Vector Algebra

Question1

Let $\vec{a} = \hat{i} + 2\hat{j} + k$, $\vec{b} = 3(\hat{i} - \hat{j} + k)$. Let $\vec{c}$ be the vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$. Then $\vec{a} \cdot ((\vec{c} \times \vec{b}) - \vec{b} - \vec{c})$ is equal to :
[27-Jan-2024 Shift 1]

Options:

A. 32

B. 24

C. 20

D. 36

Answer: B

Solution:

Solution:

$\vec{a} \cdot [(\vec{c} \times \vec{b}) - \vec{b} - \vec{c}]$

$\vec{a} \cdot (\vec{c} \times \vec{b}) - \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c}$ ....(i)

given $\vec{a} \times \vec{c} = \vec{b}$

$\Rightarrow (\vec{a} \times \vec{c}) \cdot \vec{b} = \vec{b} \cdot \vec{b} = |\vec{b}|^2 = 27$

$\Rightarrow \vec{a} \cdot (\vec{c} \times \vec{b}) = [\vec{a} \ \vec{c} \ \vec{b}] = (\vec{a} \times \vec{c}) \cdot \vec{b} = 27$ ....(ii)

Now $\vec{a} \cdot \vec{b} = 3 - 6 + 3 = 0$ ....(iii)

$\vec{a} \cdot \vec{c} = 3$ ...(iv) (given)

By (i), (ii), (iii) & (iv)

$27 - 0 - 3 = 24$

Question2

The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2k$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2k$ is acute, is___
[27-Jan-2024 Shift 1]

Answer: 5

Solution:

$$\cos\theta = \frac{(\alpha\hat{i} - 2\hat{j} + 2\hat{k}) \cdot (\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k})}{\sqrt{\alpha^2 + 4 + 4}\,\sqrt{\alpha^2 + 4\alpha^2 + 4}}$$

$$\cos\theta = \frac{\alpha^2 - 4\alpha - 4}{\sqrt{\alpha^2 + 8}\,\sqrt{5\alpha^2 + 4}}$$

$$\Rightarrow \alpha^2 - 4\alpha - 4 > 0$$

$$\Rightarrow \alpha^2 - 4\alpha + 4 > 8$$

$$\Rightarrow (\alpha - 2)^2 > 8$$

$$\Rightarrow \alpha - 2 > 2\sqrt{2} \text{ or } \alpha - 2 < -2\sqrt{2}$$

$$\alpha > 2 + 2\sqrt{2} \text{ or } \alpha < 2 - 2\sqrt{2}$$

$$\alpha \in (-\infty, -0.82) \cup (4.82, \infty)$$

Least positive integral value of $\alpha \Rightarrow 5$

Question3

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear .if $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}$, $\vec{b} + 6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a} + \alpha\vec{b} + \beta\vec{c} = \vec{0}$, then $\alpha + \beta$ is equal to
[29-Jan-2024 Shift 1]

Options:

A. 35

B. 30

C. -30

D. -25

Answer: A

Solution:

Solution:

$\vec{a} + 5\vec{b} = \lambda\vec{c}$

$\vec{b} + 6\vec{c} = \mu\vec{a}$

Eliminating $\vec{a}$

$\lambda\vec{c} - 5\vec{b} = \frac{6}{\mu}\vec{c} + \frac{1}{\mu}\vec{b}$

$\therefore \mu = \frac{-1}{5}, \lambda = -30$

$\alpha = 5, \beta = 30$

Question4

Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = 12\vec{a} + 4\vec{b}$ and $\overrightarrow{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC, then $\frac{\text{area of the quadrilateral OABC}}{\text{area of S}}$ is equal to
[29-Jan-2024 Shift 2]

Options:

A. 6

B. 10

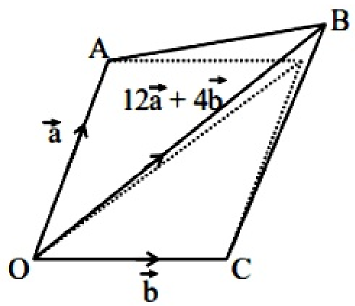
C. 7

D. 8

Answer: D

Solution:

Solution:



Area of parallelogram, $S = |\vec{a} \times \vec{b}|$

Area of quadrilateral $= \text{Area}(\triangle OAB) + \text{Area}(\triangle OBC)$

$= \frac{1}{2}\left\{ \left|\vec{a} \times (12\vec{a} + 4\vec{b})\right| + \left|\vec{b} \times (12\vec{a} + 4\vec{b})\right| \right\}$

$= 8\left|(\vec{a} \times \vec{b})\right|$

$\text{Ratio} = \frac{8\left|(\vec{a} \times \vec{b})\right|}{\left|(\vec{a} \times \vec{b})\right|} = 8$

Question5

Let $\vec{OA} = \vec{a}$, $\vec{OB} = 12\vec{a} + 4\vec{b}$ and $\vec{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC, then $\frac{\text{area of the quadrilateral OABC}}{\text{area of S}}$ is equal to
[29-Jan-2024 Shift 2]

Options:

A. 6

B. 10

C. 7

D. 8

Answer: D

Solution:

Solution:

Question6

Let $\vec{a} = a_i\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\vec{a}| = 1$; $\vec{a} \cdot \vec{b} = 2$ and $|\vec{b}| = 4$. If $\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to :
[30-Jan-2024 Shift 1]

Options:

A. $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

B. $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

C. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

D. $\cos^{-1}\left(\frac{2}{3}\right)$

Answer: C

Solution:

Given $|\vec{a}| = 1, |\vec{b}| = 4, \vec{a} \cdot \vec{b} = 2$

$\vec{c} = 2(\vec{a} \times \vec{b}) - 3\vec{b}$

Dot product with $\vec{a}$ on both sides

$\vec{c} \cdot \vec{a} = -6$ ....(1)

Dot product with $\vec{b}$ on both sides

$\vec{b} \cdot \vec{c} = -48$ .....(2)

$\vec{c} \cdot \vec{c} = 4|\vec{a} \times \vec{b}|^2 + 9|\vec{b}|^2$

$|\vec{c}|^2 = 4\left[|a|^2 |b|^2 - (a \cdot \vec{b})^2\right] + 9|\vec{b}|^2$

$|\vec{c}|^2 = 4[(1)(4)^2 - (4)] + 9(16)$

$|\vec{c}|^2 = 4[12] + 144$

$|\vec{c}|^2 = 48 + 144$

$|\vec{c}|^2 = 192$

$\therefore \cos\theta = \dfrac{\vec{b} \cdot \vec{c}}{|\vec{b}||\vec{c}|}$

$\therefore \cos\theta = \dfrac{-48}{\sqrt{192} \cdot 4}$

$\therefore \cos\theta = \dfrac{-48}{8\sqrt{3} \cdot 4}$

$\therefore \cos\theta = \dfrac{-3}{2\sqrt{3}}$

$\therefore \cos\theta = \dfrac{-\sqrt{3}}{2} \Rightarrow \theta = \cos^{-1}\left(\dfrac{-\sqrt{3}}{2}\right)$

Question7

Let a unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$ make angles $\frac{\pi}{2}$, $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$, $\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ respectively. If $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$, then $|\hat{u} - \vec{v}|^2$ is equal to

[29-Jan-2024 Shift 2]

Options:

A. $\frac{11}{2}$

B. $\frac{5}{2}$

C. 9

D. 7

Answer: B

Solution:

Unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{p}_1 = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}, \vec{p}_2 = \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$

$\vec{p}_3 = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

Now angle between $\hat{u}$ and $\vec{p}_1 = \frac{\pi}{2}$

$\hat{u} \cdot \vec{p}_1 = 0 \Rightarrow \frac{x}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0$

$\Rightarrow x + z = 0$ ....(i)

Angle between $\hat{u}$ and $\vec{p}_2 = \frac{\pi}{3}$

$\hat{u} \cdot \vec{p}_2 = |\hat{u}| \cdot |\vec{p}_2| \cos\frac{\pi}{3}$

$\Rightarrow \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = \frac{1}{2} \Rightarrow y + z = \frac{1}{\sqrt{2}}$ ....(ii)

Angle between $\hat{u}$ and $\vec{p}_3 = \frac{2\pi}{3}$

$\hat{u} \cdot \vec{p}_3 = |\hat{u}| \cdot |\vec{p}_3| \cos\frac{2\pi}{3}$

$\Rightarrow \frac{x}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \frac{-1}{2} \Rightarrow x + y = \frac{-1}{\sqrt{2}}$ ....(iii)

from equation (i), (ii) and (iii) we get

$x = \frac{-1}{\sqrt{2}}\ y = 0\ z = \frac{1}{\sqrt{2}}$

Thus $\hat{u} - \vec{v} = \frac{-1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k} - \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}$

$\hat{u} - \vec{v} = \frac{-2}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$

$\therefore |\hat{u} - \vec{v}|^2 = \left(\sqrt{\frac{4}{2} + \frac{1}{2}}\right)^2 = \frac{5}{2}$

Question8

Let A(2, 3, 5) and C(−3, 4, −2) be opposite vertices of a parallelogram ABCD if the diagonal $\vec{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$ then the area of the parallelogram is equal to
[30-Jan-2024 Shift 1]

Options:

A. $\frac{1}{2}\sqrt{410}$

B. $\frac{1}{2}\sqrt{474}$

C. $\frac{1}{2}\sqrt{586}$

D. $\frac{1}{2}\sqrt{306}$

Answer: B

Solution:

$$\text{Area} = \left|\overrightarrow{AC} \times \overrightarrow{BD}\right|$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 7 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \frac{1}{2}\left|-17\hat{i} - 8\hat{j} + 11\hat{k}\right| = \frac{1}{2}\sqrt{474}$$

Question9

Let $\vec{a} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$, $\alpha, \beta \in R$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $\left|\vec{b}\right|^2 = 6$, If $\vec{a} \cdot \vec{b} = 3\sqrt{2}$, then the value of $(\alpha^2 + \beta^2)\left|\vec{a} \times \vec{b}\right|^2$ is equal to
[30-Jan-2024 Shift 2]

Options:

A. 90

B. 75

C. 95

D. 85

Answer: A

Solution:

Solution:

$|\vec{b}|^2 = 6; |\vec{a}||\vec{b}|\cos\theta = 3\sqrt{2}$

$|\vec{a}|^2|\vec{b}|^2\cos^2\theta = 18$

$|\vec{a}|^2 = 6$

Also $1+\alpha^2+\beta^2 = 6$

$\alpha^2+\beta^2 = 5$

to find

$(\alpha^2+\beta^2)|\vec{a}|^2|\vec{b}|^2 \sin^2\theta$

$= (5)(6)(6)\left(\frac{1}{2}\right)$

$= 90$

Question10

Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{b}| = 1$ and $|\vec{b} \times \vec{a}| = 2$. Then $\left|(\vec{b} \times \vec{a}) - \vec{b}\right|^2$ is equal to
[30-Jan-2024 Shift 2]

Options:

A. 3

B. 5

C. 1

D. 4

Answer: B

Solution:

$|\vec{b}| = 1$ & $|\vec{b} \times \vec{a}| = 2$

$(\vec{b} \times \vec{a}) \cdot \vec{b} = \vec{b} \cdot (\vec{b} \times \vec{a}) = 0$

$|(\vec{b} \times \vec{a}) - \vec{b}|^2 = |\vec{b} \times \vec{a}|^2 + |\vec{b}|^2$

$= 4 + 1 = 5$

Question11

Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be three vectors. If a vectors $\vec{p}$ satisfies $\vec{p} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{p} \cdot \vec{a} = 0$, then $\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$ is

equal to
[31-Jan-2024 Shift 1]

Options:

A. 24

B. 36

C. 28

D. 32

Answer: D

Solution:

Solution:

$\vec{p} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$

$(\vec{p} - \vec{c}) \times \vec{b} = \vec{0}$

$\vec{p} - \vec{c} = \lambda\vec{b} \Rightarrow \vec{p} = \vec{c} + \lambda\vec{b}$

Now, $\vec{p} \cdot \vec{a} = 0$ (given)

So, $\vec{c} \cdot \vec{a} + \lambda\vec{a} \cdot \vec{b} = 0$

$(3 - 3 - 8) + \lambda(12 + 1 - 14) = 0$

$\lambda = -8$

$\vec{p} = \vec{c} - 8\vec{b}$

$\vec{p} = -31\hat{i} - 11\hat{j} - 52\hat{k}$

So, $\vec{p} \cdot (\hat{i} - \hat{j} - \hat{k})$

$= -31 + 11 + 52$

$= 32$

Question12

Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = 1, |\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is α, then $192\sin^2\alpha$ is equal to____
[31-Jan-2024 Shift 1]

Answer: 48

Solution:

$\vec{b} \cdot \vec{c} = \left(2\vec{a} \times \vec{b}\right) \cdot \vec{b} - 3\left|b\right|^2$

$|b||c|\cos\alpha = -3|b|^2$

$|c|\cos\alpha = -12, \quad \text{as} \quad |b| = 4$

$\vec{a} \cdot \vec{b} = 2$

$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$|c|^2 = \left|\left(2\vec{a} \times \vec{b}\right) - 3\vec{b}\right|^2$

$= 64 \times \frac{3}{4} + 144 = 192$

$|c|^2 \cos^2\alpha = 144$

$192\cos^2\alpha = 144$

$192 sin^2\alpha = 48$

Question13

Let $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c}$ be a vector such that $\left(\vec{a} + \vec{b}\right) \times \vec{c} = 2\left(\vec{a} \times \vec{b}\right) + 24\hat{j} - 6\hat{k}$ and $\left(\vec{a} - \vec{b} + \hat{i}\right) \cdot \vec{c} = -3$. Then $\left|\vec{c}\right|^2$ is equal to____
[31-Jan-2024 Shift 2]

Answer: 38

Solution:

$$\left(\vec{a}+\vec{b}\right)\times\vec{c}=2\left(\vec{a}\times\vec{b}\right)+24\hat{j}-6\hat{k}$$

$$\left(5\hat{i}+\hat{j}+4\hat{k}\right)\times\vec{c}=2\left(7\hat{i}-7\hat{j}-7\hat{k}\right)+24\hat{j}-6\hat{k}$$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & 1 & 4\\ x & y & z\end{vmatrix}=14\hat{i}+10\hat{j}-20\hat{k}$$

$$\Rightarrow \hat{i}(z-4y)-\hat{j}(5z-4x)+\hat{k}(5y-x)=14\hat{i}+10\hat{j}-20\hat{k}$$

$$z-4y=14,\ 4x-5z=10,\ 5y-x=-20$$

$$(a-b+i)\cdot\vec{c}=-3$$

$$\left(2\hat{i}+3\hat{j}-2\hat{k}\right)\cdot\vec{c}=-3$$

$$2x+3y-2z=-3$$

$$\therefore x=5,\ y=-3,\ z=2$$

$$|\vec{c}|^2=25+9+4=38$$

Question14

Let $\vec{a}=-5\hat{i}+\hat{j}-3\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-4\hat{k}$ and $\vec{c}=\left(\left(\left(\vec{a}\times\vec{b}\right)\times\hat{i}\right)\times\hat{i}\right)\times\hat{i}$. Then $\vec{c}\cdot\left(-\hat{i}+\hat{j}+\hat{k}\right)$ is equal to
[1-Feb-2024 Shift 1]

Options:

A. -12

B. -10

C. -13

D. -15

Answer: A

Solution:

Solution:

$\vec{a} = -5\hat{i} + j - 3\hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$

$(\vec{a} \times \vec{b}) \times \hat{i} = (\vec{a} \cdot \hat{i})\vec{b} - (\vec{b} \cdot \hat{i})\vec{a}$

$= -5\vec{b} - \vec{a}$

$= (((-5\vec{b} - \vec{a}) \times \hat{i}) \times \hat{i})$

$= ((-11\hat{j} + 23\hat{k}) \times \hat{i}) \times \hat{i}$

$\Rightarrow (11\hat{k} + 23\hat{j}) \times \hat{i}$

$\Rightarrow (11\hat{j} - 23\hat{k})$

$\vec{c} \cdot (-\hat{i} + \hat{j} + \hat{k}) = 11 - 23 = -12$

Question15

Consider a $\triangle ABC$ where $A(1, 2, 3)$,, $B(-2, 8, 0)$ and $C(3, 6, 7)$. If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\overrightarrow{AD}$ on the vector $\overrightarrow{AC}$ is:
[1-Feb-2024 Shift 2]

