FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Thursday 01st February, 2024) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is:

(1) $\frac{2}{5}$ (2) $\frac{2}{7}$

(3) $\frac{1}{7}$ (4) $\frac{1}{5}$

Ans. (2)

Sol.

P(4W4B/2W2B) =

$$\frac{P(4W4B)\times P(2W2B/4W4B)}{P(2W6B)\times P(2W2B/2W6B)+P(3W5B)\times P(2W2B/3W5B)+\ldots\ldots\ldots\ldots+P(6W2B)\times P(2W2B/6W2B)}$$

$$=\frac{\frac{1}{5}\times\frac{{}^4C_2\times{}^4C_2}{{}^8C_4}}{\frac{1}{5}\times\frac{{}^2C_2\times{}^6C_2}{{}^8C_4}+\frac{1}{5}\times\frac{{}^3C_2\times{}^5C_2}{{}^8C_4}+\ldots+\frac{1}{5}\times\frac{{}^6C_2\times{}^2C_2}{{}^8C_4}}$$

$$=\frac{2}{7}$$

2. The value of the integral

$$\int_0^{\frac{\pi}{4}}\frac{xdx}{\sin^4(2x)+\cos^4(2x)}\,equals:$$

(1) $\frac{\sqrt{2}\pi^2}{8}$ (2) $\frac{\sqrt{2}\pi^2}{16}$

(3) $\frac{\sqrt{2}\pi^2}{32}$ (4) $\frac{\sqrt{2}\pi^2}{64}$

Ans. (3)

Sol. $\int_0^{\frac{\pi}{4}}\frac{xdx}{\sin^4(2x)+\cos^4(2x)}$

Let $2x=t$ then $dx=\frac{1}{2}dt$

$$I=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{tdt}{\sin^4 t+\cos^4 t}$$

$$I=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{\left(\frac{\pi}{2}-t\right)dt}{\sin^4\left(\frac{\pi}{2}-t\right)+\cos^4\left(\frac{\pi}{2}-t\right)}$$

$$I=\frac{1}{4}\int_0^{\frac{\pi}{2}}\frac{\frac{\pi}{2}dt}{\sin^4 t+\cos^4 t}-I$$

$$2I=\frac{\pi}{8}\int_0^{\frac{\pi}{2}}\frac{dt}{\sin^4 t+\cos^4 t}$$

$$2I=\frac{\pi}{8}\int_0^{\frac{\pi}{2}}\frac{\sec^4 tdt}{\tan^4 t+1}$$

Let tant = y then $\sec^2$t dt = dy

$$2I=\frac{\pi}{8}\int_0^{\infty}\frac{(1+y^2)dy}{1+y^4}$$

$$=\frac{\pi}{16}\int_0^{\infty}\frac{1+\frac{1}{y^2}}{y^2+\frac{1}{y^2}}dy$$

Put $y-\frac{1}{y}=p$

$$I=\frac{\pi}{16}\int_{-\infty}^{\infty}\frac{dp}{p^2+\left(\sqrt{2}\right)^2}$$

$$=\frac{\pi}{16\sqrt{2}}\left[\tan^{-1}\left(\frac{p}{\sqrt{2}}\right)\right]_{-\infty}^{\infty}$$

$$I=\frac{\pi^2}{16\sqrt{2}}$$

3. If A = $\begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$, B = $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, C = ABA^T and X = A^TC^2A, then det X is equal to :

(1) 243

(2) 729

(3) 27

(4) 891

Ans. (2)

Sol.

$$A=\begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix} \Rightarrow \det(A)=3$$

$$B=\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow \det(B)=1$$

Now C = ABA^T $\Rightarrow$det(C) = (dct (A))^2 x det(B)

$|C|=9$

Now |X| = $|A^TC^2A|$

= $|A^T|\,|C|^2\,|A|$

= $|A|^2\,|C|^2$

= 9 x 81

= 729

4. If tanA = $\dfrac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B=\dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}$

and

$$\tan C=\left(x^{-3}+x^{-2}+x^{-1}\right)^{\frac{1}{2}}, 0<A,B,C<\frac{\pi}{2}, then$$

A + B is equal to :

(1) C

(2) $\pi - C$

(3) $2\pi - C$

(4) $\dfrac{\pi}{2}-C$

Ans. (1)

Sol.

Finding tan (A + B) we get

$\Rightarrow$ tan (A + B) =

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{\dfrac{1}{\sqrt{x(x^2+x+1)}}+\dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1-\dfrac{1}{x^2+x+1}}$$

$$\Rightarrow \tan(A+B)=\frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{(x^2+x)\left(\sqrt{x}\right)}$$

$$\frac{(1+x)\left(\sqrt{x^2+x+1}\right)}{(x^2+x)\left(\sqrt{x}\right)}$$

$$\tan(A+B)=\frac{\sqrt{x^2+x+1}}{x\sqrt{x}}=\tan C$$

$$A+B=C$$

5. If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:

(1) 47

(2) 53

(3) 51

(4) 43

Ans. (3)

Sol.

Total ways to partition 5 into 4 parts are :

5, 0, 0, 0 $\Rightarrow$1 way

4, 1, 0, 0 $\Rightarrow \dfrac{5!}{4!}=5$ ways

3, 2, 0, 0, $\Rightarrow \dfrac{5!}{3!2!}=10$ ways

$2,2,0,1 \Rightarrow \dfrac{5!}{2!2!2!}=15$ ways

$2,1,1,1 \Rightarrow \dfrac{5!}{2!(1!)^3 3!}=10$ ways

$3,1,1,0 \Rightarrow \dfrac{5!}{3!2!}=10$ ways

Total $\Rightarrow$ 1+5+10+15+10+10 = 51 ways

6. Let S = $\{z \in C : |z-1| = 1$ and $\left(\sqrt{2}-1\right)\left(z+\bar{z}\right)-i\left(z-\bar{z}\right)=2\sqrt{2}\}$. Let z_1, z_2 $\in S$ be such that $|z_1| = \max_{z\in s}|z|$ and $|z_2| = \min_{z\in s}|z|$.

Then $\left|\sqrt{2}z_1 - z_2\right|^2$ equals :

(1) 1 (2) 4

(3) 3 (4) 2

Ans. (4)

Sol. Let Z = x + iy

Then $(x-1)^2 + y^2 = 1 \quad \rightarrow (1)$

& $\left(\sqrt{2}-1\right)(2x) - i(2iy) = 2\sqrt{2}$

$\Rightarrow (\sqrt{2}-1)x + y = \sqrt{2} \rightarrow (2)$

Solving (1) & (2) we get

Either x = 1 or $x = \dfrac{1}{2-\sqrt{2}} \rightarrow (3)$

On solving (3) with (2) we get

For x = 1 $\Rightarrow$ y = 1 $\Rightarrow$ $Z_2 = 1 + i$

& for

$$x = \frac{1}{2-\sqrt{2}} \Rightarrow y = \sqrt{2} - \frac{1}{\sqrt{2}} \Rightarrow Z_1 = \left(1+\frac{1}{\sqrt{2}}\right) + \frac{i}{\sqrt{2}}$$

Now

$$\left|\sqrt{2}z_1 - z_2\right|^2$$

$$= \left|\left(\frac{1}{\sqrt{2}}+1\right)\sqrt{2} + i - (1+i)\right|^2$$

$$= \left(\sqrt{2}\right)^2$$

$$= 2$$

7. Let the median and the mean deviation about the median of 7 observation 170, 125, 230, 190, 210, a, b be 170 and $\dfrac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is :

(1) 31

(2) 28

(3) 30

(4) 32

Ans. (3)

Sol. Median = 170 $\Rightarrow$ 125, a, b, 170, 190, 210, 230

Mean deviation about

Median =

$$\frac{0+45+60+20+40+170-a+170-b}{7} = \frac{205}{7}$$

$\Rightarrow$ a + b = 300

Mean = $\dfrac{170+125+230+190+210+a+b}{7} = 175$

Mean deviation

About mean =

$$\frac{50+175-a+175-b+5+15+35+55}{7} = 30$$

8. Let $\vec{a} = -5\hat{i} + \hat{j} - 3\hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{c} = \left(\left(\left(\vec{a}\times\vec{b}\right)\times\hat{i}\right)\times\hat{i}\right)\times\hat{i}$. Then $\vec{c}\cdot\left(-\hat{i}+\hat{j}+\hat{k}\right)$ is equal to

(1) –12 (2) –10

(3) –13 (4) –15

Ans. (1)

Sol. $\vec{a} = -5\hat{i} + j - 3\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$

$(\vec{a}\times\vec{b})\times\hat{i} = \left(\vec{a}\cdot\hat{i}\right)\vec{b} - \left(\vec{b}\cdot\hat{i}\right)\vec{a}$

$= -5\vec{b} - \vec{a}$

$= \left(\left(\left(-5\vec{b} - \vec{a}\right)\times\hat{i}\right)\times\hat{i}\right)$

$= \left(\left(-11\hat{j} + 23\hat{k}\right)\times\hat{i}\right)\times\hat{i}$

$\Rightarrow \left(11\hat{k} + 23\hat{j}\right)\times\hat{i}$

$\Rightarrow \left(11\hat{j} - 23\hat{k}\right)$

$\vec{c}.\left(-\hat{i} + \hat{j} + \hat{k}\right) = 11 - 23 = -12$

9. Let S = $\{x \in R : (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10\}$.

Then the number of elements in S is :

(1) 4 (2) 0

(3) 2 (4) 1

Ans. (3)

