Binomial Theorem

Question1

${}^{n-1}C_r = (k^2 - 8)\,{}^{n}C_{r+1}$ if and only if :

[27-Jan-2024 Shift 1]

Options:

A.

$2\sqrt{2} < k \le 3$

B.

$2\sqrt{3} < k \le 3\sqrt{2}$

C.

$2\sqrt{3} < k < 3\sqrt{3}$

D.

$2\sqrt{2} < k < 2\sqrt{3}$

Answer: A

Solution:

$${}^{n-1}C_r = (k^2 - 8)\,{}^{n}C_{r+1}$$

$$\underbrace{r + 1 \ge 0, r \ge 0}_{r \ge 0}$$

$$\frac{{}^{n-1}C_r}{{}^{n}C_{r+1}} = k^2 - 8$$

$\frac{r+1}{n} = k^2 - 8$

$\Rightarrow k^2 - 8 > 0$

$(k - 2\sqrt{2})(k + 2\sqrt{2}) > 0$

$k \in (-\infty, -2\sqrt{2}) \cup (2\sqrt{2}, \infty) \quad \cdots\cdots (i)$

$\therefore n \geq r + 1, \ \frac{r+1}{n} \leq 1$

$\Rightarrow k^2 - 8 \leq 1$

$k^2 - 9 \leq 0$

$-3 \leq k \leq 3 \quad \cdots\cdots (ii)$

From equation (I) and (II) we get

$k \in [-3, -2\sqrt{2}) \cup (2\sqrt{2}, 3]$

Question2

If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then :

[27-Jan-2024 Shift 1]

Options:

A.

$A = B^3$

B.

$3A = B$

C.

$B = A^3$

D.

$A = 3B$

Answer: A

Solution:

Sum of coefficients in the expansion of

$(1-3x+10x^2)^n = A$

then $A=(1-3+10)^n = 8^n$(put $x=1$)

and sum of coefficients in the expansion of

$(1+x^2)^n = B$

then $B=(1+1)^n = 2^n$

Question3

The coefficient of x^{2012} in the expansion of $(1-x)^{2008}(1+x+x^2)^{2007}$ is equal to

[27-Jan-2024 Shift 2]

Answer: 0

Solution:

$(1-x)(1-x)^{2007}(1+x+x^2)^{2007}$

$(1-x)(1-x^3)^{2007}$

$(1-x)({}^{2007}C_0 - {}^{2007}C_1(x^3)+\ldots\ldots)$

General term

$(1-x)((-1)^{r\,2007}C_r x^{3r})$

$(-1)^{r\,2007}C_r x^{3r} - (-1)^{r\,2007}C_r x^{3r+1}$

$3r = 2012$

$r \neq \dfrac{2012}{3}$

$3r+1 = 2012$

$3r = 2011$

$r \neq \dfrac{2011}{3}$

Hence there is no term containing x^{2012}.

So coefficient of $x^{2012} = 0$

Question4

If $\frac{{}^{11}C_1}{2} + \frac{{}^{11}C_2}{3} + ... + \frac{{}^{11}C_9}{10} = \frac{n}{m}$ with $\gcd(n, m) = 1$, then $n + m$ is equal to

[29-Jan-2024 Shift 1]

Answer: 2041

Solution:

$$\sum_{r=1}^{9} \frac{{}^{11}C_r}{r+1}$$

$$= \frac{1}{12} \sum_{r=1}^{9} {}^{12}C_{r+1}$$

$$= \frac{1}{12}[2^{12} - 26] = \frac{2035}{6}$$

$$\therefore m + n = 2041$$

Question5

Remainder when $64^{32^{32}}$ is divided by 9 is equal to__

[29-Jan-2024 Shift 2]

Answer: 1

Solution:

Let $32^{32} = t$

$$64^{32^{32}} = 64^t = 8^{2t} = (9-1)^{2t}$$

$$= 9k + 1$$

Hence remainder $= 1$

Question6

Number of integral terms in the expansion of $\left\{7^{\left(\frac{1}{2}\right)} + 11^{\left(\frac{1}{6}\right)}\right\}^{824}$ is equal to

[30-Jan-2024 Shift 1]

Answer: 138

Solution:

General term in expansion of $((7)^{1/2}+(11)^{1/6})^{824}$ is $t_{r+1}={}^{824}C_r(7)^{\frac{824-r}{2}}(11)^{r/6}$

For integral term, r must be multiple of 6 .

Hence $r=0, 6, 12, \ldots\ldots..822$

Question7

Suppose 2 − p, p, 2 − α, α are the coefficient of four consecutive terms in the expansion of $(1 + x)^n$. Then the value of $p^2 - \alpha^2 + 6\alpha + 2p$ equals

[30-Jan-2024 Shift 2]

Options:

A.

4

B.

10

C.

8

D.

2

Answer: D

Solution:

$2-p, p, 2-\alpha, \alpha$

Binomial coefficients are

$\Rightarrow {}^nC_r, {}^nC_{r+1}, {}^nC_{r+2}, {}^nC_{r+3}$ respectively

$\Rightarrow {}^nC_r + {}^nC_{r+1} = 2$

${}^{n+1}C_{r+1} = 2 \quad \ldots\ldots(1)$

Also ${}^nC_{r+2} + {}^nC_{r+3} = 2$

$\Rightarrow {}^{n+1}C_{r+3} = 2 \quad \ldots\ldots(2)$

From (1) and (2)

${}^{n+1}C_{r+1} = {}^{n+1}C_{r+3}$

$2r+4 = n+1$

$n = 2r+3$

${}^{2r+4}C_{r+1} = 2$

Data Inconsistent

Question8

Let $\alpha = \sum_{k=0}^{n}\left(\frac{({}^nC_k)^2}{k+1}\right)$ and $\beta = \sum_{k=0}^{n-1}\left(\frac{{}^nC_k \quad {}^nC_{k+1}}{k+2}\right)$ If $5\alpha = 6\beta$, then n equals

[30-Jan-2024 Shift 2]

Answer: 10

Solution:

$$\alpha = \sum_{k=0}^{n} \frac{{}^nC_k \cdot {}^nC_k}{k+1} \cdot \frac{n+1}{n+1}$$

$$= \frac{1}{n+1} \sum_{k=0}^{n} {}^{n+1}C_{k+1} \cdot {}^nC_{n-k}$$

$$\alpha = \frac{1}{n+1} \cdot {}^{2n+1}C_{n+1}$$

$$\beta = \sum_{k=0}^{n-1} {}^nC_k \cdot \frac{{}^nC_{k+1}}{k+2} \frac{n+1}{n+1}$$

$$\frac{1}{n+1} \sum_{k=0}^{n-1} {}^nC_{n-k} \cdot {}^{n+1}C_{k+2}$$

$$= \frac{1}{n+1} \cdot {}^{2n+1}C_{n+2}$$

$$\frac{\beta}{\alpha} = \frac{{}^{2n+1}C_{n+2}}{{}^{2n+1}C_{n+1}} = \frac{2n+1-(n+2)+1}{n+2}$$

$$\frac{\beta}{\alpha} = \frac{n}{n+2} = \frac{5}{6}$$

$$n = 10$$

Question9

In the expansion of $(1+x)(1-x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5, x \neq 0,$ the sum of the coefficient of x^3 and x^{-13} is equal to

[31-Jan-2024 Shift 1]

Options:

Answer: 118

Solution:

$$(1+x)(1-x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$$

$$=(1+x)(1-x^2)\left(\left(1+\frac{1}{x}\right)^3\right)^5$$

$$=\frac{(1+x)^2(1-x)(1+x)^{15}}{x^{15}}$$

$$=\frac{(1+x)^{17}-x(1+x)^{17}}{x^{15}}$$

$=$ coeff(x^3) in the expansion $\approx$coeff(x^{18}) in $(1+x)^{17}-x(1+x)^{17}$

$=0-1$

$=-1$

coeff(x^{-13}) in the expansion $\approx$coeff(x^2) in $(1+x)^{17}-x(1+x)^{17}$

$$=\binom{17}{2}-\binom{17}{1}$$

$=17\times 8-17$

$=17\times 7$

$=119$

Hence Answer $=119-1=118$

Question10

If for some m, n; ${}^6C_m + 2({}^6C_{m+1}) + {}^6C_{m+2} > {}^8C_3$ and ${}^{n-1}P_3 : {}^nP_4 = 1 : 8$, then ${}^nP_{m+1} + {}^{n+1}C_m$ is equal to

[31-Jan-2024 Shift 2]

Options:

A.

380

B.

376

C.

384

D.

372

Answer: D

Solution:

${}^6C_m + 2({}^6C_{m+1}) + {}^6C_{m+2} > {}^8C_3$

${}^7C_{m+1} + {}^7C_{m+2} > {}^8C_3$

${}^8C_{m+2} > {}^8C_3$

$\therefore m = 2$

And ${}^{n-1}P_3 : {}^nP_4 = 1 : 8$

$$\frac{(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)} = \frac{1}{8}$$

$\therefore n = 8$

$\therefore {}^nP_{m+1} + {}^{n+1}C_m = {}^8P_3 + {}^9C_2$

$= 8 \times 7 \times 6 + \dfrac{9 \times 8}{2}$

$= 372$

Question11

Let the coefficient of x^r in the expansion of

$(x+3)^{n-1}+(x+3)^{n-2}(x+2)+$

$(x+3)^{n-3}(x+2)^2+\ldots\ldots+(x+2)^{n-1}$

be α_r. If $\sum_{r=0}^{n}\alpha_r=\beta^n-\gamma^n, \beta, \gamma\in N$, then the value of $\beta^2+\gamma^2$ equals

[31-Jan-2024 Shift 2]

Answer: 25

Solution:

$$(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+\ldots\ldots+(x+2)^{n-1}$$

$$\sum\alpha_r=4^{n-1}+4^{n-2}\times 3+4^{n-3}\times 3^2\ \ldots\ldots+3^{n-1}$$

$$=4^{n-1}\left[1+\frac{3}{4}+\left(\frac{3}{4}\right)^2\ldots.+\left(\frac{3}{4}\right)^{n-1}\right]$$

$$=4^{n-1}\times\frac{1-\left(\frac{3}{4}\right)^n}{1-\frac{3}{4}}$$

$$=4^n-3^n=\beta^n-\gamma^n$$

$$\beta=4, \gamma=3$$

$$\beta^2+\gamma^2=16+9=25$$

Question12

If the Coefficient of x^{30} in the expansion of $(1 + 1/x)^6(1 + x^2)^7(1 - x^3)^8$; $x \neq 0$ is α, then $|\alpha|$ equals______

[1-Feb-2024 Shift 1]

Answer: 678

Solution:

coeff of x^{30} in $\frac{(x+1)^6(1+x^2)^7(1-x^3)^8}{x^6}$

coeff. of x^{36} in $(1+x)^6(1+x^2)^7(1-x^3)^8$

General term

${}^6C_{r_1}\,{}^7C_{r_2}\,{}^8C_{r_3}(-1)^{r_3}x^{r_1+2r_2+3r_3}$

$r_1+2r_2+3r_3=36$

Case - I :

r_1	r_2	r_3
0	6	8
2	5	8
4	4	8
6	3	8

$r_1+2r_2=12$ (Taking $r_3=8$)

Case - II :

r_1	r_2	r_3
1	7	7
3	6	7
5	5	7

$r_1+2r_2=15$ (Taking $r_3=7$)

Case - III :

r_1	r_2	r_3
4	7	6
6	6	6

$r_1+2r_2=18$ (Taking $r_3=6$)

Coeff. $= 7 + (15 \times 21) + (15 \times 35) + (35) - (6 \times 8) - (20 \times 7 \times 8) - (6 \times 21 \times 8) + (15 \times 28) + (7 \times 28) = -678 = \alpha$

$|\alpha| = 678$

Question13

Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}}+\frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $(n/m)^{1/3}$ is:

[1-Feb-2024 Shift 2]

Options:

A.

4/9

B.

1/9

C.

1/4

D.

9/4

Answer: D

Solution:

$$\left(\frac{x^{\frac{1}{3}}}{3}+\frac{x^{\frac{-2}{3}}}{}\right)^{18}$$

$$t_7={}^{18}c_6\left(\frac{x^{\frac{1}{3}}}{3}\right)^{12}\left(\frac{x^{\frac{-2}{3}}}{2}\right)^{6}={}^{18}c_6\,\frac{1}{(3)^{12}}\cdot\frac{1}{2^6}$$

$$t_{13}={}^{18}c_{12}\left(\frac{x^{\frac{1}{3}}}{3}\right)^{6}\left(\frac{x^{\frac{-2}{3}}}{2}\right)^{12}={}^{18}c_{12}\,\frac{1}{(3)^{6}}\cdot\frac{1}{2^{12}}\cdot x^{-6}$$

$$m={}^{18}c_6\cdot 3^{-12}\cdot 2^{-6}:n={}^{18}c_{12}\cdot 2^{-12}\cdot 3^{-6}$$

$$\left(\frac{n}{m}\right)^{\frac{1}{3}}=\left(\frac{2^{-12}\cdot 3^{-6}}{3^{-12}\cdot 2^{-6}}\right)^{\frac{1}{3}}=\left(\frac{3}{2}\right)^{2}=\frac{9}{4}$$

Question14

The value $\sum_{r=0}^{22}{}^{22}C_r\,{}^{23}C_r$ is

[24-Jan-2023 Shift 1]

Options:

A. ${}^{45}C_{23}$

B. ${}^{44}C_{23}$

C. ${}^{45}C_{24}$

D. ${}^{44}C_{22}$

Answer: A

Solution:

Solution:

$$\sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_r = \sum_{r=0}^{22} {}^{22}C_r \cdot {}^{23}C_{23-r}$$
$$= {}^{45}C_{23}$$

Question15

Suppose $\sum_{r=0}^{2023} r^2 \ {}^{2023}C_r = 2023 \times \alpha \times 2^{2022}$. Then the value of α is
[24-Jan-2023 Shift 1]

Answer: 1012

Solution:

using result
$$\sum_{r=0}^{n} r^{2n}C_r = n(n+1) \cdot 2^{n-2}$$
Then $\sum_{r=0}^{2023} r^2 \ {}^{2023}C_r = 2023 \times 2024 \times 2^{2021}$
$= 2023 \times \alpha \times 2^{2022}$ So,
$\alpha = 1012$

Question16

If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \ldots\ldots. + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$, then α is equal to
[24-Jan-2023 Shift 2]

Options:

A. 30

B. 60

C. 15

D. 10

Answer: C

Solution:

Solution:

$S = 0 \cdot ({}^{30}C_0)^2 + 1 \cdot ({}^{30}C_1)^2 + 2 \cdot ({}^{30}C_2)^2 + \ldots\ldots \cdot + 30 \cdot ({}^{30}C_{30})^2$

$S = 30 \cdot ({}^{30}C_0)^2 + 29 \cdot ({}^{30}C_1)^2 + 28 \cdot ({}^{30}C_2)^2 + \ldots\ldots + 0 \cdot ({}^{30}C_0)^2$

$2S = 30 \cdot ({}^{30}C_0{}^2 + {}^{30}C_1{}^2 + \ldots\ldots \cdot + \cdot {}^{30}C_{30}{}^2)$

$S = 15 \cdot {}^{60}C_{30} = 15 \cdot \frac{60!}{(30!)^2}$

$\frac{15 \cdot 10!}{(30!)^2} = \frac{\alpha \cdot 60!}{(30!)^2}$

$\Rightarrow \alpha = 15$

Question17

Let the sum of the coefficients of the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n$, $x \neq 0$, $n \in N$, be 376 . Then the coefficient of x^4 is___

[24-Jan-2023 Shift 2]

Answer: 405

Solution:

Given Binomial $\left(x - \frac{3}{x^2}\right)^n$, $x \neq 0, n \in N$,

Sum of coefficients of first three terms

${}^nC_0 - {}^nC_1 \cdot 3 + {}^nC_2 3^2 = 376$

$\Rightarrow 3n^2 - 5n - 250 = 0$

$\Rightarrow (n - 10)(3n + 25) = 0$

$\Rightarrow n = 10$

Now general term ${}^{10}C_r x^{10-r}\left(\frac{-3}{x^2}\right)^r$

$= {}^{10}C_r x^{10-r}(-3)^r \cdot x^{-2r}$

$= {}^{10}C_r(-3)^r \cdot x^{10-3r}$

Coefficient of $x^4 \Rightarrow 10 - 3r = 4$

$\Rightarrow r = 2$

${}^{10}C_2(-3)^2 = 405$

Question18

If a_r is the coefficient of x^{10-r} in the Binomial expansion of $(1+x)^{10}$, then $\sum_{r=1}^{10} r^3 \left(\frac{a_r}{a_{r-1}}\right)^2$ is equal to

[25-Jan-2023 Shift 1]

Options:

A. 4895

B. 1210

C. 5445

D. 3025

Answer: B

Solution:

Solution:

$a_r = {}^{10}C_{10-r} = {}^{10}C_r$

$\Rightarrow \sum_{r=1}^{10} r^3 \left(\frac{{}^{10}C_r}{{}^{10}C_{r-1}}\right)^2$

$= \sum_{r=1}^{10} r^3 \left(\frac{11-r}{r}\right)^2$

$= \sum_{r=1}^{10} r(11-r)^2$

$= \sum_{r=1}^{10} (121r + r^3 - 22r^2) = 1210$

Question19

The constant term in the expansion of $\left(2x + \frac{1}{x^7} + 3x^2\right)^5$ is ______.

[25-Jan-2023 Shift 1]

Answer: 1080

Solution:

Solution:

General term is $\sum \frac{5!(2x)^{n_1}(x^{-7})^{n_2}(3x^2)^{n_3}}{n_1!n_2!n_3!}$

For constant term,

$n_1 + 2n_3 = 7n_2$

$\& n_1 + n_2 + n_3 = 5$

Only possibility $n_1 = 1, n_2 = 1, n_3 = 3$
$\Rightarrow$ constant term $= 1080$

Question20

$\sum_{k=0}^{6} {}^{51-k}C_3$ is equal to
[25-Jan-2023 Shift 2]

Options:

A. ${}^{51}C_4 - {}^{45}C_4$

B. ${}^{51}C_3 - {}^{45}C_3$

C. ${}^{52}C_4 - {}^{45}C_4$

D. ${}^{52}C_3 - {}^{45}C_3$

Answer: C

Solution:

Solution:
$\sum_{k=0}^{6} {}^{51-k}C_3$
$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + \ldots + {}^{45}C_3$
$= {}^{45}C_3 + {}^{46}C_3 + \ldots . + {}^{51}C_3$
$= {}^{45}C_4 + {}^{45}C_3 + {}^{46}C_3 + \ldots . + {}^{51}C_3 - {}^{45}C_4$
$({}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r)$
$= {}^{52}C_4 - {}^{45}C_4$

Question21

The remainder when $(2023)^{2023}$ is divided by 35 is ________.
[25-Jan-2023 Shift 2]

Answer: 7

Solution:

$(2023)^{2023}$
$= (2030 - 7)^{2023}$
$= (35K - 7)^{2023}$
$= {}^{2023}C_0(35K)^{2023}(-7)^0 + {}^{2023}C_1(35K)^{2022}(-7) +$
$\ldots .. + \ldots \ldots . + {}^{2023}C_{2023}(-7)^{2023}$
$= 35N - 7^{2023}$.

Now, $-7^{2023} = -7 \times 7^{2022} = -7(7^2)^{1011}$

$= -7(50-1)^{1011}$

$= -7({}^{1011}C_0 50^{1011} - {}^{1011}C_1 (50)^{1010} + \ldots\ldots {}^{1011}C_{1011})$

$= -7(5\lambda - 1)$

$= -35\lambda + 7$

$\therefore$ when $(2023)^{2023}$ is divided by 35 remainder is 7

Question22

If the co-efficient of x^9 in $\left(\alpha x^3 + \frac{1}{\beta x}\right)^{11}$ and the co-efficient of x^{-9} in $\left(\alpha x - \frac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to _______.

[29-Jan-2023 Shift 1]

Answer: 1

Solution:

Coefficient of x^9 in $\left(\alpha x^3 + \frac{1}{\beta x}\right) = {}^{11}C_6 \cdot \frac{\alpha^5}{\beta^6}$

$\because$ Both are equal

$\therefore \frac{11}{C_6} \cdot \frac{\alpha^5}{\beta^6} = -\frac{11}{C_5} \cdot \frac{\alpha^6}{\beta^5}$

$\Rightarrow \frac{1}{\beta} = -\alpha$

$\Rightarrow \alpha\beta = -1$

$\Rightarrow (\alpha\beta)^2 = 1$

Question23

Let the coefficients of three consecutive terms in the binomial expansion of $(1+2x)^n$ be in the ratio 2 : 5 : 8. Then the coefficient of the term, which is in the middle of these three terms, is _______.

[29-Jan-2023 Shift 1]

Answer: 1120

Solution:

$t_{r+1} = {}^nC_r (2x)^r$

$$\Rightarrow \frac{{}^nC_{r-1}(2)^{r-1}}{{}^nC_r(2)^r} = \frac{2}{5}$$

$$\Rightarrow \frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!(2)}{r!(n-r)!}} = \frac{2}{5}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{4}{5} \Rightarrow 5r = 4n - 4r + 4$$

$\Rightarrow 9r = 4(n+1)\ldots(1)$

$$\Rightarrow \frac{{}^nC_r(2)^r}{{}^nC_{r+1}(2)^{r+1}} = \frac{5}{8}$$

$$\Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{5}{4} \Rightarrow \frac{r+1}{n-r} = \frac{5}{4}$$

$\Rightarrow 4r + 4 = 5n - 5r \Rightarrow 5n - 4 = 9r\ldots(2)$

From (1) and (2)

$\Rightarrow 4n + 4 = 5n - 4 \Rightarrow n = 8$

(1) $\Rightarrow r = 4$

so, coefficient of middle term is

$${}^8C_4 2^4 = 16 \times \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 16 \times 70 = 1120$$

Question24

Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99}K}{a} = \frac{2^l m}{n}$, where m and n are odd numbers, then the ordered pair (ℓ, n) is equal to :
[29-Jan-2023 Shift 2]

Options:

A. (50, 51)

B. (51, 99)

C. (50, 101)

D. (51, 101)

Answer: C

Solution:

In the expansion of

$(1+x)^{99} = C_0 + C_1x + C_2x^2 + \ldots. + C_{99}x^{99}$

$K = C_1 + C_3 + \ldots. + C_{99} = 2^{98}$

a$\Rightarrow$ Middle in the expansion of $\left(2 + \frac{1}{\sqrt{2}}\right)^{200}$

$$T_{\frac{200}{2}+1} = {}^{200}C_{100}(2)^{100}\left(\frac{1}{\sqrt{2}}\right)^{100}$$

$$= {}^{200}C_{100} \cdot 2^{50}$$

So, $\frac{{}^{200}C_{99} \times 2^{98}}{{}^{200}C_{100} \times 2^{50}} = \frac{100}{101} \times 2^{48}$

So, $\frac{25}{101} \times 2^{50} = \frac{m}{n} 2^{\ell}$
$\therefore$ m, n are odd so
(ℓ, n) become (50, 101) Ans.

Question25

If the coefficient of x^{15} in the expansion of $\left(ax^3 + \frac{1}{bx^{\frac{1}{3}}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{\frac{1}{3}} - \frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then for each such ordered pair (a, b) :
[30-Jan-2023 Shift 1]

Options:

A. a = b

B. ab = 1

C. a = 3b

D. ab = 3

Answer: B

Solution:

Solution:
Option (2)
Coefficient Of x^{15} in $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$
$T_{r+1} = {}^{15}C_r (ax^3)^{15-r} \left(\frac{1}{bx^{1/3}}\right)^r$
$45 - 3r - \frac{r}{3} = 15$
$30 = \frac{10r}{3}$
$r = 9$
Coefficient of $x^{15} = {}^{15}C_9 a^6 b^{-9}$
Coefficient of x^{-15} in $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$
$T_{r+1} = {}^{15}C_r (ax^{1/3})^{15-r} \left(-\frac{1}{bx^3}\right)^r$
$5 - \frac{r}{3} - 3r = -15$
$\frac{10r}{3} = 20$
$r = 6$
Coefficient $= {}^{15}C_6 a^9 \times b^{-6}$
$\Rightarrow \frac{a^9}{b^6} = \frac{a^6}{b^9}$
$\Rightarrow a^3 b^3 = 1 \Rightarrow ab = 1$

Question26

The coefficient of x^{301} in $(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \ldots\ldots + x^{500}$ is:
[30-Jan-2023 Shift 1]

Options:

A. ${}^{501}C_{302}$

B. ${}^{500}C_{301}$

C. ${}^{500}C_{300}$

D. ${}^{501}C_{200}$

Answer: D

Solution:

Solution:
$(1+x)^{500} + x(1+x)^{499} + x^2(1+x)^{498} + \ldots. + x^{500}$

$$= (1+x)^{500} \cdot \left\{ \frac{1 - \left(\frac{x}{1+x}\right)^{501}}{1 - \frac{x}{1+x}} \right\}$$

$$= (1+x)^{500} \frac{\left((1+x)^{501} - x^{501}\right)}{(1+x)^{501}} \cdot (1+x)$$

$$= (1+x)^{501} - x^{501}$$

Coefficient of x^{301} in $(1+x)^{501} - x^{501}$ is given by
${}^{501}C_{301} = {}^{501}C_{200}$

Question27

Let $x = (8\sqrt{3} + 13)^{13}$ and $y = (7\sqrt{2} + 9)^9$. If $[t]$ denotes the greatest integer $\le t$, then
[30-Jan-2023 Shift 2]

Options:

A. $[x] + [y]$ is even

B. $[x]$ is odd but $[y]$ is even

C. $[x]$ is even but $[y]$ is odd

D. $[x]$ and $[y]$ are both odd

Answer: A

Solution:

Solution:
$x = (8\sqrt{3} + 13) = {}^{13}C_0 \cdot (8\sqrt{3})^{13} + {}^{13}C_1 (8\sqrt{3})^{12} (13)^1 + \ldots$

$x' = (8\sqrt{3} - 13)^{13} = {}^{13}C_0(8\sqrt{3})^{13} - {}^{13}C_1(8\sqrt{3})^{12}(13)^1 + \ldots$

$x - x' = 2[{}^{13}C_1 \cdot (8\sqrt{3})^{12}(13)^1 + {}^{13}C_3(8\sqrt{3})^{10} \cdot (13)^3 \ldots]$

therefore, $x - x'$ is even integer, hence [x] is even

Now, $y = (7\sqrt{2} + 9)^9 = {}^9C_0(7\sqrt{2})^9 + {}^9C_1(7\sqrt{2})^8(9)^1$

$+ {}^9C_2(7\sqrt{2})^7(9)^2 \ldots\ldots.$

$y' = (7\sqrt{2} - 9)^9 = {}^9C_0(7\sqrt{2})^9 - {}^9C_1(7\sqrt{2})^8(9)^1$

$+ {}^9C_2(7\sqrt{2})^7(9)^2 \ldots\ldots.$

$y - y' = 2[{}^9C_1(7\sqrt{2})^8(9)^1 + {}^9C_3(7\sqrt{2})^6(9)^3 + \ldots]$

$y - y' =$ Even integer, hence [y] is even

Question28

50 $^{\text{th}}$ root of a number x is 12 and 50 $^{\text{th}}$ root of another number y is 18 . Then the remainder obtained on dividing (x + y) by 25 is _______.
[30-Jan-2023 Shift 2]

Answer: 23

Solution:

$x + y = 12^{50} + 18^{50} = (150 - 6)^{25} + (325 - 1)^{25}$

$= 25K - (6^{25} + 1) = 25K - ((5 + 1)^{25} + 1)$

$= 25K_1 - 2 \quad \text{Remainder} \quad = 23$

Question29

Let $\alpha > 0$, be the smallest number such that the expansion of

$$\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$$ **has a term $\beta x^{-\alpha}$, $\beta \in N$.**

Then α is equal to _______.
[31-Jan-2023 Shift 1]

Answer: 2

Solution:

$$T_{r+1} = {}^{30}C_r(x^{2/3})^{30-r}\left(\frac{2}{x^3}\right)^r$$

$$= {}^{30}C_r \cdot 2 \cdot x^{\frac{60 - 11r}{3}}$$

$\frac{60-11r}{3} < 0 \Rightarrow 11r > 60 \Rightarrow r > \frac{60}{11} \Rightarrow r = 6$

$T_7 = {}^{30}C_6 \cdot 2^6 x^{-2}$

We have also observed $\beta = {}^{30}C_6(2)^6$ is a natural number.

