Sequences and Series

Question1

The number of common terms in the progressions 4, 9, 14, 19,......, up to 25^{th} term and 3, 6, 9, 12, up to 37^{th} term is :

[27-Jan-2024 Shift 1]

Options:

A.

9

B.

5

C.

7

D.

8

Answer: C

Solution:

$4, 9, 14, 19, \ldots$, up to 25^{th} term

$T_{25} = 4 + (25 - 1)5 = 4 + 120 = 124$

$3, 6, 9, 12, \ldots$, up to 37^{th} term

$T_{37} = 3 + (37 - 1)3 = 3 + 108 = 111$

Common difference of I^{st} series $d_1 = 5$

Common difference of II^{nd} series $d_2 = 3$

First common term $= 9$, and

their common difference $= 15(\mathrm{LCM}^{2.}$ of d_1 and $d_2)$ then common terms are

$9, 24, 39, 54, 69, 84, 99$

Question2

If

$$8 = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \ldots \infty,$$

then the value of p is_____

[27-Jan-2024 Shift 1]

Answer: 9

Solution:

$$8 = \frac{3}{1-\frac{1}{4}} + \frac{p \cdot \frac{1}{4}}{\left(1-\frac{1}{4}\right)^2}$$

(sum of infinite terms of A.G.P $= \frac{a}{1-r} + \frac{dr}{(1-r)^2}$)

$$\Rightarrow \frac{4p}{9} = 4 \Rightarrow p = 9$$

Question3

The 20th term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, -129\frac{1}{4}$ is :-

[27-Jan-2024 Shift 2]

Options:

A.

-118

B.

-110

C.

-115

D.

-100

Answer: C

Solution:

$$20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots\ldots, -129\frac{1}{4}$$

This is A.P. with common difference

$$d_1 = -1 + \frac{1}{4} = -\frac{3}{4}$$

$$-129\frac{1}{4}, \ldots\ldots\ldots\ldots\ldots 19\frac{1}{4}, 20$$

This is also A.P. $a = -129\frac{1}{4}$ and $d = \frac{3}{4}$

Required term =

$$-129\frac{1}{4} + (20-1)\left(\frac{3}{4}\right)$$

$$= -129 - \frac{1}{4} + 15 - \frac{3}{4} = -115$$

Question4

If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to

[29-Jan-2024 Shift 1]

Options:

A.

7

B.

4

C.

5

D.

6

Answer: D

Solution:

$$a + ar + ar^2 + ar^3 + \ldots + a^{63}$$

$$= 7(a + ar^2 + ar^4 \ldots . + ar^{62})$$

$$\Rightarrow \frac{a(1-r^{64})}{1-r} = \frac{7a(1-r^{64})}{1-r^2}$$

$$r = 6$$

Question5

In an A.P., the sixth terms a6 = 2. If the $a_1a_4a_5$ is the greatest, then the common difference of the A.P., is equal to

[29-Jan-2024 Shift 1]

Options:

A.

3/2

B.

8/5

C.

2/3

D.

5/8

Answer: B

Solution:

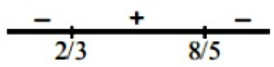
$a_6 = 2 \Rightarrow a + 5d = 2$

$a_1 a_4 a_5 = a(a+3d)(a+4d)$

$= (2-5d)(2-2d)(2-d)$

$f(d) = 8 - 32d + 34d^2 - 20d + 30d^2 - 10d^3$

$f'(d) = -2(5d-8)(3d-2)$



$d = \frac{8}{5}$

Question6

If $\log_e a$, $\log_e b$, $\log_e c$ are in an A.P. and $\log_e a - \log_e 2b$, $\log_e 2b - \log_e 3c$, $\log_e 3c - \log_e a$ are also in an A.P, then a:b:c is equal to

[29-Jan-2024 Shift 2]

Options:

A.

9 : 6 : 4

B.

16 : 4 : 1

C.

25 : 10 : 4

D.

6 : 3 : 2

Answer: A

Solution:

$\log_e a, \log_e b, \log_e c$ are in A.P.

$\therefore b^2 = ac$ ......(i)

Also

$\log_e\left(\frac{a}{2b}\right), \log_e\left(\frac{2b}{3c}\right), \log_e\left(\frac{3c}{a}\right)$ are in A.P.

$\left(\frac{2b}{3c}\right)^2 = \frac{a}{2b} \times \frac{3c}{a}$

$\frac{b}{c} = \frac{3}{2}$

Putting in eq. (i) $b^2 = a \times \frac{2b}{3}$

$\frac{a}{b} = \frac{3}{2}$

$a : b : c = 9 : 6 : 4$

Question7

If each term of a geometric progression a_1, a_2, a_3,... with a_1 = 1/8 and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + ... +$ an, then $S_{20} - S_{18}$ is equal to

[29-Jan-2024 Shift 2]

Options:

A.

2^{15}

B.

-2^{18}

C.

2^{18}

D.

-2^{15}

Answer: D

Solution:

Let r' th term of the GP be ar^{n-1}. Given,

$$2a_r = a_{r+1} + a_{r+2}$$

$$2ar^{n-1} = ar^n + ar^{n+1}$$

$$\frac{2}{r} = 1 + r$$

$$r^2 + r - 2 = 0$$

Hence, we get, $r = -2$(as $r \neq 1$)

So, $S_{20} - S_{18} =$ (Sum upto 20 terms) - (Sum upto 18 terms) $= T_{19} + T_{20}$

$$T_{19} + T_{20} = ar^{18}(1 + r)$$

Putting the values $a = \frac{1}{8}$ and $r = -2$;

we get $T_{19} + T_{20} = -2^{15}$

Question8

Let Sa denote the sum of first n terms an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then $S_{15} - S_5$ is :

[30-Jan-2024 Shift 1]

Options:

A.

395

B.

390

C.

405

D.

410

Answer: A

Solution:

$$S_{20} = \frac{20}{2}[2a + 19d] = 790$$

$2a + 19d = 79$ ....(1)

$$S_{10} = \frac{10}{2}[2a + 9d] = 145$$

$2a + 9d = 29$ ....(2)

From (1) and (2) $a = -8, d = 5$

$$S_{15} - S_5 = \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d]$$

$$= \frac{15}{2}[-16 + 70] - \frac{5}{2}[-16 + 20]$$

$$= 405 - 10$$

$$= 395$$

Question9

Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \ldots\ldots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$. **If $4\alpha - \beta = 55k + 40$, then k is equal to**

[30-Jan-2024 Shift 1]

Options:

Answer: 353

Solution:

$\alpha = 1^2 + 4^2 + 8^2 \ldots$

$t_n = a^2 + bn + c$

$1 = a + b + c$

$4 = 4a + 2b + c$

$8 = 9a + 3b + c$

On solving we get, $a = \frac{1}{2}$, $b = \frac{3}{2}$, $c = -1$

$$\alpha = \sum_{n=1}^{10} \left(\frac{n^2}{2} + \frac{3n}{2} - 1 \right)^2$$

$$4\alpha = \sum_{n=1}^{10} (n^2 + 3n - 2)^2, \beta = \sum_{n=1}^{10} n^4$$

$$4\alpha - \beta = \sum_{n=1}^{10} (6n^3 + 5n^2 - 12n + 4) = 55(353) + 40$$

Question10

Let a and b be be two distinct positive real numbers. Let 11^{th} term of a GP, whose first term is a and third term is b, is equal to p^{th} term of another GP, whose first term is a and fifth term is b. Then p is equal to

[30-Jan-2024 Shift 2]

Options:

A.

20

B.

25

C.

21

D.

24

Answer: C

Solution:

1^{st} GP $\Rightarrow t_1 = a, t_3 = b = ar^2 \Rightarrow r^2 = \frac{b}{a}$

$t_{11} = ar^{10} = a(r^2)^5 = a \cdot \left(\frac{b}{a}\right)^5$

$2^{nd}G.P. \Rightarrow T_1 = a, T_5 = ar^4 = b$

$\Rightarrow r^4 = \left(\frac{b}{a}\right) \Rightarrow r = \left(\frac{b}{a}\right)^{1/4}$

$T_p = ar^{p-1} = a\left(\frac{b}{a}\right)^{\frac{p-1}{4}}$

$t_{11} = T_p \Rightarrow a\left(\frac{b}{a}\right)^5 = a\left(\frac{b}{a}\right)^{\frac{p-1}{4}}$

$\Rightarrow 5 = \frac{p-1}{4} \Rightarrow p = 21$

Question11

Let S_n be the sum to n-terms of an arithmetic progression 3,7,11,.......

If $40 < \left(\frac{6}{n(n+1)}\sum_{k=1}^{n} S_k\right) < 42$, then n equals____

[30-Jan-2024 Shift 2]

Options:

Answer: 9

Solution:

$S_n = 3 + 7 + 11 + \ldots\ldots n$ terms

$= \frac{n}{2}(6 + (n-1)4) = 3n + 2n^2 - 2n$

$= 2n^2 + n$

$\sum_{k=1}^{n} S_k = 2\sum_{k=1}^{n} K^2 + \sum_{k=1}^{n} K$

$= 2 \cdot \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= n(n+1)\left[\frac{2n+1}{3} + \frac{1}{2}\right]$

$= \frac{n(n+1)(4n+5)}{6}$

$\Rightarrow 40 < \frac{6}{n(n+1)}\sum_{k=1}^{n} S_k < 42$

$40 < 4n + 5 < 42$

$35 < 4n < 37$

$n = 9$

Question12

The sum of the series $\frac{1}{1-3\cdot1^2+1^4} + \frac{2}{1-3\cdot2^2+2^4} + \frac{3}{1-3\cdot3^2+3^4} + \ldots$ up to 10 terms is

[31-Jan-2024 Shift 1]

Options:

A.

45/109

B.

$-\frac{45}{109}$

C.

55/109

D.

$-\frac{55}{109}$

Answer: D

Solution:

General term of the sequence,

$$T_r = \frac{r}{1-3r^2+r^4}$$

$$T_r = \frac{r}{r^4-2r^2+1-r^2}$$

$$T_r = \frac{r}{(r^2-1)^2-r^2}$$

$$T_r = \frac{r}{(r^2-r-1)(r^2+r-1)}$$

$$T_r = \frac{\frac{1}{2}[(r^2+r-1)-(r^2-r-1)]}{(r^2-r-1)(r^2+r-1)}$$

$$= \frac{1}{2}\left[\frac{1}{r^2-r-1} - \frac{1}{r^2+r-1}\right]$$

Sum of 10 terms,

$$\sum_{r=1}^{10} T_r = \frac{1}{2}\left[\frac{1}{-1} - \frac{1}{109}\right] = \frac{-55}{109}$$

Question13

Let 2^{nd}, 8^{th} and 44^{th} , terms of a non-constant A.P. be respectively the 1^{st}, 2^{nd} and 3^{rd} terms of G.P. If the first term of A.P. is 1 then the sum of first 20 terms is equal to-

[31-Jan-2024 Shift 2]

Options:

A.

980

B.

960

C.

990

D.

970

Answer: D

Solution:

$1+d,\quad 1+7d, 1+43d$ are in GP

$(1+7d)^2=(1+d)(1+43d)$

$1+49d^2+14d=1+44d+43d^2$

$6d^2-30d=0$

$d=5$

$S_{20}=\frac{20}{2}[2\times 1+(20-1)\times 5]$

$=10[2+95]$

$=970$

Question14

Let 3, a, b, c be in A.P. and 3, a−1, b+1, c+9 be in G.P. Then, the arithmetic mean of a, b and c is :

[1-Feb-2024 Shift 1]

Options:

A.

-4

B.

-1

C.

13

D.

11

Answer: D

Solution:

$3, a, b, c \to$ A.P $\Rightarrow 3, 3+d, 3+2d, 3+3d$

$3, a-1, b+1, c+9 \to$ G.P $\Rightarrow 3, 2+d, 4+2d, 12+3d$

$a=3+d \qquad (2+d)^2=3(4+2d)$

$b=3+2d \qquad d=4, -2$

$c=3+3d$

If $d=4$ G.P $\Rightarrow 3, 6, 12, 24$

$a=7$

$b=11$

$c=15$

$\frac{a+b+c}{3}=11$

Question15

Let 3, 7, 11, 15,....,403 and 2, 5, 8, 11, . . ., 404 be two arithmetic progressions. Then the sum, of the common terms in them, is equal to____

[1-Feb-2024 Shift 1]

Options:

Answer: 6699

Solution:

3, 7, 11, 15, ..., 403

2, 5, 8, 11, ..., 404

LCM(4, 3) = 12

11, 23, 35, ... let (403)

$403 = 11 + (n-1) \times 12$

$\frac{392}{12} = n - 1$

$33.66 = n$

$n = 33$

Sum $\frac{33}{2}(22 + 32 \times 12)$

$= 6699$

Question16

Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to:

[1-Feb-2024 Shift 2]

Options:

A.

800

B.

890

C.

790

D.

690

Answer: C

Solution:

$S_{10} = 390$

$\frac{10}{2}[2a + (10-1)d] = 390$

$\Rightarrow 2a + 9d = 78$

$\frac{t_{10}}{t_5} = \frac{15}{7} \Rightarrow \frac{a+9d}{a+4d} = \frac{15}{7} \Rightarrow 8a = 3d$

From (1) & (2) $a = 3$ & $d = 8$

$S_{15} - S_5 = \frac{15}{2}(6 + 14 \times 8) - \frac{5}{2}(6 + 4 \times 8)$

$= \frac{15 \times 118 - 5 \times 38}{2} = 790$

Question17

If three successive terms of a G.P. with common ratio r(r > 1) are the lengths of the sides of a triangle and [r] denotes the greatest integer less than or equal to r, then 3[r] + [−r] is equal to :

[1-Feb-2024 Shift 2]

Options:

Answer: 1

Solution:

$a, ar, ar^2 \rightarrow$ G.P.

Sum of any two sides > third side

$a + ar > ar^2, a + ar^2 > ar, ar + ar^2 > a$

$r^2 - r - 1 < 0$

$r \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$ ....(1)

$r^2 - r + 1 > 0$

always true

$r^2 + r - 1 > 0$

$r \in \left(-\infty, -\frac{1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$

Taking intersection of (1),(2)

$r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$

As $r > 1$

$r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$

$[r] = 1 [-r] = -2$

$3[r] + [-r] = 1$

Question18

For three positive integers p, q, r, $x^{pq^2} = y^{qr} = z^{p^2r}$ and $r = pq + 1$ such that $3, 3\log_y x, 3\log_z y, 7\log_x z$ are in A.P. with common difference $\frac{1}{2}$. Then $r - p - q$ is equal to
[24-Jan-2023 Shift 1]

Options:

A. 2

B. 6

C. 12

D. −6

Answer: A

Solution:

Solution:
$pq^2 = \log_x \lambda$
$qr = \log_y \lambda$
$p^2r = \log_z \lambda$
$\log_y x = \frac{qr}{pq^2} = \frac{r}{pq} \ldots (1)$
$\log_x z = \frac{pq^2}{p^2r} = \frac{q^2}{pr} \ldots\ldots\ldots (2)$
$\log_z y = \frac{p^2r}{qr} = \frac{p^2}{q} \ldots\ldots (3)$
$3, \frac{3r}{pq}, \frac{3p^2}{q}, \frac{7q^2}{pr}$ in A.P
$\frac{3r}{pq} - 3 = \frac{1}{2}$
$r = \frac{7}{6}pq \ldots\ldots (4)$
$r = pq + 1$
$pq = 6 \ldots\ldots (5)$
$r = 7 \ldots\ldots (6)$
$\frac{3p^2}{q} = 4$
After solving p = 2 and q = 3

Question19

The 4 th term of GP is 500 and its common ratio is $\frac{1}{m}$, $m \in N$. Let S_n denote the sum of the first n terms of this GP. If $S_6 > S_5 + 1$ and $S_7 < S_6 + \frac{1}{2}$, then the number of possible values of m is
[24-Jan-2023 Shift 1]

Answer: 12

Solution:

$T_4 = 500$ where a = first term,

r = common ratio $= \frac{1}{m}, m \in N$

$ar^3 = 500$

$\frac{a}{m^3} = 500$

$S_n - S_{n-1} = ar^{n-1}$

$S_6 > S_5 + 1$ and $S_7 - S_6 < \frac{1}{2}$

$S_6 - S_5 > 1$ $\frac{a}{m^6} < \frac{1}{2}$

$ar^5 > 1$ $m^3 > 10^3$

$\frac{500}{m^2} > 1$ $m > 10$

$m^2 < 500$ ......(1)

From (1) and (2)

m = 11, 12, 13…………, 22

So number of possible values of m is 12

Question20

If $\frac{1^3 + 2^3 + 3^3 + \ldots\ldots \text{ upto } n \text{ terms}}{1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \ldots\ldots \text{ upto } n \text{ terms}} = \frac{9}{5}$, then the value of n is

[24-Jan-2023 Shift 2]

Answer: 5

Solution:

Solution:

$1^3 + 2^3 + 3^3 \ldots\ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

$1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \ldots + n^n + \text{terms} =$

$\sum_{r=1}^{n} r(2r+1) = \sum_{r=1}^{n} (2r^2 + r)$

$= \frac{2 \cdot n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{6}(2(2n+1) + 3)$

$= \frac{n(n+1)}{2} \times \frac{(4n+5)}{3}$

$= \frac{n^2(n+1)^2}{4}$

$\Rightarrow \frac{\frac{5n(n+1)}{2} \times \frac{(4n+5)}{3}}{2} = \frac{9}{5}$

$\Rightarrow \frac{5n(n+1)}{2} = \frac{9(4n+5)}{3}$

$\Rightarrow 15n(n+1) = 18(4n+5)$

$\Rightarrow 15n^2 + 15n = 72n + 90$

$\Rightarrow 15n^2 - 57n - 90 = 0 \Rightarrow 5n^2 - 19n - 30 = 0$

$\Rightarrow (n-5)(5n+6) = 0$

$\Rightarrow n = \frac{-6}{5}$ or 5

$\Rightarrow n = 5.$

Question21

Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as A, A + 1, A + 2, respectively. Let a, b, c be the

7^{th}, 9^{th}, 17^{th} terms of A_1, A_2, A_3, respectively such that

$$\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$$

If $a = 29$, then the sum of first 20 terms of an AP whose first term is $c - a - b$ and common difference is $\frac{d}{12}$, is equal to ________.
[25-Jan-2023 Shift 1]

Answer: 495

Solution:

Solution:

$$\begin{vmatrix} A + 6d & 7 & 1 \\ 2(A + 1 + 8d) & 17 & 1 \\ A + 2 + 16d & 17 & 1 \end{vmatrix} + 70 = 0$$

$\Rightarrow A = -7$ and $d = 6$
$\therefore c - a - b = 20$
$S_{20} = 495$

Question22

For the two positive numbers a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in an arithmetic progression, then, $16a + 12b$ is equal to ________.
[25-Jan-2023 Shift 2]

Answer: 3

Solution:

Solution:

$a, b, \frac{1}{18} \rightarrow GP$

$\frac{a}{18} = b^2$

$\frac{1}{a}, 10, \frac{1}{b} \rightarrow AP$

$\frac{1}{a} + \frac{1}{b} = 20$

$\Rightarrow a + b = 20ab$, from eq. (i) ; we get

$\Rightarrow 18b^2 + b = 360b^3$

$\Rightarrow 360b^2 - 18b - 1 = 0 \ \{\because b \neq 0\}$

$\Rightarrow b = \frac{18 \pm \sqrt{324 + 1440}}{720}$

$\Rightarrow b = \frac{18 + \sqrt{1764}}{720} \ \{\because b > 0\}$

$\Rightarrow b = \frac{1}{12}$

$\Rightarrow = 18 \times \frac{1}{144} = \frac{1}{8}$

Now, $16a + 12b = 16\times\frac{1}{8} + 12\times\frac{1}{12} = 3$

Question23

Let a_1, a_2, a_3, ... be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24 , then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to ______.
[29-Jan-2023 Shift 1]

Answer: 60

Solution:

Solution:
$a_4 \cdot a_6 = 9 \Rightarrow (a_5)^2 = 9 \Rightarrow a_5 = 3$
$\& a_5 + a_7 = 24 \Rightarrow a_5 + a_5r^2 = 24 \Rightarrow (1 + r^2) = 8 \Rightarrow r = \sqrt{7}$
$\Rightarrow a = \frac{3}{49}$
$\Rightarrow a_1a_9 + a_2a_4a_9 + a_5 + a_7 = 9 + 27 + 3 + 21 = 60$

Question24

Let $\{a_k\}$ and $\{b_k\}$, $k \in N$, be two G.P.s with common ratio r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k$, $k \in N$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to__
[29-Jan-2023 Shift 2]

Answer: 9

Solution:

Solution:
Given that
$c_k = a_k + b_k$ and $a_1 = b_1 = 4$
also $a_2 = 4r_1$ $a_3 = 4r_1^{\,2}$
$b_2 = 4r_2$ $b_3 = 4r_2^{\,2}$
Now $c_2 = a_2 + b_2 = 5$ and $c_3 = a_3 + b_3 = \frac{13}{4}$
$\Rightarrow r_1 + r_2 = \frac{5}{4}$ and $r_1^{\,2} + r_2^{\,2} = \frac{13}{16}$
Hence $r_1r_2 = \frac{3}{8}$ which gives $r_1 = \frac{1}{2}$ & $r_2 = \frac{3}{4}$
$\sum_{k=1^{\infty}c}^{k} - (12a_6 + 8b_4)$
$= \frac{4}{1 - r_1} + \frac{4}{1 - r_2} - \left(\frac{48}{32} + \frac{27}{2}\right)$
$= 24 - 15 = 9$

Question25

Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1)$, $b_n = b_{n-1} + a_{n-1}$, $\forall n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to ______.

[29-Jan-2023 Shift 2]

Answer: 461

Solution:

Solution:

As, $S = \frac{b_1}{2} + \frac{b_2}{2^2} + \ldots\ldots + \frac{b_9}{2^9} + \frac{b_{10}}{2^{10}}$

$\Rightarrow \frac{S}{2} = \frac{b_1}{2^2} + \frac{b_2}{2^3} + \ldots\ldots + \frac{b_9}{2^{10}} + \frac{b_{10}}{2^{11}}$

subtracting

$\Rightarrow \frac{S}{2} = \frac{b_1}{2} + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} \ldots\ldots + \frac{a_9}{2^{10}} \right) - \frac{b_{10}}{2^{11}}$

$\Rightarrow S = b_1 - \frac{b_{10}}{2^{10}} + \left(\frac{a_1}{2} + \frac{a_2}{2^2} \ldots\ldots\ldots + \frac{a_9}{2^9} \right)$

$\Rightarrow \frac{S}{2} = \frac{b_1}{2} - \frac{b_{10}}{2^{11}} + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} \ldots\ldots\ldots + \frac{a_9}{2^{10}} \right)$

subtracting

$\Rightarrow \frac{s}{2} = \frac{b_1}{2} - \frac{b_{10}}{2^{11}} + \left(\frac{a_1}{2} - \frac{a_9}{2^{10}} \right) + \left(\frac{1}{2^2} + \frac{2}{2^3} + \ldots + \frac{8}{2^9} \right)$

$\Rightarrow \frac{S}{2} = \frac{a_1 + b_1}{2} - \frac{(b_{10} + 2a_9)}{2^{11}} + \frac{T}{4}$

$\Rightarrow 2S = 2(a_1 + b_1) - \frac{(b_{10} + 2a_9)}{2^9} + T$

$\Rightarrow 2^7(2S - T) = 2^8(a_1 + b_1) - \frac{(b_{10} + 2a_9)}{4}$

Also, $b_n - b_{n-1} = a_{n-1}$

$\therefore b_{10} - b_1 = a_1 + a_2 + \ldots + a_9$

$= 1 + 2 + 4 + 7 + 11 + 16 + 22 + 29 + 37$

$\Rightarrow b_{10} = 130($ As $b_1 = 1)$

$\therefore 2^7(2S - T) = 2^8(1 + 1) - (130 + 2 \times 37)$

$2^9 - \frac{204}{4} = 461$

Question26

If $a_n = \frac{-2}{4n^2 - 16n + 15}$, then $a_1 + a_2 + \ldots\ldots + a_{25}$ is equal to :

[30-Jan-2023 Shift 1]

Options:

A. $\frac{51}{144}$

B. $\frac{49}{138}$

C. $\frac{50}{141}$

D. $\frac{52}{147}$

Answer: C

Solution:

Solution:

Option (3)

If $a_n = \frac{-2}{4n^2 - 16n + 15}$ then $a_1 + a_2 + \ldots\ldots\ldots a_{25}$

$\Rightarrow \sum_{n=1}^{25} a_n = \sum \frac{-2}{4n^2 - 16n + 15}$

$= \sum \frac{-2}{4n^2 - 6n - 10n + 15}$

$= \sum \frac{-2}{2n(2n-3) - 5(2n-3)}$

$= \sum \frac{-2}{(2n-3)(2n-5)}$

$= \sum \frac{1}{2n-3} - \frac{1}{2n-5}$

$= \frac{1}{47} - \frac{1}{(-3)}$

$= \frac{50}{141}$

Question27

Let $\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)(n!)}{(n!)((2n)!)} = ae + \frac{b}{e} + c$, where $a, b, c \in \mathbb{Z}$ and $e = \sum_{n=0}^{\infty} \frac{1}{n}$ Then $a^2 - b + c$ is equal to _______.
[30-Jan-2023 Shift 1]

Answer: 26

Solution:

Solution:

$\sum_{n=0}^{\infty} \frac{n^3((2n)!) + (2n-1)(n!)}{(n!)((2n)!)}$

$= \sum_{n=0}^{\infty} \frac{1}{(n-3)!} + \sum_{n=0}^{\infty} \frac{3}{(n-2)!}$

$+ \sum_{n=0}^{\infty} \frac{1}{(n-1)!} + \sum_{n=0}^{\infty} \frac{1}{(2n-1)!} - \sum_{n=0}^{\infty} \frac{1}{(2n)!}$

$= e + 3e + e + \frac{1}{2}\left(e - \frac{1}{e}\right) - \frac{1}{2}\left(e + \frac{1}{e}\right)$

$= 5e - \frac{1}{e}$

$\therefore a^2 - b + c = 26$

Question28

Let $a, b, c > 1$, a^3, b^3 and c^3 be in A.P., and $\log_a b$, $\log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a + 4b + c}{3}$ and the common difference is $\frac{a - 8b + c}{10}$ is -444, then abc is equal to
[30-Jan-2023 Shift 2]

Options:

A. 343

B. 216

C. $\frac{343}{8}$

D. $\frac{125}{8}$

Answer: B

Solution:

Solution:
As a^3, b^3, c^3 be in A.P. $\rightarrow a^3 + c^3 = 2b^3 \ldots (1)$
$\log_a{}^b, \log_c{}^a, \log_b{}^c$ are in G.P.
$\therefore \frac{\log b}{\log a} \cdot \frac{\log c}{\log b} = \left(\frac{\log a}{\log c}\right)^2$
$\therefore (\log a)^3 = (\log c)^3 \Rightarrow a = c \ldots (2)$
From (1) and (2)
$a = b = c$
$T_1 = \frac{a + 4b + c}{3} = 2a; \ d = \frac{a - 8b + c}{10} = \frac{-6a}{10} = \frac{-3}{5}a$
$\therefore S_{20} = \frac{20}{2}\left[4a + 19\left(-\frac{3}{5}a\right)\right]$
$= 10\left[\frac{20a - 57a}{5}\right]$
$= -74a$
$\therefore -74a = -444 \Rightarrow a = 6$
$\therefore abc = 6^3 = 216$

Question29

The parabolas : $ax^2 + 2bx + cy = 0$ and $d x^2 + 2ex + f y = 0$ intersect on the line $y = 1$. If a, b, c, d , e, f are positive real numbers and a, b, c are in G.P., then [30-Jan-2023 Shift 2]

Options:

A. d, e, f are in A.P.

B. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

C. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

D. d, e, f are in G.P.

Answer: C

Solution:

Solution:
$ax^2 + 2bx + c = 0$
$\Rightarrow ax^2 + 2\sqrt{ac}x + c = 0 (\because b^2 = ac)$
$\Rightarrow (x\sqrt{a} + \sqrt{c})^2 = 0$
$x^2 - \frac{\sqrt{c}}{\sqrt{a}} \ldots\ldots$
Now, $dx^2 + 2ex + f = 0$
$\Rightarrow d\left(\frac{c}{a}\right) + 2e\left[-\frac{\sqrt{c}}{\sqrt{a}}\right] + f = 0$
$\Rightarrow \frac{dc}{a} + f = 2e\sqrt{\frac{c}{a}}$
$\Rightarrow \frac{d}{a} + \frac{f}{c} = 2e\sqrt{\frac{1}{ac}}$
$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b} [\text{ as } b = \sqrt{ae}]$

$\therefore \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Question30

The 8^{th} common term of the series
$S_1 = 3 + 7 + 11 + 15 + 19 + ...$
$S_2 = 1 + 6 + 11 + 16 + 21 + ...$
is _______.
[30-Jan-2023 Shift 2]

Answer: 151

Solution:

Solution:
$T_8 = 11 + (8 - 1) \times 20$
$= 11 + 140 = 151$

Question31

Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$. Then
$S = \left\{ x \in R : \tan^{-1}(\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x) + 1}) = \frac{\pi}{2} \right\}$:
[31-Jan-2023 Shift 1]

Options:

A. contains exactly two elements

B. contains exactly one element

C. is an infinite set

D. is an empty set

Answer: A

Solution:

Solution:
$\left(x + \frac{1}{2}\right)^2 = \left(y + \frac{1}{4}\right)$
$y = (x^2 + x)$
$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \pi / 2$
$0 \le x^2 + x + 1 \le 1$
$x^2 + x \le 0 \ . . . (1)$
Also $x^2 + x \ge 0 \ . . . (2)$
$\therefore x^2 + x = 0 \Rightarrow x = 0, -1$
S contains 2 element.

Question32

Let $a_1, a_2, \ldots\ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then

$12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \ldots . \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$

is equal to ________.

[31-Jan-2023 Shift 1]

Answer: 8

Solution:

Solution:

$2a_7 = a_5$ (given)

$2(a_1 + 6d) = a_1 + 4d$

$a_1 + 8d = 0 \ldots (1)$

$a_1 + 10d = 18 \ldots (2)$

By (1) and (2) we get $a_1 = -72, d = 9$

$a_{18} = a_1 + 17d = -72 + 153 = 81$

$a_{10} = a_1 + 9d = 9$

$$12\left(\frac{\sqrt{a_{11}}-\sqrt{a_{10}}}{d} + \frac{\sqrt{a_{12}}-\sqrt{a_{11}}}{d} + \ldots\ldots \frac{\sqrt{a_{18}}-\sqrt{a_{17}}}{d}\right)$$

$$12\left(\frac{\sqrt{a_{18}}-\sqrt{a_{10}}}{d}\right) = \frac{12(9-3)}{9} = \frac{12\times 6}{6} = 8$$

Question33

Let $a_1, a_2, a_3, \ldots.$ be an A.P. If $a_7 = 3$, the product a_1a_4 is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to :

[31-Jan-2023 Shift 2]

Options:

A. 24

B. $\frac{33}{4}$

C. $\frac{381}{4}$

D. 9

Answer: A

Solution:

Solution:

$a + 6d = 3 \ldots (1)$

$Z = a(a+3d)$

$= (3-6d)(3-3d)$

$= 18d^2 - 27d + 9$

Differentiating with respect to d

$\Rightarrow 36d - 27 = 0$

$\Rightarrow d = \frac{3}{4}$, from (1) $a = \frac{-3}{2}$, (Z = minimum)

Now, $S_n = \frac{n}{2}\left(-3 + (n-1)\frac{3}{4}\right) = 0$

$\Rightarrow n = 5$

Now,

$n! - 4a_{n(n+2)} = 120 - 4(a_{35})$

$= 120 - 4(a + (35-1)d)$

$= 120 - 4\left(\frac{-3}{2} + 34 \cdot \left(\frac{3}{4} \right) \right)$
$= 120 - 4\left(\frac{-6 + 102}{4} \right)$
$= 120 - 96 = 24$

Question34

The sum $1^2 - 2.3^2 + 3.5^2 - 4.7^2 + 5.9^2 - .... + 15.29^2$ is ________. [31-Jan-2023 Shift 2]

Answer: 6952

Solution:

Solution:
Separating odd placed and even placed terms we get
$S = (1.1^2 + 3.5^2 + ...15 \cdot (29)^2) - (2.3^2 + 4.7^{2.}$
$+ ... + 14.(27)^2$
$S = \sum_{n=1}^{8} (2n-1)(4n-3)^2 - \sum_{n=1}^{7} (2n)(4n-1)^2$
Applying summation formula we get
$= 29856 - 22904 = 6952$

Question35

The sum to 10 terms of the series $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + ...$ is:- [1-Feb-2023 Shift 1]

Options:

A. $\frac{59}{111}$

B. $\frac{55}{111}$

C. $\frac{56}{111}$

D. $\frac{58}{111}$

Answer: B

Solution:

Solution:
$T_r = \frac{(r^2+r+1) - (r^2-r+1)}{2(r^4+r^2+1)}$
$\Rightarrow T_r = \frac{1}{2}\left[\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1} \right]$
$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3} \right]$
$T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{7} \right]$
$T_3 = \frac{1}{2}\left[\frac{1}{7} - \frac{1}{13} \right]$

$T_{10} = \frac{1}{2}\left[\frac{1}{91} - \frac{1}{111}\right]$

$\Rightarrow \sum_{r=1}^{10} T_r = \frac{1}{2}\left[1 - \frac{1}{111}\right] = \frac{55}{111}$

Question36

Let $a_1 = 8, a_2, a_3, \ldots a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170 , then the product of its middle two terms is ______.
[1-Feb-2023 Shift 1]

Answer: 754

Solution:

Solution:

$a_1 + a_2 + a_3 + a_4 = 50$
$\Rightarrow 32 + 6d = 50$
$\Rightarrow d = 3$
and, $a_{n-3} + a_{n-2} + a_{n-1} + a_n = 170$
$\Rightarrow 32 + (4n - 10) \cdot 3 = 170$
$\Rightarrow n = 14$
$a_7 = 26, a_8 = 29$
$\Rightarrow a_7 \cdot a_8 = 754$

Question37

The number of 3 -digit numbers, that are divisible by either 2 or 3 but not divisible by 7 is _______.
[1-Feb-2023 Shift 1]

Answer: 514

Solution:

Solution:

Divisible by 2 → 450
Divisible by 3 → 300
Divisible by 7 → 128
Divisible by 2&7 → 64
Divisible by 3&7 → 43
Divisible by 2&3 → 150
Divisible by 2, 3&7 → 21
∴ Total numbers $= 450 + 300 - 150 - 64 - 43 + 21 = 514$

Question38

Which of the following statements is a tautology ?
[1-Feb-2023 Shift 2]

Options:

A. $p \rightarrow (p \wedge (p \rightarrow q))$

B. $(p \wedge q) \rightarrow (\sim(p) \rightarrow q)$

C. $(p \wedge (p \rightarrow q)) \rightarrow \sim q$

D. $p \vee (p \wedge q)$

Answer: B

Solution:

Solution:
(i) $p \rightarrow (p \wedge (p \rightarrow q))$
$(\sim p) \vee (p \wedge (\sim p \vee q))$
$(\sim p) \vee (f \vee (p \wedge q))$
$\sim p \vee (p \wedge q) = (\sim p \vee p) \wedge (\sim p \vee q)$
$= \sim p \vee q$
(ii) $(p \wedge q) \rightarrow (\sim p \rightarrow q)$
$\sim (p \wedge q) \vee (p \vee q) = t$
$\{a, b, d\} \vee \{a, b, c\} = V$
Tautology
(iii) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$
$\sim (p \wedge (\sim p \vee q)) \vee \sim q = \sim (p \wedge q) \vee \sim q = \sim p \vee \sim q$
Not tantology
(iv) $p \vee (p \wedge q) = p$
Not tautology.

