Matrices

Question1

Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given below are two statements :

Statement I: $f(-x)$ is the inverse of the matrix $f(x)$.

Statement II: $f(x)f(y) = f(x+y)$.

In the light of the above statements, choose the correct answer from the options given below

[27-Jan-2024 Shift 1]

Options:

A.

Statement I is false but Statement II is true

B.

Both Statement I and Statement II are false

C.

Statement I is true but Statement II is false

D.

Both Statement I and Statement II are true

Answer: D

Solution:

$$f(-x) = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x) \cdot f(-x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Hence statement- I is correct

Now, checking statement II

$$f(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f(x) \cdot f(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow f(x) \cdot f(y) = f(x+y)$

Hence statement-II is also correct.

Question2

Let A be a 2 × 2 real matrix and I be the identity matrix of order 2 . If the roots of the equation $|A - xI| = 0$ be -1 and 3 , then the sum of the diagonal elements of the matrix A^2 is...............

[27-Jan-2024 Shift 2]

Answer: 10

Solution:

$|A - xI| = 0$

Roots are -1 and 3

Sum of roots $= \operatorname{tr}(A) = 2$

Product of roots $= |A| = -3$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

We have $a + d = 2$

$ad - bc = -3$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

We need $a^2 + bc + bc + d^2$

$= a^2 + 2bc + d^2$

$= (a+d)^2 - 2ad + 2bc$

$= 4 - 2(ad - bc)$

$= 4 - 2(-3)$

$= 4 + 6$

$= 10$

Question3

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^3 = 2^{21}$ where $\alpha, \beta \in Z$, Then a value of α is

[29-Jan-2024 Shift 1]

Options:

A.

3

B.

5

C.

17

D.

9

Answer: B

Solution:

$|A| = \alpha^2 - \beta^2$

$|2A|^3 = 2^{21} \Rightarrow A| = 2^4$

$\alpha^2 - \beta^2 = 16$

$(\alpha+\beta)(\alpha-\beta) = 16 \Rightarrow \alpha = 4 \text{ or } 5$

Question4

Let A be a square matrix such that $AA^T = I$. Then $\frac{1}{2}A[(A+A^T)^2 + (A-A^T)^2]$ is equal to

[29-Jan-2024 Shift 1]

Options:

A.

$A^2 + I$

B.

$A^3 + I$

C.

$A^2 + A^T$

D.

$A^3 + A^T$

Answer: D

Solution:

$AA^T = I = A^TA$

On solving given expression, we get

$$\frac{1}{2}A[A^2 + (A^T)^2 + 2AA^T + A^2 + (A^T)^2 - 2AA^T]$$

$$= A[A^2 + (A^T)^2] = A^3 + A^T$$

Question5

Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$. The sum of the prime factors of $|P^{-1}AP - 2I|$ is equal to

[29-Jan-2024 Shift 2]

Options:

A.

26

B.

27

C.

66

D.

23

Answer: A

Solution:

$|P^{-1}AP-2I| = |P^{-1}AP-2P^{-1}P|$

$= |P^{-1}(A-2I)P|$

$= |P^{-1}|\,|A-2I|\,|P|$

$= |A-2I|$

$$= \begin{vmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{vmatrix} = 69$$

So, Prime factor of 69 is 3 & 23

So, sum $= 26$

Question6

Let R = $\begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ be a non-zero 3 × 3 matrix, where $x\sin\theta = y\sin\left(\theta+\frac{2\pi}{3}\right) = z\sin\left(\theta+\frac{4\pi}{3}\right) \neq 0, \theta \in (0, 2\pi).$ **For a square matrix M, let trace (M) denote the sum of all the diagonal entries of M. Then, among the statements:**

(I) Trace (R) = 0

(II) If trace(adj (adj(R)) = 0, then R has exactly one non-zero entry.

[30-Jan-2024 Shift 2]

Options:

A.

Both (I) and (II) are true

B.

Neither (I) nor (II) is true

C.

Only (II) is true

D.

Only (I) is true

Answer: C

Solution:

$$x\sin\theta = y\sin\left(\theta+\frac{2\pi}{3}\right) = z\sin\left(\theta+\frac{4\pi}{3}\right) \neq 0$$

$\Rightarrow x, y, z \neq 0$

Also,

$$\sin\theta + \sin\left(\theta+\frac{2\pi}{3}\right) + \sin\left(\theta+\frac{4\pi}{3}\right) = 0 \ \forall \theta \in R$$

$$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$$

$\Rightarrow xy + yz + zx = 0$

(i) Trace (R) $= x + y + z$

If $x+y+z=0$ and $xy+yz+zx=0$

$\Rightarrow x = y = z = 0$

Statement (i) is False

(ii) Let P : trace (Adj(Adj(R))) $= 0$

Q : R has exactly one non zero entry

if P is false then $P \longrightarrow Q$ is always true

Statement (ii) is True

Question7

Let A be a 3×3 matrix and $\det(A)=2$. If

$$n = \det\underbrace{(\mathrm{adj}(\mathrm{adj}(\ldots\ldots(\mathrm{adj}\,A))))}_{2024-\ \text{times}}$$

Then the remainder when n is divided by 9 is equal to______

[31-Jan-2024 Shift 2]

Answer: 7

Solution:

$|A| = 2$

$$\underbrace{\text{adj}(\text{adj}(\text{adj}\ldots..(a)))}_{2024-\text{ times}} = |A|^{(n-1)^{2024}}$$

$$= |A|^{2024}$$

$$= 2^{2^{2024}}$$

$$2^{2024} = (2^2)2^{2022} = 4(8)^{674} = 4(9-1)^{674}$$

$$\Rightarrow 2^{2024} \equiv 4(\text{mod } 9)$$

$$\Rightarrow 2^{2024} \equiv 9m + 4,\ m \leftarrow \text{ even}$$

$$2^{9m+4} \equiv 16 \cdot (2^3)^{3m} \equiv 16(\text{mod } 9)$$

$$\equiv 7$$

Question8

If $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, C = ABA^T$ and $X = A^TC^2A$, then $\det X$ is equal to :

[1-Feb-2024 Shift 1]

Options:

A.

243

B.

729

C.

27

D.

891

Answer: B

Solution:

$$A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix} \Rightarrow \det(A) = 3$$

$$B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow \det(B) = 1$$

Now $C = ABA^T \Rightarrow \det(C) = (\det(A))^2 x \det(B)$

$|C| = 9$

Now $|X| = |A^T C^2 A|$

$= |A^T| \, |C|^2 \, |A|$

$= |A|^2 \, |C|^2$

$= 9 \times 81$

$= 729$

Question9

Let A = $I_2 - 2MM^T$, where M is real matrix of order 2 × 1 such that the relation $M^TM = I_1$ holds. If λ is a real number such that the relation AX = λX holds for some non-zero real matrix X of order 2 × 1, then the sum of squares of all possible values of λ is equal to :

[1-Feb-2024 Shift 2]

Answer: 2

Solution:

$A = I_2 - 2MM^T$

$A^2 = (I_2 - 2MM^T)(I_2 - 2MM^T)$

$= I_2 - 2MM^T - 2MM^T + 4MM^TMM^T$

$= I_2 - 4MM^T + 4MM^T$

$= I_2$

$AX = \lambda X$

$A^2X = \lambda AX$

$X = \lambda(\lambda X)$

$X = \lambda^2 X$

$X(\lambda^2 - 1) = 0$

$\lambda^2 = 1$

$\lambda = \pm 1$

Sum of square of all possible values $= 2$

Question10

If A and B are two non-zero n × n matrics such that $A^2 + B = A^2B$, then [24-Jan-2023 Shift 1]

Options:

A. $AB = I$

B. $A^2B = I$

C. $A^2 = I$ or $B = I$

D. $A^2B = BA^2$

Answer: D

Solution:

$$A^2 + B = A^2B$$
$$(A^2 - I)(B - I) = I$$
$$A^2 + B = A^2B$$
$$A^2(B - I) = B$$
$$A^2 = B(B - I)^{-1}$$
$$A^2 = B(A^2 - I)$$
$$A^2 = BA^2 - B$$
$$A^2 + B = BA^2$$
$$A^2B = BA^2$$

Question11

The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1 , is
[24-Jan-2023 Shift 2]

Options:

A. 225

B. 120

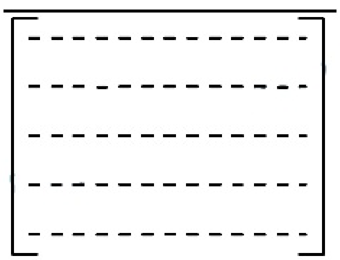
C. 150

D. 125

Answer: B

Solution:

Solution:



In each row and each column exactly one is to be placed -
$\therefore$ No. of such matrices $= 5 \times 4 \times 3 \times 2 \times 1 = 120$
Alternate :

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{array}{l} \rightarrow 5 \text{ ways} \\ \rightarrow 4 \text{ ways} \\ \rightarrow 3 \text{ ways} \\ \rightarrow 2 \text{ ways} \\ \rightarrow 1 \text{ ways} \end{array}$$

Step-1 : Select any 1 place for 1 's in row 1. Automatically some column will get filled with 0 's.
Step-2 : From next now select 1 place for 1 's. Automatically some column will get filled with 0 's. $\Rightarrow$ Each time one less place will be available for putting 1's.
Repeat step-2 till last row.
Req. ways $= 5 \times 4 \times 3 \times 2 \times 1 = 120$

Question12

Let A be a 3×3 matrix such that $|\operatorname{adj}(\operatorname{adj}(\operatorname{adj} A))| = 12^4$. Then $\left|A^{-1} \operatorname{adj} A\right|$ is equal to
[24-Jan-2023 Shift 2]

Options:

A. $2\sqrt{3}$

B. $\sqrt{6}$

C. 12

D. 1

Answer: A

Solution:

Solution:
Given $|\operatorname{adj}(\operatorname{adj}(\operatorname{adj}.A))| = 12^4$
$\Rightarrow |A|^{(n-1)^3} = 12^4$
We are asked
$|A^{-1}\cdot \operatorname{adj} A|$
$= |A^{-1}|\cdot|\operatorname{adj} A|$
$= \frac{1}{|A|}\cdot\left|A\right|^{3-1}$
$= |A| = 2\sqrt{3}$

Question13

Let x, y, z > 1 and

$$A = \begin{bmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 2 & \log_y z \\ \log_z x & \log_z y & 3 \end{bmatrix}.$$

Then $\left|\operatorname{adj}(\operatorname{adj} A^2)\right|$ is equal to
[25-Jan-2023 Shift 1]

Options:

A. 6^4

B. 2^8

C. 4^8

D. 2^4

Answer: B

Solution:

Solution:

$$|A| = \frac{1}{\log x\cdot\log y\cdot\log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & 2\log y & \log z \\ \log x & \log y & 3\log z \end{vmatrix} = 2$$

$$\Rightarrow \left|\operatorname{adj}(\operatorname{adj} A^2)\right| = \left|A^2\right|^4 = 2^8$$

Question14

Let A = $\begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ **and B =** $\begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$**, where i =** $\sqrt{-1}$**. If M =** $A^T BA$**,**

then the inverse of the matrix $AM^{2023}A^T$ **is**
[25-Jan-2023 Shift 2]

Options:

A. $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ -2023i & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Answer: D

Solution:

Solution:

$$AA^T = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} cc \frac{1}{\sqrt{10}} & \frac{-3}{\sqrt{10}} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2i \\ 0 & 1 \end{bmatrix}$$

$$B^3 = \begin{bmatrix} 1 & -3i \\ 0 & 1 \end{bmatrix}$$

.
.
.

$$B^{2023} = \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

$M = A^T BA$
$M^2 = M \cdot M = A^T BA A^T BA = A^T B^2 A$
$M^3 = M^2 \cdot M = A^T B^2 AA^T BA = A^T B^3 A$
.
.
.
$M^{2023} = \ldots \cdots\cdots\cdots \cdots \cdot A^T B^{2023} A$
$AM^{2023}A^T = AA^T B^{2023} AA^T = B^{2023}$

$$= \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$$

Inverse of $(AM^{2023}A^T)$ is $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$

Question15

Let A, B, C be 3 × 3 matrices such that A is symmetric and B and C are skew-symmetric.
Consider the statements
(S1)$A^{13}B^{26} - B^{26}A^{13}$ is symmetric
(S2) $A^{26}C^{13} - C^{13}A^{26}$ is symmetric
Then.
[25-Jan-2023 Shift 2]

Options:

A. Only S2 is true

B. Only S1 is true

C. Both S1 and S2 are false

D. Both S1 and S2 are true

Answer: A

Solution:

Given, $A^T = A$, $B^T = -B$, $C^T = -C$
Let $M = A^{13}B^{26} - B^{26}A^{13}$
Then, $M^T = (A^{13}B^{26} - B^{26}A^{13})^T$
$= (A^{13}B^{26})^T - (B^{26}A^{13})^T$
$= (B^T)^{26}(A^T)^{13} - (A^T)^{13}(B^T)^{26}$
$= B^{26}A^{13} - A^{13}B^{26} = -M$
Hence, M is skew symmetric
Let, $N = A^{26}C^{13} - C^{13}A^{26}$
then, $N^T = (A^{26}C^{13})^T - (C^{13}A^{26})^T$
$= -(C)^{13}(A)^{26} + A^{26}C^{13} = N$
Hence, N is symmetric.
∴ Only S2 is true.

Question16

Let α and β be real numbers. Consider a 3 × 3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then
[29-Jan-2023 Shift 1]

Options:

A. $\alpha = 1$

B. $\alpha = 4$

C. $\beta = 8$

D. $\beta = -8$

Answer: D

Solution:

Solution:

$A^2 = 3A + \alpha I$
$A^3 = 3A^2 + \alpha A$
$A^3 = 3(3A + \alpha I) + \alpha A$
$A^3 = 9A + \alpha A + 3\alpha I$
$A^4 = (9 + \alpha)A^2 + 3\alpha A$
$= (9 + \alpha)(3A + \alpha I) + 3\alpha A$
$= A(27 + 6\alpha) + \alpha(9 + \alpha)$
$\Rightarrow 27 + 6\alpha = 21 \Rightarrow \alpha = -1$
$\Rightarrow \beta = \alpha(9 + \alpha) = -8$

Question17

The set of all values of t ∈ ℝ, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$

is invertible, is
[29-Jan-2023 Shift 2]

Options:

A. $\left\{ (2k+1)\frac{\pi}{2}, k \in \mathbb{Z} \right\}$

B. $\left\{ k\pi + \frac{\pi}{4}, k \in \mathbb{Z} \right\}$

C. $\{k\pi, k \in \mathbb{Z}\}$

D. $\mathbb{R}$

Answer: D

Solution:

Solution:
If its invertible, then determinant value $\neq 0$
So,

$$\begin{vmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{vmatrix} \neq 0$$

$$\Rightarrow e^t \cdot e^{-t} \cdot e^{-t}\begin{vmatrix} 1 & \sin t - 2\cos t & -2\sin t - \cos t \\ 1 & 2\sin t + \cos t & \sin t - 2\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

Applying, $R_1 \to R_1 - R_2$ then $R_2 \to R_2 - R_3$

We get

$$e^{-t}\begin{vmatrix} 0 & -\sin t - \cos t & -3\sin t + \cos t \\ 0 & 2\sin t & -2\cos t \\ 1 & \cos t & \sin t \end{vmatrix} \neq 0$$

By expanding we have,

$e^{-t} \times 1(2\sin t\cos t + 6\cos^2 t + 6\sin^2 t - 2\sin t\cos t) \neq 0$

$\Rightarrow e^{-t} \times 6 \neq 0$

for $\forall t \in$

Question18

Let A be a symmetric matrix such that |A| = 2 and

$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$. If the sum of the diagonal elements of A is s,

then $\frac{\beta s}{\alpha^2}$ is equal to ________.