$\therefore \alpha = 2$

Question30

The remainder on dividing 5^{99} by 11 is ________.
[31-Jan-2023 Shift 1]

Answer: 9

Solution:

$5^{99} = 5^4 \cdot 5^{95}$
$= 625[5^5]^{19}$
$= 625[3125]^{19}$
$= 625[3124 + 1]^{19}$
$= 625[11k \times 19 + 1]$
$= 625 \times 11k \times 19 + 625$
$= 11k_1 + 616 + 9$
$= 11(k_2) + 9$
$\text{Remainder } = 9$

Question31

The Coefficient of x^{-6}, in the expansion of $\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$, is _______

[31-Jan-2023 Shift 2]

Answer: 5040

Solution:

$$\left(\frac{4x}{5} + \frac{5}{2x^2}\right)^9$$

Now, $T_{r+1} = {}^9C_r \cdot \left(\frac{4x}{5}\right)^{9-r}\left(\frac{5}{2x^2}\right)^r$

$= {}^9C_r \cdot \left(\frac{4}{5}\right)^{9-r}\left(\frac{5}{2}\right)^r \cdot x^{9-3r}$

Coefficient of x^{-6} i.e. $9 - 3r = -6 \Rightarrow r = 5$

Question32

If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^{\ell}}\right)^9$ is -84 and the Coefficient of $x^{-3\ell}$ is $2^{\alpha}\beta$, where $\beta < 0$ is an odd number, Then $|\alpha\ell - \beta|$ is equal to ________
[31-Jan-2023 Shift 2]

Answer: 98

Solution:

In, $\left(\frac{x^{\frac{5}{2}}}{2} - \frac{4}{x^{\ell}}\right)^9$

$T_{r+1} = {}^9C_r \frac{(x^{5/2})^{9-r}}{2^{9-r}}\left(\frac{-4}{x^{\ell}}\right)^r$

$= (-1)^r \frac{{}^9C_r}{2^{9-r}} 4^r x^{\frac{45}{2} - \frac{5r}{2} - r}$

$= 45 - 5r - 21r = 0$

$r = \frac{45}{5+21} \ldots (1)$

Now, according to the question, $(-1)^r \frac{{}^9C_r}{2^{9-r}} 4^r = -84$

$= (-1)^{r\,9}C_r 2^{3r-9} = 21 \times 4$

Only natural value of r possible if $3r - 9 = 0$ $r = 3$ and ${}^9C_3 = 84$

$\therefore 1 = 5$ from equation (1)

Now, coefficient of $x^{-31} = x^{\frac{45}{2} - \frac{5r}{2} - lr}$ at $1 = 5$, gives

$r = 5$

$\therefore {}^9c_5(-1)\frac{4^5}{2^4} = 2^{\alpha} \times \beta$

$= -63 \times 2^7$

$\Rightarrow \alpha = 7, \beta = -63$

$\therefore$ value of $|\alpha\ell - \beta| = 98$

Question33

The value of $\frac{1}{1!50!} + \frac{1}{3!48!} + \frac{1}{5!46!} + \ldots\ldots + \frac{1}{49!2!} + \frac{1}{51!1!}$ is
[1-Feb-2023 Shift 1]

Options:

A. $\frac{2^{50}}{50!}$

B. $\frac{2^{50}}{51!}$

C. $\frac{2^{51}}{51!}$

D. $\frac{2^{51}}{50!}$

Answer: B

Solution:

Solution:

$$\sum_{r=1}^{26} \frac{1}{(2r-1)!(51-(2r-1))!} = \sum_{r=1}^{26} {}^{51}C_{(2r-1)} \frac{1}{51!}$$
$$= \frac{1}{51!}\{{}^{51}C_1 + {}^{51}C_3 + \ldots + {}^{51}C_{51}\} = \frac{1}{51!}(2^{50})$$

Question34

The remainder when $19^{200} + 23^{200}$ is divided by 49 , is _______.
[1-Feb-2023 Shift 1]

Answer: 29

Solution:

$(21+2)^{200} + (21-2)^{200}$
$\Rightarrow 2[{}^{100}C_0 21^{200} + 200C_2 21^{198} \cdot 2^2 + \ldots.. + {}^{200}C_{198}.$
$21^2 \cdot 2^{198} + 2^{200}]$
$\Rightarrow 2[49I_1 + 2^{200}] = 49I_1 + 2^{201}$
Now , $2^{201} = (8)^{67} = (1+7)^{67} = 49I_2 + {}^{67}C_0 {}^{67}C_1 \cdot 7 =$
$49I_2 + 470 = 49I_2 + 49 \times 9 + 29$

Question35

Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2(10-3^x)}} + \sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21 . If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P., then the sum of the squares of all possible values of x is ________.
[1-Feb-2023 Shift 2]

Answer: 4

Solution:

$T_6 = {}^mC_5(10 - 3^x)^{\frac{m-5}{2}} \cdot (3^{x-2}) = 21$

${}^mC_1, {}^mC_2, {}^mC_3$ are in A.P.

$2. \; {}^mC_2 = {}^mC_1 + {}^mC_3$

Solving for m, we get

$m = 2$ (rejected), 7

Put in equation (1)

$21 \,.\, (10 - 3^x)\, \frac{3^x}{9} = 21$

$3^x = 3^0, 3^2$

$x = 0, 2$

Sum of the squares of all possible values of $x = 4$

Question36

If the term without x in the expansion of $\left(x^{\frac{2}{3}} + \frac{\alpha}{x^3}\right)^{22}$ is 7315 , then $|\alpha|$ is equal to _______.
[1-Feb-2023 Shift 2]

Answer: 1

Solution:

$T_{r+1} = {}^{22}C_r \cdot \left(x^{\frac{2}{3}}\right)^{22-r} \cdot (\alpha)^r, x^{-3r}$

$= {}^{22}C_r \cdot x^{\frac{44}{3} - \frac{2r}{3} - 3r}(\alpha)^r$

$\frac{44}{3} = \frac{11r}{3}$

$r = 4$

${}^{22}C_4 \cdot \alpha^4 = 7315$

$\frac{22 \times 21 \times 20 \times 19}{24} \cdot \alpha^4 = 7315$

$\alpha = 1$

Question37

If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left({}^4\sqrt{2} + \frac{1}{{}^4\sqrt{3}}\right)^a$ is $\sqrt{6} : 1$, then the third term from the beginning is :
[6-Apr-2023 shift 1]

Options:

A. $30\sqrt{2}$

B. $60\sqrt{2}$

C. $30\sqrt{3}$

D. $60\sqrt{3}$

Answer: D

Solution:

Solution:

$$\frac{T_5}{T_5'} = \frac{{}^nC_4 \cdot \left((2)^{\frac{1}{4}}\right)^{n-4}\left(\frac{1}{3^{\frac{1}{4}}}\right)^4}{{}^nC_4\left(\frac{1}{3^{\frac{1}{4}}}\right)^{n-4}\left(2^{\frac{1}{4}}\right)^4} = \frac{\sqrt{6}}{1}$$

$$2^{\frac{n-8}{4}} \cdot \left(3^{\frac{1}{4}}\right)^{-4-4+n} = \sqrt{6}$$

$$2^{\frac{n-8}{4}} \cdot 3^{\frac{n-8}{4}} = \sqrt{6}$$

$$\frac{n-8}{4} = \frac{1}{2} \Rightarrow n-8 = 2 \Rightarrow n = 10$$

$$T_3 = {}^{10}C_2\left(2^{\frac{1}{4}}\right)^8\left(\frac{1}{3^{\frac{1}{4}}}\right)^2$$

$$= {}^{10}C_2 \cdot 2^2 \cdot 3^{-\frac{1}{2}} = \frac{10.9}{2} \cdot 4 \cdot \frac{1}{\sqrt{3}} = 60\sqrt{3}$$

Question38

If ${}^{2n}C_3 : {}^nC_3 : 10 : 1$, then the ratio $(n^2 + 3n) : (n^2 - 3n + 4)$ is :
[6-Apr-2023 shift 1]

Options:

A. 27 : 11

B. 35 : 16

C. 2 : 1

D. 65 : 37

Answer: C

Solution:

Solution:

$$\frac{{}^{2n}C_3}{{}^nC_3} = 10 \Rightarrow \frac{2n!(n-3)!}{(2n-3)!n!} = 10$$

$$\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = 10$$
$$\frac{4(2n-1)}{n-2} = 10 \Rightarrow 8n - 4 = 10n - 20$$
$2n = 16$

Now $\frac{n^2 + 3n}{n^2 - 3n + 4}$

$= \frac{64 + 24}{64 - 24 + 4} = \frac{88}{44} = 2$

Question39

The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is ______.

[6-Apr-2023 shift 1]

Answer: 5005

Solution:

$$\left(x^4 - \frac{1}{x^3}\right)^{15}$$

$$T_{r+1} = {}^{15}C_r (x^4)^{15-r} \left(\frac{-1}{x^3}\right)^r$$

$60 - 7r = 18$

$r = 6$

Hence coeff. of $x^{18} = {}^{15}C_6 = 5005$

Question40

If the coefficients of x^7 in $\left(ax^2 + \frac{1}{2bx}\right)^{11}$ and x^{-7} in $\left(ax - \frac{1}{3bx^2}\right)^{11}$ are equal, then :

[6-Apr-2023 shift 2]

Options:

A. $64\,ab = 243$

B. $32\,ab = 729$

C. $729ab = 32$

D. $243\,ab = 64$

Answer: C

Solution:

Solution:

$\left(ax^2 + \frac{1}{2bx}\right)^{11}$

$r = \frac{11 \times 2 - 7}{3} = 5$

Coefficient of x^7 is $= {}^{11}C_5(a)^6\left(\frac{1}{2b}\right)^5$

$\left(ax - \frac{1}{3bx^2}\right)^{11}$

$r = \frac{11 \times 1 - (-7)}{3} = 6$

Coefficient of x^{-7} is $= {}^{11}C_6 \cdot \frac{a5}{3^6b^6}$

$\because {}^{11}C_5(a^6)\left(\frac{1}{2^5b^5}\right) = {}^{11}C_6 \cdot \frac{a5}{3^6b^6}$

$\Rightarrow ab = \frac{2^5}{3^6}$

$\Rightarrow 729\,ab = 32$

Ans. Opiton 3

Question41

Among the statements :
(S1) : $2023^{2022} - 1999^{2022}$ is divisible by 8
(S2) : $13(13)^n - 11n - 13$ is divisible by 144 for infinitely many $n \in \mathbb{N}$
[6-Apr-2023 shift 2]

Options:

A. only (S2) is correct

B. only (S1) is correct

C. both (S1) and (S2) are incorrect

D. both (S1) and (S2) are correct

Answer: D

Solution:

Solution:

$\because x^n - y^n = (x - y)[x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \ldots\ldots + y^{n-1}]$

$x^n - y^n$ is divisible by $x - y$

Stat 1 $\rightarrow$ $(2023)^{2022} - (1999)^{2022}$

$(2023) - (1999) = 24$

Stat 2 $\rightarrow$ $(2023)^{2022} - (1999)^{2022}$

is divisible by 8

$13(1 + 12)^n - 11n - 13$

$13[1 + {}^nC_1, (12) + {}^nC_2(12)^2 + \ldots] - 11n - 13$

$\Rightarrow (156n - 11n) + 13 \cdot {}^nC_2(12)^2 + 13 \cdot {}^nC_3(12)^3 + \ldots$

$\Rightarrow 145n + 13 \cdot {}^nC_2(12)^2 + 13 \cdot {}^nC_3(12)^3 + \ldots$

If ($n = 144m, m \in N$) then it is divisible by 144 for infinite values of n.

Ans. Option 4

Question42

Let (t) denote the greatest integer ≤t, If the constant term in the expansion of $\left(3x^2 - \frac{1}{2x^5}\right)^7$ is α, then [α] is equal to _______.
[8-Apr-2023 shift 1]

Answer: 1275

Solution:

Solution:

$\left(3x^2 - \frac{1}{2x^5}\right)^7$

$T_{r+1} = {}^7C_r(3x^2)^{7-r}\left(-\frac{1}{2x^5}\right)^r$

$14 - 2r - 5r = 14 - 7r = 0$

$\therefore r = 2$

$\therefore T_3 = {}^7C_2 \cdot 3^5\left(-\frac{1}{2}\right)^2 = \frac{21 \times 243}{4} = 1275.75$

$\therefore [\alpha] = 1275$

Question43

If a_a is the greatest term in the sequence $a_n = \frac{n^3}{n^4 + 147}$, n = 1, 2, 3, ..., then a is equal to ______.
[8-Apr-2023 shift 1]

Options:

A.

Answer: 5

Solution:

$$f(x) = \frac{x^3}{x^4 + 147}$$

$$f'(x) = \frac{(x^4 + 147)3x^2 - x^3(4x^3)}{(x^4 + 147)^2}$$

$$= \frac{3x^6 + 147 \times 3x^2 - 4x^6}{+ve} = x^2(44 - x^4)$$

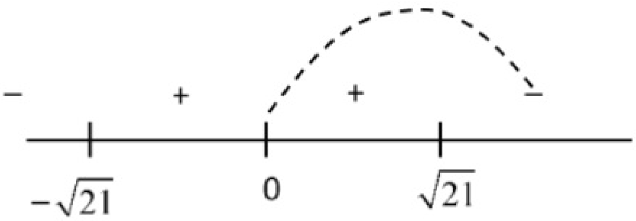
$f'(x) = 0$ at $x^6 = 147 \times 3x^2$

$x^2 = 0, x^4 = 147 \times 3$

$x = 0, x^2 = \pm\sqrt{147 \times 3}$

$x^2 = \pm 21$

$x = \pm\sqrt{21}$



fmax at f (4) or f (5)

$f(4) = \frac{64}{403} \simeq 0.158$ $f(5) = \frac{125}{772} \simeq 0.161$

$\therefore a = 5$

Question44

The largest natural number n such that 3^n divides 66 ! is _______.
[8-Apr-2023 shift 1]

Answer: 31

Solution:

Solution:

$\left[\frac{66}{3}\right] + \left[\frac{66}{9}\right] + \left[\frac{66}{27}\right]$

$22 + 7 + 2 = 31$

Question45

The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is equal to
[8-Apr-2023 shift 2]

Options:

A. $10^3 - 10$

B. $11^3 - 11$

C. $12^3 - 12$

D. $13^3 - 13$

Answer: C

Solution:

Solution:

$T_{r+1} = {}^{11}C_r(2x^2)^{11-r}\left(\frac{1}{2x}\right)^r$
$= {}^{11}C_r 2^{11-2r} x^{22-3r}$
$22 - 3r = 10$ and $22 - 3r = 7$
$r = 4$ and $r = 5$
Coefficient of $x^{10} = {}^{11}C_4 \cdot 2^3$
Coefficient of $x^7 = {}^{11}C_5 \cdot 2^1$
difference $= {}^{11}C_4 \cdot 2^3 - {}^{11}C_5 \cdot 2$
$= \frac{11 \times 10 \times 9 \times 8}{24} \times 8 - \frac{11 \times 10 \times 9 \times 8 \times 7}{120} \times 2$
$= 11 \times 10 \times 3 \times 8 - 11 \times 3 \times 4 \times 7$
$= 11 \times 3 \times 4 \times (20 - 7)$
$= 11 \times 12 \times 13$
$= 12(12 - 1)(12 + 1)$
$= 12(12^2 - 1)$
$= 12^3 - 12$(Option 3)

Question46

$25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by
[8-Apr-2023 shift 2]

Options:

A. 34 but not by 14

B. 14 but not by 34

C. Both 14 and 34

D. Neither 14 nor 34

Answer: A

Solution:

Solution:
$25^{190} - 8^{190}$ is divisible by $25 - 8 = 17$
$19^{190} - 2^{190}$ is divisible by $19 - 2 = 17$
$25^{190} - 19^{190}$ is divisible by $25 - 19 = 6$
$8^{190} - 2^{190}$ is divisible by $8 - 2 = 6$
L.C.M. of 1746 = 34
∴ divisible by 34 but not by 14

Question47

If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal, then a^4b^4 is equal to :
[10-Apr-2023 shift 1]

Options:

A. 22

B. 44

C. 11

D. 33

Answer: A

Solution:

Solution:

$\left(ax - \frac{1}{bx^2}\right)^{13}$

We have,

$T_{r+1} = {}^nC_r (p)^{n-r} (q)^r$

$T_{r+1} = {}^{13}C_r (ax)^{13-r} \left(-\frac{1}{bx^2}\right)^r$

$= {}^{13}C_r (a)^{13-r} \left(-\frac{1}{b}\right)^r (x)^{13-r} \cdot (x)^{-2r}$

$= {}^{13}C_r (a)^{13-r} \left(-\frac{1}{b}\right)^r (x)^{13-3r} \ldots (1)$

Coefficient of x^7

$\Rightarrow 13 - 3r = 7$

$r = 2$

r in equation (1)

$T_3 = {}^{13}C_2 (a)^{13-2} \left(-\frac{1}{b}\right)^2 (x)^{13-6}$

$= {}^{13}C_2 (a)^{11} \left(\frac{1}{b}\right)^2 (x)^7$

Coefficient of x^7 is ${}^{13}C_2 \frac{(a)^{11}}{b^2}$

Now, $\left(ax + \frac{1}{bx^2}\right)^{13}$

$T_{r+1} = {}^{13}C_r (ax)^{13-r} \left(\frac{1}{bx^2}\right)^r$

$= {}^{13}C_r (a)^{13-r} \left(\frac{1}{b}\right)^r (x)^{13-r} (x)^{-2r}$

$= {}^{13}C_r (a)^{13-r} \left(\frac{1}{b}\right)^r (x)^{13-3r} \ldots (2)$

Coefficient of x^{-5}

$\Rightarrow 13 - 3r = -5$

$r = 6$

r in equation

$T_7 = {}^{13}C_6 (a)^{13-6} \left(\frac{1}{b}\right)^6 (x)^{13-18}$

$T_7 = {}^{13}C_6 (a)^7 \left(\frac{1}{b}\right)^6 (x)^{-5}$

Coefficient of x^{-5} is ${}^{13}C_6 (a)^7 \left(\frac{1}{b}\right)^6$

ATQ

Coefficient of x^7 = coefficient of x^{-5}

$T_3 = T_7$

${}^{13}C_2 \left(\frac{a^{11}}{b^2}\right) = {}^{13}C_6 (a)^7 \left(\frac{1}{b}\right)^6$

$a^4 \cdot b^4 = \frac{{}^{13}C_6}{{}^{13}C_2}$

$= \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 1}{13 \times 12 \times 6 \times 5 \times 4 \times 3} = 22$

Question48

The coefficient of x^7 in $(1-x+2x^3)^{10}$ is _______.
[10-Apr-2023 shift 1]

Answer: 960

Solution:

$(1-x+2x^3)^{10}$

$T_n = \frac{10!}{a!b!c!}(-2x)^b(x^3)^c$

$= \frac{10!}{a!b!c!}(-2)^b x^{b+3c}$

$\Rightarrow b+3c=7,\ a+b+c=10$

$\therefore$ Coefficient of $x^7 = \frac{10!}{3!7!0!}(-1)^7 + \frac{10!}{5!4!1!}(-1)^4(2)$

$+ \frac{10!}{7!1!2!}(-1)^1(2)^2$

$= -120 + 2520 - 1440 = 960$

Question49

Let the number $(22)^{2022}+(2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7 . Then ($\alpha^{2\cdot}+\beta^2$) is equal to
[10-Apr-2023 shift 2]

Options:

A. 13

B. 20

C. 10

D. 5

Answer: D

Solution:

Solution:
$(22)^{2022}+(2022)^{22}$
divided byy 3
$(21+1)^{2022}+(2022)^{22}$
$=3k+1$
$(\alpha=1)$
Divided by 7
$(21+1)^{2022}+(2023-1)^{22}$
$7k+1+1\ (\beta=2)$
$7k+2$
So $\alpha^2+\beta^2 \Rightarrow 5$

Question50

If the coefficients of x and x^2 in $(1 + x)^p(1 - x)^q$ are 4 and -5 respectively, then $2p + 3q$ is equal to
[10-Apr-2023 shift 2]

Options:

A. 60

B. 63

C. 66

D. 69

Answer: B

Solution:

Solution:

$(1 + x)^p(1 - x)^q$

$\left(1 + px + \frac{p(p-1)}{2!}x^2 + \ldots\right)$

$\left(1 - qx + \frac{q(q-1)}{2!}x^2 - \ldots\right)$

$p - q = 4$

$\frac{p(p-1)}{2} + \frac{q(q-1)}{2} - pq = -5$

$p^2 + q^2 - p - q - 2pq = -10$

$(q + 4)^2 + q^2 - (q + 4) - q - 2(4 + q)q = -10$

$q^2 + 8q + 16 - q^2 - q - 4 - q - 8q - 2q^2 = -10$

$-2q = -22$

$q = 11$

$p = 15$

$2(15) + 3(11)$

$30 + 33 = 63$

Question51

The number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to :
[11-Apr-2023 shift 1]

Answer: 171

Solution:

Solution:

The number of integral term in the expression of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to

General term $= {}^{680}C_r \left(3^{\frac{1}{2}}\right)^{680-r} \left(5^{\frac{1}{4}}\right)^{r}$

$= {}^{680}C_r 3^{\frac{680-r}{2}} 5^{\frac{r}{4}}$

Values' s of r, where $\frac{r}{4}$ goes to integer

$r = 0, 4, 8, 12, \ldots\ldots 680$

All value of r are accepted for $\frac{680-r}{2}$ as well so

No of integral terms $= 171$.

Question52

The mean of the coefficients of $x, x^2, \ldots x^7$ in the binomial expansion of $(2+x)^9$ is ________.
[11-Apr-2023 shift 1]

Answer: 2736

Solution:

Solution:

Coefficient of $x = {}^9C_1 2^8$

Coef. $x^2 = {}^9C_2 2^7$

Coef. $x^7 = {}^9C_7 \cdot 2^2$

$$\text{Mean} = \frac{{}^9C_1 \cdot 2^8 + {}^9C_2 \cdot 2^7 \ldots + {}^9C_7 \cdot 2^2}{7}$$

$$= \frac{(1+2)^9 - {}^9C_0 \cdot 2^9 - {}^9C_8 \cdot 2^1 - {}^9C_9}{7}$$

$$= \frac{3^9 - 2^9 - 18 - 1}{7}$$

$$= \frac{19152}{7} = 2736$$

Question53

If the 1011^{th} term from the end in the binominal expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times 1011^{th} term from the beginning, the $|x|$ is equal to
[11-Apr-2023 shift 2]

Options:

A. 8

B. 12

C. 10

D. 15

Answer: C

Solution:

Solution:

T_{1011} from beginning $= T_{1010+1}$

$= {}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}$

T_{1011} from end

$= {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

Given: $= {}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1012}\left(\frac{4x}{5}\right)^{1010}$

$= 2^{10}\cdot{}^{2022}C_{1010}\left(\frac{-5}{2x}\right)^{1010}\left(\frac{4x}{5}\right)^{1012}$

$\left(\frac{-5}{2x}\right)^{2} = 2^{10}\left(\frac{4x}{5}\right)^{2}$

$x^4 = \frac{5^4}{2^{16}}$

$|x| = \frac{5}{16}$

Question54

The sum of the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+2}$, which are in the ratio 1 : 3 : 5, is equal to [11-Apr-2023 shift 2]

Options:

A. 63

B. 92

C. 25

D. 41

Answer: A

Solution:

Solution:

${}^{n+2}C_{r-1} : {}^{n+2}C_{r} : {}^{n+2}C_{r+1} : : 1 : 3 : 5$

$\frac{(n+2)!}{(r-1)!(n-r+3)!}\times\frac{r!(n+2-r)!}{(n+2)!} = \frac{1}{3}$

$\frac{r}{(n-r+3)} = \frac{1}{3} \Rightarrow n-r+3 = 3r$

$n = 4r-3-0$

$\frac{(n+1)!}{r!(n+2-r)!}\times\frac{(r+1)!(n-r+1)!}{(n+2)!} = \frac{3}{5}$

$\frac{r+1}{n+2-r} = \frac{3}{5}$

$8r-1 = 3n\ldots(2)$

By equation 1 and 2

$\frac{8r-1}{3} = 4r-3\ n = 4r-3$

$r = 2$ $n = 4 \times 2 - 3$
$n = 5$
Sum: ${}^{7}C_{1} + {}^{7}C_{2} + {}^{7}C_{3} = 7 + 21 + 35 = 63$

Question55

If $\frac{1}{n+1}{}^{n}C_{n} + \frac{1}{n}{}^{n}C_{n-1} + \ldots + \frac{1}{2}{}^{n}C_{1} + {}^{n}C_{0} = \frac{1023}{10}$ then n is equal to [12-Apr-2023 shift 1]

Options:

A. 7

B. 9

C. 6

D. 8

Answer: B

Solution:

Solution:

$$\sum_{T=0}^{n} \frac{{}^{n}C_{r}}{r+1} = \frac{1}{n+1} \sum_{r=0^{n}}^{n+1} C_{r+1}$$

$$= \frac{1}{n+1}(2^{n+1} - 1) = \frac{1023}{10}$$

$n + 1 = 10 \Rightarrow n = 9$

Question56

The sum, of the coefficients of the first 50 terms in the binomial expansion of $(1 - x)^{100}$, is equal to [12-Apr-2023 shift 1]

Options:

A. $-{}^{101}C_{50}$

B. ${}^{99}C_{49}$

C. ${}^{101}C_{50}$

D. $-{}^{99}C_{49}$

Answer: D

Solution:

Solution:

$(1 - x)^{100} = C_{0} - C_{1}x + C_{2}x^{2} -$

$C_3x^3 + \ldots\ldots C_{99}x^{99} + C_{100}x^{100}$
$\Rightarrow C_0 - C_1 + C_2 - C_3 + \ldots - C_{99} + C_{100} = 0$
$C_0 - C_1 + C_2 + \ldots C_{99} = -\frac{1}{2}{}^{100}C_{50}$
$-\frac{1}{2}\frac{100!}{50!50!} = -\frac{1}{2}\times\frac{100\times 99!}{50!50!} = -{}^{99}C_{49}$

Question57

Fractional part of the number is $\frac{4^{2022}}{15}$ equal to
[13-Apr-2023 shift 1]

Options:

A. $\frac{4}{15}$

B. $\frac{8}{15}$

C. $\frac{1}{15}$

D. $\frac{14}{15}$

Answer: C

Solution:

Solution:
$\left\{\frac{4^{2022}}{15}\right\} = \left\{\frac{2^{4044}}{15}\right\} = \left\{\frac{(1+15)^{1011}}{15}\right\} = \frac{1}{15}$

Question58

Let α be the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{6}{x^{\frac{3}{2}}}\right)^n$, n ≤ 15. If the sum of the coefficients of the remaining terms in the expansion is 649 and the coefficient of x^{-n} is λα, then λ is equal to _______.
[13-Apr-2023 shift 1]

Answer: 36

Solution:

$T_{k+1} = {}^nC_k(x)^{\frac{n-k}{2}}(-6)^k(x)^{\frac{-3}{2}k}$

$\frac{n-k}{2} - \frac{3}{2}k = 0$

$n - 4k = 0$

$(-5)^n - \left({}_nC_{\frac{n}{4}}(-6)^{\frac{n}{4}} \right) = 649$

By observation $(625 + 24 = 649)$, we get $n = 4$

$\because n = 4 \& k = 1$

Required is coefficient of

x^{-4} is $\left(\sqrt{4} - \frac{6}{x^{\frac{3}{2}}} \right)^4$

${}^4C_1(-6)^3$

By calculating we will get $\lambda = 36$

Question59

Let for $x \in R$, $S_0(x) = x$, $S_k(x) = C_k x + k\int_0^x S_{k-1}(t)\,dt$ where $C_0 = 1$, $C_k = 1 - \int_0^1 S_{k-1}(x)\,dx$, $k = 1, 2, 3, \ldots$ Then $S_2(3) + 6C_3$ is equal to ______.

[13-Apr-2023 shift 1]

Answer: 18

Solution:

Solution:

Given, $S_k(x) = C_k x + k\int_0^x S_{k-1}(t)\,dt$

put $k = 2$ and $x = 3$

$S_2(3) = C_2(3) + 2\int_0^3 S_1(t)\,dt$

Also, $S_1(x) = C_1(x) + \int_0^x S_0(t)\,dt \ldots\ldots (1)$

$= C_1 x + \frac{x^2}{2}$

$S_2(3) = 3C_2 + 2\int_0^3 \left(C_1 t + \frac{t^2}{2} \right) dt$

$= 3C_2 + 9C_1 + 9$

Also,

$C_1 = 1 - \int_0^1 S_0(x)\,dx = \frac{1}{2}$

$C_2 = 1 - \int_0^1 S_1(x)\,dx = 0$

$C_3 = 1 - \int_0^1 S_2(x)\,dx$

$= 1 - \int_0^1 \left(C_2 x + C_1 x^2 + \frac{x^3}{3} \right) dx = \frac{3}{4}$

$S_2(x) = C_2 x + 2\int_0^x S_1(t)\,dt$

$= C_2 x + C_1 x^2 + \frac{x^3}{3}$

$\Rightarrow S_2(3) + 6C_3 = 6C_3 + 3C_2 + 9C_1 + 9 = 18$

Question60

The coefficient of x^5 in the expansion of $\left(2x^3 - \frac{1}{3x^2}\right)^5$ is [13-Apr-2023 shift 2]

Options:

A. $\frac{80}{9}$

B. 8

C. 9

D. $\frac{26}{3}$

Answer: A

Solution:

Solution:

general term for $\left(2x^3 - \frac{1}{3x^2}\right)^5$

$T_{r+1} = {}^5C_r\left(-\frac{1}{3x^2}\right)^r (2x^3)^{5-r}$

${}^5C_r(-1)^r 3^{-r} 2^{5-r} \cdot x^{15-5r}$

$15 - 5r = 5 \Rightarrow r = 2$

Coeff. Of $x^5 = {}^5C_2(-1)^2 3^{-2} 2^3$

$= 10 \times \frac{1}{9} \times 8$

$= \frac{80}{9}$

Question61

The remainder, when 7^{103} is divided by 17 , is ______. [13-Apr-2023 shift 2]

Answer: 12

Solution:

$7^{103} = 7.7^{102}$

$= 7(7^2)^{51}$

$= 7(51-2)^{51} \rightarrow$ remainder $= 7(-2)^{51}$

$-7(2^3)(16)^{12} = -56(17-1)^{12} \rightarrow$ remainder $= -56(-1)^{12}$
Remainder $= -56 + 17k$
$= -56 + 68$
$= 12$

Question62

Let $(a + bx + cx^2)^{10} = \sum_{i=0}^{20} p_i x^i$, a, b, c $\in$ N. If $p_1 = 20$ and $p_2 = 210$, then $2(a + b + c)$ is equal to
[15-Apr-2023 shift 1]

Options:

A. 8

B. 12

C. 6

D. 15

Answer: B

Solution:

Solution:

$(a + bx + cx^2)^{10} = \sum_{i=0}^{20} p_i x^i$

Coefficient of $x^1 = 20$

$20 = \frac{10!}{9!1!} \times a^9 \times b^1$

$a^9 \cdot b = 2$

$a = 2, b = 2$

Coefficient of $x^2 = 210$

$210 = \frac{10!}{9!1!} \times a^9 \times c^1 + \frac{10!}{8!2!} \times a^8 b^2$

$210 = 10 . c + 45 \times 4$

$10c = 30$

$c = 3$

$2(a + b = c) = 12$

Question63

The remainder when 3^{2022} is divided by 5 is :
[24-Jun-2022-Shift-1]

Options:

A. 1

B. 2

C. 3

D. 4

Answer: D

Solution:

3^{2022}

$= (3^2)^{1011}$

$= (9)^{1011}$

$= (10-1)^{1011}$

$= {}^{1011}C_0(10)^{1011} + \ldots\ldots + {}^{1011}C_{1010} \cdot (10)^1 - {}^{1011}C_{1011}$

$= 10[{}^{1011}C_0(10)^{1010} + \ldots\ldots + {}^{1011}C_{1010}] - 1$

$= 10K - 1$

[As $10[{}^{1011}C_0 \cdot (10)^{1010} + \ldots\ldots + {}^{1011}C_{1010}]$ is multiple of 10]

$= 10K + 5 - 5 - 1$

$= 10K - 5 + 5 - 1$

$= 5(2K-1) + 4$

$\therefore$ Unit digit $= 4$ when divided by 5.