Options:

A. $\frac{37}{2\sqrt{38}}$

B. $\frac{\sqrt{38}}{2}$

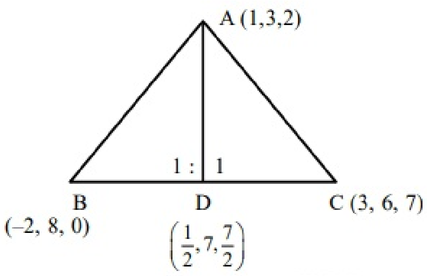
C. $\frac{39}{2\sqrt{38}}$

D. $\sqrt{19}$

Answer: A

Solution:

Solution:



$A(1, 3, 2); B(-2, 8, 0); C(3, 6, 7);$

$\overrightarrow{AC} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

$AB = \sqrt{9 + 25 + 4} = \sqrt{38}$

$AC = \sqrt{4 + 9 - 25} - \sqrt{38}$

$\overrightarrow{AD} = \frac{1}{2}\hat{i} - 4\hat{j} - \frac{3}{2}\hat{k} = \frac{1}{2}(\hat{i} - 8\hat{j} - 3\hat{k})$

Length of projection of $\overrightarrow{AD}$ on $\overrightarrow{AC}$

$$= \left| \frac{\overrightarrow{AD} \cdot \overrightarrow{AC}}{\left|\overrightarrow{AC}\right|} \right| = \frac{37}{2\sqrt{38}}$$

Question16

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$ and $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is θ , then the greatest integer less than or equal to $\tan^2\theta$ is :
[1-Feb-2024 Shift 2]

Answer: 38

Solution:

Solution:

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$

$\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$

$\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$

$\left(\vec{b} - \vec{c}\right) \times \vec{a} = 0$

$\vec{b} - \vec{c} = \lambda\vec{\alpha}$

$\vec{b} = \vec{c} + \lambda\vec{\alpha}$

$-\hat{i} - 8\hat{j} + 2\hat{k} = \left(4\hat{i} + c_2\hat{j} + c_3\hat{k}\right) + \lambda\left(\hat{i} + \hat{j} + \hat{k}\right)$

$\lambda + 4 = -1 \Rightarrow \lambda = -5$

$\lambda + c_2 = -8 \Rightarrow c_2 = -3$

$\lambda + c_3 = 2 \Rightarrow c_3 = 7$

$\vec{c} = 4\hat{i} - 3\hat{j} + 7k$

$\cos\theta = \frac{12 - 12 + 7}{\sqrt{26} \cdot \sqrt{74}} = \frac{7}{\sqrt{26} \cdot \sqrt{74}} = \frac{7}{2\sqrt{481}}$

$\tan^2\theta = \frac{625 \times 3}{49}$

$[\tan^2\theta] = 38$

Question17

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a} - \vec{b} + \vec{c}$, $\lambda\vec{a} - 3\vec{b} + 4\vec{c}$, $-\vec{a} + 2\vec{b} - 3\vec{c}$ and $2\vec{a} - 4\vec{b} + 6\vec{c}$ respectively. If $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, then λ is :
Official Ans. by NTA
[29-Jan-2023 Shift 1]

Answer: 2

Solution:

Solution:

$\overline{AB} = (\lambda - 1)\overline{a} - 2\overline{b} + 3\overline{c}$
$\overline{AC} = 2\overline{a} + 3\overline{b} - 4\overline{c}$
$\overline{AD} = \overline{a} - 3\overline{b} + 5\overline{c}$

$$\begin{vmatrix} \lambda - 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

$\Rightarrow (\lambda - 1)(15 - 12) + 2(-10 + 4) + 3(6 - 3) = 0$
$\Rightarrow (\lambda - 1) = 1 \Rightarrow \lambda = 2$

Question18

Let $\vec{u} = \hat{i} - \hat{j} - 2k$, $\vec{v} = 2\hat{i} + \hat{j} - k$, $\vec{v} \cdot \vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u} \cdot \vec{w}$ is equal to
[24-Jan-2023 Shift 1]

Options:

A. 1

B. $\frac{3}{2}$

C. 2

D. $-\frac{2}{3}$

Answer: A

Solution:

Solution:

$\vec{u} = (1, -1, -2)$, $\vec{v} = (2, 1, -1)$, $\vec{v} \cdot \vec{w} = 2$
$\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v} \ldots\ldots\ldots\ldots\ldots (1)$
Taking dot with $\vec{w}$ in (1)
$\vec{W} \cdot (\vec{v} \times \vec{w}) = \vec{u} \cdot \vec{w} + \lambda\vec{v} \cdot \vec{w}$
$\Rightarrow 0 = \vec{u} \cdot \vec{w} + 2\lambda$
Taking dot with $\vec{v}$ in (1)
$\vec{v} \cdot (\vec{v} \times \vec{w}) = \vec{u} \cdot \vec{v} + \lambda\vec{v} \cdot \vec{v}$
$\Rightarrow 0 = (2 - 1 + 2) + \lambda(6)$
$\lambda = -\frac{1}{2}$
$\Rightarrow \vec{u} \cdot \vec{w} = -2\lambda = 1$

Question19

Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot \left(\hat{i} + \hat{j} + \hat{k}\right)$ is
[24-Jan-2023 Shift 2]

Options:

A. 6

B. 11

C. 7

D. 9

Answer: C

Solution:

Solution:
Let $\vec{\beta}_1 = \lambda\vec{\alpha}$
Now $\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$
$= \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) - \lambda\left(4\hat{i} + 3\hat{j} + 5\hat{k}\right)$
$= (1 - 4\lambda)\hat{i} + (2 - 3\lambda)\hat{j} - (5\lambda + 4)\hat{k}$
$\vec{\beta}_2 \cdot \vec{\alpha} = 0$
$\Rightarrow 4(1 - 4\lambda) + 3(2 - 3\lambda) - 5(5\lambda + 4) = 0$
$\Rightarrow 4 - 16\alpha + 6 - 9\lambda - 25\lambda - 20 = 0$
$\Rightarrow 50\lambda = -10$
$\Rightarrow \lambda = \dfrac{-1}{5}$
$\vec{\beta}_2 = \left(1 + \dfrac{4}{5}\right)\hat{i} + \left(2 + \dfrac{3}{5}\right)\hat{j} - (-1 + 4)\hat{k}$
$\vec{\beta}_2 = \dfrac{9}{5}\hat{i} + \dfrac{13}{5}\hat{j} - 3\hat{k}$
$5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$
$5\vec{\beta}_2 \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 9 + 13 - 15 = 7$

Question20

Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$, $\vec{a} \cdot \vec{c} = 7$, $2\vec{b} \cdot \vec{c} + 43 = 0$, $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$. Then $\left|\vec{a} \cdot \vec{b}\right|$ is equal to
[24-Jan-2023 Shift 2]

Answer: 8

Solution:

$\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}, \vec{a}\cdot\vec{c} = 7$
$\vec{a}\times\vec{c} - \vec{b}\times\vec{c} = \vec{0}$
$(\vec{a}-\vec{b})\times\vec{c} = 0 \Rightarrow (\vec{a}-\vec{b})$ is paralleled to $\vec{c}$
$\vec{a}-\vec{b} = \mu\vec{c}$, where μ is a scalar
$-2\hat{i} + 7\hat{j} + 2\lambda\hat{k} = \mu\cdot\vec{c}$

Now $\vec{a}\cdot\vec{c} = 7$ gives $2\lambda^2 + 12 = 7\mu$
And $\vec{b}\cdot\vec{c} = -\frac{43}{2}$ gives $4\lambda^2 + 82 = 43\mu$
$\mu = 2$ and $\lambda^2 = 1$
$|\vec{a}\cdot\vec{b}| = 8$

Question21

The vector $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated through a right angle, passing through the y-axis in its way and the resulting vector is $\vec{b}$. Then the projection of $3\vec{a} + \sqrt{2}\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is
[25-Jan-2023 Shift 1]

Options:

A. $3\sqrt{2}$

B. 1

C. $\sqrt{6}$

D. $2\sqrt{3}$

Answer: A

Solution:

Solution:
$\vec{b} = \lambda\vec{a}\times(\vec{a}\times\hat{j})$
$\Rightarrow \vec{b} = \lambda(-2\hat{i} - 2\hat{j} + 2\hat{k})$
$|\vec{b}| = |\vec{a}| \quad \therefore \sqrt{6} = \sqrt{12}|\lambda| \Rightarrow \lambda = \pm\frac{1}{\sqrt{2}}$
$\left(\lambda = \frac{1}{\sqrt{2}}\right.$ rejected $\because \vec{b}$ makes acute angle with y axis $\left.\right)$
$\vec{b} = -\sqrt{2}(-\hat{i} - \hat{j} + \hat{k})$
$\left(3\vec{a} + \sqrt{2}\vec{b}\right)\cdot\frac{\vec{c}}{|\vec{c}|} = 3\sqrt{2}$

Question22

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non zero vectors such that $\vec{b}\cdot\vec{c} = 0$ and

$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} - \frac{\vec{c}}{2}$. If $\vec{d}$ be a vector such that $\vec{b} \cdot \vec{d} = \vec{a} \cdot \vec{b}$, then $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$ is equal to
[25-Jan-2023 Shift 1]

Options:

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $-\frac{1}{4}$

D. $\frac{1}{4}$

Answer: D

Solution:

Solution:

$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \vec{b} - \frac{\vec{c}}{2}$

$\vec{a} \cdot \vec{c} = \frac{1}{2}, \vec{a} \cdot \vec{b} = \frac{1}{2}$

$\therefore \vec{b} \cdot \vec{d} = \frac{1}{2}$

$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \vec{a} \cdot (\vec{b} \times (\vec{c} \times \vec{d}))$

$= \vec{a} \cdot ((\vec{b} \cdot \vec{d})\vec{c} - (\vec{b} \cdot \vec{c})\vec{d})$

$= (\vec{a} \cdot \vec{c})(\vec{b} \cdot \vec{d}) = \frac{1}{4}$

Question23

If the four points, whose position vectors are $3\hat{i} - 4\hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$, $-2\hat{i} - \hat{j} + 3\hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to
[25-Jan-2023 Shift 2]

Options:

A. $\frac{73}{17}$

B. $-\frac{107}{17}$

C. $-\frac{73}{17}$

D. $\frac{107}{17}$

Answer: A

Solution:

Let A : $(3, -4, 2)$
C : $(-2, -1, 3)$
B : $(1, 2, -1)$ D : $(5, -2\alpha, 4)$
A, B, C, D are coplanar points, then

$$\Rightarrow \begin{vmatrix} 1-3 & 2+4 & -1-2 \\ -2-3 & -1+4 & 3-2 \\ 5-3 & -2\alpha+4 & 4-2 \end{vmatrix} = 0$$

$\Rightarrow \alpha = \frac{73}{17}$

Question24

Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$, $\vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to
[25-Jan-2023 Shift 2]

Options:

A. $3\left(\hat{i} - \hat{j} - \hat{k}\right)$

B. $3\left(\hat{i} + \hat{j} + \hat{k}\right)$

C. $3\left(\hat{i} - \hat{j} + \hat{k}\right)$

D. $3\left(\hat{i} + \hat{j} - \hat{k}\right)$

Answer: B

Solution:

Solution:

$\vec{a} \times \vec{b} = \left(\hat{i} - \hat{j}\right)$

Taking cross product with $\vec{a}$

$\Rightarrow \vec{a} \times \left(\vec{a} \times \vec{b}\right) = \vec{a} \times \left(\hat{i} - \hat{j}\right)$

$\Rightarrow \left(\vec{a} \cdot \vec{b}\right)\vec{a} - \left(\vec{a} \cdot \vec{a}\right)\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$\Rightarrow \vec{a} - 3\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$

$\Rightarrow 2\vec{a} - 6\vec{b} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

$\Rightarrow \vec{a} - 6\vec{b} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

Question25

If the vectors $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the projection of $\vec{a}$ on the vector $\vec{b}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to
[29-Jan-2023 Shift 1]

Options:

A. 0

B. 6

C. 24

D. 18

Answer: C

Solution:

Solution:

$$\begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$\lambda(10) - \mu(2) + 4(-14) = 0$
$10\lambda - 2\mu = 56$
$5\lambda - \mu = 28 \ldots (1)$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \sqrt{54}$$

$-2\lambda + 4\mu - 8 / \sqrt{24} = \sqrt{54}$
$-2\lambda + 4\mu - 8 = \sqrt{54 \times 24} \ldots (2)$
By solving equation (1) & (2)
$\Rightarrow \lambda + \mu = 24$

Question26

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a} - \vec{b} + \vec{c}$, $\lambda\vec{a} - 3\vec{b} + 4\vec{c}$, $-\vec{a} + 2\vec{b} - 3\vec{c}$ and $2\vec{a} - 4\vec{b} + 6\vec{c}$ respectively. If $\vec{AB}$, $\vec{AC}$ and $\vec{AD}$ are coplanar, then λ is :
Official Ans. by NTA
[29-Jan-2023 Shift 1]

Answer: 2

Solution:

$\overline{AB} = (\lambda - 1)\overline{a} - 2\overline{b} + 3\overline{c}$
$\overline{AC} = 2\overline{a} + 3\overline{b} - 4\overline{c}$
$\overline{AD} = \overline{a} - 3\overline{b} + 5\overline{c}$

$$\begin{vmatrix} \lambda - 1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

$\Rightarrow (\lambda - 1)(15 - 12) + 2(-10 + 4) + 3(6 - 3) = 0$
$\Rightarrow (\lambda - 1) = 1 \Rightarrow \lambda = 2$

Question27

Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c}$ is a vector such that $\vec{c} \cdot (\vec{a} \times \vec{b}) + 25 = 0$, $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$ and projection of $\vec{c}$ on $\vec{a}$ is 1 , then the projection of $\vec{c}$ on $\vec{b}$ equals :
[29-Jan-2023 Shift 2]

Options:

A. $\frac{5}{\sqrt{2}}$

B. $\frac{1}{5}$

C. $\frac{1}{\sqrt{2}}$

D. $\frac{3}{\sqrt{2}}$

Answer: A

Solution:

Solution:

$\vec{a} \times \vec{b} = 15\hat{i} - 20\hat{j} - 25\hat{k}$

Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow 15x - 20y - 25z + 25 = 0$

$\Rightarrow 3x - 4y - 5z = -5$

Also $x + y + z = 4$

and $\frac{\vec{c} \cdot \vec{a}}{|\vec{a}|} = 1 \Rightarrow 4x + 3y = 5$

$\Rightarrow \vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$

Projection of $\vec{c}$ or $\vec{b}$ = $\frac{25}{5\sqrt{2}} = \frac{5}{\sqrt{2}}$

Question28

If $\vec{a} = \hat{i} + 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = 7\hat{i} - 3\hat{k} + 4\hat{k}$ $\vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$ then $\vec{r} \cdot \vec{c}$ is equal to :
[29-Jan-2023 Shift 2]

Options:

A. 34

B. 12

C. 36

D. 30

Answer: A

Solution:

$\vec{r} \times \vec{b} - \vec{c} \times \vec{b} = 0$

$\Rightarrow (\vec{r} - \vec{c}) \times \vec{b} = 0$

$\Rightarrow \vec{r} - \vec{c} = \lambda\vec{b}$

$\Rightarrow \vec{r} = \vec{c} + \lambda\vec{b}$

And given that $\vec{r} \cdot \vec{a} = 0$

$\Rightarrow (\vec{c} + \lambda\vec{b}) \cdot \vec{a} = 0$

$\Rightarrow \vec{c} \cdot \vec{a} + \lambda\vec{b} \cdot \vec{a} = 0$

$\Rightarrow \lambda = -\vec{c} \cdot \dfrac{\vec{a}}{\vec{b} \cdot \vec{a}}$

Now $\vec{r} \cdot \vec{c} = (\vec{c} + \lambda\vec{b}) \cdot \vec{c}$

$= \left(\vec{c} - \dfrac{\vec{c} \cdot \vec{a}}{\vec{b} \cdot \vec{a}}\vec{b}\right) \cdot \vec{c}$