[29-Jan-2023 Shift 2]

Answer: 5

Solution:

$$\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$$

Now $ac - b^2 = 2$ and $2a + b = 1$ and $2b + c = 2$

solving all these above equations we get

$\frac{1-b}{2} \times \left(\frac{2-2b}{1}\right) - b^2 = 2$

$\Rightarrow (1-b)^2 - b^2 = 2$

$\Rightarrow 1 - 2b = 2$

$\Rightarrow b = -\frac{1}{2}$ and $a = \frac{3}{4}$ and $c = 3$

Hence $\alpha = 3a + \frac{3b}{2} = \frac{9}{4} - \frac{3}{4} = \frac{3}{2}$

and $\beta = 3b + \frac{3c}{2} = -\frac{3}{2} + \frac{9}{2} = 3$

also $s = a + c = \frac{15}{4}$

$\therefore \frac{\beta s}{\alpha^2} = \frac{3 \times 15}{4 \times \frac{9}{4}} = 5$

Question19

Let A = $\begin{pmatrix} m & n \\ p & q \end{pmatrix}$, d = $|A| \neq 0$ $|A - d(\text{Adj}\,A)| = 0$
Then
[30-Jan-2023 Shift 1]

Options:

A. $(1+d)^2 = (m+q)^2$

B. $1+d^2 = (m+q)^2$

C. $(1+d)^2 = m^2+q^2$

D. $1+d^2 = m^2+q^2$

Answer: A

Solution:

Solution:

Sol. $A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}$, $|A - d(\text{adj}\,A)| = 0$

$\Rightarrow |A - d(\text{adj}\,A)| = \left| \begin{bmatrix} m & n \\ p & q \end{bmatrix} - d \begin{bmatrix} q & -n \\ -p & m \end{bmatrix} \right|$

$= \begin{vmatrix} m-qd & n(1+d) \\ p(1+d) & q-md \end{vmatrix} = 0$

$\Rightarrow (m-qd)(q-md) - np(1+d)^2 = 0$

$\Rightarrow mq - m^2d - q^2d + mqd^2 - np(1+d)^2 = 0$

$\Rightarrow (mq-np) + d^2(mq-np) - d(m^2+q^2+2np) = 0$

$\Rightarrow d + d^3 - d((m+q)^2 - 2d) = 0$

$\Rightarrow 1 + d^2 = (m+q)^2 - 2d$

$\Rightarrow (1+d)^2 = (m+q)^2$

Question20

If P is a 3 × 3 real matrix such that $P^T = aP + (a-1)I$, where $a > 1$, then
[30-Jan-2023 Shift 2]

Options:

A. P is a singular matrix

B. $|\text{Adj}\,P| > 1$

C. $|\text{Adj}\,P| = \frac{1}{2}$

D. $|\text{Adj}\,P| = 1$

Answer: D

Solution:

Solution:

$P^T = aP + (a-1)I$

$\Rightarrow P = aP^T + (a-1)I$

$\Rightarrow P^T - P = a(P - P^T)$

$\Rightarrow P = P^T, \quad \text{as } a \neq -1$

Now, $P = aP + (a-1)I$

$\Rightarrow P = -I \Rightarrow |P| = 1$

$\Rightarrow |\mathrm{Adj}\,P| = 1$

Question21

Let A = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$ **. Then the sum of the diagonal elements of the matrix** $(A+I)^{11}$ **is equal to:**
[31-Jan-2023 Shift 1]

Options:

A. 6144

B. 4094

C. 4097

D. 2050

Answer: C

Solution:

Solution:

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$$

$\Rightarrow A^3 = A^4 = \ldots\ldots = A$

$(A+I)^{11} = {}^{11}C_0A^{11} + {}^{11}C_1A^{10} + \ldots.\ {}^{11}C_{10}A + {}^{11}C_{11}I$

$= ({}^{11}C_0 + {}^{11}C_1 + \ldots.\ {}^{11}C_{10})A + I$

$= (2^{11} - 1)A + I = 2047A + I$

$\therefore$ Sum of diagonal elements $= 2047(1 + 4 - 3) + 3$

$= 4094 + 3 = 4097$

Question22

Let A = $[a_{ij}]$**,** $a_{ij} \in Z \cap [0, 4]$**,** $1 \le i, j \le 2$**. The number of matrices A such that the sum of all entries is a prime number** $p \in (2, 13)$ **is _______.**
[31-Jan-2023 Shift 2]

Answer: 204

Solution:

As given a + b + c + d = 3 or 5 or 7 or 11

if sum = 3

$(1 + x + x^2 + \ldots + x^4)^4 \rightarrow x^3$

$(1 - x^5)^4(1 - x)^{-4} \rightarrow x^3$

$\therefore {}^{4+3-1}C_3 = {}^6C_3 = 20$

If sum = 5

$(1 - 4x^5)(1 - x)^{-4} \rightarrow x^5$

$\Rightarrow {}^{4+5-1}C_5 - 4x^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 7

$(1 - 4x^5)(1 - x)^{-4} \rightarrow x^7$

$\Rightarrow {}^{4+5-1}C_4 - {}^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 11

$(1 - 4x^5 + 6x^{10})(1 - x)^{-4} \rightarrow x^{11}$

$\Rightarrow {}^{4+11-1}C_{11} - 4 \cdot {}^{4+6-4}C_6 + 6 \cdot {}^{4+1-1}C_1$

$= {}^{14}C_{11} - 4 \cdot {}^9C_6 + 6.4 = 364 - 336 + 24 = 52$

$\therefore$ Total matrices $= 20 + 52 + 80 + 52 = 204$

Question23

Let A be a n × n matrix such that |A| = 2. If the determinant of the matrix $\mathrm{Adj}(2 \,.\, \mathrm{Adj}(2A^{-1}))$. is 2^{84}, then n is equal to ______.
[31-Jan-2023 Shift 2]

Answer: 5

Solution:

$|\,\mathrm{Adj}(2\,\mathrm{Adj}(2A^{-1}))\,|$

$= |\,2\,\mathrm{Adj}(\mathrm{Adj}(2A^{-1}))\,|^{n-1}$

$= 2^{n(n-1)}\,|\,\mathrm{Adj}(2A^{-1})\,|^{n-1}$

$= 2^{n(n-1)}\,|\,(2A^{-1})\,|^{(n-1)(n-1)}$

$= 2^{n(n-1)}2^{n(n-1)(n-1)}\,|\,A^{-1}|^{(n-1)(n-1)}$

$= 2^{n(n-1)+n(n-1)(n-1)}\dfrac{1}{|A|^{(n-1)^2}}$

$= \dfrac{2^{n(n-1)+n(n-1)(n-1)}}{2^{(n-1)^2}}$

$= 2^{n(n-1)+n(n+1)^2-(n-1)^2}$

$= 2^{(n-1)(n^2-n+1)}$

Now, $2^{(n-1)(n^2-n+1)}$

$2^{(n-1)(n^2-n+1)} = 2^{84}$

So, n = 5

Question24

If A = $\frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then :

[1-Feb-2023 Shift 2]

Options:

A. $A^{30} - A^{25} = 2I$

B. $A^{30} + A^{25} + A = I$

C. $A^{30} + A^{25} - A = I$

D. $A^{30} = A^{25}$

Answer: C

Solution:

Solution:

$A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$

$A = \begin{bmatrix} \cos 60^\circ & \sin 60^\circ \\ -\sin 60^\circ & \cos 60^\circ \end{bmatrix}$

If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ Here $\alpha = \frac{\pi}{3}$

$= \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$

$A^{30} = \begin{bmatrix} \cos 30\alpha & \sin 30\alpha \\ -\sin 30\alpha & \cos 30\alpha \end{bmatrix}$

$A^{30} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$A^{25} = A$

$A^{25} - A = 0$

Question25

Let $A = [a_{ij}]_{2\times 2}$, where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of | all diagonal elements of A and b = | A|. Then $3a^2 + 4b^2$ is equal to :

[6-Apr-2023 shift 1]

Options:

A. 14

B. 4

C. 3

D. 7

Answer: B

Solution:

Solution:

$A^2 = I \Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1 = b$

Let $A = \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix}$

$A^2 = \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} = I$

$\begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta + \beta\delta \\ \alpha\gamma + \gamma\delta & \gamma\beta + \delta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \alpha^2 + \beta\gamma = 1$

$(\alpha + \delta)\beta = 0 \Rightarrow \alpha + \delta = 0 = a$

$(\alpha + \delta)\gamma = 0$

$\beta\gamma + \delta^2 = 0$

Now $3a^2 + 4b^2 = 3(0)^2 + 4(1) = 4$

Question26

Let P be a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma, \delta \in N$, if $P^\alpha + P^\beta = \gamma I - 29P$ and $P^\alpha - P^\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \delta$ is equal to : [6-Apr-2023 shift 2]

Options:

A. 40

B. 22

C. 24

D. 18

Answer: C

Solution:

Solution:

$P^2 = I - P$

$P^\alpha + P^\beta = \gamma I - 29P$

$P^\alpha - P^\beta = \delta I - 13P$

$P^4 = (I - P)^2 = I + P^2 - 2P$

$P^4 = I + I - P - 2P = 2I - 3P$

$P^8 = (P^4)^2 = (2I - 3P)^2 = 4I + 9P^2 - 12P$

$= 4I + 9(I - P) - 12P$

$P^8 = 13I - 21P \ldots (1)$

$P^6 = P^4 \cdot P^2 = (2I - 3P)(I - P)$

$= 2I - 5P + 3P^2$

$= 2I - 5P + 3(I - P)$

$= 5I - 8P \ldots (2)$

(1) + (2)

$P^8 + P^6 = 18I - 29P$

(1) − (2)

$P^8 - P^6 = 8I - 13P$

From (A) $\alpha = 8,\ \beta = 6$
$\gamma = 18$
$\delta = 8$
$\alpha + \beta + \gamma - \delta = 32 - 8 = 24$

Question27

Let A = $\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If $|\operatorname{adj}(\operatorname{adj}(\operatorname{adj} 2A))| = (16)^n$, then n is equal to

[8-Apr-2023 shift 1]

Options:

A. 8

B. 9

C. 12

D. 10

Answer: D

Solution:

Solution:
$|A| = 2[3] - 1[2] = 4$
$\therefore |\operatorname{adj}(\operatorname{adj}(\operatorname{adj} 2A))|$
$= |2A|^{(n-1)^3} \Rightarrow |2A|^8 = 16^n$
$\Rightarrow (2^3|A|)^8 = 16^n$
$\Rightarrow (2^3 \times 2^2)^8 = 16^n$
$= 2^{40} = 16^n$
$= 16^{10} = 16^n \Rightarrow n = 10$

Question28

Let P = $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and Q = PAP^T. If

$P^TQ^{2007}P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then 2a + b − 3c − 4d equal to

[8-Apr-2023 shift 1]

Options:

A. 2004

B. 2007

C. 2005

D. 2006

Answer: C

Solution:

Solution:

$Q = PAP^T$

$P^T \cdot Q^{2007} \cdot P = P^T \cdot Q \cdot Q \ldots Q \cdot P$

$= P^T(PAP^T)(P \cdot AP^T) \ldots (P^T)P.$

$\Rightarrow (P^TP)A(P^TP)A \ldots A(P^TP)$

$$P^T \cdot P = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} -\sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$\therefore P^T \cdot Q^{2007} \cdot P = A^{2007}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$\therefore A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$a = 1, b = 2007, c = 0, d = 1$

$2a + b - 3c - 4d = 2 + 2007 - 4 = 2005$

Question29

If $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$, $A^{-1} = \alpha A + \beta I$ and $\alpha + \beta = -2$, then $4\alpha^2 + \beta^2 + \lambda^2$ is equal to :
[8-Apr-2023 shift 2]

Options:

A. 14

B. 12

C. 19

D. 10

Answer: A

Solution:

Solution:

$$|A - xI| = 0 \Rightarrow \begin{vmatrix} 1-x & 5 \\ \lambda & 10-x \end{vmatrix} = 0 \Rightarrow x^2 - 11x + 10 - 5\lambda = 0$$

$\Rightarrow (10 - 5\lambda)A^{-1} = -A + 11I$

$\therefore \alpha = \dfrac{-1}{10 - 5\lambda}$ and $\beta = \dfrac{+11}{10 - 5\lambda}$

$\alpha + \beta = -2 \Rightarrow \frac{10}{10-5\lambda} = -2 \Rightarrow 10 - 5\lambda = -5 \Rightarrow \lambda = 3$

$\therefore \alpha = \frac{1}{5}$ & $\beta = \frac{-11}{5}$

$\therefore 4a^2 + \beta^2 + \lambda^2 = \frac{4}{25} + \frac{121}{25} + 3^2 = 14$ Ans.

