Determinants

Question1

Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $B = [B_1, B_2, B_3]$, where B_1, B_2, B_3 are column matrices, and $AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to

[27-Jan-2024 Shift 1]

Options:

Answer: 28

Solution:

$x_1 = 1, y_1 = -1, z_1 = -1$

$$AB_2 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$x_2 = 2, y_2 = 1, z_2 = -2$

$$AB_3 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$x_3 = 2, y_3 = 0, z_3 = -1$

$$B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$\alpha = |B| = 3$

$\beta = 1$

$\alpha^3 + \beta^3 = 27 + 1 = 28$

Question2

The values of α, for which $\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix}=0$, lie in the interval

[27-Jan-2024 Shift 2]

Options:

A.

$(-2, 1)$

B.

$(-3, 0)$

C.

$\left(-\frac{3}{2}, \frac{3}{2}\right)$

D.

$(0, 3)$

Answer: B

Solution:

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix}=0$$

$$\Rightarrow (2\alpha+3)\left\{\frac{7\alpha}{6}\right\}-(3\alpha+1)\left\{\frac{-7}{6}\right\}=0$$

$$\Rightarrow (2\alpha+3)\cdot\frac{7\alpha}{6}+(3\alpha+1)\cdot\frac{7}{6}=0$$

$$\Rightarrow 2\alpha^2+3\alpha+3\alpha+1=0$$

$$\Rightarrow 2\alpha^2+6\alpha+1=0$$

$$\Rightarrow \alpha=\frac{-3+\sqrt{7}}{2}, \frac{-3-\sqrt{7}}{2}$$

Hence option (2) is correct.

Question3

Let for any three distinct consecutive terms a, b, c of an A.P, the lines ax + by + c = 0 be concurrent at the point P and Q(α, β) be a point such that the system of equations

$x + y + z = 6,$

$2x + 5y + \alpha z = \beta$ and

$x + 2y + 3z = 4,$

has infinitely many solutions. Then $(PQ)^2$ is equal to____

[29-Jan-2024 Shift 2]

Answer: 113

Solution:

$\because$ a, b, c and in A.P

$\Rightarrow 2b = a + c \Rightarrow a - 2b + c = 0$

$\therefore ax + by + c$ passes through fixed point $(1, -2)$

$\therefore P = (1, -2)$

For infinite solution,

$D = D_1 = D_2 = D_3 = 0$

$$D : \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow \alpha = 8$

$$D_1 : \begin{vmatrix} 6 & 1 & 1 \\ \beta & 5 & \alpha \\ 4 & 2 & 3 \end{vmatrix} = 0 \Rightarrow \beta = 6$$

$\therefore Q = (8, 6)$

$\therefore Q^2 = 113$

Question4

If $f(x)=\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5}f'(0)$ is equal to

[30-Jan-2024 Shift 1]

Options:

A.

0

B.

1

C.

2

D.

6

Answer: B

Solution:

$$\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$$

$R_2 \longrightarrow R_2-R_1, R_3 \longrightarrow R_3-R_1$

$$\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3 & 0 & -3 \\ 0 & 3 & -3 \end{vmatrix}$$

$f(x)=45$

$f'(x)=0$

Question5

Consider the system of linear equation x + y + z = 4μ, x + 2y + 2λz = 10μ, x + 3y + 4λ^{2}z = μ2 + 15 where λ, μ∈R. Which one of the following statements is NOT correct?

[30-Jan-2024 Shift 1]

Options:

A.

The system has unique solution if λ ≠ 1/2 and μ ≠ 1, 15

B.

The system is inconsistent if λ = 1/2 and μ ≠ 1

C.

The system has infinite number of solutions if λ = 1/2 and μ = 15

D.

The system is consistent if λ ≠ 1/2

Answer: B

Solution:

$x+y+z=4\mu,\ x+2y+2\lambda z=10\mu,\ x+3y+4\lambda^2 z=\mu^2+15$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = (2\lambda-1)^2$$

For unique solution $\Delta \neq 0,\ 2\lambda - 1 \neq 0,\ \left(\lambda \neq \frac{1}{2}\right)$

Let $\Delta = 0,\ \lambda = \frac{1}{2}$

$$\Delta_y = 0,\ \Delta_x = \Delta_z = \begin{vmatrix} 4\mu & 1 & 1 \\ 10\mu & 2 & 1 \\ \mu^2+15 & 3 & 1 \end{vmatrix}$$

$= (\mu-15)(\mu-1)$

For infinite solution $\lambda = \frac{1}{2},\ \mu = 1$ or 15

Question6

Consider the system of linear equations

x + y + z = 5, x + 2y + λ^2z = 9

x + 3y + λz = μ, where λ, μ∈R. Then, which the following statement is NOT correct?

[30-Jan-2024 Shift 2]

Options:

A.

System has infinite number of solution if λ = and μ = 13

B.

System is inconsistent if λ = 1 and μ ≠ 13

C.

System is consistent if λ ≠ 1 and μ = 13

D.

System has unique solution if λ ≠ 1 and μ ≠ 13

Answer: D

Solution:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 2\lambda^2 - \lambda - 1 = 0$$

$$\lambda = 1, -\frac{1}{2}$$

$$\begin{vmatrix} 1 & 1 & 5 \\ 2 & \lambda^2 & 9 \\ 3 & \lambda & \mu \end{vmatrix} = 0 \Rightarrow \mu = 13$$

Infinite solution $\lambda = 1$ & $\mu = 13$

For unique $\mathrm{sol}^n \lambda \neq 1$

For no $\mathrm{sol}^n \lambda = 1$ & $\mu \neq 13$

If $\lambda \neq 1$ and $\mu \neq 13$

Considering the case when $\lambda = -\frac{1}{2}$ and $\mu \neq 13$ this will generate no solution case

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Question7

If the system of linear equations

$$x-2y+z=-4$$
$$2x+\alpha y+3z=5$$
$$3x-y+\beta z=3$$

has infinitely many solutions, then 12α + 13β is equal to

[31-Jan-2024 Shift 1]

Options:

A.

60

B.

64

C.

54

D.

58

Answer: D

Solution:

$$D=\begin{vmatrix} 1 & -2 & 1 \\ 2 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix}$$

$$=1(\alpha\beta+3)+2(2\beta-9)+1(-2-3\alpha)$$

$$=\alpha\beta+3+4\beta-18-2-3\alpha$$

For infinite solutions $D=0$, $D_1=0$, $D_2=0$ and

$D_3=0$

$D=0$

$\alpha\beta-3\alpha+4\beta=17 \ldots\ldots (1)$

$$D_1 = \begin{vmatrix} -4 & -2 & 1 \\ 5 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$$D_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$$\Rightarrow 1(5\beta - 9) + 4(2\beta - 9) + 1(6 - 15) = 0$$

$$13\beta - 9 - 36 - 9 = 0$$

$$13\beta = 54, \beta = \frac{54}{13} \text{ put in (1)}$$

$$\frac{54}{13}\alpha - 3\alpha + 4\left(\frac{54}{13}\right) = 17$$

$$54\alpha - 39\alpha + 216 = 221$$

$$15\alpha = 5 \quad \alpha = \frac{1}{3}$$

$$\text{Now, } 12\alpha + 13\beta = 12 \cdot \frac{1}{3} + 13 \cdot \frac{54}{13}$$

$$= 4 + 54 = 58$$

Question8

If $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all $x \in \mathbb{R}$, then $2f(0) + f'(0)$ is equal to

[31-Jan-2024 Shift 1]

Options:

A.

48

B.

24

C.

42

D.

18

Answer: C

Solution:

$$f(0)=\begin{vmatrix}0&1&1\\2&0&6\\0&4&-2\end{vmatrix}=12$$

$$f'(x)=\begin{vmatrix}3x^2&4x&3\\3x^2+2&2x&x^3+6\\x^3-x&4&x^2-2\end{vmatrix}+\begin{vmatrix}x^3&2x^2+1&1+3x\\6x&2&3x^2\\x^3-x&4&x^2-2\end{vmatrix}+\begin{vmatrix}x^3&2x^2+1&1+3x\\3x^2+2&2x&x^3+6\\3x^2-1&0&2x\end{vmatrix}$$

$$\therefore f'(0)=\begin{vmatrix}0&0&3\\2&0&6\\0&4&-2\end{vmatrix}+\begin{vmatrix}0&1&1\\0&2&0\\0&4&-2\end{vmatrix}+\begin{vmatrix}0&1&1\\2&0&6\\-1&0&0\end{vmatrix}$$

$$=24-6=18$$

$$\therefore 2f(0)+f'(0)=42$$

Question9

Let A be a 3×3 real matrix such that

$$A\begin{pmatrix}1\\0\\1\end{pmatrix}=2\begin{pmatrix}1\\0\\1\end{pmatrix},\ A\begin{pmatrix}-1\\0\\1\end{pmatrix}=4\begin{pmatrix}-1\\0\\1\end{pmatrix},\ A\begin{pmatrix}0\\1\\0\end{pmatrix}=2\begin{pmatrix}0\\1\\0\end{pmatrix}$$

Then, the system $(A-3I)\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ has

[31-Jan-2024 Shift 2]

Options:

A.

unique solution

B.

exactly two solutions

C.

no solution

D.

infinitely many solutions

Answer: A

Solution:

Let $A=\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$

Given $A\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$ ....(1)

$\therefore \begin{bmatrix} x_1+z_1 \\ x_2+z_2 \\ x_3+z_3 \end{bmatrix}=\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$

$\therefore x_1+z_1=2$ ....(2)

$x_2+z_2=0$ ....(3)

$x_3+z_3=0$ ....(4)

Given $A\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}=\begin{bmatrix} -4 \\ 0 \\ 4 \end{bmatrix}$

$\therefore \begin{bmatrix} -x_1+z_1 \\ -x_2+z_2 \\ -x_3+z_3 \end{bmatrix}=\begin{bmatrix} 4 \\ 0 \\ 4 \end{bmatrix}$

$\Rightarrow -x_1+z_1=-4$ ....(5)

$-x_2+x_2=0$ ....(6)

$-x_3+z_3=4$ ....(7)

Given $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\2\\0\end{bmatrix}$

$\therefore \begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}=\begin{bmatrix}0\\2\\0\end{bmatrix}$

$\therefore y_1=0, y_2=2, y_3=0$

$\therefore$ from (2), (3), (4), (5), (6) and (7)

$x_1=3x, x_2=0, x_3=-1$

$y_1=0, y_2=2, y_3=0$

$z_1=-1, z_2=0, z_3=3$

$\therefore A=\begin{bmatrix}3&0&-1\\0&2&0\\-1&0&3\end{bmatrix}$

$\therefore$ Now $(A-3I)\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$

$\therefore \begin{bmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$

$\begin{bmatrix}-z\\-y\\-x\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$

$[z=-1], [y=-2], [x=-3]$

Question10

If the system of equations
$2x+3y-z=5$
$x+\alpha y+3z=-4$
$3x-y+\beta z=7$

has infinitely many solutions, then $13\alpha\beta$ is equal to

[1-Feb-2024 Shift 1]

Options:

A.

1110

B.

1120

C.

1210

D.

1220

Answer: B

Solution:

Using family of planes

$2x + 3y - z - 5 = k_1(x + \alpha y + 3z + 4) + k_2(3x - y + \beta z - 7)$

$2 = k_1 + 3k_2,\ 3 = k_1\alpha - k_2,\ -1 = 3k_1 + \beta k_2,\ -5 = 4k_1 - 7k_2$

On solving we get

$k_2 = \frac{13}{19}, k_1 = \frac{-1}{19}, \alpha = -70, \beta = \frac{-16}{13}$

$13\alpha\beta = 13(-70)\left(\frac{-16}{13}\right)$

$= 1120$

Question11

Let the system of equations x + 2y + 3z = 5, 2x + 3y + z = 9, 4x + 3y + λz = μ have infinite number of solutions. Then λ + 2μ is equal to :

[1-Feb-2024 Shift 2]

Options:

A.

28

B.

17

C.

22

D.

15

Answer: B

Solution:

$x+2y+3z=5$

$2x+3y+z=9$

$4x+3y+\lambda z=\mu$

for infinite following $\Delta=\Delta_1=\Delta_2=\Delta_3=0$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = -13$$

$$\Delta_1 = \begin{vmatrix} 5 & 2 & 3 \\ 9 & 3 & 1 \\ \mu & 3 & -13 \end{vmatrix} = 0 \Rightarrow \mu = 15$$

$$\Delta_2 = \begin{vmatrix} 1 & 5 & 3 \\ 2 & 9 & 1 \\ 4 & 15 & -13 \end{vmatrix} = 0$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & 15 \end{vmatrix} = 0$$

for $\lambda=-13$, $\mu=15$ system of equation has infinite solution hence $\lambda+2\mu=17$

Question12

If the system of equations
x + 2y + 3z = 3
4x + 3y − 4z = 4
8x + 4y − λz = 9 + μ
has infinitely many solutions, then the ordered pair |(λ, μ) is equal to
[24-Jan-2023 Shift 2]

Options:

A. $\left(\frac{72}{5}, \frac{21}{5}\right)$

B. $\left(\frac{-72}{5}, \frac{-21}{5}\right)$

C. $\left(\frac{72}{5}, \frac{-21}{5}\right)$

D. $\left(\frac{-72}{5}, \frac{21}{5}\right)$

Answer: C

Solution:

Solution:

$x + 2y + 3z = 3 \ldots . (i)$
$4x + 3y - 4z = 4 \ldots . (ii)$
$8x + 4y - \lambda z = 9 + \mu \ldots . (iii)$
$(i) \times 4 - (ii) \Rightarrow 5y + 16z = 8 \ldots\ldots (iv)$
$(ii) \times 2 - (iii) \Rightarrow 2y + (\lambda - 8)z = -1 - \mu \ldots\ldots (v)$
$(iv) \times 2 - (iii) \times 5 \Rightarrow (32 - 5(\lambda - 8))z = 16 - 5(-1 - \mu)$

For infinite solutions $\Rightarrow 72 - 5\lambda = 0 \Rightarrow \lambda = \frac{72}{5}$

$21 + 5\mu = 0 \Rightarrow \mu = \frac{-21}{5}$

$\Rightarrow (\lambda, \mu) \equiv \left(\frac{72}{5}, \frac{-21}{5}\right)$

Question13

Let S_1 and S_2 be respectively the sets of all $a \in R - \{0\}$ for which the system of linear equations
$ax + 2ay - 3az = 1$
$(2a + 1)x + (2a + 3)y + (a + 1)z = 2$
$(3a + 5)x + (a + 5)y + (a + 2)z = 3$
has unique solution and infinitely many solutions. Then
[25-Jan-2023 Shift 1]

Options:

A. $n(S_1) = 2$ and S_2 is an infinite set

B. S_1 is an infinite set an $n(S_2) = 2$

C. $S_1 = \Phi$ and $S_2 = \mathbb{R} - \{0\}$

D. $S_1 = \mathbb{R} - \{0\}$ and $S_2 = \Phi$

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} a & 2a & -3a \\ 2a + 1 & 2a + 3 & a + 1 \\ 3a + 5 & a + 5 & a + 2 \end{vmatrix}$$

$= a(15a^2 + 31a + 36) = 0 \Rightarrow a = 0$
$\Delta \neq 0$ for all $a \in R - \{0\}$
Hence $S_1 = R - \{0\}$ $S_2 = \Phi$

Question14

Consider the following system of questions
$\alpha x + 2y + z = 1$
$2\alpha x + 3y + z = 1$
$3x + \alpha y + 2z = \beta$
For some $\alpha, \beta \in \mathbb{R}$. Then which of the following is NOT correct.
[29-Jan-2023 Shift 1]

Options:

A. It has no solution if $\alpha = -1$ and $\beta \neq 2$

B. It has no solution for $\alpha = -1$ and for all $\beta \in \mathbb{R}$

C. It has no solution for $\alpha = 3$ and for all $\beta \neq 2$

D. It has a solution for all $\alpha \neq -1$ and $\beta = 2$

Answer: B

Solution:

Solution:

$$D = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix} = 0 \Rightarrow \alpha = -1, 3$$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ \alpha & 2 & \beta \end{vmatrix} = 0 \Rightarrow \beta = 2$$

$$D_y = \begin{vmatrix} \alpha & 1 & 1 \\ 2\alpha & 1 & 1 \\ 3 & 2 & \beta \end{vmatrix} = 0$$

$$D_z = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & \beta \end{vmatrix} = 0$$

$\beta = 2, \alpha = -1$
$\alpha = -1, \beta = 2$ Infinite solution

Question15

Let the system of linear equations
$x + y + kz = 2$
$2x + 3y - z = 1$
$3x + 4y + 2z = k$
have infinitely many solutions. Then the system

$(k+1)x+(2k-1)y=7$
$(2k+1)x+(k+5)y=10$ has:
[30-Jan-2023 Shift 1]

Options:

A. infinitely many solutions

B. unique solution satisfying $x-y=1$

C. no solution

D. unique solution satisfying $x+y=1$

Answer: D

Solution:

Solution:

$$\begin{vmatrix} 1 & 1 & k \\ 2 & 3 & -1 \\ 3 & 4 & 2 \end{vmatrix} = 0$$

$\Rightarrow\ 1(10)-1(7)+k(-1)-0$
$\Rightarrow\ k=3$
For $k=3$, 2^{nd} system is
$4x+5y=7\ ...(1)$
and $7x+8y=10\ ...(2)$
Clearly, they have a unique solution
$(2)-(1)\Rightarrow 3x+3y=3$
$\Rightarrow x+y=1$

Question16

For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations
$x-y+z=5$
$2x+2y+\alpha z=8$
$3x-y+4z=\beta$
has infinitely many solutions. Then α and β are the roots of
[30-Jan-2023 Shift 2]

Options:

A. $x^2-10x+16=0$

B. $x^2+18x+56=0$

C. $x^2-18x+56=0$

D. $x^2+14x+24=0$

Answer: C

Solution:

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0; 8 + \alpha - 2(-4 + 1) + 3(-\alpha - 2) = 0$$

$8 + \alpha + 6 - 3\alpha - 6 = 0$
$\alpha = 4$

Question17

For the system of linear equations
$x + y + z = 6$
$\alpha x + \beta y + 7z = 3$
$x + 2y + 3z = 14$
which of the following is NOT true?
[31-Jan-2023 Shift 1]

Options:

A. If $\alpha = \beta = 7$, then the system has no solution

B. If $\alpha = \beta$ and $\alpha \neq 7$ then the system has a unique solution.

C. There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions

D. For every point $(\alpha, \beta) \neq (7, 7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions.

Answer: D

Solution:

By equation 1 and 3
$y + 2z = 8$
$y = 8 - 2z$
And $\quad x = -2 + z$
Now putting in equation 2
$\alpha(z - 2) + \beta(-2z + 8) + 7z = 3$
$\Rightarrow (\alpha - 2\beta + 7)z = 2\alpha - 8\beta + 3$
So equations have unique solution if
$\alpha - 2\beta + 7 \neq 0$
And equations have no solution if
$\alpha - 2\beta + 7 = 0$ and $2\alpha - 8\beta + 3 \neq 0$
And equations have infinite solution if
$\alpha - 2\beta + 7 = 0$ and $2\alpha - 8\beta + 3 = 0$

Question18

Let S denote the set of all real values of λ such that the system of equations
$\lambda x + y + z = 1$
$x + \lambda y + z = 1$
$x + y + \lambda z = 1$

is inconsistent, then $\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$ is equal to
[1-Feb-2023 Shift 1]

Options:

A. 2

B. 12

C. 4

D. 6

Answer: D

Solution:

$$\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$$(\lambda + 2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$(\lambda + 2)[1(\lambda^2 - 1) - 1(\lambda - 1) + (1 - \lambda)] = 0$
$(\lambda + 2)[(\lambda^2 - 2\lambda + 1) = 0.$
$(\lambda + 2)(\lambda - 1)^2 = 0 \Rightarrow \lambda = -2, \lambda = 1$
at $\lambda = 1$ system has infinite solution, for inconsistent $\lambda = -2$
so $\sum(|-2|^2 + |-2|) = 6$

Question19

For the system of linear equations $ax + y + z = 1$,
$x + ay + z = 1$, $x + y + az = \beta$, which one of the following statements is NOT correct?
[1-Feb-2023 Shift 2]

Options:

A. It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$

B. It has no solution if $\alpha = -2$ and $\beta = 1$

C. $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$

D. It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$

Answer: A

Solution:

$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

$\alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha) = 0$
$\alpha^3 - 3\alpha + 2 = 0$
$\alpha^2(\alpha - 1) + \alpha(\alpha - 1) - 2(\alpha - 1) = 0$
$(\alpha - 1)(\alpha^2 + \alpha - 2) = 0$
$\alpha = 1, \alpha = -2, 1$
For $\alpha = 1, \beta = 1$
$x + y + z = 1$
$x + y + z = b$ } infinite solution
For $\alpha = 2, \beta = 1$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 3 - 1 - 1 \Rightarrow x = \frac{1}{4}$$

$$\Delta_2 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = 2 - 1 = 1 \Rightarrow y = \frac{1}{4}$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 2 - 1 = 1 \Rightarrow z = \frac{1}{4}$$

For $\alpha = 2 \Rightarrow$ unique solution

Question20

If the system of equations
$x + y + az = b$
$2x + 5y + 2z = 6$
$x + 2y + 3z = 3$
has infinitely many solutions, then $2a + 3b$ is equal to :
[6-Apr-2023 shift 1]

Options:

A. 28

B. 20

C. 25

D. 23

Answer: D

Solution:

Solution:
$x + y + az = b$
$2x + 5y + 2z = 6$
$x + 2y + 3z = 3$
For ∞ solution
$\Delta = 0, \Delta_x = 0, \Delta_y = 0, \Delta_z = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 11 - 4 - a = 0 \Rightarrow a = 7$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & b \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 3 - 0 - b = 0 \Rightarrow b = 3$$

Hence $2a + 3b = 23$

Question21

For the system of equations
$x + y + z = 6$
$x + 2y + \alpha z = 10$
$x + 3y + 5z = \beta$, which one of the following is NOT true :
[6-Apr-2023 shift 2]

Options:

A. System has a unique solution for $\alpha = 3, \beta \neq 14$.

B. System has a unique solution for $\alpha = -3, \beta = 14$.

C. System has no solution for $\alpha = 3, \beta = 24$.

D. System has infinitely many solutions for $\alpha = 3, \beta = 14$.

Answer: A

Solution:

Solution:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 3 & 5 \end{vmatrix}$$

$= (10 - 3\alpha) - (5 - \alpha) + (3 - 2)$
$= 6 - 2\alpha$

$$\Delta x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & \alpha \\ \beta & 3 & 5 \end{vmatrix}$$

$= 6(10 - 3\alpha) - (50 - \alpha 13) + (30 - 2\beta)$
$= 40 - 18\alpha + \alpha\beta - 2\beta$

$$\Delta y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & \alpha \\ 1 & \beta & 5 \end{vmatrix}$$

$= (50 - \alpha\beta) - 6(5 - \alpha) + (\beta - 10)$
$= 10 + 6\alpha + \beta - \alpha\beta$

$$\Delta z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 3 & \beta \end{vmatrix}$$

$= (2\beta - 30) - (\beta - 10) + 6(1)$
$= \beta - 14$

for Infinite solution

$\Delta = 0,\ \Delta_x = \Delta_y = \Delta_z = 0$

$x\alpha = 3, \; x\beta = 14$
For unique solution $\alpha \neq 3$
Ans. Option 1

Question22

Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations
$x + y + \sqrt{3}z = 0$
$-x + (\tan\theta)y + \sqrt{7}z = 0$
$x + y + (\tan\theta)z = 0$
has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta \in S} \theta$ is equal to
[8-Apr-2023 shift 2]

Options:

A. 20

B. 40

C. 30

D. 10

Answer: A

Solution:

Solution:
For non trivial solutions
$D = 0$

$$\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix} = 0$$

$\tan^2\theta - (\sqrt{3} - 1) - \sqrt{3} = 0$
$\tan\theta = \sqrt{3}, -1$
$\theta = \left\{ \frac{\pi}{3}, \frac{-2\pi}{3}, \frac{-\pi}{4}, \frac{3\pi}{4} \right\}$
$\frac{120}{\pi}(\Sigma\theta) = \frac{120}{\pi} \times \frac{\pi}{6} = 20$ (Option 1)

Question23

For the system of linear equations
$2x - y + 3z = 5$
$3x + 2y - z = 7$
$4x + 5y + \alpha z = \beta$
which of the following is NOT correct ?
[10-Apr-2023 shift 1]

Options:

A. The system in inconsistent for $\alpha = -5$ and $\beta = 8$

B. The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

C. The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$

D. The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

Answer: B

Solution:

Solution:
Given
$2x - y + 3z = 5$
$3x + 2y - z = 7$
$4x + 5y + \alpha z = \beta$

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \alpha \end{vmatrix} = 7\alpha + 35$$

$\Delta = 7(\alpha + 5)$
For unique solution $\Delta \neq 0$
$\alpha \neq -5$
For inconsistent & Infinite solution
$\Delta = 0$
$\alpha + 5 = 0 \Rightarrow \alpha = -5$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & 3 \\ 7 & 2 & -1 \\ \beta & 5 & -5 \end{vmatrix} = -5(\beta - 9)$$

$$\Delta_2 = \begin{vmatrix} 2 & 5 & 3 \\ 3 & 7 & -1 \\ 4 & \beta & -5 \end{vmatrix} = 11(\beta - 9)$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 5 \\ 3 & 2 & 7 \\ 4 & 5 & \beta \end{vmatrix}$$

$\Delta_3 = 7(\beta - 9)$
For Inconsistent system : -
At least one $\Delta_1, \Delta_2 \& \Delta_3$ is not zero $\alpha = -5, \beta = 8$ option (A) True Infinite solution:
$\Delta_1 = \Delta_2 = \Delta_3 = 0$
From here $\beta - 9 = 0 \Rightarrow \beta = 9$ $\alpha = -5$& option (D) True
$\beta = 9$
Unique solution
$\alpha \neq -5, \beta = 8 \rightarrow$ option (C) True
Option (B) False
For Infinitely many solution α must be -5 .

Question24

Let S be the set of values of λ, for which the system of equations $6\lambda x - 3y + 3z = 4\lambda^2$ $2x + 6\lambda y + 4z = 1$ $3x + 2y + 3\lambda z = \lambda$ has no solution. Then $12 \sum_{1 \in S} |\lambda|$ is equal to _______.

