JEE Main 27 Jan 2024
(Shift-2)
(Memory Based)

The Actual Paper will be Updated with Solution After the Official Release

JEE (MAIN) 2024 DATE-27/01/2024 (SHIFT-2)

PHYSICS

1. An equation of real gas $\left(p-\frac{a}{V^2}\right)(V-b)=RT$

then dimension of $\left(\frac{a}{b^2}\right)$ is

P : Pressure

V = Volume

R = Gas constant

T = Temperature

(1) $[ML^{-1}T^{-2}]$ (2) $[MLT^{-2}]$ (3) $[ML^2T^{-2}]$ (4) $[MLT^{-1}]$

Ans. **(1)**

Sol. Basic theory

2. **Assertion:** There can be positive zero error in vernier calliper.

Reason: Due to mishandling or rough handling of instrument

(1) Assertion true, reason true and reason is correct explanation of assertion

(2) Assertion true, reason true and reason is not correct explanation of assertion

(3) Assertion true, reason false

(4) Assertion false, reason true

Ans. **(1)**

3. In a RLC series circuit R = 10Ω, L = $\frac{100}{\pi}$ mH, C = $\frac{10^{-3}}{\pi}$ F and frequency is 50 Hz. Find power factor.

Ans. **1**

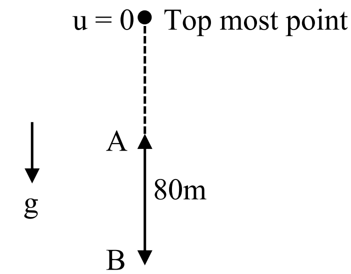
Sol. $X_L = \frac{100}{\pi}\times 2\pi\times 50\times 10^{-3} = 10\Omega$

$$X_C = \frac{1}{2\pi\times 50\times\frac{10^{-3}}{\pi}} = 10\Omega$$

$X_L = X_C$

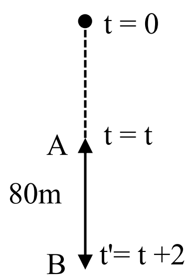
$\cos\phi = 1$

4. A stone is released and while free-fall stone covers 80 m distance in last 2 sec. Find distance of point A from top most point.



Ans. **45**

Sol.



$$S = ut + \frac{1}{2}gt^2$$

$5\,(t + 2)^2 - 5t^2 = 80 \Rightarrow t = 3\text{sec}$

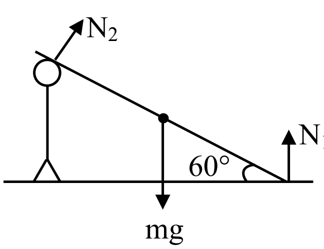
$S_A = 0 + \dfrac{1}{2} \times 10 \times 3^2 = 45\text{m}$

5. A person is standing on horizontal ground. A rod of mass 12 kg is touching a shoulder of person and other end is resting on ground. Angle made by rod with horizontal is 60°. Reaction force applied by person on rod is

(1) 60 N (2) 30 N (3) 90 N (4) 120 N

Ans. **(2)**

Sol.

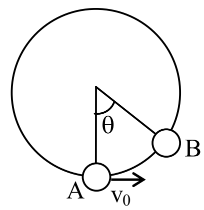


Taking torque about N_1.

$$mg.\frac{\ell}{2}\cos 60° = N_2.\ell$$

$N_2 = 30$ N

6. Point 'B' is at highest point of trajectory of object. Magnitude of acceleration at 'A' and 'B' is equal. Find the angle 'θ' as shown.



(1) $2\tan^{-1}(1/2)$ (2) $\tan^{-1}(1/2)$ (3) $\tan^{-1}(1/4)$ (4) $\tan^{-1}(2)$

Ans. (1)

Sol. Apply work energy theorem

$$mg\ell(1-\cos\theta) = \frac{1}{2}mv_0^2$$

$$\frac{v_0^2}{\ell} = 4g\sin^2\left(\frac{\theta}{2}\right) \quad ....(1)$$

$$g\sin\theta = \frac{v_0^2}{\ell} \quad ....(2)$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{1}{2}$$

$$\theta = 2\tan^{-1}\left(\frac{1}{2}\right)$$

7. In an adiabatic process, pressure is proportional to cube of temperature. Find the ratio C_p/C_v.

Ans. 3/2

Sol. $PT^{\gamma/1-\gamma}$ = constant

$P \propto T^3$

$PT^{-3} = C$

$$\frac{\gamma}{1-\gamma} = -3$$

$\gamma = -3 + 3\gamma$

$2\gamma = 3$

$\gamma = 3/2$

8. **Assertion:** Angular velocity of earth around sun is lesser than the angular velocity of moon about earth.

Reason: Time taken by moon revolve around earth is less than time taken by earth to revolve around sun.

(1) Both Assertion (A) and Reason (R) are true & correct explanation of Assertion 'A'

(2) Both 'A' and 'R' are correct but 'R' is not correct explanation of 'A'

(3) 'A' is correct and 'R' is false

(4) 'A' is false and 'R' is correct

Ans. **(1)**

Sol. $\boxed{T = \frac{2\pi}{\omega}}$ $T_{earth} = 365$ days

$T_{moom} = 27$ days

9. If wave function of a metal is 6.68eV. Find threshold frequency.

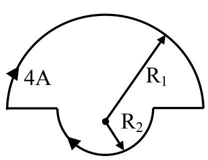
(1) 8×10^{15} Hz (2) 1.6×10^{15} Hz (3) 10×10^{15} Hz (4) 4×10^{15} Hz

Ans. **(2)**

Sol. $6.68 \times 1.6 \times 10^{-19} = 6.626 \times 10^{-34}\, v_0$

$1.6 \times 10^{15}\text{Hz} = v_0$

10. Find magnetic field strength at the centre of loop.



$R_1 = \frac{\pi}{2}$

$R_2 = \frac{\pi}{4}$

Ans. **24×10^{-7} Tesla**

Sol. $B_{centre} = \frac{\mu_0(i)}{4R_1} + \frac{\mu_0 i}{4R_2}$

$= \frac{\mu_0 \times 4}{4}\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$

$= \mu_0\left[\frac{2}{\pi} + \frac{4}{\pi}\right]$

$= 4\pi \times 10^{-7}\left[\frac{6}{\pi}\right]$

$= 24 \times 10^{-7}$ Tesla

11. **Assertion:** If external force is removed, then body will try to regain its actual shape, this is called elasticity.