Question30

If A is a 3 × 3 matrix and $|A| = 2$, then $|3\,\mathrm{adj}(|3A|A^2)|$ is equal to : [10-Apr-2023 shift 1]

Options:

A. $3^{12} \cdot 6^{10}$

B. $3^{11} \cdot 6^{10}$

C. $3^{12} \cdot 6^{11}$

D. $3^{10} \cdot 6^{11}$

Answer: B

Solution:

Solution:

Given $|A| = 2$

Now, $|3\,\mathrm{adj}(|3A|A^2)|$

$|3A| = 3^3 \cdot |A|$

$= 3^3 . (2)$

Adj. $(|3A|A^2) = \mathrm{adj}\{(3^3 \cdot 2)A^2\}$

$= (2.3^3)^2(\mathrm{adj}\,A)^2$

$= 2^2 \cdot 3^6 \cdot (\mathrm{adj}A)^2$

$|3\,\mathrm{adj}(|3A|A^2)| = |2^2 \cdot 3.3^6(\mathrm{adj}\,A)^2|$

$= (2^2.3^7)^3\,|\,\mathrm{adj}A|^2$

$= 2^6 \cdot 3^{21}(|A|^2)^2$

$= 2^6 \cdot 3^{21}(2^2)^2$

$= 2^{10} \cdot 3^{21}$

$= 2^{10} \cdot 3^{10} \cdot 3^{11}$

$|3\,\mathrm{adj}(|3A|A^2)| = 6^{10} \cdot 3^{11}$

Question31

If $A = \frac{1}{5!6!7!}\begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$, then $|\mathrm{adj}(\mathrm{adj}(2A))|$ is equal to

[10-Apr-2023 shift 2]

Options:

A. 2^{16}

B. 2^{8}

C. 2^{12}

D. 2^{20}

Answer: A

Solution:

Solution:

$$|A| = \frac{1}{5!6!7!}5!6!7! \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$$

$R_3 \to R_3 \to R_2$
$R_2 \to R_2 \to R_1$

$$|A| = \begin{vmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} = 2$$

$| \text{adjadj}(2A) | = | 2A|^{(n-1)^2}$
$= | 2A|^4$
$= (2^3 | A|)^4$
$= 2^{12} | A |^4 \Rightarrow 2^{16}$

Question32

Let $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$, where a, c $\in \mathbb{R}$. If $A^3 = A$ and the positive value of a belongs to the interval $(n-1, n]$, where $n \in \mathbb{N}$, then n is equal to _____.
[11-Apr-2023 shift 1]

Answer: 2

Solution:

$$A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$A^3 = A$

$$A^2 = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & 1 & 2+3c \end{bmatrix}$$

$$A^3 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & a & 2+3c \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 2ac+3 & a+2+3c & 2a+4+6c \\ a(a+3c)+2a & 3+2ac & 6+3a+9c \\ a+2+3c & ac+c(2+3c) & 2ac+3 \end{bmatrix}$$

Given $A^3 = A$

$2ac + 3 = 0 \ldots (1)$ and $a + 2 + 3c = 1$

$a + 1 + 3c = 0$

$a + 1 - \frac{9}{2a} = 0$

$2a^2 + 2a - 9 = 0$

$f(1) < 0, f(2) > 0$

$a \in (1, 2]$

$n = 2$

Question33

Let $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$. If $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to

[12-Apr-2023 shift 1]

Options:

A. 50

B. 75

C. 125

D. 100

Answer: D

Solution:

Solution:

Let $C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}, D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$$DC = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$B = CAD$

$B^n = (CAD)(CAD)(CAD) \ldots\ldots (CAD)_{\text{n-times}}$

$\Rightarrow B^n = CA^nD \ \ldots (1)$

$$A^3 = \begin{bmatrix} 1 & \frac{3}{51} \\ 0 & 1 \end{bmatrix}$$

Similarly $A^n = A^3 = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

$Bn \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & \frac{n}{51} + 2 \\ -1 & -\frac{n}{51} - 1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{n}{51} + 1 & \frac{n}{51} \\ -\frac{n}{51} & 1 - \frac{n}{51} \end{bmatrix}$

$\sum_{n=1}^{50} B^n = \begin{bmatrix} 25 - 50 & 25 \\ -25 & -25 - 50 \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$

Sum of the elements $= 100$

Question34

The number of symmetric matrices of order 3 , with all the entries from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, is [13-Apr-2023 shift 1]

Options:

A. 10^9

B. 10^6

C. 9^{10}

D. 6^{10}

Answer: B

Solution:

Solution:

$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$, $a, b, c, d, e, f \in \{0, 1, 2, \ldots .9\}$, Number of matrices $= 10^6$

Question35

Let $B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$, $\alpha > 2$ be the adjoint of a matrix A and $|A| = 2$. then

$\begin{bmatrix} \alpha & -2\alpha & \alpha \end{bmatrix} B \begin{bmatrix} \alpha \\ -2\alpha \\ \alpha \end{bmatrix}$ is equal to

[13-Apr-2023 shift 1]

Options:

A. 16

B. 32

C. 0

D. -16

Answer: D

Solution:

Solution:

Given, $B = \begin{bmatrix} 1 & 3 & \alpha \\ 1 & 2 & 3 \\ \alpha & \alpha & 4 \end{bmatrix}$

$|B| = 4$

$1(8 - 3\alpha) - 3(4 - 3\alpha) + \alpha(\alpha - 2\alpha) = 4$

$-\alpha^2 + 6\alpha - 8 = 0$

$\alpha = 2, 4$

Given $\alpha > 2$

So, $\alpha = 2$ is rejected

$$\begin{bmatrix} 4 & -8 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 & 4 \\ 1 & 2 & 3 \\ 4 & 4 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ -8 \\ 4 \end{bmatrix} = [-16]_{1 \times 1}$$

Question36

Let for $A = \begin{bmatrix} 1 & 2 & 3 \\ \alpha & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, $|A| = 2$. If $|2\,\mathrm{adj}(2\,\mathrm{adj}(2A))| = 32^n$, then $3n + \alpha$ is equal to

[13-Apr-2023 shift 2]

Options:

A. 10

B. 9

C. 12

D. 11

Answer: D

Solution:

$|A| = 2$
$adj(kA) = k^{m-1} adjA$ {m = order of matrix }
$adj(2A) = 2^2 adj A = 4 adj(A)$
$adj(2 adj(2A)) = adj(8 adj A)$
$= 8^2 adj adj(A)$
$|2 adj 2 adj(2A)| = |2^7 adj adj(A)|$
$= (2^7)^3 |A|^{2^2}$
$= 2^{21} |A|^4$
$= 2^{21} \cdot 2^4$
$\Rightarrow 2^{25} = (32)^n$
$\Rightarrow 2^{25} = 2^{5n}$
$\Rightarrow n = 5$
$|A| = 2$
$(6 - 1) - 2(2\alpha - 1) + 3(\alpha - 3) = 2$
$\Rightarrow 5 - 4\alpha + 2 + 3\alpha - 9$
$\Rightarrow \alpha = -4$
$3n + \alpha = 11$

Question37

Let the determinant of a square matrix A of order m be m − n, where m and n satisfy 4m + n = 22 and 17m + 4n = 93. If $\det(n\, adj(adj(mA))) = 3^a 5^b 6^c$, then a + b + c is equal to [15-Apr-2023 shift 1]

Options:

A. 101

B. 84

C. 109

D. 96

Answer: D

Solution:

Solution:
$|A| = m - n$
$4m + n = 22$
$17m + 4n = 93$
$m = 5, n = 2$
$|A| = 3$
$|2 adj(adj 5A)| = 2^5 |5A|^{16}$
$= 2^5 \cdot 5^{80} |A|^{16}$
$= 2^5 \cdot 5^{80} \cdot 3^{16}$
$= 3^{11} \cdot 5^{80} \cdot 6^5$
$a + b + c = 96$

Question38

Let S = $\left\{ \begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix} ; a, b \in \{1, 2, 3, \ldots 100\} \right\}$ and let

$T_n = \{A \in S : A^{n(n+1)} = I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is
[24-Jun-2022-Shift-2]

Answer: 100

Solution:

Solution:

$A = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix} \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$

$= \begin{bmatrix} 1 & -a + ab \\ 0 & b^2 \end{bmatrix}$

$\therefore T_n = \{A \in S; A^{n(n+1)} = I\}$

$\therefore$ b must be equal to 1

$\therefore$ In this case A^2 will become identity matrix and a can take any value from 1 to 100

$\therefore$ Total number of common element will be 100 .

Question39

Let A be a 3 × 3 real matrix such that

$A \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$; $A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and $A \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix of order 3 , then the system

$(A - 2I)X = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$

has :
[25-Jun-2022-Shift-1]

Options:

A. no solution

B. infinitely many solutions

C. unique solution

D. exactly two solutions

Answer: B

Solution:

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$$A = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} a+b=1 \\ d+e=1 \\ g+h=0 \end{matrix}$$

$$A = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \begin{matrix} a+c=-1 \\ d+f=1 \\ g+i=0 \end{matrix}$$

$$A = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \begin{matrix} rc=1 \\ f=1 \\ i=2 \end{matrix}$$

Solving will get

$a=-2,\ b=3,\ c=1,\ d=-1,\ e=2,\ f=1,\ g=-1,\ h=1,\ i=2$

$$A = \begin{bmatrix} -2 & 3 & 1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow A = 2I = \begin{bmatrix} -4 & 3 & 1 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$$(A-2I)x = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$$

$\Rightarrow -4x_1 + 3x_2 + x_3 = 4 \ldots$ (i)

$-x_1 + x_3 = 1$

$-x_1 + x_2 = 1 \ldots$ (ii)

So $3(\text{iii}) + (\text{ii}) = (\text{i})$

$\therefore$ Infinite solution

Question40

Let A = $\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. If M and N are two matrices given by M $= \sum_{k=1}^{10} A^{2k}$ and N $= \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is :
[25-Jun-2022-Shift-1]

Options:

A. a non-identity symmetric matrix

B. a skew-symmetric matrix

C. neither symmetric nor skew-symmetric matrix

D. an identity matrix

Answer: A

Solution:

Solution:

$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$

$M = A^2 + A^4 + A^6 + \ldots\ldots + A^{20}$

$= -4I + 16I - 64I + \ldots\ldots$ upto 10 terms

$= -I[4 - 16 + 64 \ldots. +$ upto 10 terms]

$= -I.4\left[\frac{(-4)^{10} - 1}{-4 - 1}\right] = \frac{4}{5}(2^{20} - 1)I$

$N = A^1 + A^3 + A^5 + \ldots\ldots + A^{19}$

$= A - 4A + 16A + \ldots\ldots$ upto 10 terms

$= A\left(\frac{(-4)^{10} - 1}{-4 - 1}\right) = -\left(\frac{2^{20} - 1}{5}\right)A$

$N^2 = \frac{(2^{20} - 1)^2}{2^5}A^2 = \frac{-4}{24}(2^{20} - 1)^2 I$

$MN^2 = \frac{-16}{125}(2^{20} - 1)^3 I = KI \quad (K \neq \pm 1)$

$(MN^2)^T = (KI)^T = KI$

$\therefore$A is correct

Question41

Let A be a 3 × 3 matrix having entries from the set {−1, 0, 1}. The number of all such matrices A having sum of all the entries equal to 5 , is

[25-Jun-2022-Shift-1]

Answer: 414

Solution:

Solution:

Case-I:

$1 \to 7$ times

and $-1 \to 2$ times

number of possible marrix $= \frac{9!}{7!2!} = 36$

Case-II:

$1 \to 6$ times,

$-1 \to 1$ times

and $0 \to 2$ times

number of possible marrix $= \frac{9!}{6!2!} = 252$

Case-III:

$1 \to 5$ times

and $0 \to 4$ times

number of possible marrix $= \frac{9!}{5!4!} = 126$

Hence total number of all such matrix $A = 414$

Question42

Let A = $\begin{pmatrix} 2 & -2 \\ 1 & -1 \end{pmatrix}$ and B = $\begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$. Then the number of elements in the set { (n, m) : n, m ∈ {1, 2, ..., 10}. and .$nA^n + mB^m = 1$ } is___
[25-Jun-2022-Shift-2]

Answer: 1

Solution:

$$A^2 = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} = A$$

$\Rightarrow A^K = A, K \in I$

$B^2 = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix} = B$

So, $B^K = B, K \in I$

$nA^n + mB^m = nA + mB$

$= \begin{bmatrix} 2n - 2n \\ n - n \end{bmatrix} + \begin{bmatrix} -m & 2m \\ -m & 2m \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

So, $2n - m = 1$, $-n + m = 0$, $2m - n = 1$

So, $(m, n) = (1, 1)$

Question43

Let A be a 3×3 invertible matrix. If $|\operatorname{adj}(24A)| = |\operatorname{adj}(3\operatorname{adj}(2A))|$, then $|A|^2$ is equal to :
[26-Jun-2022-Shift-1]

Options:

A. 6^6

B. 2^{12}

C. 2^6

D. 1

Answer: C

Solution:

We know, $|\operatorname{adj} A| = |A|^{n-1}$

Now, $|adj24A| = |adj3(adj2A)|$

$\Rightarrow |24A|^{3-1} = |3adj2A|^{3-1}$

$\Rightarrow |24A|^2 = |3adj2A|^2$

Also, we know, $|KA| = K^n|A|$

$\Rightarrow ((24)^2)^2 |A|^2 = ((3)^3)^2 |adj2A|^2$

$\Rightarrow (24)^6 |A|^2 = 3^6 \cdot (|2A|^{3-1})^2$

$\Rightarrow (24)^6 |A|^2 = 3^6 \cdot |2A|^4$

$\Rightarrow (24)^6 |A|^2 = 3^6 \cdot (2^3)^4 \cdot |A|^4$

$\Rightarrow 3^6 \cdot 8^6 \cdot |A|^2 = 3^6 \cdot 8^4 \cdot |A|^4$

$\Rightarrow 8^2 = |A|^2$

$\Rightarrow |A|^2 = 64 = 2^6$

Question44

Let X = $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$**, Y = αI + βX + γX² and**

Z = $\alpha^2 I - \alpha\beta X + (\beta^2 - \alpha\gamma)X^2$, α, β, γ ∈ R.

If $Y^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$**, then $(\alpha - \beta + \gamma)^2$ is equal to___**

[26-Jun-2022-Shift-2]

Answer: 100

Solution:

$$\because X = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore X^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore Y = \alpha I + \beta X + \gamma X^2 \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix}$$

$$\because Y \,.\, Y^{-1} = I$$

$$\therefore \begin{bmatrix} \alpha & \beta & \gamma \\ 0 & \alpha & \beta \\ 0 & 0 & \alpha \end{bmatrix} \begin{bmatrix} \frac{1}{5} & \frac{-2}{5} & \frac{1}{5} \\ 0 & \frac{1}{5} & \frac{-2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} \frac{\alpha}{5} & \frac{\beta-2\alpha}{5} & \frac{\alpha-2\beta+\gamma}{5} \\ 0 & \frac{\alpha}{5} & \frac{\beta-2\alpha}{5} \\ 0 & 0 & \frac{\alpha}{5} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \alpha = 5, \beta = 10, \gamma = 15$$

$$\therefore (\alpha - \beta + \gamma)^2 = 100$$

Question45

The positive value of the determinant of the matrix A, whose

Adj(Adj(A)) = $\begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix}$ **, is____**

[27-Jun-2022-Shift-1]

Answer: 14

Solution:

Solution:

$|adj(adj(A))| = |A|^{2^2} = |A|^4$

$$\therefore |A|^4 = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix}$$

$$= (14)^3 \begin{vmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix}$$

$= (14)^3(3 - 2(-5) - 1(-1))$

$|A|^4 = (14)^4 \Rightarrow |A| = 14$

Question46

Let A and B be two 3×3 matrices such that $AB = I$ and $|A| = \frac{1}{8}$. Then $|adj(B\ adj(2A))|$ is equal to [27-Jun-2022-Shift-2]

Options:

A. 16

B. 32

C. 64

D. 128

Answer: C

Solution:

Solution:

A and B are two matrices of order 3×3.

and $AB = I$,

$|A| = \frac{1}{8}$

Now, $|A||B| = 1$

$|B| = 8$

$\therefore |adj\left(B(adj(2A))\right| = \left|B(ad\,j(2A))\cdot\right|^{2.}$

$= |B|^2 |adj(2A)|^2$

$= 2^6 |2A|^{2 \times 2}$

$= 2^6 \cdot 2^{12} \cdot \frac{1}{2^{12}} = 64$

Question47

Let A be a matrix of order 2×2, whose entries are from the set $\{0, 1, 2, 3, 4, 5\}$. If the sum of all the entries of A is a prime number p, $2 < p < 8$, then the number of such matrices A is____ [27-Jun-2022-Shift-2]

