FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Tuesday 09th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. $\lim\limits_{x\to 0}\dfrac{e-(1+2x)^{\frac{1}{2x}}}{x}$ is equal to :

(1) e (2) $\dfrac{-2}{e}$

(3) 0 (4) $e-e^2$

Ans. (1)

Sol. $\lim\limits_{x\to 0}\dfrac{e-e^{\frac{1}{2x}\ln(1+2x)}}{x}$

$= \lim\limits_{x\to 0}(-e)\dfrac{\left(e^{\frac{\ln(1+2x)}{2x}-1}-1\right)}{x}$

$= \lim\limits_{x\to 0}(-e)\dfrac{\ln(1+2x)-2x}{2x^2}$

$= (-e)\times(-1)\,\dfrac{4}{2\times 2} = e$

2. Consider the line L passing through the points (1, 2, 3) and (2, 3, 5). The distance of the point $\left(\frac{11}{3},\frac{11}{3},\frac{19}{3}\right)$ from the line L along the line $\dfrac{3x-11}{2}=\dfrac{3y-11}{1}=\dfrac{3z-19}{2}$ is equal to :

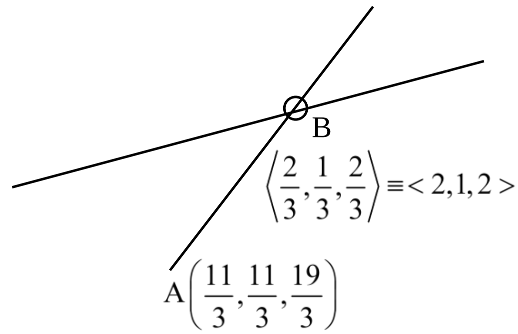
(1) 3 (2) 5

(3) 4 (4) 6

Ans. (1)

Sol. $\dfrac{x-1}{2-1}=\dfrac{y-2}{3-2}=\dfrac{z-3}{5-3}$

$\Rightarrow \dfrac{x-1}{1}=\dfrac{y-2}{1}=\dfrac{z-3}{2}=\lambda$



$B(1+\lambda, 2+\lambda, 3+2\lambda)$

D.R. of AB $= \left\langle \dfrac{3\lambda-8}{3},\dfrac{3\lambda-5}{3},\dfrac{6\lambda-10}{3}\right\rangle$

$B\left(\frac{5}{3},\frac{8}{3},\frac{13}{3}\right)$ $\dfrac{3\lambda-8}{3\lambda-5}=\dfrac{2}{1} \Rightarrow 3\lambda-8=6\lambda-10$

$3\lambda = 2$

$\lambda = \dfrac{2}{3}$

$AB = \dfrac{\sqrt{36+9+36}}{3}=\dfrac{9}{3}=3$

3. Let $\int_0^x \sqrt{1-(y'(t))^2}\,dt = \int_0^x y(t)dt$, $0\le x\le 3$, $y\ge 0$, $y(0)=0$. Then at $x = 2$, $y''+y+1$ is equal to :

(1) 1 (2) 2

(3) $\sqrt{2}$ (4) 1/2

Ans. (1)

Sol. $\sqrt{1-(y'(x))^2}=y(x)$

$1-\left(\dfrac{dy}{dx}\right)^2=y^2$

$\left(\dfrac{dy}{dx}\right)^2=1-y^2$

$$\frac{dy}{\sqrt{1-y^2}} = dx \text{ OR } \frac{dy}{\sqrt{1-y^2}} = -dx$$

$\Rightarrow \sin^{-1}y = x + c, \sin^{-1}y = -x + c$

$x = 0, y = 0 \Rightarrow c = 0$

$\sin^{-1} y = x$, as $y \geq 0$

$\sin x = y$

$$\Rightarrow \frac{dy}{dx} = \cos x$$

$$\frac{d^2y}{dx^2} = -\sin x$$

$\Rightarrow - \sin x + \sin x + 1 = 1$

4. Let z be a complex number such that the real part of $\frac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z-(6+8i)|$ is equal to :

(1) 12 (2) ∞

(3) 10 (4) 8

Ans. (1)

Sol. $$\frac{z-2i}{z+2i} + \frac{\bar{z}+2i}{\bar{z}-2i} = 0$$

$$z\bar{z} - 2i\bar{z} - 2iz + 4(-1)$$

$$+z\bar{z} + 2zi + 2\bar{z}i + 4(-1) = 0$$

$\Rightarrow 2|z|^2 = 8 \Rightarrow |z| = 2$

$$|z-(6+8i)|_{\text{maximum}} = 10 + 2 = 12$$

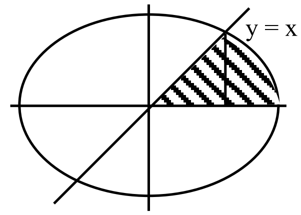
5. The area (in square units) of the region enclosed by the ellipse $x^2 + 3y^2 = 18$ in the first quadrant below the line $y = x$ is :

(1) $\sqrt{3}\pi + \frac{3}{4}$ (2) $\sqrt{3}\pi$

(3) $\sqrt{3}\pi - \frac{3}{4}$ (4) $\sqrt{3}\pi + 1$

Ans. (2)

Sol. $$\frac{x^2}{18} + \frac{y^2}{6} = 1$$



$$\frac{x^2}{18} + \frac{3x^2}{18} = 1 \Rightarrow 4x^2 = 18 \Rightarrow x^2 = \frac{9}{2}$$

$$\int_{\frac{3}{\sqrt{2}}}^{3\sqrt{2}} \frac{\sqrt{18-x^2}}{\sqrt{3}}\,dx$$

$$= \frac{1}{\sqrt{3}}\left(\frac{x\sqrt{18-x^2}}{2} + \frac{18}{2}\sin^{-1}\frac{x}{3\sqrt{2}}\right)_{\frac{3}{\sqrt{2}}}^{3\sqrt{2}}$$

$$= \frac{1}{\sqrt{3}}\left(9\times\frac{\pi}{2} - \frac{3}{2\sqrt{2}}\times\frac{3\sqrt{3}}{\sqrt{2}} - 9\times\frac{\pi}{6}\right)$$

Required Area

$$= \frac{1}{2}\times\frac{9}{2} + \left(\frac{18\pi}{6} - \frac{9\sqrt{3}}{4}\right)\frac{1}{\sqrt{3}}$$

$$= \sqrt{3}\pi$$

6. Let the foci of a hyperbola H coincide with the foci of the ellipse E : $\frac{(x-1)^2}{100} + \frac{(y-1)^2}{75} = 1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its conjugate axis is β, then $3\alpha^2 + 2\beta^2$ is equal to :

(1) 242

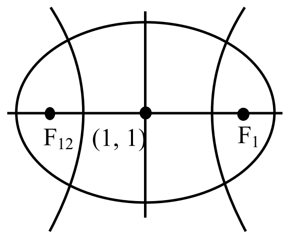
(2) 225

(3) 237

(4) 205

Ans. (2)

Sol.



$e_1 = \sqrt{1 - \frac{75}{100}} = \frac{5}{10} = \frac{1}{2}$

$e_2 = 2$

F_1 (6, 1), F_2 (–4, 1)

$2ae_2 = 10 \Rightarrow a = \frac{5}{2} \Rightarrow 2a = 5$

$\Rightarrow \alpha = 5$

$4 = 1 + \frac{b^2}{a^2} \Rightarrow b^2 = 3a^2$

$b = \sqrt{3} \times \frac{5}{2}$

$\beta = 5\sqrt{3}$

$3\alpha^2 + 2\beta^2 = 3 \times 25 + 2 \times 25 \times 3$

$= 225$

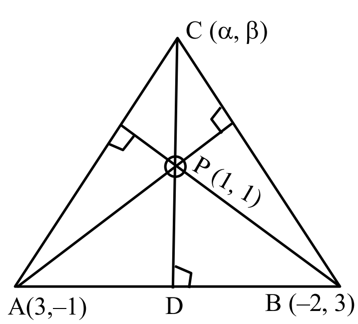
7. Two vertices of a triangle ABC are A(3, –1) and B (–2, 3), and its orthocentre is P(1, 1). If the coordinates of the point C are (α, β) and the centre of the circle circumscribing the triangle PAB is (h, k), then the value of $(\alpha + \beta) + 2(h + k)$ equals :

(1) 51 (2) 81

(3) 5 (4) 15

Ans. (3)

Sol.



$M_{AB} = \frac{4}{-5} \Rightarrow M_{DP} = \frac{5}{4}$

Equation of PC is $y - 1 = \frac{5}{4}(x-1)$ ....(1)

$M_{AP} = \frac{2}{-2} = -1 \Rightarrow M_{BC} = +1$

Equation of BC is $y - 3 = (x + 2)$ ....(2)

On solving (1) and (2)

$x + 4 = \frac{5}{4}(x-1) \Rightarrow 4x + 16 = 5x - 5 \Rightarrow \alpha = 21$

$\Rightarrow \beta = y = x + 5 = 26$

$\alpha + \beta = 47$

Equation of $\perp$ bisector of AP

$y - 0 = (x - 2)$ .... (3)

Equation of $\perp$ bisector of AB

$y - 1 = \frac{5}{4}\left(x - \frac{1}{2}\right)$ ....(4)

On solving (3) & (4)

$(x - 3)4 = 5x - \frac{5}{2}$

$x = \frac{-19}{2} = h$

$y = \frac{-23}{2} = k$

$\Rightarrow 2(h + k) = -42$

8. If the variance of the frequency distribution is 160, then the value of $c \in N$ is

x	c	2c	3c	4c	5c	6c
f	2	1	1	1	1	1

(1) 5 (2) 8

(3) 7 (4) 6

Ans. (3)

Sol.

x	C	2C	3C	4C	5C	6C
f	2	1	1	1	1	1

$\bar{x} = \frac{(2+2+3+4+5+6)C}{7} = \frac{22C}{7}$

$$\text{Var}(x) = \frac{c^2\left(2+2^2+3^2+4^2+5^2+6^2\right)}{7}$$

$$-\left(\frac{22c}{7}\right)^2$$

$$= \frac{92c^2}{7} - c^2 \times \frac{484}{49}$$

$$= \frac{(644-484)c^2}{49} = \frac{160c^2}{49}$$

$$160 = \frac{160 \times c^2}{49} \Rightarrow c = 7$$

9. Let the range of the function

$f(x) = \dfrac{1}{2+\sin 3x+\cos 3x}$, $x \in$ IR be [a, b].

If α and β are respectively the A.M. and the G.M. of a and b, then $\dfrac{\alpha}{\beta}$ is equal to :

(1) $\sqrt{2}$ (2) 2

(3) $\sqrt{\pi}$ (4) π

Ans. (1)

Sol. $f(x)\ \dfrac{1}{2+\sin 3x+\cos 3x}$

$$\left[\frac{1}{2+\sqrt{2}}, \frac{1}{2-\sqrt{2}}\right]$$

$$\frac{\alpha}{\beta} = \frac{a+b}{2\sqrt{ab}} = \frac{1}{2}\left(\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}\right)$$

$$= \frac{1}{2}\left(\sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}}} + \sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}\right)$$

$$= \frac{\left(2-\sqrt{2}\right)+\left(2+\sqrt{2}\right)}{2\times\sqrt{2}} = \sqrt{2}$$

10. Between the following two statements :

Statement-I : Let $\vec{a} = \hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{j}-\hat{k}$. Then the vector $\vec{r}$ satisfying $\vec{a}\times\vec{r} = \vec{a}\times\vec{b}$ and $\vec{a}.\vec{r} = 0$ is of magnitude $\sqrt{10}$.

Statement-II : In a triangle ABC, cos2A + cos2B + cos2C $\geq -\dfrac{3}{2}$.

