(x) be the solution of the differential equation $(1 - x^2)dy = \left(xy + (x^3 + 2)\sqrt{1 - x^2}\right)dx$, $-1 < x < 1$, and $y(0) = 0$. If $\int_{\frac{-1}{2}}^{\frac{1}{2}} \sqrt{1 - x^2}y(x)dx = k$, then $k^{-1}$ is equal to____**

**[27-Jun-2022-Shift-2]**

**Answer: 320**

## Solution:

**Solution:**

$(1 - x^2)dy = \left(xy + (x^3 + 2)\sqrt{1 - x^2}\right)dx$

$\therefore \frac{dy}{dx} - \frac{x}{1-x^2} y = \frac{x^3+3}{\sqrt{1-x^2}}$

$\therefore$ I. F. $= e^{\int -\frac{x}{1-x^2} dx} = \sqrt{1-x^2}$

Solution is

y. $\sqrt{1-x^2} = \int (x^3+3)\,dx$

$y \cdot \sqrt{1-x^2} = \frac{x^4}{4} + 3x + c$

$\because y(0) = 0 \Rightarrow c = 0$

$\therefore y(x) = \frac{x^4+12x}{4\sqrt{1-x^2}}$

$\therefore \int_{\frac{-1}{2}}^{\frac{1}{2}} \sqrt{1-x^2}\, y(x)\,dx = \int_{\frac{-1}{2}}^{\frac{1}{2}} \left( \frac{x^4+12x}{4} \right) dx = \int_0^{\frac{1}{2}} \frac{x^4}{2} dx$

$\therefore k = \frac{1}{320}$

$\therefore = k^{-1} = 320$

----------------------------------------------------------------------------------------------------

# Question52

## Let the solution curve y = y(x) of the differential equation $\left[ \frac{x}{\sqrt{x^2-y^2}} + e^{\frac{y}{x}} \right] x \frac{dy}{dx} = x + \left[ \frac{x}{\sqrt{x^2-y^2}} + e^{\frac{y}{x}} \right] y$ pass through the points (1, 0) and (2α, α), α > 0. Then α is equal to [28-Jun-2022-Shift-1]

**Options:**

A. $\frac{1}{2} \exp \left( \frac{\pi}{6} + \sqrt{e} - 1 \right)$

B. $\frac{1}{2} \exp \left( \frac{\pi}{6} + e - 1 \right)$

C. $\exp \left( \frac{\pi}{6} + \sqrt{e} + 1 \right)$

D. $2 \exp \left( \frac{\pi}{3} + \sqrt{e} - 1 \right)$

**Answer: A**

## Solution:

**Solution:**

$\left( \frac{1}{\sqrt{1-\frac{y^2}{x^2}}} + e^{\frac{y}{x}} \right) \frac{dy}{dx} = 1 + \left( \frac{1}{\sqrt{1-\frac{y^2}{x^2}}} + e^{\frac{y}{x}} \right) \frac{y}{x}$

Putting y = tx

$\left( \frac{1}{\sqrt{1-t^2}} + e^t \right) \left( t + x \frac{dt}{dx} \right) = 1 + \left( \frac{1}{\sqrt{1-t^2}} + e^t \right) t$

$\Rightarrow x \left( \frac{1}{\sqrt{1-t^2}} + e^t \right) \frac{dt}{dx} = 1$

$\Rightarrow \sin^{-1} t + e^t = \ln x + C$

$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) + e^{y/x} = \ln x + C$

at x = 1, y = 0

So, $0 + e^{0} = 0 + C \Rightarrow C = 1$

at $(2\alpha, \alpha)$

$\sin -1\left(\frac{y}{x}\right) + e^{y/x} = \ln x + 1$

$\Rightarrow \frac{\pi}{6} + e^{\frac{1}{2}} - 1 = \ln(2\alpha)$

$\Rightarrow \alpha = \frac{1}{2}e^{\left(\frac{\pi}{6} + e^{\frac{1}{2}} - 1\right)}$

----------------------------------------------------------------------------------------------------

# Question53

## Let y = y(x) be the solution of the differential equation $x(1-x^2)\frac{dy}{dx} + (3x^2y - y - 4x^3) = 0$, x > 1, with y(2) = −2. Then y(3) is equal to

## [28-Jun-2022-Shift-1]

**Options:**

A. −18

B. −12

C. −6

D. −3

**Answer: A**

## Solution:

**Solution:**

$\frac{dy}{dx} + \frac{y(3x^2-1)}{x(1-x^2)} = \frac{4x^3}{x(1-x^2)}$

$IF = e^{\int \frac{3x^2-1}{x-x^3}dx} = e^{-\ln\left|x^3-x\cdot\right|} = e^{-\ln(x^3-x)} = \frac{1}{x^3-x}$

Solution of D.E. can be given by

$y.\ \frac{1}{x^3-x} = \int \frac{4x^3}{x(1-x^2)} \cdot \frac{1}{x(x^2-1)}dx$

$\Rightarrow \frac{y}{x^3-x} = \int \frac{-4x}{(x^2-1)^2}dx$

$\Rightarrow \frac{y}{x^3-x} = \frac{2}{(x^2-1)} + c$

at x = 2, y = −2

$\frac{-2}{6} = \frac{2}{3} + c \Rightarrow c = -1$

at $x = 3 \Rightarrow \frac{y}{24} = \frac{2}{8} - 1 \Rightarrow y = -18$

----------------------------------------------------------------------------------------------------

# Question54

## Let x = x(y) be the solution of the differential equation

**$2ye^{x/y^2}dx + \left(y^2 - 4xe^{x/y^2}\right)dy = 0$ such that $x(1) = 0$. Then, $x(e)$ is equal to :**
**[28-Jun-2022-Shift-2]**

**Options:**

A. $e\log_e(2)$

B. $-\log_e(2)$

C. $e^2\log_e(2)$

D. $-e^2\log_e(2)$

**Answer: D**

## Solution:

**Solution:**
Given differential equation
$$2ye^{\frac{x}{y^2}}dx + \left(y^2 - 4xe^{\frac{x}{y^2}}\right)dy = 0,\ x(1) = 0$$
$$\Rightarrow e^{\frac{x}{y^2}}[2y\,dx - 4x\,dy] = -y^2dy$$
$$\Rightarrow e^{\frac{x}{y^2}}\left[\frac{2y^2dx - 4xy\,dy}{y^4}\right] = \frac{-1}{y}dy$$
$$\Rightarrow 2e^{\frac{x}{y^2}}d\left(\frac{x}{y^2}\right) = -\frac{1}{y}dy$$
$$\Rightarrow 2e^{\frac{x}{y^2}} = -\ln y + c\ldots..\ (i)$$
Now, using $x(1) = 0$, $c = 2$
So, for $x(e)$, Put $y = e$ in (i)
$$2e^{\frac{x}{e^2}} = -1 + 2$$
$$\Rightarrow \frac{\ }{e^2} = \ln\left(\frac{1}{2}\right) \Rightarrow x(e) = -e^2\ln 2$$

----------------------------------------------------------------------------------------------------

# Question55

**Let the solution curve of the differential equation $x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ be $y = y(x)$. Then $y(2)$ is equal to:**
**[29-Jun-2022-Shift-1]**

**Options:**

A. 15

B. 11

C. 13

D. 17

**Answer: A**

# Solution:

**Solution:**
Given,
$x\frac{dy}{dx} - y = \sqrt{y^2+16x}$
$\Rightarrow x\frac{dy}{dx} = y + \sqrt{y^2+16x}$
$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{\left(\frac{y}{x}\right)^2 + 16}$
This is a homogenous different equation.
Let $\frac{y}{x} = v$
$\Rightarrow y = vx$
$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$
$\therefore v + x\frac{dv}{dx} = v + \sqrt{v^2+16}$
$\Rightarrow x\frac{dv}{dx} = \sqrt{v^2+16}$
$\Rightarrow \frac{dv}{\sqrt{v^2+16}} = \frac{dx}{x}$
Integrating both sides, we get
$\int \frac{dv}{\sqrt{v^2+16}} = \int \frac{dx}{x}$
$\Rightarrow \ln\left|v + \sqrt{v^2+16}\right| = \ln x + \ln c$
$\Rightarrow v + \sqrt{v^2+16} = cx$
Now putting, $v = \frac{y}{x}$, we get
$\frac{y}{x} + \sqrt{\frac{y^2}{x^2} + 16} = cx$
$\Rightarrow \frac{y}{x} + \sqrt{\frac{y^2+16x^2}{x^2}} = cx$
$\Rightarrow y + \sqrt{y^2+16x^2} = cx^2$......
Given, $y(1) = 3$
∴ When $x = 1$ then $y = 3$.
Putting in equation (1) we get,
$3 + \sqrt{9+16} = c.1$
$\Rightarrow c = 8$
∴ Solution of equation,
$y + \sqrt{y^2+16x^2} = 8x^2$
Now, $y(2)$ means when $x = 2$ then $y = ?$

$\therefore y + \sqrt{y^2 + 16\times 4} = 8\times 4$
$\Rightarrow y = 15$

-----------------------------------------------------------------------------------------------

# Question56

**Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} + \frac{\sqrt{2}y}{2\cos^4 x - \cos^2 x} = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}$, $0 < x < \frac{\pi}{2}$ with $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}$ If $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18}e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to____**

**[29-Jun-2022-Shift-1]**

**Answer: 2**

## Solution:

**Solution:**

$\frac{dy}{dx} + \frac{\sqrt{2}}{2\cos^4 x - \cos 2x} y = x e^{\tan^{-1}(\sqrt{2}\cot 2x)}$

$\int \frac{dx}{2\cos^4 x - \cos 2x}$

$= \int \frac{dx}{\cos^4 x + \sin^4 x} = \int \frac{\operatorname{cosec}^4 x\, dx}{1 + \cot^4 x}$

$= -\int \frac{t^2 + 1}{t^4 + 1} dt = -\int \frac{1}{\left(t - \frac{1}{t}\right)^2 + 2} dt = \frac{-1}{\sqrt{2}} \tan^{-1}\left(\frac{t - \frac{1}{t}}{\sqrt{2}}\right)$

$\operatorname{Cot} x = t$

$= -\frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2}\cot 2x)$

$\therefore \text{IF} = e^{-\tan^{-1}(\sqrt{2}\cot 2x)}$

$y e^{-\tan^{-1}(\sqrt{2}\cot 2x)} = \int x dx$

$y e^{-\tan^{-1}(\sqrt{2}\cot 2x)} = \frac{x^2}{2} + c$

$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32} + c \Rightarrow c = 0$

$y = \frac{x^2}{2} e^{\tan^{-1}(\sqrt{2}\cot 2x)}$

$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18} e^{\tan^{-1}\left(\sqrt{2}\cot \frac{2\pi}{3}\right)}$

$= \frac{\pi^2}{18} e^{-\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)}$

$\alpha = \sqrt{\frac{2}{3}} \Rightarrow 3\alpha^2 = 2$

-----------------------------------------------------------------------------------------------

# Question57

**If $y = y(x)$ is the solution of the differential equation $(1 + e^{2x}) \frac{dy}{dx} + 2(1 + y^2)e^x = 0$ and $y(0) = 0$ then $6(y'(0) + (y(\log_e \sqrt{3}))^2)$ is equal to**
**[29-Jun-2022-Shift-2]**

**Options:**

A. 2

B. $-2$

C. $-4$

D. $-1$

**Answer: C**

## Solution:

$\frac{dy}{1+y^2} + \frac{2e^x}{1+e^{2x}} dx = 0$
on integration
$\tan^{-1}y + 2\tan^{-1}e^x = c$
$\because y(0) = 0$
so, $C = \frac{\pi}{2} \Rightarrow \tan^{-1}y + 2\tan^{-1}e^x = \frac{\pi}{4}$ from eq.(i), $\left(\frac{dy}{dx}\right)_{x=0} = -1$ arg $y(\ln\sqrt{3}) = -\frac{1}{\sqrt{3}}$
$6\left[y'(0) + (y(\ln\sqrt{3})^2\right] = 6\left[-1 + \frac{1}{3}\right] = -4.$

---------------------------------------------------------------------------------------------------

# Question58

**Let y = y(x), x > 1, be the solution of the differential equation $(x-1)\frac{dy}{dx} + 2xy = \frac{1}{x-1}$, with $y(2) = \frac{1+e^4}{2e^4}$. If $y(3) = \frac{e^{\alpha}+1}{\beta e^{\alpha}}$, then the value of $\alpha + \beta$ is equal to____**
**[29-Jun-2022-Shift-2]**

**Answer: 14**

## Solution:

**Solution:**
$\frac{dy}{dx} + \frac{2x}{x-1} \cdot y = \frac{1}{(x-1)^2}$
$y = \frac{1}{(x-1)^2}\left[\frac{e^{2x}+1}{2e^{2x}}\right]$
$y(3) = \frac{e^6+1}{8e^6}$
$\alpha + \beta = 14$

---------------------------------------------------------------------------------------------------

# Question59

**If x = x(y) is the solution of the differential equation $y\frac{dx}{dy} = 2x + y^3(y+1)e^y$, x(1) = 0; then x(e) is equal to:**
**[24-Jun-2022-Shift-1]**

**Options:**

A. $e^3(e^e - 1)$

B. $e^e(e^3 - 1)$

C. $e^2(e^e + 1)$

D. $e^e(e^2 - 1)$

**Answer: A**

## Solution:

$$\frac{dx}{dy} - \frac{2x}{y} = y^2(y+1)e^y$$

$$\text{If } = e^{\int -\frac{2}{y}dy} = e^{-2\ln y} = \frac{1}{y^2}$$

Solution is given by

$$x \cdot \frac{1}{y^2} = \int y^2(y+1)e^y \cdot \frac{1}{y^2} dy$$

$$\Rightarrow \frac{x}{y^2} = \int (y+1)e^y dy$$

$$\Rightarrow \frac{x}{y^2} = ye^y + c$$

$$\Rightarrow x = y^2(ye^y + c) \text{ at, } y = 1, x = 0$$

$$\Rightarrow 0 = 1(1 \cdot e^1 + c) \Rightarrow c = -e \text{ at } y = e$$

$$x = e^2(e \cdot e^e - e)$$

----------------------------------------------------------------------------------------------------

# Question60

**The slope of the tangent to a curve C : y = y(x) at any point (x, y) on it is $\frac{2e^{2x} - 6e^{-x} + 9}{2 + 9e^{-2x}}$. If C passes through the points $\left(0, \frac{1}{2} + \frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha, \frac{1}{2}e^{2\alpha}\right)$, then $e^{\alpha}$ is equal to :**
**[25-Jul-2022-Shift-1]**

**Options:**

A. $\frac{3 + \sqrt{2}}{3 - \sqrt{2}}$

B. $\frac{3}{\sqrt{2}}\left(\frac{3 + \sqrt{2}}{3 - \sqrt{2}}\right)$

C. $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\right)$

D. $\frac{\sqrt{2} + 1}{\sqrt{2} - 1}$

**Answer: B**

## Solution:

**Solution:**

$$\frac{dy}{dx}=\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}=e^{2x}-\frac{6e^{-x}}{2+9e^{-2x}}$$

$$\int dy=\int e^{2x}dx-3\int 1+\left(\frac{3e^{-x}}{\sqrt{2}}\right)^2 dx(\text{ put } e^{-x}=t)$$

$$=\frac{e^{2x}}{2}+3\int\frac{dt}{1+\left(\frac{3t}{\sqrt{2}}\right)^2}$$

$$=\frac{e^{2x}}{2}+\sqrt{2}\tan^{-1}\frac{3t}{\sqrt{2}}+C$$

$$y=\frac{e^{2x}}{2}+\sqrt{2}\tan^{-1}\left(\frac{3e^{-x}}{\sqrt{2}}\right)+C$$

It is given that the curve passes through

$$\left(0,\ \frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$$

$$\frac{1}{2}+\frac{\pi}{2\sqrt{2}}=\frac{1}{2}+\sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)+C$$

$$\Rightarrow\ C=\frac{\pi}{2\sqrt{2}}-\sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$$

Now if $\left(\alpha,\ \frac{1}{2}e^{2\alpha}\right)$ satisfies the curve, then

$$\frac{1}{2}e^{2\alpha}=\frac{e^{2\alpha}}{2}+\sqrt{2}\tan^{-1}\left(\frac{3e^{-\alpha}}{\sqrt{2}}\right)+\frac{\pi}{2\sqrt{2}}-\sqrt{2}\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)$$

$$\tan^{-1}\left(\frac{3}{\sqrt{2}}\right)-\tan^{-1}\left(\frac{3e^{-\alpha}}{\sqrt{2}}\right)=\frac{\pi}{2\sqrt{2}}\times\frac{1}{\sqrt{2}}=\frac{\pi}{4}$$

$$\frac{\frac{3}{\sqrt{2}}-\frac{3e^{-\alpha}}{\sqrt{2}}}{1+\frac{9}{2}e^{-\alpha}}=1$$

$$\frac{3}{\sqrt{2}}e^{\alpha}-\frac{3}{\sqrt{2}}=e^{\alpha}+\frac{9}{2}$$

$$e^{\alpha}=\frac{\frac{9}{2}+\frac{3}{\sqrt{2}}}{\frac{3}{\sqrt{2}}-1}=\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$$

----------------------------------------------------------------------------------------------------

# Question61

## The general solution of the differential equation $(x-y^2)dx+y(5x+y^2)dy=0$ is: [25-Jul-2022-Shift-1]

**Options:**

A. $(y^2+x)^4=C\,|(y^2+2x)^3|$

B. $(y^2+2x)^4=C\,|(y^2+x)^3|$

C. $|(y^2+x)^3|=C(2y^2+x)^4$

D. $|(y^2+2x)^3|=C(2y^2+x)^4$

**Answer: A**

## Solution:

$(x-y^2)dx+y(5x+y^2)dy=0$

$y\frac{dy}{dx}=\frac{y^2-x}{5x+y^2}$

Let $y^2=t$ $\frac{1}{2}\cdot\frac{dt}{dx}=\frac{t-x}{5x+t}$

Now substitute, $t=vx$

$\frac{dt}{dx}=v+x\frac{dv}{dx}$

$\frac{1}{2}\left\{v+x\frac{dv}{dx}\right\}=\frac{v-1}{5+v}$

$x\frac{dv}{dx}=\frac{2v-2}{5+v}-v=\frac{-3v-v^2-2}{5+v}$

$\int\frac{5+v}{v^2+3v+2}dv=\int-\frac{dx}{x}$

$\int\frac{4}{v+1}dv-\int\frac{3}{v+2}dv=-\int\frac{dx}{x}$

$4\ln|v+1|-3\ln|v+2|=-\ln x+\ln C$

$\left|\frac{(v+1)^4}{(v+2)^3}\right|=\frac{c}{x}$

$\left|\frac{\left(\frac{y^2}{x}+1\right)^4}{\left(\frac{y^2}{x}+2\right)^3}=\frac{c}{x}\right|$

$|(y^2+x)^4|=C\,|(y^2+2x)^3|$

---------------------------------------------------------------------------------------------------

# Question62

**Let a smooth curve y = f (x) be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the points (1, 2) and (8, 1), then $y\left(\frac{1}{8}\right)\Big|$ is equal to**

**[25-Jul-2022-Shift-2]**

**Options:**

A. $2\log_e 2$

B. 4

C. 1

D. $4\log_e 2$

**Answer: B**

## Solution:

$\frac{dy}{dx} \propto \frac{-y}{x}$
$\frac{dy}{dx} = \frac{-ky}{x} \Rightarrow \int \frac{dy}{y} = -K \int \frac{dx}{x}$
$\ln |y| = -K \ln |x| + C$
If the above equation satisfy (1, 2) and (8, 1)
$\ln 2 = -K \times 0 + C \Rightarrow C = \ln 2$
$\ln 1 = -K \ln 8 + \ln 2 \Rightarrow K = \frac{1}{3}$
So, at $x = \frac{1}{8}$
$\ln |y| = -\frac{1}{3} \ln\left(\frac{1}{8}\right) + \ln 2 = 2 \ln 2$
$|y| = 4$

----------------------------------------------------------------------------------------------------

# Question63

**Let a smooth curve y = f (x) be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the points (1, 2) and (8, 1), then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to**
**[25-Jul-2022-Shift-2]**

**Options:**

A. $2\log_e 2$

B. 4

C. 1

D. $4\log_e 2$

**Answer: B**

## Solution:

**Solution:**
$\frac{dy}{dx} \propto \frac{-y}{x}$
$\frac{dy}{dx} = \frac{-ky}{x} \Rightarrow \int \frac{dy}{y} = -K \int \frac{dx}{x}$
$\ln |y| = -K \ln |x| + C$
If the above equation satisfy (1, 2) and (8, 1)
$\ln 2 = -K \times 0 + C \Rightarrow C = \ln 2$
$\ln 1 = -K \ln 8 + \ln 2 \Rightarrow K = \frac{1}{3}$
So, at $x = \frac{1}{8}$
$\ln |y| = -\frac{1}{3} \ln\left(\frac{1}{8}\right) + \ln 2 = 2 \ln 2$
$|y| = 4$

----------------------------------------------------------------------------------------------------

# Question64

**Let y = y(x) be the solution of the differential equation**

$\frac{dy}{dx} = \frac{4y^3+2yx^2}{3xy^2+x^3}$**, y(1) = 1.**

**If for some n ∈ ℕ, y(2) ∈ [n − 1, n), then n is equal to __**
**[25-Jul-2022-Shift-2]**

**Answer: 3**

## Solution:

**Solution:**

$\frac{dy}{dx} = \frac{y}{x}\frac{(4y^2+2x^2)}{(3y^2+x^2)}$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v(4v^2+2)}{(3v^2+1)}$

$\Rightarrow x\frac{dv}{dx} = v\left(\frac{(4v^2+2-3v^2-1)}{3v^2+1}\right)$

$\Rightarrow \int(3v^2+1)\frac{dv}{v^3+v} = \int\frac{dx}{x}$

$\Rightarrow \ln|v^3+v| = \ln x + c$

$\Rightarrow \ln\left|\left(\frac{y}{x}\right)^3+\left(\frac{y}{x}\right)\right| = \ln x + C$

$\downarrow y(1) = 1$

$\Rightarrow C = \ln 2$

$\therefore$ for $y(2)$

$\ln\left(\frac{y^3}{8}+\frac{y}{2}\right) = 2\ln 2 \Rightarrow \frac{y^3}{8}+\frac{y}{2} = 4$

$\Rightarrow [y(2)] = 2$

$\Rightarrow n = 3$

----------------------------------------------------------------------------------------------------

# Question65

**If $\frac{dy}{dx} + 2y\tan x = \sin x$, $0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of y(x) is :**
**[26-Jul-2022-Shift-1]**

**Options:**

A. $\frac{1}{8}$

B. $\frac{3}{4}$

C. $\frac{1}{4}$

D. $\frac{3}{8}$

**Answer: A**

## Solution:

**Solution:**

$\frac{dy}{dx} + 2y\tan x = \sin x$

which is a first order linear differential equation.

Integrating factor (I. F.) $= e^{\int 2\tan x\,dx}$

$= e^{2\ln|\sec x|} = \sec^2 x$

Solution of differential equation can be written as

$y \cdot \sec^2 x = \int \sin x \cdot \sec^2 x\,dx = \int \sec x \cdot \tan x\,dx\ y\sec^2 x = \sec x + C$

$y\left(\frac{\pi}{3}\right) = 0,\ 0 = \sec\frac{\pi}{3} + C \Rightarrow C = -2$

$y = \frac{\sec x - 2}{\sec^2 x} = \cos x - 2\cos^2 x$

$= \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$

$y_{max} = \frac{1}{8}$

-----------------------------------------------------------------------------------------------------

# Question66

**Let a curve $y = y(x)$ pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and $x(>3)$ be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $(\alpha, 6\sqrt{10})$ in the first quadrant, then $\alpha$ is equal to __________.**
**[26-Jul-2022-Shift-1]**

**Answer: 6**

## Solution:

$\int_3^x f(x)dx = \left(\frac{f(x)}{x}\right)^3$

$x^3 \cdot \int_3^x f(x)dx = f^3(x)$

Differentiate w.r.t. x

$x^3 f(x) + 3x^2 \cdot \frac{f^3(x)}{x^3} = 3f^2(x)f'(x)$

$\Rightarrow 3y^2\frac{dy}{dx} = x^3 y + \frac{3y^3}{x}$

$3xy\frac{dy}{dx} = x^4 + 3y^2$

Let $y^2 = t$

$\frac{3}{2}\frac{dt}{dx} = x^3 + \frac{3t}{x}$

$\frac{dt}{dx} - \frac{2t}{x} = \frac{2x^3}{3}$

I. F. $= .e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

Solution of differential equation

$t.\ \frac{1}{x^2} = \int \frac{2}{3}x\,dx$

$\frac{y^2}{x^2} = \frac{x^2}{3} + C$

$y^2 = \frac{x^4}{3} + Cx^2$
Curve passes through (3, 3) ⇒ C = −2

$y^2 = \frac{x^4}{3} - 2x^2$
Which passes through $(\alpha, 6\sqrt{10})$
$\frac{\alpha^4 - 6\alpha^2}{3} = 360$
$\alpha^4 - 6\alpha^2 - 1080 = 0$
$\alpha = 6$

-----------------------------------------------------------------------------------------------

# Question67

## Let the solution curve $y = f(x)$ of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$, $x \in (-1, 1)$ pass through the origin. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is equal to
## [26-Jul-2022-Shift-2]

**Options:**

A. $\frac{\pi}{3} - \frac{1}{4}$

B. $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

C. $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$

D. $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

**Answer: B**

## Solution:

**Solution:**
$\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^4 + 2x}{\sqrt{1 - x^2}}$
which is first order linear differential equation.

Integrating factor $(I.F) = .e^{\int \frac{x}{x^2 - 1}dx}$
$= e^{\frac{1}{2}\ln|x^2 - 1|} = \sqrt{|x^2 - 1|}$
$= \sqrt{1 - x^2}$
$\because x \in (-1, 1)$
Solution of differential equation
$y\sqrt{1 - x^2} = \int(x^4 + 2x)dx = \frac{x^5}{5} + x^2 + c$
Curve is passing through origin, $c = 0$

$y = \frac{x^5 + 5x^2}{5\sqrt{1 - x^2}}$

$$\int_{\frac{-\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^5+5x^2}{5\sqrt{1-x^2}}dx = 0 + 2\int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}}dx$$
put $x = \sin\theta$
$dx = \cos\theta d\theta$
$$I = 2\int_0^{\frac{\pi}{3}} \frac{\sin^2\theta \cdot \cos\theta d\theta}{\cos\theta}$$
$$= \int_0^{\frac{\pi}{3}} (1-\cos 2\theta)d\theta$$
$$= \left(\theta - \frac{\sin 2\theta}{2}\right)\Bigg|_0^{\frac{\pi}{3}}$$

---------------------------------------------------------------------------------------------------

# Question68

**Suppose $y = y(x)$ be the solution curve to the differential equation $\frac{dy}{dx} - y = 2 - e^{-x}$ such that $\lim_{x\to\infty} y(x)$ is finite. If a and b are respectively the x - and y-intercepts of the tangent to the curve at $x = 0$, then the value of $a - 4b$ is equal to ________.**
**[26-Jul-2022-Shift-2]**

**Answer: 3**

## Solution:

**Solution:**
IF $= e^{-x}$
$y \cdot e^{-x} = -2e^{-x} + \frac{e^{-2x}}{2} + C$
$\Rightarrow y = -2 + e^{-x} + Ce^{x}$
$\lim_{x\to\infty} y(x)$ is finite so $C = 0$
$y = -2 + e^{-x}$
$\Rightarrow \frac{dy}{dx} = -.e^{-x} \Rightarrow \frac{dy}{dx}\Big|_{x=0} = -1$
Equation of tangent
$y + 1 = -1(x - 0)$
or $y + x = -1$
So $a = -1, b = -1$
$\Rightarrow a - 4b = 3$

---------------------------------------------------------------------------------------------------

# Question69

**Let $y = y_1(x)$ and $y = y_2(x)$ be two distinct solutions of the differential equation $\frac{dy}{dx} = x + y$, with $y_1(0) = 0$ and $y_2(0) = 1$ respectively. Then, the number of points of intersection of $y = y_1(x)$ and $y = y_2(x)$ is**
**[27-Jul-2022-Shift-1]**

**Options:**

A. 0

B. 1

C. 2

D. 3

**Answer: A**

## Solution:

$\frac{dy}{dx} = x + y$

Let $x + y = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dt}{dx} - 1 = t \Rightarrow \int \frac{dt}{t+1} = \int dx$

$\ln|t + 1| = x + C'$

$|t + 1| = Ce^x$

$|x + y + 1| = Ce^x$

For $y_1(x)$, $y_1(0) = 0 \Rightarrow C = 1$

For $y_2(x)$, $y_2(0) = 1 \Rightarrow C = 2$

$y_1(x)$ is given by $|x + y + 1| = e^x$

$y_2(x)$ is given by $|x + y + 1| = 2e^x$

At point of intersection

$e^x = 2e^x$

No solution

So, there is no point of intersection of $y_1(x)$ and $y_2(x)$.