$= |\vec{c}| - \left(\dfrac{\vec{c} \cdot \vec{a}}{\vec{b} \cdot \vec{a}}\right)(\vec{b} \cdot \vec{c})$

$= 74 - \left[\dfrac{15}{3}\right]8$

$= 74 - 40 = 34$

Question29

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a} = \alpha\vec{b} - \hat{n}$, $(\alpha \neq 0)$ and $\vec{b} \cdot \vec{c} = 12$, then $\left|\vec{c} \times (\vec{a} \times \vec{b})\right|$ is equal to :
[30-Jan-2023 Shift 1]

Options:

A. 15

B. 9

C. 12

D. 6

Answer: C

Solution:

$\hat{n} \perp \vec{c}$ $\vec{a} = \alpha\vec{b} - \vec{n}$

$\vec{b} \cdot \vec{c} = 12$

$\vec{a} \cdot \vec{c} = \alpha(\vec{b} \cdot \vec{c}) - \vec{n} \cdot \vec{c}$

$\vec{a} \cdot \vec{c} = \alpha(\vec{b} \cdot \vec{c})$

$\left|\vec{c} \times (\vec{a} \times \vec{b})\right| = \left|(\vec{c} \cdot \vec{b})\vec{a} - (\vec{c} \cdot \vec{a})\vec{b}\right|$

$= \left|(\vec{c} \cdot \vec{b})\vec{a} - \alpha(\vec{b} \cdot \vec{c})\vec{b}\right|$

$= \left|\left(\vec{c}\cdot\vec{b}\right)\right|\left|\vec{a}-\alpha\vec{b}\right|$
$= 12 \times \left(\left|\vec{n}\right|\right)$
$= 12 \times 1$
$= 12$

Question30

Let $\lambda \in \mathbb{R}$, $\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$.
If $\left(\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)\right)\times\left(\vec{a}-\vec{b}\right) = 8\hat{i} - 40\hat{j} - 24\hat{k}$, then $\left|\lambda\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|^2$ is equal to
[30-Jan-2023 Shift 2]

Options:

A. 140

B. 132

C. 144

D. 136

Answer: A

Solution:

Solution:
$\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$
$\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$
$\Rightarrow \left(\vec{b}-\vec{a}\right)\times\left(\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)\right) = 8\hat{i} - 40\hat{j} - 24\hat{k}$
$\Rightarrow \left(\left(\vec{a}-\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)\right)\left(\vec{a}\times\vec{b}\right) = 8\hat{i} - 40j - 24\hat{k}$
$\Rightarrow 8\left(\vec{a}\times\vec{b}\right) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

Now, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \lambda & 2 & -3 \\ 1 & -\lambda & 2 \end{vmatrix}$

$= (4-3\lambda)\hat{i} - (2\lambda+3)\hat{j} + (-\lambda^2-2)\hat{k}$
$\Rightarrow \lambda = 1$
$\therefore \vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$
$\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$
$\Rightarrow \vec{a}+\vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{a}-\vec{b} = 3\hat{j} - 5\hat{k}$

$\Rightarrow \left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 0 & 3 & -5 \end{vmatrix} = 2\hat{i} + 10\hat{j} + 6\hat{k}$

$\therefore$ required answer $= 4 + 100 + 36 = 140$

Question31

Let $\vec{a}$ and $\vec{b}$ be two vectors. Let $|\vec{a}| = 1, |\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$, then the value of $\vec{b} \cdot \vec{c}$ is
[30-Jan-2023 Shift 2]

Options:

A. -24

B. -48

C. -84

D. -60

Answer: B

Solution:

Solution:

$\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$
$\vec{b} \cdot \vec{c} = \vec{b} \cdot (2\vec{a} \times \vec{b}) - 3\vec{b} \cdot \vec{b}$
$= -3|b|^2$
$= -48$

Question32

Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$, and $\vec{b}$ and $\vec{c}$ be two nonzero vectors such that $|\vec{a} + \vec{b} + \vec{c}| = |\vec{a} + \vec{b} - \vec{c}|$ and $\vec{b} \cdot \vec{c} = 0$. Consider the following two statement:
(A) $|\vec{a} + \lambda\vec{c}| \geq |\vec{a}|$ for all $\lambda \in \mathbb{R}$.
(B) $\vec{a}$ and $\vec{c}$ are always parallel
[31-Jan-2023 Shift 1]

Options:

A. only (B) is correct

B. neither (A) nor (B) is correct

C. only (A) is correct

D. both (A) and (B) are correct.

Answer: C

Solution:

Solution:

$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a} + \vec{b} - \vec{c}|^2$
$2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a} = 2\vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{c} - 2\vec{c} \cdot \vec{a}$
$4\vec{a} \cdot \vec{c} = 0$
B is incorrect

$\left|\vec{a}+\lambda\vec{c}\right|^2 \geq \left|\vec{a}\right|^2$
$\lambda^2 c^2 \geq 0$
True $\forall\lambda \in R$ (A) is correct.

Question33

Let $\vec{a}$ and $\vec{b}$ be two vector such that $\left|\vec{a}\right| = \sqrt{14}$, $\left|\vec{b}\right| = \sqrt{6}$ and $\left|\vec{a} \times \vec{b}\right| = \sqrt{48}$. Then $\left(\vec{a} \cdot \vec{b}\right)^2$ is equal to ______.
[31-Jan-2023 Shift 1]

Answer: 36

Solution:

Solution:
$\left|\vec{a}\right| = \sqrt{14}, \left|\vec{b}\right| = \sqrt{6}\ \ \left|\vec{a} \times \vec{b}\right| = \sqrt{48}$
$\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a} \cdot \vec{b}\right|^2 = \left|\vec{a}\right|^2 \times \left|\vec{b}\right|^2$
$\Rightarrow \left(\vec{a} \cdot \vec{b}\right)^2 = 84 - 48 = 36$

Question34

Let : $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$ be there vectors. If $\vec{r}$ is a vector such that, $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$. Then $25\left|\vec{r}\right|^2$ is equal to
[31-Jan-2023 Shift 2]

Options:

A. 449

B. 336

C. 339

D. 560

Answer: C

Solution:

Solution:
Sol. $\vec{a} = i + 2j + 3k$
$\vec{b} = i - j + 2k$
$\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$
$\left(\vec{r} - \vec{c}\right) \times \vec{b} = 0, \vec{r} \cdot \vec{a} = 0$
$\Rightarrow \vec{r} - \vec{c} = \lambda\vec{b}$

Also, $\left(\vec{c}+\lambda\vec{b}\right)\cdot\vec{a}=0$

$\Rightarrow \vec{a}\cdot\vec{c}+\lambda\left(\vec{a}\cdot\vec{b}\right)=0$

$\therefore \lambda=\dfrac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}=\dfrac{-8}{5}$

$\vec{r}=\dfrac{5\left(5\hat{i}-3\hat{i}+3\hat{k}\right)-8\left(\hat{i}-\hat{j}+2\hat{k}\right)}{5}$

$\vec{r}=\dfrac{17\hat{i}-7\hat{j}+\hat{k}}{5}$

$|\vec{r}|^2=\dfrac{1}{25}(289+50)$

$25|\vec{r}|^2=339$

Question35

The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is (2, a, 4), $a\in N$. If the volume of the tetrahedron OABC is 144unit^3, then which of the following points is NOT on P? [31-Jan-2023 Shift 2]

Options:

A. (2, 2, 4)

B. (0, 4, 4)

C. (3, 0, 4)

D. (0, 6, 3)

Answer: C

Solution:

Solution:

Equation of Plane:

$\left(2\hat{i}+a\hat{j}+4\hat{k}\right)\cdot\left[(x-2)\hat{i}+(y-a)\hat{j}+(z-4)\hat{k}\right]=0$

$\Rightarrow 2x+ay+4z=20+a^2$

$\Rightarrow A\equiv\left(\dfrac{20+a^2}{2},0,0\right)$

$B\equiv\left(0,\dfrac{20+a^2}{a},0\right)$

$C\equiv\left(0,0,\dfrac{20+a^2}{4}\right)$

$\Rightarrow$ Volume of tetrahedron

$=\dfrac{1}{6}\left[\vec{a}\,\vec{b}\,\vec{c}\right]$

$=\dfrac{1}{6}\vec{a}\,.\left(\vec{b}\times\vec{c}\right)$

$\Rightarrow \dfrac{1}{6}\left(\dfrac{20+a^2}{2}\right)\cdot\left(\dfrac{20+a^2}{a}\right)\cdot\left(\dfrac{20+a^2}{4}\right)=144$

$\Rightarrow (20+a^2)^3=144\times48\times a$

$\Rightarrow a=2$

$\Rightarrow$ Equation of plane is $2x+2y+4z=24$

Or $x+y+2z=12$

$\Rightarrow$(3, 0, 4) Not lies on the Plane

$x+y+2z=12$

Question36

Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{31}, 4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$.

If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}}\right)^2$ is equal to ________.

[31-Jan-2023 Shift 2]

Answer: 3

Solution:

Sol. $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$

$\vec{a} \times (2\vec{b} + 3\vec{c}) = 0$

$\vec{a} = \lambda(2\vec{b} + 3\vec{c})$

$|\vec{a}|^2 = \lambda^2|2\vec{b} + 3\vec{c}|^2$

$|\vec{a}|^2 = \lambda^2(4|\vec{b}|^2 + 9|\vec{c}|^2 + 12\vec{b} \cdot \vec{c})$

$31 = 31\lambda^2 \Rightarrow \lambda = \pm 1$

$\vec{a} = \pm(2\vec{b} + 3\vec{c})$

$|\vec{b} \times \vec{c}|^2 = |\vec{b}|^2|\vec{c}|^2 - (\vec{b} \cdot \vec{c})^2 = \frac{3}{4}$

$\left(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}}\right)^2 = 3$

Question37

Let $\vec{v} = \alpha\hat{i} + 2j - 3k$, $\vec{w} = 2\alpha\hat{i} + j - k$, and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $[\vec{u}\vec{v}\vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u}.\hat{i}|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then $m + n$ is equal to _______.

[1-Feb-2023 Shift 1]

Answer: 3501

Solution:

$[\vec{u}\vec{v}\vec{w}] = \vec{u}\cdot(\vec{v}\times\vec{w})$

$\min.\left(|u|\left|\vec{v}\times\vec{w}\right|\cos\theta\right) = -\alpha\sqrt{3401}$

$\Rightarrow\ \cos\theta = -1$

$|u| = \alpha$ (Given)

$\left|\vec{v}\times\vec{w}\right| = \sqrt{3401}$

$$\vec{v}\times\vec{w} = \begin{vmatrix} i & j & k \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix}$$

$\vec{v}\times\vec{w} = \hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}$

$\left|\vec{v}\times\vec{w}\right| = \sqrt{1 + 25\alpha^2 + 9\alpha^2} = \sqrt{3401}$

$34\alpha^2 = 3400$

$\alpha^2 = 100$

$\alpha = 10$

(as $\alpha > 0$)

So

$\vec{u} = \lambda(\hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k})$

$\vec{u} = \sqrt{\lambda^2 + 25\alpha^2\lambda^2 + 9\alpha^2\lambda}$

$\alpha^2 = \lambda^2(1 + 25\alpha^2 + 9\alpha^2)$

$100 = \lambda^2(1 + 34\times 100)$

$\lambda^2 = \dfrac{100}{3401} = \dfrac{m}{n}$

Question38

A(2, 6, 2), B(−4, 0, λ), C(2, 3, −1) and D(4, 5, 0), |λ| ≤ 5 are the vertices of a quadrilateral ABCD. If its area is 18 square units, then 5 − 6λ is equal to ______.
[1-Feb-2023 Shift 1]

Answer: 11

Solution:

Solution:

$A(2, 6, 2)\ \ B(-4, 0, \lambda), C(2, 3, -1) D(4, 5, 0)$

$\text{Area} = \dfrac{1}{2}\left|\vec{BD}\times\vec{AC}\right| = 18$

$$\vec{AC}\times\vec{BD} = \begin{bmatrix} \hat{i} & j & k \\ 0 & -3 & -3 \\ 8 & 5 & -\lambda \end{bmatrix}$$

$= (3\lambda + 15)\hat{i} - j(-24) + k(-24)$

$\vec{AC}\times\vec{BD} = (3\lambda + 15)\hat{i} + 24j - 24k$

$= \sqrt{(3\lambda + 15)^2 + (24)^2 + (24)^2} = 36$

$= \lambda^2 + 10\lambda + 9 = 0$

$= \lambda = -1, -9$

$|\lambda| \le 5 \Rightarrow \lambda = -1$

$5 - 6\lambda = 5 - 6(-1) = 11$

Question39

Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors. Then which one of the following statements is TRUE?
[1-Feb-2023 Shift 2]

Options:

A. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the p?

B. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the p?

C. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

D. Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-13}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$

Answer: D

Solution:

Solution:

$\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$

$\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{a} \cdot \hat{b} = \frac{5 - 3 - 15}{\sqrt{35}} = -\frac{-13}{\sqrt{35}}$

Question40

Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $|\vec{r}|$ is equal to :
[1-Feb-2023 Shift 2]

Options:

A. $\frac{11}{7}\sqrt{2}$

B. $\frac{11}{7}$

C. $\frac{11}{5}\sqrt{2}$

D. $\frac{\sqrt{914}}{7}$

Answer: A

Solution:

$a = 2\hat{i} - 7\hat{j} + 5\hat{k}$
$\vec{b} = \hat{i} + \hat{k}$
$\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$
$\vec{r} \times \vec{a} = \vec{c} \times \vec{a} \Rightarrow (\vec{r} - \vec{c}) \times \vec{a} = 0$
$\therefore \vec{r} = \vec{c} + \lambda\vec{a}$
$\vec{r} \cdot \vec{b} = 0 \Rightarrow \vec{c} \cdot \vec{b} + \lambda\ \vec{b} \cdot \vec{a} = 0$
$-2 + \lambda(7) = 0 \Rightarrow \lambda = \frac{2}{7}$
$\therefore \vec{r} = \vec{c} + 2\frac{\vec{a}}{7} = \frac{1}{7}(11\hat{i} - 11\hat{k})$
$\left|\vec{r}\right| = \frac{11\sqrt{2}}{7}$

Question41

Let the position vectors of the points A, B, C and D be $5\hat{i} + 5\hat{j} + 2\lambda\hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$, $-2\hat{i} + \lambda\hat{j} + 4\hat{k}$ and $-\hat{i} + 5\hat{j} + 6\hat{k}$. Let the set $S = \{\lambda \in \mathbb{R}$: the points A, B, C and D are coplanar $\}$. Then $\sum_{\lambda \in S}(\lambda + 2)^2$ is equal to :
[6-Apr-2023 shift 1]

Options:

A. $\frac{37}{2}$

B. 13

C. 25

D. 41

Answer: D

Solution:

Solution:
A, B, C, D are coplanar

$$\Rightarrow \left[\vec{AB}\vec{AC}\vec{AD}\right] = 0 \Rightarrow \begin{bmatrix} -4 & -3 & 3-2\lambda \\ -7 & \lambda-5 & 4-2\lambda \\ -6 & 0 & 6-2\lambda \end{bmatrix} = 0$$
$\Rightarrow -6[6\lambda - 12 - (\lambda - 5)(3 - 2\lambda)] + 0[\,] + (6 - 2\lambda)[20 - 4\lambda - 21]$
$\Rightarrow -6[6\lambda - 12 + 2\lambda^2 + 15 - 13\lambda] + (6 - 2\lambda)[-4\lambda - 1] = 0$
$\Rightarrow -12\lambda^2 + 42\lambda - 18 + 8\lambda^2 - 22\lambda - 6 = 0$
$\Rightarrow -4\lambda^2 + 20\lambda - 24 = 0 \Rightarrow \lambda^2 - 5\lambda + 6 = 0$
Now $\sum_{\lambda \in s}(\lambda + 2)^2 = 16 + 25 = 41$

Question42

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, b = $\hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$. If $\vec{d}$ is a vector

perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{a} \cdot \vec{d} = 18$, then $[\vec{a} \times \vec{d}]^2$ is equal to :
[6-Apr-2023 shift 1]

Options:

A. 760

B. 640

C. 720

D. 680

Answer: C

Solution:

Solution:

$\vec{d} = \lambda(\vec{b} \times \vec{c})$

For $\lambda : \vec{a} \cdot \vec{d} = 18 \Rightarrow \lambda[\vec{a}\vec{b}\vec{c}] = 18$

$$\Rightarrow \lambda \begin{vmatrix} 2 & 3 & 4 \\ 1 & -2 & -2 \\ -1 & 4 & 3 \end{vmatrix} = 18$$

$\Rightarrow \lambda(4 - 3 + 8) = 18 \Rightarrow \lambda = 2$

$\Rightarrow \vec{d} = 2(2\hat{i} - \hat{j} + 2\hat{k})$

Hence $|\vec{a} \times \vec{d}|^2 = a^2 d^2 - (\vec{a} \cdot \vec{d})^2$

$= 29 \cdot 36 - (18)^2 = 18(58 - 18)$

$= 18 \cdot 40 = 720$

Question43

Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ represent three coterminous edges of a parallelepiped of volume V. Then the volume of the parallelepiped, whose coterminous edges are represented by $\vec{a}$, $\vec{b} + \vec{c}$ and $\vec{a} + 2\vec{b} + 3\vec{c}$ is equal to :
[6-Apr-2023 shift 2]

Options:

A. 2V

B. 6V

C. 3V

D. V

Answer: D

Solution:

$v_1 = \left[\ \vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{a}+2\vec{b}+3\vec{c}\ \right]$

$v_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} [\vec{a}\vec{b}\vec{c}]$

$v_1 = (3-2)v$

$= v$

Ans. Option 4

Question44

The sum of all values of α, for which the points whose position vectors are $\hat{i} - 2\hat{j} + 3k$, $2\hat{i} - 3\hat{j} + 4k$, $(\alpha + 1)\hat{i} + 2k$ and $9\hat{i} + (\alpha - 8)\hat{j} + 6\hat{k}$ are coplanar, is equal to :
[6-Apr-2023 shift 2]

Options:

A. -2

B. 2

C. 6

D. 4

Answer: B

Solution:

Solution:

$A = (1, -2, 3)$
$B = (2, -3, 4)$
$C = (\alpha + 1, 0, 2)$
$D = (9, \alpha - 8, 6)$

$[\vec{AB}\vec{AC}\vec{AD}] = 0$

$$\begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & \alpha-6 & 3 \end{vmatrix} = 0$$

$\Rightarrow (6 + \alpha - 6) + 1(3\alpha + 8) + (\alpha^2 - 6\alpha - 16) = 0$
$\Rightarrow \alpha^2 - 2\alpha - 8 = 0$
$\Rightarrow \alpha = 4, -2$
$\Rightarrow$ sum of all values of $\alpha = 2$
Ans. option 2

Question45

Let $\vec{a} = 6\hat{i} + 9\hat{j} + 12\hat{k}$, $\vec{b} = \alpha\hat{i} + 11\hat{j} - 2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a} \times \vec{c} - = \vec{a} \times \vec{b}$. If $\vec{a} . \vec{c} = -12$, $\vec{c} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 5$, then $\vec{c} . (\hat{i} + \hat{j} + \hat{k})$ is equal to ________.
[8-Apr-2023 shift 1]

Answer: 11

Solution:

$\vec{a} \times \vec{c} = \vec{a} \times 5$

$\Rightarrow \vec{a} \times \left(\vec{c} - \vec{b}\right) = 0$

$\vec{a} \mid^{rl} \left(\vec{c} - \vec{b}\right)$

$\therefore \vec{a} = \lambda\left(\vec{c} - \vec{b}\right)$

$(6, 9, 12) = \lambda[x - \alpha, y - 11, z + 2]$

$\frac{x - \alpha}{2} = \frac{y - 11}{3} = \frac{z + 2}{4}$

$4y - 44 = 3z + 6$

$4y - 3z = 50$

$6x + 9y + 12z = -12$

$2x + 3y + 4z = -4$

$(\because x - 2y + z = 5)$

$2x - 4y + 2z = 10$

$\underline{\ +\ \ \ -\ }$

$7y + 2z = -14 \ \ldots (2)$

$8y - 6z = 100$

$21y + 6z = -42$

$29y = 58$

$y = 2, z = -14$

$\therefore x - 4 - 14 = 5$

$\underline{x} = 23$

$\underline{c} = (23, 2, -14)$

$c \cdot (1, 1, 1) = 23 + 2 - 14 = 11$

Question46

Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}$, $\vec{\mu}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a + b)\hat{i} + cj + ck$, $\vec{v}_2 = a\hat{i} + (b + c)\hat{j} + ak\hat{k}$ and $\vec{v}_3 = b\hat{i} + b\hat{j} + (c + a)\hat{k}$ are also coplanar, then 6(a + b + c) is equal to [8-Apr-2023 shift 2]

Options:

A. 4

B. 12

C. 6

D. 0

Answer: B

Solution:

$\left[\vec{u}_1\ \ \vec{u}_2\ \ \vec{u}_3\right] = 0 \ \therefore \begin{vmatrix} 1 & 1 & a \\ 1 & b & 1 \\ c & 1 & 1 \end{vmatrix} = 0$

$\Rightarrow b - 1 + c - 1 + a(1 - bc) = 0$
$\therefore abc = a + b + c - 2$

$\left[\vec{v}_1\ \ \vec{v}_2\ \ \vec{v}_3\right] = 0 \ \therefore \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & c+a \end{vmatrix} = 0$

$\Rightarrow -2a(ac - bc - c^2) + 2c(a^2 + ab - ac) = 0$
$\Rightarrow -2a^2c + 2abc + 2ac^2 + 2a^2c + 2abc - 2ac^2 = 0$
$\Rightarrow 4abc = 0 \ \therefore abc = 0$
$\therefore a + b + c = 2 \ \therefore 6(a + b + c) = 12$ Ans.

Question47

An arc PQ of a circle subtends a right angle at its centre O. The mid point of the arc PQ is R. If $\vec{OP} = \vec{u}$, $\vec{OR} = \vec{v}$ and $\vec{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β^2 are the roots of the equation :
[10-Apr-2023 shift 1]

Options:

A. $3x^2 - 2x - 1 = 0$

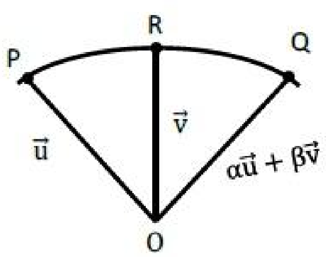
B. $3x^2 + 2x - 1 = 0$

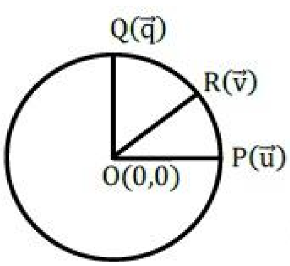
C. $x^2 - x - 2 = 0$

D. $x^2 + x - 2 = 0$

Answer: C

Solution:





Let $\vec{OP} = \vec{u} = \hat{i}$
$\vec{OQ} = \vec{q} = \hat{j}$
$\because$R is the mid point of $\vec{PQ}$

Then $\vec{OR} = \vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

Now

$\vec{OQ} = \alpha\vec{u} + \beta\vec{v}$

$\hat{j} = \alpha\hat{i} + \beta\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$

$\beta = \sqrt{2}, \alpha + \frac{\beta}{\sqrt{2}} = 0 \Rightarrow \alpha = -1$

Now equation

$x^2 - (\alpha + \beta^2)x + \alpha\beta^2 = 0$

$x^2 - (-1 + 2)x + (-1)(2) = 0$

$x^2 - x - 2 = 0$

Question48

Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} + 5\hat{k}$ and $\vec{C} = \hat{i} + \hat{j} + 2\hat{k}$, Let $\vec{d}$-be a vector which is perpendicular to both $\vec{a}$, and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 12$. The $\left(-\hat{i} + \hat{j} - \hat{k}\right) \cdot \left(\vec{c} \times \vec{d}\right)$-is equal to
[10-Apr-2023 shift 2]

Options:

A. 24

B. 42

C. 48

D. 44

Answer: D

Solution:

Solution:

$\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} + 5\hat{k}$

$\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$

$$\vec{d} = \lambda\left(\vec{a} \times \vec{b}\right) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 7 & -1 \\ 3 & 0 & 5 \end{vmatrix}$$

$\vec{d} = \lambda\left(35\hat{i} - 13\hat{j} - 21\hat{k}\right)$

$\lambda(35 + 13 - 42) = 12$

$\lambda = 2$

$\vec{d} = 2\left(35\hat{i} - 13\hat{j} - 21\hat{k}\right)$

$\left(\hat{i} + \hat{j} - \hat{k}\right)\left(\vec{c} \times \vec{d}\right)$

$$= \begin{vmatrix} -1 & 1 & -1 \\ 1 & -1 & 2 \\ 70 & -26 & -42 \end{vmatrix} = 44$$

Question49

For any vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, with $10|a_i| < 1$, $i = 1, 2, 3$, consider the following statements :

(A) : $\max\{|a_1|, |a_2|, |a_3|\} \leq |\vec{a}|$

(B) : $|\vec{a}| \leq 3\max\{|a_1|, |a_2|, |a_3|\}$

[11-Apr-2023 shift 1]

Options:

A. Only (B) is true

B. Both (A) and (B) are true

C. Neither (A) nor (B) is true

D. Only (A) is true

Answer: B

Solution:

Solution:

Without loss of generality

Let $|a_1| \leq |a_2| \leq |a_3|$

$|\vec{a}|^2 = |\vec{a}_1|^2 + |\vec{a}_2|^2 + |\vec{a}_3|^2 \geq (a_3)^2$

$\Rightarrow |\vec{a}| \geq |a_3| = \max\{|a_1|, |a_2|, |a_3|\}$

A is true

$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \leq |a_3|^2 + |a_3|^2 + |a_3|^2$

$\Rightarrow |\vec{a}|^2 \leq 3|a_3|^2$

$\Rightarrow |\vec{a}| \leq \sqrt{3}|\vec{a}_3| = \sqrt{3}\max\{|a_1|, |a_2|, |a_3|\}$

$\leq 3\max\{|a_1|, |a_2|, |a_3|\}$

(2) is true

Question50

Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two places described by $\hat{i} + \hat{j}$, $\hat{i} + \hat{k}$ and $\hat{i} - \hat{j}$, $\hat{j} - \hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a} \cdot \vec{b} = 6$, then ordered pair $\left(\theta, |\vec{a} \times \vec{b}|\right)$ is equal to :

[11-Apr-2023 shift 1]

Options:

A. $\left(\frac{\pi}{3}, 6\right)$

B. $\left(\frac{\pi}{4}, 3\sqrt{6}\right)$

C. $\left(\frac{\pi}{3}, 3\sqrt{6}\right)$

D. $\left(\frac{\pi}{4}, 6\right)$

Answer: D

Solution:

Solution:

$\vec{n}_1$ and $\vec{n}_2$ are normal vector to the plane $\hat{i}+\hat{j}$, $\hat{i}+\hat{k}$ and $\hat{i}-\hat{j}$, $\hat{i}-\hat{k}$ respectively

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i}-\hat{j}-\hat{k}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i}+\hat{j}+\hat{k}$$

$\vec{a} = \lambda\left|\vec{n}_2 \times \vec{n}_2\right|$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = \lambda\left(-2\hat{j}+2\hat{k}\right)$$

$\vec{a}\cdot\vec{b} = \lambda\left|0+4+2\right| = 6$

$\Rightarrow \lambda = 1$

$\vec{\alpha} = -2\hat{j}+2\hat{k}$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|a|\,|b|}$

$\cos\theta = \dfrac{6}{2\sqrt{2}\times 3} = \dfrac{1}{\sqrt{2}}$

$\theta = \dfrac{\pi}{4}$

Now $|\vec{a}\cdot\vec{b}|^2 + |\vec{a}\times\vec{b}|^2 = |a|^2|b|^2$

$36 + |\vec{a}\times b^2| = 8\times 9 = 72$

$|\vec{a}\times b|^2 = 36$

$|\vec{a}\times\vec{b}| = 6$

Question51

If four distinct points with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are coplanar, then $[\vec{a}\vec{b}\vec{c}]$ is equal to [11-Apr-2023 shift 2]

Options:

A. $[\vec{d}\vec{c}\vec{a}] + [\vec{b}\vec{d}\vec{a}] + [\vec{c}\vec{d}\vec{b}]$

B. $[\vec{d}\vec{b}\vec{a}] + [\vec{a}\vec{c}\vec{d}] + [\vec{d}\vec{b}\vec{c}]$

C. $[\vec{a}\vec{d}\vec{b}] + [\vec{d}\vec{c}\vec{a}] + [\vec{d}\vec{b}\vec{c}]$

D. $[\vec{b}\vec{c}\vec{d}] + [\vec{d}\vec{a}\vec{c}] + [\vec{d}\vec{b}\vec{a}]$

Answer: A

Solution:

$\vec{a}, \vec{b}, \vec{c}, \vec{d} \rightarrow$ coplanar

$[\vec{a}\vec{b}\vec{c}] = ?$

$\vec{b}-\vec{a}, \vec{c}-\vec{b}, \vec{d}-\vec{c} \rightarrow$ coplanar

$[\vec{b}-\vec{a}\,\vec{c}-\vec{b}, \vec{d}-\vec{c}] = 0$

$\Rightarrow (\vec{b}-\vec{a}) \cdot ((\vec{c}-\vec{b}) \times (\vec{d}-\vec{c})) = 0$

$(\vec{b}-\vec{a}) \cdot (\vec{c} \times \vec{b} - \vec{c} \times \vec{a} - \vec{a} \times \vec{d}) = 0$

[bcd] − [bca] − [bad] − [acd] = 0

$[\vec{a}\vec{b}\vec{c}] = [\vec{d}\vec{c}\vec{a}] + [\vec{b}\vec{d}\vec{a}] + [\vec{c}\vec{d}\vec{b}]$

Question52

Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = 11$, $\vec{b} \cdot (\vec{a} \times \vec{c}) = 27$ and $\vec{b} \cdot \vec{c} = -\sqrt{3}|\vec{b}|$, then $|\vec{a} \times \vec{c}|^2$ is equal to ______.