(1) Both Statement-I and Statement-II are incorrect

(2) Statement-I is incorrect but Statement-II is correct

(3) Both Statement-I and Statement-II are correct

(4) Statement-I is correct but Statement-II is incorrect

Ans. (2)

Sol. $\bar{a} = \hat{i}+2\hat{j}-3\hat{k}$

$\bar{a} = 2\hat{i}+\hat{j}-\hat{k}$

$\bar{a}\times\bar{r} = \bar{a}\times\bar{b};\ \bar{a}.\bar{r} = 0$

$\Rightarrow \bar{a}\times\left(\bar{r}-\bar{b}\right) = \bar{0}$

$\Rightarrow \bar{a} = \lambda\left(\bar{r}-\bar{b}\right)$

$\bar{a}.\bar{a} = \lambda\left(\bar{a}.\bar{r}-\bar{a}.\bar{b}\right)$

$14 = -7\lambda \Rightarrow \lambda = -2$

$\dfrac{-\bar{a}}{2} = \bar{r}-\bar{b} \Rightarrow \bar{r} = \bar{b}-\dfrac{\bar{a}}{2}$

$= \dfrac{2\bar{b}-\bar{a}}{2} = \dfrac{3\hat{i}+\hat{k}}{2}$

Statement (I) is incorrect

cos2A + cos 2B + cos 2c $\geq -\dfrac{3}{2}$

2A + 2B + 2C = 2π

cos2A + cos2B + cos 2C

= – 1 – 4 cosA·cosB·cosC

$\geq -1 - 4 \times \dfrac{1}{2}\times\dfrac{1}{2}\times\dfrac{1}{2}$

$= -\dfrac{3}{2}$

Statement (II) is correct.

11. $\lim_{x\to\frac{x}{2}}\left(\dfrac{\int_{x^3}^{(\pi/2)^3}\left(\sin\left(2t^{1/3}\right)+\cos\left(t^{1/3}\right)\right)dt}{\left(x-\frac{\pi}{2}\right)^2}\right)$ is equal to :

(1) $\dfrac{9\pi^2}{8}$ (2) $\dfrac{11\pi^2}{10}$

(3) $\dfrac{3\pi^2}{2}$ (4) $\dfrac{5\pi^2}{9}$

Ans. (1)

Sol. $\lim_{x\to\frac{\pi}{2}}\dfrac{0-\{\sin(2x)+\cos(x)\}.3x^2}{2\left(x-\frac{\pi}{2}\right)}$

$= \lim_{x\to\frac{\pi}{2}}\dfrac{-\{2\sin x\cos x+\cos x\}3x^2}{2\left(x-\frac{\pi}{2}\right)}$

$= \lim_{x\to\frac{\pi}{2}}\left\{\dfrac{2\sin x\sin\left(\frac{\pi}{2}-x\right)}{2\left(x-\frac{\pi}{2}\right)}+\dfrac{\sin\left(\frac{\pi}{2}-x\right)}{2\left(\frac{\pi}{2}-x\right)}\right\}3x^2$

$=\left(1(1)+\frac{1}{2}\right)3\left(\frac{\pi}{2}\right)^2$

$=\dfrac{9\pi^2}{8}$

12. The sum of the coefficient of $x^{2/3}$ and $x^{-2/5}$ in the binomial expansion of $\left(x^{2/3}+\frac{1}{2}x^{-2/5}\right)^9$ is :

(1) 21/4 (2) 69/16

(3) 63/16 (4) 19/4

Ans. (1)

Sol. $T_{r+1}={}^9C_r\left(x^{2/3}\right)^{9-r}\left(\dfrac{x^{-2/5}}{2}\right)^r$

$={}^9C_r\left(\frac{1}{2}\right)^r(r)^{\left(6-\frac{2r}{3}-\frac{2r}{5}\right)}$

for coefficient of $x^{2/3}$, put $6-\frac{2r}{3}-\frac{2r}{5}=\frac{2}{3}$

$\Rightarrow r=5$

$\therefore$ Coefficient of $x^{2/3}$ is $={}^9C_5\left(\frac{1}{5}\right)^5$

For coefficient of $x^{-2/5}$, put $6-\frac{2r}{3}-\frac{2r}{5}=-\frac{2}{5}$

$\Rightarrow r=6$

Coefficient of $x^{-2/5}$ is ${}^9C_6\left(\frac{1}{2}\right)^6$

Sum $={}^9C_5\left(\frac{1}{2}\right)^5+{}^9C_6\left(\frac{1}{2}\right)^6=\dfrac{21}{4}$

13. Let $B=\begin{bmatrix}1 & 3\\ 1 & 5\end{bmatrix}$ and A be a 2 × 2 matrix such that $AB^{-1}=A^{-1}$. If $BCB^{-1}=A$ and $C^4+\alpha C^2+\beta I=O$, then $2\beta-\alpha$ is equal to :

(1) 16 (2) 2

(3) 8 (4) 10

Ans. (4)

Sol. $BCB^{-1}=A$

$\Rightarrow (BCB^{-1})(BCB^{-1})=A.A$

$\Rightarrow BCI\,CB^{-1}=A^2$

$\Rightarrow BC^2B^{-1}=A^2$

$\Rightarrow B^{-1}(BC^2B^{-1})B=B^{-1}(A.A)B$

From equation (1)

$C^2=A^{-1}.A.B$

$C^2=B$

Also $AB^{-1}=A^{-1}$

$\Rightarrow AB^{-1}.A=A^{-1}A=I$

$\Rightarrow A^{-1}(AB^{-1}A)=A^{-1}I$

$B^{-1}A=A^{-1}$

Now characteristics equation of C^2 is

$|C_2-\lambda I|=0$

$|B-\lambda I|=0$

$\Rightarrow \begin{vmatrix} 1-\lambda & 3 \\ 1 & 5-\lambda \end{vmatrix} = 0$

$\Rightarrow (1-\lambda)(5-l) - 3 = 0 \Rightarrow (\lambda^2 - 6\lambda + 5) - 3 = 0$

$\Rightarrow \lambda^2 - 6\lambda + 2 = 0$

$\Rightarrow \beta^2 - 6B + 2I = 0$

$\Rightarrow C^4 - 6C^2 + 2I = 0$

$\alpha = -6$

$\beta = 2$

$\therefore 2\beta - \alpha = 4 + 6 = 10$

14. If $\log_e y = 3\sin^{-1}x$, then $(1-x)^2 y'' - xy'$ at $x = \frac{1}{2}$ is equal to :

(1) $9e^{\pi/6}$ (2) $3e^{\pi/6}$

(3) $3e^{\pi/2}$ (4) $9e^{\pi/2}$

Ans. (4)

Sol. $\ln(y) = 3\sin^{-1}x$

$$\frac{1}{y}\cdot y' = 3\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow y' = \frac{3y}{\sqrt{1-x^2}} \quad \text{at } x = \frac{1}{2}$$

$$\Rightarrow y' = \frac{3e^{3\left(\frac{\pi}{6}\right)}}{\frac{\sqrt{3}}{2}} = 2\sqrt{3}e^{\frac{\pi}{2}}$$

$$\Rightarrow y'' = 3\left(\frac{\sqrt{1-x^2}\,y' - y\frac{1}{2\sqrt{1-x^2}}(-2x)}{(1-x^2)}\right)$$

$$\Rightarrow (1-x^2)y'' = 3\left(3y + \frac{xy}{\sqrt{1-x^2}}\right)$$

$$\downarrow \text{at } x = \frac{1}{2},\; y = e^{3\sin^{-1}\left(\frac{1}{2}\right)} = e^{3\left(\frac{\pi}{6}\right)} = e^{\frac{\pi}{2}}$$

$$(1-x^2)y''\Big|_{\text{at } x=\frac{1}{2}} = 3\left(3e^{\frac{\pi}{2}} + \frac{\frac{1}{2}\left(e^{\frac{\pi}{2}}\right)}{\frac{\sqrt{3}}{2}}\right)$$

$$= 3e^{\frac{\pi}{2}}\left(3 + \frac{1}{\sqrt{3}}\right)$$

$$(1-x^2)y'' - xy'\Big|_{\text{at } x=\frac{1}{2}}$$

$$= 3e^{\frac{\pi}{2}}\left(3 + \frac{1}{\sqrt{3}}\right) - \frac{1}{2}\left(2\sqrt{3}e^{\frac{\pi}{2}}\right) = 9e^{\frac{\pi}{2}}$$

15. The integral $\int_{1/4}^{3/4} \cos\left(2\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)dx$ is equal to:

(1) $-1/2$ (2) $1/4$

(3) $1/2$ (4) $-1/4$

Ans. (4)

Sol. $I = \int_{1/4}^{3/4} \cos\left(2\cot^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)dx\right)$

$$\int_{1/4}^{3/4} \cos\left(2\left(\tan^{-1}\sqrt{\frac{1+x}{1+x}}\right)\right)dx$$

$$\int_{1/4}^{3/4} \frac{1-\tan^2\left(\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)}{1+\tan^2\left(\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right)}dx$$

$$= \int_{1/4}^{3/4} \frac{1-\left(\frac{1+x}{1-x}\right)}{1+\left(\frac{1+x}{1-x}\right)}dx = \int_{1/4}^{3/4} \frac{-2x}{2}dx$$

$$= \int_{1/4}^{3/4} (-x)\,dx = -\left(\frac{x^2}{2}\right)_{1/4}^{3/4}$$

$$= -\frac{1}{2}\left[\frac{9}{16} - \frac{1}{16}\right]$$

$$= -\frac{1}{4}$$

16. Let a, ar, ar^2, ...be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3r^{3n}$ = 9747, then a + 18r is equal to :

(1) 27 (2) 46

(3) 38 (4) 31

Ans. (4)

Sol. $\sum_{n=0}^{\infty} ar^n = 57$

a + ar + ar^2 + ∞ = 57

$\frac{a}{1-r} = 57$ .... (I)

$\sum_{n=0}^{\infty} a^3r^{3n} = 9747$

$a^3 + a^3 \cdot r^3 + a^3 \cdot r^6 + ... \infty = 9746$

$\frac{a^3}{1-r^3} = 9746$ .... (II)

$\frac{(I)^3}{(II)} \Rightarrow \frac{\frac{a^3}{(1-r)^3}}{\frac{a^3}{1-r^3}} = \frac{57^3}{9717} = 19$

On solving, $r = \frac{2}{3}$ and $r = \frac{3}{2}$ (rejected)

a = 19

$\therefore a + 18r = 19 + 18 \times \frac{2}{3} = 31$

17. If an unbiased dice is rolled thrice, then the probability of getting a greater number in the i^{th} roll than the number obtained in the $(i-1)^{th}$ roll, i = 2, 3, is equal to :

(1) 3/54 (2) 2/54

(3) 5/54 (4) 1/54

Ans. (3)

Sol. Favourable cases = $^6C_3$

Total out comes = 6^3

Probability of getting greater number than previous one = $\frac{^6C_3}{r^3} = \frac{20}{216} = \frac{5}{54}$

18. The value of the integral $\int_{-1}^{2} \log_e\left(x + \sqrt{x^2+1}\right)dx$ is :

(1) $\sqrt{5} - \sqrt{2} + \log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(2) $\sqrt{2} - \sqrt{5} + \log_e\left(\frac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(3) $\sqrt{5} - \sqrt{2} + \log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

(4) $\sqrt{2} - \sqrt{5} + \log_e\left(\frac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

Ans. (2)

Sol. $I = \int_{-1}^{2} 1.\log_e\left(x + \sqrt{x^2+1}\right)dx$

$= x\log_e\left(x + \sqrt{x^2+1}\right) - \int_{-1}^{2}\left(\frac{1 + \frac{x}{\sqrt{x^2+1}}}{x + \sqrt{x^2+1}}\right)dx$