Sol. $\left(\sqrt{3}+\sqrt{2}\right)^x + \left(\sqrt{3}-\sqrt{2}\right)^x = 10$

Let $\left(\sqrt{3}+\sqrt{2}\right)^x = t$

$t + \frac{1}{t} = 10$

$t^2 - 10t + 1 = 0$

$t = \frac{10 \pm \sqrt{100-4}}{2} = 5 \pm 2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^x = \left(\sqrt{3}\pm\sqrt{2}\right)^2$

x = 2 or x = –2

Number of solutions = 2

10. The area enclosed by the curves xy + 4y = 16 and x + y = 6 is equal to :

(1) $28 - 30\ \log_e 2$ (2) $30 - 28\ \log_e 2$

(3) $30 - 32\ \log_e 2$ (4) $32 - 30\ \log_e 2$

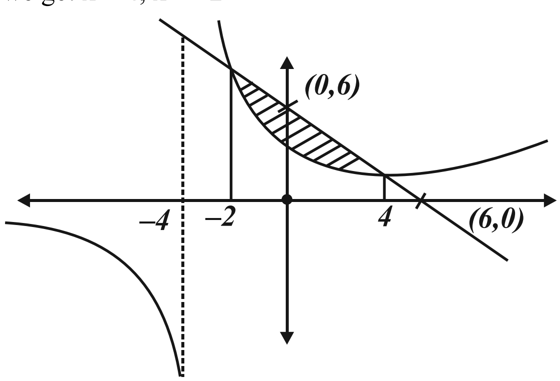
Ans. (3)

Sol. xy + 4y = 16 , x + y = 6

y(x + 4) = 16 ____(1) , x + y = 6___(2)

on solving, (1) & (2)

we get x = 4, x = –2



$$\text{Area} = \int_{-2}^{4}\left(\left(6-x\right)-\left(\frac{16}{x+4}\right)\right)dx$$

$$= 30 - 32\ln 2$$

11. Let f : **R** → **R** and g : **R** → **R** be defined as

$$f(x) = \begin{cases} \log_e x & , \quad x > 0 \\ e^{-x} & , \quad x \le 0 \end{cases} \text{ and}$$

$$g(x) = \begin{cases} x & , \quad x \ge 0 \\ e^{x} & , \quad x < 0 \end{cases}$$. Then, gof : **R** → **R** is :

(1) one-one but not onto

(2) neither one-one nor onto

(3) onto but not one-one

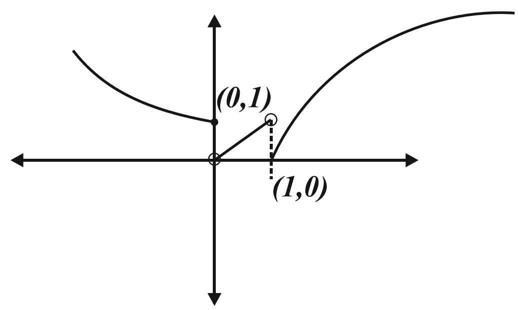
(4) both one-one and onto

Ans. (2)

Sol.

$$g(f(x)) = \begin{cases} f(x), f(x) \ge 0 \\ e^{f(x)}, f(x) < 0 \end{cases}$$

$$g(f(x)) = \begin{cases} e^{-x}, (-\infty, 0] \\ e^{\ln x}, (0,1) \\ \ln x, [1, \infty) \end{cases}$$



Graph of g(f(x))

g(f(x)) $\Rightarrow$ Many one into

12. If the system of equations

2x + 3y – z = 5

x + αy + 3z = –4

3x – y + βz = 7

has infinitely many solutions, then 13 αβ is equal to

(1) 1110 (2) 1120

(3) 1210 (4) 1220

Ans. (2)

Sol. Using family of planes

$2x + 3y - z - 5 = k_1 (x + \alpha y + 3z + 4) + k_2 (3x - y + \beta z - 7)$

$2 = k_1 + 3k_2$, $3 = k_1\alpha - k_2$, $-1 = 3k_1 + \beta k_2$, $-5 = 4k_1 - 7k_2$

On solving we get

$$k_2 = \frac{13}{19}, k_1 = \frac{-1}{19}, \alpha = -70, \beta = \frac{-16}{13}$$

$$13\alpha\ \beta = 13(-70)\left(\frac{-16}{13}\right)$$

$$= 1120$$

13. For $0 < \theta < \pi/2$, if the eccentricity of the hyperbola $x^2 - y^2\text{cosec}^2\theta = 5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^2\text{cosec}^2\theta + y^2 = 5$, then the value of θ is :

(1) $\frac{\pi}{6}$ (2) $\frac{5\pi}{12}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. (3)

Sol.

$$e_h = \sqrt{1+\sin^2\theta}$$

$$e_c = \sqrt{1-\sin^2\theta}$$

$$e_h = \sqrt{7}e_c$$

$$1+\sin^2\theta = 7(1-\sin^2\theta)$$

$$\sin^2\theta = \frac{6}{8} = \frac{3}{4}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3}$$

14. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2x(x + y)^3 - x(x + y) - 1$, $y(0) = 1$. Then, $\left(\frac{1}{\sqrt{2}} + y\left(\frac{1}{\sqrt{2}}\right)\right)^2$ equals :

(1) $\frac{4}{4+\sqrt{e}}$ (2) $\frac{3}{3-\sqrt{e}}$

(3) $\frac{2}{1+\sqrt{e}}$ (4) $\frac{1}{2-\sqrt{e}}$

Ans. (4)

Sol. $$\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$$

$$x + y = t$$

$$\frac{dt}{dx} - 1 = 2xt^3 - xt - 1$$

$$\frac{dt}{2t^3 - t} = xdx$$

$$\frac{tdt}{2t^4 - t^2} = xdx$$

Let $t^2 = z$

$$\int \frac{dz}{2(2z^2 - z)} = \int xdx$$

$$\int \frac{dz}{4z\left(z - \frac{1}{2}\right)} = \int xdx$$

$$\ln\left|\frac{z - \frac{1}{2}}{z}\right| = x^2 + k$$

$$z = \frac{1}{2-\sqrt{e}}$$

15. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} \dfrac{a - b\cos 2x}{x^2} & ; \quad x < 0 \\ x^2 + cx + 2 & ; \quad 0 \le x \le 1 \\ 2x + 1 & ; \quad x > 1 \end{cases}$$

If f is continuous everywhere in **R** and m is the number of points where f is **NOT** differential then m + a + b + c equals :

(1) 1 (2) 4

(3) 3 (4) 2

Ans. (4)

Sol. At x = 1, f(x) is continuous therefore,

$f(1^-) = f(1) = f(1^+)$

$f(1) = 3 + c$ ….(1)

$f(1^+) = \lim_{h\to 0} 2(1 + h) + 1$

$f(1^+) = \lim_{h\to 0} 3 + 2h = 3$ ….(2)

from (1) & (2)

c = 0

at x = 0, f(x) is continuous therefore,

$f(0^-) = f(0) = f(0^+)$ ….(3)

$f(0) = f(0^+) = 2$ ….(4)

$f(0^-)$ has to be equal to 2

$$\lim_{h\to 0}\frac{a-b\cos(2h)}{h^2}$$

$$\lim_{h\to 0}\frac{a-b\left\{1-\frac{4h^2}{2!}+\frac{16h^4}{4!}+...\right\}}{h^2}$$

$$\lim_{h\to 0}\frac{a-b+b\left\{2h^2-\frac{2}{3}h^4...\right\}}{h^2}$$

for limit to exist a – b = 0 and limit is 2b ….(5)

from (3), (4) & (5)

a = b = 1

checking differentiability at x = 0

$$\text{LHD}: \lim_{h\to 0}\frac{\frac{1-\cos 2h}{h^2}-2}{-h}$$

$$\lim_{h\to 0}\frac{1-\left(1-\frac{4h^2}{2!}+\frac{16h^4}{4!}...\right)-2h^2}{-h^3}=0$$

$$\text{RHD}: \lim_{h\to 0}\frac{(0+h)^2+2-2}{h}=0$$

Function is differentiable at every point in its domain

$\therefore$ m = 0

m + a + b + c = 0 + 1 + 1 + 0 = 2

16. Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b be an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is $\sqrt{14}$. Then the square of the eccentricity of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is :

(1) 3 (2) 7/2

(3) 3/2 (4) 5/2

Ans. (3)

Sol.

$$e=\frac{1}{\sqrt{2}}=\sqrt{1-\frac{b^2}{a^2}}\Rightarrow\frac{1}{2}=1-\frac{b^2}{a^2}$$

$$\frac{2b^2}{a}=14$$

$$e_H=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{1}{2}}=\sqrt{\frac{3}{2}}$$

$$(e_H)^2=\frac{3}{2}$$

17. Let 3, a, b, c be in A.P. and 3, a – 1, b + l, c + 9 be in G.P. Then, the arithmetic mean of a, b and c is :

(1) –4 (2) –1

(3) 13 (4) 11

Ans. (4)

Sol.