Question64

The remainder on dividing $1 + 3 + 3^2 + 3^3 + \ldots.. + 3^{2021}$ by 50 is__
[24-Jun-2022-Shift-2]

Answer: 4

Solution:

Given,

$1+3+3^2+3^3+\ldots\ldots+3^{2021}$

$=3^0+3^1+3^2+3^3+\ldots+3^{2021}$

This is a G.P with common ratio $=3$

$\therefore$ Sum $=\dfrac{1(3^{2022}-1)}{3-1}$

$=\dfrac{3^{2022}-1}{2}$

$=\dfrac{(3^2)^{2011}-1}{2}$

$=\dfrac{(10-1)^{1011}-1}{2}$

$=\dfrac{[{}^{1011}C_0\cdot 10^{1011}-{}^{1011}C_1\cdot 10^{1010}+\ldots\ldots-{}^{1011}C_{1009}\cdot(10)^2+{}^{1011}C_{1010}\cdot 10-{}^{1011}C_{1011}]-1}{2}$

$=\dfrac{10^2[{}^{1011}C_0\cdot(10)^{1009}-{}^{1011}C_1\cdot(1008)+\ldots\ldots{}^{1011}C_{1009}]+10110-1-1}{2}$

$=\dfrac{100k+10110-2}{2}$

$=\dfrac{100k+10108}{2}$

$=50k+5054$

$=50k+50\times 101+4$

$=50[k+101]+4$

$=50k\,.\;k$

$\therefore$ By dividing 50 we get remainder as 4 .

Question65

Let C_r denote the binomial coefficient of x^r in the expansion of $(1+x)^{10}$.
If for $\alpha, \beta \in R$, $C_1 + 3.2C_2 + 5.3C_3 + \ldots\ldots$ upto 10 terms

$= \frac{\alpha \times 2^{11}}{2^{\beta} - 1} \left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots \text{ upto 10 terms} \right)$ **then the value of $\alpha + \beta$ is equal to**
[25-Jun-2022-Shift-1]

Answer: 286

Solution:

$(1+x)^{10} = C_0 + C_1 x + C_2 x^2 + \ldots\ldots + C_{10} x^{10}$

Differentiating

$10(1+x)^9 = C_1 + 2C_2 x + 3C_3 x^2 + \ldots + 10C_{10} x^9$

replace $x \to x^2$

$10(1+x^2)^9 = C_1 + 2C_2 x^2 + 3C_3 x^4 + \ldots + 10C_{10} x^{18}$

$10 \cdot x(1+x^2)^9 = C_1 x + 2C_2 x^3 + 3C_3 x^5 + \ldots + 10C_{10} x^{19}$

Differentiating

$10((1+x^2)^9 \cdot 1 + x \cdot 9(1+x^2)^8 2x)$

$= C_1 x + 2C_2 \cdot 3x^3 + 3 \cdot 5 \cdot C_3 x^4 + \ldots + 10 \cdot 19 C_{10} x^{18}$

putting $x = 1$

$10(2^9 + 18 \cdot 2^8)$

$= C_1 + 3 \cdot 2 \cdot C_2 + 5 \cdot 3 \cdot C_3 + \ldots + 19 \cdot 10 \cdot C_{10}$

$C_1 + 3 \cdot 2 \cdot C_2 + \ldots\ldots + 19 \cdot 10 \cdot C_{10}$

$= 10 \cdot 2^9 \cdot 10 = 100 \cdot 2^9$

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots + \underbrace{\frac{C_9}{11}}_{10^{\text{th}} \text{ term}} + \underbrace{\frac{C_{10}}{11}}_{11^{\text{th}} \text{ term}} = \frac{2^{11} - 1}{11}$$

$$C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots\ldots + \frac{C_9}{11} = \frac{2^{11} - 2}{11}$$

Now, $100 \cdot 2^9 = \frac{\alpha \cdot 2^{11}}{2^{\beta} - 1} \left(\frac{2^{11} - 2}{11} \right)$

Eqn. of form $y = k(2^x - 1)$

It has infinite solutions even if we take $x, y \in N$.

Question66

The coefficient of x^{101} in the expression $(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+...+x^{500}$, $x>0$, is [25-Jun-2022-Shift-2]

Options:

A. ${}^{501}C_{101}(5)^{399}$

B. ${}^{501}C_{101}(5)^{400}$

C. ${}^{501}C_{100}(5)^{400}$

D. ${}^{500}C_{101}(5)^{399}$

Answer: A

Solution:

Given,

$(5+x)^{500}+x(5+x)^{499}+x^2(5+x)^{498}+\ldots\ldots x^{500}$

This is a G.P. with first term $(5+x)^{500}$

Common ratio $=\dfrac{x(5+x)^{499}}{(5+x)^{500}}=\dfrac{x}{5+x}$ and 501 terms present.

$$\therefore \text{ Sum } = \frac{(5+x)^{500}\left(\left(\frac{x}{5+x}\right)^{501}-1\right)}{\frac{x}{5+x}-1}$$

$$=\frac{(5+x)^{500}\left(\frac{x^{501}-(5+x)^{501}}{(5+x)^{501}}\right)}{\frac{x-5-x}{5+x}}$$

$$=\frac{\frac{x^{501}-(5+x)^{501}}{5+x}}{\frac{-5}{5+x}}$$

$$=\frac{1}{5}\left((5+x)^{501}-x^{501}\right)$$

Coefficient of x^{101} in $(5+x)^{501}$ is $={}^{501}C_{101}\cdot 5^{400}$

$\therefore$ln $\frac{1}{5}((5+x)^{500}-x^{501})$ coefficient of x^{101} is $=\frac{1}{5}\cdot {}^{501}C_{101}\cdot 5^{400}$

$={}^{501}C_{101}\cdot 5^{399}$

Question67

If the sum of the co-efficient of all the positive even powers of x in the binomial expansion of $\left(2x^3 + \frac{3}{x}\right)^{10}$ is $5^{10} - \beta.3^9$, then β is equal to____ [25-Jun-2022-Shift-2]

Answer: 83

Solution:

Given, Binomial Expansion

$\left(2x^3 + \frac{3}{x}\right)^{10}$

General term

$T_{r+1} = {}^{10}C_r \cdot (2x^3)^{10-r} \cdot \left(\frac{3}{x}\right)^r$

$= {}^{10}C_r \cdot 2^{10-r} \cdot 3^r \cdot x^{30-3r} \cdot x^{-r}$

$= {}^{10}C_r \cdot 2^{10-r} \cdot 3^r \cdot x^{30-4r}$

For positive even power of x, $30 - 4r$ should be even and positive.

For $r = 0$, $30 - 4 \times 0 = 30$ (even and positive)

For $r = 1$, $30 - 4 \times 1 = 26$ (even and positive)

For $r = 2$, $30 - 4 \times 2 = 22$ (even and positive)

For $r = 3$, $30 - 4 \times 3 = 18$ (even and positive)

For $r = 4$, $30 - 4 \times 4 = 14$ (even and positive)

For $r = 5$, $30 - 4 \times 5 = 10$ (even and positive)

For $r = 6$, $30 - 4 \times 6 = 6$ (even and positive)

For $r = 7$, $30 - 4 \times 7 = 2$ (even and positive)

For $r = 8$, $30 - 4 \times 8 = -2$ (even but not positive)

So, for $r = 1, 2, 3, 4, 5, 6$ and 7 we can get positive even power of x.

$\therefore$ Sum of coefficient for positive even power of

$x = {}^{10}C_0 \cdot 2^{10} \cdot 3^0 + {}^{10}C_{1.2}{}^9 \cdot 3^1 + {}^{10}C_2 \cdot 2^8 \cdot 3^2 + {}^{10}C_3 \cdot 2^7 \cdot 3^3 + {}^{10}C_4 \cdot 2^6 \cdot 3^4 + {}^{10}C_5 \cdot 2^5 \cdot 3^5 + {}^{10}C_6 \cdot 2^4 \cdot 3^6 + {}^{10}C_7 \cdot 2^3 \cdot 3^7$

$= {}^{10}C_{10} \cdot 2^{10} \cdot 3^0 + {}^{10}C_{1.2}{}^9 \cdot 3^1 + \ldots\ldots + {}^{10}C_{10} \cdot 2^0 \cdot 3^{10} - [{}^{10}C_8 \cdot 2^2 \cdot 3^8 + {}^{10}C_9 \cdot 2 \cdot 3^9 + {}^{10}C_{10} \cdot 2^0 \cdot 3^{10}]$

$= (2 + 3)^{10} - [45.4.3^8 + 10.2.3^9 + 1.1.3^{10}]$

$= 5^{10} - [60 \times 3^9 + 20 \cdot 3^9 + 3 \cdot 3^9]$

$= 5^{10} - (60 + 20 + 3)3^9$

$= 5^{10} - 83.3^9$

$\therefore \beta = 83$

Question68

The remainder when $(2021)^{2023}$ is divided by 7 is :
[26-Jun-2022-Shift-1]

Options:

A. 1
B.2
C.5
D.6

Answer: C

Solution:

$(2021)^{2023}$

$= (2016+5)^{2023}$[here 2016 is divisible by 7]

$= {}^{2023}C_0(2016)^{2023} + \ldots\ldots\ldots + {}^{2023}C_{2022}(2016)(5)^{2022} + {}^{2023}C_{2023}(5)^{2023}$

$= 2016[{}^{2023}C_0 \cdot (2016)^{2022} + \ldots\ldots + {}^{2023}C_{2022} \cdot (5)^{2022}] + (5)^{2023}$

$= 2016\lambda + (5)^{2023}$

$= 7 \times 288\lambda + (5)^{2023}$

$= 7K + (5)^{2023} \ldots\ldots (1)$

Now, $(5)^{2023}$

$= (5)^{2022} \cdot 5$

$= (5^3)^{674} \cdot 5$

$= (125)^{674} \cdot 5$

$= (126-1)^{674} \cdot 5$

$= (126-1)^{674} \cdot 5$

$= 5[{}^{674}C_0(126)^{674} + \ldots\ldots\ldots - {}^{674}C_{673}(126) + {}^{674}C_{674}]$

$= 5 \times 126[{}^{674}C_0(126)^{673} + \ldots\ldots - {}^{674}C_{673}] + 5$

$= 5.7.18[{}^{674}C_0(126)^{673} + \ldots\ldots - {}^{674}C_{673}] + 5$

$= 7\lambda + 5$

Replacing $(5)^{2023}$ in equation (1) with $7\lambda + 5$, we get,

$(2021)^{2023} = 7K + 7\lambda + 5$

$= 7(K+\lambda) + 5$

$\therefore$ Remainer $= 5$

Question69

If $({}^{40}C_0) + ({}^{41}C_1) + ({}^{42}C_2) + \ldots\ldots + ({}^{60}C_{20}) = \frac{m}{n}{}^{60}C_{20}$, m and n are coprime, then m + n is equal to____
[26-Jun-2022-Shift-2]

Answer: 102

Solution:

Here property used is

${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$

Given, ${}^{40}C_0 + {}^{41}C_1 + {}^{42}C_2 + \ldots + {}^{60}C_{20} = \frac{m}{n}{}^{60}C_{20}$

As ${}^{40}C_0 = {}^{41}C_0 = 1$

So, we replace ${}^{40}C_0$ with ${}^{41}C_0$.

$\Rightarrow {}^{41}C_0 + {}^{41}C_1 + {}^{42}C_2 + \ldots\ldots + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$

$\Rightarrow {}^{42}C_1 + {}^{42}C_2 + \ldots\ldots + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$

$\Rightarrow {}^{43}C_2 + {}^{43}C_3 + \ldots\ldots + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$

$\Rightarrow {}^{44}C_3 + {}^{44}C_4 + \ldots\ldots + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$

$\Rightarrow {}^{45}C_4 + {}^{45}C_5 + \ldots\ldots + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$

$\vdots$

$\Rightarrow {}^{60}C_{19} + {}^{60}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$

$\Rightarrow {}^{61}C_{20} = \frac{m}{n} \cdot {}^{60}C_{20}$

$\Rightarrow \frac{61!}{20!41!} = \frac{m}{n} \cdot \frac{60!}{20!40!}$

$\Rightarrow \frac{61}{41} = \frac{m}{n}$

$\therefore m = 61$ and $n = 41$

$\therefore m + n = 61 + 41 = 102$

Question70

If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k \cdot l$, where I, k ∈ N and I is co-prime to 5 , then k is equal to
[27-Jun-2022-Shift-1]

Answer: 5

Solution:

Given Binomial Expansion

$= \left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{x}}{5^{\frac{1}{3}}}\right)^{60}$

$\therefore$ General term

$$T_{r+1} = {}^{60}C_r \cdot \left(\frac{x^{1/2}}{5^{1/4}} \right)^{60-r} \cdot \left(\frac{5^{1/2}}{x^{1/3}} \right)^r$$

$$= {}^{60}C_r \cdot 5^{\left(\frac{r}{4} - 15 + \frac{r}{2} \right)} \cdot x^{\left(30 - \frac{r}{2} - \frac{r}{3}\right)}$$

$$= {}^{60}C_r \cdot 5^{\left(\frac{3r-60}{4} \right)} \cdot x^{\left(\frac{180-5r}{6} \right)}$$

For x^{10} term,

$$\frac{180-5r}{6} = 10$$

$\Rightarrow 5r = 120$

$\Rightarrow r = 24$

$\therefore$ Coefficient of $x^{10} = {}^{60}C_{24} \cdot 5^{\left(\frac{3 \times 24 - 60}{4} \right)}$

$= {}^{60}C_{24} \cdot 5^3$

$= \frac{60!}{24!36!} \cdot 5^3$

It is given that,

$\frac{60!}{24!36!} \cdot 5^3 = 5^k \cdot l \ldots\ldots \; (1)$

Also given that, l is coprime to 5 means l can't be multiple of 5 . So we have to find all the factors of 5 in 60!, 24 ! and 36 !

[Note : Formula for exponent or degree of prime number in n!.

Exponent of p in $n!$ = $\left\lceil \frac{n}{p} \right\rceil + \left\lceil \frac{n}{p^2} \right\rceil + \left\lceil \frac{n}{p^3} \right\rceil + \ldots..$ until 0 comes

here p is a prime number.]

$\therefore$ Exponent of 5 in 60 !

$= \left\lceil \frac{60}{5} \right\rceil + \left\lceil \frac{60}{5^2} \right\rceil + \left\lceil \frac{60}{5^3} \right\rceil + \ldots\ldots$

$= 12 + 2 + 0 + \ldots.$

$= 14$

Exponent of 5 in 24 !

$= \left\lceil \frac{24}{5} \right\rceil + \left\lceil \frac{24}{5^2} \right\rceil + \left\lceil \frac{24}{5^3} \right\rceil + \ldots\ldots$

$= 4 + 0 + 0 \ldots\ldots$

$= 4$

Exponent of 5 in 36 !

$= \left\lceil \frac{36}{5} \right\rceil + \left\lceil \frac{36}{5^2} \right\rceil + \left\lceil \frac{36}{5^3} \right\rceil + \ldots\ldots$

$= 7 + 1 + 0 \ldots\ldots$

$= 8$

$\therefore$ From equation (1), exponent of 5 overall

$\frac{5^{14}}{5^4 \cdot 5^8} \cdot 5^3 = 5^k$

$\Rightarrow 5^5 = 5^k$

$\Rightarrow k = 5$

Question71

If the sum of the coefficients of all the positive powers of x, in the Binomial expansion of $\left(x^n + \frac{2}{x^5} \right)^7$ is 939 , then the sum of all the possible integral values of n is____

[27-Jun-2022-Shift-2]

Answer: 57

Solution:

Solution:
Given, Binomial expression is
$= \left(x^n + \frac{2}{x^5}\right)^7$
∴ General term
$T_{r+1} = {}^7C_r \cdot (x^n)^{7-r} \cdot \left(\frac{2}{x^5}\right)^r$
$= {}^7C_r \cdot x^{7n - nr - 5r} \cdot 2^r$
For positive power of x,
$7n - nr - 5r > 0$
$\Rightarrow 7n > r(n+5)$
$\Rightarrow r < \frac{7n}{n+5}$
As r represent term of binomial expression so r is always integer.
Given that sum of coefficient is 939.
When $r = 0$,
sum of coefficient $= {}^7C_0 \cdot 2^0 = 1$
when $r = 1$,
sum of coefficient $= {}^7C_0 \cdot 2^0 + {}^7C_1 \cdot 2^1 = 1 + 14 = 15$
when $r = 2$,
sum of coefficient
$= {}^7C_0 \cdot 2^0 + {}^7C_1 \cdot 2^1 + {}^7C_2 \cdot 2^2$
$= 1 + 14 + 84$
$= 99$
when $r = 3$,
sum of coefficient
$= {}^7C_0 \cdot 2^0 + {}^7C_1 \cdot 2^1 + {}^7C_2 \cdot 2^2 + {}^7C_3 \cdot 2^3$
$= 1 + 14 + 84 + 280$
$= 379$
when $r = 4$,
sum of coefficient
$= {}^7C_0 \cdot 2^0 + {}^7C_1 \cdot 2^1 + {}^7C_2 \cdot 2^2 + {}^7C_3 \cdot 2^3 + {}^7C_4 \cdot 2^4$
$= 1 + 14 + 84 + 280 + 560$
$= 939$
To get value of $r = 4$, value of $\frac{7n}{n+5}$ should be between 4 and 5 .
$\therefore 4 < \frac{7n}{n+5} < 5$
$\Rightarrow 4n + 20 < 7n < 5n + 25$
$\therefore 4n + 20 < 7n$
$\Rightarrow 3n > 20$
$\Rightarrow n > \frac{20}{3}$
$\Rightarrow n > 6.66$
and
$7n < 5n + 25$
$\Rightarrow 2n < 25$
$\Rightarrow n < 12.5$
$\therefore 6.66 < n < 12.5$
∴ Possible integer values of $n = 7, 8, 9, 10, 11, 12$
∴ Sum of values of $n = 7 + 8 + 9 + 10 + 11 + 12 = 57$

Question72

If
$$\sum_{k=1}^{31} ({}^{31}C_k)({}^{31}C_{k-1}) - \sum_{k=1}^{30} ({}^{31}C_k)({}^{31}C_{k-1}) = \frac{\alpha(60!)}{(30!)(31!)}$$
where $\alpha \in R$, then the value of 16α is equal to
[28-Jun-2022-Shift-1]

Options:

A. 1411

B. 1320

C. 1615

D. 1855

Answer: A

Solution:

Solution:
Given,

$$\sum_{k=1}^{31}({}^{31}C_k)({}^{31}C_{k-1})-\sum_{k=1}^{30}({}^{30}C_k)({}^{30}C_{k-1})=\frac{\alpha(60!)}{(30!)(31!)}$$

Now,

$$\sum_{k=1}^{31}({}^{31}C_k)({}^{31}C_{k-1})$$

$$=({}^{31}C_1\cdot{}^{31}C_0+{}^{31}C_2\cdot{}^{31}C_1+{}^{31}C_3\cdot{}^{31}C_2+\ldots\ldots+{}^{31}C_{31}\cdot{}^{31}C_{30})$$

$$=({}^{31}C_0\cdot{}^{31}C_{31-1}+{}^{31}C_1\cdot{}^{31}C_{31-2}+\ldots\ldots+{}^{31}C_{30}\cdot{}^{31}C_{31-31})$$

[using ${}^{n}C_{r}={}^{n}C_{n-r}$]

$$=({}^{31}C_0\cdot{}^{31}C_{30}+{}^{31}C_1\cdot{}^{31}C_{29}+\ldots\ldots+{}^{31}C_{30}\cdot{}^{31}C_0)$$

$$={}^{62}C_{30}$$

Now, $\sum_{k=1}^{30}{}^{30}C_k\cdot{}^{30}C_{k-1}$

$$=({}^{30}C_1\cdot{}^{30}C_0+{}^{30}C_2\cdot{}^{30}C_1+\ldots\ldots+{}^{30}C_{30}\cdot{}^{30}C_{29})$$

$$=({}^{30}C_0\cdot{}^{30}C_{29}+{}^{30}C_1\cdot{}^{30}C_{28}+\ldots+{}^{30}C_{29}\cdot{}^{30}C_0)$$

$$={}^{60}C_{29}$$

$$\therefore{}^{60}C_{30}-{}^{60}C_{29}=\frac{\alpha(60!)}{30!31!}$$

$$\Rightarrow\frac{62.61.60!}{30!32!}-\frac{60!}{29!31!}=\frac{\alpha(60!)}{30!31!}$$

$$\Rightarrow\frac{62.61.60!}{30!32!}-\frac{60!}{\frac{30!}{30}\cdot31!}=\frac{\alpha(60!)}{30!31!}$$

$$\Rightarrow\frac{60!}{30!31!}\left(\frac{62.61}{32}-30\right)=\frac{\alpha(60!)}{30!31!}$$

$$\Rightarrow\alpha=\frac{62.61}{32}-30$$

$$\Rightarrow16\alpha=\frac{62.61-30\times32}{2}$$

$$\Rightarrow16\alpha=\frac{2822}{2}=1411$$

Question73

The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3+\frac{3}{x^k}\right)^{12}$, x ≠ 0 is 2^8 . I, where I is an odd integer, is___
[28-Jun-2022-Shift-1]

Answer: 2

Solution:

$\left(2x^3 + \frac{3}{x^k}\right)^{12}$

$t_{r+1} = {}^{12}C_r(2x^3)^r\left(\frac{3}{x^k}\right)^{12-r}$

$x^{3r-(12-r)k} \rightarrow$ constant

$\therefore 3r - 12k + rk = 0$

$\Rightarrow k = \frac{3r}{12-r}$

$\therefore$ possible values of r are 3, 6, 8, 9, 10 and corresponding values of k are 1, 3, 6, 9, 15

Now ${}^{12}C_r = 220, 924, 495, 220, 66$

$\therefore$ possible values of k for which we will get 2^8 are 3,6

Question74

The term independent of x in the expansion of $(1 - x^2 + 3x^3)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}, x \neq 0$ is :

[28-Jun-2022-Shift-2]

Options:

A. $\frac{7}{40}$

B. $\frac{33}{200}$

C. $\frac{39}{200}$

D. $\frac{11}{50}$

Answer: B

Solution:

Solution:

General term of Binomial expansion $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$ is

$T_{r+1} = {}^{11}C_r \cdot \left(\frac{5}{2}x^3\right)^{11-r} \cdot \left(-\frac{1}{5x^2}\right)^r$

$= {}^{11}C_r \cdot \left(\frac{5}{2}\right)^{11-r} \cdot \left(-\frac{1}{5}\right)^r \cdot x^{33-5r}$

In the term,

$(1 - x^2 + 3x^3)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$

Term independent of x is when

(1) $33 - 5r = 0$

$\Rightarrow r = \frac{33}{5} \notin$ integer

(2) $33 - 5r = -2$

$\Rightarrow 5r = 35$

$\Rightarrow r = 7 \in$ integer

(3) $33 - 5r = -3$

$\Rightarrow 5r = 36$

$\Rightarrow r = \frac{36}{5} \notin$ integer

$\therefore$ Only for $r = 7$ independent of x term possible.

$\therefore$ Independent of x term

$= -\left({}^{11}C_7\left(\frac{5}{2}\right)^4 \cdot \left(-\frac{1}{5}\right)^7\right)$

$= -\left(\frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{5^4}{2^4} \cdot -\frac{1}{5^7}\right)$

$= \frac{11 \cdot 10 \cdot 3}{2^4 \cdot 5^3}$

$= \frac{11 \cdot 3}{2^3 \cdot 5^2}$

$= \frac{33}{200}$

Question75

If the constant term in the expansion of $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$ is $2^k . I$, where I is an odd integer, then the value of k is equal to:

[29-Jun-2022-Shift-1]

Options:

A. 6

B. 7

C. 8

D. 9

Answer: D

Solution:

Solution:

Note : Multinomial Theorem :

The general term of $(x_1 + x_2 + \ldots + x_n)^n$ the expansion is

$\frac{n!}{n_1!n_2!\ldots n_n!} x_1^{n_1} x_2^{n_2} \ldots x_n^{n_n}$

where $n_1 + n_2 + \ldots.. + n_n = n$

Given,

$\left(3x^2 - 2x^2 + \frac{5}{x^5}\right)^{10}$

$= \frac{(3x^8 - 2x^7 + 5)^{10}}{x^{50}}$

Now constant term in $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10} = x^{50}$ term in $(3x^8 - 2x^7 + 5)^{10}$

General term in $(3x^8 - 2x^7 + 5)^{10}$ is

$= \frac{10!}{n_1!n_2!n_3!}(3x^8)^{n_1}(-2x^7)^{n_2}(5)^{n_3}$

$= \frac{10!}{n_1!n_2!n_3!}(3)^{n^1}(-2)^{n_2}(5)^{n^3} \cdot x^{8n_1 + 7n_2}$

∴ Coefficient of $x^{8n_1 + 7n_2}$ is

$= \frac{10!}{n_1!n_2!n_3!}(3)^{n_1}(-2)^{n_2}(5)^{n_3}$

where $n_1 + n_2 + n_3 = 0$

For coefficient of x^{50} :

$8n_1 + 7n_2 = 50$

∴ Possible values of n_1, n_2 and n_3 are

n_1	n_2	n_3
1	6	3

$\therefore$ Coefficient of x^{50}

$= \frac{10!}{1!6!3!}(3)^1(-2)^6(5)^3$

$= \frac{10 \times 9 \times 8 \times 7}{6} \times 3 \times 5^3 \times 2^6$

$= 5 \times 3 \times 8 \times 7 \times 3 \times 5^3 \times 2^6$

$= 7 \times 5^4 \times 3^2 \times 2^9$

$= 2^k \cdot l$

$\therefore l = 7 \times 5^4 \times 3^2 =$ An odd integer

and $2^k = 2^9$

$\Rightarrow k = 9$

Question76

Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\alpha$ and $6^n - 5n - 1 = 25\beta$, then $\alpha - \beta$ is equal to [29-Jun-2022-Shift-2]

Options:

A. $1 + {}^nC_2(8-5) + {}^nC_3(8^2 - 5^2) + \ldots.. + {}^nC_n(8^{n-1} - 5^{n-1})$

B. $1 + {}^nC_3(8-5) + {}^nC_4(8^2 - 5^2) + \ldots.. + {}^nC_n(8^{n-2} - 5^{n-2})$

C. ${}^nC_3(8-5) + {}^nC_4(8^2 - 5^2) + \ldots\ldots + {}^nC_n(8^{n-2} - 5^{n-2})$

D. ${}^nC_4(8-5) + {}^nC_5(8^2 - 5^2) + \ldots\ldots + {}^nC_n(8^{n-3} - 5^{n-3})$

Answer: C

Solution:

Solution:

Given,

$9^n - 8n - 1 = 64\alpha$

$\Rightarrow \alpha = \frac{(1+8)^n - 8n - 1}{64}$

$= \frac{({}^nC_0 \cdot 1 + {}^nC_1 \cdot 8^1 + {}^nC_2 \cdot 8^2 + \ldots\ldots + {}^nC_n \cdot 8^n) - 8n - 1}{8^2}$

$= \frac{1 + 8n + {}^nC_2 \cdot 8^2 + \ldots. + {}^nC_n \cdot 8^n - 8n - 1}{8^2}$

$= \frac{{}^nC_2 \cdot 8^2 + {}^nC_3 \cdot 8^3 + \ldots. + {}^nC_n \cdot 8^n}{8^2}$

$= {}^nC_2 + {}^nC_3 \cdot 8 + {}^nC_4 \cdot 8^2 + \ldots {}^nC_n \cdot 8^{n-2}$

Also given,

$6^n - 5n - 1 = 25\beta$

$\Rightarrow \beta = \frac{(1+5)^n - 5n - 1}{25}$

$= \frac{{}^nC_0 \cdot 1 + {}^nC_1 \cdot 5 + {}^nC_2 \cdot 5^2 + \ldots.. + {}^nC_n \cdot 5^n - 5n - 1}{5^2}$

$= \frac{1 + 5n + {}^nC_2 \cdot 5^2 + {}^nC_3 \cdot 5^3 + \ldots. + {}^nC_n \cdot 5^2 - 5n - 1}{5^2}$

$= \frac{{}^nC_2 \cdot 5^2 + {}^nC_3 \cdot 5^3 + {}^nC_4 \cdot 5^4 + \ldots + {}^nC_n \cdot 5^n}{5^2}$

$= {}^nC_2 + {}^nC_3 \cdot 5 + {}^nC_4 \cdot 5^2 + \ldots\ldots + {}^nC_n \cdot 5^{n-2}$

$\therefore \alpha - \beta$

$= ({}^nC_2 + {}^nC_3 \cdot 8 + {}^nC_4 \cdot 8^2 + \ldots + {}^nC_n \cdot 8^{n-2}) - ({}^nC_2 + {}^nC_3 \cdot 5 + {}^nC_4 \cdot 5^2 + \ldots + {}^nC_n \cdot 5^{n-2}.$

$= {}^nC_3 \cdot (8-5) + {}^nC_4 \cdot (8^2 - 5^2) + \ldots + {}^nC_n(8^{n-2} - 5^{n-2})$

Question77

Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}} - \frac{1}{x^{\frac{1}{5}}}\right)^{15}$, x > 0, be m and n respectively. If r is a positive integer such that $mn^2 = {}^{15}C_r \cdot 2^r$, then the value of r is equal to___
[29-Jun-2022-Shift-2]

Answer: 5

Solution:

Solution:

$T_{r+1} = (-1)^r \cdot {}^{15}C_r \cdot 2^{15-r} \times \frac{15-2r}{5}$

$m = {}^{15}C_{10}2^5$

$n = -1$

so $mn^2 = {}^{15}C_5 2^5$

Question78

If the maximum value of the term independent of t in the expansion of $\left(t^2x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, x ≥ sl ant0, is K, then 8K is equal to
[25-Jul-2022-Shift-1]

Answer: 6006

Solution:

General term of $\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$ is

$T_{r+1} = {}^{15}C_r \cdot \left(t^2 x^{\frac{1}{5}}\right)^{15-r} \cdot \left(\frac{(1-x)^{\frac{1}{10}}}{t}\right)^r$

$= {}^{15}C_r \cdot t^{30-2r} \cdot x^{\frac{15-r}{5}} \cdot (1-x)^{\frac{r}{10}} \cdot t^{-r}$

$= {}^{15}C_r \cdot t^{30-3r} \cdot x^{\frac{15-r}{5}} \cdot (1-x)^{\frac{r}{10}}$

Term will be independent of t when $30 - 3r = 0 \Rightarrow r = 10$

$\therefore T_{10+1} = T_{11}$ will be independent of t

$\therefore T_{11} = {}^{15}C_{10} \cdot x^{\frac{15-10}{5}} \cdot (1-x)^{\frac{10}{10}}$

$= {}^{15}C_{10} \cdot x^1 \cdot (1-x)^1$

T_{11} will be maximum when $x(1-x)$ is maximum.