---------------------------------------------------------------------------------------------------

# Question70

**Let $y = y(x)$ be the solution curve of the differential equation $\sin(2x^2)\log_e(\tan x^2)\,dy + \left(4xy - 4\sqrt{2}x\sin\left(x^2 - \frac{\pi}{4}\right)\right)dx = 0$, $0 < x < \sqrt{\frac{\pi}{2}}$, which passes through the point $\left(\sqrt{\frac{\pi}{6}}, 1\right)$. Then $\left|y\left(\sqrt{\frac{\pi}{3}}\right)\right|$ is equal to ________.**

**[27-Jul-2022-Shift-1]**

**Answer: 1**

## Solution:

$$\frac{dy}{dx} + y\left(\frac{4x}{\sin(2x^2)\ln(\tan x^2)}\right) = \frac{4\sqrt{2}x\sin\left(x^2 - \frac{\pi}{4}\right)}{\sin(2x^2)\ln(\tan x^2)}$$

$$I.F = e^{\int \frac{4x}{\sin(2x^2)\ln(\tan x^2)} dx}$$
$$= e^{\ln|\ln(\tan x^2)} = \ln(\tan x^2)$$
$$\therefore \int d\left(y \cdot \ln(\tan x^2)\right) = \int \frac{4\sqrt{2} x \sin\left(x^2 - \frac{\pi}{4}\right)}{\sin(2x^2)} dx$$
$$\Rightarrow y\ln(\tan x^2) = \ln\left|\frac{\sec x^2 + \tan x^2}{\operatorname{cosec} x^2 - \cot x^2}\right| + C$$
$$\ln\left(\frac{1}{\sqrt{3}}\right) = \ln\left(\frac{\frac{3}{\sqrt{3}}}{2-\sqrt{3}}\right) + C$$
$$e = \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2-\sqrt{3}}\right)$$
For $y\left(\sqrt{\frac{\pi}{3}}\right)$
$$y\ln(\sqrt{3}) = \ln\left|\frac{2+\sqrt{3}}{\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}}\right| + \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2\sqrt{3}}\right)$$
$$= \ln(2+\sqrt{3}) + \ln\left(\frac{1}{\sqrt{3}}\right) + \ln\left(\frac{1}{\sqrt{3}}\right) - \ln\left(\frac{\sqrt{3}}{2-\sqrt{3}}\right)$$
$$\Rightarrow y\ln\sqrt{3} = \ln\left(\frac{1}{\sqrt{3}}\right)$$
$$\Rightarrow \frac{y}{2}\ln 3 = -\frac{1}{2}\ln 3$$
$$\Rightarrow y = 1$$
$$\therefore \left|y\left(\sqrt{\frac{\pi}{3}}\right)\right| = 1$$

---------------------------------------------------------------------------------------------------

# Question71

**Let the solution curve of the differential equation $xdy = \left(\sqrt{x^2 + y^2} + y\right)dx$, $x > 0$, intersect the line $x = 1$ at $y = 0$ and the line $x = 2$ at $y = \alpha$. Then the value of $\alpha$ is :**
**[28-Jul-2022-Shift-1]**

**Options:**

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $-\frac{3}{2}$

D. $\frac{5}{2}$

**Answer: B**

## Solution:

**Solution:**
$$\frac{xdy - ydx}{\sqrt{x^2 + y^2}} = dx$$
$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{x^2 + y^2}}{x} + \frac{y}{x}$$

$\Rightarrow \frac{dy}{dx} = \sqrt{1 + \frac{y^2}{x^2}} + \frac{y}{x}$

Let $\frac{y}{x} = v$

$\Rightarrow v + x\frac{dv}{dx} = \sqrt{1 + v^2} + v$

$\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$

OR $\ln\left(v + \sqrt{1 + v^2}\right) = \ln x + C$

at $x = 1$, $y = 0$

$\Rightarrow C = 0$

$\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = x$

At $x = 2$

$\frac{y}{2} + \sqrt{1 + \frac{y^2}{4}} = 2$

$\Rightarrow 1 + \frac{y^2}{4} = 4 + \frac{y^2}{4} - 2y$

OR $y = \frac{3}{2}$

----------------------------------------------------------------------------------------------------

# Question72

## If $y = y(x)$, $x \in (0, \pi / 2)$ be the solution curve of the differential equation $(\sin^2 2x)\frac{dy}{dx} + (8\sin^2 2x + 2\sin 4x)y - 2e^{-4x}(2\sin 2x + \cos 2x)$, with $y\left(\frac{\pi}{4}\right) = e^{-\pi}$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

## [28-Jul-2022-Shift-1]

**Options:**

A. $\frac{2}{\sqrt{3}}e^{-2\pi / 3}$

B. $\frac{2}{\sqrt{3}}e^{2\pi / 3}$

C. $\frac{1}{\sqrt{3}}e^{-2\pi / 3}$

D. $\frac{1}{\sqrt{3}}e^{2\pi / 3}$

**Answer: A**

## Solution:

**Solution:**

$(\sin^2 2x)\frac{dy}{dx} + (8\sin^2 2x + 2\sin 4x)y$

$= 2e^{-4x}(2\sin 2x + \cos 2x)$

$\frac{dy}{dx} + (8 + 4\cot 2x)y = 2e^{-4x}\left(\frac{2\sin 2x + \cos 2x}{\sin^2 2x}\right)$

Integrating factor

$(I.F.) = e^{\int (8 + 4\cot 2x)dx}$

$= e^{8x + 2\ln \sin 2x}$

Solution of differential equation

$y.e^{8x + 2\ln \sin 2x}$

$= \int 2e^{(4x+2\ln \sin 2x)} \frac{(2\sin 2x+\cos 2x)}{\sin^2 2x} dx$

$= 2\int e^{4x}(2\sin 2x+\cos 2x) dx$

$y.\, e^{8x+2\ln \sin 2x} = e^{4x}\sin 2x + c$

$y\left(\frac{\pi}{4}\right) = e^{-\pi}$

$e^{-\pi}\cdot e^{2\pi} = e^{\pi} + c \Rightarrow c = 0$

$y\left(\frac{\pi}{6}\right) = \dfrac{e^{\frac{2\pi}{3}}\frac{\sqrt{3}}{2}}{e^{\left(\frac{4\pi}{3}+2\ln\frac{\sqrt{3}}{2}\right)}}$

$= e^{\frac{-2\pi}{3}}\cdot\frac{2}{\sqrt{3}}$

---

# Question73

**Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} + \frac{1}{x^2-1}y = \left(\frac{x-1}{x+1}\right)^{1/2}$, $x > 1$ passing through the point $\left(2, \sqrt{\frac{1}{3}}\right)$. Then $\sqrt{7}y(8)$ is equal to:**

**[28-Jul-2022-Shift-2]**

**Options:**

A. $11 + 6\log_e 3$

B. 19

C. $12 - 2\log_e 3$

D. $19 - 6\log_e 3$

**Answer: B**

## Solution:

**Solution:**

$\frac{dy}{dx} + \frac{1}{x^2-1}y = \sqrt{\frac{x-1}{x+1}}, x > 1$

Integrating factor I.F. $= e^{\int\frac{1}{x^2-1}dx} = e^{\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|}$

$= \sqrt{\frac{x-1}{x+1}}$

Solution of differential equation

$y\sqrt{\frac{x-1}{x+1}} = \int\frac{x-1}{x+1}dx = \int\left(1-\frac{2}{x+1}\right)dx$

$y\sqrt{\frac{x-1}{x+1}} = x - 2\ln|x+1| + C$

Curve passes through $\left(2, \sqrt{\frac{1}{3}}\right)$

$\frac{1}{\sqrt{3}}\times\frac{1}{\sqrt{3}} = 2 - 2\ln 3 + C$

$C = 2\ln 3 - \frac{5}{3}$

$y(8)\times\frac{\sqrt{7}}{3} = 8 - 2\ln 9 + 2\ln 3 - \frac{5}{3}$

$\sqrt{7}\cdot y(8) = 19 - 6\ln 3$

----------------------------------------------------------------------------------------------------

# Question74

## The differential equation of the family of circles passing through the points (0, 2) and (0, −2) is :
## [28-Jul-2022-Shift-2]

**Options:**

A. $2xy\frac{dy}{dx} + (x^2 - y^2 + 4) = 0$

B. $2xy\frac{dy}{dx} + (x^2 + y^2 - 4) = 0$

C. $2xy\frac{dy}{dx} + (y^2 - x^2 + 4) = 0$

D. $2xy\frac{dy}{dx} - (x^2 - y^2 + 4) = 0$

**Answer: A**

## Solution:

**Solution:**
Family of circles passing through the points (0, 2) and (0, −2)
$x^2 + (y-2)(y+2) + \lambda x = 0, \lambda \in R$
$x^2 + y^2 + \lambda x - 4 = 0 \ldots\ldots$ (1)
Differentiate w.r.t x
$2x + 2y\frac{dy}{dx} + \lambda = 0 \ldots\ldots$ (2)
Using (1) and (2), eliminate λ
$x^2 + y^2 - \left(2x + 2y\frac{dy}{dx}\right)x - 4 = 0$
$2xy\frac{dy}{dx} + x^2 - y^2 + 4 = 0$

----------------------------------------------------------------------------------------------------

# Question75

## Let the solution curve $y = y(x)$ of the differential equation $(1 + e^{2x})\left(\frac{dy}{dx} + y\right) = 1$ pass through the point $\left(0, \frac{\pi}{2}\right)$. Then, $\lim_{x \to \infty} e^x y(x)$ is equal to :
## [29-Jul-2022-Shift-1]

**Options:**

A. $\frac{\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{2}$

**Answer: B**

## Solution:

**Solution:**

D.E. $(1+e^{2x})\left(\frac{dy}{dx}+y\right)=1$

$\Rightarrow \frac{dy}{dx}+y=\frac{1}{1+e^{2x}}$

I.F. $=e^{\int 1 \cdot dx}=e^{x}$

$\therefore$ Solution

$e^{x}y(x)=\int \frac{e^{x}}{1+e^{2x}}dx$

$\Rightarrow e^{x}y(x)=\tan^{-1}(e^{x})+C$

$\because$ It passes through $\left(0, \frac{\pi}{2}\right)$, $C=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

$\therefore \lim\limits_{x\to\infty} e^{x}y(x)=\lim\limits_{x\to\infty}\tan^{-1}(e^{x})+\frac{\pi}{4}$

$=\frac{3\pi}{4}$

----------------------------------------------------------------------------------------------------

# Question76

## If the solution curve of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passes through the points (2, 1) and (k + 1, 2), k > 0, then [29-Jul-2022-Shift-2]

**Options:**

A. $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e(k^2+1)$

B. $\tan^{-1}\left(\frac{1}{k}\right)-\log_e(k^2+1)$

C. $2\tan^{-1}\left(\frac{1}{k+1}\right)=\log_e(k^2+2k+2)$

D. $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(\frac{k^2+1}{k^2}\right)$

**Answer: A**

## Solution:

**Solution:**

$\frac{dy}{dx}=\frac{x+y-2}{x-y}=\frac{(x-1)+(y-1)}{(x-1)-(y-1)}$

Let $x-1=X$, $y-1=Y$

$\frac{dY}{dX}=\frac{X+Y}{X-Y}$

Let $Y=tX \Rightarrow \frac{dY}{dX}=t+X\frac{dt}{dX}$

$t+X\frac{dt}{dX}=\frac{1+t}{1-t}$

$X\frac{dt}{dX}=\frac{1+t}{1-t}-t=\frac{1+t^2}{1-t}$

$\int \frac{1-t}{1+t^2} dt = \int \frac{dX}{X}$

$\tan^{-1}t - \frac{1}{2}\ln(1+t^2) = \ln\left|X\right| + c$

$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\left(1+\left(\frac{y-1}{x-1}\right)^2\right) = \ln\left|x-1\right| + c$

Curve passes through (2, 1)

$0 - 0 = 0 + c \Rightarrow c = 0$

If $(k+1, 2)$ also satisfies the curve

$\tan^{-1}\left(\frac{1}{k}\right) - \frac{1}{2}\ln\left(\frac{1+k^2}{k^2}\right) = \ln k$

$2\tan^{-1}\left(\frac{1}{k}\right) = \ln(1+k^2)$

----------------------------------------------------------------------------------------------------

# Question77

## Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y = \frac{(x+3)}{x+1}$, $x > -1$, which passes through the point (0, 1). Then y(1) is equal to: [29-Jul-2022-Shift-2]

**Options:**

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. $\frac{5}{2}$

D. $\frac{7}{2}$

**Answer: B**

## Solution:

**Solution:**

$\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y = \frac{(x+3)}{x+1}, x > -1$

Integrating factor I.F. $= e^{\int \frac{2x^2+11x+13}{x^3+6x^2+11x+6} dt}$

Let $\frac{2x^2+11x+13}{(x+1)(x+2)(x+3)} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+3}$ $A = 2, B = 1, C = -1$

I.F. $= e^{(2\ln|x+1| + \ln|x+2| - \ln|x+3|)}$

$= \frac{(x+1)^2(x+2)}{x+3}$

Solution of differential equation

$y \cdot \frac{(x+1)^2(x+2)}{x+3} = \int (x+1)(x+2)\,dt$

$y\frac{(x+1)^2(x+2)}{x+3} = \frac{x^3}{3} + \frac{3x^2}{2} + 2x + c$

Curve passes through (0, 1)

$1 \times \frac{1 \times 2}{3} = 0 + c \Rightarrow c = \frac{2}{3}$

So, $y(1) = \dfrac{\frac{1}{3} + \frac{3}{2} + 2 + \frac{2}{3}}{\frac{(2^2 \times 3)}{4}} = \dfrac{3}{2}$

-----------------------------------------------------------------------------------------------

# Question78

**The population P = P(t) at time ' t ' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If P(0) = 850, then the time at which population becomes zero is :**
**24 Feb 2021 Shift 1**

**Options:**

A. $\log_e 18$

B. $\log_e 9$

C. $1 / 2\log_e 18$

D. $2\log_e 18$

**Answer: D**

## Solution:

**Solution:**

Given that $\frac{dP}{dt} = 0.5P - 450$

$\Rightarrow \int_0^t \frac{dp}{P - 900} = \int_0^t \frac{dt}{2}$

$\Rightarrow [\ell n \mid P(t) - 900 \mid]_0^t = \left[\frac{t}{2}\right]_0^t$

$\Rightarrow \ell n \mid P(t) - 900 \mid - \ell n \mid P(0) - 900 \mid = \frac{t}{2}$

$\Rightarrow \ell n \mid P(t) - 900 \mid - \ell n \mid 50 \mid = \frac{t}{2}$

For P(t) = 0

$\Rightarrow \ell n \left|\frac{900}{50}\right| = \frac{t}{2}$

$\Rightarrow t = 2\ell n 18$

-----------------------------------------------------------------------------------------------

# Question79

**Let f be a twice differentiable function defined on R, such that $f(0) = 1, f'(0) = 2$ and $f''(x) \neq 0$ for all $x \in R$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all $x \in R$, then the value of $f(1)$ lies in the interval**
**[24 Feb 2021 Shift 2]**

**Options:**

A. (9, 12)

B. (6, 9)

C. (0, 3)

D. (3, 6)

**Answer: B**

## Solution:

Given, $f(0) = 1$,
$f'(0) = 2$,
$f''(x) \neq 0$

$$\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$$

$\Rightarrow f(x)f''(x) - f'(x)f'(x) = 0$

$\Rightarrow \frac{f''(x)}{f'(x)} = \frac{f'(x)}{f(x)}$

$\Rightarrow \int \frac{f''(x)}{f'(x)} dx = \int \frac{f'(x)}{f(x)} dx$

$\Rightarrow \log f'(x) = \log f(x) + \log c$

$\Rightarrow f'(x) = cf(x)$

Now, put $x = 0$, we get

$f'(0) = cf(0)$

$\Rightarrow 2 = c \times 1$

$\Rightarrow c = 2$

Putting the value of $c = 2$ in Eq. (i), we get

$\log f'(x) = \log f(x) + \log 2$

$\Rightarrow f'(x) = 2f(x) \Rightarrow \int \frac{f'(x)}{f(x)} dx = \int 2 dx$

$\Rightarrow \log f(x) = 2x + D \Rightarrow f(x) = e^{2x+D}$

$\Rightarrow f(x) = e^{D} \cdot e^{2x}$

$\Rightarrow f(x) = k \cdot e^{2x}$ [Let $k = e^{D}$ ]

Put $x = 0$, we get

$f(0) = k \cdot e^{0}$

$\Rightarrow 1 = k \Rightarrow f(x) = k \cdot e^{2x}$

$\therefore f(x) = e^{2x}$

Put $x = 1$, we get

$f(1) = e^{2}$

Clearly, $e^{2}$ lies in (6, 9).

---------------------------------------------------------------------------------------------------

# Question80

**The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$, $a > 0$ is ......... .**
**[26 Feb 2021 Shift 1]**

**Answer: 2**

## Solution:

**Solution:**

Given, $y^2 = a\left[x + \frac{\sqrt{a}}{2}\right]$, a > 0 ...(i)

Differentiating both sides w.r.t. ' x ',

$2y\frac{dy}{dx} = a[1 + 0] = a$ . . . (ii)

Use Eq. (ii) in Eq. (i) to eliminate the constant ' a '.

$y^2 = 2y\frac{dy}{dx}\left(x + \sqrt{2y}\sqrt{\frac{dy}{dx}}\right)$

$y^2 - 2xy\frac{dy}{dx} = 2\sqrt{2} \cdot y\sqrt{y} \cdot \frac{dy}{dx}\sqrt{\frac{dy}{dx}}$

Squaring on both sides,

$y^4 + 4x^2y^2\left(\frac{dy}{dx}\right)^2 - 4xy^3\frac{dy}{dx} = 8y^3\left(\frac{dy}{dx}\right)^3$

Thus, degree of above differential equation is 3 and its order is 1 .

Difference between degree and order $= 3 - 1 = 2$

------------------------------------------------------------------------------------------------

# Question81

## Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2 + y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y, for which the point (3, y) lies on the curve, is [26 Feb 2021 Shift 2]

**Options:**

A. $\frac{18}{35}$

B. $-\frac{4}{3}$

C. $-\frac{18}{19}$

D. $-\frac{18}{11}$

**Answer: C**

## Solution:

**Solution:**

Given, slope of tangent line to curve at (x, y) is $\frac{xy^2 + y}{x}$

i.e., $\frac{dy}{dx} = \frac{xy^2 + y}{x}$

$\Rightarrow \frac{dy}{dx} = y^2 + \frac{y}{x} \Rightarrow x\,dy = xy^2dx + y\,dx$

$\Rightarrow x\,dy - y\,dx = xy^2dx \Rightarrow \frac{x\,dy - y\,dx}{y^2} = x\,dx$

Integrating both sides, we get

$\frac{-x}{y} = \frac{x^2}{2} + C$ . . . (i)

The curve intersect line at $x = -2$

Then, $x = -2$, is satisfied by $x + 2y = 4$

Hence, $(-2) + 2y = 4$ Gives, $y = 3$

∴ Curve passes through $(2, -3)$.

Use $(2, -3)$ is Eq. (i), we get

$\frac{-2}{-3} = \frac{(-2)^2}{2} + C \Rightarrow C = \frac{-4}{3}$

∴ The curve is

$\frac{-x}{y} = \frac{x^2}{2} = \frac{-4}{3}$ . . . (ii)

It also passes through (3, y).

Put (3, 4) in Eq. (ii), we get

$\Rightarrow \quad \frac{-3}{y} = \frac{(3)^2}{2} - \frac{4}{3} \Rightarrow y = -\frac{18}{19}$

------------------------------------------------------------------------------------------------

# Question82

## Let $f(x) = \int_0^x e^t f(t) dt + e^x$ be a differentiable function for all $x \in R$. Then, $f(x)$ equals [26 Feb 2021 Shift 2]

**Options:**

A. $2e^{(e^x - 1)} - 1$

B. $e^{e^x} - 1$

C. $2e^{e^x} - 1$

D. $e^{(e^x - 1)}$

**Answer: A**

## Solution:

**Solution:**

Given, $f(x) = \int_0^x e^t f(t) dt + e^x$ . . . (i)

Since, $f(x)$ is differentiable function, differentiate Eq. (i)

$f'(x) = e^x f(x) + e^x$[Using Newton Leibnitz theorem ]

$f'(x) = e^x(f(x) + 1) \Rightarrow \frac{f'(x)}{f(x) + 1} = e^x$

Integrating it, $\int \frac{f'(x)}{f(x) + 1} dx = \int e^x dx + C$

Let $f(x) + 1 = u$, then $f'(x) dx = du$

$\int \frac{du}{u} = e^x + C \Rightarrow \log u = e^x + C$

$\log(f(x) + 1) = e^x + C \quad [\because u = f(x) + 1] \ldots$ (ii)

Now

$f(x) = \int_0^x e^t f(t) dt + e^x \Rightarrow f(0) = e^0 = 1$

Put x = 0, in Eq. (ii), we get

$\log(2) = e^{\circ} + C \Rightarrow C = \log(2) - 1$

From Eq. (ii), we get

$\log(f(x) + 1) \; = e^x + \log 2 - 1$

$f(x) + 1 \; = e^{e^x + \log 2 - 1} = e^{e^x} \cdot e^{\log 2} \cdot e^{-1}$

$f(x) + 1 \; = 2e^{e^x} \cdot e^{-1} = 2e^{e^x - 1}$

$\therefore \; f(x) \; = 2e^{e^x - 1} - 1$

------------------------------------------------------------------------------------------------

# Question83

**The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time t = 0. The number of bacteria is increased by 20% in 2h. If the population of bacteria is 2000 after $\frac{k}{\log_e(6/5)}$h, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to**

**[26 Feb 2021 Shift 1]**

**Options:**

A. 4

B. 8

C. 2

D. 16

**Answer: A**

## Solution:

**Solution:**

Let $x$ be the number of bacteria at any time $t$.

Given that, $\frac{dx}{dt} \propto x$ ($\because$ Rate of growth $= \frac{dx}{dt}$)

$\Rightarrow \frac{dx}{dt} = \lambda x \Rightarrow \frac{dx}{x} = \lambda dt$

After integrating it, we get

$\log x = \lambda t + C$ . . . (i)

$= 0, x = 1000$ which gives

$\log 1000 = 0 + C \Rightarrow C = \log 1000$

Given, when $t = 0$, $x =$ $\log 1000$

$\log x - \log 1000 = \lambda t$ or $\log\left(\frac{x}{1000}\right) = \lambda t$ . . . (ii)

Given that in $2h$, number of bacteria increased by $20\%$ i.e. when $t = 2h$, $x = 1200$ Put, $t = 2$ and $x = 1200$ in Eq. (ii),

$\log\left(\frac{1200}{1000}\right) = 2\lambda$ gives, $\lambda = \frac{1}{2}\log\left(\frac{6}{5}\right)$

Again, from Eq. (ii),

$\log\left(\frac{x}{1000}\right) = \frac{1}{2}\log\left(\frac{6}{5}\right)$

or $\frac{x}{1000} = e^{\frac{t}{2}\log\left(\frac{6}{5}\right)}$ . . . (iii)

Given, $x = 2000$ at $t = k/\log_e(6/5)$,

put in Eq. (iii),

$\frac{2000}{1000} = e^{\frac{k}{2}\log\left(\frac{6}{5}\right)/\log\left(\frac{6}{5}\right)}$

$2 = e^{k/2}$ or $\log 2 = k/2$

$k/\log 2 = 2$

$\therefore (k/\log_e 2)^2 = (2)^2 = 4$

-----------------------------------------------------------------------------------------------

# Question84

**If $y = y(x)$ is the solution of the equation $e^{\sin y}\cos y\frac{dy}{dx} + e^{\sin y}\cos x = \cos x$, $y(0) = 0$, then**

**$1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}}y\left(\frac{\pi}{4}\right)$ is .......... .**

**[26 Feb 2021 Shift 1]**

**Answer: 1**

## Solution:

**Solution:**

Given $e^{\sin y}\cos y\dfrac{dy}{dx}+e^{\sin y}\cos x=\cos x$ . . . (i)

Let $e^{\sin y}=t$, then $e^{\sin y}\cdot\cos y\cdot\dfrac{dy}{dx}=\dfrac{dt}{dx}$

Putting in Eq. (i),

$\cos x$ ... (ii) (Linear form)

Then, IF $=e^{\int\cos x\,dx}=e^{\sin x}$

Solution of differential Eq. (ii) is,

$t\cdot IF=\int\cos x\cdot IF\,dx+C$

$t\cdot e^{\sin x}=\int\cos x\cdot e^{\sin x}dx+C$

$=e^{u}$ i.e. let $\sin x=u$ then $\cos x\,d x=du$

$\Rightarrow\ t\cdot e^{\sin x}=\int e^{u}du+C=e^{u}+C$

Put $u=\sin x$ and $t=e^{\sin y}$

$\Rightarrow\ e^{\sin y}\cdot e^{\sin x}=e^{\sin x}+C$

Given, y(0) = 0, this gives C = 0

$\Rightarrow\ e^{\sin y}\cdot e^{\sin x}=e^{\sin x}$

$\Rightarrow\ e^{\sin y+\sin x}=e^{\sin x}$

$\Rightarrow\ \sin y+\sin x=\sin x$

$\Rightarrow\ \sin y=0$

$\Rightarrow\ y=0$

$\therefore y(\pi/6)=y(\pi/3)=y(\pi/4)=0$

Hence, $1+y\left(\dfrac{\pi}{6}\right)+\dfrac{\sqrt{3}}{2}y\left(\dfrac{\pi}{3}\right)+\dfrac{1}{\sqrt{2}}y\left(\dfrac{\pi}{4}\right)$

$=1+0+0+0=1$

----------------------------------------------------------------------------------------------------

# Question85

## If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point [25 Feb 2021 Shift 1]

**Options:**

A. (5, 4)

B. (4, 5)

C. (4, 4)

D. (5, 5)

**Answer: D**

## Solution:

**Solution:**

Given, slope $=\dfrac{x^2-4x+y+8}{x-2}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{x^2 - 4x + y + 8}{x - 2} = \dfrac{(x-2)^2 + (y+4)}{(x-2)}$

$= (x - 2) + \dfrac{y + 4}{x - 2}$

Let $(x - 2) = t \Rightarrow dx = dt$

and $(y + 4) = u \Rightarrow dy = du$

$\therefore \dfrac{dy}{dx} = \dfrac{du}{dt}$

Now, $\dfrac{dy}{dx} = (x - 2) + \dfrac{(y+4)}{(x-2)}$

$\Rightarrow \dfrac{du}{dt} = t + \dfrac{u}{t} \Rightarrow \dfrac{du}{dt} - \dfrac{u}{t} = t$

Here, integrating factor (IF) $= 1 / t$

$\Rightarrow u \cdot \left(\dfrac{1}{t}\right) = \int t \left(\dfrac{1}{t}\right) dt \Rightarrow u / t = t + c$

$\Rightarrow \dfrac{(y+4)}{(x-2)} = (x - 2) + c$

$\because$ It passes through origin [i.e. (0, 0) ], then

$\therefore \dfrac{4}{-2} = -2 + c$

$\Rightarrow -2 + 2 = c \Rightarrow c = 0$

Hence, $\dfrac{(y+4)}{(x-2)} = (x - 2) + 0$

$\Rightarrow y + 4 = (x - 2)^2$

Clearly, this curve passes through (5, 5) as it satisfies the equation.