Question39

The sum of the common terms of the following three arithmetic progressions.
3, 7, 11, 15, ..., 399
2, 5, 8, 11, ..., 359 and
2, 7, 12, 17, ..., 197, is equal to ________.
[1-Feb-2023 Shift 2]

Answer: 321

Solution:

Solution:
3, 7, 11, 15, ..., 399 $d_1 = 4$
2, 5, 8, 11, ..., 359 $d_2 = 3$
2, 7, 12, 17, ..., 197 $d_3 = 5$
$LCM(d_1, d_2, d_3) = 60$
Common terms are 47, 107, 167
Sum = 321

Question40

The sum of the first 20 terms of the series 5 + 11 + 19 + 29 + 41 + ... is :
[6-Apr-2023 shift 1]

Options:

A. 3450

B. 3420

C. 3520

D. 3250

Answer: C

Solution:

Solution:

$$s_n = 5+11+19+29+41+\ldots..+T_n$$
$$S_n = \quad 5+11+19+29+\ldots+T_{n-1}+T_n$$
$$0 = 5+\left\{\underbrace{6+8+10+12+\ldots.}_{(n-1)\text{ terms}}\right\}-T_n$$

$T_n = 5 + \frac{(n-1)}{2}[2 \cdot 6 + (n-2) \cdot 2]$

$T_n = 5 + (n-1)(n+4) = 5 + n^2 + 3n - 4 = n^2 + 3n + 1$

Now $S_{20} = \sum_{n=1}^{20} T_n = \sum_{n=1}^{20} n^2 + 3n + 1$

$S_{20} = \frac{20.21.41}{6} + \frac{3.20.21}{2} + 20$

$S_{20} = 2870 + 630 + 20$

$S_{20} = 3520$

Question41

Let $a_1, a_2, a_3, \ldots, a_n$ be n positive consecutive terms of an arithmetic progression. If d > 0 is its common difference, then :

$\lim_{n\to\infty} \sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}} + \frac{1}{\sqrt{a_2}+\sqrt{a_3}} + \ldots\ldots. + \frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$ **is**

[6-Apr-2023 shift 1]

Options:

A. $\frac{1}{\sqrt{d}}$

B. 1

C. $\sqrt{d}$

D. 0

Answer: B

Solution:

Solution:

$\underset{n\to\infty}{\text{Lt}} \sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_1}-\sqrt{a_2}}{a_1-a_2} + \frac{\sqrt{a_2}-\sqrt{a_3}}{a_2-a_3} + \ldots\ldots + \frac{\sqrt{a_{n-1}}-\sqrt{a_n}}{a_{n-1}-a_n}\right)$

$= \underset{n\to\infty}{\text{Lt}} \sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_1}-\sqrt{a_2}+\sqrt{a_2}+\sqrt{a_3}+\ldots\ldots+\sqrt{a_{n-1}}-\sqrt{a_n}}{-d}\right)$

$= \underset{n\to\infty}{\text{Lt}} \sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_n}-\sqrt{a_1}}{d}\right)$

$= \underset{n\to\infty}{\text{Lt}} \frac{1}{\sqrt{n}}\left(\frac{\sqrt{a_1+(n-1)d}-\sqrt{a_1}}{\sqrt{d}}\right)$

$= \underset{n\to\infty}{\text{Lt}} \frac{1}{\sqrt{d}}\left(\sqrt{\frac{a_1}{n}+d-\frac{d}{n}} - \frac{\sqrt{a_1}}{n}\right)$

$= 1$

Question42

If gcd(m, n) = 1 and $1^2 - 2^2 + 3^2 - 4^2 + ... + (2021)^2 - (2022)^2 + (2023)^2 = 1012m^2n$ then $m^2 - n^2$ is equal to : [6-Apr-2023 shift 2]

Options:

A. 180

B. 220

C. 200

D. 240

Answer: D

Solution:

Solution:

$(1-2)(1+2)+(3-4)(3+4)+\ldots\ldots+(2021-2022)(2021+2022)+(2023)^2=(1012)m^2n$

$\Rightarrow (-1)[1+2+3+4+\ldots+2022]+(2023)^2=(1012)m^2n$

$\Rightarrow (-1)\dfrac{(2022)(2023)}{2}+(2023)^2=(1012)m^2n$

$\Rightarrow (2023)[2023-1011]=(1012)m^2n$

$\Rightarrow (2023)(1012)=(1012)m^2n$

$\Rightarrow m^2n=2023$

$\Rightarrow m^2n=(17)^2\times 7$

$m=17, n=7$

$m^2-n^2=(17)^2-7^2=289-49=240$

Ans. Option 4

Question43

If $(20)^{19} + 2(21)(20)^{18} + 3(21)^2(20)^{17} + ... + 20(21)^{19} = k(20)^{19}$, then k is equal to ________ : [6-Apr-2023 shift 2]

Answer: 400

Solution:

Solution:

$S=(20)^{19}+2(21)(20)^{18}+\ldots\ldots+20(21)^{19}$

$\dfrac{21}{20}S=21(20)^{18}+2(21)^9(20)^{17}+\ldots\ldots.+(21)^{20}$

Subtract

$\left(1-\dfrac{21}{20}\right)S=(20)^{19}+(21)(20)^{18}+(21)^2(20)^{17}+\ldots\ldots+(21)^{19}-(21)^{20}$

$\left(\dfrac{-1}{20}\right)S=(20)^{19}\left[\dfrac{1-\left(\dfrac{21}{20}\right)^{20}}{1-\dfrac{21}{20}}\right]-(21)^{20}$

$\left(\dfrac{-1}{20}\right)S=(21)^{20}-(20)^{20}-(21)^{20}$

$S=(20)^{21}=K(20)^{19}$ (given)

$K=(20)^2$

$=400$

Question44

Let $S_K \frac{1+2+\ldots+K}{K}$ and $\sum_{j=1}^{n} S_j^{2} = \frac{n}{A}(Bn^2 + Cn + D)$, where A, B, C, D ∈ N and A has least value. Then
[8-Apr-2023 shift 1]

Options:

A. A + B is divisible by D

B. $A + B = 5(D - C)$

C. A + C + D is not divisible by B

D. A + B + D is divisible by 5

Answer: A

Solution:

Solution:

$S_k = \frac{k+1}{2}$

$S_k{}^2 = \frac{k^2+1+2k}{4}$

$\therefore \sum_{j-1}^{n} S_j{}^2 = \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + n + n(n+1)\right]$

$= \frac{n}{4}\left[\frac{(n+1)(2n+1)}{6} + 1 + n + 1\right]$

$= \frac{n}{4}\left[\frac{2n^2+3n+1}{6} + n + 2\right]$

$= \frac{n}{4}\left[\frac{2n^2+9n+13}{6}\right] = \frac{n}{24}[2n^2+9n+13]$

A = 24, B = 2, C = 9, D = 13

Question45

Let a_n be the n^{th} term of the series 5 + 8 + 14 + 23 + 35 + 50 + ... and $Sn = \sum_{k=1}^{n} a_K$. Then $S_{30} - a_{40}$ is equal to
[8-Apr-2023 shift 2]

Options:

A. 11260

B. 11280

C. 11290

D. 11310

Answer: C

Solution:

Solution:

$S_n = 5 + 8 + 14 + 23 + 35 + 50 + \ldots + a_n$

$S_n = 5 + 8 + 14 + 23 + 35 + \ldots + a_n$

$O = 5 + 3 + 6 + 9 + 12 + 15 + \ldots - a_n$

$a_n = 5 + (3 + 6 + 9 + \ldots (n-1) \text{ terms })$

$a_n = \dfrac{3n^2 - 3n + 10}{2}$

$a_{40} = \dfrac{3(40)^2 - 3(40) + 10}{2} = 2345$

$S_{30} = \dfrac{3 \sum\limits_{n=1}^{30} n^2 - 3 \sum\limits_{n=1}^{30} n + 10 \sum\limits_{n=1}^{30} 1}{2}$

$= \dfrac{\dfrac{3 \times 30 \times 31 \times 61}{6} - \dfrac{3 \times 30 \times 31}{2} + 10 \times 30}{2}$

$S_{30} = 13635$

$S_{30} - a_{40} = 13635 - 2345$

$= 11290$(Option (3))

Question46

Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x + y + z)^2$ is equal to _______.

[8-Apr-2023 shift 2]

Answer: 150

Solution:

Solution:

$\dfrac{2}{y} = \dfrac{1}{x} + \dfrac{1}{z}$

$2y^2 = xz$

$\dfrac{2}{y} = \dfrac{x+z}{xz} = \dfrac{x+z}{2y^2}$

$x + z = 4y$

$xy + yz + zx = \dfrac{3}{\sqrt{2}}xyz$

$y(x+z) + zx = \dfrac{3}{\sqrt{2}}xz \cdot y$

$4y^2 + 2y^2 = \dfrac{3}{\sqrt{2}}y \cdot 2y^2$

$6y^2 = 3\sqrt{2}y^3$

$y = \sqrt{2}$

$x + y + z = 5y = 5\sqrt{2}$

$3(x + y + z)^2 = 3 \times 50 = 150$

Question47

Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three is 33033 , then the sum of these terms is equal to :

[10-Apr-2023 shift 1]

Options:

A. 210

B. 220

C. 231

D. 241

Answer: C

Solution:

Solution:
Let a, ar, ar^2 be three terms of GP
Given : $a^2 + (ar)^2 + (ar^2)^2 = 33033$
$a^2(1 + r^2 + r^4) = 11^2.3.7.13$
$\Rightarrow a = 11$ and $1 + r^2 + r^4 = 3.7.13$
$\Rightarrow r^2(1 + r^2) = 273 - 1$
$\Rightarrow r^2(r^2 + 1) = 272 = 16 \times 17$
$\Rightarrow r^2 = 16$
$\therefore r = 4$ $[\because r > 0]$
Sum of three terms $= a + ar + ar^2 = a(1 + r + r^2)$
$= 11(1 + 4 + 16)$
$= 11 \times 21 = 231$

Question48

If $f(x) = \frac{(\tan 1^{\circ})x + \log_e(123)}{x\log_e(1234) - (\tan 1^{\circ})}$, $x > 0$, then the least value of $f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$ is:
[10-Apr-2023 shift 1]

Options:

A. 2

B. 4

C. 8

D. 0

Answer: B

Solution:

Solution:

$f(x) = \frac{(\tan 1)x + \log 123}{x\log 1234 - \tan 1}$

Let $A = \tan 1$, $B = \log 123$, $C = \log 1234$

$f(x) = \frac{Ax + B}{xC - A}$

$f(f(x)) = \frac{A\left(\frac{Ax + B}{xC - A}\right) + B}{C\left(\frac{Ax + B}{CX - A}\right) - A}$

$= \frac{A^2x + AB + xBC - AB}{ACx + BC - ACx + A^2}$

$= \frac{x(A^2 + BC)}{(A^2 + BC.} = x$

$f(f(x)) = x$

$f\left(f\left(\frac{4}{x}\right)\right) = \frac{4}{x}$

$f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$

AM $\geq$ GM

$x + \frac{4}{x} \geq 4$

Question49

The sum of all those terms, of the arithmetic progression 3, 8, 13, ..., 373, which are not divisible by 3 , is equal to _______.
[10-Apr-2023 shift 1]

Answer: 9525

Solution:

Solution:
A.P: 3, 8, 13… . .373
$T_n = a + (n - 1)d$
$373 = 3 + (n - 1)5$
$\Rightarrow n = \frac{370}{5}$
$\Rightarrow n = 75$
Now Sum $= \frac{n}{2}[a + 1]$
$= \frac{75}{2}[3 + 373] = 14100$
Now numbers divisible by 3 are,
3, 18, 33…. . .363
$363 = 3 + (k - 1)15$
$\Rightarrow k - 1 = \frac{360}{15} = 24 \Rightarrow k = 25$
Now, sum $= \frac{25}{2}(3 + 363) = 4575$ s
$\therefore$ req. sum $= 14100 - 4575$
$= 9525$

Question50

If $S_n = 4 + 11 + 21 + 34 + 50 + ...$ to n terms, then $\frac{1}{60}(S_{29} - S_9)$ is equal to
[10-Apr-2023 shift 2]

Options:

A. 220

B. 227

C. 226

D. 223

Answer: D

Solution:

Solution:
$S_n = 4 + 11 + 21 + 34 + 50 + \ldots + n$ terms
Difference are in A.P.
Let $T_n = an^2 + bn + c$
$T_1 = a + b + c = 4$
$T_2 = 4a + 2b + c = 11$
$T_3 = 9a + 3b + c = 21$
By solving these 3 equations
$a = \frac{3}{2}, b = \frac{5}{2}, c = 0$
So $T_n = \frac{3}{2}n^2 + \frac{5}{2}n$
$S_n = \Sigma T_n$
$= \frac{3}{2}\sum n^2 + \frac{5}{2}\sum n$

$= \frac{3}{2}\frac{n(n+1)(2n+1)}{6} + \frac{5}{2}\frac{(n)(n+1)}{2}$

$= \frac{n(n+1)}{4}[2n+1+5]$

$S_n = \frac{n(n+1)}{4}(2n+6) = \frac{n(n+1)(n+3)}{2}$

$\frac{1}{60}\left(\frac{29 \times 30 \times 32}{2} - \frac{9 \times 10 \times 12}{2}\right) = 223$

Question51

Suppose a_1, a_2, 2, a_3, a_4 be in an arithemetico-geometric progression. If the common ratio of the corresponding geometric progression in 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to _______.
[10-Apr-2023 shift 2]

Answer: 16

Solution:

Solution:

$\frac{(a-2d)}{4}, \frac{(a-d)}{2}, a, 2(a+d), 4(a+2d)$

$a = 2$

$\left(\frac{1}{4} + \frac{1}{2} + 1 + 6\right) \times 2 + (-1+2+8)d = \frac{49}{2}$

$2\left(\frac{3}{4} + 7\right) + 9d = \frac{49}{2}$

$9d = \frac{49}{2} - \frac{62}{4} = \frac{98-62}{4} = 9$

$d = 1$

$\Rightarrow a_4 = 4(a+2d)$

$= 16$

Question52

Let $x_1, x_2, \ldots\ldots, x_{100}$ be in an arithmetic progression, with $x_1 = 2$ and their mean equal to 200 . If $y_i = i(x_i - i)$, $1 \le i \le 100$, then the mean of $y_1, y_2, \ldots\ldots y_{100}$ is :
[11-Apr-2023 shift 1]

Options:

A. 10051.50

B. 10100

C. 10101.50

D. 10049.50

Answer: D

Solution:

Solution:

Mean $= 200$

$$\Rightarrow \frac{\frac{100}{2}(2 \times 2 + 99d)}{100} = 200$$

$\Rightarrow 4 + 99d = 400$

$\Rightarrow d = 4$

$y_i = i(xi - 1)$

$= i(2 + (i - 1)4 - i) = 3i^2 - 2i$

$$\text{Mean} = \frac{\Sigma y_i}{100}$$

$$= \frac{1}{100} \sum_{i=1}^{100} 3i^2 - 2i$$

$$= \frac{1}{100} \left\{ \frac{3 \times 100 \times 101 \times 201}{6} - \frac{2 \times 100 \times 101}{2} \right\}$$

$$= 101 \left\{ \frac{201}{2} - 1 \right\} = 101 \times 99.5$$

$= 10049.50$

Question53

Let S = $S = 109 + \frac{108}{5} + \frac{107}{5^2} + ... + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to _______.
[11-Apr-2023 shift 1]

Answer: 2175

Solution:

Solution:

$$S = 109 + \frac{108}{5} + \frac{107}{5^2} \ldots\ldots + \frac{1}{5^{108}}$$

$$\frac{S}{5} = \frac{109}{5} + \frac{108}{5^2} \ldots\ldots \cdot \frac{2}{5^{108}} + \frac{1}{5^{109}}$$

$$\frac{4S}{5} = 109 - \frac{1}{5} - \frac{1}{5^2} \ldots\ldots - \frac{1}{5^{108}} - \frac{1}{5^{109}}$$

$$= 109 - \left(\frac{1}{5} \frac{\left(1 - \frac{1}{5^{109}}\right))}{\left(1 - \frac{1}{5}\right)} \right)$$

$$= 109 - \frac{1}{4}\left(1 - \frac{1}{5^{109}}\right)$$

$$= 109 - \frac{1}{4} + \frac{1}{4} \times \frac{1}{5^{109}}$$

$$S = \frac{5}{4}\left(109 - \frac{1}{4} + \frac{1}{4.5^{109}}\right)$$

$$16S = 20 \times 109 - 5 + \frac{1}{5^{108}}$$

$$16S - (25)^{-54} = 2180 - 5 = 2175$$

Question54

Let a, b, c and d be positive real numbers such that $a + b + c + d = 11$. If the maximum value of $a^5b^3c^2d$ is 3750β, then the value of β is
[11-Apr-2023 shift 2]

Options:

A. 55

B. 108

C. 90

D. 110

Answer: C

Solution:

Solution:
Given a + b + c + d = 11
(a, b, c, d > 0}
$(a^5b^3c^2d)$ max. = ?
Let assume Numbers -
$\frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{a}{5}, \frac{b}{3}, \frac{b}{3}, \frac{b}{3}, \frac{c}{2}, \frac{c}{2},$
We know A.M. ≥ G.M.

$$\frac{\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}+d}{11} \geq \left(\frac{a^5b^3c^2d}{5^5\cdot 3^3\cdot 2^2\cdot 1}\right)^{\frac{1}{11}}$$

$$\frac{11}{11} \geq \left(\frac{a^5b^3c^2d}{5^5\cdot 3^3\cdot 2^2\cdot 1}\right)^{\frac{1}{11}}$$

$a^5\cdot b^3\cdot c^2\cdot d \leq 5^5\cdot 3^3\cdot 2^2$
$\max(a^5b^3c^2d) = 5^5\cdot 3^3\cdot 2^2 = 337500$
$= 90\times 3750 = \beta\times 3750$
$\beta = 90$
Option (C) 90 correct

Question55

For k ∈ N, if the sum of the series 1 + $\frac{4}{k}$ + $\frac{8}{k^2}$ + $\frac{13}{k^3}$ + $\frac{19}{k^4}$ + ... is 10 , then the value of k is _____
[11-Apr-2023 shift 2]

Answer: 2

Solution:

Solution:

$$10 = 1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+\ldots\ldots.\text{ upto }\infty$$

$$9 = \frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+\ldots\ldots.\text{ upto }\infty$$

$$\frac{9}{k} = \frac{4}{k^2}+\frac{8}{k^3}+\frac{13}{k^4}+\ldots\ldots\text{upto }\infty$$

$$S = 9\left(1-\frac{1}{k}\right) = \frac{4}{k}+\frac{4}{k^2}+\frac{5}{k^3}+\frac{6}{k^4}\ldots\ldots\text{ upto }\infty$$

$$\frac{S}{k} = \frac{4}{k^2}+\frac{4}{k^3}+\frac{5}{k^4}+\ldots\ldots\text{upto }\infty$$

$$\left(1-\frac{1}{k}\right)S = \frac{4}{k}+\frac{1}{k^3}+\frac{1}{k^4}+\frac{1}{k^5}+\ldots.\infty$$

$$9\left(1-\frac{1}{k}\right)^2 = \frac{4}{k}+\frac{\frac{1}{k^3}}{\left(1-\frac{1}{k}\right)}$$

$9(k-1)^3 = 4k(k-1)+1$
$k = 2$

Question56

Let <a_n> be a sequence such that $a_1 + a_2 + .. + a_n = \frac{n^2 + 3n}{(n+1)(n+2)}$. If $28\sum_{k=1}^{10}\frac{1}{a_k} = p_1p_2p_3...p_m$, where $p_1, p_2 ...... p_m$ are the first m prime numbers, then m is equal to
[12-Apr-2023 shift 1]

Options:

A. 8

B. 5

C. 6

D. 7

Answer: C

Solution:

Solution:

$a_n = S_n - S_{n-1} = \frac{n^2 + 3n}{(n+1)(1+2)} - \frac{(n-1)(n+2)}{n(n+1)}$

$\Rightarrow a_n = \frac{4}{n(n+1)(1+2)}$

$\Rightarrow 28\sum_{k-1}^{10}\frac{1}{a_k} = 28\sum_{k-1}^{10}\frac{k(k+1)(k+2)}{4}$

$= \frac{7}{4}\sum_{k-1}^{10}(k(k+1)(k+2)(k+3) - (k-1)k(k+1)(k+2))$

$= \frac{7}{4}\cdot 10\cdot 11\cdot 12\cdot 13 = 2\cdot 3\cdot 5\cdot 7\cdot 11\cdot 13$

So m = 6

Question57

Let s_1, s_2, s_3, ..., s_{10} respectively be the sum to 12 terms of 10 A.P. s_m whose first terms are 1, 2, 3, ..., 10 and the common differences are 1, 3, 5, ..., 19 respectively. Then $\sum_{i=1}^{10} s_i$ is equal to
[13-Apr-2023 shift 1]

Options:

A. 7260

B. 7380

C. 7220

D. 7360

Answer: A

Solution:

Solution:

$S_k = 6(2k + (11)(2k - 1))$

$S_k = 6(2k + 22k - 11)$

$S_k = 144k - 66$

$$\sum_{1}^{10} S_k = 144 \sum_{k=1}^{10} k - 66 \times 10$$
$$= 144 \times \frac{10 \times 11}{2} - 660$$
$$= 7920 - 660$$
$$= 7260$$

Question58

The sum to 20 terms of the series $2.2^2 - 3^2 + 2.4^2 - 5^2 + 2.6^2 - ....$ is equal to _______.
[13-Apr-2023 shift 1]

Answer: 1310

Solution:

$$(2^2 - 3^2 + 4^2 - 5^2 + 20 \text{ terms }) + (2^2 + 4^2 + ... + 10 \text{ terms })$$
$$-(2 + 3 + 4 + 5 + ... + 11) + 4[1 + 2^2 + ...10^2]$$
$$-\left[\frac{21 \times 22}{2} - 1\right] + 4 \times \frac{10 \times 11 \times 21}{6}$$
$$= 1 - 231 + 14 \times 11 \times 10$$
$$= 1540 + 1 - 231$$
$$= 1310$$

Question59

Let $a_1, a_2, a_3, ...$ be a G. P. of increasing positive numbers. Let the sum of its 6^{th} and 8^{th} terms be 2 and the product of its $3r^d$ and 5^{th} terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to
[13-Apr-2023 shift 2]

Options:

A. 2

B. 3

C. $3\sqrt{3}$

D. $2\sqrt{2}$

Answer: B

Solution:

Solution:

$$a_3 \cdot a_5 = \frac{1}{9}$$
$$\Rightarrow ar^2 \cdot ar^4 = \frac{1}{9}$$
$$\Rightarrow (ar^3)^2 = \frac{1}{9}$$
$$\Rightarrow ar^3 = \frac{1}{3} \ ...(i)$$
$$a_6 + a_8 = 2$$
$$\Rightarrow ar^5 + ar^7 = 2$$

$\Rightarrow ar^3(r^2 + r^4) = 2$
$\Rightarrow \frac{1}{3}r^2(1 + r^2) = 2$
$\Rightarrow r^2(1 + r^2) = 2 \times 3$
$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$
$a = \frac{1}{3} \times \frac{1}{r^3}$
$= \frac{1}{3} \times \frac{1}{2\sqrt{2}} = \frac{1}{6\sqrt{2}}$
$6(a_2 + a_4)(a_4 + a_6)$
$\Rightarrow 6(ar + ar^3)(ar^3 + ar^5)$
$\Rightarrow 6\left(\frac{ar^3}{r^2} + \frac{1}{3}\right)\left(\frac{1}{3} + \frac{1}{3}r^2\right) = 3$

Question60

Let $[\alpha]$ denote the greatest integer $\leq \alpha$. Then $[\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + ... + [\sqrt{120}]$ is equal to _______.
[13-Apr-2023 shift 2]

Answer: 825

Solution:

Solution:
$S = [\sqrt{1}] + [\sqrt{2}] + [\sqrt{3}] + \ldots + [\sqrt{120}]$
$[\sqrt{1}] \to [\sqrt{3}] = 1 \times 3$
$[\sqrt{4}] \to [\sqrt{8}] = 2 \times 5$
$[\sqrt{9}] \to [\sqrt{15}] = 3 \times 7$
$\vdots$
$[\sqrt{100}] \to [\sqrt{120}] = 10 \times 21$
$S = 1 \times 3 + 2 \times 5 + 3 \times 7 + \ldots + 10 \times 21$
$= \sum_{r=1}^{10} r(2r + 1)$
$= 2\sum_{r=1}^{10} r^2 + \sum_{r=1}^{10} r$
$= \frac{2 \times 10 \times 11 \times 21}{6} + \frac{10 \times 11}{2}$
$= 770 + 55$
$= 825$

Question61

Let $f(x) = \sum_{k=1}^{10} kx^k$, $x \in R$ If $2f'(2) - f'(2) = 119(2)^n + 1$ then n is equal to _______
[13-Apr-2023 shift 2]

Answer: 10

Solution:

$f(x) = \sum_{k=1}^{10} kx\,x^k$
$\Rightarrow f(x) = x + 2x^2 + 3x^3 + \cdots + 9x^9 + 10x^{10} - (i)$

$xf(x) = x^2 + 2x^3 + \ldots + 9x^{10} + 10x^{11} \ldots$ (ii)
"(i) − (ii)^1
$f(x)(1 - x) = x + x^2 + x^3 + \cdots + x^{10} - 10x^{11}$
$f(x)(1 - x) = \frac{x(1 - x^{10})}{1 - x} - 10x^{11}$
$f(x) = \frac{x(1 - x^{10})}{(1 - x)^2} - \frac{10x^{11}}{(1 - x)}$
$f(2) = 2 + g(2)^{11}$
$(1 - x)^2 f(x) = x(1 - x^{10}) - 10x^{11}(1 - x)$
diff. w.r.t. x
$(1 - x)^2 f'(2) + f(2)2(1 - x)(-1)$
$= x(-10x^9) + (1 - x^{10}) - 10x^{11}(-1) - (1 - x)(110)x^{10}$

put x = 2
$f'(2) + f(2)(2) = -10(2)^{10} + 1 - 2^{10} + 10(2)^{11} - 110(2)^{10} + 110(2)^{11}$
$= (-121)2^{10} + (120)2^{11} + 1$
$= 2^{10}(240 - 121) + 1$
$= 119(2)^{10} + 1$
$n = 10$

Question62

Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric means of two distinct positive numbers. Then $G_1^4 + G_2^4 + G_3^4 + G_1^2 G_3^2$ is equal to
[15-Apr-2023 shift 1]

Options:

A. $2(A_1 + A_2)G_1 G_3$

B. $(A_1 + A_2)^2 G_1 G_3$

C. $2(A_1 + A_2)G_1^2 G_3^2$

D. $(A_1 + A_2)G_1^2 G_3^2$

Answer: B

Solution:

Solution:
a, A_1, A_2, b are in A.P.
$d = \frac{b - a}{3}$; $A_1 = a + \frac{b - a}{3} = \frac{2a + b}{3}$
$A_2 = \frac{a + 2b}{3}$
$A_1 + A_2 = a + b$
a, G_1, G_2, G_3, b are in G.P.
$r = \left(\frac{b}{a}\right)^{\frac{1}{4}}$
$G_1 = (a^3 b)^{\frac{1}{4}}$
$G_2 = (a^2 b^2)^{\frac{1}{4}}$
$G_3 = (ab^3)^{\frac{1}{4}}$
$G_1^4 + {}_2^4 + G_3^4 + G_1^2 G_3^2 =$
$a^3 b + a^2 b^2 + ab^3 + (a^3 b)^{\frac{1}{2}} \cdot (ab^3)^{\frac{1}{2}}$
$= a^3 b + a^2 b^2 + ab^3 + a^2 \cdot b^2$
$= ab(a^2 + 2ab + b^2)$
$= ab(a + b)^2$

$= G_1 \cdot G_3 \cdot (A_1 + A_2)^2$

Question63

If the sum of the series $\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{2 \cdot 3} + \frac{1}{3^2}\right) + \left(\frac{1}{2^3} - \frac{1}{2^2 \cdot 3} + \frac{1}{2 \cdot 3^2} - \frac{1}{3^3}\right) + \left(\frac{1}{2^4} - \frac{1}{2^2 \cdot 3} + \frac{1}{2^2 \cdot 3^2} - \frac{1}{2 \cdot 3^2} + \frac{1}{3^4}\right) + \ldots\ldots.$ is $\frac{\alpha}{\beta}$, where α and β are co-prime, then $\alpha + 3\beta$ is equal to _______

[15-Apr-2023 shift 1]

Answer: 7

Solution:

Solution:

$P\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{2 \cdot 3} + \frac{1}{3^2}\right) + \left(\frac{1}{2^3} + \frac{1}{2^2 \cdot 3} + \frac{1}{2 \cdot 3^2} - \frac{1}{3^3}\right) + \ldots P\left(\frac{1}{2} + \frac{1}{3}\right) = \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{2^3} + \frac{1}{3^3}\right) + \left(\frac{1}{2^4} - \frac{1}{3^4}\right) + \ldots$

$$\frac{5P}{6} = \frac{\frac{1}{4}}{1 - \frac{1}{2}} - \frac{\frac{1}{9}}{1 + \frac{1}{3}}$$

$$\frac{5P}{6} = \frac{1}{2} - \frac{1}{12} = \frac{5}{12}$$

$$\therefore P = \frac{1}{2} = \frac{\alpha}{\beta} \quad \therefore \alpha = 1, \beta = 2$$

$\alpha + 3\beta = 7$

Question64

If $\{a_i\}_{i=1}^{n}$, where n is an even integer, is an arithmetic progression with common difference 1 , and $\sum_{i=1}^{n} a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to :

[24-Jun-2022-Shift-1]

Options:

A. 48

B. 96

C. 92

D. 104

Answer: B

Solution:

$$\sum_{i=1}^{n} a_i = 192$$

$$\Rightarrow a_1 + a_2 + a_3 + \ldots\ldots + a_n = 192$$

$$\Rightarrow \frac{n}{2}[a_1 + a_n] = 192$$

$$\Rightarrow a_1 + a_n = \frac{384}{n} \ldots\ldots \ (1)$$

Now, $\sum_{i=1}^{\frac{n}{2}} a_{2i} = 120$

$$\Rightarrow a_2 + a_4 + a_6 + \ldots\ldots + a_n = 120$$

Here total $\frac{n}{2}$ terms present.

$$\therefore \frac{\frac{n}{2}}{2}[a_2 + a_n] = 120$$

$$\Rightarrow \frac{n}{4}[a_1 + 1 + a_n] = 120$$

$$\Rightarrow a_1 + a_n + 1 = \frac{480}{n} \ldots\ldots$$

Subtracting (1) from (2), we get

$$1 = \frac{480}{n} - \frac{384}{n}$$

$$\Rightarrow 1 = \frac{96}{n}$$

$$\Rightarrow n = 96$$

Question65

If $\frac{1}{2 \cdot 3^{10}} + \frac{1}{2^2 \cdot 3^9} + \ldots + \frac{1}{2^{10} \cdot 3} = \frac{K}{2^{10} \cdot 3^{10}}$, then the remainder when K is divided by 6 is :

[25-Jun-2022-Shift-1]

Options:

A. 1

B. 2

C. 3

D. 5

Answer: D

Solution:

Solution:

$$\frac{1}{2\cdot 3^{10}}+\frac{1}{2^2\cdot 3^9}+\ldots+\frac{1}{2^{10}\cdot 3}=\frac{K}{2^{10}\cdot 3^{10}}$$

$$\Rightarrow \frac{1}{2\cdot 3^{10}}\left[\frac{\left(\frac{3}{2}\right)^{10}-1}{\frac{3}{2}-1}\right]=\frac{K}{2^{10}\cdot 3^{10}}$$

$$=\frac{3^{10}-2^{10}}{2^{10}\cdot 3^{10}}=\frac{K}{2^{10}\cdot 3^{10}}\Rightarrow K=3^{10}-2^{10}$$

Now $K=(1+2)^{10}-2^{10}$

$$={}^{10}C_0+{}^{10}C_1 2+{}^{10}C_2 2^3+\ldots+{}^{10}C_{10}2^{10}-2^{10}$$

$$={}^{10}C_0+{}^{10}C_1 2+6\lambda+{}^{10}C_9\cdot 2^9$$

$$=1+20+5120+6\lambda$$

$$=5136+6\lambda+5$$

$$=6\mu+5$$

$\lambda,\mu\in N$

$\therefore$ remainder $=5$

Question66

The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, .$ is equal to
[25-Jun-2022-Shift-1]

Options:

A.

Answer: 98

Solution:

$S = \frac{1}{3} + \frac{5}{9} + \frac{19}{27} + \frac{65}{81} + \ldots$

$= \sum_{r=1}^{100} \left(\frac{3^r - 2^r}{3^r} \right)$

$= 100 - \frac{2}{3} \frac{\left(1 - \left(\frac{2}{3}\right)^{100}\right)}{1/3}$

$= 98 + 2\left(\frac{2}{3}\right)^{100}$

$\therefore [S] = 98$

Question67

The sum $1 + 2 \cdot 3 + 3 \cdot 3^2 + \ldots\ldots\ldots + 10.3^9$ is equal to [25-Jun-2022-Shift-2]

Options:

A. $\frac{2 \cdot 3^{12} + 10}{4}$

B. $\frac{19 \cdot 3^{10} + 1}{4}$

C. $5 \cdot 3^{10} - 2$

D. $\frac{9 \cdot 3^{10} + 1}{2}$

Answer: B

Solution:

Solution:

Let $S = 1.3^0 + 2.3^1 + 3.3^2 + \ldots\ldots + 10.3^9$

$3S = 1.3^1 + 2.3^2 + \ldots\ldots\ldots + 10.3^{10}$

$-2S = (1.3^0 + 1.3^1 + 1.3^2 + \ldots\ldots + 1.3^9) - 10.3^{10}$

$\Rightarrow S = \frac{1}{2}\left[10.3^{10} - \frac{3^{10} - 1}{-3 - 1} \right]$

$\Rightarrow S = \frac{19.3^{10} + 1}{4}$

Question68

Let $A = \sum_{i=1}^{10} \sum_{j=1}^{10} \min\{i, j\}$ and $B = \sum_{i=1}^{10} \sum_{j=1}^{10} \max\{i, j\}$. Then $A + B$ is equal to [26-Jun-2022-Shift-1]

Answer: 1100

Solution:

$$A = \sum_{i=1}^{10} \sum_{j=1}^{10} \min\{i, j\}$$

$$B = \sum_{i=1}^{10} \sum_{j=1}^{10} \max\{i, j\}$$

$$A = \sum_{j=1}^{10} \min(i, 1) + \min(j, 2) + \ldots \min(i, 10)$$

$$= (\underbrace{1 + 1 + 1 + \ldots + 1}_{19 \text{ times}}) + (\underbrace{2 + 2 + 2 \ldots + 2}_{17 \text{ times}}) + (\underbrace{3 + 3 + 3 \ldots + 3}_{15 \text{ times}}) + \ldots (1)1 \text{ times}$$

$$B = \sum_{j=1}^{10} \max(i, 1) + \max(j, 2) + \ldots \max(i, 10)$$

$$= (\underbrace{10 + 10 + \ldots + 10}_{19 \text{ times}}) + (\underbrace{9 + 9 + \ldots + 9}_{17 \text{ times}}) + \ldots + 11 \text{ times}$$

$$A + B = 20(1 + 2 + 3 + \ldots + 10)$$

$$= 20 \times \frac{10 \times 11}{2} = 10 \times 110 = 1100$$

Question69

If $A = \sum_{n=1}^{\infty} \frac{1}{(3 + (-1)^n)^n}$ and $B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3 + (-1)^n)^n}$, then $\frac{A}{B}$ is equal to : [26-Jun-2022-Shift-2]

Options:

A. $\frac{11}{9}$

B. 1

C. $-\frac{11}{9}$

D. $-\frac{11}{3}$

Answer: C

Solution:

Solution:

$$A = \sum_{n=1}^{\infty} \frac{1}{(3 + (-1)^n)^n} \text{ and } B = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3 + (-1)^n)^n}$$

$$A = \frac{1}{2} + \frac{1}{4^2} + \frac{1}{2^3} + \frac{1}{4^4} + \ldots\ldots$$

$$B = \frac{-1}{2} + \frac{1}{4^2} - \frac{1}{2^3} + \frac{1}{4^4} + \ldots\ldots$$

$$A = \frac{\frac{1}{2}}{1 - \frac{1}{4}} + \frac{\frac{1}{16}}{1 - \frac{1}{16}}, B = \frac{-\frac{1}{2}}{1 - \frac{1}{4}} + \frac{\frac{1}{16}}{1 - \frac{1}{16}}$$

$$A = \frac{11}{15}, B = \frac{-9}{15}$$

$$\therefore \frac{A}{B} = \frac{-11}{9}$$

Question70

If $a_1(>0)$, a_2, a_3, a_4, a_5 are in a G.P., $a_2 + a_4 = 2a_3 + 1$ and $3a_2 + a_3 = 2a_4$, then $a_2 + a_4 + 2a_5$ is equal to____
[26-Jun-2022-Shift-2]

Answer: 40

Solution:

Solution:

Let G.P. be $a_1 = a$, $a_2 = ar$, $a_3 = ar^2$,

$\because 3a_2 + a_3 = 2a_4$

$\Rightarrow 3ar + ar^2 = 2ar^3$

$\Rightarrow 2ar^2 - r - 3 = 0$

$\therefore r = -1$ or $\frac{3}{2}$

$\because a_1 = a > 0$ then $r \neq -1$

Now, $a_2 + a_4 = 2a_3 + 1$

$ar + ar^3 = 2ar^2 + 1$

$a\left(\frac{3}{2} + \frac{27}{8} - \frac{9}{2}\right) = 1$

$\therefore a = \frac{8}{3}$

$\therefore a_2 + a_4 + 2a_5 = a(r + r^3 + 2r^4)$

$= \frac{8}{3}\left(\frac{3}{2} + \frac{27}{8} + \frac{81}{8}\right) = 40$

Question71

$x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$, $abc \neq 0$, then :
[27-Jun-2022-Shift-1]

Options:

A. x, y, z are in A.P.

B. x, y, z are in G.P.

C. $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in A.P.

D. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1 - (a + b + c)$

Answer: C

Solution:

$x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$; $y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b}$; $z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c}$
Now,
$a, b, c \rightarrow AP$
$1-a, 1-b, 1-c \rightarrow AP$
$\frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c} \rightarrow HP$
$x, y, z \rightarrow HP$
$\therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z} \rightarrow AP$

Question72

If the sum of the first ten terms of the series
$\frac{1}{5} + \frac{2}{65} + \frac{3}{325} + \frac{4}{1025} + \frac{5}{2501} + \ldots$
is $\frac{m}{n}$, where m and n are co-prime numbers, then m + n is equal to
[27-Jun-2022-Shift-1]

Answer: 276

Solution:

Solution:
$T_r = \frac{r}{(2r^2)^2 + 1}$
$= \frac{r}{(2r^2+1)^2 - (2r)^2}$
$= \frac{1}{4} \frac{4r}{(2r^2+2r+1)(2r^2-2r+1)}$
$S_{10} = \frac{1}{4} \sum_{r=1}^{10} \left(\frac{1}{(2r^2-2r+1)} - \frac{1}{(2r^2+2r+1)} \right)$
$= \frac{1}{4} \left[1 - \frac{1}{5} + \frac{1}{5} - \frac{1}{13} + \ldots + \frac{1}{181} - \frac{1}{221} \right]$
$\Rightarrow S_{10} = \frac{1}{4} \cdot \frac{220}{221} = \frac{55}{221} = \frac{m}{n}$
$\therefore m + n = 276$

Question73

Let S = 2 + $\frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots\ldots$ Then 4S is equal to
[27-Jun-2022-Shift-2]

Options:

A. $\left(\frac{7}{3}\right)^2$

B. $\frac{7^3}{3^2}$

C. $\left(\frac{7}{3}\right)^3$

D. $\frac{7^2}{3^3}$

Answer: C

Solution:

Solution:

$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots\ldots\ldots$ (i)

$\frac{1}{7}S = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \ldots\ldots\ldots$ (ii)

(i) - (ii)

$\frac{6}{7}S = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \ldots\ldots\ldots\ldots$ (iii)

$\frac{6}{7^2}S = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \ldots\ldots\ldots\ldots$ (iv)

(iii) - (iv)

$\left(\frac{6}{7}\right)^2 S = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \ldots\ldots.$

$= 2\left[\frac{1}{1 - \frac{1}{7}}\right] = 2\left(\frac{7}{6}\right)$

$\therefore 4S = 8\left(\frac{7}{6}\right)^3 = \left(\frac{7}{3}\right)^3$

Question74

If $a_1, a_2, a_3 \ldots\ldots$ and $b_1, b_2, b_3 \ldots\ldots$ are A.P., and $a_1 = 2, a_{10} = 3, a_1 b_1 = 1 = a_{10} b_{10}$, then $a_4 b_4$ is equal to - [27-Jun-2022-Shift-2]

Options:

A. $\frac{35}{27}$

B. 1

C. $\frac{27}{28}$

D. $\frac{28}{27}$

Answer: D

Solution:

Solution:

$a_1, a_2, a_3 \ldots$ are in A.P. (Let common difference is d_1)

$b_1, b_2, b_3 \ldots$ are in A.P. (Let common difference is d_2)

and $a_1 = 2, a_{10} = 3, a_1 b_1 = 1 = a_{10} b_{10}$

$\because a_1 b_1 = 1$

$\therefore b_1 = \frac{1}{2}$

$a_{10} b_{10} = 1$

$\therefore b_{10} = \frac{1}{3}$

Now, $a_{10} = a_1 + 9d_1 \Rightarrow d_1 = \frac{1}{9}$

$b_{10} = b_1 + 9d_2 \Rightarrow d_2 = \frac{1}{9}\left[\frac{1}{3} - \frac{1}{2}\right] = -\frac{1}{54}$

Now, $a_4 = 2 + \frac{3}{9} = \frac{7}{3}$

$b_4 = \frac{1}{2} - \frac{3}{54} = \frac{4}{9}$

$\therefore a_4 b_4 = \frac{28}{27}$

Question75

Let $A_1, A_2, A_3, \ldots\ldots$ be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1256}$ and $A_2 + A_4 = \frac{7}{36}$, then the value of $A_6 + A_8 + A_{10}$ is equal to
[28-Jun-2022-Shift-1]

Options:

A. 33

B. 37

C. 43

D. 47

Answer: C

Solution:

Solution:
$A_1 \cdot A_3 \cdot A_5 \cdot A_7 = \frac{1}{1296}$
$(A_4)^4 = \frac{1}{1296}$
$A_4 = \frac{1}{6}$
$A_2 + A_4 = \frac{7}{36}$
$A_2 = \frac{1}{36}$
$A_6 = 1$
$A_8 = 6$
$A_{10} = 36$
$A_6 + A_8 + A_{10} = 43$

Question76

If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1 : 7 and a + n = 33, then the value of n is :
[28-Jun-2022-Shift-2]

Options:

A. 21

B. 22

C. 23

D. 24

Answer: C

Solution:

Solution:
$a, A_1, A_2 \ldots\ldots\ldots . A_n, 100$
Let d be the common difference of above A.P. then
$\frac{a+d}{100-d} = \frac{1}{7}$
$\Rightarrow 7a + 8d = 100 \ldots\ldots.$ (i)
and $a + n = 33$

and $100 = a + (n + 1)d$
$\Rightarrow 100 = a + (34 - a)\frac{(100 - 7a)}{8}$
$\Rightarrow 800 = 8a + 7a^2 - 338a + 3400$
$\Rightarrow 7a^2 - 330a + 2600 = 0$
$\Rightarrow a = 10, \frac{260}{7}$, but $a \neq \frac{260}{7}$
$\therefore n = 23$

Question77

Let for n = 1, 2, ..., 50, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of $\frac{1}{26} + \sum_{n=1}^{50}\left(S_n + \frac{2}{n+1} - n - 1\right)$ is equal to____
[28-Jun-2022-Shift-2]

Answer: 41651

Solution:

Solution:
$S_n = \frac{n^2}{1 - \frac{1}{(n+1)^2}} = \frac{n(n+1)^2}{n+2} = (n^2 + 1) - \frac{2}{n+2}$
Now $\frac{1}{26} + \sum_{n=1}^{50}\left(S_n + \frac{2}{n+1} - n - 1\right)$
$= \frac{1}{26} + \sum_{n=1}^{50}\left\{(n^2 - n) + 2\left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right\}$
$= \frac{1}{26} + \frac{50 \times 51 \times 101}{6} - \frac{50 \times 51}{2} + 2\left(\frac{1}{2} - \frac{1}{52}\right)$
$= 1 + 25 \times 17(101 - 3)$
$= 41651$

Question78

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$. Then, $\sum_{n=2}^{\infty}\frac{a_n}{7^n}$ is equal to:
[29-Jun-2022-Shift-1]

Options:

A. $\frac{6}{343}$

B. $\frac{7}{216}$

C. $\frac{8}{343}$

D. $\frac{49}{216}$

Answer: B

Solution:

Solution:

$a_{n+2} = 2a_{n+1} - a_n + 1 \& a_0 = a_1 = 0$

$a_2 = 2a_1 - a_0 + 1 = 1$

$a_3 = 2a_2 - a_1 + 1 = 3$

$a_4 = 2a_3 - a_2 + 1 = 6$

$a_5 = 2a_4 - a_3 + 1 = 10$

$\sum_{n=2}^{\infty} \frac{a_n}{7^n} = \frac{a_2}{7^2} + \frac{a_3}{7^3} + \frac{a_4}{7^4} + \ldots$

$s = \frac{1}{7^2} + \frac{3}{7^3} + \frac{6}{7^4} + \frac{10}{7^5} + \ldots$

$\frac{1}{7}s = \frac{1}{7^3} + \frac{3}{7^4} + \frac{6}{7^5} + \ldots$

$\frac{6s}{7} = \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \ldots$

$\frac{6s}{49} = \frac{1}{7^3} + \frac{2}{7^4} + \ldots$

$\frac{36s}{49} = \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \ldots$

$\frac{36s}{49} = \frac{\frac{1}{7^2}}{1 - \frac{1}{7}}$

$\frac{36s}{49} = \frac{7}{49 \times 6}$

$s = \frac{7}{216}$

Question79

The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + \ldots\ldots$ is equal to:
[29-Jun-2022-Shift-2]

Options:

A. $\frac{425}{216}$

B. $\frac{429}{216}$

C. $\frac{288}{125}$

D. $\frac{280}{125}$

Answer: C

Solution:

Solution:

Question80

Let 3, 6, 9, 12, upto 78 terms and 5, 9, 13, 17, upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to_____
[29-Jun-2022-Shift-2]

Answer: 2223

Solution:

Solution:
1st AP :
3, 6, 9, 12, ……. upto 78 terms
$t_{78} = 3 + (78 - 1)3$
$= 3 + 77 \times 3$
$= 234$
2nd AP:
5, 9, 13, 17, …… upto 59 terms
$t_{59} = 5 + (59 - 1)4$
$= 5 + 58 \times 4$
$= 237$
Common term's AP :
First term $= 9$
Common difference of first AP $= 3$
And common difference of second AP $= 4$
∴ Common difference of common terms
$AP = LCM(3, 4) = 12$
$\therefore$ New AP $= 9, 21, 33, \ldots\ldots$
$t_n = 9 + (n - 1)12 \le 234$
$\Rightarrow n \le \frac{237}{12}$
$\Rightarrow n = 19$
$\therefore S_{19} = \frac{19}{2}[2.9 + (19 - 1)12]$
$= 19(9 + 108)$
$= 2223$

Question81

Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq \text{sl ant}2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to [25-Jul-2022-Shift-1]

Answer: 27560

Solution:

Solution:
Given,
$a_n = a_{n-1} + 2$
$\Rightarrow a_n - a_{n-1} = 2$
∴ In this series between any two consecutives terms difference is 2 . So this is an A.P. with common difference 2.
Also given $a_1 = 1$
∴ Series is $= 1, 3, 5, 7\ldots\ldots$
$\therefore a_n = 1 + (n - 1)2 = 2n - 1$
Also $b_n = a_n + b_{n-1}$
When $n = 2$ then
$b_2 - b_1 = a_2 = 3$
$. \Rightarrow b_2 - 1 = 3$ [Given $b_1 = 1$]
$\Rightarrow b_2 = 4$
When $n = 3$ then
$b_3 - b_2 = a_3$
$\Rightarrow b_3 - 4 = 5$
$\Rightarrow b_3 = 9$
∴ Series is $= 1, 4, 9\ldots\ldots$
$= 1^2, 2^2, 3^2\ldots\ldots n^2$
$\therefore b_n = n^2$
Now, $\sum_{n=1}^{15}(a_n . b_n)$
$= \sum_{n=1}^{15}[(2n - 1)n^2]$

$= \sum_{n=1}^{15} 2n^3 - \sum_{n=1}^{15} n^2$
$= 2(1^3 + 2^3 + \ldots 15^3) - (1^2 + 2^2 + \ldots 15^2)$
$= 2 \times \left(\frac{15 \times 16}{2} \right)^2 - \left(\frac{15(16) \times 31}{6} \right)$
$= 27560$

Question82

The sum $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$ is equal to
[25-Jul-2022-Shift-2]

Options:

A. $\frac{7}{87}$

B. $\frac{7}{29}$

C. $\frac{14}{87}$

D. $\frac{21}{29}$

Answer: B

Solution:

Solution:
$\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)} = \frac{3}{4} \sum_{n=1}^{21} \frac{1}{4n-1} - \frac{1}{4n+3}$
$= \frac{3}{4} \left[\left(\frac{1}{3} - \frac{1}{7} \right) + \left(\frac{1}{7} - \frac{1}{11} \right) + \ldots + \left(\frac{1}{83} - \frac{1}{87} \right) \right]$
$= \frac{3}{4} \left[\frac{1}{3} - \frac{1}{87} \right] = \frac{3}{4} \frac{84}{3.87} = \frac{7}{29}$

Question83

Consider two G.Ps. $2, 2^2, 2^3, \ldots$ and $4, 4^2, 4^3, \ldots$ of 60 and n terms respectively. If the geometric mean of all the 60 + n terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to :
[26-Jul-2022-Shift-1]

Options:

A. 560

B. 1540

C. 1330

D. 2600

Answer: C

Solution:

Solution:
Given G.P's $2, 2^2, 2^3, \ldots .60$ terms

$4, 4^2, \ldots.$ n terms

Now, G.M $= 2^{\frac{225}{8}}$

$(2.2^2 \ldots 4.4^2 \ldots)^{\frac{1}{60+n}} = 2^{\frac{225}{8}}$

$\left(2^{\frac{n^2+n+1830}{6+n}}\right) = 2^{\frac{225}{8}}$

$\Rightarrow \frac{n^2+n+1830}{60+n} = \frac{225}{8}$

$\Rightarrow 8n^2 - 217n + 1140 = 0$

$n = \frac{57}{8}, 20,\ \ $ so $\ n = 20$

$\therefore \sum_{k=1}^{20} k(20-k) = 20 \times \frac{20 \times 21}{2} - \frac{20 \times 21 \times 41}{6}$

$= \frac{20 \times 21}{2}\left[20 - \frac{41}{3}\right] = 1330$

Question84

The series of positive multiples of 3 is divided into sets :
{3}, {6, 9, 12}, {15, 18, 21, 24, 27}, ... Then the sum of the elements in the 11^{th} set is equal to ________.
[26-Jul-2022-Shift-1]

Answer: 6993

Solution:

Solution:

$\underbrace{\{3 \times 1\}}_{1-\text{term}}, \underbrace{\{3 \times 2, 3 \times 3, 3 \times 4\}}_{3\,\text{terms}}, \underbrace{\{3 \times 5, 3 \times 6, 3 \times 7, 3 \times 8, 3 \times 9\}}_{5-\text{terms}}, \ldots$

$\therefore 11^{\text{th}}$ set will have $1 + (10)2 = 21$ term
Also upto 10^{th} set total $3 \times k$ type terms will be $1 + 3 + 5 + \ldots\ldots + 19 = 100 -$ term
$\therefore$ Set 11 $= \{3 \times 101, 3 \times 102, \ldots\ldots. 3 \times 121\}$
$\therefore$ Sum of elements $= 3 \times (101 + 102 + \ldots + 121)$

$= \frac{3 \times 222 \times 21}{2} = 6993$

Question85

If $\sum_{k=1}^{10} \frac{k}{k^4+k^2+1} = \frac{m}{n}$, where m and n are co-prime, then m + n is equal to ________.
[26-Jul-2022-Shift-2]

Answer: 166

Solution:

$$\sum_{k=1}^{10} \frac{k}{k^4+k^2+1}$$

$$= \frac{1}{2}\left[\sum_{k=1}^{10}\left(\frac{1}{k^2-k+1} - \frac{1}{k^2+k+1}\right).\right.$$

$$= \frac{1}{2}\left[1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{7} + \frac{1}{7} - \frac{1}{13} + \ldots + \frac{1}{91} - \frac{1}{111}\right]$$
$$= \frac{1}{2}\left[1 - \frac{1}{111}\right] = \frac{110}{2.111} = \frac{55}{111} = \frac{m}{n}$$
$\therefore m + n = 55 + 111 = 166$

Question86

Different A.P.'s are constructed with the first term 100, the last term 199, and integral common differences. The sum of the common differences of all such A.P.'s having at least 3 terms and at most 33 terms is ________.
[26-Jul-2022-Shift-2]

Answer: 53

Solution:

Solution:
$d_1 = \frac{199 - 100}{2} \notin I$
$d_2 = \frac{199 - 100}{3} = 33$
$d_3 = \frac{199 - 100}{4} \notin I$
$d_n = \frac{199 - 100}{i + 1} \in I$
$d_i = 33 + 11, 9$
Sum of CD's $= 33 + 11 + 9$
$= 53$

Question87

Suppose $a_1, a_2, \ldots, a_n, \ldots$ be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is 5 : 17 and , $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to
[27-Jul-2022-Shift-1]

Options:

A. 290

B. 380

C. 460

D. 510

Answer: B

Solution:

Solution:
$\because a_1, a_2, \ldots . a_n$ be an A.P of natural numbers and

$$\frac{S_5}{S_9} = \frac{5}{17} \Rightarrow \frac{\frac{5}{2}[2a_1 + 4d]}{\frac{9}{2}[2a_1 + 8d]} = \frac{5}{17}$$

$\Rightarrow 34a_1 + 68d = 18a_1 + 72d$
$\Rightarrow 16a_1 = 4d$
$\therefore d = 4a_1$
And $110 < a_{15} < 120$
$\therefore 110 < a_1 + 14d < 120 \Rightarrow 110 < 57a_1 < 120$
$\therefore a_1 = 2 (\because a_i \in N)$
$d = 8$
$\therefore S_{10} = 5[4 + 9 \times 8] = 380$

Question88

Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \leq 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to _______.
[27-Jul-2022-Shift-1]

Answer: 10620

Solution:

Solution:
$\because |f(n)| \leq 800$
$\Rightarrow -800 \leq 2n^2 - n - 1 \leq 800$
$\Rightarrow 2n^2 - n - 801 \leq 0$
$\therefore n \in \left[\frac{-\sqrt{6409} + 1}{4}, \frac{\sqrt{6409} + 1}{4} \right]$ and $n \in z$
$\therefore n = -19, -18, -17, \ldots\ldots\ldots, 19, 20.$
$\therefore \sum (2x^2 - x - 1) = 2\sum x^2 - \sum x - \sum 1.$
$= 2.2 \cdot (1^2 + 2^2 + \ldots + 19^2) + 2.20^2 - 20 - 40$
$= 10620$

Question89

Let the sum of an infinite G.P., whose first term is a and the common ratio is r, be 5 . Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is 10ar, n^{th} term is a_n and the common difference is $10ar^2$, is equal to :
[27-Jul-2022-Shift-2]

Options:

A. $21a_{11}$

B. $22a_{11}$

C. $15a_{16}$

D. $14a_{16}$

Answer: A

Solution:

Solution:
Let first term of G.P. be a and common ratio is r Then, $\frac{a}{1-r} = 5 \ldots\ldots$ (i)

$a\frac{(r^5-1)}{(r-1)} = \frac{98}{25} \Rightarrow 1-r^5 = \frac{98}{125}$

$\therefore r^5 = \frac{27}{125}, r = \left(\frac{3}{5}\right)^{\frac{3}{5}}$

$\therefore$ Then, $S_{21} = \frac{21}{2}[2\times 10ar + 20\times 10ar^2]$

$= 21[10ar + 10.10ar^2]$

$= 21a_{11}$

Question90

$\frac{2^3-1^3}{1\times 7} + \frac{4^3-3^3+2^3-1^3}{2\times 11} + \frac{6^3-5^3+4^3-3^3+2^3-1^3}{3\times 15} + \cdots + \frac{30^3-29^3+28^3-27^3+\ldots+2^3-1^3}{15\times 63}$ is equal to ________.

[27-Jul-2022-Shift-2]

Answer: 120

Solution:

Solution:

$T_n = \frac{\sum_{k=1}^{n}[(2k)^3-(2k-1)^3]}{n(4n+3)}$

$= \frac{\sum_{k=1}^{n} 4k^2+(2k-1)^2+2k(2k-1)}{n(4n+3)}$

$= \frac{\sum_{k=1}^{n}(12k^2-6k+1)}{n(4n+3)}$

$= \frac{2n(2n^2+3n+1)-3n^2-3n+n}{n(4n+3)}$

$= \frac{n^2(4n+3)}{n(4n+3)} = n$

$\therefore T_n = n$

$S_n = \sum_{n=1}^{15} T_n = \frac{15\times 16}{2} = 120$

Question91

Consider the sequence $a_1, a_2, a_3, \ldots$ such that $a_1 - 1, a_2 - 2$ and $a_{n+2} = \frac{2}{a_{n+1}} + a_{-n}$ for $n - 1, 2, 3, \ldots$ If $\left(\frac{a_1+\frac{1}{a_2}}{a_3}\right)\cdot\left(\frac{a_2+\frac{1}{a_3}}{a_4}\right)\cdot\left(\frac{a_3+\frac{1}{a_4}}{a_5}\right)\ldots\left(\frac{a_{30}+\frac{1}{a_{31}}}{a_{32}}\right) = 2^{\alpha}({}^{61}C_{31})$, then α is equal to :

[28-Jul-2022-Shift-1]

Options:

A. -30

B. -31

C. -60

D. −61

Answer: C

Solution:

Solution:

$a_{n+2} = \frac{2}{a_{n+1}} + a_n$

$\Rightarrow a_n a_{n+1} + 1 = a_{n+1} a_{n+2} - 1$

$\Rightarrow a_{n+2} a_{n+1} - a_n \cdot a_{n+1} = 2$

For

$n = 1\ a_3 a_2 - a_1 a_2 = 2$

$n = 2\ a_4 a_3 - a_3 a_2 = 2$

$n = 3\ a_5 a_4 - a_4 a_3 = 2$

.

.

.

$n = n \quad \frac{a_{n+2} a_{n+1} - a_n a_{n+1} = 2}{a_{n+2} a_{n+1} = 2n + a_1 a_2}$

Now,

$\frac{(a_1 a_2 + 1)}{a_2 a_3} \cdot \frac{(a_2 a_3 + 1)}{a_3 a_4} \cdot \frac{(a_3 a_4 + 1)}{a_4 a_5} \ldots\ldots \cdot \frac{(a_{30} a_{31} + 1)}{a_{31} a_{32}}$

$= \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8} \ldots\ldots \cdot \frac{61}{62}$

$= 2^{-60} ({}^{61}C_{31})$

Question92

For p, q ∈ R_s consider the real valued function $f(x) = (x - p)^2 - q$, $x \in R$ and $q > 0$. Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $f(a_i)$ | = 500 for all i = 1, 2, 3, 4, then the absolute difference between the roots of $f(x) = 0$ is _______.
[28-Jul-2022-Shift-1]

Answer: 50

Solution:

Solution:

$\because a_1, a_2, a_3, a_4$

$\therefore a_2 = p - 3d$, $a_2 = p - d$, $a_3 = p + d$ and $a_4 = p + 3d$

Where $d > 0$

$\because |f(a_i)| = 500$

$\Rightarrow |9d^2 - q| = 500$

and $|d^2 - q| = 500$

either $9d^2 - q = d^2 - q$

$\Rightarrow d = 0$ not acceptable

$\therefore 9d^2 - q = q - d^2$

$\therefore 5d^2 - q = 0$

Roots of $f(x) = 0$ are $p + \sqrt{q}$ and $p - \sqrt{q}$

$\therefore$ absolute difference between roots $= |2\sqrt{q}| = 50$

Question93

Let $x_1, x_2, x_3, \ldots, x_{20}$ be in geometric progression with $x_1 = 3$ and the common

ratio $\frac{1}{2}$. A new data is constructed replacing each x_i by $(x_i - i)^2$. If $\bar{x}$ is the mean of new data, then the greatest integer less than or equal to $\bar{x}$ is ________.
[28-Jul-2022-Shift-1]

Answer: 142

Solution:

Solution:
$x_1, x_2, x_3, \ldots, x_{20}$ are in G.P.
$x_1 = 3, r = \frac{1}{2}$
$\bar{x} = \frac{\sum x_i^2 - 2x_i i + i^2}{20}$
$= \frac{1}{20}\left[12\left(1 - \frac{1}{2^{40}}\right) - 6\left(4 - \frac{11}{2^{18}}\right) + 70 \times 41\right]$

$$\left\{\begin{array}{l} S = 1 + 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2^2} + \ldots \\ \frac{S}{2} = \frac{1}{2} + \frac{2}{2^2} + \ldots . \end{array}\right.$$

$\cdot \frac{S}{2} = 2\left(1 - \frac{1}{2^{20}}\right) - \frac{20}{2^{20}} = 4 - \frac{11}{2^{18}}\}$
$\therefore [\bar{x}] = \left[\frac{2858}{20} - \left(\frac{12}{240} - \frac{66}{2^{18}}\right) \cdot \frac{1}{20}\right]$
$= 142$

Question94

$\frac{6}{3^{12}} + \frac{10}{3^{11}} + \frac{20}{3^{10}} + \frac{40}{3^9} + \ldots + \frac{10240}{3} = 2^n \cdot m$, where m is odd, then m . n is equal to_______.
[28-Jul-2022-Shift-2]

Answer: 12

Solution:

Solution:
$\frac{1}{3^{12}} + 5\left(\frac{2^0}{3^{12}} + \frac{2^1}{3^{11}} + \frac{2^2}{3^{10}} + \ldots . + \frac{2^{11}}{3}\right) = 2^n \cdot m$
$\Rightarrow \frac{1}{3^{12}} + 5\left(\frac{1}{3^{12}} \frac{((6)^2 - 1)}{(6 - 1)}\right) = 2^n \cdot m$
$\Rightarrow \frac{1}{3^{12}} + \frac{5}{5}\left(\frac{1}{3^{12}} \cdot 2^{12} \cdot 3^{12} - \frac{1}{3^{12}}\right) = 2^n \cdot m$
$\Rightarrow \frac{1}{3^{12}} + 2^{12} - \frac{1}{3^{12}} = 2^n \cdot m$
$\Rightarrow 2^n \cdot m = 2^{12}$
$\Rightarrow m = 1$ and $n = 12$
$m \cdot n = 12$

Question95

If $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + \ldots + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$, then the maximum value of a is :
[29-Jul-2022-Shift-1]

Options:

A. 198

B. 202

C. 212

D. 218

Answer: C

Solution:

Solution:
$\frac{1}{20}\left(\frac{1}{20-a} - \frac{1}{40-a} + \frac{1}{40-a} - \frac{1}{60-a} + \ldots + \frac{1}{180-a} - \frac{1}{200-a}\right) = \frac{1}{256}$
$\Rightarrow \frac{1}{20}\left(\frac{1}{20-a} - \frac{1}{200-a}\right) = \frac{1}{256}$
$\Rightarrow \frac{1}{20}\left(\frac{180}{(20-a)(200-a)}\right) = \frac{1}{256}$
$\Rightarrow (20-a)(200-a) = 9.256$
OR $a^2 - 220a + 1696 = 0$
$\Rightarrow a = 212, 8$

Question96

Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to ________.
[29-Jul-2022-Shift-1]

Answer: 16

Solution:

Solution:
Given
$S = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \frac{a_4}{2^4} + \ldots\ldots\infty$

$\frac{1}{2}S = \frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots\ldots\ldots\infty$

$\frac{S}{2} = \frac{a_1}{2} + \frac{(a_2 + a_1)}{2^2} + \frac{(a_3 + a_2)}{2^3} + \ldots\ldots\infty$

$\Rightarrow \frac{S}{2} = \frac{a_1}{2} + \frac{d}{2}$
$\Rightarrow a_1 + d = a_2 = 4 \Rightarrow 4a_2 = 16$

Question97

If $\frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + \frac{1}{4\times5\times6} + \ldots + \frac{1}{100\times101\times102} = \frac{k}{101}$, then 34k is equal to ________.
[29-Jul-2022-Shift-1]

Answer: 286

Solution:

Solution:

$S = \frac{1}{2 \times 3 \times 4} + \frac{1}{3 \times 4 \times 5} + \frac{1}{4 \times 5 \times 6} + \ldots + \frac{1}{100 \times 101 \times 102}$

$= \frac{1}{(3-1) \cdot 1}\left[\frac{1}{2 \times 3} - \frac{1}{101 \times 102}\right]$

$= \frac{1}{2}\left(\frac{1}{6} - \frac{1}{101 \times 102}\right)$

$= \frac{143}{102 \times 101} = \frac{k}{101}$

$\therefore 34k = 286$

Question98

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 - a_1 - 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1$, $\forall n \geq 0$. Then $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$ is equal to [29-Jul-2022-Shift-2]

Options:

A. 483

B. 528

C. 575

D. 624

Answer: B

Solution:

Solution:

$a_0 = 0, a_1 = 0$

$a_{n+2} = 3a_{n+1} - 2a_{n+1} : n \geq 0$

$a_{n+2} - a_{n+1} = 2(a_{n+1} - a_n) + 1$

$n = 0 \ \ a_2 - a_1 = 2(a_1 - a_0) + 1$

$n = 1 \ \ a_3 - a_2 = 2(a_2 - a_1) + 1$

$n = 2 \ \ a_4 - a_3 = 2(a_3 - a_2) + 1$

$n = n \ \ a_{n+2} - a_{n+1} = 2(a_{n+1} - a_n) + 1$

$(a_{n+2} - a_1) - 2(a_{n+1} - a_0) - (n+1) = 0$

$a_{n+2} = 2a_{n+1} + (n+1)$

$n \rightarrow n - 2$

$a_n - 2a_{n-1} = n - 1$

Now $a_{25}a_{23} - 2a_{25}a_{22} - 2a_{23}a_{24} + 4a_{22}a_{24}$

$= (a_{25} - 2a_{24})(a_{23} - 2a_{22}) = (24)(22) = 528$

Question99

$\sum_{r=1}^{20}(r^2 + 1)(r!)$ is equal to [29-Jul-2022-Shift-2]

Options:

A. $22!-21!$

B. $22!-2(21!)$

C. $21!-2(20!)$

D. $21!-20!$

Answer: B

Solution:

Solution:
Given,
$\sum_{r=1}^{20}(r^2+1)(r!)$
Let, $f(r)=(r^2+1)(r!)$
$=(r^2)(r!)+r!$
$=r(rr!)+r!$
$=r[(r+1-1)r!]+r!$
$=r[(r+1)r!-r!]+r!$
$=r[(r+1)!-(r!)]+r!$
$=r(r+1)!-r(r!)+r!=(r+2-2)(r+1)!-r(r!)+r!$
$=(r+2)(r+1)!-2(r+1)!-[(r+1-1)(r!)]+r!$
$=(r+2)!-2(r+1)!-(r+1)!+r!+r!$
$=(r+2)!-3(r+1)!+2r!$
$=[(r+2)!-(r+1)!]-2[(r+1)!-r!]$
$\therefore \sum_{r=1}^{20} f(r)$
$=\sum_{r=1}^{20}[(r+2)!-(r+1)!]-2\sum_{r=1}^{20}[(r+1)!-r!]$
$=[(22!+21!+20!+\ldots.+4!+3!)-(21!+20!+19!+\ldots.+3!+2!]-2[(21!+20!+\ldots+3!+2!)-(20!+19!+\ldots\ldots.1!)]$
$=[(22!)-(2!)]-2[(21)!-(1!)]$
$=22!-2!-2\cdot(21)!+2.1!$
$=22!-2\cdot(21)!$

Question100

If $e^{(\cos^2 x+\cos^4 x+\cos^6 x+\ldots\infty)\log_e 2}$ satisfies the equation $t^2-9t+8=0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0<x<\frac{\pi}{2}\right)$ is
[24-Feb-2021 Shift 1]

Options:

A. $2\sqrt{3}$

B. $\frac{3}{2}$

C. $\sqrt{3}$

D. $\frac{1}{2}$

Answer: D

Solution:

Solution:
$e^{(\cos^2 x+\cos^4 x+\ldots\infty)\ell n2}=2^{\cos^2 x+\cos^4 x+\ldots\infty}=2^{\cot^2 x}$
Now $t^2-9t+9=0\Rightarrow t=1,8$
$\Rightarrow 2^{\cot^2 x}=1,8\Rightarrow \cot^2 x=0,3$
$\Rightarrow \frac{2\sin x}{\sin x+\sqrt{3}\cos x}=\frac{2}{1+\sqrt{3}\cot x}=\frac{2}{4}=\frac{1}{2}$

Question101

Let A = { x : x is 3-digit number] B = {x : x = 9k + 2, k ∈ I } and C = {x : x = 9k + ℓ, k ∈ I , ℓ ∈ I , O < ℓ < 9} for some ℓ(0 < ℓ < 9) If the sum of all the elements of the set A ∩ (B ∪ C) is 274 × 400, then ℓ is equal to [24-Feb-2021 Shift 1]

Answer: 5

Solution:

Solution:
B and C will contain three digit numbers of the form $9k+2$ and $9k+\ell$ respectively. We need to find sum of all elements in the set $B \cup C$ effectively.
Now, $S(B \cup C) = S(B) + S(C) - S(B \cap C)$ where $S(k)$ denotes sum of elements of set k .
Also $B = \{101, 110, \ldots\ldots 992\}$
$\therefore S(B) = \frac{100}{2}(101 + 992) = 54650$

Case-I : If $\ell = 2$
then $B \cap C = B$
$\therefore S(B \cup C) = S(B)$
which is not possible as given sum is
274 × 400=109600

Case-II : If $\ell \neq 2$
then $B \cap C = \varphi$
$\therefore S(B \cup C) = S(B) + S(C) = 400 \times 274$

$\Rightarrow 54650 + \sum_{k=11}^{110} 9k + \ell = 109600$

$\Rightarrow 9\sum_{k=11}^{110} k + \sum_{k=11}^{110} \ell = 54950$
$\Rightarrow 9\left(\frac{100}{2}(11 + 110)\right) + \ell(100) = 54950$
$\Rightarrow 54450 + 100\ell = 54950$
$\Rightarrow \ell = 5$

Question102

The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1 and the third term is α_1 then 2α is [2021, 24 Feb. Shift-II]

Answer: 9

Solution:

Let four numbers in GP be a, ar, ar^2, ar^3 .
According to the question,
$a + ar + ar^2 + ar^3 = \frac{65}{12} \quad \cdots\cdots (i)$
and $\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$

$\Rightarrow \frac{1}{a}\left(\frac{1+r+r^2+r^3}{r^3}\right) = \frac{65}{18}$ .... (ii)

Dividing Eq. (i) by (ii), we get

$\frac{a(1+r+r^2+r^3)}{\frac{1}{a}\frac{(1+r+r^2+r^3)}{r^3}} = \frac{65/12}{65/18}$

$\Rightarrow a^2r^3 = \frac{18}{12}$

$\Rightarrow a^2r^3 = \frac{3}{2}$

Also, product of first three terms $= 1$

$a \times ar \times ar^2 = 1$

$\Rightarrow a^3r^3 = 1$

$\Rightarrow a^3 \times \frac{3}{2a^2} = 1 \left[\because r^3 = \frac{3/2}{a^2}\right]$

$\Rightarrow a = \frac{2}{3}$

and $r^3 = \frac{3/2}{(2/3)^2} = \left(\frac{3}{2}\right)^3 \Rightarrow r = \frac{3}{2}$

According to the question, third term

$\alpha = ar^2 = \frac{2}{3} \times \frac{3}{2} \times \frac{3}{2} = \frac{3}{2} \therefore 2\alpha = 2 \times \frac{3}{2} = 3$ third term

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Question103

The sum of the series $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$ is equal to [2021, 26 Feb. Shift-II]

Options:

A. $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

B. $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$

C. $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

D. $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

Answer: B

Solution:

Solution:

Let $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!} = S$

$= \sum_{n=1}^{\infty} \frac{4n^2+24n+40}{4(2n+1)!}$

$= \sum_{n=1}^{\infty} \frac{(2n+1)^2+(2n+1)\cdot 10+29}{4(2n+1)!}$

$= \frac{1}{4}\left[\sum_{n=1}^{\infty} \frac{(2n+1)^2}{(2n+1)(2n)!} + \sum_{n=1}^{\infty} \frac{(2n+1)\cdot 10}{(2n+1)(2n)!} + \sum_{n=1}^{\infty} \frac{29}{(2n+1)!}\right]$

$= \frac{1}{4}\left[\sum_{n=1}^{\infty} \frac{(2n+1)}{(2n)!} + \sum_{n=1}^{\infty} \frac{10}{(2n)!} + \sum_{n=1}^{\infty} \frac{29}{(2n+1)!}\right]$ .... (1)

Now,

$= \sum_{n=1}^{\infty} \frac{(2n+1)}{(2n)!} = \sum_{n=1}^{\infty} \frac{2n}{(2n)!} + \sum_{n=1}^{\infty} \frac{1}{(2n)!}$

$= \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} + \sum_{n=1}^{\infty} \frac{1}{(2n)!}$

Now,

$$= \sum_{n=1}^{\infty} \frac{1}{(2n-1)!} = \frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \ldots = \frac{e - \frac{1}{e}}{2} \quad \cdots\cdots (ii)$$

and $\sum_{n=1}^{\infty} \frac{1}{(2n)!} = \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots = \frac{e + \frac{1}{e} - 2}{2} \quad \cdots\cdots (iii)$

and $\sum_{n=1}^{\infty} \frac{1}{(2n+1)!} = \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \ldots = \frac{e - \frac{1}{e} - 2}{2} \quad \cdots\cdots (iv)$

Using Eqs. (ii), (iii), (iv) in (i),

$$S = \frac{1}{4}\left[\frac{e - \frac{1}{e}}{2} + 11\& \left(\frac{e + \frac{1}{e} - 2}{2} \right) \right.$$

$$\left. +29\& \left(\frac{e - \frac{1}{e} - 2}{2} \right) \right]$$

$$= \frac{1}{4}\left[\frac{e}{2} - \frac{1}{2e} + \frac{11e}{2} + \frac{11}{2e} + \frac{29e}{2} - \frac{29}{2e} - 4 \right]$$

$$= \frac{41e}{8} - \frac{19}{8e} - 10$$

Question104

The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + \ldots$ is equal to [2021, 26 Feb. Shift-1]

Options:

A. $\frac{13}{4}$

B. $\frac{9}{4}$

C. $\frac{15}{4}$

D. $\frac{11}{4}$

Answer: A

Solution:

Solution:

Given, $S = 1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \ldots$

Let, $S_1 = \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \ldots$

Multiply 1 / 3 in series Eq. (i),

$$\frac{S_1}{3} = \frac{2}{3^2} + \frac{7}{3^3} + \frac{12}{3^4} + \ldots$$

Subtract Eq. (ii) from Eq. (i), we get

$$S_1 - \frac{S_1}{3} = \frac{2}{3} + \frac{5}{3^2} + \frac{5}{3^3} + \ldots$$

$$\Rightarrow \frac{2S_1}{3} = \frac{2}{3} + \left[\frac{5}{3^2} + \frac{5}{3^3} + \ldots \right]$$

$= \frac{2}{3} + \left[\frac{5/3^2}{1 - 1/3} \right] \left[\because \frac{5}{3^2} + \frac{5}{3^3} + \ldots \right.$ is a geometric series with $r = 1/3$, sum upto infinity of this series is $\frac{a}{1-r}$, where $a =$ first term]

$$= \frac{2}{3} + \left[\frac{5}{6} \right] = \frac{9}{6} = \frac{3}{2}$$

$$\Rightarrow S_1 = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$$

$$\therefore S = 1 + S_1$$

$$= 1 + \frac{9}{4} = \frac{13}{4}$$

Question105

If the arithmetic mean and geometric mean of the pth and qth terms of the sequence $-16, 8, -4, 2, \ldots$ satisfy the equation $4x^2 - 9x + 5 = 0$, then $p + q$ is equal to
[2021, 26 Feb. Shift-II]

Answer: 10

Solution:

Solution:
If AM and GM satisfy the equation $4x^2 - 9x + 5 = 0$, then AM and GM are nothing but roots of this quadratic equation,
$4x^2 - 9x + 5 = 0$
$\Rightarrow 4x^2 - 4x - 5x + 5 = 0$
$\Rightarrow 4x(x-1) - 5(x-1) = 0$
$\Rightarrow (x-1)(4x-5) = 0$
$\Rightarrow x = 1, \frac{5}{4}$
Then, $AM = \frac{5}{4}$ and $GM = 1$ $[\because AM \geq GM]$
Again, the given series is
$-16, 8, -4, 2 \ldots\ldots$
which is a geometric progression series with common ratio $\frac{-1}{2}$, then
pth term $= -16\left(\frac{-1}{2}\right)^{p-1} = t_p$
qth term $= -16\left(\frac{-1}{2}\right)^{q-1} = t_q$
Arithmetic mean $= \frac{5}{4}$
$\Rightarrow \frac{t_p + t_q}{2} = \frac{5}{4}$
Geometric mean $= 1$
$\Rightarrow \sqrt{t_p t_q} = 1$
$\because \sqrt{t_p t_q} = 1$
$\Rightarrow (-16)\left(\frac{-1}{2}\right)^{p-1}(-16)\left(\frac{-1}{2}\right)^{q-1} = 1$
$\Rightarrow (-16)^2\left(\frac{-1}{2}\right)^{p+q-2} = 1$
$\Rightarrow (-2^4)^2\left(\frac{-1}{2}\right)^{p+q-2} = 1$
$\Rightarrow (-2)^8 \frac{(+1)^{p+q-2}}{(-2)^{p+q-2}} = 1$
$\Rightarrow (-2)^8(+1)^{p+q-2} = (-2)^{p+q-2}$
$\Rightarrow (-2)^8 = (-2)^{p+q-2}$
$\Rightarrow p + q - 2 = 8$
$\Rightarrow p + q = 10$

Question106

In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25 . Then, the sum of 4th, 6th and 8th terms is equal to
[2021,26 Feb. Shift-1]

Options:

A. 30

B. 26

C. 35

D. 32

Answer: C

Solution:

Solution:
Let the first term of geometric series be' a' and common ratio be 'r'.
Then, n th term of given series is given as
$T_n = ar^{n-1}$
Now, given that sum of second and sixth term is 25/2.
i.e. $T_2 + T_6 = 25 / 2$
$\Rightarrow ar + ar^5 = 25 / 2$
$\Rightarrow ar(1 + r^4) = 25 / 2 \quad \cdots\cdots (i)$
Also, given that product of third and fifth term is 25.
$l\,rl$ i.e. $(T_3)(T_5) = 25$
$\Rightarrow (ar^2)(ar^4) = 25$
$\Rightarrow a^2r^6 = 25 \quad \cdots\cdots (ii)$
Squaring Eq. (i), we get$a^2r^2(1 + r^4)^2 = \left(\frac{25}{2}\right)^2 \quad \cdots\cdots (iii)$
Divide Eq. (ii) by (iii),
$\Rightarrow \frac{a^2r^2(1 + r^4)^2}{a^2r^6} = \frac{(25)^2}{4(25)}$
$\Rightarrow \frac{(1 + r^4)^2}{r^4} = \frac{25}{4}$
$\Rightarrow 4(1 + r^4)^2 = 25r^4$
$\Rightarrow 4(1 + r^8 + 2r^4) = 25r^4$
$\Rightarrow 4r^8 - 17r^4 + 4 = 0$
$\Rightarrow 4r^8 - 16r^4 - r^4 + 4 = 0$
$\Rightarrow 4r^4(r^4 - 4) - 1(r^4 + (-4)) = 0$
$\Rightarrow (r^4 - 4)(4r^4 - 1) = 0$
Gives, $r^4 = 40rr^4 = 1 / 4$
We have to find sum of 4 th, 6th and 8th term, i.e.
$T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7$
$= ar(r^2 + r^4 + r^6)$
$= ar^3(1 + r^2 + r^4) \quad \cdots\cdots (iv)$
Using Eq. (ii),
$(ar^3)^2 = 25$
$\Rightarrow ar^3 = 5$
Also, we take $r^4 = 4$ because given series is increasing and $r^2 = 2$.
$\therefore T_4 + T_6 + T_8 = 5(1 + 2 + 4)$
$= 5(7) = 35$

Question107

The minimum value of $f(x) = a^{a^x} + a^{1 - a^x}$, where $a, x \in R$ and $a > 0$, is equal to [2021, 25 Feb. Shift-II]

Options:

A. $a + 1$

B. $a + \frac{1}{a}$

C. $2\sqrt{a}$

D. $2a$

Answer: C

Solution:

Solution:
We already know, Arithmetic mean ≥ Geometric mean,

Let us take AM and GM of two terms $a^{a^{*}}$ and $a^{1-a^{x}}$,

$\Rightarrow$ AM $= \dfrac{a^{a^x} + a^{1-a^x}}{2}$

and GM $= \sqrt{a^{a^x} \cdot a^{1-a^x}}$

$\because$AM $\geq$ GM $\Rightarrow \dfrac{a^{a^x} + a^{1-a^x}}{2} \geq \sqrt{a^{a^x} \cdot a^{1-a^x}}$

$\Rightarrow a^{a^x} + a^{1-a^x} \geq 2\sqrt{a^1}$

$\therefore$ Minimum value of $f(x) = a^{a^x} + a^{1-a^x}$ is $2\sqrt{a}$.