[10-Apr-2023 shift 2]

Answer: 24

Solution:

Solution:

$$\Delta = \begin{vmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{vmatrix} = 0$$

$2\lambda(9\lambda^2 - 4) + (3\lambda - 6) + (2 - 9\lambda) = 0$

$18\lambda^3 - 14\lambda - 4 = 0$

$(\lambda - 1)(3\lambda + 1)(3\lambda + 2) = 0$

$\Rightarrow \lambda = 1, -1/3, -2/3$

$$\text{For each values of } \lambda, \Delta_1 = \begin{vmatrix} 6\lambda & -3 & 4\lambda^2 \\ 2 & 6\lambda & 1 \\ 3 & 2 & \lambda \end{vmatrix} \neq 0$$

$$12\left(1 + \frac{1}{3} + \frac{2}{3}\right) = 24$$

Question25

Let A be a 2 × 2 matrix with real entries such that $A' = \alpha A + I$, where $a \in \mathbb{R} - \{-1, 1\}$. If $\det(A^2 - A) = 4$, then the sum of all possible values of α is equal to :
[11-Apr-2023 shift 1]

Options:

A. 0

B. $\frac{5}{2}$

C. 2

D. $\frac{3}{2}$

Answer: B

Solution:

Solution:

$A^T = \alpha A + I$

$A = \alpha A^T + I$

$A = \alpha(\alpha A + I) + I$

$A = \alpha^2 A + (\alpha + 1)I$

$A(1 - \alpha^2) = (\alpha + 1)I$

$A = \dfrac{I}{1 - \alpha} \ldots (1)$

$|A| = \dfrac{1}{(1 - \alpha)^2} \ldots (2)$

$|A^2 - A| = |A|\,|A - 1| \ldots (3)$

$A - I = \dfrac{I}{I - \alpha} - I = \dfrac{\alpha}{1 - \alpha} I$

$$|A - I| = \left(\frac{\alpha}{1-\alpha}\right)^2 \ldots (4)$$

Now $|A^2 - A| = 4$

$|A||A - I| = 4$

$$\Rightarrow \frac{1}{(1-\alpha)^2}\frac{\alpha^2}{(1-\alpha^2)} = 4$$

$$\Rightarrow \frac{\alpha}{(1-\alpha)^2} = \pm 2$$

$\Rightarrow 2(1-\alpha)^2 = \pm\alpha$

$(C_1) 2(1-\alpha)^2 = \alpha \qquad (C_2) 2(1-\alpha)^3 = -\alpha$

$$\alpha_1 + \alpha_2 = \frac{5}{2}$$

Question26

If the system of linear equations
$7x + 11y + \alpha z = 13$
$5x + 4y + 7z = \beta$
$175x + 194y + 57z = 361$
has infinitely many solutions, then $\alpha + B + 2$ is equal to:
[11-Apr-2023 shift 2]

Options:

A. 3

B. 6

C. 5

D. 4

Answer: D

Solution:

Solution:
$7x + 11y + \alpha z = 13$
$5x + 4y + 7z = \beta$
$175x + 194y + 57z = 361$
4 sc condition of Infinite Many solution
$\Delta = 0$& $\Delta x, \Delta y, \Delta z = 0$ check.
After solving we get $\alpha + 13 + 2 = 4$

Question27

If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x + 81)$, then $\lambda, \frac{\lambda}{3}$ are the roots of the equation
[11-Apr-2023 shift 2]

Options:

A. $4x^2 - 24x - 27 = 0$

B. $4x^2 + 24x + 27 = 0$

C. $4x^2 - 24x + 27 = 0$

D. $4x^2 + 24x - 27 = 0$

Answer: C

Solution:

Solution:

$$\begin{vmatrix} x+1 & x & x \\ x & x+d & x \\ x & x & x+d^2 \end{vmatrix} = \frac{9}{8}(103x+81)$$

Put $x = 0$

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9}{8} \times 81$$

$\lambda^3 = \frac{9^3}{8}$

$\lambda = \frac{9}{2}$

$\frac{\lambda}{3} = \frac{9}{2 \times 3} \Rightarrow \frac{3}{2}$

$\frac{\lambda}{3} = \frac{3}{2}$

Option (C) $4x^2 - 24x + 27 = 0$

has Root $\frac{3}{2}, \frac{9}{2}$

Question28

Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. If $\sum_{k=1}^{n} D_k = 96$, then n is equal to

[12-Apr-2023 shift 1]

Answer: 6

Solution:

Solution:

$$D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

$\sum_{k=1}^{n} D_k = 96 \Rightarrow$

$$\begin{vmatrix} \sum_{k=1}^{n} 1 & \sum 2k & \sum (2k-1) \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$$

$$\Rightarrow \begin{vmatrix} n & n^2+n & n^2 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\begin{vmatrix} n & n^2+n & n^2 \\ 0 & 2 & 0 \\ 0 & 0 & n+2 \end{vmatrix} = 96$$

$\Rightarrow n(2n+4) = 96 \Rightarrow n(n+2) = 48 \Rightarrow n = 6$

Question29

For the system of linear equations $2x + 4y + 2az = b$ $x + 2y + 3z = 4$ $2x - 5y + 2z = 8$ which of the following is NOT correct? [13-Apr-2023 shift 1]

Options:

A. It has infinitely many solutions if $a = 3, b = 8$

B. It has unique solution if $a = b = 8$

C. It has unique solution if $a = b = 6$

D. It has infinitely many solutions if $a = 3, b = 6$

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & 4 & 2a \\ 1 & 2 & 3 \\ 2 & -5 & 2 \end{vmatrix} = 18(3-a)$$

$$\Delta_x = \begin{vmatrix} b & 4 & 2a \\ 4 & 2 & 3 \\ 8 & -5 & 2 \end{vmatrix} = (64 + 19b - 72a)$$

For unique solution $\Delta = 0$

$\Rightarrow a \neq 3$ and $b \in R$

For infinitely many solution :

$\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

$\Rightarrow a = 3 \quad \because \Delta = 0$

and $b = 8 \because \Delta_x = 0$

Question30

If the system of equations
$2x + y - z = 5$
$2x - 5y + \lambda z = \mu$
$x + 2y - 5z = 7$
has infinitely many solutions, then $(\lambda + \mu)^2 + (\lambda - \mu)^2$ is equal to
[13-Apr-2023 shift 2]

Options:

A. 904

B. 916

C. 912

D. 920

Answer: B

Solution:

Solution:

$\Delta = 0$

$$\Rightarrow \begin{vmatrix} 2 & 1 & -1 \\ 2 & -5 & \lambda \\ 1 & 2 & -5 \end{vmatrix} = 0$$

$\Rightarrow 2(25 - 2\lambda) - 1(-10 - \lambda) - 1(4 + 5) = 0$

$\Rightarrow 51 - 3x = 0$

$\Rightarrow \lambda = 17$

$\Delta_x = 0$

$$\begin{vmatrix} 5 & 1 & -1 \\ \mu & -5 & 17 \\ 7 & 2 & -5 \end{vmatrix} = 0$$

$\Rightarrow 5(25 - 34) - 1(-5\mu - 119) - 1(2\mu + 35) = 0$

$\Rightarrow -45 + 5\mu + 119 - 2\mu - 35 = 0$

$\Rightarrow 39 + 3\mu = 0 \Rightarrow \mu = -13$

$(\lambda + \mu)^2 + (\lambda - \mu)^2 = 4^2 + (30)^2$

$= 916$

Question31

Let the system of linear equations
$-x + 2y - 9z = 7$
$-x + 3y + 7z = 9$
$-2x + y + 5z = 8$
$-3x + y + 13z = \lambda$
has a unique solution $x = \alpha, y = \beta, z = \gamma$. Then the distance of the point (α, β, γ) from the plane $2x - 2y + z = \lambda$ is

[15-Apr-2023 shift 1]

Options:

A. 7

B. 9

C. 13

D. 11

Answer: A

Solution:

Solution:

$-x + 2y - 9z = 7 - (1)$
$-x + 3y - 7z = 9 - (2)$
$-2x + y + 5z = 8 - (3)$
$(2) - (1)$
$y + 16z = 2(4)$
$(3) - 2 \times (1)$
$-3y + 23z = -6 - (5)$
$3 \times (4) + (5)$
$71z = 0 \Rightarrow z = 0$
$y = 2$
$x = -3$
$(-3, 2, 0) \rightarrow (\alpha, \beta, \gamma)$
Put in $-3x + y + 13z = 1$
$\lambda = 9 + 2 = 11$

$$d = \left| \frac{-6 - 4 - 11}{3} \right| = 7$$

Question32

Let the system of linear equations
$x + y + \alpha z = 2$
$3x + y + z = 4$
$x + 2z = 1$
have a unique solution (x^*, y^*, z^*). If (α, x^*), (y^*, α) and $(x^*, -y^*)$ are collinear points, then the sum of absolute values of all possible values of α is
[24-Jun-2022-Shift-2]

Options:

A. 4

B. 3

C. 2

D. 1

Answer: C

Solution:

Given system of equations

$x+y+\alpha z=2 \ldots\ldots$ (i)

$3x-y-z=4 \ldots\ldots$ (ii)

$x+2z=1 \ldots\ldots$ (iii)

Solving (i), (ii) and (iii), we get

$x=1, y=1, z=0$(and for unique solution $\alpha \neq -3$)

Now, $(\alpha, 1), (1, \alpha)$ and $(1, -1)$ are collinear

$$\therefore \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$$

$\Rightarrow \alpha(\alpha+1)-1(0)+1(-1-\alpha)=0$

$\Rightarrow \alpha^2-1=0$

$\therefore \alpha = \pm 1$

$\therefore$ Sum of absolute values of $\alpha = 1-1=2$

Question33

The system of equations $-kx+3y-14z=25$ $-15x+4y-kz=3$ $-4x+y+3z=4$ is consistent for all k in the set [25-Jun-2022-Shift-2]

Options:

A. R

B. $R-\{-11, 13\}$

C. $R-\{13\}$

D. $R-\{-11, 11\}$

Answer: D

Solution:

Solution:
The system may be inconsistent if

$$\begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} = 0 \Rightarrow k = \pm 11$$

Hence if system is consistent then $k \in R-\{11, -11\}$.

Question34

The ordered pair (a, b), for which the system of linear equations
$3x - 2y + z = b$
$5x - 8y + 9z = 3$
$2x + y + az = -1$
has no solution, is :
[26-Jun-2022-Shift-1]

Options:

A. $\left(3, \frac{1}{3}\right)$

B. $\left(-3, \frac{1}{3}\right)$

C. $\left(-3, -\frac{1}{3}\right)$

D. $\left(3, -\frac{1}{3}\right)$

Answer: C

Solution:

Solution:

$$\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0 \Rightarrow -14a - 42 = 0 \Rightarrow a = -3$$

Now 3 (equation (1)) - (equation (2)) −2 (equation (3)) is

$$3(3x - 2y + z - b) - (5x - 8y + 9z - 3) - 2(2x + y + az + 1) = 0$$

$$\Rightarrow -3b + 3 - 2 = 0 \Rightarrow b = \frac{1}{3}$$

So for no solution $a = -3$ and $b \neq \frac{1}{3}$

Question35

If the system of equations
$\alpha x + y + z = 5$, $x + 2y + 3z = 4$, $x + 3y + 5z = \beta$
has infinitely many solutions, then the ordered pair (α, β) is equal to :
[26-Jun-2022-Shift-2]

Options:

A. $(1, -3)$

B. $(-1, 3)$

C. $(1, 3)$

D. $(-1, -3)$

Answer: C

Solution:

Solution:

Given system of equations

$\alpha x+y+z=5$

$x+2y+3z=4$, has infinite solution

$x+3y+5z=\beta$

$$\therefore \Delta=\begin{vmatrix}\alpha & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & 5\end{vmatrix}=0 \Rightarrow \alpha(1)-1(2)+1(1)=0$$

$\Rightarrow \alpha=1$

$$\text{and } \Delta_1=\begin{vmatrix}5 & 1 & 1\\ 4 & 2 & 3\\ \beta & 3 & 5\end{vmatrix}=0$$

$\Rightarrow 5(1)-1(20-3\beta)+1(12-2\beta)=0$

$\Rightarrow \beta=3$

$$\text{and } \Delta_2=\begin{vmatrix}1 & 5 & 1\\ 1 & 4 & 3\\ 1 & \beta & 5\end{vmatrix}=0 \Rightarrow (20-3\beta)-5(2)+1(\beta-4)=0$$

$\Rightarrow -2\beta+6=0$

$\Rightarrow \beta=3$

Similarly,

$$\Rightarrow \Delta_3=\begin{vmatrix}1 & 1 & 5\\ 1 & 2 & 4\\ 1 & 3 & \beta\end{vmatrix}=0 \Rightarrow \beta=3$$

$\therefore (\alpha, \beta)=(1, 3)$

Question36

Let the system of linear equations
$x + 2y + z = 2$
$\alpha x + 3y - z = \alpha$
$-\alpha x + y + 2z = -\alpha$
be inconsistent. Then α is equal to:
[27-Jun-2022-Shift-1]

Options:

A. $\frac{5}{2}$

B. $-\frac{5}{2}$

C. $\frac{7}{2}$

D. $-\frac{7}{2}$

Answer: D

Solution:

Solution:
$x + 2y + z = 2$
$\alpha x + 3y - z = \alpha$
$-\alpha x + y + 2z = -\alpha$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 1(6+1) - 2(2\alpha - \alpha) + 1(\alpha + 3\alpha)$$

$= 7 + 2\alpha$

$\Delta = 0 \Rightarrow \alpha = -\frac{7}{2}$

$$\Delta_1 = \begin{vmatrix} 2 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 14 + 2\alpha \neq 0 \text{ for } \alpha = -\frac{7}{2}$$

$\therefore$ For no solution $\alpha = -\frac{7}{2}$

Question37

Let for some real numbers α and β, $a = \alpha - i\beta$. If the system of equations $4ix + (1+i)y = 0$ and $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \overline{a}y = 0$ has more than one solution, then $\frac{\alpha}{\beta}$ is equal to
[27-Jun-2022-Shift-2]

Options:

A. $-2\sqrt{3}$

B. $2 - \sqrt{3}$

C. $2 + \sqrt{3}$

D. $-2 - \sqrt{3}$

Answer: B

Solution:

Solution:
Given $a = \alpha - i\beta$ and
$4ix + (1+i)y = 0$……. (i)
$8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \overline{a}y = 0$
By (i)
$\frac{x}{y} = \frac{-(1+i)}{4i}$…… (iii)
By (ii)
$\frac{x}{y} = \frac{-\overline{a}}{8\left(\frac{-1}{2} + \frac{\sqrt{3}i}{2}\right)}$….
Now by (iii) and (iv)

$$\frac{1+i}{4i} = \frac{\bar{a}}{4(-1+\sqrt{3}i)}$$
$\Rightarrow \bar{a} = (\sqrt{3}-1) + (\sqrt{3}+1)i$
$\Rightarrow \alpha + i\beta = (\sqrt{3}-1) + (\sqrt{3}+1)i$
$\therefore \frac{\alpha}{\beta} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3}$

Question38

If the system of linear equations $2x + 3y - z = -2$ $x + y + z = 4$ $x - y + |\lambda| z = 4\lambda - 4$ where, $\lambda \in R$, has no solution, then [28-Jun-2022-Shift-1]

Options:

A. $\lambda = 7$

B. $\lambda = -7$

C. $\lambda = 8$

D. $\lambda^2 = 1$

Answer: B

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} \lambda \end{vmatrix} = 7$$
But at $\lambda = 7$, $D_x = D_y = D_z = 0$
$P_1 : 2x + 3y - z = -2$
$P_2 : x + y + z = 4$
$P_3 : x - y + |\lambda| z = 4\lambda - 4$
So clearly $5P_2 - 2P_1 = P_3$, so at $\lambda = 7$, system of equation is having infinite solutions.
So $\lambda = -7$ is correct answer.

Question39

If the system of linear equations $2x - 3y = \gamma + 5$, $\alpha x + 5y = \beta + 1$, where $\alpha, \beta, \gamma \in R$ has infinitely many solutions then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to_____ [28-Jun-2022-Shift-2]

Answer: 58

Solution:

Solution:

If $2x - 3y = \gamma + 5$ and $\alpha x + 5y = \beta + 1$ have infinitely many solutions then

$$\frac{2}{\alpha} = \frac{-3}{5} = \frac{\gamma + 5}{\beta + 1}$$

$\Rightarrow \alpha = -\frac{10}{3}$ and $3\beta + 5\gamma = -28$

So $|9\alpha + 3\beta + 5\gamma| = |-30 - 28| = 58$

Question40

If the system of linear equations
$2x + y - z = 7$
$x - 3y + 2z = 1$
$x + 4y + \delta z = k$, where $\delta, k \in R$ has infinitely many solutions, then $\delta + k$ is equal to:
[29-Jun-2022-Shift-1]

Options:

A. -3

B. 3

C. 6

D. 9

Answer: B

Solution:

Solution:

$2x + y - z = 7$

$x - 3y + 2z = 1$

$x + 4y + \delta z = k$

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = -7\delta - 21 = 0$$

$\delta = -3$

$$\Delta_1 = \begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ k & 4 & -3 \end{vmatrix}$$

$\Rightarrow 6 - k = 0 \Rightarrow k = 6$

$\delta + k = -3 + 6 = 3$

Question41

The number of values of α for which the system of equations:
$x + y + z = \alpha$
$\alpha x + 2\alpha y + 3z = -1$
$x + 3\alpha y + 5z = 4$
is inconsistent, is
[24-Jun-2022-Shift-1]

Options:

A. 0

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & 2\alpha & 3 \\ 1 & 3\alpha & 5 \end{vmatrix}$$

$= 1(10\alpha - 9\alpha) - 1(5\alpha - 3) + 1(3\alpha^2 - 2\alpha)$

$= \alpha - 5\alpha + 3 + 3\alpha^2 - 2\alpha$

$= 3\alpha^2 - 6\alpha + 3$

For inconsistency $\Delta = 0$ i.e. $\alpha = 1$

Now check for $\alpha = 1$

$x + y + z = 1 \quad \ldots (i)$

$x + 2y + 3z = -1 \quad \ldots (ii)$

By (ii) $\times 2 -$ (i) $\times 1$

$x + 3y + 5z = -3$

so equations are inconsistent for $\alpha = 1$

Question42

The number of $\theta \in (0, 4\pi)$ for which the system of linear equations
$3(\sin 3\theta)x - y + z = 2$
$3(\cos 2\theta)x + 4y + 3z = 3$
$6x + 7y + 7z = 9$
has no solution, is :
[25-Jul-2022-Shift-1]

Options:

A. 6

B. 7

C. 8

D. 9

Answer: B

Solution:

Solution:
Given,
$3(\sin 3\theta)x - y + z = 2$
$3(\cos 2\theta)x + 4y + 3z = 3$
$6x + 7y + 7z = 9$
For no solutions determinant of coefficient will be $= 0$

$$\therefore D = \begin{vmatrix} 3\sin 3\theta & -1 & 1 \\ 3\cos 2\theta & 4 & 3 \\ 6 & 7 & 7 \end{vmatrix} = 0$$

$\Rightarrow 3\sin 3\theta(28 - 21) + 1(21\cos 2\theta - 18) + 1(21\cos 2\theta - 24) = 0$
$\Rightarrow 21\sin 3\theta + 42\cos 2\theta - 42 = 0$
$\Rightarrow \sin 3\theta + 2\cos 2\theta - 2 = 0$
$\Rightarrow 3\sin\theta - 4\sin^3\theta + 2(1 - 2\sin^2\theta) - 2 = 0$
$\Rightarrow 3\sin\theta - 4\sin^3\theta - 4\sin^2\theta = 0$
$\Rightarrow 4\sin^3\theta + 4\sin^2\theta - 3\sin\theta = 0$
$\Rightarrow \sin\theta(4\sin^2\theta + 4\sin\theta - 3) = 0$
$\therefore \sin\theta = 0$
$\Rightarrow \theta = \pi, 2\pi, 3\pi$ when $\theta \in (0, 4\pi)$
or,
$4\sin^2\theta + 4\sin\theta - 3 = 0$
$\Rightarrow 4\sin 2\theta + 6\sin\theta - 2\sin\theta - 3 = 0$
$\Rightarrow 2\sin\theta(2\sin\theta + 3) - 1(2\sin\theta + 3) = 0$
$\Rightarrow (2\sin\theta - 1)(2\sin\theta + 3) = 0$
$\therefore \sin\theta = \frac{1}{2}$
or,
$\sin\theta = -\frac{3}{2}$ [not possible as $\sin \in [-1, 1]$]
$\therefore \sin\theta = \frac{1}{2}$
$\Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$
$\therefore$ Possible values of $\theta = \pi, 2\pi, 3\pi, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$
$\therefore$ Total 7 values of θ possible.

Question43

The number of real values of λ, such that the system of linear equations
$2x - 3y + 5z = 9$
$x + 3y - z = -18$
$3x - y + (\lambda^2 - |\lambda|)z = 16$
has no solutions, is
[25-Jul-2022-Shift-2]

Options:

A. 0

B. 1

C. 2

D. 4

Answer: C

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 1 & 3 & -1 \\ 3 & -1 & \lambda^2 - |\lambda| \end{vmatrix} = 2(3\lambda^2 - 3|\lambda| - 1) + 3(\lambda^2 - |\lambda| - 3) + 5(-1 - 9)$$

$= 9\lambda^2 - 9|\lambda| - 43$

$= 9|\lambda|^2 - 9|\lambda| - 43$

$\Delta = 0$ for 2 values of $|\lambda|$ out of which one is $-$ve and other is $+$ve

So, 2 values of λ satisfy the system of equations to obtain no solution.

Question44

If the system of linear equations.
$8x + y + 4z = -2$
$x + y + z = 0$
$\lambda x - 3y = \mu$
has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane
$8x + y + 4z + 2 = 0$ is :
[26-Jul-2022-Shift-1]

Options:

A. $3\sqrt{5}$

B. 4

C. $\frac{26}{9}$

D. $\frac{10}{3}$

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix}$$

$= 8(3) - 1(-\lambda) + 4(-3-\lambda)$

$= 24 + \lambda - 12 - 4\lambda$

$= 12 - 3\lambda$

So for $\lambda = 4$, it is having infinitely many solutions.

$$\Delta_x = \begin{vmatrix} -2 & 1 & 4 \\ 0 & 1 & 1 \\ \mu & -3 & 0 \end{vmatrix}$$

$= -2(3) - 1(-\mu) + 4(-\mu)$

$= -6 - 3\mu = 0$

For $\mu = -2$

Distance of $\left(4, -2, \frac{-1}{2}\right)$ from $8x - y - 4z + 2 = 0$ $= \frac{32-2-2-2}{\sqrt{64+1+16}} = \frac{10}{3}$ units

---

Question45

Let p and p + 2 be prime numbers and let $\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$

Then the sum of the maximum values of α and β, such that p^{α} and $(p+2)^{\beta}$ divide Δ, is ________.
[29-Jul-2022-Shift-1]

Answer: 4

Solution:

Solution:

$$\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$$

$$= p!\,.\,(p+1)! \cdot (p+2)! \begin{vmatrix} 1 & (p+1) & (p+1)(p+2) \\ 1 & (p+2) & (p+2)(p+3) \\ 1 & (p+3) & (p+3)(p+4) \end{vmatrix}$$

$$= p! \cdot (p+1)! \cdot (p+2)! \begin{vmatrix} 1 & p+1 & p^2+3p+2 \\ 0 & 1 & 2p+4 \\ 0 & 1 & 2p+6 \end{vmatrix}$$

$= 2(p!) \cdot ((p+1)!) \cdot ((p+2)!)$
$= 2(p+1) \cdot (p!)^2 \cdot ((p+2)!)$
$= 2(p+1)^2 \cdot (p!)^3 \cdot ((p+2)!)$
$\therefore$ Maximum value of α is 3 and β is 1.
$\therefore \alpha + \beta = 4$

Question46

If the system of equations
$x + y + z = 6$
$2x + 5y + \alpha z = \beta$
$x + 2y + 3z - 14$
has infinitely many solutions, then $\alpha + \beta$ is equal to
[29-Jul-2022-Shift-2]

Options:

A. 8

B. 36

C. 44

D. 48

Answer: C

Solution:

Solution:
Given,
$x + y + z = 6 \ldots . (1)$
$2x + 5y + \alpha z = \beta \ldots . (2)$
$x + 2y + 3z = 14 \ldots \ldots (3)$
System of equation have infinite many solutions.
$\therefore \Delta_x = \Delta_y = \Delta_z = 0$ and $\Delta = 0$

$$\text{Now, } \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$C_1 \to C_1 - C_3$
$C_2 \to C_2 - C_3$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ 2-\alpha & 5-\alpha & \alpha \\ -2 & -1 & 3 \end{vmatrix} = 0$$

$\Rightarrow -2 + \alpha + 10 - 2\alpha = 0$
$\Rightarrow 8 - \alpha = 0$
$\Rightarrow \alpha = 8$
Now, $x + y + z = 6$
$2x + 5y + 8z = \beta$
$x + 2y + 3z = 14$

$$\therefore \Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ \beta & 5 & 8 \\ 14 & 2 & 3 \end{vmatrix} = 0$$

$C_1 \rightarrow C_1 - 6C_3$
$C_2 \rightarrow C_2 - C_3$

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ \beta - 48 & -3 & 8 \\ -4 & -1 & 3 \end{vmatrix} = 0$$

$\Rightarrow -\beta + 48 - 12 = 0$
$\Rightarrow \beta = 36$
$\therefore \alpha + \beta = 8 + 36 = 44$

Question47

The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is

[26 Feb 2021 Shift 1]

Options:

A. $(a+2)(a+3)(a+4)$

B. -2

C. $(a+1)(a+2)(a+3)$

D. 0

Answer: B

Solution:

Solution:

Given, $A = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$

Apply $R_2 \rightarrow R_2 - R_1$
Apply $R_3 \rightarrow R_3 - R_1$

$$A = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix}$$

Now, expanding along third column,
$A = 1[4(a+2) - (4a+10)] = 4a + 8 - 4a - 10$
$= -2$

Question48

Let A be a 3 × 3 matrix with $\det(A) = 4$. Let R_i denote the ith row of A. If

a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on 2A, then det (B) is equal to
[25 Feb 2021 Shift 2]

Options:

A. 16

B. 80

C. 64

D. 128

Answer: C

Solution:

Solution:

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Then, $2A = \begin{bmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 2g & 2h & 2i \end{bmatrix}$

Now, perform the operation
$R_2 \to 2R_2 + 5R_3$ on 2A, we get

$B = \begin{bmatrix} 2a & 2b & 2c \\ 4d + 10g & 4e + 10h & 4f + 10i \\ 2g & 2h & 2i \end{bmatrix}$

Using property of invariance to calculate |B|, apply
$R_2 \to R_2 - 5R_3$

$|B| = \begin{bmatrix} 2a & 2b & 2c \\ 4d & 4e & 4f \\ 2g & 2h & 2i \end{bmatrix} = 2 \times 4 \times 2 \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ $[\because \det(A) = 4]$

$= 16 \times \det(A)$
$= 16 \times 4 = 64$

Question49

Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers, such that $x + y + z > 0$ and $xyz = 2$. If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is ....
[25 Feb 2021 Shift 1]

Answer: 7

Solution:

Solution:

Here, $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$

$\therefore \ |A| = \begin{vmatrix} x & y & z \\ y & z & x \\ z & x & y \end{vmatrix} = x^3 + y^3 + z^3 - 3xyz$

$|A^2| = |I_3| = 1$

$\therefore \ |(x^3 + y^3 + z^3 - 3xyz)^2| = 1$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 1$

$\Rightarrow x^3 + y^3 + z^3 = 1 + 3xyz$ [$\because x + y + z > 0$]

$\Rightarrow \ = 1 + 3(2)$

$= 7$ [$\because xyz = 2$]

Question50

Consider the following system of equations

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c$

where, a, b and c are real constants. Then, the system of equations [26 Feb 2021 Shift 2]

Options:

A. has a unique solution, when $5a = 2b + c$

B. has infinite number of solutions when $5a = 2b + c$

C. has no solution for all a, b and c

D. has a unique solution for all a, b and c

Answer: B

Solution:

Solution:

Given, system of equation can be written as $AX = B$, where

$A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ Then,

$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -1 \\ 1 & -2 & 7 \end{vmatrix}$

$= 1(42 - 22) - 2(14 + 11) - 3(-4 - 6)$

$= 20 - 50 + 30 = 0$

$$|A_1| = \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix}$$

$= a(42 - 22) - 2(7b + 11c)$
$- 3(-2b - 6c)$
$= 20a - 14b - 22c + 6b + 18c$
$= 20a - 8b - 4c = 4(5a - 2b - c)$

$$|A_2| = \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix}$$

$= 1(7b + 11c) - a(14 + 11) - 3(2c - b)$
$= -25a + 10b + 5c = -5(5a - 2b - c)$

$$|A_3| = \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix}$$

$= 1(42 - 22) - 2(14 + 11) - 3(-4 - 6)$
$= 20 - 50 + 30 = 0$

$$|A_1| = \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix}$$

$= a(42 - 22) - 2(7b + 11c)$
$- 3(-2b - 6c)$
$= 20a - 14b - 22c + 6b + 18c$
$= 20a - 8b - 4c = 4(5a - 2b - c)$

$$|A_2| = \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix}$$

$= 1(7b + 11c) - a(14 + 11) - 3(2c - b)$
$= -25a + 10b + 5c = -5(5a - 2b - c)$

$$|A_3| = \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix}$$

$= -10a + 4b + 2c$
$= -2(5a - 2b - c)$

For infinite solution,

$|A| = |A_1| = |A_2| = |A_3| = 0$

$\Rightarrow 5a - 2b - c = 0 \Rightarrow 5a = 2b + c$

Question51

The following system of linear equations
$2x + 3y + 2z = 9$
$3x + 2y + 2z = 9$
$x - y + 4z = 8$
[25 Feb 2021 Shift 2]

Options:

A. does not have any solution

B. has a unique solution

C. has infinitely many solutions

D. has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

Answer: B

Solution:

Solution:
The given system of equations is non-homogeneous and it can be written as,

$$\begin{bmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ 8 \end{bmatrix}$$

i.e,. $AX = B$
Now, $|A| = 2(8+2) - 3(12-2) + 2(-3-2)$
$= 20 - 30 - 10 = -20 \neq 0$
$\because |A| \neq 0$, then this system have unique solution.