Reason: Due to intermolecular force, this happens

(1) Assertion True, Reason True & Reason is correct explanation of assertion

(2) Assertion True, Reason True & Reason is not correct explanation of assertion

(3) Assertion True, Reason false

(4) Assertion false, Reason True

Ans. **(1)**

12. A bullet gets embedded in a fixed target. It is found that bullet losses 1/3rd of its velocity in traveling 4 cm into target and losses remaining kinetic energy while traveling further $d \times 10^{-3}$ m. Find d.

Ans. **32**

Sol. $v^2 = u^2 + 2ax$

$$\left(\frac{2u}{3}\right)^2 = u^2 + 2(-a)(4cm) \qquad \ldots(1)$$

for next

$$O = \left(\frac{2u}{3}\right)^2 + 2(-a)(x)] \qquad \ldots(2)$$

using equation (i) &(ii)

$x = 32 \times 10^{-3}$ m

So d = 32

13. 1 mole of an ideal O_2 gas is at 27ºC. Find its total kinetic energy?

(1) 1250 J (2) 6250 J (3) 645 J (4) 1025 J

Ans. **(2)**

Sol. **Kinetic Energy =** $\frac{n}{2}fRT$

KE = $\frac{1}{2} \times 5 \times \frac{25}{3} \times 300$

= 6250 J

14. Light of intensity $I = 6 \times 10^8 \ \frac{W}{m^2}$ is incident on an object kept in medium of refractive index, $\mu = 3$ assuming 100% absorption. Find radiation pressure (N/m^2)?

Ans. **6**

Sol. Radiation $= \frac{IA}{\left(\frac{hv}{\lambda}\right)}\left(\frac{h}{\lambda}\right)\frac{1}{A} = \frac{I}{v} = \frac{I}{C}\mu = \frac{6 \times 10^8 \times 3}{3\times10^8} = 6$

15. A ring and a solid sphere of same mass and radius are released from same point of inclined plane. Find the ratio of their KE when they reach to bottom without slipping

(1) 1 : 7 (2) 1 : 3 (3) 1 : 5 (4) 1 : 1

Ans. **(4)**

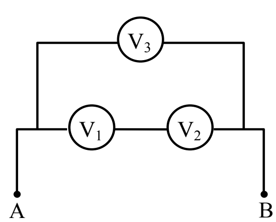
Sol. EC

$mgh = k_f - k_i$

$k_f = mgh$

so $KE_{Ring} = K.E._{solid\ sphere}$

16. Three voltmeters of different internal resistances are connected as shown in figure and a certain voltage is applied across AB. State which is true?



(1) $V_1 + V_2 > V_3$ (2) $V_1 + V_2 \neq V_3$ (3) $V_1 + V_2 = V_3$ (4) $V_1 + V_2 < V_3$

Ans. **(3)**

Sol. By series and parallel combination

$V_1 + V_2 = V_3$

17. Specific resistance S is given as $S = \frac{RA}{\ell}$. If length is doubled, find corresponding change in S.

(1) S is halved

(2) S is doubled

(3) S is quadrupled

(4) No change in S

Ans. (4)

18. **Assertion :** Static friction depends on area of contact but independent of material.

Reason : Kinetic friction is independent of area of contact but depends on material.

(1) Assertion true, reason true and reason is correct explanation of assertion.

(2) Assertion true, reason true and reason is not correct explanation of assertion.

(3) Assertion true, reason false.

(4) Assertion false, reason true.

Ans. **(4)**

19. **Assertion :** Work done by electrostatics force on an object when moved on equipotential surface is always zero.

Reason : Electric field lines falls perpendicular to the equipotential surface

(1) Assertion true, reason true and reason is correct explanation of assertion.

(2) Assertion true, reason true and reason is not correct explanation of assertion.

(3) Assertion true, reason false.

(4) Assertion false, reason true.

Ans. **(1)**

Sol. **Assertion is true and reason is true and correct explanation.**

20. A nucleus of C^{13} breaks into C^{12} and neutron. Find energy released.

Atomic mass of $C^{12} = 12.000$ u

$C^{13} = 13.013975$ u

$n = 1.008665$ u

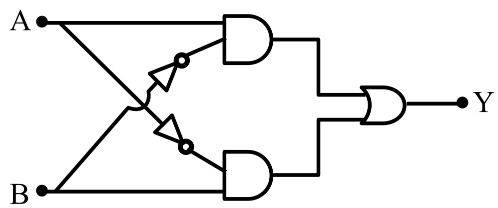
(1) 3.04 MeV (2) 4.1 MeV (3) 4.94 MeV (4) 6 MeV

Ans. **(3)**

Sol. Mass defect = 13.013975 – (12 + 1.008665) = 0.00531 U

Energy released = 0.00531 × 931 = 4.94 MeV

21. For given logic circuit. The truth table will be



(1)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(3)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Ans. **(4)**

Sol. Using Boolean algebra

$$Y = A\overline{B} + \overline{A}B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

22. In a transformer, ratio of turns in primary to secondary coil is 10 : 1. If primary side voltage is 230 volt and frequency is 50 Hz and resistance of secondary side is 46 Ω, then find power output.

(1) 11.5 W (2) 12 W (3) 12.5 W (4) 23 W

Ans. **(1)**

Sol. $\frac{N_1}{N_2} = \frac{V_1}{V_2}$

$\frac{10}{1} = \frac{230}{V_2}$

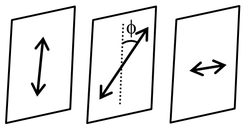
$V_2 = 23$ V

$P_2 = \frac{V_2^2}{R_2} = \frac{23 \times 23}{46} = \frac{23}{2} = 11.5\text{W}$

23. Between two polaroid placed in crossed position, a third polaroid is introduced. By what angle (in degree) the introduced polaroid placed should be rotated to get maximum intensity of the out coming light.

Ans. **45**

Sol.



$I = I_0 \cos^2\phi \sin^2\phi$

I_{max} at $\phi = 45°$

24. If their fundamental frequencies are sounded together, beat frequency is 7 Hz. Find velocity (in m/s) of sound in air?

Ans. **24**

Sol. $f_1 = \dfrac{v}{4\ell_1}$ $\qquad f_2 = \dfrac{v}{2\ell_2}$

$f_1 = \dfrac{v \times 100}{4 \times 150} \qquad f_2 = \dfrac{v \times 100}{2 \times 350}$

$f_1 = \dfrac{v}{6} \qquad f_2 = \dfrac{v}{7}$

$\dfrac{v}{6} - \dfrac{v}{7} = 7$

$\dfrac{v}{42} = 7, \qquad v = 42 \times 7$

$v = 294$ m/sec

25. For 200 μA current galvanometer deflects by π/3 radians. For what value of current, it will deflect by π/10 radians ?