[11-Apr-2023 shift 2]

Answer: 285

Solution:

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} + \hat{j} - \hat{k}$

$\vec{b} \cdot (\vec{a} \times \vec{c}) = 27, \vec{a} \cdot \vec{b} = 0$

$\vec{b} \times (\vec{a} \times \vec{c}) = -3\vec{a}$

Let θ be angle between $\vec{b}, \vec{a} \times \vec{c}$

Then $|\vec{b}| \cdot |\vec{a} \times \vec{c}| \sin\theta = 3\sqrt{14}$

$|\vec{b}| \cdot |\vec{a} \times \vec{c}| \cos\theta = 27$

$\Rightarrow \sin\theta = \frac{\sqrt{14}}{\sqrt{95}}$

$\therefore |\vec{b}| \times |\vec{a} \times \vec{c}| = 3\sqrt{95}$

$\Rightarrow |\vec{a} \times \vec{c}| = \sqrt{3} \times \sqrt{95}$

Question53

Let a, b, c be three distinct real numbers, none equal to one. If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ are coplanar, then $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is equal to

[12-Apr-2023 shift 1]

Options:

A. 1

B. 2

C. -2

D. -1

Answer: A

Solution:

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$$

$a(b-1)(c-1) - (1-a)(c-1) + (1-a)(1-b) = 0$

$a(1-b)(1-c) + (1-a)(1-c) + (1-a)(1-b) = 0$

$\frac{a}{1-a} + \frac{a}{1-b} + \frac{a}{1-c} = 0$

$\Rightarrow -1 + \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$

$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

Question54

Let $\lambda \in Z$, $\vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. Let $\vec{c}$ be a vector such that $\left(\vec{a} + \vec{b} + \vec{c}\right) \times \vec{c} = \vec{0}$, $\vec{a} \cdot \vec{c} = -17$ and $\vec{b} \cdot \vec{c} = -20$. Then $\left|\vec{c} \times \left(\lambda\hat{i} + \hat{j} + \hat{k}\right)\right|^2$ is equal to
[12-Apr-2023 shift 1]

Options:

A. 62

B. 53

C. 49

D. 46

Answer: D

Solution:

$\left(\vec{a} + \vec{b} + \vec{c}\right)_{\times}\vec{c} = 0$

$\left(\vec{a}+\vec{b}\right)\times\vec{c}=0$

$\vec{c}=\alpha\left(\vec{a}+\vec{b}\right)=\alpha(\lambda+3)\hat{i}+\alpha\hat{k}$

$\vec{b}\cdot\vec{c}=-20\Rightarrow 3\alpha(\lambda+3)+2\alpha=-20$

$\vec{a}\cdot\vec{c}=-17\Rightarrow \alpha\lambda(\lambda+3)-\alpha=-17$

$\Rightarrow \alpha(3\lambda+9+2)=-20$

$\alpha(\lambda^2+3\lambda-1)=-17$

$17(3\lambda+11)=20(\lambda^2+3\lambda-1)$

$20\lambda^2+9\lambda-207=0$

$\lambda=3\ \ (\lambda\in Z)$

$\Rightarrow \alpha=-1\Rightarrow \vec{c}=-\left(6\hat{i}+\hat{k}\right)$

$\vec{v}=\vec{c}\times\left(3\hat{i}+\hat{j}+\hat{k}\right)$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -6 & 0 & -1\\ 3 & 1 & 1\end{vmatrix}=\hat{i}+3\hat{j}-6\hat{k}$$

$|\vec{v}|^2=(-1)^2+3^2+6^2=46$

Question55

Let $\vec{a}=3\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=2\hat{i}-3\hat{j}+3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a}=\vec{b}\times\vec{c}$ and $\left|\vec{b}\right|^2=50$, then $|72-|\vec{b}+\vec{c}|^2|$ is equal to _______.
[13-Apr-2023 shift 1]

Answer: 66

Solution:

Solution:

$\left|\vec{a}\right|=\sqrt{11},\left|\vec{c}\right|=\sqrt{22}$

$\left|\vec{a}\right|=\left|\vec{b}\times\vec{c}\right|=\left|\vec{b}\right|\left|\vec{c}\right|\sin\theta$

$\sqrt{11}=\sqrt{50}\sqrt{22}\sin\theta$

$\Rightarrow \sin\theta=\dfrac{1}{10}$

$\left|\vec{b}+\vec{c}\right|^2=\left|\vec{b}\right|^2+\left|\vec{c}\right|^2+2\vec{b}\cdot\vec{c}$

$=\left|\vec{b}\right|^2+\left|\vec{c}\right|^2+2\left|\vec{b}\right|\left|\vec{c}\right|\cos\theta$

$=50+22+2\times\sqrt{50}\times\sqrt{22}\times\dfrac{\sqrt{99}}{10}$

$=72+66$

$|72-|\vec{b}+\vec{c}|^2|=66$

Question56

Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$, $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{d}\cdot\vec{a}=24$, then $\left|\vec{d}\right|^2$ is equal to -
[13-Apr-2023 shift 1]

Options:

A. 323

B. 423

C. 413

D. 313

Answer: C

Solution:

Solution:

$\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$

$\Rightarrow (\vec{d} - \vec{c}) \times \vec{b} = 0$

$\Rightarrow \vec{d} = \vec{c} + \lambda\vec{b}$

Also $\vec{d}ä = 24$

$\Rightarrow (\vec{c} + \lambda\vec{b}) \cdot \vec{a} = 24$

$\lambda = \dfrac{24 - \vec{a} \cdot \vec{c}}{\vec{b} \cdot \vec{a}} = \dfrac{24 - 6}{9} = 2$

$\Rightarrow \vec{d} = \vec{c} + 2(\vec{b})$

$= 8\hat{i} - 5\hat{j} + 18\hat{k}$

$\Rightarrow |\vec{d}|^2 = 64 + 25 + 324 = 413$

Question57

Let $|\vec{a}| = 2, |\vec{b}| = 3$ and the angle between the vectors $\vec{a}$ and $\vec{b}$ be $\frac{\pi}{4}$. Then $|(\vec{a} + 2\vec{b}) \times (2\vec{a} - 3\vec{b})|^2$ is equal to
[13-Apr-2023 shift 2]

Options:

A. 482

B. 841

C. 882

D. 441

Answer: C

Solution:

$\cos\left(\frac{\pi}{4}\right) = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$\Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{\vec{a} \cdot \vec{b}}{(2)(3)} \Rightarrow \vec{a} \cdot \vec{b} = 3\sqrt{2}$

Let $\vec{p} = \vec{a} + 2\vec{b}$

$\vec{q} = 2\vec{a} - 3\vec{b}$
$|\vec{p}|^2 = |\vec{a}|^2 + 4|\vec{b}|^2 + 4(\vec{a} \cdot \vec{b})$
$= 4 + 36 + 12\sqrt{2}$
$= 40 + 12\sqrt{2}$
$|\vec{q}|^2 = 4|\vec{a}|^2 + 9|\vec{b}|^2 - 12(\vec{a} - \vec{b})$
$= 16 + 81 - 36\sqrt{2}$
$= 97 - 36\sqrt{2}$
$\vec{p} \cdot \vec{q} = 2|\vec{a}|^2 - 6|\vec{b}|^2 + \vec{a} \cdot \vec{b}$
$= 8 - 54 + 3\sqrt{2}$
$= -46 + 3\sqrt{2}$
$|\vec{p} \times \vec{q}| = (|\vec{p}||\vec{q}|)^2 - (\vec{p} \cdot \vec{q})^2$
$= (40 + 12\sqrt{2}(97 - 36\sqrt{2})) - (3\sqrt{2} - 46)^2$
$= (3016 - 276\sqrt{2}) - (2134 - 276\sqrt{2})$
$= 882$

Question58

Let S be the set of all (λ, μ) for which the vectors $\lambda\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + \mu\hat{k}$ and $3\hat{i} - 4\hat{j} + 5\hat{k}$, where $\lambda - \mu = 5$, are coplanar, then $\sum_{(\lambda, \mu) \in S} 80(\lambda 2 + \mu^2)$ is equal to
[15-Apr-2023 shift 1]

Options:

A. 2130

B. 2210

C. 2290

D. 2370

Answer: C

Solution:

Solution:

$$\begin{vmatrix} \lambda & -1 & 1 \\ 1 & 2 & \mu \\ 3 & -4 & 5 \end{vmatrix} = 0 \ \& \lambda - \mu = 5$$

$\lambda(10 + 4\mu) + (5 - 3\mu) + (-10) = 0$
$(\mu + 5)(4\mu + 10) + 5 - 3\mu - 10 = 0$
$\mu = -15; \lambda = 5 / 4$
$\mu = -3; \lambda = 2$

Hence $\sum_{(\lambda, \mu) \in S80(\lambda}^{2 + \mu^2)}$

$= 80\left(\frac{250}{16} + 13\right)$
$= 1250 + 1040$
$= 2290$

Question59

Let ABCD be a quadrilateral. If E and F are the mid points of the diagonals AC and BD respectively and $\left(\overrightarrow{AB} - \overrightarrow{BC}\right) + \left(\overrightarrow{AD} - \overrightarrow{DC}\right) = k\overrightarrow{FE}$, then k is equal to
[15-Apr-2023 shift 1]

Options:

A. 4

B. 2

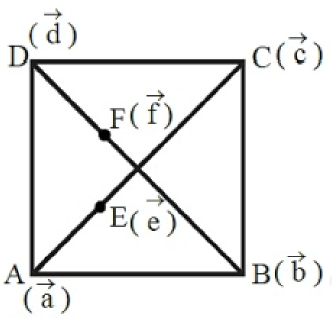
C. -2

D. -4

Answer: D

Solution:

Solution:



$\overrightarrow{AB} - \overrightarrow{BC} + \overrightarrow{AB} - \overrightarrow{DC} = k\overrightarrow{FE}$
$\left(\vec{b} - \vec{a}\right) - \left(\vec{c} - \vec{b}\right) + \left(\vec{d} - \vec{a}\right) - \left(\vec{c} - \vec{d}\right) = k\overrightarrow{FE}$
$2\left(\vec{b} + \vec{d}\right) - 2\left(\vec{a} - \vec{c}\right) = k\overrightarrow{FE}$
$2\left(2\vec{f}\right) - 2\left(2\vec{e}\right) = k\overrightarrow{FE}$
$4\left(\vec{f} - \vec{e}\right) = k\overrightarrow{FE}$
$-4\overrightarrow{FE} = k\overrightarrow{FE}$
$k = -4$

Question60

Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $\left|\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right| = 2$. If $\theta \in (0, \pi)$ is the angle between $\hat{a}$ and $\hat{b}$, then among the statements :
(S1) : $2\left|\hat{a} \times \hat{b}\right| = \left|\hat{a} - \hat{b}\right|$
(S2) : The projection of $\hat{a}$ on $\left(\hat{a} + \hat{b}\right)$ is $\frac{1}{2}$
[24-Jun-2022-Shift-2]

Options:

A. Only (S1) is true.

B. Only (S2) is true.

C. Both (S1) and (S2) are true.

D. Both (S1) and (S2) are false.

Answer: C

Solution:

$$\left|\hat{a}+\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right| = 2,\ \theta \in (0, \pi)$$

$$\Rightarrow \left|\hat{a}+\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right|^2 = 4$$

$$\Rightarrow |\hat{a}|^2 + |\hat{b}|^2 + 4|\hat{a}\times\hat{b}|^2 + 2\hat{a}\cdot\hat{b} = 4$$

$$\therefore \cos\theta = \cos 2\theta$$

$$\therefore \theta = \frac{2\pi}{3}$$

where θ is angle between $\hat{a}$ and $\hat{b}$.

$$\therefore 2\left|\hat{a}\times\hat{b}\right| = \sqrt{3} = \left|\hat{a}-\hat{b}\right|$$

(S1) is correct.

And projection of $\hat{a}$ on $\left(\hat{a}+\hat{b}\right) = \left|\dfrac{\hat{a}\cdot\left(\hat{a}+\hat{b}\right)}{\left|\hat{a}+\hat{b}\right|}\right| = \dfrac{1}{2}$ (S2) is correct.

Question61

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3$ widehat k $a_i > 0$, i = 1, 2, 3 be a vector which makes equal angles with the coordinate axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i} + 4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90°. If $\vec{a}$, $\vec{b}$ and x-axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is equal to:
[25-Jun-2022-Shift-1]

Options:

A. $\sqrt{7}$

B. $\sqrt{2}$

C. 2

D. 7

Answer: B

Solution:

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1 \Rightarrow \cos^2\alpha=\frac{1}{3} \Rightarrow \cos\alpha=\frac{1}{\sqrt{3}}$

$\vec{a}=\frac{\lambda}{3}\left(\hat{i}+\hat{j}+\text{widehat } k\right), \lambda>0$

$\frac{\lambda}{\sqrt{3}}\frac{\left(\hat{i}+\hat{j}+\text{widehat } k\right)\cdot\left(3\hat{i}+4\hat{j}\right)}{\sqrt{3^2+4^2}}=7$

$\Rightarrow \frac{\lambda}{\sqrt{3}}(3+4)=7\times5$

$\therefore \lambda=5\sqrt{3}$

$\vec{a}=5\left(\hat{i}+\hat{j}+\text{widehat } k\right)$

Let $\vec{b}=p\hat{i}+q\hat{j}+r\text{ widehat } k$

$\vec{a}\cdot\vec{b}=0$ and $\left[\vec{a}\,\vec{b}\,\hat{i}\right]=0$

$\Rightarrow p+q+r=0$

$\& \begin{vmatrix} p & q & r \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix}=0 \Rightarrow q=r$

$\vec{b}=-2r\hat{i}+r\hat{j}+r\text{ widehat } k$

$\vec{b}=r\left(-2\hat{i}+\hat{j}+\text{widehat } k\right)$

Now $|\vec{a}|=|\vec{b}|$

$5\sqrt{3}=|r|\sqrt{b}\Rightarrow|r|=\frac{5}{\sqrt{2}}$

$\Rightarrow$ Projection of $\vec{b}$ on $3\hat{i}+4\hat{j}=\left|\frac{\vec{b}\cdot\left(3\hat{i}+4\hat{j}\right)}{\sqrt{3^2+4^2}}\right|$

$=|r|\frac{(-6+4)}{5}=\left|\frac{-2r}{5}\right|$

Projection $=\frac{2}{5}\times\frac{5}{\sqrt{2}}=\sqrt{2}$

$\therefore B$ is correct.

Question62

Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $\vec{a}|\,|=4, |\vec{b}|=,3$ and $\theta\in\left(\frac{\pi}{4},\frac{\pi}{3}\right)$. Then $\left|\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)\right|^2+4\left(\vec{a}\cdot\vec{b}\right)^2$ is equal to__

[25-Jun-2022-Shift-1]

Answer: 576

Solution:

$\left|\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)\right|^2+4\left(\vec{a}\cdot\vec{b}\right)^2$

$\Rightarrow\left|\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}\right|^2+4\left(\vec{a}\cdot\vec{b}\right)^2$

$\Rightarrow\left|2\left(\vec{a}\times\vec{b}\right)\right|^2+4\left(\vec{a}\cdot\vec{b}\right)^2$

$\Rightarrow 4\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}\cdot\vec{b}\right)^2$

. $\Rightarrow 4\left|\vec{a}\right|^2\left|\vec{b}\right|^2=4.16.9=576$

Question63

Let $\vec{b}=\hat{i}+\hat{j}+\lambda\hat{k}$, $\lambda\in R$. If $\vec{a}$ is a vector such that $\vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$ and $\vec{a}\cdot\vec{b}+21=0$, then $\left(\vec{b}-\vec{a}\right)\cdot\left(\hat{k}-\hat{j}\right)+\left(\vec{b}+\vec{a}\right)\cdot\left(\hat{i}-\hat{k}\right)$ is equal to____
[25-Jun-2022-Shift-2]

Answer: 14

Solution:

Solution:

Let $\vec{a}=x\hat{i}=y\hat{j}+z\hat{k}$

So, $\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ 1 & 1 & \lambda\end{vmatrix}=\hat{i}(\lambda y-z)+\hat{j}(z-\lambda x)+\hat{k}(x-y)$

$\Rightarrow\lambda y-z=13,\ z-\lambda x=-1,\ x-y=-4$
and $x+y+\lambda z=-21$
$\Rightarrow$ Clearly, $\lambda=3$, $x=-2$, $y=2$ and $z=-7$
So, $\vec{b}-\vec{a}=3\hat{i}-\hat{j}+10\hat{k}$
and $\vec{b}+\vec{a}=-\hat{i}+3\hat{j}-4\hat{k}$
$\Rightarrow\left(\vec{b}-\vec{a}\right)\cdot\left(\hat{k}-\hat{j}\right)+\left(\vec{b}+\vec{a}\right)\cdot\left(\hat{i}-\hat{k}\right)=11+3=14$

Question64

If $\vec{a}\cdot\vec{b}=1$, $\vec{b}\cdot\vec{c}=2$ and $\vec{c}\cdot\vec{a}=3$, then the value of $\left[\vec{a}\times\left(\vec{b}\times\vec{c}\right),\vec{b}\times\left(\vec{c}\times\vec{a}\right),\vec{c}\times\left(\vec{b}\times\vec{a}\right)\right]$ is :
[26-Jun-2022-Shift-1]

Options:

A. 0

B. $-6\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

C. $-12\vec{c}\cdot(\vec{a}\times\vec{b})$

D. $-12\vec{b}\cdot(\vec{c}\times\vec{a})$

Answer: A

Solution:

$\because \vec{a}\times(\vec{b}\times\vec{c})=3\vec{b}-\vec{c}=\vec{u}$

$\vec{b}\times(\vec{c}\times\vec{a})=\vec{c}-2\vec{a}=\vec{v}$

$\vec{c}\times(\vec{b}\times\vec{a})=3\vec{b}-2\vec{a}=\vec{w}$

$\therefore \vec{u}+\vec{v}=\vec{w}$

So, vectors $\overrightarrow{u, v}$ and $\vec{w}$ are coplanar, hence their Scalar triple product will be zero.