Question52

If the system of equations
$kx + y + 2z = 1$
$-2x - 2y - 4z = 3$
$3x - y - 2z = 2$ has infinitely many solutions, then k is equal to ....
[25 Feb 2021 Shift 1]

Answer: 21

Solution:

Given equations, $kx + y + 2z = 1$
$3x - y - 2z = 2$
$-2x - 2y - 4z = 3$
For infinitely many solutions,
$\Delta = \Delta x = \Delta y = \Delta z = 0$

Here, $\Delta y = \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = 0$

$\Rightarrow k(-8+6) - 1(-12-4) + 2(9+4) = 0$
$\Rightarrow -2k + 16 + 26 = 0$
$\Rightarrow 2k = 42$
$\therefore k = 21$

Question53

Let A and B be 3×3 real matrices, such that A is symmetric matrix and B is skew-symmetric matrix. Then, the system of linear equations $(A^2B^2 - B^2A^2)X = 0$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has
[24 Feb 2021 Shift 2]

Options:

A. no solution

B. exactly two solutions

C. infinitely many solutions

D. a unique solution

Answer: C

Solution:

Solution:
Given, A be a 3×3 matrix. A is symmetric and B is skew-symmetric.
$\therefore A^T = A, B^T = -B$
Let $A^2B^2 - B^2A^2 = P$
$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$
$= (B^2)^T(A^2)^T - (A^2)^T(B^2)^T$
$= B^2A^2 - A^2B^2 = -(A^2B^2 - B^2A^2) = -P$
$P^T = -P$
P is skew-symmetric. $\therefore |P| = 0$
Hence, $PX = 0$ have infinite solutions.

Question54

For the system of linear equations
$x - 2y = 1$, $x - y + kz = -2$, $ky + 4z = 6$, $k \in R$, consider the following statements
(A) The system has unique solution, if $k \neq 2, k \neq -2$
(B) The system has unique solution, if $k = -2$
(C) The system has unique solution, if $k = 2$
(D) The system has no solution, if $k = 2$
(E) The system has infinite number of solutions, if $k \neq -2$
Which of the following statements are correct?
[24 Feb 2021 Shift 2]

Options:

A. (C) and (D)

B. (B) and (E)

C. (A) and (E)

D. (A) and (D)

Answer: D

Solution:

Solution:
Given, $x - 2y + 0z = 1$
$x - y + kx = -2$
$0x + ky + 4z = 6$

Here, $\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 1(-4-k^2)+2(4)$

$= -4-k^2+8 = 4-k^2$

$\Delta_x = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & k \\ 6 & k & 4 \end{vmatrix} = 1(-4-k^2)+2(-8-6k)$

$= -4-k^2-16-12k = -k^2-12k-20$

If $\Delta \neq 0$, then it has unique solution i.e. $4-k^2 \neq 0$

$\Rightarrow k \neq \pm 2$ for unique solution.

Also at $k = 2$

$\Delta_x = -2^2 - 12 \times 2 - 20 = -48 \neq 0$

Then, in this case it has no solution.

Hence, statement (A) and statement (D) both are correct.

$\Rightarrow k \neq \pm 2$ for unique solution. Also at $k = 2$

Then, in this case it has no solution.

Hence, statement (A) and statement (D) both are correct.

Question55

The system of linear equations : $3x - 2y - kz = 10$; $2x - 4y - 2z = 6$; $x + 2y - z = 5m$ is inconsistent if : 24 Feb 2021 Shift 1

Options:

A. $k = 3, m = \frac{4}{5}$

B. $k \neq 3, m \in R$

C. $k \neq 3, m \neq \frac{4}{5}$

D. $k = 3, m \neq \frac{4}{5}$

Answer: D

Solution:

Solution:

$\Delta = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$\Rightarrow 24 + 2(0) - k(8) = 0 \Rightarrow k = 3$

$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix}$

$= 10(8) - 2(-10m+6) - 3(12+20m)$

$= 8(4-5m)$

$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix}$

$= 3(-6+10m) - 10(0) - 3(10m-6)$

$= 0$

$$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix}$$

$= 3(-20m - 12) - 2(6 - 10m) + 10(8)$

$= -40m + 32 = 8(4 - 5m)$

For inconsistent,

$k = 3 \& m \neq \frac{4}{5}$

Question56

Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to__
24 Feb 2021 Shift 1

Answer: 17

Solution:

Solution:

Given that

$PQ = kI$

$|P| \cdot |Q| = k^3$

$\Rightarrow |P| = 2k \; 0 \Rightarrow P$ is an invertible matrix

$\because PQ = kI$

$\therefore Q = kP^{-1}I$

$\therefore Q = \frac{\text{adj}.P}{2}$

$\because q_{23} = -\frac{k}{8}$

$\therefore \frac{(3\alpha + 4)}{2} = -\frac{k}{8} \Rightarrow k = 4$

$\therefore |P| = 2k \Rightarrow k = 10 + 6\alpha$

Put value of k in (i)... we get $\alpha = -1$

$\therefore \alpha^2 + k^2 = 1 + 16 = 17.$

Question57

The solutions of the equation $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0, (0 < x < \pi)$, are
[18 Mar 2021 Shift 1]

Options:

A. $\frac{\pi}{12}, \frac{\pi}{6}$

B. $\frac{\pi}{6}, \frac{5\pi}{6}$

C. $\frac{5\pi}{12}, \frac{7\pi}{12}$

D. $\frac{7\pi}{12}, \frac{11\pi}{12}$

Answer: D

Solution:

Solution:

Given, $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$

$(0 < x < \pi)$

Applying $R_1 \rightarrow R_1 + R_2$,

$\begin{vmatrix} 2 & 2 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$

Applying $C_1 \rightarrow C_1 - C_2$,

$\begin{vmatrix} 0 & 2 & 1 \\ -1 & 1+\cos^2 x & \cos^2 x \\ 0 & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$

$\Rightarrow\ 2 + 8\sin 2x - 4\sin 2x = 0$ (expanding along C_1)

$\Rightarrow\ 4\sin 2x = -2 \Rightarrow \sin 2x = -\frac{1}{2}$

$\Rightarrow\ 2x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6} \Rightarrow 2x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$\Rightarrow\ x = \frac{7\pi}{12}, \frac{11\pi}{12}$

[Note You can also solve by applying $C_1 \rightarrow C_1 - C_3$ and $C_2 \rightarrow C_2 - C_3$]

Question58

If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}f\right) = 0$, then a possible value of α is

[17 Mar 2021 Shift 1]

Options:

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: C

Solution:

Solution:

$A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$

and $\det\left(A^2 - \frac{1}{2}I\right) = 0$

$\therefore\ A^2 = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}\begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$

$A^2 = \begin{pmatrix} \sin^2\alpha & 0 \\ 0 & \sin^2\alpha \end{pmatrix}$

$\Rightarrow\ \frac{I}{2} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$

$\left(A^2 - \frac{1}{2}\right) = \begin{pmatrix} \sin^2\alpha - 1/2 & 0 \\ 0 & \sin^2\alpha - 1/2 \end{pmatrix}$

$\therefore\ \det\left(A^2 - \frac{1}{2}I\right) = \begin{vmatrix} \sin^2\alpha - 1/2 & 0 \\ 0 & \sin^2\alpha - 1/2 \end{vmatrix}$

$\left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0$

$\sin\alpha = \pm\frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4}$ is one possibility.

Question59

If x, y, z are in arithmetic progression with common difference d , $x \neq 3d$, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is
[17 Mar 2021 Shift 2]

Options:

A. 72

B. 12

C. 36

D. 6

Answer: A

Solution:

Solution:
Method (I)
Given, x, y and z are in AP with common difference $= d$
$\therefore$ $x =$ First term
$y =$ Second term of AP = First term + Common difference
$\Rightarrow$ $y = x + d$ ...(i)
and $z =$ Third term of AP = Second term + Common difference
$\Rightarrow$ $z = (x + d) + d = x + 2d$ ...(ii)
Also, given $x \neq 3d$ .....(iii)

and $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix} = 0$

Applying $R_2 \to R_1 + R_3 - 2R_2$, we have

$\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 0 & k - 6\sqrt{2} & 0 \\ 5 & k & z \end{vmatrix} = 0$

$\Rightarrow (k - 6\sqrt{2})(3z - 5x) = 0$ (Expanding along R_2)
Either $k - 6\sqrt{2} = 0$ or $3z - 5x = 0$
$\Rightarrow k = 6\sqrt{2}$ or $3(x + 2d) - 5x = 0$ [from Eq. (ii)]
$\Rightarrow x = 3d$ which is not possible as in Eq. (iii).
$\therefore$ $k = 6\sqrt{2}$ is only one solution.
Hence, $k^2 = (6\sqrt{2})^2$
$\Rightarrow$ $k^2 = 72$

Question60

If 1, $\log_{10}(4^x - 2)$ and $\log_{10}\left(4^x + \frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant

$\begin{vmatrix} 2\left(x - \frac{1}{2}\right) & x - 1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$ **is**

[17 Mar 2021 Shift 2]

Answer: 2

Solution:

Given 1, $\log_{10}(4^x - 2)$, $\log_{10}\left(4^x + \frac{18}{5}\right)$ are in A.P.

$\therefore 2\log_{10}(4^x - 2) = 1 + \log_{10}\left(4^x + \frac{18}{5}\right) = \log_{10}10 + \log_{10}\left(4^x + \frac{18}{5}\right)$

$\Rightarrow \log_{10}(4^x - 2)^2 = \log_{10}\left(10 \times \left(4^x + \frac{18}{5}\right)\right)$

$\Rightarrow$ $(4^x - 2)^2 = 10 \times 4^x + 36$

$\Rightarrow (4^x)^2 - 4(4^x) + 4 = 10 \times 4^x + 36$

$\Rightarrow (4^x)^2 - 14(4^x) - 32 = 0 \Rightarrow (4^x - 16)(4^x + 2) = 0$

$\Rightarrow 4^x = 16$ or $4^x = -2$(Rejected because $4^x > 0,\ \forall x \in R$)
$\Rightarrow\ 4^x = 4^2 \Rightarrow x = 2$

$$\therefore \begin{vmatrix} 2\left(x - \frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix} = \begin{vmatrix} 3 & 1 & 4 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix} = 2$$

Question61

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, such that AB = B and $a + d = 2021$, then the value of $ad - bc$ is equal to [17 Mar 2021 Shift 2]

Answer: 2020

Solution:

Solution:
Given,
$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
i.e. $B \neq 0$
and $AB = B$
$\Rightarrow\ AB - B = 0 \Rightarrow B(A - I) = 0$
$\Rightarrow\ |(A - I)B| = 0$
$\because\ B \neq 0$
$\therefore\ |A - I| = 0 \Rightarrow \begin{vmatrix} (a-1) & b \\ c & (d-1) \end{vmatrix} = 0$
$\Rightarrow\ (a-1)(d-1) - bc = 0 \Rightarrow ad - bc = 2020$

Question62

The maximum value of $f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$, $x \in R$ is

[16 Mar 2021 Shift 2]

Options:

A. $\sqrt{7}$

B. $\frac{3}{4}$

C. $\sqrt{5}$

D. 5

Answer: C

Solution:

Solution:

Given, $f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$

On applying $C_1 \to C_1 + C_2$, we get

$f(x) = \begin{vmatrix} \sin^2 x + 1 + \cos^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x + \cos^2 x & \cos^2 x & \cos 2x \\ \sin^2 x + \cos^2 x & \cos^2 x & \sin 2x \end{vmatrix}$

$f(x) = \begin{vmatrix} 2 & 1+\cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$

On applying $R_1 \to R_1 - R_2$

$f(x) = \begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$

$f(x) = -1(2\sin 2x - \cos 2x)$

As, we know that, if $f(\theta) = A\sin\theta + B\cos\theta$

Then, $-\sqrt{A^2+B^2} \le f(\theta) \le \sqrt{A^2+B^2}$

Here, we have, $f(x) = \cos 2x - 2\sin 2x$

$-\sqrt{2^2+1^2} \le f(x) \le \sqrt{2^2+1^2}$

$-\sqrt{5} \le f(x) \le \sqrt{5}$

So, maximum value of $f(x)$ is $\sqrt{5}$.

Question63

Let α, β, γ be the real roots of the equation, $x^3 + ax^2 + bx + c = 0$, ($a, b, c \in R$ and $a, b \neq 0$). If the system of equations (in u, v, w) given by $\alpha u + \beta v + \gamma w = 0$, $\beta u + w + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is [18 Mar 2021 Shift 1]

Options:

A. 5

B. 3

C. 1

D. 0

Answer: B

Solution:

Solution:
Given, α, β, γ are the real roots of $x^3 + ax^2 + bx + c = 0$, where $a, b, c \in R$ and $a, b \neq 0$
$\therefore \alpha + \beta + \gamma =$ Sum of roots taken one at a time $= -a$
$\alpha\beta + \beta\gamma + \gamma\alpha =$ Sum of roots taken two at a time $= b$
$\alpha\beta\gamma =$ Product of roots $= -c$
Also, given system of equations in u, v, w

$$\left.\begin{aligned} \alpha u + \beta v + \gamma w = 0 \\ \beta u + \gamma v + \alpha w = 0 \\ \gamma u + \alpha v + \beta w = 0 \end{aligned}\right\}$$

has non-trivial solution.

$$\therefore \Delta = 0 \Rightarrow \begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$$

$\Rightarrow \alpha(\beta\gamma - \alpha^2) - \beta(\beta^2 - \gamma\alpha) + \gamma(\alpha\beta - \gamma^2) = 0$ (expanding along R_1)
$\Rightarrow \alpha\beta\gamma - \alpha^3 - \beta^3 + \alpha\beta\gamma + \alpha\beta\gamma - \gamma^3 = 0$
$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$
Then (using standard result),
Either $\alpha + \beta + \gamma = 0$ or $\alpha = \beta = \gamma$
If $\alpha + \beta + \gamma = 0$, $\text{then} \ -a = 0$
$\Rightarrow a = 0$ which is not possible according to given condition.
$\therefore \alpha + \beta + \gamma = 0$ (not possible)
Now,
$\alpha + \beta + \gamma = -a$
$\Rightarrow \alpha + \alpha + \alpha = -a \quad (\because \alpha = \beta = \gamma)$
$\Rightarrow a = -3\alpha$...(i)
$\alpha\beta + \beta\gamma + \gamma\alpha = b$
$\Rightarrow b = 3\alpha^2$...(ii)
Using Eqs. (i) and (ii),

$$\frac{a^2}{b} = 3$$

Question64

Let the system of linear equations
$4x + \lambda y + 2z = 0$
$2x - y + z = 0$
$\mu x + 2y + 3z = 0, \lambda, \mu \in R$
has a non-trivial solution. Then which of the following is true?
[18 Mar 2021 Shift 2]

Options:

A. $\mu = 6, \lambda \in R$

B. $\lambda = 2, \mu \in R$

C. $\lambda = 3, \mu \in R$

D. $\mu = -6, \lambda \in R$

Answer: A

Solution:

Solution:
Given, system of linear equations
$4x + \lambda y + 2z = 0$
$2x - y + z = 0$
$\mu x + 2y + 3z = 0$
For non-trivial solution, $\Delta = 0$

$$\begin{vmatrix} 4 & \lambda & 2 \\ 2 & -1 & 1 \\ \mu & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 4(-3-2) - \lambda(6-\mu) + 2(4+\mu) = 0$
$\Rightarrow -\lambda(6-\mu) - 2(6-\mu) = 0$
$\Rightarrow (6-\mu)(\lambda+2) = 0$
$\Rightarrow \lambda = -2 \text{ and } \mu \in R \text{ or } \mu = 6 \text{ and } \lambda \in R.$

Question65

The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + zk = k^2$ has no solution, if k is equal to
[17 Mar 2021 Shift 1]

Options:

A. 0

B. 1

C. -1

D. -2

Answer: D

Solution:

Solution:
$kx + y + z = 1$
$x + ky + z = k$
$x + y + kz = k^2$
For this set of equations to have no solution, $\Delta = 0$

$$\begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = k(k^2-1) - 1(k-1) + (1-k)$$

$= k^3 - k - k + 1 + 1 - k = k^3 - 3k + 2$
Now, $\Delta = 0$
$\Rightarrow k^3 - 3k + 2 = 0 \Rightarrow (k-1)(k^2+k-2) = 0$
$\Rightarrow (k-1)(k-1)(k+2) = 0$
$\therefore k = 1, -2$

$$\text{If } k = 1 \quad \left.\begin{array}{l} x+y+z=1 \\ x+y+z=1 \\ x+y+z=1 \end{array}\right\}$$

There are same equations and they will have infinite solutions.
So, $K = -2$

Question66

Let

$$\mathbf{f(x) =} \begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix},$$

$x \in [0, \pi]$Then the maximum value of $f(x)$ is equal to
[27 Jul 2021 Shift 1]

Answer: 6

Solution:

Solution:

$$\begin{vmatrix} -2 & -2 & 0 \\ 2 & 0 & -1 \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}$$

$(R_1 \to R_1 - R_2 \ \& \ R_2 \to R_2 - R_3)$
$-2(\cos^2 x) + 2(2 + 2\cos 2x + \sin^2 x)$
$4 + 4\cos 2x - 2(\cos^2 x - \sin^2 x)$
$f(x) = 4 + 2\underbrace{\cos 2x}_{\max = 1}$
$f(x)_{\max} = 4 + 2 = 6$

Question67

Let

$$\mathbf{M = \left\{ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\} \right\}.}$$

Define $f : M \to Z$, as $f(A) = \det(A)$, for all $A \in M$ where Z is set of all integers. Then the number of $A \in M$ such that $f(A) = 15$ is equal to ________.
[25 Jul 2021 Shift 1]

Answer: 16

Solution:

$|A| = ad - bc = 15$
where $a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\}$
Case I $ad = 9$ & $bc = -6$
For ad possible pairs are $(3, 3), (-3, -3)$
For bc possible pairs are $(3, -2), (-3, 2), (-2, 3), (2, -3)$
So total matrix $= 2 \times 4 = 8$

Case II $ad = 6$ & $bc = -9$
Similarly total matrix $= 2 \times 4 = 8$
$\Rightarrow$ Total such matrices are $= 16$

Question68

The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is:
[25 Jul 2021 Shift 2]

Options:

A. 4

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0, -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

Apply : $R_1 \rightarrow R_1 - R_2$ & $R_2 \rightarrow R_2 - R_3$

$$\begin{vmatrix} \sin x - \cos x & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$(\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$(\sin x - \cos x)^2(\sin x + 2\cos x) = 0$

$\therefore x = \frac{\pi}{4}$

Question69

Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in R$ be written as $P + Q$ where P is a symmetric matrix and Q is skew symmetric matrix. If $\det(Q) = 9$, then the modulus of the sum of all possible values of determinant of P is equal to :
[20 Jul 2021 Shift 1]

Options:

A. 36

B. 24

C. 45

D. 18

Answer: A

Solution:

Solution:

$A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}, a \in R$

and $P = \frac{A + A^T}{2} = \begin{bmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{bmatrix}$

and $P = \frac{A - A^T}{2} = \begin{bmatrix} 0 & \frac{3-a}{2} \\ \frac{a-3}{2} & 0 \end{bmatrix}$

As, $\det(Q) = 9$

$\Rightarrow (a-3)^2 = 36$

$\Rightarrow a = 3 \pm 6$

$\therefore a = 9, -3$

$\therefore \quad \det.(P) = \begin{vmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{vmatrix}$

$= 0 - \frac{(a-3)^2}{4} = 0$, for $a = -3$

$= 0 - \frac{(a-3)^2}{4} = -\frac{1}{4}(12)(12)$, for $a = 9$

$\therefore$ Modulus of the sum of all possible values ofdet. (P) = $|-36| + |0| = 36$ Ans.

$\Rightarrow$ Option (1) is correct

Question70

Let $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = \begin{cases} 1 & , \text{if } i = j \\ -x & , \text{if } |i-j| = 1 \\ 2x+1 & , \text{otherwise} \end{cases}$

Let a function $f : R \to R$ be defined as $f(x) = \det(A)$Then the sum of maximum and minimum values of f on mathbf R is equal to:
[20 Jul 2021 Shift 1]

Options:

A. $-\frac{20}{27}$

B. $\frac{88}{27}$

C. $\frac{20}{27}$

D. $-\frac{88}{27}$

Answer: D

Solution:

Solution:

$A = \begin{bmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{bmatrix}$

$|A| = 4x^3 - 4x^2 - 4x = f(x)$

$f'(x) = 4(3x^2 - 2x - 1) = 0$

$\Rightarrow x = 1;\ x = \frac{-1}{3}$

$\therefore f(1) = -4;\ f\left(-\frac{1}{3}\right) = \frac{20}{27}$

Sum $= -4 + \frac{20}{27} = -\frac{88}{27}$

Question71

Let a, b, c, d be in arithmetic progression with common difference λ. If

$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix}$ **= 2then value of λ^2 is equal to ___________.**

[20 Jul 2021 Shift 1]

Answer: 1

Solution:

Solution:

$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$

$C_2 \rightarrow C_2 - C_3$

$\Rightarrow \begin{vmatrix} x-2\lambda & \lambda & x+a \\ x-1 & \lambda & x+b \\ x+2\lambda & \lambda & x+c \end{vmatrix} = 2$

$R_2 \rightarrow R_2 - R_1, \quad R_3 \rightarrow R_3 - R_1$

$$\Rightarrow\lambda\begin{vmatrix} x-2\lambda & 1 & x+a \\ 2\lambda-1 & 0 & \lambda \\ 4\lambda & 0 & 2\lambda \end{vmatrix} = 2$$

$\Rightarrow 1(4\lambda^2 - 4\lambda^2 + 2\lambda) = 2$

$\Rightarrow \lambda^2 = 1$

Question72

For real numbers α and β, consider the following system of linear equations :

$x + y - z = 2$, $x + 2y + \alpha z = 1$, $2x - y + z = \beta$If the system has infinite solutions, then $\alpha + \beta$ is equal to ________ .

[27 Jul 2021 Shift 1]

Answer: 5

Solution:

Solution:

For infinite solutions

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} 3 & 0 & 0 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$\Delta = 3(2 + \alpha) = 0$

$\Rightarrow \alpha = -2$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & -2 \\ 2 & \beta & 1 \end{vmatrix} = 0$$

$1(1 + 2\beta) - 2(1 + 4) - (\beta - 2) = 0$

$\beta - 7 = 0$

$\beta = 7$

$\therefore \alpha + \beta = 5$ Ans.

Question73

The values of a and b, for which the system of equations

$2x + 3y + 6z = 8$

$x + 2y + az = 5$

$3x + 5y + 9z = b$

has no solution, are :

[25 Jul 2021 Shift 1]

Options:

A. $a = 3, b \neq 13$

B. $a \neq 3, b \neq 13$

C. $a \neq 3, b = 3$

D. $a = 3, b = 13$

Answer: A

Solution:

Solution:

$$D = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 3 - a$$

$$D = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = b - 13$$

If $a = 3, b \neq 13$, no solution.

Question74

The values of λ and μ such that the system of equations $x + y + z = 6, 3x + 5y + 5z = 26, x + 2y + \lambda z = \mu$ has no solution, are : [22 Jul 2021 Shift 2]

Options:

A. $\lambda = 3, \mu = 5$

B. $\lambda = 3, \mu \neq 10$

C. $\lambda \neq 2, \mu = 10$

D. $\lambda = 2, \mu \neq 10$

Answer: D

Solution:

Solution:

$x + y + z = 6$ .....(i)

$3x + 5y + 5z = 26$ ....(ii)

$x + 2y + \lambda z = \mu$ ....(iii)

$5 \times (i) - (ii) \Rightarrow 2x = 4 \Rightarrow x = 2$

$\therefore$ from (i) and (iii)

$y + z = 4$ ....(iv)

$2y + \lambda z = \mu - 2$

$(v) - 2 \times (iv)$

$\Rightarrow (\lambda - 2)z = \mu - 10$

$\Rightarrow z = \dfrac{\mu - 10}{\lambda - 2}$ & $y = 4 - \dfrac{\mu - 10}{\lambda - 2}$

$\therefore$ For no solution $\lambda = 2$ and $\mu \neq 10$.