---------------------------------------------------------------------------------------------------

# Question86

## If the curve $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines $2x - 3y = 1$ and$3x + 2y = 8$, then $|y(1)|$ is equal to ........... . [25 Feb 2021 Shift 2]

**Answer: 1**

## Solution:

**Solution:**

Given, $(2xy^2 - y)dx + xdy = 0$

$\Rightarrow \dfrac{dy}{dx} - \dfrac{y}{x} = -2y^2$

$\Rightarrow \dfrac{-1}{y^2}\dfrac{dy}{dx} + \dfrac{1}{xy} = 2$ ...(i) [divide by $y^2$ ]

Let $\dfrac{1}{y} = v$, then $-\dfrac{1}{y^2} \cdot \dfrac{dy}{dx} = \dfrac{dv}{dx}$, putting in Eq. (i)

$\dfrac{dv}{dx} + v\left(\dfrac{1}{x}\right) = 2$ (this is a linear form)

Now, integrating factor (IF) $= e^{\int \frac{1}{x} dx} = e^{\log x} = x$

$\therefore (IF)v = \int 2 . (IF)dx = \int 2xdx = 2\dfrac{x^2}{2} + C$

$\therefore (IA)v = x^2 + C$

Put $v = \dfrac{1}{y}$, this gives

$x^2 + c = \dfrac{x}{y}$

Now, first find point of intersection of lines

$2x - 3y = 1$ and $3x = -2y + 8$ by elimination method, we get $x = 2, y = 1$

∴ The curve $x^2 + c = \frac{1}{y}$ passes through (2, 1).

Put $x = 2, y = 1$, we get $c = -2$

$\frac{x}{y} = x^2 - 2$

or $y = \frac{x}{x^2 - 2}$

Put $x = 1$, we get $y(1) = \frac{1}{1-2} = -1$

∴ $|y(1)| = 1$

---------------------------------------------------------------------------------------------------

# Question87

## If a curve $y = f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$ ? [24 Feb 2021 Shift 2]

**Options:**

A. 5

B. 10

C. $\frac{62}{5}$

D. $\frac{31}{5}$

**Answer: B**

## Solution:

**Solution:**

Given, curve $y = f(x)$ passes through (1, 2) and satisfies

$x\frac{dy}{dx} + y = bx^4$

$\Rightarrow x\frac{dy}{dx} + y = bx^4$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = bx^3$

$\text{IF} = e^{\int \frac{1}{x} \cdot dx} = x$

$yx = \int bx^4 dx = \frac{bx^5}{5} + C$

$\therefore \; y = \frac{bx^4}{5} + \frac{C}{x} = f(x) \ldots$ (i)

∵ This curve passes through (1, 2).

$\therefore \; 2 \times 1 = \frac{b \times (1)^5}{5} + C$

$\Rightarrow \; 2 = \frac{b}{5} + C \ldots$ (ii)

Also, $\int_1^2 f(x)dx = \frac{62}{5}$

$\Rightarrow \int_1^2 \left( \frac{bx^4}{5} + \frac{C}{x} \right) dx = \frac{62}{5}$

$\Rightarrow \left[ b \times \frac{x^5}{25} + C\log x \right]_1^2 = \frac{62}{5}$

$\Rightarrow \left[ \left( \frac{b \times 32}{25} + C\log 2 \right) - \left( \frac{b}{25} + c\log 1 \right) \right] = \frac{62}{5}$

$\Rightarrow \frac{b \times 32}{25} + C\log 2 - \frac{b}{25} = \frac{62}{5} + 0\log 2$

[from Eq. (i)] On comparing, we get

$\frac{b}{25} \times 31 = \frac{62}{5}$ and $c = 0$

$b = \frac{62 \times 25}{31 \times 5}$

$b = 10$

Hence, the required value of $b = 10$.

---------------------------------------------------------------------------------------------------

# Question88

## The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is [18 Mar 2021 Shift 1]

**Options:**

A. $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

B. $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

C. $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

D. $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

**Answer: C**

## Solution:

**Solution:**

Given, equation of curve is $y^2 = 4a(x + a)$

$\Rightarrow y^2 = 4ax + 4a^2$...(i)

Differentiating Eq. (i) w.r.t. x, we get

$2y\frac{dy}{dx} = 4a$

$\Rightarrow a = \left(\frac{y}{2}\right) \cdot \frac{dy}{dx}$....(ii)

∴ Required differential equation is

$y^2 = 4 \times \frac{y}{2} \times \frac{dy}{dx}x + 4\left(\frac{y}{2} \cdot \frac{dy}{dx}\right)^2$ [From Eqs. (i) and (ii)]

$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + 2xy\left(\frac{dy}{dx}\right) - y^2 = 0$

$\Rightarrow y\left[y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y\right] = 0$

As, $y \neq 0$

$\therefore y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

---------------------------------------------------------------------------------------------------

# Question89

## Let $y = y(x)$ be the solution of the differential equation $x\,dy - y\,dx = \sqrt{(x^2 - y^2)}\,dx$, $x \geq 1$, with $y(1) = 0$. If the area bounded by the line $x = 1$, $x = e^{\pi}$, $y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + \beta$, then the value of

**$10(\alpha+\beta)$ is equal to**
**[18 Mar 2021 Shift 2]**

**Answer: 4**

## Solution:

**Solution:**

$x\,dy - y\,dx = \sqrt{x^2 - y^2}\,dx$

$\Rightarrow \dfrac{x\,dy - y\,dx}{x^2} = \dfrac{1}{x}\sqrt{1 - \dfrac{y^2}{x^2}}\,dx \Rightarrow \dfrac{d\left(\frac{y}{x}\right)}{\sqrt{1-\left(\frac{y}{x}\right)^2}} = \dfrac{1}{x}dx$

On integrating. $\int \dfrac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}} \cdot d(y/x) = \int \dfrac{1}{x}dx$

$\Rightarrow \sin^{-1}(y/x) = \log|x| + C$

Now, at $x = 1$, $y = 0$

$\therefore C = 0$

Hence, $y = x\sin(\log x)$

$\therefore A = \int_1^{e^{x}} x\sin(\log x)\,dx$

Put $x = e^t \Rightarrow dx = e^t dt$

$\therefore A = \int_0^{\pi} e^{2t}\sin(t)\,dt$

$\left(\text{using} \int e^{ax}\cdot\sin bx = \dfrac{e^{2x}}{a^2+b^2}(a\sin bx - b\cos bx)\right)$

$\alpha e^{2\pi} + \beta = \left(\dfrac{e^{2t}}{5}(2\sin t - \cos t)\right)_0^{\pi} = \dfrac{1+e^{2\pi}}{5}$

$\therefore \alpha = \dfrac{1}{5}, \beta = \dfrac{1}{5}$

$\therefore 10(\alpha+\beta) = 4$

----------------------------------------------------------------------------------------------------

# Question90

**Which of the following is true for y(x) that satisfies the differential equation $\frac{dy}{dx} = xy - 1 + x - y$; $y(0) = 0$**
**[17 Mar 2021 Shift 1]**

**Options:**

A. $y(1) = e^{-\frac{1}{2}} - 1$

B. $y(1) = e^{\frac{1}{2}} - e^{-\frac{1}{2}}$

C. $y(1) = 1$

D. $y(1) = e^{\frac{1}{2}} - 1$

**Answer: A**

## Solution:

**Solution:**

$\frac{dy}{dx} = xy - 1 + x - y$

$\frac{dy}{dx} = x(y+1) - 1(y+1) \Rightarrow \frac{dy}{dx} = (x-1)(y+1)$

$\int \frac{dy}{y+1} = \int (x-1)dx$

$\log_e(y+1) = \frac{x^2}{2} - x + c \quad [\because \log_e 1 = 0]$

$\log_e 1 = 0 + C \Rightarrow C = 0$

$y(1) \Rightarrow \log_e(y+1) = \frac{1}{2} - 1$

$y + 1 = e^{-1/2}$

$y(1) = -1 + e^{-1/2}$

-----------------------------------------------------------------------------------------------

# Question91

**Let the curve y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 2(x+1)$.**
**If the numerical value of area bounded by the curve y = y(x) and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of y(1) is equal to.........**
**[16 Mar 2021 Shift 1]**

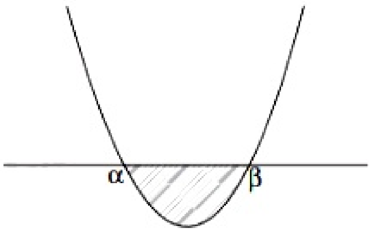
**Answer: 2**

## Solution:

We have, $dy/dx = 2(x+1)$

$y = \int 2(x+1)dx = 2(x^2/2 + x) + C$

$y = x^2 + 2x + C$...(i)




Now, $\int_\alpha^\beta y\,dx = \frac{4\sqrt{8}}{3}$

$\int_\alpha^\beta (x^2 + 2x + c)dx = \frac{4\sqrt{8}}{3}$

$\Rightarrow \left[\frac{x^3}{3} + x^2 + cx\right]_\alpha^\beta = \frac{4\sqrt{8}}{3}$

$\Rightarrow \left(\frac{\beta^3 - \alpha^3}{3}\right) + (\beta^2 - \alpha^2) + c(\beta - \alpha) = \frac{4\sqrt{8}}{3}$....(ii)

From Eq. (i),

$\alpha + \beta = -2$

$\alpha\beta = c$

$\therefore\ \beta - \alpha = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

$= \sqrt{4 - 4c} = 2\sqrt{1-c}$

From Eq. (ii), $\frac{1}{3}(\beta-\alpha)(\alpha^2+\beta^2+\alpha\beta)+(\beta-\alpha)(\beta+\alpha)+c(\beta-\alpha) = \frac{4\sqrt{8}}{3}$

$\Rightarrow \frac{1}{3}(2\sqrt{1-c})(4-c)+(2\sqrt{1-c})(-2+c) = \frac{4\sqrt{8}}{3}$

$\Rightarrow (2\sqrt{1-c})[4-c+(-6+3c)] = 4\sqrt{8}$

$\Rightarrow (2\sqrt{1-c})[2(c-1)] = 4\sqrt{8}$

$\Rightarrow (1-c)^{3/2} = -\sqrt{8}$

$\Rightarrow (1-c)^3 = 8$

$\Rightarrow 1-c = 2$

$\therefore c = -1$

Now, $y = x^2+2x-1$

$\therefore y(1) = 1^2+2\cdot 1-1 = 2$

-----------------------------------------------------------------------------------------------------

# Question92

**Let $C_1$ be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx} = y^2-x^2, x>0$. Let the curve $C_2$ be the solution of $\frac{2xy}{x^2-y^2} = \frac{dy}{dx}$. If both the curves pass through (1, 1), then the area enclosed by the curves $C_1$ and $C_2$ is equal to**

**[16 Mar 2021 Shift 2]**

**Options:**

A. $\pi-1$

B. $\frac{\pi}{2}-1$

C. $\pi+1$

D. $\frac{\pi}{4}+1$

**Answer: B**

## Solution:

**Solution:**

Given, $2xy\frac{dy}{dx} = y^2-x^2, x>0$

$\frac{dy}{dx} = \left(\frac{y^2-x^2}{2xy}\right), x>0$

Let $y = vx$

$\frac{dy}{dx} = v+x\frac{dv}{dx}$

$v+x\frac{dv}{dx} = \frac{v^2x^2-x^2}{2x\cdot vx} = \frac{v^2-1}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{v^2-1}{2v}-v \Rightarrow x\frac{dv}{dx} = \frac{v^2-1-2v^2}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{-(v^2+1)}{2v} \Rightarrow \int -\left(\frac{2v}{v^2+1}\right)dv = \int\frac{dx}{x}$

$\Rightarrow -\log|v^2+1| = \log x+c$

$\Rightarrow \log|x|+\log|v^2+1| = c$

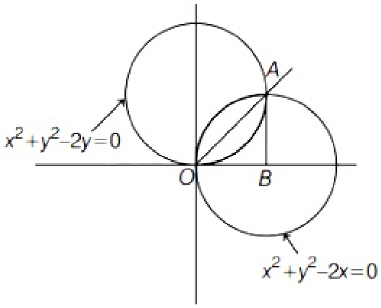
$\Rightarrow \log|(v^2+1)x| = c \Rightarrow \left(\frac{y^2}{x^2}+1\right)x = c$

$\Rightarrow \frac{(y^2+x^2)}{x} = c \Rightarrow y^2+x^2 = cx$

Similarly, for second curve, $x^2+y^2 = cy$

Both passes through (1, 1),

$C_1 \Rightarrow 1 + 1 = C_1$
$\Rightarrow\ C_1 = 2$
$C_1 \Rightarrow x^2 + y^2 = 2x$
$C_2 \Rightarrow x^2 + y^2 = C_2 y$
$\Rightarrow\ 1 + 1 = C_2 \Rightarrow C_2 = 2$
$C_2 \Rightarrow x^2 + y^2 = 2y$




$\therefore$ Required area $= 2\left[\dfrac{\pi\cdot 1^2}{4} - \dfrac{1}{2}\cdot 1\cdot 1\right]$
$= 2\left[\left(\dfrac{\pi-2}{4}\right)\right] = \dfrac{\pi}{2} - 1$

----------------------------------------------------------------------------------------------------

# Question93

## Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = (y+1)\left[(y+1)e^{x^2/2} - x\right]$ 0 < x < 2. 1, with y(2) = 0. Then the value of $\frac{dy}{dx}$ at x = 1 is equal to [18 Mar 2021 Shift 2]

**Options:**

A. $\dfrac{-e^{3/2}}{(e^2+1)^2}$

B. $-\dfrac{2e^2}{(1+e^2)^2}$

C. $\dfrac{e^{5/2}}{(1+e^2)^2}$

D. $\dfrac{5e^{1/2}}{(e^2+1)^2}$

**Answer: A**

## Solution:

**Solution:**
Given, $\dfrac{dy}{dx} = (y+1)\left[(y+1)e^{x^2/2} - x\right]$...(i)
$\Rightarrow\ \dfrac{dy}{dx} = (y+1)^2 e^{x^2/2} - x(y+1)$
$\Rightarrow\ \dfrac{dy}{dx} + x(y+1) = (y+1)^2 e^{x^2/2}$
$\Rightarrow\ \dfrac{1}{(y+1)^2}\cdot\dfrac{dy}{dx} + \dfrac{1}{y+1}\cdot x = e^{x^2/2}$...(ii)
Let $\dfrac{1}{y+1} = t \Rightarrow \dfrac{-1}{(y+1)^2}\cdot\dfrac{dy}{dx} = \dfrac{dt}{dx}$

$\Rightarrow \; -\dfrac{dt}{dx} + tx = e^{x^2/2}$

$\Rightarrow \; \dfrac{dt}{dx} + (-x)t = -e^{x^2/2}$

which is linear differential equation

$IF = e^{\int -x dx} = e^{-x^2/2}$

Now, solution of differential equation is,

$t \cdot e^{-x^2/2} = -\int e^{-x^2/2} \cdot e^{x^2/2} dx$

$\Rightarrow \left(\dfrac{1}{y+1}\right) e^{-x^2/2} = -x + c$, where $c$ is constant of integration....(iii)

Given, $y(2) = 0$ i.e. when $x = 2$, then $y = 0$.

From Eq. (iii),

$e^{-2} = -2 + c \Rightarrow c = 2 + e^{-2}$

Now, at $x = 1$

$\dfrac{1}{y+1} e^{-1/2} = -1 + e^{-2} + 2$

$\Rightarrow \; (y+1) = \dfrac{e^{-1/2}}{1 + e^{-2}}$

Now, putting the value of $(y + 1)$ in Eq. (i), we get

$y'(1) = \dfrac{e^{-1/2}}{1+e^{-2}} \left( \dfrac{e^{-1/2}}{1+e^{-2}} \cdot e^{1/2} - 1 \right)$

$= \dfrac{e^{-1/2}}{1+e^{-2}} \left( \dfrac{1}{1+e^{-2}} - 1 \right)$

$= \dfrac{e^{-1/2}}{1+e^{-2}} \left( \dfrac{1 - 1 - e^{-2}}{1+e^{-2}} \right)$

$= \dfrac{-e^{-5/2}}{(1+e^{-2})^2} = \dfrac{-e^{-5/2}}{\dfrac{(e^2+1)^2}{e^4}} = \dfrac{-e^{3/2}}{(e^2+1)^2}$

------------------------------------------------------------------------------------------------

# Question94

## Let $y = y(x)$ be the solution of the differential equation $\cos x(3\sin x + \cos x + 3)\, dy = [1 + y\sin x(3\sin x + \cos x + 3)]dx$ $0 \le x \le \frac{\pi}{2}$, $y(0) = 0$. Then, $y\left(\frac{\pi}{3}\right)$ is equal to

## [17 Mar 2021 Shift 2]

**Options:**

A. $2\log_e\left(\dfrac{2\sqrt{3}+9}{6}\right)$

B. $2\log_e\left(\dfrac{2\sqrt{3}+10}{11}\right)$

C. $2\log_e\left(\dfrac{\sqrt{3}+7}{2}\right)$

D. $2\log_e\left(\dfrac{3\sqrt{3}-8}{4}\right)$

**Answer: B**

## Solution:

**Solution:**

Given, $\cos x(3\sin x + \cos x + 3)\, dy$

$= [1 + y\sin x(3\sin x + \cos x + 3)]dx$

$\Rightarrow\ \cos x\, dy = \left(\frac{1}{3\sin x + \cos x + 3} + y\sin x\right)dx$

$\Rightarrow\ \cos x \cdot \frac{dy}{dx} = y\sin x + \frac{1}{3\sin x + \cos x + 3}$

$\Rightarrow\ \frac{dy}{dx} = y\frac{\sin x}{\cos x} + \frac{1}{\cos x(3\sin x + \cos x + 3)}$

$\Rightarrow \frac{dy}{dx} - (\tan x)y = \frac{1}{\cos x(3\sin x + \cos x + 3)}$....(i)

Which is linear differential equation.

$\therefore$ Integrating factor (I F ) = $e^{\int(-\tan x)dx}$

$= e^{\log|\cos x|} = |\cos X|$

$\because |\cos x| > 0,\ \forall x \in [0, \pi/2)$

$\therefore |\cos x| = \cos x$

Hence, solution of Eq. (i) is

$y(\cos x) = \int(\cos x)\cdot\frac{1}{\cos x(3\sin x + \cos x + 3)}dx$

$\Rightarrow\ y\cos x = \int\frac{1}{3\sin x + \cos x + 3}dx$

$\Rightarrow y\cos x = \int\frac{\sec^2\frac{x}{2}dx}{2\tan^2\frac{x}{2} + 6\tan\frac{x}{2} + 4}dx = \int\frac{\sec^2\frac{x}{2}dx}{2\left(\tan^2\frac{x}{2} + 3\tan\frac{x}{2} + 2\right)}dx$

Putting $\tan\frac{x}{2} = z$

$\Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dz$

$\therefore y\cos x = \int\frac{dz}{z^2 + 3z + 2} = \int\frac{dz}{(z+1)(z+2)}$

$= \int\frac{1}{z+1}dz - \int\frac{1}{z+2}dz = \log(z+1) - \log(z+2) + c$

$\Rightarrow y\cos x = \log\left|\frac{z+1}{z+2}\right| + c = \log\left|\frac{\tan\frac{x}{2} + 1}{\tan\frac{x}{2} + 2}\right| + c$...(i)

Since, y(0) = 0

$\therefore\ C = -\log\left(\frac{1}{2}\right) = \log 2$

From Eq. (i), $y\cos x = \log\left|\frac{\tan\frac{x}{2} + 1}{\tan\frac{x}{2} + 2}\right| + \log 2$

$\therefore\ y\left(\frac{\pi}{3}\right) = 2\left[\log\left|\frac{\frac{1}{\sqrt{3}} + 1}{\frac{1}{\sqrt{3}} + 2}\right| + \log 2\right] = 2\log\left|2\left(\frac{\sqrt{3}+1}{2\sqrt{3}+1}\right)\right|$

$= 2\log\left|2\left(\frac{\sqrt{3}+1}{2\sqrt{3}+1}\times\frac{2\sqrt{3}-1}{2\sqrt{3}-1}\right)\right| = 2\log\left|\frac{2\sqrt{3}+10}{11}\right|$

----------------------------------------------------------------------------------------------------

# Question95

**If the curve y = y(x) is the solution of the differential equation $2(x^2 + x^{5/4})dy - y(x + x^{1/4})dx = 2x^{9/4}dx$, x > 0 which passes through the point $\left(1, 1 - \frac{4}{3}\log_e 2\right)$, then the value of y(16) is equal to**

**[17 Mar 2021 Shift 2]**

**Options:**

A. $4\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

B. $\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

C. $4\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$

D. $\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$

**Answer: C**

## Solution:

**Solution:**

Given, $2(x^2+x^{5/4})dy - y\cdot(x+x^{1/4})dx =$
$2x^{9/4}dx$, where $x>0$

After rearranging, we get

$$\frac{dy}{dx}-\frac{y}{2x}=\frac{x^{9/4}}{x^{5/4}(x^{3/4}+1)}$$

This is of the form $\frac{dy}{dx}+Py=Q$, where P and Q are constants or function of x.

$\therefore$ Integrating factor (IF) $=e^{\int P\,dx}$

$$=e^{\int -\frac{1}{2x}dx}=e^{\frac{-1}{2}\int x\,dx}$$

$$=e^{-\frac{1}{2}\log x}=e^{\log(x)^{-1/2}}=\frac{1}{x^{1/2}}$$

Its solution is

$y\times(IF)=\int Q\times(IF)dx$

$$\Rightarrow\ y\times\frac{1}{x^{1/2}}=\int\frac{x^{9/4}}{x^{5/4}(x^{3/4}+1)}\times x^{-1/2}dx=\int\frac{(x)^{\frac{9}{4}-\frac{5}{4}-\frac{1}{2}}}{(x^{3/4}+1)}dx$$

$$\Rightarrow\ y\times\frac{1}{x^{1/2}}=\int\frac{x^{1/2}dx}{(x^{3/4}+1)}\ldots\text{(i)}$$

Putting $x=z^4$

$\Rightarrow\ dx=4z^3\cdot dz$

RHS of Eq. (i) becomes,

$$\int\frac{z^2\cdot 4z^3}{(z^3+1)}\cdot dz=4\int\frac{z^2(z^3+1-1)}{(z^3+1)}dz$$

$$=4\left[\int\frac{z^2(z^3+1)}{(z^3+1)}dz-\int\frac{z^2}{(z^3+1)}dz\right]$$

$$=4\left[\frac{z^3}{3}-\frac{1}{3}\cdot\int\frac{3z^2}{z^3+1}dz\right]$$

$$=4\left[\frac{z^3}{3}-\frac{1}{3}\cdot\log|z^3+1|\right]$$

$$\left(\because\int\frac{f'(x)}{f(x)}dx=\log|f(x)|+C\right)$$

$$=\frac{4z^3}{3}-\frac{4}{3}\log|z^3+1|+C$$

$$=\frac{4x^{3/4}}{3}-\frac{4}{3}\log|x^{3/4}+1|+C\quad\left(\begin{matrix}\because x=z^4\\ \therefore x^{3/4}=z^3\end{matrix}\right)$$

$\therefore$ Eq. (i) becomes,

$$y\times x^{-1/2}=\frac{4x^{3/4}}{3}-\frac{4}{3}\log|x^{3/4}+1|+C$$

Since, this passes through $\left(1,1-\frac{4}{3}\log_e 2\right)$

Then,

$$\left(1-\frac{4}{3}\log_e 2\right)\times 1=\frac{4\times 1}{3}-\frac{4}{3}\log\left|1+1\right|+C$$

$$\Rightarrow C=1-\frac{4}{3}\Rightarrow C=\frac{-1}{3}$$

Hence,

$$y=\frac{4}{3}x^{5/4}-\frac{4}{3}\sqrt{x}\log x^{3/4}+1\Big|-\frac{\sqrt{x}}{3}$$

$\because x>0$

$\therefore x^{3/4}>0\Rightarrow x^{3/4}+1>0$ i.e., $x^{3/4}+1\,|\ =x^{3/4}+1$

$\therefore\ y = \frac{4}{3}x^{5/4} - \frac{4}{3}\sqrt{x}\log(x^{3/4}+1) - \frac{\sqrt{x}}{3}$

Now, putting $x = 16$, we get

$y(16) = \frac{4}{3} \times 32 - \frac{4}{3} \times 4\log 9 - \frac{4}{3}$

$= \frac{124}{3} - \frac{32}{3}\log 3$

$= 4\left(\frac{31}{3} - \frac{8}{3}\log 3\right)$

---------------------------------------------------------------------------------------------------

# Question96

## If $y = y(x)$ is the solution of the differential equation, $\frac{dy}{dx} + 2y\tan x = \sin x$, $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of the function $y(x)$ over R is equal to [16 Mar 2021 Shift 1]

**Options:**

A. 8

B. $\frac{1}{2}$

C. $-\frac{15}{4}$

D. $\frac{1}{8}$

**Answer: D**

## Solution:

**Solution:**

Given, $\frac{dy}{dx} + 2y\tan x = \sin x$

This differential equation is of the form $\frac{dy}{dx} + Py = Q$ where P and Q is function of x.

which is a linear differential equation.

Here, $P = 2\tan x$ and $Q = \sin x$

The integrating factor of linear differential equation is $e^{\int P dx}$.