Ans. **60**

Sol. $i \propto \theta$ (angle of deflection)

$\dfrac{i_1}{i_2} = \dfrac{\theta_1}{\theta_2}$

$\dfrac{200\mu A}{i_2} = \dfrac{\pi/3}{\pi/10} = \dfrac{10}{3}$

$60\ \mu A = i_2$

26. Two charges of magnitude –4 μC kept at (1, 0, 4) and another charge of +4 μC kept at (2, –1, 5) in the presence of external electric field E = 0.2 $\hat{i}$ V/cm. The torque on the system of charges is $8\sqrt{\alpha}\times10^{-5}$ N – m. Find α.

Ans. **2**

Sol. $\vec{\tau} = \vec{P}\times\vec{E}$

$$\vec{P} = P\hat{r} = 4\times10^{-6}\times\sqrt{3}\frac{(\hat{i}-\hat{j}+\hat{k})}{\sqrt{3}}$$

$$\vec{P} = 4\times10^{-6}(\hat{i}-\hat{j}+\hat{k})$$

$$\vec{E} = 0.2\times10^{2}\,\hat{i} = 20\,\hat{i}\text{ V/m}$$

$$\vec{\tau} = 4\times10^{-6}\times20[(\hat{i}-\hat{j}+\hat{k})\times\hat{i}]$$

$$\vec{\tau} = 8\times10^{-5}(\hat{k}+\hat{j})$$

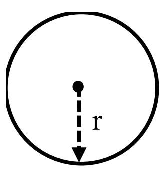
$$|\vec{\tau}| = 8\sqrt{2}\times10^{-5}\text{ Nm}$$

$$\alpha = 2$$

27. A nucleus with atomic number '50' and having radius of nucleus is 9 × 10^{-13} cm. Calculate the potential (in MV) at the surface of the nucleus.

Ans. **8**

Sol.



$$V_{surface} = \frac{kQ}{r} = \frac{9\times10^{9}\times50\times1.6\times10^{-19}}{9\times10^{-15}}$$

= 80 × 10^{5} volt

$V_{surface}$ = 8 MV

28. A pressure inside wall pipe before hole is 4.5 × 10^{4} N/m^{2}. When a small hole is made in pipe, pressure is changed to 2.0 × 10^{4} N/m^{2}. If speed of water flux after hole is $\sqrt{v}$ m/s. Find out v :

Ans. **50**

Sol. $\Delta P = \frac{1}{2}\rho v^{2}$

$$2.5\times10^{4} = \frac{1}{2}\times10^{3}v_0^{2}$$

$$v_0 = \sqrt{50}\text{ m/s}$$

$$v = 50$$

CHEMISTRY

1. For Ist order reaction, time required for 99.9% completion is :

(1) $2t_{1/2}$ (2) $4t_{1/2}$ (3) $5t_{1/2}$ (4) $10t_{1/2}$

Ans. **(4)**

Sol. $$\frac{t_{99.9\%}}{t_{1/2}} = \frac{\frac{1}{k}\ln\left(\frac{100}{100-99.9}\right)}{\frac{1}{k}\ln 2} = \frac{\ln(10^3)}{\ln 2} = \frac{3}{0.3} = 10$$

$t_{99.9\%} = 10t_{1/2}$

2. Number of non polar molecules among following are :

HF, H_2O, CO_2, NH_3, SO_2, H_2, CH_4, BF_3

Ans. **(4)**

Sol. CO_2, H_2, CH_4, BF_3

3. 3M NaOH solution is to be prepared using 84 g NaOH, then the volume of solution in litre is ____ $\times 10^{-1}$

Ans. **(7)**

Sol. $$3 = \frac{84/40}{V_{sol(L)}}$$

$\therefore V_{solution} = 0.7$ L

4. Select **incorrect** match :

(1) Haber process : Fe

(2) Polythene : Ziegler-Natta catalyst [$Al_2(CH_3)_6 + TiCl_4$]

(3) Wacker's process : $PtCl_2$

(4) Photography : AgBr

Ans. **(3)**

Sol. Wacker's process : $PdCl_2$

5. 1 mole PbS is oxidised by x mole O_3 liberating y mole O_2.

Determine (x + y).

Ans. (8)

Sol. $PbS + 4O_3 \longrightarrow PbSO_4 + 4O_2$

x = 4 ; y = 4

6. Spin only magnetic moment of $[Pt(NH_3)_2Cl(CH_3NH_2)]Cl$ is :

Ans. (0)

Sol. Pt^{+2} : $5d^8 \Rightarrow dsp^2$ & unpaired $e^- = 0 \Rightarrow$ Magnetic moment = 0

7. **S-1:** Formation of Ce^{4+} is favoured by inert gas configuration.

S-2: Ce^{4+} acts as strong oxidising agent & converts to Ce^{3+}.

Ans. Both S-1 & S-2 are correct.

8. Which of the following can't act as oxidising agent ?

(1) MnO_4^- (2) N^{3-} (3) BrO_3^- (4) SO_4^{2-}

Ans. (2)

Sol. In N^{-3}, nitrogen is present in minimum O.N. & hence it cannot act as oxidising agent.

9. The quantity which changes with temperature is:

(1) Molarity (2) Molality (3) Mole fraction (4) Mass %

Ans. (1)

Sol. Quantities involving volume are temperature dependent.

10. Reduction potential of hydrogen electrode at pH = 3 is.........

$$\left(\frac{2.303RT}{F} = 0.059\right)$$

Ans. (–0.177 volt)

Sol. $H^+ (aq) + e^- \longrightarrow \frac{1}{2} H_2(g)$

$$R.P. = -\frac{0.059}{1}\log\left(\frac{1}{H^+}\right) = -0.059\ \log(10^{+3})$$

$$= -0.059 \times 3 = -0.177 \text{ volt}$$

11. Identify the species in which central atom is in d^2sp^3 hybridisation :

(1) SF_6 (2) BrF_5 (3) $[PtCl_4]^{2-}$ (4) $[Co(NH_3)_6]^{3+}$

Ans. **(4)**

Sol.

SF_6	:	sp^3d^2
BrF_5	:	sp^3d^2
$[PtCl_4]^{2-}$	:	dsp^2
$[Co(NH_3)_6]^{3+}$	:	d^2sp^3

12. $\Delta H° = +77.2$ kJ, $\Delta S° = 122$ J/mol-K, T = 300 K, logK = ?

