Statistics

Question1

Let $a_1, a_2, \ldots a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k \cdot a_j = 1100.$ Then the standard deviation of $a_1, a_2, \ldots, a_{10}$ is equal to :

[27-Jan-2024 Shift 1]

Options:

A.

5

B.

√5

C.

10

D.

√115

Answer: B

Solution:

$\sum_{k=1}^{10} a_k = 50$

$a_1 + a_2 + \ldots + a_{10} = 50$ ....(i)

$\sum_{\forall k<j} a_k a_j = 1100$ ....(ii)

If $a_1 + a_2 + \ldots + a_{10} = 50$.

$(a_1 + a_2 + \ldots + a_{10})^2 = 2500$

$\Rightarrow \sum_{i=1}^{10} a_i^2 + 2\sum_{k<j} a_k a_j = 2500$

$\Rightarrow \sum_{i=1}^{10} a_i^2 = 2500 - 2(1100)$

$\sum_{i=1}^{10} a_i^2 = 300$, Standard deviation ' σ '

$= \sqrt{\frac{\sum a_i^2}{10} - \left(\frac{\sum a_i}{10}\right)^2} = \sqrt{\frac{300}{10} - \left(\frac{50}{10}\right)^2}$

$= \sqrt{30 - 25} = \sqrt{5}$

Question2

The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking it was found that an observation was read as 10 in place of 12 . If μ and σ^2 denote the mean and variance of the correct observations respectively, then $15(\mu + \mu^2 + \sigma^2)$ is equal to___

[27-Jan-2024 Shift 2]

Answer: 2521

Solution:

Let the incorrect mean be μ' and standard deviation be σ'

We have

$$\mu' = \frac{\Sigma x_i}{15} = 12 \Rightarrow \Sigma x_i = 180$$

As per given information correct $\Sigma x_i = 180 - 10 + 12$

$$\Rightarrow \mu(\text{ correct mean }) = \frac{182}{15}$$

Also

$$\sigma' = \sqrt{\frac{\Sigma x_i^2}{15} - 144} = 3 \Rightarrow \Sigma x_i^2 = 2295$$

Correct $\Sigma x_i^2 = 2295 - 100 + 144 = 2339$

$$\sigma^2(\text{ correct variance }) = \frac{2339}{15} - \frac{182 \times 182}{15 \times 15}$$

Required value

$$= 15(\mu + \mu^2 + \sigma^2)$$

$$= 15\left(\frac{182}{15} + \frac{182 \times 182}{15 \times 15} + \frac{2339}{15} - \frac{182 \times 182}{15 \times 15}\right)$$

$$= 15\left(\frac{182}{15} + \frac{2339}{15}\right)$$

$$= 2521$$

Question3

If the mean and variance of the data 65, 68, 58, 44, 48, 45, 60, α, β, 60 where α > β are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to

[29-Jan-2024 Shift 1]

Answer: 6344

Solution:

$\bar{x} = 56$

$\sigma^2 = 66.2$

$\Rightarrow \dfrac{\alpha^2 + \beta^2 + 25678}{10} - (56)^2 = 66.2$

$\therefore \alpha^2 + \beta^2 = 6344$

Question4

If the mean and variance of five observations are 24/5 and 194/25 respectively and the mean of first four observations is 7/2, then the variance of the first four observations in equal to

[29-Jan-2024 Shift 2]

Options:

A.

4/5

B.

77/12

C.

5/4

D.

105/4

Answer: C

Solution:

$\overline{X} = \frac{24}{5}; \sigma^2 = \frac{194}{25}$

Let first four observation be x_1, x_2, x_3, x_4

Here, $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{24}{5}$.... (1)

Also, $\frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{7}{2}$

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 14$

Now from eqn -1

$x_5 = 10$

Now, $\sigma^2 = \frac{194}{25}$

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \frac{576}{25} = \frac{194}{25}$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 54$$

Now, variance of first 4 observations

$$\text{Var} = \frac{\sum_{i=1}^{4} x_i^2}{4} - \left(\frac{\sum_{i=1}^{4} x_i}{4} \right)^2$$

$$= \frac{54}{4} - \frac{49}{4} = \frac{5}{4}$$

Question5

Let M denote the median of the following frequency distribution.

Class	0−4	4−8	8−12	12−16	16−20
Frequency	3	9	10	8	6

Then 20M is equal to :

[30-Jan-2024 Shift 1]

Options:

A.

416

B.

104

C.

52

D.

208

Answer: D

Solution:

Class	Frequency	Cumulativefrequency
0-4	3	3
4-8	9	12
8-12	10	22
12-16	8	30
16-20	6	36

$$M = 1 + \left(\frac{\frac{N}{2} - C}{f} \right) h$$

$$M = 8 + \frac{18 - 12}{10} \times 4$$

$$M = 10.4$$

$$20M = 208$$

Question6

The variance σ^2 of the data

xi	0	1	5	6	10	12	17
fi	3	2	3	2	6	3	3

Is

[30-Jan-2024 Shift 2]

Answer: 29

Solution:

x_i	f_i	$f_i x_i$	$f_i x_i^2$
0	3	0	0
1	2	2	2
5	3	15	75
6	2	12	72
10	6	60	600
12	3	36	432
17	3	51	867
	$\sum f_i = 22$		$\sum f_i x_i^2 = 2048$

$\therefore \ \Sigma f_i x_i = 176$

So $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{176}{22} = 8$

for $\sigma^2 = \frac{1}{N}\sum f_i x_i^2 - (\bar{x})^2$

$= \frac{1}{22} \times 2048 - (8)^2$

$= 93.090964$

$= 29.0909$

Question7

Let the mean and the variance of 6 observation a, b, 68, 44, 48, 60 be 55 and 194 , respectively if a > b, then a + 3b is

[31-Jan-2024 Shift 2]

Options:

A.

200

B.

190

C.

180

D.

210

Answer: C

Solution:

a, b, 68, 44, 48, 60

Mean $= 55 \quad a > b$

Variance $= 194 \quad a + 3b$

$$\frac{a+b+68+44+48+60}{6} = 55$$

$\Rightarrow 220 + a + b = 330$

$\therefore a + b = 110 \ldots\ldots (1)$

Also,

$$\sum \frac{(x_i - \bar{x})^2}{n} = 194$$

$\Rightarrow (a-55)^2 + (b-55)^2 + (68-55)^2 + (44-55)^2$

$+ (48-55)^2 + (60-55)^2 = 194 \times 6$

$\Rightarrow (a-55)^2 + (b-55)^2 + 169 + 121 + 49 + 25 = 1164$

$\Rightarrow (a-55)^2 + (b-55)^2 = 1164 - 364 = 800$

$a^2 + 3025 - 110a + b^2 + 3025 - 110b = 800$

$\Rightarrow a^2 + b^2 = 800 - 6050 + 12100$

$a^2 + b^2 = 6850 \ldots\ldots\ldots (2)$

Solve (1) & (2);

$a = 75, b = 35$

$\therefore a + 3b = 75 + 3(35) = 75 + 105 = 180$

Question8

Let the median and the mean deviation about the median of 7 observation 170,125,230,190,210, a, b be 170 and 205/7 respectively. Then the mean deviation about the mean of these 7 observations is :

[1-Feb-2024 Shift 1]

Options:

A.

31

B.

28

C.

30

D.

32

Answer: D

Solution:

Median $= 170 \Rightarrow 125, a, b, 170, 190, 210, 230$

Mean deviation about

Median =

$$\frac{0+45+60+20+40+170-a+170-b}{7} = \frac{205}{7}$$

$$\Rightarrow a+b=300$$

$$\text{Mean} = \frac{170+125+230+190+210+a+b}{7} = 175$$

Mean deviation

About mean =

$$\frac{50+175-a+175-b+5+15+35+55}{7} = 30$$

Question9

Consider 10 observation $x_1, x_{2, \ldots}, x_{10}$. such that $\sum_{i=1}^{10}(x_i-\alpha)=2$ and $\sum_{i=1}^{10}(x_i-\beta)^2=40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. The $\frac{\beta}{\alpha}$ is equal to :

[1-Feb-2024 Shift 2]

Options:

A.

2

B.

3/2

C.

5/2

D.

1

Answer: A

Solution:

$x_1, x_2 \ldots\ldots x_{10}$

$$\sum_{i=1}^{10}(x_i-\alpha)=2 \Rightarrow \sum_{i=1}^{10}x_i-10\alpha=2$$

$$\text{Mean } \mu=\frac{6}{5}=\frac{\Sigma x_i}{10}$$

$$\therefore \Sigma x_i=12$$

$$10\alpha+2=12 \quad \therefore \alpha=1$$

$$\text{Now } \sum_{i=1}^{10}(x_i-\beta)^2=40 \text{ Let } y_i=x_i-\beta$$

$$\therefore \sigma_y{}^2=\frac{1}{10}\sum y_i{}^2-(\bar{y})^2$$

$$\sigma_x{}^2=\frac{1}{10}\sum(x_i-\beta)^2-\left(\frac{\sum_{i=1}^{10}(x_i-\beta)}{10}\right)^2$$

$$\frac{84}{25}=4-\left(\frac{12-10\beta}{10}\right)^2$$

$$\therefore \left(\frac{6-5\beta}{5}\right)^2=4-\frac{84}{25}=\frac{16}{25}$$

$$6-5\beta=\pm4 \Rightarrow \beta=\frac{2}{5} \text{ (not possible) or } \beta=2$$

$$\text{Hence } \frac{\beta}{\alpha}=2$$

Question10

Let the six numbers a_1, a_2, a_3, a_4, a_5, a_6 be in A.P. and $a_1+a_3=10$. If the mean of these six numbers $\frac{19}{2}$ and their variance is σ^2, then $8\sigma^2$ is equal to
[24-Jan-2023 Shift 2]

Options:

A. 220

B. 210

C. 200

D. 105

Answer: B

Solution:

Solution:

$a_1+a_3=10=a_1+d \Rightarrow 5$
$a_1+a_2+a_3+a_4+a_5+a_6=57$
$\Rightarrow \frac{6}{2}[a_1+a_6]=57$

$\Rightarrow a_1 + a_6 = 19$
$\Rightarrow 2a_1 + 5d = 19$ and $a_1 + d = 5$
$\Rightarrow a_1 = 2, d = 3$
Numbers : 2, 5, 8, 11, 14, 17
Variance $= \sigma^2 =$ mean of squares-square of mean

$$= \frac{2^2 + 5^2 + 8^2 + (11)^2 + (14)^2 + (17)^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4} = \frac{105}{4}$$

$8\sigma^2 = 210$

Question11

The mean and variance of the marks obtained by the students in a test are 10 and 4 respectively. Later, the marks of one of the students is increased from 8 to 12 . If the new mean of the marks is 10.2. then their new variance is equal to:
[25-Jan-2023 Shift 1]

Options:

A. 4.04

B. 4.08

C. 3.96

D. 3.92

Answer: C

Solution:

Solution:

$$\sum_{i=1}^{n} x_i = 10n$$

$$\sum_{i=1}^{n} x_i - 8 + 12 = (10.2)n \quad \therefore n = 20$$

$$\text{Now} \quad \frac{\sum_{i=1}^{20} x_i^2}{20} - (10)^2 = 4 \Rightarrow \sum_{i=1}^{20} x_i^2 = 2080$$

$$\frac{\sum_{i=1}^{20} x_i^2 - 8^2 + 12^2}{20} - (10.2)^2$$

$$= 108 - 104.04 = 3.96$$

Question12

There rotten apples are mixed accidently with seven good apples and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent mean and variance of X , respectively, then $10(\mu^2 + \sigma^2)$ is equal to
[29-Jan-2023 Shift 1]

Options:

A. 20

B. 250

C. 25

D. 30

Answer: A

Solution:

Solution:

$\sum xP(x) = \frac{6}{2} = \mu$

$\sigma^2 = \sum x^2P(x) - \mu^2$

$\sigma^2 + \mu^2 = 0 + \frac{1}{2} + \frac{12}{10} + \frac{9}{30} = 2$

$10(\sigma^2 + \mu^2) = 20$ Ans.

Question13

Let X = {11, 12, 13, ..., 40, 41} and Y = { 61, 62, 63, ..., 90, 91 } be the two sets of observations. If $\overline{x}$ and $\overline{y}$ are their respective means and σ^2 is the variance of all the observations in X ∪ Y, then $|\overline{x} + \overline{y} - \sigma^2|$ is equal to _______.
[29-Jan-2023 Shift 2]

Answer: 603

Solution:

$\overline{x} = \frac{\sum_{i=11}^{41} i}{31} = \frac{11 + 41}{2} = 26$ (31 elements)

$\overline{x} = \frac{\sum_{j=61}^{91} i}{31} = \frac{61 + 91}{2} = 76$ (31 elements)

Combined mean, $\mu = \frac{31 \times 26 + 31 \times 76}{31 + 31}$

$= \frac{26 + 76}{2} = 51$

$\sigma^2 = \frac{1}{62} \times \left(\sum_{i=1}^{31} (x_i - \mu)^2 + \sum_{i=1}^{31} (y_i - \mu)^2 \right) = 705$

Since, $x_i \in X$ are in A.P. with 31 elements & common difference 1 , same is $y_i \in y$, when written in increasing order.

$\therefore \sum_{i=1}^{31} (x_i - \mu)^2 = \sum_{i=1}^{31} (y_i - \mu)^2$

$= 10^2 + 11^2 + ... + 40^2$

$= \frac{40 \times 41 \times 81}{6} - \frac{9 \times 10 \times 19}{6} = 21855$

$\therefore |\bar{x} + \bar{y} - \sigma^2| = |26 + 76 - 705| = 603$

Question14

The mean and variance of 7 observations are 8 and 16 respectively. If one observation 14 is omitted a and b are respectively mean and variance of remaining 6 observation, then a + 3b − 5 is equal to _______.
[30-Jan-2023 Shift 1]

Answer: 37

Solution:

Solution:

$\frac{x_1 + x_2 + \ldots.. + x_7}{7} = 8$

$\frac{x_1 + x_2 + x_3 \ldots + x_6 + 14}{7} = 8$

$\Rightarrow x_1 + x_2 + \ldots. + x_6 = 42$

$\therefore \frac{x_1 + x_2 \ldots. + x_6}{6} = \frac{42}{6} = 7 = a$

$\frac{\sum x_i^2}{7} - 8^2 = 16$

$\Sigma xi^2 = 560$

$\Rightarrow x_1^2 + x_2^2 + \ldots + x_6^2 = 364$

$b = \frac{x_1^2 + x_2^2 + \ldots + x_6^2}{6} - 7^2$

$= \frac{364}{6} - 49$

$b = \frac{70}{6}$

$a + 3b - 5 = 7 + 3 \times \frac{70}{6} - 5$

$= 37$

Question15

Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots., a_{100}$ is 25 . Then S is
[30-Jan-2023 Shift 2]

Options:

A. φ

B. {99}

C. ℕ

D. {9}

Answer: C

Solution:

Solution:
let a_1 be any natural number
$a_1, a_1+1, a_1+2, \ldots, a_1+99$ are values of $a_i'S$

$$\bar{x} = \frac{a_1 + (a_1+1) + (a_1+2) + \ldots + a_1 + 99}{100}$$

$$= \frac{100a_1 + (1+2+\ldots+99)}{100} = a_1 + \frac{99 \times 100}{2 \times 100}$$

$$= a_1 + \frac{99}{2}$$

$$\text{Mean deviation about mean} = \frac{\sum_{1=1}^{100} \left| x_i - \bar{x} \right|}{100}$$

$$= \frac{2\left(\frac{99}{2} + \frac{97}{2} + \frac{95}{2} + \ldots + \frac{1}{2}\right)}{100}$$

$$= \frac{1+3+\ldots+99}{100}$$

$$= \frac{\frac{50}{2}[1+99]}{100}$$

$$= 25$$

So, it is true for every natural no. ' a_1'

Question16

If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

[31-Jan-2023 Shift 1]

Answer: 5

Solution:

Solution:

$$\sigma_x^2 = \sigma_d^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2$$

$$= \frac{150}{45+\alpha} - 0 = 3$$

$$\Rightarrow 150 = 135 + 3\alpha$$

$$\Rightarrow 3\alpha = 15 \Rightarrow \alpha = 5$$

Question17

Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and $\alpha(>0)$, and the mean and standard deviation of marks of class B of n students be respectively 55 and $30-\alpha$. If the mean and variance of the marks of the combined class of $100+n$ students are respectively 50 and 350 , then the sum of variances of classes A and B is:
[31-Jan-2023 Shift 2]

Options:

A. 500

B. 650

C. 450

D. 900

Answer: 0

Solution:

Solution:

$\bar{x} = \frac{100\times 40+55n}{100+n}$

$5000+50n = 4000+55n$

$1000 = 5n$

$n = 200$

$\sigma_1{}^2 = \frac{\sum x_i{}^2}{100} - 40^2$

$\sigma_2{}^2 = \frac{\sum x_j{}^2}{100} - 55^2$

$350 = \sigma^2 = \frac{\sum x_i{}^2+\sum x_j{}^2}{300} - (\bar{x})^2$

$2850 = \frac{(1600+\alpha^2)\times 100+[(30-\alpha)^2+3025]\times 200}{300} - (50)^2$

$8550 = \alpha^2+2(30-\alpha)^2+7650$

$\alpha^2+2(30-\alpha)^2 = 900$

$\alpha^2-40\alpha+300 = 0$

$\alpha = 10, 30$

$\sigma_1{}^2+\sigma_2{}^2 = 10^2+20^2 = 500$

Question18

The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5, then the sum of cubes of the remaining two observations is
[1-Feb-2023 Shift 1]

Options:

A. 1072

B. 1792

C. 1216

D. 1456

Answer: A

Solution:

Solution:

$$\frac{1+3+5+a+b}{5}=5$$

$a+b=16\ldots\ldots(1)$

$$\sigma^2=\frac{\sum x_1{}^2}{5}-\left(\frac{\sum x}{5}\right)^2$$

$$8=\frac{1^2+3^2+5^2+a^2+b^2}{5}-25$$

$a^2+b^2=130\ldots(2)$

by (1), (2)

$a=7, b=9$

or $a=9, b=7$

Question19

Let $9=x_1<x_2<\ldots<x_7$ be in an A.P. with common difference d. If the standard deviation of $x_1, x_2, \ldots, x_7$ is 4 and the mean is $\bar{x}$, then $\bar{x}+x_6$ is equal to :
[1-Feb-2023 Shift 2]

Options:

A. $18\left(1+\frac{1}{\sqrt{3}}\right)$

B. 34

C. $2\left(9+\frac{8}{\sqrt{7}}\right)$

D. 25

Answer: B

Solution:

Solution:

$9=x_1<x_2<\ldots\ldots<x_7$

$9, 9+d, 9+2d, \ldots\ldots.9+6d$

$0, d, 2d, \ldots\ldots.6d$

$$\bar{x}_{\text{new}}=\frac{21d}{7}=3d$$

$$16=\frac{1}{7}(0^2+1^2+\ldots\ldots+6^2)d^2-9d^2$$

$$=\frac{1}{\text{not}}\left(\frac{\text{not}\,6\times\text{not}\,t\times 13}{\text{not}\,6}\right)d^2-9d^2$$

$16=4d^2$

$d^2=4$

$d=2$

$\bar{x} + x_6 = 6 + 9 + 10 + 9$

Question20

The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13 , then σ^2 is equal to :
[6-Apr-2023 shift 1]

Options:

A. 12

B. 10

C. 11

D. 9

Answer: B

Solution:

Solution:

$$\text{Combine var.} = \frac{n_1\sigma^2 + n_2\sigma^2}{n_1 + n_2} + \frac{n_1 n_2 (m_1 - m_2)^2}{(n_1 + n_2)}$$

$$13 = \frac{15 \cdot 14 + 15 \cdot \sigma^2}{30} + \frac{15 \cdot 15(12 - 14)^2}{30 \times 30}$$

$$13 = \frac{14 + \sigma^2}{2} + \frac{4}{4}$$

$$\sigma^2 = 10$$

Question21

If the mean and variance of the frequency distribution.