Here, $e^{\int 2\tan x\, dx} = e^{\int \frac{2\sin x}{\cos x}dx} = e^{-2\log(\cos x)} = \sec^2 x$

Now, $\frac{dy}{dx} + 2y\tan x = \sin x$

On multiplying $\sec^2 \times$ both the sides,

$\sec^2 x\frac{dy}{dx} + 2y\sec^2 x\tan x = \sin x\sec^2 x$

$\Rightarrow\ \frac{d}{dx}(y\sec^2 x) = \sin x\sec^2 x$

$\Rightarrow\ y\sec^2 x = \int \sin x\sec^2 x\, dx$

$\Rightarrow\ y\sec^2 x = \int \frac{\sin x}{\cos^2 x}dx$

Let $\cos x = t$

$(-\sin x)dx = dt$

$\int \frac{-dt}{t^2} = \frac{1}{t} + c$

So, $y\sec^2 x = \frac{1}{\cos x} + c$

$y\sec^2 x = \sec x + c$

Now, $x = \pi / 3, y = 0$
$0 = 2 + c$
$\Rightarrow \ c = -2$
So, $y\sec^2 x = \sec x - 2$
$y = \cos^2 x\left(\frac{1}{\cos x} - 2\right) = -2\cos^2 x + \cos x$
$\Rightarrow \ y = -2\left(\cos^2 x - \frac{\cos x}{2}\right)$
$\Rightarrow \ y = -2\left[\left(\cos^2 x - \frac{\cos x}{2} + \frac{1}{16}\right) - \frac{1}{16}\right]$
$\Rightarrow \ y = -2\left[(\cos x - 1 / 4)^2 - \frac{1}{16}\right]$
$\Rightarrow \ y = \frac{1}{8} - 2(\cos x - 1 / 4)^2$
So, $y_{min} = \frac{1}{8}$

----------------------------------------------------------------------------------------------------

# Question97

## If y = y(x) is the solution of the differential equation $\frac{dy}{dx} + (\tan x)y = \sin x,\ 0 \le x \le \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ is equal to [16 Mar 2021 Shift 2]

**Options:**

A. $\frac{1}{4}\log_e 2$

B. $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

C. $\log_e 2$

D. $\frac{1}{2}\log_e 2$

**Answer: B**

## Solution:

**Solution:**
Given, $\frac{dy}{dx} + (\tan x)y = \sin x,\ x \in \left[0, \frac{\pi}{3}\right]$
which is a linear differential equation of the form of $\frac{dy}{dx} + Py = Q$
Here, $P = \tan x$
$\therefore \quad IF = e^{\int P dx}$
$\Rightarrow e^{\int \tan x\, dx} = e^{\log(\sec x)} = \sec x$
Multiplying by $\sec x$ on both sides
$\frac{dy}{dx} + (\tan x)y = \sin x$
$\sec x \frac{dy}{dx} + (\tan x \sec x)y = \sin x \sec x$
$\Rightarrow \quad \frac{d}{dx}(y \sec x) = \tan x \Rightarrow y \sec x = \int \tan x\, dx$
$\Rightarrow \quad y \sec x = \log(\sec x) + c$
$y = \cos x \log(\sec x) + c \cdot \cos x$
$y(0) = 0$
$\Rightarrow \ 0 = 1 \cdot 0 + c \cdot 1 \Rightarrow c = 0$
$\therefore \ y = \cos x \cdot \log(\sec x)$

$\Rightarrow\ y\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)\cdot\log\left(\sec\frac{\pi}{4}\right)$

$= \frac{1}{\sqrt{2}}\log(\sqrt{2})$

$= \frac{1}{2\sqrt{2}}\log 2$

---------------------------------------------------------------------------------------------------

# Question98

**In a triangle ABC, if $\left|\overrightarrow{BC}\right| = 8, \left|\overrightarrow{CA}\right| = 7, \left|\overrightarrow{AB}\right| = 10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to**
**[18 Mar 2021 Shift 2]**

**Options:**

A. $\frac{25}{4}$

B. $\frac{85}{14}$

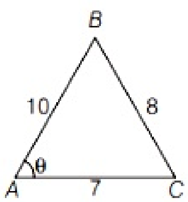
C. $\frac{127}{20}$

D. $\frac{115}{16}$

**Answer: B**

## Solution:

Projection of AB or AC = $AB\cos theta = 10\,cdot\cos theta$




$= 10\,.\left(\frac{10^2+7^2-8^2}{2\times7\times10}\right) = \frac{85}{14}\left(\text{using } \cos\theta = \frac{c^2+b^2-a^2}{2bc}\right)$

---------------------------------------------------------------------------------------------------

# Question99

**Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 1 + xe^{y-x}, -\sqrt{2} < x < \sqrt{2}, y(0) = 0$then, the minimum value of $y(x), x \in (-\sqrt{2}, \sqrt{2})$ is equal to:**
**[25 Jul 2021 Shift 1]**

**Options:**

A. $(2-\sqrt{3}) - \log_e 2$

B. $(2+\sqrt{3}) + \log_e 2$

C. $(1+\sqrt{3})-\log_e(\sqrt{3}-1)$

D. $(1-\sqrt{3})-\log_e(\sqrt{3}-1)$

**Answer: D**

## Solution:

**Solution:**

$\frac{dy-dx}{e^{y-x}}=xdx$

$\Rightarrow\frac{dy-dx}{e^{y-x}}=xdx$

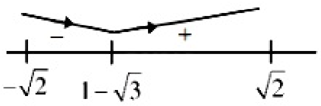
$\Rightarrow -e^{x-y}=\frac{x^2}{2}+c$

At $x=0, y=0\Rightarrow c=-1$

$\Rightarrow e^{x-y}=\frac{2-x^2}{2}$

$\Rightarrow y=x-ln\left(\frac{2-x^2}{2}\right)$

$\Rightarrow\frac{dy}{dx}=1+\frac{2x}{2-x^2}=\frac{2+2x-x^2}{2-x^2}$




So minimum value occurs at $x=1-\sqrt{3}$

$y(1-\sqrt{3})=(1-\sqrt{3})-\ln\left(\frac{2-(4-2\sqrt{3})}{2}\right)$

$=(1-\sqrt{3})-\ln(\sqrt{3}-1)$

-----------------------------------------------------------------------------------------------

# Question100

**Let $y=y(x)$ be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right)=3x+4y$, with $y(0)=0$ If $y\left(-\frac{2}{3}\log_e 2\right)=\alpha\log_e 2$, then the value of $\alpha$ is equal to: [27 Jul 2021 Shift 1]**

**Options:**

A. $-\frac{1}{4}$

B. $\frac{1}{4}$

C. 2

D. $-\frac{1}{2}$

**Answer: A**

## Solution:

$\frac{dy}{dx} = e^{3x}.e^{4y} \Rightarrow \int e^{-4y} dy = \int e^{3x} dx$

$\frac{e^{-4y}}{-4} = e^{3x}3 + C \Rightarrow -\frac{1}{4} - \frac{1}{3} = C \Rightarrow C = -\frac{7}{12}$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12} \Rightarrow e^{-4y} = \frac{4e^{3x} - 7}{-3}$

$e^{4y} = \frac{3}{7 - 4e^{3x}} \Rightarrow 4y = \ln\left(\frac{3}{7 - 4e^{3x}}\right)$

$4y = \ln\left(\frac{3}{6}\right)$ when $x = -\frac{2}{3}\ln 2$

$y = \frac{1}{4}\ln\left(\frac{1}{2}\right) = -\frac{1}{4}\ln 2$

---------------------------------------------------------------------------------------------------

# Question101

**If $y = y(x), y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation $\sec y \frac{dy}{dx} - \sin(x + y) - \sin(x - y) = 0$, with $y(0) = 0$ then $5y'\left(\frac{\pi}{2}\right)$ is equal to __________.**
**[27 Jul 2021 Shift 1]**

**Answer: 2**

## Solution:

**Solution:**

$\sec y \frac{dy}{dx} = 2\sin x \cos y$

$\sec^2 y\, dy = 2\sin x\, dx$

$\tan y = -2\cos x + c$

$c = 2$

$\tan y = -2\cos x + 2 \Rightarrow$ at $x = \frac{\pi}{2}$

$\tan y = 2$

$\sec^2 y \frac{dy}{dx} = 2\sin x$

$5\frac{dy}{dx} = 2$

---------------------------------------------------------------------------------------------------

# Question102

**Let $y = y(x)$ be the solution of the differential equation $(x - x^3)dy = (y + yx^2 - 3x^4)dx$, $x > 2$ If $y(3) = 3$, then $y(4)$ is equal to :**
**[27 Jul 2021 Shift 2]**

**Options:**

A. 4

B. 12

C. 8

D. 16

**Answer: B**

## Solution:

**Solution:**

$(x-x^3)dy=(y+yx^2-3x^4)dx$

$\Rightarrow xdy-ydx=(yx^2-3x^4)dx+x^3dy$

$\Rightarrow \frac{xdy-ydx}{x^2}=(ydx+xdy)-3x^2dx$

$\Rightarrow d\left(\frac{y}{x}\right)=d(xy)-d(x^3)$

Integrate $\Rightarrow \frac{y}{x}=xy-x^3+c$

given $f(3)=3$

$\Rightarrow \frac{3}{3}=3\times 3-3^3+c$

$\Rightarrow c=19$

$\therefore \frac{y}{x}=xy-x^3+19$

at $x=4$, $\frac{y}{4}=4y-64+19$

$15y=4\times 45$

$\Rightarrow y=12$

------------------------------------------------------------------------------------------------

# Question103

**Let $y=y(x)$ be the solution of the differential equation $dy=e^{\alpha x+y}dx$; $\alpha\in N$ . If $y(\log_e 2)=\log_e 2$ and $y(0)=\log_e\left(\frac{1}{2}\right)$, then the value of $\alpha$ is equal to ____.**
**[27 Jul 2021 Shift 2]**

**Answer: 2**

## Solution:

$\int e^{-y}dy=\int e^{ex}dx$

$\Rightarrow e^{-y}=\frac{e^{\alpha x}}{\alpha}+c$ ........(i)

Put $(x,y)=(l\,n2,\ \ln 2)$

$\frac{-1}{2}=\frac{2^{\alpha}}{\alpha}+C$ ........(ii)

Put $(x,y)\equiv(0,-\ln 2)$ in (i)

$-2=\frac{1}{\alpha}+C$ ......(iii)

(ii) - (iii)

$\frac{2^{\alpha}-1}{\alpha}=\frac{3}{2}$

$\Rightarrow \alpha=2$ (as $\alpha\in N$ )

# Question104

**Let $y = y(x)$ be the solution of the differential equation $x\,dy = (y + x^3 \cos x)dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to:**
**[25 Jul 2021 Shift 2]**

**Options:**

A. $\frac{\pi^2}{4} + \frac{\pi}{2}$

B. $\frac{\pi^2}{2} + \frac{\pi}{4}$

C. $\frac{\pi^2}{2} - \frac{\pi}{4}$

D. $\frac{\pi^2}{4} - \frac{\pi}{2}$

**Answer: A**

## Solution:

**Solution:**
$x\,dy = (y + x^3 \cos x)dx$
$x\,dy = y\,dx + x^3 \cos x\,dx$
$\frac{x\,dy - y\,dx}{x^2} = \frac{x^3 \cos x\,dx}{x^2}$
$\frac{d}{dx}\left(\frac{y}{x}\right) = \int x \cos x\,dx$
$\Rightarrow \frac{y}{x} = x \sin x - \int 1 \,.\, \sin x\,dx$
$\frac{y}{x} = x \sin x + \cos x + C$
$\Rightarrow 0 = -1 + C \Rightarrow C = 1,\ x = \pi,\ y = 0$
so $\frac{y}{x} = x \sin x + \cos x + 1$
$y = x^2 \sin x + x \cos x + x \quad x = \frac{\pi}{2}$
$y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$

---------------------------------------------------------------------------------------------------

# Question105

**Let a curve $y = f(x)$ pass through the point $(2, (\log_e 2)^2)$ and have slope $\frac{2y}{x\log_c x}$ for all positive real value of x. Then the value of $f(e)$ is equal to ________.**
**[25 Jul 2021 Shift 2]**

**Answer: 1**

## Solution:

**Solution:**

$y' = \frac{2y}{x \ln x}$

$\Rightarrow \frac{dy}{y} = \frac{2dx}{x \ln x}$

$\Rightarrow \ln|y| = 2\ln|\ln x| + C$

put $x = 2$, $y = (\ln 2)^2$

$\Rightarrow c = 0$

$\Rightarrow y = (\ln x)^2$

$\Rightarrow f(e) = 1$

---------------------------------------------------------------------------------------------------

# Question106

**Let y = y(x) be the solution of the differential equation $\left((x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right)dx = (x+2)dy, y(1) = 1$. If the domain of $y = y(x)$ is an open interval $(\alpha, \beta)$, then $|\alpha + \beta|$ is equal to _______.**
**[22 Jul 2021 Shift 2]**

**Answer: 4**

## Solution:

**Solution:**

$y + 1 = Y \Rightarrow dy = dY$

$x + 2 = X \Rightarrow dx = dX$

$\Rightarrow \left(Xe^{\frac{Y}{X}} + Y\right)dX = XdY$

$\Rightarrow XdY - YdX = Xe^{Y/x}dX$

$\Rightarrow d\left(\frac{Y}{X}\right)e^{-\frac{Y}{x}} = \frac{dX}{X}$

$-e^{-Y/x} = l|X| + c$

$(3, 2) \rightarrow -e^{-\frac{2}{3}} = l|3| + c$

$-e^{-\frac{Y}{x}} = \ln|X| - e^{-\frac{2}{3}} - \ln 3$

$e^{-\frac{Y}{x}} = e^{2/3} + \ln 3 - \ln|X| > 0$

$\ln|X| < (e^{2/3} + \ln 3)$

Let $\lambda = (e^{2/3} + \ln 3)$

$|x + 2| < e^{\lambda}$

$-e^{\lambda} < x + 2 < e^{\lambda}$

$-e^{\lambda} - 2 < x < e^{\lambda} - 2$

$\alpha + \beta = -4 \Rightarrow |\alpha + \beta| = 4$

Although $x = -2$ should be excluded from domain but according to the given problem it will the most appropriate solution.

---------------------------------------------------------------------------------------------------

# Question107

**Let y = y(x) be the solution of the differential equation $x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx - 1 \le x \le 1$, $y\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Then the area of the region bounded by the curves x = 0, $x = \frac{1}{\sqrt{2}}$ and y = y(x) in the upper half plane is:**
**[20 Jul 2021 Shift 1]**

**Options:**

A. $\frac{1}{8}(\pi - 1)$

B. $\frac{1}{12}(\pi - 3)$

C. $\frac{1}{4}(\pi - 2)$

D. $\frac{1}{6}(\pi - 1)$

**Answer: A**

## Solution:

**Solution:**
We have

$$\frac{dy}{dx} = \frac{x\left(\frac{y}{x}.\tan\frac{y}{x} - 1\right)}{x\tan\frac{y}{x}}$$

$\therefore \frac{dy}{dx} = \frac{y}{x} - \cot\left(\frac{y}{x}\right)$

Put $\frac{y}{x} = v$

$\Rightarrow y = vn$

$\therefore \frac{dy}{dx} = v + \frac{ndv}{dx}$

Now, we get $v + n\frac{dv}{dx} = v - \cot(v)$

$\Rightarrow \int(\tan)dv = -\int\frac{dx}{x}$

$\therefore \ln\left|\sec\left(\frac{y}{x}\right)\right| = -\ln|x| + c$

As $\left(\frac{1}{2}\right) = \left(\frac{y}{x}\right) \Rightarrow C = 0$

$\therefore \sec\left(\frac{y}{x}\right) = \frac{1}{x}$

$\Rightarrow \cos\left(\frac{y}{x}\right) = x$

$\therefore y = x\cos^{-1}(x)$

So, required bounded area

$= \int_0^{1/\sqrt{2}} x(\cos^{-1}x)dx = \left(\frac{\pi - 1}{8}\right)$

∴ option (1) is correct.

----------------------------------------------------------------------------------------------------

# Question108

**Let $y = y(x)$ be the solution of the differential equation**
**$e^x \sqrt{1-y^2} dx + \left(\frac{y}{x}\right) dy = 0$, $y(1) = -1$.Then the value of $(y(3))^2$ is equal to:**
**[20 Jul 2021 Shift 1]**

**Options:**

A. $1 - 4e^3$

B. $1 - 4e^6$

C. $1 + 4e^3$

D. $1 + 4e^6$

**Answer: B**

## Solution:

**Solution:**
$e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0$
$\Rightarrow e^x \sqrt{1-y^2} dx + \frac{-y}{x} dy$
$\Rightarrow \int \frac{-y}{\sqrt{1-y^2}} dy = \int e^x x dx$
$\Rightarrow \sqrt{1-y^2} = e^x(x-1) + c$
Given : At $x = 1$, $y = -1$
$\Rightarrow 0 = 0 + c \Rightarrow c = 0$
$\therefore \sqrt{1-y^2} = e^x(x-1)$
At $x = 3$ $1 - y^2 = (e^3 2)^2 \Rightarrow y^2 = 1 - 4e^6$

-----------------------------------------------------------------------------------------------------

# Question109

**Let a curve $y = y(x)$ be given by the solution of the differential equation**
**$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right) dx = \sqrt{e^{2x} - 1} dy$**
**If it intersects y-axis at $y = -1$, and the intersection point of the curve with x-axis is $(\alpha, 0)$, then $e^\alpha$ is equal to ________.**
**[20 Jul 2021 Shift 2]**

**Answer: 2**

## Solution:

**Solution:**
$\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right) dx = \sqrt{e^{2x} - 1} dy$
Put $\cos^{-1}(e^{-x})\theta$, $\theta \in [0, \pi]$
$\cos\theta = e^{-x} \Rightarrow 2\cos^2\theta 2 - 1 = e^{-x}$

$\cos\frac{\theta}{2} = \sqrt{\frac{e^{-x}+1}{2}} = \sqrt{\frac{e^{x}+1}{2c^{x}}}$

$\sqrt{\frac{e^{x}+1}{2c^{x}}}dx = \sqrt{e^{2x}-1}dy$

$\frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{e^{x}}\sqrt{e^{x}-1}} = \int dy$

Put $e^{x} = t$, $\frac{dt}{dx} = e^{x}$

$\frac{1}{\sqrt{2}}\int\frac{dt}{e^{x}\sqrt{e^{x}}\sqrt{e^{x}-1}} = \int dy$

$\int\frac{dt}{t\sqrt{t^{2}-t}} = \sqrt{2}y$

Put $t = \frac{1}{z}$, $\frac{dt}{dz} = -\frac{1}{z^{2}}$

$\int\frac{-\frac{dz}{z^{2}}}{\frac{1}{z}\sqrt{\frac{1}{z^{2}}-\frac{1}{z}}} = \frac{\sqrt{2}}{y}$

$-\int\frac{dz}{\sqrt{1-z}} = \sqrt{2}y$

$\frac{-2(1-z)^{1/2}}{-1} = \sqrt{2}y + c$

$2\left(1-\frac{1}{t}\right)^{1/2} = \sqrt{2}y + c$

$2(1-e^{-x})^{1/2} = \sqrt{2y} + c \xrightarrow{(0,-1)} \Rightarrow c = \sqrt{2}$

$2(1-e^{-x})^{1/2} = \sqrt{2}(y+1)$, passes through $(\alpha, 0)$

$2(1-e^{-\alpha})^{1/2} = \sqrt{2}$

$\sqrt{1-e^{-\alpha}} = \frac{1}{\sqrt{2}} \Rightarrow 1-e^{-\alpha} = \frac{1}{2}$

$e^{-\alpha} = \frac{1}{2} \Rightarrow e^{\alpha} = 2$

---------------------------------------------------------------------------------------------------------

# Question110

**Let $y = y(x)$ be solution of the following differential equation$e^{y}\frac{dy}{dx} - 2e^{y}\sin x + \sin x\cos^{2}x = 0$, $y\left(\frac{\pi}{2}\right) = 0$ If $y(0) = \log_{e}(\alpha + \beta e^{-2})$, then $4(\alpha+\beta)$ is equal to ________.**
**[25 Jul 2021 Shift 1]**

**Answer: 4**

## Solution:

**Solution:**

Let $e^{y} = t$

$\Rightarrow\frac{dt}{dx} - (2\sin x)t = -\sin x\cos^{2}x$

I.F. $= e^{2\cos x}$

$\Rightarrow t.e^{2\cos x} = \int e^{2\cos x}.(-\sin x\cos^{2}x)dx$

$\Rightarrow e^{y}.e^{2\cos x} = \int e^{2z}.z^{2}dz$, $z = e^{2\cos x}$

at $x = \frac{\pi}{2}$, $y = 0 \Rightarrow C = \frac{3}{4}$

$\Rightarrow e^{y}=\frac{1}{2}\cos^{2}x-\frac{1}{2}\cos x+\frac{1}{4}+\frac{3}{4}\cdot e^{-2\cos x}$

$\Rightarrow y=\log\left[\frac{\cos^{2}x}{2}-\frac{\cos x}{2}+\frac{1}{4}+\frac{3}{4}e^{-2\cos x}\right]$

Put x = 0

$\Rightarrow y=\log\left[\frac{1}{4}+\frac{3}{4}e^{-2}\right]\ \Rightarrow\alpha=\frac{1}{4},\beta=\frac{3}{4}$

-----------------------------------------------------------------------------------------------

# Question111

**Let y = y(x) be the solution of the differential equation $\mathrm{cosec}^{2}x\,dy+2dx=(1+y\cos 2x)\,\mathrm{cosec}^{2}x\,dx$, with $y\left(\frac{\pi}{4}\right)=0$. Then, the value of $(y(0)+1)^{2}$ is equal to:**
**[22 Jul 2021 Shift 2]**

**Options:**

A. $e^{1/2}$

B. $e^{-1/2}$

C. $e^{-1}$

D. e

**Answer: C**

## Solution:

**Solution:**

$\frac{dy}{dx}+2\sin^{2}x=1+y\cos 2x$

$\Rightarrow\frac{dy}{dx}+(-\cos 2x)y=\cos 2x$

I.F. $=e^{\int-\cos 2x\,dx}=e^{-\frac{\sin 2x}{2}}$

Solution of D.E.

$y\left(e^{-\frac{\sin 2x}{2}}\right)=\int(\cos 2x)\left(e^{-\frac{\sin 2x}{2}}\right)dx+c$

$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right)=-e^{-\frac{\sin 2x}{2}}+c$

Given

$y\left(\frac{\pi}{4}\right)=0$

$\Rightarrow 0=-e^{-\frac{1}{2}}+c\Rightarrow c=e^{-\frac{1}{2}}$

$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right)=-e^{-\frac{\sin 2x}{2}}+e^{-1/2}$

at x = 0

$y=-1+e^{-\frac{1}{2}}$

$\Rightarrow y(0)=-1+e^{-\frac{1}{2}}\Rightarrow(y(0)+1)^{2}=e^{-1}$

-----------------------------------------------------------------------------------------------

# Question112

**Let y = y(x) satisfies the equation $\frac{dy}{dx} - |A| = 0$,for all x > 0, where**

**A =** $\begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x} \end{bmatrix}$ **. If $y(\pi) = \pi + 2$, then the value of $y\left(\frac{\pi}{2}\right)$ is :**

**[20 Jul 2021 Shift 2]**

**Options:**

A. $\frac{\pi}{2} + \frac{4}{\pi}$

B. $\frac{\pi}{2} - \frac{1}{\pi}$

C. $\frac{3\pi}{2} - \frac{1}{\pi}$

D. $\frac{\pi}{2} - \frac{4}{\pi}$

**Answer: A**

## Solution:

**Solution:**

$|A| = -\frac{y}{x} + 2\sin x + 2$

$\frac{dy}{dx} = |A|$

$\frac{dy}{dx} = -\frac{y}{x} + 2\sin x + 2$

$\frac{dy}{dx} + \frac{y}{x} = 2\sin x + 2$

I.F. $= e^{\int \frac{1}{x} dx} = x$

$\Rightarrow yx = \int x(2\sin x + 2) dx$

$xy = x^2 - 2x\cos x + 2\sin x + c$ .......(i)

Now $x = \pi$, $y = \pi + 2$

Use in (i) $c = 0$

Now (i) be comes

$xy = x^2 - 2x\cos x + 2\sin x$

put $x = \pi / 2$

$\frac{\pi}{2}y = \left(\frac{\pi}{2}\right)^2 - 2 . \frac{\pi}{2}\cos\frac{\pi}{2} + 2\sin\frac{\pi}{2}$

$\frac{\pi}{2}y = \frac{\pi^2}{4} + 2$

---------------------------------------------------------------------------------------------------

# Question113

**Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = -\alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers.**

**If $\vec{a} . \vec{b} = -1$ and $\vec{b} . \vec{c} = 10$, then $(\vec{a} \times \vec{b}) . \vec{c}$ is equal to ____ .**

**[27 Jul 2021 Shift 2]**

**Answer: 9**

## Solution:

**Solution:**
$\vec{a} = (1, -\alpha, \beta)$
$\vec{b} = (3, \beta, -\alpha)$
$\vec{c} = (-\alpha, -2, 1)$; $\alpha, \beta \in I$
$\vec{a} \cdot \vec{b} = -1 \Rightarrow 3 - \alpha\beta - \alpha\beta = -1$
$\Rightarrow \alpha\beta = 2$

| | |
|---|---|
| 1 | 2 |
| 2 | 1 |
| −1 | −2 |
| −2 | −1 |

$\vec{b} \cdot \vec{c} = 10$
$\Rightarrow -3\alpha - 2\beta - \alpha = 10$
$\Rightarrow 2\alpha + \beta + 5 = 0$
$\therefore \alpha = -2; \beta = -1$

$$\left[\begin{array}{ccc} \vec{a} & \vec{b} & \vec{c} \end{array}\right] = \left|\begin{array}{ccc} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{array}\right|$$

$= 1(-1 + 4) - 2(3 - 4) - 1(-6 + 2)$
$= 3 + 2 + 4 = 9$

----------------------------------------------------------------------------------------------------

# Question114

## A differential equation representing the family of parabolas with axis parallel to Y-axis and whose length of latus rectum is the distance of the point (2, −3) from the line 3x + 4y = 5, is given by [27 Aug 2021 Shift 2]

**Options:**

A. $10\frac{d^2y}{dx^2} = 11$

B. $11\frac{d^2x}{dy^2} + 10$

C. $10\frac{d^2x}{dy^2} = 11$

D. $11\frac{d^2y}{dx^2} = 10$

**Answer: D**

## Solution:

Let (h, k) be the vertex of parabola. Then, equation of parabola parallel to Y-axis is
$(x - h)^2 = 4a(y - k)$ ...(i)
Also,

Length of latusrectum = Distance of point (2, − 3) from the line
$3x + 4y = 5$

$\Rightarrow 4a = \frac{|6-12-5|}{\sqrt{3^2+4^2}}$

$\Rightarrow 4a = \frac{11}{5}$

∴ From Eq. (i),

$(x-h)^2 = \frac{11}{5}(y-k)$

Differentiating w.r.t. x, we get

$2(x-h) = \frac{11}{5}\frac{dy}{dx}$

Again, differentiating w.r.t. x

$2 = \frac{11}{5}\frac{d^2y}{dx^2}$

$\Rightarrow 11\frac{d^2y}{dx^2} = 10$

---------------------------------------------------------------------------------------------------

# Question115

**Let f be a non-negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt$, $0 \le x \le 1$ and $f(0) = 0$, then**

$\lim_{x \to 0} \frac{1}{x^2}\int_0^x f(t)\,dt$

**[31 Aug 2021 Shift 1]**

**Options:**

A. equals 0

B. equals 1

C. does not exist

D. equals $\frac{1}{2}$

**Answer: D**

## Solution:

**Solution:**

$\int_0^x [1-(f'(t))^2]^{\frac{1}{2}} = \int_0^x f(t)$

Differentiating on both sides,

$\sqrt{1-[f'(x)]^2} = [f(x)]$

$\Rightarrow 1-[f'(x)]^2 = [f(x)]^2$

$\Rightarrow 1-[f(x)]^2 = [f'(x)]^2$

$\Rightarrow f'(x) = \sqrt{1-[f(x)]^2}$

$\Rightarrow \int \frac{f'(x)\,dx}{\sqrt{1-[f(x)]^2}} = \int 1\,dx$

$\Rightarrow \sin^{-1}f(x) = x + C$

$\because f(0) = 0$

$C = 0$

$f(x) = \sin x$

$\Rightarrow \lim_{x \to 0}\frac{\int_0^x f(t)\,dt}{x^2} = \lim_{x \to 0}\frac{\int_0^x \sin x\,dt}{x^2}$ [applying L’Hopital Rule]

$= \lim_{x \to 0}\left(\frac{\sin x}{2x}\right) = \frac{1}{2}$

---

# Question116

**If $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$, y(0) = 1, then y(1) is equal to**
**[31 Aug 2021 Shift 1]**

**Options:**

A. $\log_2(2 + e)$

B. $\log_2(1 + e)$

C. $\log_2(2e)$

D. $\log_2(1 + e^2)$

**Answer: B**

## Solution:

**Solution:**

$\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y} = \frac{2^x(2^y - 1)}{2^y}$

$\int \frac{2^y}{2^y - 1} dy = \int 2^x dx$

$\frac{\ln(2^y - 1)}{\ln 2} = \frac{2^x}{\ln 2} + C$

As, $y(0) = 1$

$\Rightarrow 0 = \frac{1}{\log 2} + C$

For y(1), $\ln_2(2^y - 1) = 2^1 - 1$

$\Rightarrow 2^y - 1 = e$

$y = \log_2(e + 1)$

---

# Question117

**If $\frac{dy}{dx} = \frac{2^x y + 2^y . 2^x}{2^x + 2^{x+y} \log_e 2}$, y(0) = 0, then for y = 1 , the value of x lies in the interval**
**[31 Aug 2021 Shift 2]**

**Options:**

A. (1, 2)

B. $\left(\frac{1}{2}, 1\right]$

C. (2, 3)

D. $\left(0, \frac{1}{2}\right]$

**Answer: A**

# Solution:

**Solution:**
$\frac{dy}{dx} = \frac{2^x y + 2^y . 2^x}{2^x + 2^{x+y} \log_e 2}$, $y(0) = 0$
$\Rightarrow \frac{dy}{dx} = \frac{y + 2^y}{1 + 2^y \log_e 2}$
$\Rightarrow \int \frac{1 + 2^y . \log_e 2 dy}{y + 2^y} = \int dx$
$\Rightarrow \log | y + 2^y | = x + C$
$\because y(0) = 0$
$\Rightarrow C = 0$
$\therefore \ln | y + 2^y | = x$
For $y = 1$
$x = \ln | 1 + 2 | = \ln 3$
$\Rightarrow x \in (1, 2)$

---------------------------------------------------------------------------------------------------

# Question118

**If $y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, $x > 0$, $\phi > 0$, and $y(1) = -1$, then $\phi\left(\frac{y^2}{4}\right)$ is equal to**
**[31 Aug 2021 Shift 2]**

**Options:**

A. $4\ \phi\ (2)$

B. $4\ \phi\ (1)$

C. $2\ \phi\ (1)$

D. $\phi\ (1)$

**Answer: B**

# Solution:

**Solution:**
Given,
$y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$ ...(i)

Let $t = \frac{y}{x}$
$\Rightarrow y = xt$
$\Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$
$\therefore$ Eq. (i) becomes
$t\left(t + x\frac{dt}{dx}\right) = \left(t^2 + \frac{\phi(t^2)}{\phi'(t^2)}\right)$

$\Rightarrow xt\frac{dt}{dx} = \frac{\phi(t^2)}{\phi'(t^2)}$

$\Rightarrow \frac{t\phi'(t^2)}{\phi(t^2)}dt = \frac{dx}{x}$

Integrating both sides

$\int \frac{t\phi'(t^2)}{\phi t^{2)}}dt = \int \frac{dx}{x}$

Let $\phi(t^2) = u$

$\Rightarrow t\phi'(t^2)dt = \frac{du}{2}$

$\therefore \frac{1}{2}\int \frac{du}{u} = \int \frac{dx}{x}$

$\Rightarrow \frac{1}{2}\ln u = \ln x + C$

$\Rightarrow \frac{1}{2}\ln \phi(t^2) = \ln x + C$

$\Rightarrow \frac{1}{2}\ln\left(\phi\left(\frac{y^2}{x^2}\right)\right) = \ln x + C$