Ans. **(–7.07)**

Sol. $\Delta G° = -2.303RT\log k$

$$77.2 - \frac{300 \times 122}{1000} = \frac{-2.303 \times 8.314 \times 300 \log K}{1000}$$

$\therefore$ logK = –7.07

13. In group 16

Statement-I : Oxygen shows only –2 oxidation state.

Statement-II : On moving top to bottom, stability of +4 oxidation state decreases, whereas that of +6 oxidation state increases.

(1) Both Statement I and Statement II are correct.

(2) Both Statement I and Statement II are incorrect.

(3) Statement I is correct but Statement II is incorrect.

(4) Statement I is incorrect but Statement II is correct.

Ans. **(2)**

Sol. **Statement-I :** Since electronegativity of oxygen is very high, it shows only negative oxidation state as –2 except in the case of OF_2 where its oxidation state is + 2.

Statement-II : The stability of + 6 oxidation state decreases down the group and stability of + 4 oxidation state increases (inert pair effect).

14. How many of following has/have noble gas configuration ?

Sr^{2+}, Cs^{+}, Yb^{+2}, La^{2+}

Ans. **(2)**

Sol. (Sr^{2+}, Cs^{+})

15. Which of the following has d^{10} configuration ?

(1) Cr, Cd, Cu, Ag (2) Cd, Cr, Ag, Zn

(3) Ag, Cr, Cu, Zn (4) Cu, Cd, Zn, Ag

Ans. **(4)**

Sol. Cr : [Ar] $3d^{5}$ $4s^{1}$

Cu : [Ar] $3d^{10}$ $4s^{1}$

Ag : [Kr] $4d^{10}$ $5s^{1}$

Zn : [Ar] $3d^{10}$ $4s^{2}$

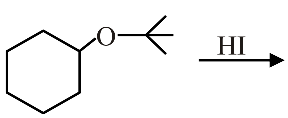
Cd : [Kr] $4d^{10}$ $5s^{2}$

16. Which of the following is used to identify the phenolic group test?

(1) Carbylamine test (2) Lucas test

(3) Tollen's test (4) Phthalein dye test

Ans. **(4)**

17. 

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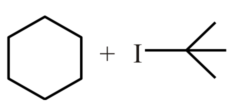
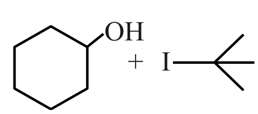
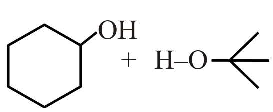
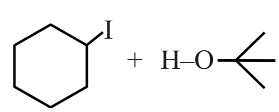
(1) 

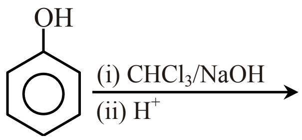
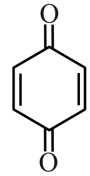
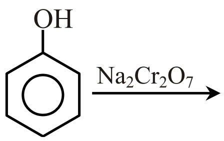
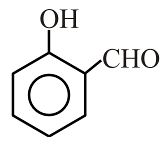
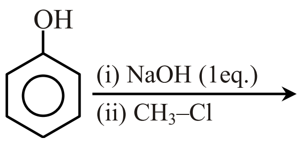
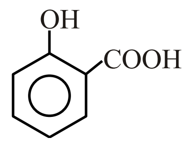
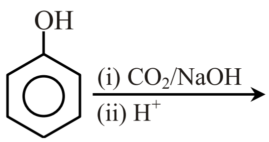
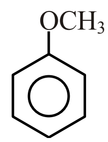
(2) 

(3) 

(4) 

Ans. **(2)**

18. Match the column

(P) 	(1) 
(Q) 	(2) 
(R) 	(3) 
(S) 	(4) 

Ans. (P) – (2) ; (Q) – (1) ; (R) – (4) ; (S) – (3)

19. When egg is boiled then which of the following structure of protein remains intact?

(1) Quaternary structure (2) Primary structure

(3) Secondary structure (4) Tertiary structure

Ans. (2)

20. Which of the following compound will not give S_N1 reaction?

(1) $CH_2=CH–CH_2Cl$ (2) $Ph–CH_2–Cl$

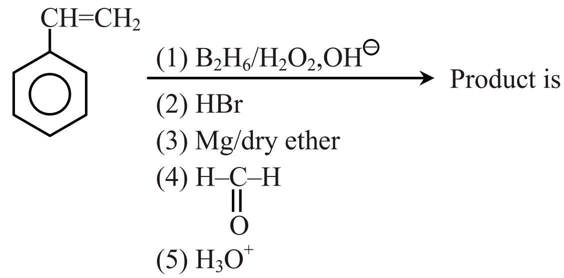
(3) $\begin{matrix} H_3C \\ H_3C \end{matrix}\!\!>\!CH–Cl$ (4) $CH_3–CH=CH–Cl$

Ans. (4)

21. The second homologue of monocarboxylic acid is

(1) HCOOH (2) CH_3COOH (3) CH_3CH_2COOH (4) $CH_3CH_2CH_2–COOH$

Ans. (2)

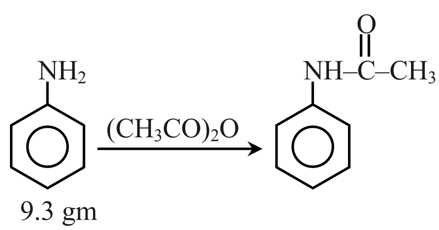
22. 

(1) $Ph-\underset{\displaystyle |}{CH}-CH_3$ with CH_2-OH on the CH   (2) $Ph-CH_2-CH_2-CH_2-OH$

(3) $Ph-CH_2-CH_2-O-CH_3$ (4) $Ph-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_2-CH_3$

Ans. (1)

23. When 9.3 gm of aniline in reacted with acetic anhydride then mass of acetanilide formed is [X] gm. Report your answer as 10X.

Sol. 

Mole of Aniline = $\frac{9.3}{93} = 0.1$

Mole of acetanilide = 0.1

Mass of acetanilide = 0.1 × 135 = 13.5 gm

10x = 13.5 × 10 = 135 gm

24. The correct stability order of following resonating structures is

(I) $CH_2=CH-CH=O$ (II) $\overset{\oplus}{C}H_2-CH=\overset{\overset{\displaystyle O^{\ominus}}{|}}{C}-H$ (III) $\overset{\ominus}{C}H_2-CH=\overset{\overset{\displaystyle O^{\oplus}}{|}}{C}-H$

(1) II > III > I   (2) I > II > III   (3) I > III > II   (4) III > II > I

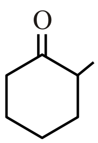
Ans. (2)

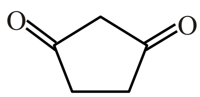
25. Steam volatile and water immiscible substances are separated by

(1) Steam distillation
(2) Fractional distillation under reduced pressure
(3) Fractional distillation
(4) Distillation.