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

are 9 and 15.08 respectively, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is ________ :
[6-Apr-2023 shift 2]

Answer: 25

Solution:

x_i	f_i	$f_i x_i$	$f_i x_i^2$
2	4	8	16
2	4	16	64
6	α	6α	36α
8	15	120	960
10	8	80	800
12	β	12β	144β
14	4	56	784
16	5	80	1280

$N = \sum f_i = 40 + \alpha + \beta$

$\sum f_i x_i = 360 + 6\alpha + 12\beta$

$\sum f_i x_i^2 = 3904 + 36\alpha + 144\beta$

Mean $(\bar{x}) = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = 9$

$\Rightarrow 360 + 6\alpha + 12\beta = 9(40 + \alpha + \beta)$

$3\alpha = 3\beta \Rightarrow \alpha = \beta$

$\sigma^2 = \dfrac{\Sigma f_i x_1^{\ 2}}{\Sigma f_i} - \left(\dfrac{\Sigma f_i x_i}{\Sigma f_i}\right)^2$

$\Rightarrow \dfrac{3904 + 36\alpha + 144\beta}{40 + \alpha + \beta} - (\bar{x})^2 = 15.08$

$\Rightarrow \dfrac{3904 + 180\alpha}{40 + 2\alpha} - (9)^2 = 15.08$

$\Rightarrow \alpha = 5$

Now, $\alpha^2 + \beta^2 - \alpha\beta = \alpha^2 = 25$

Question22

Let the mean and variance of 8 numbers x, y, 10, 12, 6, 12, 4, 8 be 9 and 9.25 respectively. If x > y, then 3x − 2y is equal to ________. [8-Apr-2023 shift 1]

Answer: 25

Solution:

Solution:

$\dfrac{x + y + 52}{8} = 9 \Rightarrow x + y = 20$

For variance

$\underline{x} - 9, y - 9, 3, 3, 1, -5, -1, -3$

$\bar{x} = 0$

$\therefore \dfrac{(x-9)^2 + (y-9)^2 + 54}{8} - \bar{0}^2 = 9.25$

$(x-9)^2 + (11-x)^2 = 20$

$x = 7$ or $13 \therefore y = 13, 7$

$3x - 2y = 3 \times 13 - 2 \times 7 = 25$

Question23

Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are coprime, then m + n is equal to
[8-Apr-2023 shift 2]

Options:

A. 316

B. 317

C. 315

D. 314

Answer: B

Solution:

Solution:

$\frac{\Sigma x}{12} = \frac{9}{2}$

$\sum x = 54$

$\frac{\Sigma x^2}{12} - \left(\frac{9}{2}\right)^2 = 4$

$\sum x^2 = 291$

$\sum x_{new} = 54 - (9 + 10) + 7 + 14 = 56$

$\sum x_{new}{}^2 = 291 - (81 + 100) + 49 + 196 = 355$

$\sigma_{new}{}^2 = \frac{355}{12} - \left(\frac{56}{12}\right)^2$

$\sigma_{new}{}^2 = \frac{281}{36} = \frac{m}{n}$

$m + n = 317$ Option (2)

Question24

If the mean of the frequency distribution

Class:	0-10	10-20	20-30	30-40	40-50
Frequency:	2	3	x	5	4

is 28 , then its variance is _______.
[10-Apr-2023 shift 1]

Answer: 151

Solution:

C.I.	f	x	$f_i x_i$	x_i^2
0-10	2	5	10	25
10-20	3	15	45	225
20-30	x	25	25x	625
30-40	5	35	175	1225
40-50	4	45	180	2025

$\overline{x} = \dfrac{\Sigma f_i x_i}{N}$

$28 = \dfrac{10 + 45 + 25x + 175 + 130}{14 + x}$

$28 \times 14 + 28x = 410 + 25x$

$\Rightarrow 3x = 410 - 392$

$\Rightarrow x = \dfrac{18}{3} = 6$

$\therefore$ Variance $= \dfrac{1}{N} \sum f_i x_i^2 - (\overline{x})^2$

$= \dfrac{1}{20} 18700 - (28)^2$

$= 935 - 784 = 151$

Question25

Let m be the mean and sigma be the standard deviation of the distribution

x_i	0	1	2	3	4	5
f_i	$k+2$	2k	k^2-1	k^2-1	k^2+1	$k-3$

where $\sum f_i = 62$. If [x] denotes the greatest integer ≤x, then $[\mu 2 + \sigma^2]$ is equal to
[10-Apr-2023 shift 2]

Options:

A. 8

B. 7

C. 6

D. 9

Answer: A

Solution:

$\sum f_i = 62$
$3k^2 + 16k - 12k - 64 = 0$
$k = 4 \text{ or } -\frac{16}{3} (\text{ rejected })$
$\mu = \frac{\Sigma f_i x_i}{\Sigma f_i}$
$\mu = \frac{8 + 2(15) + 3(15) + 4(17) + 5}{62} = \frac{156}{62}$
$\sigma^2 = \sum f_i x_i^2 - (\sum f_i x_i)^2$
$= \frac{8 \times 1^2 + 15 \times 13 + 17 \times 16 + 25}{62} - \left(\frac{156}{62}\right)^2$
$\sigma^2 = \frac{500}{62} - \left(\frac{156}{62}\right)^2$
$\sigma^2 + \mu^2 = \frac{500}{62}$
$[\sigma^2 + \mu^2] = 8$

Question26

Let sets A and B have 5 elements each. Let mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively. A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is _______.
[11-Apr-2023 shift 1]

Options:

A. 36

B. 40

C. 32

D. 38

Answer: D

Solution:

Solution:
$\omega A = \{a_1, a_2, a_3, a_4, a_5\}$
$B = \{b_1, b_2, b_3, b_4, b_5\}$

$$\frac{\sum_{i=1}^{5} a_i^2}{5} - \left(\frac{\sum_{i=1}^{5} a_i}{5}\right)^2 = 12, \quad \frac{\sum_{i=1}^{5} b_i^2}{5} - \left(\frac{\sum_{i=1}^{5} b_i}{5}\right)^2 = 20$$

$$\Rightarrow \sum_{i=1}^{5} a_i^2 = 185, \quad \sum_{i=1}^{5} b_i^2 = 420$$

Now, $C = \{C_1, C_2, \ldots C_{10}\}$

$\therefore$ Mean of C, $\overline{C} = \frac{(\Sigma a_i - 15) + (\Sigma b_i - 10)}{10}$

$\overline{C} = \frac{10 + 50}{10} = 6$

$$\therefore \sigma^2 = \frac{\sum_{i=1}^{10} C_i^2}{10} = (\overline{C})^2$$

$= \frac{\Sigma(a_i - 3)^2 + \Sigma(b_i - 2)^2 +}{10} - (6)^2$

$= \frac{\Sigma a_i^2 + \Sigma b_i^2 - 6\Sigma a_i + 4\Sigma b_i + 65}{10} - 36$

$= \frac{185 + 420 - 150 + 160 + 65}{10} - 36$

$= 32$

$\therefore$ Mean + Variance $= \overline{C} + \sigma^2 = 6 + 32 = 38$

Question27

Let the mean of 6 observations 1, 2, 4, 5, x and y be 5 and their variance be 10 . Then their mean deviation about the mean is equal to
[11-Apr-2023 shift 2]

Options:

A. $\frac{7}{3}$

B. $\frac{10}{3}$

C. $\frac{8}{3}$

D. 3

Answer: C

Solution:

Solution:
Mean of 1, 2, 4, 5, x, y is 5
and variance is 10

$\Rightarrow$ mean $\Rightarrow \frac{12 + x + y}{6} = 5$

$12 + x + y = 30$

$x + y = 18$

and by variance $\frac{x^2 + y^2 + 46}{6} - 5^2 = 10$

$x^2 + y^2 = 164$

$x = 8 \ \ y = 10$

mean daviation $= \frac{|x - \overline{x}|}{6}$

$\Rightarrow \frac{4 + 3 + 1 + 0 + 3 + 5}{6} = \frac{16}{6} = \frac{8}{3}$

Question28

Let the positive numbers a_1, a_2, a_3, a_4 and a_5 be in a G.P. Let their mean and variance be $\frac{31}{10}$ and $\frac{m}{n}$ respectively, where m and n are co-prime. If the mean of their reciprocal is $\frac{31}{40}$ and $a_3 + a_4 + a_5 = 14$, then $m + n$ is equal to ______.
[12-Apr-2023 shift 1]

Answer: 211

Solution:

Solution:

Let $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

Given $\frac{a}{r^2} + \frac{a}{r} + a + ar + ar^2 = 5 \times \frac{31}{10} \ldots (1)$

And $\frac{r^2}{a} + \frac{r}{a} + \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} = 5 \times \frac{31}{40} \ldots (2)$

$(1) \div (2) a^2 = 4 \Rightarrow a = 2$

$\therefore r + \frac{1}{r} = 5/2 \ (a \neq -2)$

$\Rightarrow r = 2$

$\therefore$ Now $\frac{1}{2}, 1, 2.4, 8$

$\therefore$ Now $\frac{1}{2}, 1, 2.4, 8$

$\therefore \sigma^2 = \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2$

$= \frac{186}{25} = \frac{M}{N} \Rightarrow 211 = m + n$

Question29

Let the mean of the data

x	1	3	5	7	9
Frequency (f)	4	24	28	α	8

be 5 . If m and σ^2 are respectively the mean deviation about the mean and the variance of the data, then $\frac{3\alpha}{m+\sigma^2}$ is equal to _____.

[13-Apr-2023 shift 1]

Answer: 8

Solution:

$$5 = \bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{4 + 72 + 140 + 7\alpha + 72}{64 + \alpha}$$

$\Rightarrow 320 + 5\alpha = 288 + 7\alpha \Rightarrow 2\alpha = 32 \Rightarrow \alpha = 16$

$$\text{M.D. } (\bar{x}) = \Sigma \frac{f_i}{|x_i - \bar{x}|} \sum f_i \text{ where } \sum f_i = 64 + 16 = 80$$

$$\text{M.D. } (\bar{x}) = \frac{4 \times 4 + 24 \times 2 + 28 \times 0 + 16 \times 2 + 8 \times 4}{80} = \frac{8}{5}$$

$$\text{Variance} = \frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}$$

$$= \frac{4 \times 16 + 24 \times 4 + 0 + 16 \times 4 + 8 \times 16}{80} = \frac{352}{80}$$

$$\therefore \frac{3\alpha}{m + \sigma^2} = \frac{3 \times 16}{\frac{128}{80} + \frac{352}{80}} = 8$$

Question30

The mean and standard deviation of the marks of 10 students were found to be 50 and 12 respectively, Later, it was observed that two marks 20 and 25 were wrongly read as 45 and 50 respectively. Then the correct variance is _______.
[13-Apr-2023 shift 2]

Answer: 269

Solution:

Solution:

$$\text{Mean} = \frac{\Sigma x_i}{10}$$

$$\Rightarrow 50 = \frac{\Sigma x_i}{10}$$

$$\Rightarrow \Sigma x_i = 500$$

$$\text{correct } \sum x_i = 500 - 45 - 50 + 20 + 25 = 450$$

$$\sigma^2 = \frac{\Sigma {x_i}^2}{10} = (\bar{x})^2$$

$$\Rightarrow 144 = \frac{\Sigma {x_i}^2}{10} - 2500$$

$$\Rightarrow \Sigma {x_i}^2 = 26440$$

$$\text{correct } \sum {x_i}^2 = 26440 - (45)^2 - (50)^2 + (20)^2 + (25)^2$$

$$= 26440 - 2025 - 2500 + 400 + 625$$

$$= 22940$$

$$\sigma^2 = \frac{\text{correct } \Sigma {x_i}^2}{10} - \left(\frac{\text{correct } \Sigma x_i}{10} \right)^2$$

$$= \frac{22940}{10} - \left(\frac{450}{10} \right)^2 = 2294 - 2025$$

$$= 269$$

Question31

The mean and standard deviation of 10 observations are 20 and 8 respectively. Later on, it was observed that one observation was recorded as 50 instead of 40 . Then the correct variance is
[15-Apr-2023 shift 1]

Options:

A. 14

B. 11

C. 12

D. 13

Answer: D

Solution:

Solution:

$\mu = 20, \sigma = 8$

$\mu_{\text{Corrected}} = \frac{200 - 50 + 40}{10} = 19$

$\sigma^2 = \frac{1}{10}\sum x_i^{\ 2} - 20^2$

$(64 + 400)10 = \sum x_i^{\ 2}$

$\sigma_{\text{Corrected}}^{\ \ 2} = \frac{1}{10}[(64 + 400)10 - 2500 + 1600] - 19^2$

$= 374 - 361 = 13$

Question32

Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting atmost two heads is :
[26-Jun-2022-Shift-1]

Options:

A. $\frac{275}{6^5}$

B. $\frac{36}{5^4}$

C. $\frac{181}{5^5}$

D. $\frac{46}{6^4}$

Answer: D

Solution:

Coin is tossed 5 times, so $n = 5$

Let, $p =$ probability of getting heads

$q =$ probability of getting tails.

$\therefore p+q=1 \ldots\ldots (1)$

$\therefore$ Probability of getting 4 heads

$= {}^5C_4 \cdot p^4 \cdot q$

And probability of getting 5 heads

$= {}^5C_5 \cdot p^5$

Given, ${}^5C_4 \cdot p^4 \cdot q = {}^5C_5 \cdot p^5$

$\Rightarrow 5q = p \ldots\ldots$ (2)

From equation (1) and (2), we get,

$5q + q = 1$

$\Rightarrow 6q = 1$

$\Rightarrow q = \frac{1}{6}$

$\therefore p = 1 - \frac{1}{6} = \frac{5}{6}$

Now, probability of getting atmost two heads

$= p(x=0) + p(x=1) + p(x=2)$

$p(x=0) =$ Getting zero head in 5 trials

$= {}^5C_0 \cdot p^0 \cdot q^5$

$p(x=1) =$ Getting one head in 5 trials

$= {}^5C_1 \cdot p^1 \cdot q^4$

$p(x=2) =$ Getting two heads in 5 trials

$= {}^5C_2 \cdot p^2 \cdot q^3$

$= {}^5C_0 \cdot q^5 + {}^5C_1 \cdot pq^4 + {}^5C_2 \cdot p^2 q^3$

$= \left(\frac{1}{6}\right)^5 + 5 \cdot \frac{5}{6} \cdot \left(\frac{1}{6}\right)^4 + 10 \cdot \left(\frac{5}{6}\right)^2 \cdot \left(\frac{1}{6}\right)^3$

$= \frac{1+25+250}{6^5} = \frac{276}{6^5} = \frac{46}{6^4}$

Question33

If the probability that a randomly chosen 6-digit number formed by using digits 1 and 8 only is a multiple of 21 is p, then 96p is equal to___ [26-Jun-2022-Shift-2]

Answer: 33

Solution:

Solution:

Total number of numbers from given

Condition $= n(s) = 2^6$

Every required number is of the form

$A = 7.\left(10^{a_1} + 10^{a_2} + 10^{a_3} + \ldots\ldots\right) + 111111$

Here 111111 is always divisible by 21.