3, a, b, c $\to$ A.P $\Rightarrow$ 3, 3+d, 3+2d, 3+3d

3, a–1,b +1, c + 9 $\to$ G.P $\Rightarrow$3, 2+d, 4+2d, 12+3d

a = 3 + d $(2+d)^2=3(4+2d)$

b = 3 + 2d d = 4, -2

c = 3 + 3d

If d = 4 G.P $\Rightarrow$3, 6, 12, 24

a = 7

b = 11

c = 15

$$\frac{a+b+c}{3}=11$$

18. Let C : $x^2 + y^2 = 4$ and C' : $x^2 + y^2 - 4\lambda x + 9 = 0$ be two circles. If the set of all values of λ so that the circles C and C' intersect at two distinct points, is **R**– [a, b], then the point (8a + 12, 16b – 20) lies on the curve :

(1) $x^2 + 2y^2 - 5x + 6y = 3$

(2) $5x^2 - y = -11$

(3) $x^2 - 4y^2 = 7$

(4) $6x^2 + y^2 = 42$

Ans. (4)

Sol. $x^2 + y^2 = 4$

C (0, 0) $r_1 = 2$

C' $(2\lambda, 0)$ $r_2 = \sqrt{4\lambda^2 - 9}$

$|r_1 - r_2| < CC' < |r_1 + r_2|$

$$\left|2-\sqrt{4\lambda^2-9}\right| < |2\lambda| < 2+\sqrt{4\lambda^2-9}$$

$4 + 4\lambda^2 - 9 - 4\sqrt{4\lambda^2 - 9} < 4\lambda^2$

True $\lambda \in R$…. (1)

$4\lambda^2 < 4 + 4\lambda^2 - 9 + 4\sqrt{4\lambda^2 - 9}$

$5 < 4\sqrt{4\lambda^2-9}$ and $\lambda^2 \geq \frac{9}{4}$

$\frac{25}{16} < 4\lambda^2 - 9$ $\lambda \in \left(-\infty, -\frac{3}{2}\right] \cup \left[\frac{3}{2}, \infty\right)$

$\frac{169}{64} < \lambda^2$

$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$ …(2)

from (1) and (2) $\lambda \in$

$\lambda \in \left(-\infty, -\frac{13}{8}\right) \cup \left(\frac{13}{8}, \infty\right) \Rightarrow R - \left[-\frac{13}{8}, \frac{13}{8}\right]$

as per question $a = -\frac{13}{8}$ and $b = \frac{13}{8}$

$\therefore$ required point is (–1, 6) with satisfies option (4)

19. If $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2, \forall x \neq 0$ and $y = 9x^2 f(x)$, then y is strictly increasing in :

(1) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(2) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(3) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

(4) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

Ans. (2)

Sol. $5\,f(x) + 4\,f\left(\frac{1}{x}\right) = x^2 - 2, \ \forall x \neq 0$ …(1)

Substitute $x \to \frac{1}{x}$

$$5f\left(\frac{1}{x}\right) + 4f(x) = \frac{1}{x^2} - 2 \quad \text{…(2)}$$

On solving (1) and (2)

$$f(x) = \frac{5x^4 - 2x^2 - 4}{9x^2}$$

$y = 9x^2 f(x)$

$y = 5x^4 - 2x^2 - 4$ …(3)

$\frac{dy}{dx} = 20x^3 - 4x$

for strictly increasing

$\frac{dy}{dx} > 0$

$4x(5x^2 - 1) > 0$

$x \in \left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

20. If the shortest distance between the lines $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ is 1, then the sum of all possible values of λ is :

(1) 0

(2) $2\sqrt{3}$

(3) $3\sqrt{3}$

(4) $-2\sqrt{3}$

Ans. (2)

Sol. Passing points of lines L_1 & L_2 are

$(\lambda, 2, 1) \& (\sqrt{3}, 1, 2)$

$$S.D = \frac{\begin{vmatrix} \sqrt{3}-\lambda & -1 & 1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}$$

$$1 = \left|\frac{\sqrt{3}-\lambda}{\sqrt{3}}\right|$$

$\lambda = 0, \lambda = 2\sqrt{3}$

SECTION-B

21. If x = x(t) is the solution of the differential equation $(t + 1)dx = (2x + (t + 1)^4)\,dt$, $x(0) = 2$, then, x(1) equals ___________.

Ans. (14)

Sol. $(t + 1)dx = (2x + (t + 1)^4)dt$

$$\frac{dx}{dt} = \frac{2x + (t+1)^4}{t+1}$$

$$\frac{dx}{dt} - \frac{2x}{t+1} = (t+1)^3$$

$$I \cdot F = e^{-\int \frac{2}{t+1}dt} = e^{-2\ln(t+1)} = \frac{1}{(t+1)^2}$$

$$\frac{x}{(t+1)^2} = \int \frac{1}{(t+1)^2}(t+1)^3 dt + c$$

$$\frac{x}{(t+1)^2} = \frac{(t+1)^2}{2} + c$$

$$\Rightarrow c = \frac{3}{2}$$

$$x = \frac{(t+1)^4}{2} + \frac{3}{2}(t+1)^2$$

put, t = 1

$x = 2^3 + 6 = 14$

22. The number of elements in the set

$S = \{(x, y, z) : x, y, z \in \mathbf{Z}, x + 2y + 3z = 42, x, y, z \geq 0\}$ equals __________.

Ans. (169)

Sol. $x + 2y + 3z = 42, \quad x, y, z \geq 0$

$z = 0 \quad x + 2y = 42 \Rightarrow 22$

$z = 1 \quad x + 2y = 39 \Rightarrow 20$

$z = 2 \quad x + 2y = 36 \Rightarrow 19$

$z = 3 \quad x + 2y = 33 \Rightarrow 17$

$z = 4 \quad x + 2y = 30 \Rightarrow 16$

$z = 5 \quad x + 2y = 27 \Rightarrow 14$

$z = 6 \quad x + 2y = 24 \Rightarrow 13$

$z = 7 \quad x + 2y = 21 \Rightarrow 11$

$z = 8 \quad x + 2y = 18 \Rightarrow 10$

$z = 9 \quad x + 2y = 15 \Rightarrow 8$

$z = 10 \quad x + 2y = 12 \Rightarrow 7$

$z = 11 \quad x + 2y = 9 \Rightarrow 5$

$z = 12 \quad x + 2y = 6 \Rightarrow 4$

$z = 13 \quad x + 2y = 3 \Rightarrow 2$

$z = 14 \quad x + 2y = 0 \Rightarrow 1$

Total : 169

23. If the Coefficient of x^{30} in the expansion of $\left(1+\frac{1}{x}\right)^6 (1 + x^2)^7 (1 - x^3)^8$; $x \neq 0$ is α, then $|\alpha|$ equals ___________.

Ans. (678)

Sol. coeff of x^{30} in $\dfrac{(x+1)^6(1+x^2)^7(1-x^3)^8}{x^6}$

coeff. of x^{36} in $(1+x)^6(1+x^2)^7(1-x^3)^8$

General term

$^6C_{r_1}\,^7C_{r_2}\,^8C_{r_3}(-1)^{r_3}x^{r_1+2r_2+3r_3}$

$r_1+2r_2+3r_3=36$

Case-I : $r_1+2r_2=12$ (Taking r_3 = 8)

r_1	r_2	r_3
0	6	8
2	5	8
4	4	8
6	3	8

Case-II : $r_1+2r_2=15$ (Taking r_3 = 7)

r_1	r_2	r_3
1	7	7
3	6	7
5	5	7

Case-III : $r_1+2r_2=18$ (Taking r_3 = 6)

r_1	r_2	r_3
4	7	6
6	6	6

Coeff. = 7 + (15 × 21) + (15 × 35) + (35)

–(6 × 8) – (20 × 7 × 8) – (6 × 21 × 8) + (15 × 28)

+ (7 × 28) = –678 = α

|α| = 678

24. Let 3, 7, 11, 15, ..., 403 and 2, 5, 8, 11, . . ., 404 be two arithmetic progressions. Then the sum, of the common terms in them, is equal to _________.

Ans. (6699)

Sol. 3, 7, 11, 15, ….., 403

2, 5, 8, 11, …., 404

LCM (4, 3) = 12

11, 23, 35,….. let (403)

403 = 11 + (n - 1) x 12

$\frac{392}{12}=n-1$

$33\cdot 66=n$

$n=33$

Sum $\frac{33}{2}(22+32\times 12)$

=6699

25. Let {x} denote the fractional part of x and $f(x)=\dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}$, $x\neq 0$. If L and R respectively denotes the left hand limit and the right hand limit of f(x) at x = 0, then $\frac{32}{\pi^2}(L^2+R^2)$ is equal to _____________.

Ans. (18)

Sol. Finding right hand limit

$$\lim_{x\to 0^+}f(x)=\lim_{h\to 0}f(0+h)$$

$$=\lim_{h\to 0}f(h)$$

$$=\lim_{h\to 0}\frac{\cos^{-1}(1-h^2)\sin^{-1}(1-h)}{h(1-h^2)}$$

$$=\lim_{h\to 0}\frac{\cos^{-1}(1-h^2)}{h}\left(\frac{\sin^{-1}1}{1}\right)$$

Let $\cos^{-1}(1-h^2)=\theta\Rightarrow\cos\theta=1-h^2$

$$=\frac{\pi}{2}\lim_{\theta\to 0}\frac{\theta}{\sqrt{1-\cos\theta}}$$

$$=\frac{\pi}{2}\lim_{\theta\to 0}\frac{1}{\sqrt{\frac{1-\cos\theta}{\theta^2}}}$$

$$=\frac{\pi}{2}\frac{1}{\sqrt{1/2}}$$

$$R=\frac{\pi}{\sqrt{2}}$$

Now finding left hand limit

$$L = \lim_{x\to 0^-} f(x)$$

$$= \lim_{h\to 0} f(-h)$$

$$= \lim_{h\to 0} \frac{\cos^{-1}\left(1-\{-h\}^2\right)\sin^{-1}\left(1-\{-h\}\right)}{\{-h\}-\{-h\}^3}$$

$$= \lim_{h\to 0} \frac{\cos^{-1}\left(1-(-h+1)^2\right)\sin^{-1}\left(1-(-h+1)\right)}{(-h+1)-(-h+1)^3}$$

$$= \lim_{h\to 0} \frac{\cos^{-1}\left(-h^2+2h\right)\sin^{-1} h}{(1-h)\left(1-(1-h)^2\right)}$$

$$= \lim_{h\to 0} \left(\frac{\pi}{2}\right)\frac{\sin^{-1} h}{\left(1-(1-h)^2\right)}$$

$$= \frac{\pi}{2}\lim_{h\to 0}\left(\frac{\sin^{-1} h}{-h^2+2h}\right)$$

$$= \frac{\pi}{2}\lim_{h\to 0}\left(\frac{\sin^{-1} h}{h}\right)\left(\frac{1}{-h+2}\right)$$

$$L = \frac{\pi}{4}$$

$$\frac{32}{\pi^2}\left(L^2+R^2\right) = \frac{32}{\pi^2}\left(\frac{\pi^2}{2}+\frac{\pi^2}{16}\right)$$

$= 16 + 2$

$= 18$

26. Let the line L : $\sqrt{2}\,x + y = \alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles C_1 and C_2 of equal radius $2\sqrt{3}$. If the centres Q_1 and Q_2 of the circles C_1 and C_2 lie on the y-axis, then the square of the area of the triangle PQ_1Q_2 is equal to ____________.