Question65

Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}-3\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$ be three given vectors. Let $\vec{v}$ be a vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ is $\frac{2}{\sqrt{3}}$. If $\vec{v}\cdot\hat{j}=7$, then $\vec{v}\cdot(\hat{i}+\hat{k})$ is equal to :
[26-Jun-2022-Shift-2]

Options:

A. 6

B. 7

C. 8

D. 9

Answer: D

Solution:

Let $\vec{v}=\lambda_1\vec{a}+\lambda_2\vec{b}$, where $\lambda_1,\lambda_2\in R$ $=(\lambda_1+2\lambda_2)\hat{i}+(\lambda_1-3\lambda_2)\hat{j}+(2\lambda_1+\lambda_2)\hat{k}$

$\because$ Projection of $\vec{v}$ on $\vec{c}$ is $\frac{2}{\sqrt{3}}$

$\therefore \frac{\lambda_1+2\lambda_2-\lambda_1+3\lambda_2+2\lambda_1+\lambda_2}{\sqrt{3}}=\frac{2}{\sqrt{3}}$

$\therefore \lambda_1+3\lambda_2=1$... (i)

and $\vec{v}\cdot\hat{j}=7\Rightarrow\lambda_1-3\lambda_2=7$... (ii) from equation (i) and (ii)

$\lambda_1 = 4, \lambda_2 = -1$

$\therefore \vec{v} = 2\hat{i} + 7\hat{j} + 7\hat{k}$

$\therefore \vec{v} \cdot (\hat{i} + \hat{k}) = 2 + 7$

$= 9$

Question66

Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1, 2, ..., 10\}$ is :
[27-Jun-2022-Shift-1]

Options:

A. 0

B. 1

C. 2

D. 3

Answer: A

Solution:

$\vec{a} = \hat{i} + \hat{j} - \hat{k}$
$\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$
Now, $\vec{b} \times \vec{c} = \vec{a}$
$\vec{c} \cdot (\vec{b} \times \vec{c}) = \vec{c} \cdot \vec{a}$
$\vec{c} \cdot \vec{a} = 0$
$\Rightarrow (\hat{i} + \hat{j} - \hat{k})(2\hat{i} - 3\hat{j} + 2\hat{k}) = 0$
$= 2 - 3 - 2 = 0$
$\Rightarrow -3 = 0$ (Not possible)
$\Rightarrow$ No possible value of $\vec{b}$ is possible.

Question67

Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute, $|\vec{a}| = 1$, and $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$. If $\vec{c} = 2\sqrt{2}(\vec{a}\times\vec{b}) - 2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is
[27-Jun-2022-Shift-2]

Options:

A. $\frac{\pi}{4}$

B. $-\frac{\pi}{4}$

C. $\frac{5\pi}{6}$

D. $\frac{3\pi}{4}$

Answer: D

Solution:

Solution:

$\because \vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$.

$\therefore \frac{1}{2}|\vec{a}\times\vec{b}| = 2\sqrt{2}$

$|\vec{a}||\vec{b}|\sin\theta = 4\sqrt{2}$

$\Rightarrow |\vec{b}|\sin\theta = 4\sqrt{2}\ldots.$ (i)

and $\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$

$|\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$

$\Rightarrow \tan\theta = 1$

$\therefore \theta = \frac{\pi}{4}$

By (i) $|\vec{b}| = 8$

Now $\vec{c} = 2\sqrt{2}(\vec{a}\times\vec{b}) - 2\vec{b}$

$\Rightarrow \vec{c}\cdot\vec{b} = -2|\vec{b}|^2 = -128$

and $\vec{c}\cdot\vec{c} = 8|\vec{a}\times\vec{b}|^2 + 4|\vec{b}|^2$

$\Rightarrow |c|^2 = 8.32 + 4.64$

$\Rightarrow |\vec{c}| = 16\sqrt{2}$

From (ii) and (iii)

$||\vec{c}||\vec{b}|\cos\alpha = -128$

$\Rightarrow \cos\alpha = \frac{-1}{\sqrt{2}}$

$\alpha = \frac{3\pi}{4}$

Question68

If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a}\cdot\vec{c} = 5$, $\vec{b}\perp\vec{c}$, then $122(c_1 + c_2 + c_3)$ is equal to
[28-Jun-2022-Shift-1]

Answer: 150

Solution:

Solution:

$2C_1 + C_2 + 3C_3 = 5 \ldots\ldots.$ (i)

$3C_1 + 3C_2 + C_3 = 0$

$$[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 2 & 1 & 3 \\ 3 & 3 & 1 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$= 2(3C_3 - C_2) - 1(3C_3 - C_1) + 3(3C_2 - 3C_1)$

$= 3C_3 + 7C_2 - 8C_1$

$\Rightarrow 8C_1 - 7C_2 - 3C_3 = 0 \ldots\ldots$ (iii)

$C_1 = \frac{10}{122}, C_2 = \frac{-85}{122}, C_3 = \frac{225}{122}$

So $122(C_1 + C_2 + C_3) = 150$

Question69

Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$, where $\alpha \in R$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2 + 4)}$, then the value of $2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$ is equal to : [28-Jun-2022-Shift-2]

Options:

A. 10

B. 7

C. 9

D. 14

Answer: D

Solution:

Solution:

$\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -1 \\ -2 & \alpha & 1 \end{vmatrix} = (2 + \alpha)\hat{i} - (\alpha - 2)\hat{j} + (\alpha^2 + 4)\hat{k}$$

Now $|\vec{a} \times \vec{b}| = \sqrt{15(\alpha^2 + 4)}$

$\Rightarrow (2 + \alpha)^2 + (\alpha - 2)^2 + (\alpha^2 + 4)^2 = 15(\alpha^2 + 4)$

$\Rightarrow \alpha^4 - 5\alpha^2 - 36 = 0$

$\therefore \alpha = \pm 3$

Now, $2|\vec{a}|^2 + (\vec{a} - \vec{b})|\vec{b}|^{-2} = 2.14 - 14 = 14$

Question70

Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is :
[28-Jun-2022-Shift-2]

Options:

A. $\frac{1}{3}$

B. 1

C. $\frac{5}{3}$

D. $\frac{7}{3}$

Answer: C

Solution:

Solution:
Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
and $\vec{a} \cdot \left(3\hat{i} - \frac{1}{2}\hat{j} + 2\hat{k}\right) = 0 \Rightarrow 3a_1 + \frac{a_2}{2} + 2a_3 = 0 \ldots\ldots$ (i)
and $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$
$\Rightarrow a_2\hat{i} + (2a_3 - a_1)\hat{j} - 2a_2\hat{k} = 2\hat{i} - 13\hat{j} - 4\hat{k}$
$\therefore a_2 = 2 \ldots\ldots$ (ii)
and $a_1 - 2a_3 = 13 \ldots\ldots$ (iii)
From eq. (i) and (iii) : $a_1 = 3$ and $a_3 = -5$
$\therefore \vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$
$\therefore$ projection of $\vec{a}$ on $2\hat{i} + 2\hat{j} + \hat{k} = \frac{6 + 4 - 5}{3} = \frac{5}{3}$

Question71

Let $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in R$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ is equal to:
[29-Jun-2022-Shift-1]

Options:

A. 3

B. 4

C. 5

D. 6

Answer: A

Solution:

Solution:

$\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$

$\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$

Projection of $\vec{a}$ on $\vec{c}$ is

$\frac{\vec{a} \cdot \vec{c}}{|\vec{b}|} = \frac{10}{3}$

$\frac{\alpha + 6 + 2}{\sqrt{1^2 + 2^2 + (-2)^2}} = \frac{\alpha + 8}{3} = \frac{10}{3}$

$\therefore \alpha = 2$

$\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -\beta & 4 \\ 1 & 2 & -2 \end{vmatrix} = (2\beta - 8)\hat{i} + 10\hat{j} + (6 + \beta)\hat{k} = -6\hat{i} + 10\hat{j} + 7\hat{k}$$

$2\beta - 8 = -6 \& 6 + \beta = 7$

$\therefore \beta = 1$

$\alpha + \beta = 2 + 1 = 3$

Question72

Let A, B, C be three points whose position vectors respectively are

$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in R$

$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$

If α is the smallest positive integer for which $\vec{a}, \vec{b}, \vec{c}$ are noncollinear, then the length of the median, in ΔABC, through A is :
[29-Jun-2022-Shift-2]

Options:

A. $\frac{\sqrt{82}}{2}$

B. $\frac{\sqrt{62}}{2}$

C. $\frac{\sqrt{69}}{2}$

D. $\frac{\sqrt{66}}{2}$

Answer: A

Solution:

Solution:
$\overrightarrow{AB} | \overrightarrow{AC}$ if $\frac{1}{2} = \frac{\alpha - 4}{-6} = \frac{1}{2} \Rightarrow \alpha = 1$

$\vec{a}, \vec{b}, \vec{c}$ are non-collinear for $\alpha = 2$ (smallest positive integer)

Mid-point of BC = $M\left(\frac{5}{2}, 0, \frac{9}{2}\right)$

$AM = \sqrt{\frac{9}{4} + 16 + \frac{9}{4}} = \frac{\sqrt{82}}{2}$

Question73

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} + \left(\vec{b} \times \vec{c}\right) = \vec{0}$ and $\vec{b} \cdot \vec{c} = 5$. Then the value of $3\left(\vec{c} \cdot \vec{a}\right)$ is equal to____
[29-Jun-2022-Shift-2]

Answer: 0

Solution:

Solution:
$\vec{a} + \vec{b} \times \vec{c} = 0$
$\vec{a} \times \vec{b} + |\vec{b}|^2 \vec{c} - 5\vec{b} = 0$
It gives $\vec{c} = \frac{1}{3}\left(10\hat{i} + 3\hat{j} + 2\hat{k}\right)$
so $3\vec{a} \cdot \vec{c} = 10$
But it does not satisfy $\vec{a} + \vec{b} \times \vec{c} = 0$.
This question has data error.
Alternate (Explanation) :
According to given $\vec{a}$&$\vec{b}$ $\vec{a} \cdot \vec{b} = 1 - 2 + 3 = 2$... (i)
but given equation
$\vec{a} = -\left(\vec{b} \times \vec{c}\right)$
$\Rightarrow \vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$
which contradicts.

Question74

Let $\hat{a}$, $\hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2\left(\vec{c} \times \hat{a}\right)$, then $|6\vec{c}|^2$ is equal to :
[24-Jun-2022-Shift-1]

Options:

A. $6(3 - \sqrt{3})$

B. $3 + \sqrt{3}$

C. $6(3 + \sqrt{3})$

D. $6(\sqrt{3}+1)$

Answer: C

Solution:

$$\because \hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$$

$$\Rightarrow \hat{b} \cdot \vec{c} = |\vec{c}|^2$$

$$\therefore \hat{b} - \vec{c} = 2(\vec{c} \times \vec{a})$$

$$\Rightarrow |\hat{b}|^2 + |\vec{c}|^2 - 2\hat{b} \cdot \vec{c} = 4|\vec{c}|^2 |\vec{a}|^2 \sin^2 \frac{\pi}{12}$$

$$\Rightarrow 1 + |\vec{c}|^2 - 2|c|^2 = 4|\vec{c}|^2 \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2$$

$$\Rightarrow 1 = |\vec{c}|^2 (3 - \sqrt{3})$$

$$\Rightarrow 36|\vec{c}|^2 = \frac{36}{3-\sqrt{3}} = 6(3+\sqrt{3})$$

Question75

If the shortest distance between the lines
$\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(\hat{i} - a\hat{j})$
and $\vec{r} = (-\hat{j} + 2\hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$ is $\sqrt{\frac{2}{3}}$, then the integral value of a is equal to
[24-Jun-2022-Shift-1]

Answer: 2

Solution:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix} = -a\hat{i} - \hat{j} + (a-1)\hat{k}$$

$$\vec{a}_1 - \vec{a}_2 = -\hat{i} + \hat{j} + \hat{k}$$

$$\text{Shortest distance} = \left| \frac{(\vec{a}_1 - \vec{a}_2) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\Rightarrow \sqrt{\frac{2}{3}} = \frac{2(a-1)}{\sqrt{a^2 + 1 + (a-1)^2}}$$

$$\Rightarrow 6(a^2 - 2a + 1) = 2a^2 - 2a + 2$$

$$\Rightarrow (a-2)(2a-1) = 0 \Rightarrow a = 2 \text{ because } a \in z.$$

Question76

Let ABC be a triangle such that
$\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AB} = \vec{c}$, a$| = 6\sqrt{2}$, overrightarrow$|\vec{b}| = 2\sqrt{3}$ and $\vec{b} \cdot \vec{c} = 12$.
Consider the statements :
(S1) : $\left|\left(\vec{a} \times \vec{b}\right) + \left(\vec{c} \times \vec{b}\right)\right| - |\vec{c}| = 6(2\sqrt{2} - 1)$
(S2) : $\angle ACB = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$
Then
[25-Jul-2022-Shift-1]

Options:

A. both (S1) and (S2) are true

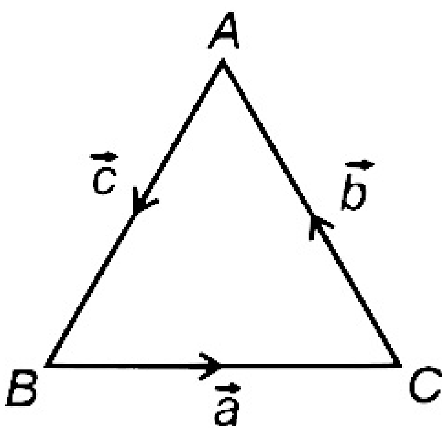
B. only (S1) is true

C. only (S2) is true

D. both (S1) and (S2) are false

Answer: D

Solution:



$\because \vec{a} + \vec{b} + \vec{c} = 0$
then $\overline{a} + \vec{c} = -\vec{b}$
then $\left(\vec{a} + \vec{c}\right) \times \vec{b} = -\vec{b} \times \overline{b}$

$\therefore\ \vec{a}\times\vec{b}+\vec{c}\times\vec{b}=\bar{0}\ \ldots$ (i)

For (S1): $\left|\vec{a}\times\vec{b}+\vec{c}\times\vec{b}\right|-\left|\vec{c}\right|=6(2\sqrt{2}-1)$

$\left|(\vec{a}+\vec{c})\times\vec{b}\right|-\left|\vec{c}\right|=6(2\sqrt{2}-1)$

$\left|\vec{c}\right|=6-12\sqrt{2}$ (not possible)

For (S2) : from (i) $\vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow \vec{b}\cdot\vec{b}+\vec{c}\cdot\vec{b}=-\vec{a}\cdot\vec{b}$

$\Rightarrow\ 12+12=-6\sqrt{2}\cdot 2\sqrt{3}\cos(\pi-\angle ACB)$

$\therefore\ \cos(\angle ACB)=\sqrt{\frac{2}{3}}$

$\therefore\ \angle ACB=\cos^{-1}\sqrt{\frac{2}{3}}$

$\therefore$ S(2) is correct.