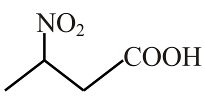
Ans. (1)

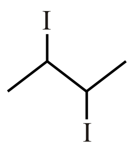
26. How many of the following compounds contain chiral centre ?

(I) 

(II) 

(III) $CH_3-CH_2-\underset{\displaystyle OH}{\underset{|}{CH}}-COOH$

(IV) 

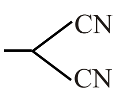
(V) 

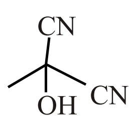
(VI) $CH_3-CH_2-\underset{\displaystyle OH}{\underset{|}{CH}}-C_2H_5$

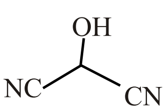
Ans. 4 (I, III, IV, V)

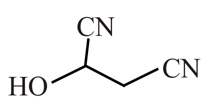
27. The bond line representation of following compound is

$CH(OH)(CN)_2$

(1) 

(2) 

(3) 

(4) 

Ans. (3)

MATHEMATICS

1. An urn contains 6 black and 9 red balls. Four balls are drawn from the urn twice without replacement. The probability that first four balls are black & 2^{nd} four balls are red in colour is:

(1) $\frac{3}{765}$ (2) $\frac{6}{715}$ (3) $\frac{3}{715}$ (4) $\frac{6}{615}$

Ans. **(3)**

Sol. $\frac{{}^6C_4}{{}^{15}C_4} \times \frac{{}^9C_4}{{}^{11}C_4} = \frac{3}{715}$

2. A line x + y = 0 touches the circle $(x - \alpha)^2 + (y - \beta)^2 = 50$, α, β, > 0. The distance of origin from its points of contact is $4\sqrt{2}$. Find $\alpha^2 + \beta^2$.

Ans. 82

Sol. Point of contact is $(0 + 4\sqrt{2}\cos 135^\circ, 0 + 4\sqrt{2}\ \sin 135^\circ) = (-4, 4)$

$\left|\frac{\alpha+\beta}{\sqrt{2}}\right| = 5\sqrt{2}$ $\alpha + \beta = 10$ ….(i)

$(-4 - \alpha)^2 + (4 - \beta)^2 = 50$

$(\alpha + 4)^2 + (4 - 10 + \alpha)^2 = 50$

$(\alpha + 4)^2 + (\alpha - 6)^2 = 50$

$\alpha = 1, \beta = 9$

$\alpha^2 + \beta^2 = 82$

Also point of contact is (4, –4)

Satisfying this point of contact in the equation of circle we get

$(4 - \alpha)^2 + (-4 - \beta)^2 = 50$

$(4 - \alpha)^2 + (\beta + 4)^2 = 50$

$(4 - \alpha)^2 + (14 - \alpha)^2 = 50$

$\Rightarrow \alpha = 9, \beta = 1$

$\alpha^2 + \beta^2 = 82$

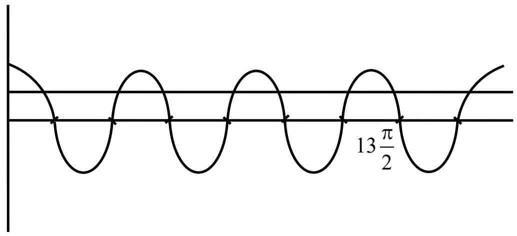
3. Let $2\tan^2x - 5\sec x - 1 = 0$ has 7 solutions in $x \in \left[0, \frac{n\pi}{2}\right]$, then the minimum value of n is N find

$\sum_{k=1}^{N} \frac{k}{2^k}$

(1) $2.\left(\frac{2^{13}-1}{2^{13}}\right) - \frac{13}{2^{13}}$ (2) $\left(\frac{2^{13}-1}{2^{13}}\right) - \frac{13}{2^{13}}$

(3) $2.\left(\frac{2^{13}-1}{2^{13}}\right) + \frac{13}{2^{14}}$ (4) $2.\left(\frac{2^{13}-1}{2^{13}}\right) - \frac{13}{2^{14}}$

Ans. **(1)**

Sol. 

$2\tan^2 x - 5\sec x - 1 = 0$

$2\sec^2 x - 5\sec x - 3 = 0$

$2\sec^2 x - 6\sec x + \sec x - 3 = 0$

$(2\sec x + 1)(\sec x - 3) = 0$

$\sec x = 3, -\frac{1}{2}$

$\Rightarrow \sec x = 3$

$\Rightarrow \cos x = \frac{1}{3}$

For 7 solutions, n = 13 = N

so $\sum_{k=1}^{13} \frac{k}{2^k}$

let $S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \ldots\ldots + \frac{13}{2^{13}}$

$\frac{1}{2} S = \frac{1}{2^2} + \frac{2}{2^3} + \ldots\ldots.. + \frac{13}{2^{14}}$

$\frac{1}{2} S = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots. \frac{1}{2^{13}} - \frac{13}{2^{14}}$

$$\frac{1}{2} S = \frac{1}{2} . \frac{\left(1 - \frac{1}{2^{13}}\right)}{1 - \frac{1}{2}} - \frac{13}{2^{14}}$$

$$S = 2.\left(\frac{2^{13} - 1}{2^{13}}\right) - \frac{13}{2^{13}}$$

4. The vertices of a triangle are A(1, 2, 2), B(2, 1, 2) & C(2, 2, 1). The perpendicular distance of its orthocentre from the given sides are ℓ_1, ℓ_2 & ℓ_3. Find the value of $\ell_1^2 + \ell_2^2 + \ell_3^2$.