Ans. (72)

Sol. $x^2 + y^2 = 3$ and $x^2 = 2y$

$y^2 + 2y - 3 = 0 \Rightarrow (y + 3)(y - 1) = 0$

$y = -3$ or $y = 1$

$y = 1$ $x = \sqrt{2} \Rightarrow P(\sqrt{2}, 1)$

p lies on the line

$$\sqrt{2}x + y = \alpha$$
$$\sqrt{2}(\sqrt{2}) + 1 = \alpha$$
$$\alpha = 3$$

For circle C_1

Q_1 lies on y axis

Let Q_1 $(0, \alpha)$ coordinates

$R_1 = 2\sqrt{3}$ (Given

Line L act as tangent

Apply P = r (condition of tangency)

$$\Rightarrow \left|\frac{\alpha-3}{\sqrt{3}}\right| = 2\sqrt{3}$$
$$\Rightarrow |\alpha - 3| = 6$$

$\alpha - 3 = 6$ or $\alpha - 3 = -6$

$\Rightarrow \alpha = 9$ $\alpha = -3$

$$\triangle PQ_1Q_2 = \frac{1}{2}\begin{vmatrix} \sqrt{2} & 1 & 1 \\ 0 & 9 & 1 \\ 0 & -3 & 1 \end{vmatrix}$$

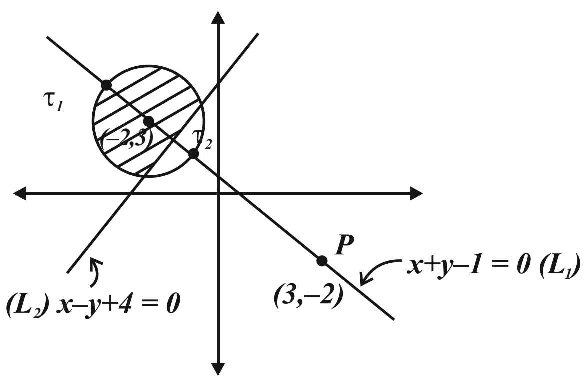
$$= \frac{1}{2}\left(\sqrt{2}(12)\right) = 6\sqrt{2}$$

$$\left(\triangle PQ_1Q_2\right)^2 = 72$$

27. Let P = {z ∈ : |z + 2 – 3i | ≤ 1} and Q = {z ∈ ℂ : z (l + i) + $\bar{z}$ (1 – i) ≤ –8} . Let in P ⋂ Q, |z – 3 + 2i| be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then α + β equals ________.

Ans. (36)

Sol.



Clearly for the shaded region z_1 is the intersection of the circle and the line passing through P (L_1) and z_2 is intersection of line L_1 & L_2

Circle : $(x + 2)^2 + (y - 3)^2 = 1$

$L_1 : x + y – 1 = 0$

$L_2 : x – y + 4 = 0$

On solving circle & L_1 we get

$z_1 : \left(-2-\frac{1}{\sqrt{2}}, 3+\frac{1}{\sqrt{2}}\right)$

On solving L_1 and z_2 is intersection of line L_1 & L_2 we get $z_2 : \left(\frac{-3}{2}, \frac{5}{2}\right)$

$$|z_1|^2 + 2|z_2|^2 = 14 + 5\sqrt{2} + 17$$
$$= 31 + 5\sqrt{2}$$

So $\alpha = 31$

$\beta = 5$

$\alpha + \beta = 36$

28. If $\int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x dx}{(1+e^{\sin x})(1+\sin^4 x)} = \alpha\pi + \beta \log_e(3 + 2\sqrt{2})$, where α, β are integers, then $\alpha^2 + \beta^2$ equals ___________.

Ans. (8)

Sol. $$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{\left(1+e^{\sin x}\right)\left(1+\sin^4 x\right)} dx$$

Apply king

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x\left(e^{\sin x}\right)}{\left(1+e^{\sin x}\right)\left(1+\sin^4 x\right)} dx \quad ....(2)$$

adding (1) & (2)

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{1+\sin^4 x} dx$$

$$I = \int_{0}^{\frac{\pi}{2}} \frac{8\sqrt{2}\cos x}{1+\sin^4 x} dx,$$

$\sin x = t$

$$I = \int_0^1 \frac{8\sqrt{2}}{1+t^4} dx$$

$$I = 4\sqrt{2}\int_0^1 \left(\frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}} - \frac{1-\frac{1}{t^2}}{t^2+\frac{1}{t^2}}\right) dt$$

$$I = 4\sqrt{2}\int_0^1 \frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+2} - \frac{\left(1-\frac{1}{t^2}\right)}{\left(t+\frac{1}{t}\right)^2-2} dt$$

$$Let\ t-\frac{1}{t} = z \ \&\ t+\frac{1}{t} = k$$

$$= 4\sqrt{2}\left[\int_{-\infty}^{0} \frac{dz}{z^2+2} - \int_{\infty}^{2} \frac{dk}{k^2-2}\right]$$

$$= 4\sqrt{2}\left[\frac{1}{\sqrt{2}}\tan^{-1}\frac{z}{\sqrt{2}}\right]_{-\infty}^{0} - \left[\frac{1}{2\sqrt{2}}\ln\left(\frac{k-\sqrt{2}}{k+\sqrt{2}}\right)\right]_{\infty}^{2}$$

$$= 4\sqrt{2}\left[\frac{\pi}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\left[\ln\frac{2-\sqrt{2}}{2+\sqrt{2}}\right]\right]$$

$$= 2\pi + 2\ln(3+2\sqrt{2})$$

$\alpha = 2$

$\beta = 2$

29. Let the line of the shortest distance between the lines

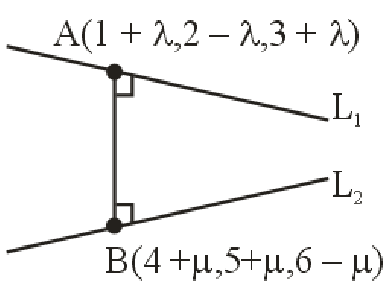
$L_1 : \vec{r} = \left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$ and

$L_2 : \vec{r} = \left(4\hat{i}+5\hat{j}+6\hat{k}\right)+\mu\left(\hat{i}+\hat{j}-\hat{k}\right)$

intersect L_1 and L_2 at P and Q respectively. If (α, β, γ) is the midpoint of the line segment PQ, then $2(\alpha + \beta + \gamma)$ is equal to ___________.

Ans. (21)

Sol.



$\vec{b} = \hat{i} - \hat{j} + \hat{k}$ (DR's of L_1)

$\vec{d} = \hat{i} + \hat{j} - \hat{k}$ (DR's of L_2)

$$\vec{b}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$= 0\hat{i} + 2\hat{j} + 2\hat{k}$ (DR's of Line perpendicular to L_1 and L_2)

DR of AB line

$= (0,2,2) = (3+\mu-\lambda, 3+\mu+\lambda, 3-\mu-\lambda)$

$$\frac{3+\mu-\lambda}{0} = \frac{3+\mu+\lambda}{2} = \frac{3-\mu-\lambda}{2}$$

Solving above equation we get $\mu = -\frac{3}{2}$ and $\lambda = \frac{3}{2}$

point $A = \left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right)$

$B = \left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$

Point of AB $= \left(\frac{5}{2}, 2, 6\right) = (\alpha, \beta, \gamma)$

$2(\alpha + \beta + \gamma) = 5 + 4 + 12 = 21$

30. Let A= {1, 2, 3, . . 20}. Let R_1 and R_2 two relation on A such that

R_1 = {(a, b) : b is divisible by a}

R_2 = {(a, b) : a is an integral multiple of b}.

Then, number of elements in $R_1 - R_2$ is equal to ________.

Ans. (46)

Sol. $n(R_1) = 20 + 10 + 6 + 5 + 4 + 3 + 2 + 2 + 2$

$+2 + \underbrace{1 + \ldots + 1}_{10\,\text{times}}$

$n(R_1) = 66$

$$R_1 \cap R_2 = \{(1,1),(2,2),\ldots(20,20)\}$$

$$n(R_1 \cap R_2) = 20$$

$$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$$

$$= n(R_1) - 20$$

$= 66 - 20$

$R_1 - R_2 = 46$ Pair

SECTION-A

31. With rise in temperature, the Young's modulus of elasticity

(1) changes erratically

(2) decreases

(3) increases

(4) remains unchanged

Ans. **(2)**

Sol. Conceptual questions

32. If R is the radius of the earth and the acceleration due to gravity on the surface of earth is $g = \pi^2$ m/s^2, then the length of the second's pendulum at a height h = 2R from the surface of earth will be,:

(1) $\frac{2}{9}$m

(2) $\frac{1}{9}$m

(3) $\frac{4}{9}$m

(4) $\frac{8}{9}$m

Ans. **(2)**

Sol. $g' = \frac{GMe}{(3R)^2} = \frac{1}{9}g$

$T = 2\pi\sqrt{\frac{\ell}{g'}}$

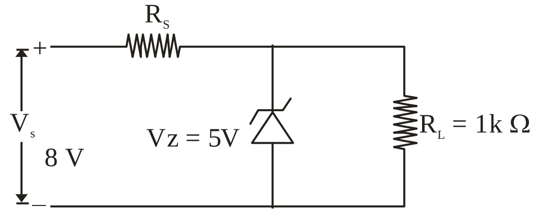
Since the time period of second pendulum is 2 sec.