$\therefore$ If A is divisible by 21 then

$10^{a_1} + 10^{a_2} + 10^{a_3} + \ldots\ldots$ must be divisible by 3.

For this we have ${}^6C_0 + {}^6C_3 + {}^6C_6$ cases are there

$\therefore n(E) = {}^6C_0 + {}^6C_3 + {}^6C_6 = 22$

$\therefore$ Required probability $= \frac{22}{2^6} = p$

$\therefore \frac{11}{32} = p$

$\therefore 96p = 33$

Question34

The mean of the numbers a, b, 8, 5, 10 is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then 25M is equal to : [26-Jun-2022-Shift-1]

Options:

A. 60

B. 55

C. 50

D. 45

Answer: A

Solution:

$$\sigma^2 = \frac{\sum_{i=1}^{5}(x_i - \bar{x})^2}{n}$$

Mean $= 6$

$$\frac{a+b+8+5+10}{5} = 6 \Rightarrow a+b=7$$

$a+b=7$

$b = 7 - a$

$$6.8 = \frac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5}$$

$34 = (a-6)^2 + (7-a-6)^2 + 4 + 1 + 18$

$a^2 - 7a + 12 = 0 \Rightarrow a = 4 \text{ or } a = 3$

$a = 4 \quad a = 3$

$b = 3 \quad b = 4$

$$M = \frac{\sum_{i=1}^{5}|x_i - x|}{n}$$

$$M = \frac{|a-6| + |b-6| + |8-6| + |5-6| + |10-6|}{5}$$

when $a = 3, b = 4$ when $a = 4, b = 3$

$$M = \frac{3+2+2+1+4}{5} \quad M = \frac{2+3+2+1+7}{5}$$

$$M = \frac{12}{5} \quad M = \frac{12}{5} \Rightarrow 25M = 25 \times \frac{12}{5} = 60$$

Question35

The mean and standard deviation of 50 observations are 15 and 2 respectively. It was found that one incorrect observation was taken such that the sum of correct and incorrect observations is 70 . If the correct mean is 16 , then the correct variance is equal to :
[26-Jun-2022-Shift-2]

Options:

A. 10

B. 36

C. 43

D. 60

Answer: C

Solution:

Solution:

Given $\bar{x} = 15$, $\sigma = 2 \Rightarrow \sigma^2 = 4$

$\therefore x_2 + x_2 + \ldots\ldots + x_{50} = 15 \times 50 = 750$

$$4 = \frac{x_1^2 + x_2^2 + \ldots. + x_{50}^2}{50} - 225$$

$\therefore x_1^2 + x_2^2 + \ldots\ldots + x_{50}^2 = 50 \times 229$

Let a be the correct observation and b is the incorrect observation

then $a + b = 70$

and $16 = \dfrac{750 - b + a}{50}$

$\therefore a - b = 50 \Rightarrow a = 60, b = 10$

$\therefore$ Correct variance $= \dfrac{50 \times 229 + 60^2 - 10^2}{50} - 256 = 43$

Question36

The mean and variance of the data 4, 5, 6, 6, 7, 8, x, y, where x < y, are 6 and $\frac{9}{4}$ respectively. Then $x^4 + y^2$ is equal to
[27-Jun-2022-Shift-2]

Options:

A. 162

B. 320

C. 674

D. 420

Answer: B

Solution:

Solution:

$$\text{Mean} = \frac{4+5+6+6+7+8+x+y}{8} = 6$$

$\therefore x + y = 12 \ldots.$ (i)

And variance

$$= \frac{2^2 + 1^2 + 0^2 + 0^2 + 1^2 + 2^2 + (x-6)^2 + (y-6)^2}{8}$$

$$= \frac{9}{4}$$

$\therefore (x-6)^2 + (y-6)^2 = 8 \ldots.$ (ii)

From (i) and (ii)

x = 4 and y = 8

$\therefore x^4 + y^2 = 320$

Question37

The mean and standard deviation of 15 observations are found to be 8 and 3 respectively. On rechecking it was found that, in the observations, 20 was misread as 5 . Then, the correct variance is equal to____
[28-Jun-2022-Shift-1]

Answer: 17

Solution:

$$\frac{\sum x_i^{\,2}}{15} - 8^2 = 9 \Rightarrow \sum x_i^{\,2} = 15 \times 73 = 1095$$

Let $\overline{x}_c$ be corrected mean $\overline{x}_c = 9$

$$\sum x_c^{\,2} = 1095 - 25 + 400 = 1470$$

Correct variance $= \frac{1470}{15} - (9)^2 = 98 - 81 = 17$

Question38

Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively. If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, then $(4a + x_5)$ is equal to:
[29-Jun-2022-Shift-1]

Options:

A. 13

B. 15

C. 17

D. 18

Answer: B

Solution:

Solution:

Mean $(\bar{x}) = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$

Given, $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{24}{5}$

$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 24$

Now, Mean of first 4 observation

$= \frac{x_1 + x_2 + x_3 + x_4}{4}$

Given, $= \frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{7}{2}$

$\Rightarrow x_1 + x_2 + x_3 + x_4 = 14$

From equation (1) and (2), we get

$14 + x_5 = 24$

$\Rightarrow x_5 = 10$

Now, variance of first 5 observation

$= \frac{\sum x_i^2}{n} - (\bar{x})^2$

$= \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \left(\frac{24}{5}\right)^2$

Given,

$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{24}$

$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 5\left(\frac{194}{25} + \frac{576}{25}\right)$

$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 154$

$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + (10)^2 = 154$

$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 54$

Now, variance of first 4 observation

$= \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \left(\frac{7}{2}\right)^2$

Given,

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \left(\frac{7}{2}\right)^2 = a$$

Given,

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \left(\frac{7}{2}\right)^2 = a$$

$\Rightarrow \frac{54}{4} - \frac{49}{4} = a$

$\Rightarrow a = \frac{5}{4}$

$\therefore 4a + x_5$

$= 4 \times \frac{5}{4} + 10 = 15$

Question39

The number of values of $a \in N$ such that the variance of 3, 7, 12, a, $43 - a$ is a natural number is : [29-Jun-2022-Shift-2]

Options:

A. 0

B. 2

C. 5

D. infinite

Answer: A

Solution:

Solution:

Mean $= 13$

Variance $= \frac{9 + 49 + 144 + a^2 + (43 - a)^2}{5} - 13^2 \in N$

$\Rightarrow \frac{2a^2 - a + 1}{5} \in N$

$\Rightarrow 2a^2 - a + 1 - 5n = 0$ must have solution as natural numbers

its $D = 40n - 7$ always has 3 at unit place

$\Rightarrow D$ can't be perfect square

So, a can't be integer.

Question40

If the mean deviation about the mean of the numbers 1, 2, 3, n, where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to___ [25-Jun-2022-Shift-2]

Answer: 21

Solution:

Mean $= \dfrac{n\frac{(n+1)}{2}}{n} = \dfrac{n+1}{2}$

M.D. $= \dfrac{2\left(\frac{n-1}{2} + \frac{n-3}{2} + \frac{n-5}{2} + \ldots 0\right)}{n} = \dfrac{5(n+1)}{n}$

$\Rightarrow ((n-1) + (n-3) + (n-5) + \ldots 0) = 5(n+1)$

$\Rightarrow \left(\dfrac{n+1}{4}\right) \cdot (n-1) = 5(n+1)$

So, $n = 21$

Question41

If the mean deviation about median for the numbers 3, 5, 7, 2k, 12, 16, 21, 24, arranged in the ascending order, is 6 then the median is
[25-Jul-2022-Shift-2]

Options:

A. 11.5

B. 10.5

C. 12

D. 11

Answer: D

Solution:

Solution:

Median $= \dfrac{2k+12}{2} = k+6$

Mean deviation $= \Sigma \dfrac{|x_i - M|}{n} = 6$

$\Rightarrow \dfrac{(k+3)+(k+1)+(k-1)+(6-k)+(6-k)+(10-k)+(15-k)+(18-k)}{8}$

$\therefore \dfrac{58-2k}{8} = 6$

$k = 5$

Median $= \dfrac{2 \times 5 + 12}{2} = 11$

Question42

The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, then $38\sigma^2$ is equal

to ________.
[26-Jul-2022-Shift-2]

Answer: 238

Solution:

Solution:

$\mu = \frac{\sum x_i}{40} = 30 \Rightarrow \sum x_i = 1200$

$\sigma^2 = \frac{\sum x_i^2}{40} - (30)^2 = 25 \Rightarrow \sum x_i^2 = 37000$

After omitting two wrong observations

$\sum y_i = 1200 - 12 - 10 = 1178$

$\sum y_i^2 = 37000 - 144 - 100 = 36756$

Now $\sigma^2 = \frac{\sum y_i^2}{38} - \left(\frac{\sum y_i}{38}\right)^2$

$= \frac{36756}{38} - \left(\frac{1178}{38}\right)^2 = -31^2$

$= 38\sigma^2 = 36756 - 36518 = 238$

Question43

The mean and variance of 10 observations were calculated as 15 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then, the correct standard deviation is ________.
[27-Jul-2022-Shift-1]

Answer: 2

Solution:

Solution:

Given $\frac{\sum_{i=1}^{10} x_i}{10} = 15 \ldots\ldots$

$\Rightarrow \sum_{i=1}^{10} x_i = 150$

and $\frac{\sum_{i=1}^{10} x_i^2}{10} - 15^2 = 15$

$\Rightarrow \sum_{i=1}^{10} x_i^2 = 2400$

Replacing 25 by 15 we get

$\sum_{i=1}^{9} x_i + 25 = 150$

$\Rightarrow \sum_{i=1}^{9} x_i = 125$

$\therefore$ Correct mean $= \dfrac{\sum\limits_{i=1}^{9} x_i + 15}{10} = \dfrac{125+15}{10} = 14$

Similarly, $\sum\limits_{i=1}^{2} x_i^2 = 2400 - 25^2 = 1775$

$\therefore$ Correct variance $= \dfrac{\sum\limits_{i=1}^{9} x_i^2 + 15^2}{10} - 14^2 = \dfrac{1775+225}{10} - 14^2 = 4$

$\therefore$ Correct S . D $= \sqrt{4} = 2.$

Question44

Let the mean and the variance of 20 observations $x_1, x_2, \ldots, x_{20}$ be 15 and 9 , respectively. For $\alpha \in R$, if the mean of $(x_1+\alpha)^2, (x_2+\alpha)^2, \ldots, (x_{20}+\alpha)^2$ is 178 , then the square of the maximum value of α is equal to ______.
[29-Jul-2022-Shift-1]

Answer: 4

Solution:

Solution:

Given $\dfrac{\sum\limits_{i=1}^{20} x_i}{20} = 15 \Rightarrow \sum\limits_{i=1}^{20} x_i = 300$

and $\dfrac{\sum\limits_{i=1}^{20} x_i^2}{20} - (\bar{x})^2 = 9 \Rightarrow \sum\limits_{i=1}^{20} x_i^2 = 4680$

Mean $= \dfrac{(x_i+\alpha)^2 + (x_2+\alpha)^2 + \ldots\ldots + (x_{20}+\alpha)^2}{20} = 178$

$\Rightarrow \dfrac{\sum\limits_{i=1}^{20} x_i^2 + 2\alpha \sum\limits_{i=1}^{20} x_i + 20\alpha^2}{20} = 178$

$\Rightarrow 4680 + 600\alpha + 20\alpha^2 = 3560$

$\Rightarrow \alpha^2 + 30\alpha + 56 = 0$

$\Rightarrow \alpha^2 + 28\alpha + 2\alpha + 56 = 0$

$\Rightarrow (\alpha+28)(\alpha+2) = 0$

$\alpha_{max} = -2 \Rightarrow \alpha^2{}_{max} = 4$

Question45

If the variance of 10 natural numbers 1, 1, 1, ..., 1, k is less than 10 , then the maximum possible value of k is ....
[2021, 24 Feb. Shift-11]

Answer: 11

Solution:

Solution:
Given, 10 natural numbers $= 1, 1, 1, \ldots 1, k$
According to the question, variance
$(\sigma^2) < 10$

$\because\ \sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$

$\sigma^2 = \frac{(9 + k^2)}{10} - \left(\frac{9 + k}{10}\right)^2 < 10$

$\Rightarrow 10(9 + k^2) - (81 + k^2 + 18k) < 1000$
$\Rightarrow 90 + 10k^2 - 81 - k^2 - 18k < 1000$
$\Rightarrow 9k^2 - 18k + 9 < 1000$

$\Rightarrow (k - 1)^2 < \frac{1000}{9}$

$\Rightarrow k - 1 < \frac{10\sqrt{10}}{3}$

$\text{and} \quad k - 1 < \frac{-10\sqrt{10}}{3}$

$\Rightarrow\ k < \frac{10\sqrt{10}}{3} + 1$

It is not possible because $k \in N$.
$\therefore$ Maximum possible integral value of k is 11.

Question46

Let $X_1, X_2, \ldots, X_{18}$ be eighteen observations, such that $\sum_{i=1}^{18}(X_i - \alpha) = 36$ and $\sum_{i=1}^{18}(X_i - \beta)^2 = 90$,
where α and β are distinct real numbers. If the standard deviation of these observations is 1 , then the value of $|\alpha - \beta|$ is
[2021, 26 Feb. Shift-II]

Answer: 4

Solution:

Given, $\sum_{i=1}^{18}[x_i - \alpha] = 36$

and $\sum_{i=1}^{18} x_i - \beta|^2 = 90$

Now, $\sum_{i=1}^{18}(x_i - \alpha) = 36$

$\sum_{i=1}^{18} x_i - 18\alpha = 36$

$\sum_{i=1}^{18} x_i = 36 + 18\alpha \ldots$ (i)

Again,

$\sum_{i=1}^{18} x_i^2 + 18\beta^2 - 2\beta\sum_{i=1}^{18} x_i = 90$

$\Rightarrow \sum_{i=1}^{18} x_i^2 + 18\beta^2 - 2\beta(36 + 18\alpha) = 90$

[Using Eq. (i)]

$\Rightarrow \sum_{i=1}^{18} x_i^2 = 90 - 18\beta^2 + 2\beta(36 + 18\alpha)$

Given, standard deviation = 1

i.e $\sigma^2 = 1$

$\Rightarrow \frac{\sum_{i=1}^{18} x_i^2}{18} - \left(\frac{\sum_i^{18} x_i}{18}\right)^2 = 1$

$\Rightarrow \frac{1}{18}(90 - 18\beta^2 + 36\alpha\beta + 72\beta)$

$-\left(\frac{36 + 18\alpha}{18}\right)^2 = 1$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - (2 + \alpha)^2 = 1$

$\Rightarrow 5 - \beta^2 + 2\alpha\beta + 4\beta - 4 - \alpha^2 + (-4\alpha) = 1$

$\Rightarrow -\beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4\alpha = 0$

$\Rightarrow \alpha^2 + \beta^2 + (-2\alpha\beta) - 4\beta + 4\alpha = 0$

$\Rightarrow (\alpha - \beta)^2 + 4(\alpha - \beta) = 0$

$\Rightarrow (\alpha - \beta)(\alpha - \beta + 4) = 0$

$\Rightarrow \alpha - \beta = -4$ $(\because \alpha \neq \beta)$

$|\alpha - \beta| = |-4| = 4$

Question47

The mean age of 25 teachers in a school is 40yr. A teacher retires at the age of 60y r and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39yr, then the age (in years) of the newly appointed teacher is ....
[2021, 18 March Shift-I]

Answer: 35

Solution:

Solution:

Given, $n = 25$, $\overline{x} = 40$

$\therefore \overline{x} = \frac{\sum x}{n}$

$\Rightarrow \sum x = 25 \times 40 = 1000$

New sum $= 1000 - 60 + x$

$= 940 + x$

New average = 39

$\therefore 39 = \frac{940 + x}{25}$

$\Rightarrow 975 = 940 + x$

$\Rightarrow x = 35$

Question48

Let in a series of 2n observations, half of them are equal to a and remaining half are equal to −a. Also, by adding a constant b in each of these observations, the mean and standard deviation of new set become

5 and 20 , respectively. Then, the value of $a^2 + b^2$ is equal to
[2021, 18 March Shift-11]

Options:

A. 425

B. 650

C. 250

D. 925

Answer: A

Solution:

Solution:
Let observations are denoted by x_i for $1 \le i < 2n$.