Question108

If $0 < \theta, \varphi < \frac{\pi}{2}$, $x = \sum\limits_{n=0}^{\infty} \cos^{2n}\theta_1$ $y = \sum\limits_{n=0}^{\infty} \sin^{2n}\varphi$ and $z = \sum\limits_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\varphi$, then [2021, 25 Feb. Shift-1]

Options:

A. $xy - z = (x + y)z$

B. $xy + yz + zx = z$

C. $xyz = 4$

D. $xy + z = (x + y)z$

Answer: D

Solution:

Solution:

Given, $x = \sum\limits_{n=0}^{\infty} \cos^{2n}\theta$

$y = \sum\limits_{n=0}^{\infty} \sin^{2n}\varphi$

$z = \sum\limits_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\varphi$

$\Rightarrow x = 1 + \cos^2\theta + \cos^4\theta + \ldots\infty$

$\therefore x = \dfrac{1}{1 - \cos^2\theta} = \text{cosec}^2\theta$

$\Rightarrow y = 1 + \sin^2\varphi + \sin^4\varphi + \ldots\infty$

$\therefore x = \dfrac{1}{1 - \cos^2\theta} = \text{cosec}^2\theta \quad \cdots\cdots (i)$

$\Rightarrow y = 1 + \sin^2\varphi + \sin^4\varphi + \ldots\infty$

$\therefore y = \dfrac{1}{1 - \sin^2\varphi} = \sec^2\varphi \quad \cdots\cdots (ii)$

$\Rightarrow z = 1 + \cos^2\theta \cdot \sin^2\varphi + \cos^4\theta\sin^4\varphi + \ldots\infty$

$\therefore z = \dfrac{1}{1 - \cos^2\theta\sin^2\varphi} \ \ldots (iii)$

From Eqs. (i), (ii) and (iii), we get

$$z = \frac{1}{1 - \left(1 - \frac{1}{x}\right)\left(1 - \frac{1}{y}\right)} \quad \left[\begin{array}{l} \because \cos^2\theta = 1 - \frac{1}{x} \\ \because \sin^2\varphi = 1 - \frac{1}{y} \end{array}\right]$$

$z = \dfrac{xy}{xy - (x-1)(y-1)}$

$z = \dfrac{xy}{xy - xy + x + y - 1}$

$\Rightarrow xz + yz - z = xy$

$\Rightarrow xy + z = (x + y)z$

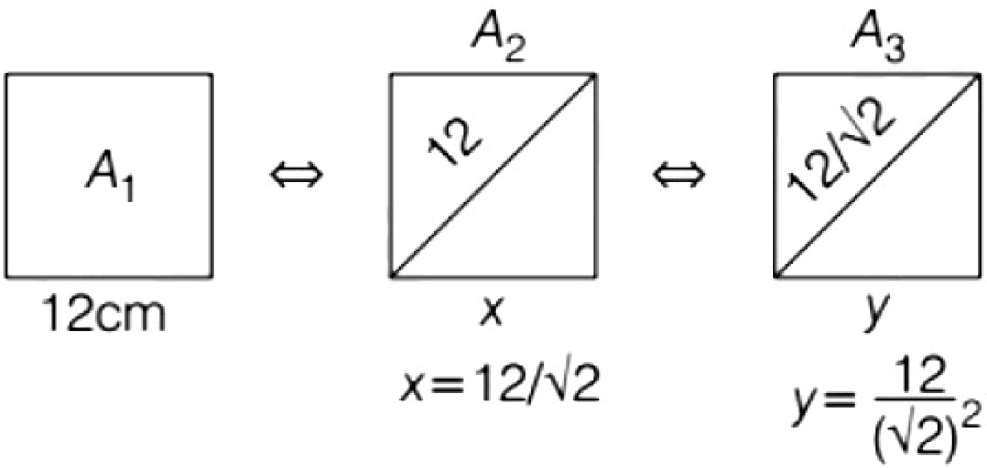
Question109

Let A_1, A_2, A_3, ... be squares, such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12cm, then the smallest value of n for which area of A_n is less than one, is ....
[2021, 25 Feb. Shift-I]

Answer: 9

Solution:

Solution:
According to the question, length of side of A_1 square is $12 \mathrm{mathrm} \sim \mathrm{cm}$.



$\because$ Side lengths are in GP.

$\therefore \ T_n = \dfrac{12}{(\sqrt{2})^{n-1}}$

(Side of nth square i.e. A_n)

$\therefore$ Area $= (\text{ Side })^2 = \left(\dfrac{12}{(\sqrt{2})^{n-1}}\right)^2 = \dfrac{144}{2^{n-1}}$

According to the question, the area of A_n square <1

$\Rightarrow \ 2^{n-1} > 144$

Here, the smallest possible value of is $= 9$.

Question110

Consider an arithmetic series and a geometric series having four initial terms from the set { 11, 8, 21, 16, 26, 32, 4 }. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to.........
[2021, 16 March Shift-1]

Answer: 3

Solution:

Solution:
Given, set $\{11, 8, 21, 16, 26, 32, 4\}$
By observation, we can say that
$AP = \{11, 16, 21, 26, \ldots\}$
$GP = \{4, 8, 16, 32, \ldots\}$
$5m + 6 = 4.2^{n-1}$
$5m + 6 = 2^{n+1}$
So, $(2^{n+1} - 6)$ should be a multiple of 5 . The unit digit of 2^k is 2, 4, 6, 8. So, when 6 is subtracted from 2^{n+1}, the possible unit digits will be 6, 8, 0, 2. Only 0 is divisible by 5. Hence, 2^{n+1} unit digit has to be 6 .
$2^{n+1} = 2^4, 2^8, 2^{12}, 2^{16}\ldots$
As, 2^{16} will not be a 4 digit number, so, common terms $= \{16, 256, 4096\}$
$\therefore$ Number of common terms $= 3$

Question111

Let $\frac{1}{16}$, a and b be in G. P. and $\frac{1}{a}$, $\frac{1}{b}$, 6 be in (a) P., where a, b > 0. Then, 72(a + b) is equal to ....
[2021, 16 March Shift-II]

Answer: 14

Solution:

Solution:
Given, $GP = \frac{1}{16}, a, b$
$\Rightarrow \quad a^2 = \frac{b}{16}$
and given, $AP = 1/a, 1/b, 6$
$\Rightarrow \quad \frac{2}{b} = \frac{1}{a} + 6$
$\Rightarrow \quad \frac{2}{16a^2} = \frac{1}{a} + 6$
$\Rightarrow \quad \frac{1}{8a^2} = \frac{1 + 6a}{a}$
$\Rightarrow \quad 1 = 8a(1 + 6a)$
$\Rightarrow \quad 48a^2 + 8a - 1 = 0$
$\Rightarrow \quad (4a + 1)(12a - 1) = 0$
$\Rightarrow \quad a = -1/4$ or $1/12$
As per the question, $a > 0$
$\therefore \; a = 1/12$
$b = 16a^2 = 16 \cdot \frac{1}{144} = \frac{1}{9}$
$\therefore \; 72(a + b) = 72\left(\frac{1}{12} + \frac{1}{9}\right)$
$= 6 + 8$
$= 14$

Question112

If α, β are natural numbers, such that $100^\alpha - 199\beta = (100)(100) + (99)(101) + (98)(102) + \ldots + (1)(199)$, then the slope of the line passing through (α, β) and origin is
[2021,18 March Shift-1]

Options:

A. 540

B. 550

C. 530

D. 510

Answer: B

Solution:

Solution:
Given,
$100^{\alpha} - 199 . \beta = (100)(100) + (99)(101)$
$+(98)(102) + \ldots + (1)(199)$
$\Rightarrow 100^{\alpha} - 199\beta = \sum_{x=0}^{99} (100 - x)(100 + x)$
$= \sum_{x=0}^{99} (100^2 - x^2)$
$= \sum_{x=0}^{99} (100)^2 - \sum_{x=0}^{99} (x)^2$
$= (100)^3 - \dfrac{99 \times 100 \times 199}{6}$
$\Rightarrow (100)^{\alpha} - (199)\beta = (100)^3 - (199)(1650)$
On comparing, we get $\alpha = 3$, $\beta = 1650$
Then, the slope of the line passing through (α, β) and origin is
$= \dfrac{\beta - 0}{\alpha - 0} = \dfrac{\beta}{\alpha} = \dfrac{1650}{3} = 550$

Question113

$\frac{1}{3^2 - 1} + \frac{1}{5^2 - 1} + \frac{1}{7^2 - 1} + \ldots + \frac{1}{(201)^2 - 1}$
is equal to
[2021, 18 March Shift-1]

Options:

A. $\frac{101}{404}$

B. $\frac{25}{101}$

C. $\frac{101}{408}$

D. $\frac{99}{400}$

Answer: B

Solution:

Solution:
$\frac{1}{3^2 - 1} + \frac{1}{5^2 - 1} + \frac{1}{7^2 - 1} + \ldots + \frac{1}{(201)^2 - 1}$
$= \sum_{r=1}^{100} \frac{1}{(2r + 1)^2 - 1}$
$= \sum_{r=1}^{100} \frac{1}{4r^2 + 4r + 1 - 1}$
$= \sum_{r=1}^{100} \frac{1}{2r(2r + 2)} = \sum_{r=1}^{100} \frac{1}{4(r)(r + 1)}$

$= \frac{1}{4}\sum_{r=1}^{100}\left(\frac{1}{r} - \frac{1}{r+1}\right)$
$= \frac{1}{4}\left[\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots + \left(\frac{1}{100} - \frac{1}{101}\right)\right]$
$= \frac{1}{4}\left[1 - \frac{1}{101}\right] = \frac{1}{4} \times \frac{100}{101}$
$= \frac{25}{101}$

Question114

Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000 , then the sum of the first 6n terms of the arithmetic progression is equal to
[2021, 18 March Shift-11]

Options:

A. 1000

B. 7000

C. 5000

D. 3000

Answer: D

Solution:

Solution:
Given, $S_1 = S_{2n}$ and $S_2 = S_{4n}$
and $S_2 - S_1 = 1000 \Rightarrow S_{4n} - S_{2n} = 1000$
$\Rightarrow \frac{4n}{2}[2a + (4n - 1)d] - \frac{2n}{2}[2a + (2n - 1)d] = 1000$
$\Rightarrow 2n[2a + (4n - 1)d] - n[2a + (2n - 1)d] = 1000$
$\Rightarrow 2an + n(8n - 2 - 2n + 1)d = 1000$
$\Rightarrow 2an + n(6n - 1)d = 1000$
$\Rightarrow n[2a + (6n - 1)d] = 1000$
$\Rightarrow S_{6n} = \frac{6n}{2}[2a + (6n - 1)d]$
$= \frac{6}{2} \times (1000)$
$= 6 \times 500 = 3000$

Question115

If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to
[2021, 27 July Shift-1]

Answer: 3

Solution:

$\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right) \rightarrow AP$

$\Rightarrow 2\log_3(2^x - 5) = \log_3 2 + \log_3\left(2^x - \frac{7}{2}\right)$

$\Rightarrow \log_3(2^x - 5)^2 = \log_3\left[2 \cdot \left(2^x - \frac{7}{2}\right)\right]$

$\Rightarrow (2^x - 5)^2 = 2 \cdot 2^x - 7$

$\Rightarrow (2^x)^2 + 25 - 10 \cdot 2^x - 2 \cdot 2^x + 7 = 0$

$\Rightarrow (2^x)^2 - 12 \cdot 2^x + 32 = 0$

$\Rightarrow (2^x - 4)(2^x - 8) = 0$

$\Rightarrow 2^x = 4$ or $8 \Rightarrow x = 2$ or 3

If $x = 2$, then $\log_3(2^x - 5) = \log_3(2^2 - 5)$

Here, argument is negative, so, $x \neq 2$.

Hence, $x = 3$

Question116

If $\tan\left(\frac{\pi}{9}\right)$, x, $\tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right)$, y, $\tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to
[2021, 27 July Shift-II]

Options:

A. 4

B. 3

C. 0

D. 1

Answer: C

Solution:

Solution:

If $\tan\left(\frac{\pi}{9}\right)$, x, $\tan\left(\frac{7\pi}{18}\right)$ are in AP.

So, $x = \frac{1}{2}\left[\tan\frac{\pi}{9} + \tan\left(\frac{7\pi}{18}\right)\right]$

$\left(\because\right.$ if a, b, c are in AP, so, $\left.b = \frac{a+c}{2}\right)$

And $\tan\left(\frac{\pi}{9}\right)$, $y_1 \tan\left(\frac{5\pi}{18}\right)$ are in AP.

Now, $x - 2y = \frac{1}{2}\left[\tan\frac{\pi}{9} + \tan\frac{7\pi}{18}\right] - \left(\tan\frac{\pi}{9} + \tan\frac{5\pi}{18}\right)$

$$\Rightarrow |x - 2y| = \left|\frac{\cot\frac{\pi}{9} - \tan\frac{\pi}{9}}{2} - \tan\frac{5\pi}{18}\right| \quad \left\{\begin{array}{c}\because \tan\frac{5\pi}{18} = \cot\frac{2\pi}{9} \\ \text{and } \tan\frac{7\pi}{18} = \cot\frac{\pi}{9}\end{array}\right\}$$

$= \left|\cot\frac{2\pi}{9} - \cot\frac{2\pi}{9}\right| = 0$

$\left[\because \cot 2A = \frac{2\cot^2 A^{-1}}{2\cot A}\right]$

Question117

Let S_n be the sum of the first n terms of an arithmetic progression, If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is
[2021, 25 July Shift-1]

Options:

A. 6

B. 4

C. 2

D. 8

Answer: A

Solution:

Solution:
Let $S_n = An^2 + Bn = n(An + B) S_{3n} = 3S_{2n}$
$\Rightarrow 3n[A(3n) + B] = 3 \cdot 2n \cdot [A(2n) + B]$
$\Rightarrow \ 3An + B = 4An + 2B$
$\Rightarrow \ An + B = 0$
$\therefore \ \frac{S_{4n}}{S_{2n}} = \frac{4n[A(4n) + B]}{2n[A(2n) + B]}$
$= 2\left(\frac{4An - An}{2An - An}\right)$
$= 2 \times 3 = 6$

Question118

Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to [2021,22 July Shift-II]

Options:

A. 1862

B. 1842

C. 1852

D. 1872

Answer: A

Solution:

Solution:
$S_n = An^2 + Bn$
$S_{10} = 100A + 10B = 530$
$S_5 = 25A + 5B = 140$ Solving both equations, we get $B = 3$ and
$A = 5$
$\therefore \ S_n = 5n^2 + 3n$
$\therefore S_{20} - S_6 = 5(20^2 - 6^2) + 3(20 - 6)$
$= 5 \cdot 26.14 + 3 \cdot 14$
$= 14(130 + 3) = 14 \times 133 = 1862$

Question119

The sum of all the elements in the set $\{ n \in \{1, 2, ... 100\}$: HCF of n and 2040 is 1$\}$ is equal to [2021, 22 July Shift-II]

Answer: 1251

Solution:

Solution:
$n \in \{1, 2, 3, \ldots\ldots 100\}$
$2040 = 2^3 \times 3 \times 5 \times 17$
If HCF of n and 2040 is 1, n should not be a multiple of $2, 3, 5, 17$.
$n \in \{ 1, 7, 11, 13, 19, 23, 29, 31, 37, 41, 43, 47 , 53, 59, 61, 67, 71, 73, 77, 79, 83, 89, 91, 97 \}$
$\sum_n = |1251|$

Question120

If sum of the first 21 terms of the series $\log_{9^{1/2}}x + \log_{9^{1/3}}x + \log_{9^{1/4}}x + \ldots\ldots$ where $x > 0$ is 504 , then x is equal to [2021, 20 July Shift-II]

Options:

A. 243

B. 9

C. 7

D. 81

Answer: D

Solution:

Solution:
Let $S = \log_{9^{1/2}}x + \log_{9^{1/3}}x + \ldots$
Using property, $\log_{a^b}x = \frac{1}{b}\log_a x$
$S = 2\log_9 x + 3\log_9 x + \ldots + 22\log_9 x$
$= \log_9 x(2 + 3 + 4 + \ldots + 22)$
$= \log_9 x\left[\frac{21}{2}(4 + 20)\right] = \log_9 x(21 \times 12)$
$\therefore S = 252\log_9 x$
Given, $S = 504$, then
$252\log_9 x = 504$
$\Rightarrow \log_9 x = 2$
$\Rightarrow x = (9)^2 = 81$

Question121

For $k \in N$, let $\frac{1}{\alpha(\alpha+1)(\alpha+2)\ldots.(\alpha+20)} = \sum_{k=0}^{20} \frac{A_k}{\alpha+k}$ where $\alpha > 0$. Then the value of $100\left(\frac{A_{14}+A_{15}}{A_{13}}\right)^2$ [2021, 20 July Shift-II]

Answer: 9

Solution:

Solution:
Given,

$$\frac{1}{\alpha(\alpha+1)\ldots+(\alpha+20)}=\sum_{k=0}^{20}\frac{A_k}{\alpha+k}, \alpha>0$$

$$\Rightarrow \frac{1}{\alpha(\alpha+1)...(\alpha+20)}=\frac{A_0}{\alpha}+\frac{A_1}{\alpha+1}$$

$$+\ldots+\frac{A_{20}}{\alpha+20}$$

$$A_{14}=\frac{1}{(-14)(-13)...(-1)(1)...(6)}=\frac{1}{14!6!}$$

$$A_{15}=\frac{1}{(-15)(-14)...(-1)(1)...(5)}=\frac{-1}{15!5!}$$

$$A_{13}=\frac{1}{(-13)...(-1)(1)...(7)}=\frac{-1}{13!7!}$$

$$\frac{A_{14}}{A_{13}}=\frac{-13!7!}{14!6!}=\frac{-7}{14}=\frac{-1}{2}$$

$$\Rightarrow \frac{A_{15}}{A_{13}}=\frac{13!7!}{15!5!}=\frac{42}{15\times 14}=\frac{1}{5}$$

$$\therefore 100\left(\frac{A_{14}}{A_{13}}+\frac{A_{15}}{A_{13}}\right)^2=100\left(-\frac{1}{2}+\frac{1}{5}\right)^2$$

$$=100\left(\frac{-3}{10}\right)^2=9$$

Question122

If the value of

$$\left(\begin{array}{c}1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}\\ +\ldots\text{ upto }\infty\end{array}\right)^{\log_{(0.25)}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\ldots\text{ upto }\infty\right)}$$

is l , then l^2 is equal to .... [2021, 25 July Shift-I]

Answer: 3

Solution:

Let $l=\alpha^{\beta}$

$$\alpha=1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+\ldots\infty \quad \cdots\cdots\text{(i)}$$

$$\frac{\alpha}{3}=\frac{1}{3}+\frac{2}{3^2}+\frac{6}{3^3}+\ldots. \quad \cdots\cdots\text{(ii)}$$

Subtracting Eq. (ii) from Eq. (i),

$$\frac{2\alpha}{3}=1+\frac{1}{3}+\frac{4}{3^2}+\frac{4}{3^3}+\ldots..$$

$$=\frac{4}{3}\left(\frac{1}{1-\frac{1}{3}}\right)=2$$

$\therefore \ \alpha=3$

$$\beta=\log_{0.25}\left(\frac{1}{3}+\frac{1}{3^2}+\ldots.\right)$$

$$=\log_{0.25}\left(\frac{\frac{1}{3}}{1-\frac{1}{3}}\right)=\log_{\frac{1}{4}}\frac{1}{2}=\frac{1}{2}$$

$\therefore \ L=3^{1/2}$

$\Rightarrow \; L^2 = 3$

Question123

Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1$, $a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47 \sum_{n=1}^{\infty} \frac{a_n}{2^{3n}}$ is equal to ....
[2021, 20 July Shift-II]

Answer: 7

Solution:

Solution:

Let $\sum_{n=1}^{\infty} \frac{a_n}{2^{3n}} = x$ i.e. $\sum_{n=1}^{\infty} \frac{a_n}{8^n} = x$

Given, $a_{n+2} = 2a_{n+1} + a_n$

Divide the whole by 8^n,

$\frac{a_{n+2}}{8^n} = \frac{2a_{n+1}}{8^n} + \frac{a_n}{8^n}$

$\Rightarrow \; 8^2 \cdot \frac{a_{n+2}}{8^{n+2}} = 8 \cdot 2 \frac{a_{n+1}}{8^{n+1}} + \frac{a_n}{8^n} \Rightarrow$

$64\left(\frac{a_{n+2}}{8^{n+2}}\right) = 16\left(\frac{a_{n+1}}{8^{n+1}}\right) + \frac{a_n}{8^n}$

Now, take the summation,

$64 \sum_{n=1}^{\infty} \frac{a_{n+2}}{8^{n+2}} = 16 \sum_{n=1}^{\infty} \frac{a_{n+1}}{8^{n+1}} + \sum_{n=1}^{\infty} \frac{a_n}{8^n} \quad \cdots\cdots \text{(i)}$

$\because \; \sum_{n=1}^{\infty} \frac{a_n}{8^n} = x$

i.e. $\frac{a_1}{8} + \frac{a_2}{8^2} + \frac{a_3}{8^3} + \frac{a_4}{8^4} + \ldots = x$

$\Rightarrow \; \frac{a_3}{8^3} + \frac{a_4}{8^4} + \ldots = x - \frac{a_1}{8} - \frac{a_2}{8^2}$

$\Rightarrow \; \sum_{n=1}^{\infty} \frac{a_{n+2}}{8^{n+2}} = x - \frac{a_1}{8} - \frac{a_2}{8^2} \quad \cdots\cdots \text{(ii)}$

Again, $\frac{a_2}{8^2} + \frac{a_3}{8^3} + \ldots = x - \frac{a_1}{8}$

$\Rightarrow \; \sum_{n=1}^{\infty} \frac{a_{n+1}}{8^{n+1}} = x - \frac{a_1}{8} \quad \cdots\cdots \text{(iii)}$

From Eqs. (i), (ii) and (iii),

$64\left(x - \frac{a_1}{8} - \frac{a_2}{64}\right) = 16\left(x - \frac{a_1}{8}\right) + x$

Use $a_1 = 1 = a_2$

$64\left(x - \frac{1}{8} - \frac{1}{64}\right) = 16\left(x - \frac{1}{8}\right) + x$

$\Rightarrow 64x - 9 = 2(8x - 1) + x$
$\Rightarrow 64x - 16x - x = 9 - 2 \Rightarrow 47x = 7$

$\therefore 47 \sum \frac{a_n}{2^{3n}} = 7$

Question124

Let a_1, a_2, a_3...... be an (a)P.

If $\frac{a_1+a_2+\ldots+a_{10}}{a_1+a_2+\ldots..+a_p} = \frac{100}{p^2}$, p ≠ 10, then $\frac{a_{11}}{a_{10}}$ is equal to
[2021,31 Auq. Shift-II]

Options:

A. $\frac{19}{21}$

B. $\frac{100}{121}$

C. $\frac{21}{19}$

D. $\frac{121}{100}$

Answer: C

Solution:

Solution:

$$\frac{a_1+a_2+\ldots+a_{10}}{a_1+a_2+\ldots+a_p} = \frac{100}{p^2}$$

$$\Rightarrow \frac{S_{10}}{S_p} = \frac{100}{p^2} \Rightarrow S_p = \frac{S_{10}\cdot p^2}{100}$$

$$\frac{a_{11}}{a_{10}} = \frac{S_{11}-S_{10}}{S_{10}-S_9} = \frac{S_{10}\cdot\frac{121}{100}-S_{10}}{S_{10}-S_{10}\cdot\frac{81}{100}}$$

$$= \frac{\frac{121}{100}-1}{1-\frac{81}{100}} = \frac{21}{19}$$

Question125

The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is
[2021, 31 Aug. Shift-II]

Answer: 5143

Solution:

Total 4-digit number
$= 9 \times 10 \times 10 \times 10 = 9000$
4 -digit number divisible by 7
1001, 1008, …, 9996
Number of 4-digit number divisible by 7
$= \frac{9996-1001}{7} + 1 = 1286$
4 -digit number divisible by 3
1002, 1005, …, 9999
Number of 4-digit number divisible by 3
$= \frac{9999-1002}{3} + 1 = 3000$
4 digit number divisible by 21
1008, 1031, …, 9996
Number of 4 - digit number divisible by 21
$= \frac{9996-1008}{21} + 1 = 429$
∴ Number of 4-digit numbers neither divisible by 7 nor 3

$= 9000 - 1286 - 3000 + 429 = 5143$

Question126

Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d . If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to
[2021, 31 Aug. Shift-I]

Options:

A. $7 - 7\sqrt{3}$

B. $7 + \sqrt{3}$

C. $7 - \sqrt{3}$

D. $7 + 3\sqrt{3}$

Answer: B

Solution:

Solution:

Let three numbers be $\frac{a}{r}$, a, ar.

According to the question, $\frac{a}{r}, 2a, ar \rightarrow AP$

$4a = ar + \frac{a}{r} \Rightarrow r + \frac{1}{r} = 4$

$\Rightarrow r^2 - 4r + 1 = 0 \Rightarrow r = 2 \pm \sqrt{3}$

T_4 of GP $= 3r^2$

$3r^2 = ar^2$

$a = 3$

$r = 2 + \sqrt{3}$

$d = 2a - \frac{a}{r} = 3\sqrt{3}$

$r^2 - d = (2 + \sqrt{3})^2 - 3\sqrt{3}$

$= 7 + 4\sqrt{3} - 3\sqrt{3} = 7 + \sqrt{3}$

Question127

If 0 < x < 1, then
$\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \ldots\ldots$, is equal to
[2021, 27 Aug. Shift-1]

Options:

A. $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

B. $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$

C. $\frac{1-x}{1+x} + \log_e(1-x)$

D. $\frac{1+x}{1-x} + \log_e(1-x)$

Answer: A

Solution:

Solution:
We have,

$\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \ldots$

$= \left(2 - \frac{1}{2}\right)x^2 + \left(2 - \frac{1}{3}\right)x^3 + \left(2 - \frac{1}{4}\right)x^4 + \ldots$

$= 2(x^2 + x^3 + x^4 + \ldots) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots\right)$

$= 2 \cdot \frac{x^2}{1 - x} - [-\log_e(1 - x) - x]$

[using sum of infinite GP = $\frac{a}{1 - r}$ and logarithmic series]

$= \frac{2x^2}{1 - x} + x + \log_e(1 - x)$

$= \frac{2x^2 + x - x^2}{1 - x} + \log_e(1 - x)$

$= \frac{x^2 + x}{1 - x} + \log_e(1 - x)$

$= x\left(\frac{1 + x}{1 - x}\right) + \log_e(1 - x)$

Question128

If x, y ∈ R, x > 0 $y = \log_{10}x + \log_{10}x^{1/3} + \log_{10}x^{1/9} + \ldots$ upto ∞ terms and $\frac{2 + 4 + 6 + \ldots + 2y}{3 + 6 + 9 + \ldots + 3y} = \frac{4}{\log_{10}x}$, then the ordered pair (x, y) is equal to [2021, 27 Aug. Shift-1]

Options:

A. $(10^6, 6)$

B. $(10^4, 6)$

C. $(10^2, 3)$

D. $(10^6, 9)$

Answer: D

Solution:

Solution:

Given, $\frac{2 + 4 + 6 + \ldots + 2y}{3 + 6 + 9 + \ldots + 3y} = \frac{4}{\log_{10}x}$

$\Rightarrow \frac{2(1 + 2 + 3 + \ldots + y)}{3(1 + 2 + 3 + \ldots + y)} = \frac{4}{\log_{10}x}$

$\Rightarrow \frac{2}{3} = \frac{4}{\log_{10}x} \Rightarrow \log_{10}x = 6$

$\Rightarrow x = 10^6$

Now, $y = \log_{10}x + \log_{10}x^{\frac{1}{3}} + \log_{10}x^{\frac{1}{9}} + \ldots$ upto ∞ terms.

$= \log_{10}\left(x \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{9}} \ldots \infty \text{ terms}\right)$

$= \log_{10}x^{1 + \frac{1}{3} + \frac{1}{9} + \ldots \infty \text{ terms}}$

$$= \log_{10} x^{\frac{1}{1-\frac{1}{3}}} = \log_{10} x^{3/2}$$

$$= \log_{10}(10^6)^{\frac{3}{2}} \quad [\because x = 10^6]$$

$$\Rightarrow y = 6 \times \frac{3}{2} = 9$$

$$\therefore x = 10^6, y = 9$$

$$(x, y) = (10^6, 9)$$

Question129

If $0 < x < 1$ and $y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^{4+\ldots}$ then the value of e^{1+y} at $x = \frac{1}{2}$ is [2021, 27 Aug. Shift-II]

Options:

A. $\frac{1}{2}e^2$

B. 2e

C. $\frac{1}{2}\sqrt{e}$

D. $2e^2$

Answer: A

Solution:

Solution:

$$y = \frac{1}{2}x^2 + \frac{2}{3}x^3 + \frac{3}{4}x^4 + \ldots$$

$$\Rightarrow y = \left(1 - \frac{1}{2}\right)x^2 + \left(1 - \frac{1}{3}\right)x^3 + \left(1 - \frac{1}{4}\right)x^4 +$$

$$= (x^2 + x^3 + x^4 + \ldots) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots\right)$$

$$= \frac{x^2}{1-x} + x - \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} \ldots\right)$$

$$\therefore y = \frac{x}{1-x} + \ln(1-x)$$

Put $x = \frac{1}{2}$, we get

$y = 1 - \ln 2$

Then, $e^{1+y} = e^{1+1-\ln 2} = e^{2-\ln 2}$

$$= e^2 \cdot e^{\ln 2^{-1}}$$

$$= \frac{1}{2}e^2$$

Question130

If the sum of an infinite GP a, ar, ar^2, ar^3, ... is 15 and the sum of the squares of its each term is 150 , then the sum of ar^2, ar^4, ar^6, ... is [2021, 26 Aug. Shift-1]

Options:

A. 5 / 2

B. 1 / 2

C. 25 / 2

D. 9 / 2

Answer: B

Solution:

Solution:
We have, sum of infinite GP a, ar, ar^2, … is

$S_\infty = \frac{a}{1-r} = 15 \quad \cdots\cdots \text{(i)}$

and sum of infinite GP a^2, a^2r^2, a^2r^4, … is

$cS_\infty' = \frac{a^2}{1-r^2} = 150$

$\Rightarrow \left(\frac{a}{1-r}\right)\left(\frac{a}{1+r}\right) = 150 \quad \cdots\cdots \text{(ii)}$

Divide Eq. (ii) by Eq. (i)

$\frac{a}{1+r} = 10 \quad \cdots\cdots \text{(iii)}$

Divide Eq. (iii) by Eq. (i)

$\frac{1-r}{1+r} = \frac{10}{15} = \frac{2}{3}$

$\Rightarrow 3 - 3r = 2 + 2r$

$\Rightarrow 1 = 5r \Rightarrow r = \frac{1}{5}$

Now, putting $r = \frac{1}{5}$ in Eq. (iii), we get

$\frac{a}{1+\frac{1}{5}} = 10$

$\Rightarrow \frac{5a}{6} = 10 \Rightarrow a = 12$

Now, sum of ar^2, ar^4, ar^6, …, ∞

$S_\infty'' = \frac{ar^2}{1-r^2} = \frac{12 \cdot \left(\frac{1}{25}\right)}{\frac{24}{25}} = \frac{1}{2}$

Question131

Let $a_1, a_2, \ldots\ldots, a_{10}$ be an AP with common difference −3 and $b_1, b_2, \ldots\ldots, b_{10}$ be a GP with common ratio 2.
Let $c_k = a_k + b_k$, k = 1, 2, ……, 10. If $c_2 = 12$ and $c_3 = 13$, then $\sum_{k=1}^{10} c_k$ is equal to
[2021, 26 Aug. Shift-II]

Answer: 2021

Solution:

$a_1, a_2, a_3, \ldots, a_{10}$ arein APcommon
difference $= -3$
$b_1, b_2, b_3, \ldots, b_{10}$ arein GPcommon ratio $= 2$
Since,c $= a_k + b_k$, k = 1, 2, 3……, 10

$\therefore\ c_2 = a_2 + b_2 = 12$

$c_3 = a_3 + b_3 = 13$

Now, $c_3 - c_2 = 1$

$\Rightarrow (a_3 - a_2) + (b_3 - b_2) \neq 1$

$\Rightarrow -3 + (2b_2 - b_2) \neq 1$

$\Rightarrow b_2 = 4$

$\therefore\ a_2 = 8$

So, AP is 11, 8, 5, ….

and GP is 2, 4, 8, ...