T = 2 sec

$2 = 2\pi\sqrt{\frac{\ell}{g}9}$

$\boxed{\ell = \frac{1}{9}m}$

33. In the given circuit if the power rating of Zener diode is 10 mW, the value of series resistance R_s to regulate the input unregulated supply is :

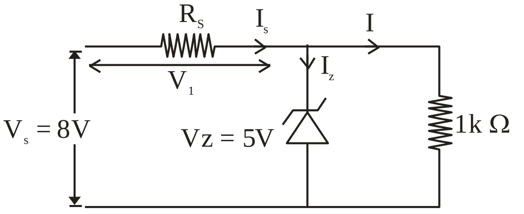


(1) 5kΩ (2) 10Ω

(3) 1kΩ (4) 10kΩ

Ans. **(BONUS)**

Sol.



Pd across R_s

$V_1 = 8 - 5 = 3V$

Current through the load resistor

$I = \frac{5}{1 \times 10^3} = 5mA$

Maximum current through Zener diode

$I_{z\ max.} = \frac{10}{5} = 2mA$

And minimum current through Zener diode

$I_{z\ min.} = 0$

$\therefore I_{s\ max.} = 5 + 2 = 7mA$

And $R_{s\ min} = \frac{V_1}{I_{s\ max.}} = \frac{3}{7}k\Omega$

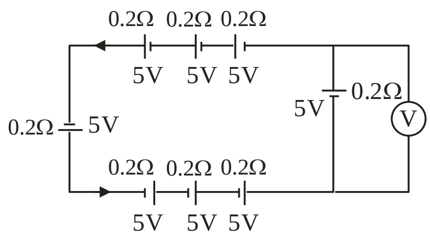
Similarly

$I_{s\ min.} = 5mA$

And $R_{s\ max.} = \frac{V_1}{I_{s\ min.}} = \frac{3}{5}k\Omega$

$\therefore \frac{3}{7}k\Omega < R_s < \frac{3}{5}k\Omega$

34. The reading in the ideal voltmeter (V) shown in the given circuit diagram is :



(1) 5V (2) 10V

(3) 0 V (4) 3V

Ans. **(3)**

Sol. $i = \frac{E_{eq}}{r_{eq}} = \frac{8 \times 5}{8 \times 0.2}$

$I = 25A$

$V = E - ir$

$= 5 - 0.2 \times 25$

$= 0$

35. Two identical capacitors have same capacitance C. One of them is charged to the potential V and other to the potential 2V. The negative ends of both are connected together. When the positive ends are also joined together, the decrease in energy of the combined system is :

(1) $\frac{1}{4}CV^2$

(2) $2\ CV^2$

(3) $\frac{1}{2}CV^2$

(4) $\frac{3}{4}CV^2$

Ans. **(1)**

Sol. $V_C = \frac{q_{net}}{C_{net}} = \frac{CV + 2CV}{2C}$

$V_C = \frac{3V}{2}$

Loss of energy

$= \frac{1}{2}CV^2 + \frac{1}{2}C(2V)^2 - \frac{1}{2}2C\left(\frac{3V}{2}\right)^2$

$= \left(\frac{CV^2}{4}\right)$

36. Two moles a monoatomic gas is mixed with six moles of a diatomic gas. The molar specific heat of the mixture at constant volume is :

(1) $\frac{9}{4}R$ (2) $\frac{7}{4}R$

(3) $\frac{3}{2}R$ (4) $\frac{5}{2}R$

Ans. **(1)**

Sol. $C_V = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$

$= \frac{2 \times \frac{3}{2}R + 6 \times \frac{5}{2}R}{2 + 6}$

$= \frac{9}{4}R$

37. A ball of mass 0.5 kg is attached to a string of length 50 cm. The ball is rotated on a horizontal circular path about its vertical axis. The maximum tension that the string can bear is 400 N. The maximum possible value of angular velocity of the ball in rad/s is,:

(1) 1600 (2) 40

(3) 1000 (4) 20

Ans. **(2)**

Sol. $T = m\omega^2\ell$

$400 = 0.5\omega^2 \times 0.5$

$\omega = 40$ rad/s.

38. A parallel plate capacitor has a capacitance C = 200 pF. It is connected to 230 V ac supply with an angular frequency 300 rad/s. The rms value of conduction current in the circuit and displacement current in the capacitor respectively are :

(1) 1.38 μA and 1.38 μA

(2) 14.3 μA and 143 μA

(3) 13.8 μA and 138 μA

(4) 13.8 μA and 13.8 μA

Ans. **(4)**

Sol. $I = \frac{V}{X_C} = 230 \times 300 \times 200 \times 10^{-12} = 13.8\ \mu A$

39. The pressure and volume of an ideal gas are related as $PV^{3/2} = K$ (Constant). The work done when the gas is taken from state A (P_1, V_1, T_1) to state B (P_2, V_2, T_2) is :

(1) $2(P_1V_1 - P_2V_2)$

(2) $2(P_2V_2 - P_1V_1)$

(3) $2(\sqrt{P_1}V_1 - \sqrt{P_2}V_2)$

(4) $2(P_2\sqrt{V_2} - P_1\sqrt{V_1})$

Ans. **(1 or 2)**

Sol. For PV^x = constant

If work done by gas is asked then

$$W = \frac{nR\Delta T}{1-x}$$

Here $x = \frac{3}{2}$

$$\therefore W = \frac{P_2V_2 - P_1V_1}{-\frac{1}{2}}$$

$= 2(P_1V_1 - P_2V_2)$ ….. Option (1) is correct

If work done by external is asked then

$W = -2(P_1V_1 - P_2V_2)$ ….. Option (2) is correct

40. A galvanometer has a resistance of 50 Ω and it allows maximum current of 5 mA. It can be converted into voltmeter to measure upto 100 V by connecting in series a resistor of resistance

(1) 5975 Ω

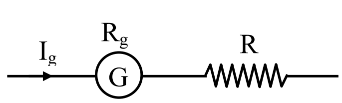
(2) 20050 Ω

(3) 19950 Ω

(4) 19500 Ω

Ans. **(3)**

Sol.



$$R = \frac{V}{I_g} - R_g = \frac{100}{5\times10^{-3}} - 50$$

$= 20000 - 50$

$= 19950\Omega$

41. The de Broglie wavelengths of a proton and an α particle are λ and 2 λ respectively. The ratio of the velocities of proton and α particle will be :

(1) 1 : 8

(2) 1 : 2

(3) 4 : 1

(4) 8 : 1

Ans. **(4)**

Sol. $\lambda = \frac{h}{p} = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda}$

$$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} \times \frac{\lambda_\alpha}{\lambda_p}$$

$= 4 \times 2 = 8$

42. 10 divisions on the main scale of a Vernier calliper coincide with 11 divisions on the Vernier scale. If each division on the main scale is of 5 units, the least count of the instrument is :

(1) $\frac{1}{2}$

(2) $\frac{10}{11}$

(3) $\frac{50}{11}$

(4) $\frac{5}{11}$

Ans. **(4)**

Sol. 10 MSD = 11 VSD

$1 \text{ VSD} = \frac{10}{11}\text{MSD}$

LC = 1MSD – 1VSD

$= 1 \text{ MSD} - \frac{10}{11}\text{MSD}$

$= \frac{1\text{MSD}}{11}$

$= \frac{5}{11}$ units

43. In series LCR circuit, the capacitance is changed from C to 4C. To keep the resonance frequency unchanged, the new inductance should be :

(1) reduced by $\frac{1}{4}L$

(2) increased by 2L

(3) reduced by $\frac{3}{4}L$

(4) increased to 4L

Ans. **(3)**

Sol. $\omega' = \omega$

$$\frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{LC}}$$

$\therefore$ $L'C' = LC$

$L'(4C) = LC$

$L' = \frac{L}{4}$

$\because$ Inductance must be decreased by $\frac{3L}{4}$

44. The radius (r), length (l) and resistance (R) of a metal wire was measured in the laboratory as

$r = (0.35 \pm 0.05)$ cm

$R = (100 \pm 10)$ ohm

$l = (15 \pm 0.2)$ cm

The percentage error in resistivity of the material of the wire is :

(1) 25.6% (2) 39.9%

(3) 37.3% (4) 35.6%

Ans. **(2)**

Sol. $\rho = R\,\frac{\rho}{\ell}$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta R}{R} + 2\frac{\Delta r}{r} + \frac{\Delta\ell}{\ell}$$

$$= \frac{10}{100} + 2\times\frac{0.05}{0.35} + \frac{0.2}{15}$$

$$= \frac{1}{10} + \frac{2}{7} + \frac{1}{75}$$

$$\frac{\Delta\rho}{\rho} = 39.9\%$$

45. The dimensional formula of angular impulse is :

(1) $[M\,L^{-2}\,T^{-1}]$ (2) $[M\,L^{2}\,T^{-2}]$

(3) $[M\,L\,\,T^{-1}]$ (4) $[M\,L^{2}\,T^{-1}]$

Ans. **(4)**

Sol. Angular impulse = change in angular momentum.

[Angular impulse] = [Angular momentum] = [mvr]

$= [M\,L^2\,T^{-1}]$

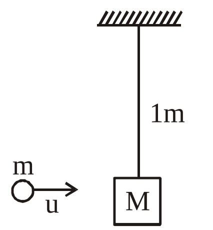
46. A simple pendulum of length 1 m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of 2×10^2 ms^{-1}. The bullet gets embedded into the bob. The height to which the bob rises before swinging back is. (use $g = 10\ m/s^2$)

(1) 0.30 m (2) 0.20 m

(3) 0.35 m (4) 0.40 m

Ans. **(2)**

Sol.