Question75

The value of $k \in R$, for which the following system of linear equations
$3x - y + 4z = 3$
$x + 2y - 3z = -2$
$6x + 5y + kz = -3$
has infinitely many solutions, is :
[20 Jul 2021 Shift 2]

Options:

A. 3

B. -5

C. 5

D. -3

Answer: B

Solution:

Solution:

$$\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & K \end{vmatrix} = 0$$

$\Rightarrow 3(2K + 15) + K + 18 - 28 = 0$
$\Rightarrow 7K + 35 = 0 \Rightarrow K = -5$

Question76

Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where [t] denotes the greatest integer less than or equal to t. If det(A) = 192, then the set of values of x is the interval
[27 Aug 2021 Shift 2]

Options:

A. [68, 69]

B. [62, 63]

C. [65, 66]

D. [60, 61]

Answer: B

Solution:

Solution:

Given, $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$

$A = \begin{pmatrix} [x]+1 & [x]+2 & [x]+3 \\ [x] & [x]+3 & [x]+3 \\ [x] & [x]+2 & [x]+4 \end{pmatrix}$ (∵ [x + n] = n + [x], n ∈ I)

Applying $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$

$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{bmatrix}$

det(A) = 1([x] + 4 + [x] + 2) − 1(−[x])
= 3[x] + 6
∵ det(A) = 192
⇒ 3[x] + 6 = 192
⇒ [x] = 62
⇒ 62 ≤ x < 6
⇒ x ∈ [62,63)

Question77

Let A(a, 0), B(b, 2b + 1) and C(0, b), b ≠ 0, | b | ≠1, be points such that the area of ΔABC is1 sq. unit, then the sum of all possible values of a is [27 Aug 2021 Shift 2]

Options:

A. $\frac{-2b}{b+1}$

B. $\frac{2b}{b+1}$

C. $\frac{2b^2}{b+1}$

D. $\frac{-2b^2}{b+1}$

Answer: D

Solution:

Solution:
A(a, 0), B(b, 2b + 1), C(0, b)

Area of ΔABC $= \frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = \pm 1$

$\Rightarrow \frac{1}{2}[a(b+1) + b^2] = \pm 1$

$\Rightarrow a = \frac{2-b^2}{b+1}$ or $\frac{-2-b^2}{b+1}$

Sum of all possible values of a = $\frac{-2b^2}{b+1}$

Question78

If the following system of linear equations $2x + y + z = 5$, $x - y + z = 3$ and $x + y + az = b$ has no solution, then [31 Aug 2021 Shift 1]

Options:

A. $a = -\frac{1}{3}$ and $b \neq \frac{7}{3}$

B. $a \neq \frac{1}{3}$ and $b = \frac{7}{3}$

C. $a \neq -\frac{1}{3}$ and $b = \frac{7}{3}$

D. $a = \frac{1}{3}$ and $b \neq \frac{7}{3}$

Answer: D

Solution:

Solution:

$\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} = 2(-a-1) - 1(a-1) + (1+1)$

$= 1 - 3a$

$\Delta_3 = \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} = 2(-b-3) - 1$

$(b-3) + 5(1+1) = 7 - 3b$

Now, $z = \frac{\Delta_3}{\Delta}$

If $\Delta = 0$ and $\Delta_3 \neq 0$, then no solution

$1 - 3a = 0$

$\Rightarrow 7 - 3b \neq 0$

$a = \frac{1}{3}, b \neq \frac{7}{3}$

Question79

If $\alpha + \beta + \gamma = 2\pi$, then the system of equations
$x + (\cos\gamma)y + (\cos\beta)z = 0$
$(\cos\gamma)x + y + (\cos\alpha)z = 0$
$(\cos\beta)x + (\cos\alpha)y + z = 0$
has :
[31 Aug 2021 Shift 2]

Options:

A. no solution

B. infinitely many solution

C. exactly two solutions

D. a unique solution

Answer: B

Solution:

Solution:
Given $\alpha + \beta + \gamma = 2\pi$

$$\Delta = \begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$$

$= 1 - \cos^2\alpha - \cos\gamma(\cos\gamma - \cos\alpha\cos\beta) + \cos\beta(\cos\alpha\cos\gamma - \cos\beta)$
$= 1 - \cos^2\alpha - \cos^2\beta - \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma$
$= -\cos(\alpha+\beta)\cos(\alpha-\beta) - \cos\gamma(\cos(2\pi - (\alpha+\beta)) - 2\cos\alpha\cos\beta)$
$= -\cos(2\pi-\gamma)\cos(\alpha-\beta) - \cos\gamma(\cos(\alpha+\beta) - 2\cos\alpha\cos\beta)$
$= -\cos\gamma\cos(\alpha-\beta) + \cos\gamma(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$
$= -\cos\gamma\cos(\alpha-\beta) + \cos\gamma\cos(\alpha-\beta)$
$= 0$

Question80

If the system of linear equations
$2x + y - z = 3$
$x - y - z = \alpha$
$3x + 3y + \beta z = 3$
has infinitely many solution, then $\alpha + \beta - \alpha\beta$ is equal to
[27 Aug 2021 Shift 1]

Answer: 5

Solution:

Solution:
Given, system of equation
$2x + y - z = 3$
$x - y - z = \alpha$
$3x + 3y + \beta z = 3$
has infinitely many solutions,
if $\Delta = 0$, $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\text{Now}, \Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & -1 \\ 3 & 3 & \beta \end{vmatrix} = 0$$

$\Rightarrow 2(-\beta + 3) - 1(\beta + 3) - 1(3 + 3) = 0$
$\Rightarrow \beta = -1$

$$\Delta_1 = \begin{vmatrix} 3 & 1 & -1 \\ \alpha & -1 & -1 \\ 3 & 3 & -1 \end{vmatrix} = 0$$
$\Rightarrow 3(1+3) - 1(-\alpha+3) - 1(3\alpha+3) = 0$
$\Rightarrow 12 + \alpha - 3 - 3\alpha - 3 = 0$
$\Rightarrow \alpha = 3$

Also, $\Delta_2 = \begin{vmatrix} 2 & 3 & -1 \\ 1 & \alpha & -1 \\ 3 & 3 & -1 \end{vmatrix} = 0$
$\Rightarrow 2(-\alpha+3) - 3(-1+3) - 1(3-3\alpha) = 0$
$\Rightarrow \alpha = 3$

and $\Delta_3 = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & \alpha \\ 3 & 3 & 3 \end{vmatrix} = 0$
$\Rightarrow 2(-3-3\alpha) - 1(3-3\alpha) + 3(3+3) = 0$
$\Rightarrow -3\alpha + 9 = 0$
$\Rightarrow \alpha = 3$
$\therefore \alpha = 3, \beta = -1$
So, $\alpha + \beta - \alpha\beta = 3 - 1 - 3(-1) = 5$

Question81

Let $|\lambda|$ be the greatest integer less than or equal to λ. The set of all values of lambda for which the system of linear equations $x + y + z = 4$, $3x + 2y + 5z = 3$, $9x + 4y + (28 + |\lambda|)z = |\lambda|$ has a solution is [27 Aug 2021 Shift 2]

Options:

A. R

B. $(-\infty, -9) \cup (-9, \infty)$

C. $[-9, -8)$

D. $(-\infty, -9) \cup [-8, \infty)$

Answer: A

Solution:

Solution:
Given, system of equations
$x + y + z = 4$
$3x + 2y + 5z = 3$
$9x + 4y + (28 + |\lambda|)z = |\lambda|$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28 + |\lambda| \end{vmatrix}$$
$= 1(56 + 2|\lambda| - 20) - 1(84 + 3|\lambda| - 45) + 1(-6)$
$= -(|\lambda| + 9)$
If $\Delta \neq 0$ i.e $|\lambda| + 9 \neq 0$, then system of equation has unique solution.
If $|\lambda| + 9 = 0$, then $\Delta_1 = \Delta_2 = \Delta_3 = 0$, the system of equation has infinite solution.
$\Rightarrow \lambda \in R$

Question82

Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations
$(1+\cos^2\theta)x + \sin^2\theta y + 4\sin 3\theta z = 0$
$\cos^2\theta x + (1+\sin^2\theta)y + 4\sin 3\theta z = 0$
$\cos^2\theta x + \sin^2\theta y + (1+4\sin 3\theta)z = 0$
has a non-trivial solution, then the value of θ is
[26 Aug 2021 Shift 1]

Options:

A. $\frac{4\pi}{9}$

B. $\frac{7\pi}{18}$

C. $\frac{\pi}{18}$

D. $\frac{5\pi}{18}$

Answer: B

Solution:

Solution:

$$\Rightarrow \begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

Applying $R_1 \to R_1 - R_3$, we get

$$\begin{vmatrix} 1 & 0 & -1 \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_3$, we get

$$\begin{vmatrix} 1 & 0 & 0 \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta+\cos^2\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta+\cos^2\theta \end{vmatrix} = 0$$

$\Rightarrow (1+\sin^2\theta)(1+4\sin 3\theta+\cos^2\theta) - \sin^2\theta(4\sin 3\theta+\cos^2\theta) = 0$

$\Rightarrow 1+4\sin 3\theta+\cos^2\theta+\sin^2\theta+4\sin^2\theta\sin 3\theta+\sin^2\theta\cos^2\theta-4\sin^2\theta\sin 3\theta-\sin^2\theta\cos^2\theta = 0$

$\Rightarrow 1+4\sin 3\theta+\cos^2\theta+\sin^2\theta = 0$

$\Rightarrow 1+4\sin 3\theta+1 = 0$

$\Rightarrow 4\sin 3\theta+2 = 0$

$\Rightarrow \sin 3\theta = \frac{-1}{2}$

$\Rightarrow 3\theta = \left(\pi+\frac{\pi}{6}\right)$

$\Rightarrow \theta = \frac{7\pi}{18}$

Question83

Two fair dice are thrown. The numbers on them are taken as λ and μ , and a system of linear equations
$x + y + z = 5$, $x + 2y + 3z = \mu$ and
$x + 3y + \lambda z = 1$
is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then
[26 Aug 2021 Shift 2]

Options:

A. $p = \frac{1}{6}$ and $q = \frac{1}{36}$

B. $p = \frac{5}{6}$ and $q = \frac{5}{36}$

C. $p = \frac{5}{6}$ and $q = \frac{1}{36}$

D. $p = \frac{1}{6}$ and $q = \frac{5}{36}$

Answer: B

Solution:

Solution:
$x + y + z = 5$
$x + 2y + 3z = \mu$
$x + 3y + \lambda z = 1$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = (2\lambda - 9) + (3 - \lambda) + (3 - 2) = \lambda - 5$$

For unique solution $\Delta \neq 0$
$\Rightarrow \lambda \neq 5$
And Δ_1 or Δ_2 or $\Delta_3 \neq 0$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & \mu \\ 1 & 3 & 1 \end{vmatrix} = (2 - 3\mu) + (\mu - 1) + 5 = 6 - 2\mu$$

If $\Delta_3 \neq 3$ and $\Delta = 0$, then no solution
$\mu \neq 3$ and $\lambda = 5$
$p =$ Probability of unique solution $= \frac{5}{6}$
$q =$ Probability of no solution $= \frac{5}{6} \cdot \frac{1}{6} = \frac{5}{36}$

Question84

Consider the system of linear equations
$-x + y + 2z = 0$
$3x - ay + 5z = 1$
$2x - 2y - az = 7$
Let S_1 be the set of all $a \in R$ for which the system is inconsistent and S_2 be the set of all $a \in R$ for which the system hasinfinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2

respectively, then
[1 Sep 2021 Shift 2]

Options:

A. $n(S_1) = 2$ and $n(S_2) = 2$

B. $n(S_1) = 1$ and $n(S_2) = 0$

C. $n(S_1) = 2$ and $n(S_2) = 0$

D. $n(S_1) = 0$ and $n(S_2) = 2$

Answer: C

Solution:

Solution:
For in consistent system of equations
[$\Delta = 0$ and atleast one is non-zero in Δ_1, Δ_2 and .Δ_3]

$$\Delta = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix} = 0$$

$\Rightarrow a^2 - 7a + 12 = 0$
$\Rightarrow a = 3, 4$

$$\Delta_x = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix} = 15a + 31$$

$\Delta_x \neq 0$ for $a = 3, 4$
$\Rightarrow n(S_1) = 2$
Now, for infinitely many solutions.
If $\Delta = 0$ also $\Delta_x = \Delta_y = \Delta_z = 0$
Which is not possible for any real value of a
$\Rightarrow n(S_2) = 0$

Question85

If the system of linear equations,
$x + y + z = 6$
$x + 2y + 3z = 10$
$3x + 2y + \lambda z = \mu$
has more than two solutions, then $\mu - \lambda^2$ is equal to ________.
[NA Jan. 7, 2020 (II)]

Answer: 13

Solution:

$x + y + z = 6$ ....(i)
$x + 2y + 3z = 10$ .....(ii)
$3x + 2y + \lambda z = \mu$ ......(iii)
From (i) and (ii),
If $z = 0 \Rightarrow x + y = 6$ and $x + 2y = 10$
$\Rightarrow y = 4, x = 2$
(2, 4, 0)
If $y = 0 \Rightarrow x + z = 6$ and $x + 3z = 10$
$\Rightarrow z = 2$ and $x = 4$
(4, 0, 2)
So, $3x + 2y + \lambda z = \mu$, must pass through (2,4,0) and (4,0,2)
So, $6 + 8 = \mu \Rightarrow \mu = 14$
and $12 + 2\lambda = \mu$
$12 + 2\lambda = 14 \Rightarrow \lambda = 1$
So, $\mu - \lambda^2 = 14 - 1 = 13$

Question86

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i + j - 2)}a_{ij}$, where i, j = 1, 2, 3. If the determinant of B is 81, then the determinant of A is:
[Jan. 7, 2020 (II)]

Options:

A. 1/3

B. 3

C. 1/81

D. 1/9

Answer: D

Solution:

Solution:
It is given that |B| = 81

$$\therefore |B| = \begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = \begin{vmatrix} 3^0a_{11} & 3^1a_{12} & 3^2a_{13} \\ 3^1a_{21} & 3^2a_{22} & 3^3a_{23} \\ 3^2a_{31} & 3^3a_{32} & 3^4a_{33} \end{vmatrix}$$

$\Rightarrow 81 = 3^3 . 3^2 . 3^1 |A|$
$\Rightarrow 3^4 = 3^6|A| \Rightarrow |A| = \frac{1}{9}$

Question87

Let two points be $A(l, -1)$ and $B(0, 2)$. If a point $P(x', y')$ be such that the area of ΔPAB = 5 sq. units and it lies on the line, $3x + y - 4\lambda = 0$, then a value of λ is:
[Jan.8,2020 (I)]

Options:

A. 4

B. 3

C. 1

D. -3

Answer: B

Solution:

Solution:

$D = \frac{1}{2}\begin{vmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ x' & y' & 1 \end{vmatrix} = 5$

$\Rightarrow -2(1 - x') + (y' + x') = \pm 10$

$\Rightarrow -2 + 2x' + y' + x' = \pm 10$

$\Rightarrow 3x' + y' = 12$ or $3x' + y' = -8$

$\therefore \lambda = 3, -2$

Question88

If the matrices $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \operatorname{adj} A$ and $C = 3A$, then $\frac{|\operatorname{adj} B|}{|C|}$ is equal to:
[Jan. 9, 2020 (I)]

Options:

A. 8

B. 16

C. 72

D. 2

Answer: A

Solution:

Solution:

$|A| = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix} = ((9 + 4) - 1(3 - 4) + 2(-1 - 3))$

$= 13 + 1 - 8 = 6$

$|\operatorname{ad} jB| = |\operatorname{ad} j(\operatorname{ad} jA)| = |A|^{(n-1)^2} = |A|^4 = (36)^2$

$|C| = |3A| = 3^3 \times 6$

Hence, $\frac{|\operatorname{ad} jB|}{|C|} = \frac{36 \times 36}{3^3 \times 6} = 8$

Question89

The following system of linear equations
$7x + 6y - 2z = 0$
$3x + 4y + 2z = 0$
$x - 2y - 6z = 0$, has
[Jan. 9, 2020 (II)]

Options:

A. infinitely many solutions, (x, y, z) satisfying y = 2z.

B. no solution.

C. infinitely many solutions, (x, y, z) satisfying x = 2z.

D. only the trivial solution.

Answer: C

Solution:

Solution:
The given system of linear equations
$7x + 6y - 2z = 0$ ......(i)
$3x + 4y + 2z = 0$ ....(ii)
$x - 2y - 6z = 0$ ....(iii)
Now, determinant of coefficient matrix

$$\Delta = \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}$$

$= 7(-20) - 6(-20) - 2(-10)$
$= -140 + 120 + 20 = 0$
So, there are infinite non-trivial solutions.
From eqn. (i) $+3 \times$ (iii); we get
$10x - 20z = 0 \Rightarrow x = 2z$
Hence, there are infinitely many solutions (x, y, z) satisfying $x = 2z$

Question90

For which of the following ordered pairs (μ, δ), the system of linear equations
$x + 2y + 3z = 1$
$3x + 4y + 5z = \mu$
$4x + 4y + 4z = \delta$
is inconsistent?
[Jan. 8, 2020 (I)]

Options:

A. (4,3)

B. (4,6)

C. (1,0)

D. (3,4)

Answer: A

Solution:

Solution:
From the given linear equation, we get

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 4 & 4 \end{vmatrix} \quad (R_3 \rightarrow R_3 - 2R_2 + 3R_3)$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

Now, let $P_3 = 4x + 4y + 4z - \delta = 0$. If the system has solutions it will have infinite solution.
So, $P_3 \equiv \alpha P_1 + \beta P_2$
Hence, $3\alpha + \beta = 4$ and $4\alpha + 2\surd = 4$
$\Rightarrow \alpha = 2$ and $\beta = -2$
So, for infinite solution $2\mu - 2 = \delta$
$\Rightarrow$ For $2\mu \neq \delta + 2$ system is inconsistent

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Question91

The system of linear equations
$\lambda x + 2y + 2z = 5$
$2\lambda x + 3y + 5z = 8$
$4x + \lambda y + 6z = 10$ has :
[Jan. 8, 2020 (II)]

Options:

A. no solution when $\lambda = 8$

B. a unique solution when $\lambda = -8$

C. no solution when $\lambda = 2$

D. infinitely many solutions when $\lambda = 2$

Answer: C

Solution:

Solution:

$$D = \begin{vmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{vmatrix}$$

$D = \lambda^2 + 6\lambda - 16$
$D = (\lambda + 8)(2 - \lambda)$
For no solutions, $D = 0$
$\Rightarrow \lambda = -8, 2$
when $\lambda = 2$

$$D_1 = \begin{vmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{vmatrix}$$

$= 5[18 - 10] - 2[48 - 50] + 2(16 - 30]$
$= 40 + 4 - 28 \neq 0$
There exist no solutions for $\lambda = 2$

Question92

If the system of linear equations
$2x + 2ay + az = 0$
$2x + 3by + bz = 0$
$2x + 4cy + cz = 0$
where $a, b, c \in R$ are non-zero and distinct; has a non-zero solution, then:
[Jan. 7, 2020 (I)]

Options:

A. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

B. a, b, c are in G.P.

C. $a + b + c = 0$

D. a, b, c are in A.P.

Answer: A

Solution:

Solution:
For non-zero solution

$$\begin{vmatrix} 2 & 2a & a \\ 2 & 3b & b \\ 2 & 4c & c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

$\Rightarrow (3bc - 4bc) - (2ac - 4ac) + (2ab - 3ab) = 0$
$\Rightarrow -bc + 2ac - ab = 0$
$\Rightarrow ab + bc + 2ac$
$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$
$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ in A.P.

Question93

Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$. If $B = A + A^4$, then det (B):

[Sep. 06, 2020 (II)]

Options:

A. is one

B. lies in (2,3)

C. is zero

D. lies in (1,2)

Answer: D

Solution:

Solution:

$\because A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\therefore A = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in N$

$\therefore B = A + A^4$

$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

$\therefore B = \begin{bmatrix} \cos\frac{\pi}{5} + \cos\frac{4\pi}{5} & \sin\frac{\pi}{5} + \sin\frac{4\pi}{5} \\ -\sin\frac{\pi}{5} - \sin\frac{4\pi}{5} & \cos\frac{\pi}{5} + \cos\frac{4\pi}{5} \end{bmatrix}$

Then, $\det(B) = 2\sin\left(\frac{\pi}{5}\right) \cdot \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix}$

$= \frac{\sqrt{10 - 2\sqrt{5}}}{2} \approx 2.352 \approx 1.175$

$\therefore \det B \in (1, 2)$

Question94

If $\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix} = Ax^3 + Bx^2 + Cx + D$ then B + C is equal to :

[Sep. 03, 2020 (I)]

Options:

A. -1

B. 1

C. -3

D. 9

Answer: C

Solution:

Solution:

$$\Delta = \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 2x-3 & 3x-4 & 4x-5 \\ 3x-5 & 5x-8 & 10x-17 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x-2 & x-1 & x-1 \\ 2x-3 & x-1 & x-1 \\ 3x-5 & 2x-3 & 5x-9 \end{vmatrix} \begin{bmatrix} C_3 \to C_3 - C_2 \\ C_2 \to C_2 - C_1 \end{bmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x-2 & x-1 & x-1 \\ x-1 & 0 & 0 \\ 3x-5 & 2x-3 & 5x-9 \end{vmatrix} [R_2 \to R_2 - R_1]$$

$\Rightarrow \Delta = -(x-1)[(x-1)(5x-9) - (x-1)(2x-3)]$

$\Rightarrow \Delta = -(x-1)[(5x^2 - 14x + 9) - (2x^2 - 5x + 3)]$

$= -3x^3 + 12x^2 - 15x + 6$

So, $B + C = -3$

Question95

If the minimum and the maximum values of the function $f : \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \to R$, defined by $f(\theta) = \begin{vmatrix} -\sin^2\theta & -1-\sin^2\theta & 1 \\ -\cos^2\theta & -1-\cos^2\theta & 1 \\ 12 & 10 & -2 \end{vmatrix}$ are m and M respectively, then the ordered pair (m, M) is equal to :
[Sep. 05, 2020 (I)]

Options:

A. $(0, 2\sqrt{2})$

B. (-4,0)

C. (-4,4)

D. (0,4)

Answer: B

Solution:

Solution:
Applying $C_2 \to C_2 - C_1$

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1 & 1 \\ -\cos^2\theta & -1 & 1 \\ 12 & -2 & -2 \end{vmatrix}$$

$= 4(\cos^2\theta - \sin^2\theta)$

$= 4\cos 2\theta, \theta \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$
Max. $f(\theta) = M = 0$
Min. $f(\theta) = m = -4$
So, $(m, M) = (-4, 0)$

Question96

Let $a - 2b + c = 1$.

If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then:

[Jan. 9,2020 (II)]

Options:

A. $f(-50) = 501$

B. $f(-50) = -1$

C. $f(50) = -501$

D. $f(50) = 1$

Answer: D

Solution:

Solution:

$f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

$R_1 = R_1 + R_3 - 2R_2$

$\Rightarrow f(x) = \begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

$\Rightarrow f(x) = 1 \Rightarrow f(50) = 1$

Question97

If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to

[Sep. 05, 2020 (II)]

Options:

A. $y(b - a)$

B. $y(a-b)$

C. 0

D. $y(a-c)$

Answer: B

Solution:

Solution:
Use properties of determinant

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix} = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} + y \begin{vmatrix} x & 1 & x+a \\ y & 1 & y+b \\ z & 1 & z+c \end{vmatrix}$$

$$= 0 + y \begin{vmatrix} x & 1 & x+a \\ y-x & 0 & 0 \\ z-x & 0 & -1 \end{vmatrix} \quad \begin{bmatrix} R_2 \to R_2 - R_1, \\ R_3 \to R_3 - R_1 \end{bmatrix}$$

$$= -y(x-y) = -y(b-a) = y(a-b)$$

Question98

Let A bea 3 × 3 matrix such that adj A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & -2 & -1 \end{bmatrix}$ **and**

B = adj(adj A). If |A| = λ and $|(B^{-1})^T| = \mu$, then the ordered pair, (|λ|, μ) is equal to:
[Sep. 03, 2020 (II)]

Options:

A. $\left(3, \frac{1}{81}\right)$

B. $\left(9, \frac{1}{9}\right)$

C. (3,81)

D. $\left(9, \frac{1}{81}\right)$

Answer: A

Solution:

Solution:
$|\text{ad}\,\text{j}A| = |A|^2 = 9$
$[\because |\text{ad}\,\text{j}A| = |A|^{n-1}]$
$\Rightarrow |A| = \pm 3 = \lambda \Rightarrow |\lambda| = 3$
$\Rightarrow |B| = |\text{ad}\,\text{j}A|^2 = 81$
$\mu = |(B^{-1})^T| = |B^{-1}| = |B|^{-1} = \frac{1}{|B|} = \frac{1}{81}$

Question99

The values of λ and μ for which the system of linear equations
$x + y + z = 2$
$x + 2y + 3z = 5$
$x + 3y + \lambda z = \mu$
has infinitely many solutions are, respectively:
[Sep. 06, 2020 (I)]

Options:

A. 6 and 8

B. 5 and 7

C. 5 and 8

D. 4 and 9

Answer: C

Solution:

Solution:

$$D = \begin{vmatrix} 1 & 1 & \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow \lambda = 5$$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 5 & 2 & 3 \\ \mu & 3 & 5 \end{vmatrix} = 0 \Rightarrow \lambda = 8$$

Question100

The sum of distinct values of λ for wheih the system of equations
$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$
$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$
$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$
has non-zero solutions, is __________.
[NA Sep. 06, 2020 (II)]

Answer: 3

Solution:

For non-zero solution, $\Delta = 0$

$$\Rightarrow \begin{vmatrix} \lambda-1 & 3\lambda+1 & 2\lambda \\ \lambda-1 & 4\lambda-2 & \lambda+3 \\ 2 & 3\lambda+1 & 3(\lambda-1) \end{vmatrix} = 0$$

$\Rightarrow 6\lambda^3 - 36\lambda^2 + 54\lambda = 0$
$\Rightarrow 6\lambda[\lambda^2 - 6\lambda + 9] = 0$
$\Rightarrow \lambda = 0, \lambda = 3$ [Distinct values]
Then, the sum of distinct values of $\lambda = 0 + 3 = 3$.