Now, $\sum_{k=1}^{10} c_k = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$

$= \left(\frac{10}{2}\right)[22 + 9(-3)] + 2\left(\frac{2^{10}-1}{2-1}\right)$

$= 5(22 - 27) + 2(1023)$

$= 2046 - 25 = 2021$

Question132

The sum of 10 terms of the series $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + ...$ is [2021, 31 Aug. Shift-1]

Options:

A. 1

B. 120/121

C. 99/100

D. 143 / 144

Answer: B

Solution:

Solution:

$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \dots$

$= \frac{2^2 - 1^2}{1^2 \times 2^2} + \frac{3^2 - 2^2}{2^2 \times 3^2} + \frac{4^2 - 3^2}{3^2 \times 4^2} + \dots.$

$= \left(\frac{1}{1^2} - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{3^2} - \frac{1}{4^2}\right) + \dots. \left(\frac{1}{10^2} - \frac{1}{11^2}\right)$

$= \frac{1}{1^2} - \frac{1}{11^2} = 1 - \frac{1}{121} = \frac{120}{121}$

Question133

If S = $\frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + ...$, then 160 Sis equal to [2021, 31 Aug. Shift-II]

Answer: 305

Solution:

$$S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \dots + \infty \quad \cdots\cdots (i)$$

$$\frac{S}{5} = \frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + \frac{19}{5^5} + \dots \infty \quad \cdots\cdots (ii)$$

Subtracting Eq. (ii) from Eq. (i),

$$c\, \frac{4S}{5} = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \frac{8}{5^5} + \dots \infty$$

$$\frac{4S}{5} - \frac{7}{5} = \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \frac{8}{5^5} + \dots \infty = K \quad \cdots\cdots (iii)$$

$$\frac{K}{5} = \frac{2}{5^3} + \frac{4}{5^4} + \frac{6}{5^5} + \frac{8}{5^6} + \dots \infty \quad \dots \ (iv)$$

Subtracting Eq. (iv) from Eq (iii),

$$\frac{4K}{5} = \frac{2}{5^2} + \frac{2}{5^3} + \frac{2}{5^4} + \frac{2}{5^5} + \ldots \infty$$

$$\frac{4K}{5} = \frac{2}{25}\left(\frac{1}{1-1/5}\right) = \frac{1}{10} \Rightarrow K = \frac{1}{8}$$

From Eq. (iii),

$$\frac{4S}{5} - \frac{7}{5} = \frac{1}{8} \Rightarrow S = \frac{61}{32}$$

Now, $106S = 160 \times \frac{61}{32} = 305$

Question134

Let $a_1, a_2, \ldots\ldots, a_{21}$ be an AP such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$. If the sum of this AP is 189 , then $a_6 a_{16}$ is equal to
[2021, 01 Sep. Shift-II]

Options:

A. 57

B. 72

C. 48

D. 36

Answer: B

Solution:

Solution:
Let d be the common difference of an AP

$a_1, a_2, \ldots, a_{21}$ and $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$

$$\Rightarrow \sum_{n=1}^{20} \frac{1}{a_n(a_n + d)} = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\sum_{n=1}^{20}\left(\frac{1}{a_n} - \frac{1}{a_n + d}\right) = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left[\frac{1}{a_1} - \frac{1}{a_2} + \frac{1}{a_3} - \frac{1}{a_4} + \ldots + \frac{1}{a_{20}} - \frac{1}{a_{21}}\right] = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left(\frac{1}{a_1} - \frac{1}{a_{21}}\right) = \frac{4}{9}$$

$$\Rightarrow \frac{1}{d}\left(\frac{a_{21} - a_1}{a_1 a_{21}}\right) = \frac{4}{9} \Rightarrow a_1 a_{21} = 45$$

$$\Rightarrow a_1(a_1 + 20d) = 45 \quad \cdots\cdots (i)$$

$$\Rightarrow \frac{21}{2}(2a_1 + 20d) = 189$$

Also sum of first 21 terms $= 189$

$$\Rightarrow \frac{21}{2}(2a_1 + 20d) = 189$$

$$\Rightarrow a_1 + 10d = 9 \quad \cdots\cdots (ii)$$

By Eqs. (i) and (ii), we get $a_1 = 3, d = 3/5$

or $a_1 = 15, d = -\frac{3}{5}$

So, $a_6 a_{16} = (a_1 + 5d)(a_1 + 15d) = 72$

Question135

Let $S_n = 1 \cdot (n-1) + 2 \cdot (n-2) + 3.$
$(n-3) + \ldots + (n-1) \cdot 1, n \geq 4$.

The sum $\sum_{n=4}^{\infty}\left(\frac{2S_n}{n!}-\frac{1}{(n-2)!}\right)$ is equal to
[2021, 01 Sep. Shift-II]

Options:

A. $\frac{e-1}{3}$

B. $\frac{e-2}{6}$

C. $\frac{e}{3}$

D. $\frac{e}{6}$

Answer: A

Solution:

Solution:
$S_n = 1\cdot(n-1)+2(n-2)+3(n-3)+\ldots+(n-1)\cdot 1,\ n\ge 4$
$=\sum_{r=1}^{n-1} r(n-r)=\frac{n(n^2-1)}{6}$
$=\frac{n(n-1)(n+1)}{6}$
$\frac{2S_n}{n!}=\frac{(n+1)}{3(n-2)!}$
$\Rightarrow \sum_{n=4}^{\infty}\left(\frac{2s_n}{n!}-\frac{1}{(n-2)!}\right)$
$=\sum_{n=4}^{\infty}\frac{n-2}{3(n-2)!}=\frac{1}{3}\sum_{n=4}^{\infty}\frac{1}{(n-3)!}$
$=\frac{1}{3}\left(\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots\right)=\frac{e-1}{3}$

Question136

The number of terms common to the two A.P.'s 3, 7, 11, ... 407 and 2, 9, 16, ..., 709 is ______.
[NA Jan. 9, 2020 (II)]

Answer: 14

Solution:

Solution:
First common term of both the series is 23 and common difference is $7\times 4=28$
$\because$ Last term $\le 407 \Rightarrow 23+(n-1)\times 28\le 407$
$\Rightarrow (n-1)\times 28\le 384$
$\Rightarrow n\le \frac{384}{28}+1$
$\Rightarrow n\le 14.71$
Hence, $n=14$

Question137

If the 10^{th} term of an A.P. is $\frac{1}{20}$ and its 20^{th} term is $\frac{1}{10}$, then the sum of its first 200 terms is:
[Jan. 8, 2020 (II)]

Options:

A. 50

B. 50 $\frac{1}{4}$

C. 100

D. 100 $\frac{1}{2}$

Answer: D

Solution:

Solution:

$T_{10} = \frac{1}{20} = a + 9d \ \ ...\text{(i)}$

$T_{20} = \frac{1}{10} = a + 19d \ \ ...\text{(ii)}$

Solving equations (i) and (ii), we get

$a = \frac{1}{200}, d = \frac{1}{200}$

$\Rightarrow S_{200} = \frac{200}{2}\left[\frac{2}{200} + \frac{199}{200}\right] = \frac{201}{2} = 100\frac{1}{2}$

Question138

Let $f: R \to R$ be such that for all $x \in R$, $(2^{1+x} + 2^{1-x})$, $f(x)$ and $(3^x + 3^{-x})$ are in A.P., then the minimum value of $f(x)$ is:
[Jan. 8, 2020 (I)]

Options:

A. 2

B. 3

C. 0

D. 4

Answer: B

Solution:

Solution:

If $2^{1-x} + 2^{1+x}$, $f(x)$, $3^x + 3^{-x}$ are in A.P., then

$f(x) = \left(\frac{2^{1+x} + 2^{1-x} + 3^x + 3^{-x}}{2}\right)$

$2f(x) = 2\left(2^x + \frac{1}{2^x}\right) + \left(3^x + \frac{1}{3^x}\right)$

Using AM $\geq$ GM

$f(x) \geq 3$

Question139

Five numbers are in A.P., whose sum is 25 and product is 2520 . If one of these five numbers is $-\frac{1}{2}$, then the greatest number amongst them is:

[Jan. 7, 2020 (I)]

Options:

A. 27

B. 7

C. $\frac{21}{2}$

D. 16

Answer: D

Solution:

Solution:
Let 5 terms of A.P. be
$a - 2d, a - d, a, a + d, a + 2d$
Sum $= 25 \Rightarrow 5a = 25 \Rightarrow a = 5$
Product $= 2520$
$(5 - 2d)(5 - d)5(5 + d)(5 + 2d) = 2520$
$\Rightarrow (25 - 4d^2)(25 - d^2) = 504$
$\Rightarrow 625 - 100d^2 - 25d^2 + 4d^4 = 504$
$\Rightarrow 4d^4 - 125d^2 + 625 - 504 = 0$
$\Rightarrow 4d^4 - 125d^2 + 121 = 0$
$\Rightarrow 4d^4 - 121d^2 - 4d^2 + 121 = 0$
$\Rightarrow (d^2 - 1)(4d^2 - 121) = 0$
$\Rightarrow d = \pm 1, d = \pm \frac{11}{2}$
$d = \pm 1$ and $d = -\frac{11}{2}$, does not give $\frac{-1}{2}$ as a term
$\therefore d = \frac{11}{2}$
$\therefore$ Largest term $= 5 + 2d = 5 + 11 = 16$

Question140

The product $2^{\frac{1}{4}} \cdot 4^{\frac{1}{16}} \cdot 8^{\frac{1}{48}} \cdot 16^{\frac{1}{128}} \ldots$ to ∞ is equal to:
[Jan. 9, 2020 (I)]

Options:

A. $2^{\frac{1}{2}}$

B. $2^{\frac{1}{4}}$

C. 1

D. 2

Answer: A

Solution:

Solution:
$2^{\frac{1}{4} + \frac{2}{16} + \frac{3}{48} + \ldots \infty}$
$= 2^{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots \infty} = \sqrt{2}$

Question141

Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$, then $\sum_{n=1}^{200} a_n$ is equal to : [Jan. 9, 2020 (II)]

Options:

A. 300

B. 225

C. 175

D. 150

Answer: D

Solution:

Solution:

Let G.P. be a, ar, ar^2......

$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \ldots\ldots + a_{201} = 200$

$\Rightarrow \frac{ar^2(r^{200}-1)}{r^2-1} = 200$

$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \ldots\ldots + a_{200} = 100$

$\Rightarrow \frac{ar(r^{200}-1)}{r^2-1} = 100$

From equations (i) and (ii), r = 2 and

$a_2 + a_3 + \ldots\ldots + a_{200} + a_{201} = 300$

$\Rightarrow r(a_1 + \ldots\ldots + a_{200}) = 300$

$\Rightarrow \sum_{n=1}^{200} a_n = \frac{300}{r} = 150$

Question142

If $x = \sum_{n=0}^{\infty} (-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty} \cos^{2n}\theta$, for $0 < \theta < \frac{\pi}{4}$, then : [Jan. 9, 2020 (II)]

Options:

A. $x(1 + y) = 1$

B. $y(1 - x) = 1$

C. $y(1 + x) = 1$

D. $x(1 - y) = 1$

Answer: B

Solution:

Solution:

$y = 1 + \cos^2\theta + \cos^4\theta + \ldots.$

$\Rightarrow y = \frac{1}{1-\cos^2\theta} \Rightarrow \frac{1}{y} = \sin^2\theta$

$x = 1 - \tan^2\theta + \tan^4\theta + \ldots.$

$x = \frac{1}{1-(-\tan^2\theta)} = \frac{1}{\sec^2\theta} \Rightarrow x = \cos^2\theta$

$y = \frac{1}{\sin^2\theta} \Rightarrow y = \frac{1}{1-x}$
$\therefore\ y(1-x) = 1$

Question143

The greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + ... + 49^2 + 49 + 1$, is:
[Jan. 7, 2020 (I)]

Options:

A. 32

B. 63

C. 60

D. 65

Answer: B

Solution:

Solution:
$\frac{(49)^{126}-1}{48} = \frac{((49)^{63}+1)(49^{63}-1)}{48}\left[\because S_n = \frac{a(r^n-1)}{r-1}\right]$
$\therefore\ K = 63$

Question144

Let $a_1, a_2, a_3, ...$ be a G. P. such that $a_1 < 0$, $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $\sum_{i=1}^{9} a_i = 4\lambda$, then λ is equal to:
[Jan. 7, 2020 (II)]

Options:

A. -513

B. -171

C. 171

D. $\frac{511}{3}$

Answer: B

Solution:

Solution:
Since, $a_1 + a_2 = 4 \Rightarrow a_1 + a_1r = 4 \ldots$ (i)
$a_3 + a_4 = 16 \ \ \Rightarrow a_1r^2 + a_1r^3 = 16 \ldots$ (ii)
From eqn. (i), $a_1 = \frac{4}{1+r}$ and substituting the value of a_1, in eqn (ii),
$\left(\frac{4}{1+r}\right)^{r^2} + \left(\frac{4}{1+r}\right)^{r^3} = 16$
$\Rightarrow 4r^2(1+r) = 16(1+r)$
$\Rightarrow r^2 = 4 \ \ \therefore r = \pm 2$
$r = 2, a_1(1+2) = 4 \Rightarrow a_1 = \frac{4}{3}$
$r = -2, a_1(1-2) = 4 \Rightarrow a_1 = -4$

$$\sum_{i=1}^{a} a_i = \frac{a_1(r^q - 1)}{r - 1} = \frac{(-4)((-2)^9 - 1)}{-2 - 1}$$
$$= \frac{4}{3}(-513) = 4\lambda \Rightarrow \lambda = -171$$

Question145

The coefficient of x^7 in the expression $(1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + ... + x^{10}$ is: [Jan. 7, 2020 (II)]

Options:

A. 210

B. 330

C. 120

D. 420

Answer: B

Solution:

Solution:

The given series is in G.P. then $S_n = \frac{a(1 - r^n)}{1 - r}$

$$\frac{(1 + x)^{10}\left[1 - \left(\frac{x}{1 + x}\right)^{11}\right]}{\left(1 - \frac{x}{1 + x}\right)}$$

$$\Rightarrow \frac{(1 + x)^{10}[(1 + x)^{11} - x^{11}]}{(1 + x)^{11} \times \frac{1}{(1 + x)}} = (1 + x)^{11} - x^{11}$$

$\therefore$ Coefficient of x^7 is ${}^{11}C_7 = {}^{11}C_{11-7} = {}^{11}C_4 = 330$

Question146

The sum, $\sum_{n=1}^{7} \frac{n(n + 1)(2n + 1)}{4}$ is equal to __________. [Jan. 8, 2020 (II)]

Answer: 504

Solution:

$$\left[\sum_{n=1}^{7} \frac{n(n + 1)(2n + 1)}{4}\right] \frac{1}{4}\left[\sum_{n=1}^{7} (2n^3 + 3n^2 + n)\right]$$
$$= \frac{1}{4}\left[2\left(\frac{7.8}{2}\right)^2 + 3\left(\frac{7.8.15}{6}\right) + \frac{7.8}{2}\right]$$
$$\Rightarrow \frac{1}{4}[2 \times 49 \times 16 + 28 \times 15 + 28]$$
$$= \frac{1}{4}[1568 + 420 + 28] = 504$$

Question147

The sum $\sum_{k=1}^{20}(1+2+3+...+k)$ is
[Jan. 8, 2020 (I)]

Answer: 1540

Solution:

Solution:
Given series can be written as

$$\sum_{k=1}^{20}\frac{k(k+1)}{2}=\frac{1}{2}\sum_{k=1}^{20}(k^2+k)$$

$$=\frac{1}{2}\left[\frac{20(21)(41)}{6}+\frac{20(21)}{2}\right]$$

$$=\frac{1}{2}\left[\frac{420\times 41}{6}+\frac{20\times 21}{2}\right]=\frac{1}{2}[2870+210]=1540$$

Question148

If the sum of the first 40 terms of the series, 3 + 4 + 8 + 9 +13 + 14 + 18 + 19 + ... is (102)m, then m is equal to:
[Jan. 7, 2020 (II)]

Options:

A. 20

B. 25

C. 5

D. 10

Answer: A

Solution:

Solution:
$S = 3+4+8+9+13+14+18+19......40$ terms
$S = 7+17+27+37+47+.....20$ terms

$$S_{40}=\frac{20}{2}[2\times 7+(19)10]=10[14+190]$$
$$=10[2040]=(102)(20)$$
$\Rightarrow m=20$

Question149

If the sum of first 11 terms of an A.P., $a_1, a_2, a_3, ...$ is 0 ($a_1 \neq 0$), then the sum of the A.P., $a_1, a_3, a_5, ..., a_{23}$ is ka_1, where k is equal to :
[Sep. 02, 2020 (II)]

Options:

A. $-\frac{121}{10}$

B. $\frac{121}{10}$

C. $\frac{72}{5}$

D. $-\frac{72}{5}$

Answer: D

Solution:

Solution:
Let common difference be d
$\because S_{11}=0 \quad \therefore \frac{11}{2}\{2a_1+10\cdot d\}=0$
$\Rightarrow a_1+5d=0 \Rightarrow d=-\frac{a_1}{5}$ . . . (i)
Now, $S=a_1+a_3+a_5+\ldots+a_{23}$
$=a_1+(a_1+2d)+(a_1+4d)+\ldots.+(a_1+22d)$
$=12a_1+2d\ \frac{11\times 12}{2}$
$=12\left[a_1+11\cdot\left(-\frac{a_1}{5}\right)\right]$ (From (i))
$=12\times\left(-\frac{6}{5}\right)a_1=-\frac{72}{5}a_1$

Question150

If the first term of an A.P. is 3 and the sum of its first 25 terms is equal to the sum of its next 15 terms, then the common difference of this A.P. is: [Sep. 03, 2020 (I)]

Options:

A. $\frac{1}{6}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{1}{7}$

Answer: A

Solution:

Solution:
Given $a=3$ and $S_{25}=S_{40}-S_{25}$
$\Rightarrow 2S_{25}=S_{40}$
$\Rightarrow 2\times\frac{25}{2}[6+24d]=\frac{40}{2}[6+39d]$
$\Rightarrow 25[6+24d]=20[6+39d]$
$\Rightarrow 5(2+8d)=4(2+13d)$
$\Rightarrow 10+40d=8+52d$
$\Rightarrow d=\frac{1}{6}$

Question151

In the sum of the series $20+19\frac{3}{5}+19\frac{1}{5}+18\frac{4}{5}+\ldots$ upto n^{th} term is 488 and

then n^{th} term is negative, then :
[Sep. 03, 2020 (II)]

Options:

A. $n = 60$

B. n^{th} term is -4

C. $n = 41$

D. n^{th} term is $-4\frac{2}{5}$

Answer: B

Solution:

Solution:
$S_n = 20 + 19\frac{3}{5} + 19\frac{1}{5} + 18\frac{4}{5} + \ldots.$
$\because S_n = 488$
$488 = \frac{n}{2}\left[2\left(\frac{100}{5}\right) + (n-1)\left(-\frac{2}{5}\right)\right]$ $488 = \frac{n}{2}(101 - n) \Rightarrow n^2 - 101n + 2440 = 0$
$\Rightarrow n = 61$ or 40
For $n = 40 \Rightarrow T_n > 0$
For $n = 61 \Rightarrow T_n < 0$
n^{th} term $= T_{61} = \frac{100}{5} + (61-1)\left(-\frac{2}{5}\right) = -4$

Question152

Let $a_1, a_2, \ldots., a_n$ be a given A.P. whose common difference is an integer and $S_n = a_1 + a_2 + \ldots. + a_n$. If $a_1 = 1$, $a_n = 300$ and $15 \leq n \leq 50$, then the ordered pair (S_{n-4}, a_{n-4}) is equal to :
[Sep. 04, 2020 (II)]

Options:

A. (2490,249)

B. (2480,249)

C. (2480,248)

D. (2490,248)

Answer: D

Solution:

Solution:
Given that $a_1 = 1$ and $a_n = 300$ and $d \in Z$
$\therefore 300 = 1 + (n-1)d$
$\Rightarrow d = \frac{299}{(n-1)} = \frac{23 \times 13}{(n-1)}$
$\because d$ is an integer
$\therefore n - 1 = 13$ or 23
$\Rightarrow n = 14$ or 24 $(\because 15 \leq n \leq 50)$
$\Rightarrow n = 24$ and $d = 13$
$a_{20} = 1 + 19 \times 13 = 248$
$s_{20} = \frac{20}{2}(2 + 19 \times 13) = 2490$

Question153

If $3^{2\sin 2\alpha - 1}$, 14 and $3^{4 - 2\sin 2\alpha}$ are the first three terms of an A.P. for some α, then the sixth term of this A.P is:
[Sep. 05, 2020 (I)]

Options:

A. 66

B. 81

C. 65

D. 78

Answer: A

Solution:

Solution:
Given that $3^{2\sin 2\alpha - 1}$, 14, $3^{4 - 2\sin 2\alpha}$ are in A.P.
So, $3^{2\sin 2\alpha - 1} + 3^{4 - 2\sin 2\alpha} = 28$
$\Rightarrow \frac{3^{2\sin 2\alpha}}{3} + \frac{81}{3^{2\sin 2\alpha}} = 28$
Let $3^{2\sin 2\alpha} = x$
$\Rightarrow \frac{x}{3} + \frac{81}{x} = 28$
$\Rightarrow x^2 - 84x + 243 = 0 \Rightarrow x = 81, x = 3$
When $x = 81 \Rightarrow \sin 2\alpha = 2$ (Not possible)
When $x = 3 \Rightarrow \alpha = \frac{\pi}{12}$
$\therefore a = 3^0 = 1, d = 14 - 1 = 13$
$a_6 = a + 5d = 1 + 65 = 66$

Question154

If the sum of the first 20 terms of the series $\log_{(7^{1/2})}x + \log_{(7^{1/3})}x + \log_{(7^{1/4})}x + \ldots$ is 460, then x is equal to :
[Sep. 05, 2020 (II)]

Options:

A. 7^2

B. $7^{\frac{1}{2}}$

C. e^2

D. $7^{\frac{46}{21}}$

Answer: A

Solution:

Solution:
$S = \log_7 x^2 + \log_7 x^3 + \log_7 x^4 + \ldots$ 20 terms
$\because S = 460$
$\Rightarrow \log_7(x^2 \cdot x^3 \cdot x^4 \cdot \ldots\ldots x^{21}) = 460$
$\Rightarrow \log_7 x^{(2 + 3 + 4 \ldots\ldots 21)} = 460$
$\Rightarrow (2 + 3 + 4 + \ldots\ldots + 21)\log_7 x = 460$

$\Rightarrow \frac{20}{2}(2+21)\log_7 x = 460$

$\Rightarrow \log_7 x = \frac{460}{230} = 2 \Rightarrow x = 7^2 = 49$

Question155

If $f(x+y) = f(x)f(y)$ and $\sum_{x=1}^{\infty} f(x) = 2$, $x, y \in N$, where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is :
[Sep. 06, 2020 (I)]

Options:

A. $\frac{2}{3}$

B. $\frac{1}{9}$

C. $\frac{1}{3}$

D. $\frac{4}{9}$

Answer: D

Solution:

Solution:
Let $f(1) = k$, then $f(2) = f(1+1) = k^2$
$f(3) = f(2+1) = k^3$
$\sum_{x=1}^{\infty} f(x) = 2 \Rightarrow k + k^2 + k^3 + \ldots . \infty = 2$
$\Rightarrow \frac{k}{1-k} = 2 \Rightarrow k = \frac{2}{3}$
Now, $\frac{f(4)}{f(2)} = \frac{k^4}{k^2} = k^2 = \frac{4}{9}$.

Question156

Let a, b, c, d and p be any non zero distinct real numbers such that $(a^2+b^2+c^2)p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) = 0$. Then :
[Sep. 06, 2020 (I)]

Options:

A. a, c, p are in A.P.

B. a, c, p are in G.P.

C. a, b, c, d are in G.P.

D. a, b, c, d are in A.P.

Answer: C

Solution:

Solution:
Rearrange given equation, we get
$(a^2p^2 - 2abp + b^2) + (b^2p^2 - 2bcp + c^2)$
$+(c^2p^2 - 2cd\,p + d^2) = 0$

$\Rightarrow (ap-b)^2 + (bp-c)^2 + (cp-d)^2 = 0$
$\therefore ap-b = bp-c = cp-d = 0$
$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ ∴ a, b, c, d are in G.P.

Question157

The common difference of the A.P. $b_1, b_2, ..., b_m$ is 2 more than the common difference of A.P. $a_1, a_2, ..., a_n$. If $a_{40} = -159$, $a_{100} = -399$ and $b_{100} = a_{70}$, then b_1 is equal to:
[Sep. 06, 2020 (II)]

Options:

A. 81

B. -127

C. -81

D. 127

Answer: C

Solution:

Solution:
Let common difference of series
$a_1, a_2, a_3, \ldots\ldots, a_n$ be d
$\because a_{40} = a_1 + 39d = -159$ . . . (i)
and $a_{100} = a_1 + 99d = -399$ . . . (ii)
From equations (i) and (ii),
$d = -4$ and $a_1 = -3$
Since, the common difference of $b_1, b_2, \ldots\ldots, b_n$ is 2 more than common difference of $a_1, a_2, \ldots\ldots, a_n$
∴ Common difference of $b_1, b_2, b_3, \ldots\ldots$ is (-2)
$\because b_{100} = a_{70}$
$\Rightarrow b_1 + 99(-2) = (-3) + 69(-4)$
$\Rightarrow b_1 = 198 - 279 \Rightarrow b_1 = -81$

Question158

Suppose that a function $f : R \to R$ satisfies $f(x+y) = f(x)f(y)$ for all $x, y \in R$ and $f(a) = 3$. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to ______.
[NA Sep. 06, 2020 (II)]

Answer: 5

Solution:

$\because f(x+y) = f(x) \cdot f(y) \quad \forall x \in R$ and $f(1) = 3$
$\Rightarrow f(x) = 3^x \Rightarrow f(i) = 3^i$
$$\Rightarrow \sum_{i=1}^{n} f(i) = 363$$
$\Rightarrow 3 + 3^2 + 3^3 + \ldots. + 3^n = 363$

$\Rightarrow \frac{3(3^n - 1)}{3 - 1} = 363 \quad \left[\because S_n = \frac{a(r^n - 1)}{(r - 1)}\right]$

$\Rightarrow 3^n - 1 = \frac{363 \times 2}{3} = 242$

$\Rightarrow 3^n = 243 = 3^5 \Rightarrow n = 5$

Question159

If $2^{10} + 2^9 \cdot 3^1 + 2^8 \cdot 3^2 + ... + 2 \times 3^9 + 3^{10} = S - 2^{11}$ then S is equal to: [Sep. 05, 2020 (I)]

Options:

A. $3^{11} - 2^{12}$

B. 3^{11}

C. $\frac{3^{11}}{2} + 2^{10}$

D. $2 \cdot 3^{11}$

Answer: B

Solution:

Solution:
Given sequence are in G.P. and common ratio 3

$\therefore \frac{2^{10}\left(\left(\frac{3}{2}\right)^{11} - 1\right)}{\left(\frac{3}{2} - 1\right)} = S - 2^{11}$

$\Rightarrow 2^{10} \frac{\left(\frac{3^{11} - 2^{11}}{2^{11}}\right)}{\frac{1}{2}} = S - 2^{11}$

$\Rightarrow 3^{11} - 2^{11} = S - 2^{11} \Rightarrow S = 3^{11}$

Question160

If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is: [Sep. 05, 2020 (II)]

Options:

A. $\frac{1}{26}(3^{49} - 1)$

B. $\frac{1}{26}(3^{50} - 1)$

C. $\frac{2}{13}(3^{50} - 1)$

D. $\frac{1}{13}(3^{50} - 1)$

Answer: B

Solution:

Let the first term be ' a ' and common ratio be ' r '.
$\because ar(1 + r + r^2) = 3$ . . . (i)
and $ar^5(1 + r + r^2) = 243$ . . . (ii)
From (i) and (ii),
$r^4 = 81 \Rightarrow r = 3$ and $a = \frac{1}{13}$
$\therefore S_{50} = \frac{a(r^{50} - 1)}{r - 1} = \frac{3^{50} - 1}{26}$ $\left[\because S_n = \frac{a(r^n - 1)}{(r - 1)}\right]$

Question161

Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If α, β, γ, δ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is :
[Sep. 04, 2020 (I)]

Options:

A. 3: 1

B. 9: 7

C. 5: 3

D. 33: 31

Answer: B

Solution:

Solution:
Let α, β, γ, δ be in G.P., then $\alpha\delta = \beta\gamma$
$\Rightarrow \frac{\alpha}{\beta} = \frac{\gamma}{\delta} \Rightarrow \left|\frac{\alpha - \beta}{\alpha + \beta}\right| = \left|\frac{\gamma - \delta}{\gamma + \delta}\right|$
$\Rightarrow \frac{\sqrt{9 - 4p}}{3} = \frac{\sqrt{36 - 4q}}{6}$
$\Rightarrow 36 - 16p = 36 - 4q \Rightarrow q = 4p$
$\therefore \frac{2q + p}{2q - p} = \frac{8p + p}{8p - p} = \frac{9p}{7p} = \frac{9}{7}$

Question162

The value of $(0.16)^{\log_{2.5}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots \text{ to } \infty\right)}$ is equal to ________.
[NA Sep. 03, 2020 (I)]

Answer: 4

Solution:

$(0.16)^{\log_{2.5}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots . \infty\right)}$

$0.16^{\log_{2.5}\left(\frac{\frac{1}{3}}{1 - \frac{1}{3}}\right)}$ $\left[\because S_\infty = \frac{a}{1 - r}\right]$

$= 0.16^{\log_{2.5}\left(\frac{1}{2}\right)}$
$= (2.5)^{-2\log_{2.5}\left(\frac{1}{2}\right)} = \left(\frac{1}{2}\right)^{-2} = 4$

Question163

The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in :
[Sep. 02, 2020 (I)]

Options:

A. $(-\infty, -9] \cup [3, \infty)$

B. $[-3, \infty)$

C. $(-\infty, -3] \cup [9, \infty)$

D. $(-\infty, 9]$

Answer: C

Solution:

Solution:
Let terms of G.P. be $\frac{a}{r}$, a, ar
$\therefore a\left(\frac{1}{r} + 1 + r\right) = S$ . . . (i)
and $a^3 = 27$
$\Rightarrow a = 3$ . . . (ii)
Put a = 3 in eqn. (1), we get
$S = 3 + 3\left(r + \frac{1}{r}\right)$
If $f(x) = x + \frac{1}{x}$, then $f(x) \in (-\infty, -2] \cup [2, \infty)$
$\Rightarrow 3f(x) \in (-\infty, -6] \cup [6, \infty)$
$\Rightarrow 3 + 3f(x) \in (-\infty, -3] \cup [9, \infty)$
Then, it concludes that
$S \in (-\infty, -3] \cup [9, \infty)$

Question164

If |x| < 1, | y | <1 and x ≠ y, then the sum to infinity of the following series $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + ...$ is :
[Sep. 02, 2020 (I)]

Options:

A. $\frac{x + y - xy}{(1 + x)(1 + y)}$

B. $\frac{x + y + xy}{(1 + x)(1 + y)}$

C. $\frac{x + y - xy}{(1 - x)(1 - y)}$

D. $\frac{x + y + xy}{(1 - x)(1 - y)}$

Answer: C

Solution:

Solution:

$S = (x + y) + (x^2 + y^2 + xy) + (x^3 + x^2y + xy^2 + y^3) + \ldots \infty$

$= \frac{1}{x - y}[(x^2 - y^2) + (x^3 - y^3) + (x^4 - y^4 + \ldots \infty]$

$= \frac{1}{x - y}\left[\frac{x^2}{1 - x} - \frac{y^2}{1 - y}\right] = \frac{(x - y)(x + y - xy)}{(x - y)(1 - x)(1 - y)}$

$\left[\because S_\infty = \frac{a}{1 - r}\right]$

$= \frac{x + y - xy}{(1 - x)(1 - y)}$

Question165

Let S be the sum of the first 9 terms of the series: $\{x + ka\} + \{x^2 + (k + 2)a\} + \{x^3 + (k + 4)a\} + \{x^4 + (k + 6)a\} + \ldots$ where $a \neq 0$ and $x \neq 1$. If $S = \frac{x^{10} - x + 45a(x - 1)}{x - 1}$, then k is equal to: [Sep. 02, 2020 (II)]

Options:

A. -5

B. 1

C. -3

D. 3

Answer: C

Solution:

Solution:

$S = (x + x^2 + x^3 + \ldots 9 \text{ terms})$

$+ a[k + (k + 2) + + (k + 4) + \ldots 9 \text{ terms}] \Rightarrow S = \frac{x(x^9 - 1)}{x - 1} + \frac{9}{2}[2ak + 8 \times (2a)]$

$\Rightarrow S = \frac{x^{10} - x}{x - 1} + \frac{9a(k + 8)}{1} = \frac{x^{10} - x + 45a(x - 1)}{x - 1}$ (Given)

$\Rightarrow \frac{x^{10} - x + 9a(k + 8)(x - 1)}{x - 1} = \frac{x^{10} - x + 45a(x - 1)}{x - 1}$

$\Rightarrow 9a(k + 8) = 45a \Rightarrow k + 8 = 5 \Rightarrow k = -3$

Question166

If m arithmetic means (A.Ms) and three geometric means (G.Ms) are inserted between 3 and 243 such that 4^{th} A.M. is equal to 2^{nd} G.M., then m is equal to _______ [Sep. 03, 2020 (II)]

Answer: 39

Solution:

Let m arithmetic mean be $A_1, A_2 \ldots A_m$ and G_1, G_2, G_3 be geometric mean.

The A.P. formed by arithmetic mean is,
$3, A_1, A_2, A_3, \ldots\ldots A_m, 243$

$\therefore d = \frac{243 - 3}{m + 1} = \frac{240}{m + 1}$

The G.P. formed by geometric mean
$3, G_1, G_2, G_3, 243$

$r = \left(\frac{243}{3}\right)^{\frac{1}{3+1}} = (81)^{1/4} = 3$

$\because A_4 = G_2$

$\Rightarrow 3 + 4\left(\frac{240}{m+1}\right) = 3(3)^2$

$\Rightarrow 3 + \frac{960}{m+1} = 27 \Rightarrow m + 1 = 40 \Rightarrow m = 39.$

Question167

If $1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \ldots\ldots + (1 - 20^2)\ 19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to :
[Sep. 04, 2020 (I)]

Options:

A. (10,97)

B. (11,103)

C. (10,103)

D. (11,97)

Answer: B

Solution:

Solution:
The given series is
$1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \ldots (1 - 20^2 \cdot 19)$

$S = 1 + \sum_{r=1}^{10} [1 - (2r)^2(2r - 1)]$

$= 1 + \sum_{r=1}^{10} (1 - 8r^3 + 4r^2) = 1 + 10 - \sum_{r=1}^{10} (8r^3 - 4r^2)$

$= 11 - 8\left(\frac{10 \times 11}{2}\right)^2 + 4 \times \left(\frac{10 \times 11 \times 21}{6}\right)$

$= 11 - 2 \times (110)^2 + 4 \times 55 \times 7$
$= 11 - 220(110 - 7)$
$= 11 - 220 \times 103 = \alpha - 220\beta$
$\Rightarrow \alpha = 11, \beta = 103$
$\therefore (\alpha, \beta) = (11, 103)$

Question168

Let $f : R \to R$ be a function which satisfies $f(x + y) = f(x) + f(y)$, $\forall x, y \in R$. If $f(a) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k)$, $n \in N$, then the value of n, for which $g(n) = 20$, is :
[Sep. 02, 2020 (II)]

Options:

A. 5

B. 20

C. 4

D. 9

Answer: A

Solution:

Solution:
Given: $f(x+y) = f(x) + f(y),\ \forall x, y \in R,\ f(1) = 2$
$\Rightarrow f(2) = f(1) + f(1) = 2 + 2 = 4$
$f(3) = f(1) + f(2) = 2 + 4 = 6$
$f(n-1) = 2(n-1)$
Now, $g(n) = \sum_{k=1}^{n-1} f(k)$
$= f(1) + f(2) + f(3) + \ldots.\ f(n-1)$
$= 2 + 4 + 6 + \ldots.. + 2(n-1)$
$= 2[1 + 2 + 3 + \ldots\ldots + (n-1)]$
$= 2 \times \frac{(n-1)(n)}{2} = n^2 - n$
$\because g(n) = 20$ (given)
So, $n^2 - n = 20$
$\Rightarrow n^2 - n - 20 = 0$
$\Rightarrow (n-5)(n+4) = 0$
$\Rightarrow n = 5$ or $n = -4$ (not possible)

Question169

Let a, b and c be the 7^{th}, 11^{th} and 13^{th} terms respectively of a non-constant A.P. If these are also the three consecutive terms of a G.P., then $\frac{a}{c}$ is equal to:
[Jan. 09, 2019 (II)]

Options:

A. 2

B. $\frac{1}{2}$

C. $\frac{7}{13}$

D. 4

Answer: D

Solution:

Solution:
Let first term and common difference be A and D respectively.
$\therefore a = A + 6D$, $b = A + 10D$ and $c = A + 12D$
Since, a, b, c are in G.P.
Hence, relation between a, b and c is,
$\therefore b^2 = a\,.\,c.$
$\therefore (A + 10D)^2 = (A + 6D)(A + 12D)$
$\therefore 14D + A = 0$
$\therefore A = -14D$
$\therefore a = -8D$, $b = -4D$ and $c = -2D$
$\therefore \frac{a}{c} = \frac{-8D}{-2D} = 4$

Question170

If a, b and c be three distinct real numbers in G.P. and a + b + c = xb, then x

cannot be:
[Jan. 09, 2019 (I)]

Options:

A. -2

B. -3

C. 4

D. 2

Answer: D

Solution:

Solution:
$\because a, b, c$, are in G.P.
$\Rightarrow b^2 = ac$
Since, $a + b + c = xb$
$\Rightarrow a + c = (x - 1)b$
Take square on both sides, we get
$a^2 + c^2 + 2ac = (x - 1)^2 b^2$
$\Rightarrow a^2 + c^2 = (x - 1)^2 ac - 2ac [\because b^2 = ac]$
$\Rightarrow a^2 + c^2 = ac[(x - 1)^2 - 2]$
$\Rightarrow a^2 + c^2 = ac[x^2 - 2x - 1]$
$\because a^2 + c^2$ are positive and $b^2 = ac$ which is also positive.
Then, $x^2 - 2x - 1$ would be positive but for $x = 2$, $x^2 - 2x - 1$ is negative.
Hence, x cannot be taken as 2 .