$mu = (M + m)V$

$10^{-2} \times 2 \times 10^2 \cong 1 \times V$

$V \cong 2m/s$

$h = \frac{V^2}{2g} = 0.2$ m

47. A particle moving in a circle of radius R with uniform speed takes time T to complete one revolution. If this particle is projected with the same speed at an angle θ to the horizontal, the maximum height attained by it is equal to 4R. The angle of projection θ is then given by :

(1) $\sin^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (2) $\sin^{-1}\left[\frac{\pi^2 R}{2gT^2}\right]^{\frac{1}{2}}$

(3) $\cos^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (4) $\cos^{-1}\left[\frac{\pi R}{2gT^2}\right]^{\frac{1}{2}}$

Ans. **(1)**

Sol. $\frac{2\pi R}{T} = V$

Maximum height $H = \frac{v^2 \sin^2\theta}{2g}$

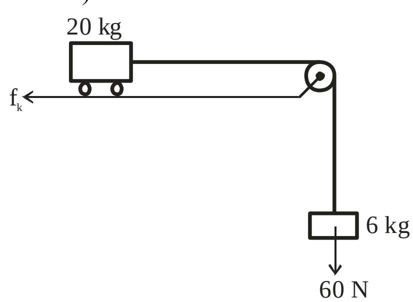
$4R = \frac{4\pi^2 R^2}{T^2 2g}\sin^2\theta$

$\sin\theta = \sqrt{\frac{2gT^2}{\pi^2 R}}$

$\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{\frac{1}{2}}$

48. Consider a block and trolley system as shown in figure. If the coefficient of kinetic friction between the trolley and the surface is 0.04, the acceleration of the system in ms^{-2} is :

(Consider that the string is massless and unstretchable and the pulley is also massless and frictionless) :



(1) 3 (2) 4

(3) 2 (4) 1.2

Ans. **(3)**

Sol. $f_k = \mu N = 0.04 \times 20g = 8$ Newton

$a = \frac{60-8}{26} = 2m/s^2$

49. The minimum energy required by a hydrogen atom in ground state to emit radiation in Balmer series is nearly :

(1) 1.5 eV (2) 13.6 eV

(3) 1.9 eV (4) 12.1 eV

Ans. **(4)**

Sol. Transition from n = 1 to n = 3

$\Delta E = 12.1$eV

50. A monochromatic light of wavelength 6000Å is incident on the single slit of width 0.01 mm. If the diffraction pattern is formed at the focus of the convex lens of focal length 20 cm, the linear width of the central maximum is :

(1) 60 mm

(2) 24 mm

(3) 120 mm

(4) 12 mm

Ans. **(2)**

Sol. Linear width

$W = \frac{2\lambda d}{a} = \frac{2 \times 6 \times 10^{-7} \times 0.2}{1 \times 10^{-5}}$

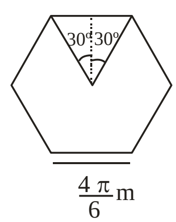
$= 2.4 \times 10^{-2} = 24$ mm

SECTION-B

51. A regular polygon of 6 sides is formed by bending a wire of length 4 π meter. If an electric current of $4\pi\sqrt{3}$ A is flowing through the sides of the polygon, the magnetic field at the centre of the polygon would be x × 10^{-7} T. The value of x is ____.

Ans. **(72)**

Sol.



$B = 6\left(\frac{\mu_0 I}{4\pi r}\right)(\sin 30^\circ + \sin 30^\circ)$

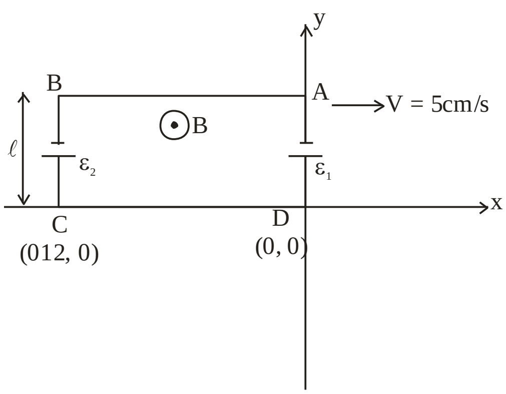
$= 6\frac{10^{-7} \times 4\pi\sqrt{3}}{\left(\frac{\sqrt{3} \times 4\pi}{2 \times 6}\right)}$

$= 72 \times 10^{-7}$T

52. A rectangular loop of sides 12 cm and 5 cm, with its sides parallel to the x-axis and y-axis respectively moves with a velocity of 5 cm/s in the positive x axis direction, in a space containing a variable magnetic field in the positive z direction. The field has a gradient of 10^{-3}T/cm along the negative x direction and it is decreasing with time at the rate of 10^{-3} T/s. If the resistance of the loop is 6 mΩ, the power dissipated by the loop as heat is _______ × 10^{-9} W.

Ans. **(216)**

Sol.



B_0 is the magnetic field at origin

$$\frac{dB}{dx} = -\frac{10^{-3}}{10^{-2}}$$

$$\int_{B_0}^{B} dB = -\int_0^x 10^{-1} dx$$

$$B - B_0 = -10^{-1}x$$

$$B = \left(B_0 - \frac{x}{10}\right)$$

Motional emf in AB = 0

Motional emf in CD = 0

Motional emf in AD = $\varepsilon_1 = B_0 \ell v$

Magnetic field on rod BC B

$$= \left(B_0 - \frac{(-12 \times 10^{-2})}{10}\right)$$

Motional emf in BC = $\varepsilon_2 = \left(B_0 + \frac{12 \times 10^{-2}}{10}\right)\ell \times v$

$\varepsilon_{eq} = \varepsilon_2 - \varepsilon_1 = 300 \times 10^{-7}$ V

For time variation

$(\varepsilon_{eq})' = A \frac{dB}{dt} = 60 \times 10^{-7}$ V

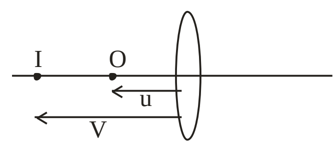
$(\varepsilon_{eq})_{net} = \varepsilon_{eq} + (\varepsilon_{eq})' = 360 \times 10^{-7}$ V

$$\text{Power} = \frac{\left(\varepsilon_{eq}\right)_{net}^2}{R} = 216 \times 10^{-9} \text{ W}$$

53. The distance between object and its 3 times magnified virtual image as produced by a convex lens is 20 cm. The focal length of the lens used is __________ cm.

Ans. **(15)**

Sol.



v = 3u

v – u = 20 cm

2u = 20 cm

u = 10 cm

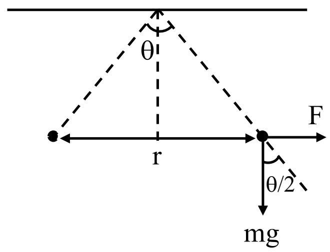
$$\frac{1}{(-30)} - \frac{1}{(-10)} = \frac{1}{f}$$

f = 15 cm

54. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle θ with each other. When suspended in water the angle remains the same. If density of the material of the sphere is 1.5 g/cc, the dielectric constant of water will be ________

(Take density of water = 1 g/cc)

Ans. **(3)**

Sol. 

In air $\tan\frac{\theta}{2} = \frac{F}{mg} = \frac{q^2}{4\pi\varepsilon_0 r^2 mg}$

In water $\tan\frac{\theta}{2} = \frac{F'}{mg'} = \frac{q^2}{4\pi\varepsilon_0\varepsilon_r r^2 mg_{eff}}$

Equate both equations

$\varepsilon_0 g = \varepsilon_0\ \varepsilon_r\ g\left[1-\frac{1}{1.5}\right]$

$\varepsilon_r = 3$

55. The radius of a nucleus of mass number 64 is 4.8 fermi. Then the mass number of another nucleus having radius of 4 fermi is $\frac{1000}{x}$, where x is _____.

Ans. (27)

Sol. $R = R_0 A^{1/3}$

$R^3 \propto A$

$\left(\frac{4.8}{4}\right)^3 = \frac{64}{A}$

$= \frac{64}{A} = (1.2)^3$

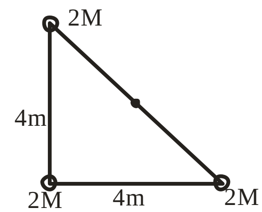
$\frac{64}{A} = 1.44\times 1.2$

$A = \frac{64}{1.44\times 1.2} = \frac{1000}{x}$

$x = \frac{144\times 12}{64} = 27$

56. The identical spheres each of mass 2M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 4 m each. Taking point of intersection of these two sides as origin, the magnitude of position vector of the centre of mass of the system is $\frac{4\sqrt{2}}{x}$, where the value of x is ___________

Ans. (3)

Sol. 

Position vector $\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1+m_2+m_3}$

$\vec{r}_{COM} = \frac{2M\times 0 + 2M\times 4\hat{i} + 2M\times 4\hat{j}}{6M}$

$\vec{r} = \frac{4}{3}\hat{i} + \frac{4}{3}\hat{j}$

$|\vec{r}| = \frac{4\sqrt{2}}{3}$

$x = 3$

57. A tuning fork resonates with a sonometer wire of length 1 m stretched with a tension of 6 N. When the tension in the wire is changed to 54 N, the same tuning fork produces 12 beats per second with it. The frequency of the tuning fork is _______ Hz.

Ans. (6)

Sol. $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$f_1 = \frac{1}{2}\sqrt{\frac{6}{\mu}}$ $\qquad$ $f_2 = \frac{1}{2}\sqrt{\frac{54}{\mu}}$

$\frac{f_1}{f_2} = \frac{1}{3}$ $\qquad$ $f_2 - f_1 = 12$

$f_1 = 6HZ$

58. A plane is in level flight at constant speed and each of its two wings has an area of 40 m^2. If the speed of the air is 180 km/h over the lower wing surface and 252 km/h over the upper wing surface, the mass of the plane is ________kg. (Take air density to be 1 kg m^{-3} and g = 10 ms^{-2})

Ans. **(9600)**

Sol. A = 80 m^2

Using Bernonlli equation

$$A(P_2 - P_1) = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)A$$

$$mg = \frac{1}{2} \times 1\,(70^2 - 50^2) \times 80$$

mg = 40 × 2400

m = 9600 kg

59. The current in a conductor is expressed as $I = 3t^2 + 4t^3$, where I is in Ampere and t is in second. The amount of electric charge that flows through a section of the conductor during t = 1s to t = 2s is ____________ C.