If $x = 1$, $y = -1$, then $C = \frac{1}{2}\ln(\phi(1))$

$\therefore \frac{1}{2}\ln\left(\phi\left(\frac{y^2}{x^2}\right)\right) = \ln x + 2\ln(\phi(1))$

If $x = 2$, then $\ln\left(\phi\left(\frac{y^2}{4}\right)\right) = \ln 4 + \ln[\phi(1)]$

Or $\phi\left(\frac{y^2}{4}\right) = 4\phi(1)$

----------------------------------------------------------------------------------------------------

# Question119

## Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2\sin x - 5)x - 2\cos x$ such that, $y(0) = 7$. Then $y(\pi)$ is equal to [27 Aug 2021 Shift 1]

**Options:**

A. $2e^{\pi^2} + 5$

B. $e^{\pi^2} + 5$

C. $3e^{\pi^2} + 5$

D. $7e^{\pi^2} + 5$

**Answer: A**

## Solution:

**Solution:**

Given, $\frac{dy}{dx} = 2(y + 2\sin x - 5)x - 2\cos x$, $y(0) = 7$

$\Rightarrow \frac{dy}{dx} + 2\cos x = 2(y + 2\sin x - 5)x$ ...(i)

Let $y + 2\sin x - 5 = t$

$\Rightarrow \frac{dy}{dx} + 2\cos x = \frac{dt}{dx}$

Then, Eq. (i) becomes

$\frac{dt}{dx} = 2tx$

$\Rightarrow \frac{dt}{t} = 2x\,dx$
On integrating
$\ln t = x^2 + C$
$\Rightarrow \ln(y + 2\sin x - 5) = x^2 + C$ ...(ii)
$\therefore y(0) = 7$
$\Rightarrow \ln(7 + 0 - 5) = 0 + C$
$\Rightarrow C = \ln 2$
$\therefore$ From Eq. (ii),
$\ln(y + 2\sin x - 5) = x^2 + \ln 2$
Now, at $x = \pi$
$\ln(y(\pi) + 2\sin\pi - 5) = \pi^2 + \ln 2$
$\Rightarrow \ln(y(\pi) - 5) = \pi^2 + \ln 2$
$\Rightarrow y(\pi) - 5 = e^{\pi^2 + \ln 2}$
$\Rightarrow y(\pi) = 2e^{\pi^2} + 5$

----------------------------------------------------------------------------------------------------

# Question120

## Let y(x) be the solution of the differential equation $2x^2 dy + (e^y - 2x)dx = 0$, $x > 0$. If $y(e) = 1$, then $y(1)$ is equal to [26 Aug 2021 Shift 2]

**Options:**

A. 0

B. 2

C. $\log_e 2$

D. $\log_e(2e)$

**Answer: C**

## Solution:

**Solution:**
We have, $2x^2 dy + (e^y - 2x)dx = 0$
$\frac{dy}{dx} + \frac{e^y - 2x}{2x^2} = 0$
$\frac{dy}{dx} + \frac{e^y}{2x^2} - \frac{1}{x} = 0$
$e^{-y}\frac{dy}{dx} - \frac{e^{-y}}{x} = -\frac{1}{2x^2}$ ...(i)
$e^{-y} = t$ ...(ii)
$-e^{-y}dy = dt$
$dy = -\left(\frac{dt}{t}\right)$...(iii)
$\frac{-dt}{dx} - \frac{t}{x} = -\frac{1}{2x^2}$ [From Eq. (i)]
$x\,dt + t\,dx = \frac{dx}{2x}$
$\int d(xt) = \int \frac{dx}{2x}$
$xt = \frac{1}{2}\log(x) + \frac{C}{2}$
$2xe^{-y} = \log x + c$
When $x = e$, $y = 1$
$2e \,.\, e^{-1} = \log e + c$
$c = 1$

$\therefore 2xe^{-y} = \log x + 1$
When $x = 1$,
$e^{-y} = 0 + 1$
$e^{y} = 2$
$\Rightarrow y = \log_e 2$

----------------------------------------------------------------------------------------------------

# Question121

## Let us consider a curve, $y = f(x)$ passing through the point $(-2, 2)$ and the slope of the tangent to the curve at any point $(x, f(x))$ is given by $f(x) + xf'(x) = x^2$ Then [27 Aug 2021 Shift 1]

**Options:**

A. $x^2 + 2xf(x) - 12 = 0$

B. $x^3 + 2xf(x) + 12 = 0$

C. $x^3 - 3xf(x) - 4 = 0$

D. $x^2 + 2xf(x) + 4 = 0$

**Answer: C**

## Solution:

**Solution:**
Given, $f(x) + xf'(x) = x^2$
$\Rightarrow f'(x) + \frac{f(x)}{x} = x$
$\Rightarrow \frac{dy}{dx} + \frac{1}{x}y = x \left[\because y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)\right]$
This is linear differential equation.
$\therefore$ Integrating factor IF $= e^{\int \frac{1}{x} dx} = x$
Solution, $y \cdot x = \int x \cdot x dx + C$
or $xy = \frac{x^3}{3} + C$
$\because$ It passes through $(-2, 2)$.
$\therefore -2.2 = \frac{(-2)^3}{3} + C$
$C = -\frac{4}{3}$
Hence, $xf(x) = \frac{x^3}{3} - \frac{4}{3}$
or $x^3 - 3f(x) \cdot x - 4 = 0$

----------------------------------------------------------------------------------------------------

# Question122

## If the solution curve of the differential equation $(2x - 10y^3)dy + ydx = 0$, passes through the points $(0, 1)$ and $(2, \beta)$, then $\beta$ is a root of the equation [27 Aug 2021 Shift 2]

**Options:**

A. $y^5 - 2y - 2 = 0$

B. $2y^5 - 2y - 1 = 0$

C. $2y^5 - y^2 - 2 = 0$

D. $y^5 - y^2 - 1 = 0$

**Answer: D**

## Solution:

**Solution:**
Given, differential equation
$(2x - 10y^3)dy + ydx = 0$
$\Rightarrow \frac{dx}{dy} + \frac{2x}{y} = 10y^2$ ...(i)
This is Linear differential equation
Integrating factor IF $= e^{2\int \frac{1}{y} dy} = y^2$
Solution of differential Eq. (i),
$x \cdot y^2 = \int 10y^2 \cdot y^2 dy + C$
$\Rightarrow xy^2 = 2y^5 + C$ ...(ii)
Solution Eq. (ii) passes through (0, 1)
$\Rightarrow 0.1^2 = 21^5 + C$
$\Rightarrow C = -2$
$\therefore$ Solution of Eq. (i) is
$xy^2 = 2y^5 - 2$
Now, this equation passes through (2, β).
$\therefore 2 \cdot \beta^2 = 2\beta^5 - 2$
$\Rightarrow \beta^5 - \beta^2 - 1 = 0$
$\Rightarrow$ β is root of the equation $y^5 - y^2 - 1 = 0$

----------------------------------------------------------------------------------------------------

# Question123

**Let $y = y(x)$ be a solution curve of the differential equation $(y + 1)\tan^2 x dx + \tan x dy + y dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $\lim_{x \to 0^+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is**
**[26 Aug 2021 Shift 1]**

**Options:**

A. $-\frac{\pi}{4}$

B. $\frac{\pi}{4} - 1$

C. $\frac{\pi}{4} + 1$

D. $\frac{\pi}{4}$

**Answer: D**

## Solution:

**Solution:**

We have, $(y+1)\tan^2 x\,dx + \tan x\,dy + y\,dx = 0$

$\Rightarrow [(y+1)\tan^2 x + y]dx + \tan x\,dy = 0$

$\Rightarrow \frac{dy}{dx} + (y+1)\tan x + \frac{y}{\tan x} = 0$

$\Rightarrow \frac{dy}{dx} + \frac{y\tan^2 x + \tan^2 x + y}{\tan x} = 0$

$\Rightarrow \frac{dy}{dx} + \frac{y\sec^2 x}{\tan x} + \tan^2 x = 0$

$\Rightarrow \frac{dy}{dx} + \left(\frac{\sec^2 x}{\tan x}\right)y = -\tan x$

This is a linear differential equation

$\therefore IF = e^{\int \frac{\sec^2 x}{\tan x}dx} = e^{\ln(\tan x)} = \tan x$

So, solution is given by

$(y\tan x) = \int -\tan^2 x\,dx = \int (1-\sec^2 x)dx = x - \tan x + C$

$y = x\cot x - 1 + C\cot x$

Now, $\lim_{x\to 0^+} x \cdot y = 1$

$\Rightarrow \lim_{x\to 0^+} (x^2\cot x - x + Cx\cot x) = 1$

$\Rightarrow \lim_{x\to 0} + \left(x \cdot \frac{x}{\tan x} - x + \frac{Cx}{\tan x}\right) = 1$

$\Rightarrow 0 - 0 + C = 1$

$\Rightarrow C = 1$

$\therefore y = x\cot x - 1 + \cot x$

Now, $x = \frac{\pi}{4}$

$y = \frac{\pi}{4} - 1 + 1 = \frac{\pi}{4}$

----------------------------------------------------------------------------------------------------

# Question124

**Let a, b and c be three vectors mutually perpendicular to each other and have same magnitude. If a vector r satisfies.**
**a × {(r - b) × a} + b × {(r - c) × b} + c × {(r - a) × c} = 0, then r is equal to**
**[31 Aug 2021 Shift 2]**

**Options:**

A. $\frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$

B. $\frac{1}{3}(2\mathbf{a} + \mathbf{b} - \mathbf{c})$

C. $\frac{1}{2}(\mathbf{a} + \mathbf{b} + \mathbf{c})$

D. $\frac{1}{2}(\mathbf{a} + \mathbf{b} + 2\mathbf{c})$

**Answer: C**

## Solution:

$\mathbf{a} \times [(\mathbf{r} - \mathbf{b}) \times \mathbf{a}] + \mathbf{b} \times [(\mathbf{r} - \mathbf{c}) \times \mathbf{b}] + \mathbf{c} \times [(\mathbf{r} - \mathbf{a}) \times \mathbf{c}] = 0$
$\Rightarrow \mathbf{a} \cdot \mathbf{a} (\mathbf{r} - \mathbf{b}) - (\mathbf{a} \cdot (\mathbf{r} - \mathbf{b})) \mathbf{a} + \mathbf{b} \cdot \mathbf{b} (\mathbf{r} - \mathbf{c}) - (\mathbf{b} \cdot (\mathbf{r} - \mathbf{c})) \mathbf{b} + \mathbf{c} \cdot \mathbf{c} (\mathbf{r} - \mathbf{a}) - (\mathbf{c} \cdot (\mathbf{r} - \mathbf{a})) \mathbf{c} = 0$
$\Rightarrow |a|^2 (r - b) - (r . a)a + |b|^2 (r - c) - (r . b)b + |c|^2 (r - a) - (r . c)c = 0$ [$\because$ **a b, c** are mutually perpendicular; $\therefore$ a . **b** = **b** . **c** = **c** . **a** = 0]
$\Rightarrow |a|^2 [3r - (a + b + c)] - [(r . a)a + (r . b)b + (r . c)c] = 0$ [$\because |\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}|$]
$\Rightarrow |a|^2 [3r - (a + b + c) - r] = 0$
$\therefore 3\mathbf{r} - (\mathbf{a} + \mathbf{b} + \mathbf{c}) - \mathbf{r} = 0$
$\Rightarrow r = \frac{a + b + c}{2}$

------------------------------------------------------------------------------------------------

# Question125

## If y = y(x) is the solution curve of the differential equation $x^2 dy + \left(y - \frac{1}{x}\right) dx = 0$ ; $x > 0$ and y(1) = 1, then $y\left(\frac{1}{2}\right)$ is equal to [1 Sep 2021 Shift 2]

**Options:**

A. $\frac{3}{2} - \frac{1}{\sqrt{e}}$

B. $3 + \frac{1}{\sqrt{e}}$

C. $3 + e$

D. $3 - e$

**Answer: D**

## Solution:

**Solution:**

$x^2 dy + \left(y - \frac{1}{x}\right) dx = 0$

$\Rightarrow x^2 dy + y dx = \frac{dx}{x}$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x^2} = \frac{1}{x^3}$

This in linear differential equations

$\text{IF} = e^{\int \frac{1}{x^2} dx} = e^{-\frac{1}{x}}$

$\Rightarrow y . e^{-1/x} = \int e^{-1/x} . \frac{1}{x^3} dx + c$

Let $\frac{-1}{x} = t$

$\Rightarrow \frac{1}{x^2} dx = dt$

$\Rightarrow y . e^{-1/x} = \int -te^t dt + c = -(te^t - e^t) + c$

$\Rightarrow ye^{-1/x} = \frac{1}{x} e^{-\frac{1}{x}} + e^{-\frac{1}{x}} + c$

Put $x = 1, y = 1$

$(1) e^{-1} = \frac{e^{-1}}{1} + e^{-1} + c$

$\Rightarrow c = -e^{-1}$

Solution $y . e^{-1/x} = \frac{1}{x} e^{-\frac{1}{x}} + e^{-\frac{1}{x}} - e^{-1}$

$\Rightarrow y = \frac{1}{x} + 1 - \frac{e^{1/x}}{e}$

Put $x = \frac{1}{2}$

$y = 2 + 1 - \frac{e^2}{e}$

$\Rightarrow y = 3 - e$

---------------------------------------------------------------------------------------------------

# Question126

**If for $x \geq 0$, $y = y(x)$ is the solution of the differential equation,$(x+1)dy = ((x+1)^2 + y - 3)dx$ ,$y(2) = 0$ then $y(3)$ is equal to ________.**

**[NA Jan. 09,2020 (I)]**

**Answer: 3**

## Solution:

**Solution:**

$(x+1)dy = ((x+1)^2 + (y-3))\,dx = 0$

$\Rightarrow \frac{dy}{dx} = (1+x) + \left(\frac{y-3}{1+x}\right)$

$\frac{dy}{dx} - \frac{1}{(1+x)}y = (1+x) - \frac{3}{(1+x)}$

I.F. $= e^{-\int \frac{1}{(1+x)}dx} = \frac{1}{(1+x)}$

$\therefore \frac{d}{dx}\left(\frac{y}{1+x}\right) = 1 - \frac{3}{(1+x)^2}$

$y = (1+x)\left[x + \frac{3}{(1+x)} + C\right]$

$\because$ At $x = 2$, $y = 0$

$\therefore 0 = 3(2 + 1 + C) \Rightarrow C = -3$

Then, $y = (1+x)\left[x + \frac{3}{1+x} - 3\right]$

Now, at $x = 3$, $y = (1+3)\left[3 + \frac{3}{1+3} - 3\right] = 3$

---------------------------------------------------------------------------------------------------

# Question127

**Let $y = y(x)$ be the solution curve of the differential equation, $(y^2 - x)\frac{dy}{dx} = 1$, satisfying $y(0) = 1$. This curve intersects the x -axis at a point whose abscissa is:**

**[Jan. 7,2020 (II)]**

**Options:**

A. $2 - e$

B. $-e$

C. 2

D. $2+e$

**Answer: A**

## Solution:

**Solution:**
The given differential equation is $\frac{dx}{dy}+x=y^2$
Comparing with $\frac{dx}{dy}+Px=Q$, where $P=1$, $Q=y^2$
Now, I.F. $=e^{\int 1 \, dy}=e^y$
$x \, . \, e^y=\int (y^2)e^y \, . \, dy=y^2 \, . \, e^y-\int 2y \, . \, e^y \, . \, dy$
$=y^2e^y-2(y \, . \, e^y-e^y)+C$
$\Rightarrow x \, . \, e^y=y^2e^y-2ye^y+2e^y+C$
$\Rightarrow x=y^2-2y+2+C \, . \, e^{-y}$ .......(i)
As $y(0)=1$, satisfying the given differential eqn,
$\therefore$ put $x=0$, $y=1$ in eqn. (i)
$0=1-2+2+\frac{C}{e}$
$C=-e$
$y=0, x=0-0+2+(-e)(e^{-0})$
$x=2-e$

---------------------------------------------------------------------------------------------------

# Question128

## The differential equation of the family of curves,$x^2=4b(y+b)$, $b \in R$, is: [Jan. 8, 2020 (II)]

**Options:**

A. $x(y')^2=x+2yy'$

B. $x(y')^2=2yy'-x$

C. $xy''=y'$

D. $x(y')^2=x-2yy'$

**Answer: A**

## Solution:

**Solution:**
Since, $x^2=4b(y+b)$
$x^2=4by+4b^2$
$2x=4by'$
$\Rightarrow b=\frac{x}{2y'}$
So, differential equation is
$x^2=\frac{2x}{y'} \, . \, y+\left(\frac{x}{y'}\right)^2$
$x(y')^2=2yy'+x$

---------------------------------------------------------------------------------------------------

# Question129

**If $f\,2(x) = \tan^{-1}(\sec x + \tan x), -\frac{\pi}{2} < x < \frac{\pi}{2}$and $f(0) = 0$, then $f(1)$ is equal to:**
**[Jan. 9, 2020 (I)]**

**Options:**

A. $\frac{\pi + 1}{4}$

B. $\frac{1}{4}$

C. $\frac{\pi - 1}{4}$

D. $\frac{\pi + 2}{4}$

**Answer: A**

## Solution:

**Solution:**
$f'(x) = \tan^{-1}(\sec x + \tan x)$

$= \tan^{-1}\left(\frac{1 + \sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1 - \cos\left(\frac{\pi}{2} + x\right)}{\sin\left(\frac{\pi}{2} + x\right)}\right)$

$= \tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4} + \frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right) = \frac{\pi}{4} + \frac{x}{2}$

Integrate both sides, we get

$\int (f'(x))dx = \int \left(\frac{\pi}{4} + \frac{x}{2}\right)dx$

$f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + C$

$\because f(0) = 0$

$C = 0 \Rightarrow f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$

So, $f(1) = \frac{\pi + 1}{4}$

-----------------------------------------------------------------------------------------------

# Question130

**If $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$; $y(1) = 1$; then a value of x satisfying $y(x) = e$ is:**
**[Jan. 9, 2020 (II)]**

**Options:**

A. $\frac{1}{2}\sqrt{3}e$

B. $\frac{e}{\sqrt{2}}$

C. $\sqrt{2}e$

D. $\sqrt{3}e$

**Answer: D**

## Solution:

**Solution:**
The given differential equation,
$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$
Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$
Then, $v + x\frac{dv}{dx} = \frac{vx^2}{x^2 + v^2x^2} = \frac{v}{1 + v^2}$
$\Rightarrow \frac{1 + v^2}{v^3} dv = -\frac{1}{x} dx$
$\Rightarrow \int \left(\frac{1}{v^3} + \frac{1}{v}\right) dv = \int \frac{-1}{x} dx$
$\Rightarrow \frac{-1}{2}\left(\frac{1}{v^2}\right) + \ln v = -\ln x + c$
$\Rightarrow -\frac{x^2}{2y^2} = -\ln y + c \quad \left[\because v = \frac{y}{x}\right]$
When $x = 1, y = 1$, then $-\frac{1}{2} = c$
$\Rightarrow x^2 = y^2(1 + 2\ln y)$
At $y = e, x^2 = e^2(3)$
$\Rightarrow x = \pm\sqrt{3}e$
So, $x = \sqrt{3}e$

-----------------------------------------------------------------------------------------------

# Question131

## Let $f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1, |x| > 1$. If $\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\sin^{-1}(f(x)))$ and $y(\sqrt{3}) = \frac{\pi}{6}$, then $y(-\sqrt{3})$ is equal to:
## [Jan. 8, 2020 (I)]

**Options:**

A. $\frac{2\pi}{3}$

B. $-\frac{\pi}{6}$

C. $\frac{5\pi}{6}$

D. $\frac{\pi}{3}$

**Answer: B**

## Solution:

**Solution:**
$\frac{dy}{dx} = \frac{1}{2}\frac{d}{dx}(\sin^{-1}f(x))$

$2y = \sin^{-1}f(x) + C = \sin^{-1}(\sin(2\tan^{-1}x)) + C$

$\Rightarrow 2\left(\frac{\pi}{6}\right) = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) + C$

$\frac{\pi}{3} = \frac{\pi}{3} + C \quad \therefore C = 0$

for $x = -\sqrt{3}$, $2y = \sin^{-1}\left(\sin\left(\frac{-2\pi}{6}\right)\right) + 0$

$\Rightarrow 2y = \frac{-\pi}{3} \Rightarrow y = \frac{-\pi}{6}$

----------------------------------------------------------------------------------------------------

# Question132

**Let y = y(x) be a solution of the differential equation,**
$\sqrt{1 - x^2}\frac{dy}{dx} + \sqrt{1 - y^2} = 0$, **$|x| < 1$**
**If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$, then $y\left(\frac{-1}{\sqrt{2}}\right)$ is equal to:**
**[Jan. 8, 2020 (I)]**

**Options:**

A. $\frac{\sqrt{3}}{2}$

B. $-\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. $-\frac{\sqrt{3}}{2}$

**Answer: C**

## Solution:

**Solution:**
The given differential eqn. is
$\frac{dy}{\sqrt{1 - y^2}} + \frac{dx}{\sqrt{1 - x^2}} = 0 \Rightarrow \sin^{-1}y + \sin^{-1}x = c$

At $x = \frac{1}{2}$, $y = \frac{\sqrt{3}}{2} \Rightarrow c = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}y = \cos^{-1}x$

Hence, $y\left(-\frac{1}{\sqrt{2}}\right) = \sin\left(\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right)$

$= \sin\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) = \frac{1}{\sqrt{2}}$

----------------------------------------------------------------------------------------------------

# Question133

**If y = y(x) is the solution of the differential equation, $e^y = e^x$ such that y(0) = 0, then y(l ) is equal to:**
**[Jan. 7, 2020 (I)]**

**Options:**

A. $1+\log_e 2$

B. $2+\log_e 2$

C. $2e$

D. $\log_e 2$

**Answer: A**

## Solution:

**Solution:**
Let $e^y = t$
$e^y \frac{dy}{dx} = \frac{dt}{dx}$
$\therefore \frac{dt}{dx} - t = e^x \left[\because e^y \frac{dy}{dx} - e^y = e^x\right]$
$I.F. = e^{\int -1 . dx} = e^{-x}$
$t(e^{-x}) = \int e^x . e^{-x} dx \Rightarrow e^{y-x} = x + c$
Put $x = 0$, $y = 0$, then we get $c = 1$
$e^{y-x} = x + 1$
$y = x + \log_e(x+1)$
Put $x = 1$ $\quad \therefore y = 1 + \log_e 2$

------------------------------------------------------------------------------------------------------

# Question134

## The general solution of the differential equation $\sqrt{1+x^2+y^2+x^2y^2} + xy\frac{dy}{dx} = 0$ is: (where C is a constant of integration) :[Sep. 06, 2020 (I)]

**Options:**

A. $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

B. $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

C. $\sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

D. $\sqrt{1+y^2} - \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right) + C$

**Answer: A**

## Solution:

$\sqrt{1+x^2}\,.\,\sqrt{1+y^2} = -xy\frac{dy}{dx}$

$\int \frac{\sqrt{1+x^2}}{x}dx = -\int \frac{y}{\sqrt{1+y^2}}dy$

Let $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

$\Rightarrow \int \frac{\sec^3\theta d\theta}{\tan\theta} = -\int \frac{2y}{2\sqrt{1+y^2}}dy$

$\Rightarrow \int \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\,.\,\cos^2\theta}d\theta = -\sqrt{1+y^2}$

$\Rightarrow \int (\tan\theta\,.\,\sec\theta + \operatorname{cosec}\theta)d\theta = -\sqrt{1+y^2}$

$\Rightarrow \sec\theta + \log_e|\operatorname{cosec}\theta - \cot\theta| = -\sqrt{1+y^2} + C$

$\therefore \sqrt{1+x^2} + \log_e\left|\frac{\sqrt{1+x^2}-1}{x}\right| = -\sqrt{1+y^2} + C$

$\Rightarrow \sqrt{1+y^2} + \sqrt{1+x^2} = \frac{1}{2}\log_e\left(\frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}-1}\right) + C$

-----------------------------------------------------------------------------------------------

# Question135

**If $y = \left(\frac{2}{\pi}x - 1\right)\operatorname{cosec}x$ is the solution of the differentialequation, $dy dx + p(x)y = \frac{2}{\pi}\operatorname{cosec}x$, $0 < x < \frac{\pi}{2}$, then the function $p(x)$ is equal to: [Sep. 06, 2020 (II)]**

**Options:**

A. $\cot x$

B. $\operatorname{cosec} x$

C. $\sec x$

D. $\tan x$

**Answer: A**

## Solution:

**Solution:**

$\because y = \left(\frac{2}{\pi}x - 1\right)\operatorname{cosec}x$

$\frac{dy}{dx} = \frac{2}{\pi}\operatorname{cosec}x - \left(\frac{2}{\pi}x - 1\right)\operatorname{cosec}x\,.\,\cot x$

$= \operatorname{cosec}x\left[\frac{2}{\pi} - \left(\frac{2}{\pi}x - 1\right)\cot x\right]$

$\Rightarrow \frac{dy}{dx} - \frac{2}{\pi}\operatorname{cosec}x = y\cot x$ .........(i)

It is given that,

$\Rightarrow \frac{dy}{dx} - \frac{2}{\pi}\operatorname{cosec}x = -yp(x)$ ........(ii)

By comparision of (i) and (ii), we get

$p(x) = \cot x$

-----------------------------------------------------------------------------------------------

# Question136

**If y = y(x) is the solution of the differential equation $\frac{5+e^x}{2+y} \cdot \frac{dy}{dx} + e^x = 0$ satisfying y(0) = 1, then a value of $y(\log_e 13)$ is :**
**[Sep. 05, 2020 (I)]**

**Options:**

A. 1

B. -1

C. 0

D. 2

**Answer: B**

## Solution:

**Solution:**
$\frac{5+e^x}{2+y} \cdot \frac{dy}{dx} = -e^x$
$\int \frac{dy}{2+y} = -\int \frac{e^x}{5+e^x} dx$
$\Rightarrow \log_e |2+y| . \log_e |5+e^x| = \log_e C$
$\Rightarrow |(2+y)(5+e^x)| = C \because y(0) = 1$
$C = 18$
$\therefore (2+y).(5+e^x) = 18$
When $x = \log_e 13$ then $(2+y).18 = 18$
$\Rightarrow 2+y = \pm 1$
$\therefore y = -1, -3$
$\therefore y(\ln 13) = -1$

-----------------------------------------------------------------------------------------------

# Question137

**The solution of the differential equation $\frac{dy}{dx} - \frac{y+3x}{\log_e(y+3x)} + 3 = 0$ is (where C is a constant of integration.)**
**[Sep. 04, 2020 (II)]**

**Options:**

A. $x - \frac{1}{2}(\log_e(y+3x))^2 = C$

B. $x - \log_e(y+3x) = C$

C. $y + 3x - \frac{1}{2}(\log_e x)^2 = C$

D. $x - 2\log_e(y+3x) = C$

**Answer: A**

## Solution:

Let $y + 3x = t$

$\Rightarrow \frac{dy}{dx} + 3 = \frac{dt}{dx}$

Putting these value in given differential equation

$\frac{dt}{dx} = \frac{t}{\log_e t}$

$\Rightarrow \int \frac{\log_e t}{t} dt = \int dx$

$\Rightarrow \frac{(\log_e t)^2}{2} = x - C$

$\Rightarrow x - \frac{1}{2}(\ln(y + 3x))^2 = C$

----------------------------------------------------------------------------------------------------

# Question138

## Let $f : (0, \infty) \to (0, \infty)$ be a differentiable function such that $f(1) = e$ and $\lim_{t \to x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$

## If $f(x) = 1$, then x is equal to :

## [Sep. 04, 2020 (II)]

**Options:**

A. $\frac{1}{e}$

B. 2e

C. $\frac{1}{2e}$

D. e

**Answer: A**

## Solution:

**Solution:**

$\lim_{t \to x} \frac{t^2 f^2(x) - x^2 f^2(t)}{t - x} = 0$

$\Rightarrow \lim_{t \to x} \frac{2t f^2(x) - 2x^2 f(t) . f'(t)}{1} = 0$

Using L'Hospital's rule

$\Rightarrow f(x) = x f'(x)$

$\int \frac{f'(x)}{f(x)} dx = \int \frac{1}{x} dx$

$\log_e f(x) = \log_e x + \log_e C$

$\Rightarrow f(x) = Cx \quad \because f(1) = e$

$\Rightarrow C = e$; so $f(x) = ex$

When $f(x) = 1 = ex \Rightarrow x = \frac{1}{e}$

----------------------------------------------------------------------------------------------------