$= x\log_e\left(x + \sqrt{x^2+1}\right) - \int_{-1}^{2} \frac{x}{\sqrt{x^2+1}}dx$

$= x\log_e\left(x + \sqrt{x^2+1}\right) - \sqrt{x^2+1}\Big|_{-1}^{2}$

$= \left(2\log_e\left(2+\sqrt{5}\right) - \sqrt{5}\right)$

$-\left(-\log_e\left(-1+\sqrt{2}\right) - \sqrt{2}\right)$

$= \log_e\left(2+\sqrt{5}\right)^2 - \sqrt{5} + \log_e\left(\sqrt{2}-1\right) + \sqrt{2}$

$= \log_e\left(2+\sqrt{5}\right)^2 - \sqrt{5} + \log_e\left(\sqrt{2}-1\right) + \sqrt{2}$

$$=\sqrt{2}-\sqrt{5}+\log_e\left(\frac{\left(2+\sqrt{5}\right)^2}{\sqrt{2+1}}\right)$$

$$=\sqrt{2}-\sqrt{5}+\log_e\left(\frac{9+4\sqrt{5}}{\sqrt{2+1}}\right)$$

19. Let α, β; $\alpha > \beta$, be the roots of the equation $x^2-\sqrt{2}x-\sqrt{3}=0$. Let $P_n = \alpha^n-\beta^n$, $n \in N$. Then $\left(11\sqrt{3}-10\sqrt{2}\right)$ P_{10} + $\left(11\sqrt{2}+10\right)$ P_{11} – $11P_{12}$ is equal to :

(1) $10\sqrt{2}P_9$

(2) $10\sqrt{3}P_9$

(3) $11\sqrt{2}P_9$

(4) $11\sqrt{3}P_9$

Ans. (2)

Sol. $x^2-\sqrt{2x}-\sqrt{3}=0 \left\langle\begin{matrix}\alpha\\ \beta\end{matrix}\right.$

$\alpha^{n+2}-\sqrt{2}\alpha^{n+1}-\sqrt{3}\alpha^n=0$

and $\beta^{n+2}-\sqrt{2}\beta^{n+1}-\sqrt{3}\beta^n=0$

Subtracting

$\left(\alpha^{n+2}-\beta^{n+2}\right)-\sqrt{2}\left(\alpha^{n+1}-\beta^{n+1}\right)-\sqrt{3}\left(\alpha^n-\beta^n\right)=0$

$\Rightarrow P_{n+2}-\sqrt{2}P_{n+1}-\sqrt{3}P_n=0$

Put $n=10$

$P_{12}-\sqrt{2}P_{11}-\sqrt{3}P_{10}=0$

$n=9$

$P_{11}-\sqrt{2}P_{10}-\sqrt{3}P_9=0$

$11\left(\sqrt{3}.P_{10}+\sqrt{2}P_{11}-P_{11}\right)-10\left(\sqrt{2}P_{10}-P_{11}\right)$

$=0-10\left(-\sqrt{3}P_9\right)=10\sqrt{3}P_9$

20. Let $\vec{a}=2\hat{i}+\alpha\hat{j}+\hat{k}$, $\vec{b}=-\hat{i}+\hat{k}$, $\vec{c}=\beta\hat{j}-\hat{k}$, where α and β are integers and $\alpha\beta$ = –6. Let the values of the ordered pair (α, β) for which the area of the parallelogram of diagonals $\vec{a}+\vec{b}$ and $\vec{b}+\vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1, β_1) and (α_2, β_2).

Then $\alpha_1^2+\beta_1^2-\alpha_2\beta_2$ is equal to

(1) 17 (2) 24

(3) 21 (4) 19

Ans. (4)

Sol. Area of parallelogram = $\frac{1}{2}|\vec{d}_1\times\vec{d}_2$

$A=\frac{1}{2}\left|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})\right|=\frac{\sqrt{21}}{2}$

so, $\vec{a}+\vec{b}=\hat{i}+\alpha\hat{j}+2\hat{k}$

$\vec{b}+\vec{c}=-\hat{i}+\beta\hat{j}$

$\left(\vec{a}+\vec{b}\right)\times\left(\vec{b}+\vec{c}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \alpha & 2\\ -1 & \beta & 0\end{vmatrix}$

$=\hat{i}(-2\beta)-\hat{j}(2)+\hat{k}(\beta+\alpha)$

$\left|(\vec{a}+\vec{b})\times(\vec{b}+\vec{c})\right|=\sqrt{4\beta^2+4+(\alpha+\beta)^2}=\sqrt{21}$

$4\beta^2+4+\alpha^2+\beta^2+2\alpha\beta=21$

$\alpha^2+5\beta^2-12=17$

$\alpha^2+5\beta^2=29$

and $\alpha\beta=-6$

and given $\alpha_i\beta$ are integers

so,

$\alpha=-3,\ \beta=2$

or

$\alpha=3,\ \beta=-2$

$\left(\alpha_1,\beta_1\right)=\left(-3,2\right)$

$\left(\alpha_2,\beta_2\right)=\left(3,-2\right)$

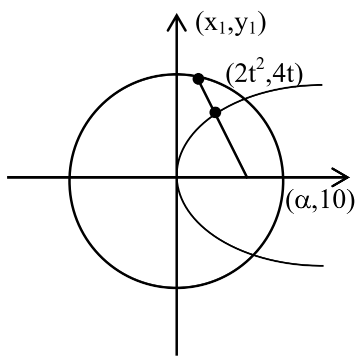
$\alpha_1^2+\beta_1^2-\alpha_2\beta_2=9+4+6=19$

SECTION-B

21. Consider the circle C : $x^2 + y^2 = 4$ and the parabola P : $y^2 = 8x$. If the set of all values of α, for which three chords of the circle C on three distinct lines passing through the point $(\alpha, 0)$ are bisected by the parabola P is the interval (p, q), then $(2q - p)^2$ is equal to _____.

Ans. (80)

Sol.



$T = S_1$

$xx_1 + yy_1 = x_1^2 + y_1^2$

$\alpha x_1 = x_1^2 + y_1^2$

$\alpha(2t^2) = 4t^4 + 16t^2$

$\alpha = 2t^2 + 8$

$\frac{\alpha - 8}{2} = t^2$

Also, $4t^4 + 16t^2 - 4 < 0$

$t^2 = -2 + \sqrt{5}$

$\alpha = 4 + 2\sqrt{5}$

$\therefore \alpha \in (8, 4 + 2\sqrt{5})$

$\therefore (2q - p)^2 = 80$

22. Let the set of all values of p, for which $f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2 - p)x + 7$ does not have any critical point, be the interval (a, b). Then 16ab is equal to _____ .

Ans. (252)

Sol. $f(x) = -(p^2 - 6p + 8)\cos 4n + 2(2-p)n + 7$

$f^{I}(x) = +4(p^2 - 6p + 8)\sin 4x + (4-2p) \neq 0$

$\sin 4x \neq \frac{2p - 4}{4(p-4)(p-2)}$

$\sin 4x \neq \frac{2(p-2)}{4(p-4)(p-2)}$

$p \neq 2$

$\sin 4x \neq \frac{1}{2(p-4)}$

$\Rightarrow \left|\frac{1}{2(p-4)}\right| > 1$

on solving we get

$\therefore p \in \left(\frac{7}{2}, \frac{9}{2}\right)$

Hence $a = \frac{7}{2}, b = \frac{9}{2}$

$\therefore 16ab = 252$

23. For a differentiable function $f : IR \to IR$, suppose $f'(x) = 3f(x) + \alpha$, where $\alpha \in IR$, $f(0) = 1$ and $\lim_{x \to -\infty} f(x) = 7$. Then $9f(-\log_e 3)$ is equal to______.

Ans. (61)

Sol. $\frac{dy}{dx} - 3y = \alpha$

$If = e^{\int -3dx} = e^{-3x}$

$\therefore y - e^{-3x} = \int e^{-3x} \cdot \alpha\, dx$

$y e^{-3x} = \frac{\alpha e^{-3x}}{-3} + c$

$(* e^{3x})$

$y = \frac{\alpha}{-3} + C \cdot e^{3x}$

on substituting x = 0, y = 1

$x \to -\infty$, y = 7

we get $y = 7 - 6e^{3x}$

$9f(-\log_e 3) = 61$

24. The number of integers, between 100 and 1000 having the sum of their digits equals to 14, is ______.

Ans. (70)

Sol. N = a b c

(i) All distinct digits

$a + b + c = 14$

$a \geq 1$

$b, c \in \{0 \text{ to } 9\}$

by hit & trial : 8 cases

(6, 5, 3)	(8, 6, 0)	(9, 4, 1)
(7, 6, 1)	(8, 5, 1)	(9, 3, 2)
(7, 5, 2)	(8, 4, 2)	
(7, 4, 3)	(9, 5, 0)	

(ii) 2 same, 1 diff a = b ; c

$2a + c = 14$

by values :

$$\left.\begin{matrix}(3,8)\\(4,6)\\(5,4)\\(6,2)\\(7,0)\end{matrix}\right\} \begin{matrix}\text{Total}\\ \frac{3!}{2!}\times 5-1\end{matrix}$$

= 14 cases

(iii) all same :

$3a = 14$

$a = \frac{14}{3} \times$ rejected

0 cases

Hence, Total cases :

$8 \times 3! + 2 \times (4) + 14$

$= 48 + 22$

$= 70$

25. Let $A = \{(x, y) : 2x + 3y = 23, x, y \in N\}$ and $B = \{x : (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to _____.

Ans. (24)

Sol. $2x + 3y = 23$

x = 1	y = 7
x = 4	y = 5
x = 7	y = 3
x = 10	y = 1

A	B
(1, 7)	1
(4, 5)	4
(7, 3)	7
(10, 1)	10

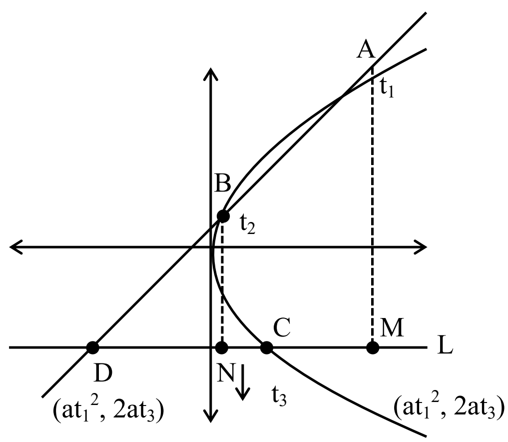
The number of one-one functions from A to B is equal to 4!

26. Let A, B and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line L through C parallel to the x-axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L.

Then $\left(\frac{AM \cdot BN}{CD}\right)^2$ is equal to _______.

Ans. (36)

Sol.



Sol.