Ans. **(22)**

Sol. $q = \int_1^2 i\,dt = \int_1^2 (3t^2 + 4t^3)dt$

$$q = \left(t^3 + t^4\right)\Big|_1^2$$

q = 22C

60. A particle is moving in one dimension (along x axis) under the action of a variable force. It's initial position was 16 m right of origin. The variation of its position (x) with time (t) is given as $x = -3t^3 + 18t^2 + 16t$, where x is in m and t is in s. The velocity of the particle when its acceleration becomes zero is _________ m/s.

Ans. **(52)**

Sol. $x = 3t^3 + 18t^2 + 16t$

$\mathbf{v} = -9t^2 + 36 + 16$

$a = -18t + 36$

a = 0 at t = 2s

$v = -9(2)^2 + 36 \times 2 + 16$

v = 52 m/s

SECTION-A

61. If one strand of a DNA has the sequence ATGCTTCA, sequence of the bases in complementary strand is:

(1) CATTAGCT (2) TACGAAGT

(3) GTACTTAC (4) ATGCGACT

Ans. **(2)**

Sol. Adenine base pairs with thymine with 2 hydrogen bonds and cytosine base pairs with guanine with 3 hydrogen bonds.

A T G C T T C A ⟶ DNA strand
|| || ||| ||| || || ||| || ⟶ Hydrogen bonds
T A C G A A G T ⟶ Complementary strand

62. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Haloalkanes react with KCN to form alkyl cyanides as a main product while with AgCN form isocyanide as the main product.

Reason (R) : KCN and AgCN both are highly ionic compounds.

In the light of the above statement, choose the most appropriate answer from the options given below:

(1) (A) is correct but (R) is not correct

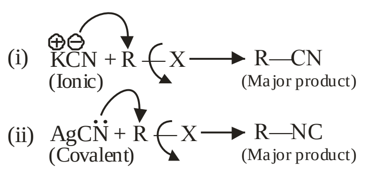
(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(3) (A) is not correct but (R) is correct

(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Ans. (1)

Sol.



AgCN is mainly covalent in nature and nitrogen is available for attack, so alkyl isocyanide is formed as main product.

63. In acidic medium, $K_2Cr_2O_7$ shows oxidising action as represented in the half reaction

$$Cr_2O_7^{2-} + XH^+ + Ye^- \rightarrow 2A + ZH_2O$$

X, Y, Z and A are respectively are:

(1) 8, 6, 4 and Cr_2O_3 (2) 14, 7, 6 and Cr^{3+}

(3) 8, 4, 6 and Cr_2O_3 (4) 14, 6, 7 and Cr^{3+}

Ans. **(4)**

Sol. The balanced reaction is,

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$$

X = 14

Y = 6

A = 7

64. Which of the following reactions are disproportionation reactions?

(A) $Cu^+ \rightarrow Cu^{2+} + Cu$

(B) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

(C) $2KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$

(D) $2MnO_4^- + 3Mn^{2+} + 2H_2O \rightarrow 5MnO_2 + 4H^+$

Choose the correct answer from the options given below:

(1) (A), (B) (2) (B), (C), (D)

(3) (A), (B), (C) (4) (A), (D)

Ans. **(1)**

Sol. When a particular oxidation state becomes less stable relative to other oxidation state, one lower, one higher, it is said to undergo disproportionation.

$Cu^+ \rightarrow Cu^{2+} + Cu$

$3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

65. In case of isoelectronic species the size of F^-, Ne and Na^+ is affected by:

(1) Principal quantum number (n)

(2) None of the factors because their size is the same

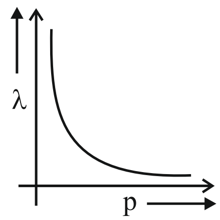
(3) Electron-electron interaction in the outer orbitals

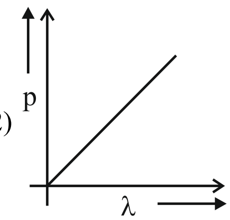
(4) Nuclear charge (z)

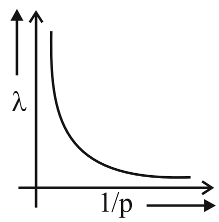
Ans. **(4)**

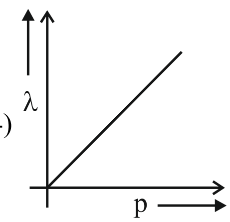
Sol. In F^-, Ne, Na^+ all have $1s^2$, $2s^2$, $2p^6$ configuration. They have different size due to the difference in nuclear charge.

66. According to the wave-particle duality of matter by de-Broglie, which of the following graph plot presents most appropriate relationship between wavelength of electron (λ) and momentum of electron (p)?

(1) 

(2) 

(3) 

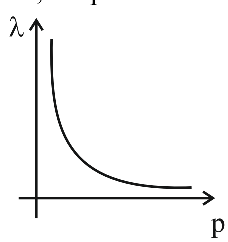
(4) 

Ans. **(1)**

Sol. $\lambda = \frac{h}{p}\left[\lambda \propto \frac{1}{p}\right]$

$\Rightarrow \lambda p = h$ (constant)

So, the plot is a rectangular hyperbola.



67. Given below are two statements:

Statement (I): A solution of $[Ni(H_2O)_6]^{2+}$ is green in colour.

Statement (II): A solution of $[Ni(CN)_4]^{2-}$ is colourless.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are incorrect

(2) Both Statement I and Statement II are correct

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Ans. **(2)**

Sol. $[Ni(H_2O)_6]^{+2} \rightarrow$ Green colour solution due to d-d transition.

$[Ni(CN)_4]^{-2} \rightarrow$ is diamagnetic and it is colourless.

68. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : PH_3 has lower boiling point than NH_3.

Reason (R) : In liquid state NH_3 molecules are associated through vander waal's forces, but PH_3 molecules are associated through hydrogen bonding.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both (A) and (R) are correct and (R) is not the correct explanation of (A)

(2) (A) is not correct but (R) is correct

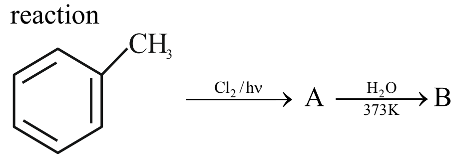
(3) Both (A) and (R) are correct but (R) is the correct explanation of (A)

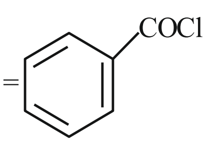
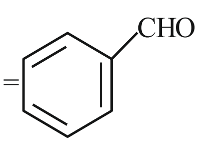
(4) (A) is correct but (R) is not correct

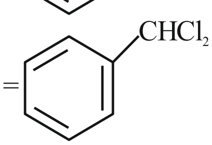
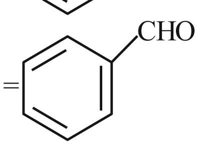
Ans. **(4)**

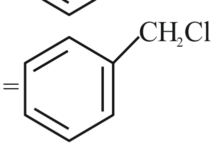
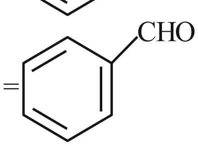
Sol. Unlike NH_3, PH_3 molecules are not associated through hydrogen bonding in liquid state. That is why the boiling point of PH_3 is lower than NH_3.

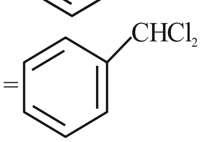
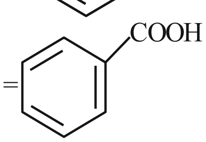
69. Identify A and B in the following sequence of reaction



(1) (A) =  (B) = 

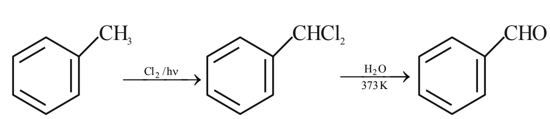
(2) (A) =  (B) = 

(3) (A) =  (B) = 

(4) (A) =  (B) = 

Ans. **(2)**

Sol.



Toluene → Benzal chloride → Benzaldehyde

70. Given below are two statements:

Statement (I) : Aminobenzene and aniline are same organic compounds.

Statement (II) : Aminobenzene and aniline are different organic compounds.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both Statement I and Statement II are correct

(2) Statement I is correct but Statement II is incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are incorrect

Ans. (2)

Sol. Aniline is also known as amino benzene.

71. Which of the following complex is homoleptic?

(1) $[Ni(CN)_4]^{2-}$

(2) $[Ni(NH_3)_2Cl_2]$

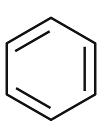
(3) $[Fe(NH_3)_4Cl_2]^+$

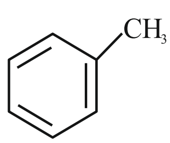
(4) $[Co(NH_3)_4Cl_2]^+$

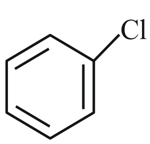
Ans. (1)

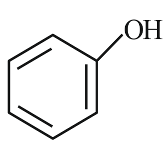
Sol. In Homoleptic complex all the ligand attached with the central atom should be the same. Hence $[Ni(CN)_4]^{2-}$ is a homoleptic complex.

72. Which of the following compound will most easily be attacked by an electrophile?

(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. Higher the electron density in the benzene ring more easily it will be attacked by an electrophile. Phenol has the highest electron density amongst all the given compound.

73. Ionic reactions with organic compounds proceed through:

(A) Homolytic bond cleavage

(B) Heterolytic bond cleavage

(C) Free radical formation

(D) Primary free radical

(E) Secondary free radical

Choose the correct answer from the options given below:

(1) (A) only

(2) (C) only

(3) (B) only

(4) (D) and (E) only

Ans. (3)

Sol. Heterolytic cleavage of Bond lead to formation of ions.

74. Arrange the bonds in order of increasing ionic character in the molecules. LiF, K_2O, N_2, SO_2 and ClF_3.

(1) $ClF_3 < N_2 < SO_2 < K_2O < LiF$

(2) $LiF < K_2O < ClF_3 < SO_2 < N_2$

(3) $N_2 < SO_2 < ClF_3 < K_2O < LiF$

(4) $N_2 < ClF_3 < SO_2 < K_2O < LiF$

Ans. (3)

Sol. Increasing order of ionic character

$N_2 < SO_2 < ClF_3 < K_2O < LiF$

Ionic character depends upon difference of electronegativity (bond polarity).