$\therefore Mean(\bar{x}) = \frac{\sum x_i}{2n}$

$= \frac{(a + a + a + \ldots + a) - (a + a + a + \ldots + a)}{2n}$

$\Rightarrow \bar{x} = 0$

and $\sigma_x{}^2 = \frac{\sum x_1{}^2}{2n} - (\bar{x})^2$

$= \frac{a^2 + a^2 + a^2 + \ldots. + a^{22n \text{ times}}}{2n} - 0 = a^2$

$\Rightarrow \sigma_x{}^2 = a^2 \Rightarrow \sigma_x = a$

According to the question, adding a constant b, then new mean $(\bar{y}) = \bar{x} + b = 5$

$\Rightarrow 0 + b = 5$

$\Rightarrow b = 5$

and new SD $(\sigma_y) = \sigma_x$

$\Rightarrow \sigma_y = \sigma_x = 20$

$\Rightarrow a = 20$

$\therefore a^2 + b^2 = 400 + 25 = 425$

Question49

Consider a set of 3n numbers having variance 4 . In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3 . A new set is constructed by adding 1 into each of first 2n numbers and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to
[2021, 17 March Shift-II]

Answer: 68

Solution:

Solution:
$x : x_1, x_2, x_3, \ldots x_{n+1}{}'$

$x_{n+2}, \cdots x_{2n} x_{2n+1} \cdots x_{3n}$

$\frac{1}{3n} \sum_{i=1}^{3n} x_i^2 - (\bar{x})^2 = 4$

[where, $\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2} = \frac{2n(6) + n(3)}{3n} = 5$]

$\frac{1}{3n} \sum_{i=1}^{3n} x_i^2 = 25 + 4 = 29$

$\Rightarrow \sum_{i=1}^{3n} = 87n$

$x' : x_1 + 1, x_2 + 1, \ldots, x_{2n} + 1, x_{(2n+1)} - 1 \ldots,$
$x_{3n} - 1$

$\bar{x} = \frac{2n(6+1) + n(3-1)}{3n} = \frac{16}{3}$

$k = \frac{1}{3n} \sum_{i=1}^{3n} x_i^2 - \frac{256}{9}$

$= \frac{1}{3n} \left[\sum_{i=1}^{3n} x_i'^2 + 2 \sum_{i=1}^{2n} x_i' - 2 \sum_{i=2n+1}^{3n} x_i' + 3n \right]$
$- \frac{256}{9}$

$= \frac{1}{3n}[87n + 2(12n) - 2 \cdot 3n + 3n] - \frac{256}{9}$

$\Rightarrow k = 36 - \frac{256}{9}$

$\Rightarrow 9k = 324 - 256 = 68$

Question50

Consider three observations a, b and c, such that b = a + c. If the standard deviation of a + 2, b + 2, c + 2 is d , then which of the following is true?
[2021, 16 March Shift-1]

Options:

A. $b^2 = 3(a^2 + c^2) + 9d^2$

B. $b^2 = a^2 + c^2 + 3d^2$

C. $b^2 = 3(a^2 + c^2 + d^2)$

D. $b^2 = 3(a^2 + c^2) - 9d^2$

Answer: D

Solution:

Solution:
Given, three observations = a, b, c
$b = a + c$
Standard deviation = σ

$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$

When $x_1 = a + 2, x_2 = b + 2, x_3 = c + 2$

$\mu = \frac{a + 2 + b + 2 + c + 2}{3} = \frac{2b}{3} + 2$

$x_1 - \mu = (a + 2) - (2b/3 + 2) = a - 2b/3$
$x_2 - \mu = (b + 2) - (2b/3 + 2) = b - 2b/3$
$x_3 - \mu = (c + 2) - (2b/3 + 2) = c - 2b/3$

$$d = \sqrt{\frac{(a - 2b/3)^2 + (b - 2b/3)^2 + (c - 2b/3)^2}{3}}$$

$3d^2 = a^2 + b^2 + c^2 + \frac{12b^2}{9}$

$- \frac{4}{3}(ab + b^2 + bc)$

$\Rightarrow 3d^2 = a^2 + b^2 + c^2 + \frac{4}{3}b^2 - \frac{4}{3}$

$[b(a + c) + b^2]$

$\Rightarrow 3d^2 = a^2 + b^2 + c^2 + \frac{4}{3}$

$b^2 - \frac{4}{3}[b \cdot b + b^2]$

$\Rightarrow 3d^2 = a^2 + b^2 + c^2 + \frac{4}{3}b^2 - \frac{4}{3} \cdot 2b^2$

$\Rightarrow 3d^2 = a^2 + c^2 - \frac{b^2}{3}$

$\Rightarrow b^2 = 3a^2 + 3c^2 - 9d^2$

Question51

Consider the statistics of two sets of observations as follows Size Mean Variance

	Size	Mean	Varience
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is 17 then the value of nis equal to $\frac{17}{9}$, then the value of n is equal to [2021, 16 March Shift-1]

Answer: 5

Solution:

Solution:

	Size	Mean	Varience
Observation I	10	2	2
Observation II	n	3	1

Variance of combined set is 17 / 9. Let the observations for group I be set A : $\{x_1, x_2, x_3, \ldots, x_{10}\}$

$\Rightarrow \frac{\sum x_i}{10} = 2$

$\Rightarrow \sum_{x_i} = 20$

and $\Sigma\left(\frac{x_i^2}{10}\right) - \left(\frac{\sum x_i}{10}\right)^2 = 2$

$\Rightarrow \frac{\sum x_i^2}{10} - \frac{400}{100} = 2$

$\Rightarrow \frac{\sum x_i^2}{10} = 6$

$\Rightarrow \sum x_i^2 = 60$

Let the observation for group II be set B : $\{y_1, y_2, y_3, \ldots, y_{10}\}$

$\frac{\sum y_i}{n} = 3 \Rightarrow \sum y_i = 3n$

and $\frac{\sum y_i^2}{n} - \left(\frac{\sum y_i}{n}\right)^2 = 1$

$\frac{\sum y_i^2}{n} - 9 = 1 \Rightarrow \sum y_i^2 = 10n$

Combined variance $= 17 / 9$

$\Rightarrow \sigma^2 = \frac{\sum(x_i^2 + y_i^2)}{n+10} - \left[\sum\left(\frac{x_i + y_i}{n+10}\right)\right]^2$

$\Rightarrow \frac{17}{9} = \frac{60+10n}{n+10} - \left[\frac{20+3n}{n+10}\right]^2$

$\Rightarrow \frac{17}{9} = \frac{-(10n+60)(n+10) - (3n+20)^2}{(n+10)^2}$

$\Rightarrow \frac{17}{9} = \frac{n^2+40n+200}{n^2+20n+100}$

$\Rightarrow n^2 + 340n + 1700 = 9n^2 + 360n + 1800$

$\Rightarrow 8n^2 - 20n - 100 = 0$

$\Rightarrow 2n^2 - 5n - 25 = 0$

$\Rightarrow 2n^2 - 10n + 5n - 25 = 0$

$\Rightarrow 2n(n-5) + 5(n-5) = 0$

$\Rightarrow (2n+5)(n-5) = 0$

$\therefore n = 5$

Question52

If the mean and variance of the following data : 6, 10, 7, 13, a, 12, b, 12 are 9 and $\frac{37}{4}$ respectively, then $(a-b)^2$ is equal to [2021, 27 July Shift-1]

Options:

A. 24

B. 12

C. 32

D. 16

Answer: D

Solution:

Solution:

Given, data $= \{6, 10, 7, 13, a, 12, b, 12\}$

Mean $= 9$, Variance $= \frac{37}{4}$

Now, mean

$= \frac{\sum x_i}{n} = \frac{6+10+7+13+a+12+b+12}{8}$

or $9 = \frac{60+a+b}{8}$

$\Rightarrow 72 = 60 + a + b$

$\Rightarrow$ $a + b = 12 \ldots$ (i)

$\text{Variance} = \frac{\sum x_i^2}{n} - \Sigma\left(\frac{x_i}{n}\right)^2$

$\frac{37}{4} = \frac{6^2 + 10^2 + 7^2 + 13^2 + a^2 + 12^2 + b^2 + 12^2}{8}$

$\Rightarrow \frac{37}{4} = \frac{642 + a^2 + b^2}{8} - 81$

$\Rightarrow \frac{37}{4} = \frac{642 + a^2 + b^2 - 648}{8}$

$\Rightarrow$ $74 = a^2 + b^2 - 6$

$\Rightarrow$ $a^2 + b^2 = 80 \ldots$ (ii)

$\Rightarrow$ $(a + b)^2 = a^2 + b^2 + 2ab$

Putting the values from Eqs. (i) and (ii), we get

$2ab = (a + b)^2 - (a^2 + b^2)$

$\Rightarrow 2ab = 12^2 - 80 = 64$

Now, $(a - b)^2 = a^2 + b^2 - 2ab$

$= 80 - 64 = 16$

Question53

Let the mean and variance of the frequency distribution

x	$x_1 = 2$	$x_2 = 6$	$x_3 = 8$	$x_4 = 9$
f	4	4	α	β

be 6 and 6.8, respectively. If x_3 is changed from 8 to 7 , then the mean for the new data will be [2021, 27 July Shift-II]

Options:

A. 4

B. 5

C. $\frac{17}{3}$

D. $\frac{16}{3}$

Answer: C

Solution:

Solution:

Given,

x	$x_1 = 2$	$x_2 = 6$	$x_3 = 8$	$x_4 = 9$
f	4	4	α	β

Mean $= 6$

$\Rightarrow \frac{8 + 24 + 8\alpha + 9\beta}{8 + \alpha + \beta} = 6$

$\Rightarrow \left(\because \text{Mean} = \frac{\sum f_i x_i}{\sum f_i}\right)$

$\Rightarrow$ $2\alpha + 3\beta = 16 \ldots$ (i)

Also, variance $= 6.8$

$$\frac{4\times16+4\times\alpha+9\times\beta}{8+\alpha+\beta}=6.8$$

$$\left(\because \text{ Variance } = \frac{\sum f_i(x_i-\bar{x})^2}{\sum f_i}\right)$$

$\Rightarrow 64+4\alpha+9\beta=(8+\alpha+\beta)6.8$

$\Rightarrow$ Multiply both sides by 10,

$640+40\alpha+90\beta=544+68\alpha+68\beta)$

$\Rightarrow 28\alpha-22\beta=96$

$\Rightarrow 14\alpha-11\beta=48.\ldots$ (ii)

From Eqs. (i) and (ii),

$\alpha=5$ and $\beta=2$

If x_3 is changed from 8 to 7 then,

x	$x_1=2$	$x_2=6$	$x_3=7$	$x_4=9$
f	4	4	5	2

New mean $=\frac{8+24+35+18}{15}=\frac{85}{15}=\frac{17}{3}$

Question54

10 Consider the following frequency distribution

Class	$10-20$	$20-30$	$30-40$	$40-50$	$50-60$
Frequency	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45 , then $|\alpha-\beta|$ is equal to ....
[2021, 25 July Shift-1]

Answer: 164

Solution:

Solution:

Sum of frequencies $=584$

$\alpha+\beta+110+54+30=584$

$\alpha+\beta=390$

Median is at $\frac{584}{2}=292$ th

$\because$ Median $=45$ (lies in class $40-50$)

$\Rightarrow \alpha+110+54+15=292$

$\Rightarrow \alpha=113$

$\Rightarrow \beta=390-113=277$

$\Rightarrow \alpha-\beta=113-277=-164$

$\therefore |\alpha-\beta|=164$

Question55

The first of the two samples in a group has 100 items with mean 15 and standard deviation 3 . If the whole group has 250 items with mean 15.6

and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is
[2021, 25 July Shift-II]

Options:

A. 8

B. 6

C. 4

D. 5

Answer: C

Solution:

Solution:
Combined mean $= 15.6$

$\Rightarrow \dfrac{100 \times 15 + 150 \times \overline{x}_B}{250} = 15.6$

$\Rightarrow 1500 + 150\overline{x}_B = 3900$

$\Rightarrow 150\overline{x}_B = 2400$

$\Rightarrow \overline{x}_B = \dfrac{2400}{150} = 16$

Combined SD $= \sqrt{13.44}$

$\Rightarrow$ Combined variance $(\sigma^2) = 13.44$

$\sigma^2 = \dfrac{\sum x_i^2}{n} - (\overline{x})^2$

$\Rightarrow 13.44 = \dfrac{\sum x_i^2}{250} - (15.6)^2$

$\Rightarrow \sum \sum x_i^2 = 64200 \ldots$ (i)

For sample A_1

$\Rightarrow \sum \sum (x_i)_A{}^2 = 23400$

Now, $\sum (x_i)_B{}^2 = 64200 - 23400 = 40800$

SD of sample B $= \sqrt{\dfrac{\sum (x_i)_B{}^2}{n_B} - (\overline{x}_B)^2}$

$= \sqrt{\dfrac{40800}{150} - 256} = 4$

Question56

12 Consider the following frequency distribution

Class	$0-6$	$6-12$	$12-18$	$18-24$	$24-30$
Frequency	a	b	12	9	5

If mean $= \frac{309}{22}$ and median $= 14$, then the value $(a-b)^2$ is equal to ....
[2021, 22 July Shift-II]

Answer: 4

Solution:

Solution:

$$\text{Mean} = \frac{\sum f_i x_i}{\sum n} = \frac{3a + 9b + 504}{a + b + 26} = \frac{309}{22}$$

$\Rightarrow 66a + 198b + 11088 = 309a + 309b + 8034$

$\Rightarrow\ 243a + 11bb = 3054$

$\Rightarrow\ \ 81a + 37b = 1018$

$$\text{Median} = 12 + \frac{\frac{a+b+26}{2} - (a+b)}{12} \times 6 = 14$$

$$\Rightarrow\ \frac{13}{2} - \left(\frac{a+b}{4}\right) = 2 \Rightarrow \frac{a+b}{4} = \frac{9}{2}$$

$\Rightarrow\ a + b = 18$

So, $81a + 37b = 1018$

$a + b = 18$

$a = 8 \text{and } b = 10$

$\therefore\ \ (a - b)^2 = 4$

Question57

The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7 , then the remaining two observations are [2021, 20 July Shift-1]

Options:

A. 10, 11

B. 3,18

C. 8, 13

D. 1, 20

Answer: A

Solution:

Solution:

Mean $= 6.5$

$\therefore\ \ \text{Total} = 6 \times 6.5 = 39$

$$\text{Variance} = \frac{\sum x_i^2}{n} - \sum\left(\frac{x_i}{n}\right)^2$$

$$\Rightarrow\ 10.25 = \frac{\sum x_i^2}{6} - 6.5^2$$

$\therefore\ \sum x_i^2 = 6(10.25 + 42.25) = 52.5 \times 6$

$\Rightarrow\ 2 + 4 + 5 + 7 + x_1 + x_2 = 39$

$\Rightarrow\ x_1 + x_2 = 21$

$\sum x_i^2 = 52.5 \times 6$

$4 + 16 + 25 + 49 + x_1^2 + x_2^2 = 315$

$\Rightarrow\ x_1^2 + x_2^2 = 221$

$\Rightarrow\ x_1^2 + (21 - x_1)^2 = 221$

$\Rightarrow\ 2x_1^{\ 2} - 42x_1 + 441 = 221$
$\Rightarrow\ x_1^{\ 2} - 21x_1 + 110 = 0$
$\Rightarrow\ (x_1 - 11)(x_1 - 10) = 0$
$\Rightarrow\ x_1 = 10$ or 11

Question58

If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of $|a - b|$ is equal to [2021, 20 July Shift-II]

Options:

A. 9

B. 11

C. 7

D. 1

Answer: D

Solution:

Solution:
Given, Mean $= 10$
i.e. $\frac{7 + 10 + 11 + 15 + a + b}{6} = 10$
$\Rightarrow\ a + b = 17 \ldots$ (i)
Also, variance $= \frac{20}{3}$
i.e. $\frac{(7)^2 + (10)^2 + (11)^2 + (15)^2 + a^2 + b^2}{6}$
$-(\text{ Mean })^2 = \frac{20}{3}$
$\Rightarrow\ \frac{495 + a^2 + b^2}{6} - 100 = \frac{20}{3}$
$\Rightarrow\ a^2 + b^2 = 145 \ldots$ (ii)
Using Eqs. (i)and (ii), put $b = 17 - a$
$a^2 + (17 - a)^2 = 145$
$\Rightarrow\ a^2 + 289 + a^2 - 34a = 145$
$\Rightarrow\ 2a^2 - 34a + 144 = 0$
$\Rightarrow\ a^2 - 17a + 72 = 0$
$\Rightarrow\ (a - 9)(a - 8) = 0$
$\therefore\ a = 9$ or $a = 8$
Using $b = 17 - a$
$\therefore\ b = 8$ or $b = 9$
$\therefore\ |a - b| = 1$

Question59

The mean of 10 numbers $7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, \ldots$ is [2021, 31 Aug. Shift-1]

Answer: 398

Solution:

Solution:
$(7 \times 8), (10 \times 10), (13 \times 12), \ldots\ldots$
$7, 10, 13, \ldots\ldots$
$a_n = 7 + (n-1)3 = 3n + 4$
$8, 10, 12, \ldots\ldots.$
$b_n = 8 + (n-1)2 = 2n + 6$
So, $T_n = (3n+4)(2n+6)$
$= 6n^2 + 26n + 24$
Sum $S_n = \Sigma T_n$
$= 6 \cdot \left[\frac{n(n+1)(2n+1)}{6}\right] + 26\left[\frac{n(n+1)}{2}\right] + 24n$
Mean $= \frac{\text{sum}}{10} = \frac{10 \cdot 11 \cdot 21}{10} + \frac{13 \cdot 10 \cdot 11}{10} + \frac{24 \cdot 10}{10}$
$= 398$

Question60

The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8 , then the variance of the remaining 5 observations is [2021, 31 Aug. Shift-II]

Options:

A. $\frac{92}{5}$

B. $\frac{134}{5}$

C. $\frac{536}{25}$

D. $\frac{112}{5}$

Answer: C

Solution:

Solution:
Let a, b, c, d and e are 5 remaining observations $n = 7$, Mean $= 8$, Variance $= 16$
Sum of observations $= 7 \times 8 = 56$
Mean of remaining observations
$= \frac{56 - 8 - 6}{5} = \frac{42}{5}$
and Variance $= \frac{\Sigma x^2}{n} - (\bar{x})^2$
$16 = \frac{\Sigma x^2}{7} - 64 \Rightarrow \Sigma x^2 = 560$
$\Rightarrow a^2 + b^2 + c^2 + d^2 + e^2 = 560 - 8^2 - 6^2$
$= 460$
Variance of 5 remaining observations