# Question139

## The solution curve of the differential equation, $(1 + e^{-x})(1 + y^2)\frac{dy}{dx} = y^2$, which passes through the point (0, 1), is :

**[Sep. 03, 2020 (I)]**

**Options:**

A. $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{-x}}{2}\right) + 2\right)$

B. $y^2 + 1 = y\left(\log_e\left(\frac{1+e^{x}}{2}\right) + 2\right)$

C. $y^2 = 1 + y\log_e\left(\frac{1+e^{x}}{2}\right)$

D. $y^2 = 1 + y\log_e\left(\frac{1+e^{-x}}{2}\right)$

**Answer: C**

## Solution:

**Solution:**

$\int\left(\frac{y^2+1}{y^2}\right)dy = \int\frac{e^x dx}{e^x+1}$

$\Rightarrow y - \frac{1}{y} = \log_e\left|e^x + 1\right| + c$

$\because$ Passes through (0,1)

$\therefore c = -\log_e 2$

$\Rightarrow y^2 - 1 = y\log_e\left(\frac{e^x+1}{2}\right)$

$\Rightarrow y^2 = 1 + y\log_e\left(\frac{e^x+1}{2}\right)$

---------------------------------------------------------------------------------------------------

# Question140

## If $x^3 dy + xy dx = x^2 dy + 2y dx$; $y(2) = e$ and $x > 1$, then $y(4)$ is equal to : [Sep. 03,2020 (II)]

**Options:**

A. $\frac{3}{2} + \sqrt{e}$

B. $\frac{3}{2}\sqrt{e}$

C. $\frac{1}{2} + \sqrt{e}$

D. $\frac{\sqrt{e}}{2}$

**Answer: B**

## Solution:

**Solution:**

$x^3 dy + xy\,dyx = 2y\,dx + x^2 dy$

$\Rightarrow (x^3 - x^2) d y = (2 - x) y d x$
$\Rightarrow \frac{d y}{y} = \frac{2 - x}{x^2 (x - 1)} d x$
$\Rightarrow \int \frac{d y}{y} = \int \frac{2 - x}{x^2 (x - 1)} d x$
Let $\frac{2 - x}{x^2 (x - 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x - 1}$
$\Rightarrow 2 - x = A(x - 1) + B(x - 1) + Cx^2$
Compare the coefficients of $x$, $x^2$ and constant term.
$C = 1$, $B = -2$ and $A = -1$
$\therefore \int \frac{d y}{y} = \int \left\{ \frac{-1}{x} - \frac{2}{x^2} + \frac{1}{x - 1} \right\} d x$
$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln \left| x - 1 \right| + C$
$\therefore y(2) = e$
$\Rightarrow 1 = -\ln 2 + 1 + 0 + C$ [$\because$ log e = 1]
$\Rightarrow C = \ln 2$
$\Rightarrow \ln y = -\ln x + \frac{2}{x} + \ln \left| x - 1 \right| + \ln 2$
At $x = 4$,
$\Rightarrow \ln y(4) = -\ln 4 \frac{1}{2} + \ln 3 + \ln 2$
$\Rightarrow \ln y(4) = \ln \left( \frac{3}{2} \right) + \frac{1}{2} = \ln \left( \frac{3}{2} e^{1/2} \right)$ [$\because \log m + \log n = \log(mn)$]
$\Rightarrow y(4) = \frac{3}{2} e^{1/2}$

----------------------------------------------------------------------------------------------------

# Question141

**Let $y = y(x)$ be the solution of the differential equation, $2 + \sin x y + 1 . \frac{d y}{d x} = -\cos x$, $y > 0$, $y(0) = 1$. If $y(\pi) = a$ and $\frac{d y}{d x}$ at $x = \pi$ is $b$, then the ordered pair $(a, b)$ is equal to :**
**[Sep. 02, 2020 (I)]**

**Options:**

A. $\left( 2, \frac{3}{2} \right)$

B. (1,-1)

C. (1,1)

D. (2,1)

**Answer: C**

## Solution:

**Solution:**
The given differential equation is
$\frac{2 + \sin x}{y + 1} \frac{d y}{d x} = -\cos x, y > 0$
$\Rightarrow \frac{d y}{y + 1} = -\frac{\cos x}{2 + \sin x} d x$
Integrate both sides,
$\int \frac{d y}{y + 1} = \int \frac{(-\cos x) d x}{2 + \sin x}$
$\ln | y + 1 | = -\ln | 2 + \sin x | + \ln C$
$\Rightarrow \ln | y + 1 | + \ln | 2 + \sin x | = \ln C$
$\Rightarrow \ln | (y + 1)(2 + \sin x) | = \ln C$

$\because y(0) = 1 \Rightarrow \ln 4 = \ln C \Rightarrow C = 4$
$\therefore (y + 1)(2 + \sin x) = 4$
$\Rightarrow y = \frac{4}{2 + \sin x} - 1$
$\therefore y = \frac{2 - \sin x}{2 + \sin x} \Rightarrow y(\pi) = \frac{2 - \sin \pi}{2 + \sin \pi} = 1$
$\Rightarrow a = 1$
Now, $\frac{dy}{dx} = \frac{(2 + \sin x)(-\cos x) - (2 - \sin x) \cdot \cos x}{(2 + \sin x)^2}$
$\left.\frac{dy}{dx}\right|_{x = \pi} = 1 \Rightarrow b = 1$
Ordered pair $(a, b) = (1, 1)$

----------------------------------------------------------------------------------------------------

# Question142

**If a curve $y = f(x)$, passing through the point (1,2) , is the solution of the differential equation,$2x^2 dy = (2xy + y^2)dx$, then $f\left(\frac{1}{2}\right)$ is equal to [Sep. 02, 2020 (II)]**

**Options:**

A. $\frac{1}{1 + \log_e 2}$

B. $\frac{1}{1 - \log_e 2}$

C. $1 + \log_e 2$

D. $\frac{-1}{1 + \log_e 2}$

**Answer: A**

## Solution:

**Solution:**
$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$
It is homogeneous differential equation.
$\therefore$ Put $y = vx$
$\Rightarrow v + x\frac{dv}{dx} = v + \frac{v^2}{2} \Rightarrow \int 2\frac{dv}{v^2} = \int \frac{dx}{x}$
$\Rightarrow \frac{-2}{v} = \log_e x + c \Rightarrow \frac{-2x}{y} = \log_e x + c$
Put $x = 1$, $y = 2$, we get $c = -1$
$\Rightarrow \frac{-2x}{y} = \log_e x - 1$
Hence, put $x = \frac{1}{2} \Rightarrow y = \frac{1}{1 + \log_e 2}$

----------------------------------------------------------------------------------------------------

# Question143

**Let $y = y(x)$ be the solution of the differential equation $\cos x \frac{dy}{dx} + 2y \sin x = \sin 2x$, $x \in \left(0, \frac{\pi}{2}\right)$**

**If $y(\pi / 3) = 0$, then $y(\pi / 4)$ is equal to :**
**[Sep. 05, 2020 (II)]**

**Options:**

A. $2-\sqrt{2}$

B. $2+\sqrt{2}$

C. $\sqrt{2}-2$

D. $\frac{1}{\sqrt{2}}-1$

**Answer: C**

## Solution:

**Solution:**
$\frac{dy}{dx}+2y\tan x=2\sin x$
I.F. $=e^{\int 2\tan x\,dx}=\sec^2 x$
The solution of the differential equation is
$y\times I.F=\int I.F\times 2\sin x\,dx+C$
$\Rightarrow y.\sec^2 x=\int 2\sin x.\sec^2 x\,dx+C$
$\Rightarrow y\sec^2 x=2\sec x+C$ ......(i)
When $x=\frac{\pi}{3}$, $y=0$; then $C=-4$
$\therefore$ From(i), $y\sec^2 x=2\sec x-4$
$\Rightarrow y=\frac{2\sec x-4}{\sec^2 x}\Rightarrow y\left(\frac{\pi}{4}\right)=\sqrt{2}-2$

----------------------------------------------------------------------------------------------------

# Question144

**Let $y = y(x)$ be the solution of the differential equation, $xy'-y=x^2(x\cos x+\sin x)$, $x>0$. If $y(\pi)=\pi$, then $y''\left(\frac{\pi}{2}\right)+y\left(\frac{\pi}{2}\right)$ is equal to :**
**[Sep. 04, 2020 (I)]**

**Options:**

A. $2+\frac{\pi}{2}$

B. $2+\frac{\pi}{2}+\frac{\pi^2}{4}$

C. $2+\frac{\pi}{2}+\frac{\pi^2}{4}$

D. $1+\frac{\pi}{2}$

**Answer: A**

## Solution:

**Solution:**

$\frac{dy}{dx} - \frac{y}{x} = x(x\cos x + \sin x)$

I.F. $= e^{-\int \frac{1}{x} dx} = \frac{1}{x}$

$\therefore \int d\left(\frac{y}{x}\right) = \int (x\cos x + \sin x) dx$

$\Rightarrow \frac{y}{x} = x\sin x + C \because y(\pi) = \pi \Rightarrow C = 1$

$y = x^2 \sin x + x \Rightarrow y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$

$y' = 2x\sin x + x^2 \cos x + 1$

$y'' = 2\sin x - x^2 \sin x \Rightarrow y''\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4}$

$\therefore y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right) = 2 - \frac{\pi^2}{4} + \frac{\pi^2}{4} + \frac{\pi}{2} = 2 + \frac{\pi}{2}$

-----------------------------------------------------------------------------------------------

# Question145

## If y = y(x) is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying y(1) = 1, then $y\left(\frac{1}{2}\right)$ is equal to:

## [Jan. 09, 2019 (I)]

### Options:

A. $\frac{7}{64}$

B. $\frac{1}{4}$

C. $\frac{49}{16}$

D. $\frac{13}{16}$

**Answer: C**

## Solution:

**Solution:**

Since, $x\frac{dy}{dx} + 2y = x^2$

$\Rightarrow \frac{dy}{dx} + \frac{2}{x}y = x$

I . F . $= e^{\int \frac{2}{x} dx} = e^{2\ln x} = e^{\ln x^2} = x^2$

Solution of differential equation is:

$y \cdot x^2 = \int x \cdot x^2 dx$

$y \cdot x^2 = \frac{x^4}{4} + C$ .........(1)

$\because y(1) = 1$

$\therefore C = \frac{3}{4}$

Then, from equation (1)

$y \cdot x^2 = \frac{x^4}{4} + \frac{3}{4}$

$\therefore y = \frac{x^2}{4} + \frac{3}{4x^2}$

$\therefore y\left(\frac{1}{2}\right) = \frac{1}{16} + 3 = \frac{49}{16}$

-----------------------------------------------------------------------------------------------

# Question146

## Let $f : [0, 1] \to R$ be such that $f(xy) = f(x), f(y)$, for all $x, y \in [0, 1]$, and $f(0) \neq 0$. If $y = y(x)$ satisfies the differential equation, $\frac{dy}{dx} = f(x)$ with $y(0) = 1$, then $y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right)$ is[ Jan. 09,2019 equal to:

## [Jan. 09, 2019 (II)]

**Options:**

A. 3

B. 4

C. 2

D. 5

**Answer: A**

## Solution:

**Solution:**

$f(xy) = f(x)f(y)$ ......(1)

Put $x = y = 0$ in (1) to get $f(0) = 1$

Put $x = y = 1$ in (1) to get $f(1) = 0$ or $f(1) = 1$

$f(1) = 0$ is rejected else $y = 1$ in (1) gives $f(x) = 0$

imply $f(0) = 0$

Hence, $f(0) = 1$ and $f(1) = 1$

By first principle derivative formula,

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} f(x)\left(\frac{f\left(1 + \frac{h}{x}\right) - f(1)}{h}\right)$

$\Rightarrow f'(x) = \frac{f(x)}{x} f'(1)$

$\Rightarrow \frac{f'(x)}{f(x)} = \frac{k}{x} \Rightarrow \ln f(x) = k \ln x + c$

$f(1) = 1 \Rightarrow \ln 1 = k \ln 1 + c \Rightarrow c = 0$

$\Rightarrow \ln f(x) = k \ln x \Rightarrow f(x) = x^k$ but $f(0) = 1$

$\Rightarrow k = 0$

$\therefore f(x) = 1$

$\frac{dy}{dx} = f(x) = 1 \Rightarrow y = x + c, y(0) = 1 \Rightarrow c = 1$

$\Rightarrow y = x + 1$

$\therefore y\left(\frac{1}{4}\right) + y\left(\frac{3}{4}\right) = \frac{1}{4} + 1 + \frac{3}{4} + 1 = 3$

-----------------------------------------------------------------------------------------------

# Question147

## If $\frac{dy}{dx} + \frac{3}{\cos^2 x} y = \frac{1}{\cos^2 x}$, $x \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)$, and $y\left(\frac{\pi}{4}\right) = \frac{4}{3}$,then $y\left(-\frac{\pi}{4}\right)$ equals:

**[10 Jan 2019 I]**

**Options:**

A. $\frac{1}{3}+e^6$

B. $\frac{1}{3}$

C. $-\frac{4}{3}$

D. $\frac{1}{3}+e^3$

**Answer: A**

## Solution:

**Solution:**

Given, $\frac{dy}{dx}+\frac{3}{\cos^2 x}y=\frac{1}{\cos^2 x}$

$\frac{dy}{dx}=\sec^2 x(1-3y)$

$\Rightarrow\int\frac{dy}{(1-3y)}=\int\sec^2 x\,dx$

$\Rightarrow-\frac{1}{3}\ln\left|1-3y\right|=\tan x+C$ ...... (i)

$\because y\left(\frac{\pi}{4}\right)=\frac{4}{3}\left(\text{ Given }\right)$

$\Rightarrow-\frac{1}{3}\ln\left|1-4\right|=\tan\frac{\pi}{4}+C$

$\Rightarrow-\frac{1}{3}\ln 3=C+1\Rightarrow C=-1-\frac{1}{3}\ln 3$

$\therefore$ in eq. (i), we get

$\frac{-1}{3}\ln\left|1-3y\right|=\tan x-1-\frac{1}{3}\ln 3$

Put, $x=-\frac{\pi}{4}$

$\Rightarrow-\frac{1}{3}\ln\left|1-3y\right|=\tan\left(-\frac{\pi}{4}\right)-1-\frac{1}{3}\ln 3$

$=-1-1-\frac{1}{3}\ln 3$

$\Rightarrow\ln|1-3y|=6+\ln 3$

$\Rightarrow\ln|13-y|=6\Rightarrow\left|\frac{1}{3}-y\right|=e^6\Rightarrow y=\frac{1}{3}\pm e^6$

---------------------------------------------------------------------------------------------------

# Question148

## The curve amongst the family of curves represented by the differential equation, $(x^2-y^2)dx+2xy\,dy=0$ which passes through (1, 1), is: [Jan. 10, 2019 (II)]

**Options:**

A. a circle with centre on the x-axis.

B. an ellipse with maor axis along the y-axis.

C. a circle with centre on the y-axis.

D. a hyperbola with transverse axis along the x-axis

**Answer: A**

## Solution:

**Solution:**
$(x^2 - y^2)dx + 2xydy = 0$
$y^2dx - 2xydy = x^2dx$
$2xydy - y^2dx = -x^2dx$
$d(xy^2) = -x^2dx$
$\frac{xd(y^2) - y^2d(x)}{x^2} = -dx$
$d\left(\frac{y^2}{x}\right) = -dx$
$\int d\left(\frac{y^2}{x}\right) = -\int dx$
$\frac{y^2}{x} = -x + C$
Since, the above curve passes through the point (1,1)
Then, $\frac{1^2}{1} = -1 + C \Rightarrow C = 2$
Now, the curve (1) becomes
$y^2 = -x^2 + 2x$
$\Rightarrow y^2 = -(x-1)^2 + 1$
$(x-1)^2 + y^2 = 1$
The above equation represents a circle with centre (1,0) and centre lies on $x$ -axis.

---------------------------------------------------------------------------------------------------

# Question149

## Let f be a differentiable function such that $f'(x) = 7 - \frac{3}{4}\frac{f(x)}{x}$, $(x > 0)$ and $f(1) \neq 4$. Then $\lim_{x \to 0^+} xf\left(\frac{1}{x}\right)$

## [Jan. 10, 2019 (II)]

**Options:**

A. exists and equals $\frac{4}{7}$.

B. exists and equals 4 .

C. does not exist.

D. exists and equals 0 .

**Answer: B**

## Solution:

**Solution:**
Let $y = f(x)$
$\frac{dy}{dx} + \left(\frac{3}{4x}\right)y = 7$
I.F. $= e^{\int \frac{3}{4x}dx} = e^{\frac{3}{4}\ln x} = x^{\left(\frac{3}{4}\right)}$
Solution of differential equation

$y \cdot x^{\frac{3}{4}} = \int 7 \cdot x^{\frac{3}{4}} dx + C$

$y \cdot x^{\frac{3}{4}} = 7 \cdot \frac{x^{\frac{7}{4}}}{\left(\frac{7}{4}\right)} + C \;= 4x^{\frac{7}{4}} + C$

$y = 4x + Cx^{-\frac{3}{4}}$

$\Rightarrow f\left(\frac{1}{x}\right) = \frac{4}{x} + Cx^{\frac{3}{4}}$

$\Rightarrow \lim_{x \to 0^-} x \cdot f\left(\frac{1}{x}\right) = \lim_{x \to 0^+} \left(4 + Cx^{\frac{7}{4}}\right) = 4$

---

# Question150

## The solution of the differential equation, $\frac{dy}{dx} = (x - y)^2$, when $y(1) = 1$, is: [Jan. 11, 2019(II)]

### Options:

A. $\log_e \left| \frac{2 - x}{2 - y} \right| = x - y$

B. $-\log_e \left| \frac{1 - x + y}{1 + x - y} \right| = 2(x - 1)$

C. $-\log_e \left| \frac{1 + x - y}{1 - x + y} \right| = x + y - 2$

D. $\log_e \left| \frac{2 - y}{2 - x} \right| = 2(y - 1)$

**Answer: B**

## Solution:

**Solution:**
The given differential equation

$\frac{dy}{dx} = (x - y)^2$ ........(1)

Let $x - y = t \Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$

Now, from equation (1)

$\left(1 - \frac{dt}{dx}\right) = (t)^2$

$\Rightarrow 1 - t^2 = \frac{dt}{dx} \Rightarrow \int dx = \int \frac{dt}{1 - t^2}$

$\Rightarrow -x = \frac{1}{2 \times 1} \ln \left| \frac{t - 1}{t + 1} \right| + c$

$\Rightarrow -x = \frac{1}{2} \ln \left| \frac{x - y - 1}{x - y + 1} \right| + c$

$\because$ The given condition $y(1) = 1$

$-1 = \frac{1}{2} \ln \left| \frac{1 - 1 - 1}{1 - 1 + 1} \right| + c \Rightarrow c = -1$

Hence, $2(x - 1) = -\ln \left| \frac{1 - x + y}{1 - y + x} \right|$

------------------------------------------------------------------------------------------------

# Question151

**If y(x) is the solution of the differential equation $\frac{dy}{dx}+\left(\frac{2x+1}{x}\right)y=e^{-2x}$, $x>0$, where $y(1)=\frac{1}{2}e^{-2}$, then:**
**[Jan. 11, 2019 (I)]**

**Options:**

A. $y(\log_e 2)=\log_e 4$

B. $y(\log_e 2)=\frac{\log_e 2}{4}$

C. y(x) is decreasing in $\left(\frac{1}{2},1\right)$

D. y(x) is decreasing in (0,1)

**Answer: C**

## Solution:

**Solution:**
Given differential equation is,
$\frac{dy}{dx}+\left(2+\frac{1}{x}\right)y=e^{-2x}, x>0$
$IF = e^{\int\left(2+\frac{1}{x}\right)dx} = e^{2x+\ln x}=xe^{2x}$
Complete solution is given by
$y(x).xe^{2x}=\int xe^{2x}.e^{-2x}dx+c$
$=\int x dx+c$
$y(x).e^{2x}.x=\frac{x^2}{2}+c$
Given, $y(1)=\frac{1}{2}e^{-2}$
$\therefore \frac{1}{2}e^{-2}.e^2.1 = \frac{1}{2}+c \Rightarrow c=0$
$\therefore y(x)=\frac{x^2}{2}.\frac{e^{-2x}}{x}$
$y(x)=\frac{x}{2}.e^{-2x}$
Differentiate both sides with respect to x,
$y'(x)=\frac{e^{-2x}}{2}(1-2x)<0\ \forall x\in\left(\frac{1}{2},1\right)$
Hence, y(x) is decreasing in $\left(\frac{1}{2},1\right)$

------------------------------------------------------------------------------------------------

# Question152

**Let y = y(x) be the solution of the differential equation,**
**$x\frac{dy}{dx}+y=x\log_e x$, $(x>1)$. If $2y(2)=\log_e 4-1$, then y(e) is equal to :**
**[Jan. 12, 2019 (I)]**

**Options:**

A. $-\frac{e}{2}$

B. $-\frac{e^2}{2}$

C. $\frac{e}{4}$

D. $\frac{e^2}{4}$

**Answer: C**

## Solution:

**Solution:**

Consider the differential equation,

$\frac{dy}{dx} + \frac{y}{x} = \log_e x$

$\because IF = e^{\int \frac{1}{x} dx} = x$

$\therefore yx = \int x \ln x \, dx$

$\Rightarrow xy = \ln x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$

$\Rightarrow xy = \frac{x^2}{2} \cdot \ln x - \frac{x^2}{4} + c$

Given, $2y(2) = \log_e 4 - 1$

$\therefore 2y = 2 \ln 2 - 1 + c$

$\Rightarrow \ln 4 - 1 = \ln 4 - 1 + c$

i.e. $c = 0$

$\Rightarrow xy = \frac{x^2}{2} \ln x - \frac{x^2}{4}$

$\Rightarrow y = \frac{x}{2} \ln x - \frac{x}{4}$

$\Rightarrow y(e) = \frac{e}{2} - \frac{e}{4} = \frac{e}{4}$

---------------------------------------------------------------------------------------------------

# Question153

## If a curve passes through the point (1,-2) and has slope of the tangent at any point (x, y) on it as $\frac{x^2 - 2y}{x}$, then the curve also passes through the point : [Jan. 12, 2019 (II)]

**Options:**

A. (3,0)

B. $(\sqrt{3}, 0)$

C. (-1,2)

D. $(-\sqrt{2}, 1)$

**Answer: B**

## Solution:

**Solution:**

$\because$ Slope of the tangent $= \frac{x^2 - 2y}{x}$

$\because \frac{dy}{dx} = \frac{x^2 - 2y}{x}$

$\frac{dy}{dx} + \frac{2}{x}y = x$

$I.F = e^{\int \frac{2}{x}} = e^{2\ln x} = x^2$

Solution of equation

$y . x^2 = \int x . x^2 dx$

$x^2 y = \frac{x^4}{4} + C$

$\therefore$ curve passes through point (1,-2)

$(1)^2(-2) = \frac{1^4}{4} + C$

$\Rightarrow C = \frac{-9}{4}$

Then, equation of curve

$y = \frac{x^2}{4} - \frac{9}{4x^2}$

Since, above curve satisfies the point.

Hence, the curve passes through $(\sqrt{3}, 0)$

----------------------------------------------------------------------------------------------------

# Question154

**The general solution of the differential equation $(y^2 - x^3) dx - xy dy = 0 (x \neq 0)$ is:**
**(where c is a constant of integration)**
**[April 12, 2019 (II)]**

**Options:**

A. $y^2 - 2x^2 + cx^3 = 0$

B. $y^2 + 2x^3 + cx^2 = 0$

C. $y^2 + 2x^2 + cx^3 = 0$

D. $y^2 - 2x^3 + cx^2 = 0$

**Answer: B**

## Solution:

**Solution:**

Given differential equation can be written as,

$y^2 dx - xy dy = x^3 dx$

$\Rightarrow \frac{(y dx - x dy)y}{x^2} = x dx \Rightarrow -y d\left(\frac{y}{x}\right) = x dx$

$\Rightarrow -\frac{y}{x} . d\left(\frac{y}{x}\right) = dx \Rightarrow -\frac{1}{2}\left(\frac{y}{x}\right)^2 = x + c_1$

$\Rightarrow 2x^3 + cx^2 + y^2 = 0$ [Here, $c = 2c_1$ ]

----------------------------------------------------------------------------------------------------

# Question155

**If $\cos x \frac{dy}{dx} - y\sin x = 6x$, $\left(0 < x < \frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{3}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to:**
**[April. 09, 2019 (II)]**

**Options:**

A. $\frac{\pi^2}{2\sqrt{3}}$

B. $-\frac{\pi^2}{2}$

C. $-\frac{\pi^2}{2\sqrt{3}}$

D. $-\frac{\pi^2}{4\sqrt{3}}$

**Answer: C**

## Solution:

**Solution:**
$\cos x\, dy - (\sin x)y\, dx = 6x\, dx$
$\Rightarrow \int d(y\cos x) = \int 6x\, dx \Rightarrow y\cos x = 3x^2 + C$ .....(1)
Given, $y\left(\frac{\pi}{3}\right) = 0$
Putting $x = \frac{\pi}{3}$ and $y = 0$ in eq. (1), we get
$(10) \times \left(\frac{1}{2}\right) = \frac{3\pi^2}{9} + C \Rightarrow C\frac{-\pi^2}{3}$
So, from(1) $y\cos x = 3x^2 - \frac{\pi^2}{3}$
Now, put $x = \frac{\pi}{6}$ in the above equation,
$y\frac{\sqrt{3}}{2} = \frac{3\pi^2}{36} - \frac{\pi^2}{3} \Rightarrow \frac{\sqrt{3}y}{2} = \frac{-3\pi^2}{12} \Rightarrow y = \frac{-\pi^2}{2\sqrt{3}}$

---------------------------------------------------------------------------------------------------

# Question156

**Given that the slope of the tangent to a curve $y = y(x)$ at any point $(x, y)$ is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is:**
**[April. 08,2019 (II)]**

**Options:**

A. $x\log_e |y| = 2(x-1)$

B. $x\log_e |y| = -2(x-1)$

C. $x^2\log_e |y| = -2(x-1)$

D. $x\log_e |y| = x - 1$

**Answer: A**

## Solution:

**Solution:**
Given $\frac{dy}{dx}=\frac{2y}{x^2}$

Integrating both sides, $\int\frac{dy}{y}=2\int\frac{dx}{x^2}$

$\Rightarrow \ln|y| = -\frac{2}{x}+C$

Equation (i) passes through the centre of the circle $x^2+y^2-2x-2y=0$, i.e. ,$(1, 1)$
$\therefore C=2$

Now, $\ln|y| = -\frac{2}{x}+2$

$x\ln|y| = -2(1-x) \Rightarrow x\ln|y| = 2(x-1)$

-----------------------------------------------------------------------------------------------

# Question157

### Consider the differential equation, $y^2dx+\left(x-\frac{1}{y}\right)dy=0$If value of y is 1 when x = 1, then the value of x for which y = 2, is : [April 12, 2019 (I)]

**Options:**

A. $\frac{5}{2}+\frac{1}{\sqrt{e}}$

B. $\frac{3}{2}-\frac{1}{\sqrt{e}}$

C. $\frac{1}{2}+\frac{1}{\sqrt{e}}$

D. $\frac{3}{2}-\sqrt{e}$

**Answer: B**

## Solution:

**Solution:**
Consider the differential equation,

$y^2dx+\left(x-\frac{1}{y}\right)dy=0$

$\Rightarrow\frac{dx}{dy}+\left(\frac{1}{y^2}\right)x=\frac{1}{y^3}$

$I.F.=e^{\int\frac{1}{y^2}dy}=e^{-\frac{1}{y}}$

$\therefore x.e^{-\frac{1}{y}}=\int e^{-\frac{1}{y}}\frac{1}{y^3}dy+c$

Put $-\frac{1}{y}=u\Rightarrow\frac{1}{y^2}dy=du$

$\Rightarrow x.e^{-\frac{1}{y}}=-\int ue^u du+c = -ue^u+e^u+c$

$\Rightarrow x.e^{-\frac{1}{y}}=e^{-\frac{1}{y}}\left(\frac{1}{y}+1\right)+c$

At $y=1$, $x=1$

$1 = 2 + ce \Rightarrow c = -\frac{1}{e} \Rightarrow x = \left(1 + \frac{1}{y}\right) - \frac{1}{e}e^{\frac{1}{y}}$

On putting y = 2, we get $x = \frac{3}{2} - \frac{1}{\sqrt{e}}$

----------------------------------------------------------------------------------------------------

# Question158

## If y = y(x) is the solution of the differential equation $\frac{dy}{dx} = (\tan x - y)\sec^2 x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y(0) = 0then $y\left(-\frac{\pi}{4}\right)$ is equal to:

## [April 10, 2019 (I)]

**Options:**

A. $e - 2$

B. $\frac{1}{2} - e$

C. $2 + \frac{1}{e}$

D. $\frac{1}{e} - 2$

**Answer: A**

## Solution:

**Solution:**

$\frac{dy}{dx} + y\sec^2 x = \sec^2 \times \tan x$

Given equation is linear differential equation.