Question101

Let $\lambda \in R$. The system of linear equations $2x_1 - 4x_2 + \lambda x_3 = 1$ $x_1 - 6x_2 + x_3 = 2$ $\lambda x_1 - 10x_2 + 4x_3 = 3$ [Sep. 05, 2020 (I)]

Options:

A. exactly one negative value of λ

B. exactly one positive value of λ

C. every value of λ

D. exactly two value of λ

Answer: A

Solution:

Solution:

$$\because \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = 0 \Rightarrow 3\lambda^2 - 7\lambda - 12 = 0$$

$\Rightarrow \lambda = 3$ or $-\frac{2}{3}$

$$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -10 & 4 \end{vmatrix} = 2(3-\lambda)$$

$\therefore$ When $\lambda = -\frac{2}{3}$, $D_1 \neq 0$.

Hence, equations will be inconsistent when $\lambda = -\frac{2}{3}$.

Question102

If the system of linear equations $x + y + 3z = 0$ $x + 3y + k^2z = 0$ $3x + y + 3z = 0$

has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to :
[Sep. 05, 2020 (II)]

Options:

A. -3

B. 9

C. 3

D. -9

Answer: A

Solution:

Solution:
Since, system of linear equations has non-zero solution
$\therefore \Delta = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 1(9 - k^2) - 1(3 - 3k^2) + 3(1 - 9) = 0$
$\Rightarrow 9 - k^2 - 3 + 3k^2 - 24 = 0$
$\Rightarrow 2k^2 = 18 \Rightarrow k^2 = 9, k = \pm 3$
So, equations are
$x + y + 3z = 0$ ....(i)
$x + 3y + 9z = 0$ ....(ii)
$3x + y + 3z = 0$ ....(iii)
Now, from equation (i) – (ii),
$-2y - 6z = 0 \Rightarrow y = -3z \Rightarrow \frac{y}{z} = -3$ ....(iv)
Now, from equation (i) – (iii),
$-2x = 0 \Rightarrow x = 0$
So, $x + \frac{y}{z} = 0 - 3 = -3$

Question103

If the system of equations $x - 2y + 3z = 9$, $2x + y + z = bx - 7y + az = 24$, has infinitely many solutions, then $a - b$ is equal to _______.
[NA Sep. 04, 2020 (I)]

Answer: 5

Solution:

For infinitely many solutions,
$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ 1 & -7 & a \end{vmatrix} = 0$$

$\Rightarrow (a + 7) - 2(1 - 2a) + 3(-15) = 0$
$\Rightarrow a = 8$

$$\Delta_3 = \begin{vmatrix} 1 & -2 & 9 \\ 2 & 1 & b \\ 1 & -7 & 24 \end{vmatrix} = 0$$

$\Rightarrow (24 + 7b) - 2(b - 48) + 9(-15) = 0$
$\Rightarrow b = 3$
$\therefore a - b = 5$

Question104

Suppose the vectors x_1, x_2 and x_3 are the solutions of the system of linear equations, $Ax = b$ when the vector b on the right side is equal to b_1, b_2 and b_3 respectively. If

$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $x_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$, $x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $b_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

and $b_3 = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$, then the determinant of A is equal to:

[Sep. 04, 2020 (II)]

Options:

A. 4

B. 2

C. $\frac{1}{2}$

D. $\frac{3}{2}$

Answer: B

Solution:

Given that $Ax = b$ has solutions x_1, x_2, x_3 and b is equal to b_1, b_2 and b_3
$\therefore x_1 + y_1 + z_1 = 1$
$\Rightarrow 2y_1 + z_1 = 2 \Rightarrow z_1 = 2$
Determinant of coefficient matrix

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2$$

Question105

If the system of equations
$x + y + z = 2$
$2x + 4y - z = 6$
$3x + 2y + \lambda z = \mu$
has infinitely many solutions, then :
[Sep. 04, 2020 (II)]

Options:

A. $\lambda + 2\mu = 14$

B. $2\lambda - \mu = 5$

C. $\lambda - 2\mu = -5$

D. $2\lambda + \mu = 14$

Answer: D

Solution:

Solution:

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -1 \\ 3 & 2 & \lambda \end{vmatrix} = 0 \text{ [} \because \text{ Equation has many solutions]}$$

$\Rightarrow -15 + 6 + 2\lambda = 0 \Rightarrow \lambda = \frac{9}{2}$

$$\therefore D_z = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 2\mu \end{vmatrix} = 0 \Rightarrow \mu = 5$$

$\therefore 2\lambda + \mu = 14$

Question106

Let S be the set of all integer solutions, (x, y, z), of the system of equations
$x - 2y + 5z = 0$
$-2x + 4y + z = 0$
$-7x + 14y + 9z = 0$
such that $15 \le x^2 + y^2 + z^2 \le 150$. Then, the number of elements in the set S is equal to ________.
[NA Sep. 03, 2020 (II)]

Answer: 8

Solution:

The given system of equations
$x - 2y + 5z = 0$ ....(i)
$-2x + 4y + z = 0$ ....(ii)
$-7x + 14y + 9z = 0$ ......(iii)
From equation, $2 \times$ (i) + (ii) $\Rightarrow z = 0$
Put $z = 0$ in equation (i), we get $x = 2y$
$\because 15 \le x^2 + y^2 + z^2 \le 150$
$\Rightarrow 15 \le 4y^2 + y^2 \le 150$
$[\because x = 2y, z = 0]$
$\Rightarrow 3 \le y^2 \le 30$
$\Rightarrow y = \pm 2, \pm 3, \pm 4, \pm 5$
$\Rightarrow$8 solutions.

Question107

Let S be the set of all $\lambda \in R$ for which the system of linear equations
$2x - y + 2z = 2$
$x - 2y + \lambda z = -4$
$x + \lambda y + z = 4$
has no solution. Then the set S
[Sep. 02, 2020 (I)]

Options:

A. contains more than two elements.

B. is an empty set.

C. is a singleton.

D. contains exactly two elements.

Answer: D

Solution:

Solution:

$$\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = -(\lambda - 1)(2\lambda + 1)$$

$$\Delta_1 = \begin{vmatrix} 2 & -1 & 2 \\ -4 & -2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = -2(\lambda^2 + 6\lambda - 4)$$

For no solution $\Delta = 0$ and at least one of Δ_1, Δ_2 and Δ_3 is non-zero.

$\therefore \Delta = 0 \Rightarrow \lambda = 1, -\frac{1}{2}$ and $\Delta_1 \ne 0$

Hence, S = $\left\{ 1, -\frac{1}{2} \right\}$

Question108

Let A = $\{ X = (x, \ y, \ z)^T : PX = 0 \text{ and } x^2 + y^2 + z^2 = 1 \}$where

P = $\begin{bmatrix} 1 & 2 & 1 \\ -2 & 3 & -4 \\ 1 & 9 & -1 \end{bmatrix}$ **, then the set A :**

[Sep. 02, 2020 (II)]

Options:

A. is a singleton

B. is an empty set

C. contains more than two elements

D. contains exactly two elements

Answer: D

Solution:

Solution:
$\because |P| = 1(-3+36) - 2(2+4) + 1(-18-3) = 0$
Given that $PX = 0$
$\therefore$ System of equations
$x + 2y + z = 0; 2x - 3y + 4z = 0$
and $x + 9y - z = 0$ has infinitely many solution.
Let $z = k \in R$ and solve above equations, we get
$x = -\frac{11k}{7}, y = \frac{2k}{7}, z = k$
But given that $x^2 + y^2 + z^2 = 1$
$\therefore k = \pm\frac{7}{\sqrt{174}}$
$\therefore$ Two solutions only.

Question109

If A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ **, then the matrix A^{-50} when$\theta = \frac{\pi}{12}$, is equal to:**

[Jan 09, 2019 (I)]

Options:

A. $\begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

B. $\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

C. $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

D. $\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$

Answer: C

Solution:

Solution:

$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \Rightarrow |A| = 1$

$adj(A) = \begin{bmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & +\cos\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = B$

$B^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$\Rightarrow B^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

$\Rightarrow A^{-50} = B^{50} = \begin{bmatrix} \cos(50\theta) & \sin(50\theta) \\ -\sin(50\theta) & \cos(50\theta) \end{bmatrix}$

$(A^{-50})_{\theta = \frac{\pi}{12}} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$

$\left[\because \cos\left(\frac{50\pi}{12}\right) = \cos\left(4\pi\frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \right]$

Question110

If

$$\mathbf{A} = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$$

then A is:
[Jan. 09, 2019 (II)]

Options:

A. invertible for all $t \in R$.

B. invertible only if $t = \pi$.

C. not invertible for any $t \in R$.

D. invertible only if $t = \frac{\pi}{2}$.

Answer: A

Solution:

Solution:

$d\,et(A) = |A|$

$$A = \begin{vmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{vmatrix}$$

$$= e^t \cdot e^{-t} \cdot e^{-t} \begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$$= e^{-t} \begin{vmatrix} 0 & 2\cos t + \sin t & 2\sin t - \cos t \\ 0 & -\cos t - 3\sin t & -\sin t + 3\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 + R_3$

$$= e^{-t} \begin{vmatrix} 0 & -5\sin t & 5\cos t \\ 0 & -\cos t - 3\sin t & -\sin t + 3\cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$$

$R_1 \to R_1 + 2R_2$

$= e^{-t}[(-5\sin t)(-\sin t + 3\cos t) - 5\cos t(-\cos t - 3\sin t)$

$= 5e^{-t} \neq 0,\ \forall t \in R$

$\therefore$A is invertible.

Question111

Let $d \in R$, and

$$\mathbf{A} = \begin{bmatrix} -2 & 4 + d & (\sin\theta) - 2 \\ 1 & (\sin\theta) + 2 & d \\ 5 & (2\sin\theta) - d & (-\sin\theta) + 2 + 2d \end{bmatrix}$$

$\theta \in [0, 2\pi]$. If the minimum value of det (A) is 8 , then a value of d is: [Jan 10, 2019 (I)]

Options:

A. -5

B. -7

C. $2(\sqrt{2} + 1)$

D. $2(\sqrt{2} + 2)$

Answer: A

Solution:

Solution:

det A = $\begin{bmatrix} -2 & 4+d & \sin\theta^{-2} \\ 1 & \sin\theta+2 & d \\ 5 & (2\sin\theta)-d & -\sin\theta+2+2d \end{bmatrix}$

Applying $R_3 \to R_3 - 2R_2 + R_1$

we get d et(A) $= \begin{vmatrix} -2 & 4+d & \sin\theta-2 \\ 1 & \sin\theta+2 & d \\ 1 & 0 & 0 \end{vmatrix}$

$= d(4+d) - (\sin^2\theta - 4)$

$\Rightarrow \det(A) = d^2 + 4d + 4 - \sin^2\theta = (d+2)^2 - \sin^2\theta$

Minimum value of d et(A) is attained when $\sin^2\theta = 1$

$\therefore (d+2)^2 - 1 = 8 \Rightarrow (d+2)^2 = 9 \Rightarrow d + 2 = \pm 3$

$\Rightarrow d = -5$ or 1

Question112

Let $a_1, a_2, a_3, \ldots, a_{10}$ be in G.P. with $a_i > 0$ for i = 1, 2, ..., 10 andS be the set of pairs (r, k), r, k ∈ N (the set of natural numbers) for which

$$\begin{vmatrix} \log_e a_1^r a_2^k & \log_e a_2^r a_3^k & \log_e a_3^r a_4^k \\ \log_e a_4^r a_5^k & \log_e a_5^r a_6^k & \log_e a_6^r a_7^k \\ \log_e a_7^r a_8^k & \log_e a_8^r a_9^k & \log_e a_9^r a_{10}^k \end{vmatrix} = 0$$

Then the number of elements in S, is :
[Jan. 10, 2019 (II)]

Options:

A. 4

B. infinitely many

C. 2

D. 10

Answer: B

Solution:

Solution:
Let common ratio of G.P. be R
$\Rightarrow a_2 = a_1R, a_3 = a_1R^2, \ldots . a^{10} = a_1R^9$
$C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} \ln\left(\frac{a_1{}^r a_2{}^k}{a_2{}^r a_3{}^k}\right) & \ln\left(\frac{a_2{}^r a_3{}^k}{a_3{}^r a_4{}^k}\right) & \ln a_3{}^r a_4{}^k \\ \ln\left(\frac{a_4{}^r a_5{}^k}{a_5{}^r a_6{}^k}\right) & \ln\left(\frac{a_5{}^r a_6{}^k}{a_6{}^r a_7{}^k}\right) & \ln a_6{}^r a_7{}^k \\ \ln\left(\frac{a_7{}^r a_8{}^k}{a_8{}^r a_9{}^k}\right) & \ln\left(\frac{a_8{}^r a_9{}^k}{a_9{}^r a_{10}{}^k}\right) & \ln a_9{}^r a_{10}{}^k \end{vmatrix}$$

$$\Delta = \begin{vmatrix} \ln 1 R^{r+k} & \ln \frac{1}{R^{r+k}} & \ln a_3{}^r a_4{}^k \\ \ln \frac{1}{R^{r+k}} & \ln \frac{1}{R^{r+k}} & \ln a_6{}^r a_7{}^k \\ \ln \frac{1}{R^{r+k}} & \ln \frac{1}{R^{r+k}} & \ln a_9{}^r a_{10}{}^k \end{vmatrix} = 0$$

$\forall r, K \in N$

Hence, number of elements in S is infinitely many.

Question113

Let A = $\begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where b > 0. Then the minimum value of $\frac{\det(A)}{b}$ is:

[Jan. 10, 2019 (II)]

Options:

A. $2\sqrt{3}$

B. $-2\sqrt{3}$

C. $-\sqrt{3}$

D. $\sqrt{3}$

Answer: A

Solution:

Solution:

$|A| = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$

$= 2(2b^2 + 2 - b^2) - b(2b - b) + 1(b^2 - b^2 - 1)$

$= 2b^2 + 4 - b^2 - 1 = b^2 + 3$

$\frac{|A|}{b} = b + \frac{3}{b}$

$\because \frac{b + \frac{3}{b}}{2} \geq \left(b\frac{3}{b}\right)^{\frac{1}{2}} \Rightarrow b + \frac{3}{b} \geq 2\sqrt{3}$

$\therefore \frac{|A|}{b} \geq 2\sqrt{3}$

Minimum value of $\frac{|A|}{b}$ is $2\sqrt{3}$.

Question114

If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ **$= (a+b+c)(x+a+b+c)^2$, $x \neq 0$ and $a+b+c \neq 0$, then xis equal to: [Jan. 11,2019 (II)]**

Options:

A. abc

B. $-(a+b+c)$

C. $2(a+b+c)$

D. $-2(a+b+c)$

Answer: D

Solution:

Solution:

$\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$R_1 \rightarrow R_1 + R_2 + R_3$

$\Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

$C_1 \rightarrow C_1 - C_3, C_2 \rightarrow C_2 - C_3$

$\Delta = (a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ 0 & -b-c-a & 2b \\ c+a+b & c+a+b & c-a-b \end{vmatrix}$

$= (a+b+c)(a+b+c)^2$

Hence, $x = -2(a+b+c)$

Question115

If A = $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$ then for all $\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$,det (A) lies in the interval :
[Jan. 12, 2019 (II)]

Options:

A. $\left(1, \frac{5}{2}\right]$

B. $\left[\frac{5}{2}, 4\right)$

C. $\left(0, \frac{3}{2}\right]$

D. $\left(\frac{3}{2}, 3\right]$

Answer: D

Solution:

Solution:

$A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 2 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix} R_1 \to R_1 + R_3$

$= 2(\sin^2\theta + 1)$

Since ,$\theta \in \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right) \Rightarrow \sin^2\theta \in \left(0, \frac{1}{2}\right)$

$\therefore \det(A) \in [2, 3)$

$[2, 3) \subset \left(\frac{3}{2}, 3\right]$

Question116

An ordered pair (α, β) for which the system of linear equations
$(1 + \alpha)x + \beta y + z = 2$
$ax + (1 + \beta)y + z = 3$
$\alpha x + \beta y + 2z = 2$
has a unique solution, is :
[Jan. 12, 2019 (I)]

Options:

A. (2,4)

B. (-3,1)

C. (-4,2)

D. (1,-3)

Answer: A

Solution:

Solution:
∵ The system of linear equations has a unique solution.
$\therefore \Delta \neq 0$

$$\Delta = \begin{vmatrix} 1+\alpha & \beta & 1 \\ \alpha & 1+\beta & 1 \\ \alpha & \beta & 2 \end{vmatrix} \neq 0$$

$$\Rightarrow \begin{vmatrix} 1+\alpha+\beta+1 & \beta & 1 \\ \alpha+1+\beta+1 & 1+\beta & 1 \\ \alpha+\beta+2 & \beta & 2 \end{vmatrix} \neq 0 \ [C_1 \to C_1 + C_2 + C_3]$$

$$\Rightarrow (\alpha+\beta+2) \begin{vmatrix} 1 & \beta & 1 \\ 1 & 1+\beta & 1 \\ 1 & \beta & 2 \end{vmatrix} \neq 0$$

$$\Rightarrow (\alpha+\beta+2) \begin{vmatrix} 1 & \beta & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \neq 0$$

$R_2 \to R_2 - R_1$
$R_3 \to R_3 - R_1$
$\Rightarrow (\alpha+\beta+2)1(1) \neq 0$
$\Rightarrow \alpha+\beta+2 \neq 0$
∵ Ordered pair (2,4) satisfies this condition
$\therefore \alpha = 2$ and $\beta = 4$.

Question117

The set of all values of λ for which the system of linear equations
$x - 2y - 2z = \lambda x$
$x + 2y + z = \lambda y$
$-x - y = \lambda 2$
has a non-trivial solution :
[Jan. 12, 2019 (II)]

Options:

A. is a singleton

B. contains exactly two elements

C. is an empty set

D. contains more than two elements

Answer: A

Solution:

Solution:
Consider the given system of linear equations
$x(1-\lambda)-2y-2z=0$
$x+(2-\lambda)y+z=0$
$-x-y-\lambda z=0$
Now, for a non-trivial solution, the determinant of coefficient matrix is zero.

$$\begin{vmatrix} 1-\lambda & -2 & -2 \\ 1 & 2-\lambda & 1 \\ -1 & -1 & -\lambda \end{vmatrix}=0$$

$\Rightarrow(1-\lambda)^3=0$
$\lambda=1$

Question118

If the system of linear equations
$2x+2y+3z=a$
$3x-y+5z=b$
$x-3y+2z=c$
where, a, b, c are non-zero real numbers, has more than one solution, then :
[Jan. 11, 2019 (I)]

Options:

A. $b-c+a=0$

B. $b-c-a=0$

C. $a+b+c=0$

D. $b+c-a=0$

Answer: B

Solution:

Solution:
$\because$ System of equations has more than one solution $\therefore \Delta=\Delta_1=\Delta_2=\Delta_3=0$ for infinite solution

$$\Delta_1=\begin{vmatrix} a & 2 & 3 \\ b & -1 & 5 \\ c & -3 & 2 \end{vmatrix}=a(13)+2(5c-2b)+3(-3b+c)$$

$=13a-13b+13c=0$
i.e, $a-b+c=0$
or $b-c-a=0$

Question119

The number of values of $\theta\in(0,\pi)$ for which the system of linear equations
$x+3y+7z=0$
$-x+4y+7z=0$
$(\sin 3\theta)x+(\cos 2\theta)y+2z=0$

has a non-trivial solution, is:
[Jan. 10, 2019 (II)]

Options:

A. three

B. two

C. four

D. one

Answer: B

Solution:

Solution:
Since, the system of linear equations has, non-trivial solution then determinant of coefficient matrix = 0

i.e., $\begin{vmatrix} \sin 3\theta & \cos 2\theta & 2 \\ 1 & 3 & 7 \\ -1 & 4 & 7 \end{vmatrix} = 0$

$\sin 3\theta(21 - 28) - \cos 2\theta(7 + 7) + 2(4 + 3) = 0$
$\sin 3\theta + 2\cos 2\theta - 2 = 0$
$3\sin\theta - 4\sin^3\theta + 2 - 4\sin^2\theta - 2 = 0$
$4\sin^3\theta + 4\sin^2\theta - 3\sin\theta = 0$
$\sin\theta(4\sin^2\theta + 4\sin\theta - 3) = 0$
$\sin\theta(4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3) = 0$
$\sin\theta[2\sin\theta(2\sin\theta - 1) + 3(2\sin\theta - 1)] = 0$
$\sin\theta(2\sin\theta - 1)(2\sin\theta + 3) = 0$
$\sin\theta = 0, \sin\theta = \frac{1}{2}\left(\because \sin\theta \neq -\frac{3}{2}\right)$
$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$
Hence, for two values of θ, system of equations has non-trivial solution

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Question120

If the system of equations
$x + y + z = 5$
$x + 2y + 3z = 9$
$x + 3y + \alpha z = \beta$
has infinitely many solutions, then $\beta - \alpha$ equals:
[Jan 10, 2019 (I)]

Options:

A. 21

B. 8

C. 18

D. 5

Answer: B

Solution:

Solution:

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \alpha \end{vmatrix} = 2\alpha - 9 - \alpha + 3 + 1 = \alpha - 5$

$\Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 3 \\ \beta & 3 & \alpha \end{vmatrix} = 5(2\alpha - 9) - 1(9\alpha - 3\beta) + (27 - 2\beta)$

$= 10\alpha - 45 - 9\alpha + 3\beta + 27 - 2\beta$

$= \alpha + \alpha - 18$

$\Delta_2 = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 9 & 3 \\ 1 & \beta & \alpha \end{vmatrix} = 9\alpha - 3\beta - 5(\alpha - 3) + 1(\beta - 9)$

$= 9\alpha - 3\beta - 5\alpha + 15 + \beta - 9 = 4\alpha - 2\beta + 6$

$\Delta_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & \beta \end{vmatrix} = 2\beta - 27 - \beta + 9 + 5 = \beta - 13$

Since, the system of equations has infinite many solutions.
Hence,
$\Delta_1 = \Delta_2 = \Delta_3 = \Delta = 0$
$\Rightarrow \alpha = 5, \beta = 13 \Rightarrow \beta - \alpha = 8$

Question121

If the system of linear equations
$x - 4y + 7z = g$
$3y - 5z = h$
$-2x + 5y - 9z = k$
is consistent, then :
[Jan. 09, 2019 (II)]

Options:

A. $g + 2h + k = 0$

B. $g + h + 2k = 0$

C. $2g + h + k = 0$

D. $g + h + k = 0$

Answer: C

Solution:

Solution:

Consider the system of linear equations
$x - 4y + 7z = g$ .....(i)
$3y - 5z = h$ ....(ii)
$-2x + 5y - 9z = k$ ....(iii)
Multiply equation (i) by 2 and add equation (i), equation (ii) and equation (iii)
$\Rightarrow 0 = 2g + h + k. \therefore 2g + h + k = 0$
then system of equation is consistent.

Question122

Let A and B be two invertible matrices of order 3 × 3. If $\det(ABA^T) = 8$ and $\det(AB^{-1}) = 8$, then $\det(BA^{-1}B^T)$ is equal to :
[Jan. 11, 2019 (II)]

Options:

A. $\frac{1}{4}$

B. 1

C. $\frac{1}{16}$

D. 16

Answer: C

Solution:

Solution:
Let $|A| = a, |B| = b$
$\Rightarrow |A^T| = a |A^{-1}| = \frac{1}{a}, |B^T| = b, |B^{-1}| = \frac{1}{b}$
$\because |ABA^T| = 8 \Rightarrow |A||B||A^T| = 8$ .....(1)
$\Rightarrow a.b.a = 8 \Rightarrow a^2b = 8$
$\because |AB^{-1}| = 8 \Rightarrow |A||B^{-1}| = 8 \Rightarrow a.\frac{1}{b} = 8$ .....(2)
From (1) & (2)
$a = 4, b = \frac{1}{2}$
Then, $|BA^{-1}B^T| = |B||A^{-1}||B^T| = b.\frac{1}{a}.b = \frac{b^2}{a} = \frac{1}{16}$

Question123

If $\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ and

$\Delta_2 = \begin{vmatrix} x & \sin 2\theta & \cos 2\theta \\ -\sin 2\theta & -x & 1 \\ \cos 2\theta & 1 & x \end{vmatrix}$,

$x \neq 0$ then for all $\theta \in \left(0, \frac{\pi}{2}\right)$:
[April 10, 2019 (I)]

Options:

A. $\Delta_1 - \Delta_2 = -2x^3$

B. $\Delta_1 - \Delta_2 = x(\cos 2\theta - \cos 4\theta)$

C. $\Delta_1 \times \Delta_2 = -2(x^3 + x - 1)$

D. $\Delta_1 + \Delta_2 = -2x^3$

Answer: D

Solution:

Solution:

$$\Delta_1 = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

$= (x - x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$
$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \cos\theta\sin\theta + x\cos^2\theta$
$= -x^3 - x + x = -x^3$
Similarly, $\Delta_2 = -x^3$
Then ,$\Delta_1 + \Delta_2 = -2x^3$

Question124

The sum of the real roots of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$ is equal to

[April 2019]

Options:

A. 0

B. 6

C. -4

D. 1

Answer: A

Solution:

Solution:
Given equation

$$\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$$

On expansion of determinant along R , we get
$x[(-3x)(x+2) - 2x(x-3)] + 6[2(x+2) + 3(x-3)]$
$-1[9(2x) - (-3x)(-3)] = 0$
$\Rightarrow x[-3x^2 - 6x - 2x^2 + 6x] + 6[2x + 4 + 3x - 9] - 1[4x - 9x] = 0$

$\Rightarrow x(-5x^2) + 6(5x - 5) - 1(-5x) = 0$
$\Rightarrow -5x^3 + 30x - 30 + 5x = 0$
$\Rightarrow 5x^3 - 35x + 30 = 0 \Rightarrow x^3 - 7x + 6 = 0.$
Since all roots are real

$\therefore$ Sum of roots $= -\dfrac{\text{coefficient of } x^2}{\text{coefficient of } x^3} = 0$

Question125

A value of θ ∈ (0, π / 3), for which

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\cos 6\theta \\ \cos^2\theta & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0,$$

is
[April 12, 2019 (II)]

Options:

A. $\frac{\pi}{9}$

B. $\frac{\pi}{18}$

C. $\frac{7\pi}{24}$

D. $\frac{7\pi}{36}$

Answer: A

Solution:

Solution:
$C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\cos 6\theta \\ 2 & 1+\sin^2\theta & 4\cos 6\theta \\ 1 & \sin^2\theta & 1+4\cos 6\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & (1+4\cos 6\theta) \end{vmatrix} = 0$$

On expanding, we get $2 + 4\cos 6\theta = 0$
$\cos 6\theta = -\frac{1}{2} \because \theta \in \left(0, \frac{\pi}{3}\right) \Rightarrow 6\theta(0, 2\pi)$
Therefore, $6\theta = \frac{2\pi}{3}$ or $\frac{4\pi}{3} \Rightarrow \theta = \frac{\pi}{9}$ or $\frac{2\pi}{9}$

Question126

Let α and beta be the roots of the equation $x^2 + x + 1 = 0$. Then for y"´0

in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ **is equal to:**

[April 09,2019 (I)]

Options:

A. $y(y^2 - 1)$

B. $y(y^2 - 3)$

C. y^3

D. $y^3 - 1$

Answer: C

Solution:

Solution:
Let $\alpha = \omega$ and $\beta = \omega^2$ are roots of $x^2 + x + 1 = 0$

& Let $\Delta = \begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & y+\omega^2 & 1 \\ \omega^2 & 1 & y+\omega \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$

$$\Delta = \begin{vmatrix} y+1+\omega+\omega^2 & \omega & \omega^2 \\ y+1+\omega+\omega^2 & y+\omega^2 & 1 \\ 1+\omega+\omega^2+y & 1 & y+\omega \end{vmatrix}$$

$$\Delta = \begin{vmatrix} y & \omega & \omega^2 \\ y & y+\omega^2 & 1 \\ y & 1 & y+\omega \end{vmatrix} \quad (\because 1+\omega+\omega^2 = 0)$$

$$\Delta = y\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & y+\omega^2 & 1 \\ 1 & 1 & y+\omega \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\Delta = y\begin{vmatrix} y+\omega^2-\omega & 1-\omega^2 \\ 1-\omega & y+\omega-\omega^2 \end{vmatrix}$$

$\Rightarrow \Delta = y[\,y-(\omega-\omega^2)(\,y+(\omega-\omega^2)\,) - (1-\omega)(1-\omega^2)\,]$

$\Rightarrow \Delta = y[\,y^2-(\omega-\omega^2)^2 - 1 + \omega^2 + \omega - \omega^3\,]$

$\Rightarrow \Delta = y[\,y^2-\omega^2-\omega^4+2\omega^3 - 1 + \omega^2 + \omega^4 - \omega^3\,]\ (\because \omega^4 = \omega)$

$\Rightarrow \Delta = y(y^2) = y^3$

Question127

Let the numbers 2, b, c be in an A.P. and A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det(A)?[2, 16], then c lies in the interval :
[April 08, 2019 (II)]

Options:

A. [2,3)

B. $(2 + 2^{3/4}, 4)$

C. [4,6]

D. $[3, 2 + 2^{3/4}]$

Answer: C

Solution:

Solution:

Consider, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$

$C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$

$|A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & (b-2)(b+2) & (c-2)(c+2) \end{vmatrix}$

$= (b-2)(c-2)(c-b)$

$\therefore 2, b, c$ are in A.P.