$= \frac{460}{5} - \left(\frac{42}{5} \right)^2 = \frac{536}{25}$

Question61

Let n be an odd natural number such that the variance of 1, 2, 3, 4, ..., n is 14 . Then n is equal to [2021, 27 Aug. Shift-1]

Answer: 13

Solution:

Solution:

Variance $= \frac{\Sigma x^2}{n} - (\overline{x})^2$ [$\because \overline{x} =$ means]

$14 = \frac{1^2 + 2^2 + 3^3 + \ldots + n^2}{n}$

$- \left(\frac{1 + 2 + 3 + \ldots + n}{n} \right)^2$

$\Rightarrow 14 = \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n(n+1)^2}{2n} \right)$

$\Rightarrow 14 = \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$

$\Rightarrow 14 = \left(\frac{n+1}{12} \right) [2(2n+1) - 3(n+1)]$

$\Rightarrow 168 = (n+1)(n-1)$

$\Rightarrow n^2 - 1 = 168$

$\Rightarrow n^2 = 169 \Rightarrow n = 13$

Question62

An online exam is attempted by 50 candidates out of which 20 are boys. The average marks obtained by boys is 12 with a variance 2 . The variance of marks obtained by 30 girls is also 2 . The average marks of all 50 candidates is 15 . If μ is the average marks of girls and σ^2 is the variance of marks of 50 candidates, then $\mu + \sigma^2$ is equal to [2021, 27 Aug. Shift-II]

Answer: 25

Solution:

Solution:

Number of boys $= 20$

Number of girls $= 30$
$\sigma_B^2 = 2, \overline{X}_B = 12, \sigma_G^2 = 2$
$\overline{X}_G = \frac{50 \times 15 - 20 \times 12}{30}$
$= \frac{510}{30} = 17 = \mu$
Variance of 50 candidates
$\sigma^2 = \frac{20\sigma_B^2 + 30\sigma_G^2}{50} + \frac{20 \cdot 30}{(20+30)^2}$
$(\overline{X}_B - \overline{X}_G)^2$
$= \frac{20 \times 2 + 30 \times 2}{50} + \frac{600}{2500} \times 25 = 8$
$\therefore \mu + \sigma^2 = 17 + 8 = 25$

Question63

The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is
[2021, 26 Aug. Shift-1]

Options:

A. (11, 26)

B. (10.5, 25)

C. (11, 25)

D. (10.5, 26)

Answer: D

Solution:

Solution:
$\overline{x} = \frac{\sum x_i}{20} = 10$
$\Rightarrow \sum x_i = 200$
$\sigma^2 = \frac{\sum x_i^2}{20} - 100 = 6.25$
$\sum x_i^2 = 20 \times 106.25 = 2125$
Now, replacing 25 with 35 as one data, then
$\sum x_i - 25 + 35 = 210$
$\sum x_i^2 - 25^2 + 35^2 = 2725$
New mean $= \frac{210}{20} = 10.5 = (\alpha)$
New SD $= \sqrt{\frac{2725}{20} - (10.5)^2}$
SD $= \sqrt{136.25 - 110.25}$
$= \sqrt{26} = \sqrt{\beta}$
$(\alpha, \beta) = (10.5, 26)$

Question64

Let the mean and variance of four numbers 3, 7, x and y(x > y) be 5 and 10 respectively. Then, the mean of four numbers $3 + 2x_1 7 + 2y_1 x + y$ and $x - y$ is
[2021, 26 Aug. Shift-II]

Answer: 12

Solution:

Solution:
Given, Mean of 3, 7x and y is 5
So, $3 + 7 + x + y = 20$
$\Rightarrow\ x + y = 10 \ldots$ (i)
and Variance of 3, 7, x and y is 10
$\Rightarrow\ \frac{3^2 + 7^2 + x^2 + y^2}{4} - (5)^2$
$\Rightarrow\ 9 + 49 + x^2 - y^2 - 100 = 40$
$\Rightarrow\ x^2 + y^2 = 140 - 58$
$\Rightarrow\ x^2 + y^2 = 82$
$\Rightarrow x^2 + (10 - x)^2 = 82$ [Using Eq. (i)]
$\Rightarrow x^2 + 100 + x^2 - 20x = 82$ (ii)
$\Rightarrow\ 2x^2 - 20x + 18 = 0$
$\Rightarrow\ x^2 - 10x + 9 = 0$
$\Rightarrow\ (x - 9)(x - 1) = 0$
$\Rightarrow\ x = 1, 9\ (x, y) = (9, 1)$ [as $x > y$]
$\text{Mean} = \frac{3 + 2x + 7 + 2y + x + y + x - y}{4}$
$= \frac{10 + 4x + 2y}{4} = \frac{48}{4} = 12$

Question65

Let X be a random variable with distribution.

x	-2	-1	3	4	5
$P(X = X)$	$\frac{1}{5}$	a	$\frac{1}{3}$	$\frac{1}{5}$	b

If the mean of X is 2.3 and variance of X is σ^2, then $100\sigma^2$ is equal to
[2021, 01 Sep. Shift-II]

Answer: 781

Solution:

Solution:
Given, mean$\mu = 2.3$
$\Rightarrow\ \Sigma P \cdot x = 2 \cdot 3$

$\Rightarrow \frac{-2}{5} - a + 1 + \frac{4}{5} + 6b = 2 \cdot 3$
$\Rightarrow 6b - a = 0 \cdot 9...$ (i)
Also, $\Sigma P = 1$
$\frac{1}{5} + a + \frac{1}{3} + \frac{1}{5} + b = 1$
$\Rightarrow a + b = \frac{4}{15}...$ (ii)
$\Rightarrow$ Solving Eqs. (i) and (ii), we get
$a = \frac{1}{10}, b = \frac{1}{6}$
Variance $\sigma^2 = \Sigma P_i x_i^2 - \mu^2$
$= \frac{1}{5}(4) + \frac{1}{10}(1) + \frac{1}{3}(9) + \frac{1}{5}(16)$
$+ \frac{1}{6}(36) - 5.29$
$\sigma^2 = 7.81$
$\Rightarrow 100\sigma^2 = 781$

Question66

Let the observations $x_i (1 \leq i \leq 10)$ satisfy the equations, $\sum_{i=1}^{10} (x_i - 5) = 10$ and $\sum_{i=1}^{10} (x_i - 5)^2 = 40$. If μ and lambda are the mean and the variance of the observations, $x_1 - 3, x_2 - 3, ..., x_{10} - 3$, then the ordered pair (μ, λ) is equal to:
[Jan. 9, 2020 (I)]

Options:

A. (3, 3)

B. (6, 3)

C. (6, 6)

D. (3, 6)

Answer: A

Solution:

Solution:

Mean of the observation $(x_i - 5) = \frac{\sum (x_i - 5)}{10} = 1$
$\therefore \lambda = \{\text{Mean}(x_i - 5)\} + 2 = 3$
Variance of the observation
$\mu = \text{var}(x_i - 5) = \frac{\sum (x_i - 5)^2}{10} - \frac{\sum (x_i - 5)}{10} = 3$

Question67

The mean and the standard deviation (s.d.) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where p ≠ 0 and q ≠ 0. If the new mean and new s.d.

become half of their original values, then q is equal to:
[Jan. 8, 2020 (I)]

Options:

A. –5

B. 10

C. –20

D. –10

Answer: C

Solution:

Solution:
Let $\overline{x}$ and σ be the mean and standard deviations of given observations.
If each observation is multiplied with p and then q is subtracted.
New mean $(\overline{x}_1) = p\overline{x} - q$
$\Rightarrow 10 = p(20) - q$ ......(i)
and new standard deviations $\sigma_1 = |p|\sigma$
$\Rightarrow 1 = |p|(2) \Rightarrow |p| = \frac{1}{2} \Rightarrow p = \pm\frac{1}{2}$
If $p = \frac{1}{2}$, then $q = 0$ (from equation (i))
If $p = -\frac{1}{2}$, then $q = -20$

Question68

The mean and variance of 20 observations are found to be 10 and 4, respectively. On rechecking, it was found that an observation 9 was incorrect and the correct observation was 11. Then the correct variance is:
[Jan. 8, 2020 (II)]

Options:

A. 3.99

B. 4.01

C. 4.02

D. 3.98

Answer: A

Solution:

Solution:
Let $x_1, x_2, \ldots\ldots, x_{20}$ be 20 observations, then

Mean $= \frac{x_1 + x_2 + \ldots.. + x_{20}}{20} = 10$

$$\Rightarrow \frac{\sum_{i=1}^{20} x_i}{20} = 10 \text{ ......(i)}$$

Variance $= \sum x_i^2 n - (\bar{x})^2$

$$\Rightarrow \frac{\sum x_i^2}{20} - 100 = 4 \text{ ......(ii)}$$

$\sum x_i^2 = 104 \times 20 = 2080$

Actual mean $= \frac{200 - 9 + 11}{20} = \frac{202}{20}$

Variance $= \frac{2080 - 81 + 121}{20} - \left(\frac{202}{20}\right)^2$

$= \frac{2120}{20} - (10.1)^2 = 106 - 102.01 = 3.99$

Question69

If the variance of the first n natural numbers is 10 and the variance of the first m even natural numbers is 16, then m + n is equal to ________. [7-Jan-2020 Shift 1]

Answer: 18

Solution:

Solution:
Step -1: Find n using variance of first n natural number is .
$var(1, 2, 3, \ldots, n) = 10$
Using formula for variance we have,

$$\Rightarrow \frac{1^2 + 2^2 + \ldots + n^2}{n} - \left(\frac{1 + 2 + \ldots + n}{n}\right)^2 = 10$$

$$\left[\text{ Since, } \sigma^2 = \frac{\Sigma f_i d_i}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2\right]$$

$$\Rightarrow \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = 10$$

$$\Rightarrow \frac{n^2 - 1}{12} = 10$$

$\Rightarrow n^2 - 1 = 120$
$\Rightarrow n^2 = 121$
$\Rightarrow n = 11$

Step -2: The variance of the first m even natural number:
$\Rightarrow var(2, 4, 6 \ldots 2m) = 16$
$\Rightarrow 2^2 var(1, 2, \ldots, m) = 16$

$$\Rightarrow var(1, 2, \ldots, m) = \frac{16}{2^2} = 4$$

$$\Rightarrow \frac{m^2 - 1}{12} = 4$$

$\Rightarrow m^2 - 1 = 48$
$\Rightarrow m^2 = 49$
$\Rightarrow m = 7$

Step -3: Evaluate $m + n$.
$m + n = 7 + 11$
$\Rightarrow m + n = 18.$
The value of $m + n = 18.$

Question70

If the mean and variance of eight numbers 3, 7, 9, 12, 13, 20, x and y be 10 and 25 respectively, then x . y is equal to ________.
[NA Jan. 7, 2020 (II)]

Answer: 52

Solution:

Solution:

Mean $= \bar{x} = \dfrac{3 + 7 + 9 + 12 + 13 + 20 + x + y}{8} = 10$

$\Rightarrow x + y = 16$ ......(i)

Variance $= \sigma^2 = \dfrac{\sum (x_i)^2}{8} - (\bar{x})^2 = 25$

$\sigma^2 = \dfrac{9 + 49 + 81 + 144 + 169 + 400 + x^2 + y^2}{8} - 100 = 25$

$\Rightarrow x^2 + y^2 = 148$

From eqn. (i), $(x + y)^2 = (16)^2$

$\Rightarrow x^2 + y^2 + 2xy = 256$

Using eqn. (ii), $148 + 2xy = 256$

$\Rightarrow xy = 52$

Question71

Consider the data on x taking the values 0, 2, 4, 8, ..., 2^n with frequencies ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ..., ${}^nC_n$ respectively. If the mean of this data is $\frac{728}{2^n}$, then n is equal to ______.
[NA Sep. 06, 2020 (II)]

Answer: 6

Solution:

Solution:

Mean $= \dfrac{\sum x_i f_i}{\sum f_i} = \dfrac{0 \,.\, {}^nC_0 + 2 \,.\, {}^nC_1 + 2^2 \,.\, {}^nC_2 + \ldots + 2^n \,.\, {}^nC_n}{{}^nC_0 + {}^nC_1 + \ldots + {}^nC_n}$

To find sum of numerator consider

$(1 + x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \ldots\ldots + {}^nC_n x^n$ ......(i)

Put $x = 2 \Rightarrow 3^n - 1 = 2 \,.\, {}^nC_1 + 2^2 \,.\, {}^nC_2 + \ldots\ldots + 2^n \,.\, {}^nC_n$

To find sum of denominator, put x = 1 in (i), we get

$2^n = {}^nC_0 + {}^nC_1 + \ldots\ldots + {}^nC_n$

$\therefore \dfrac{3^n - 1}{2^n} = \dfrac{728}{2^n} \Rightarrow 3^n = 729 \Rightarrow n = 6$

Question72

The minimum value of $2^{\sin x} + 2^{\cos x}$ is:
[Sep. 04, 2020 (II)]

Options:

A. $2^{-1+\frac{1}{\sqrt{2}}}$

B. $2^{-1+\sqrt{2}}$

C. $2^{1-\sqrt{2}}$

D. $2^{1-\frac{1}{\sqrt{2}}}$

Answer: D

Solution:

Solution:

$\frac{2^{\sin x} + 2^{\cos x}}{2} \geq (2^{\sin x + \cos x})^{\frac{1}{2}}$ ($\because$AM $\geq$ GM)

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2 \, . \, 2^{\frac{\sin x + \cos x}{2}}$

Since, $-2 \leq \sin x + \cos x \leq \sqrt{2}$

$\therefore$ Minimum value of $2^{\frac{\sin x + \cos x}{2}} = 2^{-\frac{1}{\sqrt{2}}}$

$\Rightarrow 2^{\sin x} + 2^{\cos x} \geq 2^{1-\frac{1}{\sqrt{2}}}$.

Question73

If $\sum_{i=1}^{n} (x_i - a) = n$ and $\sum_{i=1}^{n} (x_i - a)^2 = na$, $(n, a > 1)$, then the standard deviation of n observations $x_1, x_2, \ldots\ldots, x_n$ is :
[Sep. 06, 2020 (I)]

Options:

A. $a - 1$

B. $n\sqrt{a-1}$

C. $\sqrt{n(a-1)}$

D. $\sqrt{a-1}$

Answer: D

Solution:

Solution:

Standard deviation $= \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - a)^2}{n} - \left(\dfrac{\sum_{i=1}^{n}(x_i - a)}{n}\right)^2}$ $[\because n, a > 1]$

$= \sqrt{\dfrac{na}{n} - \left(\dfrac{n}{n}\right)^2} = \sqrt{a - 1}$

Question74

The mean and variance of 7 observations are 8 and 16, respectively. If five observations are 2, 4, 10, 12, 14, then the absolute difference of the remaining two observations is : [Sep. 05, 2020 (I)]

Options:

A. 1

B. 4

C. 2

D. 3

Answer: C

Solution:

Solution:

Let two remaining observations are x_1, x_2.

So, $\bar{x} = \dfrac{2 + 4 + 10 + 12 + 14 + x_1 + x_2}{7} = 8(\text{ given })$

$\Rightarrow x_1 + y_1 = 14$ ......(i)

Now, $\sigma^2 = \dfrac{\sum x_i^2}{N} - \left(\dfrac{\sum x_i}{N}\right)^2 = 16(\text{ given })$

$= \dfrac{4 + 16 + 100 + 144 + 196 + x_1^2 + x_2^2}{7} - 64 = 16$

$\Rightarrow 460 + x_1^2 + x_2^2 = (16 + 64) \times 7$

$\Rightarrow x_1^2 + x_2^2 = 100$ ....(ii)

$\because (x + y)^2 = x^2 + y^2 + 2xy \Rightarrow xy = 48$ ....(iii)

$\because (x - y)^2 = (x + y)^2 - 4xy = 196 - 192 = 4$

$\Rightarrow x - y = 2 \Rightarrow |x - y| = 2$

Question75

If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation : [Sep. 05, 2020 (II)]

Options:

A. $x^2 - 10x + 18 = 0$

B. $2x^2 - 20x + 19 = 0$

C. $x^2 - 10x + 19 = 0$

D. $x^2 - 20x + 18 = 0$

Answer: C

Solution:

Solution:

Mean $= \dfrac{3+5+7+a+b}{5} = 5 \Rightarrow a+b = 10$

Variance $= \dfrac{3^2+5^2+7^2+a^2+b^2}{5} - (5)^2 = 4$

$\Rightarrow a^2 + b^2 = 62$

$\Rightarrow (a+b)^2 - 2ab = 62$

$\Rightarrow ab = 19$

Hence, a and b are the roots of the equation,

$x^2 - 10x + 19 = 0$

Question76

The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are 5, 7, 10, 12, 14, 15, then the absolute difference of the remaining two observations is : [Sep. 04, 2020 (I)]

Options:

A. 9

B. 5

C. 3

D. 7

Answer: D

Solution:

Solution:

Let the two remaining observations be x and y.