Let $f(x) = x(1-x) = x - x^2$

$f(x)$ is maximum or minimum when $f'(x) = 0$

$\therefore f'(x) = 1 - 2x$

For maximum / minimum $f'(x) = 0$

$\therefore 1 - 2x = 0$

$\Rightarrow x = \frac{1}{2}$

Now, $f''(x) = -2 < 0$

$\therefore$ At $x = \frac{1}{2}$, $f(x)$ maximum

$\therefore$ Maximum value of T_{11} is

$= {}^{15}C_{10} \cdot \frac{1}{2}\left(1 - \frac{1}{2}\right)$

$= {}^{15}C_{10} \cdot \frac{1}{4}$

Given $K = {}^{15}C_{10} \cdot \frac{1}{4}$

Now, $8K = 2({}^{15}C_{10})$

$= 6006$

Question79

The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is [25-Jul-2022-Shift-2]

Options:

A. 1

B. 4

C. 6

D. 8

Answer: D

Solution:

Solution:

$\operatorname{Re}\left(\frac{(11)^{1011} + (1011)^{11}}{9}\right) = \operatorname{Re}\left(\frac{2^{1011} + 3^{11}}{9}\right)$

For $\operatorname{Re}\left(\frac{2^{1011}}{9}\right)$

$2^{1011} = (9-1)^{337} = {}^{337}C_0 9^{337}(-1)^0 + {}^{337}C_1 9^{336}(-1)^1 + {}^{337}C_2 9^{335}(-1)^2 + \ldots\ldots + {}^{337}C_{337} 9^0 (-1)^{337}$

So, remainder is 8

and $\mathrm{Re}\left(\frac{3^{11}}{9}\right) = 0$

So, remainder is 8

Question80

If the coefficients of x and x^2 in the expansion of $(1+x)^p(1-x)^q$, p, q ≤ 15, are −3 and −5 respectively, then the coefficient of x^3 is equal to ________.
[26-Jul-2022-Shift-1]

Answer: 23

Solution:

Solution:

Coefficient of x in $(1+x)^p(1-x)^q$

${}^pC_0{}^qC_1 + {}^pC_1{}^qC_0 = -3 \Rightarrow p - q = -3$

Coefficient of x^2 in $(1+x)^p(1-x)^q$

${}^pC_0{}^qC_2 - {}^pC_1{}^qC_1 + {}^pC_2{}^qC_0 = -5$

$\frac{q(q-1)}{2} - pq + \frac{p(q-1)}{2} = -5$

$\frac{q^2-q}{2} - (q-3)q + \frac{(q-3)(q-4)}{2} = -5$

$\Rightarrow q = 11,\ p = 8$

Coefficient of x^3 in $(1+x)^8(1-x)^{11}$

$= -{}^{11}C_3 + {}^8C_1{}^{11}C_2 - {}^8C_2{}^{11}C_1 + {}^8C_3 = 23$

Question81

$\sum_{\substack{i,j=0 \\ i \neq j}}^{n} {}^nC_i{}^nC_j$ **is equal to**

[26-Jul-2022-Shift-2]

Options:

A. $2^{2n} - {}^{2n}C_n$

B. $2^{2n-1} - {}^{2n-1}C_{n-1}$

C. $2^{2n} - \frac{1}{2}{}^{2n}C_n$

D. $2^{2n-1} + 2n - 1C_n$

Answer: B

Solution:

Solution:

$$\sum_{i,j=0, i\neq j}^{n} {}^nC_i\ {}^nC_j = \sum_{i,j=0}^{n} {}^nC_i\ {}^nC_j - \sum_{i=j}^{n} {}^nC_i\ {}^nC_j$$

$$= \sum_{j=0}^{n} {}^nC_i \sum_{j=0}^{n} {}^nC_j - \sum_{i=0}^{n} {}^nC_iC_i$$

$$= 2^n \cdot 2^n - {}^{2n}C_n$$

$$= 2^{2n} - {}^{2n}C_n$$

Question82

The remainder when $(2021)^{2022} + (2022)^{2021}$ is divided by 7 is [27-Jul-2022-Shift-1]

Options:

A. 0

B. 1

C. 2

D. 6

Answer: A

Solution:

Solution:

$(2021)^{2022} + (2022)^{2021}$

$= (7k-2)^{2022} + (7k_1-1)^{2021}$

$= [(7k-2)^3]^{674} + (7k_1)^{2021} - 2021(7k_1)^{2020} + \ldots - 1$

$= (7k_2-1)^{674} + (7m-1)$

$= (7n+1) + (7m-1) = 7(m+n)$ (multiple of 7)

$\therefore$ Remainder $= 0$

Question83

Let for the 9^{th} term in the binomial expansion of $(3+6x)^n$, in the increasing powers of $6x$, to be the greatest for $x = \frac{3}{2}$, the least value of n is n_0. If k is the ratio of the coefficient of x^6 to the coefficient of x^3, then $k + n_0$ is equal to: [27-Jul-2022-Shift-2]

Answer: 24

Solution:

Solution:

$(3+6x)^n = 3^n(1+2x)^n$

If T_9 is numerically greatest term

$\therefore T_8 \le T_9 \le T_{10}$

${}^nC_7 3^{n-7}(6x)^7 \le {}^nC_8 3^{n-8}(6x)^8 \ge {}^nC_9 3^{n-9}(6x)^9$

$\Rightarrow \frac{n!}{(n-7)!7!}9 \le \frac{n!}{(n-8)!8!}3\cdot(6x) \ge \frac{n!}{(n-9)!9!}(6x)^2$

$\Rightarrow \frac{9}{(n-7)(n-8)} \le \frac{18\left(\frac{3}{2}\right)}{(n-8)8} \ge \frac{36}{9.8}\frac{9}{4}$

$72 \le 27(n-7)$ and $27 \ge 9(n-8)$

$\frac{29}{3} \le n$ and $n \le 11$

$\therefore n_0 = 10$

For $(3+6x)^{10}$

$T_{r+1} = {}^{10}C_r$

$3^{10-r}(6x)^r$

For coeff. of x^6

$r = 6 \Rightarrow {}^{10}C_6 3^4 \cdot 6^6$

For coeff. of x^3

$r = 3 \Rightarrow {}^{10}C_3 3^7 \cdot 6^3$

$\therefore k = \frac{{}^{10}C_6}{{}^{10}C_3}\cdot\frac{3^4\cdot6^6}{3^7\cdot6^3} = \frac{10!7!3!}{6!4!10!}\cdot 8$

$\Rightarrow k = 14$

$\therefore k + n_0 = 24$

Question84

The remainder when $7^{2022} + 3^{20222}$ is divided by 5 is : [28-Jul-2022-Shift-1]

Options:

A. 0

B. 2

C. 3

D. 4

Answer: C

Solution:

Solution:

$7^{2022} + 3^{2022}$

$= (49)^{1011} + (9)^{1011}$

$= (50-1)^{1011} + (10-1)^{1011}$

$= 5\lambda - 1 + 5k - 1$

$= 5m - 2$

Remainder $= 5 - 2 = 3$

Question85

Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}}+\beta x\right)^4$, $(1-3\beta x)^2$ and $\left(1-\frac{\beta}{2}x\right)^6$, $\beta>0$, respectively form the first three terms of an A.P. If d is the common difference of this A.P. , then $50-\frac{2d}{\beta^2}$ is equal to ________.

[28-Jul-2022-Shift-2]

Answer: 57

Solution:

Solution:

Coefficients of middle terms of given expansions are ${}^4C_2\frac{1}{6}\beta^2$, ${}^2C_1(-3\beta)$, ${}^6C_3\left(\frac{-\beta}{2}\right)^3$ form an A.P.

$\therefore 2.2(-3\beta)=\beta^2-\frac{5\beta^3}{2}$

$\Rightarrow -24=2\beta-5\beta^2$

$\Rightarrow 5\beta^2-2\beta-24=0$

$\Rightarrow 5\beta^2-12\beta+10\beta-24=0$

$\Rightarrow \beta(5\beta-12)+2(5\beta-12)=0$

$\beta=\frac{12}{5}$

$d=-6\beta-\beta^2$

$\therefore 50-\frac{2d}{\beta^2}=50-2\frac{(-6\beta-\beta^2)}{\beta^2}=50+\frac{12}{\beta}+2=57$

Question86

If $1+(2+{}^{49}C_1+{}^{49}C_2+\ldots+{}^{49}C_{49})({}^{50}C_2+{}^{50}C_4+\ldots+{}^{50}C_{50})$ is equal to $2^n\cdot m$, where m is odd, then n + m is equal to _______.

[28-Jul-2022-Shift-2]

Answer: 99

Solution:

Solution:

$l=1+(1+{}^{49}C_0+{}^{49}C_1+\ldots+{}^{49}C_{49})({}^{50}C_2+{}^{50}C_4+\ldots+{}^{50}C_{50})$

As ${}^{49}C_0+{}^{49}C_1+\ldots+{}^{49}C_{49}=2^{49}$

and ${}^{50}C_0+{}^{50}C_2+\ldots+{}^{50}C_{50}=2^{49}$

$\Rightarrow {}^{50}C_2+{}^{50}C_4+\ldots+{}^{50}C_{50}=2^{49}-1$

$\therefore l=1+(2^{49}+1)(2^{49}-1)$

$=2^{98}$

$\therefore m=1$ and $n=98$

m + n = 99

Question87

Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^{n}$, in the increasing powers of $\frac{1}{\sqrt[4]{3}}$ be $\sqrt[4]{6}$: 1. If the sixth term from the beginning is $\frac{\alpha}{\sqrt[4]{3}}$, then α is equal to ________.
[29-Jul-2022-Shift-1]

Answer: 84

Solution:

Solution:

Fifth term from beginning $= {}^{n}C_{4}\left(2^{\frac{1}{4}}\right)^{n-4}\left(3^{\frac{-1}{4}}\right)^{4}$

Fifth term from end $= (n-5+1)^{\text{th}}$ term from begin $= {}^{n}C_{n-4}\left(2^{\frac{1}{4}}\right)^{3}\left(3^{\frac{-1}{4}}\right)^{n-4}$

Given $\dfrac{{}^{n}C_{4}2^{\frac{n-4}{4}}\cdot 3^{-1}}{{}^{n}C_{n-3}2^{\frac{4}{4}}\cdot 3^{-\left(\frac{n-4}{4}\right)}} = 6^{\frac{1}{4}}$

$\Rightarrow 6^{\frac{n-8}{4}} = 6^{\frac{1}{4}}$

$\Rightarrow \frac{n-8}{4} = \frac{1}{4} \Rightarrow n = 9$

$T_{6} = T_{5+1} = {}^{9}C_{5}\left(2^{\frac{1}{4}}\right)^{4}\left(3^{\frac{-1}{4}}\right)^{5}$

$= \dfrac{{}^{9}C_{5}\cdot 2}{3^{\frac{1}{4}}\cdot 3} = \dfrac{84}{3^{\frac{1}{4}}} = \dfrac{\alpha}{3^{\frac{1}{4}}}$

$\Rightarrow \alpha = 84$

Question88

If $\sum_{k=1}^{10} K^{2}\left(10_{C_{K}}\right)^{2} = 22000L$, then L is equal to _______.
[29-Jul-2022-Shift-2]

Answer: 221

Solution:

Solution:
Given,
$\sum_{k=1}^{10} k^2({}^{10}C_k)^2 = 2200L$
$\Rightarrow \sum_{k=1}^{10} (k \cdot {}^{10}C_k)^2 = 22000L$
$\Rightarrow \sum_{k=1}^{10} \left(k \cdot \frac{10}{k} \cdot {}^{9}C_{k-1}\right)^2 = 22000L$
$\Rightarrow \sum_{k=1}^{10} (10 \cdot {}^{9}C_{k-1})^2 = 22000L$
$\Rightarrow 100 \cdot \sum_{k=1}^{10} ({}^{9}C_{k-1})^2 = 22000L$
$\Rightarrow 100(({}^{9}C_0)^2 + ({}^{9}C_1)^2 + \ldots + ({}^{9}C_9)^2) = 22000L$
$\Rightarrow 100({}^{18}C_9) = 22000L$
[Note : $({}^{n}C_1)^2 + ({}^{n}C_2)^2 + \ldots + ({}^{n}C_n)^2 = {}^{2n}C_n$]
$\Rightarrow 100 \times \frac{18!}{9!9!} = 22000L$
$\Rightarrow L = 221$

Question89

If n ≥ 2 is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2({}^{2}C_2 + {}^{3}C_2 + {}^{4}C_2 + \ldots + {}^{n}C_2)$ is
[2021, 24 Feb. Shift-II]

Options:

A. $\frac{n(n-1)(2n+1)}{6}$

B. $\frac{n(n+1)(2n+1)}{6}$

C. $\frac{n(2n+1)(3n+1)}{6}$

D. $\frac{n(n+1)^2(n+2)}{12}$

Answer: B

Solution:

Solution:
Given, $n \geq 2$
Let $S = {}^{2}C_2 + {}^{3}C_2 + \ldots + {}^{n}C_2 = {}^{n+1}C_3$
Now, ${}^{n+1}C_2 + 2 \times ({}^{2}C_2 + {}^{3}C_2 + \ldots + {}^{n}C_2)$
$= {}^{n+1}C_2 + 2 \times {}^{n+1}C_3$
$= ({}^{n+1}C_2 + {}^{n+1}C_3) + {}^{n+1}C_3$
$= {}^{n+2}C_3 + {}^{n+1}C_3 = \frac{(n+2)!}{3!(n-1)!} + \frac{(n+1)!}{3!(n-2)!}$
$= \frac{(n+2)(n+1)n(n-1)!}{3 \times 2 \times 1 \times (n-1)!}$
$+ \frac{(n+1) \times n \times (n-1) \times (n-2)!}{3 \times 2 \times 1 \times (n-2)!}$
$= \frac{n(n+1)}{6}[n+2+n-1]$
$= \frac{n(n+1)(2n+1)}{6}$

Question90

For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r & \text{if } n \geq r \geq 0 \\ 0 & \text{otherwise .} \end{cases}$ **The maximum value of k for which the sum,**

$$\sum_{i=0}^{k} \binom{10}{i}\binom{5}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$$

exists, is equal to
[2021, 24 Feb Shift-II]

Answer: 1

Solution:

Solution:

Given, $\binom{n}{r} = \begin{cases} {}^nC_r & \text{if } n \geq r \geq 0 \\ 0 & \text{otherwise .} \end{cases}$

and $\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\left(\frac{13}{k+1-i}\right)$

$\because (1+x)^{10} = {}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + \ldots$
$+ {}^{10}C_{10} x^{10}$

and $(1+x)^{15} = {}^{15}C_0 + {}^{15}C_1 x + {}^{15}C_2 x^2$
$+ \ldots + {}^{15}C_{15} x^{15}$

$\therefore \sum_{i=0}^{k} ({}^{10}C_i)({}^{15}C_{k-i}) = {}^{10}C_0 \cdot {}^{15}C_k$
$+ {}^{15}C_1 \cdot {}^{15}C_{k-1} + \ldots + {}^{10}C_k \cdot {}^{15}C_0$

$\Rightarrow$ Coefficient of x^k in $(1+x)^{25} = {}^{25}C_k$

Also, $\sum_{i=0}^{k+1} ({}^{12}C_i)({}^{13}C_{k+1-i}) = {}^{12}C_0 \cdot {}^{13}C_{k+1}$
$+ {}^{12}C_1 \cdot {}^{13}C_k + \ldots + {}^{12}C_{k+1} \cdot {}^{13}C_0$

$\Rightarrow$ Coefficient of x^{k+1} in $(1+x)^{25} = {}^{25}C_{k+1}$

$\Rightarrow {}^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$

As, we know by the definition of ${}^nC_r$, the maximum value of ${}^{26}C_{k+1}$ is possible for any possible large value of k.
Hence, k can have any large value.

Question91

The value of $-{}^{15}C_1 + 2 \cdot {}^{15}C_2 - 3 \cdot {}^{15}C_3 + \ldots - 15 \cdot {}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots + {}^{14}C_{11}$ **is**
[2021, 24 Feb. Shift-l]

Options:

A. $2^{16}-1$

B. $2^{13}-14$

C. $2^{13}-13$

D. 2^{14}

Answer: B

Solution:

Solution:
Given,
$(-{}^{15}C_1+2\cdot{}^{15}C_2-3\cdot{}^{15}C_3+\ldots-15\cdot{}^{15}C_{15})+({}^{14}C_1+{}^{14}C_3+\ldots+{}^{14}C_{11})$
Let
$S_1=-{}^{15}C_1+2\cdot{}^{15}C_2-3\cdot{}^{15}C_3+\ldots-15\cdot{}^{15}C_{15}$
$=\sum_{r=1}^{15}(-1)^r\cdot r\cdot{}^{15}C_r=15\sum_{r=1}^{15}(-1))^{r\,14}C_{r-1}$
$=15(-{}^{14}C_0+{}^{14}C_1-{}^{14}C_2+\ldots-{}^{14}C_{14})$
$=15(0)=0$
$S_2={}^{14}C_1+{}^{14}C_3+{}^{14}C_5+\ldots+{}^{14}C_{11}$
$=({}^{14}C_1+{}^{14}C_3+\ldots+{}^{14}C_{13})-{}^{14}C_{13}$
$=2^{13}-14$
Now, the required value is
$(-{}^{15}C_1+2\cdot{}^{15}C_2-3\cdot{}^{15}C_3+\ldots-15\cdot{}^{15}C_{15})$
$+({}^{14}C_1+{}^{14}C_3+\ldots+{}^{14}C_{11})$
$=S_1+S_2$
$=0+2^{13}-14$
$=2^{13}-14$

Question92

If the remainder when x is divided by 4 is 3 , then the remainder when $(2020+x)^{2022}$ is divided by 8 is [2021, 25 Feb. Shift-II]

Answer: 1

Solution:

Solution:
Given, when x is divided by 4 , the remainder is 3 .
Let $x=4p+3$, then
$(2020+x)^{2022}=(2020+4p+3)^{2022}$
$=(2024+4p-1)^{2022}$
$=(4k-1)^{2022}$
($\because$2024 is divisible by 4) $={}^{2022}C_0(4K)^{2022}(-1)^0+{}^{2022}C_1(4K)^{2021}(-1)^1+\ldots.+{}^{2022}C_{2022}(4A)^0(-1)^{2022}$
On expansion $(2020+x)^{2022}$, we get the form of $8\lambda+1$. Since, each terms have 2022 and $4k_1$ so if we take 2 common from 2022 we get 8 . Thus, each term have 8 in common.
Hence, remainder is 1.

Question93

Let $m, n \in N$ and $\gcd(2, n) = 1$. If

$$30\binom{30}{0} + 29\binom{30}{28} + \ldots + 2\binom{30}{28} + 1\binom{30}{29} = n \cdot 2^m\text{, then } n + m \text{ is}$$

.... (Here, $\binom{n}{k} = {}^nC_k$)

[2021, 26 Feb. Shift-I]

Answer: 45

Solution:

Solution:
Given,
$30 \cdot {}^{30}C_0 + 29 \cdot {}^{30}C_1 + \ldots + 2 \cdot {}^{30}C_{28} + {}^{30}C_{29}$
$= n \cdot 2^m$
This can be written as,
$\sum_{r=0}^{29} (30 - r)^{30}C_r = n \cdot 2^m$
or $\sum_{r=0}^{30} (30 - r) \cdot {}^{30}C_r = n \cdot 2^m$
$\Rightarrow \sum_{r=0}^{30} 30 \cdot {}^{30}C_r - \sum_{r=0}^{30} r \cdot {}^{30}C_r = n \cdot 2^m$
$\Rightarrow 30 \sum_{r=0}^{30} C_r{}^{30} - \sum_{r=0}^{30} r \cdot {}^{30}C_r = n \cdot 2^m$Using combination properties,
$\Rightarrow 30 \cdot (2)^{30} - 30 \cdot (2)^{29} = n \cdot 2^m$
$\Rightarrow 30 \cdot (2)^{29}(2 - 1) = n \cdot 2^m$
$\Rightarrow 2 \cdot 15 \cdot (2)^{29} = n \cdot 2^m$
$\Rightarrow 15 \cdot (2)^{30} = n \cdot 2^m$Comparing both sides,
$n = 15$ and $m = 30$
$\Rightarrow n + m = 15 + 30 = 45$

Question94

The maximum value of the term independent of ' t ' in the expansion of $\left(tx^{1/5} + \frac{(1 - x)^{1/10}}{t}\right)^{10}$, where $x \in (0, 1)$ is

[2021, 26 Feb. Shift-l]

Options:

A. $\frac{10!}{\sqrt{3}(5!)^2}$

B. $\frac{2 \cdot 10!}{3\sqrt{3}(5!)^2}$

C. $\frac{2 \cdot 10!}{3(5!)^2}$

D. $\frac{10!}{3(5!)^2}$

Answer: B

Solution:

Solution:
Using Binomial expansion, its (r + 1) th term be,

$T_{r+1} = {}^{10}C_r(tx^{1/5})^{10-r}\left\{\frac{(1-x)^{1/10}}{t}\right\}^r$

$= {}^{10}C_r\frac{(t)^{10-r}}{(t)^r}(x^{1/5})^{10-r}(1-x)^{r/10}$

$= {}^{10}C_r(t)^{10-2r}(x)\frac{10-r}{5}(1-x)^{r/10}$

If this term is independent of ' t ', then we have $10-2r=0$ gives, $r=5$

$\therefore\ T_6 = {}^{10}C_5(x)^1(1-x)^{1/2}$

Let $f(x) = x(1-x)^{1/2}$, to obtain its maximum value, we have to differentiate it and equate it to 0 .

i.e. $f'(x) = 0 \Rightarrow \frac{x}{2\sqrt{1-x}}(-1)+\sqrt{1-x} = 0$

$\Rightarrow -x+2(1-x) = 0$

$\Rightarrow\ -3x+2 = 0$

$\Rightarrow\ x = 2/3$

(Maximum value)

Thus, greatest term will be

$T_6 = {}^{10}C_5\left(\frac{2}{3}\right)\left(1-\frac{2}{3}\right)^{1/2}$

$= {}^{10}C_5\frac{2}{3\sqrt{3}} = \frac{10!\cdot 2}{(5!)^2(3\sqrt{3})}$

Question95

The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}$, x ≠ 1, is equal to [2021, 18 March Shift-II]

Answer: 210

Solution:

Solution:

$\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}$

$= \left[\frac{(x^{1/3})^3+(1)^3}{x^{2/3}-x^{1/3}+1}-\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$

$= \left[(x^{1/3}+1)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right]^{10}$

[use $(a^3+b^3) = (a+b)(a^2-ab+b^2)$]

$= (x^{1/3}-x^{-1/2})^{10}$

General term, $T_{r+1} = {}^{10}C_r(x^{1/3})^{10-r}(-x^{1/2})^r$

$= {}^{10}C_r(x)^{\frac{10-r}{3}} . (-x)^{-\frac{r}{2}}$

For term independent of x , we must put

$\frac{10-r}{3} - \frac{r}{2} = 0$

$\Rightarrow 20 - 2r - 3r = 0$

$\Rightarrow r = 4$

$\therefore$

$T_{4+1} = T_5 = {}^{10}C_4 = \frac{10!}{4!6!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{4 \times 3 \times 2 \times 1 \times 6!}$

$= 210$

Question96

Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1+x)^n$.
If $\sum_{k=0}^{10} (2^2 + 3k)^nC_k = \alpha \cdot 3^{10} + \beta \cdot 2^{10}$
$\alpha, \beta \in R$, then $\alpha + \beta$ is equal to
[2021, 18 March Shift-II]

Answer: 19

Solution:

Solution:

$\sum_{k=0}^{10} (2^2 + 3k) \cdot {}^nC_k = 4 \sum_{k=0}^{10} {}^nC_k + 3 \cdot \sum_{k=0}^{10} k \cdot {}^nC_k$

$= 4 \times 2^n + 3 \times n \sum_{k=1}^{n-1} C_{k-1}$

$= 4 \times 2^n + 3n \times 2^{n-1} \left[\because {}^nC_r = \frac{n}{r} \ {}^{n-1}C_{r-1} \right]$

$= 2^n \left(4 + \frac{3n}{2} \right)$

$= \left(4 + \frac{3n}{2} \right) \cdot 2^n + 0 \times 3^n \sum_{k=0}^{10} (2^2 + 3k) \cdot {}^nC_k = 4 \sum_{k=0}^{10} {}^nC_k + 3 \cdot \sum_{k=0}^{10} k \cdot {}^nC_k$

$= 4 \times 2^n + 3 \times n \sum_{k=1}^{n-1} C_{k-1}$

$= 4 \times 2^n + 3n \times 2^{n-1} \left[\because {}^nC_r = \frac{n}{r} \ {}^{n-1}C_{r-1} \right]$

$= 2^n \left(4 + \frac{3n}{2} \right)$

$= \left(4 + \frac{3n}{2} \right) \cdot 2^n + 0 \times 3^n$

On comparing, $\left[0 \times 3^n + \left(4 + \frac{3n}{2} \right) \cdot 2^n \right] + 0$

$[\alpha \cdot 3^{10} + \beta \cdot 2^{10}]$,

we get $n = 10, \alpha = 0, \beta = 19$

$\therefore \alpha + \beta = 0 + 19 = 19$

Question97

If the fourth term in the expansion of $\left(x + x^{\log_2 x}\right)^7$ is 4480 , then the value of x, where $x \in N$ is equal to [2021, 17 March Shift-I]

Options:

A. 2

B. 4

C. 3

D. 1

Answer: A

Solution:

Solution:

$\left(x + x^{\log_2 x}\right)^7$

$T_4 = 4480$

$T_r = {}^nC_r\left(x^{\log_2 x}\right)^r x^{(n-r)}$

$T_4 = {}^7C_4\left(x^{\log_2 x}\right)^4 x^3 = 35x^{4\log_2 x}x^3$

$T_4 = 4480$

$35x^{3+4\log_2 x} = 4480$

$x^{3+4\log_2 x} = 128 = 2^7$

Taking log on both sides,

$\log_2 x^{3+4\log_2 x} + 4\log_2 x = \log_2 2^7$

$\Rightarrow (3 + 4\log_2 x)(\log_2 x) = 7$

$\Rightarrow 4(\log_2 x)^2 + 3\log_2 x - 7 = 0$

$\Rightarrow (\log_2 x - 1)(4\log_2 x + 7) = 0$

$\Rightarrow \log_2 x = 1 \ (\because x \in N)$

$\therefore x = 2$

Question98

If $(2021)^{3762}$ is divided by 17 , then the remainder is [2021, 17 March Shift-I]

Answer: 4

Solution:

Solution:

$(2021)^{3762}$

$2021 = (17 \times 119 - 2) \Rightarrow (17\lambda - 2)$

$(2021)^{3762} = (17\lambda - 2)^{3762} = C_0(17\lambda)^{3762}$

$-C_1(17\lambda)^{3761}2^1 + \ldots C_n 2^{3762}$

Now, $(2021)^{3762}$ will be divisible by 17 all the terms except the last one for last one.
$\therefore\ (2021)^{3762} = 17\mu - 2^{3762}$
$= 17\mu - 2^2(2^{3760})$
$= 17\mu - 4(16)^{235}$
$= 17\mu - 4 \cdot (17-1)^{235}$
$(17-1)^{235} = (-1)(1-17)^{235}$
$= -(C_0 - C_1 17 + C_2 17^2 - \ldots)$
$= -C_0 + 17\gamma = -1 + 17\gamma$
$17\mu - 4(17-1)^{235} = 17\mu - 4[-1 + 17\gamma]$
$= 17(\mu - 4\gamma) + 4$
$\therefore\ (2021)^{3762} = 17k + 4$
Hence, 4 is the remainder.