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Ans. (2)

Sol. ΔABC is equilateral

$\therefore$ orthocentre & centroid will be same $\left(\frac{5}{3},\frac{5}{3},\frac{5}{3}\right)$

midpoint of AB is $\left(\frac{3}{2},\frac{3}{2},2\right)$

$\Rightarrow \ell_1 = \sqrt{\frac{1}{36}+\frac{1}{36}+\frac{1}{9}}$

$= \frac{1}{\sqrt{6}} = \ell_2 = \ell_3$

5. Let two sets A and B having 'm' & 'n' elements respectively such that difference of the number of subsets of A and that of B is 56, then (m, n) is

(1) (8, 3) (2) (8, 5) (3) (6, 3) (4) (7, 4)

Ans. **(3)**

Sol. $2^m - 2^n = 56; m > n$

$\Rightarrow 2^n(2^{m-n} - 1) = 8\,(2^3 - 1)$

$\Rightarrow m = 6, n = 3$

6. If 'A' is a square matrix of order '2' such that roots of the equation $\det(A - \lambda I) = 0$ are 1 and –3, then sum of diagonal elements of matrix 'A^2' is

(1) 2 (2) –3 (3) 9 (4) 10

Ans. **(4)**

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore |A - \lambda I| = \begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^2 - (a + d)\lambda + ad - bc = 0$

Sum of the roots = a + d = 2

Product of roots = ad – bc = –3

Now $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & b(a+d) \\ c(a+d) & d^2+bc \end{bmatrix}$

$\therefore tr(A^2) = a^2 + d^2 + 2bc = (a + d)^2 - 2(ad - bc) = 4 + 6 = 10$

7. Let $\tan^{-1}x + \tan^{-1}2x = \frac{\pi}{4}$; x > 0, then number of positive values of x is/are

(1) 0 (2) 1 (3) 2 (4) 3

Ans. **(2)**

Sol. $\tan^{-1}\left(\dfrac{x+2x}{1-2x^2}\right) = \dfrac{\pi}{4}$

$\Rightarrow \dfrac{3x}{1-2x^2} = 1 \qquad \Rightarrow x = \dfrac{-3 \pm \sqrt{17}}{4}$

$\therefore x = \dfrac{\sqrt{17}-3}{4}$

8. Find the coefficient of x^{2012} in $(1-x)^{2008}.(1+x+x^2)^{2007}$

Ans. **(0)**

Sol. Coefficient of x^{2012} in $(1-x^3)^{2007}.(1-x)$

Coefficient of x^{2012} in $(1-x^3)^{2007} - x(1-x^3)^{2007}$

Coefficient of x^{2012} in ${}^{2007}C_{r_1}\left(-x^3\right)^{r_1} + {}^{2007}C_{r_2}\left(-1\right)^{r_2} x^{3r_2+1}$

$3r_1 = 2012$ Which is not possible for any $r_1 \in w$

and $3r_2 + 1 = 2012$ also not possible for any $r_2 \in w$

$\therefore$ no term containing x^{2012}

$\therefore$ Coefficient of x^{2012} is 0

9. An ellipse is passing through focii of hyperbola $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$ and product of their eccentricities is 1, then the length of chord of ellipse passing through (0, 2) and parallel to x-axis is

(1) $\dfrac{5\sqrt{5}}{3}$ (2) $\dfrac{3}{5\sqrt{5}}$ (3) $\dfrac{10\sqrt{5}}{3}$ (4) $\dfrac{20\sqrt{5}}{3}$

Ans. **(3)**

Sol. $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1 \qquad \Rightarrow e_H = \dfrac{5}{4} \qquad \Rightarrow e_E = \dfrac{4}{5}$

Ellipse is passing through (±5, 0)

$\therefore$ ellipse : $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

and points of chord : $\left(\pm\dfrac{5\sqrt{5}}{3}, 2\right)$

$\therefore$ Length of chord = $\dfrac{10\sqrt{5}}{3}$

10. If α and $\dfrac{1}{\alpha}$ are two complex numbers which satisfy the equations $|z - z_0|^2 = 4$ and $|z - z_0|^2 = 16$ respectively, where $z_0 = 1 + i$, then the value of $5|\alpha|^2$ is

Ans. (1)

Sol. $|\alpha - z_0|^2 = 4$

$\Rightarrow (\alpha - z_0)\ (\bar{\alpha} - \bar{z}_0) = 4$

$|\alpha|^2 - \alpha\bar{z}_0 - \bar{\alpha}z_0 + |z_0|^2 = 4$

$|\alpha|^2 - \alpha\bar{z}_0 - \bar{\alpha}z_0 = 2$ …(i)

(ii) $\left|\frac{1}{\bar{\alpha}} - z_0\right|^2 = 16 \Rightarrow (1 - \bar{\alpha}\, z_0)\ (1 - \alpha\,\bar{z}_0) = 16|\alpha|^2$

$\Rightarrow 1 - \alpha\bar{z}_0 - \bar{\alpha}z_0 + |\alpha|^2 . 2 = 16|\alpha|^2$ ….(ii)

from (i) & (ii) $- 1 - |\alpha|^2 = 2 - 16\,|\alpha|^2 \Rightarrow 15|\alpha|^2 = 3 \Rightarrow 5|\alpha|^2 = 1$

11. Let $x^2 - x - 1 = 0$ has roots α and β such that $S_n = 2023\ \alpha^n + 2024\ \beta^n$, then

(1) $S_{12} = S_{11} - S_{10}$ (2) $S_{12} = S_{10} - S_{11}$

(3) $S_{12} = S_{10} + S_{11}$ (4) $S_{12} = -S_{10} - S_{11}$

Ans. **(3)**

Sol. $S_n = 2023\ \alpha^n + 2024\ \beta^n$

$\Rightarrow S_n - S_{n-1} - S_{n-2} = 0$

$\Rightarrow S_{12} = S_{11} + S_{10}$

12. For the series 20, $19\frac{1}{4}$, $18\frac{1}{2}$, ……., $-129\frac{1}{4}$, the 20th term from end is

(1) –115 (2) – 119 (3) –117 (4) –120

Ans. **(1)**

Sol. T_{20} for $a = -129\frac{1}{4} = -\frac{517}{4}$, $d = \frac{3}{4}$

$T_{20} = -\frac{517}{4} + 19 . \frac{3}{4} = -\frac{460}{4} = -115$

13. $\int \frac{x^8 - x^2}{\left(x^{12} + 3x^6 + 1\right)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}\,dx =$

(1) $\frac{1}{3}\ \ell n \tan^{-1}\left(x^3 + \frac{1}{x^3}\right) + C$ (2) $\ell n \tan^{-1}\left(x^3 + \frac{1}{x^3}\right) + C$

(3) $\tan^{-1}\left(x^3 + \frac{1}{x^3}\right) + C$ (4) None of these

Ans. **(1)**

Sol. $\int \frac{x^8 - x^2}{(x^{12} + 3x^6 + 1)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)} dx =$