Question171

The sum of the following series $1 + 6 + \frac{9(1^2 + 2^2 + 3^2)}{7} + \frac{12(1^2 + 2^2 + 3^2 + 4^2)}{9} + \frac{15(1^2 + 2^2 + \ldots + 5^2)}{11} + \ldots$ up to 15 terms, is:
[Jan. 09, 2019 (II)]

Options:

A. 7520

B. 7510

C. 7830

D. 7820

Answer: D

Solution:

Solution:
$S = 1 + 6 + \frac{9(1^2 + 2^2 + 3^2)}{7} + \frac{12(1^2 + 2^2 + 3^2 + 4^2)}{9} + \frac{15(1^2 + 2^2 + 3^2 + 4^2 + 5^2)}{11} + \ldots$
$S = \frac{3 \cdot (1)^2}{3} + \frac{6 \cdot (1^2 + 2^2)}{5} + \frac{9 \cdot (1^2 + 2^2 + 3^2)}{7} + \frac{12 \cdot (1^2 + 2^2 + 3^2 + 4^2)}{9} + \ldots$
Now, n^{th} term of the series,
$t_n = \frac{3n \cdot (1^2 + 2^2 + \ldots + n^2)}{(2n + 1)}$
$\Rightarrow t_n = \frac{3n \cdot n(n + 1)(2n + 1)}{6(2n + 1)} = \frac{n^3 + n^2}{2}$
$\therefore S_n = \Sigma t_n = \frac{1}{2}\left\{ \left(\frac{n(n + 1)}{2} \right)^2 + \frac{n(n + 1)(2n + 1)}{6} \right\}$
$= \frac{n(n + 1)}{4}\left(\frac{n(n + 1)}{2} + \frac{2n + 1}{3} \right)$

Hence, sum of the series upto 15 terms is,

$S_{15} = \frac{15 \times 16}{4}\left\{ \frac{15 \cdot 16}{2} + \frac{31}{3} \right\}$

$= 60 \times 120 + 60 \times \frac{31}{3}$

$= 7200 + 620 = 7820$

Question172

The sum of all values of $\theta \in \left(0, \frac{\pi}{2}\right)$ satisfying $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$ is:

[Jan. 10, 2019 (I)]

Options:

A. π

B. $\frac{5\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{3\pi}{8}$

Answer: C

Solution:

Solution:

$\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$\Rightarrow \cos^2 2\theta(1 - \cos^2 2\theta) = \frac{1}{4}$ . . . (i)

$\because$ G.M. $\leq$ A.M.

$\therefore (\cos^2 2\theta)(1 - \cos^2 2\theta) \leq \left(\frac{\cos^2 2\theta + (1 - \cos^2 2\theta)}{2} \right)^2$

$= \frac{1}{4}$ . . . (ii)

So, from equation (i) and (ii), we get.

G.M. = A.M.

It is possible only if

$\cos^2 2\theta = 1 - \cos^2 2\theta$

$\Rightarrow \cos^2 2\theta = \frac{1}{2} \Rightarrow \cos 2\theta = \pm \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8}$ $\therefore$ Sum $= \frac{\pi}{8} + \frac{3\pi}{8} = \frac{\pi}{2}$

Question173

Let α and β be the roots of the quadratic equation $x^2 \sin\theta - x(\sin\theta\cos\theta + 1) + \cos\theta = 0 (0 < \theta < 45^\circ)$, and $\alpha < \beta$. Then $\sum_{n=0}^{\infty} \left(\alpha^n + \frac{(-1)^n}{\beta^n} \right)$ is equal to :

[Jan. 11, 2019 (II)]

Options:

A. $\frac{1}{1 - \cos\theta} - \frac{1}{1 + \sin\theta}$

B. $\frac{1}{1+\cos\theta}+\frac{1}{1-\sin\theta}$

C. $\frac{1}{1-\cos\theta}+\frac{1}{1+\sin\theta}$

D. $\frac{1}{1+\cos\theta}-\frac{1}{1-\sin\theta}$

Answer: C

Solution:

Solution:
$x^2\sin\theta - x(\sin\theta\cdot\cos\theta+1)+\cos\theta=0.$
$x^2\sin\theta - x\sin\theta\cdot\cos\theta - x+\cos\theta=0$
$x\sin\theta(x-\cos\theta)-1(x-\cos\theta)=0$
$(x-\cos\theta)(x\sin\theta-1)=0$
$\therefore x=\cos\theta, \operatorname{cosec}\theta, \theta\in(0,45^\circ)$
$\therefore\ \alpha=\cos\theta, \beta=\operatorname{cosec}\theta$
$\sum_{n=0}^{\infty}\alpha^n=1+\cos\theta+\cos^2\theta+\ldots\infty=\frac{1}{1-\cos\theta}$
$\sum_{n=0}^{\infty}\frac{(-1)^n}{\beta^n}=1-\frac{1}{\operatorname{cosec}\theta}+\frac{1}{\operatorname{cosec}^2\theta}-\frac{1}{\operatorname{cosec}^3\theta}+\ldots\infty$
$=1-\sin\theta+\sin^2\theta-\sin^3\theta+\ldots\infty$
$=\frac{1}{1+\sin\theta}$
$\therefore\ \sum_{n=0}^{\infty}\left(\alpha^n+\frac{(-1)^n}{\beta^n}\right)=\sum_{n=0}^{\infty}\alpha^n+\sum_{n=0}^{\infty}\frac{(-1)^n}{\beta^n}$
$=\frac{1}{1-\cos\theta}+\frac{1}{1+\sin\theta}$

Question174

Let $a_1, a_2, \ldots, a_{10}$ be a G.P. If $\frac{a_3}{a_1}=25$, then $\frac{a_9}{a_5}$ equals: [Jan. 11, 2019 (I)]

Options:

A. 5^4

B. $4(5^2)$

C. 5^3

D. $2(5^2)$

Answer: A

Solution:

Solution:
Let $a_1=a, a_2=ar, a_3=ar^2\ldots a_{10}=ar^9$
where r = common ratio of given G.P.
Given, $\frac{a_3}{a_1}=25$
$\Rightarrow\frac{ar^2}{a}=25$
$\Rightarrow r=\pm5$
Now, $\frac{a_9}{a_5}=\frac{ar^8}{ar^4}=r^4=(\pm5)^4=5^4$

Question175

The sum of an infinite geometric series with positive terms is 3 and the sum of the cubes of its terms is $\frac{27}{19}$. Then the common ratio of this series is :
[Jan. 11, 2019 (I)]

Options:

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{2}{9}$

D. $\frac{4}{9}$

Answer: B

Solution:

Solution:
Let the terms of infinite series are a, ar, ar^2, ar^3, ...
So, $\frac{a}{1-r} = 3$
Since, sum of cubes of its terms is $\frac{27}{19}$ that is sum of a^3,
$a^3r^3, \ldots \infty$ is $\frac{27}{19}$
So, $\frac{a^3}{1-r^3} = \frac{27}{19}$
$\Rightarrow \frac{a}{1-r} \times \frac{a^2}{(1+r^2+r)} = \frac{27}{19}$
$\Rightarrow \frac{9(1+r^2-2r)\times 3}{1+r^2+r} = \frac{27}{19}$
$\Rightarrow 6r^2 - 13r + 6 = 0$
$\Rightarrow (3r-2)(2r-3) = 0$
$\Rightarrow r = \frac{2}{3}$, or $\frac{3}{2}$
As $|r| < 1$
So, $r = \frac{2}{3}$

Question176

Let $S_n = 1 + q + q^2 + ... + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + ... + \left(\frac{q+1}{2}\right)^n$ where q is a real number and $q \neq 1$. If
${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + ... + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$, then α is equal to :
[Jan. 11, 2019 (II)]

Options:

A. 2^{99}

B. 202

C. 200

D. 2^{100}

Answer: D

Solution:

$S_n = \left(\frac{1-q^{n+1}}{1-q} \right), T_n = \frac{1-\left(\frac{q+1}{2}\right)^{n+1}}{1-\left(\frac{q+1}{2}\right)}$

$\Rightarrow T_{100} = \frac{1-\left(\frac{q+1}{2}\right)^{101}}{1-\left(\frac{q+1}{2}\right)}$

$Sn = \frac{1}{1-q} - \frac{q^{n+1}}{1-q}, T_{100} = \frac{2^{101}-(q+1)^{101}}{2^{100}(1-q)}$

Now, ${}^{101}C_1 + {}^{101}C_2S_1 + {}^{101}C_3S_2 + \ldots + {}^{101}C_{101}S_{100}$

$= \left(\frac{1}{1-q}\right)({}^{101}C_2 + \ldots + {}^{101}C_{101})$

$- \frac{1}{1-q}({}^{101}C_2q^2 + {}^{101}C_3q^3 + \ldots + {}^{101}C_{101}q^{101}) + 101$

$= \frac{1}{1-q}(2^{101} - 1 - 101) - \left(\frac{1}{1-q}\right)((1+q)^{101} - 1$

$-{}^{101}C_1q) + 101$

$= \frac{1}{1-q}[2^{101} - 102 - (1+q)^{101} + 1 + 101q] + 101$

$= \frac{1}{1-q}[2^{101} - 101 + 101q - (1+q)^{101}] + 101$

$= \left(\frac{1}{1-q}\right)[2^{101} - (1+q)^{101}] = 2^{100}T_{100}$

Hence, by comparison $\alpha = 2^{100}$

Question177

The product of three consecutive terms of a G.P. is 512 . If 4 is added to each of the first and the second of these terms, the three terms now form an A.P. Then the sum of the original three terms of the given G.P. is : [Jan. 12, 2019 (I)]

Options:

A. 36

B. 32

C. 24

D. 28

Answer: D

Solution:

Let three terms of a G.P. be $\frac{a}{r}, a, ar$

$\frac{a}{r} \cdot a \cdot ar = 512$

$a^3 = 512 \Rightarrow a = 8$

4 is added to each of the first and the second of three terms then three terms are, $\frac{8}{r} + 4, 8 + 4, 8r.$

$\because \frac{8}{r} + 4, 12, 8r$ form an A.P.

$\therefore\ 2 \times 12 = \frac{8}{r} + 8r + 4$

$\Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow (2r-1)(r-2) = 0$

$\Rightarrow r = \frac{1}{2}$ or 2

Therefore, sum of three terms $= \frac{8}{2} + 8 + 16 = 28$

Question178

Let $S_k = \frac{1+2+3+.....+k}{k}$ If $S_1{}^2 + S_2{}^2 + ... + S_{10}{}^2 = \frac{5}{12}$A. Then A is equal to
[Jan. 12, 2019 (I)]

Options:

A. 283

B. 301

C. 303

D. 156

Answer: C

Solution:

$\because 1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2}$

$\therefore S_k = \frac{k(k+1)}{2k} = \frac{k+1}{2}$

$\Rightarrow \frac{5}{12}A = \frac{1}{4}[2^2 + 3^3 + \ldots + 11^2]$

$= \frac{1}{4}[1^2 + 2^2 + \ldots + 11^2 - 1]$

$= \frac{1}{4}\left[\frac{11(11+1)(2 \times 11 + 1)}{6} - 1\right]$

$\frac{1}{4}\left[\frac{11 \times 12 \times 23}{6} - 1\right]$

$= \frac{1}{4}[505]$

$A = \frac{505}{4} \times \frac{12}{5} = 303$

Question179

If the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3 + \left(1\frac{1}{2}\right)^3 + \left(2\frac{1}{4}\right)^3 + 3^3 + \left(3\frac{3}{4}\right)^3 + ...$ is equal to 225k then k is equal to :
[Jan. 12, 2019 (II)]

Options:

A. 108

B. 27

C. 54

D. 9

Answer: B

Solution:

Solution:

$S = \left(\frac{3}{4}\right)^3 + \left(\frac{3}{2}\right)^3 + \left(\frac{9}{4}\right)^3 + (3)^3 + \ldots$

$S = \left(\frac{3}{4}\right)^3 + \left(\frac{6}{4}\right)^3 + \left(\frac{9}{4}\right)^3 + \left(\frac{12}{4}\right)^3 + \ldots$

Let the general term of S be

$T_r = \left(\frac{3r}{4}\right)^3$, then

$255K = \sum_{r=1}^{15} T_r = \left(\frac{3}{4}\right)^3 \sum_{r=1}^{15} r^3$

$255K = \frac{27}{64} \times \left(\frac{15 \times 16}{2}\right)^2$

$\Rightarrow K = 27$

Question180

I f $^nC_4$, $^nC_5$ and $^nC_6$ are in A.P., then n can be [Jan. 12, 2019 (II)]

Options:

A. 9

B. 14

C. 11

D. 12

Answer: B

Solution:

Solution:
Since $^nC_4$, $^nC_5$ and $^nC_6$ are in A.P.

$2\,^nC_5 = {}^nC_4 + {}^nC_6$

$2 = \frac{^nC_4}{^nC_5} + \frac{^nC_6}{^nC_5}$

$2 = \frac{5}{n-4} + \frac{n-5}{5}$

$\Rightarrow 12(n-4) = 30 + n^2 - 9n + 20$

$\Rightarrow n^2 - 21n + 98 = 0$

$(n-7)(n-14) = 0$

$(n-7)(n-14) = 0$

$n = 7, n = 14$

Question181

If 1 9th term of a non-zero A.P. is zero, then its (49th term): (29th term) is : [Jan. 11, 2019 (II)]

Options:

A. 4: 1

B. 1: 3

C. 3: 1

D. 2: 1

Answer: C

Solution:

Solution:
Let first term and common difference of AP be a and d respectively, then

$t_n = a + (n-1)d$

$\therefore\ t_{19} = a + 18d = 0 \therefore\ a = -18d$

$\therefore \ \frac{t_{49}}{t_{29}} = \frac{a + 48d}{a + 28d}$

$= \frac{-18d + 48d}{-18d + 28d} = \frac{30d}{10d} = 3$

$t_{49} : t_{29} = 3 : 1$

Question182

The sum of all two digit positive numbers which when divided by 7 yield 2 or 5 as remainder is:
[Jan. 10, 2019 (I)]

Options:

A. 1256

B. 1465

C. 1365

D. 1356

Answer: D

Solution:

Solution:

Two digit positive numbers which when divided by 7 yield 2 as remainder are 12 terms i.e, 16, 23, 30, …, 93

Two digit positive numbers which when divided by 7 yield
5 as remainder are 13 terms i.e ,12, 19, 26, …, 96

By using AP sum of 16, 23, …, 93, we get

$S_1 = 16 + 23 + 30 + \ldots + 93 = 654$

By using AP sum of 12, 19, 26, …, 96, we get

$S_1 = 12 + 19 + 26 + \ldots + 96 = 702$

$\therefore$ required $\text{Sum} = S_1 + S_2 = 654 + 702 = 1356$

Question183

Let $a_1, a_2, \ldots\ldots, a_{30}$ be an A.P., $S = \sum_{i=1}^{30} a_i$ and $T = \sum_{i=1}^{15} a_{(2i-1)}$ If $a_5 = 27$ and $S - 2T = 75$, then a_{10} is equal to:
[Jan. 09, 2019 (I)]

Options:

A. 52

B. 57

C. 47

D. 42

Answer: A

Solution:

Solution:

$S = \sum_{i=1}^{30} a_i = \frac{30}{2}[2a_1 + 29d]$

$T = \sum_{i=1}^{15} a_{(2i-1)} = \frac{15}{2}[2a_1 + 28d]$

Since, $S - 2T = 75$

$\Rightarrow 30a_1 + 435d - 30a_1 - 420d = 75$
$\Rightarrow d = 5$
Also, $a_5 = 27 \Rightarrow a_1 + 4d = 27 \Rightarrow a_1 = 7$
Hence, $a_{10} = a_1 + 9d = 7 + 9 \times 5 = 52$

Question184

If three distinct numbers a, b, c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then which one of the following statements is correct?
[April 08, 2019 (II)]

Options:

A. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

B. d, e, f are in A.P.

C. d, e, f are in G.P.

D. $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

Answer: A

Solution:

Solution:
Since a, b, c are in G.P.
$\therefore b^2 = acc$
Given equation is, $ax^2 + 2bx + c = 0$
$\Rightarrow ax^2 + 2\sqrt{ac}x + c = 0 \Rightarrow (\sqrt{a}x + \sqrt{c})^2 = 0$
$\Rightarrow x = -\sqrt{\frac{c}{a}}$
Also, given that $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root.
$\Rightarrow x = -\sqrt{\frac{c}{a}}$ must satisfy $dx^2 + 2ex + f = 0$
$\Rightarrow d \cdot \frac{c}{a} + 2e\left(-\sqrt{\frac{c}{a}}\right) + f = 0$
$\frac{d}{a} - \frac{2e}{\sqrt{ac}} + \frac{f}{c} = 0 \Rightarrow \frac{d}{a} - \frac{2e}{b} + \frac{f}{c} = 0$
$\Rightarrow \frac{2e}{b} = \frac{d}{a} + \frac{f}{c} \Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Question185

For $x \in R$, let $[x]$ denote the greatest integer $\leq x$, then the sum of the series $\left[-\frac{1}{3}\right] + \left[-\frac{1}{3} - \frac{1}{100}\right] + \left[-\frac{1}{3} - \frac{2}{100}\right] + \cdot s + \left[-\frac{1}{3} - \frac{99}{100}\right]$ is
[April 12, 2019 (I)]

Options:

A. -153

B. -133

C. -131

D. -135

Answer: B

Solution:

Solution:

$\because [x]+\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]\ldots.\left[x+\frac{n-1}{n}\right]=[nx]$

and $[x]+[-x]=-1(x\notin z)$

$\therefore\left[-\frac{1}{3}\right]+\left[-\frac{1}{3}-100\right]+\ldots+\left[-\frac{1}{3}-\frac{99}{100}\right]$

$=-100-\left\{\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{100}\right]+\ldots\left[\frac{1}{3}+\frac{99}{100}\right]\right\}$

$=-100-\left[\frac{100}{3}\right]=-133$

Question186

The sum $\frac{3\times1^3}{1^2}+\frac{5\times(1^3+2^3)}{1^2+2^2}+\frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2}\Big)+\ldots\ldots$ upto 10^{th} term, is :
[April 10, 2019 (I)]

Options:

A. 680

B. 600

C. 660

D. 620

Answer: C

Solution:

Solution:

r^{th} term of the series,

$T_r=\frac{(2r+1)(1^3+2^3+3^3+\ldots+r^3)}{1^2+2^2+3^2+\ldots+r^2}$

$T_r=(2r+1)\left(\frac{r(r+1)}{2}\right)^2\times\frac{6}{r(r+1)(2r+1)}=\frac{3r(r+1)}{2}$

$\therefore$ sum of 10 terms is $=S=\sum_{r=1}^{10}T_r=\frac{3}{2}\sum_{r=1}^{10}(r^2+r)$

$=\frac{3}{2}\left\{\frac{10\times(10+1)(2\times10+1)}{6}+\frac{10\times11}{2}\right\}$

$=\frac{3}{2}\times5\times11\times8=660$

Question187

The sum $1+\frac{1^3+2^3}{1+2}+\frac{1^3+2^3+3^3}{1+2+3}+\ldots\ldots+\frac{1^3+2^3+3^3+\ldots+15^3}{1+2+3+\ldots+15}-\frac{1}{2}(1+2+3+\ldots+15$ is equal to :
[April 10, 2019 (II)]

Options:

A. 620

B. 1240

C. 1860

D. 660

Answer: A

Solution:

Solution:

Let, $S = 1 + \frac{1^3 + 2^3}{1 + 2} + \frac{1^3 + 2^3 + 3^3}{1 + 2 + 3} + \ldots$ 15 terms

$T_n = \frac{1^3 + 2^3 + \ldots n^3}{1 + 2 + \ldots n} = \frac{\left(\frac{n(n+1)}{2}\right)^2}{\frac{n(n+1)}{2}} = \frac{n(n+1)}{2}$

Now, $S = \frac{1}{2}\left(\sum_{n=1}^{15} n^2 + \sum_{n=1}^{15} n\right) = \frac{1}{2}\left(\frac{15(16)(31)}{6} + \frac{15(16)}{2}\right)$

$= 680$

$\therefore$ required sum is, $680 - \frac{1}{2}\frac{15(16)}{2} = 680 - 60 = 620$

Question188

The sum of the series $1 + 2 \times 3 + 3 \times 5 + 4 \times 7 + \ldots\ldots$ upto 11^{th} term is: [April 09, 2019 (II)]

Options:

A. 915

B. 946

C. 945

D. 916

Answer: B

Solution:

Solution:

$1 + 2.3 + 3.5 + 4.7 + \ldots\ldots.$ Let, $S = (2.3 + 3.5 + 4.7 + \ldots\ldots)$

Now, $S_{10} = \sum_{n=1}^{10} (n+1)(2n+1) = \sum_{n=1}^{10} (2n^2 + 3n + 1)$

$= \frac{2n(n+1)(2n+1)}{6} + \frac{3n(n+1)}{2} + n$

Put $n = 10$

$= \frac{2.10.11.21}{6} + \frac{3.10.11}{2} + 10 = 945$

Hence required sum of the series $= 1 + 945 = 946$

Question189

Some identical balls are arranged in rows to form an equilateral triangle. The first row consists of one ball, the second row consists of two balls and so on. If 99 more identical balls are added to the total number of balls used in forming the equilateral triangle, then all these balls can be arranged in a square whose each side contains exactly 2 balls less than the number of balls each side of the triangle contains. Then the number of balls used to form the equilateral triangle is: [April 09, 2019 (II)]

Options:

A. 157

B. 262

C. 225

D. 190

Answer: D

Solution:

Solution:
Number of balls used in equilateral triangle
$= \dfrac{n(n+1)}{2}$
$\because$ side of equilateral triangle has n -balls
$\therefore$ no. of balls in each side of square is $= (n-2)$
According to the question, $\dfrac{n(n+1)}{2} + 99 = (n-2)^2$
$\Rightarrow n^2 + n + 198 = 2n^2 - 8n + 8$
$\Rightarrow n^2 - 9n - 190 = 0 \Rightarrow (n-19)(n+10) = 0$
$\Rightarrow n = 19$
Number of balls used to form triangle
$= \dfrac{n(n+1)}{2} = \dfrac{19 \times 20}{2} = 190$

Question190

The sum $\sum_{k=1}^{20} k \frac{1}{2^k}$ is equal to :

[April 08, 2019 (II)]

Options:

A. $2 - \dfrac{3}{2^{17}}$

B. $1 - \dfrac{11}{2^{20}}$

C. $2 - \dfrac{11}{2^{19}}$

D. $2 - \dfrac{21}{2^{20}}$

Answer: C

Solution:

Solution:
Let, $S = \sum_{k=1}^{20} k \cdot \frac{1}{2^k}$
$S = \frac{1}{2} + 2 \cdot \frac{1}{2^2} + 3 \cdot \frac{1}{2^3} + \ldots. + 20 \cdot \frac{1}{2^{20}}$ . . . (i)
$\frac{1}{2}S = \frac{1}{2^2} + 2 \cdot \frac{1}{2^3} + \ldots. + 19 \frac{1}{2^{20}} + 20 \frac{1}{2^{21}}$ . . . (ii)
On subtracting equations (ii) by (i),
$\frac{S}{2} = \left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots. + \frac{1}{2^{20}} \right) - 20 \frac{1}{2^{21}}$
$= \dfrac{\frac{1}{2}\left(1 - \frac{1}{2^{20}}\right)}{1 - \frac{1}{2}} - 20 \cdot \frac{1}{2^{21}} = 1 - \frac{1}{2^{20}} - 10 \cdot \frac{1}{2^{20}}$
$\frac{S}{2} = 1 - 11 \cdot \frac{1}{2^{20}} \Rightarrow S = 2 - 11 \cdot \frac{1}{2^{19}} = 2 - \frac{11}{2^{19}}$

Question191

Let S_n denote the sum of the first n terms of an A.P. If $S_4 = 16$ and $S_6 = -48$, then S_{10} is equal to : [April 12, 2019 (I)]

Options:

A. -260

B. -410

C. -320

D. -380

Answer: C

Solution:

Solution:
Given, $S_4 = 16$ and $S_6 = -48$
$\Rightarrow\ 2(2a + 3d) = 16 \Rightarrow 2a + 3d = 8 \ldots$ (i)
And $3[2a + 5d] = -48 \Rightarrow 2a + 5d = -16$
$\Rightarrow 2d = -24$ [using equation(i)]
$\Rightarrow d = -12$ and $a = 22$
$\therefore\ S_{10}\ \frac{10}{2} = (44 + 9(-12)) = -320$

Question192

If $a_1, a_2, a_3, \ldots\ldots$ are in A.P. such that $a_1 + a_7 + a_{16} = 40$, then the sum of the first 15 terms of this A.P. is : [April 12, 2019 (II)]

Options:

A. 200

B. 280

C. 120

D. 150

Answer: A

Solution:

Solution:
Let the common difference of the A.P. is 'd'.
Given, $a_1 + a_7 + a_{16} = 40$
$\Rightarrow\ a_1 + a_1 + 6d + a_1 + 15d = 40$
$\Rightarrow\ 3a_1 + 21d = 40$
$\Rightarrow\ a_1 + 7d = \frac{40}{3}$
Now, sum of first 15 terms of this A.P. is,
$S_{15} = \frac{15}{2}[2a_1 + 14d] = 15(a_1 + 7d)$
$= 15\left(\frac{40}{3}\right) = 200$
[Using(i)]

Question193

If $a_1, a_2, a_3, \ldots\ldots a_n$ are in A.P. and $a_1 + a_4 + a_7 + \ldots\ldots + a_{16} = 114$ then $a_1 + a_6 + a_{11} + a_{16}$ is equal to :
[April 10, 2019 (I)]

Options:

A. 98

B. 76

C. 38

D. 64

Answer: B

Solution:

Solution:
$a_1 + a_4 + a_7 + \ldots + a_{16} = 114$
$\Rightarrow 3(a_1 + a_{16}) = 114$
$\Rightarrow a_1 + a_{16} = 38$
Now, $a_1 + a_6 + a_{11} + a_{16} = 2(a_1 + a_{16}) = 2 \times 38 = 76$

Question194

Let the sum of the first n terms of a non-constant A.P. $a_1, a_2, a_3, \ldots\ldots\ldots\ldots\ldots$ be $50n + \frac{n(n-7)}{2}A$, where A is a constant. If d is the common difference of this A.P., then the ordered pair (d, a_{50}) is equal to:
[April 09, 2019 (I)]

Options:

A. (50, 50 + 46A)

B. (50, 50 + 45A)

C. (A, 50 + 45A)

D. (A, 50 + 46A)

Answer: D

Solution:

Solution:
$\because S_n = \left(50 - \frac{7A}{2}\right)n + n^2 \times \frac{A}{2} \Rightarrow a_1 = 50 - 3S$
$\therefore\ d = a_2 - a_1 = S_{n_2} - S_{n_1} - S_{n_1} \Rightarrow d = \frac{A}{2} \times 2 = A$
Now, $a_{50} = a_1 + 49 \times d$
$= (50 - 3A) + 49A = 50 + 46A$
So, $(d, a_{50}) = (A, 50 + 46A)$

Question195

Let $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$, where the function f satisfies

$f(x+y) = f(x)f(y)$ for all natural numbers x, y and $f(a) = 2$ Then the natural number 'a ' is:
[April 09, 2019 (I)]

Options:

A. 2

B. 16

C. 4

D. 3

Answer: D

Solution:

$\because f(x+y) = f(x) \cdot f(y)$
$\Rightarrow \text{Let} f(x) = t^x$
$\because f(1) = 2$
$\therefore t' = 2$
$\Rightarrow f(x) = 2^x$
Since, $\sum_{k=1}^{10} f(a+k) = 16(2^{10} - 1)$
Then, $\sum_{k=1}^{10} 2^{a+k} = 16(2^{10} - 1)$
$\Rightarrow 2^a \sum_{k=1}^{10} 2^k = 16(2^{10} - 1)$
$\Rightarrow 2^a \times \frac{((2^{10}) - 1) \times 2}{(2-1)} = 16 \times (2^{10} - 1) 2.2^a = 16$
$\Rightarrow a = 3$

Question196

If the sum and product of the first three terms in an A.P. are 33 and 1155, respectively, then a value of its 11^{th} term is:
[April 09, 2019 (II)]

Options:

A. -35

B. 25

C. -36

D. -25

Answer: D

Solution:

Solution:
Let three terms of A.P. are $a-d, a, a+d$
Sum of terms is, $a-d+a+a+d = 33 \Rightarrow a = 11$
Product of terms is, $(a-d)a(a+d) = 11(121 - d^2) = 1155$
$\Rightarrow 121 - d^2 = 105 \Rightarrow d = \pm 4$ if $d = 4$
$T_{11} = T_1 + 10d = 7 + 10(4) = 47$
if $d = -4$
$T_{11} = T_1 + 10d = 15 + 10(-4) = -25$

Question197

The sum of all natural numbers ' n ' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is :
[April 08, 2019 (I)]

Options:

A. 3203

B. 3303

C. 3221

D. 3121

Answer: D

Solution:

Solution:
$\because 91 = 13 \times 7$
Then, the required numbers are either divisible by 7 or 13 .
$\therefore$ Sum of such numbers = Sum of no. divisible by 7+
sum of the no. divisible by 13− Sum of the numbers divisible by 91
$= (105 + 112 + \ldots + 196) + (104 + 117 + \ldots + 195) - 182$
$= 2107 + 1196 - 182 = 3121$

Question198

If α, β and γ are three consecutive terms of a nonconstant G.P. such that the equations $\alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root, then $\alpha(\beta + \gamma)$ is equal to :
[April 12, 2019 (II)]

Options:

A. 0

B. $\alpha\beta$

C. $\alpha\gamma$

D. $\beta\gamma$

Answer: D

Solution:

Solution:
$\because \alpha, \beta, \gamma$ are three consecutive terms of a non- constant G.P.
$\therefore \beta^2 = \alpha\gamma$
So roots of the equation $\alpha x^2 + 2\beta x + \gamma = 0$ are
$\frac{-2\beta \pm 2\sqrt{\beta^2 - \alpha\gamma}}{2\alpha} = \frac{\beta}{\alpha}$
$\because \alpha x^2 + 2\beta x + \gamma = 0$ and $x^2 + x - 1 = 0$ have a common root.
$\therefore$ this root satisfy the equation $x^2 + x - 1 = 0$
$\beta^2 - \alpha\beta - \alpha^2 = 0$
$\Rightarrow \alpha\gamma - \alpha\beta - \alpha^2 = 0 \Rightarrow \alpha + \beta = \gamma$
Now, $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$
$= \alpha\beta + \beta^2 = (\alpha + \beta)\beta = \beta\gamma$

Question199

Let a, b and c be in G.P. with common ratio r, where a ≠ 0 and 0 < r ≤ $\frac{1}{2}$. If 3a, 7b and 15c are the first three terms of an A.P., then the 4^{th} term of this A.P. is:
[April 10, 2019 (II)]

Options:

A. $\frac{2}{3}a$

B. 5a

C. $\frac{7}{3}a$

D. a

Answer: D

Solution:

Solution:
$\because a, b, c$ are in G.P. $\Rightarrow b = ar,\ c = ar^2$
$\because 3a, 7b, 15c$ are in A.P.
$\Rightarrow 3a,\ 7ar,\ 15ar^2$ are in A.P.
$\therefore 14ar = 3a + 15ar^2$
$\Rightarrow 15r^2 - 14r + 3 = 0 \Rightarrow r = \frac{1}{3}$ or $\frac{3}{5}$
$\because r < \frac{1}{2}\quad \therefore r = \frac{3}{5}$ rejected
Fourth term $= 15ar^2 + 7ar - 3a$
$= a(15r^2 + 7r - 3) = a\left(\frac{15}{9} + \frac{7}{3} - 3\right) = a$

Question200

Let $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \ldots, (x_i \neq 0$ for $i = 1, 2, \ldots, n)$ be in A.P. such that $x_1 = 4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$, then $\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)$ is equal to.
[Online April 16, 2018]

Options:

A. 3

B. $\frac{13}{8}$

C. $\frac{13}{4}$

D. $\frac{1}{8}$

Answer: C

Solution:

$\because \frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \ldots, \frac{1}{x_n}$ are in A.P

$x_1 = 4$ and $x_{21} = 20$

Let 'd' be the common difference of this A.P

its 21^{st} term $= \frac{1}{x_{21}} = \frac{1}{x_1} + [(21-1) \times d]$

$\Rightarrow d = \frac{1}{20} \times \left(\frac{1}{20} - \frac{1}{4}\right) \Rightarrow d = -\frac{1}{100}$

Also $x_n > 50$ (given).

$\therefore \frac{1}{x_n} = \frac{1}{x_1} + [(n-1) \times d]$

$\Rightarrow x_n = \frac{x_1}{1 + (n-1) \times d \times x_1}$

$\therefore \frac{x_1}{1 + (n-1) \times d \times x_1} > 50$

$\Rightarrow \frac{4}{1 + (n-1) \times \left(-\frac{1}{100}\right) \times 4} > 50$

$\Rightarrow 1 + (n-1) \times \left(-\frac{1}{100}\right) \times 4 < \frac{4}{50}$

$\Rightarrow -\frac{1}{100}(n-1) < -\frac{23}{100}$

$\Rightarrow n - 1 > 23 \Rightarrow n > 24$

Therefore, $n = 25$.

$\Rightarrow \sum_{i=1}^{25} \frac{1}{x_i} = \frac{25}{2}\left[\left(2 \times \frac{1}{4}\right) + (25-1) \times \left(-\frac{1}{100}\right)\right] = \frac{13}{4}$

Question201

If $x_1, x_2, \ldots, x_n$ and $\frac{1}{h_1}, \frac{1}{h^2}, \ldots \cdot \frac{1}{h_n}$ are two A.P's such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 . h_{10}$ equals.
[Online April 15, 2018]

Options:

A. 2560

B. 2650

C. 3200

D. 1600

Answer: A

Solution:

Solution:

Suppose d_1 is the common difference of the A.P. $x_1, x_2, \ldots . x_n$ then

$\because x_8 - x_3 = 5d_1 = 12 \Rightarrow d_1 = \frac{12}{5} = 2.4$

$\Rightarrow x_5 = x_3 + 2d_1 = 8 + 2 \times \frac{12}{5} = 12.8$

Suppose d_2 is the common difference of the A.P

$\frac{1}{h_1}, \frac{1}{h_2}, \ldots \cdot \frac{1}{h_n}$ then

$5d_2 = \frac{1}{20} - \frac{1}{8} = \frac{-3}{40} \Rightarrow d_2 = \frac{-3}{200}$

$\because \frac{1}{h_{10}} = \frac{1}{h_7} + 3d_2 = \frac{1}{200} \Rightarrow h_{10} = 200$

$\Rightarrow x_5 \cdot h_{10} = 12.8 \times 200 = 2560$

Question202

If b is the first term of an infinite G. P whose sum is five, then b lies in the interval.
[Online April 15, 2018]

Options:

A. $(-\infty, -10)$

B. $(10, \infty)$

C. (0,10)

D. (-10,0)

Answer: C

Solution:

Solution:
First term $= b$ and common ratio $= r$
For infinite series, $\text{Sum} = \frac{b}{1-r} = 5$
$\Rightarrow b = 5(1-r)$
So, interval of $b = (0, 10)$ as, $-1 < r < 1$ for infinite G.P.

Question203

Let $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$. Then, the least odd natural number p, so that $B_n > A_n$, for all $n \geq p$ is
[Online April 15, 2018]

Options:

A. 5

B. 7

C. 11

D. 9

Answer: B

Solution:

Solution:
$A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$
Which is a G.P. with $a = \frac{3}{4}$, $r = \frac{-3}{4}$ and number of terms $= n$
$\therefore A_n = \frac{\frac{3}{4} \times \left(1 - \left(\frac{-3}{4}\right)^n\right)}{1 - \left(\frac{-3}{4}\right)} = \frac{\frac{3}{4} \times \left(1 - \left(\frac{-3}{4}\right)^n\right)}{\frac{7}{4}}$
$\Rightarrow A_n = \frac{3}{7}\left[1 - \left(\frac{-3}{4}\right)^n\right] \;...\; \text{(i)}$
As, $B_n = 1 - A_n$
For least odd natural number p, such that $B_n > A_n$
$\Rightarrow 1 - A_n > A_n \;\; \Rightarrow 1 > 2 \times A_n \Rightarrow A_n < \frac{1}{2}$
From eqn. (i), we get
$\frac{3}{7} \times \left[1 - \left(\frac{-3}{4}\right)^n\right] < \frac{1}{2} \Rightarrow 1 - \left(\frac{-3}{4}\right)^n < \frac{7}{6}$
$\Rightarrow 1 - \frac{7}{6} < \left(\frac{-3}{4}\right)^n \Rightarrow \frac{-1}{6} < \left(\frac{-3}{4}\right)^n$

As n is odd, then $\left(\frac{-3}{4}\right)^n = -\frac{3^n}{4}$

So $\frac{-1}{6} < -\left(\frac{3}{4}\right)^n \Rightarrow \frac{1}{6} > \left(\frac{3}{4}\right)^n$

$\log\left(\frac{1}{6}\right) = n\log\left(\frac{3}{4}\right) \Rightarrow 6.228 < n$

Hence, n should be 7 .