$m_{AB} = m_{AD}$

$\Rightarrow \quad \frac{2}{t_1 + t_2} = \frac{2a(t_1 - t_3)}{at_1^2 - \alpha}$

$\Rightarrow \quad at_1^2 - \alpha = a\{t_1^2 - t_1t_3 + t_1t_2 - t_2t_3\}$

$\Rightarrow \quad \alpha = a(t_1t_3 + t_2t_3 - t_1t_2)$

$AM = |2a(t_1 - t_3)|$, $BN = |2a(t_2 - t_3)|$,

$CD = |at_3^2 - \alpha|$

$$CD = \left|at_3^2 - a(t_1t_3 + t_2t_3 - t_1t_2)\right|$$

$$= a\left|t_3^2 - t_1t_3 - t_2t_3 + t_1t_2\right|$$

$$= a\left|t_3(t_3 - t_1) - t_2(t_3 - t_1)\right|$$

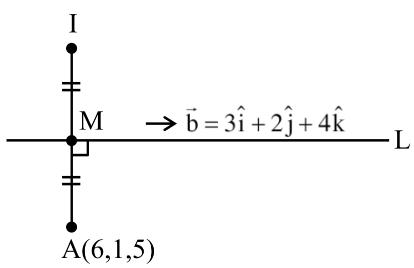
$$CD = a\left|(t_3 - t_2)(t_3 - t_1)\right|$$

$$\left(\frac{AM \cdot BN}{CD}\right)^2 = \left\{\frac{2a(t_1 - t_3) \cdot 2a(t_2 - t_3)}{a(t_3 - t_2)(t_3 - t_1)}\right\}^2$$

$$16a^2 = 16 \times \frac{9}{4} = 36$$

27. The square of the distance of the image of the point (6, 1, 5) in the line $\frac{x-1}{3} = \frac{y}{2} = \frac{z-2}{4}$, from the origin is ______.

Ans. (62)



Sol.

Let M$(3\lambda + 1, 2\lambda, 4\lambda + 2)$

$\overrightarrow{AM} \cdot \vec{b} = 0$

$\Rightarrow \quad 9\lambda - 15 + 4\lambda - 2 + 16\lambda - 12 = 0$

$\Rightarrow \quad 29\lambda = 29$

$\Rightarrow \quad \lambda = 1$

M (4, 2, 6), I = (2, 3, 7)

Required Distance = $\sqrt{4+9+49} = \sqrt{62}$

Ans. 62

28. If $\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + ... + \frac{1}{\alpha+1012}\right)$

$- \left(\frac{1}{2 \cdot 1} + \frac{1}{4 \cdot 3} + \frac{1}{6 \cdot 5} + ... + \frac{1}{2024 \cdot 2023}\right)$

$= \frac{1}{2024}$, then α is equal to-

Ans. (1011)

Sol. $\left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + ... + \frac{1}{\alpha+2012}\right)$

$$-\left\{\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + ... + \left(\frac{1}{2023} - \frac{1}{2024}\right)\right\} = \frac{1}{2024}$$

$$\Rightarrow \left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + ... + \frac{1}{\alpha+2012}\right)$$

$$-\left\{\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) + ... + \frac{1}{2023}\right.$$

$$\left. - \frac{1}{2024} - 2\left(\frac{1}{2} + \frac{1}{4} + ... + \frac{1}{2022}\right)\right\} = \frac{1}{2024}$$

$$\Rightarrow \left(\frac{1}{\alpha+1} + \frac{1}{\alpha+2} + ... + \frac{1}{\alpha+2012}\right)$$

$$-\left(\frac{1}{1} + \frac{1}{2} + ... + \frac{1}{2023}\right)$$

$$+\frac{1}{2024} + \left(\frac{1}{1} + \frac{1}{2} + ... + \frac{1}{1011}\right) = \frac{1}{2024}$$

$$\Rightarrow \frac{1}{\alpha+1} + \frac{1}{\alpha+2} + ... + \frac{1}{\alpha+2012}$$

$$= \frac{1}{1012} + \frac{1}{1013} + ... + \frac{1}{2023}$$

$\Rightarrow \alpha = 1011$

29. Let the inverse trigonometric functions take principal values. The number of real solutions of the equation $2\sin^{-1}x + 3\cos^{-1}x = \frac{2\pi}{5}$, is ______.

Ans. (0)

Sol. $2\sin^{-1}x + 3\cos^{-1}x = \frac{2\pi}{5}$

$\Rightarrow \quad \pi + \cos^{-1}x = \frac{2\pi}{5}$

$\Rightarrow \quad \cos^{-1}x = \frac{-3\pi}{5}$

Not possible

Ans. 0

30. Consider the matrices : $A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}$, $B = \begin{bmatrix} 20 \\ m \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Let the set of all m, for which the system of equations AX = B has a negative solution (i.e., x < 0 and y < 0), be the interval (a, b).

Then $8\int_a^b |A|\,dm$ is equal to ________.

Ans. (450)

Sol. $A = \begin{pmatrix} 2 & -5 \\ 3 & m \end{pmatrix}$, $B = \begin{pmatrix} 20 \\ m \end{pmatrix}$

$X = \begin{pmatrix} x \\ y \end{pmatrix}$

$2x - 5y = 20 \qquad \ldots(1)$

$3x + my = m \qquad \ldots(2)$

$\Rightarrow y = \dfrac{2m - 60}{2m + 15}$

$y < 0 \Rightarrow m \in \left(\dfrac{-15}{2}, 30\right)$

$x = \dfrac{25m}{2m + 15}$

$x < 0 \Rightarrow m \in \left(\dfrac{-15}{2}, 0\right)$

$\Rightarrow m \in \left(\dfrac{-15}{2}, 0\right)$

$|A| = 2m + 15$

Now,

$$8\int_{\frac{-15}{2}}^{0} (2m + 15)\,dm = 8\left\{m^2 + 15m\right\}_{\frac{-15}{2}}^{0}$$

$$\Rightarrow 8\left\{-\left(\frac{225}{4} - \frac{225}{2}\right)\right\}$$

$$= 8 \times \frac{225}{4} = 450$$

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

31. A nucleus at rest disintegrates into two smaller nuclei with their masses in the ratio of 2:1. After disintegration they will move :-

(1) In opposite directions with speed in the ratio of 1:2 respectively

(2) In opposite directions with speed in the ratio of 2:1 respectively

(3) In the same direction with same speed.

(4) In opposite directions with the same speed.

Ans. **(1)**

Sol. By conservation of momentum

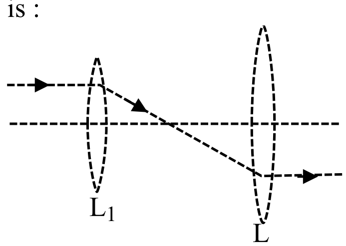
$p_i = p_f$

$O = m_1 u_1 + m_2 u_2$

$\frac{u_1}{u_2} = -\left[\frac{1}{2}\right]$ as $\frac{m_1}{m_2} = \frac{2}{1}$

move in opposite direction with speed ratio 1 : 2

32. The following figure represents two biconvex lenses L_1 and L_2 having focal length 10 cm and 15 cm respectively. The distance between L_1 & L_2 is :

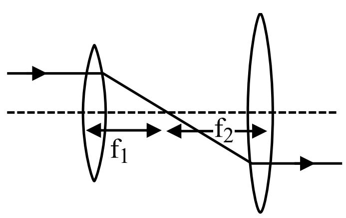


(1) 10 cm (2) 15 cm

(3) 25 cm (4) 35 cm

Ans. **(3)**

Sol.



$D = f_1 + f_2 = 25$ cm

Paraxial parallel rays pass through focus and ray from focus of convex lens will become parallel

33. The temperature of a gas is –78° C and the average translational kinetic energy of its molecules is K. The temperature at which the average translational kinetic energy of the molecules of the same gas becomes 2K is :

(1) –39°C (2) 117°C

(3) 127°C (4) –78°C

Ans. **(2)**

Sol. $K.E = \frac{nf_1RT}{2}$

$T_i = -78°C \rightarrow 273 + [-78°C] = 195K$

K.E α T

To double the K.E energy temp also become double

$T_f = 390$ K

$T_f = 117°C$

34. A hydrogen atom in ground state is given an energy of 10.2 eV. How many spectral lines will be emitted due to transition of electrons ?

(1) 6 (2) 3

(3) 10 (4) 1

Ans. **(4)**

Sol. Hydrogen will be in first excited state therefore it will emit one spectral line corresponding to transition b/w energy level 2 to 1

35. The magnetic field in a plane electromagnetic wave is $B_y = (3.5 \times 10^{-7}) \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)$T. The corresponding electric field will be

(1) $E_y = 1.17 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)Vm^{-1}$

(2) $E_z = 105 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)Vm^{-1}$

(3) $E_z = 1.17 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)Vm^{-1}$

(4) $E_y = 10.5 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)Vm^{-1}$

Ans. **(2)**

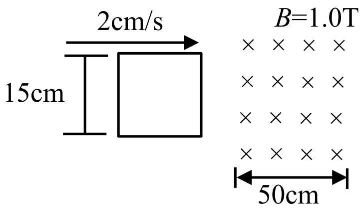
Sol. $E_0 = B_0 C$

$E_0 = 3 \times 10^8 \times (3.5 \times 10^{-7}) \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)$

$E_0 = 105 \sin(1.5 \times 10^3 x + 0.5 \times 10^{11} t)Vm^{-1}$

Data inconsistent while calculating speed of wave. You can challenge for data.

36. A square loop of side 15 cm being moved towards right at a constant speed of 2 cm/s as shown in figure. The front edge enters the 50 cm wide magnetic field at t = 0. The value of induced emf in the loop at t = 10 s will be :

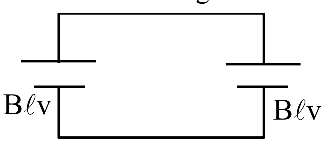


(1) 0.3 *m*V (2) 4.5 *m*V

(3) zero (4) 3 *m*V

Ans. **(3)**

Sol. At t = 10 sec complete loop is in magnetic field therefore no change in flux



$$e = \frac{d\phi}{dt} = 0$$

e = 0 for complete loop

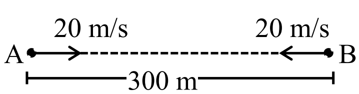
37. Two cars are travelling towards each other at speed of 20 m s^{-1} each. When the cars are 300 m apart, both the drivers apply brakes and the cars retard at the rate of 2 m s^{-2} . The distance between them when they come to rest is :

(1) 200 m (2) 50 m

(3) 100 m (4) 25 m

Ans. **(3)**

Sol.



$$|\vec{u}_{BA}| = 40 \text{ m/s}$$

$$|\vec{a}_{BA}| = 4 \text{ m/s}$$

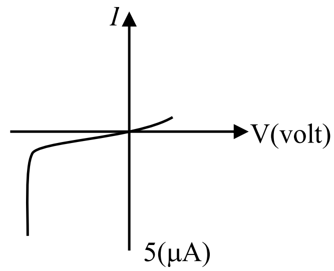
Apply $(v^2 = u^2 + 2as)_{\text{relative}}$

$O = (40)^2 + 2(-4)(S)$

S = 200 m

Remaining distance = 300 – 200 = 100 m

38. The *I-V* characteristics of an electronic device shown in the figure. The device is :



(1) a solar cell

(2) a transistor which can be used as an amplifier

(3) a zener diode which can be used as voltage regulator

(4) a diode which can be used as a rectifier

Ans. (3)

Sol. Theory

Zener diode used as voltage regulator

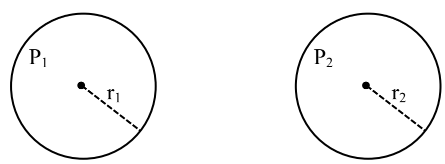
39. The excess pressure inside a soap bubble is thrice the excess pressure inside a second soap bubble. The ratio between the volume of the first and the second bubble is :

(1) 1 : 9 (2) 1 : 3

(3) 1 : 81 (4) 1 : 27

Ans. **(4)**

Sol.



$$P_1 - P_0 = \frac{4T}{r_1} \qquad P_2 - P_0 = \frac{4T}{r_2}$$

$$P_1 - P_0 = 3(P_2 - P_0)$$

$$\frac{4T}{r_1} = 3\frac{4T}{r_2}$$

$$r_2 = 3r_1$$

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{1}{27}$$

40. The de-Broglie wavelength associated with a particle of mass m and energy E is $h/\sqrt{2mE}$. The dimensional formula for Planck's constant is :

(1) $[ML^{-1}T^{-2}]$ (2) $[ML^{2}T^{-1}]$

(3) $[MLT^{-2}]$ (4) $[M^{2}L^{2}T^{-2}]$

Ans. (2)

Sol. $\lambda = \dfrac{h}{\sqrt{2mE}}$ or $E = h\nu$

$[ML^{2}T^{-2}] = h[T^{-1}]$

$h = [ML^{2}T^{-1}]$

41. A satellite of 10^{3} kg mass is revolving in circular orbit of radius 2R. If $\dfrac{10^{4}R}{6}J$ energy is supplied to the satellite, it would revolve in a new circular orbit of radius :

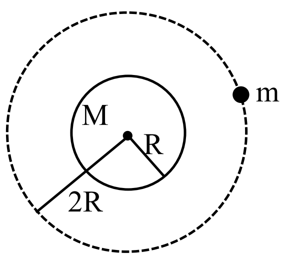
(use $g = 10 m/s^{2}$, R = radius of earth)

(1) 2.5 R (2) 3 R

(3) 4 R (4) 6 R

Ans. (4)

Sol.