$I\,F = e^{\int \sec^2 x dx} = e^{\tan x}$

$\Rightarrow y . e^{\tan x} = \int e^{\tan x} \sec^2 \times \tan x d\,x$

Put $\tan x = u = \sec^2 x d x = d u$

$ye^{\tan x} = \int e^u u d u \Rightarrow ye^{\tan x} = ue^u - e^u + c$

$\Rightarrow ye^{\tan x} = (\tan x - 1)e^{\tan x} + c$

$\Rightarrow y = (\tan x - 1) + c . e^{-\tan x}$

$\therefore y(0) = 0$ (given ) $\Rightarrow 0 = -1 + c \Rightarrow c = 1$

Hence, solution of differential equation,

$y\left(-\frac{\pi}{4}\right) = -1 - 1 + e = -2 + e$

----------------------------------------------------------------------------------------------------

# Question159

## Let y = y(x) be the solution of the differential equation, $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, such that y = 1. Then :

## [April 10, 2019 (II)]

**Options:**

A. $y\left(\frac{\pi}{4}\right)+y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{2}+2$

B. $y'\left(\frac{\pi}{4}\right)+y'\left(-\frac{\pi}{4}\right)=-\sqrt{2}$

C. $y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=\sqrt{2}$

D. $y'\left(\frac{\pi}{4}\right)-y'\left(-\frac{\pi}{4}\right)=\pi-\sqrt{2}$

**Answer: D**

## Solution:

**Solution:**
Given differential equation is,
$\frac{dy}{dx}+y\tan x=2x+x^2\tan x$
Here, $P=\tan x$, $Q=2x+x^2\tan x$
I.F. $=e^{\int\tan x\,dx}=e^{\ln|\sec x|}=|\sec x|$
$\therefore y(\sec x)=\int(2x+x^2\tan x)\sec x\,dx$
$=\int x^2\tan x\sec x\,dx+\int 2x\sec x\,dx=x^2\sec x+c$
Given $y(0)=1\Rightarrow c=1$
$\therefore y=x^2+\cos x$
Now put $x=\frac{\pi}{4}$ and $x=\frac{-\pi}{4}$ in equation (i),
$y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$ and $y\left(-\frac{\pi}{4}\right)=\frac{\pi^2}{16}+\frac{1}{\sqrt{2}}$
$\Rightarrow y\left(\frac{\pi}{4}\right)-y\left(-\frac{\pi}{4}\right)=0$
$\frac{dy}{dx}=2x-\sin x$
$\therefore y'\left(\frac{\pi}{4}\right)=\frac{\pi}{2}-\frac{1}{\sqrt{2}}$ and $y'\left(-\frac{\pi}{4}\right)=-\frac{\pi}{2}+\frac{1}{\sqrt{2}}$
$\Rightarrow y'\left(\frac{\pi}{4}\right)-y'\left(-\frac{\pi}{4}\right)=\pi-\sqrt{2}$

----------------------------------------------------------------------------------------------------

# Question160

**The solution of the differential equation $x\frac{dy}{dx}+2y=x^2$ $(x\neq 0)$ with $y(1)=1$, is:**
**[April 09, 2019 (I)]**

**Options:**

A. $y=\frac{4}{5}x^3+\frac{1}{5x^2}$

B. $y=\frac{x^3}{5}+\frac{1}{5x^2}$

C. $y=\frac{x^2}{4}+\frac{3}{4x^2}$

D. $y=\frac{3}{4}x^2+\frac{1}{4x^2}$

**Answer: C**

## Solution:

**Solution:**

$\frac{dy}{dx}+\frac{2}{x}y=xy(1)=1(\text{ given })$

Since, the above differential equation is the linear differential equation, then $I.F=e^{\int\frac{2}{x}dx}=x^2$

Now, the solution of the linear differential equation

$y\times x^2=\int x^3 dx$

$\Rightarrow yx^2=\frac{x^4}{4}+C\because y(1)=1$

$\therefore 1\times 1=\frac{1}{4}+C\Rightarrow C=\frac{3}{4}$

∴ Solution becomes.

$y=\frac{x^2}{4}+\frac{3}{4x^2}$

----------------------------------------------------------------------------------------------------

# Question161

## Let y = y(x) be the solution of the differential equation, $(x^2+1)^2\frac{dy}{dx}+2x(x^2+1)y=1$ such that $y(0)=0$. If $\sqrt{a}y(1)=\frac{\pi}{32}$, then the value of ´a ' is: quad [April [April 08, 2019 (I)]

**Options:**

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. $\frac{1}{16}$

**Answer: D**

## Solution:

**Solution:**

$(x^2+1)^2\frac{dy}{dx}+2x(x^2+1)y=1$

$\Rightarrow\frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{1}{(1+x^2)^2}$

Since, the above differential equation is a linear differential equation

$\therefore$ I.F. $=\int e^{\int 2x.1+x^2dx}=e^{\log(1+x^2)}=1+x^2$

Then, the solution of the differential equation

$\Rightarrow y(1+x^2)=\int\frac{dx}{1+x^2}+c$

$\Rightarrow y(1+x^2)=\tan^{-1}x+c$ ....(1)

If x = 0 then y = 0 (given)

$\Rightarrow 0=0+c$

$\Rightarrow c=0$

Then, equation (1) becomes,

$\Rightarrow y(1+x^2)=\tan^{-1}x$

Now put x = 1 in above equation, then

$2y = \frac{\pi}{4}$

$\Rightarrow 2\left(\frac{\pi}{32\sqrt{a}}\right) = \frac{\pi}{4}\left[\sqrt{a}y(1) = \frac{\pi}{32}\right]$

$\Rightarrow \sqrt{a} = \frac{1}{4}$

$\Rightarrow a = \frac{1}{16}$

---------------------------------------------------------------------------------------------------

# Question162

## The differential equation representing the family of ellipse having foci either on the x -axis or on the y-axis centre at the origin and passing through the point (0,3) is:
## [Online April 16, 2018]

**Options:**

A. $xyy' + y^2 - 9 = 0$

B. $x + yy'' = 0$

C. $xyy'' + x(y')^2 - yy' = 0$

D. $xyy' - y^2 + 9 = 0$

**Answer: C**

## Solution:

**Solution:**

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since, it passes through (0,3)

$\therefore \frac{0}{a^2} + \frac{9}{b^2} = 1$

$\Rightarrow b^2 = 9$

$\therefore$ eq. of ellipse becomes:

$\frac{x^2}{a^2} + \frac{y^2}{9} = 1$

differential w.r.t. x, we get;

$\frac{2x}{a^2} + \frac{2y}{9}\frac{dy}{dx} = 0$

$\Rightarrow \frac{y}{x}\left(\frac{dy}{dx}\right) = \frac{-9}{a^2}$

Again differentiating w.r.t. x, we get;

$\frac{y}{x}\frac{d^2y}{dx^2} + \frac{x\frac{dy}{dx} - y}{x^2}\frac{dy}{dx} = 0$

$\Rightarrow xyy'' + x(y')^2 - yy' = 0$

---------------------------------------------------------------------------------------------------

# Question163

## The curve satisfying the differential equation, $(x^2 - y^2)dx + 2xydy = 0$ and passing through the point (1,1) is

**[Online April 15, 2018]**

**Options:**

A. a circle of radius two

B. a circle of radius one

C. a hyprbola

D. an ellipse

**Answer: B**

## Solution:

**Solution:**

$(x^2 - y^2)dx + 2xydy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

Let $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2vx^2} \Rightarrow v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{-v^2 - 1}{2v}$

$\Rightarrow \frac{2vdv}{v^2 + 1} = -\frac{dx}{x}$

After integrating, we get

$\ln|v^2 + 1| = -\ln|x| + \ln c$

$\frac{y^2}{x^2} + 1 = \frac{c}{x}$

As curve passes through the point (1, 1), so $1 + 1 = c$

$\Rightarrow c = 2$

$x^2 + y^2 - 2x = 0$, which is a circle of radius one.

---------------------------------------------------------------------------------------------------

# Question164

**Let $y - y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y\cos x = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then$y\left(\frac{\pi}{6}\right)$ is equal to: [2018]**

**Options:**

A. $\frac{-8}{9\sqrt{3}}\pi^2$

B. $-\frac{8}{9}\pi^2$

C. $-\frac{4}{9}\pi^2$

D. $\frac{4}{9\sqrt{3}}\pi^2$

**Answer: B**

## Solution:

**Solution:**
Consider the given differential equation the
$\sin x\, d\ y + y \cos x\, d\ x = 4x d\, x$
$\Rightarrow d\,(y\,.\,\sin x) = 4x d\, x$
Integrate both sides
$\Rightarrow y\,.\,\sin x = 2x^2 + C$ .......(1)
$\Rightarrow y(x) = \frac{2x^2}{\sin x} + c$ ........(2)
$\because$ eq. (2) passes through $\left(\frac{\pi}{2}, 0\right)$
$\Rightarrow 0 = \frac{\pi^2}{2} + C$
$\Rightarrow C = -\frac{\pi^2}{2}$
Now, put the value of C in (1)
Then, $y \sin x = 2x^2 - \frac{\pi^2}{2}$ is the solution
$\therefore y\left(\frac{\pi}{6}\right) = \left(2\,.\,\frac{\pi^2}{36} - \frac{\pi^2}{2}\right)2 \ = -\frac{8\pi^2}{9}$

----------------------------------------------------------------------------------------------------

# Question165

## Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 2y = f(x)$,

## where $f(x) = \begin{cases} 1, & x \in [0, 1] \\ 0, & \text{otherwise} \end{cases}$

## If $y(0) = 0$, then $y\left(\frac{3}{2}\right)$ is

## [Online April 15, 2018]

### Options:

A. $\frac{e^2 - 1}{2e^3}$

B. $\frac{e^2 - 1}{e^3}$

C. $\frac{1}{2e}$

D. $\frac{e^2 + 1}{2e^4}$

**Answer: A**

## Solution:

**Solution:**
When $x \in [0, 1]$, then $\frac{dy}{dx} + 2y = 1 \Rightarrow y = \frac{1}{2} + C_1 e^{-2x}$
$\because y(0) = 0 \Rightarrow y(x) = \frac{1}{2} - \frac{1}{2}e^{-2x}$
Here ,$y(1) = \frac{1}{2} - \frac{1}{2}e^{-2} = \frac{e^2 - 1}{2e^2}$

When $x \notin [0, 1]$, then $\frac{dy}{dx} + 2y = 0 \Rightarrow y = c_2 e^{-2x}$

$\because y(1) = \frac{e^2 - 1}{2} \Rightarrow \frac{e^2 - 1}{2} = c^2 e^{-2} \Rightarrow C_2 = \frac{e^2 - 1}{2}$

$\therefore y(x) \left( \frac{e^2 - 1}{2} \right) e^{-2x} \Rightarrow y\left( \frac{3}{2} \right) = \frac{e^2 - 1}{2e^3}$

----------------------------------------------------------------------------------------------------

# Question166

**If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y\left( \frac{\pi}{2} \right)$ is equal to : [2017]**

**Options:**

A. $\frac{4}{3}$

B. $\frac{1}{3}$

C. $-\frac{2}{3}$

D. $-\frac{1}{3}$

**Answer: B**

## Solution:

**Solution:**

We have $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$

$\Rightarrow \frac{d}{dx}(2 + \sin x)(y + 1) = 0$

On integrating, we get

$(2 + \sin x)(y + 1) = C$

At $x = 0$, $y = 1$ we have

$(2 + \sin 0)(1 + 1) = C$

$\Rightarrow C = 4$

$\Rightarrow y + 1 = \frac{4}{2 + \sin x}$

$y = \frac{4}{2 + \sin x} - 1$

Now $y\left( \frac{\pi}{2} \right) = \frac{4}{2 + \sin \frac{\pi}{2}} - 1 = \frac{4}{3} - 1 = \frac{1}{3}$

----------------------------------------------------------------------------------------------------

# Question167

**The curve satisfying the differential equation, $y\,dx - (x + 3y^2)\,dy = 0$ and passing through the point (1, 1), also passes through the point : [Online April 8, 2017]**

**Options:**

A. $\left(\frac{1}{4}, -\frac{1}{2}\right)$

B. $\left(-\frac{1}{3}, \frac{1}{3}\right)$

C. $\left(\frac{1}{3}, -\frac{1}{3}\right)$

D. $\left(\frac{1}{4}, \frac{1}{2}\right)$

**Answer: B**

## Solution:

**Solution:**
$y\,dx - x\,dy - 3y^2\,dy = 0$
$\Rightarrow \frac{dx}{dy} = \frac{x}{y} + 3y \Rightarrow \frac{dx}{dy} - \frac{x}{y} = 3y$
if $= e^{-\int \frac{1}{y} dy} = e^{-\ln y} = \frac{1}{y}$
$\therefore$ solution is $\frac{x}{y} = \int 3y \cdot \frac{1}{y} dy$
$\Rightarrow \frac{x}{y} = 3y + c$
which passes through (1,1)
$\therefore 1 = 3 + c \Rightarrow c = -2$
$\therefore$ solution becomes $\Rightarrow x = 3y^2 - 2y$
which also passes through $\left(-\frac{1}{3}, \frac{1}{3}\right)$

---------------------------------------------------------------------------------------------------------

# Question168

## If a curve $y = f(x)$ passes through the point (1,-1) and satisfies the differential equation, $y(1 + xy)\,dx = x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to: [2016]

**Options:**

A. $\frac{2}{5}$

B. $\frac{4}{5}$

C. $-\frac{2}{5}$

D. $-\frac{4}{5}$

**Answer: B**

## Solution:

**Solution:**
$y(1 + xy)\,dx = x\,dy$

$\frac{xdy - ydx}{y^2} = xdx$

$\Rightarrow \int -d\left(\frac{x}{y}\right) = \int xdx$

$-\frac{x}{y} = \frac{x^2}{2} + C$ as $y(1) = -1 \Rightarrow C = \frac{1}{2}$

Hence, $y = \frac{-2x}{x^2+1} \Rightarrow f\left(\frac{-1}{2}\right) = \frac{4}{5}$

---------------------------------------------------------------------------------------------------

# Question169

**If $f(x)$ is a differentiable function in the interval $((0, \infty)$ such that $f(a) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$, for each $x > 0$,then $f\left(\frac{3}{2}\right)$ is equal to:**
**[Online April 9, 2016]**

**Options:**

A. $\frac{23}{18}$

B. $\frac{13}{6}$

C. $\frac{25}{9}$

D. $\frac{31}{18}$

**Answer: D**

## Solution:

**Solution:**

Let $L = \lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

Applying L.H. rule

$L = \lim_{t \to x} \frac{2tf(x) - x^2 f'(t)}{1} = 1$

$2xf(x) - x^2 f'(x) = 1$

solving above differential equation, we get

$f(x) = \frac{2}{3}x^2 + \frac{1}{3x}$

Put $x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = \frac{2}{3} \times \left(\frac{3}{2}\right)^2 + \frac{1}{3} \times \frac{2}{3} = \frac{3}{2} + \frac{2}{9} = \frac{27+4}{18} = \frac{31}{18}$

---------------------------------------------------------------------------------------------------

# Question170

**The solution of the differential equaiton $\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y}$, where $0 \le x < \frac{\pi}{2}$, and $y(0) = 1$, is given by :**
**[Online April 10, 2016]**

**Options:**

A. $y^2 = 1 + \frac{x}{\sec x + \tan x}$

B. $y = 1 + \frac{x}{\sec x + \tan x}$

C. $y = 1 - \frac{x}{\sec x + \tan x}$

D. $y^2 = 1 - \frac{x}{\sec x + \tan x}$

**Answer: D**

## Solution:

**Solution:**
$\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y}$
$2y\frac{dy}{dx} + y^2 \sec x = \tan x$
Put $y^2 = t \Rightarrow 2y\frac{dy}{dx} = \frac{dt}{dx}$
$\frac{dt}{dx} + t\sec x = \tan x$
$I f = e^{\int \sec x\, dx} = e^{\ln(\sec x + \tan x)} = \sec x + \tan x$
$\frac{dt}{dx}(\sec x + \tan x) + t\sec x(\sec x + \tan x)$
$= \tan x(\sec x + \tan x)$
$\int d(t(\sec x + \tan x)) = \int \tan x(\sec x + \tan x)\, dx$
$t(\sec x + \tan x) = \sec x + \tan x - x$
$t = 1 - \frac{x}{\sec x + \tan x} \Rightarrow y^2 = 1 - \frac{x}{\sec x + \tan x}$

----------------------------------------------------------------------------------------------------

# Question171

## The solution of the differential equation $y\,dx - (x + 2y^2)\, dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then $f(a)$ is equal to : [Online April 11, 2015]

**Options:**

A. 4

B. 3

C. 1

D. 2

**Answer: B**

## Solution:

**Solution:**
Given differential equation is
$y\,dx - (x + 2y^2)dy = 0$
$\Rightarrow y\,dx - x\,dy - 2y^2 dy = 0$
$\Rightarrow \frac{y\,dx - x\,dy}{y^2} = 2dy$

$\Rightarrow d\left(\frac{x}{y}\right) = 2dy$

Integrate both the side

$\Rightarrow \frac{x}{y} = 2y + c$

using $f(-1) = 1$, we get

$c = 1$

$\Rightarrow \frac{x}{y} = 2y + 1$

put $y = 1$, we get $f(a) = 3$

------------------------------------------------------------------------------------------------

# Question172

## If y(x) is the solution of the differential equation$(x+2)\frac{dy}{dx} = x^2 + 4x - 9$, $x \neq -2$ and $y(0) = 0$, then $y(-4)$is equal to :
## [Online April 10, 2015]

### Options:

A. 0

B. 2

C. 1

D. -1

**Answer: A**

## Solution:

**Solution:**

$(x+2)\frac{dy}{dx} = x^2 + 4x - 9x \neq -2$

$\frac{dy}{dx} = \frac{x^2 + 4x - 9}{x+2}$

$dy = \frac{x^2 + 4x - 9}{x+2}dx$

$\int dy = \int \frac{x^2 + 4x - 9}{x+2}dx$

$y = \int \left(x + 2 - \frac{13}{x+2}\right)dx$

$y = \int (x+2)dx - 13\int \frac{1}{x+2}dx$

$y = \frac{x^2}{2} + 2x - 13\log\left|x+2\right| + c$

Given that $y = (0) = 0$

$0 = -13\log 2 + c$

$y = \frac{x^2}{2} + 2x - 13\log|x+2| + 13\log 2$

$y(-4) = 8 - 8 - 13\log 2 + 13\log 2 = 0$

------------------------------------------------------------------------------------------------

# Question173

## Let y(x) be the solution of the differential equation

$(x \log x)\frac{dy}{dx} + y = 2x \log x,\ (x \geq 1)$. **Then y(e) is equal to:**
**[2015]**

**Options:**

A. 2

B. 2e

C. e

D. 0

**Answer: A**

## Solution:

**Solution:**
Given, $\frac{dy}{dx} + \left(\frac{1}{x \log x}\right) y = 2$
$I.F. = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$
$y . \log x = \int 2 \log x \, dx + c$
$y \log x = 2[x \log x - x] + c$
Put $x = 1, y.0 = -2 + c$
$c = 2$
Put $x = e$
$y \log e = 2e(\log e - 1) + c$
$y(e) = c = 2$

------------------------------------------------------------------------------------------------

# Question174

## If the differential equation representing the family of all circles touching x -axis at the origin is $(x^2 - y^2)\frac{dy}{dx} = g(x)y$, then g(x) equals:
## [Online April 9, 2014]

**Options:**

A. $\frac{1}{2}x$

B. $2x^2$

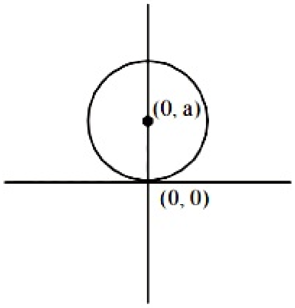
C. $2x$

D. $\frac{1}{2}x^2$

**Answer: C**

## Solution:

**Solution:**
Since family of all circles touching x-axis at the origin




$\therefore$ Eqn is $(x)^2 + (y - a)^2 = a^2$
where (0, a) is the centre of circle.
$\Rightarrow x^2 + y^2 + a^2 - 2ay = a^2$
$\Rightarrow x^2 + y^2 - 2ay = 0$ .......(1)
Differentiate both side w.r.t 'x', we get
$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0$
$\Rightarrow x + y\frac{dy}{dx} - a\frac{dy}{dx} = 0$
$\Rightarrow \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}} = a$
Put value of 'a' in eqn (1), we get
$x^2 + y^2 - 2y\left[\frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}\right] = 0$
$\Rightarrow (x^2 + y^2)\frac{dy}{dx} - 2y^2\frac{dy}{dx} - 2xy = 0$
$\Rightarrow (x^2 + y^2 - 2y^2)\frac{dy}{dx} = 2xy$
$\Rightarrow (x^2 - y^2)\frac{dy}{dx} = 2xy \equiv g(x)y$
Hence, $g(x) = 2x$

----------------------------------------------------------------------------------------------------

# Question175

## Let the population of rabbits surviving at time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If p(0) = 100, then p(t) equals: [2014]

**Options:**

A. $600 - 500e^{t/2}$

B. $400 - 300e^{-t/2}$

C. $400 - 300e^{t/2}$

D. quad $300 - 200e^{-t/2}$

**Answer: C**

## Solution:

**Solution:**
Given differential equation is
$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$

By separating the variable, we get

$d\,p(t) = \left[\frac{1}{2}p(t) - 200\right] d\,t$

$\Rightarrow \frac{d\,p(t)}{\frac{1}{2}p(t) - 200} = d\,t$

Integrate on both the sides,

$\int \frac{d\,(p(t))}{\frac{1}{2}p(t) - 200} = \int d\,t$

Let $\frac{1}{2}p(t) - 200 = s \Rightarrow \frac{d\,p(t)}{2} = d\,s$

So, $\int \frac{d\,p(t)}{\left(\frac{1}{2}p(t) - 200\right)} = \int d\,t$

$\Rightarrow \int \frac{2d\,s}{s} = \int d\,t \Rightarrow 2\log s = t + c$

$\Rightarrow 2\log\left(\frac{p(t)}{2} - 200\right) = t + c$

$\Rightarrow \frac{p(t)}{2} - 200 = e^{\frac{t}{2}}k$

Using given condition $p(t) = 400 - 300e^{t/2}$

----------------------------------------------------------------------------------------------------

# Question176

## If the general solution of the differential equation $y' = \frac{y}{x} + \Phi\left(\frac{x}{y}\right)$, for some function $\Phi$, is given by $y\ln|cx| = x$, where c is an arbitrary constant, then $\Phi(2)$ is equal to: [Online April 11, 2014]

**Options:**

A. 4

B. $\frac{1}{4}$

C. -4

D. $-\frac{1}{4}$

**Answer: D**

## Solution:

**Solution:**

Given $\frac{d\,y}{d\,x} = \frac{y}{x} + \phi\left(\frac{y}{x}\right)$ .....(1)

Let $\left(\frac{y}{x}\right) = v$ so that $y = xv$

or $\frac{d\,y}{d\,x} = x\frac{d\,v}{d\,x} + v$ ......(2)

from (1) & (2), $x\frac{d\,v}{d\,x} + v = v + \phi\left(\frac{1}{v}\right)$

or, $\frac{d\,v}{\phi\left(\frac{1}{v}\right)} = \frac{d\,x}{x}$

Integrating both sides, we get

$\int \frac{dx}{x} = \int \frac{dv}{\phi\left(\frac{1}{v}\right)} \Rightarrow \ln x + c = \int \frac{dv}{\phi\left(\frac{1}{v}\right)}$ (where c being constant of integration)

But, given $y = \frac{x}{\ln|cx|}$ is the general solution

so that $\frac{x}{y} = \frac{1}{v} = \ln\left|cx\right| = \int \frac{dv}{\phi\left(\frac{1}{v}\right)}$

Differentiating w.r.t $v$ both sides, we get

$\phi\left(\frac{1}{v}\right) = \frac{-1}{v^2} \Rightarrow \phi\left(\frac{x}{y}\right) = -\frac{y^2}{x^2}$

when $\frac{x}{y} = 2$ i.e. $\phi(2) = -\left(\frac{y}{x}\right)^2 = -\left(\frac{1}{2}\right)^2 = \left(\frac{-1}{4}\right)$

-----------------------------------------------------------------------------------------------

# Question177

## If $\frac{dy}{dx} + y\tan x = \sin 2x$ and $y(0) = 1$, then $y(\pi)$ is equal to: [Online April 19, 2014]

**Options:**

A. 1

B. -1

C. -5

D. 5

**Answer: C**

## Solution:

**Solution:**

$\frac{dy}{dx} + y\tan x = \sin 2x$

$I.F = e^{\int \tan x\,dx} = e^{-\log\cos x} = \sec x$

Required solution is

$y(\sec x) = \int \sin 2x \sec x\,dx + c$

$y(\sec x) = \int \frac{2\sin x\cos x}{\cos x}dx + c$

$y(\sec x) = 2\int \sin x\,dx + c$

$y(\sec x) = -2\cos x + c$ .........(1)

Given $y(0) = 1$

$\therefore$ put $x = 0$ and $y = 1$, we get

$1(\sec 0) = -2\cos 0 + c$

$\Rightarrow c = 1 + 2 \Rightarrow c = 3$

$\therefore$ from eqn (1), we have

$y\sec x = -2\cos x + 3$

To find $y(\pi)$, put $x = \pi$ in eqn (2), we get

$y(\sec\pi) = -2\cos\pi + 3$

$y = -2(-1)(-1) + 3(-1) = -2 - 3 = -5$

-----------------------------------------------------------------------------------------------

# Question178

## The general solution of the differential equation, $\sin 2x\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$, is

**[Online April 12, 2014]**

**Options:**

A. $y\sqrt{\tan x} = x + c$

B. $y\sqrt{\cot x} = \tan x + c$

C. $y\sqrt{\tan x} = \cot x + c$

D. $y\sqrt{\cot x} = x + c$

**Answer: D**

## Solution:

**Solution:**

Given, $\sin 2x\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$

or , $\frac{dy}{dx} = \frac{y}{\sin 2x} + \sqrt{\tan x}$

or, $\frac{dy}{dx} - y \operatorname{cosec} 2x = \sqrt{\tan x}$ ......(1)

Now, integrating factor (I.F) $= e^{\int -\operatorname{cosec} 2x}$

or, $I.F = e^{-\frac{1}{2}\log|\tan x|} = e^{\log(\sqrt{\tan x})^{-1}}$

$= \frac{1}{\sqrt{\tan x}} = \sqrt{\cot x}$

Now, general solution of eq. (1) is written as

$y(I.F) = \int Q(I.F)dx + c$

$\therefore y\sqrt{\cot x} = \int \sqrt{\tan x}.\sqrt{\cot x}dx + c$

$\therefore y\sqrt{\cot x} = \int 1 . dx + c$

$\therefore y\sqrt{\cot x} = x + c$

---------------------------------------------------------------------------------------------------

# Question179

**Statement-1: The slope of the tangent at any point P on a parabola, whose axis is the axis of x and vertex is at the origin, is inversely proportional to the ordinate of the point P.**
**Statement-2: The system of parabolas $y^2 = 4ax$ satisfies a differential equation of degree 1 and order 1.**
**[Online April 9, 2013]**

**Options:**

A. Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for statement-1.

B. Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for statement-1.

C. Statement-1 is true; Statement-2 is false.

D. Statement-1 is false; Statement-2 is true.

**Answer: B**

## Solution:

**Solution:**
**Statement -1:** $y^2 = \pm 4ax$
$\Rightarrow \frac{dy}{dx} = \pm 2a \cdot \frac{1}{y} \Rightarrow \frac{dy}{dx} \propto \frac{1}{y}$
**Statement - 2:** $y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$
Thus both statements are true but statement- 2 is not a correct explanation for statement-1.