Answer: 180

Solution:

Solution:
∵ Sum of all entries of matrix A must be prime p such that $2 < p < 8$ then sum of entries may be 3,5 or 7 .
If sum is 3 then possible entries are $(0, 0, 0, 3)$, $(0, 0, 1, 2)$ or $(0, 1, 1, 1)$.
∴ Total number of matrices $= 4 + 4 + 12 = 20$
If sum of 5 then possible entries are
$(0, 0, 0, 5)$, $(0, 0, 1, 4)$, $(0, 0, 2, 3)$, $(0, 1, 1, 3)$, $(0, 1, 2, 2)$ and $(1, 1, 1, 2)$.
∴ Total number of matrices $= 4 + 12 + 12 + 12 + 12 + 4 = 56$
If sum is 7 then possible entries are
$(0, 0, 2, 5)$, $(0, 0, 3, 4)$, $(0, 1, 1, 5)$, $(0, 3, 3, 1)$, $(0, 2, 2, 3)$, $(1, 1, 1, 4)$, $(1, 2, 2, 2)$, $(1, 1, 2, 3)$ and $(0, 1, 2, 4)$.
Total number of matrices with sum 7 = 104
∴ Total number of required matrices
$= 20 + 56 + 104 = 180$

Question48

Let A be a matrix of order 3 × 3 and det(A) = 2. Then $\det(\det(A)\,\mathrm{adj}(5\,\mathrm{adj}(A^3)))$ is equal to [28-Jun-2022-Shift-1]

Options:

A. 512×10^6

B. 256×10^6

C. 1024×10^6

D. 256×10^{11}

Answer: A

Solution:

Solution:
$|A| = 2$

$|A| = \mathrm{adj}(5\,\mathrm{adj}\,A^3)\cdot\Big|$

$= |\ 25\ |\ A\Big|\mathrm{adj}(\mathrm{adj}\,A^3)\cdot\Big|$

$= 25^3\ |\ A\ |^3\cdot\Big|\mathrm{ad\,j}A^3\cdot\Big|^2$

$= 25^3\cdot 2^3\cdot\Big|A^3\cdot\Big|^4$

$= 25^3\cdot 2^3\cdot 2^{12} = 10^6\cdot 512$

Question49

Let A = $\begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, ..., 100\} : A^n = A\}$ is____
[28-Jun-2022-Shift-2]

Answer: 25

Solution:

Solution:
$\therefore A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1+i & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix}$
$A^4 = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix} \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix} = I$
So $A^5 = A, A^9 = A$ and so on.
Clearly $n = 1, 5, 9, ..., 97$
Number of values of $n = 25$

Question50

The probability that a randomly chosen 2 × 2 matrix with all the entries from the set of first 10 primes, is singular, is equal to :
[29-Jun-2022-Shift-1]

Options:

A. $\frac{133}{10^4}$

B. $\frac{18}{10^3}$

C. $\frac{19}{10^3}$

D. $\frac{271}{10^4}$

Answer: C

Solution:

Solution:
Let matrix A is singular then $|A| = 0$
Number of singular matrix = All entries are same + only two prime number are used in matrix
$= 10 + 10 \times 9 \times 2$
$= 190$
Required probability $= \frac{190}{10^4} = \frac{19}{10^3}$

Question51

Let A = $[a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all i, j = 1, 2, 3. Then, the matrix $A^2 + A^3 + ... + A^{10}$ is equal to :
[29-Jun-2022-Shift-1]

Options:

A. $\left(\frac{3^{10}-3}{2}\right)A$

B. $\left(\frac{3^{10}-1}{2}\right)A$

C. $\left(\frac{3^{10}+1}{2}\right)A$

D. $\left(\frac{3^{10}+3}{2}\right)A$

Answer: A

Solution:

Solution:
Given, $a_{ij} = 2^{j-i}$

Now, $A = \begin{bmatrix} 2^0 & 2^1 & 2^2 \\ 2^{-1} & 2^0 & 2^1 \\ 2^{-2} & 2^{-1} & 2^0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+1+1 & 2+2+2 & 4+4+4 \\ \frac{1}{2}+\frac{1}{2}+\frac{1}{2} & 1+1+1 & 2+2+2 \\ \frac{1}{4}+\frac{1}{4}+\frac{1}{4} & \frac{1}{2}+\frac{1}{2}+\frac{1}{2} & 1+1+1 \end{bmatrix}$

$= \begin{bmatrix} 3 & 6 & 12 \\ \frac{3}{2} & 3 & 6 \\ \frac{3}{4} & \frac{3}{2} & 3 \end{bmatrix}$

$$= 3\begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$= 3A$

Similarly, $A^3 = 3^2A$

$A^4 = 3^3A$

$\therefore A^2 + A^3 + \ldots\ldots + A^{10}$

$= 3A + 3^2A + 3^3A + \ldots\ldots + 3^9A$

$= A(3 + 3^2 + 3^3 + \ldots\ldots + 3^9)$

$= A\left(\frac{3(3^9 - 1)}{3 - 1}\right) = \frac{3(3^9 - 1)}{2}A = \left(\frac{3^{10} - 3}{2}\right)A$

Question52

Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If $B = I - {}^5C_1(\text{adj } A) + {}^5C_2(\text{adj } A)^2 - \ldots\ldots - {}^5C_5(\text{adj } A)^5$, then the sum of all elements of the matrix B is
[29-Jun-2022-Shift-2]

Options:

A. -5

B. -6

C. -7

D. -8

Answer: C

Solution:

Solution:

Given $A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$ and

$B = I - 5_{C_1}(\text{adj } A) + 5_{C_2}(\text{adj } A)^2 - 5_{C_3}(\text{adj } A)^3 + 5_{C_4}(\text{adj } A)^4 - 5_{C_5}(\text{adj } A)^5$

$= (I - (\text{adj } A))^5$

Cofactor of $A = \begin{bmatrix} (-1)^{1+1} \cdot 2 & (-1)^{1+2} \cdot 0 \\ (-1)^{2+1} \cdot (-1) & (-1)^{2+2} \cdot 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

Transpose of cofactor of $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

$\therefore \text{adj } A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

Now, $I - \text{ad jA}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$

Now let,

$P = I - \text{adj} A = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$

$\therefore P^2 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$P^4 = P^2 \cdot P^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

$P^5 = P^4 \cdot P = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$

$\therefore B = \begin{bmatrix} -1 & -5 \\ 0 & -1 \end{bmatrix}$

Now sum of elements $= -1 - 5 - 1 + 0 = -7$

Question53

Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where α is a non-zero real number an $N = \sum_{k=1}^{49} M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is____
[29-Jun-2022-Shift-2]

Answer: 1

Solution:

Solution:

$M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}; M^2 = \begin{bmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{bmatrix} = -\alpha^2 I$

$N = M^2 + M^4 + \ldots\ldots + M^{98} = [-\alpha^2 + \alpha^4 - \alpha^6 + \ldots]I$

$= -\alpha^2 \frac{(1 - (-\alpha^2)^{49})}{1 + \alpha^2} \cdot I$

$I - M^2 = (1 + \alpha^2)I$

$(I - M^2)N = -\alpha^2(\alpha^{98} + 1) = -2$

$\alpha = 1$

Question54

Let $S = \{\sqrt{n} : 1 \le n \le 50$ and n is odd $\}$.

Let $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$.

If $\sum_{a \in S} \det(\operatorname{adj} A) = 100\lambda$, then λ is equal to : ion:
[24-Jun-2022-Shift-1]

Options:

A. 218

B. 221

C. 663

D. 1717

Answer: B

Solution:

Solution:

Given, $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}_{3 \times 3}$ $S = \{ \sqrt{n} : 1 \le n \le 50 \text{ and } n \text{ is odd} \}$

$\therefore S = \{1, \sqrt{3}, \sqrt{5}, \sqrt{7}, \ldots, \sqrt{49}\}$

We know,

$|\operatorname{adj} A| = |A|^{n-1}$

Here, $n =$ order of matrix.

Here, $n = 3$

$\therefore |adjA| = |A|^{3-1} = |A|^2$

Now, $|A| = \begin{vmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{vmatrix}$

$= 1(1-0) - 0 + a(0-(-a))$

$= a^2 + 1$

$\therefore |adjA| = |A|^2 = (a^2+1)^2$

Now, $\sum_{a \in S} \det(adjA)$

$= \sum_{a \in S} (a^2+1)^2$

$= (1^2+1)^2 + ((\sqrt{3})^2+1)^2 + ((\sqrt{5})^2+1)^2 + \ldots + ((\sqrt{49})^2+1)^2$

$= (1^2+1)^2 + (3+1)^2 + (5+1)^2 + \ldots + (49+1)^2$

$= 2^2 + 4^2 + 6^2 + \ldots + 50^2$

$= 2^2(1^2 + 2^2 + 3^2 + \ldots. + 25^2)$

$= 4. \frac{25.26.51}{6} = 100.221$

$\therefore 100K = 100.221$

$\Rightarrow K = 221$

Question55

Let $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$ and $B = A - I$. If $\omega = \frac{\sqrt{3}i - 1}{2}$, then the number of elements in the set $\{n \in \{1, 2, ..., 100\} : A^n + (\omega B)^n = A + B\}$ is equal to
[25-Jul-2022-Shift-1]

Answer: 17

Solution:

Solution:

Here $A = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$

We get $A^2 = A$ and similarly for

$B = A - I = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$

We get $B^2 = -B \Rightarrow B^3 = B$

$\therefore A^n + (\omega B)^n = A + (\omega B)^n \quad \text{for } n \in N$

For ω^n to be unity n shall be multiple of 3 and for B^n to be B. n shell be $3, 5, 7, \ldots 99$

$\therefore n = \{3, 9, 15, \ldots .99\}$

Number of elements $= 17$

Question56

Let $A = \begin{bmatrix} 1 & a & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$, $a, b \in C$. If for some $n \in \mathbb{N}$, $A^n = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 96 \\ 0 & 0 & 1 \end{bmatrix}$ then $n + a + b$ is equal to
[25-Jul-2022-Shift-2]

Answer: 24

Solution:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = I + B$$

$$B^2 = \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & a & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & ab \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$B^3 = 0$

$\therefore A^n = (1 + B)^n = {}^nC_0 I + {}^nC_1 B + {}^nC_2 B^2 + {}^nC_3 B^3 + \ldots$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & na & na \\ 0 & 0 & nb \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \frac{n(n-1)ab}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & na & na + \frac{n(n-1)}{2}ab \\ 0 & 1 & nb \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 48 & 2160 \\ 0 & 1 & 48 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing we get $na = 48$, $nb = 96$ and

$na + \frac{n(n-1)}{2}ab = 2160$

$\Rightarrow a = 4$, $n = 12$ and $b = 8$

$n + a + b = 24$

Question57

Let A be a 2×2 matrix with $\det(A) = -1$ and $\det((A + I)(\operatorname{Adj}(A) + I)) = 4$. Then the sum of the diagonal elements of A can be :
[26-Jul-2022-Shift-1]

Options:

A. -1

B. 2

C. 1

D. $-\sqrt{2}$

Answer: B

Solution:

$|(A+I)(adjA+I)|=4$

$\Rightarrow |A \operatorname{adj} A+A+\operatorname{adj} A+I|=4$

$\Rightarrow |(A)I+A+\operatorname{adj} A+I|=4$

$|A|=-1 \Rightarrow |A+\operatorname{adj} A|=4$

$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} adjA=\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$

$\Rightarrow \begin{vmatrix} (a+d) & 0 \\ 0 & (a+d) \end{vmatrix}=4$

$\Rightarrow a+d=\pm 2$

Question58

Let A = $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and B = $\begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix}$, then the value of $A'BA$ is:

[26-Jul-2022-Shift-2]

Options:

A. 1224

B. 1042

C. 540

D. 539

Answer: D

Solution:

Solution:

$A'BA = [\,1 \quad 1 \quad 1\,]\begin{bmatrix} 9^2 & -10^2 & 11^2 \\ 12^2 & 13^2 & -14^2 \\ -15^2 & 16^2 & 17^2 \end{bmatrix} A$

$= [\,9^2+12^2-15^2 \quad -10^2+13^2+16^2 \quad 11^2-14^2+17^2\,]\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$= [9^2+12^2-15^2-10^2+13^2+16^2+11^2-14^2+17^2]$

$= [(9^2-10^2)+(11^2+12^2)+(13^2-14^2)+(16^2-15^2)+17^2]$

$= [-19+265+(-27)+31+289]$

$= [585-46] = [539]$

Question59

The number of matrices A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where a, b, c, d $\in \{-1, 0, 1, 2, 3, \ldots, ., 10\}$, such that A = A^{-1}, is ________.
[26-Jul-2022-Shift-2]

Answer: 50

Solution:

Solution:

$\because A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^2 = \begin{bmatrix} a^2 + bc & b(a + d) \\ c(a + d) & bc + d^2 \end{bmatrix}$

For A^{-1} must exist $ad - bc \neq 0 \ldots\ldots$ (i)

and $A = A^{-1} \Rightarrow A^2 = I$

$\therefore a^2 + bc = d^2 + bc = 1$

and $b(a + d) = c(a + d) = 0$

Case I : When $a = d = 0$, then possible values of (b, c) are (1, 1), (−1, 1) and (1, −1) and (−1, 1).

Total four matrices are possible.

Case II: When $a = -d$ then (a, d) be (1, −1) or (−1, 1).

Then total possible values of (b, c) are $(12 + 11) \times 2 = 46$.

$\therefore$ Total possible matrices $= 46 + 4 = 50$.

Question60

Let A = $\begin{pmatrix} 1 & 2 \\ -2 & -5 \end{pmatrix}$. Let $\alpha, \beta \in \mathbb{R}$ be such that $\alpha A^2 + \beta A = 2I$. Then $\alpha + \beta$ is equal to
[27-Jul-2022-Shift-1]

Options:

A. −10

B. -6

C. 6

D. 10

Answer: D

Solution:

Solution:

$$A^2 = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix}$$

$$\alpha A^2 + \beta A = \begin{bmatrix} -3\alpha & -8\alpha \\ 8\alpha & 21\alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -2\beta & -5\beta \end{bmatrix}$$

$$= \begin{bmatrix} -3\alpha+\beta & -8\alpha+2\beta \\ 8\alpha-2\beta & 21\alpha-5\beta \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

On Comparing
$8\alpha = 2\beta$, $-3\alpha + \beta = 2$, $21\alpha - 5\beta = 2$
$\Rightarrow \alpha = 2, \beta = 8$
So, $\alpha + \beta = 10$

Question61

Let S be the set containing all 3 × 3 matrices with entries from {−1, 0, 1}. The total number of matrices $A \in S$ such that the sum of all the diagonal elements of $A^T A$ is 6 is ________. [27-Jul-2022-Shift-1]

Answer: 5376

Solution:

Solution:
Sum of all diagonal elements is equal to sum of square of each element of the matrix.

i.e., $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ then $t_r(A \cdot A^T)$

$= a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 + c_1^2 + c_2^2 + c_3^2$

$\because a_i, b_i, c_i \in \{-1, 0, 1\}$ for $i = 1, 2, 3$

$\therefore$ Exactly three of them are zero and rest are 1 or -1.