Total energy $= \dfrac{-GMm}{2(2R)}$

if energy $= \dfrac{10^{4}R}{6}$ is added then

$$\frac{-GMm}{4R} + \frac{10^{4}R}{6} = \frac{-GMm}{2r}$$

where r is new radius of revolving and $g = \dfrac{GM}{R^{2}}$

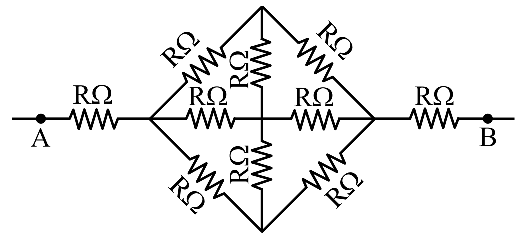
$$-\frac{mgR}{4} + \frac{10^{4}R}{6} = -\frac{mgR^{2}}{2r} \quad (m = 10^{3}\text{ kg})$$

$$-\frac{10^{3}\times 10\times R}{4} + \frac{10^{4}R}{6} = -\frac{10^{3}\times 10\times R^{2}}{2r}$$

$$-\frac{1}{4} + \frac{1}{6} = -\frac{R}{2r}$$

r = 6R

42. The effective resistance between A and B, if resistance of each resistor is R, will be



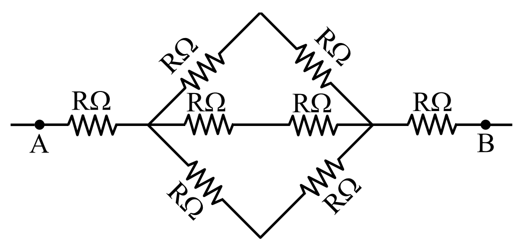
(1) $\dfrac{2}{3}R$ (2) $\dfrac{8R}{3}$

(3) $\dfrac{5R}{3}$ (4) $\dfrac{4R}{3}$

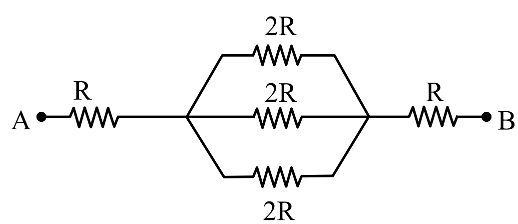
Ans. (2)

Sol. From symmetry we can remove two middle resistance.

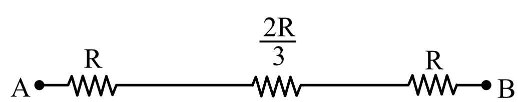
New circuit is



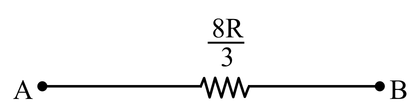
⇓



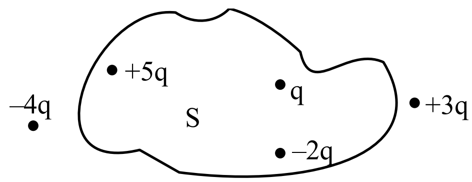
⇓



⇓



43. Five charges +q, +5q, –2q, +3q and –4q are situated as shown in the figure. The electric flux due to this configuration through the surface S is :



(1) $\frac{5q}{\epsilon_0}$ (2) $\frac{4q}{\epsilon_0}$

(3) $\frac{3q}{\epsilon_0}$ (4) $\frac{q}{\epsilon_0}$

Ans. (2)

Sol. As per gauss theorem,

$$\phi = \frac{q_{in}}{\epsilon_0} = \frac{q+(-2q)+5q}{\epsilon_0}$$

$$\frac{4q}{\epsilon_0}$$

44. A proton and a deutron (q= +e, m = 2.0u) having same kinetic energies enter a region of uniform magnetic field $\vec{B}$, moving perpendicular to $\vec{B}$. The ratio of the radius r_d of deutron path to the radius r_p of the proton path is :

(1) 1 : 1 (2) $1:\sqrt{2}$

(3) $\sqrt{2}:1$ (4) $1:2$

Ans. (3)

Sol. In uniform magnetic field,

$$R = \frac{mv}{qB} = \frac{\sqrt{2m(K.E)}}{qB}$$

Since same K.E

$$R \propto \frac{\sqrt{m}}{q}$$

$$\therefore \frac{R_{deutron}}{R_{proton}} = \sqrt{\frac{m_d}{m_p}} \times \frac{q_p}{q_d}$$

$$= \sqrt{2} \times 1$$

$$\therefore \gamma_d : \gamma_p = \sqrt{2}:1$$

45. UV light of 4.13 eV is incident on a photosensitive metal surface having work function 3.13 eV. The maximum kinetic energy of ejected photoelectrons will be :

(1) 4.13 eV (2) 1 eV

(3) 3.13 eV (4) 7.26 eV

Ans. (2)

Sol. E_{photon} = (work function) + $K.E_{max}$

$\therefore 4.13 = 3.13 + K.E_{max}$

$\therefore K.E_{max} = 1$ eV

46. The energy released in the fusion of 2 kg of hydrogen deep in the sun is E_H and the energy released in the fission of 2 kg of ^{235}U is E_U. The ratio $\frac{E_H}{E_U}$ is approximately :

(Consider the fusion reaction as $4\,^1_1H + 2e^- \rightarrow\, ^4_2He + 2v + 6\gamma + 26.7\,MeV$, energy released in the fission reaction of ^{235}U is 200 *MeV* per fission nucleus and $N_A = 6.023\times10^{23}$)

(1) 9.13 (2) 15.04

(3) 7.62 (4) 25.6

Ans. (3)

Sol. In each fusion reaction, 4 ^1_1H nucleus are used.

Energy released per Nuclei of $^1_1H = \frac{26.7}{4}$ MeV

$\therefore$ Energy released by 2 kg hydrogen (E_H)

$$= \frac{2000}{1} \times N_A \times \frac{26.7}{4} MeV$$

&

$\therefore$ Energy released by 2 kg Vranium (E_V)

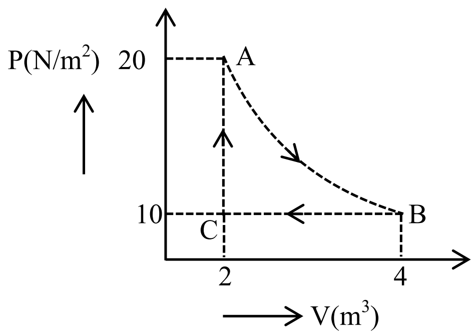
$$= \frac{2000}{235} \times N_A \times 200 MeV$$

So,

$$\frac{E_H}{E_V} = 235 \times \frac{26.7}{4\times200} = 7.84$$

$\therefore$ Approximately close to 7.62

47. A real gas within a closed chamber at 27°C undergoes the cyclic process as shown in figure. The gas obeys $PV^3 = RT$ equation for the path A to B. The net work done in the complete cycle is (assuming $R = 8$J/molK):



(1) 225 J (2) 205 J

(3) 20 J (4) –20 J

Ans. **(2)**

Sol. $W_{AB} = \int PdV$ (Assuming T to be constant)

$= \int \frac{RTdV}{V^3}$

$= RT\int_2^4 V^{-3}dV$

$= 8 \times 300 \times \left(-\frac{1}{2}\left[\frac{1}{4^2} - \frac{1}{2^2}\right]\right)$

$= 225$ J

$W_{BC} = P\int_4^2 dV = 10(2-4) = -20J$

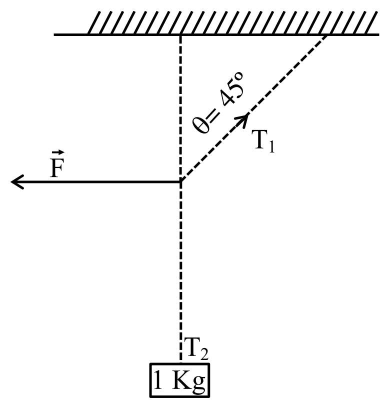
$W_{CA} = 0$

$\therefore W_{cycle} = 205$ J

Note : Data is inconsistent in process AB.

So needs to be challenged.

48. A 1 kg mass is suspended from the ceiling by a rope of length 4m. A horizontal force 'F' is applied at the mid point of the rope so that the rope makes an angle of 45° with respect to the vertical axis as shown in figure. The magnitude of F is :



(1) $\frac{10}{\sqrt{2}}$N (2) 1 N

(3) $\frac{1}{10\times\sqrt{2}}$N (4) 10 N

Ans. **(4)**

Sol. $T_1\sin45° = F$

$T_1 \cos45° = T_2 = 1\times g$

$\therefore \tan45° = \frac{F}{g}$

$\therefore F = 10N$

49. A spherical ball of radius 1×10^{-4} m and density 10^5 kg/m^3 falls freely under gravity through a distance h before entering a tank of water, If after entering in water the velocity of the ball does not change, then the value of h is approximately :

(The coefficient of viscosity of water is 9.8×10^{-6} N s/m^2**)**

(1) 2296 m (2) 2249 m

(3) 2518 m (4) 2396 m

Ans. **(3)**

Sol. $V_T = \dfrac{2g}{9}\dfrac{R^2[\rho_B - \rho_L]}{\eta}$

$\Rightarrow V_T = \dfrac{2}{9}\times\dfrac{10\times(10^{-4})^2}{9.8\times10^{-6}}\left[10^5 - 10^3\right]$

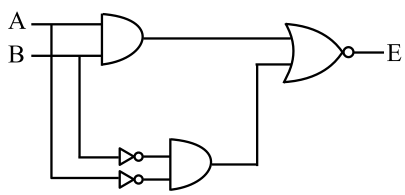
$\Rightarrow V_T = 224.5$

when ball fall from height (h)

$V = \sqrt{2gh}$

$h = \left(\dfrac{V^2}{2g}\right) = 2518\text{m}$

50.



A	B	E
0	0	0
0	1	X
1	0	Y
1	1	0

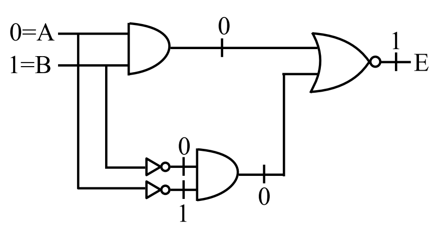
In the truth table of the above circuit the value of X and Y are :

(1) 1, 1 (2) 1, 0

(3) 0, 1 (4) 0, 0

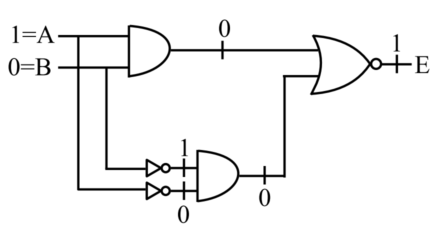
Ans. (1)

Sol. For x



$\boxed{x = 1}$

For y



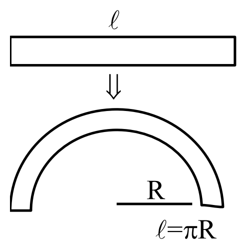
SECTION-B

51. A straight magnetic strip has a magnetic moment of 44 Am^2. If the strip is bent in a semicircular shape, its magnetic moment will be ………. Am^2.