75. We have three aqueous solutions of NaCl labelled as ‘A’, ‘B’ and ‘C’ with concentration 0.1 M, 0.01M & 0.001 M, respectively. The value of van t’ Haft factor (i) for these solutions will be in the order.

(1) $i_A < i_B < i_C$

(2) $i_A < i_C < i_B$

(3) $i_A = i_B = i_C$

(4) $i_A > i_B > i_C$

Ans. (1)

Sol.

Salt	Values of i (for different conc. of a Salt)		
	0.1 M	0.01 M	0.001 M
NaCl	1.87	1.94	1.94

i approach 2 as the solution become very dilute.

76. In Kjeldahl's method for estimation of nitrogen, $CuSO_4$ acts as :

(1) Reducing agent (2) Catalytic agent

(3) Hydrolysis agent (4) Oxidising agent

Ans. (2)

Sol. Kjeldahl's method is used for estimation of Nitrogen where $CuSO_4$ acts as a catalyst.

77. Given below are two statements :

Statement (I) : Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution.

Statement (II) : In this titration phenolphthalein can be used as indicator.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both Statement I and Statement II are correct

(2) Statement I is correct but Statement II is incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are incorrect

Ans. (1)

Sol. **Statement (I) :** Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution as it is economical and its concentration does not changes with time.

Phenophthalin can acts as indicator in acid base titration as it shows colour in pH range 8.3 to 10.1

78. Match List – I with List –II.

List – I (Reactions)		List – II (Reagents)	
(A)	$CH_3(CH_2)_5-\underset{\underset{O}{\parallel}}{C}-OC_2H_5 \rightarrow CH_3(CH_2)_5CHO$	(I)	CH_3MgBr, H_2O
(B)	$C_6H_5COC_6H_5 \rightarrow C_6H_5CH_2C_6H_5$	(II)	Zn(Hg) and conc. HCl
(C)	$C_6H_5CHO \rightarrow C_6H_5CH(OH)CH_3$	(III)	$NaBH_4$, H^+
(D)	$CH_3COCH_2COOC_2H_5 \rightarrow CH_3\underset{\underset{H}{\mid}}{C}(OH)CH_2COOC_2H_5$	(IV)	DIBAL-H, H_2O

Choose the correct answer from options given below:

(1) A-(III), (B)-(IV), (C)-(I), (D)-(II)

(2) A-(IV), (B)-(II), (C)-(I), (D)-(III)

(3) A-(IV), (B)-(II), (C)-(III), (D)-(I)

(4) A-(III), (B)-(IV), (C)-(II), (D)-(I)

Ans. (2)

Sol. $CH_3(CH_2)_5COOC_2H_5 \xrightarrow{\text{DIBAL-H, } H_2O} CH_3(CH_2)_5CHO$

$C_6H_5COC_6H_5 \xrightarrow{\text{Zn(Hg) \& conc. HCl}} C_6H_5CH_2C_6H_5$

$C_6H_5CHO \xrightarrow[H_2O]{CH_3MgBr} C_6H_5CH(OH)CH_3$

$CH_3COCH_2COOC_2H_5 \xrightarrow{NaBH_4,\ H^+} CH_3CH(OH)CH_2COOC_2H_5$

79. Choose the correct option for free expansion of an ideal gas under adiabatic condition from the following :

(1) $q = 0, \Delta T \neq 0, w = 0$

(2) $q = 0, \Delta T < 0, w \neq 0$

(3) $q \neq 0, \Delta T = 0, w = 0$

(4) $q = 0, \Delta T = 0, w = 0$

Ans. (4)

Sol. During free expansion of an ideal gas under adiabatic condition $q = 0, \Delta T = 0, w = 0$.

80. Given below are two statements:

Statement (I) : The NH_2 group in Aniline is ortho and para directing and a powerful activating group.

Statement (II) : Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation).

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both Statement I and Statement II are correct

(2) Both Statement I and Statement II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Ans. (1)

Sol. The NH_2 group in Aniline is ortho and para directing and a powerful activating group as NH_2 has strong +M effect.

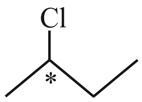
Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation) as Aniline will form complex with $AlCl_3$ which will deactivate the benzene ring.

SECTION-B

81. Number of optical isomers possible for

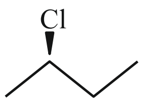
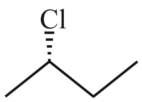
2 – chlorobutane ………

Ans. (2)

Sol. 

There is one chiral centre present in given compound.

So, Total optical isomers = 2

 and 

82. The potential for the given half cell at 298K is (-)……….. × 10^{-2} V.

$2H^+_{(aq)} + 2e^- \rightarrow H_2\,(g)$

$[H^+] = 1M,\ P_{H_2} = 2$ atm

(Given: 2.303 RT/F = 0.06 V, log2 = 0.3)

Ans. (1)

Sol. $E = E^o_{H^+/H_2} - \dfrac{0.06}{2}\log\dfrac{P_{H_2}}{[H^+]^2}$

$E = 0.00 - \dfrac{0.06}{2}\log\dfrac{2}{[1]^2}$

$E = -0.03 \times 0.3 = -0.9 \times 10^{-2}\,V$

83. The number of white coloured salts among the following is …………….

(A) $SrSO_4$ (B) $Mg(NH_4)PO_4$ (c) $BaCrO_4$

(D) $Mn(OH)_2$ (E) $PbSO_4$ (F) $PbCrO_4$

(G) AgBr (H) PbI_2 (I) CaC_2O_4

(J) $[Fe(OH)_2(CH_3COO)]$

Ans. (5)

Sol. $SrSO_4$ – white

$Mg(NH_4)PO_4$ – white

$BaCrO_4$ – yellow

$Mn(OH)_2$ – white

$PbSO_4$ – white

$PbCrO_4$ – yellow

AgBr – pale yellow

PbI_2 – yellow

CaC_2O_4 – white

$[Fe(OH)_2(CH_3COO)]$ – Brown Red

84. The ratio of $\dfrac{^{14}C}{^{12}C}$ in a piece of wood is $\dfrac{1}{8}$ part that of atmosphere. If half life of ^{14}C is 5730 years, the age of wood sample is ….. years.

Ans. (17190)

Sol. $\lambda t = \ln\dfrac{(^{14}C/^{12}C)_{atmosphere}}{(^{14}C/^{12}C)_{wood\ sample}}$

As per the question,

$\dfrac{(^{14}C/^{12}C)_{wood}}{(^{14}C/^{12}C)_{atmosphere}} = \dfrac{1}{8}$

So, $\lambda t = \ln 8$

$\dfrac{\ln 2}{t_{1/2}}t = \ln 8$

$t = 3 \times t_{1/2} = 17190$ years

85. The number of molecules/ion/s having trigonal bipyramidal shape is …….. .

PF_5, BrF_5, PCl_5, $[PtCl_4]^{2-}$, BF_3, $Fe(CO)_5$

Ans. (3)

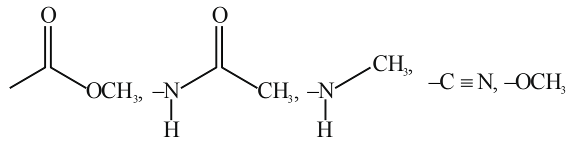
Sol. PF_5, PCl_5, $Fe(CO)_5$; Trigonal bipyramidal

BrF_5 ; square pyramidal

$[PtCl_4]^{-2}$; square planar

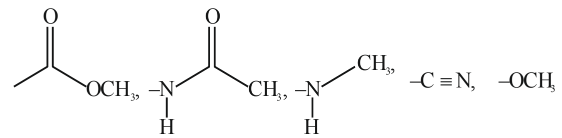
BF_3 ; Trigonal planar

86. Total number of deactivating groups in aromatic electrophilic substitution reaction among the following is



Ans. (2)

Sol.



(–M group) (+M group) (+M group) (–M group) (+M group)

87. Lowest Oxidation number of an atom in a compound A_2B is -2. The number of an electron in its valence shell is

Ans. (6)

Sol. $A_2B \rightarrow 2A^{+} + B^{-2}$, B^{-2} has complete octet in its di-anionic form, thus in its atomic state it has 6 electrons in its valence shell. As it has negative charge, it has acquired two electrons to complete its octet.

88. Among the following oxide of p - block elements, number of oxides having amphoteric nature is

Cl_2O_7, CO, PbO_2, N_2O, NO, Al_2O_3, SiO_2, N_2O_5, SnO_2

Ans. (3)

Sol. Acidic oxide: Cl_2O_7, SiO_2, N_2O_5

Neutral oxide: CO, NO, N_2O

Amphoteric oxide: Al_2O_3 , SnO_2 , PbO_2

89. Consider the following reaction:

$3PbCl_2 + 2(NH_4)_3PO_4 \rightarrow Pb_3(PO_4)_2 + 6NH_4Cl$

If 72 mmol of $PbCl_2$ is mixed with 50 mmol of $(NH_4)_3PO_4$, then amount of $Pb_3(PO_4)_2$ formed is …… mmol. (nearest integer)

Ans. (24)

Sol. Limiting Reagent is $PbCl_2$

mmol of $Pb_3(PO_4)_2$ formed

$= \frac{\text{mmol of } PbCl_2 \text{ reacted}}{3}$

= 24 mmol

90. K_a for CH_3COOH is 1.8×10^{-5} and K_b for NH_4OH is 1.8×10^{-5}. The pH of ammonium acetate solution will be

Ans. (7)

Sol. $pH = \frac{pK_w + pK_a - pK_b}{2}$

$pK_a = pK_b$

$\Rightarrow pH = \frac{pK_w}{2} = 7$