Let $\tan^{-1}\left(x^3 + \frac{1}{x^3}\right) = t$

$$\frac{1}{1 + \left(x^3 + \frac{1}{x^3}\right)^2}\left(3x^2 - \frac{3}{x^4}\right) dx = dt$$

$$\frac{x^6}{x^{12} + 2x^6 + 1 + x^6} \times \frac{3x^6 - 3}{x^4} \; dx \; = dt$$

$$\frac{1}{3}\left|\frac{1}{t} dt = \frac{1}{3}\ell n(t) + C\right.$$

$$= \frac{1}{3}\ell n \tan^{-1}\left(x^3 + \frac{1}{x^3}\right) + C$$

14. If $\lim\limits_{x \to 0} \frac{\alpha \sin x + \beta \cos x + \ln(1-x) + 3}{3\tan^2 x} = \frac{1}{3}$ find $2\alpha - \beta$

(1) 10 (2) 7 (3) 8 (4) 5

Ans. **(4)**

Sol. $\lim\limits_{x \to 0} \frac{\alpha \sin x + \beta \cos x + \ln(1-x) + 3}{\frac{3\tan^2 x}{x^2}} = \frac{1}{3}$

$\beta + 3 = 0 \Rightarrow \beta = -3$

$$\lim_{x \to 0} \frac{\alpha \cos x - \beta \sin x - \frac{1}{1-x}}{2x}$$

$\alpha - 1 = 0 \Rightarrow \alpha = 1$

so $2\alpha - \beta = 5$

15. Let $a_1, a_2, a_3 \ldots\ldots. a_{15}$ are 15 observations having mean and variance as 12 & 9 respectively. One of the observation which was 12, misread as 10. The correct mean and variance are μ and σ^2 respectively, then $15(\mu + \mu^2 + \sigma^2)$

(1) 2521 (2) 2522 (3) 2518 (4) 2621

Ans. **(1)**

Sol. old mean $12 = \frac{\Sigma x_i}{n} \Rightarrow 12 = \frac{a_1 + a_2 + ...a_{14} + 10}{15}$

$$\sum_{i=1}^{14} a_i = 170$$

old variance = 9 $\Rightarrow 9 + (12)^2 = \frac{a_1^2 + a_2^2 + ...+ a_{14}^2 + 10^2}{15}$

$$\sum_{i=1}^{14} a_i^2 = 2195$$

new mean (μ) = $\frac{\sum_{i=1}^{14} a_i + 12}{15} = \frac{170 + 12}{15} = \frac{182}{15}$

new variance (σ^2)

$\sigma^2 + \mu^2 = \frac{\sum_{i=1}^{14} a_i^2 + 12^2}{15} = \frac{2339}{15}$

$\sigma^2 + \mu^2 + \mu = \frac{2339}{15} + \frac{182}{15} = \frac{2521}{15}$

$15(\sigma^2 + \mu^2 + \mu) = 2521$

16. Values of α for which $\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$ lies in the interval

(1) (0, 3) (2) (–3, 0) (3) (–2, 1) (4) (–2, 0)

Ans. **(2)**

Sol. $C_3 \to C_3 - (\alpha C_1 + C_2)$

$$\begin{vmatrix} 1 & \frac{3}{2} & 0 \\ 1 & \frac{1}{3} & 0 \\ 2\alpha + 3 & 3\alpha + 1 & -(2\alpha^2 + 6\alpha + 1) \end{vmatrix} = 0$$

$\Rightarrow \frac{7}{6}(2\alpha^2 + 6\alpha + 1) = 0$

$\Rightarrow \alpha = \frac{-3 + \sqrt{7}}{2}, \frac{-3 - \sqrt{7}}{2}$

17. Let $g(x) = 3f\left(\frac{x}{3}\right) + f(3 - x)$, where $f''(x) > 0$ and $x \in (0, 3)$, $g(x)$ is decreasing in $x \in (0, \alpha)$ and increasing in $(\alpha, 3)$, then 8α is

Ans. **(18)**

Sol. $g'(x) = 3 \cdot \frac{1}{3} f'\left(\frac{x}{3}\right) - f'(3 - x) = f'\left(\frac{x}{3}\right) - f'(3 - x)$

$g(x)$ is decreasing $g'(x) < 0$

$f'\left(\frac{x}{3}\right) < f'(3 - x)$

$\because f''(x) > 0 \Rightarrow f'(x)$ is increasing

$\frac{x}{3} < 3 - x$

$\frac{4x}{3} < 3 \Rightarrow x < \frac{9}{4}$

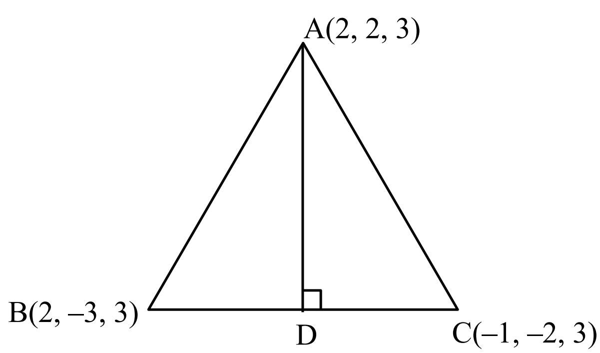
$\alpha = \frac{9}{4}$

$\therefore 8\alpha = 18$

18. Let ΔABC have vertices $A(2, 2, 3)$, $B(2, -3, 3)$ $C(-1, -2, 3)$ and length of internal angle bisector of angle A is ℓ, then the value of $2\ell^2$ is

Ans. **(45)**

Sol.



$\overrightarrow{AB} = -5j$

$\overrightarrow{AC} = -3i - 4j$

$\left|\overrightarrow{AB}\right| = \left|\overrightarrow{AC}\right|$

$D\left(\frac{-1+2}{2}, \frac{-2-3}{2}, \frac{3+3}{2}\right)$

$D\left(\frac{1}{2}, \frac{-5}{2}, 3\right)$

$AD = \sqrt{\left(\frac{1}{2}-2\right)^2 + \left(2+\frac{5}{2}\right)^2 + (3-3)^2}$