(Given $\pi = \dfrac{22}{7}$)

Ans. (28)

Sol. Magnetic moment of straight wire = $m\times\ell = 44$



Magnetic moment of arc

$= m \times 2\ r$

$= m \times \dfrac{2\ell}{\pi}$

$= \dfrac{44\times2}{\pi} = \dfrac{88}{\pi} = 28$

52. A particle of mass 0.50 kg executes simple harmonic motion under force $F = -50(Nm^{-1})x$. The time period of oscillation is $\dfrac{x}{35}$s. The value of x is ……………..

(Given $\pi = \dfrac{22}{7}$)

Ans. (22)

Sol. $m = 0.5$ kg

$F = -50\ (x)$

$ma = (-50x)$

$0.5\ a = -50x$

$a = (-100x)$

$W^2 = 100 \Rightarrow (w = 10)$

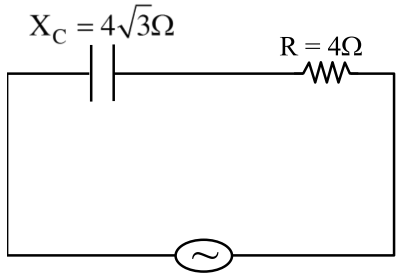
$T = \dfrac{2\pi}{10} = \left(\dfrac{\pi}{5}\right) = \dfrac{22}{7\times15} = \left(\dfrac{22}{35}\right)$

$\dfrac{\pi}{35} = \dfrac{22}{35} \Rightarrow \boxed{x = 22}$

53. A capacitor of reactance $4\sqrt{3}\Omega$ and a resistor of resistance 4Ω are connected in series with an ac source of peak value $8\sqrt{2}$V. The power dissipation in the circuit is ……………..W.

Ans. (4)

Sol.



$Z = \sqrt{R^2 + X^2L}$

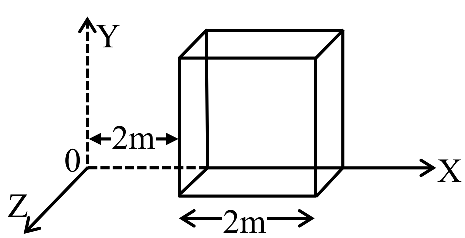
$Z = \sqrt{4^2 + \left(4\sqrt{3}\right)^2} = 8\Omega$

$V_{rms} = \dfrac{V}{\sqrt{2}} = \dfrac{8\sqrt{2}}{\sqrt{2}} = (8V)$

$I_{rms} = \dfrac{V_{rms}}{z} = \dfrac{8}{8} = 1A$

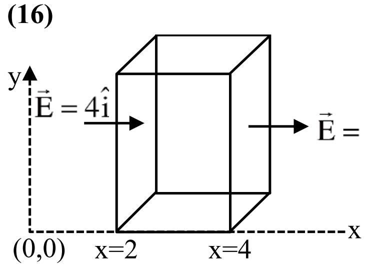
Power dissipated = $I^2_{rms} \times R = 1 \times 4 = (4W)$

54. An electric field $\vec{E} = (2x\hat{i})NC^{-1}$ exists in space. A cube of side 2m is placed in the space as per figure given below. The electric flux through the cube is ……………… Nm^2/C.



Ans. (16)

Sol.



$\vec{E} = 2x\hat{i}$

$\phi = \vec{E}.\vec{A}$

$\phi_{in} = -4 \times 4 = -16\,Nm^2/c$

$\phi_{out} = 8 \times 4 = 32 Nm^2/c$

$d_{net} = \phi_{in} + \phi_{out} = -16 + 32 = 16\ Nm^2/c$

55. A circular disc reaches from top to bottom of an inclined plane of length *l*. When it slips down the plane, if takes t s. When it rolls down the plane then it takes $\left(\dfrac{\alpha}{2}\right)^{1/2}$ t s, where α is ………..

Ans. (3)

Sol. For slipping

$a = g\sin\theta$

$\ell = \dfrac{1}{2}at^2 \Rightarrow t = \sqrt{\dfrac{2\ell}{g\sin\theta}}$

For rolling

$a' = \dfrac{g\sin\theta}{1 + \dfrac{k^2}{R^2}} \left[k = \dfrac{R}{\sqrt{2}}\right]$

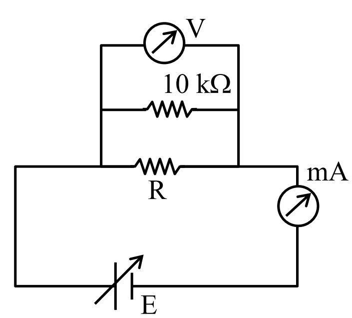
$\Rightarrow a' = \dfrac{2g\sin\theta}{3}$

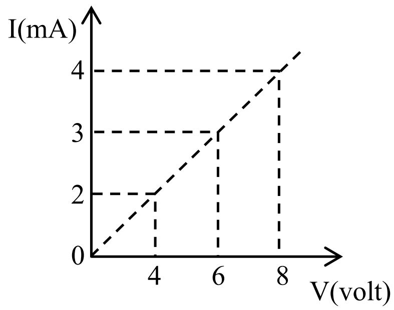
$\ell = \dfrac{1}{2}a'(t')^2$

$\Rightarrow t' = \sqrt{\dfrac{6\ell}{2g\sin\theta}} = \sqrt{\dfrac{\alpha}{2}}\sqrt{\dfrac{2\ell}{g\sin\theta}}$

$\Rightarrow \boxed{\alpha = 3}$

56. To determine the resistance (R) of a wire, a circuit is designed below, The V-I characteristic curve for this circuit is plotted for the voltmeter and the ammeter readings as shown in figure. The value of R is ………… Ω.





Ans. **(2500)**

Sol. $Req = \dfrac{10^4 R}{10^4 + R}$

$E = 4V, I = 2mA$

$I = \dfrac{E}{Req} \Rightarrow 2 \times 10^{-3} = \dfrac{4(10^4 + R)}{10^4 R}$

$\Rightarrow 20R = 40000 + 4R$

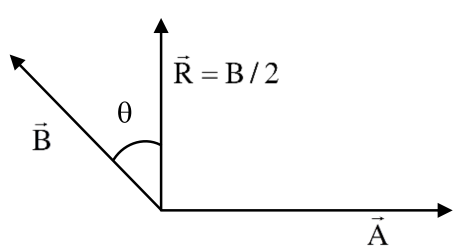
$16R = 40000$

$R = 2500\Omega$

57. The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$ and its magnitude is half that of $\vec{B}$. The angle between vectors $\vec{A}$ and $\vec{B}$ is …………

Ans. **(150)**

Sol.



$B\cos\theta = \dfrac{B}{2}$

$\Rightarrow \theta = 60°$

So, angle between $\vec{A}$ & $\vec{B}$ is $90° + 60° = 150°$

58. Monochromatic light of wavelength 500 nm is used in Young's double slit experiment. An interference pattern is obtained on a screen When one of the slits is covered with a very thin glass plate (refractive index = 1.5), the central maximum is shifted to a position previously occupied by the 4^{th} bright fringe. The thickness of the glass-plate is ……………….µm.

Ans. **(4)**

Sol. $(\mu - 1)\, t = n\lambda$

$(1.5 - 1)\, t = 4 \times 500 \times 10^{-9}$ m

$t = 4000 \times 10^{-9}$ m

$t = 4\mu m$

59. A force $(3x^2 + 2x - 5)$ N displaces a body from x = 2 m to x = 4m. Work done by this force is …………J.

Ans. **(58)**

Sol. $W = \int_{x_1}^{x_2} F dx$

$W = \int_{2}^{4} (3x^2 + 2x - 5) dx$

$W = \left[x^3 + x^2 - 5x\right]_2^4$

$W = [60 - 2] J = 58J$

60. At room temperature (27ºC), the resistance of a heating element is 50Ω. The temperature coefficient of the material is 2.4×10^{-4} ºC^{-1}. The temperature of the element, when its resistance is 62 Ω, is …………. ºC.

Ans. **(1027)**

Sol. $R = R_0(1 + \alpha\Delta T)$

$62 = 50\,[1 + 2.4 \times 10^{-4} \Delta T]$

$\Delta T = 1000°C$

$\Rightarrow T - 27° = 1000°C$

$\boxed{T = 1027°C}$

SECTION-A

61. The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 'A' $\times 10^{12}$ hertz and that has a radiant intensity in that direction of $\frac{1}{'B'}$ watt per steradian. 'A' and 'B' are respectively

(1) 540 and $\frac{1}{683}$

(2) 540 and 683

(3) 450 and $\frac{1}{683}$

(4) 450 and 683

Ans. (2)

Sol. The candela is the luminous intensity of a source that emits monochromatic radiation of frequency radiation of frequency 540×10^{12} Hz and has a radiant intensity in that direction of $\frac{1}{683}$ w/sr. It is unit of Candela.

62. The correct stability order of the following resonance structures of $CH_3-CH=CH-CHO$ is

$$\underset{\text{I}}{CH_3-\overset{\ominus}{C}H-CH=\overset{:\ddot{O}:^{\oplus}}{\overset{|}{C}}-H} \leftrightarrow \underset{\text{II}}{CH_3-\overset{\oplus}{C}H-CH=\overset{:\ddot{O}:^{\ominus}}{\overset{|}{C}}-H}$$

$$\leftrightarrow \underset{\text{III}}{CH_3-CH=CH-\overset{:\ddot{O}:}{\overset{||}{C}}-H}$$

(1) II > III > I
(2) III > II > I
(3) I > II > III
(4) II > I > III

Ans. (2)

Sol. $CH_3-CH=CH-\overset{O}{\overset{||}{C}}H$ (III)

$\downarrow$

Non Polar R.S.
More No of covalent bond

$CH_3-\overset{\oplus}{C}H-CH=\overset{O^{\ominus}}{\overset{|}{C}}H$ (II)

$\downarrow$

Having –ve charge on more electronegative atom

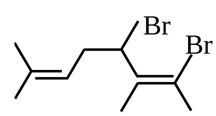
$CH_3-\overset{\ominus}{C}H-CH=\overset{O^{\oplus}}{\overset{|}{C}}H$ (I)

$\downarrow$

Having –ve charge on less electronegative atom

Stability order III > II > I

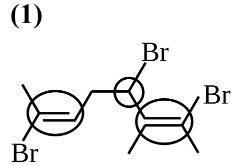
63. Total number of stereo isomers possible for the given structure:



(1) 8 (2) 2
(3) 4 (4) 3

Ans. (1)

Sol.



There are three stereo center
So No of stereoisomer = $2^3 = 8$

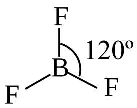
64. The correct increasing order for bond angles among BF_3, PF_3 and $C\ell F_3$ is :

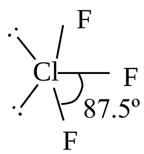
(1) $PF_3 < BF_3 < C\ell F_3$ (2) $BF_3 < PF_3 < C\ell F_3$

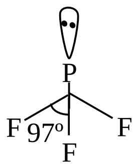
(3) $C\ell F_3 < PF_3 < BF_3$ (4) $BF_3 = PF_3 < C\ell F_3$

Ans. (3)

Sol.