$\therefore (b-2) = (c-b) = d$ and $c - 2 = 2d$

$\Rightarrow |A| = d .2d . d = 2d^3$

$\because |A| \in [2, 16] \Rightarrow 1 \le d^3 \le 8 \Rightarrow 1 \le d \le 2$

$4 \le 2d + 2 \le 6 \Rightarrow 4 \le c \le 6$

Question128

If B = $\begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix}$ is the inverse of a 3 × 3 matrix A, then the sum of all values of α for which det(A) + 1 = 0, is:
[April 12, 2019 (I)]

Options:

A. 0

B. -1

C. 1

D. 2

Answer: C

Solution:

Solution:

$\because B = A^{-1} = \Rightarrow |B| = \frac{1}{|A|}$

Now, $|B| = \begin{bmatrix} 5 & 2\alpha & 1 \\ 0 & 2 & 1 \\ \alpha & 3 & -1 \end{bmatrix} = 2\alpha^2 - 2\alpha - 25$

Given, $d\,et(A) + 1 = 0$

$\Rightarrow \frac{1}{2\alpha^2 - 2\alpha - 25} + 1 = 0$

$\Rightarrow \frac{2\alpha^2 - 2\alpha - 24}{2\alpha^2 - 2\alpha - 25} = 0$

$\Rightarrow \alpha = 4, -3 \Rightarrow$ Sum of values $= 1$

Question129

If $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \ldots\ldots. \begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$ then the inverse of $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is:

[April 09, 2019 (II)]

Options:

A. $\begin{bmatrix} 1 & 0 \\ 12 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & -12 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 \\ 13 & 1 \end{bmatrix}$

Answer: B

Solution:

Solution:

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\ldots\ldots\begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1+2+3+\ldots+(n-1) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix} \Rightarrow (n-1)\frac{n}{2} = 78 \Rightarrow n^2 - n - 15 = 0$$

$\Rightarrow n = 13$

Now, the matrix $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix}$

Then, the required inverse of $\begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

Question130

If the system of linear equations
$x + y + z = 5$
$x + 2y + 2z = 6$
$x + 3y + \lambda z = \mu$, $(\lambda, \mu \in R)$, has infinitely many solutions, then the value of $\lambda + \mu$ is:
[April 10, 2019 (I)]

Options:

A. 12

B. 9

C. 7

D. 10

Answer: D

Solution:

Solution:
Given system of linear equations: $x + y + z = 5$; $x + 2y + 2z = 6$ and $x + 3y + \lambda z = \mu$ have infinite solution.
$\therefore \Delta = 0, \Delta x = \Delta y = \Delta z = 0$

Now, $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & \lambda \end{vmatrix} = 0$

$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 2) + 1(3 - 2) = 0$
$\Rightarrow 2\lambda - 6 - \lambda + 2 + 1 = 0 \Rightarrow \lambda = 3$

$\Delta y = \begin{vmatrix} 1 & 5 & 1 \\ 1 & 6 & 2 \\ 1 & \mu & 3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 5 & 1 \\ 0 & 1 & 1 \\ 0 & \mu - 5 & 2 \end{vmatrix} = 0$

$\Rightarrow 1(2 - \mu + 5) = 0 \Rightarrow \mu = 7$
$\therefore \lambda + \mu = 10$

Question131

Let λ be a real number for which the system of linear equations:

$x+y+z=6$
$4x+\lambda y-\lambda z=\lambda-2$
$3x+2y-4z=-5$
has infinitely many solutions. Then λ is a root of the quadratic equation :
[April 10,2019 (II)]

Options:

A. $\lambda^2+3\lambda-4=0$

B. $\lambda^2-3\lambda-4=0$

C. $\lambda^2+\lambda-6=0$

D. $\lambda^2-\lambda-6=0$

Answer: D

Solution:

Solution:
$\because$ system of equations has infinitely many solutions.
$\therefore \Delta=\Delta_1=\Delta_2=\Delta_3=0$

Here, $\Delta=\begin{vmatrix}1 & 1 & 1\\ 4 & \lambda & -\lambda\\ 3 & 2 & -4\end{vmatrix}=0$

$C_1\to C_1-C_2$ & $C_2\to C_2-C_3$

$\Rightarrow \begin{vmatrix}0 & 0 & 1\\ 4-\lambda & 2\lambda & -\lambda\\ 1 & 6 & -4\end{vmatrix}=0 \Rightarrow \lambda=3$

Now, for $\lambda=3$, $\Delta_1=\begin{vmatrix}6 & 1 & 1\\ \lambda-2 & \lambda & -\lambda\\ -5 & 2 & -4\end{vmatrix}=0$

For $\lambda=3$, $\Delta_2=\begin{vmatrix}1 & 6 & 1\\ 4 & \lambda-2 & -\lambda\\ 3 & -5 & -4\end{vmatrix}=0$

For $\lambda=3$, $\Delta_3=\begin{vmatrix}1 & 1 & 6\\ 4 & \lambda & \lambda-2\\ 3 & 2 & -5\end{vmatrix}=0$

$\therefore$ for $\lambda=3$, system of equations has infinitely manysolutions.

Question132

If the system of equations $2x+3y-z=0$, $x+ky-2z=0$ and $2x-y+z=0$ has a non-trivial solution (x, y, z), then $\frac{x}{y}+\frac{y}{z}+\frac{z}{x}+k$ is equal to:
[April 09, 2019 (II)]

Options:

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $-\frac{1}{4}$

D. -4

Answer: B

Solution:

Solution:
Given system of equations has a non-trivial solution.

$$\Rightarrow \Delta = 0 \Rightarrow \begin{vmatrix} 2 & 3 & -1 \\ 1 & k & -2 \\ 2 & -1 & 1 \end{vmatrix} = 0 \Rightarrow k = \frac{9}{2}$$

$\therefore$ equations are $2x + 3y - z = 0$ .....(i)
$2x - y + z = 0$ ......(ii)
$2x + 9y - 4z = 0$ .....(iii)
By (i) $-$ (ii), $2y = z$
$\therefore z = -4x$ and $2x + y = 0$
$\therefore \frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k = \frac{-1}{2} + \frac{1}{2} - 4 + \frac{9}{2} = \frac{1}{2}$

Question133

The greatest value of $c \in R$ for which the system of linear equations $x - cy - cz = 0$; $cx - y + cz = 0$; $cx + cy - z = 0$has a non-trivial solution, is:
[April 08, 2019 (I)]

Options:

A. -1

B. $\frac{1}{2}$

C. 2

D. 0

Answer: B

Solution:

Solution:
If the system of equations has non-trivial solutions, then the determinant of coefficient matrix is zero

$$\begin{vmatrix} 1 & -c & -c \\ c & -1 & c \\ c & c & -1 \end{vmatrix} = 0$$

$\Rightarrow (1 - c^2) + c(-c - c^2) - c(c^2 + c) = 0$
$\Rightarrow (1 + c)(1 - c) - 2c^2(1 + c) = 0$

$\Rightarrow(1+c)(1-c-2c^2)=0$
$\Rightarrow(1+c)^2(1-2c)=0$
$\Rightarrow c=-1$ or $\frac{1}{2}$
Hence, the greatest value of c is $\frac{1}{2}$ for which the system of linear equations has non-trivial solution.

Question134

If the system of linear equations
$x-2y+kz=1$
$2x+y+z=2$
$3x-y-kz=3$
has a solution (x, y, z), z ≠ 0, then (x, y) lies on the straight line whose equation is :
[April 08, 2019 (II)]

Options:

A. $3x-4y-1=0$

B. $4x-3y-4=0$

C. $4x-3y-1=0$

D. $3x-4y-4=0$

Answer: B

Solution:

Solution:
Given system of linear equations,
$x-2y+kz=1,\ 2x+y+z=2,\ 3x-y-kz=3$

$$\Delta=\begin{vmatrix}1 & -2 & k\\ 2 & 1 & 1\\ 3 & -1 & -k\end{vmatrix}$$

$=1(-k+1)+2(-2k-3)+k(-2-3)$
$=-k+1-4k-6-5k=-10k-5=-5(2k+1)$

$$\Delta_1=\begin{vmatrix}1 & -2 & k\\ 2 & 1 & 1\\ 3 & -1 & -k\end{vmatrix}=-5(2k+1)$$

$$\Delta_2\begin{vmatrix}1 & 1 & k\\ 2 & 2 & 1\\ 3 & 3 & -k\end{vmatrix}=0,\ \Delta_3\begin{vmatrix}1 & -2 & 1\\ 2 & 1 & 2\\ 3 & -1 & 3\end{vmatrix}$$

$\because z\neq 0\Rightarrow\Delta=0$
$\Rightarrow-5(2k+1)=0\Rightarrow k=-\frac{1}{2}$
$\therefore$ System of equation has infinite many solutions.
Let $z=\lambda\neq 0$ then $x=\frac{10-3\lambda}{10}$ and $y=-\frac{2\lambda}{5}$
$\therefore(x, y)$ must lie on line $4x-3y-4=0$

Question135

If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to :
[2018]

Options:

A. (-4,3)

B. (-4,5)

C. (4,5)

D. (-4,-5)

Answer: B

Solution:

Solution:

Here, $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$

Put x = 0 ⇒ $\begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3 \Rightarrow A^3 = (-4)^3$

$\Rightarrow A = -4$

$\Rightarrow \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (Bx-4)(x+4)^2$

Now take x common from both the sides

$$\therefore \begin{vmatrix} 1-\frac{4}{x} & 2x & 2x \\ 2x & 1-\frac{4}{x} & 2x \\ 2x & 2x & 1-\frac{4}{x} \end{vmatrix} = \left(B-\frac{4}{x}\right)\left(1+\frac{4}{x}\right)^2$$

Now take $x \to \infty$, then $\frac{1}{x} \to 0$

$\Rightarrow \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B = 5$

∴ ordered pair (A, B) is (-4,5)

Question136

Let A be a matrix such that A . $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and |3A| = 108.

Then A^2 equals
[Online April 15, 2018]

Options:

A. $\begin{bmatrix} 4 & -32 \\ 0 & 36 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 0 \\ -32 & 36 \end{bmatrix}$

C. $\begin{bmatrix} 36 & 0 \\ -32 & 4 \end{bmatrix}$

D. $\begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

Answer: D

Solution:

Solution:

Since A . $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$

suppose the scalar matrix is $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$\therefore A . \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^{-1}$

$[\therefore AB = C \Rightarrow ABB^{-1} = CB^{-1} \Rightarrow A = CB^{-1}]$

$\Rightarrow A = \frac{1}{3} \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} 1 & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} k & -\frac{2}{3}k \\ 0 & \frac{k}{3} \end{bmatrix}$ ....(1)

$\because |\ 3A\ | = 108$

$\Rightarrow 108 = \begin{vmatrix} 3k & -2k \\ 0 & k \end{vmatrix}$

$\Rightarrow 3k^2 = 108 \Rightarrow k^2 = 36 \Rightarrow k = \pm 6$

For k = 6

$A = \begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix}$ ..... From (1)

$\Rightarrow A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

For $k = -6$

$\Rightarrow A = \begin{bmatrix} -6 & 4 \\ 0 & -2 \end{bmatrix}$......From (1)

$\Rightarrow A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

Question137

Suppose A is any 3×3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I_3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$ then $\alpha + \beta$ is equal to [Online April 15, 2018]

Options:

A. 8

B. 12

C. 13

D. 7

Answer: A

Solution:

Solution:

We have

$(A - 3I)(A - 5I) = O$

$\Rightarrow A^2 - 8A + 15I = O$

Multiplying both sides by A^{-1}, we get;

$A^{-1}A \cdot A - 8A^{-1}A + 15A^{-1}I = A^{-1}O$

$\Rightarrow A - 8I + 15A^{-1} = O$

$A + 15A^{-1} = 8I$

$\frac{A}{2} + \frac{15A^{-1}}{2} = 4I$

$\therefore \alpha + \beta = \frac{1}{2} + \frac{15}{2} = \frac{16}{2} = 8$

Question138

If the system of linear equations
$x + ky + 3z = 0$
$3x + ky - 2z = 0$
$2x + 4y - 3z = 0$
has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to :

[2018]

Options:

A. 10

B. – 30

C. 30

D. – 10

Answer: A

Solution:

Solution:
For non zero solution of the system of linear equations;

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$\Rightarrow k = 11$
Now equations become
$x + 11y + 3z = 0$ ......(1)
$3x + 11y - 2z = 0$ ......(2)
$2x + 4y - 3z = 0$ ......(3)
Adding equations (1) & (3) we get
$3x + 15y = 0$
$\Rightarrow x = -5y$
Now put $x = -5y$ in equation (1), we get
$-5y + 11y + 3z = 0$
$\Rightarrow z = -2y$
$\therefore \frac{xz}{y^2} = \frac{(-5y)(-2y)}{y^2} = 10$

Question139

The number of values of k for which the system of linear equations, $(k+2)x + 10y = k$, $kx + (k+3)y = k - 1$ has no solution, is [Online April 16, 2018]

Options:

A. Infinitely many

B. 3

C. 1

D. 2

Answer: C

Solution:

Solution:
Here, the equations are;
$(k+2)x + 10y = k$
& $kx + (k+3)y = k - 1$.
These equations can be written in the form of $Ax = B$ as

$$\begin{bmatrix} k+2 & 10 \\ k & k+3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ k-1 \end{bmatrix}$$

For the system to have no solution
$|A| = 0$
$\Rightarrow \begin{bmatrix} k+2 & 10 \\ k & k+3 \end{bmatrix} = 0 \Rightarrow (k+2)(k+3) - k \times 10 = 0$
$\Rightarrow k^2 - 5k + 6 = (k-2)(k-3) = 0$
$\therefore k = 2, 3$
For $k = 2$, equations become:
$4x + 10y = 2$
& $2x + 5y = 1$
& hence infinite number of solutions.
For $k = 3$, equations becomes;
$5x + 10y = 3$
$3x + 6y = 2$
& hence no solution.
$\therefore$ required number of values of k is 1

Question140

Let S be the set of all real values of k for which the system of linear equations
$x + y + z = 2$
$2x + y - z = 3$
$3x + 2y + kz = 4$
has a unique solution. Then S is
[Online April 15, 2018]

Options:

A. an empty set

B. equal to $R - \{0\}$

C. equal to $\{0\}$

D. equal to R

Answer: B

Solution:

Solution:
The system of inear equations is:
$x + y + z = 2$
$2x + y - z = 3$
$3x + 2y + kz = 4$
As, system has unique solution.
So, $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix} \neq 0$
$\Rightarrow k + 2 - (2k + 3) + 1 \neq 0$
$\Rightarrow k \neq 0$
Hence, $k \in R - \{0\} \equiv S$

Question141

If the system of linear equations
$x + ay + z = 3$
$x + 2y + 2z = 6$
$x + 5y + 3z = b$
has no solution, then
[Online April 15, 2018]

Options:

A. $a = 1, b \neq 9$

B. $a \neq -1, b = 9$

C. $a = -1, b = 9$

D. $a = -1, b \neq 9$

Answer: D

Solution:

Solution:
As the system of equations has no solution then Δ should be zero and at least one of Δ_1, Δ_2 and Δ_3 should not be zero.

$$\therefore \Delta = \begin{vmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{vmatrix} = 0$$

$\Rightarrow -a - 1 = 0 \Rightarrow a = -1$

$$\Delta_2 = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 6 & 2 \\ 1 & b & 3 \end{vmatrix} \neq 0$$

$\Rightarrow b \neq 9$

Question142

If

$$S = \left\{ x \in [0, 2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0 \right\},$$

then $\sum_{x \in S} \tan\left(\frac{\pi}{3} + x\right)$ is equal to
[Online April 8, 2017]

Options:

A. $4 + 2\sqrt{3}$

B. $-2 + \sqrt{3}$

C. $-2 - \sqrt{3}$

D. $-4 - 2\sqrt{3}$

Answer: C

Solution:

Solution:
Since the given determinant is equal to zero.
$\Rightarrow 0(0 - \cos x \sin x) - \cos x(0 - \cos^2 x) - \sin x(\sin^2 x - 0) = 0$
$\Rightarrow \cos^3 x - \sin^3 x = 0$
$\Rightarrow \tan^3 = 1 \Rightarrow \tan x = 1$
$\therefore \sum_{x \in S} \tan\left(\frac{\pi}{3} + x\right) = \sum_{x \in S} \frac{\tan \pi / 3 + \tan x}{1 - \tan \pi / 3 . \tan x}$
$\sum_{x \in S} \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$
$\sum_{x \in S} \frac{\sqrt{3} + 1}{1 - \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}} \Rightarrow \sum_{x \in S} \frac{1 + 3 + 2\sqrt{3}}{-2}$
$= -2 - \sqrt{3}$

Question143

Let k be an integer such that triangle with vertices (k, −3k), (5, k) and (−k, 2) has area 28sq. units. Then the orthocentre of this triangle is at the point:
[2017]

Options:

A. $\left(2, \frac{1}{2}\right)$

B. $\left(2, -\frac{1}{2}\right)$

C. $\left(1, \frac{3}{4}\right)$

D. $\left(1, -\frac{3}{4}\right)$

Answer: A

Solution:

Solution:
Let A (k, –3k), B(5, k) and C(–k + 2),
we have
$$\frac{1}{2}\left|\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix}\right| = 28$$
$\Rightarrow 5k^2 + 13k - 46 = 0$
or $5k^2 + 13k + 66 = 0$
Now, $5k^2 + 13k - 46 = 0$
$\Rightarrow k = \frac{-13 \pm \sqrt{1089}}{10} \therefore k = \frac{-23}{5}; k = 2$
since k is an integer, $\therefore k = 2$
Also $5k^2 + 13k + 66 = 0$
$\Rightarrow k = \frac{-13 \pm \sqrt{-1151}}{10}$

So no real solution exist
A(2, −6), B(5, 2) and C(−2, 2)
For orthocentre H (α, β)
BH ⊥ AC
$\therefore \left(\frac{\beta - 2}{\alpha - 5}\right)\left(\frac{8}{-4}\right) = -1$
$\Rightarrow \alpha - 2\beta = 1$ ....(1)
Also CH ⊥ AB
$\therefore \left(\frac{\beta - 2}{\alpha + 2}\right)\left(\frac{8}{3}\right) = -1$
$\Rightarrow 3\alpha + 8\beta = 1$ ......(2)
Solving (1) and (2), we get
$\alpha = 2, \beta = \frac{1}{2}$
orthocentre is $\left(2, \frac{1}{2}\right)$

Question144

Let ω be a complex number such that 2ω + 1 = z where z = $\sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix}$$ **= 3k, then k is equal to**

[2017]

Options:

A. 1

B. $-z$

C. z

D. -1

Answer: B

Solution:

Solution:
Given $2\omega + 1 = z$;
and $z = \sqrt{3}i \Rightarrow \omega = \frac{\sqrt{3}i - 1}{2}$
$\Rightarrow \omega$ is complex cube root of unity Applying $R_1 \to R_1 + R_2 + R_3$

$$= \begin{vmatrix} 3 & 0 & 0 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix}$$

$= 3(-1 - \omega - \omega) = -3(1 + 2\omega) = -3z$
$\Rightarrow k = -z$

Question145

If A = $\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj($3A^2 + 12A$) is equal to:

[2017]

Options:

A. $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

B. $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

C. $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

D. $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$

Answer: C

Solution:

Solution:

We have A = $\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix} \Rightarrow 3A^2 = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix}$

Also $12A = \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$

$\therefore 3A^2 + 12A \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$

$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

$\text{adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Question146

Let A be any 3 × 3 invertible matrix. Then which one of the following is not always true?
[Online April 8, 2017]

Options:

A. $\text{adj}(A) = |A|.A^{-1}$

B. $\text{adj}(\text{adj}(A)) = |A|.A$

C. $\text{adj}(\text{adj}(A)) = |A|^2.(\text{adj}(A))^{-1}$

D. $\operatorname{adj}(\operatorname{adj}(A)) = |A| . (\operatorname{adj}(A))^{-1}$

Answer: B

Question147

If S is the set of distinct values of ' b ' for which the following system of linear equations
$x + y + z = 1$
$x + ay + z = 1$
$ax + by + z = 0$
has no solution, then S is :
[2017]

Options:

A. a singleton

B. an empty set

C. an infinite set

D. a finite set containing two or more elements

Answer: A

Solution:

Solution:

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$$

$\Rightarrow 1[a - b] - 1[1 - a] + 1[b - a^2] = 0$
$\Rightarrow (a - 1)^2 = 0$
$\Rightarrow a = 1$
For $a = 1$, First two equations are identical
i.e., $x + y + z = 1$
To have no solution with $x + by + z = 0$
$b = 1$
So $b = \{1\} \Rightarrow$ It is singleton set.

Question148

The number of real values of λ for which the system of linear equations
$2x + 4y - \lambda z = 0$
$4x + \lambda y + 2z = 0$
$\lambda x + 2y + 2z = 0$

has infinitely many solutions, is :
[Online April 8, 2017]

Options:

A. 0

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:
Since the given system of linear equations has infinitely many solutions.

$$\therefore \begin{vmatrix} 2 & 4 & -\lambda \\ 4 & \lambda & 2 \\ \lambda & 2 & 2 \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 + 4\lambda - 40 = 0$
λ has only 1 real root.