Question99

Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x + \frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then, the term independent of x in the expansion, is equal to
[2021, 17 March Shift-II]

Answer: 4

Solution:

Solution:

General term, $T_{r+1} = {}^nC_r \cdot x^{n-r} \cdot \left(\frac{a}{x^2}\right)^r$
$= {}^nC_r \cdot a^r \cdot x^{n-3r}$
$\therefore\ T_3 = {}^nC_2 \cdot a^2 \cdot x^{n-6}$
$T_4 = {}^nC_3 a^3 \cdot x^{n-9}$
$T_5 = {}^nC_4 \cdot a^4 \cdot x^{n-12}$
Now, $\frac{\text{coefficient of } T_3}{\text{coefficient of } T_4} = \frac{{}^nC_2 \cdot a^2}{{}^nC_3 \cdot a^3}$
$= \frac{3}{a(n-2)} = \frac{3}{2}$
$\Rightarrow\ a(n-2) = 2 \quad \cdots\cdots \text{(i)}$
Also, $\frac{\text{coefficient of } T_4}{\text{coefficient of } T_5} = \frac{{}^nC_3 \cdot a^2}{{}^nC_3 \cdot a^3}$
$= \frac{4}{a(n-3)} = \frac{8}{3}$
$\Rightarrow\ a(n-3) = \frac{3}{2} \quad \cdots\cdots \text{(ii)}$
From Eqs. (i) and (ii), $n = 6$, $a = \frac{1}{2}$
For the term independent of ' x '
$n - 3r = 0$
$\Rightarrow r = \frac{n}{3}$
$\Rightarrow r = \frac{6}{3}$
$\Rightarrow\ r = 2$
$\therefore$ Independent term is T_3.
Now, $T_3 = {}^6C_2 \cdot \left(\frac{1}{2}\right)^2 \cdot (x)^0$

$= \frac{15}{4} = 3.75 \simeq 4$

Question100

If n is the number of irrational terms in the expansion of $(3^{1/4} + 5^{1/8})^{60}$, then $(n - 1)$ is divisible by [2021, 16 Mar Shift-I]

Options:

A. 26

B. 30

C. 8

D. 7

Answer: A

Solution:

Solution:
$(3^{1/4} + 5^{1/8})^{60}$
By using Binomial expansion, $(r + 1)$ th term,
$T_{r+1} = {}^{60}C_r(3^{1/4})^r(5^{1/8})^{60-r}$
$= {}^{60}C_r 3^{r/4} 5^{\frac{60-r}{8}}$
For this term to be a rational number, r should be a multiple of 4 and $(60 - r)$ should be a multiple of 8 .
Let A be a set when r is the multiple of 4 .
$A = \{4, 8, 12, \ldots, 56, 60\}$
$n(A) = 15$
Let B be a set of r, when $(60 - r)$ is the multiple of 8 .
$B = \{4, 12, 20, 28, 36, 44, 52, 60\}$
$n(B) = 8$
Now, $n(A \cap B) = 8$
So, there are only 8 terms out of 61 terms which will be rational numbers.
53 terms will be irrational.
So, $n = 53$

and $n - 1 = 52$ which is divisible only by 26 among the given options.

Question101

Let [x] denote greatest integer less than or equal to x. If for $n \in N$, $(1 - x + x^3)^n = \sum_{j=0}^{3n} a_j x^j$, then $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4 \sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j} + 1$ is equal to [2021, 16 March Shift-I]

Options:

A. 2

B. 2^{n-1}

C. 1

D. n

Answer: C

Solution:

Solution:

Given, $(1-x+x^3)^n = \sum_{j=0}^{3n} a_j \times j$

$(1-x+x^3)^n$

$= a_0 + a_1x + a_2x^2 + \ldots + a_{3n}x^{3n}$

Putting $x = 1$,

$(1-1+1)^n = a_0 + a_1 + a_2 + \ldots + a_{3n}$

$1 = a_0 + a_1 + a_2 + \ldots + a_{3n} \ldots$ (i)

Putting $x = -1$,

$(1+1-1)^n = a_0 - a_1 + a_2 - a_3 + \ldots (-1)^{3n}a_n$

$1 = a_0 - a_1 + a_2 - a_3 + \ldots (-1)^{3n}a_n \ldots$ (ii)

Adding Eqs. (i) and (ii), we get

$2 = 2(a_0 + a_2 + a_4 + a_6 \ldots)$

$a_0 + a_2 + a_4 + \ldots = 1$

On subtracting Eq. (ii) from Eq. (i), we get

$0 = 2(a_1 + a_3 + a_5 + \ldots)$

$a_1 + a_2 + a_5 + \ldots = 0$

$\left[\frac{3n}{2}\right] \left[\frac{3n-1}{2}\right]$

Now, $\sum_{j=0} a_{2j} + 4\sum_{j=0} a_{2j} + 1 = [a_0 + a_2 + a_4 + \ldots] + 4$

$= 1 + 4 \times 0$

$= 1$

Question102

The value of $\sum_{r=0}^{6}({}^6C_r \cdot {}^6C_{6-r})$ is equal to [2021, 17 March Shift-II]

Options:

A. 1124

B. 1324

C. 1024

D. 924

Answer: D

Solution:

Method (1) (Proper Method)

$\sum_{r=0}^{6} {}^6C_r \cdot {}^6C_{6-r}$
$= {}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + {}^6C_2 \cdot {}^6C_4 +$
$C_3 \cdot {}^6C_3 + {}^6C_4 \cdot {}^6C_2 + {}^6C_5 \cdot {}^6C_1 + {}^6C_6 \cdot {}^6C_0$
Now,
$(1+x)^6 \cdot (1+x)^6 = ({}^6C_0 + {}^6C_1 x + {}^6C_2 x^2 +$
${}^6C_3 x^3 + \ldots + {}^6C_6 x^6)({}^6C_0 + {}^6C_1 x + {}^6C_2 x^2$
$+ {}^6C_3 x^3 + \ldots + {}^6C_6 x^6)$
On comparing the coefficients of x^6 from both sides, we have
${}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + {}^6C_2 \cdot {}^6C_4 + \ldots +$
$= \frac{{}^6C_6 \cdot C_0 = {}^{12}C_6}{6!(12-6)!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6!}{6! \times (6 \times 5 \times 4 \times 3 \times 2 \times 1)}$
$= 924$

Method (2) (Short-cut Method)
As, we know that,
$r^nC_0{}^2 + {}^nC_1{}^2 + {}^nC_2{}^2 + {}^nC_3{}^2 + \ldots + {}^nC_n{}^2 = {}^{2n}C_n$
$\Rightarrow {}^nC_0 \cdot {}^nC_0 + {}^nC_1 \cdot {}^nC_1 + {}^nC_2 \cdot {}^nC_2 + \ldots +$
${}^nC_n \cdot {}^nC_n = {}^{2n}C_n$

$\Rightarrow {}^nC_0 \cdot {}^nC_n + {}^nC_1 \cdot {}^nC_{n-1} + {}^nC_2 \cdot {}^nC_{n-2}$
$+ \ldots + {}^nC_n {}^nC_0 = {}^{2n}C_n (\because {}^nC_r = {}^nC_{n-r})$
Putting $n = 6$, we get,
${}^6C_0 \cdot {}^6C_6 + {}^6C_1 \cdot {}^6C_5 + {}^6C_2 \cdot {}^6C_4 + \ldots +$
${}^6C_6 \cdot C_0 = {}^{12}C_6$

Question103

Let n be a positive integer. Let

$$A = \sum_{k}^{n} (-1)^k {}_nC^k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k \right]$$

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to
[2021, 16 March Shift-II]

Answer: 6

Solution:

Solution:
Given, $A = \sum_{k=0}^{n} (-1)^k \cdot {}^nC_k$
$\left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k \right]$
$\Rightarrow A = \sum_{k=0}^{n} (-1)^k$

$$\left[\begin{array}{c} {}^nC_k\left(\frac{1}{2}\right)^k + {}^nC_k\left(\frac{3}{4}\right)^k + {}^nC_k\left(\frac{7}{8}\right)^k \\ +{}^nC_k\left(\frac{15}{16}\right)^k + {}^nC_k\left(\frac{31}{32}\right)^k \end{array}\right]$$

$$\Rightarrow (1-x)^n = \sum_{r=0}^{n} (-1)^r \cdot C_r x^r$$

$$x = \frac{1}{2}$$

$$\Rightarrow \left(1-\frac{1}{2}\right)^n = C_0 - C_1\left(\frac{1}{2}\right) + C_2\left(\frac{1}{2}\right)^2 \cdots$$

$$\Rightarrow x = \frac{3}{4}$$

$$\left(1-\frac{3}{4}\right)^n = C_0 - C_1\left(\frac{3}{4}\right) + C_2\left(\frac{3}{4}\right)^2 \cdots$$

Similarly, we will get

$$A = \sum_{k=0}^{n} (-1)^k \left[\begin{array}{c} {}^nC_k\left(\frac{1}{2}\right)^k + {}^nC_k\left(\frac{3}{4}\right)^k + {}^nC_k\left(\frac{7}{8}\right)^k \\ +{}^nC_k\left(\frac{15}{16}\right)^k + {}^nC_k\left(\frac{31}{32}\right)^k \end{array}\right]$$

$$\Rightarrow A = \left(1-\frac{1}{2}\right)^n + \left(1-\frac{3}{4}\right)^n + \left(1-\frac{7}{8}\right)^n$$

$$+\left(1-\frac{15}{16}\right)^n + \left(1-\frac{31}{32}\right)^n$$

$$\Rightarrow A = \frac{1}{2^n} + \frac{1}{2^{2n}} + \frac{1}{2^{3n}} + \frac{1}{2^{4n}} + \frac{1}{2^{5n}} \Rightarrow A = \frac{1}{2^n} \cdot \left[\frac{1-\frac{1}{(2^n)^5}}{1-\frac{1}{2^n}}\right]$$

$$\Rightarrow A = \left(\frac{1}{2^n}\right)\left(\frac{2^{5n}-1}{2^n-1}\right)\left(\frac{2^n}{2^{5n}}\right)$$

$$\Rightarrow A = \left(\frac{2^{5n}-1}{2^n-1}\right)\left(\frac{1}{2^{5n}}\right)$$

$$\because\ 63A = 1 - \frac{1}{2^{30}} \Rightarrow 63\,\frac{(2^{5n}-1)}{2^{5n}(2^n-1)} = \frac{2^{30}-1}{2^{30}}$$

$$\Rightarrow \left(\frac{63}{2^n-1}\right)\left(1-\frac{1}{2^{5n}}\right) = \left(1-\frac{1}{2^{30}}\right)$$

$n = 6$, satisfies the equation.

Question104

The term independent of x in the expansion of $\left(\frac{x+1}{x^{23}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x \neq 0, 1$ is equal to ....
[2021, 2 July Shift I]

Answer: 210

Solution:

Solution:

$$\left[\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}\right) - \left(\frac{x-1}{x-x^{1/2}}\right)\right]^{10}$$

$$= \left(x^{1/3} + 1 - \frac{x^{1/2}+1}{x^{1/2}}\right)^{10}$$
$$= \left(x^{1/3} - \frac{1}{x^{1/2}}\right)^{10}$$
General term,
$$\Rightarrow T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r}\left(\frac{-1}{x^{1/2}}\right)^r$$
For independent term,
$$\frac{10-r}{3} - \frac{r}{2} = 0$$
$\Rightarrow r = 4$
$\therefore T_5 = {}^{10}C_4 = 210$

Question105

If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity $\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + ... + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3$, then the value of γ is [2021, 25 July Shift-I]

Options:

A. $\frac{a^2+b}{3a^3}$

B. $\frac{a+b}{3a^2}$

C. $\frac{b^2}{3a^3}$

D. $\frac{a+b^2}{3a^3}$

Answer: C

Solution:

Solution:

$(a-b)^{-n} = a^{-n}\left(1 - \frac{b}{a}\right)^{-n}$

$\left(1 - \frac{b}{a}\right)^{-n} = \left[1 + n\left(\frac{b}{a}\right) + \frac{n(n+1)}{2}\left(\frac{b}{a}\right)^2\right]$

As, we can ignore the powers greater than or equal to 3.

$$a^{-n}\left(1 - \frac{b}{a}\right)^{-n}$$
$$= \frac{1}{a^n} + \frac{n \cdot b}{a^{n+1}} + \frac{n(n+1)}{2} \cdot \frac{b^2}{a^{n+2}}$$
$$= \frac{1}{a^n} + \frac{(a-b)^{-n}}{a^{n+1}} + \frac{n(n+1)}{2} \cdot \frac{b^2}{a^{n+2}}$$

When $n = 1$

$c(a-b)^{-1} = \frac{1}{a} + \frac{b}{a^2} + \frac{b^2}{a^3}b \to rb$

$$\sum_{r=b}^{n}(a-rb)^{-1} = \Sigma\left[\frac{1}{a} + r\left(\frac{b}{a^2}\right) + r^2\left(\frac{b^2}{a^3}\right)\right]$$

$S = \frac{n}{a} + \frac{n(n+1)b}{2a^2} + \frac{n(n+1)(2n+1)b^2}{6a^3}$

Coefficient of $n^3 = \frac{2b^2}{6a^3} = \frac{b^2}{3a^3}$

Question106

The sum of all those terms which are rational numbers in the expansion of $(2^{1/3} + 3^{1/4})^{12}$ is
[2021, 25 July Shift-II]

Options:

A. 89

B. 27

C. 35

D. 43

Answer: D

Solution:

Solution:

In the given expansion of $\left(2^{\frac{1}{3}} + 3^{\frac{1}{4}}\right)^{12}$

General term $\Rightarrow T_{r+1} = {}^{12}C_r(2^{1/3})^r(3^{1/4})^{12-r}$ $\left(2^{\frac{1}{3}}\right)^r$

will be a rational number when $r = 0, 3, 6, 9, 12$ and $\left(3^{\frac{1}{4}}\right)^{12-r}$

will be rational number when $r = 0, 4, 8, 12 \Rightarrow r = 0, 12$

If $r = 0$, then

$T_1 = {}^{12}C_0\left(2^{\frac{1}{3}}\right)^0\left(3^{\frac{1}{4}}\right)^{12} = 27$

If $r = 12$, then $T_{13} = {}^{12}C_{12}\left(2^{\frac{1}{3}}\right)^{12}\left(3^{\frac{1}{4}}\right)^0 = 16$

So, $T_1 + T_{13} = 27 + 16 = 43$

Question107

If the greatest value of the term independent of x in the expansion of $\left(x\sin\alpha + a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of a is equal to
[2021, 25 July Shift-II]

Options:

A. -1

B. 1

C. -2

D. 2

Answer: D

Solution:

Solution:

In the expansion of $\left(x \sin \alpha + a \dfrac{\cos \alpha}{x}\right)^{10}$

$T_{r+1} = {}^{10}C_r (x \sin \alpha)^{10-r} \left(\dfrac{a \cos \alpha}{x}\right)^r$

$= {}^{10}C_r (x)^{10-2r} (\sin \alpha)^{10-r} (a \cos \alpha)^r$

T_{r+1} is independent of x, when $10 - 2r = 0$

$\therefore$ cr = 5

$T_6 = {}^{10}C_5 (\sin \alpha)^5 (a^5)(\cos \alpha)^5$

$= {}^{10}C_5 a^5 \cdot \dfrac{1}{2^5} (\sin 2\alpha)^5$

For greatest value, $\sin 2\alpha = 1$

$= {}^{10}C_5 (a)^5 \cdot \dfrac{1}{2^5}$

Given, that the greatest value is $\dfrac{10!}{(5!)^2}$.

So, ${}^{10}C_5 \dfrac{a^5}{2^5} = \dfrac{10!}{(5!)^2}$

$\Rightarrow \dfrac{10!}{(5!)^2} \cdot \dfrac{a^5}{2^5} = \dfrac{10!}{(5!)^2}$

$\Rightarrow$ a = 2

Question108

For the natural numbers m, n, if $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \ldots\ldots + a_{m+n} y^{m+n}$ and $a_1 = a_2 = 10$, then the value of $(m + n)$ is equal to [2021, 20 July Shift-II]

Options:

A. 88

B. 64

C. 100

D. 80

Answer: D

Solution:

Solution:

Given, $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \ldots + a_{m+n} y^{m+n}$

Now,
$(1-y)^m(1+y)^n = (1 - my + {}^mC_2y^2 + \ldots +$
$(-1)^m \cdot {}^mC_my^m)$
$\times(1 + ny + {}^nC_2y^2 + \ldots + {}^nC_ny^n)$
On expanding,
$(1-y)^m(1+y)^n = \ 1 + (n-m)y$
$+({}^nC_2 - mn + {}^mC_2)y^2 + \ldots$

$\therefore$ Coefficient of $y = n - m$
Coefficient of $y^2 = {}^nC_2 - mn + {}^mC_2$
Given expression have
Coefficient of $y = a_1 = 10$
Coefficient of $y^2 = a_2 = 10$
$\therefore\ n - m = 10 \quad \cdots\cdots (i)$
and ${}^nC_2 + {}^mC_2 - mn = 10 \quad \cdots\cdots (ii)$
From Eq. (ii),
$$\frac{n!}{2!(n-2)!} + \frac{m!}{2!(m-2)!} - mn = 10$$
$$\frac{n(n-1)}{2} + \frac{m(m-1)}{2} - mn = 10$$
$\Rightarrow\ n(n-1) + m(m-1) - 2mn = 20$
$\Rightarrow\ n^2 - n + m^2 - m - 2mn = 20$
$\Rightarrow\ (m^2 + n^2 - 2mn) - (m+n) = 20$
$\Rightarrow\ (m-n)^2 - (m+n) = 20$
$\Rightarrow\ (-10)^2 - (m+n) = 20$ [using Eq. (i)]
$\Rightarrow\ 100 - (m+n) = 20$
$\Rightarrow\ m + n = 100 - 20 = 80$

Question109

The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is [2021, 20 July Shift-I]

Options:

A. ${}^{100}C_{16}$

B. ${}^{100}C_{15}$

C. $-{}^{100}C_{16}$

D. $-{}^{100}C_{15}$

Answer: B

Solution:

Solution:
$(1-x)^{101}(x^2+x+1)^{100}$
Coefficient of x^{256}
$= [(1-x)(1+x+x^2)]^{100}(1-x) =$
$(1-x^3)^{100}(1-x)$
$\Rightarrow({}^{100}C_0 - {}^{100}C_1x^3 + {}^{100}C_2x^6 - {}^{100}C_3x^9 \ldots)$
$(1-x)$
$\sum(-1)^{r\,100}C_rx^{3r}(1-x)$
$\Rightarrow\ 3r = 256$ or $255 \Rightarrow r = \frac{256}{3}\Big($ Reject)
$r = 85$
Coefficient $= {}^{100}C_{85} = {}^{100}C_{15}$

Question110

The number of rational terms in the binomial expansion of $(4^{1/4}+5^{1/6})^{120}$ is ....
[2021, 20 July Shift-I]

Answer: 21

Solution:

Solution:

$\left(4^{\frac{1}{4}}+5^{\frac{1}{6}}\right)^{120}$

General term $= {}^{120}C_r\left(4^{\frac{1}{4}}\right)^r\left(5^{\frac{1}{6}}\right)^{120-r}$

$= {}^{120}C_r 4^{\frac{r}{4}} 5^{20-\frac{r}{6}}$

$= {}^{120}C_r 2^{\frac{r}{2}} 5^{20-\frac{r}{6}}$

For this term to be rational, r should be a multiple of 2 and 6 i.e. r should be a multiple of 6 .
$r = \{0, 6, 12, 18, \ldots, 120\}$
Number of terms $= 21$

Question111

If the constant term, in Binomial expansion of $\left(2x^r+\frac{1}{x^2}\right)^{10}$ is 180 , then r is equal to
[2021, 22 July Shift-II]

Answer: 8

Solution:

Solution:

$\left(2x^r+\frac{1}{x^2}\right)^{10}$

General term $= {}^{10}C_k(2x^r)^{10-k}x^{-2k}$,
${}^{10}C_k(2^{10-k})(x^{10r-rk-2k})$
$10r-rk-2k=0$
$\Rightarrow\ k=\frac{10r}{r+2}\Rightarrow r=\frac{2k}{10-k}$
$\Rightarrow\ r=-2+\frac{20}{10-k}\Rightarrow k<10$
If $k=8$ and $r=-2+10=8$

$rk = 5$ and $r = 2$
${}^{10}C_k 2^{10-k} = 180$
At, $r = 8$ only this is satisfied.

Question112

The number of elements in the set $\{n \in \{1, 2, 3, ..., 100\} : (11)^n > (10)^n + (9)^n\}$ is [2021, 22 July Shift-II]

Answer: 96

Solution:

Solution:
$11^n > 10^n + 9^n$
$\Rightarrow 11^n - 9^n > 10^n$
$\Rightarrow (10+1)^n - (10-1)^n > 10^n$
${}^nC_0 10^n + {}^nC_1 10^{n-1} + {}^nC_2 10^{n-2} + ... \}$
$-\{C_0 {}^nC_0 10^n - {}^nC_1 10^{n-1}\}$
$\Rightarrow 2({}^nC_1 10^{n-1} + {}^nC_3 10^{n-3}) > 10^n$
For, $n = 1 \Rightarrow 2.1 \times 10$
For, $n = 2 \Rightarrow 2(2) \times 100$
For, $n = 3 \Rightarrow 2(3 \cdot 10^2 + 1) \times 1000$
For, $n = 5 \Rightarrow$
$2({}^5C_1 10^n + {}^5C_3 10^2 + {}^5C_5) \times 10^5$
Hence, $n \in \{5, 6, 7, ... 100\}$
$\therefore$ Number of elements $= 96$

Question113

The lowest integer which is greater than $\left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$ is

[2021, 25 July Shift-11]

Options:

A. 3

B. 4

C. 2

D. 1

Answer: A

Solution:

Solution:

Let $P = \left(1 + \frac{1}{10^{100}}\right)^{10^{100}}$

Let $x = 10^{100} \Rightarrow P = \left(1 + \frac{1}{x}\right)^{x}$

$P = {}^xC_0 + {}^xC_1 + {}^xC_1\left(\frac{1}{x}\right) + {}^xC_2\left(\frac{1}{x}\right)^2$

$+ {}^xC_3\left(\frac{1}{x}\right)^3 + \ldots$ up to $(10^{100} + 1)$ term

$\Rightarrow P = 1 + x\left(\frac{1}{x}\right) + \frac{x(x-1)}{2!} \cdot \frac{1}{x^2}$

$+ \frac{x(x-1)(x-2)}{3!} \cdot \frac{1}{x^3}$

$+ \ldots \text{ upto } (10^{100} + 1) \text{ terms}$

$\Rightarrow P = 1 + 1 + \left[\left(\frac{1}{2!} - \frac{1}{2!x^2}\right) + \left(\frac{1}{3!} + \ldots\right) + \ldots\right]$

$\Rightarrow P = 2 +$

(Positive value less than $\frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots$)

Now

$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots$

$\Rightarrow \frac{1}{2!} + \frac{1}{3!} + \ldots = e - 2$

⇒ P=2+[Positive value less than (e-2)]

$\Rightarrow P \in (2, 3)$

So, lowest integer which is greater than P is 3.

Question114

If the co-efficient of x^7 and x^8 in the expansion of $\left(2 + \frac{x}{3}\right)^n$ are equal, then the value of n is equal to ....
[2021, 25 July Shift-II]

Answer: 55

Solution:

Solution:

The coefficient of x^7 in the expansion of $\left(2 + \frac{x}{3}\right)^7 = {}^nC_7 2^{n-7} \frac{1}{3^7}$

and the coefficient of $x^8 = {}^nC_8 2^{n-8} \frac{1}{3^8}$

According to the question,

${}^nC_7 2^{n-7} \cdot \frac{1}{3^7} = {}^nC_8 2^{n-8} \frac{1}{3^8}$

$\frac{n!}{7!(n-7)!} \cdot 2 \cdot 2^{n-8} \frac{1}{3^7} = \frac{n!}{8!(n-8)!} 2^{n-8} \times \frac{1}{3^8}$

$\Rightarrow \frac{1}{7!(n-7)(n-8)!} \times 2 = \frac{1}{8.7!(n-8)!} \times \frac{1}{3}$

$\Rightarrow \frac{2}{n-7} = \frac{1}{24}$

$\Rightarrow n - 7 = 48 \Rightarrow n = 55$

Question115

$\sum_{k=0}^{20} ({}^{20}C_k)^2$ is equal to
[2021, 27 Aug. Shift-I]

Options:

A. ${}^{40}C_{21}$

B. ${}^{40}C_{19}$

C. ${}^{40}C_{20}$

D. ${}^{41}C_{20}$

Answer: C

Solution:

Solution:
$\sum_{k=0}^{20} ({}^{20}C_k)^2 = ({}^{20}C_0)^2 + ({}^{20}C_1)^2$
$+ ({}^{20}C_2)^2 + \ldots + ({}^{20}C_{20})^2$
$\left[\because C_0^{\ 2} + C_1^{\ 2} + \ldots + C_n^{\ 2} = \frac{(2n)!}{(n!)^2} \right]$
$= \frac{40!}{(20!)^2} = {}^{40}C_{20}$

Question116

If ${}^{20}C_r$ is the coefficient of x^r in the expansion of $(1 + x)^{20}$, then the value of $\sum_{r=0}^{20} r^2\,{}^{20}C_r$ is equal to
[2021, 26 Aug. Shift-I]

Options:

A. 420×2^{19}

B. 380×2^{18}

C. 380×2^{19}

D. 420×2^{18}

Answer: D

Solution:

Solution:
${}^nC_r = \left(\frac{n}{r} \right) {}^{n-1}C_{r-1}$

$r^nC_r = n^{n-1}C_{r-1}$
Similarly, $(r-1)^{n-1}C_{r-1} = (n-1)(^{n-2}C_{r-2})$ ······ (i)
Multiplying Eq. (i) with $(r-1)$
$r(r-1)^nC_r = n(r-1)^{n-1}C_{r-1}$
$\Rightarrow r(r-1)^nC_r = n\cdot(n-1)^{n-2}C_{r-2}$
$r^{2n}C_r = [r(r-1)+r]^nC_r$
$= r(r-1)^nC_r + r^nC_r$
$= n(n-1)^{n-2}C_{r-2} + n\cdot{}^{n-1}C_{r-1}$
$\sum r^{2n}C_r = n(n-1)\sum_{r=2}{}^{n-2}C_{r-2} + n\sum_{r=1}^{n-1}C_{r-1}$
Now, when $n = 20$
$\sum r^{2n}C_r = (20\times 19)\sum^{18}C_r + 20\sum^{19}C_r$
$= (20\times 19)2^{18} + 20\cdot 2^{19} = 420\cdot 2^{18}$

Question117

Let $\begin{pmatrix} n \\ k \end{pmatrix}$ denotes ^nC_k and

$$\begin{bmatrix} n \\ k \end{bmatrix} = \begin{cases} \begin{pmatrix} n \\ k \end{pmatrix} & \text{if } 0 \le k \le n \\ 0 & \text{otherwise} \end{cases}.$$

If $A_k = \sum_{i=0}^{9}\begin{pmatrix} 9 \\ i \end{pmatrix}\begin{bmatrix} 12 \\ 12-k+i \end{bmatrix}$
$+ \sum_{i=0}^{8}\begin{pmatrix} 8 \\ i \end{pmatrix}\begin{bmatrix} 13 \\ 13-k+i \end{bmatrix}$
and $A_4 - A_3 = 190p$, then p is equal to
[2021, 26 Aug. Shift-II]

Answer: 49

Solution:

Given, $a_k = \sum_{i=0}^{9}(^9C_i \times {}^{12}C_{12-k} + i) + \sum_{i=0}^{8}(^8C_j \times {}^{13}C_{13-k} + i)$
$\Rightarrow A_k = \sum_{i=0}^{9}{}^9C_i{}^{12}C_{k-i} + \sum_{i=0}^{8}{}^8C_i{}^{13}C_{k-i}$
$^9C_0{}^{12}C_k + {}^9C_1{}^{12}C_{k-1} + {}^9C_2{}^{12}C_{k-2} + \ldots +$
$^9C_9{}^{12}C_{k-9} = {}^{21}C_k{}^k \quad \left[\because \sum_{r=0}^{\alpha}{}^nC_r \times {}^mC_{\alpha-r} = {}^{m+n}C_\alpha\right]$
Similarly,
$\sum_{i=0}^{8}{}^8C_i{}^{13}C_{k-i} = {}^{21}C_k$
$A_k = {}^{21}C_k + {}^{21}C_k 2^{21}C_k$
$A_4 - A_3 = 2\cdot(^{21}C_4 - {}^{21}C_3)$

$= 2(5985 - 1330)$
$\Rightarrow 190p = 9310$
$p = 49$

Question118

If the coefficients of x^7 in $\left(x^2 + \frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x - \frac{1}{bx^2}\right)^{11}$, $b \neq 0$, are equal, then the value of b is equal to
[2021, 27 July Shift-1]

Options:

A. 2

B. -1

C. 1

D. -2

Answer: C

Solution:

Solution:

Coefficient of x^7 in $\left(x^2 + \frac{1}{bx}\right)^{11}$

General term $= {}^{11}C_r (x^2)^r \left(\frac{1}{bx}\right)^{11-r}$

$= {}^{11}C_r x^{3r-11} b^{r-11}$

$\Rightarrow 3r - 11 = 7 \Rightarrow r = 6$

$\therefore$ Coefficient of $x^7 = {}^{11}C_6 b^{-5}$

Now, coefficient of x^{-7} in $\left(x - \frac{1}{bx^2}\right)^{11}$

General term $= {}^{11}C_r x^r \left(\frac{-1}{bx^2}\right)^{11-r}$

$= {}^{11}C_r \left(\frac{-1}{b}\right)^{11-r} x^r \cdot \frac{1}{x^{22-2r}}$

$= {}^{11}C_r \left(\frac{-1}{b}\right)^{11-r} x^{3r-22}$

l rl $\Rightarrow 3r - 22 = -7$

$\Rightarrow r = 5$

Coefficient $= {}^{11}C_5 \left(\frac{-1}{b}\right) = {}^{11}C_5 b^{-6}$

Now, according to the question,

${}^{11}C_6 b^{-5} = {}^{11}C_5 b^{-6}$

$b = 1$

Question119

A possible value of ' x, for which the ninth term in the expansion of

$$\left\{ 3^{\log_3 \sqrt{25^{x-1}+7}} + 3\left(-\frac{1}{8}\right)^{\log_3(5^{x-1}+1)} \right\}^{10}$$

is equal to 180 , is
[2021, 27 July Shift-II]

Options:

A. 0

B. −1

C. 2

D. 1

Answer: D

Solution:

Solution:
We have,

$$\left\{ 3^{\log_3 \sqrt{25^{x-1}+7}} + 3^{-\frac{1}{8}\cdot\log_3(5^{x-1}+1)} \right\}^{10}$$

$$= \left\{ \sqrt{25^{x-1}+7} + (5^{x-1}+1)^{\frac{-1}{8}} \right\}^{10}$$

Ninth term in the expansion is 180.

c So, ${}^{10}C_8\left(\sqrt{25^{x-1}+7}^{10-8}\left[(5^{x-1}+1)^{\frac{-1}{8}}\right]^8\right.$

$= 180\{\because (r+1)$ th term or expansion $(x+a)^n$,,
$T_{r+1} = {}^nC_r x^{n-r} a^r\}$

$\Rightarrow {}^{10}C_8(25^{x-1}+7)(5^{x-1}+1)^{-1} = 180$

$\Rightarrow 45(25^{x-1}+7)(5^{x-1}+1)^{-1} = 180$

$\Rightarrow \dfrac{25^{x-1}+7}{5^{x-1}+1} = 4$

Let $5^{x-1} = t$

$\Rightarrow \dfrac{t^2+7}{t+1} = 4$

$\Rightarrow t^2+7 = 4t+4$

$\Rightarrow t^2-4t+3 = 0$

$\Rightarrow (t-3)(t-1) = 0$

$\Rightarrow t = 3$ or $t = 1$

Whent $= 3$
$5^{x-1} = 3$
$5^x = 15$
$x = \log_5 15$
When, $t = 1$
rrr $5^{x-1} = 1$ $[t = 5^{x-1}]$
$\Rightarrow x-1 = 0$
$\Rightarrow x = 1$

Question120

The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is ....
[2021, 25 July Shift-I]

Answer: 1

Solution:

Solution:
Coefficient of middle term of $(1+x)^{20}$ is ${}^{20}C_{10}$.
Coefficient of middle term of $(1+x)^{19}$ is ${}^{19}C_9$ and ${}^{19}C_{10}$
According to the question

$$\left(\frac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}}\right) = \frac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$$

Question121

If $\left(\frac{3^6}{4^4}\right)$k is the term, independent of x, in the binomial expansion of $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$, then k is equal to
[2021, 31 Aug. Shift-I]

Answer: 55

Solution:

Solution:

$$\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$$

$$T_{r+1} = (-1)^{r\,12}C_r\left(\frac{x}{4}\right)^{12-r}\left(\frac{12}{x^2}\right)^r$$

$$= (-1)^r\left(\frac{{}^{12}C_r \cdot 12^r}{4^{12-r}}\right)x^{(12-r-2r)}$$

Term independent of r
$12 - 3r = 0$
$\Rightarrow\ r = 4$

$$T_5 = (-1)^4\left(\frac{{}^{12}C_4 \cdot 12^4}{4^8}\right) = \frac{3^6}{4^4}k$$

$k = 55$

Question122

$3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder
[2021, 27 Aug. Shift-II]

Answer: 15

Solution:

Solution:

$3 \times 7^{22} + 2 \times 10^{22} - 44$

$= 3 \times (6+1)^{22} + 2 \times (9+1)^{22} - 44$

Now,

$(1+6)^{22} = 1 + {}^{22}C_1 6 + {}^{22}C_2 \cdot 6^2 + \ldots. + {}^{22}C_{22} 6^{22}$

$= (1 + 6\lambda)$

$(1+9)^{22} = 1 + {}^{22}C_1 9 + {}^{22}C_2 \cdot 9^2 + \ldots. + {}^{22}C_{22} 6^{22}$

$= (1 + 9\mu)$

$\therefore \; = 3(1+6\lambda) + 2(1+9\mu) - 44$

$= 18\lambda + 3 + 18\mu + 2 - 44$

$= 18\delta - 39 = 18\alpha + 15$

$3 \times 7^{22} + 2 \times 10^{22} - 44$, when divided by 18
leaves remainder 15 .

Question123

If the coefficient of a^7b^8 in the expansion of $(a + 2b + 4ab)^{10}$ is $k \cdot 2^{16}$, then k is equal to
[2021, 31 Aug. Shift-II]

Answer: 315

Solution:

Solution:

$(a + 2b + 4ab)^{10} = a^{10}b^{10}\left(\frac{1}{b} + \frac{2}{a} + 4\right)^{10}$

Generalterm

$= a^{10}b^{10} \frac{10!\left(\frac{1}{b}\right)^{r_1}\left(\frac{2}{a}\right)^{r_2} 4^{10-r_1-r_2}}{r_1! \cdot r_2!(10 - r_1 - r_2)!}$

So, $r_1 = 2, r_2 = 3$

Coefficient of $a^7b^8 = \frac{10! \cdot 2^3 \cdot 4^{10-2-3}}{2!3!(10-2-3)!}$

$= \frac{2^{13} \cdot 10!}{2!3!5!} = 2^{16} \cdot 315$

$\therefore \; k = 315$

Question124

If the sum of the coefficients of all even powers of x in the product $(1 + x + x^2 + \ldots + x^{2n})(1 - x + x^2 - x^3 + \ldots + x^{2n})$ is 61, then n is equal to ________.

[NA Jan. 7, 2020 (I)]

Answer: 30

Solution:

Solution:
Let $(1 - x + x^2 \ldots\ldots x^{2n})(1 + x + x^2 \ldots\ldots x^{2n})$
$= a_0 + a_1 x + a_2 x^2 + \ldots.$
put $x = 1$
$1(2n + 1) = a_0 + a_1 + a_2 + \ldots.\ a_{2n}$ . . . (i)
put $x = -1$
$(2n + 1) \times 1 = a_0 - a_1 + a_2 + \ldots\ldots a_{2n}$ . . . (ii)
Adding (i) and (ii), we get,
$4n + 2 = 2(a_0 + a_2 + \ldots..) = 2 \times 61$
$\Rightarrow\ 2n + 1 = 61 \Rightarrow n = 30$

Question125

The coefficient of x^4 in the expansion of $(1 + x + x^2)^{10}$ is ________.
[NA Jan. 9, 2020 (I)]

Answer: 615

Solution:

Solution:
General term of the expansion $= \dfrac{10!}{\alpha!\beta!\gamma!}x^{\beta + 2\gamma}$
For coefficient of x^4; $\beta + 2\gamma = 4$
Here, three cases arise
Case-1 : When $\gamma = 0, \beta = 4, \alpha = 6$
$\Rightarrow \dfrac{10!}{\alpha!\beta!\gamma!}x^{\beta + 2\gamma}$
Case-2 : When $\gamma = 1, \beta = 2, \alpha = 7$
$\Rightarrow\ \dfrac{10!}{7!2!1!} = 360$
Case-3 : When $\gamma = 2, \beta = 0, \alpha = 8$
$\Rightarrow\ \dfrac{10!}{8!0!2!} = 45$
Hence, total $= 615$

Question126

If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $\left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$, then:

[Jan. 8, 2020 (II)]

Options:

A. $\alpha + \beta = 60$

B. $\alpha + \beta = -30$

C. $\alpha - \beta = 60$

D. $\alpha - \beta = -132$

Answer: D

Solution:

Solution:
Using Binomial expansion
$(x + a)^n + (x - a)^n = 2(T_1 + T_3 + T_5 + T_7 \ldots)$
$\therefore \left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6 = 2(T_1 + T_3 + T_5 + T_7)$
$2[{}^6C_0 x^5 + {}^6C_2 x^4 (x^2 - 1) + {}^6C_4 x^2 (x^2 - 1)^2 + {}^6C_6 (x^2 - 1)^3]$
$= 2[x^6 + 15(x^6 - x^4) + 15x^2(x^4 - 2x^2 + 1) + (-1 + 3x^2 - 3x^4 + x^6)]$
$= 2(32x^6 - 48x^4 + 18x^2 - 1)$
$\alpha = -96$ and $\beta = 36$
$\therefore\ \alpha - \beta = -132$

Question127

In the expansion of $\left(\frac{x}{\cos\theta} + \frac{1}{x\sin\theta} \right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8} \le \theta \le \frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16} \le \theta \le \frac{\pi}{8}$, then the ratio $l_2 : l_1$ is equal to :
[Jan. 9, 2020 (II)]

Options:

A. 1: 8

B. 16: 1

C. 8: 1

D. 1: 16

Answer: B

Solution:

Solution:
General term of the given expansion
$T_{r+1} = {}^{16}C_r \left(\frac{x}{\sin\theta} \right)^{16-r} \left(\frac{1}{x\cos\theta} \right)^r$
For $r = 8$ term is free from ' x '
$T_9 = {}^{16}C_8 \frac{1}{\sin^8\theta\cos^8\theta}$

$T_9 = {}^{16}C_8 \frac{2^8}{(\sin 2\theta)^8}$

When $\theta \in \left[\frac{\pi}{8}, \frac{\pi}{4}\right]$, then least value of the term independent of x

$l_1 = {}^{16}C_8 2^8$ [$\because$ min. value of l_1 at $\theta = \pi / 4$]

When $\theta \in \left[\frac{\pi}{16}, \frac{\pi}{8}\right]$, then least value of the term independent of x,

$l_2 = {}^{16}C_8 = \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 \cdot 2^8 \cdot 2^4$

[$\because$ min. value of l_2 at $\theta = \pi / 8$]

Now, $\frac{l_2}{l_1} = \frac{{}^{16}C_8 \cdot 2^8 \cdot 2^4}{{}^{16}C_8 \cdot 2^8} = 16$

Question128

If {p} denotes the fractional part of the number p, then $\left\{ \frac{3^{200}}{8} \right\}$, is equal to :
[Sep. 06, 2020 (I)]

Options:

A. $\frac{5}{8}$

B. $\frac{7}{8}$

C. $\frac{3}{8}$

D. $\frac{1}{8}$

Answer: D

Solution:

Solution:

$\frac{3^{200}}{8} = \frac{1}{8}(9^{100})$

$= \frac{1}{8}(1+8)^{100} = \frac{1}{8}\left[1 + n \cdot 8 + \frac{n(n+1)}{2} \cdot 8^2 + \ldots\right]$

$= \frac{1}{8} + \text{ Integer}$

$\therefore \left\{ \frac{3^{200}}{8} \right\} = \left\{ \frac{1}{8} + \text{ integer } \right\} = \frac{1}{8}$

Question129

The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m + \frac{1}{x^2}\right)^{22}$ is 1540, is ______.
[NA Sep. 05, 2020 (I)]

Answer: 13

Solution:

Solution:

$T_{r+1} = {}^{22}C_r \cdot (x^m)^{22-r} \cdot \left(\frac{1}{x^2}\right)^r$

$T_{r+1} = {}^{22}C_r \cdot x^{22m - mr - 2r}$

$\because 22m - mr - 2r = 1$

$\Rightarrow r = \frac{22m-1}{m+2} \Rightarrow r = 22 - \frac{3 \cdot 3 \cdot 5}{m+2}$

So, possible value of $m = 1, 3, 7, 13, 43$

But ${}^{22}C_r = 1540$

$\therefore$ Only possible value of $m = 13$.

Question130

The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is __________.
[NA Sep. 04, 2020 (I)]

Answer: 120

Solution:

Solution:

Coefficient of x^4 in $\left(\frac{1-x^4}{1-x}\right)^6 =$ coefficient of x^4 in $(1-6x^4)(1-x)^{-6}$

= coefficient of x^4 in $(1-6x^4)[1 + {}^6C_1 x + {}^7C_2 x^2 + \ldots.]$

$= {}^9C_4 - 6 \cdot 1 = 126 - 6 = 120$

Question131

Let $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{12}}$ is equal to __________.
[NA Sep. 04, 2020 (I)]

Answer: 8

Solution:

Solution:

The given expression is $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^7$

General term $= \frac{10!}{r_1!r_2!r_3!}(2x^2)^{n}(3x)^{1/2}(4)^{r_3}$

Since, $a_7 =$ Coeff. of x^7

$2r_1 + r_2 = 7$ and $r_1 + r_2 + r_3 = 10$

Possibilities are

r_1	r_2	r_3
0	7	3
1	5	4
2	3	5
3	1	6

$a_7 = \frac{10!3\,]^7\,4^3}{7!3!} + \frac{10!(2)(3)^5(4)^4}{5!4!}$

$+ \frac{10!(2)^2(3)^3(4)^5}{2!3!5!} + \frac{10!(2)^3(3)(4)^6}{3!6!}$

$a_{13} =$ Coeff. of x^{13}

$2r_1 + r_2 = 13$ and $r_1 + r_2 + r_3 = 10$

Possibilities are

r_1	r_2	r_3
3	7	0
4	5	1
5	3	2
6	1	3

$a_{13} = \frac{10!(2^3)(3^7)}{3!7!} + \frac{10!(2^4)(3^5)(4)}{4!5!}$

$+ \frac{10!(2^5)(3^3)(4^2)}{5!3!2!} + \frac{10!(2^6)(3)(4^3)}{6!1!3!}$

$\therefore \frac{a_7}{a_{13}} = 2^3 = 8$

Question132

If the constant term in the binomial expansion of $\left(\sqrt{x}\ \frac{k}{x^2}\right)^{10}$ is 405, then |k| equals:
[Sep. 06, 2020 (II)]

Options:

A. 9

B. 1

C. 3

D. 2

Answer: C

Solution:

Solution:

General term $= T_{r+1} = {}^{10}C_r(\sqrt{x})^{10-r} \cdot \left(-\frac{k}{x^2}\right)^r$

$= {}^{10}C_r(-k)^r \cdot x^{\frac{10-r}{2} - 2r}$

$= {}^{10}C_r(-k)^r \cdot x^{\frac{10-5r}{2}}$

Since, it is constant term, then

$\frac{10-5r}{2} = 0 \Rightarrow r = 2$

$\therefore {}^{10}C_2(-k)^2 = 405$

$\Rightarrow k^2 = \frac{405 \times 2}{10 \times 9} = \frac{81}{9} = 9$

$\therefore |k| = 3$

Question133

If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is : [Sep. 04, 2020 (II)]

Options:

A. 462

B. 330

C. 792

D. 252

Answer: A

Solution:

Solution:

Consider the three consecutive coefficients of $(1+x)^{n+5}$ be ${}^{n+5}C_r, {}^{n+5}C_{r+1}, {}^{n+5}C_{r+2}$

$\because \frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \frac{1}{2}$ (Given)

$\Rightarrow \frac{r+1}{n+5-r} = \frac{1}{2} \Rightarrow 3r = n+3 \ldots$ (i)

and $\frac{n+5}{n+5}C_{r+1} = \frac{5}{7}$

$\Rightarrow \frac{r+2}{n+4-r} = \frac{5}{7} \Rightarrow 12r = 5n + 6 \ldots$(ii)

Solving (i) and (ii) we get $r = 4$ and $n = 6$

$\therefore$ Largest coefficient in the expansion is ${}^{11}C_6 = 462$.

Question134

If the number of integral terms in the expansion of $(3^{1/2} + 5^{1/8})^n$ is exactly 33, then the least value of n is : [Sep. 03, 2020 (I)]

Options:

A. 264

B. 128

C. 256

D. 248

Answer: C

Solution:

Solution:

Here $\left(3^{\frac{1}{2}} + 5^{\frac{1}{8}}\right)^n$

$T_{r+1} = {}^nC_r(3)^{\frac{n-r}{2}}(5)^{\frac{r}{8}}$

$\because \frac{n-r}{2}$ and $\frac{r}{8}$ are integer

So, r must be 0, 8, 16, 24……

Now $n = t_{33} = a + (n-1)d = 0 + 32 \times 8 = 256$

$\Rightarrow n = 256$

Question135

If the term independent of x in the expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ is k, then 18k is equal to : [Sep. 03, 2020 (II)]

Options:

A. 5

B. 9

C. 7

D. 11

Answer: C

Solution:

Solution:

General term $= T_{r+1} = {}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$= {}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$

The term is independent of x, then

$18 - 3r = 0 \Rightarrow r = 6$

$\therefore T_7 = {}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6 = {}^9C_3\left(\frac{1}{6}\right)^3$

$= \frac{9\times 8\times 7}{3\times 2\times 1}\left(\frac{1}{6}\right)^3 = \left(\frac{7}{18}\right)$

$\therefore 18k = 18\times\frac{7}{18} = 7$

Question136

Let $\alpha > 0$, $\beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{\frac{1}{9}} + \beta x^{-\frac{1}{6}}\right)^{10}$ is 10k, then k is equal to :
[Sep. 02, 2020 (I)]

Options:

A. 336

B. 352

C. 84

D. 176

Answer: A

Solution:

Solution:

General term of

$\left(\alpha x^{\frac{1}{9}} + \beta x^{\frac{-1}{6}}\right)^{10} = {}^{10}C_r\left(\alpha x^{\frac{1}{9}}\right)^{10-r}\left(\beta x^{\frac{-1}{6}}\right)^r$

$= {}^{10}C_r\alpha^{10-r}\beta^r(x)^{\frac{10-r}{9}-\frac{r}{6}}$

Term independent of x if $\frac{10-r}{9} - \frac{r}{6} = 0 \Rightarrow r = 4$

$\therefore$ Term independent of $x = {}^{10}C_4\alpha^6\beta^4$

Since $\alpha^3 + \beta^2 = 4$

Then, by AM-GM inequality

$\frac{\alpha^3 + \beta^2}{2} \geq (\alpha^3 b^2)^{\frac{1}{2}}$

$\Rightarrow (2)^2 \geq \alpha^3\beta^2 \Rightarrow \alpha^6\beta^4 \leq 16$

$\because$ The maximum value of the term independent of $x = 10k$

$\therefore 10k = {}^{10}C_4 \cdot 16 \Rightarrow k = 336$

Question137

For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to ________.
[NA Sep. 02, 2020 (II)]

Answer: 118

Solution:

Solution:
According to the question,
${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2 : 5 : 12$
$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{5}{2} \Rightarrow \frac{n-r+1}{r} = \frac{5}{2}$
$\Rightarrow 2n - 7r + 2 = 0$ . . . (i)
$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{12}{5} \Rightarrow \frac{n-r}{r+1} = \frac{12}{5}$
$\Rightarrow 5n - 17r - 12 = 0$ . . . (ii)
Solving eqns. (i) and (ii),
$n = 118, r = 34$

Question138

The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to:
[Sep. 04, 2020 (I)]

Options:

A. ${}^{51}C_7 - {}^{30}C_7$

B. ${}^{50}C_7 - {}^{30}C_7$

C. ${}^{50}C_6 - {}^{30}C_6$

D. ${}^{51}C_7 + {}^{30}C_7$

Answer: A

Solution:

Solution:
The given series, $\sum_{r=0}^{20} {}^{50-r}C_6$
$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + {}^{47}C_6 + \ldots + {}^{32}C_6 + {}^{31}C_6 + {}^{30}C_6$
$= ({}^{30}C_7 + {}^{30}C_6) + {}^{31}C_6 + {}^{32}C_6 + \ldots. + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$
$= ({}^{31}C_7 + {}^{31}C_6) + {}^{32}C_6 + \ldots. + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$
$= ({}^{32}C_7 + {}^{32}C_6) + \ldots. + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$
....

....
....
$= {}^{51}C_7 - {}^{30}C_7$

Question139

Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1x + a_2x^2 + ... + a_{50}x^{50}$ for all $x \in R$;
then $\frac{a_2}{a_0}$ is equal to:
[Jan. 11, 2019 (II)]

Options:

A. 12.50

B. 12.00

C. 12.25

D. 12.75

Answer: C

Solution:

Solution:
$(x + 10)^{50} + (x - 10)^{50}$
$= a_0 + a_1x + a_2x^2 + \ldots + a_{50}x^{50}$
$\therefore\ a_0 + a_1x + a_2x^2 + \ldots + a_{50}x^{50}$
$= 2({}^{50}C_0x^{50} + {}^{50}C_2x^{48} \cdot 10^2 + {}^{50}C_4x^{46} \cdot 10^4 + \ldots)$
$\therefore\ a_0 = 2 \cdot {}^{50}C_{50}10^{50}$
$a_2 = 2 \cdot {}^{50}C_2 \cdot 10^{48}$
$\therefore\ \frac{a_2}{a_0} = \frac{{}^{50}C_2 \times 10^{48}}{{}^{50}C_{50}10^{50}}$
$= \frac{50 \times 49}{2 \times 100} = \frac{49}{4} = 12.25$

Question140

If the third term in the binomial expansion of $\left(1 + x^{\log_2 x}\right)^5$ equals 2560 ,
then a possible value of x is:
[Jan. 10, 2019 (I)]

Options:

A. $\frac{1}{4}$

B. $4\sqrt{2}$

C. $\frac{1}{8}$

D. $2\sqrt{2}$

Answer: A

Solution:

Solution:

Third term of $\left(1+x^{\log_2 x}\right)^5 = {}^5C_2\left(x^{\log_2 x}\right)^{5-3}$

$= {}^5C_2\left(x^{\log_2 x}\right)^2$

Given, ${}^5C_2\left(x^{\log_2 x}\right)^2 = 2560$

$\Rightarrow \left(x^{\log_2 x}\right)^2 = 256 = (\pm 16)^2$

$\Rightarrow x^{\log_2 x} = 16$ or $x^{\log_2 x} = -16$ (rejected)

$\Rightarrow x^{\log_2 x} = 16 \Rightarrow \log_2 x \log_2 x = \log_2 16 = 4$

$\Rightarrow \log_2 x = \pm 2 \Rightarrow x = 2^2$ or 2^{-2}

$\Rightarrow x = 4$ or $\frac{1}{4}$

Question141

The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is:

[Jan. 10, 2019 (II)]

Options:

A. 4

B. $2\sqrt{2}$

C. $\sqrt{5}$

D. 3

Answer: A

Solution:

Solution:

Since, coefficient of x^2 in the expression x^2

$\left(\sqrt{x}+\frac{\lambda}{x^2}\right)$ is a constant term, then

Coefficient of x^2 in $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

= co-efficient of constant term in $\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

General term in $\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10} = {}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{\lambda}{x^2}\right)^{\prime}$

$= {}^{10}C_r(x)^{\frac{10-r}{2}2r} \cdot \lambda^2$

Then, for constant term,

$\frac{10-r}{2} - 2r = 0 \Rightarrow r = 2$

Co-efficient is x^2 in expression $= {}^{10}C_2\lambda^2 = 720$

$\Rightarrow \lambda^2 = \frac{720}{5 \times 9} = 16 \Rightarrow \lambda = 4d$

Question142

If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to: [Jan. 9, 2019 (I)]

Options:

A. 6

B. 8

C. 4

D. 14

Answer: B

Solution:

Solution:
$2^{403} = 2^{400} \cdot 2^{3}$
$= 2^{4 \times 100} \cdot 2^{3}$
$= (2^{4})^{100} \cdot 8$
$= 8(2^{4})^{100} = 8(16)^{100}$
$= 8(1 + 15)^{100}$
$= 8 + 15\mu$
When 2^{403} is divided by 15, then remainder is 8
Hence, fractional part of the number is $\frac{8}{15}$
Therefore value of k is 8

Question143

The total number is irrational terms in the binomial expansion of $\left(7^{\frac{1}{5}} - 3^{\frac{1}{10}}\right)^{60}$ is : Jan. 12, 2019 (II)]

Options:

A. 55

B. 49

C. 48

D. 54

Answer: D

Solution:

Solution:
Let the general term of the expansion

$T_{r+1} = {}^{60}C_r \left(7^{\frac{1}{5}}\right)^{60-r} \left(-3^{\frac{1}{10}}\right)^r$

$= {}^{60}C_r \cdot (7)^{12-\frac{r}{5}} (-1)^r \cdot (3)^{\frac{r}{10}}$

Then, for getting rational terms, r should be multiple of L.C.M. of (5,10)
Then, r can be 0,10,20,30,40,50,60 .
Since, total number of terms $= 61$
Hence, total irrational terms $= 61 - 7 = 54$

Question144

A ratio of the 5^{th} term from the begining to the 5 th term from the end in the binomial expansion of $\left(2^{\frac{1}{3}} + \frac{1}{2(3)^{\frac{1}{3}}}\right)^{10}$ is:

[Jan. 12, 2019 (I)]

Options:

A. $1 : 2(6)^{\frac{1}{3}}$

B. $1 : 4(16)^{\frac{1}{3}}$

C. $4(36)^{\frac{1}{3}} : 1$

D. $2(36)^{\frac{1}{3}} : 1$

Answer: C

Solution:

Solution:

$\left(2^{\frac{1}{3}} + \frac{1}{2(3)^{\frac{1}{3}}}\right)^{10} = {}^{10}C_0 \left(2^{\frac{1}{3}}\right)^0 \left(\frac{1}{2(3)^{1/3}}\right)^{10} +$

$\cdot s + {}^{10}C_{10} \left(2^{\frac{1}{3}}\right)^{10} \left(\frac{1}{2(3)^{1/3}}\right)^0$

5th term from beginning $T_5 = {}^{10}C_4 \left(2^{\frac{1}{3}}\right)^6 \frac{1}{\left(2.3^{\frac{1}{3}}\right)^4}$

and 5^{th} term from end $T_{11-5+1} = {}^{10}C_6 \left(2^{\frac{1}{3}}\right)^4 \left(\frac{1}{2.3^{\frac{1}{3}}}\right)^6$

$\therefore T_5 : T_7 = {}^{10}C_4 \left(2^{\frac{1}{3}}\right)^6 \left(\frac{1}{2.3^{\frac{1}{3}}}\right)^4 : {}^{10}C_6 \left(2^{\frac{1}{3}}\right)^4 \left(\frac{1}{2.3^{\frac{1}{3}}}\right)^6$

$= \left(2^{\frac{1}{3}}\right)^2 : \left(\frac{1}{2.3^{\frac{1}{3}}}\right)^2$

$$= \frac{2^{\frac{2}{3}} \cdot 2^2 \cdot 3^{\frac{2}{3}}}{1} = 4(6)^{\frac{2}{3}} : 1 = 4 \cdot (36)^{\frac{1}{3}} : 1$$

Question145

The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3} + \frac{3}{x} \right)^8$ equals 5670 is : [Jan. 11, 2019 (I)]

Options:

A. 0

B. 6

C. 4

D. 8

Answer: A

Solution:

Solution:

Middle Term, $\left(\frac{n}{2} + 1 \right)^{\text{th}}$ term in the binomial expansion of $\left(\frac{x^3}{3} + \frac{3}{x} \right)^8$ is,

$T_{4+1} = {}^8C_4 \left(\frac{x^3}{3} \right)^4 \left(\frac{3}{x} \right)^4 = 5670$

$\Rightarrow \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2} \times x^{12-4} = 5670$

$\Rightarrow x^8 = 81$

$\Rightarrow x^8 - 81 = 0$

$\therefore$ sum of all values of $x =$ sum of roots of equation $(x^8 - 81 = 0)$.

Question146

The value of r for which ${}^{20}C_r {}^{20}C_0 + {}^{20}C_{r-1} {}^{20}C_1 + {}^{20}C_{r-2} {}^{20}C_2 + ... + {}^{20}C_0 {}^{20}C_r$ is maximum, is : [Jan. 11, 2019 (I)]

Options:

A. 15

B. 20

C. 11

D. 10

Answer: B

Solution:

Solution:
Consider the expression
${}^{20}C_r\,{}^{20}C_0 + {}^{20}C_{r-1}\,{}^{20}C_1 + {}^{20}C_{r-2}\,{}^{20}C_2 + \ldots + {}^{20}C_0 \cdot {}^{20}C_r$
For maximum value of above expression r should be equal to 20 .
as ${}^{20}C_{20} \cdot {}^{20}C_0 + {}^{20}C_{19} \cdot {}^{20}C_1 + \ldots + {}^{20}C_{20} \cdot {}^{20}C_0$
$= ({}^{20}C_0)^2 + ({}^{20}C_1)^2 + \ldots + ({}^{20}C_{20})^2 = {}^{40}C_{20}$
Which is the maximum value of the expression, So, $r = 20$.