Order of bond angle is
$ClF_3 < PF_3 < BF_3$

65. Match List I with List II

LIST-I (Test)		**LIST-II (Observation)**	
A.	Br_2 water test	I.	Yellow orange or orange red precipitate formed
B.	Ceric ammonium nitrate test	II.	Reddish orange colour disappears
C.	Ferric chloride test	III.	Red colour appears
D.	2, 4-DNP test	IV.	Blue, Green, Violet or Red colour appear

Choose the correct answer from the options given below:
(1) A-I, B-II, C-III, D-IV
(2) A-II, B-III, C-IV, D-I
(3) A-III, B-IV, C-I, D-II
(4) A-IV, B-I, C-II, D-III

Ans. **(2)**

Sol. (A) Br_2 water test is test of unsaturation in which reddish orange colour of bromine water disappears.
(B) Alcohols given Red colour with ceric ammonium nitrate.
(C) Phenol gives Violet colour with natural ferric chloride.
(D) Aldehyde & Ketone give Yellow/Orange/Red Colour compounds with 2, 4-DNP i.e., 2, 4-Dinitrophenyl hydrazine.

66. Match List I with List II

LIST-I (Cell)		**LIST-II (Use/Property/Reaction)**	
A.	Leclanche cell	I.	Converts energy of combustion into electrical energy
B.	Ni-Cd cell	II.	Does not involve any ion in solution and is used in hearing aids
C.	Fuel cell	III.	Rechargeable
D.	Mercury cell	IV.	Reaction at anode $Zn \rightarrow Zn^{2+} + 2e^-$

Choose the correct answer from the options given below:
(1) A-I, B-II, C-III, D-IV
(2) A-III, B-I, C-IV, D-II
(3) A-IV, B-III, C-I, D-II
(4) A-II, B-III, C-IV, D-I

Ans. **(3)**

Sol. A-IV, B-III, C-I, D-II

67. Match List I with List II

LIST-I		**LIST-II**	
A.	$K_2[Ni(CN)_4]$	I.	sp^3
B.	$[Ni(CO)_4]$	II.	sp^3d^2
C.	$[Co(NH_3)_6]Cl_3$	III.	dsp^2
D.	$Na_3[CoF_6]$	IV.	d^2sp^3

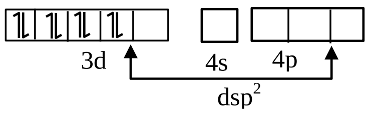
Choose the correct answer from the options given below:
(1) A-III, B-I, C-II, D-IV
(2) A-III, B-II, C-IV, D-I
(3) A-I, B-III, C-II, D-IV
(4) A-III, B-I, C-IV, D-II

Ans. **(4)**

Sol. **(A)** $K_2\overset{+2}{[Ni(CN)_4]}$

Ni^{2+} : $[Ar]3d^8\,4s^o$,($CN^-$is S.F.L)

Pre hybridization state of Ni^{+2}

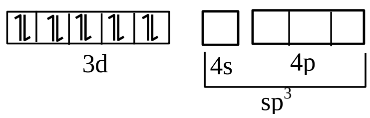


(B) $[Ni(CO)_4]$

Ni : $[Ar]\ 3d^8\,4s^2$

CO is S.F.L , so pairing occur

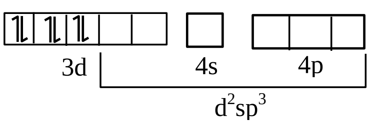
Pre hybridization state of Ni



(C) $\left[Co(NH_3)_6\right]Cl_3$

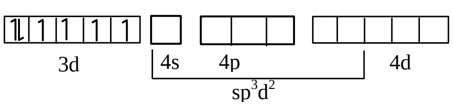
Co^{+3} :$[Ar]3d^6\,4s^0$

With Co^{3+}, NH_3 act as S.F.L



(d) $Na_3\,[CoF_6]$

Co^{3+} : [Ar] $3d^6$ ($F^{\ominus}$: W.F.L)



68. The coordination environment of Ca^{2+} ion in its complex with $EDTA^{4-}$ is :

(1) tetrahedral

(2) octahedral

(3) square planar

(4) trigonal prismatic

Ans. **(2)**

Sol. $EDTA^{4-} \rightarrow$ Hexadentate ligand

$[Ca(EDTA)]^{2-}$

So Coordination environment is octahedral

69. The **incorrect** statement about Glucose is :

(1) Glucose is soluble in water because of having aldehyde functional group

(2) Glucose remains in multiple isomeric form in its aqueous solution

(3) Glucose is an aldohexose

(4) Glucose is one of the monomer unit in sucrose

Ans. **(1)**

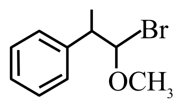
Sol. Glucose is soluble in water due to presence of alcohol functional group and extensive hydrogen bonding.

Glucose exist is open chain as well as cyclic forms in its aqueous solution.

Glucose having 6C atoms so it is hexose and having aldehyde functional group so it is aldose.

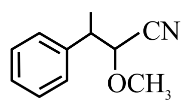
Thus, aldohexose.

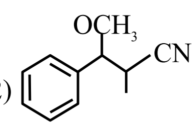
Glucose is monomer unit in sucrose with fructose.

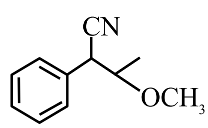
70. 

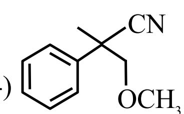
$\xrightarrow[\Delta]{KCN(alc)}$ Major Product 'P'

In the above reaction product 'P' is

(1) 

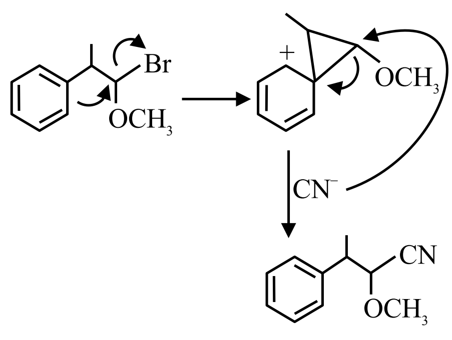
(2) 

(3) 

(4) 

Ans. **(1)**

Sol.



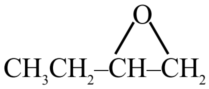
Due to NGP effect of phenyl ring Nucleophilic substitution of Br will occurs.

71. Which of the following compound can give positive iodoform test when treated with aqueous KOH solution followed by potassium hypoiodite.

(1) $CH_3CH_2-\overset{\overset{\large O}{||}}{C}-CH_2CH_3$

(2) $CH_3CH_2-\underset{\underset{\large Cl}{|}}{\overset{\overset{\large Cl}{|}}{C}}-CH_3$

(3) $CH_3CH_2CH_2CHO$

(4) 

Ans. **(2)**

Sol.

$$CH_3-CH_2-\underset{\underset{\large Cl}{|}}{\overset{\overset{\large Cl}{|}}{C}}-CH_3 \xrightarrow{\text{aq. KOH}} CH_3-CH_2-\underset{\underset{\large OH}{|}}{\overset{\overset{\large OH}{|}}{C}}-CH_3$$

$$\xrightarrow{-H_2O} CH_3-CH_2-\overset{\overset{\large O}{||}}{C}-CH_3$$

$$\xrightarrow{\text{KOI}} CH_3-CH_2-COOK + \underset{\text{Yellow ppt}}{CHI_3\downarrow}$$

72. For a sparingly soluble salt AB_2, the equilibrium concentrations of A^{2+} ions and B^- ions are 1.2×10^{-4} M and 0.24×10^{-3} M, respectively. The solubility product of AB_2 is :

(1) 0.069×10^{-12}

(2) 6.91×10^{-12}

(3) 0.276×10^{-12}

(4) 27.65×10^{-12}

Ans. **(2)**

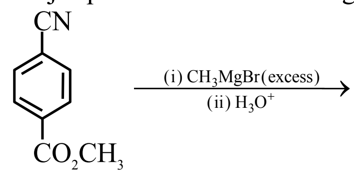
Sol. $AB_{2(s)} \rightleftharpoons A^{+2}{}_{(aq)} + 2B^-{}_{(aq)}$

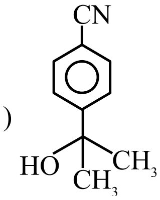
$$K_{sp} = [A^{+2}][B^-]^2$$

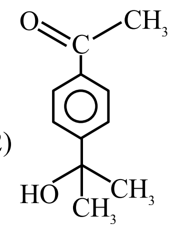
$$= 1.2 \times 10^{-4} \times (2.4 \times 10^{-4})^2$$

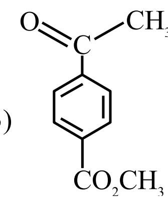
$$= 6.91 \times 10^{-12}\ M^3$$

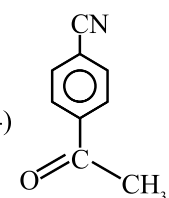
73. Major product of the following reaction is



(1) 

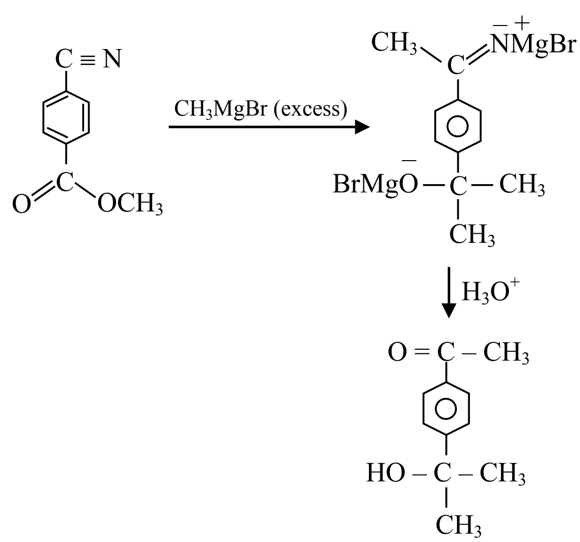
(2) 

(3) 

(4) 

Ans. **(2)**

Sol.



74. Given below are two statements :

Statement I : The higher oxidation states are more stable down the group among transition elements unlike p-block elements.

Statement II : Copper can not liberate hydrogen from weak acids.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Statement I is true but Statement II is false

Ans. **(3)**

Sol. On moving down the group in transition elements, stability of higher oxidation state increases, due to increase in effective nuclear charge.

$\Rightarrow E^{o}_{Cu^{+2}/Cu} = 0.34\ V$

$\Rightarrow E^{o}_{H^{+}/H_2} = 0$

SRP : $Cu^{2+} > H^{+}$

Cu can't liberate hydrogen gas from weak acid.

75. The **incorrect** statement regarding ethyne is

(1) The C–C bonds in ethyne is shorter than that in ethene

(2) Both carbons are sp hybridised

(3) Ethyne is linear

(4) The carbon-carbon bonds in ethyne is weaker than that in ethene

Ans. **(4)**

Sol. The carbon-carbon bonds in ethyne is stronger than that in ethene.

(H–C≡C–H) Ethyne is linear and carbon atoms are SP hybridised.