$\because \bar{x} = \dfrac{5+7+10+12+14+15+x+y}{8}$

$\Rightarrow 10 = \dfrac{63+x+y}{8}$

$\Rightarrow x + y = 80 - 63$

$\Rightarrow x + y = 17$ ......(i)

$\because \text{var}(x) = 13.5$

$= \dfrac{25+49+100+144+196+225+x^2+y^2}{8} - (10)^2$

$\Rightarrow x^2 + y^2 = 169$ ......(ii)

From (i) and (ii) we get

$(x, y) = (12, 5)$ or $(5,12)$

So, $|x - y| = 7$

Question77

If a variance of the following frequency distribution :

Class	10-20	20-30	30-40
Frequency	2	x	2

is 50, then x is equal to ____________.
[NA Sep. 04, 2020 (II)]

Answer: 4

Solution:

Solution:

x_i	15	25	35
f_i	2	x	2

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{30 + 70 + 25x}{4 + x} = 25$

$\sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - (\bar{x})^2$

$\Rightarrow 50 = \frac{450 + 625x + 2450}{4 + x} - 625$

$\Rightarrow 675 = \frac{2900 + 625x}{4 + x} \Rightarrow 50x = 200$

$\therefore x = 4$

Question78

For the frequency distribution :

Variate (x) :	x_1	x_2	$x_1 \dots x_{15}$
Frequency (f) :	f_1	f_2	$f_3 \dots f_{15}$

where $0 < x_1 < x_2 < x_3 < ..... < x_{15} = 10$ and $\sum_{i=1}^{15} f_i > 0$, the standard deviation cannot be :
[Sep. 03, 2020 (I)]

Options:

A. 4

B. 1

C. 6

D. 2

Answer: C

Solution:

Solution:
If variate varries from a to b then variance

$\operatorname{var}(x) \leq \left(\frac{b-a}{2}\right)^2$

$\Rightarrow \operatorname{var}(x) < \left(\frac{10-0}{2}\right)^2$

$\Rightarrow \operatorname{var}(x) < 25$

$\Rightarrow$ standard deviation <5

It is clear that standard deviation cann't be 6 .

Question79

Let $x_i (1 \leq i \leq 10)$ be ten observations of a random variable X. If $\sum_{i=1}^{10}(x_i - p) = 3$ and $\sum_{i=1}^{10}(x_i - p)^2 = 9$ where $0 \neq p \in R$, then the standard deviation of these observations is :
[Sep. 03,2020 (II)]

Options:

A. $\sqrt{\frac{3}{5}}$

B. $\frac{4}{5}$

C. $\frac{9}{10}$

D. $\frac{7}{10}$

Answer: C

Solution:

Solution:

$$\text{S.D.} = \sqrt{\frac{\sum_{i=1}^{10}(x_i-p)^2}{10} - \left(\frac{\sum_{i=1}^{10}(x_i-p)}{10}\right)^2}$$

$$= \sqrt{\frac{9}{10} - \left(\frac{3}{10}\right)^2} = \frac{9}{10}$$

Question80

Let $X = \{x \in N : 1 \leq x \leq 17\}$ and $Y = \{ax + b : x \in X$ and $a, b \in R, a > 0\}$. If mean and variance of elements of Y are 17 and 216 respectively then $a + b$ is equal to :

[Sep. 02, 2020 (I)]

Options:

A. 7

B. –7

C. –27

D. 9

Answer: B

Solution:

Solution:

$\because \bar{x} = \frac{1+2+3+.....+17}{17} = \frac{17 \times 18}{17 \times 2} = 9$

$\bar{y} = a\bar{x} + b = \frac{a(1+2+3+.....+17)}{17} + b = 17$

$\Rightarrow \frac{a\,.\,(17\,.\,18)}{17\,.\,2} + b = 17 \Rightarrow 9a + b = 17$ .....(i)

$V\,ar(x) = \sigma A^2 = \frac{\sum x^2}{n} - (\bar{x})^2$

$= \frac{1^2 + 2^2 + ... + 17^2}{17} - (9)^2$

$= \frac{17\,.\,18\,.\,35}{6\,.\,17} - (9)^2 = 105 - 81 = 24$

$V\,ar(y) = a^2 V\,ar(x) = a^2\,.\,24 = 216$

$a^2 = \frac{216}{24} = 9 \Rightarrow a = 3$

$\therefore$ From (i), $b = 17 - 9a = 17 - 27 = -10$

$\therefore a + b = 3 + (-10) = -7$

Question81

If the variance of the terms in an increasing A.P. $b_1, b_2, b_3, ..., b_{11}$ is 90, then the common difference ofthis A.P. is ________.
[NA Sep. 02, 2020 (II)]

Answer: 3

Solution:

Solution:

$$\text{Variance} = \frac{\sum_{i=1}^{11} b_i^2}{11} - \left(\frac{\sum_{i=1}^{11} b_i}{11}\right)^2$$

Let common difference of A.P. be d

$$= \frac{\sum_{r=0}^{10} (b_1 + rd)^2}{11} - \left(\frac{\sum_{r=0}^{10} (b_1 + rd)}{11}\right)^2$$

$$= \frac{11b_1^{\,2} + 2b_1 d\left(\frac{10 \times 11}{2}\right) + d^{\,2}\left(\frac{10 \times 11 \times 21}{6}\right)}{11} - \left(\frac{11b_1 + \frac{10 \times 11}{2}d}{11}\right)^2$$

$= (b_1^{\,2} + 10b_1 d + 35d^{\,2}) - (b_1 + 5d)^2 = 10d^{\,2}$

∵ Variance = 90 (Given)

$\Rightarrow 10d^{\,2} = 90 \Rightarrow d = 3$

Question82

If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is : [Jan. 12, 2019 (I)]

Options:

A. 30

B. 51

C. 50

D. 31

Answer: D

Solution:

Solution:

Given, $\sum_{i=1}^{50}(x_i - 30) = 50$

$\sum_{i=1}^{50} x_i - 50(30) = 50$

$\Rightarrow \sum_{i=1}^{50} x_i = 1550$

Mean, $\overline{x} = \sum x_i 50$

$= \frac{1550}{50} = 31$

Question83

The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then the absolute value of the difference of the other two observations, is : [Jan. 12, 2019 (II)]

Options:

A. 7

B. 5

C. 1

D. 3

Answer: A

Solution:

Solution:

Let two observations be x_1 and x_2, then

$\frac{x_1 + x_2 + 3 + 4 + 4}{5} = 4$

$\Rightarrow x_1 + x_2 = 9$ ......(i)

Variance $= \frac{\sum x_i^2}{N} - (\bar{x})^2$

$(5 . 20) = \frac{9 + 16 + 16 + x_1^2 + x_2^2}{5} - 16$

$26 = 41 + x_1^2 + x_2^2 - 80$

$x_1^2 + x_2^2 = 65$ ......(ii)

From (i) and (ii);

$x_1 = 8, x_2 = 1$

Hence, the required value of the difference of other two observations $= |x_1 - x_2| = 7$

Question84

The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then $|d|$ equals:
[Jan. 11, 2019 (I)]

Options:

A. $\frac{2}{3}$

B. 2

C. $\frac{\sqrt{5}}{2}$

D. $\sqrt{2}$

Answer: D

Solution:

Solution:

Outcomes are $\left(\frac{1}{2} - d\right), \left(\frac{1}{2} - d\right), 0\ldots.., 10$ times, $\frac{1}{2}, \frac{1}{2}, \ldots., 10$ times $, \frac{1}{2} + d, \frac{1}{2} + d, \ldots., 10$ times

Mean $= \frac{1}{30}\left(\frac{1}{2} \times 30\right) = \frac{1}{2}$

Variance of the outcomes is,

$\sigma^2 = \frac{1}{30}\sum x_i^2 - (\bar{x})^2$

$= \frac{1}{30}\left[\left(\frac{1}{2} - d\right)^2 \times 10 + \left(\frac{1}{2}\right)^2 \times 10 + \left(\frac{1}{2} + d\right)^2 \times 10\right] - \frac{1}{4}$

$\Rightarrow \frac{4}{3} = \frac{1}{30}\left[30 \times \frac{1}{4} + 20d^2\right] - \frac{1}{4}$

$\Rightarrow \frac{4}{3} = \frac{1}{4} + \frac{2}{3}d^2 - \frac{1}{4}$

$\Rightarrow d^2 = 2 \Rightarrow | d | = \sqrt{2}$

Question85

A data consists of n observations: $x_1, x_2, \ldots\ldots, x_n$. If $\sum_{i=1}^{n}(x_i + 1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i - 1)^2 = 5n$ then the standard deviation of this data is: [Jan. 09, 2019 (II)]

Options:

A. 2

B. $\sqrt{5}$

C. 5

D. $\sqrt{7}$

Answer: B

Solution:

Solution:
Variance is given by,

$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2 - \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2$

$\sigma^2 = \frac{1}{n}A - \frac{1}{n^2}B^2$ .....(i)

Here, $A = \sum_{i=1}^{n} x_i^2$ and $B = \sum_{i=1}^{n} x_i$

$\because \sum_{i=1}^{n}(x_i + 1)^2 = 9n$

$\Rightarrow A + n + 2B = 9n \Rightarrow A + 2B = 8n$ ......(ii)

$\because \sum_{i=1}^{n}(x_i - 1)^2 = 5n$

$\Rightarrow A + n - 2B = 5n \Rightarrow A - 2B = 4n$ ......(iii)

From (ii) and (iii),

$A = 6n, B = n$

$\Rightarrow \sigma^2 = \frac{1}{n} \times 6n - \frac{1}{n^2} \times n^2 = 6 - 1 = 5$

$\Rightarrow \sigma = \sqrt{5}$

Question86

The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is: [Jan. 10, 2019 (I)]

Options:

A. 10 : 3

B. 4 : 9

C. 5 : 8

D. 6 : 7

Answer: B

Solution:

Solution:
Since mean of x_1, x_2, x_3, x_4 and x_5 is 5
$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 25$
$\Rightarrow 1 + 3 + 8 + x_4 + x_5 = 25$
$\Rightarrow x_4 + x_5 = 13$ ....(i)
$\because \sum_{i=1}^{5} x_i^2 5 - (5)^2 = 9.2 \Rightarrow \sum_{i=1}^{5} x_i^2 = 5(25 + 9.2)$
$= 125 + 46 = 171$
$\Rightarrow (1)^2 + (3)^2 + (8)^2 + x_4^2 + x_5^2 = 171$
$\Rightarrow x_4^2 + x_5^2 = 97$ ......(ii)
$\Rightarrow (x_4 + x_5)^2 - 2x_4x_5 = 97$
$\Rightarrow 2x_4x_5 = 13^2 - 97 = 72 \Rightarrow x_4x_5 = 36$ ......(iii)
(i) and (iii) $\Rightarrow x_4 : x_5 = \frac{4}{9}$ or $\frac{9}{4}$

Question87

If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5 are 10 and 3 , respectively, then the variance of 6 observations x_1, x_2, ..., x_5 and -50 is equal to:
[Jan. 10,2019 (II)]

Options:

A. 509.5

B. 586.5

C. 582.5

D. 507.5

Answer: D

Solution:

Solution:
$\because \bar{x} = \frac{\sum_{i=1}^{5} x_i}{5} \Rightarrow \sum_{i=1}^{5} x_i = 10 \times 5 = 50 \Rightarrow \sum_{i=1}^{6} xi = 50 - 50 = 0$
$\frac{\sum_{i=1}^{5} x_i^2}{5} - (10)^2 = 3^2 = 9$
$\Rightarrow \sum_{i=1}^{5} x_i^2 = 545$
Then,

$$\Rightarrow \sum_{i=1}^{6} x_i^2 = \sum_{i=1}^{5} x_i^2 + (-50)^2$$
$$= 545 + (-50)^2 = 3045$$
$$\text{Variance } = \frac{\sum_{i=1}^{6} x_i^2}{6} - \left(\sum_{i=1}^{6} x_i 6 \right)^2 = \frac{3045}{6} - 0 = 507.5$$

Question88

5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is : [9-Jan-2019 Shift 1]

Options:

A. 22

B. 20

C. 16

D. 18

Answer: B

Solution:

Solution:

$$\text{Given } \bar{x} = \frac{\Sigma x_i}{5} = 150$$
$$\Rightarrow \sum_{i=1}^{5} x_i = 750 \ldots. \text{ (i)}$$
$$\sigma^2 = 18$$
$$\frac{\Sigma x_i^2}{5} - (\bar{x})^2 = 18$$
$$\frac{\Sigma x_i^2}{5} - (150)^2 = 18$$
$$\sum x_i^2 = 112590 \ldots. \text{ (ii)}$$
Given height of new student
$$x_6 = 156$$
$$\text{Now, } \bar{x}_{new} = \frac{\sum_{i=1}^{5} x_i}{6} = \frac{750 + 156}{6} = 151$$
$$\text{Also, } \sigma_{new}{}^2 = \frac{\sum_{i=1}^{5} x_i^2}{6} - (\bar{x}_{new})^2$$
$$= \frac{112590 + (156)^2}{6} - (151)^2$$
$$= 22821 - 22801 = 20.$$

Question89

If for some x ∈ R, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequency	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is :
[April 10, 2019 (I)]

Options:

A. 3.2

B. 3.0

C. 2.5

D. 2.8

Answer: D

Solution:

Solution:
Number of students are,
$(x+1)^2 + (2x-5) + (x^2-3x) + x = 20$
$\Rightarrow 2x^2 + 2x - 4 = 20 \Rightarrow x^2 + x - 12 = 0$
$\Rightarrow (x+4)(x-3) = 0 \Rightarrow x = 3$
$\therefore$

Marks	2	3	5	7
No. of students	16	1	0	3

Average marks $= \frac{32+3+21}{20} = \frac{56}{20} = 2.8$

Question90

The mean and the median of the following ten numbers in increasing order 10, 22, 26, 29, 34, x, 42, 67, 70, y are 42 and 35 respectively, then $\frac{y}{x}$ is equal to:
[April. 09, 2019 (II)]

Options:

A. 9/4

B. 7/2

C. 8/3

D. 7/3

Answer: D

Solution:

Solution:
Ten numbers in increasing order are 10, 22, 26, 29, 34, x, 42 ,67, 70, y

Mean $= \frac{\sum x_i}{n} = \frac{x + y + 300}{10} = 42 \Rightarrow x + y = 120$

Median $= \frac{T_5 + T_6}{2} = 35 = \frac{34 + x}{2} \Rightarrow x = 36$ and $y = 84$

Hence ,$\frac{y}{x} = \frac{84}{36} = \frac{7}{3}$

Question91

If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11 , the mean of the remaining six is 16 and the sum of squares of all of these is 2,000 ; then the standard deviation of this data is :
[April 12, 2019 (I)]

Options:

A. $2\sqrt{2}$

B. 2

C. 4

D. $\sqrt{2}$

Answer: B

Solution:

Solution:
According to the question,

$\frac{x_1 + x_2 + x_3 + x_4}{4} = 11 \Rightarrow x_1 + x_2 + x_3 + x_4 = 44$

$\frac{x_5 + x_6 + ... + x_{10}}{6} = 16 \Rightarrow x_5 + x_6 + ... + x_{10} = 96$

and $x_1^2 + x_2^2 + ... x_{10}^2 = 2000$

$\because$ standard deviation, $\sigma^2 = \frac{\because x_i^2}{N} - (\overline{x})^2$

$= \frac{2000}{10} - \left(\frac{44 + 96}{10}\right)^2 = 4 \Rightarrow \sigma = 2$

Question92

If both the mean and the standard deviation of 50 observations $x_1, x_2, ..., x_{50}$ are equal to 16, then the mean of $(x_1 - 4)^2, (x_2 - 4)^2, ..., (x_{50} - 4)^2$ is :
[April 10, 2019 (II)]

Options:

A. 400

B. 380

C. 525

D. 480

Answer: A

Solution:

Solution:
Given, mean and standard deviation are equal to 16.

$\therefore \frac{x_1 + x_2 + \ldots\ldots x_{50}}{50} = 16$

and $16^2 = \frac{x_1{}^2 + x_2{}^2 \ldots\ldots x_{50}{}^2}{50} - 16^2$

$\Rightarrow 2(16)^2 50 = x_1{}^2 + x_2{}^2 + \ldots\ldots x_{50}{}^2$

Required mean $= \frac{(x_1 - 4)^2 + (x_2 - 4)^2 + \ldots\ldots (x_{50} - 4)^2}{50}$

$= \frac{x_1{}^2 + x_2{}^2 + \ldots\ldots + x_{50}{}^2 + 50 \times 16 - 8(x_1 + x_2 + \ldots\ldots + x_{50})}{50}$

$= \frac{16^2(100) + (50) - 8(16 \times 50)}{50} = 400$

Question93

If the standard deviation of the numbers $-1, 0, 1, k$ is $\sqrt{5}$ where $k > 0$, then k is equal to: [April 09, 2019 (I)]

Options:

A. $2\sqrt{6}$

B. $2\sqrt{\frac{10}{3}}$

C. $4\sqrt{\frac{5}{3}}$

D. $\sqrt{6}$

Answer: A

Solution:

Solution:
Mean of given observation $= \frac{k}{4}$

$\because$ Standard deviation $= 5$

$\therefore \sigma^2 = 5$

$\Rightarrow \sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$

$= \frac{\left(\frac{k}{4} + 1\right)^2 + \left(\frac{k}{4}\right)^2 + \left(\frac{k}{4} - 1\right)^2 + \left(\frac{3k}{4}\right)^2}{4} = 5$

$$\Rightarrow \frac{\frac{12k^2}{16} + 2}{4} = 5 \Rightarrow k = 2\sqrt{6}$$

Question94

The mean and variance of seven observations are 8 and 16, respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the product of the remaining two observations is : [April 08, 2019 (I)]

Options:

A. 45

B. 49

C. 48

D. 40

Answer: C

Solution:

Solution:
Let the remaining numbers are a and b.