Question77

Let $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ and let $\vec{b}$ be a vector such that $\vec{a}\times\vec{b}=2\hat{i}-\hat{k}$ and $\vec{a}\cdot\vec{b}=3$. Then the projection of $\vec{b}$ on the vector $\vec{a}-\vec{b}$ is:
[25-Jul-2022-Shift-2]

Options:

A. $\frac{2}{\sqrt{21}}$

B. $2\sqrt{\frac{3}{7}}$

C. $\frac{2}{3}\sqrt{\frac{7}{3}}$

D. $\frac{2}{3}$

Answer: A

Solution:

$\vec{a}=\hat{i}-\hat{j}+2\hat{k}$

$\vec{a}\times\vec{b}=2\hat{i}-\hat{k}$

$\vec{a}\cdot\vec{b}=3$

$\left|\vec{a}\times\vec{b}\right|^2+\left|\vec{a}\cdot\vec{b}\right|^2=\left|\vec{a}\right|^2\cdot|\vec{b}|^2$

$\Rightarrow 5+9=6\left|\vec{b}\right|^2$

$\Rightarrow |b|^2=\frac{7}{3}$

$\left|\vec{a}-\vec{b}\right|=\sqrt{|\vec{a}|^2+.|\vec{b}|^2-2\vec{a}\cdot\vec{b}}=\sqrt{\frac{7}{3}}$

projection of $\vec{b}$ on $\vec{a}-\vec{b}=\dfrac{\vec{b}\cdot\left(\vec{a}-\vec{b}\right)}{\left|\vec{a}-\vec{b}\right|}$

$=\dfrac{\vec{b}\cdot\vec{a}-\left|\vec{b}\right|^2}{\vec{a}-\vec{b}|}=\dfrac{3-\frac{7}{3}}{\sqrt{\frac{7}{3}}}$

$=\frac{2}{\sqrt{21}}$

Question78

Let $\vec{a} = \alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$, $\alpha > 0$. If the projection of $\vec{a} \times \vec{b}$ on the vector $-\hat{i} + 2\hat{j} - 2\hat{k}$ is 30 , then α is equal to :
[26-Jul-2022-Shift-1]

Options:

A. $\frac{15}{2}$

B. 8

C. $\frac{13}{2}$

D. 7

Answer: D

Solution:

Solution:

Given : $\vec{a} = (\alpha, 1, -1)$ and $\vec{b} = (2, 1, -\alpha)$

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & -1 \\ 2 & 1 & -\alpha \end{vmatrix}$$

$= (-\alpha + 1)\hat{i} + (\alpha^2 - 2)\hat{j} + (\alpha - 2)\hat{k}$

Projection of $\vec{c}$ on $\vec{d} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$. = \left| \vec{c} \cdot \frac{\vec{d}}{|d|} \right| = 30$ Given $\Big\}$

$\Rightarrow = \left| \frac{\alpha - 1 - 4 + 2\alpha^2 - 2\alpha + 4}{\sqrt{1 + 4 + 4}} \right| = 30$

On solving $\alpha = \frac{-13}{2}$ (Rejected as $.\alpha > 0$) and $\alpha = 7$

Question79

Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{i} + 12\hat{k}$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to
[27-Jul-2022-Shift-1]

Options:

A. 2

B. $\frac{39}{5}$

C. 9

D. $\frac{46}{5}$

Answer: D

Solution:

Solution:

$\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$

$\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & \beta \\ 3 & -5 & 4 \end{vmatrix} = -\hat{i} + 9\hat{j} + 12\hat{k}$$

$4 + 5\beta = -1 \Rightarrow \beta = -1$

$-5\alpha - 3 = 12 \Rightarrow \alpha = -3$

$\vec{b} - 2\vec{a} = 3\hat{i} - 5\hat{j} + 4\hat{k} - 2\left(-3\hat{i} + \hat{j} - \hat{k}\right)$

$\vec{b} - 2\vec{a} = 9\hat{i} - 7\hat{j} + 6\hat{k}$

$\vec{b} + \vec{a} = \left(3\hat{i} - 5\hat{j} + 4\hat{k}\right) + \left(-3\hat{i} + \hat{j} - \hat{k}\right)$

$\vec{b} + \vec{a} = -4\hat{j} + 3\hat{k}$

Projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is $= \frac{\left(\vec{b} - 2\vec{a}\right) \cdot \left(\vec{b} + \vec{a}\right)}{\left|\vec{b} + \vec{a}\right|}$

$= \frac{28 + 18}{5} = \frac{46}{5}$

Question80

Let $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$. If $\left(\left(\vec{a} \times \vec{b}\right) \times \hat{i}\right) \cdot \hat{k} = \frac{23}{2}$, then $\left|\vec{b} \times 2\hat{j}\right|$ is equal to
[27-Jul-2022-Shift-1]

Options:

A. 4

B. 5

C. $\sqrt{21}$

D. $\sqrt{17}$

Answer: B

Solution:

Solution:

Given, $\vec{a} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{b} = \alpha\hat{i} + \beta\hat{j} + 2\hat{k}$

Also, $\left(\left(\vec{a} \times \vec{b}\right) \times i\right) \cdot \hat{k} = \frac{23}{2}$

$\Rightarrow \left(\left(\vec{a} \cdot \hat{i}\right)\vec{b} - \left(\vec{b} \cdot \hat{i}\right) \cdot \bar{a}\right) \cdot \hat{k} = \frac{23}{2}$

$\Rightarrow \left(2 \cdot \vec{b} - \alpha \cdot \vec{a}\right) \cdot \hat{k} = \frac{23}{2}$

$\Rightarrow 2 \cdot 2 - 5\alpha = \frac{23}{2} \Rightarrow \alpha = \frac{-3}{2}$

Now, $\left|\vec{b} \times 2j\right| = \left|\left(\alpha\hat{i} + \beta\hat{j} + 2\hat{k}\right) \times 2\hat{j}\right|$

$= \left|2\alpha\hat{k} + 0 - 4\hat{i}\right|$

$= \sqrt{4\alpha^2 + 16}$

$= \sqrt{4\left(\frac{-3}{2}\right)^2 + 16} = 5$

Question81

Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-coplanar vectors such that $\vec{a} \times \vec{b} = 4\vec{c}$, $\vec{b} \times \vec{c} = 9\vec{a}$ and $\vec{c} \times \vec{a} = \alpha\vec{b}$, $\alpha > 0$

If $\left|\vec{a}\right| + \left|\vec{b}\right| + \left|\vec{c}\right| = \frac{1}{36}$, then α is equal to________.

[27-Jul-2022-Shift-2]

Answer: 36

Solution:

Given,

$\vec{a} \times \vec{b} = 4 \cdot \vec{c}.....$ (i)

$\vec{b} \times \vec{c} = 9 \cdot \vec{a}.....$ (ii)

$\vec{c} \times \vec{a} = \alpha \cdot \vec{b}....$ (iii)

Taking dot products with c, $\vec{a}$, $\vec{\vec{b}}$ we get

$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

Hence,

(i) $\Rightarrow\left|\vec{a}\right| \cdot \left|\vec{b}\right| = 4 . \vec{c}|.....$ (iv)

(ii) $\Rightarrow|\vec{b}| \cdot \left|\vec{c}\right| = 9 \cdot \left|\vec{a}\right|....$ (v)

(iii) $\Rightarrow \vec{c}| \cdot \left|\vec{a}\right| = \alpha \cdot |\vec{b}|...$ (vi)

Multiplying (iv), (v) and (vi)

$\Rightarrow\left|\vec{a}\right| \cdot |\vec{b}| \cdot \left|\vec{c}\right| = 36\alpha.....$ (vii)

Dividing (vii) by (iv) $\Rightarrow\left|\vec{c}\right|^2 = 9\alpha \Rightarrow |\vec{c} = 3\sqrt{\alpha}....$ (viii)

Dividing (vii) by (v) $\Rightarrow\left|\vec{a}\right|^2 = 4\alpha \Rightarrow \left|\vec{a}\right| = 2\sqrt{\alpha}$

Dividing (viii) by (vi) $\Rightarrow$overrightarrow $\left|\vec{b}\right|^2 = 36 \Rightarrow \vec{b}| = 6$

Now, as given, $3\sqrt{\alpha} + 2\sqrt{\alpha} + 6 = \frac{1}{36} \Rightarrow \sqrt{\alpha} = \frac{-43}{36}$

Question82

Let the vectors $\vec{a} = (1+t)\hat{i} + (1-t)\hat{j} + \hat{k}$, $\vec{b} = (1-t)\hat{i} + (1+t)\hat{j} + 2\hat{k}$ and $\vec{c} - t\hat{i} - t\hat{j} + \hat{k}$, $t \in R$ be such that for $\alpha, \beta, \gamma \in R$, $\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c} = \vec{0} \Rightarrow \alpha = \beta = \gamma = 0$. Then, the setof all values of t

is :
[28-Jul-2022-Shift-1]

Options:

A. a non-empty finite set

B. equal to N

C. equal to $R-\{0\}$

D. equal to R.

Answer: C

Solution:

Solution:

Clearly $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar

$$\begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix} \neq 0$$

$\Rightarrow(1+t)(1+t+2t)-(1-t)(1-t-2t)+1(t^2-t-t-t^2) \neq 0$
$\Rightarrow(3t^2+4t+1)-(1-t)(1-3t)-2t \neq 0$
$\Rightarrow(3t^2+4t+1)-(3t^2-4t+1)-2t \neq 0$
$\Rightarrow t \neq 0$

Question83

Let a vector $\vec{a}$, has magnitude 9. Let a vector $\vec{b}$ be such that for every $(x, y) \in R \times R-\{(0,0)\}$, the vector $(x\vec{a}+y\vec{b})$ is perpendicular to the vector $(6y\vec{a}-18x\vec{b})$. Then the value of $|\vec{a}\times\vec{b}|$ is equal to:
[28-Jul-2022-Shift-1]

Options:

A. $9\sqrt{3}$

B. $27\sqrt{3}$

C. 9

D. 81

Answer: B

Solution:

Solution:

$(x\vec{a}+y\vec{b})\cdot(6y\vec{a}-18x\vec{b})=0$
$\Rightarrow(6xy|\vec{a}|^2-18xy|\vec{b}|^2)+(6y^2-18x^2)\vec{a}\cdot\vec{b}=0$
As given equation is identity
Coefficient of x^2 = coefficient of y^2 = coefficient of $xy=0$

$\Rightarrow |\vec{a}|^2 = .3|\vec{b}|^2 \Rightarrow |\vec{b}| = 3\sqrt{3}$

and $\vec{a} . \vec{b} = 0$

$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$

$= 9.3\sqrt{3}.1 = 27\sqrt{3}$

Question84

Let S be the set of all $a \in R$ for which the angle between the vectors $\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}$, $(b > 1)$ is acute. Then S is equal to :
[28-Jul-2022-Shift-2]

Options:

A. $\left(-\infty, -\frac{4}{3}\right)$

B. Φ

C. $\left(-\frac{4}{3}, 0\right)$

D. $\left(\frac{12}{7}, \infty\right)$

Answer: B

Solution:

Solution:

$\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$

$\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}$

For acute angle $\vec{u} \cdot \vec{v} > 0$

$\Rightarrow a(\log_e b)^2 - 12 + 6a(\log_e b) > 0$

$\because b > 1$

Let $\log_e b = t \Rightarrow t > 0$ as $b > 1$

$at^2 + 6at - 12 > 0 \quad \forall t > 0$

$\Rightarrow a \in \varphi$

Question85

Let $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. Let $\vec{c}$ be a vector satisfying $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \vec{c}$. If $\vec{b}$ and $\vec{c}$ are non-parallel, then the value of λ is:
[29-Jul-2022-Shift-1]

Options:

A. -5

B. 5

C. 1

D. -1

Answer: A

Solution:

Solution:

$\vec{a}=3\hat{i}+\hat{j}\,\&\,\vec{b}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\vec{b}+\lambda\vec{c}$

If $\vec{b}\,\&\,\vec{c}$ are non-parallel

then $\vec{a}\cdot\vec{c}=1\,\&\,\vec{a}\cdot\vec{b}=-\lambda$

but $\vec{a}\cdot\vec{b}=5\Rightarrow\lambda=-5$

Question86

Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{4}$. If θ is the angle between the vectors $\left(\hat{a}+\hat{b}\right)$ and $\left(\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right)$, then the value of $164\cos^2\theta$ is equal to :
[29-Jul-2022-Shift-1]

Options:

A. $90+27\sqrt{2}$

B. $45+18\sqrt{2}$

C. $90+3\sqrt{2}$

D. $54+90\sqrt{2}$

Answer: A

Solution:

Solution:

$\hat{a}\cdot\hat{b}=\frac{1}{\sqrt{2}}$ and $\left|\hat{a}\times\hat{b}\right|=\frac{1}{\sqrt{2}}$

$$\frac{\left(\hat{a}+\hat{b}\right)\cdot\left(\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right)}{\left|\hat{a}+\hat{b}\right|\left|\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right|}=\cos\theta$$

$$\Rightarrow\cos\theta=\frac{1+3\hat{a}\hat{b}+2}{\left|\hat{a}+\hat{b}\right|\left|\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right|}$$

$\left|\hat{a}+\hat{b}\right|^2=2+\sqrt{2}$

$\left|\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right|^2=1+4+4\left|\hat{a}\times\hat{b}\right|^2+4\hat{a}\hat{b}$

$=5+4\cdot\frac{1}{2}+\frac{4}{\sqrt{2}}=7+2\sqrt{2}$

So, $\cos^2\theta=\frac{\left(3+\frac{3}{\sqrt{2}}\right)^2}{(2+\sqrt{2})(7+2\sqrt{2})}=\frac{9\sqrt{2}(5\sqrt{2}+3)}{164}$

$\Rightarrow 164\cos^2\theta=90+27\sqrt{2}$

Question87

If (2, 3, 9), (5, 2, 1), (1, λ, 8) and (λ, 2, 3) are coplanar, then the product of all possible values of λ is:
[29-Jul-2022-Shift-2]

Options:

A. $\frac{21}{2}$

B. $\frac{59}{8}$

C. $\frac{57}{8}$

D. $\frac{95}{8}$

Answer: D

Solution:

Solution:
$\because A(2, 3, 9), B(5, 2, 1), C(1, \lambda, 8)$ and $D(\lambda, 2, 3)$ are coplanar.
$\therefore [\overrightarrow{ABACAD}] = 0$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda - 3 & -1 \\ \lambda - 2 & -1 & -6 \end{vmatrix} = 0$$

$\Rightarrow [-6(\lambda - 3) - 1] - 8(1 - (\lambda - 3)(\lambda - 2)) + (6 + (\lambda - 2) = 0$
$\Rightarrow 3(-6\lambda + 17) - 8(-\lambda^2 + 5\lambda - 5) + (\lambda + 4) = 8$
$\Rightarrow 8\lambda^2 - 57\lambda + 95 = 0$
$\therefore \lambda_1\lambda_2 = \frac{95}{8}$

Question88

Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}) \cdot (\vec{c} \times \vec{a}) + (\vec{c} \times \vec{a}) \cdot (\vec{a} \times \vec{b}) = 168$, then $|\vec{a}| + |\vec{b}| + |\vec{c}|$ is equal to:
[29-Jul-2022-Shift-2]

Options:

A. 10

B. 14

C. 16

D. 18

Answer: C

Solution:

Solution:

$\left|\vec{a}\right|\left|\vec{b}\right|\left|\vec{c}\right| = 14$

$\vec{a} \wedge \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \wedge \vec{a} = \theta = \dfrac{2\pi}{3}$

So, $\vec{a} \cdot \vec{b} = -\dfrac{1}{2}ab$, $\vec{b} \cdot \vec{c} = -\dfrac{1}{2}bc$, $\vec{a} \cdot \vec{c}. = -\dfrac{1}{2}ac$

(let)

$\left(\vec{a} \times \vec{b}\right) \cdot \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \cdot \vec{b}\right)\left(\vec{b} \cdot \vec{c}\right) - \left(\vec{a} \cdot \vec{c}\right)\left(\vec{b} \cdot \vec{b}\right)$

$= \dfrac{1}{4}ab^2c + \dfrac{1}{2}ab^2c = \dfrac{3}{4}ab^2c$

Similarly

$\left(\vec{b} \times \vec{c}\right) \cdot \left(\vec{c} \times \vec{a}\right)$

$= \dfrac{3}{4}abc^2 \left(\vec{c} \times \vec{a}\right) \cdot \left(\vec{a} \times \vec{b}\right)$

$= \dfrac{3}{4}a^2bc\ 168$

$= \dfrac{3}{4}abc(a + b + c)$

So, $(a + b + c) = 16$

Question89

Let $\vec{a}$ and $\vec{b}$ be two vectors such that $\left|\vec{a} + \vec{b}\right|^2 = \left|\vec{a}\right|^2 + 2\left|\vec{b}\right|^2$, $\vec{a} \cdot \vec{b} = 3$ and $\left|\vec{a} \times \vec{b}\right|^2 = 75$. Then $\left|\vec{a}\right|^2$ is equal to _______.
[29-Jul-2022-Shift-2]

Answer: 14

Solution:

Solution:

$\because \left|\vec{a} + \dot{}b\right|^2 = \left|\vec{a}\right|^2 + 2\left|b\right|^2$

or $\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\vec{a} \cdot \vec{b} = \left|\vec{a}\right|^2 + 2\left|\vec{b}\right|^2$

$\therefore \left|\vec{b}\right|^2 = 6$

Now $\left|\vec{a} \times \vec{b}\right|^2 = \left|\vec{a}\right|^2\left|\vec{b}\right|^2 - \left(\vec{a} \cdot \vec{b}\right)^2$

$75 = \left|\vec{a}\right|^2 \cdot 6 - 9$

$\therefore \left|\vec{a}\right|^2 = 14$

Question90

If (1, 5, 35), (7, 5, 5), (1, λ, 7) and (2λ, 1, 2) are coplanar, then the sum of all possible values of λ is
[26 Feb 2021 Shift 1]

Options:

A. $\frac{39}{5}$

B. $-\frac{39}{5}$

C. $\frac{44}{5}$

D. $-\frac{44}{5}$

Answer: C

Solution:

Solution:
Let $P(1, 5, 35)$, $Q(7, 5, 5)$, $R(1, \lambda, 7)$, $S(2\lambda, 1, 2)$
Given P, Q, R, S are coplanar. Then, PQ, PR, PS lie on the same plane.