-----------------------------------------------------------------------------------------------

# Question180

## At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is [2013]

**Options:**

A. 2500

B. 3000

C. 3500

D. 4500

**Answer: C**

## Solution:

**Solution:**
Given, Rate of change is $\frac{dP}{dx} = 100 - 12\sqrt{x}$
$\Rightarrow dP = (100 - 12\sqrt{x})dx$
By integrating
$\int dP = \int (100 - 12\sqrt{x})dx$
$P = 100x - 8x^{3/2} + C$
Given when $x = 0$ then $P = 2000$
$\Rightarrow C = 2000$
Now when $x = 25$ then
$P = 100 \times 25 - 8 \times (25)^{\frac{3}{2}} + 2000 \; = 4500 - 1000$
$\Rightarrow P = 3500$

-----------------------------------------------------------------------------------------------

# Question181

## If a curve passes through the point $\left(2, \frac{7}{2}\right)$ and has slope $\left(1 - \frac{1}{x^2}\right)$ at any point (x, y) on it, then the ordinate of the point on the curve whose abscissa is -2 is : [Online April 23, 2013]

**Options:**

A. $-\frac{3}{2}$

B. $\frac{3}{2}$

C. $\frac{5}{2}$

D. $-\frac{5}{2}$

**Answer: A**

## Solution:

**Solution:**
Slope $= \frac{dy}{dx} = 1 - \frac{1}{x^2}$

$\Rightarrow \int dy = \int \left(1 - \frac{1}{x^2}\right) dx$

$\Rightarrow y = x + \frac{1}{x} + C$, which is the equation of the curve since curve passes through the point $\left(2, \frac{7}{2}\right)$

$\therefore \frac{7}{2} = 2 + \frac{1}{2} + C \Rightarrow C = 1$

$\therefore y = x + \frac{1}{x} + 1$

when $x = -2$, then $y = -2 + \frac{1}{-2} + 1 = \frac{-3}{2}$

----------------------------------------------------------------------------------------------------

# Question182

## Consider the differential equation :
$\frac{dy}{dx} = \frac{y^3}{2(xy^2 - x^2)}$

## Statement-1: The substitution $z = y^2$ transforms the above equation into a first order homogenous differential equation.

## Statement-2: The solution of this differential equation is $y^2 e^{-y^2/x} = C$ [Online April 22, 2013]

**Options:**

A. Both statements are false.

B. Statement- 1 is true and statement- 2 is false.

C. Statement- 1 is false and statement- 2 is true.

D. Both statements are true.

**Answer: D**

## Solution:

**Solution:**
Given differential equation is

$\frac{dy}{dx} = \frac{y^3}{2(xy^2 - x^2)}$

By substituting $z = y^2$, we get diff. eqn. as

$$\frac{dz}{dx} = \frac{2z^2}{2(xz - x^2)} = \frac{z^2}{xz - x^2}$$

Now, $\frac{dx}{dz} = \frac{x}{z} - \frac{x^2}{z^2} = \frac{x}{z}\left[1 - \frac{x}{z}\right] \approx F\left(\frac{x}{z}\right)$

Hence, statement- 1 is true.

Now, $y^2 e^{-y^2/x} = C$ satisfies the given diff. equation

∴ It is the solution of given diff. equation.

Thus, statement- 2 is also true.

----------------------------------------------------------------------------------------------------

# Question183

## The equation of the curve passing through the origin and satisfying the differential equation$(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$ is
## [Online April 25, 2013]

### Options:

A. $(1 + x^2)y = x^3$

B. $3(1 + x^2)y = 2x^3$

C. $(1 + x^2)y = 3x^3$

D. $3(1 + x^2)y = 4x^3$

### Answer: D

## Solution:

**Solution:**

Given differential equation is

$(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$

$\Rightarrow \frac{dy}{dx} + \left(\frac{2x}{1 + x^2}\right)y = \frac{4x^2}{1 + x^2}$

This is linear diff. equation

$I.F = e^{\int \frac{2x}{1 + x^2}dx} = e^{\log(1 + x^2)} = 1 + x^2$

∴ Solution is

$y(1 + x^2) = \int \frac{4x^2}{1 + x^2} \times 1 + x^2 + C$

$\Rightarrow y(1 + x^2) = \frac{4x^3}{3} + C$

⇒ Required curve is

$3y(1 + x^2) = 4x^3 (\because C = 0)$

----------------------------------------------------------------------------------------------------

# Question184

## Statement 1: The degrees of the differential equations$\frac{dy}{dx} + y^2 = x$ and $\frac{d^2y}{dx^2} + y = \sin x$ are equal.
## Statement 2: The degree of a differential equation, when it is a polynomial equation in derivatives, is the highest positive integral

**power of the highest order derivative involved in the differential equation, otherwise degree is not defined.**
**[Online May 12, 2012]**

**Options:**

A. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

B. Statement 1 is false, Statement 2 is true.

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.

**Answer: D**

## Solution:

**Solution:**
**Statement - 1**
Given differential equations are $\frac{dy}{dx} + y^2 = x$ and $\frac{d^2y}{dx^2} + y = \sin x$
Their degrees are 1 .
Both have equal degree.
Also, Statement -2 is the correct explanation for Statement -1

----------------------------------------------------------------------------------------------------

# Question185

## The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is: [2012]

**Options:**

A. $2\ln 18$

B. $\ln 9$

C. $\frac{1}{2}\ln 18$

D. $\ln 18$

**Answer: A**

## Solution:

**Solution:**
Given differential equation is
$\frac{dp(t)}{dt} = 0.5p(t) - 450$
$\Rightarrow \frac{dp(t)}{dt} = \frac{1}{2}p(t) - 450$
$\Rightarrow \frac{dp(t)}{dt} = \frac{p(t) - 900}{2}$

$\Rightarrow 2\frac{d\,p(t)}{d\,t} = -[900 - p(t)]$

$\Rightarrow 2\frac{d\,p(t)}{900 - p(t)} = -d\,t$

Integrate both the side, we get

$-2\int \frac{d\,p(t)}{900 - p(t)} = \int d\,t$

Let $900 - p(t) = u$

$\Rightarrow -d\,p(t) = d\,u$

$2\int \frac{d\,u}{u} = \int d\,t \Rightarrow 2\ln u = t + c$ ......(i)

$\Rightarrow 2\ln[900 - p(t)] = t + c$

Given $t = 0$, $p(0) = 850$

$2\ln(50) = c$

Putting in (i)

$\Rightarrow 2\left[\ln\left(\frac{900 - p(t)}{50}\right)\right] = t$

$\Rightarrow 900 - p(t) = 50e^{\frac{t}{2}}$

$\Rightarrow p(t) = 900 - 50e^{\frac{t}{2}}$

let $p(t_1) = 0$

$0 = 900 - 50e^{\frac{t_1}{2}} \therefore t_1 = 2\ln 18$

----------------------------------------------------------------------------------------------------

# Question186

## Let y(x) be a solution of $\frac{(2+\sin x)}{(1+y)}\frac{d\,y}{d\,x} = \cos x$. If $y(0) = 2$,then $y\left(\frac{\pi}{2}\right)$ equals [Online May 7, 2012]

**Options:**

A. $\frac{5}{2}$

B. 2

C. $\frac{7}{2}$

D. 3

**Answer: C**

## Solution:

**Solution:**

Given differential equation is

$\frac{(2+\sin x)}{(1+y)} \cdot \frac{d\,y}{d\,x} = \cos x$

which can be rewritten as

$\frac{d\,y}{1+y} = \frac{\cos x}{2+\sin x}d\,x$

Integrate both the sides, we get

$\int \frac{d\,y}{1+y} = \int \frac{\cos x\,d\,x}{2+\sin x}$

$\Rightarrow \log(1+y) = \log(2+\sin x) + \log C$

$\Rightarrow 1 + y = C(2+\sin x)$

Given $y(0) = 2$

$\Rightarrow 1 + 2 = C[2+\sin 0] \Rightarrow C = \frac{3}{2}$

Now, $y\left(\frac{\pi}{2}\right)$ can be found as

$1+y=\frac{3}{2}\left(2+\sin\frac{\pi}{2}\right) \Rightarrow 1+y=\frac{9}{2}$

$\Rightarrow y=\frac{7}{2}$

Hence, $y\left(\frac{\pi}{2}\right)=\frac{7}{2}$

-------------------------------------------------------------------------------------------

# Question187

## The integrating factor of the differential equation $(x^2-1)\frac{dy}{dx}+2xy=x$ is [Online May 26, 2012]

**Options:**

A. $\frac{1}{x^2-1}$

B. $x^2-1$

C. $\frac{x^2-1}{x}$

D. $\frac{x}{x^2-1}$

**Answer: B**

## Solution:

**Solution:**

Given differential equation is $(x^2-1)\frac{dy}{dx}+2xy=x$

$\Rightarrow \frac{dy}{dx}+\frac{2x}{x^2-1}.y=\frac{x}{x^2-1}$

This is in linear form.

Integrating factor $=\underset{e}{\int}\frac{2x}{x^2-1}dx=\underset{e}{\int}\frac{dt}{t}$ where $t=x^2-1$

$=e^{\log t}=x^2-1$

Hence, required integrating factor $=x^2-1$

-------------------------------------------------------------------------------------------

# Question188

## The general solution of the differential equation $\frac{dy}{dx}+\frac{2}{x}y=x^2$ is [Online May 19, 2012]

**Options:**

A. $y=cx^{-3}-\frac{x^2}{4}$

B. $y=cx^3-\frac{x^2}{4}$

C. $y=cx^2+\frac{x^3}{5}$

D. $y = cx^{-2} + \frac{x^3}{5}$

**Answer: D**

## Solution:

**Solution:**
Given differential equation is
$\frac{dy}{dx} + \frac{2}{x} . y = x^2$
This is of the linear form.
$\therefore P = \frac{2}{x}, Q = x^2$
$I . F = e^{\int \frac{2}{x} dx} = e^{\log x^2} = x^2$
Solution is
$y . x^2 = \int x^2 . x^2 dx + c = \frac{x^5}{5} + c$
$y = \frac{x^3}{5} + cx^{-2}$

-----------------------------------------------------------------------------------------------

# Question189

## The curve that passes through the point (2, 3), and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact is given by: [2011RS]

**Options:**

A. quad $2y - 3x = 0$

B. $y = \frac{6}{x}$

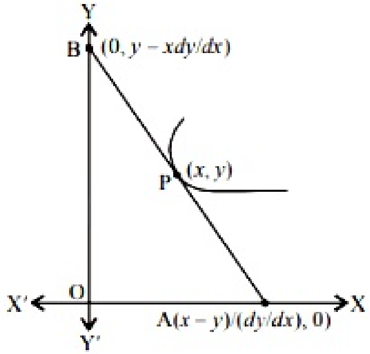
C. $x^2 + y^2 = 13$

D. $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$

**Answer: B**

## Solution:

Equation of tangent at P
$Y - y = \frac{dy}{dx}(X - x)$



X -intercept $= x - \frac{y}{dy/dx}$

Y -intercept $= y - \frac{x dy}{dx}$

Since P is mid-point of A and B

$x - \frac{y}{dy} = 2x$ and $y - \frac{x dy}{dx} = 2y$

$\Rightarrow \frac{-y}{\frac{dy}{dx}} = x$ and $\frac{-x dy}{dx} = y$

$\Rightarrow \frac{dx}{x} = -\frac{dy}{y}$

$\ln y = -\ln x + \ln c$

$y = \frac{c}{x}$

Since the above line passes through the point (2,3) .

$\therefore c = 6$

Hence $y = \frac{6}{x}$ is the required equation.

-----------------------------------------------------------------------------------------------

# Question190

**Let I be the purchase value of an equipment and V (t) be the value after it has been used for t years. The value V (t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T - t)$, where k > 0 is a constant and T is the total life in years of the equipment. Then the scrap value V (T ) of the equipment is [2011]**

**Options:**

A. $I - \frac{kT^2}{2}$

B. $I - \frac{k(T - t)^2}{2}$

C. $e^{-kT}$

D. $T^2 - \frac{1}{k}$

**Answer: A**

## Solution:

**Solution:**

$\frac{dV(t)}{dt} = -k(T - t)$

$\Rightarrow \int dV(t) = -k \int (T - t) dt$

$V(t) = \frac{k(T - t)^2}{2} + c$

at $t = 0$, $V(t) = I$

$I = \frac{kT^2}{2} + c$

$\Rightarrow c = I - \frac{kT^2}{2}$

$\Rightarrow V(t) = I + \frac{k}{2}(t^2 - 2tT)$

$V(T) = I + \frac{k}{2}(T^2 - 2T^2) = I - \frac{k}{2}T^2$

----------------------------------------------------------------------------------------------------

# Question191

## If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to:
## [2011]

**Options:**

A. 5

B. 13

C. -2

D. 7

**Answer: D**

## Solution:

**Solution:**
$\frac{dy}{dx} = y + 3 \Rightarrow \int \frac{dy}{y+3} = \int dx$
$\Rightarrow \ln|y+3| = x + c$
Given $y(0) = 2$, $\therefore \ln 5 = c$
$\Rightarrow \ln|y+3| = x + \ln 5$
Put $x = \ln 2$, then $\ln|y+3| = \ln 2 + \ln 5$
$\Rightarrow \ln|y+3| = \ln 10$
$\therefore y + 3 = \pm 10 \Rightarrow y = 7, -13$

----------------------------------------------------------------------------------------------------

# Question192

## Consider the differential equation $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$. If $y(1) = 1$, then $x$ is given by:
## [2011RS]

**Options:**

A. $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$

B. $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

C. $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$

D. $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

**Answer: C**

## Solution:

$\frac{dx}{dy} + \frac{x}{y^2} = \frac{1}{y^3}$
It is linear differential eqn.
$I.F. = e^{\int \frac{1}{y^2} dy} = e^{-\frac{1}{y}}$
So $x . e^{-\frac{1}{y}} = \int \frac{1}{y^3} e^{-\frac{1}{y}} dy$
Let $\frac{-1}{y} = t$
$\Rightarrow \frac{1}{y^2} dy = dt$
$\Rightarrow I = -\int te^t dt = e^t - te^t = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$
$\Rightarrow xe^{-\frac{1}{y}} = e^{-\frac{1}{y}} + \frac{1}{y} e^{-\frac{1}{y}} + c$
$\Rightarrow x = 1 + \frac{1}{y} + c . e^{1/y}$
Given y(1) = 1
$\therefore c = -\frac{1}{e}$
$\Rightarrow x = 1 + \frac{1}{y} - \frac{1}{e} . e^{1/y}$

----------------------------------------------------------------------------------------------------

# Question193

## Solution of the differential equation $\cos x \, dy = y(\sin x - y) dx$, $0 < x < \frac{\pi}{2}$ is [2010]

### Options:

A. $y \sec x = \tan x + c$

B. $y \tan x = \sec x + c$

C. $\tan x = (\sec x + c) y$

D. $\sec x = (\tan x + c) y$

**Answer: D**

## Solution:

**Solution:**
$\cos x \, dy = y(\sin x - y) dx$
$\frac{dy}{dx} = y \tan x - y^2 \sec x$
$\frac{1}{y^2} \frac{dy}{dx} - \frac{1}{y} \tan x = -\sec x$ .......(i)
Let $\frac{1}{y} = t \Rightarrow -\frac{1}{y^2} \frac{dy}{dx} = \frac{dt}{dx}$
Putting in (i)
$-\frac{dt}{dx} - t \tan x = -\sec x$
$\Rightarrow \frac{dt}{dx} + (\tan x) t = \sec x$
I.F. $= e^{\int \tan x \, dx} = e^{\log |\sec x|} = \sec x$
Solution : $t \sec x = \int \sec x \sec x \, dx$
$\Rightarrow \frac{1}{y} \sec x = \tan x + c$

----------------------------------------------------------------------------------------------------

# Question194

**The differential equation which represents the family of curves $y = c_1 e^{c_2 x}$, where $c_1$, and $c_2$ are arbitrary constants, is**
**[2009]**

**Options:**

A. $y'' = y'y$

B. $yy'' = y'$

C. $yy'' = (y')^2$

D. $y' = y^2$

**Answer: C**

## Solution:

**Solution:**
We have $y = c_1 e^{c_2 x}$
Differentiate it w.r.t. to $x$
$\Rightarrow y' = c_1 c_2 e^{c_2 x} = c_2 y$
$\Rightarrow \frac{y'}{y} = c_2$ Differentiate it w.r.t. to $x$
$\Rightarrow \frac{y''y - (y')^2}{y^2} = 0 \Rightarrow y''y = (y')^2$

----------------------------------------------------------------------------------------------------

# Question195

**The differential equation of the family of circles with fixed radius 5 units and centre on the line y = 2 is**
**[2009]**

**Options:**

A. $(x - 2)y'^2 = 25 - (y - 2)^2$

B. $(y - 2)y'^2 = 25 - (y - 2)^2$

C. $(y - 2)^2 y'^2 = 25 - (y - 2)^2$

D. $(x - 2)^2 y'^2 = 25 - (y - 2)^2$

**Answer: C**

## Solution:

**Solution:**
Let the centre of the circle be $(h, 2)$
$\therefore$ Equation of circle is

$(x-h)^2 + (y-2)^2 = 25$ ......(1)
Differentiating with respect to $x$, we get
$2(x-h) + 2(y-2)\frac{dy}{dx} = 0$
$\Rightarrow x - h = -(y-2)\frac{dy}{dx}$
Substituting in equation (1) we get
$(y-2)^2\left(\frac{dy}{dx}\right)^2 + (y-2)^2 = 25$
$\Rightarrow (y-2)^2(y')^2 = 25 - (y-2)^2$

------------------------------------------------------------------------------------------------

# Question196

## The solution of the differential equation $\frac{dy}{dy} = \frac{x+y}{x}$ satisfying the condition $y(1) = 1$ is
## [2008]

**Options:**

A. $y = \ln x + x$

B. $y = x\ln x + x^2$

C. $y = xe^{(x-1)}$

D. $y = x\ln x + x$

**Answer: D**

## Solution:

**Solution:**
$\frac{dy}{dx} = \frac{x+y}{x} = 1 + \frac{y}{x}$
It is homogeneous differential eqn.
Putting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$
we get $v + x\frac{dv}{dx} = 1 + v \Rightarrow \int\frac{dx}{x} = \int dv$
$\Rightarrow v = \ln x + c \Rightarrow y = x\ln x + cx$
As $y(1) = 1$
$\therefore c = 1$ So solution is $y = x\ln x + x$

------------------------------------------------------------------------------------------------

# Question197

## The differential equation of all circles passing through the origin and having their centres on the x -axis is
## [2007]

**Options:**

A. $y^2 = x^2 + 2xy\frac{dy}{dx}$

B. $y^2 = x^2 - 2xy\frac{dy}{dx}$

C. $x^2 = y^2 + xy\frac{dy}{dx}$

D. $x^2 = y^2 + 3xy\frac{dy}{dx}$

**Answer: A**

## Solution:

**Solution:**
General equation of circles passing through origin and having their centres on the $x$ -axis is
$x^2 + y^2 + 2gx = 0$ ......(i)
On differentiating w.r.t $x$, we get
$2x + 2y . \frac{dy}{dx} + 2g = 0 \Rightarrow g = -\left(x + y\frac{dy}{dx}\right)$
Putting in (i)
$x^2 + y^2 + 2\left\{-\left(x + y\frac{dy}{dx}\right)\right\} . x = 0$
$\Rightarrow x^2 + y^2 - 2x^2 - 2x\frac{dy}{dx} . y = 0$
$\Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$

-----------------------------------------------------------------------------------------------

# Question198

## The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a [2007]

**Options:**

A. circle

B. hyperbola

C. ellipse

D. parabola.

**Answer: B**

## Solution:

**Solution:**
Equation of normal at $P(x, y)$ is
$Y - y = -\frac{dx}{dy}(X - x)$
Coordinate of $G$ at $X$ axis is $(X, 0)$ (let)
$\therefore 0 - y = -\frac{dx}{dy}(X - x)$
$\Rightarrow y\frac{dy}{dx} = X - x$
$\Rightarrow X = x + y\frac{dy}{dx}$
$\therefore$ Co-ordinate of $G\left(x + y\frac{dy}{dx}, 0\right)$
Given distance of $G$ from origin = twice of the abscissa of $P$
$\because$ distance cannot be $-ve$, therefore abscissa $x$ should be +ve
$\therefore x + y\frac{dy}{dx} = 2x \Rightarrow y\frac{dy}{dx} = x$

$\Rightarrow y\,dy = x\,dx$
On Integrating, we have $\frac{y^2}{2} = \frac{x^2}{2} + c_1$
$\Rightarrow x^2 - y^2 = -2c_1$
$\therefore$ the curve is a hyperbola

----------------------------------------------------------------------------------------------------

# Question199

## The differential equation whose solution is $Ax^2 + By^2 = 1$ where A and B are arbitrary constants is of
## [2006]

**Options:**

A. second order and second degree

B. first order and second degree

C. first order and first degree

D. second order and first degree

**Answer: D**

## Solution:

**Solution:**
$Ax^2 + By^2 = 1$ ......(i)
Differentiate w.r. to $x$
$Ax + By\frac{dy}{dx} = 0$ ......(ii)
Again differentiate w.r. to $x$
$A + By\frac{d^2y}{dx^2} + B\left(\frac{dy}{dx}\right)^2 = 0$ ......(iii)
From (ii) and (iii)
$x\left\{-By\frac{d^2y}{dx^2} - B\left(\frac{dy}{dx}\right)^2\right\} + By\frac{dy}{dx} = 0$
Dividing both sides by $-B$, we get
$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$
Therefore order 2 and degree 1 .

----------------------------------------------------------------------------------------------------

# Question200

## The differential equation representing the family of curves$y^2 = 2c(x + \sqrt{c})$, where $c > 0$, is a parameter, is of order and degree as follows:
## [2005]

**Options:**

A. order 1, degree 2

B. order 1, degree 1

C. order 1, degree 3

D. order 2, degree 2

**Answer: C**

## Solution:

**Solution:**
$y^2 = 2c(x + \sqrt{c})$ ........ (i)Differentite it w.r.t. to $x$
$2yy' = 2c.1$ or $yy' = c$ ........ (ii)
[On putting value of $c$ from (ii) in (i)]
$\Rightarrow y^2 = 2yy'(x + \sqrt{yy'})$
On simplifying, we get
$(y - 2xy')^2 = 4yy^3$ ........ (iii)
Hence equation (iii) is of order 1 and degree 3 .

----------------------------------------------------------------------------------------------------

# Question201

## If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is [2005]

**Options:**

A. $y \log\left(\frac{x}{y}\right) = cx$

B. $x \log\left(\frac{y}{x}\right) = cy$

C. $\log\left(\frac{y}{x}\right) = cx$

D. $\log\left(\frac{x}{y}\right) = cy$

**Answer: C**

## Solution:

**Solution:**
$\frac{xdy}{dx} = y(\log y - \log x + 1)$
$\frac{dy}{dx} = \frac{y}{x}\left(\log\left(\frac{y}{x}\right) + 1\right)$
Put $y = vx$
$\frac{dy}{dx} = v + \frac{xdv}{dx} \Rightarrow v + \frac{xdv}{dx} = v(\log v + 1)$
$\frac{xdv}{dx} = v \log v \Rightarrow \int \frac{dv}{v \log v} = \int \frac{dx}{x}$
Put $\log v = z$
$\frac{1}{v}dv = dz \Rightarrow \int \frac{dz}{z} = \int \frac{dx}{x}$
$\ln z = \ln x + \ln c$
$x = cx$ or $\log v = cx$ or $\log\left(\frac{y}{x}\right) = cx$.

----------------------------------------------------------------------------------------------------

# Question202

## The differential equation for the family of circle $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is
## [2004]

**Options:**

A. $(x^2 + y^2)y' = 2xy$

B. $2(x^2 + y^2)y' = xy$

C. $(x^2 - y^2)y' = 2xy$

D. $2(x^2 - y^2)y' = xy$

**Answer: C**

## Solution:

**Solution:**
$x^2 + y^2 - 2ay = 0 \ldots\ldots(1)$
Differentiate w.r.t. to x
$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0 \Rightarrow a = \frac{x + yy'}{y'}$
Putting in (1) we get, $x^2 + y^2 - 2\left(\frac{x + yy'}{y'}\right)y = 0$
$\Rightarrow (x^2 + y^2)y' - 2xy - 2y^2y' = 0$
$\Rightarrow (x^2 - y^2)y' = 2xy$

----------------------------------------------------------------------------------------------------

# Question203

## Solution of the differential equation $y\,dx + (x + x^2y)dy = 0$ is
## [2004]

**Options:**

A. $\log y = Cx$

B. $-\frac{1}{xy} + \log y = C$

C. $\frac{1}{xy} + \log y = C$

D. $-\frac{1}{xy} = C$

**Answer: B**

## Solution:

**Solution:**
$y\,dx + (x + x^2y)dy = 0$
$\Rightarrow \frac{dx}{dy} = -\frac{x}{y} - x^2 \Rightarrow \frac{dx}{dy} + \frac{x}{y} = -x^2$
It is Bernoulli form. Divide by $x^2$

$x^{-2}\frac{dx}{dy} + x^{-1}\left(\frac{1}{y}\right) = -1$
put $x^{-1} = t$, $-x^{-2}\frac{dx}{dy} = \frac{dt}{dy}$ we get,
$-\frac{dt}{dy} + t\left(\frac{1}{y}\right) = -1 \Rightarrow \frac{dt}{dy} - \left(\frac{1}{y}\right)t = 1$
It is linear differential eqn. in t.
$I.F = e^{\int -\frac{1}{y}dy} = e^{-\log y} = y^{-1}$
$\therefore$ Solution is $t(y^{-1}) = \int (y^{-1})dy + C$
$\Rightarrow \frac{1}{x}\cdot\frac{1}{y} = \log y + C \Rightarrow \log y - \frac{1}{xy} = C$

-----------------------------------------------------------------------------------------------

# Question204

## The degree and order of the differential equation of the family of all parabolas whose axis is x - axis, are respectively. [2003]

**Options:**

A. 2, 3

B. 2, 1

C. 1, 2

D. 3, 2.

**Answer: C**

## Solution:

**Solution:**
$y^2 = 4a(x - h)$
Differentiating $2yy_1 = 4a \Rightarrow yy_1 = 2a$
Again differentiating, we get $\Rightarrow y_1{}^2 + yy_2 = 0$
Degree $= 1$, order $= 2$

-----------------------------------------------------------------------------------------------

# Question205

## The solution of the differential equation $(1 + y^2) + \left(x - e^{\tan^{-1}y}\right)\frac{dy}{dx} = 0$, is [2003]

**Options:**

A. $xe^{2\tan^{-1}y} = e^{\tan^{-1}y} + k$

B. $(x - 2) = ke^{2\tan^{-1}y}$

C. $2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$

D. $xe^{\tan^{-1}y} = \tan^{-1}y + k$

**Answer: C**

## Solution:

**Solution:**

$(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$

$\Rightarrow \frac{dx}{dy}+\frac{x}{(1+y^2)}=\frac{e^{\tan^{-1}y}}{(1+y^2)}$

It is form of linear differential equation.

$I.F=e^{\int\frac{1}{(1+y^2)}dy}=e^{\tan^{-1}y}$

$x\left(e^{\tan^{-1}y}\right)=\int\frac{e^{\tan^{-1}y}}{1+y^2}e^{\tan^{-1}y}dy$

$x\left(e^{\tan^{-1}y}\right)=\frac{e^{2\tan^{-1}y}}{2}+C\left[\because\int e^{2x}dx=\frac{e^{2x}}{2}\right]$

$\therefore 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

---

# Question206

**The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3}=4\frac{d^3y}{dx^3}$ are**
**[2002]**

**Options:**

A. $\left(1,\frac{2}{3}\right)$

B. (3,1)

C. (3,3)

D. (1,2)

**Answer: C**

## Solution:

**Solution:**

$\left(1+3\frac{dy}{dx}\right)^2=\left(\frac{4d^3y}{dx^3}\right)^3$

$\Rightarrow\left(1+3\frac{dy}{dx}\right)^2=16\left(\frac{d^3y}{dx^3}\right)^3$

Order $=3$, degree 3

---

# Question207

**The solution of the equation $\frac{d^2y}{dx^2}=e^{-2x}$**
**[2002]**

**Options:**

A. $\frac{e^{-2x}}{4}$

B. $\frac{e^{-2x}}{4} + cx + d$

C. $\frac{1}{4}e^{-2x} + cx^2 + d$

D. $\frac{1}{4}e^{-4x} + cx + d$

**Answer: B**

## Solution:

$\frac{d^2y}{dx^2} = e^{-2x}$; on integration $\frac{dy}{dx} = \frac{e^{-2x}}{-2} + c$;

Again integrate we get $y = \frac{e^{-2x}}{4} + cx + d$

---------------------------------------------------------------------------------------------------