Total number of possible matrices ${}^9C_3 \times 2^6$

$= \frac{9 \times 8 \times 7}{6} \times 64$

$= 5376$

Question62

Let $A = \begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}$.

If $A^2 + \gamma A + 18I = O$, then det(A) is equal to ________. [27-Jul-2022-Shift-2]

Options:

A. -18

B. 18

C. -50

D. 50

Answer: B

Solution:

Solution:
Characteristic equation of A is given by
$|A - \lambda I| = 0$
$$\begin{vmatrix} 4-\lambda & -2 \\ \alpha & \beta-\lambda \end{vmatrix} = 0$$
$\Rightarrow \lambda^2 - (4+\beta)\lambda + (4\beta + 2\alpha) = 0$
So, $A^2 - (4+\beta)A + (4\beta + 2\alpha)I = 0$
$|A| = 4\beta + 2\alpha = 18$

Question63

Consider a matrix $A = \begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$, where α, β, γ are three distinct natural numbers. If $\frac{\det(\operatorname{adj}(\operatorname{adj}(\operatorname{adj}(\operatorname{adj} A))))}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}} = 2^{32} \times 3^{16}$, then the number of such 3 - tuples (α, β, γ) is ________.
[27-Jul-2022-Shift-2]

Answer: 42

Solution:

Solution:
$$\det(A) = \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix}$$
$R_3 \to R_3 + R_1$
$$\Rightarrow (\alpha+\beta+\gamma) \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$$
$\therefore \det(A) = (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$
Also, $\det(\operatorname{adj}(\operatorname{adj}(\operatorname{adj}(\operatorname{adj}(A)))))$
$= (\det(A))^{2^4} = (\det(A))^{16}$.
$\therefore \frac{(\alpha+\beta+\gamma)^{16}(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}} = (4.13)^{16}$
$\Rightarrow \alpha+\beta+\gamma = 12$
$\Rightarrow (\alpha, \beta, \gamma)$ distinct natural triplets
$= {}^{11}C_2 - 1 - {}^{3}C_2(4) = 55 - 1 - 12 = 42$

Question64

Let the matrix A = $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ **and the matrix** $B_0 = A^{49} + 2A^{98}$**. If** $B_n = Adj(B_{n-1})$ **for all n > 1, then** $\det(B_4)$ **is equal to :**
[28-Jul-2022-Shift-1]

Options:

A. 3^{28}

B. 3^{30}

C. 3^{32}

D. 3^{36}

Answer: C

Solution:

Solution:

$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

Now $B_0 = A^{49} + 2A^{98} = (A^3)^{16} \cdot A + 2(A^3)^{32} \cdot A^2$

$B_0 = A + 2A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{bmatrix}$

$|B_0| = 9$

Since, $B_n = Adj\,|B_{n-1}| \Rightarrow |B_n| = |B_{n-1}|^2$

Hence $|B_4| = |B_3|^2 = |B_2|^4 = |B_1|^8 = |B_0|^{16} = |3^2|^{16} = 3^{32}$

Question65

Let A = $\begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$ **and B =** $\begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}$**,** $\alpha, \beta \in R$**. Let** α_1 **be the value of** α **which satisfies** $(A+B)^2 - A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ **and** α_2 **be the value of** α **which satisfies** $(A+B)^2 - B^2$**. Then** $|\alpha_1 - \alpha_2|$ **is equal to ______.**
[28-Jul-2022-Shift-1]

Answer: 2

Solution:

Solution:

$(A+B)^2 = A^2 + B^2 + AB + BA$

$= A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$\therefore B^2 + AB + BA = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \ldots\ldots (1)$

$AB = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \beta - 1 & 1 \\ \alpha + 2\beta & 2 \end{bmatrix}$

$BA = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} \beta + 2 & \alpha - \beta \\ 1 & -1 \end{bmatrix}$

$B^2 = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \beta^2 + 1 & \beta \\ \beta & 1 \end{bmatrix}$

By (1) we get

$\begin{bmatrix} \beta^2 + 2\beta & \alpha + 1 \\ \alpha + 3\beta + 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

$\therefore \alpha = 1 \beta = 0 \Rightarrow \alpha_1 = 1$

Similarly if $A^2 + AB + BA = 0$ then

$\left(A^2 = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix} = \begin{bmatrix} -1 & -1 - \alpha \\ 2 + 2\alpha & \alpha^2 - 2 \end{bmatrix} \right)$

$\begin{bmatrix} 2\beta & \alpha - \beta + 1 - 1 - \alpha \\ \alpha + 2\beta + 1 + 2 + 2\alpha & \alpha^2 - 2 + 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \beta = 0$ and $\alpha = -1 \Rightarrow \alpha_2 = -1$

$\therefore |\alpha_1 - \alpha_2| = |2| = 2$

Question66

Let A and B be any two 3 × 3 symmetric and skew symmetric matrices respectively. Then which of the following is NOT true? [28-Jul-2022-Shift-2]

Options:

A. $A^4 - B^4$ is a smmetric matrix

B. $AB - BA$ is a symmetric matrix

C. $B^5 - A^5$ is a skew-symmetric matrix

D. $AB + BA$ is a skew-symmetric matrix

Answer: C

Solution:

Solution:
(A) $M = A^4 - B^4$
$M^T = (A^4 - B^4)^T = (A^T)^4 - (B^T)^4$
$= A^4 - (-B)^4 = A^4 - B^4 = M$
(B) $M = AB - BA$
$M^T = (AB - BA)^T = (AB)^T - (BA)^T$
$= B^T A^T - A^T B^T$
$= -BA - A(-B)$
$= AB - BA = M$
(C) $M = B^5 - A^5$
$M^T = (B^T)^5 - (A^T)^5 = -(B^5 + A^5) \neq -M$
(D) $M = AB + BA$
$M^T = (AB)^T + (BA)^T$
$= B^T A^T + A^T B^T = -BA - AB = -M$

Question67

Let A and B be two 3×3 non-zero real matrices such that AB is a zero matrix. Then
[29-Jul-2022-Shift-1]

Options:

A. the system of linear equations $AX = 0$ has a unique solution

B. the system of linear equations $AX = 0$ has infinitely many solutions

C. B is an invertible matrix

D. adj(A) is an invertible matrix

Answer: B

Solution:

Solution:
AB is zero matrix
$\Rightarrow |A| = |B| = 0$
So neither A nor B is invertible
If $|A| = 0$
$\Rightarrow |\operatorname{adj} A| = 0$ so $\operatorname{adj} A$
$AX = 0$ is homogeneous system and $|A| = 0$
So, it is having infinitely many solutions

Question68

The number of matrices of order 3×3, whose entries are either 0 or 1 and the sum of all the entries is a prime number, is ________.
[29-Jul-2022-Shift-1]

Answer: 282

Solution:

Solution:
In a 3 × 3 order matrix there are 9 entries.
These nine entries are zero or one.
The sum of positive prime entries are 2, 3, 5 or 7 .
Total possible matrices $= \frac{9!}{2!.7!} + \frac{9!}{3!.6!} + \frac{9!}{5!.4!} + \frac{9!}{7!.2!} = 36 + 84 + 126 + 36 = 282$

Question69

Which of the following matrices can NOT be obtained from the matrix $\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$ by a single elementary row operation?

[29-Jul-2022-Shift-2]

Options:

A. $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

C. $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$

D. $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

Answer: C

Solution:

Solution:
$A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

(1) $R_1 \rightarrow R_1 + R_2$; $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ possible

(2) $R_1 \leftrightarrow R_2$; $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ possible

(3) Option is not possible

(4) $R_2 \rightarrow R_2 + 2R_1$; $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ possible

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Question70

Let $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A - \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$. For $k \in N$, if $X^{'}A^{k}X - 33$, then k is equal to ________.
[29-Jul-2022-Shift-2]

Answer: 10

Solution:

Solution:

$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$; $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$

$X^{T}A^{K}X = 33$

$[\ 1 \quad 1 \quad 1\ ] \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}^{k} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$

$[\ 1 \quad 1 \quad 1\ ] \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$

As $A^{2} = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^{4} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^{8} = \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^{10} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

for $K \rightarrow \text{Even}\, A^{K} = \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$X^{T}A^{K}X = 33$ (This is not correct)

$[\ 1 \quad 1 \quad 1\ ] \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$= [\ 1 \quad 1 \quad 3K + 1\ ] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [3K + 3]$

$\therefore 3K + 3 = 33 \therefore K = 10$

But it should be dropped as 33 is not matrix

If K is odd

$X^T A^K X = 33$
$X^T A A^{K-1} X = 33$

$$[\,1\ \ 1\ \ 1\,]\begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 3k-3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$[\,-1\ \ 3\ \ 8\,]\begin{bmatrix} 3k-2 \\ 1 \\ 1 \end{bmatrix} = [33]$$

$[-3k + 13] = [33]$
$k = 20 / 3$ (not possible)

Question71

Let M be any 3×3 matrix with entries from the set $\{0, 1, 2\}$. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is
24 Feb 2021 Shift 1

Answer: 540

Solution:

Solution:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$
$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case-I : Seven (1's) and two (0's)
$\frac{9!}{7!2!} = 36$
Case-II: One (2) and three (1's) and five (0's)
$\frac{9!}{5!3!} = 504$
$\therefore$ Total $= 540$

Question72

If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is ....
[25 Feb 2021 Shift 1]

Answer: 13

Solution:

Solution:

$A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow |(I_2 + A)(I_2 - A)^{-1}| = a^2 + b^2$

$\Rightarrow a^2 + b^2 = \frac{|I_2 + A|}{|I_2 - A|}$ . . . (i)

Now, $I_2 + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$

Similarly, $I_2 - A = \begin{bmatrix} 1 & \tan\left(\frac{\theta}{2}\right) \\ -\tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$

Here, $|I_2 + A| = |I_2 - A| = \left(1 + \tan^2\left(\frac{\theta}{2}\right)\right)$

$\Rightarrow \frac{|I_2 + A|}{|I_2 - A|} = 1$ . . . (ii)

From Eqs. (i) and (ii),

$a^2 + b^2 = 1$

Now, $13(a^2 + b^2) = 13 \times 1 = 13$

Question73

If for the matrix, A = $\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$,

$AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is

[2021, 25 Feb. Shift-II]

Options:

A. 4

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:

Given, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix}$

Given, $AA^\top = I_2$ i.e.

$\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix} \begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1+\alpha^2 & \alpha - \alpha\beta \\ \alpha - \alpha\beta & \alpha^2 + \beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Equating these matrices,
$c1 = \alpha^2 + 1$ gives, $\alpha = 0$
$\alpha(1-\beta) = 0$
$\alpha^2 + \beta^2 = 1$

Put $\alpha = 0$ in $\alpha^2 + \beta^2 = 1$, we get $0 + \beta^2 = 1$, gives $\beta = \pm 1$
where we take $\beta^4 = 1$
$\therefore\ \alpha^4 + \beta^4 = 0^4 + 1 = 1$

Question74

Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1 , then the possible number of such matrices is [2021, 26 Feb. Shift-I]

Options:

A. 4

B. 1

C. 6

D. 12

Answer: A

Solution:

Solution:

Let A be the matrix as follows, $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$, since A is symmetric matrix.

Now, $A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$= \begin{bmatrix} a^2 + b^2 & ab + bc \\ ab + bc & b^2 + c^2 \end{bmatrix}$

Given that, diagonal entries of A^2 is 1 . i.e. $a^2 + b^2 + b^2 + c^2 = 1$
or $a^2 + 2b^2 + c^2 = 1$
Case 1a $= 0$
Then, $2b^2 + c^2 = 1$

(a) $c = 0$ gives, $b^2 = \frac{1}{2}$ or $b = \pm\frac{1}{\sqrt{2}}$ $\therefore a = 0, b = 1/\sqrt{2}, c = 0$ (2 matrices) $a = 0, b = -1/\sqrt{2}, c = 0$

(b) $b = 0$, gives $c^2 = 1$ or $c = \pm 1$ $\therefore a = 0, b = 0, c = 1$
and $a = 0, b = 0, c = -1$ (2 matrices)
Case $2b = 0$, then $a^2 + c^2 = 1$
(a) $a = 0$, then $c = \pm 1$ $a = 0, b = 0, c = 1$ and $a = 0, b = 0, c = -1$
This is repeated case.
(b) $c = 0$, then $a = \pm 1$ $a = 1, b = 0, c = 0$ and $a = -1, b = 0, c = 0$ Again 2 matrices.

Thus, only acceptable matrices are as follows

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix}$$

Then possible number of such matrices are 4.

Question75

If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation

$A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers α and β, then $\beta - \alpha$ is equal to
[2021, 26 Feb. Shift-II]

Answer: 4

Solution:

Solution:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

And so on,

$$A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}$$

$$A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c So, " $A^{20} + \alpha A^{19} + \beta A$

$$= \begin{bmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20}+\alpha 2^{19}+2\beta & 0 \\ 3\alpha+2\beta & 0 & 1-\alpha-\beta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing, we get
$1 - \alpha - \beta = 1$
$\Rightarrow\ \alpha + \beta = 0$
and $2^{20} + \alpha 2^{19} + 2\beta = 4$
$\Rightarrow 2^{20} + \alpha(2^{19} - 2) = 4$ [use, $\alpha + \beta = 0$]
$\Rightarrow\ \alpha = \dfrac{4 - 2^{20}}{2^{19} - 2} = -2$
$\Rightarrow\ \beta = 2$
$\therefore\ \beta - \alpha = 2 - (-2) = 2 + 2 = 4$

Question76

Let $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$.