$\ell = \sqrt{\frac{9}{4}+\frac{81}{4}}$

$2\ell^2 = 45$

$= \sqrt{\frac{90}{4}} = \sqrt{\frac{45}{2}}$

19. Let $S_1 = \frac{\lfloor 4!}{(4!)^{3!}}$ and $S_2 = \frac{\lfloor 5!}{(5!)^{4!}}$, then

(1) $S_1 \in N$ and $S_2 \notin N$

(2) $S_1 \in N$ and $S_2 \in N$

(3) $S_1 \notin N$ and $S_2 \in N$

(2) $S_1 \notin N$ and $S_2 \notin N$

Ans. **(2)**

Sol. Make 6 groups of 4 each

$24 \rightarrow (4, 4, 4, 4, 4, 4, 4)$

Number of ways of making groups $= \frac{24!}{(4!)^6 . 6!} = I_1$

$\frac{(24)!}{(4!)^6} = \frac{\lfloor 4!}{(4!)^{3!}} = (6! I_1)$

$S_1 \in N$

$(5!) \rightarrow (5, 5, 5, 5 \ldots\ldots, 5(24 \text{ times}))$

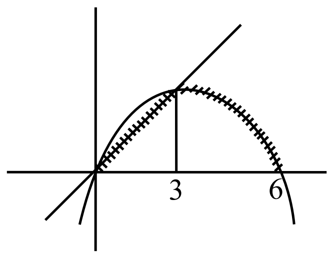
$\frac{\lfloor 5!}{(5!)^{24} . 24!} = I_2 \Rightarrow S_2 = (24!)\, I_2$

Hence $S_2 \in N$

20. Let area bounded by $y = \min.(3x, 6x - x^2)$; $y \geq 0$ is A, then 2A is

Ans. **(63)**

Sol.



$2x = 6x - x^2$

$A = \frac{1}{2} \times 3 \times 9 + \int_3^6 \sqrt{6x - x^2}\,dx$

$A = \frac{27}{2} + \int_3^6 \sqrt{9 - (x-3)^2}\,dx$

$A = \frac{27}{2} + \left(\frac{x-3}{2}\sqrt{9-(x-3)^2} + \frac{9}{2}\sin^{-1}\left(\frac{x-3}{3}\right) \right)_4^6$

$A = \frac{27}{2} + \frac{9\pi}{4}$

$12A = 162 + 27\pi$

21. Let $(x^2 - 4)dy = y(y-3)dx$ satisfying $y(4) = \frac{3}{2}$ then $y(10)$ is equal to

(1) $\frac{3}{1-8^{\frac{1}{4}}}$ (2) $\frac{3}{1+8^{\frac{1}{4}}}$ (3) $\frac{3}{1+2^{\frac{1}{4}}}$ (4) $\frac{3}{1-2^{\frac{1}{4}}}$

Ans. **(2)**

Sol. $= \frac{1}{3}\int \frac{y-(y-3)}{y(y-3)}dy = \frac{1}{4}\int \frac{(x+2)-(x-2)}{(x+2)(x-2)}dx$

$\frac{1}{3}(\ell n|y-3| - \ell n|y|) = \frac{1}{4}(\ell n|x-2| - \ell n|x+2|) + C$

$\frac{1}{3}\ell n\left|\frac{y-3}{y}\right| = \frac{1}{4}\left(\ell n\left|\frac{x-2}{x+2}\right|\right) + C$

$\frac{1}{3}\ell n\left|\frac{\frac{3}{2}-3}{\frac{3}{2}}\right| = \frac{1}{4}\ell n\left(\frac{4-2}{4+2}\right) + C$

$C = \frac{1}{4}\ell n 3$

$$\frac{1}{3}\ell n\left|\frac{y-3}{y}\right| = \frac{1}{4}\ell n\left|\frac{x-2}{x+2}\right| + \frac{1}{4}\ell n 3$$

$\Rightarrow x = 10$

$$\frac{1}{3}\ell n\left|\frac{y-3}{y}\right| = \frac{1}{4}\ell n\left|\frac{2}{3}\right| + \frac{1}{4}\ell n 3$$

$$\frac{1}{3}\ell n\left|\frac{y-3}{y}\right| = \frac{1}{4}\ell n 2$$

$$\ell n\left|\frac{y-3}{y}\right| = \ell n 2^{\frac{3}{4}}$$

$$\left|\frac{y-3}{y}\right| = 2^{\frac{3}{4}}$$

$$-y + 3 = 8^{\frac{1}{4}} y$$

$$y = \frac{3}{1+8^{\frac{1}{4}}}$$

22. Three lines $2x - y - 3 = 0$, $6x + 3y + 4 = 0$, $\alpha x + 2y + 4 = 0$ does not form triangle then find $[\Sigma\alpha^2]$ (where [.] denotes the greatest integer function)

Ans. **(32)**

Sol. If two lines are parallel

$$\frac{2}{\alpha} = \frac{-1}{2} \Rightarrow \alpha = -4$$

$$\frac{6}{\alpha} = \frac{3}{2} \Rightarrow \alpha = 4$$

If lines are concurrent

$$\begin{vmatrix} 2 & -1 & -3 \\ 6 & 3 & 4 \\ \alpha & 2 & 4 \end{vmatrix} = 0$$

$2(12 - 8) + 1(24 - 4\alpha) - 3(12 - 3\alpha) = 0$

$8 + 24 - 4\alpha - 36 + 9\alpha = 0$

$5\alpha = 4 \Rightarrow \alpha = \dfrac{4}{5}$

$\Sigma\alpha^2 = 16 + 16 + \dfrac{16}{25}$

$[\Sigma\alpha^2] = 32$

23. Let $f(x) = \int_0^x g(t)\ \log\left(\frac{1-t}{1+t}\right) dt$, (where g(x) is cont. odd function).

If $\int_{-\frac{\pi}{2}}^{\pi/2}\left(f(x)+\frac{x^2\cos x}{1+e^x}\right)dx\ = \left(\frac{\pi}{\alpha}\right)^2 - \alpha$, then find α

Ans. $\alpha = 2$

Sol. $I = \int_0^{\pi/2}\left(f(x)+f(-x)+\frac{x^2\cos x}{1+e^x}+\frac{x^2\cos x}{1+e^{-x}}\right) dx = \int_0^{\pi/2}(f(x) + f(-x) + x^2\cos x)\ dx$ ….(i)

Now $f(-x) = \int_0^{-x} g(t)\ \log\left(\frac{1-t}{1+t}\right) dt$

$t = -p$

$= \int_0^x -g(-p)\log\left(\frac{1+p}{1-p}\right) dp = -f(x)$

$\therefore$ (i) becomes $I = \int_0^{\pi/2} x^2\cos x\ dx = x^2\sin x - 2\int x\ \sin x\ dx = (x^2\sin x - 2)\ (-x\cos x + \sin x))_0^{\pi/2}$

$= \frac{\pi^2}{4} - 2 \Rightarrow \frac{\pi^2}{4} - 2$