Mean $(\bar{x}) = \frac{\sum x_i}{N} = \frac{2 + 4 + 10 + 12 + 14 + a + b}{7} = 8$

$\Rightarrow a + b = 14$ ......(i)

Variance $(\sigma^2) = \frac{\sum x_i{}^2}{N} - (\bar{x})^2 = 16$

$\Rightarrow \frac{2^2 + 4^2 + 10^2 + 12^2 + 14^2 + a^2 + b^2}{7} - (8)^2 = 16 \Rightarrow a^2 + b^2 = 100$ ......(ii)

From (i) and (ii) ,$(14 - b)^2 + b^2 = 100$

$\Rightarrow 196 + b^2 - 28b + b^2 = 100$

$\Rightarrow b^2 - 14b + 48 = 0$

$\Rightarrow b = 6, 8$

$\therefore (a, b) = (6, 8)$ or $(8, 6)$

Hence, the product of the remaining two observations $= ab = 48$

Question95

A student scores the following marks in five tests: 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is : [April. 08, 2019 (II)]

Options:

A. $\frac{10}{\sqrt{3}}$

B. $\frac{100}{3}$

C. $\frac{10}{3}$

D. $\frac{100}{\sqrt{3}}$

Answer: A

Solution:

Solution:
$\because$ Mean score $= 48$
Let unknown score be x,
$\therefore \bar{x} = \frac{41 + 45 + 54 + 57 + 43 + x}{6} = 48$
$\Rightarrow x + 240 = 288 \Rightarrow x = 48$
Now, $\sigma^2 = \frac{1}{6}[(48 - 41)^2 + (48 - 45)^2 + (48 - 54)^2 + (48 - 57)^2 + (48 - 43)^2 + (48 - 48)^2$
$= \frac{1}{6}(49 + 9 + 36 + 81 + 25) = \frac{200}{6} = \frac{100}{3}$
$\sigma = \frac{10}{\sqrt{3}}$

Question96

The mean of a set of 30 observations is 75. If each other observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. The l is equal to [Online April 15, 2018]

Options:

A. $\frac{10}{3}$

B. $\frac{4}{3}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

Answer: B

Solution:

Solution:
As mean is a linear operation, so if each observation is multiplied by lambda and decreased by 25 then the mean becomes $75\lambda - 25$.
According to the question,
$75\lambda - 25 = 75 \Rightarrow \lambda = \frac{4}{3}$

Question97

The mean and the standard deviation (s.d.) of five observations are 9 and 0, respectively.
If one of the observations is changed such that the mean of the new set of five observations becomes 10, then their s.d. is?
[Online April 16, 2018]

Options:

A. 0

B. 4

C. 2

D. 1

Answer: C

Solution:

Solution:
Here mean $= \overline{x} = 9$
$\Rightarrow \overline{x} = \frac{\sum x_i}{n} = 9$
$\Rightarrow \sum x_i = 9 \times 5 = 45$
Now, standard deviation $= 0$
$\therefore$ all the five terms are same i.e.; 9.
Now for changed observation
$\overline{x}_{new} = \frac{36 + x_5}{5} = 10$
$\Rightarrow x_5 = 14$
$\therefore \sigma_{new} = \sqrt{\frac{\sum (x_i - \overline{x}_{new})^2}{n}}$
$= \sqrt{\frac{4(9-10)^2 + (14-10)^2}{5}} = 2$

Question98

If the mean of the data :7, 8, 9, 7, 8, 7, λ, 8 is 8, then the variance of this data is
[Online April 15, 2018]

Options:

A. $\frac{9}{8}$

B. 2

C. $\frac{7}{8}$

D. 1

Answer: D

Solution:

Solution:

$\bar{x} = \frac{7+8+9+7+8+7+\lambda+8}{8} = 8$

$\Rightarrow \frac{54+\lambda}{8} = 8 \Rightarrow \lambda = 10$

Now variance $= \sigma^2$

$= \frac{(7-8)^2+(8-8)^2+(9-8)^2+(7-8)^2+(8-8)^2+(7-8)^2+(10-8)^2+(8-8)^2}{8}$

$\Rightarrow \sigma^2 = \frac{1+0+1+1+0+1+4+0}{8} = \frac{8}{8} = 1$

Hence, the variance is 1 .

Question99

If $\sum_{i=1}^{9}(x_i-5) = 9$ and $\sum_{i=1}^{9}(x_i-5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots\ldots, x_9$ is: [2018]

Options:

A. 4

B. 2

C. 3

D. 9

Answer: B

Solution:

Solution:

Given $\sum_{i=1}^{9}(x_i-5) = 9 \Rightarrow \sum_{i=1}^{9} x_i = 54$ ......(i)

Also, $\sum_{i=1}^{9}(x_i-5)^2 = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 - 10\sum_{i=1}^{9} x_i + 9(25) = 45$ ......(ii)

From (i) and (ii) we get,

$\sum_{i=1}^{9} x_i^2 = 360$

Since, variance $= \frac{\sum x_i^2}{9} - \left(\frac{\sum x_i}{9}\right)^2$

$= \frac{360}{9} - \left(\frac{54}{9}\right)^2 = 40 - 36 = 4$

$\therefore$ Standard deviation $= \sqrt{\text{Variance}} = 2$

Question100

The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If now the mean age of the teachers in this school is 39 years, then the age (in

years) of the newly appointed teacher is :
[Online April 8, 2017]

Options:

A. 25

B. 30

C. 35

D. 40

Answer: C

Solution:

Solution:

Let; $\frac{x_1 + x_2 + \ldots\ldots + x_{25}}{25} = \bar{x} = 40$

$\Rightarrow x_1 + x_2 + \ldots\ldots + x_{25} = 1000$

$\therefore x_2 + x_2 + \ldots\ldots + x_{25} - 60 + A = 39 \times 25$

Let A be the age of new teacher.

$\Rightarrow 1000 - 60 + A = 975$

$\Rightarrow A = 975 - 940 = 35$

Question101

The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3, 4 and 5, were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is : [Online April 9, 2017]

Options:

A. 8.25

B. 8.50

C. 8.00

D. 9.00

Answer: D

Solution:

Solution:

$\sum_{i=1}^{100} x_i = 400 \quad \sum_{i=1}^{100} x_i^2 = 2475$

Variance $= \sigma^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$

$= \frac{2475}{97} - \left(\frac{388}{97}\right)^2$

$= \frac{2425 - 1552}{97} = \frac{873}{97} = 9$

Question102

If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?
[2016]

Options:

A. $3a^2 - 34a + 91 = 0$

B. $3a^2 - 23a + 44 = 0$

C. $3a^2 - 26a + 55 = 0$

D. $3a^2 - 32a + 84 = 0$

Answer: D

Solution:

Solution:

$\bar{x} = \frac{2 + 3 + a + 11}{4} = \frac{a}{4} + 4$

$\sigma = \sqrt{\sum \frac{x_i^2}{n} - (\bar{x})^2}$

$\Rightarrow 3.5 = \sqrt{\frac{4 + 9 + a^2 + 121}{4} - \left(\frac{a}{4} + 4\right)^2}$

$\Rightarrow \frac{49}{4} = \frac{4(134 + a^2) - (a^2 + 256 + 32a)}{16}$

$\Rightarrow 3a^2 - 32a + 84 = 0$

Question103

The mean of 5 observations is 5 and their variance is 124. If three of the observations are 1, 2 and 6; then the mean deviation from the mean of the data is :
[Online April 10, 2016]

Options:

A. 2.5

B. 2.6

C. 2.8

D. 2.4

Answer: C

Solution:

Solution:

$n = 5$
$\bar{x} = 5$
variance = 124
$x_1 = 1, x_2 = 2, x_3 = 6$
$\bar{x} = 5$

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 5$$

$\Rightarrow x_4 + x_5 + 9 = 25$
$\Rightarrow x_4 + x_5 = 16$
$\Rightarrow x_4 + x_5 + 10 - 10 = 16$
$\Rightarrow (x_4 - 5) + (x_5 - 5) = 16 - 10$
$\Rightarrow (x_4 - 5) + (x_5 - 5) = 6$

Mean deviation $= \frac{\sum x_i - \bar{x}|}{N}$

$$= |x_1 - 5| + |x_2 - 5| + |x_3 - 5| + \frac{|x_4 - 5| + |x_5 - 5|}{5}$$

$$= \frac{4 + 3 + 1 + 6}{5} = \frac{14}{5} = 2.8$$

Question104

If the mean deviation of the numbers 1, 1 + d, ..., 1 + 100d from their mean is 255, then a value of d is : [Online April 9, 2016]

Options:

A. 10.1

B. 5.05

C. 20.2

D. 10

Answer: A

Solution:

Solution:

$$\bar{x} = \frac{1}{101}[1 + (1 + d) + (1 + 2d)] \ldots\ldots.(1 + 100d)]$$

$$= \frac{1}{101} \times \frac{101}{2}[1 + (1 + 100d)] = 1 + 50d$$

mean deviation from mean

$$= \frac{1}{101}[\,|1 - (1 + 50d)| + |(1 + d) - (1 + 50d)| \ldots\ldots\ldots|[1 + 100d] - (1 + 50d)]\,]$$

$$= \frac{2|d|}{101}(1 + 2 + 3 \ldots\ldots + 50)$$

$$= \frac{2|d|}{101} \times \frac{50 \times 51}{2} = \frac{2550}{101}|d|$$

$$= \frac{2550}{101}|d| = 225 \Rightarrow |d| = 10.1$$

Question105

The mean of the data set comprising of 16 observations is 16. If one of

the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is: [2015]

Options:

A. 15.8

B. 14.0

C. 16.8

D. 16.0

Answer: B

Solution:

Solution:
Sum of 16 observations $= 16 \times 16 = 256$
Sum of resultant 18 observations
$= 256 - 16 + (3 + 4 + 5) = 252$
Mean of observations $= \frac{252}{18} = 14$

Question106

Let the sum of the first three terms of an A. P, be 39 and the sum of its last four terms be 178. If the first term of this A.P. is 10, then the median of the A.P. is : [Online April 10, 2015]

Options:

A. 28

B. 26.5

C. 29.5

D. 31

Answer: C

Solution:

Solution:
$a_1 + a_2 + a_3 = 39$
$\Rightarrow a_1 + (a_1 + d) + (a_1 + 2d) = 39$
$\Rightarrow 3a_1 + 3d = 39 \; [\because a_1 = 10]$
$\Rightarrow d = 3$
Sum of last four term $= 178$
Their mean $= \frac{178}{4} = 44.5$
$a_n = 44.5 + 1.5 + 3 = 49$
Median $= \frac{10 + 49}{2} = \frac{59}{2} = 29.5$

Question107

A factory is operating in two shifts, day and night, with 70 and 30 workers respectively. If per day mean wage of the day shift workers is Rs.54 and per day mean wage of all the workers is Rs.60, then per day mean wage of the night shift workers (in Rs.) is :
[Online April 10, 2015]

Options:

A. 69

B. 66

C. 74

D. 75

Answer: C

Solution:

Solution:
Let average wage of Night shift worker is X
$70 \times 54 + 30 \times x = 60 \times 100$
$x = 74$

Question108

In a set of 2n distinct observations, each of the observations below the median of all the observations is increased by 5 and each of the remaining observations is decreased by 3. Then the mean of the new set of observations:
[Online April 9, 2014]

Options:

A. increases by 1

B. decreases by 1

C. decreases by 2

D. increases by 2

Answer: A

Solution:

Solution:
There are 2n observations $x_1, x_2, \ldots.., x_{2n}$

So, mean $= \sum_{i=1}^{2n} \frac{x_i}{2n}$

Let these observations be divided into two parts $x_1, x_2, \ldots\ldots, x_n$ and $x_{n+1}, \ldots\ldots., x_{2n}$

Each in 1st part 5 is added, so total of first part is $\sum_{i=1}^{n} x_i + 5n$.

In second part 3 is subtracted from each

So, total of second part is $\sum_{i=n+1}^{2n} x_i - 3n$

Total of $2n$ terms are

$\sum_{i=1}^{n} x_i + 5n + \sum_{i=n+1}^{2n} x_i - 3n = \sum_{i=1}^{2n} x_i + 2n$

Mean $= \sum_{i=1}^{2n} \frac{x_i + 2n}{2n} = \sum_{i=1}^{2n} \frac{x_i}{2n} + 1$

So, it increase by 1 .

Question109

The variance of first 50 even natural numbers is [2014]

Options:

A. 437

B. $\frac{437}{4}$

C. $\frac{833}{4}$

D. 833

Answer: D

Solution:

Solution:

First 50 even natural numbers are $2, 4, 6 \ldots\ldots, 100$

Variance $= \frac{\sum x_i^2}{N} - (\overline{x})^2$

$\Rightarrow \sigma^2 = \frac{2^2 + 4^2 + \ldots. + 100^2}{50} - \left(\frac{2 + 4 + \ldots.. + 100}{50}\right)^2$

$= \frac{4(1^2 + 2^2 + 3^2 + \ldots.. + 50^2)}{50} - (51)^2$

$= 4\left(\frac{50 \times 51 \times 101}{50 \times 6}\right) - (51)^2$

$= 3434 - 2601 \Rightarrow \sigma^2 = 833$

Question110

Let $\overline{x}$, M and sigma^2 be respectively the mean, mode and variance of n observations $x_1, x_2, \ldots\ldots., x_n$ and $d_i = -x_i - a$, $i = 1, 2, \ldots\ldots., n$, where a is any number.

Statement I: Variance of $d_1, d_2, \ldots\ldots d_n$ is σ^2

Statement II: Mean and mode of $d_1, d_2, \ldots\ldots d_n$ are $-\overline{x} - a$and $-M - a$,

respectively.
[Online April 19, 2014]

Options:

A. Statement I and Statement II are both false

B. Statement I and Statement II are both true

C. Statement I is true and Statement II is false

D. Statement I is false and Statement II is true

Answer: B

Solution:

Solution:

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots. + x_n}{n}$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Mean of $d_1, d_2, d_3, \ldots., d_n$

$$= \frac{d_1 + d_2 + d_3 + \ldots. + d_n}{n}$$

$$= \frac{(-x_1 - a) + (-x_2 - a) + (-x_3 - a) + \ldots\ldots + (-x_n - a)}{n}$$

$$= -\left[\frac{x_1 + x_2 + x_3 + \ldots. + x_n}{n}\right] - \frac{na}{n} = -\bar{x} - a$$

Since, $d_i = -x_i - a$ and we multiply or subtract each observation by any number the mode remains the same. Hence mode of $-x_i - a$ i.e. d_i and x_i are same.

Now variance of $d_1, d_2, \ldots., d_n$

$$= \frac{1}{n}\sum_{i=1}^{n}[d_i - (-\bar{x} - a)]^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}[-x_i - a + \bar{x} + a]^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}(-x_i + \bar{x})^2 = \frac{1}{n}\sum_{i=1}^{n}(\bar{x} - x_i)^2 = \sigma^2$$

Question111

Let $\overline{X}$ and M.D. be the mean and the mean deviation about $\overline{X}$ of n observations x_i, i = 1, 2, ..., n. If each of the observations is increased by 5, then the new mean and the mean deviation about the new mean, respectively, are:
[Online April 12, 2014]

Options:

A. $\overline{X}$, M . D

B. $\overline{X} + 5$, M . D

C. $\overline{X}$, M . D. + 5

D. $\overline{X} + 5$, M . D. + 5

Answer: B

Solution:

Solution:
Let x_i be n observations, $i = 1, 2, \ldots\ldots n$
Let $\overline{X}$ be the mean and M.D be the mean deviation about $\overline{X}$.
If each observation is increased by 5 then new mean will be $\overline{X} + 5$ and new M.D. about new mean will be M.D.
$\left(\because \text{Mean} = \sum_{i=1}^{n} \frac{x_i}{n}\right)$

Question112

If the median and the range of four numbers {x, y, 2x + y, x – y}, where 0 < y < x < 2y, are 10 and 28 respectively, then the mean of the numbers is : [Online April 23, 2013]

Options:

A. 18

B. 10

C. 5

D. 14

Answer: D

Solution:

Solution:
Since $0 < y < x < 2y$
$\therefore y > \frac{x}{2} \Rightarrow x - y < \frac{x}{2}$
$\therefore x - y < y < x < 2x + y$
Hence median $= \frac{y + x}{2} = 10$
$\Rightarrow x + y = 20$ ....(i)
And range $= (2x + y) - (x - y) = x + 2y$
But range $= 28$
$\therefore x + 2y = 28$ ....(ii)
From equations (i) and (ii),
$x = 12, y = 8$
Mean $= \frac{(x - y) + y + x + (2x + y)}{4} = \frac{4x + y}{4}$
$= x + \frac{y}{4} = 12 + \frac{8}{4} = 14$

Question113

The mean of a data set consisting of 20 observations is 40. If one observation 53 was wrongly recorded as 33, then the correct mean will be :

[Online April 9, 2013]

Options:

A. 41

B. 49

C. 40.5

D. 42.5

Answer: A

Solution:

Solution:

Correct mean $= \frac{20 \times 40 - 33 + 55}{20} = 41.1$

Nearest option : (a) 41

Question114

All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?
[2013]

Options:

A. mean

B. median

C. mode

D. variance

Answer: D

Solution:

Solution:

If initially all marks were x_i then

$$\sigma_1{}^2 = \frac{\sum_i (x_i - \overline{x})^2}{N}$$

Now each is increased by 10

$$\sigma_1{}^2 = \frac{\sum_i [(x_i + 10) - (\overline{x} + 10)]^2}{N} = \frac{\sum_i (x_i - \overline{x})^2}{N} = \sigma_1{}^2$$

Hence, variance will not change even after the grace marks were given.