If $Tr(A)$ denotes the sum of all diagonal elements of the matrix A, then $Tr(A) - Tr(B)$ has value equal to
[2021, 18 March Shift-1]

Options:

A. 1

B. 2

C. 0

D. 3

Answer: B

Solution:

Solution:

Given, $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} \ldots (i)$

and $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix} \ldots (ii)$

Multiply with 2 in Eq. (ii), we get

$4A - 2B = \begin{bmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{bmatrix} \cdots\cdots (iii)$

Adding Eqs. (i) and (iii),

$5A = \begin{bmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{bmatrix}$

$\therefore\ A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$

$\therefore T\,r(A) = 1 - 1 + 1 = 1$

From Eq. (i),

$B = \frac{1}{2}\left\{ \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix} \right\}$

$= \begin{bmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{bmatrix}$

$\therefore T\,r(B) = 0 - 1 + 0 = -1$

Hence, $T\,r(A) - T\,r(B) = 1 - (-1) = 2$

Question77

Let I be an identity matrix of order 2 × 2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n = 51 - 8P$ is equal to
[2021, 18 March Shift-II]

Answer: 6

Solution:

Solution:
Method (1)

Given, $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

Now, $P^n = 5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix}$

$\Rightarrow P^n = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} \cdots\cdots (i)$

Now, $P^2 = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$= \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$

Again $P^3 = P \cdot P^2 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix}$ Now,

$P^6 = P^3 \cdot P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix}$

$\Rightarrow P^6 = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = P^n$ [from Eq. (i)]

$\therefore n = 6$
Method (2)

Given $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

Characteristics equation is $|P - \lambda| = 0$

$\begin{vmatrix} 2-\lambda & -1 \\ 5 & -3-\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)(-3-\lambda) + 5 = 0$
$\Rightarrow \lambda^2 + \lambda - 1 = 0$
By Cayley-Hamilton Theorem,
$P^2 + P - I = 0$
$\Rightarrow P^2 = 1 - P$
$\Rightarrow P^3 + P^2 - P = 0$
$\Rightarrow P^3 = P - P^2 = P - (I - P)$ [from Eq. (i)]
$\Rightarrow P^3 = 2P - 1$
$\Rightarrow P^3 = 2P - 1$
Now, $P \cdot P^3 = P(2P - 1)$
$\Rightarrow P^4 = 2P^2 - P = 2(I - P) - P$[from Eq. (i)]
$\Rightarrow P^4 = -3P + 2I$
Again, $P \cdot (P^4) = P(-3P + 2I)$
$\Rightarrow P^5 = -3P^2 + 2P$
$= -3(I - P) + 2P$[from Eq. (i)]
$= 5P - 3I$

and $P(P^5) = P(5P - 3I)$
$I I \Rightarrow P^6 = 5P^2 - 3P = 5(I - P) - 3P$
$\Rightarrow p^6 = 5I - 8P = P^n$ (given)
$\therefore n = 6$

Question78

Let A = $\begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, $i = \sqrt{-1}$. Then, the system of linear equations

$A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has

[2021, 16 March Shift-I]

Options:

A. a unique solution.

B. infinitely many solutions.

C. no solution.

D. exactly two solutions.

Answer: C

Solution:

Solution:

Let A $= \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$A^2 = A \cdot A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$= \begin{bmatrix} i^2 + i^2 & -i^2 - i^2 \\ -i^2 - i^2 & i^2 + i^2 \end{bmatrix}$

$= \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$

$A^4 = A^2 \cdot A^2$

$= \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$

$= \begin{bmatrix} 4+4 & -4-4 \\ -4-4 & 4+4 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$

Similarly, $A^8 = A^4 \cdot A^4$

$= \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$

$= \begin{bmatrix} 64+64 & -64-64 \\ -64-64 & 64+64 \end{bmatrix}$

$= \begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix}$ Now, $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\begin{bmatrix} 128 & -128 \\ -128 & 128 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$128x - 128y = 8$

$\Rightarrow x - y = \frac{1}{16} \quad \cdots\cdots (i)$

and $-128x + 128y = 64$

$\Rightarrow x - y = -\frac{1}{2} \quad \cdots\cdots (ii)$

From Eqs. (i) and (ii), we get these two lines are parallel. So, there will be no solution.

Question79

The total number of 3 × 3 matrices A having entries from the set (0, 1, 2, 3), such that the sum of all the diagonal entries of AA^T is 9 , is equal to
[2021, 16 March Shift-I]

Answer: 766

Solution:

Solution:

Set S : {0, 1, 2, 3}

$$\text{Let } A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} a \cdot a & a \cdot b & a \cdot c \\ b \cdot a & b \cdot b & b \cdot c \\ c \cdot a & c \cdot b & c \cdot c \end{bmatrix}$$

where $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$b = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$c = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

Now, as per question

$ca \cdot a + b \cdot b + c \cdot c = 9$

$\Rightarrow (a_1^2 + a_2^2 + a_3^2) + (b_1^2 + b_2^2 + b_3^2)$

$+(c_1^2 + c_2^2 + c_3^2) = 9$

$[a_i, b_i, c_i \in S]$

$9 = (1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1)$

or $9 = (1 + 4 + 4 + 0 + 0 + 0 + 0 + 0 + 0)$

or $9 = (9 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0)$

or $9 = (4 + 1 + 1 + 1 + 1 + 1 + 0 + 0 + 0)$

Total permutations in case 1 = 1

Total permutations in case 2 = $\frac{9!}{6!2!} = 252$

In case 3 = $\frac{9!}{8!} = 9$

In case 4 = $\frac{9!}{5!3!} = 504$

Total permutations = $1 + 252 + 9 + 504 = 766$

Question80

Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $a = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2×1 matrices with real entries such that $A = XB$, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$ and $k \in R$. If $a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2)$ and $(k^2 + 1)b_2^2 \neq -2b_1b_2$, then the value of k is [2021, 16 March Shift-II]

Answer: 1

Solution:

Solution:

$A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$A = XB$

$X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$

$XB = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$XB = \frac{1}{\sqrt{3}} \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$

As, $A = XB$

So, $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$

$\sqrt{3}a_1 = b_1 - b_2 \quad \cdots\cdots (i)$

$\sqrt{3}a_2 = b_1 + kb_2 \quad \cdots\cdots (ii)$

And as given, $a_1{}^2 + a_2{}^2 = \frac{2}{3}(b_1{}^2 + b_2{}^2)$

Adding, Eq. (i) ^{2} + Eq. (ii) ^{2},

$3a_1{}^2 + 3a_2{}^2 = (b_1 - b_2)^2 + (b_1 + kb_2)^2$

$\Rightarrow 2(b_1{}^2 + b_2{}^2)$

$= 2b_1{}^2 + b_2{}^2(k^2 + 1) + 2b_1b_2(k - 1)$

$\Rightarrow b_2{}^2(k^2 + 1 - 2) + 2b_1b_2(k - 1) = 0$

$\Rightarrow (k - 1)[b_2{}^2(k + 1) + 2b_1b_2] = 0$

So, $k = 1$

Question81

Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ **and** $A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix}$

where, $\omega = \frac{-1 + i\sqrt{3}}{2}$**, and** I_3 **be the identity matrix of order 3 . If the determinant of the matrix** $(P^{-1}AP - I_3)^2$ **is** $\alpha\omega^2$**, then the value of** α **is equal to.........**
[16 Mar 2021 Shift 1]

Answer: 36

Solution:

Solution:

Given, $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix}$

$|(P^{-1}AP - I_3)|^2 = \alpha\omega^2$

$\Rightarrow \ |(P^{-1}AP - I_3)(P^{-1}AP - I_3)| = \alpha\omega^2$

$\Rightarrow P^{-1}APP^{-1}AP - P^{-1}API_3 - I_3P^{-1}AP + I_3 \cdot I_3| = \alpha\omega^2$

$\Rightarrow \ P^{-1}A^2P - P^{-1}AP - P^{-1}AP + I_3| = \alpha\omega^2$

$[\because PP^{-1} = I \text{ and } IA = A]$

$\Rightarrow \ |P^{-1}A^2P - 2P^{-1}AP + PP^{-1}| = \alpha\omega^2$

$\Rightarrow \ |P^{-1}(A^2 - 2A + I_3)P| = \alpha\omega\omega^2$

$\Rightarrow \ |P^{-1}||A - I_3|^2|P| = \alpha\omega^2$

$\Rightarrow \ |P^{-1}P||A - I_3|^2 = \alpha\omega^2$

$\Rightarrow |A - I_3|^2 = \alpha\omega^2$

Consider,

$A - I_3 = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega + 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$|A - I_3| = \begin{vmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega - 1 & 1 \\ 0 & -\omega & -\omega \end{vmatrix}$

On applying $C_2 \to C_2 - C_3$, we get

$|A - I_3| = \begin{vmatrix} 1 & 7 - \omega^2 & \omega^2 \\ -1 & -\omega - 2 & 1 \\ 0 & 0 & -\omega \end{vmatrix}$

On applying $C_2 \to C_2 - C_3$, we get

$|A - I_3| = \begin{vmatrix} 1 & 7 - \omega^2 & \omega^2 \\ -1 & -\omega - 2 & 1 \\ 0 & 0 & -\omega \end{vmatrix}$

$= -\omega[(-\omega - 2) - (-7 + \omega^2)]$

$= -\omega(-\omega - 2 + 7 - \omega^2)$

$= -\omega(1 - 2 + 7)$

$= -6\omega$

$|A - I_3| = -6\omega$

$|A - I_3|^2 = 36\omega^2 = \alpha\omega^2$

$\therefore \ \alpha = 36$

Question82

If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of $\det(A^4) + \det[A^{10} - \text{Adj}(2A)^{10}]$ is equal to .....
[17 Mar 2021 Shift 1]

Answer: 16

Solution:

Solution:

If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$

$\det(A^4) + \det[A^{10} - [\mathrm{Adj}(2A)]^{10}]$

$A \cdot A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}, A^2 = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 9 \\ 0 & -1 \end{bmatrix}$

$A^4 = A^3 \cdot A$

$= \begin{bmatrix} 8 & 9 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 16 & 15 \\ 0 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 2^n & 2^n - (-1)^n \\ 0 & (-1)^n \end{bmatrix}$

$A^{10} = \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix} \Rightarrow 2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$

$\mathrm{adj}(2A) = \begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$

$\mathrm{adj}(2A) = -2 \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \left[\begin{array}{c} \because x = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \\ \mathrm{adj}(x) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \end{array} \right]$

$[\mathrm{adj}(2A)]^2 = 4 \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = 4 \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix}$

$[\mathrm{adj}(2A)]^3 = 4 \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix} \times (-2) \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = -8 \begin{pmatrix} 1 & 9 \\ 0 & -8 \end{pmatrix}$

$[\mathrm{adj}(2A)]^3 = 4 \begin{pmatrix} 1 & -3 \\ 0 & 4 \end{pmatrix} \times (-2) \begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix} = -8 \begin{pmatrix} 1 & 9 \\ 0 & -8 \end{pmatrix}$

$[\mathrm{adj}(2A)]^n = (-2)^n \begin{bmatrix} 1 & (-1)^n[2^n - (-1)^n] \\ 0 & (-1)^n 2^n \end{bmatrix}$

$[\mathrm{adj}(2A)]^{10} = 2^{10} \begin{pmatrix} 1 & -(2^{10} - 1) \\ 0 & 2^{10} \end{pmatrix}$

Now, $A^{10} - [\mathrm{adj}(2A)]^{10}$

$= \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2^{10} & -(2^{20} - 2^{10}) \\ 0 & 2^{20} \end{bmatrix}$

$= \begin{bmatrix} 0 & 2 \cdot 2^{10} - 2^{20} - 1 \\ 0 & 1 - 2^{20} \end{bmatrix}$

$\det | A^{10} - \mathrm{adj}(2A)^{10} | = 0$

$\therefore \det(A^4) + \det[A^{10} - \mathrm{adj}(2A)^{10}] = (16)^4 + 0 = 16$

Question83

If A = $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and M = A + A^2 + A^3 + ... + A^{20}, then the sum of all the elements of the matrix M is equal to
[2021, 27 July Shift-II]

Answer: 2020

Solution:

Solution:

We have, $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

....
......

$$A^n = \begin{bmatrix} 1 & n & \sum n \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

Now, $M = A + A^2 + A^3 + \ldots + A^{20}$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} +$$

$$\ldots + \begin{bmatrix} 1 & n & \sum n \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 20 & \sum n & \sum(\sum r) \\ 0 & 20 & \sum n \\ 0 & 0 & 20 \end{bmatrix}$$

Now, $\sum\limits_{r=1}^{20} n = 1 + 2 + \ldots + 20 = \frac{20 \times 21}{2} = 210$

$\sum\limits_{r=1}^{20} (\sum r) = \sum\limits_{r=1}^{20} \frac{r(r+1)}{2} = \frac{1}{2} \sum\limits_{r=1}^{20} (r^2 + r)$

$= \frac{1}{2}\left[\frac{20 \times 21 \times 41}{6} + \frac{20 \times 21}{2}\right]$

$= \frac{1}{2}[2870 + 210] = 1540$

Hence, $M = \begin{bmatrix} 20 & 210 & 1540 \\ 0 & 20 & 210 \\ 0 & 0 & 20 \end{bmatrix}$

Sum of all elements = 2020.

Question84

$$S = \left\{ n \in N \;\middle|\; \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^2 \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \; \forall a, b, c, d \in R \right\}$$

, where $i = \sqrt{-1}$. Then the number of 2-digit numbers in the set S is
[2021, 25 July Shift-1]

Answer: 11

Solution:

Solution:

Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $A = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n$

$\Rightarrow AX = 1X$

$A = 1$

$A^2 = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$

$\Rightarrow A = i \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\Rightarrow A^8 = i^4 l = l$

$\therefore n = 8$

$\Rightarrow n$ is a multiple of 8 .

16, 24, ... .96

Number of elements $= \dfrac{96 - 16}{8} + 1 = 11$

Question85

If $P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$, then P^{50} is

[2021, 25 July Shift-II]

Options:

A. $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

Answer: A

Solution:

Solution:

Given, $P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

$\Rightarrow P^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow P^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$

$\Rightarrow P^4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$\vdots$

$\Rightarrow P^n = \begin{bmatrix} 1 & 0 \\ \frac{n}{2} & 1 \end{bmatrix}$

Hence, $P^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

Question86

Let $A = [a_{ij}]$ be a real matrix of order 3×3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for i = 1, 2, 3. Then, the sum of all the entries of the matrix A^3 is equal to [2021, 22 July Shift-II]

Options:

A. 2

B. 1

C. 3

D. 9

Answer: C

Solution:

Solution:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = A$$

Sum of elements of each row is 1 .