Question149

If A = $\begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is:
[Online April 10, 2016]

Options:

A. -175

B. 2014

C. 2016

D. -25

Answer: D

Solution:

Solution:

$$A = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} -4 & -1 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 3 \\ -9 & -2 \end{bmatrix} \text{ and } |A| = 1$$

Now, $A^{2016} - 2A^{2015} - A^{2014} = A^{2014}(A^2 - 2A - I)$
$\Rightarrow A^{2016} - 2A^{2015} - A^{2014}| = A^{2014} | A^2 - 2A - I |$

$= |A|^{2014}\begin{vmatrix} 20 & 5 \\ -15 & -5 \end{vmatrix} = -25$

Question150

The number of distinct real roots of the equation, $\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$

in the interval $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$ is

[Online April 9, 2016]

Options:

A. 1

B. 4

C. 2

D. 3

Answer: C

Solution:

Solution:

$$\begin{vmatrix} \cos x & \sin x & \sin x \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$R_2 \to -R_2 - R_3$

$$\begin{vmatrix} \cos x - \sin x & \sin x - \cos x & 0 \\ 0 & \cos x - \sin x & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

$C_2 \to C_2 + C_3$

$$\begin{vmatrix} \cos x - \sin x & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$$

Expanding using second row

$2\sin x(\sin x - \cos x)^2 = 0$

$\sin x = 0$ or $\sin x = \cos x$

$x = 0$ or $x = \frac{\pi}{4}$

Question151

If A = $\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A\,\text{adj}\,A = AA^T$, then 5a + b is equal to:

[2016]

Options:

A. 4

B. 13

C. –1

D. 5

Answer: D

Solution:

Solution:
Given that $A(\text{ad}\,jA) = AA^T$
Pre-multiply by A^{-1} both side, we get
$\Rightarrow A^{-1}A(\text{ad}\,jA) = A^{-1}AA^T$
$\text{ad}\,jA = A^T$
$\Rightarrow \begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$
$\Rightarrow a = \frac{2}{5}$ and $b = 3$
$\Rightarrow 5a + b = 5$

Question152

Let A be a 3 × 3 matrix such that $A^2 - 5A + 7I = 0$.
Statement-I: $A^{-1} = \frac{1}{7}(5I - A)$
Statement-I: the polynomial $A^3 - 2A^2 - 3A + \alpha$ can be reduced to $5(A - 4I)$
Then :
[Online April 10, 2016]

Options:

A. Both the statements are true.

B. Both the statements are false.

C. Statement–I is true, but Statement-II is false.

D. Statement I is false, but Statement-II is true.

Answer: A

Solution:

Solution:
$A^2 - 5A = -7I$

$AAA^{-1} - 5AA^{-1} = -7I\,A^{-1}$
$AI - 5I = -7A^{-1}$
$A - 5I = -7A^{-1}$
$A^{-1} = \frac{1}{7}(5I - A)$
$A^3 - 2A^2 - 3A + I = A(5A - 7I) - 2A^2 - 3A + I$
$= 5A^2 - 7A - 2A^2 - 3A + I = 3A^2 - 10A + I$
$= 3(5A - 7I) - 10A + I = 5A - 20I = 5(A - 4I)$

Question153

The system of linear equations
$x + \lambda y - z = 0$
$\lambda x - y - z = 0$
$x + y - \lambda z = 0$
has a non-trivial solution for:
[2016]

Options:

A. exactly two values of λ.

B. exactly three values of λ.

C. infinitely many values of λ.

D. exactly one value of λ.

Answer: B

Solution:

Solution:
For non-trivial solution,

$$\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$-\lambda(\lambda + 1)(\lambda - 1) = 0 \Rightarrow \lambda = 0, +1, -1$

Question154

if $\begin{vmatrix} x^2 + x & x + 1 & x - 2 \\ 2x^2 + 3x - 1 & 3x & 3x - 3 \\ x^2 + 2x + 3 & 2x - 1 & 2x - 1 \end{vmatrix} = ax - 12$, then 'a' is equal to:
[Online April 11, 2015]

Options:

A. 24

B. -12

C. -24

D. 12

Answer: A

Solution:

Solution:

Let $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = ax - 12$

Put $x = -1$, we get

$\begin{vmatrix} 0 & 0 & -3 \\ -2 & -3 & 0 \\ 2 & -3 & -3 \end{vmatrix} = -a - 12$

$\Rightarrow -3(6+6) = -a - 12 \Rightarrow -36 + 12 = a$

$\Rightarrow a = 24$

Question155

The least value of the product xyz for which the determinant $\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix}$ **is non-negative, is:**
[Online April 10, 2015]

Options:

A. $-2\sqrt{2}$

B. -1

C. $-16\sqrt{2}$

D. -8

Answer: D

Solution:

Solution:

$\begin{vmatrix} x & 1 & 1 \\ 1 & y & 1 \\ 1 & 1 & z \end{vmatrix} \geq 0$

$xyz - x - y - z + 2 \geq 0$

$xyz + 2 \geq x + y + z \geq 3(xyz)^{1/3}$

$xyz + 2 - 3(xyz)^{1/3} \geq 0$

$ut(xyz) = t^3$

$t^3 - 3t + 2 \geq 0$

$(t+2)(t-1)^2 \geq 0$

$[t = -2]t^3 = -8$

Question156

If A is a 3×3 matrix such that $|5 \,.\, \operatorname{adj} A| = 5$, then $|A|$ is equal to :
[Online April 11, 2015]

Options:

A. $\pm\frac{1}{5}$

B. $\pm\frac{1}{25}$

C. ±1

D. ±5

Answer: A

Solution:

Solution:
$|5 \,.\, \operatorname{ad} jA| = 5 \Rightarrow 5^3 . \mid A \mid^{3-1} = 5$
$\Rightarrow 125 \mid A \mid^2 = 5 \Rightarrow \mid A \Big| = \pm\frac{1}{5}$

Question157

The set of all values of λ for which the system of linear equations:
$\mathbf{2x_1 - 2x_2 + x_3 = \lambda x_1}$
$\mathbf{2x_1 - 3x_2 + 2x_3 = \lambda x_2}$
$\mathbf{-x_1 + 2x_2 = \lambda x_3}$
has a non-trivial solution,
[2015]

Options:

A. contains two elements.

B. contains more than two elements

C. is an empty set.

D. is a singleton

Answer: A

Solution:

$$\left.\begin{array}{r} 2x_1 - 2x_2 + x_3 = \lambda x_1 \\ 2x_1 - 3x_2 + 2x_3 = \lambda x_2 \\ -x_1 + 2x_2 = \lambda x_3 \end{array}\right\}$$

$\Rightarrow (2-\lambda)x_1 - 2x_2 + x_3 = 0$
$2x_1 - (3+\lambda)x_2 + 2x_3 = 0$
$-x_1 + 2x_2 - \lambda x_3 = 0$
For non-trivial solution,
$\Delta = 0$

i.e. $\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$

$\Rightarrow (2-\lambda)[\lambda(3+\lambda) - 4] + 2[-2\lambda + 2] + 1[4 - (3+\lambda)] = 0$
$\Rightarrow \lambda^3 + \lambda^2 - 5\lambda + 3 = 0$
$\Rightarrow \lambda = 1, 1, 3$
Hence λ has 2 values.

Question158

If $f(\theta) = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$ and A and B are respectively the maximum and the minimum values of $f(\theta)$, then (A, B) is equal to:
[Online April 12, 2014]

Options:

A. (3,-1)

B. $(4, 2-\sqrt{2})$

C. $(2+\sqrt{2}, 2-\sqrt{2})$

D. $(2+\sqrt{2}, -1)$

Answer: C

Solution:

Solution:

Let $f(\theta) = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\sin\theta & 1 & -\cos\theta \\ -1 & \sin\theta & 1 \end{vmatrix}$

$= (1 + \sin\theta\cos\theta) - \cos\theta(-\sin\theta - \cos\theta) + 1(-\sin^2\theta + 1)$
$= 1 + \sin\theta\cos\theta + \sin\theta\cos\theta + \cos^2\theta - \sin^2\theta + 1$
$= 2 + 2\sin\theta\cos\theta + \cos 2\theta$
$= 2 + \sin 2\theta + \cos 2\theta$
Now, maximum value of (1)
is $2 + \sqrt{1^2 + 1^2} = 2 + \sqrt{2}$
and minimum value of (1) is
$2 - \sqrt{1^2 + 1^2} = 2 - \sqrt{2}$

Question159

If B is a 3×3 matrix such that $B^2 = 0$, then det. $[(I + B)^{50} - 50B]$ is equal to:
[Online April 9, 2014]

Options:

A. 1

B. 2

C. 3

D. 50

Answer: A

Solution:

Solution:
$\det[(I + B)^{50} - 50B]$
$= \det[{}^{50}C_0 I + {}^{50}C_1 B + {}^{50}C_2 B^2 + {}^{50}C_3 B^3 + \ldots\ldots + {}^{50}C_{50} B^{50} B^{50} - 50B]$
{ All terms having B^n, $2 \le n \le 50$
will be zero because given that $B^2 = 0$ }
$= \det[I + 50B - 50B] = \det[I] = 1$

Question160

If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$ $= K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to:
[2014]

Options:

A. 1

B. -1

C. $\alpha\beta$

D. $\frac{1}{\alpha\beta}$

Answer: A

Solution:

Consider $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$

$= \begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$ $[\because |A| = |A^1|]$

$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2 = [(1-\alpha)(1-\beta)(\alpha-\beta)]^2$

So, $K = 1$

Question161

If

$$\Delta_r = \begin{vmatrix} r & 2r-1 & 3r-2 \\ \frac{n}{2} & n-1 & a \\ \frac{1}{2}n(n-1) & (n-1)^2 & \frac{1}{2}(n-1)(3n-4) \end{vmatrix}$$

then the value of $\sum_{r=1}^{n-1} \Delta_r$
[Online April 19, 2014]

Options:

A. depends only on a

B. depends only on n

C. depends both on a and n

D. is independent of both a and n

Answer: D

Solution:

Solution:

$\sum_{r=1}^{n-1} r = 1+2+3+\ldots..+(n-1) = \frac{n(n-1)}{2}$ $\sum_{r=1}^{n-1}(2r-1) = 1+3+5+\ldots..+[2(n-1)-2] = (n-1)^2$

$\sum_{r=1}^{n-1}(3r-2) = 1+4+7+\ldots.+(3n-3-2) = \frac{(n-1)(3n-4)}{2}$

$$\therefore \sum_{r=1}^{n-1} \Delta_r = \begin{vmatrix} \sum r & \sum(2r-1) & \sum(3r-2) \\ \frac{n}{2} & n-1 & a \\ n(n-1) & 2 & \frac{(n-1)^2}{(n-1)(3n-4)}2 \end{vmatrix}$$

$\sum_{r=1}^{n-1} \Delta_r$ consists of (n – 1) determinants in L.H.S. and in R.H.S every constituent of first row consists of (n – 1) elements and hence it can be splitted into sum of (n – 1) determinants.

$$\therefore \sum_{r=1}^{n-1} \Delta_r = \begin{vmatrix} \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \\ \frac{n}{2} & n-1 & a \\ \frac{n(n-1)}{2} & (n-1)^2 & \frac{(n-1)(3n-4)}{2} \end{vmatrix} = 0$$

($\because R_1$ and R_3 are identical)

Hence, value of $\sum_{r=1}^{n-1} \Delta_r$ is independent of both and a'and 'n'n′ a'and 'n'$

Question162

If $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$ **= kλ** $\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ **,λ ≠ 0then k is equal to**

[Online April 12, 2014]

Options:

A. $4\lambda abc$

B. $-4\lambda abc$

C. $4\lambda^2$

D. $-4\lambda^2$

Answer: C

Solution:

Solution:

Let $\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$

Apply $R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2-(a-\lambda)^2 & (b+\lambda)^2-(b-\lambda)^2 & (c+\lambda)^2-(c-\lambda)^2 \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a\lambda & 4b\lambda & 4c\lambda \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix} \quad (\because (x+y)^2-(x-y)^2 = 4xy)$$

Taking out 4 common from R_2

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a\lambda & b\lambda & c\lambda \\ a^2+\lambda^2-2a\lambda & b^2+\lambda^2-2b\lambda & c^2+\lambda^2-2c\lambda \end{vmatrix}$$

Apply $R_3 \to [R_3-(R_1-2R_2)]$

$$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a\lambda & b\lambda & c\lambda \\ \lambda^2 & \lambda^2 & \lambda^2 \end{vmatrix}$$

Taking out λ common from R_2 and λ^2 from R_3

$$= 4\lambda(\lambda^2)\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} = k\lambda\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow k = 4\lambda^2$

Question163

If A is an 3×3 non-singular matrix such that $AA' = A'A$ and $B = A^{-1}A'$, then BB' equals: [2014]

Options:

A. B^{-1}

B. $(B^{-1})'$

C. $I + B$

D. I

Answer: D

Solution:

Solution:

$BB' = B(A^{-1}A')' = B(A')'(A^{-1})'$

$= BA(A^{-1})' = (A^{-1}A')(A(A^{-1})')$

$= A^{-1}A \,.\, A' \,.\, (A^{-1})' \{$ as $AA' = A'A \}$

$= I\,(A^{-1}A)' = I \,.\, I = I^2 = I$

Question164

Let A be a 3 × 3 matrix such that

$$A\begin{bmatrix}1&2&3\\0&2&3\\0&1&1\end{bmatrix}=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$$ **Then A^{-1} is:**

[Online April 11, 2014]

Options:

A. $\begin{bmatrix}3&1&2\\3&0&2\\1&0&1\end{bmatrix}$

B. $\begin{bmatrix}3&2&1\\3&2&0\\1&1&0\end{bmatrix}$

C. $\begin{bmatrix}0&1&3\\0&2&3\\1&1&1\end{bmatrix}$

D. $\begin{bmatrix}1&2&3\\0&1&1\\0&2&3\end{bmatrix}$

Answer: A

Solution:

Solution:

Given $A\begin{bmatrix}1&2&3\\0&2&3\\0&1&1\end{bmatrix}=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$

Applying $C_1 \leftrightarrow C_3$

$$A\begin{bmatrix}3&2&1\\3&2&0\\1&1&0\end{bmatrix}=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}$$

Again Applying $\rightarrow C_2 \leftrightarrow C_3$

$$A\begin{bmatrix}3&1&2\\3&0&2\\1&0&1\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

pre-multiplying both sides by A^{-1}

$$A^{-1}A\begin{bmatrix}3&1&2\\3&0&2\\1&0&1\end{bmatrix}=A^{-1}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$I \begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix} = A^{-1}I = A^{-1}$$

($\because A^{-1}A = I$ and I= Identity matrix)

Hence, $A^{-1} = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

Question165

If a, b, c are non-zero real numbers and if the system of equations $(a-1)x = y + z$ $(b-1)y = z + x$ $(c-1)z = x + y$ has a non-trivial solution, then ab + bc + ca equals: [Online April 9, 2014]

Options:

A. $a + b + c$

B. abc

C. 1

D. -1

Answer: B

Solution:

Solution:
Given system of equations can be written as
$(a-1)x - y - z = 0$
$-x + (b-1)y - z = 0$
$-x - y + (c-1)z = 0$
For non-trivial solution, we have

$$\begin{vmatrix} a-1 & -1 & -1 \\ -1 & b-1 & -1 \\ -1 & -1 & c-1 \end{vmatrix} = 0$$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} a-1 & -1 & -1 \\ 0 & b & -c \\ -1 & -1 & c-1 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_3$

$$\begin{vmatrix} a-1 & 0 & -1 \\ 0 & b+c & -c \\ -1 & -c & c-1 \end{vmatrix} = 0$$

Apply $R_3 \to R_3 - R_1$

$$\begin{vmatrix} a-1 & 0 & -1 \\ 0 & b+c & -c \\ -a & -c & c \end{vmatrix} = 0$$

$\Rightarrow (a-1)[bc+c^2-c^2]-1[a(b+c)]=0$
$\Rightarrow (a-1)[bc]-ab-ac=0$
$\Rightarrow abc-bc-ab-ac=0$
$\Rightarrow ab+bc+ca=abc$

Question166

Let

$$S=\left\{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{pmatrix} : a_{ij}\in\{0,1,2\},\ a_{11}=a_{22}\right\}$$

Then the number of non-singular matrices in the set S is : [Online April 25, 2013]

Options:

A. 27

B. 24

C. 10

D. 20

Answer: D

Solution:

Solution:

The matrices in the form $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix} : a_{ij}\in\{0,1,2\},\ a_{11}=a_{22}$ are $\begin{bmatrix} 0 & 0/1/2 \\ 0/1/2 & 0\end{bmatrix}$, $\begin{bmatrix} 1 & 0/1/2 \\ 0/1/2 & 1\end{bmatrix}$, $\begin{bmatrix} 2 & 0/1/2 \\ 0/1/2 & 2\end{bmatrix}$
At any place, $0/1/2$ means 0,1 or 2 will be the element at that place.
Hence there are total $27 = 3\times 3+3\times 3+3\times 3$) matrices of the above form. Out of which the matrices which are singular are $\begin{bmatrix} 0 & 0/1/2 \\ 0 & 0\end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 1/2 & 0\end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ 1 & 1\end{bmatrix}$, $\begin{bmatrix} 2 & 2 \\ 2 & 2\end{bmatrix}$
Hence there are total $7(=3+2+1+1)$ singular matrices. Therefore number of all non-singular matrices in the given form $=27-7=20$

Question167

Let A, other than I or $-I$, be a 2×2 real matrix such that $A^2=I$, I being the unit matrix. Let Tr(A) be the sum of diagonal elements of A. Statement-1: Tr(A) = 0 Statement-2: det(A) = -1 [Online April 23, 2013]

Options:

A. Statement-1 is true; Statement-2 is false.

B. Statement-1 is true; Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.

C. Statement-1 is true; Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

D. Statement-1 is false; Statement-2 is true.

Answer: B

Solution:

Solution:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$b(a+d)=0$, $b=0$ or $a=-d$ .....(1)
$c(a+d)=0$, $c=0$ or $a=-d$ ....(2)
$a^2+bc=1$, $bc+d^2=1$ ....(3)
'a' and 'd' are diagonal elements $a+d=0$
statement-1 is correct.
Now, $\det(A)=ad-bc$
(3) $a^2+bc=1$ and $d^2+bc=1$
Now, from So, $a^2-d^2=0$
Adding $a^2+d^2+2bc=2$
$\Rightarrow (a+d)^2-2ad+2bc=2$
or $0-2(ad-bc)=2$
So, $ad-bc=1 \Rightarrow \det(A)=-1$
So, statement -2 is also true.
But statement -2 is not the correct explanation of statement- 1

Question168

If a, b, c are sides of a scalene triangle, then the value of $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ **is:**

[Online April 9, 2013]

Options:

A. non - negative

B. negative

C. positive

D. non-positive

Answer: B

Solution:

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix}$$

$= (a+b+c)[ab+bc+ca-a^2-b^2-c^2]$
$= -(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$
Since a, b, c are sides of a scalene triangle, therefore at least two of the a, b, c will be unequal.
$\therefore (a-b)^2+(b-c)^2+(c-a)^2 > 0$
Also $a+b+c > 0$
$\therefore -(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2] < 0$

Question169

If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to :
[2013]

Options:

A. 4

B. 11

C. 5

D. 0

Answer: B

Solution:

Solution:
$|P| = 1(12-12) - \alpha(4-6) + 3(4-6) = 2\alpha - 6$
Now, $\text{adj } A = P \Rightarrow |\text{adj} A| = |P|$
$\Rightarrow |A|^2 = |P|$
$\Rightarrow |P| = 16$
$\Rightarrow 2\alpha - 6 = 16$
$\Rightarrow \alpha = 11$

Question170

The number of values of k, for which the system of equations:$(k+1)x + 8y = 4k$
$kx + (k+3)y = 3k - 1$

has no solution, is
[2013]

Options:

A. infinite

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:
Since, system of equations have no solution
$\therefore \frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$ ($\because$ System has no solution)
$\Rightarrow k^2 + 4k + 3 = 8k \Rightarrow k^2 - 4k + 3 = 0$
$\Rightarrow k = 1, 3$
If $k = 1$ then $\frac{8}{1+3} \neq \frac{4.1}{2}$ which is false
and if $k = 3$ then $\frac{8}{6} \neq \frac{4.3}{9-1}$ which is true, therefore $k = 3$
Hence for only one value of k. System has no solution.

Question171

Consider the system of equations:
$x + ay = 0$, $y + az = 0$ and $z + ax = 0$. Then the set of all real values of 'a' for which the system has a unique solution is:
[Online April 25, 2013]

Options:

A. $R - \{1\}$

B. $R - \{-1\}$

C. $\{1, -1\}$

D. $\{1, 0, -1\}$

Answer: B

Solution:

Solution:
Given system of equations is homogeneous which is
$x + ay = 0$
$y + az = 0$
$z + ax = 0$
It can be written in matrix form as

$$A = \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{pmatrix}$$

Now, $|A| = [1 - a(-a^2)] = 1 + a^3 \neq 0$

So, system has only trivial solution.

Now, $|A| = 0$ only when $a = -1$

So, system of equations has infinitely many solutions which is not possible because it is given that system has a unique solution.

Hence set of all real values of ^{c}a ' is $R - \{-1\}$

Question172

Statement-1: The system of linear equations

$x + (\sin\alpha)y + (\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

$x - (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution for only one value of α lying in the interval $\left(0, \frac{\pi}{2}\right)$.

Statement- 2 : The equation in α

$\begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha \\ \sin\alpha & \cos\alpha & \sin\alpha \\ \cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$ has only one solution lying in the interval $\left(0, \frac{\pi}{2}\right)$.

[Online April 23, 2013]

Options:

A. Statement-1 is true, Statement-2 is true, Statement-2 is not correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

C. Statement-1 is true, Statement-2 is false.

D. Statememt-1 is false, Statement-2 is tru

Answer: C

Solution:

Solution:

$$\Delta_1 = \begin{vmatrix} 1 & \sin\alpha & \cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & -\sin\alpha & \cos\alpha \end{vmatrix} = 0$$

$$= \begin{vmatrix} 0 & \sin\alpha - \cos\alpha & \cos\alpha - \sin\alpha \\ 0 & \cos\alpha + \sin\alpha & \sin\alpha - \cos\alpha \\ 1 & -\sin\alpha & \cos\alpha \end{vmatrix} = (\sin\alpha - \cos\alpha)^2 - (\cos^2\alpha - \sin^2\alpha)$$

$$= \sin^2\alpha + \cos^2\alpha - 2\sin\alpha \, . \cos\alpha - \cos^2\alpha + \sin^2\alpha$$

$= 2\sin^2\alpha - 2\sin\alpha \cdot \cos\alpha$
$= 2\sin\alpha(\sin\alpha - \cos\alpha)$
Now, $\sin\alpha - \cos\alpha = 0$ for only
$\alpha = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$
$\therefore \Delta_1 = 2(\sin\alpha) \times 0 = 0$,
since value of $\sin\alpha$ is finite for $\alpha \in \left(0, \frac{\pi}{2}\right)$
Hence non-trivivial solution for only one value of $\alpha \in \left(0, \frac{\pi}{2}\right)$

$$\begin{vmatrix} \cos\alpha & \sin\alpha & \cos\alpha \\ \sin\alpha & \cos\alpha & \sin\alpha \\ \cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & \sin\alpha & \cos\alpha \\ 0 & \cos\alpha & \sin\alpha \\ 2\cos\alpha & -\sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$\Rightarrow 2\cos\alpha(\sin^2\alpha - \cos^2\alpha) = 0$
$\therefore \cos\alpha = 0$ or $\sin^2\alpha - \cos^2\alpha = 0$
But $\cos\alpha = 0$ not possible for any value of $\alpha \in \left(0, \frac{\pi}{2}\right)$
$\therefore \sin^2\alpha - \cos^2\alpha = 0 \Rightarrow \sin\alpha = -\cos\alpha$, which is also not possible for any value of $\alpha \in \left(0, \frac{\pi}{2}\right)$
Hence, there is no solution.

Question173

If the system of linear equations :
$x_1 + 2x_2 + 3x_3 = 6$
$x_1 + 3x_2 + 5x_3 = 9$
$2x_1 + 5x_2 + ax_3 = b$
is consistent and has infinite number of solutions, then :
[Online April 22, 2013]

Options:

A. $a = 8$, b can be any real number

B. $b = 15$, a can be anyreal number

C. $a \in R - \{8\}$ and $b \in R - \{15\}$

D. $a = 8, b = 15$

Answer: D

Solution:

Solution:
Given system of equations can be written in matrix form as AX = B where

$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & a \end{pmatrix}$ and $B = \begin{pmatrix} 6 \\ 9 \\ b \end{pmatrix}$

Since, system is consistent and has infinitely many solutions
$\therefore (\text{adj. } A)B = 0$

$$\Rightarrow \begin{pmatrix} 3a-25 & 15-2a & 1 \\ 10-a & a-6 & -2 \\ -1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 6 \\ 9 \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow -6-9+b=0 \Rightarrow b=15$
and $6(10-a)+9(a-6)-2(b)=0$
$\Rightarrow 60-6a+9a-54-30=0$
$\Rightarrow 3a=24 \Rightarrow a=8$
Hence, $a=8$, $b=15$.

Question174

If a, b, c, are non zero complex numbers satisfying $a^2+b^2+c^2=0$ and

$$\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = ka^2b^2c^2$$

, then k is equal to

[Online May 19, 2012]

Options:

A. 1

B. 3

C. 4

D. 2

Answer: C

Solution:

Solution:

Let $\Delta = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix}$

Multiply C_1 by a, C_2 by b and C_3 by c and hence divide by abc.

$$= \frac{1}{abc} \begin{vmatrix} a(b^2+c^2) & ab^2 & ac^2 \\ a^2b & b(c^2+a^2) & bc^2 \\ a^2c & b^2c & c(a^2+b^2) \end{vmatrix}$$

Take out a, b, c common from R_1, R_2 and R_3 respectively.

$$\therefore \Delta = \frac{abc}{abc} \begin{vmatrix} b^2+c^2 & b^2 & c^2 \\ a^2 & c^2+a^2 & c^2 \\ a^2 & b^2 & a^2+b^2 \end{vmatrix}$$

Apply $C_1 \to C_1 - C_2 - C_3$

$$\Delta = \begin{vmatrix} 0 & b^2 & c^2 \\ -2c^2 & c^2+a^2 & c^2 \\ -2b^2 & b^2 & a^2+b^2 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 0 & b^2 & c^2 \\ c^2 & c^2 + a^2 & c^2 \\ b^2 & b^2 & a^2 + b^2 \end{vmatrix}$$

Apply $C_2 - C_1$ and $C_3 - C_1$

$$= -2 \begin{vmatrix} 0 & b^2 & c^2 \\ c^2 & a^2 & 0 \\ b^2 & 0 & a^2 \end{vmatrix} = -2[-b^2(c^2a^2) + c^2(-a^2b^2)]$$

$= 2a^2b^2c^2 + 2a^2b^2c^2 = 4a^2b^2c^2$

But $\Delta = ka^2b^2c^2 \therefore k = 4$

Question175

If $\begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix} = \alpha(a+b)(b+c)(c+a) \neq 0$then α is equal to

[Online May 12, 2012]

Options:

A. a + b + c

B. abc

C. 4

D. 1

Answer: C

Solution:

Solution:

Let $\Delta = \begin{vmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & b+c & -2c \end{vmatrix}$

Applying $C_1 + C_3$ and $C_2 + C_3$

$$\Delta = \begin{vmatrix} -a+c & 2a+b+c & a+c \\ 2b+a+c & -b+c & b+c \\ a-c & b-c & -2c \end{vmatrix}$$

Now, applying $R_1 + R_3$ and $R_2 + R_3$

$$\Delta = \begin{vmatrix} 0 & 2(a+b) & a-c \\ 2(a+b) & 0 & b-c \\ a-c & b-c & -2c \end{vmatrix}$$

On expanding, we get

$\Delta = -2(a+b)\{-2c[2(a+b)] - (a-c)(b-c)\} + (a-c)[2(a+b)(b-c)]$

$\Delta = 8c(a+b)(a+b) + 4(a+b)(a-c)(b-c)$

$= 4(a+b)[2ac + 2bc + ab - bc - ac + c^2]$

$= 4(a+b)[ac + bc + ab + c^2]$

$= 4(a+b)[c(a+c) + b(a+c)]$

$= 4(a+b)(b+c)(c+a)$

$= \alpha(a+b)(b+c)(c+a)$

Hence, $\alpha = 4$

Question176

The area of the triangle whose vertices are complex numbers z, iz, z + iz in the Argand diagram is [Online May 12, 2012]

Options:

A. $2\,|\,z|^2$

B. $1/2\,|\,z|^2$

C. $4\,|\,z|^2$

D. $|z|^2$

Answer: B

Solution:

Solution:
Vertices of triangle in complex form is
$z, iz, z + iz$
In cartesian form vertices are
$(x, y), (-y, x)$ and $(x - y, x + y)$

$\therefore$ Area of triangle $= \frac{1}{2}\begin{vmatrix} x & y & 1 \\ -y & x & 1 \\ x-y & x+y & 1 \end{vmatrix}$

$= \frac{1}{2}[\,x(x - x - y) - y(-y - x + y) + 1(-yx - y^2 - x^2 + xy)\,]$

$= \frac{1}{2}[-xy + xy - y^2 - x^2] = \frac{1}{2}(x^2 + y^2)$

($\because$ Area can not be negative)

$= \frac{1}{2}|\,z\,|^2 \quad (\because z = x + iy, |\,z|^2 = x^2 + y^2)$

Question177

The area of triangle formed by the lines joining the vertex of the parabola, $x^2 = 8y$, to the extremities of its latus rectum is [Online May 12, 2012]

Options:

A. 2

B. 8

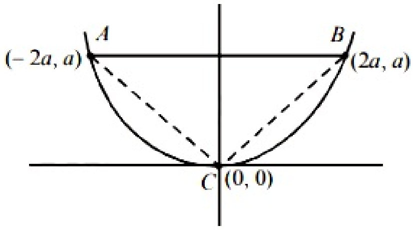
C. 1

D. 4

Answer: B

Solution:

Solution:
Given parabola is $x^2 = 8y$
$\Rightarrow 4a = 8 \Rightarrow a = 2$
To find: Area of ΔABC
$A = (-2a, a) = (-4, 2)$
$B = (2a, a) = (4, 2)$
$C = (0, 0)$



$\therefore \text{Area} = \frac{1}{2}\begin{vmatrix} -4 & 2 & 1 \\ 4 & 2 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{2}[-4(2) - 2(4) + 1(0)]$

$= \frac{-16}{2} = -8 \approx 8$ sq. unit ($\because$ area cannot be negative)

Question178

Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$ then determinant of $(P^2 + Q^2)$ is equal to : [2012]

Options:

A. -2

B. 1

C. 0

D. -1

Answer: C

Solution:

Solution:
Given that $P^3 = Q^3$ ....(1)
and $P^2Q = Q^2P$ ......(2)
Subtracting (1) and (2), we get
$P^3 - P^2Q = Q^3 - Q^2P$
$\Rightarrow P^2(P - Q) + Q^2(P - Q) = 0$
$\Rightarrow (P^2 + Q^2)(P - Q) = 0$
$\because P \neq Q, \therefore P^2 + Q^2 = 0$
Hence $|P^2 + Q^2| = 0$

Question179

Let A = $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that

$Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to :

[2012]

Options:

A. $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

B. $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

C. $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

D. $\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$

Answer: D

Solution:

Solution:

Let $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

Then, $Au_1 + Au_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\Rightarrow A(u_1 + u_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ ......(1)

Given that $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$

$\Rightarrow |A| = 1(1) - 0(2) + 0(4-3) = 1$

$C_{11} = 1 \quad C_{21} = 0 \quad C_{31} = 0$

$C_{12} = -2 \quad C_{22} = 1 \quad C_{32} = 0$

$C_{13} = 1 \quad C_{23} = -2 \quad C_{33} = 1$

$$\therefore \text{adj } A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

We know,

$A^{-1} = \frac{1}{|A|} \text{ad jA}$

$\Rightarrow A^{-1} = \text{ad j}(A)(\because |A| = 1)$

Now, from equation (1), we have

$$u_1 + u_2 = A^{-1} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

Question180

If A^T denotes the transpose of the matrix $A = \begin{bmatrix} 0 & 0 & a \\ 0 & b & c \\ d & e & f \end{bmatrix}$, where a, b, c, d, e and f are integers such that $abd \neq 0$, then the number of such matrices for which $A^{-1} = A^T$ is
[Online May 19, 2012]

Options:

A. $2(3!)$

B. $3(2!)$

C. 2^3

D. 3^2

Answer: C

Solution:

Solution:

$$A = \begin{bmatrix} 0 & 0 & a \\ 0 & b & c \\ d & e & f \end{bmatrix}, |A| = -abd \neq 0$$

$c_{11} = +(bf - ce), c_{12} = -(-\infty) = cd, c_{13} = +(-bd) = -bd$

$c_{21} = -(-ea) = ae, c_{22} = +(-ad) = -ad, c_{23} = -(0) = 0$

$c_{31} = +(-ab) = -ab, c_{32} = -(0) = 0, c_{33} = 0$

$$\text{Adj}A = \begin{bmatrix} (bf-ce) & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj}A) = \frac{1}{abd}\begin{bmatrix} (bf-ce) & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 0 & d \\ 0 & b & e \\ a & c & f \end{bmatrix}$$

Now $A^{-1} = A^T$

$$\Rightarrow \frac{1}{-abd}\begin{bmatrix} (bf-ce) & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & d \\ 0 & b & e \\ a & c & f \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} (bf-ce) & ae & -ab \\ cd & -ad & 0 \\ -bd & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -abd^2 \\ 0 & -ab^2d & -abde \\ -a^2bd & -abcd & -abdf \end{bmatrix}$$

$\therefore bf - ce = ae = cd = 0$ ....(i)

$abd^2 = ab, ab^2d = ad, a^2bd = bd$ ......(ii)

$abde = abcd = abdf = 0$ .....(iii)

From (ii),

$(abd^2) . (ab^2d) . (a^2bd) = ab . ad . bd$

$\Rightarrow (abd)^4 - (abd)^2 = 0$

$\Rightarrow (abd)^2[(abd)^2 - 1] = 0$

because $abd \neq 0, \therefore abd = \pm 1$ ....(iv)

From (iii) and (iv),

$e = c = f = 0$ ......(v)

From (i) and (v),

$bf = ae = cd = 0$ ......(iv)

From (iv), (v) and (vi), it is clear that a, b, d can be any non-zero integer such that $abd = \pm 1$

But it is only possible, if $a = b = d = \pm 1$

Hence, there are 2 choices for each a, b and d . there fore, there are $2 \times 2 \times 2$ choices for a, b and d . Hence number of required matrices $= 2 \times 2 \times 2 = (2)^3$

Question181

Let A and B be real matrices of the form $\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$ and $\begin{bmatrix} 0 & \gamma \\ \delta & 0 \end{bmatrix}$, respectively.
Statement 1: AB – BA is always an invertible matrix.
Statement 2: AB – BA is never an identity matrix.
[Online May 12, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is false, Statement 2 is true.

C. Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation of Statement 1.

D. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

Answer: A

Solution:

Solution:

Let A and B be real matrices such that $\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$ and $\begin{bmatrix} 0 & \gamma \\ \delta & 0 \end{bmatrix}$

Now, $AB = \begin{bmatrix} 0 & \alpha\gamma \\ \beta\delta & 0 \end{bmatrix}$

and $BA = \begin{bmatrix} 0 & \gamma\beta \\ \delta\alpha & 0 \end{bmatrix}$

Statement - 1 :

$AB - BA = \begin{bmatrix} 0 & \gamma(\alpha - \beta) \\ \delta(\beta - \alpha) & 0 \end{bmatrix}$

$|AB - BA| = (\alpha - \beta)^2\gamma\delta \neq 0$

$\therefore AB - BA$ is always an invertible matrix.

Hence, statement -1 is true.

But $AB - BA$ can be identity matrix if $\gamma = -\delta$ or $\delta = -\gamma$

So, statement -2 is false.

Question182

Statement 1: If the system of equations $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$ has a non-trivial solution, then the value of k is $\frac{31}{2}$.

Statement 2: A system of three homogeneous equations in three variables has a non trivial solution if the determinant of the coefficient matrix is zero.

[Online May 26, 2012]

Options:

A. Statement 1 is false, Statement 2 is true.

B. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

C. Statement 1 is true, Statement 2 is true, , Statement 2 is not a correct explanation for Statement 1.

D. Statement 1 is true, Statement 2 is false.

Answer: A

Solution:

Solution:

(a) Given system of equations is $x + ky + 3z = 03x + ky - 2z = 02x + 3y - 4z = 0$Since, system has non-trivial solution$\therefore$ $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0$

$\Rightarrow 1(-4k + 6) - k(-12 + 4) + 3(9 - 2k) = 0$

$\Rightarrow 4k + 33 - 6k = 0 \Rightarrow k = \frac{33}{2}$

Hence, statement - 1 is false.

Statement- 2 is the property.
It is a true statement.

Question183

If the system of equations
$x + y + z = 6$
$x + 2y + 3z = 10$
$x + 2y + \lambda z = 0$
has a unique solution, then λ is not equal to
[Online May 7, 2012]

Options:

A. 1

B. 0

C. 2

D. 3

Answer: D

Solution:

Solution:
Given system of equations is
$x + y + z = 6$
$x + 2y + 3z = 10$
$x + 2y + \lambda z = 0$
It has unique solution.

$$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} \neq 0$$

$\Rightarrow 1(2\lambda - 6) - 1(\lambda - 3) + 1(2 - 2) \neq 0$
$\Rightarrow 2\lambda - 6 - \lambda + 3 \neq 0 \Rightarrow \lambda - 3 \neq 0 \Rightarrow \lambda \neq 3$

Question184

Statement -1:
Determinant of a skew-symmetric matrix of order 3 is zero.
Statement -2:
For anymatrix A, $\det(A)^T = \det(A)$ and $\det(-A) = -\det(A)$.
Where det (B) denotes the determinant of matrix B. Then :
[2011RS]

Options:

A. Both statements are true

B. Both statements are false

C. Statement-1 is false and statement-2 is true

D. Statement-1 is true and statement-2 is false

Answer: D

Solution:

Solution:
We know that determinant of skew symmetric matrix of odd order is zero.
So, statement-1 is true.
We know that $\det(A^T) = \det(A)$
$\det(-A) = -(-1)^n \det(A)$
where A is a $n \times n$ order matrix.
So, statement- 2 is false.

Question185

Consider the following relation R on the set of real square matrices of order 3.
$R = \{ (A, B) \mid A = P^{-1}BP$ for some invertible matrix $P \}$
Statement- 1: R is equivalence relation.
Statement- 2 : For any two invertible 3×3 matrices M and N, $(MN)^{-1} = N^{-1}M^{-1}$.
[2011 RS]

Options:

A. Statement-1 is true, statement-2 is true and statement 2 is a correct explanation for statement-1.

B. Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.

C. Statement-1 is true, statement-2 is false.

D. Statement-1 is false, statement-2 is true.

Answer: B

Solution:

Solution:
For reflexive
$A = P^{-1}AP$ is true
For $P = I$, which is an invertible matrix.
$(A, A) \in R$
$\therefore R$ is reflexive.
For symmetry
As $(A, B) \in R$ for matrix P
$A = P^{-1}BP$
$\Rightarrow PAP^{-1} = B$
$\Rightarrow B = PAP^{-1}$
$\Rightarrow B = (P^{-1})^{-1}A(P^{-1})$
$\therefore (B, A) \in R$ for matrix P^{-1}
$\therefore R$ is symmetric.
For transitivity

$A = P^{-1}BP$
and $B = P^{-1}CP$
$\Rightarrow A = P^{-1}(P^{-1}CP)P$
$\Rightarrow A = (P^{-1})^2CP^2$
$\Rightarrow A = (P^2)^{-1}C(P^2)$
$\therefore (A, C) \in R$ for matrix P^2
$\therefore R$ is transitive.
So R is equivalence.
So, statement- 1 is true.
We know that if A and B are two invertible matrices of order n, then
$(AB)^{-1} = B^{-1}A^{-1}$
So, statement- 2 is true.

Question186

If the trivial solution is the only solution of the system of equations $x - ky + z = 0$ $kx + 3y - kz = 0$ $3x + y - z = 0$ then the set of all values of k is [2011 RS]

Options:

A. $R - \{2, -3\}$

B. $R - \{2\}$

C. $R - \{-3\}$

D. $\{2, -3\}$

Answer: A

Solution:

Solution:
$x - ky + z = 0$
$kx + 3y - kz = 0$
$3x + y - z = 0$
The given that system of equations have trivial solution,

$$\therefore \begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} \neq 0$$

$\Rightarrow 1(-3 + k) + k(-k + 3k) + 1(k - 9) \neq 0$
$\Rightarrow k - 3 + 2k^2 + k - 9 \neq 0$
$\Rightarrow k^2 + k - 6 \neq 0 \Rightarrow k = -3, k \neq 2$
So, the equation will have only trivial solution,
when $k \in R - \{2, -3\}$

Question187

The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ possess a non-zero solution is

[2011]

Options:

A. 2

B. 1

C. zero

D. 3

Answer: A

Solution:

Solution:
Given that system of equations have non-zero solution
$\Delta = 0$

$$\Rightarrow \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 4(4-2) - k(k-2) + 2(2k-8) = 0$
$\Rightarrow 8 - k^2 + 2k + 4k - 16 = 0$
$k^2 - 6k + 8 = 0$
$\Rightarrow (k-4)(k-2) = 0 \Rightarrow k = 4, 2$

Question188

Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define
$Tr(A) =$ sum of diagonal elements of A and
$|A| =$ determinant of matrix A
Statement -1: $Tr(A) = 0$
Statement -2 : $|A| = 1$
[2010]

Options:

A. Statement-1 is true, Statement-2 is true ; Statement-2 is not a correct explanation for Statement -1.

B. Statement -1 is true, Statement -2 is false.

C. Statement -1 is false, Statement -2 is true .

D. Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

Answer: B

Solution:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a, b, c, d \neq 0$

$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = I$

$\Rightarrow A^2 = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a^2 + bc = 1, bc + d^2 = 1$

$ab + bd = ac + cd = 0$

$c \neq 0$ and $b \neq 0 \Rightarrow a + d = 0$

$|A| = ad - bc = -a^2 - bc = -1$

Also if $A \neq I$, then $tr(A) = a + d = 0$.

∴ Statement- 1 true and statement- 2 false.

Question189

Consider the system of linear equations;
$x_1 + 2x_2 + x_3 = 3$
$2x_1 + 3x_2 + x_3 = 3$
$3x_1 + 5x_2 + 2x_3 = 1$
The system has
[2010]

Options:

A. exactly 3 solutions

B. a unique solution

C. no solution

D. infinite number of solutions

Answer: C

Solution:

Solution:

$D_x = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

⇒ Given system, does not have any solution.

⇒ No solution

Question190

Let a, b, c be such that $b(a + c) \neq 0$

if $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix}$ **= 0 then the value of n is :**
[2009]

Options:

A. any even integer

B. any odd integer

C. any integer

D. ero

Answer: B

Solution:

Solution:

$$\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & a-1 & (-1)^{n+2}a \\ b+1 & b-1 & (-1)^{n+1}b \\ c-1 & c+1 & (-1)^{n}c \end{vmatrix} = 0$$

(Taking transpose of second determinant)
$C_1 \Leftrightarrow C_3$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} - \begin{vmatrix} (-1)^{n+2}a & a-1 & a+1 \\ (-1)^{n+2}(-b) & b-1 & b+1 \\ (-1)^{n+2}c & c+1 & c-1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+2} \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$$\Rightarrow [1+(-1)^{n+2}] \Rightarrow \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} = 0$$

$C_2 - C_1$, $C_3 - C_1$

$$\Rightarrow [1+(-1)^{n+2}] \begin{vmatrix} a & 1 & -1 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix} = 0 R_1 + R_3$$

$$\Rightarrow[1+(-1)^{n+2}]\begin{vmatrix} a+c & 0 & 0 \\ -b & 2b+1 & 2b-1 \\ c & -1 & 1 \end{vmatrix}=0$$

$\Rightarrow[1+(-1)^{n+2}](a+c)\,(2b+1+2b-1)=0$
$\Rightarrow 4b(a+c)[1+(-1)^{n+2}]=0$
$\Rightarrow 1+(-1)^{n+2}=0$ as $b(a+c)\neq 0$
$\Rightarrow n$ should be an odd integer.er.

Question191

Let A be a 2×2 matrix
Statement -1: $\operatorname{adj}(\operatorname{adj} A)=A$
Statement -2: $|\operatorname{adj} A|=|A|$
[2009]

Options:

A. Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is false.

C. Statement -1 is false, Statement-2 is true.

D. Statement-1 is true, Statement -2 is true. Statement-2 is a correct explanation for Statement-1.

Answer: A

Solution:

Solution:
We know that if A is square matrix of order n then
$\operatorname{ad j}(\operatorname{ad j}A)=|A|^{n-2}A$
$=|A|^{0}A=A$
Also $|\operatorname{ad j}A|=|A|^{n-1}=|A|^{2-1}=|A|$
∴ Both the statements are true but statement- 2 is not a correct explanation for statement-1

Question192

Let A be a2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by tr(A), the sum of diagonal entries of a. Assume that $A^2=I$
Statement-1 : If $A\neq I$ and $A\neq -I$, then $\det(A)=-1$
Statement-2 : If $A\neq I$ and $A\neq -I$, then $\operatorname{tr}(A)\neq 0$
[2008]

Options:

A. Statement -1 is false, Statement-2 is true

B. Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

C. Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1

D. Statement -1 is true, Statement-2 is false

Answer: D

Solution:

Solution:

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given that $A^2 = I$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a^2 + bc = 1$ and $ab + bd = 0$

$ac + cd = 0$ and $bc + d^2 = 1$

From these four equations,

$a^2 + bc = bc + d^2 \Rightarrow a^2 = d^2$

and $b(a + d) = 0 = c(a + d) \Rightarrow a = -d$

$|A| = ad - bc = -a^2 - bc = -1$

Also if $A \neq I$ then $tr(A) = a + d = 0$

$\therefore$ Statement 2 is false..

Question193

Let A be a square matrix all of whose entries are integers. Then which one of the following is true? [2008]

Options:

A. If $\det A = \pm 1$, then A^{-1} exists but all its entries are not necessarily integers

B. If $\det A \neq \pm 1$, then A^{-1} exists and all its entries are non integers

C. If $\det A = \pm 1$, then A^{-1} exists but all its entries are integers

D. If $\det A = \pm 1$, then A^{-1} need not exists

Answer: C

Solution:

Solution:

Given that all entries of square matrix A are integers, therefore all cofactors should also be integers.

If $\det A = \pm 1$ then A^{-1} exists. Also all entries of A^{-1} are integers.

Question194

Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that x = cy + bz, y = az + cx, and z = bx + ay.

Then $a^2 + b^2 + c^2 + 2abc$ is equal to
[2008]

Options:

A. 2

B. -1

C. 0

D. 1

Answer: D

Solution:

Solution:
The given equations are
$-x + cy + bz = 0$
$cx - y + az = 0$
$bx + ay - z = 0$
Given that x, y, z are not all zero
∴ The above system have non-zero solution

$$\Rightarrow \Delta = 0 \Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$\Rightarrow -1(1 - a^2) - c(-c - ab) + b(ac + b) = 0$
$\Rightarrow -1 + a^2 + b^2 + c^2 + 2abc = 0$
$\Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

Question195

Let $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals

[2007]

Options:

A. 1 / 5

B. 5

C. 5^2

D. 1

Answer: A

Solution:

Solution:

Given that $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. and $|A^2| = 25$

$\therefore A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha + 25\alpha^2 + 5\alpha \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$

$\therefore |A^2| = 25(25\alpha^2)$

$\therefore 25 = 25(25\alpha^2) \Rightarrow |\alpha| = \frac{1}{5}$

Question196

If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is

[2007]

Options:

A. divisible by x but not y

B. divisible by y but not x

C. divisible by neither x nor y

D. divisible by both x and y

Answer: D

Solution:

Solution:

Given that, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$\therefore D = \begin{vmatrix} 1 & 1 & 1 \\ 0 & x & 0 \\ 0 & 0 & y \end{vmatrix} = xy$

Hence, D is divisible by both x and y

Question197

If $a^2 + b^2 + c^2 = -2$ and

$f(x) = \begin{vmatrix} 1+a^2x & (1+b^2)x & (1+c^2)x \\ (1+a^2)x & 1+b^2x & (1+c^2)x \\ (1+a^2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$

then f (x) is a polynomial of degree [2005]

Options:

A. 1

B. 0

C. 3

D. 2

Answer: D

Solution:

Solution:
Applying, $C_1 \to C_1 + C_2 + C_3$, we get

$$f(x) = \begin{vmatrix} 1+(a^2+b^2+c^2+2)x & (1+b^2)x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & 1+b^2x & (1+c^2)x \\ 1+(a^2+b^2+c^2+2)x & (1+b^2)x & 1+c^2x \end{vmatrix}$$

$[\because a^2+b^2+c^2 = -2]$

$$= \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 1 & 1+b^2x & (1+c^2)x \\ 1 & (1+b^2)x & 1+c^2x \end{vmatrix}$$

Applying, $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\therefore f(x) = \begin{vmatrix} 1 & (1+b^2)x & (1+c^2)x \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{vmatrix}$$

$f(x) = (x-1)^2$
Hence degree $= 2$

Question198

If $a_1, a_2, a_3, \ldots, a_n, \ldots$ are in G. P., then the determinant

$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$

is equal to

[2005]

Options:

A. 1

B. 0

C. 4

D. 2

Answer: B

Solution:

Solution:
Let r be the common ratio of an G.P., then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^{n} & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_2 + (n+7)\log t \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$, we get

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & 2[\log a_1 + n\log r] \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & 2[\log a_1 + (n+3)\log r] \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & 2[\log a_1 + (n+6)\log r] \end{vmatrix}$$

$= 0$

Question199

If $A^2 - A + I = 0$, then the inverse of A is [2005]

Options:

A. $A + I$

B. A

C. $A - I$

D. $I - A$

Answer: D

Solution:

Solution:
Given that $A^2 - A + I = 0$
Pre-multiply by A^{-1} both side, we get
$A^{-1}A^2 - A^{-1}A + A^{-1} . I = A^{-1} . 0$
$\Rightarrow A - I + A^{-1} = 0$ or $A^{-1} = I - A$.

Question200

The system of equations
$\alpha x + y + z = \alpha - 1$
$x + \alpha y + z = \alpha - 1$
$x + y + \alpha z = \alpha - 1$
has infinite solutions, if α is
[2005]

Options:

A. – 2

B. either – 2 or 1

C. not – 2

D. 1

Answer: A

Solution:

Solution:
$\alpha x + y + z = \alpha - 1$
$x + \alpha y + z = \alpha - 1$
$x + y + z\alpha = \alpha - 1$

$$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix}$$

$= \alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha)$
$= \alpha(\alpha - 1)(\alpha + 1) - 1(\alpha - 1) - 1(\alpha - 1)$
$= (\alpha - 1)[\alpha^2 + \alpha - 1 - 1]$
$= (\alpha - 1)[\alpha^2 + \alpha - 2]$
$= (\alpha - 1)[\alpha^2 + 2\alpha - \alpha - 2]$
$= (\alpha - 1)[\alpha(\alpha + 2) - 1(\alpha + 2)]$
$= (\alpha - 1)^2(\alpha + 2)$
∵ Equations has infinite solutions
$\therefore \Delta = 0$
$\Rightarrow (\alpha - 1) = 0, \alpha + 2 = 0$
$\Rightarrow \alpha = -2, 1;$
But $\alpha \neq 1.$
$\therefore \alpha = -2$

Question201

If a_1, a_2, a_3, ..., a_n, ... are in G.P., then the value of the determinant

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$ **, is**

[2004]

Options:

A. -2

B. 1

C. 2

D. 0

Answer: D

Solution:

Solution:
Let r be the common ratio of an G.P., then

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 r^{n-1} & \log a_1 r^{n} & \log a_1 r^{n+1} \\ \log a_1 r^{n+2} & \log a_1 r^{n+3} & \log a_1 r^{n+4} \\ \log a_1 r^{n+5} & \log a_1 r^{n+6} & \log a_1 r^{n+7} \end{vmatrix}$$

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & \log a_1 + (n+1)\log r \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & \log a_1 + (n+4)\log r \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & \log a_2 + (n+7)\log r \end{vmatrix}$$

Applying $C_3 \to C_3 + C_1$, we get

$$= \begin{vmatrix} \log a_1 + (n-1)\log r & \log a_1 + n\log r & 2[\log a_1 + n\log r] \\ \log a_1 + (n+2)\log r & \log a_1 + (n+3)\log r & 2[\log a_1 + (n+3)\log r] \\ \log a_1 + (n+5)\log r & \log a_1 + (n+6)\log r & 2[\log a_1 + (n+6)\log r] \end{vmatrix}$$

$= 0$

Question202

Let A = $\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ **and B =** $\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$ **If B is the inverse of**

matrix A, then α is
[2004]

Options:

A. 5

B. -1

C. 2

D. -2

Answer: A

Solution:

Solution:

Given that $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

$\Rightarrow B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$

Given that $B = A^{-1} \Rightarrow AB = I$

$\Rightarrow \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-2 \\ 0 & 10 & -5+\alpha \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \frac{5-\alpha}{10} = 0 \Rightarrow \alpha = 5$

Question203

Let $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is
[2004]

Options:

A. $A^2 = I$

B. $A = (-1)I$, where I is a unit matrix

C. A^{-1} does not exist

D. A is a zeromatrix

Answer: A

Solution:

Solution:
Given that

$$A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

clearly $A \neq 0$. Also $|A| = -1 \neq 0$

$\therefore A^{-1}$ exists, further $(-1)I = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \neq A$

Also $A^2 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

Question204

If 1, ω, ω^2 are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to
[2003]

Options:

A. ω^2

B. 0

C. 1

D. ω

Answer: B

Solution:

Solution:

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$$

Expand through R_1

$= 1(\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^{4n})$

$= \omega^{3n} - 1 - 0 + \omega^{3n} - \omega^{6n}$

$= 1 - 1 + 1 - 1 = 0$ $[\because \omega^{3n} = 1]$

Question205

If the system of linear equations $x + 2ay + az = 0; x + 3by + bz = 0; x + 4cy + cz = 0$ has a non - zero solution, then a, b, c. [2003]

Options:

A. satisfy $a + 2b + 3c = 0$

B. are in A.P

C. are in G..P

D. are in H.P.

Answer: D

Solution:

Solution:
For homogeneous system of equations to have nonzero solution, $\Delta = 0$

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - 2C_3$

$$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 1 & b & b \\ 1 & 2c & c \end{vmatrix} = 0 \quad R_3 \to R_3 - R_1, R_2 \to R_2 - R_1$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & a \\ 0 & b & b-a \\ 0 & 2c & c-a \end{vmatrix} = 0$$

$\Rightarrow bc - ab = 2bc - 2ac$

$\Rightarrow \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\therefore$a, b, c are in Harmonic Progression.

Question206

If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is $-$ve, then

$$\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$$ **is equal to**

[2002]

Options:

A. +ve

B. $(ac - b^2)(ax^2 + 2bx + c)$

C. -ve

D. 0

Answer: C

Solution:

Solution:

Given that $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$

Applying $R_3 \to R_3 - (xR_1 + R_2)$

$= \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+c) \end{vmatrix}$

$= (ax^2 + 2bx + c)(b^2 - ac) = (+)(-) = -ve.$

[Given that discriminant of $ax^2 + 2bx + c$ is $-ve$

$\therefore 4b^2 - 4ac < 0 \Rightarrow b^2 - ac < 0$]

Question207

l , m, n are the pth, qth and rth term of a G. P. all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals

[2002]

Options:

A. -1

B. 2

C. 1

D. 0

Answer: D

Solution:

$l = AR^{p-1} \Rightarrow \log l = \log A + (p-1)\log R$
$m = AR^{q-1} \Rightarrow \log m = \log A + (q-1)\log R$
$n = AR^{r-1} \Rightarrow \log n = \log A + (r-1)\log R$

Now, $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} = \begin{vmatrix} \log A + (p-1)\log R & p & 1 \\ \log A + (q-1)\log R & q & 1 \\ \log A + (r-1)\log R & r & 1 \end{vmatrix}$

Operating

$C_1 - (\log R)C_2 + (\log R - \log A)C_3 = \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix} = 0$