Question115

In a set of 2n observations, half of them are equal to 'a' and the remaining half are equal to ' –a'. If the standard deviation of all the observations is 2 ; then the value of | a | is :
[Online April 25, 2013]

Options:

A. 2

B. $\sqrt{2}$

C. 4

D. $2\sqrt{2}$

Answer: A

Solution:

Solution:
Clearly mean $A = 0$

Now, standard deviation $\sigma = \sqrt{\frac{\sum(x - A)^2}{2n}}$

$2 = \sqrt{\frac{(a-0)^2 + (a-0)^2 + \ldots\ldots + (0-a)^2 + \ldots\ldots}{2n}}$

$= \sqrt{\frac{a^2 \cdot 2n}{2n}} = |a|$

Hence, $|a| = 2$

Question116

Mean of 5 observations is 7. If four of these observations are 6, 7, 8, 10 and one is missing then the variance of all the five observations is : [Online April 22, 2013]

Options:

A. 4

B. 6

C. 8

D. 2

Answer: D

Solution:

Solution:
Let 5 th observation be x.
Given mean $= 7$

$\therefore 7 = \frac{6+7+8+10+x}{5}$

$\Rightarrow x = 4$

Now, Variance

$$= \sqrt{\frac{(6-7)^2 + (7-7)^2 + (8-7)^2 + (10-7)^2 + (4-7)^2}{5}}$$

$$= \sqrt{\frac{1^2 + 0^2 + 1^2 + 3^2 + 3^2}{5}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2$$

Question117

The median of 100 observations grouped in classes of equal width is 25. If the median class interval is 20 - 30 and the number of observations less than 20 is 45, then the frequency of median class is [Online May 19, 2012]

Options:

A. 10

B. 20

C. 15

D. 12

Answer: A

Solution:

Solution:
Median is given as

$$M = l + \frac{\frac{N}{2} - F}{f} \times C$$

where
l = lower limit of the median - class
f = frequency of the median class
N = total frequency
F = cumulative frequency of the class just before the median class
C = length of median class
Now, given, $M = 25, N = 100, F = 45, C = 20 - 30 = 10, l = 20$
∴ Byusing formula, we have

$$25 = 20 + \frac{50-45}{f} \times 10$$

$$25 - 20 = \frac{50}{f} \Rightarrow 5 = \frac{50}{f} \Rightarrow f = 10$$

Question118

The frequency distribution of daily working expenditureof families in a locality is as follows:

Expenditurein Rs. (x):	0-50	50-100	100-150	150-200	200-250
No. offamilies (f):	24	33	37	b	25

If the mode of the distribution is Rs.140, then the value of bis [Online May 7, 2012]

Options:

A. 34

B. 31

C. 26

D. 36

Answer: D

Solution:

Solution:
Frequency distribution is given as

Expenditure	No. of families (f)
0-50	24
50-100	33
100-150	37
150-200	b
200-250	25

Clearly, modal class is 100-150, as the maximum frequency occurs in this class.
Given, Mode = 140
We have

Mode $= l + \frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_1} \times i$

where
$l = 100, f_0 = 37, f_{-1} = 33, f_1 = b$
$i = 50$
Thus, we get

$140 = 100 + \left[\frac{37 - 33}{2(37) - 33 - b}\right] \times 50$

$= 100 + \left[\frac{4}{74 - 33 - b}\right] \times 50 = 100 + \frac{200}{41 - b}$

$\Rightarrow 5740 = 4300 + 40b \Rightarrow b = 36$

Question119

Let $x_1, x_2, ..., x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance.
Statement-1 : Variance of $2x_1, 2x_2, ..., 2x_n$ is $4\sigma^2$
Statement- 2 : Arithmetic mean $2x_1, 2x_2, ..., 2x_n$ is $4\bar{x}$.
[2012]

Options:

A. Statement-1 is false, Statement-2 is true.

B. Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

C. Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for Statement-1.

D. Statement-1 is true, statement-2 is false.

Answer: D

Solution:

Solution:
A.M. of $2x_1, 2x_2, \ldots, 2x_n$ is

$$\frac{2x_1 + 2x_2 + \ldots + 2x_n}{n} = 2\left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right) = 2\bar{x} \left(\because \text{Mean} = \frac{\text{Sum of observations}}{\text{Number of observations}}\right)$$

So statement- 2 is false.
If each observations is multiply by 2 then mean multiply by 2 and variance multiply by 2^2.
variance $(2x_i) = 2^2$ variance $(x_i) = 4\sigma^2$ where $i = 1, 2, \ldots . n$
So statement- 1 is true.

Question120

Statement 1: The variance of first n odd natural numbers is $\frac{n^2 - 1}{3}$
Statement 2 : The sum of first n odd natural number is n^2 and the sum of square of first n odd natural numbers is $\frac{n(4n^2 + 1)}{3}$
[Online May 26, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

C. Statement 1 is false, Statement 2 is true.

D. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

Answer: A

Solution:

Solution:
Statement 2: Sum of first n odd natural numbers is not equal to n^2.
So, statement - 2 is false.

Question121

If the mean of 4, 7, 2, 8, 6 and a is 7, then the mean deviation from the median of these observations is
[Online May 12, 2012]

Options:

A. 8

B. 5

C. 1

D. 3

Answer: D

Solution:

Solution:
Given observations are 4, 7, 2, 8, 6, a and mean is 7 .
We know

Mean $= \frac{4+7+2+8+6+a}{6}$

$\Rightarrow 7 = \frac{4+7+2+8+6+a}{6} \Rightarrow a = 15$

Now, given observations can be written in ascending order which is 2,4,6,7,8,15
Since, No. of observation is even

$\therefore$ Median $= \frac{\left(\frac{6}{2}\right) \text{ th observation } + \left(\frac{6}{2}+1\right) \text{ th observation}}{2}$

$= \frac{3 \text{ rd observation } + 4 \text{ th observation}}{2} = \frac{6+7}{2} = \frac{13}{2}$

Now, Mean deviation $= \frac{\sum_{i=1}^{6} \left| x_i - \frac{13}{2} \right|}{6}$

$= \frac{\left|4-\frac{13}{2}\right| + \left|7-\frac{13}{2}\right| + \left|2-\frac{13}{2}\right| + \left|8-\frac{13}{2}\right| + \left|6-\frac{13}{2}\right| + \left|15-\frac{13}{2}\right|}{6}$

$= \frac{\frac{5}{2}+\frac{1}{2}+\frac{9}{2}+\frac{3}{2}+\frac{1}{2}+\frac{17}{2}}{6} = \frac{18}{6} = 3$

Question122

A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standarion deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively : [2011 RS]

Options:

A. 32, 2

B. 32, 4

C. 28, 2

D. 28, 4

Answer: A

Solution:

Solution:
We know that if each observation is increase by 2 then mean is increase by 2 but S.D. remains same.
Correct mean = observed mean +2 = 30 + 2 = 32
Correct S. D. = observed S . D = 2

Question123

If the mean deviation about the median of the numbers a, 2a,.......,50a is 50, then |a| equals
[2011]

Options:

A. 3

B. 4

C. 5

D. 2

Answer: B

Solution:

Solution:
$\because n = 50$ (even)
Median $= \frac{25^{th} \text{ obs.} + 26^{th} \text{ obs}}{2}$
$\therefore M = \frac{25a + 26a}{2} = 25.5a$
$M . D(M) = \frac{\sum | x_i - M |}{N}$
$\Rightarrow 50 = \frac{1}{50}[2 \times a | \times (0.5 + 1.5 + 2.5 + \ldots . . 24.5)]$
$\Rightarrow 2500 = 2|a| \times \frac{25}{2}(25)$
$\Rightarrow |a| = 4$

Question124

For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is
[2010]

Options:

A. $\frac{11}{2}$

B. 6

C. $\frac{13}{2}$

D. $\frac{5}{2}$

Answer: A

Solution:

Solution:
$\sigma_x{}^2 = 4$, $\sigma_y{}^2 = 5$, $\overline{x} = 2$, $\overline{y} = 4$

$\sigma_x{}^2 = \frac{1}{5}\sum x_i{}^2 - (2)^2 = 4 \Rightarrow \sum x_i{}^2 = 40$

$\sigma_y{}^2 = \frac{1}{5}\sum y_i{}^2 - (4)^2 = 5 \Rightarrow \sum y_i{}^2 = 105$

$\Rightarrow \sum x_i{}^2 + \sum y_i{}^2 = \sum (x_i{}^2 + y_i{}^2) = 145$

$\Rightarrow \sum x_i + \sum y_i = \sum (x_i + y_i) = 5(2) + 5(4) = 30$

Variance of combined data

$= \frac{1}{10}\sum (x_i{}^2 + y_i{}^2) - \left(\frac{1}{10}\sum (x_i + y_i)\right)^2$

$= \frac{145}{10} - 9 = \frac{11}{2}$

Question125

If the mean deviation of the numbers 1, 1 + d , 1 + 2d , ... 1 + 100d from their mean is 255, then d is equal to: [2009]

Options:

A. 20.0

B. 10.1

C. 20.2

D. 10.0

Answer: B

Solution:

Solution:
Mean $= \frac{101 + d(1 + 2 + 3 + \ldots + 100)}{101}$

$= 1 + \frac{d \times 100 \times 101}{101 \times 2} = 1 + 50d$

Given that mean deviation from the mean = 255

$\Rightarrow \frac{1}{101}[\,|1 - (1 + 50d)| + |(1 + d) - (1 + 50d)| + |(1 + 2d) - (1 + 50d)| + \ldots + |(1 + 100d) - (1 + 50d)|\,] = 255$

$\Rightarrow 2d[1 + 2 + 3 + \ldots + 50] = 101 \times 255$

$\Rightarrow 2d \times \frac{50 \times 51}{2} = 101 \times 255$

$\Rightarrow d = \frac{101 \times 255}{50 \times 51} = 10.1$

Question126

Statement-1 : The variance of first n even natural numbers is $\frac{n^2-1}{4}$
Statement-2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$.
[2009]

Options:

A. Statement-1 is true, Statement-2 is true. Statement-2 is not a correct explanation for Statement-1.

B. Statement-1 is true, Statement-2 is false.

C. Statement-1 is false, Statement-2 is true.

D. Statement-1 is true, Statement-2 is true. Statement-2 is a correct explanation for Statement-1.

Answer: C

Solution:

Solution:
First n even natural numbers be 2, 4, 6, 8, …., 2n

$\therefore \bar{x} = \frac{2(1+2+3+\ldots..+n)}{n} = \frac{2[n(n+1)]}{2n} = (n+1)$

And $Var = \frac{\sum(x-\bar{x})^2}{2n} = \frac{\sum x^2}{n} - (\bar{x})^2$

$= \frac{4\sum n^2}{n} - (n+1)^2 = \frac{4n(n+1)(2n+1)}{6n} - (n+1)^2$

$= \frac{2(2n+1)(n+1)}{3} - (n+1)^2 = (n+1)\left[\frac{4n+2-3n-3}{3}\right]$

$= \frac{(n+1)(n-1)}{3} = \frac{n^2-1}{3}$

therefore Statement- 1 is false. Clearly, statement -2 is true.

Question127

The mean of the numbers a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the following gives possible values of a and b?
[2008]

Options:

A. a = 0, b = 7

B. a = 5, b = 2

C. a = 1, b = 6

D. a = 3, b = 4

Answer: D

Solution:

Solution:
Mean of a, b, 8, 5, 10 is 6
$\Rightarrow \frac{a+b+8+5+10}{5} = 6$
$\Rightarrow a + b = 6$ ......(i)
Variance of a, b, 8, 5, 10 is 6.80
$\Rightarrow \frac{(a-6)^2 + (b-6)^2 + (8-6)^2 + (5-6)^2 + (10-6)^2}{5} = 6.80$
$\Rightarrow a^2 - 12a + 36 + (1-a)^2 + 21 = 34$ [using eq.(i)]
$\Rightarrow 2a^2 - 14a + 24 = 0 \Rightarrow a^2 - 7a + 12 = 0$
$\Rightarrow a = 3$ or $4 \Rightarrow b = 4$ or 3
$\therefore$ The possible values of a and b are $a = 3$ and $b = 4$ or, $a = 4$ and $b = 3$

Question128

The average marks of boys in class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is
[2007]

Options:

A. 80

B. 60

C. 40

D. 20

Answer: A

Solution:

Solution:
Let the number of boys be x and girls be y.
$\Rightarrow 52x + 42y = 50(x+y)$
$\Rightarrow 52x - 50x = 50y - 42y$
$\Rightarrow 2x = 8y \Rightarrow \frac{x}{y} = \frac{4}{1} \Rightarrow \frac{x}{x+y} = \frac{4}{5}$
$\therefore$ Required % of boys $= \frac{x}{x+y} \times 100$
$= \frac{4}{5} \times 100 = 80\%$

Question129

Suppose a population A has 100 observations 101, 102, ... .200 and another population B has 100 observations 151, 152, ... .250. If V_A and V_B represent the variancesof the two populations, respectively then $\frac{V_A}{V_B}$ is
[2006]

Options:

A. 1

B. $\frac{9}{4}$

C. $\frac{4}{9}$

D. $\frac{2}{3}$

Answer: A

Solution:

Solution:

$\sigma_x^{\ 2} = \frac{\sum d_i^{\ 2}}{n}$ (Here d_i = deviations are taken from the mean). Since population A and population B both have 100 consecutive integers, therefore both have same standard deviation and hence the variance is also same. $\therefore \frac{V_A}{V_B} = 1$

Question130

Let $x_1, x_2, \ldots\ldots x_n$ be n observations such that $\sum x_i^{\ 2} = 400$ and $\sum x_i = 80$. Then the possible value of n among the following is [2005]

Options:

A. 15

B. 18

C. 9

D. 12

Answer: B

Solution:

Solution:

We know that for positive real numbers $x_1, x_2, \ldots.., x_n$,

A.M. of k^{th} powers of $x_i \geq k^{th}$ the power of A.M. of x_i

$\Rightarrow \frac{\sum x_1^{\ 2}}{n} \geq \left(\frac{\sum x_1}{n}\right)^2 \Rightarrow \frac{400}{n} \geq \left(\frac{80}{n}\right)^2$

$\Rightarrow n \geq 16$. So only possible value for

$n = 18$

Question131

If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately

[2005]

Options:

A. 22.0

B. 20.5

C. 25.5

D. 24.0

Answer: D

Solution:

Solution:
We know that Mode = 3 Median -2 Mean
$3 \times 22 - 2 \times 21 = 66 - 42 = 24$

Question132

In a series of 2n observations, half of them equal a and remaining half equal −a. If the standard deviation of the observations is 2, then |a| equals.
[2004]

Options:

A. $\frac{\sqrt{2}}{n}$

B. $\sqrt{2}$

C. 2

D. $\frac{1}{n}$

Answer: C

Solution:

Solution:
Clearly sum of observations = 0,
∴ mean A = 0

Standard deviation σ = $\sqrt{\frac{\sum (x - A)^2}{2n}}$

$2 = \sqrt{\frac{(a-0)^2 + (a-0)^2 + \ldots . (0-a)^2 + \ldots}{2n}}$ [∵ σ = 2]

$= \sqrt{\frac{a^2 . 2n}{2n}} = |a|$

Hence, |a| = 2

Question133

Consider the following statements :
(A) Mode can be computed from histogram
(B) Median is not independent of change of scale
(C) Variance is independent of change of origin and scale.
Which of these is / are correct ?
[2004]

Options:

A. (A), (B) and (C)

B. Only (B)

C. Only (A) and (B)

D. Only (A)

Answer: C

Solution:

Solution:
Only first statement (A) and second statements (B) are correct.

Question134

The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set
[2003]

Options:

A. remains the same as that of the original set

B. is increased by 2

C. is decreased by 2

D. is two times the original median.

Answer: A

Solution:

Solution:
Number of terms(n) $= 9$
As n is odd Median $= \frac{n+1}{2}$ thterm
Median is 5 th term
If each of the largest 4 observations of the set is increased by 2 , then it doesn't affect the 5 th term or order of the terms.
The median remains same that is it will be 20 .

Question135

In an experiment with 15 observations on x, the following results were available:
$\sum x^2 = 2830, \sum x = 170$
One observation that was 20 was found to be wrong and was replaced by the correct value 30. The corrected variance is
[2003]

Options:

A. 8.33

B. 78.00

C. 188.66

D. 177.33

Answer: B

Solution:

Solution:
$\sum x = 170, \sum x^2 = 2830$
New, $\sum x' = 170 + (30 - 20) = 180$
New, $\sum x'^2 = 2830 + (900 - 400)$
$= 2830 + 500 = 3330$
Now, Variance $= \frac{1}{n}\sum x'^2 - \left(\frac{1}{n}\sum x'\right)^2$
$= \frac{1}{15} \times 3330 - \left(\frac{1}{15} \times 180\right)^2 = 222 - 144 = 78.$

Question136

In a class of 100 students there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls?
[2002]

Options:

A. 73

B. 65

C. 68

D. 74

Answer: B

Solution:

Total student $= 100$
Total marks of 70 boys $= 75 \times 70 = 5250$
$\Rightarrow$ Total marks of girls $= 7200 - 5250 = 1950$
Number of girls $= 100 - 70 = 30$
Average of girls $= \frac{1950}{30} = 65$