Question147

If $\sum_{r=0}^{25}\{{}^{50}C_r \cdot {}^{50-r}C_{25-r}\} = K({}^{50}C_{25})$, then K is equal to: [Jan. 10, 2019 (II)]

Options:

A. $(25)^2$

B. $2^{25}-1$

C. 2^{24}

D. 2^{25}

Answer: D

Solution:

Solution:

$\sum_{r=0}^{25}({}^{50}C_r \cdot {}^{50-r}C_{25-r}) = \sum_{r=0}^{25}\left(\frac{\underline{|50}}{\underline{|50-r}\,\underline{|r}}\ \frac{\underline{|50-r}}{\underline{|25}\,\underline{|25-r}}\right)$

$= \sum_{r=0}^{25}\left(\frac{\underline{|50}}{\underline{|25}} \times \frac{1}{\underline{|25}} \times \left(\frac{\underline{|25}}{\underline{|25-r}\,\underline{|r}}\right)\right)$

$= {}^{50}C_{25}\sum_{r=0}^{25}{}^{25}C_r = {}^{50}C_{25}(2^{25})$

Then, by comparison, $K = 2^{25}$

Question148

The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ [Jan. 09, 2019 (II)]

Options:

A. 14

B. 15

C. 10

D. 12

Answer: B

Solution:

Solution:
Consider the expression

$\left(\frac{1-t^6}{1-t}\right)^3 = (1-t^6)^3(1-t)^{-3}$

$= (1-3t^6+3t^{12}-t^{18})\left(1+3t+\frac{3\cdot4}{2!}t^2\right.$

$\left.+\frac{3\cdot4\cdot5}{3!}t^3+\frac{3\cdot4\cdot5\cdot6}{4!}t^4+\ldots\infty\right)$

Hence, the coefficient of $t^4 = 1\cdot\frac{3\cdot4\cdot5\cdot6}{4!}$

$= \frac{3\times4\times5\times6}{4\times3\times2\times1} = 15$

Question149

The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is :

[April 10, 2019 (II)]

Options:

A. 38

B. 58

C. 23

D. 35

Answer: A

Solution:

Solution:

$\left(x^2+\frac{1}{x^2}\right)^n$

General term $T_{r+1} = {}^nC_r(x^2)^{n-r}\left(\frac{1}{x^3}\right)^r = {}^nC_r\cdot x^{2n-5r}$

To find coefficient of x, $2n-5r=1$

Given ${}^nC_r = {}^nC_{23} \Rightarrow r=23$ or $n-r=23$

$\therefore n=58$ or $n=38$

Minimum value is $n=38$

Question150

If the fourth term in the Binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6 (x>0)$ is 20×8^7, then a value of x is:
[April 9, 2019 (I)]

Options:

A. 8^3

B. 8^2

C. 8

D. 8^{-2}

Answer: B

Solution:

Solution:
$\because T_4 = 20\times 8^7$
$\Rightarrow {}^6C_3\left(\frac{2}{x}\right)^3\times\left(x^{\log_8 x}\right)^3 = 20\times 8^7$
$\Rightarrow 8\times 20\times\left(\frac{x^{\log_8 x}}{x}\right)^3 = 20\times 8^7 \Rightarrow \frac{x^{\log_8 x}}{x} = 64$
Now, take $\log_8$ on both sides, then
$(\log_8 x)^2 - (\log_8 x) = 2$
$\Rightarrow \log_8 x = -1$ or $\log_8 x = 2$
$\Rightarrow x = \frac{1}{8}$ or $x = 8^2$

Question151

If some three consecutive coefficients in the binomial expansion of $(x+1)^n$ in powers of x are in the ratio 2: 15: 70 , then the average of these three coefficients is:
[April 09, 2019 (II)]

Options:

A. 964

B. 232

C. 227

D. 625

Answer: B

Solution:

Solution:
Given ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2 : 15 : 70$

$\Rightarrow \frac{^nC_{r-1}}{^nC_r} = \frac{2}{15}$ and $\frac{^nC_r}{^nC_{r+1}} = \frac{15}{70}$

$\Rightarrow \frac{r}{n-r+1} = \frac{2}{15}$ and $\frac{r+1}{n-r} = \frac{3}{14}$

$\Rightarrow 17r = 2n + 2$ and $17r = 3n - 14$

i.e., $2n + 2 = 3n - 14 \Rightarrow n = 16$ & $r = 2$

$\therefore$ Average $= \frac{^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16 + 120 + 560}{3}$

$= \frac{696}{3} = 232$

Question152

The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x + \sqrt{x^3 - 1}\right)^6 + \left(x - \sqrt{x^3 - 1}\right)^6$, $(x > 1)$ is equal to :
[April 8, 2019 (I)]

Options:

A. 29

B. 32

C. 26

D. 24

Answer: D

Solution:

Solution:

(d) $\left(x + \sqrt{x^3 - 1}\right)^6 + \left(x - \sqrt{x^3 - 1}\right)^6$

$= 2[\,^6C_0x^6 + {}^6C_2x^4(x^3 - 1) + {}^6C_4x^2(x^3 - 1)^2 + {}^6C_6(x^3 - 1)^3\,]$

$= 2[\,x^6 + 15x^7 - 15x^4 + 15x^8 - 30x^5 + 15x^2 + x^9 - 3x^6 \quad + 3x^3 - 1\,]$

Hence, the sum of coefficients of even powers of x $= 2[1 - 15 + 15 + 15 - 3 - 1] = 24$

Question153

If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1 + \log_{10}x}}} + x^{\frac{1}{12}}\right)^6$ is equal to 200, and x > 1, then the value of x is:
[April 08, 2019 (II)]

Options:

A. 100

B. 10

C. 10^3

D. 10^4

Answer: B

Solution:

Solution:

$\therefore$ fourth term is equal to 200 .

$T_4 = {}^6C_3\left(\sqrt{\left(\frac{1}{1+\log_{10}x}\right)}\right)^3\left(x^{\frac{1}{12}}\right)^3 = 200$

$\Rightarrow\ 20x^{\frac{3}{2(1+\log_{10}x)}} \cdot x^{\frac{1}{4}} = 200$

$x^{\frac{1}{4}+\frac{3}{2(1+\log_{10}x)}} = 10$

Taking $\log_{10}$ on both sides and putting $\log_{10}x = t$

$\left(\frac{1}{4}+\frac{3}{2(1+t)}\right)t = 1 \Rightarrow t^2 + 3t - 4 = 0$

$\Rightarrow t^2 + 4t - t - 4 = 0 \Rightarrow t(t+4) - 1(t+4) = 0$

$\Rightarrow t = 1$ or $t = -4$

$\log_{10}x = 1 \Rightarrow x = 10$

or $\log_{10}x = -4 \Rightarrow x = 10^{-4}$

According to the question $x > 1$, $\therefore x = 10$

Question154

The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to : [NA April 12, 2019 (II)]

Options:

A. -72

B. 36

C. -36

D. -108

Answer: D

Solution:

Solution:

Given expression is,

$\left(\frac{1}{60}-\frac{x^8}{81}\right)\left(2x^2-\frac{3}{x^2}\right)^6$

$= \frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6 - \frac{x^8}{81}\left(2x^2-\frac{3}{x^2}\right)^6$

Term independent of x

= Coefficient of x° in $\frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6 - \frac{1}{81}$

coefficient of x^{-8} in $\left(2x^2-\frac{3}{x^2}\right)^6$

$= \frac{-1}{60} {}^6C_3(2)^3(3)^3 + \frac{1}{81} {}^6C_5(2)(3)^5$
$= -72 + 36 = -36$

Question155

If ${}^{20}C_1 + (2^2)^{20}C_2 + (3^2)^{20}C_3 + ... + (20^2)^{20}C_{20} = A(2^\beta)$, then the ordered pair (A, β) is equal to :
[April 12, 2019 (II)]

Options:

A. (420,19)

B. (420,18)

C. (380,18)

D. (380,19)

Answer: B

Solution:

Solution:
Given, ${}^{20}C_1 + 2^2 \cdot {}^{20}C_2 + 3^2 \cdot {}^{20}C_3 + \ldots + 20^2 \cdot {}^{20}C_{20}$
$= A(2^\beta)$
Taking L.H.S.,
$= \sum_{r=1}^{20} r^2 \cdot {}^{20}C_r = 20 \sum_{r=1}^{20} r \cdot {}^{19}C_{r-1}$
$= 20\left[\sum_{r=1}^{20} (r-1)^{19}C_{r-1} + \sum_{r=1}^{20} {}^{19}C_{r-1}\right]$
$= 20\left[19 \sum_{r=2}^{20} {}^{18}C_{r-2} + 2^{19}\right] = 20[19.2^{18} + 2^{19}]$
$= 420 \times 2^{18}$
Now, compare it with R.H.S., A = 420 and β = 18

Question156

The coefficient of x^{18} in the product $(1 + x)(1 - x)^{10}\ (1 + x + x^2)^9$ is :
[April 12, 2019 (I)]

Options:

A. 84

B. -126

C. -84

D. 126

Answer: A

Solution:

Solution:
Given expression,
$(1-x)^{10}(1+x+x^2)^9(1+x)=(1-x^3)^9(1-x^2)$
$=(1-x^3)^9-x^2(1-x^3)^9$
⇒ Coefficient of x^{18} in $(1-x^3)^9$ − coeff. of x^{16} in $(1-x^3)^9$
$={}^9C_6-0=\frac{9!}{6!3!}=\frac{7\times8\times9}{6}=84$

Question157

If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1+ax+bx^2)(1-3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to:
[April 10, 2019 (I)]

Options:

A. (28,861)

B. (-54,315)

C. (28,315)

D. (-21,714)

Answer: C

Solution:

Solution:
Given expression is $(1+ax+bx^2)(1-3x)^{15}$
Co-efficient of $x^2=0$
$\Rightarrow {}^{15}C_2(-3)^2+a\cdot{}^{15}C_1(-3)+b\cdot{}^{15}C_0=0$
$\Rightarrow \frac{15\times14}{2}\times9-15\times3a+b=0$
$\Rightarrow 945-45a+b=0$ . . .(i)
Now, co-efficient of $x^3=0$
$\Rightarrow {}^{15}C_3(-3)^3+a\cdot{}^{15}C_2(-3)^2+b\cdot{}^{15}C_1(-3)=0$
$\Rightarrow \frac{15\times14\times13}{3\times2}\times(-3\times3\times3)+a\times\frac{15\times14\times9}{2}-b\times3\times15=0$
$\Rightarrow 15\times3[-3\times7\times13+a\times7\times3-b]=0$
$\Rightarrow 21a-b=273$ . . . (ii)
From (i) and (ii), we get,
$a=28, b=315 \Rightarrow (a,b)\equiv(28,31,5)$

Question158

The sum of the series $2^{\cdot 20}C_0+5\cdot{}^{20}C_1+8\cdot{}^{\cdot 20}C_2+11^{20}C_3+\ldots+62^{\cdot 20}C_{20}$ is equal to :
[April 8, 2019 (I)]

Options:

A. 2^{26}

B. 2^{25}

C. 2^{23}

D. 2^{24}

Answer: B

Solution:

Solution:

$2 \cdot {}^{20}C_0 + 5 \cdot {}^{20}C_1 + 8 . {}^{20}C_2 + \ldots\ldots + 62 . {}^{20}C_{20}$

$= \sum_{r=0}^{20} (3r + 2)^{20}C_r = 3 \sum_{r=0}^{20} r . {}^{20}C_r + 2 \sum_{r=0}^{20} {}^{20}C_r$

$= 60 \sum_{r=1}^{20} {}^{19}C_{n-1} + 2 \sum_{r=0}^{20} {}^{20}C_r$

$= 60 \times 2^{19} + 2 \times 2^{20} = 2^{21}[15 + 1] = 2^{25}$

Question159

The coefficient of x^{10} in the expansion of $(1 + x)^2(1 + x^2)^3 (1 + x^3)^4$ is equal to [Online April 15, 2018]

Options:

A. 52

B. 44

C. 50

D. 56

Answer: A

Solution:

Solution:

$\because (1 + x)^2 = 1 + 2x + x^2$,

$(1 + x^2)^3 = 1 + 3x^2 + 3x^4 + x^6$

and $(1 + x^3)^4 = 1 + 4x^3 + 6x^6 + 4x^9 + x^{12}$

So, the possible combinations for x^{10} are:

$x \cdot x^9$, $x \cdot x^6 \cdot x^3$, $x^2 \cdot x^2 \cdot x^6$, $x^4 \cdot x^6$

Corresponding coefficients are 2×4, $2 \times 1 \times 4$, $1 \times 3 \times 6$, 3×6 or 8,8,18,18

$\therefore$ Sum of the coefficient is $8 + 8 + 18 + 18 = 52$

Therefore, the coefficient of x^{10} in the expansion of $(1 + x)^2(1 + x^2)^3(1 + x^3)^4$ is 52.

Question160

If n is the degree of the polynomial,

$\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8 + \left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$ **and m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to**
[Online April 15, 2018]

Options:

A. $(12, (20)^4)$

B. $(8, 5(10)^4)$

C. $(24, (10)^8)$

D. $(12, 8(10)^4)$

Answer: D

Solution:

$$\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8 + \left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$$

After rationalise the polynomial we get

$$\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1}+\sqrt{5x^3-1}}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$$

$$+\left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1}-\sqrt{5x^3-1}}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8$$

$$= \left[\frac{\sqrt{5x^3+1}+\sqrt{5x^3-1}}{(5x^3+1)-(5x^3-1)}\right]^8 + \left[\frac{\sqrt{5x^3+1}-\sqrt{5x^3-1}}{(5x^3+1)-(5x^3-1)}\right]^8$$

$$= \frac{1}{2^8}\left[\left(\sqrt{5x^3+1}+\sqrt{5x^3-1}\right)^8 + \left(\sqrt{5x^3+1}-\sqrt{5x^3-1}\right)^8\right]^8$$

$$= \frac{1}{2^8}\left[\begin{array}{c} {}^8C_0\left(\sqrt{5x^3+1}\right)^8 + {}^8C_2\left(\sqrt{5x^3+1}\right)^6\left(\sqrt{5x^3-1}\right)^2 \\ +{}^8C_4\left(\sqrt{5x^3+1}\right)^4\left(\sqrt{5x^3-1}\right)^4 \\ +{}^8C_6\left(\sqrt{5x^3+1}\right)^2\left(\sqrt{5x^3-1}\right)^6 + {}^8C_8\left(\sqrt{5x^3-1}\right)^8 \end{array}\right]$$

$$= \frac{1}{2^8}\left[\begin{array}{c} {}^8C_0(5x^3+1)^4 + {}^8C_2(5x^3+1)^3(5x^3-1) + 8_{C_4} \\ (5x^3+1)^2(5x^3-1)^2 \\ +{}^8C_6(5x^3+1)(5x^3-1)^3 + 8_{C_8}(5x^3-1)^4 \end{array}\right]$$

So, the degree of polynomial is 12 .
Now, coefficient of $x^{12} = [\,{}^8C_0 5^4 + {}^8C_2 5^4 + {}^8C_4 5^4 + {}^8C_6 5^4 + {}^8C_8 5^4\,]$

$$= 5^4 \times \frac{2^8}{2} = 5^4 \times 2^4 \times \frac{2^4}{2}$$

$$= 10^4 \times 2^3 = 8(10)^4$$

Question161

The coefficient of x^2 in the expansion of the product $(2 - x^2) \cdot ((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ is
[Online April 16, 2018]

Options:

A. 106

B. 107

C. 155

D. 108

Answer: A

Solution:

Solution:
Let $a = ((1+2x+3x^2)^6 + (1-4x^2)^6)$
$\therefore$ Coefficient of x^2 in the expansion of the product
$(2-x^2)((1+2x+3x^2)^6 + (1-4x^2)^6)$
$= 2($ Coefficient of x^2 in a $) - 1$ (Constant of expansion)
In the expansion of $((1+2x+3x^2)^6 + (1-4x^2)^6)$.
Constant $= 1 + 1 = 2$
Coefficient of x^2 = [Coefficient of x^2 in $({}^6C_0(1+2x)^6(3x^2)^0)$]
+[Cofficient of x^2 in $({}^6C_1(1+2x)^5(3x^2)^1)$] $-[{}^6C_1(4x^2)]$
$= 60 + 6 \times 3 - 24 = 54$
$\therefore$ The coefficient of x^2 in $(2-x^2)((1+2x+3x^2)^6 + (1-4x^2)^6)$
$= 2 \times 54 - 1(2) = 108 - 2 = 106$

Question162

The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is : [2018]

Options:

A. 0

B. 1

C. 2

D. -1

Answer: C

Solution:

Solution:
Since we know that,
$(x+a)^5 + (x-a)^5$
$= 2[{}^5C_0x^5 + {}^5C_2x^3 \cdot a^2 + {}^5C_4x \cdot a^4]$
$\therefore \left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$
$= 2[{}^5C_0x^5 + {}^5C_2x^3(x^3-1) + {}^5C_4x(x^3-1)^2]$
$\Rightarrow 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$
$\therefore$ Sum of coefficients of odd degree terms $= 2$

Question163

If $(27)^{999}$ is divided by 7, then the remainder is:
[Online April 8, 2017]

Options:

A. 1

B. 2

C. 3

D. 6

Answer: D

Solution:

Solution:

$$\frac{(28-1)^{999}}{7} = \frac{28\lambda - 1}{7}$$

$$\Rightarrow \frac{28\lambda - 7 + 7 - 1}{7} = \frac{7(4\lambda - 1) + 6}{7}$$

$\therefore$ Remainder $= 6$

Question164

The coefficient of x^{-5} in the binomial expansion of $\left(\frac{x+1}{x^{\frac{2}{3}} - x^{\frac{1}{3}} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}}\right)^{10}$ where $x \neq 0, 1$, is
[Online April 9, 2017]

Options:

A. 1

B. 4

C. -4

D. -1

Answer: A

Solution:

Solution:

$$\left[\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{(x^{2/3}-x^{1/3}+1)} - \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$

$= (x^{1/3} + 1 - 1 - 1 / x^{1/2})^{10} = (x^{1/3} - 1 / x^{1/2})^{10}$

$T_{r+1} = {}^{10}C_r x^{\frac{20-5r}{6}}$

for r = 10

$T_{11} = {}^{10}C_{10}x^{-5}$
Coefficient of $x^{-5} = {}^{10}C_{10}(1)(-1)^{10} = 1$

Question165

The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4)$ $+ \ldots. + ({}^{21}C_{10} - {}^{10}C_{10})$ is:
[2017]

Options:

A. $2^{20} - 2^{10}$

B. $2^{21} - 2^{11}$

C. $2^{21} - 2^{10}$

D. $2^{20} - 2^{9}$

Answer: A

Solution:

Solution:
We have $({}^{21}C_1 + {}^{21}C_2 \ldots\ldots + {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 \ldots .\ {}^{10}C_{10})$
$= \frac{1}{2}[({}^{21}C_1 + \ldots. + {}^{21}C_{10}) + ({}^{21}C_{11} + \ldots .\ {}^{21}C_{20})] - (2^{10} - 1)$
$(\because {}^{10}C_1 + {}^{10}C_2 + \ldots. + {}^{10}C_{10} = 2^{10} - 1)$
$= \frac{1}{2}[2^{21} - 2] - (2^{10} - 1)$
$= (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10}$

Question166

If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}} + \frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, $(x > 0)$, are m and n respectively, then $\frac{m}{n}$ is equal to :
[Online April 10, 2016]

Options:

A. 27

B. 182

C. frac 5 4

D. frac 4 5

Answer: B

Solution:

Solution:

$T_{r+1} = {}^{18}C_r \left(x^{\frac{1}{3}}\right)^{18-r} \left(\frac{1}{2x^{\frac{1}{3}}}\right)^r = {}^{18}C_r x^{6-\frac{2r}{3}} \frac{1}{2^r}$

$$\left\{\begin{array}{l} 6-\frac{2r}{3} = -2 \Rightarrow r = 12 \\ 6-\frac{2r}{3} = -4 \Rightarrow r = 15 \end{array}\right\}$$

$\Rightarrow \frac{\text{coefficient of } x^{-2}}{\text{coefficient of } x^{-4}} = \frac{{}^{18}C_{12}\frac{1}{2^{12}}}{{}^{18}C_{15}\frac{1}{2^{15}}} = 182$

Question167

If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is: [2016]

Options:

A. 243

B. 729

C. 64

D. 2187

Answer: B

Solution:

Solution:

Total number of terms $= {}^{n+2}C_2 = 28$

$(n+2)(n+1) = 56; n = 6$

$\therefore$ Put $x = 1$ in expansion $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^6$

we get sum of coefficient $= (1-2+4)^6$

$= 3^6 = 729$

Question168

If the coefficients of the three successive terms in the binomial expansion of $(1+x)^n$ are in the ratio 1 : 7 : 42, then the first of these terms in the expansion is:
[Online April 10, 2015]

Options:

A. 8^{th}

B. 6^{th}

C. 7^{th}

D. 9^{th}

Answer: C

Solution:

Solution:

$\frac{{}^nC_r}{1} = \frac{{}^nC_{r+1}}{7} = \frac{{}^nC_{r+2}}{42}$

By solving we get $r = 6$

so, it is 7^{th} term.

Question169

The term independent of x in the binomial expansion of $\left(1 - \frac{1}{x} + 3x^5\right)\left(2x^2 - \frac{1}{x}\right)^8$ is : [Online April 11, 2015]

Options:

A. 496

B. -496

C. 400

D. -400

Answer: C

Solution:

Solution:

General term of $\left(2x^2 - \frac{1}{x}\right)^8$ is ${}^8C_r(2x^2)^{8-r}\left(\frac{-1}{x}\right)^r$

$\therefore$ Given expression is equal to

$\left(1 - \frac{1}{x} + 3x^5\right){}^8C_r(2x^2)^{8-r}\left(-\frac{1}{x}\right)^r = {}^8C_r(2x^2)^{8-r}\left(-\frac{1}{x}\right)^r - \frac{1}{x^8}C_r(2x^2)^{8-r}\left(-\frac{1}{x}\right)^r$

$+3x^5 \cdot {}^8C_r(2x^2)^{8-r}\left(-\frac{1}{x}\right)^r$

$= {}^8C_r2^{8-r}(-1)^rx^{16-3r} - {}^8C_r2^{8-r}(-1)^rx^{15-3r}$

$+3 \, . \, {}^8C_r2^{(8-r)}\left(-\frac{1}{x}\right)^r(-1)^rx^{21-3r}$

For the term independent of x, we should have

$16 - 3r = 0$, $15 - 3r = 0$, $21 - 3r = 0$

From the simplification we get $r = 5$ and $r = 7$

$\therefore -{}^8C_5(2^3)(-1)^5 - 3 \, . \, {}^8C_7.2$

$+\left[\frac{8!}{5!3!}\times 8\right] - 3\times\left[\frac{8!}{7!1!}\times 2\right]$
$= (56 \times 8) - 48$
$= 448 - 6 \times 8 = 448 - 48 = 400$

Question170

The sum of coefficients of integral power of x in the binomial expansion $(1-2\sqrt{x})^{50}$ is :
[2015]

Options:

A. $\frac{1}{2}(3^{50}-1)$

B. $\frac{1}{2}(2^{50}+1)$

C. $\frac{1}{2}(3^{50}+1)$

D. $\frac{1}{2}(3^{50})$

Answer: C

Solution:

Solution:
We know that $(a+b)^n + (a-b)^n$
$= 2[{}^nC_0 a^n b^0 + {}^nC_2 a^{n-2} b^2 + {}^nC_4 a^{n-4} b^4 \ldots]$
$(1-2\sqrt{x})^{50} + (1+2\sqrt{x})^{50}$
$2[{}^{50}C_0 + {}^{50}C_2(2\sqrt{x})^2 + {}^{50}C_4(2\sqrt{x})^4 \ldots]$
$= 2[{}^{50}C_0 + {}^{50}C_2 2^2 x + {}^{50}C_4 2^4 x^2 + \ldots]$
Putting $x = 1$, we get
${}^{50}C_0 + {}^{50}C_2 2^2 + {}^{50}C_4 2^4{}_{\cdot s} = \frac{3^{50}+1}{2}$

Question171

If the coefficents of x^3 and x^4 in the expansion of $(1+ax+bx^2)(1-2x)^{18}$ in powers of x are both zero, then (a, b) is equal to:
[2014]

Options:

A. $\left(14, \frac{272}{3}\right)$

B. $\left(16, \frac{272}{3}\right)$

C. $\left(16, \frac{251}{3}\right)$

D. $\left(14, \frac{251}{3}\right)$

Answer: B

Solution:

Solution:
Consider $(1 + ax + bx^2)(1 - 2x)^{18}$
$= (1 + ax + bx^2)[{}^{18}C_0 - {}^{18}C_1(2x)].$
$+ {}^{18}C_2(2x)^2 - {}^{18}C_3(2x)^3 + {}^{18}C_4(2x)^4 - \ldots\ldots]$
Coeff. of $x^3 = {}^{18}C_3(-2)^3 + a \cdot (-2)^2 \cdot {}^{18}C_2 + b(-2) \cdot {}^{18}C_1 = 0$
Coeff. of $x^3 = -{}^{18}C_3 \cdot 8 + a \times 4 \,.\, {}^{18}C_2 - 2b \times 18 = 0$
$= -\frac{18 \times 17 \times 16}{6}.8 + \frac{4a + 18 \times 17}{2} - 36b = 0$
$= -51 \times 16 \times 8 + a \times 36 \times 17 - 36b = 0$
$= -34 \times 16 + 51a - 3b = 0 \;\; = 51a - 3b = 34 \times 16 = 544$
$= 51a - 3b = 544 \,.\,.\,.\,.(i)$
Only option number (b) satisfies the equation number (i)

Question172

If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to: [2014]

Options:

A. X

B. Y

C. N

D. $Y - X$

Answer: B

Solution:

Solution:
$4^n - 3n - 1 = (1 + 3)^n - 3n - 1$
$= [{}^nC_0 + {}^nC_1 \cdot 3 + {}^nC_2 \cdot 3^2 + \ldots\ldots + {}^nC_n 3^n] - 3n - 1$
$= 9[{}^nC_2 + {}^nC_3 \cdot 3 + \ldots. + {}^nC_n \cdot 3^{n-2}]$
$\therefore 4^n - 3n - 1$ is a multiple of 9 for all n
$\therefore X = \{x : x$ is a multiple of $9\}$
Also, $Y = \{9(n - 1) : n \in N\}$
$= \{$ All multiples of $9\}$
Clearly $X \subset Y$. $\therefore X \cup Y = Y$

Question173

The number of terms in the expansion of $(1 + x)^{101}(1 + x^2 - x)^{100}$ in powers of x is:

[Online April 9, 2014]

Options:

A. 302

B. 301

C. 202

D. 101

Answer: C

Solution:

Solution:
Given expansion is
$(1+x)^{101}(1-x+x^2)^{100}$
$= (1+x)(1+x)^{100}(1-x+x^2)^{100}$
$= (1+x)[(1+x)(1-x+x^2)]^{100}$
$= (1+x)[(1-x^3)^{100}]$
Expansion $(1-x^3)^{100}$ will have $100+1=101$ terms
So, $(1+x)(1-x^3)^{100}$ will have $2\times 101=202$ terms

Question174

If $1+x^4+x^5=\sum_{i=0}^{5} a_i(1+x)^i$, for all x in R, then a_2 is:
[Online April 12, 2014]

Options:

A. -4

B. 6

C. -8

D. 10

Answer: A

Solution:

Solution:
$1+x^4+x^5=\sum_{i=0}^{5} a_i(1+x)^i$
$= a_0+a_1(1+x)^1+a_2(1+x)^2+a_3(1+x)^3$
$+a_4(1+x)^4+a_5(1+x)^5$
$\Rightarrow 1+x^4+x^5$
$= a_0+a_1(1+x)+a_2(1+2x+x^2)+a_3(1+3x+3x^2+x^3)$
$+a_4(1+4x+6x^2+4x^3+x^4)+a_5(1+5x+10x^2+10x^3+5x^4+x^5)$
$\Rightarrow 1+x^4+x^5$
$= a_0+a_1+a_1x+a_2+2a_2x+a_2x^2+a_3+3a_3x$
$+3a_3x^2+a_3x^3+a_4+4a_4x+6a_4x^2+4a_4x^3+a_4x^4+a_5$
$+5a_5x+10a_5x^2+10a_5x^3+5a_5x^4+a_5x^5$

$\Rightarrow 1 + x^4 + x^5$
$= (a_0 + a_1 + a_2 + a_3 + a_4 + a_5) + x(a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5)$
$+x^2(a_2 + 3a_3 + 6a_4 + 10a_5) + x^3(a_3 + 4a_4 + 10a_5)$
$+x^4(a_4 + 5a_5) + x^5(a_5)$
On comparing the like coefficients, we get
$a_5 = 1 \ldots$ (i) ; $a_4 + 5a_5 = 1$ ....(ii) ;
$a_3 + 4a_4 + 10a_5 = 0$ ....(iii)
and $a_2 + 3a_3 + 6a_4 + 10a_5 = 0 \ldots$ (iv)
from (i) & (ii), we get
$a_4 = -4 \ \ldots$ (v)
from (i), (iii) & (v), we get
$a_3 = +6$ .... (vi)
Now, from (i), (v) and (vi), we get
$a_2 = -4$

Question175

If $\left(2 + \frac{x}{3}\right)^{55}$ is expanded in the ascending powers of x and the coefficients of powers of x in two consecutive terms of the expansion are equal, then these terms are:
[Online April 12, 2014]

Options:

A. 7^{th} and 8^{th}

B. 8^{th} and 9^{th}

C. 28^{th} and 29^{th}

D. 27^{th} and 28^{th}

Answer: A

Solution:

Solution:
Let r^{th} and $(r+1)^{\text{th}}$ term has equal coefficient
$\left(2 + \frac{x}{3}\right)^{55} = 2^{55}\left(1 + \frac{x}{6}\right)^{55}$
r^{th} term $= 2^{5555}C_r\left(\frac{x}{6}\right)^r$
Coefficient of x^r is $2^{5555}C_r \frac{1}{6^r}$
$(r+1)^{\text{th}}$ term $= 2^{5555}C_{r+1}\left(\frac{x}{6}\right)^{r+1}$
Coefficient of x^{r+1} is $2^{5555}C_{r+1} \cdot \frac{1}{6^{r+1}}$
Both coefficients are equal
$2^{5555}C_r \frac{1}{6^r} = 2^{5555}C_{r+1} \frac{1}{6^{r+1}}$
$\frac{1}{|r|55-r} = \frac{1}{|r+1|54-r} \cdot \frac{1}{6}$
$6(r+1) = 55 - r$
$6r + 6 = 55 - r$
$7r = 49$
$r = 7\ (r+1) = 8$
Coefficient of 7^{th} and 8^{th} terms are equal.