Question204

If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that a < b < c and $a + b + c = \frac{3}{4}$, then the value of a is [Online April 15, 2018]

Options:

A. $\frac{1}{4} - \frac{1}{3\sqrt{2}}$

B. $\frac{1}{4} - \frac{1}{4\sqrt{2}}$

C. $\frac{1}{4} - \frac{1}{\sqrt{2}}$

D. $\frac{1}{4} - \frac{1}{2\sqrt{2}}$

Answer: D

Solution:

Solution:

$\because$a, b, c are in A.P. then $a + c = 2b$

also it is given that,

$a + b + c = \frac{3}{4}$ . . . (i)

$\Rightarrow 2b + b = \frac{3}{4} \Rightarrow b = \frac{1}{4}$ . . . (ii)

Again it is given that, a^2, b^2, c^2 are in G.P. then

$(b^2)^2 = a^2c^2 \Rightarrow ac = \pm\frac{1}{16}$ . . . (iii)

From (i), (ii) and (iii), we get;

$a \pm \frac{1}{16a} = \frac{1}{2} \Rightarrow 16a^2 - 8a \pm 1 = 0$

Case I: $16a^2 - 8a + 1 = 0$

$\Rightarrow a = \frac{1}{4}\left(\text{ not possible as } a < b\right)$

Case II: $16a^2 - 8a - 1 = 0 \Rightarrow a = \frac{8 \pm \sqrt{128}}{32}$

$\Rightarrow a = \frac{1}{4} \pm \frac{1}{2\sqrt{2}}$

$\therefore a = \frac{1}{4} - \frac{1}{2\sqrt{2}}$ $(\because a < b)$

Question205

The sum of the first 20 terms of the series $1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + ...$ is? [Online April 16, 2018]

Options:

A. $38 + \frac{1}{2^{20}}$

B. $39+\frac{1}{2^{19}}$

C. $39+\frac{1}{2^{20}}$

D. $38+\frac{1}{2^{19}}$

Answer: D

Solution:

Solution:

The general term of the given series $=\frac{2\times2^{r}-1}{2^{r}}$, where $r\geq0$

$\therefore$ req. sum $=1+\sum_{r=1}^{19}\frac{2\times2^{r}-1}{2^{r}}$

Now, $\sum_{r=1}^{19}\left(\frac{2\times2^{r}-1}{2^{r}}\right)=\sum_{r=1}^{19}\left(2-\frac{1}{2^{r}}\right)$

$=2(19)-\frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{19}\right)}{1-\frac{1}{2}}=38+\frac{\left(\frac{1}{2}\right)^{19}-1}{1}$

$=38+\left(\frac{1}{2}\right)^{19}-1=37+\left(\frac{1}{2}\right)^{19}$

$\therefore$ req. sum $=1+37+\left(\frac{1}{2}\right)^{19}=38+\left(\frac{1}{2}\right)^{19}$

Question206

Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series $1^{2}+2\cdot2^{2}+3^{2}+2\cdot4^{2}+5^{2}+2\cdot6^{2}+\ldots$ If $B-2A=100\lambda$, then λ is equal to
[2018]

Options:

A. 248

B. 464

C. 496

D. 232

Answer: A

Solution:

Solution:
Here, $B-2A$
$=\sum_{n=1}^{40}a_{n}-2\sum_{n=1}^{20}a_{n}=\sum_{n=21}^{40}a_{n}-2\sum_{n=1}^{20}a_{n}$
$B-2A=(21^{2}+2.22^{2}+23^{2}+2.24^{2}+\ldots..+40^{2})$
$-(1^{2}+2.2^{2}+3^{2}+2.4^{2}\ldots..+20^{2})$
$=20[22+2.24+26+2.28+\ldots..+60]$
$=20[(22+24+26\ldots60)_{20\text{ terms}}+(24+28+\ldots.+60)_{10\text{ terms}}]$
$20\left[\frac{20}{2}(22+60)+\frac{10}{2}(24+60)\right]$
$=10[20.82+10.84]$
$=100[164+84]=100.248$

Question207

Let $a_1, a_2, a_3, ..., a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + ... + a_{17}^2 = 140m$, then m is equal to
[2018]

Options:

A. 68

B. 34

C. 33

D. 66

Answer: B

Solution:

Solution:
$\because \sum_{k=0}^{12} a_{4k+1} = 416 \Rightarrow \frac{13}{2}[2a_1 + 48d] = 416$
$\Rightarrow a_1 + 24d = 32$
N ow, $a_9 + a_{43} = 66 \Rightarrow 2a_1 + 50d = 66$
From eq. (i) & (ii) we get; $d = 1$ and $a_1 = 8$
Also, $\sum_{r=1}^{17} a_r^2 = \sum_{r=1}^{17} [8 + (r-1)1]^2 = 140m$
$\Rightarrow \sum_{r=1}^{17} (r+7)^2 = 140m$
$\Rightarrow \sum_{r=1}^{17} (r^2 + 14r + 49) = 140m$
$\Rightarrow \left(\frac{17 \times 18 \times 35}{6} \right) + 14 \left(\frac{17 \times 18}{2} \right) + (49 \times 17) = 140$
$\Rightarrow m = 34$

Question208

For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then [2017]

Options:

A. a, b and c are in G.P.

B. b, c and a are in G.P.

C. b, c and a are in A.P.

D. a, b and c are in A.P.

Answer: C

Solution:

Solution:
\We have
$9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$
$\Rightarrow 225a^2 + 9b^2 + 25c^2 - 75ac = 45ab + 15bc$
$\Rightarrow (15a)^2 + (3b)^2 + (5c)^2 - 75ac - 45ab - 15bc = 0$
$\frac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$
It is possible when $15a - 3b = 0$, $3b - 5c = 0$ and $5c - 15a = 0$
$\Rightarrow 15a = 3b \Rightarrow b = 5a$

$\Rightarrow b = \frac{5c}{3}, a = \frac{c}{3}$
$\Rightarrow a + b = \frac{c}{3} + \frac{5c}{3} = \frac{6c}{3}$
$\Rightarrow a + b = 2c$
$\Rightarrow$b, c, a are in A . P.

Question209

If three positive numbers a, b and c are in A.P. such that abc = 8, then the minimum possible value of b is : [Online April 9, 2017]

Options:

A. 2

B. $4^{\frac{1}{3}}$

C. $4^{\frac{2}{3}}$

D. 4

Answer: A

Solution:

By Arithmetic Mean:
$a + c = 2b$
Consider $a = b = c = 2$
$\Rightarrow abc = 8$
$\Rightarrow a + b = 2b$
$\therefore$ minimum possible value of $b = 2$

Question210

If the arithmetic mean of two numbers a and b, a > b > 0, is five times their geometric mean, then $\frac{a+b}{a-b}$ is equal to: [Online April 8, 2017]

Options:

A. $\frac{\sqrt{6}}{2}$

B. $\frac{3\sqrt{2}}{4}$

C. $\frac{7\sqrt{3}}{12}$

D. $\frac{5\sqrt{6}}{12}$

Answer: D

Solution:

Solution:
A.T.Q.
A.M. = 5 G.M.

$\frac{a+b}{2} = 5\sqrt{ab}$

$\frac{a+b}{\sqrt{ab}} = 10$

$\therefore\ \frac{a}{b} = \frac{10+\sqrt{96}}{10-\sqrt{96}} = \frac{10+4\sqrt{6}}{10-4\sqrt{6}}$

Use Componendo and Dividendo

$\frac{a+b}{a-b} = \frac{20}{8\sqrt{6}} = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$

Question211

Let a, b, c $\in$ R . I f f (x) = $ax^2 + bx + c$ is such that a + b + c = 3 and f (x + y) = f (x) + f (y) + xy, $\forall$x, y $\in$ R, then $\sum_{n=1}^{10} f(n)$ is equal to : [2017]

Options:

A. 255

B. 330

C. 165

D. 190

Answer: B

Solution:

Solution:

$f(x) = ax^2 + bx + c$

$f(1) = a + b + c = 3 \Rightarrow f(1) = 3$

Now $f(x+y) = f(x) + f(y) + xy$ ...

Put x = y = 1 in eqn (i)

$f(2) = f(1) + f(1) + 1 = 2f(1) + 1$

$f(2) = 7$

$\Rightarrow\ f(3) = 12$

$S_n = 3 + 7 + 12 + \ldots\ldots. + t_n$

$S_n = \ \ 3 + 7 + 12 + \ldots\ldots. + t_{n-1} + t_n$

$0 = 3 + 4 + 5\ldots\ldots\ldots$ to n term $-t_n$

$t_n = 3 + 4 + 5 + \ldots$ upto n terms

$t_n = \frac{(n^2+5n)}{2}$

$S_n = \sum t_n = \sum \frac{(n^2+5n)}{2}$

$S_n = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{5n(n+1)}{2}\right]$

$= \frac{n(n+1)(n+8)}{6}$

$S_{10} = \frac{10\times11\times18}{6} = 330$

Question212

Let $S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + \ldots. + \frac{1+2+\ldots\ldots+n}{1^3+2^3+\ldots\ldots+n^3}$, If $100S_n = n$, then n is equal to : [Online April 9, 2017]

Options:

A. 199

B. 99

C. 200

D. 19

Answer: A

Solution:

Solution:

$T_n = \dfrac{\frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2}$

$\Rightarrow T_n = \dfrac{2}{n(n+1)}$

$\Rightarrow S_n = \sum T_n = 2\sum_{n=1}^{n}\left(\dfrac{1}{n} - \dfrac{1}{n+1}\right) = 2\left\{1 - \dfrac{1}{n+1}\right\}$

$\Rightarrow S_n = \dfrac{2n}{n+1}$

$\because 100S_n = n$

$\Rightarrow 100 \times \dfrac{2n}{n+1} = n$

$\Rightarrow n + 1 = 200$

$\Rightarrow n = 199$

Question213

If the sum of the first n terms of the series $\sqrt{3} + \sqrt{75} + \sqrt{243} + \sqrt{507} + ...$ is $435\sqrt{3}$, then n equals:
[Online April 8, 2017]

Options:

A. 18

B. 15

C. 13

D. 29

Answer: B

Solution:

Solution:

$\because \sqrt{3}[1 + \sqrt{25} + \sqrt{81} + \sqrt{69} + ...] = 435\sqrt{3}$

$\Rightarrow \sqrt{3}[1 + 5 + 9 + 13 + ... + T_n] = 435\sqrt{3}$

$\Rightarrow \sqrt{3} \times \dfrac{n}{2}[2 + (n-1)_4] = 435\sqrt{3}$

$\Rightarrow 2n + 4n^2 - 4n = 870$

$\Rightarrow 4n^2 - 2n - 870 = 0$

$\Rightarrow 2n^2 - n - 435 = 0$

$n = \dfrac{1 \pm \sqrt{1 + 4 \times 2 \times 435}}{4} = \dfrac{1 \pm 59}{4}$

$\therefore n = \dfrac{1 + 59}{4} = 15$; or $n = \dfrac{1 - 59}{4} = 14.5$

Question214

Let $a_1, a_2, a_3, ..., a_n$, be in A.P. If $a_3 + a_7 + a_{11} + a_{15} = 72$, then the sum of its

first 17 terms is equal to :
[Online April 10, 2016]

Options:

A. 306

B. 204

C. 153

D. 612

Answer: A

Solution:

Solution:
$a_3 + a_7 + a_{11} + a_{15} = 72$
$(a_3 + a_{15}) + (a_7 + a_{11}) = 72$
$a_3 + a_{15} + a_7 + a_{11} = 2(a_1 + a_{17})$
$a_1 + a_{17} = 36$
$S_{17} = \frac{17}{2}[a_1 + a_{17}] = 17 \times 18 = 306$

Question215

If the 2^{nd} , 5^{th} and 9^{th} terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is :
[2016]

Options:

A. 1

B. $\frac{7}{4}$

C. $\frac{8}{5}$

D. $\frac{4}{3}$

Answer: D

Solution:

Solution:
Let the GP be a, ar and ar^2 then $a = A + d$; $ar = A + 4d$;
$ar^2 = A + 8d$
$\Rightarrow \frac{ar^2 - ar}{ar - a} = \frac{(A + 8d) - (A + 4d)}{(A + 4d) - (A + d)}$
$r = \frac{4}{3}$

Question216

Let $z = 1 + ai$ be a complex number, $a > 0$, such that z^3 is areal number. Then the sum $1 + z + z^2 + ... + z^{11}$ is equal to:
(Online April 10, 2016)

Options:

A. $1365\sqrt{3}i$

B. $-1365\sqrt{3}i$

C. $-1250\sqrt{3}i$

D. $1250\sqrt{3}\ i$

Answer: B

Solution:

Solution:
$z = 1 + ai$
$z^2 = 1 - a^2 + 2ai$
$z^2 \cdot z = \{(1 - a^2) + 2ai\}\ \{1 + ai\}$
$= (1 - a^2) + 2ai + (1 - a^2)\ ai - 2a^2$
$\because z^3$ is real $\Rightarrow 2a + (1 - a^2)a = 0$
$a(3 - a^2) = 0 \Rightarrow a = \sqrt{3}(a > 0)$
$1 + z + z^2 \ldots\ldots\ldots . z^{11} = \dfrac{z^{12} - 1}{z - 1} = \dfrac{(1 + \sqrt{3}i)^{12} - 1}{1 + \sqrt{3}i - 1}$
$= \dfrac{(1 + \sqrt{3}i)^{12} - 1}{\sqrt{3}i}$
$(1 + \sqrt{3}i)^{12} = 2^{12}\left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right)^{12}$
$= 2^{12}\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)^{12} = 2^{12}(\cos 4\pi + i\sin 4\pi) = 2^{12}$
$\Rightarrow \dfrac{2^{12} - 1}{\sqrt{3}i} = \dfrac{4095}{\sqrt{3}i} = -\dfrac{4095}{3}\sqrt{3}i = -1365\sqrt{3}i$

Question217

If $A > 0$, $B > 0$ and $A + B = \frac{\pi}{6}$, then the minimum value of $\tan A + \tan B$ is : [Online April 10, 2016]

Options:

A. $\sqrt{3} - \sqrt{2}$

B. $4 - 2\sqrt{3}$

C. $\dfrac{2}{\sqrt{3}}$

D. $2 - \sqrt{3}$

Answer: B

Solution:

Solution:
$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{y}{1 - \tan A \tan B}$ where $y = \tan A + \tan B$
$\Rightarrow \tan A \tan B = 1 - \sqrt{3}y$
Also AM $\geq$ GM
$\Rightarrow \dfrac{\tan A + \tan B}{2} \geq \sqrt{\tan A \tan B}$
$\Rightarrow\ y \geq 2\sqrt{1 - \sqrt{3}y}$
$\Rightarrow\ y^2 \geq 4 - 4\sqrt{3}y$
$\Rightarrow y^2 + 4\sqrt{3}y - 4 \geq 0$
$\Rightarrow\ y \leq -2\sqrt{3} - 4$ or $y \geq -2\sqrt{3} + 4$

($y \le -2\sqrt{3} - 4$ is not possible as $\tan A \tan B > 0$)

Question218

Let x, y, z be positive real numbers such that $x + y + z = 12$ and $x^3y^4z^5 = (0.1)(600)^3$. Then $x^3 + y^3 + z^3$ is equal to :
[Online April 9, 2016]

Options:

A. 342

B. 216

C. 258

D. 270

Answer: B

Solution:

Solution:
$x + y + z = 12$
AM $\ge$ GM
$$\frac{3\left(\frac{x}{3}\right) + 4\left(\frac{y}{4}\right) + 5\left(\frac{z}{5}\right)}{12} \ge 12\sqrt{\left(\frac{x}{3}\right)^3\left(\frac{y}{4}\right)^4\left(\frac{z}{5}\right)^5}$$
$$\frac{x^3y^4z^5}{3^34^45^5} \le 1$$
$x^3y^4z^5 \le 3^3 \cdot 4^4 \cdot 5^5$
$x^3y^4z^5 \le (0.1)(600)^3$
But, given $x^3y^4z^5 = (0.1)(600)^3$
$\therefore$ all the number are equal
$\therefore \frac{x}{3} = \frac{y}{4} = \frac{z}{5}(= k)$
$x = 3k; y = 4k; z = 5k$
$x + y + z = 12$
$3k + 4k + 5k = 12$
$k = 1 \therefore x = 3; y = 4; z = 5$
$\therefore x^3 + y^3 + z^3 = 216$

Question219

If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots\ldots$, is $\frac{16}{5}m$ then m is equal to :
[2016]

Options:

A. 100

B. 99

C. 102

D. 101

Answer: D

Solution:

$\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 \dots + \left(\frac{44}{5}\right)^2$

$S = \frac{16}{25}(2^2 + 3^2 + 4^2 + \dots + 11^2)$

$= \frac{16}{25}\left(\frac{11(11+1)(22+1)}{6} - 1\right) = \frac{16}{25} \times 505 = \frac{16}{5} \times 101$

$\Rightarrow \frac{16}{5}m = \frac{16}{5} \times 101$

$\Rightarrow m = 101$

Question220

For $x \in R$, $x \neq -1$, if $(1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + \dots + x^{2016} = \sum_{i=0}^{2016} a_i x^i$, then a_{17} is equal to :

[Online April 9, 2016]

Options:

A. $\frac{2017!}{17!2000!}$

B. $\frac{2016!}{17!1999!}$

C. $\frac{2016!}{16!}$

D. $\frac{2017!}{2000!}$

Answer: A

Solution:

Solution:

$S = (1+x)^{2016} + x(1+x)^{2015} + x^2(1+x)^{2014} + \dots + x^{2015}(1+x) + x^{2016} \dots$ (i)

$\left(\frac{x}{1+x}\right)S = x(1+x)^{2015} + x^2(1+x)^{2014} + \dots + x^{2016} + \frac{x^{2017}}{1+x} \dots$ (ii)

Subtracting (i) from (ii)

$\frac{S}{1+x} = (1+x)^{2016} - \frac{x^{2017}}{1+x}$

$\therefore S = (1+x)^{2017} - x^{2017}$

$a_{17} =$ coefficient of $x^{17} = {}^{2017}C_{17} = \frac{2017!}{17!2000!}$

Question221

If m is the A.M. of two distinct real numbers l and $n (l, n > 1)$ and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals. [2015]

Options:

A. $4l mn^2$

B. $4l^2m^2n^2$

C. $4l^2mn$

D. $4 l m^2 n$

Answer: D

Solution:

Solution:
$m = \frac{l + n}{2}$ and common ratio of G.P.

$= r = \left(\frac{n}{l}\right)^{\frac{1}{4}}$

$\therefore\ G_1 = l^{3/4} n^{1/4},\ G_2 = l^{1/2} n^{1/2},\ G_3 = l^{1/4} n^{3/4}$

$G_1^{\ 4} + 2G_2^{\ 4} + G_3^{\ 4} = l^3 n + 2l^2 n^2 + l n^3$

$= ln(l + n)^2 = ln \times (2m)^2 = 4l m^2 n$

Question222

The sum of the 3rd and the 4th terms of a G.P. is 60 and the product of its first three terms is 1000 . If the first term of this G.P. is positive, then its 7th term is :
[Online April 11, 2015]

Options:

A. 7290

B. 640

C. 2430

D. 320

Answer: D

Solution:

Solution:
Let a, ar and ar ^{2} be the first three terms of G.P According to the question

$a(ar)(ar^2) = 1000 \Rightarrow (ar)^3 = 1000 \Rightarrow ar = 10$

and $ar^2 + ar^3 = 60 \Rightarrow ar(r + r^2) = 60$

$\Rightarrow r^2 + r - 6 = 0$

$\Rightarrow r = 2, -3$

$a = 5, a = -\frac{10}{3}$ (reject)

Hence, $T_7 = ar^6 = 5(2)^6 = 5 \times 64 = 320$

Question223

The sum of first 9 terms of the series.
$\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 3} + \frac{1^3 + 2^3 + 3^3}{1 + 3 + 5} + \ldots$
[2015]

Options:

A. 142

B. 192

C. 71

D. 96

Answer: D

Solution:

Solution:

n^{th} term of series $= \dfrac{\left[\dfrac{n(n+1)}{2}\right]^2}{n^2} = \dfrac{1}{4}(n+1)^2$

Sum of n term $= \Sigma \dfrac{1}{4}(n+1)^2 = \dfrac{1}{4}[\Sigma n^2 + 2\Sigma n + n]$

$= \dfrac{1}{4}\left[\dfrac{n(n+1)(2n+1)}{6} + \dfrac{2n(n+1)}{2} + n\right]$

Sum of 9 terms

$= \dfrac{1}{4}\left[\dfrac{9 \times 10 \times 19}{6} + \dfrac{18 \times 10}{2} + 9\right] = \dfrac{384}{4} = 96$

Question224

If $\sum\limits_{n=1}^{5} \dfrac{1}{n(n+1)(n+2)(n+3)} = \dfrac{k}{3}$, then k is equal to [Online April 11, 2015]

Options:

A. $\dfrac{1}{6}$

B. $\dfrac{17}{105}$

C. $\dfrac{55}{336}$

D. $\dfrac{19}{112}$

Answer: C

Solution:

Solution:

General term of given expression can be written as

$T_r = \dfrac{1}{3}\left[\dfrac{1}{n(n+1)(n+2)} - \dfrac{1}{(n+1)(n+2)(n+3)}\right]$

on taking summation both the side, we get

$\sum\limits_{r=1}^{5} T_r = \dfrac{1}{3}\left[\dfrac{1}{6} - \dfrac{1}{6.7.8}\right] = \dfrac{k}{3}$

$\Rightarrow \dfrac{1}{3} \times \dfrac{1}{6}\left(1 - \dfrac{1}{56}\right) = \dfrac{k}{3} \Rightarrow \dfrac{1}{3} \times \dfrac{1}{6} \times \dfrac{55}{56} = \dfrac{k}{3}$

$\Rightarrow k = \dfrac{55}{336}$

Question225

The value of $\sum\limits_{r=16}^{30} (r+2)(r-3)$ is equal to : [Online April 10, 2015]

Options:

A. 7770

B. 7785

C. 7775

D. 7780

Answer: D

Solution:

Solution:

$$\sum_{r=1}^{20} (r^2 - r - 6) = 7780$$

Question226

Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$ If p, q, r are in A . P and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is: [2014]

Options:

A. $\frac{\sqrt{34}}{9}$

B. $\frac{2\sqrt{13}}{9}$

C. $\frac{\sqrt{61}}{9}$

D. $\frac{2\sqrt{17}}{9}$

Answer: B

Solution:

Solution:
Let p, q, r are in AP
$\Rightarrow 2q = p + r \ldots$ (i)
Given $\frac{1}{\alpha} + \frac{1}{\beta} = 4$
$\Rightarrow \frac{\alpha + \beta}{\alpha\beta} = 4$
We have $\alpha + \beta = -q / p$ and $\alpha\beta = \frac{r}{p}$
$-xq$
$\Rightarrow \frac{\frac{p}{r}}{p} = 4 \Rightarrow q = -4r \ldots$ (ii)
From (i), we have
$2(-4r) = p + r$
$p = -9r \ldots$(iii)
Now, $|\alpha - \beta| = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$
$= \sqrt{\left(\frac{-q}{p}\right)^2 - \frac{4r}{p}} = \frac{\sqrt{q^2 - 4pr}}{|p|}$
From (ii) and (iii)
$= \frac{\sqrt{16r^2 + 36r^2}}{|-9r|} = \frac{2\sqrt{13}}{9}$

Question227

The sum of the first 20 terms common between the series 3 +7 + 11 + 15 + ... and 1 + 6 + 11 + 16 + ..., is

[Online April 11, 2014]

Options:

A. 4000

B. 4020

C. 4200

D. 4220

Answer: B

Solution:

Solution:
Given $n = 20$; $S_{20} = ?$
Series (1) → 3, 7, x11, 15, 19, 23, 27, x31, 35, 39, 43, 47 51, 55, 59…
Series (2) → 1, 6, x11, 16, 21, 26, x31, 36, 41, 46, x51, 56 61,66,71
The common terms between both the series are 11, 31, 51, 71…
Above series forms an Arithmetic progression (A.P).
Therefore, first term (a) = 11 and
common difference (d) = 20
Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_{20} = \frac{20}{2}[2 \times 11 + (20 - 1)20]$
$S_{20} = 10[22 + 19 \times 20]$
$S_{20} = 10 \times 402 = 4020$
$\therefore S_{20} = 4020$

Question228

Given an A.P. whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its 4$^{\text{th}}$ term is:
[Online April 9, 2014]

Options:

A. 8

B. 16

C. 20

D. 24

Answer: C

Solution:

Solution:
Let a be the first term and d be the common difference of given A.P.
Second term, $a + d = 12$ . . . (i)
Sum of first nine terms,
$S_9 = \frac{9}{2}(2a + 8d) = 9(a + 4d)$
Given that S_9 is more than 200 and less than 220
$\Rightarrow 200 < S_9 < 220$
$\Rightarrow 200 < 9(a + 4d) < 220$
$\Rightarrow 200 < 9(a + d + 3d) < 220$
Putting value of $(a + d)$ from equation (i)
$200 < 9(12 + 3d) < 220$
$\Rightarrow 200 < 108 + 27d < 220$
$\Rightarrow 200 - 108 < 108 + 27d - 108 < 220 - 108$

$\Rightarrow 92 < 27d < 112$
Possible value of d is 4
$27 \times 4 = 108$
Thus, $92 < 108 < 112$
Putting value of d in equation (i)
$a + d = 12$
$a = 12 - 4 = 8$
4^{th} term $= a + 3d = 8 + 3 \times 4 = 20$

Question229

Three positive numbers form an increasing G. P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ratio of the G.P. is:
[2014]

Options:

A. $2 - \sqrt{3}$

B. $2 + \sqrt{3}$

C. $\sqrt{2} + \sqrt{3}$

D. $3 + \sqrt{2}$

Answer: B

Solution:

Solution:
Let a, ar, ar^2 are in G.P.
According to the question
$a, 2ar, ar^2$ are in A.P.
$\Rightarrow 2 \times 2ar = a + ar^2$
$\Rightarrow 4r = 1 + r^2 \Rightarrow r^2 - 4r + 1 = 0$
$r = \dfrac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$
Since $r > 1$
$\therefore$ $r = 2 - \sqrt{3}$ is rejected
Hence, $r = 2 + \sqrt{3}$

Question230

The least positive integer n such that $1 - \frac{2}{3} - \frac{2}{3^2} - .... - \frac{2}{3^{n-1}} < \frac{1}{100}$, is:
[Online April 12, 2014]

Options:

A. 4

B. 5

C. 6

D. 7

Answer: B

Solution:

$1-\frac{2}{3}-\frac{2}{3^2} \cdots \frac{2}{3^{n-1}}<\frac{1}{100}$

$\Rightarrow 1-\frac{2}{3}\left[\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots \frac{1}{3^{n-1}}\right]<\frac{1}{100}$

$\Rightarrow \frac{1-2\left[\frac{1}{3}\left(\frac{1}{3^n}-1\right)\right]}{\frac{1}{3}-1}<\frac{1}{100}$

$\Rightarrow 1-2\left[\frac{3^n-1}{2.3^n}\right]<\frac{1}{100}$

$\Rightarrow 1-\left[\frac{3^n-1}{3^n}\right]<\frac{1}{100}$

$\Rightarrow 1-1+\frac{1}{3^n}<\frac{1}{100} \Rightarrow 100<3^n$

Thus, least value of n is 5

Question231

In a geometric progression, if the ratio of the sum of first 5 terms to the sum of their reciprocals is 49, and the sum of the first and the third term is 35 . Then the first term of this geometric progression is:
[Online April 11, 2014]

Options:

A. 7

B. 21

C. 28

D. 42

Answer: C

Solution:

Solution:

According to Question

$\Rightarrow \frac{S_5}{S_5} = 49$ \{here, $S_5 =$ Sum of first 5 terms and $S_5 =$ Sum of their reciprocals)

$\Rightarrow \frac{\frac{a(r^5-1)}{(r-1)}}{\frac{a^{-1}(r^{-5}-1)}{(r^{-1}-1)}} = 49$

$\Rightarrow \frac{a(r^5-1)\times(r^{-1}-1)}{a^{-1}(r^{-5}-1)\times(r-1)} = 49$

or $\frac{a^2(1-r^5)\times(1-r)\times r^5}{(1-r^5)\times(1-r)\times r} = 49$

$\Rightarrow a^2r^4 = 49 \Rightarrow a^2r^4 = 7^2$

$\Rightarrow ar^2 = 7 \ldots$ (i)

Also, given, $S_1 + S_3 = 35$

$a + ar^2 = 35 \ldots$ (ii)

Now substituting the value of eq. (i) in eq. (ii)

$a + 7 = 35$

$a = 28$

Question232

The coefficient of x^{50} in the binomial expansion of $(1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \ldots + x^{1000}$ is:

[Online April 11, 2014]

Options:

A. $\frac{(1000)!}{(50)!(950)!}$

B. $\frac{(1000)!}{(49)!(951)!}$

C. $\frac{(1001)!}{(51)!(950)!}$

D. $\frac{(1001)!}{(50)!(951)!}$

Answer: D

Solution:

Solution:
Let given expansion be
$S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \ldots + \ldots + x^{1000}$
Put $1 + x = t$
$S = t^{1000} + xt^{999} + x^2(t)^{998} + \ldots + x^{1000}$
This is a G.P with common ratio $\frac{x}{t}$

$$S = \frac{t^{1000}\left[1 - \left(\frac{x}{t}\right)^{1001}\right]}{1 - \frac{x}{t}}$$

$$= \frac{(1+x)^{1000}\left[1 - \left(\frac{x}{1+x}\right)^{1001}\right]}{1 - \frac{x}{1+x}}$$

$$= \frac{(1+x)^{1001}[(1+x)^{1001} - x^{1001}]}{(1+x)^{1001}}$$

$= [(1+x)^{1001} - x^{1001}]$
Now coeff of x^{50} in above expansion is equal to coeff of x^{50} in $(1+x)^{1001}$ which is ${}^{1001}C_{50}$

$$= \frac{(1001)!}{50!(951)!}$$

Question233

Let G be the geometric mean of two positive numbers a and b, and M be the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$. If $\frac{1}{M}$: G is 4 : 5, then a : b can be:
[Online April 12, 2014]

Options:

A. 1: 4

B. 1: 2

C. 2: 3

D. 3: 4

Answer: A

Solution:

Solution:
$G = \sqrt{ab}$

$$M = \frac{\frac{1}{a} + \frac{1}{b}}{2}$$

$M = \frac{a+b}{2ab}$

Given that $\frac{1}{M} : G = 4 : 5$

$\frac{2ab}{(a+b)\sqrt{ab}} = \frac{4}{5}$

$\Rightarrow \frac{a+b}{2\sqrt{ab}} = \frac{5}{4}$

$\Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{5+4}{5-4}$

{Using Componendo & Dividendo}

$\Rightarrow \frac{(\sqrt{a})^2+(\sqrt{b})^2+2\sqrt{ab}}{(\sqrt{a})^2+(\sqrt{b})^2-2\sqrt{ab}} = \frac{9}{1}$

$\Rightarrow \left(\frac{\sqrt{b}+\sqrt{a}}{\sqrt{b}-\sqrt{a}}\right)^2 = \frac{9}{1} \Rightarrow \frac{\sqrt{b}+\sqrt{a}}{\sqrt{b}-\sqrt{a}} = \frac{3}{1}$

$\Rightarrow \frac{\sqrt{b}+\sqrt{a}+\sqrt{b}-\sqrt{a}}{\sqrt{b}+\sqrt{a}-\sqrt{b}+\sqrt{a}} = \frac{3+1}{3-1}$

{Using Componendo & Dividendo}

$\sqrt{\frac{b}{a}} = \frac{4}{2} = 2$

$\frac{b}{a} = \frac{4}{1}$

$\frac{a}{b} = \frac{1}{4} \Rightarrow a : b = 1 : 4$

Question234

If $(10)^9 + 2(11)^1(10^8) + 3(11)^2(10)^7 + ... +10(11)^9 = k(10)^9$, then k is equal to: [2014]

Options:

A. 100

B. 110

C. $\frac{121}{10}$

D. $\frac{441}{100}$

Answer: A

Solution:

Solution:

Given that $10^9 + 2\cdot(11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9 = k(10)^9$

Let $x = 10^9 + 2\cdot(11)(10)^8 + 3(11)^2(10)^7 + \ldots + 10(11)^9$ . . . (i)

Multiplied by $\frac{11}{10}$ on both the sides

$\frac{11}{10}x = 11.10^8 + 2\,.\,(11)^2\cdot(10)^7 + \ldots + 9(11)^9 + 11^{10}$ . . . (ii)

Subtract (ii) from (i), we get

$x\left(1-\frac{11}{10}\right) = 10^9 + 11(10)^8 + 11^2\times(10)^7 + \ldots + 11^9 - 11^{10}$

$\Rightarrow\ -\frac{x}{10} = 10^9\left[\frac{\left(\frac{11}{10}\right)^{10}-1}{\frac{11}{10}-1}\right] - 11^{10}$

$\Rightarrow\ -\frac{x}{10} = (11^{10}-10^{10}) - 11^{10} = -10^{10}$

$\Rightarrow x = 10^{11} = k\cdot 10^9$ Given

$\Rightarrow k = 100$

Question235

The number of terms in an A.P. is even; the sum of the odd terms in it is 24 and that the even terms is 30 . If the last term exceeds the first term by 10 $\frac{1}{2}$, then the number of terms in the A.P. is:
[Online April 19, 2014

Options:

A. 4

B. 8

C. 12

D. 16

Answer: B

Solution:

Solution:
Let a, d and $2n$ be the first term, common difference and total number of terms of an A.P. respectively i.e. a $+(a+d)+(a+2d)+\ldots+(a+(2n-1)d)$
No. of even terms $=n$, No. of odd terms $=n$
Sum of odd terms :
$S_o = \frac{n}{2}[2a+(n-1)(2d)] = 24$
$\Rightarrow n[a+(n-1)d] = 24 \ldots$ (i)
Sum of even terms:
$S_e = \frac{n}{2}[2(a+d)+(n-1)2d] = 30$
$\Rightarrow n[a+d+(n-1)d] = 30 \ldots$ (ii)
Subtracting equation (i) from (ii), we get
$nd = 6 \ldots$ (iii)
Also, given that last term exceeds the first term by $\frac{21}{2}$
$a+(2n-1)d = a+\frac{21}{2}$
$2nd - d = \frac{21}{2}$
$\Rightarrow 2\times 6 - \frac{21}{2} = d \quad (\because nd = 6)$
$d = \frac{3}{2}$
Putting value of d in equation (3) $n = \frac{6\times 2}{3} = 4$
Total no. of terms $= 2n = 2\times 4 = 8$

Question236

If the sum $\frac{3}{1^2}+\frac{5}{1^2+2^2}+\frac{7}{1^2+2^2+3^2}+\ldots\ldots+$ up to 20 terms is equal to $\frac{k}{21}$, then k is equal to:
[Online April 9, 2014]

Options:

A. 120

B. 180

C. 240

D. 60

Answer: A

Solution:

Solution:
n^{th} term of given series is
$$\frac{2n+1}{\frac{n(n+1)(2n+1)}{6}} = \frac{6}{n(n+1)}$$
Let n^{th} term, $a_n = 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$
Sum of 20 terms, $S_{20} = a_1 + a_2 + a_3 + \ldots. + a_{20}$
$$S_{20} = 6\left(\frac{1}{1} - \frac{1}{2}\right) + 6\left(\frac{1}{2} - \frac{1}{3}\right) + 6\left(\frac{1}{3} - \frac{1}{4}\right) + \ldots$$
$$+6\left(\frac{1}{18} - \frac{1}{19}\right) + 6\left(\frac{1}{19} - \frac{1}{20}\right) + 6\left(\frac{1}{20} - \frac{1}{21}\right)$$
$$S_{20} = \left[\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots\right.$$
$$\left.+\left(\frac{1}{18} - \frac{1}{19}\right) + \left(\frac{1}{19} - \frac{1}{20}\right) + \left(\frac{1}{20} - \frac{1}{21}\right)\right]$$
$$S_{20} = 6\left(1 - \frac{1}{21}\right) = \frac{120}{21} \ldots \text{(i)}$$
Given that $S_{20} = \frac{k}{21} \ldots$ (ii)
On comparing (i) and (ii), we get
$k = 120$

Question237

Let $a_1, a_2, a_3, \ldots$ be an A.P, such that $\frac{a_1 + a_2 + \ldots + a_p}{a_1 + a_2 + a_3 + \ldots + a_q} = \frac{p^3}{q^3}$; $p \neq q$. Then $\frac{a_6}{a_{21}}$ is equal to:
[Online April 9, 2013]

Options:

A. $\frac{41}{11}$

B. $\frac{31}{121}$

C. $\frac{11}{41}$

D. $\frac{121}{1861}$

Answer: B

Solution:

Solution:
$$\frac{a_1 + a_2 + a_3 + \ldots\ldots + a_p}{a_1 + a_2 + a_3 + \ldots\ldots + a_q} = \frac{p^3}{q^3}$$
$$\Rightarrow \frac{a_1 + a_2}{a_1} = \frac{8}{1} \Rightarrow a_1 + (a_1 + d) = 8a_1$$
$\Rightarrow d = 6a_1$
Now $\frac{a_6}{a_{21}} = \frac{a_1 + 5d}{a_1 + 20d}$
$$= \frac{a_1 + 5 \times 6a_1}{a_1 + 20 \times 6a_1} = \frac{1 + 30}{1 + 120} = \frac{31}{121}$$

Question238

The sum of the series: $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots\ldots$ upto 10 terms, is
[Online April 9, 2013]

Options:

A. $\frac{18}{11}$

B. $\frac{22}{13}$

C. $\frac{20}{11}$

D. $\frac{16}{9}$

Answer: C

Solution:

Solution:

$T_r = \frac{1}{1+2+3+\ldots+r} = \frac{2}{r(r+1)}$ $S_{10} = 2\sum_{r=1}^{10}\frac{1}{r(r+1)} = 2\sum_{r=1}^{10}\left[\frac{r+1}{r(r+1)} - \frac{r}{r(r+1)}\right]$

$= 2\sum_{r=1}^{10}\left(\frac{1}{r} - \frac{1}{r+1}\right)$

$= 2\left[\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots + \left(\frac{1}{10} - \frac{1}{11}\right)\right]$

$= 2\left[1 - \frac{1}{11}\right] = 2 \times \frac{10}{11} = \frac{20}{11}$

Question239

If $a_1, a_2, a_3, \ldots, a_n, \ldots.$ are in A.P. such that $a_4 - a_7 + a_{10} = m$, then the sum of first 13 terms of this A.P., is :
[Online April 23, 2013]

Options:

A. 10m

B. 12m

C. 13m

D. 15m

Answer: C

Solution:

Solution:
If d be the common difference, then
$m = a_4 - a_7 + a_{10} = a_4 - a_7 + a_7 + 3d = a_7$
$S_{13} = \frac{13}{2}[a_1 + a_{13}] = \frac{13}{2}[a_1 + a_7 + 6d]$
$= \frac{13}{2}[2a_7] = 13a_7 = 13m$

Question240

Given sum of the first n terms of an A.P. is $2n + 3n^2$. Another A.P. is formed with the same first term and double of the common difference, the sum of n terms of the new A.P. is:
[Online April 22, 2013]

Options:

A. $n + 4n^2$

B. $6n^2 - n$

C. $n^2 + 4n$

D. $3n + 2n^2$

Answer: B

Solution:

Solution:
Given $S_n = 2n + 3n^2$
Now, first term $= 2 + 3 = 5$
second term $= 2(2) + 3(4) = 16$
third term $= 2(3) + 3(9) = 33$
Now, sum given in option (b) only has the same first term and difference between $2nd$ and 1 st term is double also.