76. Match List I with List II

List-I (Element)		**List-II (Electronic Configuration)**	
A.	N	I.	$[Ar]\ 3d^{10}4s^2\ 4p^5$
B.	S	II.	$[Ne]\ 3s^2\ 3p^4$
C.	Br	III.	$[He]\ 2s^2\ 2p^3$
D	Kr	IV.	$[Ar]\ 3d^{10}\ 4s^2\ 4p^6$

Choose the correct answer from the options given below :

(1) A-IV, B-III, C-II, D-I

(2) A-III, B-II, C-I, D-IV

(3) A-I, B-IV, C-III, D-II

(4) A-II, B-I, C-IV, D-III

Ans. **(2)**

Sol. (A) ${}_{7}N : [He]2s^2 2p^3$

(B) ${}_{16}S : [Ne]2s^2 3p^4$

(C) ${}_{35}Br : [Ar]3d^{10} 4s^2 4p^5$

(D) ${}_{36}Kr : [Ar]3d^{10} 4s^2 4p^6$

77. Match List I with List II

List-I		**List-II**	
A.	Melting point [K]	I.	Tl > In > Ga > Al > B
B.	Ionic Radius [M^{+3}/pm]	II.	B > Tl > Al ≈ Ga > In
C.	$\Delta_i H_1$ [kJ mol^{-1}]	III.	Tl > In > Al > Ga > B
D	Atomic Radius [pm]	IV.	B > Al > Tl > In > Ga

Choose the correct answer from the options given below :

(1) A-III, B-IV, C-I, D-II

(2) A-II, B-III, C-IV, D-I

(3) A-IV, B-I, C-II, D-III

(4) A-I, B-II, C-III, D-IV

Ans. **(3)**

Sol. Melting point : B > $A\ell$ > $T\ell$ > In > Ga

Ionic radius (M^{+3}/pm) : $T\ell$ > In > Ga > $A\ell$ > B

$(\Delta_{IE}H)_1\left[\frac{kJ}{mol}\right]$: B > $T\ell$ > $A\ell$ ≈ Ga > In

Atomic radius (in pm) : $T\ell$ > In > $A\ell$ > Ga > B

78. Which of the following compounds will give silver mirror with ammoniacal silver nitrate?

(A) Formic acid

(B) Formaldehyde

(C) Benzaldehyde

(D) Acetone

Choose the correct answer from the options given below :

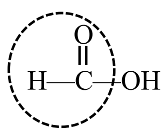
(1) C and D only

(2) A, B and C only

(3) A only

(4) B and C only

Ans. **(2)**

Sol. Apart from aldehyde, Formic acid 

also gives silver mirror test with ammonical silver nitrate.

79. Which out of the following is a correct equation to show change in molar conductivity with respect to concentration for a weak electrolyte, if the symbols carry their usual meaning :

(1) $\Lambda^2{}_mC - K_a\Lambda_m^{\circ 2} + K_a\Lambda_m\Lambda^\circ_m = 0$

(2) $\Lambda_m - \Lambda^\circ{}_m + AC^{\frac{1}{2}} = 0$

(3) $\Lambda_m - \Lambda^\circ{}_m - AC^{\frac{1}{2}} = 0$

(4) $\Lambda^2{}_mC + K_a\Lambda_m^{\circ 2} - K_a\Lambda_m\Lambda^\circ_m = 0$

Ans. **(1)**

Sol. $HA(aq) \rightleftharpoons H^+(aq) + A^-(aq)$

$$K_a = \frac{\alpha^2 C}{1-\alpha}$$

$$\alpha^2C + K_a\alpha - K_a = 0$$

$$\left(\frac{\lambda_m}{\lambda_m^\infty}\right)^2 C + K_a\frac{\lambda_m}{\lambda_m^\infty} - K_a = 0$$

$$\lambda_m^2C + K_a\lambda_m\lambda_m^\infty - K_a\left(\lambda_m^\infty\right)^2 = 0$$

80. The electronic configuration of Einsteinium is :

(Given atomic number of Einsteinium = 99)

(1) [Rn] $5f^{12}\ 6d^0\ 7s^2$ (2) [Rn] $5f^{11}\ 6d^0\ 7s^2$

(3) [Rn] $5f^{13}\ 6d^0\ 7s^2$ (4) [Rn] $5f^{10}\ 6d^0\ 7s^2$

Ans. **(2)**

Sol. Einsteinium (atomic No = 99) : [Rn] $5f^{11}\ 6d^0\ 7s^2$

SECTION-B

81. Number of oxygen atoms present in chemical formula of fuming sulphuric acid is _______.

Ans. **(7)**

Sol. Fuming sulphuric acid is a mixture of

conc. $H_2SO_4 + SO_3$ Or $H_2S_2O_7$

So, Number of Oxygen atoms = 7

82. A transition metal 'M' among Sc, Ti, V , Cr, Mn and Fe has the highest second ionisation enthalpy. The spin only magnetic moment value of M^+ ion is ______ BM (Near integer)

(Given atomic number Sc : 21, Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26)

Ans. **(6)**

Sol. Among given metals, Cr has maximum IE_2

because Second electron is removed from stable configuration $3d^5$

Cr^+ : [Ar] $3d^5\ 4s^0$

$\therefore$ No of unpaired e^- in Cr^+ is 5, n = 5

So, Magnetic moment = $\sqrt{n(n+2)}$ B.M

$= \sqrt{5(5+2)} = 5.92$ BM ≈ 6

83. The vapour pressure of pure benzene and methyl benzene at 27°C is given as 80 Torr and 24 Torr, respectively. The mole fraction of methyl benzene in vapour phase, in equilibrium with an equimolar mixture of those two liquids (ideal solution) at the same temperature is____ $\times 10^{-2}$ (nearest integer)

Ans. **(23)**

Sol. $X_{methylbenzene} = 0.5$

$$Y_{methylbenzene} = \frac{P_{methylbenzene}}{P_{total}}$$

$$Y_{methylbenzene} = \frac{0.5 \times 24}{0.5 \times 80 + 0.5 \times 24}$$

$$= \frac{12}{40 + 12} = 0.23 = 23 \times 10^{-2}$$

84. Consider the following test for a group-IV cation.

$M^{2+} + H_2S \rightarrow$ A (Black precipitate) + byproduct

A + aqua regia $\rightarrow$ B + NOCl + S + H_2O

B + KNO_2 + CH_3COOH $\rightarrow$ C + byproduct

The spin only magnetic moment value of the metal complex C is ______BM.

(Nearest integer)

Ans. **(0)**

Sol. $Co^{2+} + H_2S \rightarrow CoS\downarrow$ (Black)

(A)

CoS + Aqua-regia $\rightarrow Co^{2+}$ (aq) + NOCl + S + H_2O

(A) (B)

Co^{2+} (aq) + KNO_2 + CH_3COOH

$\downarrow$

$K_3[Co(NO_2)_6]$ + NO + S + H_2O

In $K_3[Co(NO_2)_6]$, Co^{+3} : $3d^6\ 4s^0$

Co^{3+} : d^2sp^3 Hybridisation

Number of unpaired e^- = 0

Magnetic moment = $\sqrt{n(n+2)} = 0$ B.M

85. Consider the following first order gas phase reaction at constant temperature

$A(g) \rightarrow 2B(g) + C(g)$

If the total pressure of the gases is found to be 200 torr after 23 sec. and 300 torr upon the complete decomposition of A after a very long time, then the rate constant of the given reaction is____ $\times 10^{-2}$ s^{-1} (nearest integer)

[Given : $\log_{10}(2) = 0.301$]

Ans. **(3)**

Sol. $A(g) \rightarrow 2B(g) + C(g)$

$$P_{23} = P_0 + 2x = 200$$

$$P_\infty = 3P_0 = 300$$

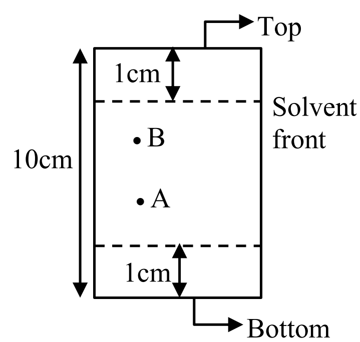
$P_0 = 100$

$$K = \frac{1}{t}\ln\frac{P_\infty - P_0}{P_\infty - P_t}$$

$$K = \frac{2.3}{23}\log\frac{300-100}{300-200}$$

$$= \frac{2.3 \times 0.301}{23} = 0.0301 = 3.01 \times 10^{-2}\,\text{sec}^{-1}$$

86.



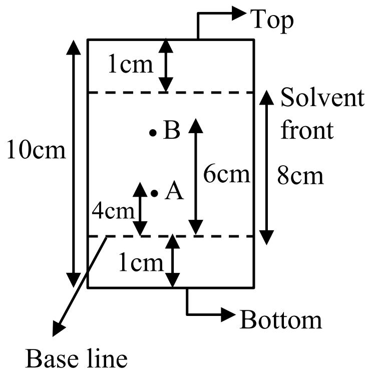
In the given TLC, the distance of spot A & B are 5 cm & 7 cm, from the bottom of TLC plate, respectively.

R_f value of B is x $\times 10^{-1}$ times more than A. The value of x is_____.

Ans. **(15)**

Sol.

$$R_f = \frac{\text{Distance moved by substance from base line}}{\text{Distance moved by solvent from base line}}$$



$$(R_f)_A = \frac{4}{8} \qquad (R_f)_B = \frac{6}{8}$$

$$\frac{(R_f)_B}{(R_f)_A} = \frac{6}{8} \times \frac{8}{4}$$

$(R_f)_B = 1.5\ (R_f)_A$

$x = 15$

87. Based on Heisenberg's uncertainty principle, the uncertainty in the velocity of the electron to be found within an atomic nucleus of diameter 10^{-15} m is ______ $\times 10^9$ ms^{-1} (nearest integer)

[Given : mass of electron $= 9.1 \times 10^{-31}$ kg, Plank's constant (h) $= 6.626 \times 10^{-34}$ Js]

(Value of $\pi = 3.14$)

Ans. **(58)**

Sol. $m\Delta V.\Delta x = \dfrac{h}{4\pi}$

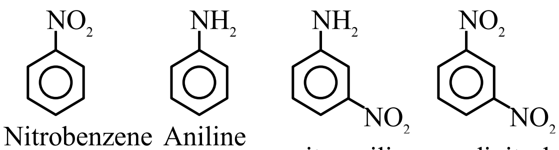
$$\Delta V = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-15} \times 4 \times 3.14}$$

$$= 57.97 \times 10^{+9}\ \text{m/sec}$$

88. Number of compounds from the following which **cannot** undergo Friedel-Crafts reactions is :____
toluene, nitrobenzene, xylene, cumene, aniline, chlorobenzene, m-nitroaniline, m-dinitrobenzene

Ans. **(4)**

Sol. Compounds which can not undergo Friedel Crafts reaction are



Nitrobenzene Aniline m-nitroaniline m-dinitrobenzene

89. Total number of electron present in (π^*) molecular orbitals of O_2, O_2^+ and O_2^- is_______.

Ans. **(6)**

Sol. O_2 (16e) : $(\sigma_{1s})^2 (\sigma^*_{1s})^2 (\sigma_{2s})^2 (\sigma^*_{2s})^2$

$$(\sigma_{2p})^2 \left[(\pi_{2p})^2 = (\pi_{2p})^2\right], \left[(\pi^*_{2p})^1 = (\pi^*_{2p})^1\right]$$

Number of e^- present in (π^*) of $O_2 = 2$

Number of e^- present in (π^*) of $O_2^+ = 1$

Number of e^- present in (π^*) of $O_2^- = 3$

So total e^- in $(\pi^*) = 2 + 1 + 3 = 6$

90. When $\Delta H_{vap} = 30$ kJ/mol and $\Delta S_{vap} = 75$ J mol^{-1} K^{-1}, then the temperature of vapour, at one atmosphere is ______K.

Ans. **(400)**

Sol. At equilibrium $\Delta G_{PT} = 0$

$\Delta H_{vap} = T\Delta S_{vap}$

$30 \times 1000 = T \times 75$

$T = 400K$