Question176

The coefficient of x^{1012} in the expansion of $(1 + x^n + x^{253})^{10}$, (where $n \le 22$ is any positive integer), is
[Online April 19, 2014]

Options:

A. 1

B. $^{10}C_4$

C. 4n

D. $^{253}C_4$

Answer: B

Solution:

Solution:
Given expansion $(1 + x^n + x^{253})^{10}$
Let $x^{1012} = (1)^a(x^n)^b \cdot (x^{253})^c$
Here a, b, c, n are all + ve integers and $a \le 10$, $b \le 10$, $c \le 4$, $n \le 22$, $a + b + c = 10$
Now $bn + 253c = 1012$
$\Rightarrow bn = 253(4 - c)$
For $c < 4$ and $n \le 22$; $b > 10$, which is not possible.
$\therefore c = 4$, $b = 0$, $a = 6$
$\therefore x^{1012} = (1)^6 \cdot (x^n)^0 \cdot (x^{253})^4$
Hence the coefficient of x^{1012} = $\frac{10!}{6!0!4!} = {}^{10}C_4$

Question177

If the 7 th term in the binomial expansion of $\left(\frac{3}{\sqrt[3]{84}} + \sqrt{3}\ln x\right)^9$, $x > 0$, is equal to 729, then x can be
[Online April 22, 2013

Options:

A. e^2

B. e

C. $\frac{e}{2}$

D. 2e

Answer: B

Solution:

Solution:
Let $r+1=7 \Rightarrow r=6$
Given expansion is
$\left(\frac{3}{\sqrt{[3]84}}+\sqrt{3}\ln x\right)^{9}, x>0$
We have
$T_{r+1}={}^{n}C_{r}(x)^{n-r}a^{r}$ for $(x+a)^{n}$
∴ According to the question
$729={}^{9}C_{6}\left(\frac{3}{\sqrt{[3]84}}\right)^{3}\cdot(\sqrt{3}\ln x)^{6}$
$\Rightarrow 3^{6}=84\times\frac{3^{3}}{84}\times 3^{3}\times(6\ln x)$
$\Rightarrow (\ln x)^{6}=1 \Rightarrow (\ln x)^{6}=(\ln e)^{6}$
$\Rightarrow x=e$

Question178

If for positive integers $r>1, n>2$, the coefficients of the $(3r)^{th}$ and $(r+2)^{th}$ powers of x in the expansion of $(1+x)^{2n}$ are equal, then n is equal to :
[Online April 25, 2013]

Options:

A. $2r+1$

B. $2r-1$

C. $3r$

D. $r+1$

Answer: A

Solution:

Solution:
Expansion of $(1+x)^{2n}$ is $1+{}^{2n}C_{1}x+{}^{2n}C_{2}x^{2}+\ldots\ldots+$
${}^{2n}C_{r}x^{r}+{}^{2n}C_{r+1}x^{r+1}+\ldots\ldots+{}^{2n}C_{2n}x^{2n}$
As given ${}^{2n}C_{r+2}={}^{2n}C_{3}$
$\Rightarrow \frac{(2n)!}{(r+2)!(2n-r-2)!}=\frac{(2n)!}{(3r)!(2n-3r)!}$
$\Rightarrow (3r)!(2n-3r)!=(r+2)!(2n-r-2)!\ldots(1)$
Now, put value of n from the given choices.
Choice (a) put $n=2r+1$ in (1)
$LHS:(3r)!(4r+2-3r)!=(3r)!(r+2)!$
$RHS:(r+2)!(3r)!$
$\Rightarrow LHS=RHS$

Question179

The sum of the rational terms in the binomial expansion of $\left(2^{\frac{1}{2}}+3^{\frac{1}{5}}\right)^{10}$ is:
[Online April 23, 2013]

Options:

A. 25

B. 32

C. 9

D. 41

Answer: D

Solution:

Solution:
$(2^{1/2}+3^{1/5})^{10} = {}^{10}C_0(2^{1/2})^{10}$
$+{}^{10}C_1(2^{1/2})^9(3^{1/5}) + \ldots\ldots + {}^{10}C_{10}(3^{1/5})^{10}$
There are only two rational terms - first term and last term.
Now sum of two rational terms
$= (2)^5 + (3)^2 = 32 + 9 = 41$

Question180

The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ is [2013]

Options:

A. 4

B. 120

C. 210

D. 310

Answer: C

Solution:

Solution:
Given expression can be written as
$$\left[\frac{(x^{1/3})^3+1^3}{x^{2/3}-x^{1/3}+1} - \frac{(\sqrt{x})^2-1^2}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$$
$$= \left((x^{1/3}+1) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10} = \left(x^{1/3}+1-1-\frac{1}{\sqrt{x}}\right)^{10}$$
$$= (x^{1/3}-x^{-1/2})^{10}$$
General term $= T_{r+1}$
$$= {}^{10}C_r(x^{1/3})^{10-r}(-x^{-1/2})^r = {}^{10}C_r x^{\frac{10-r}{3}} \cdot (-1)^r \cdot x^{-\frac{r}{2}}$$
$$= {}^{10}C_r(-1)^r \cdot x^{\frac{10-r}{3}-\frac{r}{2}}$$
Term will be independent of x when $\frac{10-r}{3} - \frac{r}{2} = 0$
$\Rightarrow r = 4$
So, required term $= T_5 = {}^{10}C_4 = 210$

Question181

The ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(x^2 + \frac{2}{x}\right)^{15}$ is :
[Online April 9, 2013]

Options:

A. 7: 16

B. 7: 64

C. 1: 4

D. 1: 32

Answer: D

Solution:

Solution:
$T_{r+1} = {}^{15}C_r(x^2)^{15-r} \cdot (2x^{-1})^r = {}^{15}C_r \times (2)^r \times x^{30-3r}$
For independent term, $30 - 3r = 0 \Rightarrow r = 10$
Hence the term independent of x,
$T_{11} = {}^{15}C_{10} \times (2)^{10}$
For term involve x^{15}, $30 - 3r = 15 \Rightarrow r = 5$
Hence coefficient of $x^{15} = {}^{15}C_5 \times (2)^5$

Required ratio $= \dfrac{{}^{15}C_5 \times (2)^5}{{}^{15}C_{10} \times (2)^{10}} = \dfrac{\frac{15!}{10!5!}}{\frac{15!}{5!10!} \times (2)^5}$

$= 1 : 32$

Question182

If n is a positive integer, then $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$ is
[2012]

Options:

A. an irrational number

B. an odd positive integer

C. an even positive integer

D. a rational number other than positive integers

Answer: A

Solution:

Solution:
Consider $(\sqrt{3}+1)^{2n}-(\sqrt{3}-1)^{2n}$

$$= 2\left[{}^{2n}C_1(\sqrt{3})^{2n-1} + {}^{2n}C_3(\sqrt{3})^{2n-3} + {}^{2n}C_5(\sqrt{3})^{2n-5} + \ldots \right]$$

$\because (a+b)^n-(a-b)^n$
$= 2[{}^nC_1a^{n-1}b + {}^nC_3a^{n-3}b^3 \ldots]$
= which is an irrational number.

Question183

If $f(y) = 1-(y-1)+(y-1)^2-(y-1)^3 + \ldots - (y-1)^{17}$ then the coefficient of y^2 in it is [O [Online May 7, 2012]

Options:

A. ${}^{17}C_2$

B. ${}^{17}C_3$

C. ${}^{18}C_2$

D. $18C_3$

Answer: D

Solution:

Solution:
Given function is
$f(y) = 1-(y-1)+(y-1)^2-(y-1)^3 + \ldots\ldots\ldots - (y-1)^{17}$
In the expansion of $(y-1)^n$
$T_{r+1} = {}^nC_ry^{n-r}(-1)^r$
coeff of y^2 in $(y-1)^2 = {}^2C_0$
coeff of y^2 in $(y-1)^3 = -{}^3C_1$
coeff of y^2 in $(y-1)^4 = {}^4C_2$
So, coeff of termwise is
${}^2C_0 + {}^3C_1 + {}^4C_2 + {}^5C_3 + \ldots\ldots\ldots + {}^{17}C_{15}$
$= 1 + {}^3C_1 + {}^4C_2 + {}^5C_3 + \ldots\ldots\ldots + {}^{17}C_{15}$
$= ({}^3C_0 + {}^3C_1) + {}^4C_2 + {}^5C_3 + \ldots\ldots\ldots + {}^{17}C_{15}$
$= {}^4C_1 + {}^4C_2 + {}^5C_3 + \ldots\ldots\ldots + {}^{17}C_{15}$
$= {}^5C_2 + {}^5C_3 + \ldots\ldots\ldots + {}^{17}C_{15}$
$= {}^{18}C_{15} = {}^{18}C_3$

Question184

The number of terms in the expansion of $(y^{1/5}+x^{1/10})^{55}$, in which powers of x and y are free from radical signs are [Online May 12, 2012]

Options:

A. six

B. twelve

C. seven

D. five

Answer: A

Solution:

Solution:
Given expansion is $\left(y^{\frac{1}{5}}+x^{\frac{1}{10}}\right)^{55}$
The general term is
$T_{r+1}={}^{55}C_r\left(y^{\frac{1}{5}}\right)^{55-r}.\left(x^{\frac{1}{10}}\right)^{r}$
T_{r+1} would free from radical sign if powers of y and x are integers.
i.e. $\frac{55-r}{5}$ and $\frac{r}{10}$ are integer.
$\Rightarrow r$ is multiple of 10 .
Hence, $r=0, 10, 20, 30, 40, 50$
It is an A.P.
Thus, $50=0+(k-1)10$
$50=10k-10 \Rightarrow k=6$
Thus, the six terms of the given expansion in which x and y are free from radical signs.

Question185

The middle term in the expansion of $\left(1-\frac{1}{x}\right)^{n}(1-x)^{n}$ in powers of x is [Online May 26, 2012]

Options:

A. $-{}^{2n}C_{n-1}$

B. $-{}^{2n}C_{n}$

C. ${}^{2n}C_{n-1}$

D. ${}^{2n}C_{n}$

Answer: D

Solution:

Solution:
Given expansion can be written as
$\left(\frac{x-1}{x}\right)^{n}\cdot(1-x)^{n}=(-1)^{n}x^{-n}(1-x)^{2n}$
Total number of terms will be $2n+1$ which is odd ($\because$: $2n$ is always even)
$\therefore$ Middle term $=\frac{2n+1+1}{2}=(n+1)$ th
Now, $T_{r+1}={}^{n}C_r(1)^{r}x^{n-r}$

So, $\frac{{}^{2n}C_n \cdot x^{2n-n}}{x^n \cdot (-1)^n} = {}^{2n}C_n \cdot (-1)^n$
Middle term is an odd term. So, $n+1$ will be odd. So, n will be even.
∴ Required answer is ${}^{2n}C_n$

Question186

Statement -1: For each natural number n, $(n+1)^7 - 1$ is divisible by 7 .
Statement -2: For each natural number n, $n^7 - n$ is divisible by 7 .
[2011 RS]

Options:

A. Statement- 1 is true, Statement- 2 is true; Statement- 2 is a correct explanation for Statement-1.

B. Statement- 1 is true, Statement- 2 is true; Statement-2 is NOT a correct explanation for Statement-1

C. Statement- 1 is true, Statement-2 is false

D. Statement-l is false, Statement- 2 is true

Answer: A

Solution:

Solution:
Statement 2:
$P(n) : n^7 - n$ is divisible by 7
Put $n = 1$, $1 - 1 = 0$ is divisible by 7 , which is true
Let $n = k$, $P(k) : k^7 - k$ is divisible by 7, true
Put $n = k + 1$
∴ $P(k+1) : (k+1)^7 - (k+1)$ is div. by 7 $P(k+1) : k^7 + {}^7C_1 k^6 + {}^7C_2 k^2 + \ldots\ldots + {}^7C_6 k + 1 - k - 1$, is div. by 7 .
$P(k+1) : (k^7 - k) + ({}^7C_1 k^6 + {}^7C_2 k^5 + \ldots\ldots\ldots + {}^7C_6 k)$ is div. by 7
Since 7 is coprime with 1,2,3,4,5,6 .
So ${}^7C_1, {}^7C_2, \ldots\ldots {}^7C_6$ are all divisible by 7
∴ $P(k+1)$ is divisible by 7
Hence $P(n) : n^7 - n$ is divisible by 7
Statement 1: $n^7 - n$ is divisible by 7
⇒ $(n+1)^7 - (n+1)$ is divisible by 7
⇒ $(n+1)^7 - n^7 - 1 + (n^7 - n)$ is divisible by 7
⇒ $(n+1)^7 - n^7 - 1$ is divisible by 7
Hence both Statements 1 and 2 are correct and Statement 2 is the correct explanation of Statement -1

Question187

The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is
[2011]

Options:

A. -132

B. -144

C. 132

D. 144

Answer: B

Solution:

Solution:

$(1 - x - x^2 + x^3)^6 = [(1 - x) - x^2(1 - x)]^6$

$= (1 - x)^6(1 - x^2)^6$

$= (1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6) \times (1 - 6x^2 + 15x^4 - 20x^6 + 15x^8 - 6x^{10} + x^{12})$

Coefficient of $x^7 = (-6)(-20) + (-20)(15) + (-6)(-6)$

$= -144$

Question188

Let $S_1 = \sum_{j=1}^{10} j(j-1)\,^{10}C_J$, $S_2 = \sum_{j=1}^{10} j\,^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,^{10}C_j$.
Statement -1: $S_3 = 55 \times 2^9$
Statement -2: $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$
[2010]

Options:

A. Statement - 1 is true, Statement -2 is true; Statement -2 is not a correct explanation for Statement -1.

B. Statement -1 is true, Statement -2 is false.

C. Statement -1 is false, Statement -2 is true.

D. Statement - 1 is true, Statement 2 is true; Statement -2 is a correct explanation for Statement -1 .

Answer: B

Solution:

Solution:

$S_2 = \sum_{j=1}^{10} j\,^{10}C_j = \sum_{j=1}^{10} 10\,^9C_{j-1}$

$\left[\because {}^nC_r = \frac{n}{r}\,^{n-1}C_{r-1}\right]$

$= 10[^9C_0 + {}^9C_1 + {}^9C_2 + \ldots + {}^9C_9] = 10.2^9$

Question189

The remainder left out when $8^{2n} - (62)^{2n+1}$ is divided by 9 is:
[2009]

Options:

A. 2

B. 7

C. 8

D. 0

Answer: A

Solution:

Solution:
$(8)^{2n} - (62)^{2n+1}$
$= (64)^n - (62)^{2n+1}$
$= (63+1)^n - (63-1)^{2n+1}$
$= [\,^nC_0(63)^n + {}^nC_1(63)^{n-1} + {}^nC_2(63)^{n-2} + \ldots\ldots + {}^nC_{n-1}(63) + {}^nC_n] + \ {}^{2n+1}C_2(63)^{2n-1} - \ldots\ldots.. + (-1)^{2n+1\,2n+1}C_{2n+1}]$
$= 63 \times [\,^nC_0(63)^{n-1} + {}^nC_1(63)^{n-2} + {}^nC_2(63)^{n-3} + \ldots\ldots + {}^nC_{n-1}] + 1 - 63 \times$
$[\,^{2n+1}C_0(63)^{2n} - {}^{2n+1}C_1(63)^{2n-1} + \ldots\ldots + {}^{2n+1}C_{2n}] + 1$
$= 63 \times$ some integral value $+ 2$
Hence, when divided by 9 leaves 2 as the remainder.

Question190

Statement -1: $\sum_{r=0}^{n}(r+1)^nC_r = (n+2)2^{n-1}$

Statement-2: $\sum_{r=0}^{n}(r+1)^nC_rx^r = (1+x)^n + nx(1+x)^{n-1}$.

[2008]

Options:

A. Statement -1 is false, Statement- 2 is true

B. Statement - 1 is true, Statement- 2 is true; Statement -2 is a correct explanation for Statement-1

C. Statement -1 is true, Statement- 2 is true; Statement -2 is not a correct explanation for Statement-1

D. Statement -1 is true, Statement- 2 is false

Answer: B

Solution:

Solution:
From statement 2:
$\sum_{r=0}^{n}(r+1)^nC_rx^r = \sum_{r=0}^{n} r \cdot {}^nC_rx^r + \sum_{r=0}^{n} {}^nC_rx^r$
$= \sum_{r=1}^{n} r \cdot \frac{n}{r^{n-1}}C_{r-1}x^r + (1+x)^n = nx\sum_{r=1}^{n} {}^{n-1}C_{r-1}x^{r-1} + (1+x)^n$
$= nx(1+x)^{n-1} + (1+x)^n = RHS$
∴ Statement 2 is correct.
Putting $x = 1$, we get
$\sum_{r=0}^{n}(r+1)^nC_r = n \cdot 2^{n-1} + 2^n = (n+2) \cdot 2^{n-1}$.
∴ Statement 1 is also true and statement 2 is a correct explanation for statement 1

Question191

In a shop there are five types of ice-creams available. A child buys six ice-creams.
Statement-1 : The number of different ways the child can buy the six ice-creams is $^{10}C_5$
Statement -2: The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6A 's and 4B 's in a row.
[2008]

Options:

A. Statement -1 is false, Statement-2 is true

B. Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

C. Statement -1 is true, Statement-2 is true; Statement - 2 is not a correct explanation for Statement-1

D. Statement -1 is true, Statement-2 is false

Answer: A

Solution:

Solution:
The given situation in statement 1 is equivalent to find the non negative integral solutions of the equation
$x_1 + x_2 + x_3 + x_4 + x_5 = 6$
which is coeff. of x^6 in the expansion of
$(1 + x + x^2 + x^3 + \ldots\ldots \infty)^5 =$ coeff. of x^6 in $(1 - x)^{-5}$
= coeff. of x^6 in $1 + 5x + \frac{5.6}{2!}x^2 \ldots\ldots\ldots$
$= \frac{5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}{6!} = \frac{10!}{6!4!} = {}^{10}C_6$
∴ Statement 1 is wrong.
Number of ways of arranging 6A 's and 4B 's in a row
$= \frac{10!}{6!4!} = {}^{10}C_6$ which is same as the number of ways the child can buy six icecreams.
∴ Statement 2 is true.

Question192

In the binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5^{th} and 6^{th} terms is zero, then a/b equals
[2007]

Options:

A. $\frac{n-5}{6}$

B. $\frac{n-4}{5}$

C. $\frac{5}{n-4}$

D. $\frac{6}{n-5}$.

Answer: B

Solution:

Solution:

$T_{r+1} = (-1)^r \cdot {}^nC_r(a)^{n-r} \cdot (b)^r$ is an expansion of $(a-b)^n$

$\therefore$ 5 th term $= t_5 = t_{4+1}$

$= (-1)^4 \cdot {}^nC_4(a)^{n-4} \cdot (b)^4 = {}^nC_4 \cdot a^{n-4} \cdot b^4$

6th term $= t_6 = t_{5+1} = (-1)^{5n}C_5(a)^{n-5}(b)^5$

Given $t_5 + t_6 = 0$

$\therefore {}^nC_4 \cdot a^{n-4} \cdot b^4 + (-{}^nC_5 \cdot a^{n-5} \cdot b^5) = 0$

$\Rightarrow \frac{n!}{4!(n-4)!} \cdot \frac{a^n}{a^4} \cdot b^4 - \frac{n!}{5!(n-5)!} \cdot \frac{a^n b^5}{a^5} = 0$

$\Rightarrow \frac{n! \cdot a^n b^4}{4!(n-5)! a^4}\left[\frac{1}{(n-4)} - \frac{b}{5 \cdot a}\right] = 0[\because a \neq 0, b \neq 0]$

$\Rightarrow \frac{1}{n-4} - \frac{b}{5a} = 0 \Rightarrow \frac{a}{b} = \frac{n-4}{5}$

Question193

The sum of the series ${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots - \ldots\ldots + {}^{20}C_{10}$ is [2007]

Options:

A. 0

B. ${}^{20}C_{10}$

C. $-{}^{20}C_{10}$

D. $\frac{1}{2^{20}}C_{10}$

Answer: D

Solution:

Solution:

We know that, $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2 x^2$

$+\ldots\ldots {}^{20}C_{10}x^{10} + \ldots\ldots {}^{20}C_{20}x^{20}$

Put $x = -1$, $(0) = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots$

$+{}^{20}C_{10} - {}^{20}C_{11} \ldots\ldots + {}^{20}C_{20}$

$\Rightarrow + \ldots\ldots - {}^{20}C_9] + {}^{20}C_{10}$

$\Rightarrow^{20} = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3$

$+ \quad \frac{1}{2}{}^{20}C_{10}$

Question194

For natural numbers m, n if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \ldots\ldots.$ and $a_1 = a_2 = 10$, then (m, n) is
[2006]

Options:

A. (20,45)

B. (35,20)

C. (45,35)

D. (35,45)

Answer: D

Solution:

Solution:
$(1-y)^m(1+y)^n$
$= [1 - {}^mC_1y + {}^mC_2y^2 - \ldots\ldots][1 + {}^nC_1y + {}^nC_2y^2 + \ldots\ldots]$
$= 1 + (n-m)y + \left\{ \frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn \right\} y^2 + \ldots.$
$\therefore a_1 = n - m = 10$
and $a_2 = \frac{m^2 + n^2 - m - n - 2mn}{2} = 10$
$(m-n)^2 - (m+n) = 20$
$\Rightarrow m + n = 80$ ...(ii) [from (i)]
Solving (i) and (ii), we get
$\therefore m = 35, n = 45$

Question195

If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation
[2005]

Options:

A. $a - b = 1$

B. $a + b = 1$

C. $\frac{a}{b} = 1$

D. $ab = 1$

Answer: D

Solution:

Solution:
T_{r+1} in the expansion
$\left[ax^2 + \frac{1}{bx}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$
$= {}^{11}C_r(a)^{11-r}(b)^{-r}(x)^{22-2r-r}$
For the Coefficient of x^7, we have
$22 - 3r = 7 \Rightarrow r = 5$
$\therefore$ Coefficient of x^7
$= {}^{11}C_5(a)^6(b)^{-5}$
Again T_{r+1} in the expansion
$\left[ax - \frac{1}{bx^2}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(-\frac{1}{bx^2}\right)^r$
$= {}^{11}C_r(a)^{11-r}(-1)^r \times (b)^{-r}(x)^{-2r+11-r}$
For the Coefficient of x^{-7}, we have
Now $11 - 3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$
$\therefore$ Coefficient of x^{-7}
$= {}^{11}C_6a^5 \times 1 \times (b)^{-6}$
$\because$ Coefficient of $x^7 =$ Coefficient of x^{-7}
From (i) and (ii),
$\therefore {}^{11}C_5(a)^6(b)^{-5} = {}^{11}C_6a^5 \times (b)^{-6}$
$\Rightarrow ab = 1$

Question196

If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+x)^{\frac{3}{2}} - \left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as

[2005]

Options:

A. $1 - \frac{3}{8}x^2$

B. $3x + \frac{3}{8}x^2$

C. $-\frac{3}{8}x^2$

D. $\frac{x}{2} - \frac{3}{8}x^2$

Answer: C

Solution:

Solution:
$\because x^3$ and higher powers of x may be neglected
$\therefore \frac{(1+x)^{\frac{3}{2}} - \left(1+\frac{x}{2}\right)^3}{\left(1-x^{\frac{1}{2}}\right)}$

$$= (1-x)\frac{-1}{2}\left[\left(1+\frac{3}{2}x+\frac{\frac{3}{2}\cdot\frac{1}{2}}{2!}x^2\right)-\left(1+\frac{3x}{2}+\frac{3.2}{2!}\frac{x^2}{4}\right)\right]$$

$$= \left[1+\frac{x}{2}+\frac{\frac{1}{2}\cdot\frac{3}{2}}{2!}x^2\right]\left[\frac{-3}{8}x^2\right] = \frac{-3}{8}x^2$$

Question197

The coefficient of x^n in expansion of $(1 + x)(1 - x)^n$ is [2004]

Options:

A. $(-1)^{n-1}n$

B. $(-1)^n(1-n)$

C. $(-1)^{n-1}(n-1)^2$

D. $(n-1)$

Answer: B

Solution:

Solution:
Coeff. of x^n in $(1+x)(1-x)^n$
= coeff of x^n in
$(1+x)(1-{}^nC_1x+{}^nC_2x^2-\ldots.+(-1)^n{}^nC_nx^n)$
$= (-1)^n{}^nC_n+(-1)^{n-1}{}^nC_{n-1}$
$= (-1)^n+(-1)^{n-1}\cdot n$
$= (-1)^n(1-n)$

Question198

The coefficient of the middle term in the binomial expansion in powers of x of $(1 + \alpha x)^4$ and of $(1 - \alpha x)^6$ is the same if α equals [2004]

Options:

A. $\frac{3}{5}$

B. $\frac{10}{3}$

C. $\frac{-3}{10}$

D. $\frac{-5}{3}$

Answer: C

Solution:

Solution:
The middle term in the expansion of
$(1+\alpha x)^4 = T_3 = {}^4C_2(\alpha x)^2 = 6\alpha^2 x^2$
The middle term in the expansion of
$(1-\alpha x)^6 = T_4 = {}^6C_3(-\alpha x)^3 = -20\alpha^3 x^3$
According to the question
$6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = -\frac{3}{10}$

Question199

The number of integral terms in the expansion of $(\sqrt{3} + {}^8\sqrt{5})^{256}$ is [2003]

Options:

A. 35

B. 32

C. 33

D. 34

Answer: C

Solution:

Solution:
$T_{r+1} = {}^{256}C_r(\sqrt{3})^{256-r}(\sqrt{[8]}5)^r$
$= {}^{256}C_r(3)^{\frac{256-r}{2}}(5)^{r/8}$
Terms will be integral if $\frac{256-r}{2}$ & $\frac{r}{8}$ both are +ve integer.
It is possible if r is an integral multiple of 8 and $0 \le r \le 256$
$\therefore r = 33$

Question200

If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is [2003]

Options:

A. 6 th term

B. 7 th term

C. 5 th term

D. 8 th term

Answer: D

Solution:

Solution:

$T_{r+1} = \frac{n(n-1)(n-2)........(n-r+1)}{r!}(x)^r$

For first negative term, $n - r + 1 < 0 \Rightarrow r > n + 1$

$\Rightarrow r > \frac{32}{5} \therefore r = 7 \cdot \left(\because n = \frac{27}{5}\right)$

Therefore, first negative term is T_8

Question201

r and n are positive integers r > 1, n > 2 and coefficient of $(r+2)^{th}$ term and $3r^{th}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals [2002]

Options:

A. 3r

B. 3r + 1

C. 2r

D. 2r + 1

Answer: C

Solution:

Solution:

$t_{r+2} = {}^{2n}C_{r+1}x^{r+1}$; $t_{3r} = {}^{2n}C_{3r-1}x^{3r-1}$

Given that, ${}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$;

$\Rightarrow r + 1 + 3r - 1 = 2n$

$\Rightarrow 2n = 4r \Rightarrow n = 2r$

Question202

The coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ are [2002]

Options:

A. equal

B. equal with opposite signs

C. reciprocals of each other

D. none of these

Answer: A

Solution:

Solution:

We know that $t_{p+1} = p + qC_p x^p$ and $t_{q+1} = p^{+q}C_q x^q$

$\because\ {}^{p+q}C_p = {}^{p+q}C_q \cdot$ [Remember ${}^nC_r = {}^nC_{n-r}$]

Question203

The positive integer just greater than $(1 + 0.0001)^{10000}$ is [2002]

Options:

A. 4

B. 5

C. 2

D. 3

Answer: D

Solution:

Solution:

$(1 + 0.0001)^{10000} = \left(1 + \frac{1}{n}\right)^n, n = 10000$

$= 1 + n \cdot \frac{1}{n} + \frac{n(n-1)}{2!}\frac{1}{n^2} + \frac{n(n-1)(n-2)}{3!}\frac{1}{n^3} + \ldots + \frac{1}{n^n}$

$= 1 + 1 + \frac{1}{2!}\left(1 - \frac{1}{n}\right) + \frac{1}{3!}\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right) + \ldots + \frac{1}{n^n}$

$< 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots + \frac{1}{(9999)!}$

$= 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots\ldots\infty = e < 3$

Question204

If the sum of the coefficients in the expansion of $(a + b)^n$ is 4096, then the greatest coefficient in the expansion is [2002]

Options:

A. 1594

B. 792

C. 924

D. 2924

Answer: C

Solution:

Solution:

Take $a = 1$ and $b = 1$ in $(a + b)^n$.

$2^n = 4096 = 2^{12} \Rightarrow n = 12$

The greatest coeff = coeff of middle term.

So middle term $= t_7$

$\Rightarrow t_7 = t_{6+1} = {}^{12}C_6 a^6 b^6$

$\Rightarrow$ Coeff of $t_7 = {}^{12}C_6 = \dfrac{12!}{6!6!} = 924$