Question241

Given a sequence of 4 numbers, first three of which are in G.P. and the last three are in A.P. with common difference six. If first and last terms of this sequence are equal, then the last term is : [Online April 25, 2013]

Options:

A. 16

B. 8

C. 4

D. 2

Answer: B

Solution:

Solution:
Let a, b, c, d be four numbers of the sequence.
Now, according to the question $b^2 = ac$ and $c - b = 6$ and $a - c = 6$
Also, given $a = d$
$\therefore b^2 = ac \Rightarrow b^2 = a\left[\frac{a+b}{2}\right]$ $(\because 2c = a + b)$
$\Rightarrow a^2 - 2b^2 + ab = 0$
Now, $c - b = 6$ and $a - c = 6$
gives $a - b = 12$
$\Rightarrow b = a - 12$
$\therefore a^2 - 2b^2 + ab = 0$
$\Rightarrow a^2 - 2(a - 12)^2 + a(a - 12) = 0$
$\Rightarrow a^2 - 2a^2 - 288 + 48a + a^2 - 12a = 0$
$\Rightarrow 36a = 288 \Rightarrow a = 8$
Hence, last term is $d = a = 8$.

Question242

The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ... is [2013]

Options:

A. $\frac{7}{81}(179 - 10^{-20})$

B. $\frac{7}{9}(99 - 10^{-20})$

C. $\frac{7}{81}(179 + 10^{-20})$

D. $\frac{7}{9}(99 + 10^{-20})$

Answer: C

Solution:

Solution:

Let S = $\frac{7}{10} + \frac{77}{100} + \frac{777}{10^3} + \ldots\ldots +$ up to 20 terms

$= 7\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{10^3} + \ldots\ldots + \text{ up to 20 terms }\right]$

Multiply and divide by 9

$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \ldots\ldots \text{ up to 20 terms }\right]$

$$= \frac{7}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \ldots\ldots \text{ up to 20 terms}\right]$$

$$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^{20}\right)}{1 - \frac{1}{10}}\right]$$

$$= \frac{7}{9}\left[\frac{179}{9} + \frac{1}{9}\left(\frac{1}{10}\right)^{20}\right] = \frac{7}{81}[179 + (10)^{-20}]$$

Question243

The value of $1^2 + 3^2 + 5^2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + 25^2$ is [Online April 25, 2013]

Options:

A. 2925

B. 1469

C. 1728

D. 1456

Answer: A

Solution:

Solution:

Consider $1^2 + 3^2 + 5^2 + \ldots\ldots + 25^2$

n^{th} term $T_n = (2n-1)^2$, $n = 1, \ldots\ldots 13$

Now, $S_n = \sum_{n=1}^{13} T_n = \sum_{n=1}^{13}(2n-1)^2$

$= \sum_{n=1}^{13} 4n^2 + \sum_{n=1}^{13} 1 - \sum_{n=1}^{13} 4n = 4\sum n^2 + 13 - 4\sum n$

$= 4\left[\frac{n(n+1)(2n+1)}{6}\right] + 13 - 4\frac{n(n+1)}{2}$

Put $n = 13$, we get

$S_n = 26 \times 14 \times 9 + 13 - 26 \times 14$

$= 3276 + 13 - 364 = 2925$

Question244

The sum of the series :
$(b)^2 + 2(d\,)^2 + 3(6)^2 + ...$ upto 10 terms is :
[Online April 23, 2013]

Options:

A. 11300

B. 11200

C. 12100

D. 12300

Answer: C

Solution:

Solution:
$2^2 + 2(4)^2 + 3(6)^2 + \ldots\ldots$ upto 10 terms
$= 2^2[\,1^3 + 2^3 + 3^3 + \ldots\ldots$ upto 10 terms $]$
$= 4 \cdot \left(\frac{10 \times 11}{2}\right)^2 = 12100$

Question245

The sum $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots.$ upto 11 -terms is:
[Online April 22, 2013]

Options:

A. $\frac{7}{2}$

B. $\frac{11}{4}$

C. $\frac{11}{2}$

D. $\frac{60}{11}$

Answer: C

Solution:

Solution:
Given sum is
$\frac{3}{12} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \ldots.$
n th term $= T_n$
$= \frac{\frac{2n+1}{n(n+1)(2n+1)}}{6} = \frac{6}{n(n+1)}$
or $T_n = 6\left[\frac{1}{n} - \frac{1}{n+1}\right]$
$\therefore\ S_n = \sum T_n = 6\sum\frac{1}{n} - 6\sum\frac{1}{n+1} = \frac{6n}{n} - \frac{6}{n+1}$
$= 6 - \frac{6}{n+1} = \frac{6n}{n+1}$
So, sum upto 11 terms means
$S_{11} = \frac{6 \times 11}{11+1} = \frac{66}{12} = \frac{33}{6} = \frac{11}{2}$

Question246

The sum of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ... + 2(2m)^2$ is [Online May 7, 2012]

Options:

A. $m(2m+1)^2$

B. $m^2(m+2)$

C. $m^2(2m+1)$

D. $m(m+2)^2$

Answer: A

Solution:

Solution:
The sum of the given series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2$
$+2.6^2 + \ldots\ldots + 2(2m)^2$ is $\frac{2m(2m+1)^2}{2} = m(2m+1)^2$

Question247

The difference between the fourth term and the first term of a Geometrical Progresssion is 52 . If the sum of its first three terms is 26, then the sum of the first six terms of the progression is [Online May 7, 2012]

Options:

A. 63

B. 189

C. 728

D. 364

Answer: C

Solution:

Solution:
Let a, ar, ar^2, ar^3, ar^4, ar^5 be six terms of a G.P. where ' a ' is first term and r is common ratio.
According to given conditions, we have
$ar^3 - a = 5 \Rightarrow a(r^3 - 1) = 52$ . . . (i)
and $a + ar + ar^2 = 26$
$\Rightarrow a(1 + r + r^2) = 26$ . . . (ii)
To find: $a(1 + r + r^2 + r^3 + r^4 + r^5)$
Consider
$a[1 + r + r^2 + r^3 + r^4 + r^5]$
$= a[1 + r + r^2 + r^3(1 + r + r^2)]$
$= a[1 + r + r^2][1 + r^3]$ . . . (iii)
Divide (i) by (ii), we get
$\frac{r^3 - 1}{1 + r + r^2} = 2$
we know $r^3 - 1 = (r-1)(1 + r + r^2)$
$\therefore r - 1 = 2 \Rightarrow r = 3$ and $a = 2$
$\therefore a(1 + r + r^2 + r^3 + r^4 + r^5) = a(1 + r + r^2)(1 + r^3)$

$= 2(1 + 3 + 9)(1 + 27) = 26 \times 28 = 728$

Question248

If a, b, c, d and p are distinct real numbers such that $(a^2 + b^2 + c^2)p^2 - 2p(ab + bc + cd) + (b^2 + c^2 + d^2) \leq 0$ then [Online May 12, 2012]

Options:

A. a, b, c, d are in A.P.

B. $ab = cd$

C. $ac = bd$

D. a, b, c, d are in G.P.

Answer: D

Solution:

Solution:
The given relation can be written as
$(a^2p^2 - 2abp + b^2) + (b^2p^2 + c^2 - 2bpc) + (c^2p^2 + d^2 - 2pcd) \leq 0$
or $(ap - b)^2 + (bp - c)^2 + (cp - d)^2 \leq 0 \ldots$ (i)
Since a, b, c, d and p are all real, the inequality (i) is possible only when each of factor is zero.
i.e., $ap - b = 0$, $bp - c = 0$ and $cp - d = 0$
or $p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$
or a, b, c, d are in G.P.

Question249

The sum of the series $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + .$ upto 15 terms is [Online May 12, 2012]

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

Solution:
Given series is $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots.$
n^{th} term $= \frac{1}{\sqrt{n}+\sqrt{n+1}}$
$\therefore 15^{th}$ term $= \frac{1}{\sqrt{15}+\sqrt{16}}$
Thus, given series upto 15 terms is
$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots + \frac{1}{\sqrt{15}+\sqrt{16}}$
This can be re-written as

$\frac{1-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} + \frac{\sqrt{3}-\sqrt{4}}{-1} + \ldots.. + \frac{\sqrt{15}-\sqrt{16}}{-1}$
(Byrationalization)
$= -1 + \sqrt{2} - \sqrt{2} + \sqrt{3} - \sqrt{3} + \sqrt{4} + \ldots.. - \sqrt{14} + \sqrt{15}$
$= -1 + \sqrt{16} = -1 + 4 = 3$
Hence, the required sum $= 3$

Question250

Suppose θ and $\varphi(\neq 0)$ are such that $\sec(\theta+\varphi)$, $\sec\theta$ and $\sec(\theta-\varphi)$ are in A.P. If $\cos\theta = k\cos\left(\frac{\varphi}{2}\right)$ for some k, then k is equal to
[Online May 19, 2012]

Options:

A. $\pm\sqrt{2}$

B. ± 1

C. $\pm\frac{1}{\sqrt{2}}$

D. ± 2

Answer: A

Solution:

Solution:
Since, $\sec(\theta-\varphi)$, $\sec\theta$ and $\sec(\theta+\varphi)$ are in A.P., $\therefore 2\sec\theta = \sec(\theta-\varphi) + \sec(\theta+\varphi)$
$\Rightarrow \frac{2}{\cos\theta} = \frac{\cos(\theta+\varphi)+\cos(\theta-\varphi)}{\cos(\theta-\varphi)\cos(\theta+\varphi)}$
$\Rightarrow 2(\cos^2\theta - \sin^2\varphi) = \cos\theta[2\cos\theta\cos\varphi]$
$\Rightarrow \cos^2\theta(1-\cos\varphi) = \sin^2\varphi = 1-\cos^2\varphi$
$\Rightarrow \cos^2\theta = 1+\cos\varphi = 2\cos^2\frac{\varphi}{2}$
$\therefore \cos\theta = \sqrt{2}\cos\frac{\varphi}{2}$
But given $\cos\theta = k\cos\frac{\varphi}{2}$
$\therefore k = \sqrt{2}$

Question251

The sum of the series $1 + \frac{4}{3} + \frac{10}{9} + \frac{28}{27} + \ldots$ upto n terms is
[Online May 19, 2012]

Options:

A. $\frac{7}{6}n + \frac{1}{6} - \frac{2}{3.2^{n-1}}$

B. $\frac{5}{3}n - \frac{7}{6} + \frac{1}{2.3^{n-1}}$

C. $n + \frac{1}{2} - \frac{1}{2.3^{n}}$

D. $n - \frac{1}{3} - \frac{1}{3.2^{n-1}}$

Answer: C

Solution:

Given series is $1+\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+\ldots.\text{n terms}$

$=1+\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{9}\right)+\left(1+\frac{1}{27}\right)+\ldots.\text{n terms}$

$=(1+1+1+\ldots.+\text{n terms})$

$+\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots.\text{n terms}\right)$

$=n+\dfrac{\frac{1}{3}\left(1-\frac{1}{3^n}\right)}{1-\frac{1}{3}}=n+\frac{1}{3}\times\frac{3}{2}[1-3^{-n}]$

$=n+\frac{1}{2}[1-3^{-n}]=n+\frac{1}{2}-\frac{1}{2.3^n}$

Question252

If the A.M. between p^{th} and q^{th} terms of an A.P. is equal to the A.M. between r^{th} and s^{th} terms of the same A.P. then $p+q$ is equal to [Online May 26, 2012]

Options:

A. $r+s-1$

B. $r+s-2$

C. $r+s+1$

D. $r+s$

Answer: D

Solution:

Solution:

Given : $\frac{a_p+a_q}{2}=\frac{a_r+a_S}{2}$

$\Rightarrow a+(p-1)d+a+(q-1)d$

$=a+(r-1)d+a+(s-1)d$

$\Rightarrow 2a+(p+q)d-2d=2a+(r+s)d-2d$

$\Rightarrow(p+q)d=(r+s)d\Rightarrow p+q=r+s$

Question253

If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is: [2012]

Options:

A. -150

B. 150 times its 50^{th} term

C. 150

D. Zero

Answer: D

Solution:

Solution:
Let ' a ' is the first term and 'd ' is the common difference of an A . P.
Now, According to the question $100a_{100} = 50a_{50}$
$100(a + 99d) = 50(a + 49d)$
$\Rightarrow 2a + 198d = a + 49d \Rightarrow a + 149d = 0$
Hence, $T_{150} = a + 149d = 0$

Question254

Statement-1: The sum of the series 1 + (1 + 2 + 4)+ (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000
Statement-2: $\sum_{k=1}^{n}(k^3 - (k-1)^3) = n^3$, for any natural number n.
[2012]

Options:

A. Statement- 1 is false, Statement- 2 is true.

B. Statement- 1 is true, statement- 2 is true; statement- 2 is a correct explanation for Statement- 1 .

C. Statement- 1 is true, statement- 2 is true; statement- 2 is not a correct explanation for Statement- 1 .

D. Statement- 1 is true, statement- 2 is false.

Answer: B

Solution:

Solution:
n th term of the given series
$= T_n = (n-1)^2 + (n-1)n + n^2$
$= \frac{((n-1)^3 - n^3)}{(n-1) - n} = n^3 - (n-1)^3$
$\Rightarrow S_n = \sum_{k=1}^{n}[k^3 - (k-1)^3] \Rightarrow 8000 = n^3$
$\Rightarrow n = 20$ which is a natural number.
Hence, both the given statements are true.
and statement 2 is correct explanation for statement 1.

Question255

Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is
[2011]

Options:

A. $\alpha - \beta$

B. $\frac{\alpha - \beta}{100}$

C. $\beta - \alpha$

D. $\frac{\alpha - \beta}{200}$

Answer: B

Solution:

Solution:
Let A.P. be a, a + d , a + 2d , ...
$a_2 + a_4 + \dots\dots\dots + a_{200} = \alpha$
$\Rightarrow \frac{100}{2}[2(a + d) + (100 - 1)2d] = \alpha$. . . (i)
and $a_1 + a_3 + a_5 + \dots\dots\dots + a_{199} = \beta$
$\Rightarrow \frac{100}{2}[2a + (100 - 1)2d] = \beta$ . . . (ii)
Subtracting (ii) from (i), we get
$d = \frac{\alpha - \beta}{100}$

Question256

If the sum of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + \dots 2.6^2 + \dots$ upto n terms, when n is even, is $\frac{n(n+1)^2}{2}$, then the sum of the series, when n is odd, is [Online May 26, 2012]

Options:

A. $n^2(n + 1)$

B. $\frac{n^2(n - 1)}{2}$

C. $\frac{n^2(n + 1)}{2}$

D. $n^2(n - 1)$

Answer: C

Solution:

Solution:
If n is odd, the required sum is
$1^2 + 2.2^2 + 3^2 + 2.4^2 + \dots\dots + 2(n - 1)^2 + n^2$
$= \frac{(n-1)(n-1+1)^2}{2} + n^2$ ($\because n - 1$ is even)
$= \left(\frac{n-1}{2} + 1\right)n^2 = \frac{n^2(n+1)}{2}$

Question257

A man saves ₹200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹40 more than the saving of immediately previous month. His total saving from the start of service will be ₹11040 after [2011]

Options:

A. 19 months

B. 20 months

C. 21 months

D. 18 months

Answer: C

Solution:

Solution:
Let number of months $= n$
$\therefore 200 \times 3 + (240 + 280 + 320 + \ldots + (n-3)^{th}$ term $) = 11040$
$\Rightarrow \frac{n-3}{2}[2 \times 240 + (n-4) \times 40] = 11040 - 600$
$\Rightarrow (n-3)[240 + 20n - 80] = 10440$
$\Rightarrow (n-3)(20n + 160) = 10440$
$\Rightarrow (n-3)(n+8) = 522$
$\Rightarrow n^2 + 5n - 546 = 0$
$\Rightarrow (n+26)(n-21) = 0$
$\therefore n = 21$

Question258

A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in an AP with common difference-2, then the time taken by him to count all notes is
[2010]

Options:

A. 34 minutes

B. 125 minutes

C. 135 minutes

D. 24 minutes

Answer: A

Solution:

Solution:
Till 10^{th} minute number of counted notes $= 1500$
Remaining notes $= 4500 - 1500 = 3000$
$3000 = \frac{n}{2}[2 \times 148 + (n-1)(-2)] = n[148 - n + 1]$
$n^2 - 149n + 3000 = 0$
$\Rightarrow n = 125, 24$
But $n = 125$ is not possible
$\therefore$ Total time $= 24 + 10 = 34$ minutes.

Question259

The sum to infinite term of the series $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \cdot s$ is
[2009]

Options:

A. 3

B. 4

C. 6

D. 2

Answer: A

Solution:

Solution:

Let $S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + \ldots\ldots\infty \ldots$ (i)

Multiplying both sides by $\frac{1}{3}$, we get

$\frac{1}{3}S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + \ldots\ldots\ldots\infty \ldots$ (ii)

Subtracting eqn. (ii) from eqn. (i), we get

$\frac{2}{3}S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \ldots\ldots\ldots\infty$

$\Rightarrow \frac{2}{3}S = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + \ldots\ldots\ldots\infty$

$\Rightarrow \frac{2}{3}S = \frac{\frac{4}{3}}{1 - \frac{1}{3}} = \frac{4}{3} \times \frac{3}{2} \Rightarrow S = 3$

Question260

The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48 . If the terms of the geometric progression are alternately positive and negative, then the first term is [2008]

Options:

A. -4

B. -12

C. 12

D. 4

Answer: B

Solution:

Solution:

AT Q

$a + ar = 12$

$ar^2 + ar^3 = 48$

$\Rightarrow \frac{ar^2(1 + r)}{a(1 + r)} = \frac{48}{12} \Rightarrow r^2 = 4, \Rightarrow r = -2$

($\because$ terms are alternately + ve and $-$ve)

$\Rightarrow a = -12$

Question261

In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression is equals [2007]

Options:

A. $\sqrt{5}$

B. $\frac{1}{2}(\sqrt{5}-1)$

C. $\frac{1}{2}(1-\sqrt{5})$

D. $\frac{1}{2}\sqrt{5}$

Answer: B

Solution:

Solution:
Let the series $a, ar, ar^2, \ldots$ are in geometric progression.
Given that, $a = ar + ar^2$
$\Rightarrow 1 = r + r^2 \quad \Rightarrow r^2 + r - 1 = 0$
$\Rightarrow r = \frac{-1 \pm \sqrt{1-4\times-1}}{2} = \frac{-1 \pm \sqrt{5}}{2}$
$\Rightarrow r = \frac{\sqrt{5}-1}{2}$[$\because$ terms of G.P. are positive $\therefore r$ should be positive]

Question262

The sum of series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots\ldots$ upto infinity is [2007]

Options:

A. $e^{-\frac{1}{2}}$

B. $e^{+\frac{1}{2}}$

C. e^{-2}

D. e^{-1}

Answer: D

Solution:

Solution:
We know that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\infty$
Put $x = -1$
$\therefore e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots\ldots\infty$
$\therefore e^{-1} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} \ldots\ldots\ldots\infty$

Question263

Let $a_1, a_2, a_3 \ldots\ldots\ldots\ldots$ be terms on A.P. If $\frac{a_1 + a_2 + \ldots\ldots\ldots a_p}{a_1 + a_2 + \ldots\ldots\ldots\ldots + a_q} = \frac{p^2}{q^2}$, $p \neq q$, then $\frac{a_6}{a_{21}}$ equals [2006]

Options:

A. $\frac{41}{11}$

B. $\frac{7}{2}$

C. $\frac{2}{7}$

D. $\frac{11}{41}$

Answer: D

Solution:

Solution:
Given that

$$\frac{S_p}{S_q} = \frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2}$$

$$\Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$$

Put p = 11 and q = 41

$$\frac{a_1 + 5d}{a_1 + 20d} = \frac{11}{41} \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$$

Question264

The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is [2006]

Options:

A. i

B. 1

C. -1

D. $-i$

Answer: D

Solution:

Solution:

$$\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$$

$$= i\sum_{k=1}^{10}\left(\cos\frac{2k\pi}{11} - i\sin\frac{2k\pi}{11}\right) \quad [\because e^{i\theta} = \cos\theta + i\sin\theta]$$

$$= i\sum_{k=1}^{10} e^{-\frac{2k\pi}{11}i} = i\left\{\sum_{k=0}^{10} e^{-\frac{2k\pi}{11}i} - 1\right\}$$

$$= i\left[1 + e^{-\frac{2\pi}{11}i} + e^{-\frac{4\pi}{11}i} + \ldots 11 \text{ terms}\right] - i$$

$$= i\left[\frac{1 - \left(e^{-\frac{2\pi}{11}i}\right)^{11}}{1 - e^{-\frac{2\pi}{11}i}}\right] - i = i\left[\frac{1 - e^{-2\pi i}}{1 - e^{-\frac{2\pi}{11}i}}\right] - i$$

$$= i \times 0 - i \quad [\because e^{-2\pi i} = 1]$$

$$= -i$$

Question265

If the expansion in powers of x of the function $\frac{1}{(1-ax)(1-bx)}$ is $a_0 + a_1x + a_2x^2 + a_3x^3 ......$ then a_n is
[2006]

Options:

A. $\frac{b^n - a^n}{b - a}$

B. $\frac{a^n - b^n}{b - a}$

C. $\frac{a^{n+1} - b^{n+1}}{b - a}$

D. $\frac{b^{n+1} - a^{n+1}}{b - a}$

Answer: D

Solution:

Solution:
$(1-ax)^{-1}(1-bx)^{-1}$
$= (1 + ax + a^2x^2 + \ldots)(1 + bx + b^2x^2 + \ldots)$
$\therefore$ Coefficient of x^n
$x^n = b^n + ab^{n-1} + a^2b^{n-2} + \ldots\ldots + a^{n-1}b + a^n$
{ which is a G.P. with $r = \frac{a}{b}$

Its sum is $= \dfrac{b^n\left[1 - \left(\frac{a}{b}\right)^{n+1}\right]}{1 - \frac{a}{b}}$ }

$= \frac{b^{n+1} - a^{n+1}}{b - a}$ $\therefore a_n = \frac{b^{n+1} - a^{n+1}}{b - a}$

Question266

If $a_1, a_2, \ldots\ldots\ldots, a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + \ldots\ldots\ldots\ldots + a_{n-1}a_n$ is equal to
[2006]

Options:

A. $n(a_1 - a_n)$

B. $(n-1)(a_1 - a_n)$

C. na_1a_n

D. $(n-1)a_1a_n$

Answer: D

Solution:

Solution:
$\because a_1, a_2, a_3 \ldots\ldots . a_n$ are in H.P.
$\therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3} \ldots \frac{1}{a_n}$ are in A.P.
$\therefore \frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = \ldots\ldots\ldots = \frac{1}{a_n} - \frac{1}{a_{n-1}} = d$ (say)

Then $a_1a_2 = \frac{a_1 - a_2}{d}$, $a_2a_3 = \frac{a_2 - a_3}{d}$,

.... $a_{n-1}a_n = \frac{a_{n-1} - a_n}{d}$

Adding all equations, we get

$\therefore$ $a_1a_2 + a_2a_3 + \ldots\ldots\ldots + a_{n-1}a_n$

$= \frac{a_1 - a_2}{d} + \frac{a_2 - a_3}{d} + \ldots. + \frac{a_{n-1} - a_n}{d}$

$= \frac{1}{d}[a_1 - a_2 + a_2 - a_3 + \ldots. + a_{n-1} - a_n] = \frac{a_1 - a_n}{d}$

Also, $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$

$\Rightarrow \frac{a_1 - a_n}{a_1a_n} = (n-1)d$

$\Rightarrow \frac{a_1 - a_n}{d} = (n-1)a_1a_n$

Which is the required result.

Question267

If the coefficients of rth, (r + 1) th , and (r + 2) th terms in the the binomial expansion of $(1 + y)^m$ are in A.P., then m and r satisfy the equation [2005]

Options:

A. $m^2 - m(4r - 1) + 4r^2 - 2 = 0$

B. $m^2 - m(4r + 1) + 4r^2 + 2 = 0$

C. $m^2 - m(4r + 1) + 4r^2 - 2 = 0$

D. $m^2 - m(4r - 1) + 4r^2 + 2 = 0$

Answer: C

Solution:

Solution:

Coeficient of r^{th} , $(r+1)^{th}$ and $(r+2)^{th}$ terms is ${}^mC_{r-1}$, ${}^mC_r$ and ${}^mC_{r+1}$ resp.

Given that ${}^mC_{r-1}$, ${}^mC_r$, ${}^mC_{r+1}$ are in A.P.

$2\,{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$

$\Rightarrow 2 = \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r} = \frac{r}{m-r+1} + \frac{m-r}{r+1}$

$\Rightarrow m^2 - m(4r+1) + 4r^2 - 2 = 0$

Question268

If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$ where a, b, c are in A.P and $|a| < 1$, $|b| < 1$, $|c| < 1$ then x, y, z are in [2005]

Options:

A. G.P.

B. A.P.

C. Arithmetic - Geometric Progression

D. H.P.

Answer: D

Solution:

Solution:

$x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \Rightarrow a = 1 - \frac{1}{x}$

$y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b} \Rightarrow b = 1 - \frac{1}{y}$

$z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c} \Rightarrow c = 1 - \frac{1}{z}$

a, b, c are in A.P. $\Rightarrow 2b = a + c$

$2\left(1 - \frac{1}{y}\right) = 1 - \frac{1}{x} + 1 - \frac{1}{z}$

$\frac{2}{y} = \frac{1}{x} + \frac{1}{z} \Rightarrow$ x, y, z are in H.P.

Question269

The sum of the series $1 + \frac{1}{4.2!} + \frac{1}{16.4!} + \frac{1}{64.6!} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots$ ad inf. is
[2005]

Options:

A. $\frac{e-1}{\sqrt{e}}$

B. $\frac{e+1}{\sqrt{e}}$

C. $\frac{e-1}{2\sqrt{e}}$

D. $\frac{e+1}{2\sqrt{e}}$

Answer: D

Solution:

Solution:
We know that

$\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} \ldots\ldots\ldots\ldots$

Putting $x = \frac{1}{2}$, we get

$1 + \frac{1}{4.2!} + \frac{1}{16.4!} + \frac{1}{64.6!} + \ldots\ldots\infty = \frac{e^{\frac{1}{2}} + e^{\frac{-1}{2}}}{2}$

$= \frac{\sqrt{e} + \frac{1}{\sqrt{e}}}{2} = \frac{e+1}{2\sqrt{e}}$

Question270

Let T_r be the rth term of an A.P. whose first term is a and common difference is d . If for some positive integers m, n, m ≠ n, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then a − d equals
[2004]

Options:

A. $\frac{1}{m} + \frac{1}{n}$

B. 1

C. $\frac{1}{mn}$

D. 0

Answer: D

Solution:

Solution:

$T_m = a + (m-1)d = \frac{1}{n} \ldots$ (i)

$T_n = a + (n-1)d = \frac{1}{m} \ldots$ (ii)

Subtracting (ii) from (i), we get

$(m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow d = \frac{1}{mn}$

From (i) $a = \frac{1}{mn} \Rightarrow a - d = 0$

Question271

Let two numbers have arithmetic mean 9 and geometric mean 4 . Then these numbers are the roots of the quadratic equation [2004]

Options:

A. $x^2 - 18x - 16 = 0$

B. $x^2 - 18x + 16 = 0$

C. $x^2 + 18x - 16 = 0$

D. $x^2 + 18x + 16 = 0$

Answer: B

Solution:

Solution:

Let two numbers be a and b then $\frac{a+b}{2} = 9$

$\Rightarrow a + b = 18$ and $\sqrt{ab} = 4 \Rightarrow ab = 16$

$\therefore$ Equation with roots a and b is

$x^2 - (a+b)x + ab = 0 \Rightarrow x^2 - 18x + 16 = 0$

Question272

The sum of series $\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\ldots$ is [2004]

Options:

A. $\frac{(e^2 - 2)}{e}$

B. $\frac{(e-1)^2}{2e}$

C. $\frac{(e^2-1)}{2e}$

D. $\frac{(e^2-1)}{2}$

Answer: B

Solution:

We know that

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots . \infty$

$\therefore e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots$

and $e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots.$

$\therefore e + e^{-1} = 2\left[1 + \frac{1}{2!} + \frac{1}{4!} + \ldots\right]$

$\therefore \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\ldots = \frac{e+e^{-1}}{2} - 1$

$= \frac{e^2+1-2e}{2e} = \frac{(e-1)^2}{2e}$

Question273

The sum of the first n terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum
[2004]

Options:

A. $\left[\frac{n(n+1)}{2}\right]^2$

B. $\frac{n^2(n+1)}{2}$

C. $\frac{n(n+1)^2}{4}$

D. $\frac{3n(n+1)}{2}$

Answer: B

Solution:

Solution:
If n is odd, the required sum is
$1^2 + 2.2^2 + 3^2 + 2.4^2 + \ldots\ldots + 2\cdot(n-1)^2 + n^2$
$= \frac{(n-1)(n-1+1)^2}{2} + n^2$
[$\because (n-1)$ is even
$\therefore$ using given formula for the sum of $(n-1)$ terms.
$= \left(\frac{n-1}{2} + 1\right)n^2 = \frac{n^2(n+1)}{2}$

Question274

If $S_n = \sum_{r=0}^{n} \frac{1}{{}^nC_r}$ and $t_n = \sum_{r=0}^{n} \frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n}$ is equal to
[2004]

Options:

A. $\frac{2n-1}{2}$

B. $\frac{1}{2}n-1$

C. $n-1$

D. $\frac{1}{2}n$

Answer: D

Solution:

Solution:

$S_n = \frac{1}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{1}{{}^nC_2} + \ldots + \frac{1}{{}^nC_n}$

$t_n = \frac{0}{{}^nC_0} + \frac{1}{{}^nC_1} + \frac{2}{{}^nC_2} + \ldots + \frac{n}{{}^nC_n}$

$t_n = \frac{n}{{}^nC_n} + \frac{n-1}{{}^nC_{n-1}} + \frac{n-2}{{}^nC_{n-2}} + \ldots + \frac{0}{{}^nC_0}$

Adding (i) and (ii), we get,

$2t_n = (n)\left[\frac{1}{{}^nC_0} + \frac{1}{{}^nC_1} + \ldots \frac{1}{{}^nC_n}\right] = nS_n$

$\because {}^nC_r = {}^nC_{n-r}$

$\therefore \frac{t_n}{S_n} = \frac{n}{2}$

Question275

If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{a}{c}$, $\frac{b}{a}$ and $\frac{c}{b}$ are in [2003]

Options:

A. Arithmetic - Geometric Progression

B. Arithmetic Progression

C. Geometric Progression

D. Harmonic Progression.

Answer: D

Solution:

Solution:

$ax^2 + bx + c = 0, \alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

AT Q, $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$

$\alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} \Rightarrow -\frac{b}{a} = \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c^2}{a^2}}$

On simplification $2a^2c = ab^2 + bc^2$

$\Rightarrow \frac{2a}{b} = \frac{c}{a} + \frac{b}{c}$ [Divide both side by abc]

$\Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P.

$\therefore \frac{a}{c}, \frac{b}{a}, \& \frac{c}{b}$ are in H.P.

Question276

The sum of the series $\frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4}$............ up to ∞ is equal to [2003]

Options:

A. $\log_e\left(\frac{4}{e}\right)$

B. $2\log_e 2$

C. $\log_e 2 - 1$

D. $\log_e 2$

Answer: A

Solution:

Solution:

Let $S = \frac{1}{1.2} - \frac{1}{2.3} + \frac{1}{3.4} \ldots\ldots\ldots\ldots. \infty$

$T_n = \frac{1}{n(n+1)} = \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$\therefore S = \left(\frac{1}{1} - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) - \left(\frac{1}{4} - \frac{1}{5}\right) \ldots.$

$= 1 - 2\left[\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} \ldots\ldots\ldots\ldots. \infty\right]$

$\left[\because \log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots \infty\right]$

$= 1 - 2[-\log(1+1) + 1] = 2\log 2 - 1 = \log\left(\frac{4}{e}\right)$

Question277

If 1, $\log_9(3^{1-x} + 2)$, $\log_3(4.3^x - 1)$ are in A.P. then x equals [2002]

Options:

A. $\log_3 4$

B. $1 - \log_3 4$

C. $1 - \log_4 3$

D. $\log_4 3$

Answer: B

Solution:

Solution:

1, $\log_9(3^{1-x} + 2)$, $\log_3(4.3^x - 1)$ are in A.P.

$\because$a, b, c are in A . P then b = a + c

$\Rightarrow 2\log_9(3^{1-x} + 2) = 1 + \log_3(4.3^x - 1)$

$\because\ \log_{b^q} a^p = \frac{p}{q}\log_b a$
$\Rightarrow \log_3(3^{1-x}+2) = \log_3 3 + \log_3(4.3^x - 1)$
$\Rightarrow \log_3(3^{1-x}+2) = \log_3[3(4 \cdot 3^x - 1)]$
$\Rightarrow 3^{1-x} + 2 = 3(4.3^x - 1)$
$\Rightarrow 3.3^{-x} + 2 = 12.3^x - 3$ Put $3^x = t$
$\Rightarrow \frac{3}{t} + 2 = 12t - 3 \Rightarrow 12t^2 - 5t - 3 = 0$
Hence $t = -\frac{1}{3}, \frac{3}{4}$
$\Rightarrow 3^x = \frac{3}{4}\left(\text{ as } 3^x \neq -\text{ve}\right)$
$\Rightarrow x = \log_3\left(\frac{3}{4}\right)$ or $x = \log_3 3 - \log_3 4$
$\Rightarrow x = 1 - \log_3 4$

Question278

Sum of infinite number of terms of GP is 20 and sum of their square is 100 . The common ratio of GP is [2002]

Options:

A. 5

B. $\frac{3}{5}$

C. $\frac{8}{5}$

D. $\frac{1}{5}$

Answer: B

Solution:

Solution:
Let $a =$ first term of G.P. and $r =$ common ratio of G.P.
Then G.P. is a, ar, ar^2
Given $S_\infty = 20 \Rightarrow \frac{a}{1-r} = 20$
$\Rightarrow a = 20(1-r) \ldots$ (i)
Also $a^2 + a^2r^2 + a^2r^4 + \ldots$ to $\infty = 100$
$\Rightarrow \frac{a^2}{1-r^2} = 100 \Rightarrow \frac{[20(1-r)]^2}{1-r^2} = 100$ [from (i)]
$\Rightarrow \frac{400(1-r)^2}{(1-r)(1+r)} = 100 \Rightarrow 4(1-r) = 1+r$
$\Rightarrow 1 + r = 4 - 4r \Rightarrow 5r = 3 \Rightarrow r = 3/5$

Question279

Fifth term of a GP is 2, then the product of its 9 terms is [2002]

Options:

A. 256

B. 512

C. 1024

D. none of these

Answer: B

Solution:

Solution:
$\because a_4 = 2 \Rightarrow ar^4 = 2$
Now, $a \times ar \times ar^2 \times ar^3 \times ar^4 \times ar^5 \times ar^6 \times ar^7 \times ar^8$
$= a^9 r^{36} = (ar^4)^9 = 2^9 = 512$

Question280

$1^3 - 2^3 + 3^3 - 4^3 + ... + 9^3 =$ [2002]

Options:

A. 425

B. -425

C. 475

D. -475

Answer: A

Solution:

Solution:
$1^3 - 2^3 + 3^3 - 4^3 + \ldots\ldots + 9^3$
$= 1^3 + 2^3 + 3^3 + \ldots\ldots + 9^3 - 2(2^3 + 4^3 + 6^3 + 8^3)$
$\left[\because \Sigma n^3 = \left(\frac{n(n+1)}{2}\right)^2\right]$
$= \left[\frac{9 \times 10}{2}\right]^2 - 2.2^3[1^3 + 2^3 + 3^3 + 4^3]$
$= (45)^2 - 16 \cdot \left[\frac{4 \times 5}{2}\right]^2 = 2025 - 1600 = 425$

Question281

The value of $2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} ... \infty$ is [2002]

Options:

A. 1

B. 2

C. $\frac{3}{2}$

D. 4

Answer: B

Solution:

Let $P = 2^{1/4} \cdot 2^{2/8} \cdot 2^{3/16} \ldots\ldots.\infty$

$= 2^{1/4 + 2/8 + 3/16 + \ldots\ldots.\infty}$

Now, let $S = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \ldots\ldots.\infty$ ...... (i)

$\frac{1}{2}S = \frac{1}{8} + \frac{2}{16} + \ldots\ldots.\infty$ . . . ..(ii)

Subtracting (ii) from (i)

$\Rightarrow \frac{1}{2}S = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\ldots.\infty$

or $\frac{1}{2}S = \frac{a}{1-r} = \frac{1/4}{1-1/2} = \frac{1}{2} \Rightarrow S = 1$

$\therefore P = 2^S = 2$