Let X be $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Then, $AX = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$$AX = \begin{bmatrix} a_{11} + & a_{12} + & a_{13} \\ a_{21} + & a_{22} + & a_{23} \\ a_{31} + & a_{32} + & a_{33} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$AX = X$
Replace X with AX.
$A \cdot AX = AX \Rightarrow A^2X = X$
Again, replace X with AX,
$A^2(AX) = AX$
$A^3X = X$

Let $A^3 = \begin{bmatrix} a_{11}' & a_{12}' & a_{13}' \\ a_{21}' & a_{22}' & a_{23}' \\ a_{31}' & a_{32}' & a_{33}' \end{bmatrix}$

Then,

$$A^3X = \begin{bmatrix} a_{11}' & a_{12}' & a_{13}' \\ a_{21}' & a_{22}' & a_{23}' \\ a_{31}' & a_{32}' & a_{33}' \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A^3X = \begin{bmatrix} a_{11}' + & a_{12}' + & a_{13}' \\ a_{21}' + & a_{22}' + & a_{23}' \\ a_{31}' + & a_{32}' + & a_{33}' \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

So, $a_{11}' + a_{12}' + a_{13}' = 1$
$a_{21}' + a_{22}' + a_{23}' = 1$
$a_{31}' + a_{32}' + a_{33}' = 1$
$\therefore$ Sum $= 3$

Question87

Let A = $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then, the number of 3×3 matrices B with entries from the set {1, 2, 3, 4, 5} and satisfying AB = BA is
[2021, 22 July Shift-II]

Answer: 3125

Solution:

Solution:

$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & a_1 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$

AB = BA

$$\begin{pmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{pmatrix} = \begin{pmatrix} a_2 & a_1 & a_3 \\ b_2 & b1 & b_3 \\ c_2 & c1 & c_3 \end{pmatrix}$$

$$\begin{cases} b_1 = a_2 \\ b_2 = a_1 \\ b_3 = a_3. \end{cases} \quad \begin{cases} a_1 = b_2 \\ a_2 = b_1 \\ a_3 = b_3. \end{cases} \quad \begin{cases} C_1 = C_2 \\ C_2 = C_1 \\ C_3 = C_3. \end{cases}$$

$B = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_2 & a_1 & a_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$

Number of distinct elements in B is 5 $\{a_1, a_2, a_3, c_1, c_3\}$ and according to question, $a_{ij} \in \{1, 2, 3, 4, 5\}$.
So, number of matrices $= 5 \times 5 \times 5 \times 5 \times 5 = 3125$

Question88

Let A = $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^{7} + 2I$, where I is an identity matrix of order 3×3. If $B = [b_{ij}]$, then b_{13} is equal to ...
[2021, 20 July Shift-1]

Answer: 910

Solution:

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$B = 7A^{20} - 20A^{7} + 2I$

$$\therefore A^2 = A \cdot A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^2 = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^4 = A^3 A = \begin{pmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^4 = \begin{pmatrix} 1 & -4 & 6 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Rightarrow A^n = \begin{pmatrix} 1 & -n & \frac{n^2 - n}{2} \\ 0 & 1 & -n \\ 0 & 0 & 1 \end{pmatrix}$$

$\therefore\ b_{13} = 7\left(\frac{20 \times 19}{2}\right) - 20\left(\frac{7 \times 6}{2}\right) + 2(0)$

$\Rightarrow b_{13} = 1330 - 420 = 910$

Question89

If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ -\frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$, and $Q = A^T BA$, then the inverse of the matrix $AQ^{2021}A^T$ is equal to
[26 Aug 2021 Shift 1]

Options:

A. $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$

B. $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

C. $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

D. $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

Answer: B

Solution:

Solution:

$AA^T = \begin{vmatrix} \frac{1}{5} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{vmatrix} \begin{vmatrix} \frac{1}{\sqrt{5}} & -\frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{vmatrix}$

$AA^T \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = l$

Now, $Q^2 = A^TBAA^TBA$

$\Rightarrow Q^2 = A^TB^2A$

Again, $Q^3 = (A^TBA)(A^TB^2A) = A^TB^3A$

Similarly,

$Q^{2021} = A^TB^{2021}A$

$AQ^{2021}A^T = A(A^TB^{2021}A)A^T$

$= (AA^T)B^{2021}(AA^T) = B^{2021}$

$B = \begin{vmatrix} 1 & 0 \\ i & 1 \end{vmatrix}$

$B^2 = \begin{vmatrix} 1 & 0 \\ 2i & 1 \end{vmatrix}$, similarly $B^{2021} = \begin{vmatrix} 1 & 0 \\ 2021i & 1 \end{vmatrix}$

$(B^{2021})^{-1} = \frac{\text{adj}(B^{2021})}{|B^{2021}|} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

Question90

Let A be a 3×3 real matrix. If $\det(2\,\text{Adj}(2\,\text{Adj}(\text{Adj}(2A)))) = 2^{41}$, then the value of $\det(A^2)$ equal
[26 Aug 2021 Shift 2]

Answer: 4

Solution:

Solution:
We have, A is a square matrix of 3×3. Now,
$2\,\text{Adj}(2\,\text{Adj}(\text{Adj}(2A)))$

$= 2\,\mathrm{Adj}(2\,\mathrm{Adj}(2^{3-1}\,\mathrm{adj}\,A))$ [$\because \mathrm{adj}(KA) = K^{n-1}\,\mathrm{adj}\,A$]
$= 2\,\mathrm{Adj}(2\,\mathrm{Adj}(4\,\mathrm{Adj}\,A))$
$= 2\,\mathrm{Adj}(2 \times 4^{3-1}\,\mathrm{Adj}\,\mathrm{Adj}\,A)$
$= 2\,\mathrm{Adj}(32\,\mathrm{Adj}\,\mathrm{Adj}\,A)$
$= 2 \times 32^{3-1}\,\mathrm{Adj}\,\mathrm{Adj}\,\mathrm{Adj}\,A$
$= 2^{11}\,\mathrm{Adj}\,\mathrm{Adj}\,\mathrm{Adj}\,A$
$= 2^{11}\,\mathrm{Adj}(|A|^{3-2}A)$
$= 2^{11}\,\mathrm{Adj}(|A|A)$
$= 2^{11} \times |A|^{3-1}\,\mathrm{Adj}\,A$
$= 2^{11} \times |A|^{2}\,\mathrm{Adj}\,A$
Now, $|2\,\mathrm{Adj}(2\,\mathrm{Adj}(\mathrm{Adj}(2A)))| = 2^{41}$
$\Rightarrow |\,2^{11} \times |A|^{2}\,\mathrm{Adj}\,A = 2^{41}$
$\Rightarrow (2^{11})^{3}(|A|^{2})^{3}|\mathrm{Adj}\,A| = 2^{41}$
$\Rightarrow 2^{33}\,|A|^{6}\,|A|^{3-1} = 2^{41}$
$\Rightarrow |A|^{6} \times |A|^{2} = 2^{8}$
$\Rightarrow |A|^{8} = 2^{8}$
$\Rightarrow |A| = \pm 2$
Now, $|A^{2}| = |A|^{2} = (\pm 2)^{2} = 4$

Question91

Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in R$, I is a 2×2 identity matrix, then $4(\alpha - \beta)$ is equal to :
[27 Jul 2021 Shift 1]

Options:

A. 5

B. $\frac{8}{3}$

C. 2

D. 4

Answer: D

Solution:

Solution:

$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$, $|A| = 6$

$$A^{-1} = \frac{\mathrm{ad\,j}A}{|A|} = \frac{1}{6}\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -\beta & 4\beta \end{bmatrix}$$

$$\left.\begin{array}{l} \alpha + \beta = \frac{2}{3} \\ \beta = -\frac{1}{6} \end{array}\right\} \Rightarrow \alpha = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}$$

$4(\alpha - \beta) = 4(1) = 4$

Question92

Let A and B be two 3×3 real matrices such that$(A^2 - B^2)$ is invertible matrix. If $A^5 = B^5$ and$A^3B^2 = A^2B^3$, then the value of the determinant ofthe matrix $A^3 + B^3$ is equal to :
[27 Jul 2021 Shift 2]

Options:

A. 2

B. 4

C. 1

D. 0

Answer: D

Solution:

Solution:
$C = A^2 - B^2; \mid C \mid \neq 0$
$A^5 = B^5$ and $A^3B^2 = A^2B^2$
Now, $A^5 - A^3B^2 = B^5 - A^2B^3$
$\Rightarrow A^3(A^2 - B^2) + B^3(A^2 - B^2) = 0$
$\Rightarrow (A^3 + B^3)(A^2 - B^2) = 0$
Post multiplying inverse of $A^2 - B^2$:
$A^3 + B^3 = 0$

Question93

Let $A = \{a_{ij}\}$ be a 3×3 matrix, where$a_{ij} = \begin{cases} (-1)^{j-1} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$ then

$\det(3Adj(2A^{-1}))$ is equal to ______.
[20 Jul 2021 Shift 2]

Answer: 108

Solution:

Solution:

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$|A| = 4$

$|3\operatorname{adj}(2A^{-1})| = |3.2^2\operatorname{adj}(A^{-1})|$

$= 12^3 |\operatorname{adj}(A^{-1})| = 12^3 |A^{-1}|^2 = \frac{12^3}{|A|^2} = \frac{12^3}{16} = 108$

Question94

If the matrix A = $\begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of K is
[27 Aug 2021 Shift 1]

Options:

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. -1

D. 1

Answer: A

Solution:

Solution:
Given matrix,

$A = \begin{bmatrix} 0 & 2 \\ K & -1 \end{bmatrix}$

Characteristic equation of A is
$|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} -\lambda & 2 \\ K & -1 & -\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda(1 + \lambda) - 2k = 0$
$\Rightarrow \lambda^2 + \lambda - 2k = 0$
∵ Every square matrix satisfied its own characteristic equation.
$\therefore A^2 + A - 2KI = 0$
$\Rightarrow A^2 = 2KI - A$
$\Rightarrow A^4 = 4K^2I + A^2 - 2(2KI)(A)$
$\Rightarrow A^4 = 4K^2I + 2KI - A - 4KA$
$\Rightarrow A^4 = (4K^2 + 2K)I - (1 + 4K)A$ ...(i)
Now, $A(A^3 + 3I) = 2I$
$\Rightarrow A^4 = 2I - 3A$ ...(ii)
Comparing Eqs. (i) and (ii), we get
$1 + 4K = 3$
$\Rightarrow K = \frac{1}{2}$

Question95

Let A = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to

[2021, 26 Aug. Shift-II]

Options:

A. $A^6 - A$

B. A^5

C. $A^5 - A$

D. A^6

Answer: A

Solution:

Solution:

$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

Now,

$$A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

......

......

$$A^n = \begin{pmatrix} 1 & 0 & 0 \\ n-2 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$A^{2025} - A^{2020} = \begin{pmatrix} 1 & 0 & 0 \\ 2024 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 2019 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now,

$$A^6 - A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore A^{2025} - A^{2020} = A^6 - A$

Question96

Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix

$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to:

[Jan. 7, 2020 (I)]

Options:

A. A

B. I_3

C. A^2

D. A^3

Answer: D

Solution:

Solution:
Solution of $x^2 + x + 1 = 0$ is ω, ω^2
So, $\alpha = \omega$ and
$\omega^4 = \omega^3 . \omega = 1 . \omega = \omega$

$$A^2 = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow A^4 = I$
$\Rightarrow A^{30} = A^{28} \times A^3 = A^3$

Question97

The number of all 3 × 3 matrices A, with enteries from the set {−1, 0, 1} such that the sum of the diagonal elements of AA^T is 3, is______.
[NA Jan. 8, 2020 (I)]

Answer: 672

Solution:

Let $A = [a_{ii}]_{3 \times 3}$

It is given that sum of diagonal elements of AA^T is 3 i.e., $tr(AA^T) = 3$
$a_{11}^2 + a_{12}^2 + a_{13}^2 + a_{21}^2 + \ldots\ldots + a_{33}^2 = 3$
Possible cases are

$$\left.\begin{array}{cc} 0,0,0,0,0,0,1,1,1 & \rightarrow 1 \\ 0,0,0,0,0,0,-1,-1,-1, & \rightarrow 1 \\ 0,0,0,0,0,0,1,1,-1 & \rightarrow 3 \\ 0,0,0,0,0,0,-1,1,-1 & \rightarrow 3 \end{array}\right\} {}^9C_6 \times 8 = 84 \times 8 = 672$$

Question98

If $A = \begin{pmatrix} 2 & 2 \\ 9 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $10A^{-1}$ is equal to:
[Jan. 8, 2020 (II)]

Options:

A. $A - 4I$

B. $6I - A$

C. $A - 6I$

D. $4I - A$

Answer: C

Solution:

Solution:
Characteristics equation of matrix ' A ' is $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 2 \\ 9 & 4-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 6\lambda - 10 = 0$$

$\therefore A^2 - 6A - 10I = 0$
$\Rightarrow A^{-1}(A^2) - 6A^{-1} - 10I\,A^{-1} = 0$
$\Rightarrow 10A^{-1} = A - 6I$

Question99

If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true?
[Sep. 04, 2020 (I)]

Options:

A. $0 \le a^2 + b^2 \le 1$

B. $a^2 - d^2 = 0$

C. $a^2 - c^2 = 1$

D. $a^2 - b^2 = \frac{1}{2}$

Answer: D

Solution:

Solution:

$\because A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$

$\therefore A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}, n \in N$

$\because A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore A^5 = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore a = \cos 5\theta, b = i\sin 5\theta = c, d = \cos 5\theta$

$\therefore a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

$a^2 - c^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

$a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 1$

$a^2 + b^2 = \cos^2 5\theta - \sin^2 5\theta = \cos 10\theta = \cos\frac{10\pi}{24}$

and $0 < \cos\frac{5\pi}{12} < 1 \Rightarrow 0 \le a^2 + b^2 \le 1$

$\therefore a^2 - b^2 = \frac{1}{2}$ is wrong.

Question100

Let $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, $x \in R$ and $A^4 = [a_{ij}]$. If $a_{11} = 109$, then a_{22} is equal to ______.

[NA Sep. 03, 2020 (I)]

Answer: 10

Solution:

Solution:

$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$x^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$

$= \begin{bmatrix} (x^2+1)^2 + x^2 & x(x^2+1) + x \\ x(x^2+1) + x & x^2+1 \end{bmatrix}$

Given that $(x^2+1)^2 + x^2 = 109$

$x^4 + 3x^2 - 108 = 0$

$\Rightarrow (x^2+12)(x^2-9) = 0$

$\therefore x^2 = 9$
$a_{22} = x^2 + 1 = 9 + 1 = 10$

Question101

Let a, b, c ∈ R be all non-zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ satisfies $A^T A = I$, then a value of abc can be:

[Sep. 02, 2020 (II)]

Options:

A. $-\frac{1}{3}$

B. $\frac{1}{3}$

C. 3

D. $\frac{2}{3}$

Answer: B

Solution:

Solution:
Given : $A^T A = I$

$$\Rightarrow = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \sum a^2 & \sum ab & \sum ab \\ \sum ab & \sum a^2 & \sum ab \\ \sum ab & \sum ab & \sum a^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, $\sum a^2 = 1$ and $\sum ab = 0$
Now, $a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$= (a + b + c)(1 - 0)$
$= \sqrt{(a + b + c)^2} = \sqrt{\sum a^2 + 2\sum ab} = \pm 1$

$\Rightarrow 2 - 3abc = 1 \Rightarrow abc = \frac{1}{3}$
or $2 - 3abc = -1 \Rightarrow abc = 1.$

Question102

Let A = $\begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then |p| is :

[Jan. 11, 2019 (I)]

Options:

A. $\frac{1}{\sqrt{5}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{1}{\sqrt{6}}$

Answer: C

Solution:

Solution:

$$A = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}$$

$$\therefore A \,.\, A^T = \begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix} \times \begin{bmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r \end{bmatrix}$$

$$= \begin{bmatrix} 4q^2 + r^2 & 2q^2 - r^2 & -2q^2 + r^2 \\ 2q^2 - r^2 & p^2 + q^2 + r^2 & p^2 - q^2 - r^2 \\ -2q^2 + r^2 & p^2 - q^2 - r^2 & p^2 + q^2 + r^2 \end{bmatrix}$$

Given, $AA^T = I$

$\therefore 4q^2 + r^2 = p^2 + q^2 + r^2 = 1$

$\Rightarrow p^2 - 3q^2 = 0$ and $r^2 = 1 - 4q^2$

and $2q^2 - r^2 = 0 \Rightarrow r^2 = 2q^2$

$\therefore p^2 = \frac{1}{2}$, $q^2 = \frac{1}{6}$ and $r^2 = \frac{1}{3}$

$\therefore |p| = \frac{1}{\sqrt{2}}$

Question103

Let P = $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and Q = $[q_{ij}]$ be two 3 × 3 matrices such that $Q - P^5 = I_3$. Then $\frac{q_{21} + q_{31}}{q_{32}}$ is equal to:

[Jan. 12, 2019 (I)]

Options:

A. 10

B. 135

C. 15

D. 9

Answer: A

Solution:

Solution:

$$P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix}$$

$$\Rightarrow P^4 = \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 27 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix}$$

$$\Rightarrow P^5 = = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 90 & 12 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$$

$\because Q - P^5 = I_3$

$$\therefore Q = I_3 + P^6 = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix}$$

$$\frac{q_{21} + q_{31}}{q_{32}} = \frac{15 + 135}{15} = 10$$

Question104

Let A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, ($\alpha \in R$) such that $A^{32} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ Then a value of α is :
[April 8, 2019 (I)]

Options:

A. $\frac{\pi}{32}$

B. 0

C. $\frac{\pi}{64}$

D. $\frac{\pi}{16}$

Answer: C

Solution:

Solution:

$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$

Similarly, $A^4 = A^2 . A^2 = \begin{bmatrix} \cos 4\alpha & -\sin 4\alpha \\ \sin 4\alpha & \cos 4\alpha \end{bmatrix}$

and so on $A^{32} = \begin{bmatrix} \cos 32\alpha & -\sin 32\alpha \\ \sin 32\alpha & \cos 32\alpha \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Then $\sin 32\alpha = 1$ and $\cos 32\alpha = 0$

$\Rightarrow 32\alpha = n\pi + (-1)^n\frac{\pi}{2}$ and $32\alpha = 2n\pi + \frac{\pi}{2}$

$\Rightarrow \alpha = \frac{n\pi}{32} + (-1)^n\frac{\pi}{64}$ and $\alpha = \frac{n\pi}{16} + \frac{\pi}{64}$ where $n \in z$

Put $n = 0$, $\alpha = \frac{\pi}{64}$

Hence, required value of α is $\frac{\pi}{64}$

Question105

The total number of matrices $A = \begin{pmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{pmatrix}$, $(x, y \in R, x \neq y)$ for which $A^T A = 3I_3$ is:
[April 09, 2019 (II)]

Options:

A. 2

B. 3

C. 6

D. 4

Answer: D

Solution:

Solution:
Given, $A^T A = 3I_3$

$\begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix} = 31$

$\Rightarrow \begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\Rightarrow 8x^2 = 3$ and $6y^2 = 3 \Rightarrow x = \pm\sqrt{\frac{3}{8}}$ and $y = \pm\sqrt{\frac{1}{2}}$

Number of combinations of $(x, y) = 2 \times 2 = 4$

Question106

If A is a symmetric matrix and B is a skew-symmetrix matrix such that $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to:

[April 12, 2019 (I)]

Options:

A. $\begin{bmatrix} -4 & -1 \\ -1 & 4 \end{bmatrix}$

B. $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

C. $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$

D. $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$

Answer: B

Solution:

Solution:

Let $A = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$ and $B = \begin{bmatrix} 0 & d \\ -d & 0 \end{bmatrix}$

Then, $A + B = \begin{bmatrix} a & c+d \\ c-d & b \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$

On comparing each term,

$a = 2, b = -1, c - d = 5, c + d = 3$

$\Rightarrow a = 2, b = -1, c = 4, d = -1$

Now, $AB = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

Question107

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is?

[Online April 16, 2018]

Options:

A. 211

B. 210

C. 231

D. 251

Answer: C

Solution:

Solution:

Here $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

also $A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix}$

and, $A^4 = A^3 \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 10 & 4 & 1 \end{bmatrix}$

On observing the pattern, we come to a conclusion that,

$A = \begin{bmatrix} 1 & 0 & 0 \\ n & 1 & 0 \\ \frac{n(n+1)}{2} & n & 1 \end{bmatrix}$

$\therefore A^{20} = [1, 0, 0; 20, 1, 0; 210, 20, 1]$

Therefore, sum of first column of $A^{20} = [1 + 20 + 210] = 231$

Question108

For two 3 × 3 matrices A and B, let $A + B = 2B^T$ and $3A + 2B = I_3$, where B^T is the transpose of B and I_3 is 3 × 3 identity matrix. Then : quad

[Online April 9, 2017]

Options:

A. $5A + 10B = 2I_3$

B. $10A + 5B = 3I_3$

C. $B + 2A = I_3$

D. $3A + 6B = 2I_3$

Answer: B

Solution:

Solution:

$A^T + B^T = 2B$

$\because [(A+B)^T = (2B^T)^T]$

$\Rightarrow B = \frac{A^T + B^T}{2} = A + \left(\frac{B^T + A^T}{2}\right) = 2B^T$

$\Rightarrow 2A + A^T = 3B^T \Rightarrow A = \frac{3B^T - A^T}{2}$

Also, $3A + 2B = I_3$ ....(i)

$\Rightarrow 3(3B^T - A^T 2) + 2\left(\frac{A^T + B^T}{2}\right) = I_3$

$\Rightarrow 11B^T - A^T = 2I_3$ ......(ii)

Add (i) and (ii)

$35\sim B = 7I_3$

$\Rightarrow B = \frac{I_3}{5} \Rightarrow 11\frac{I_3}{5} - A = 2I_3$

$\Rightarrow 11\frac{I_3}{5} - 2I_3 = A \Rightarrow A = \frac{I_3}{5}$

$\because 5A = 5B = I_3$

$\Rightarrow 10A + 5B = 3I_3$

Question109

If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then$P^T Q^{2015} P$ is ;

[Online April 9, 2016]

Options:

A. $\begin{bmatrix} 0 & 2015 \\ 0 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 2015 & 0 \\ 1 & 2015 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 2015 & 1 \\ 0 & 2015 \end{bmatrix}$

Answer: C

Solution:

Solution:

$P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$ $P^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$PP^T = P^TP = I$

$Q^{2015} = (PAP^T)(PAP^T)$———(2015 terms)

$= PA^{2015}P^T$

$P^TQ^{2015}P = A^{2015}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$\therefore A^{2015} = \begin{bmatrix} 1 & 2015 \\ 0 & 1 \end{bmatrix}$

Question110

If A = $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then which one of the following statements is not correct?
[Online April 10, 2015]

Options:

A. $A^2 + I = A(A^2 - I)$

B. $A^4 - I = A^2 + I$

C. $A^3 + I = A(A^3 - I)$

D. $A^3 - I = A(A - I)$

Answer: A

Solution:

Solution:
Given that

$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \Rightarrow A^2 = -I$

$A^3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$A^2 + I = A^3 - A$
$-I + I = A^3 - A$
$A^3 \neq A$

Question111

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3×3 identity matrix, then the ordered pair (a, b) is equal to: [2015]

Options:

A. (2,1)

B. (-2,-1)

C. (2,-1)

D. (-2,1)

Answer: B

Solution:

Solution:
Given that $AA^T = 9I$

$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -1 & b \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1+4+4 & 2+2-4 & a+4+2b \\ 2+2-4 & 4+1+4 & 2a+2-2b \\ a+4+2b & 2a+2-2b & a^2+4+b^2 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$\Rightarrow a + 4 + 2b = 0 \Rightarrow a + 2b = -4$ ......(i)
$2a + 2 - 2b = 0 \Rightarrow 2a - 2b = -2$ .....(ii)
$\Rightarrow a - b = -1$
Subtract (ii) from (i)
$a + 2b = -4$
$a - b = -1$
$- \quad + \quad +$

$3b = -3$
$b = -1$
and $a = -2$
(a, b) = (–2, –1)

Question112

If A = $\begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ be such that AB = $\begin{bmatrix} 6 \\ 8 \end{bmatrix}$,then:

[Online April 12, 2014]

Options:

A. $y = 2x$

B. $y = -2x$

C. $y = x$

D. $y = -x$

Answer: A

Solution:

Solution:

Let $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} y + 2x + x \\ 3y - x + 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} y + 3x \\ 3y - x + 2 \end{bmatrix}$

$\Rightarrow y + 3x = 6$ and $3y - x = 6$

On solving, we get

$x = \frac{6}{5}$ and $y = \frac{12}{5}$

$\Rightarrow y = 2x$

Question113

Let A and B be any two 3 × 3 matrices. If A is symmetric and B is skewsymmetric, then the matrix AB – BA is:
[Online April 19, 2014]

Options:

A. skewsymmetric

B. symmetric

C. neither symmetric nor skewsymmetric

D. I or – I, where I is an identity matrix

Answer: B

Solution:

Solution:
Let A be symmetric matrix and B be skew symmetric matrix.
$\therefore A^T = A$ and $B^T = -B$
Consider $(AB - BA)^T = (AB)^T - (BA)^T$
$= B^T A^T - A^T B^T = (-B)(A) - (A)(-B)$
$= -BA + AB = AB - BA$
This shows $AB - BA$ is symmetric matrix.

Question114

If p, q, r are 3 real numbers satisfying the matrix equation,

$$\begin{bmatrix} p & q & r \end{bmatrix} \begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$ **then $2p + q - r$ equals :**

[Online April 22, 2013]

Options:

A. – 3

B. – 1

C. 4

D. 2

Answer: A

Solution:

Solution:
Given

$$\begin{bmatrix} p & q & r \end{bmatrix} \begin{bmatrix} 3 & 4 & 1 \\ 3 & 2 & 3 \\ 2 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 3p + 3q + 2r & 4p + 2q & p + 3q + 2r \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \end{bmatrix}$$

$\Rightarrow 3p + 3q + 2r = 3$ ......(i)
$4p + 2q = 0 \Rightarrow q = -2p$ ......(ii)
$p + 3q + 2r = 1$ ......(iii)
On solving (i), (ii) and (iii), we get
$p = 1, q = -2, r = 3$
$\therefore 2p + q - r = 2(1) + (-2) - (3) = -3.$

Question115

The matrix $A^2 + 4A - 5I$, where I is identity matrix and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, equals
[Online April 9, 2013]

Options:

A. $4\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

B. $4\begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$

C. $32\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

D. $32\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Answer: A

Solution:

Solution:

$A^2 + 4A - 5I = A \times A + 4A - 5I$

$= \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 9+4-5 & -4+8-0 \\ -8+16-0 & 17-12-5 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix} = 4\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

Question116

If $A = \begin{pmatrix} \alpha - 1 \\ 0 \\ 0 \end{pmatrix}$, $B = \begin{pmatrix} \alpha + 1 \\ 0 \\ 0 \end{pmatrix}$ be two matrices, then AB^T is anon-zero matrix for |a| not equal to
[Online May 7, 2012]

Options:

A. 2

B. 0

C. 1

D. 3

Answer: C

Solution:

Solution:

Let $A = \begin{pmatrix} \alpha - 1 \\ 0 \\ 0 \end{pmatrix}$, $B = \begin{pmatrix} \alpha + 1 \\ 0 \\ 0 \end{pmatrix}$

be two matrices.

$$AB^T = \begin{pmatrix} \alpha - 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \alpha + 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \alpha^2 - 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Thus, AB^T is non-zero matrix for $|\alpha| \neq 1$

Question117

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 7 & -2 & 1 \end{bmatrix}$ then AB equals

[Online May 26, 2012]

Options:

A. I

B. A

C. B

D. 0

Answer: A

Solution:

Solution:

$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 7 & -2 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Question118

If $\omega \neq 1$ is the complex cube root of unity and matrix H $= \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to
[2011RS]

Options:

A. 0

B. $-H$

C. H^2

D. H

Answer: D

Solution:

Solution:

$H^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$

We observed that $H^k = \begin{bmatrix} \omega^k & 0 \\ 0 & \omega^k \end{bmatrix}$

$\therefore H^{70} = \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega^{69}\omega & 0 \\ 0 & \omega^{69}\omega \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = H \; [\because \omega^{3n} = 1]$

Question119

Let A and B be two symmetric matrices of order 3.
Statement-1: A(BA) and (AB)A are symmetric matrices.
Statement-2: AB is symmetric matrix if matrix multiplication of A with B is commutative.
[2011]

Options:

A. Statement-1 is true, Statement-2 is true; Statement-2 isnot a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is false.

C. Statement-1 is false, Statement-2 is true.

D. Statement-1 is true, Statement-2 is true; Statement-2 isa correct explanation for Statement-1.

Answer: A

Solution:

Solution:
Given that A and B are symmetric matrix
A' = A
B' = B
Now (A(BA))' = (BA)'A' = (A'B')A' = (AB)A = A(BA)(∵ product of matrices are associative)
Similarly, ((AB)A)' = A'(B'A') = A (BA) = (AB)A
So, A(BA) and (AB)A are symmetric matrices.
Again (AB)' = B'A' = BA
Now if BA = AB, then AB is symmetric matrix

Question120

The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is
[2010]

Options:

A. 5

B. 6

C. at least 7

D. less than 4

Answer: C

Solution:

Solution:
$\begin{bmatrix} 1 & \dots & \dots \\ \dots & 1 & \dots \\ \dots & \dots & 1 \end{bmatrix}$ are 6 non-singular matrices because 6 blanks will be filled by 5 zeros and 1 one.
$\begin{bmatrix} 1 & \dots & \dots \\ \dots & 1 & \dots \\ \dots & \dots & 1 \end{bmatrix}$ are 6 non-singular matrices.
Total = 6 + 6 = 12
So, required cases are more than 7, non-singular 3 × 3 matrices.

Question121

Let A = $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and B = $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, a, b ∈ N . Then
[2006]

Options:

A. there cannot exist any B such that AB = BA

B. there exist more than one but finite number of B's such that AB = BA

C. there exists exactly one B such that AB = BA

D. there exist infinitely many B¢s such that AB = BA

Answer: D

Solution:

Solution:

Given that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$AB = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

$BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

Hence, AB = BA only when a = b

∴ There can be infinitely many B's for which AB = BA

Question122

If A and B are square matrices of size n × n such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true? [2006]

Options:

A. A = B

B. AB = BA

C. either of A or B is a zero matrix

D. either of A or B is identity matrix

Answer: B

Solution:

Solution:

Given that $A^2 - B^2 = (A - B)(A + B)$

$A^2 - B^2 = A^2 + AB - BA - B^2$

$\Rightarrow AB = BA$

Question123

If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all n ≥ 1, by the principle of mathematical induction [2005]

Options:

A. $A^n = nA - (n-1)I$

B. $A^n = 2^{n-1}A - (n-1)I$

C. $A^n = nA + (n-1)I$

D. $A^n = 2^{n-1}A + (n-1)I$

Answer: A

Solution:

Solution:

Given that $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $A^3 = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

Therefore we observed that $A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$

Now $nA - (n-1)I = \begin{bmatrix} n & 0 \\ n & n \end{bmatrix} - \begin{bmatrix} n-1 & 0 \\ 0 & n-1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} = A^n$

$\therefore nA - (n-1)I = A^n$

Question124

If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

[2003]

Options:

A. $\alpha = 2ab, \beta = a^2 + b^2$

B. $\alpha = a^2 + b^2, \beta = ab$

C. $\alpha = a^2 + b^2, \beta = 2ab$

D. $\alpha = a^2 + b^2, \beta = a^2 - b^2$.

Answer: C

Solution:

Solution:

$A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} \Rightarrow A.A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$

$= \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$

$\alpha = a^2 + b^2$; $\beta = 2ab$