$PQ = (7-1)\hat{i} + (5-5)\hat{j} + (5-35)\hat{k} = 6\hat{i} - 30\hat{k}$

$PR = (1-1)\hat{i} + (\lambda-5)\hat{j} + (7-35)\hat{k} = (\lambda-5)\hat{j} - 28\hat{k}$

$PS = (2\lambda-1)\hat{i} + (1-5)\hat{j} + (2-35)\hat{k} = (2\lambda-1)\hat{i} - 4\hat{j} - 33\hat{k}$

∵ PQ,PR and PS lie on same plane, then

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

Expand along first row,

$6[-33(\lambda-5) - 112] + 30[(2\lambda-1)(\lambda-5)] = 0$

$\Rightarrow 6(-33\lambda + 53) + 30(2\lambda^2 - 11\lambda + 5) = 0$

$\Rightarrow 60\lambda^2 - 528\lambda + 468 = 0$

$\Rightarrow 10\lambda^2 - 88\lambda + 78 = 0$

$\Rightarrow 5\lambda^2 - 44\lambda + 39 = 0$ . . . (i)

Possible value of λ are roots of Eq. (i).
Then, sum of all possible values of $\lambda =$ Sum of roots of Eq. (i)

$= \frac{-(-44)}{5} = \frac{44}{5}$

[$\because ax^2 + bx + + c = 0$, sum of roots $= -b / a$ }

Question91

If a and b are perpendicular, then a × (a × (a × (a × b))) is equal to [26 Feb 2021 Shift 1]

Options:

A. 0

B. $\frac{1}{2}\left|a\right|^4 b$

C. $a \times b$

D. $|a|^4 b$

Answer: D

Solution:

$a \times [a \times \{a \times (a \times b)\}]$
$= a \times (a \times [(a \cdot b)a - (a \cdot a)b])$
$[\text{U sing}, a \times (b \times c) = (a \cdot cb - (a \cdot b)c]$
$= a \times [a \times ((a \cdot b)a - |a|^2 b)]$
$= a \times [(a \times (a \cdot b)a) - |a|^2 (a \times b)]$
$= a \times [0 - |a|^2 (a \times b)]$
$= -|a|^2 [a \times (a \times b)]$
$= -|a|^2 [(a \cdot b)a - (a \cdot a)b]$
$= -(a \cdot b)a|a|^2 + |a|^4 b$
$= 0 + |a|^4 b$
$= |a|^4 b$

Question92

Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2\left|\vec{a} + \vec{b} + \vec{c}\right|^2$ is
24 Feb 2021 Shift 1

Answer: 75

Solution:

Solution:
Let $\vec{c} = \lambda\left(\vec{b} \times \left(\vec{a} \times \vec{b}\right)\right)$
$= \lambda\left(\left(\vec{b} \cdot \vec{b}\right)\vec{a} - \left(\vec{b} \cdot \vec{a}\right)\vec{b}\right)$
$= \lambda\left(5\left(-\hat{i} + \hat{j} + \hat{k}\right) + 2\hat{i} + \hat{k}\right)$
$= \lambda\left(-3\hat{i} + 5\hat{j} + 6\hat{k}\right)$
$\vec{c} \cdot \vec{a} = 7 \Rightarrow 3\lambda + 5\lambda + 6\lambda = 7$
$\lambda = \frac{1}{2}$
$\therefore 2\left|\left(\frac{-3}{2} - 1 + 2\right)\hat{i} + \left(\frac{5}{2} + 1\right)\hat{j} + (3 + 1 + 1)\hat{k}\right|^2$
$= 2\left(\frac{1}{4} + \frac{49}{4} + 25\right) = 25 + 50 = 75$

Question93

If vectors $a_1 = x\hat{i} - \hat{j} + \hat{k}$ and $a_2 = \hat{i} + \hat{y}j + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + \hat{j} + z\hat{k}$ is
[26 Feb 2021 Shift 2]

Options:

A. $\frac{1}{\sqrt{2}}\left(-\hat{j} + \hat{k}\right)$

B. $\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}\right)$

C. $\frac{1}{\sqrt{3}}\left(\hat{i}+\hat{j}-\hat{k}\right)$

D. $\frac{1}{\sqrt{3}}\left(\hat{i}-\hat{j}+\hat{k}\right)$

Answer: D

Solution:

Solution:

Given, $a_1 = x\hat{i}-\hat{j}+\hat{k}$ and $a_2 = \hat{i}+y\hat{j}+z\hat{k}$ are collinear, then $\frac{x}{1} = \frac{-1}{y} = \frac{1}{z} = \lambda$ (Say)

This gives $x = \lambda$, $y = -\frac{1}{\lambda}$, $z = \frac{1}{\lambda}$

Then, unit vector parallel to vector $x\hat{i}+y\hat{j}+zk$ will be

$= \pm\left((\lambda)\hat{i}-\left(\frac{1}{\lambda}\right)\hat{j}+\left(\frac{1}{\lambda}\right)\frac{\hat{k})}{\sqrt{(\lambda)^2+\left(\frac{-1}{\lambda}\right)^2+\left(\frac{1}{\lambda}\right)^2}}\right.$

$= \pm\left(\lambda^2\hat{i}-\hat{j}+\frac{\hat{k})\lambda}{\lambda\sqrt{\lambda^4+2}}\right. = \pm\left(\lambda^2\hat{i}-\hat{j}+\frac{\hat{k})}{\sqrt{\lambda^4+2}}\right.$

Take, $\lambda = 1 = \pm\left(\hat{i}-\hat{j}+\frac{\hat{k})}{\sqrt{3}}\right.$

Question94

Let a = $\hat{i}+\alpha\hat{j}+3\hat{k}$ and b = $3\hat{i}-\alpha\hat{j}+\hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors a and b is $8\sqrt{3}$ square units, then a · b is equal to .... [25 Feb 2021 Shift 2]

Answer: 2

Solution:

Solution:

Area of parallelogram = $|a\times b|$

$= \left|\left(\hat{i}+\alpha\hat{j}+3\hat{k}\right)\times\left(3\hat{i}-\alpha\hat{j}+\hat{k}\right)\right|$

$(64)(3) = 16\alpha^2+64+16\alpha^2$ (given, area $= 8\sqrt{3}$)

$= \left|\left(\hat{i}+\alpha\hat{j}+3\hat{k}\right)\times\left(3\hat{i}-\alpha\hat{j}+\hat{k}\right)\right|$

$(64)(3) = 16\alpha^2+64+16\alpha^2$ (given, area $= 8\sqrt{3}$)

(squaring on both sides)

$\Rightarrow \alpha^2 = 4$

Now, a. b $= 3-\alpha^2+3$

$= 6-\alpha^2 = 6-4 = 2$

Question95

Let a = $\hat{i} + 2\hat{j} - \hat{k}$, b = $\hat{i} - \hat{j}$ and c = $\hat{i} - \hat{j} - \hat{k}$ be three given vectors. If r is a vector such that r × a = c × a and r · b = 0, then r · a is equal to ....
[25 Feb 2021 Shift 1]

Answer: 12

Solution:

$b = \hat{i} - \hat{j}$
$c = \hat{i} - \hat{j} - \hat{k}$
$r \times a = c \times a$
$\Rightarrow\ r \times a - c \times a = 0$
$\Rightarrow\ (r - c) \times a = 0$
$\therefore\ r - c = \lambda a$
$\Rightarrow\ r = \lambda a + c$
$\Rightarrow\ r \cdot b = \lambda a \cdot b + c \cdot b$ (taking dot w)
$\Rightarrow\ 0 = \lambda a \cdot b + c \cdot b$
$\Rightarrow \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right) \cdot \left(\hat{i} - \hat{j}\right) + \left(\hat{i} - \hat{j} - \hat{k}\right) \cdot \left(\hat{i} - \hat{j}\right) = 0$
$\Rightarrow\ \lambda(1 - 2) + 2 = 0$
$\Rightarrow\ \lambda = 2$
$\therefore\ r = 2a + c$
$\Rightarrow\ r \cdot a = 2a \cdot a + c \cdot a$ [taking dot with a]
$= 2\,|\,a\,|^2 + a \cdot c$
$= 2(1 + 4 + 1) + (1 - 2 + 1)$
$r \cdot a = 12$

Question96

A vector a has components 3p and 1 with respect to rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, a has components p + 1 and $\sqrt{10}$, then a value of p is equal to
[18 Mar 2021 Shift 1]

Options:

A. 1

B. $-\frac{5}{4}$

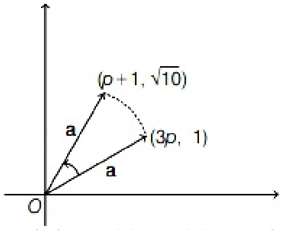
C. $\frac{4}{5}$

D. -1

Answer: D

Solution:

Solution:
After counter clockwise (or anti-clockwise) rotation, the length of the vector a remains constant.



i.e. |a| at old position = | a| at new position
$\Rightarrow (3p)^2 + (1)^2 = (p+1)^2 + (\sqrt{10})^2$
$\Rightarrow 9p^2 + 1 = p^2 + 1 + 2p + 10$
$\Rightarrow 8p^2 - 2p - 10 = 0 \Rightarrow 4p^2 - p - 5 = 0$
$\Rightarrow (p+1)(4p-5) = 0$
$\Rightarrow p = \frac{5}{4}, -1$

Question97

Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to [16 Mar 2021 Shift 1]

Options:

A. $\frac{1}{2}$

B. 1

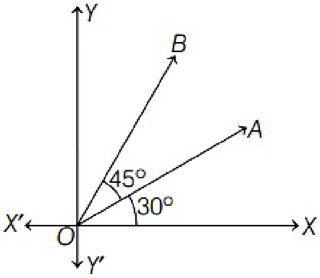
C. $\frac{1}{\sqrt{2}}$

D. $2\sqrt{2}$

Answer: A

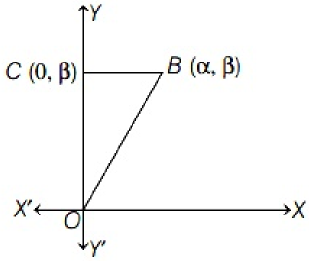
Solution:

Solution:
Let OA be $\sqrt{3}\hat{i} + \hat{j}$ and OB be $\alpha\hat{i} + \beta\hat{j}$.



As, we can notice in OA, $\frac{1}{\sqrt{3}} = \tan 30^\circ$. So, it makes an angle of 30° with the X-axis.
Now, when OA is rotated further by 45° anticlockwise, the resultant vector OB makes an angle of 75° with the X-axis.

So, OB = $|\text{OA}|\left(\cos 75^\circ \hat{i} + \sin 75^\circ \hat{j}\right)$



Let $\triangle$OBC be the required triangle whose area we have to determine.
Area of $\triangle$OBC = (1/2) × (Base) × (Height)
 = 1/2 × β × α
 = $\frac{1}{2}(2\sin 75^\circ)(2\cos 75^\circ) = 2\sin 75^\circ \cos 75^\circ$
 = $\sin 150^\circ = \sin 30^\circ$
 = 1/2
Hence, the area is 1/2sq. unit.

Question98

Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to
[17 Mar 2021 Shift 2]

Answer: 486

Solution:

Solution:
Let $x = \lambda a + \mu b$, where λ and μ are scalars.
$\Rightarrow\ x = \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} - \hat{k}\right)$
$x = \hat{i}(2\lambda + \mu) + \hat{j}(2\mu - \lambda) + \hat{k}(\lambda - \mu)$
Since, x is perpendicular to $\left(3\hat{i} + 2\hat{j} - \hat{k}\right)$.
Then, $x \cdot \left(3\hat{i} + 2\hat{j} - \hat{k}\right) = 0$
$\Rightarrow\ 3\lambda + 8\mu = 0$
Also, given projection of x on a is $\frac{17\sqrt{6}}{2}$.....(i)
$\therefore \frac{x \cdot a}{|a|} = \frac{17\sqrt{6}}{2}$
$\Rightarrow 2(2\lambda + \mu) + (\lambda - 2\mu) + (\lambda - \mu) = 51$
$6\lambda - \mu = 51$ ...(ii)
From Eqs. (i) and (ii),
$\lambda = 8, \mu = -3$
$\therefore\ x = 13\hat{i} - 14\hat{j} + 11\hat{k}$
$\Rightarrow\ \ |x| = \sqrt{(13)^2 + (-14)^2 + (11)^2}$
$\therefore\ \ |x|\ = (13)^2 + (-14)^2 + (11)^2 = 486$

Question99

Let a and b be two non-zero vectors perpendicular to each other and |a| = | b|. If |a × b| = | a|, then the angle between the vectors [a + b + (a × b)] and a is equal to
[18 Mar 2021 Shift 2]

Options:

A. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

B. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

C. $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

D. $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

Answer: B

Solution:

Solution:
Given, $a \perp b$...(i)
$|a| = |b|$...(ii)
and $|a \times b| = |a|$
$\Rightarrow |a||b|\sin 90^\circ = |a|$ [from Eq. (i)]
$\Rightarrow |b| = 1 = |a|$...(iii) [from Eq. (ii)]
From Eq. (iii), we can say that
$a \times b$ are mutually perpendicular unit vectors.
Let $a = \hat{i}$ and $b = \hat{j}$
$\therefore a \times b = \hat{k}$
Now, $[a + b + (a \times b)] = (\hat{i} + \hat{j} + \hat{k})$
$\therefore \cos\theta = (\hat{i} + \hat{j} + \hat{k}) \cdot \frac{\hat{i}}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}}$
$\therefore \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Question100

Let a = $2\hat{i} - 3\hat{j} + 4\hat{k}$ and b = $7\hat{i} + \hat{j} - 6\hat{k}$. If r × a = r × b, r · $(\hat{i} + 2\hat{j} + \hat{k}) = -3$, then. r · $(2\hat{i} - 3\hat{j} + \hat{k})$ is equal to
[17 Mar 2021 Shift 1]

Options:

A. 12

B. 8

C. 13

D. 10

Answer: A

Solution:

Solution:

$a = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$b = 7\hat{i} + \hat{j} - 6\hat{k}$

If $r \times a = r \times b$

$\Rightarrow\ r = \lambda(a - b) = \lambda\left(5\hat{i} + 4\hat{j} - 10k\right)$

Now, $r \cdot \left(\hat{i} + 2\hat{j} + \hat{k}\right) = -3$

$\Rightarrow\ \lambda\left(5\hat{i} + 4\hat{j} + 10\hat{k}\right) \cdot \left(\hat{i} + 2\hat{j} + \hat{k}\right) = -3$

$\Rightarrow\ \lambda(5 + 8 + 10) = -3$

$\Rightarrow\ \lambda = +1$

$\therefore r = -5\hat{i} - 4\hat{j} + 10\hat{k}$

So, $r \cdot \left(2\hat{i} - 3\hat{j} + \hat{k}\right)$

$= \left(-5\hat{i} - 4\hat{j} + 10\hat{k}\right)\left(2\hat{i} - 3\hat{j} + \hat{k}\right)$

$= -10 + 12 + 10$

$= 12$

Question101

Let $a = \hat{i} + 2\hat{j} - 3\hat{k}$ and $b = 2\hat{i} - 3\hat{j} + 5\hat{k}$. If $r \times a = b \times r$, $r \cdot \left(\alpha\hat{i} + 2\hat{j} + \hat{k}\right) = 3$ and $r.\left(2\hat{i} + 5\hat{j} - a\hat{k}\right) = -1$, $\alpha \in R$, then the value of $\alpha + |r|^2$ is equal to [16 Mar 2021 Shift 2]

Options:

A. 9

B. 15

C. 13

D. 11

Answer: B

Solution:

Solution:

Given, $a = \left(\hat{i} + 2\hat{j} - 3\hat{k}\right)$

and $b = \left(2\hat{i} - 3\hat{j} + 5\hat{k}\right)$

If $r \times a = b \times r$, $r \cdot \left(\alpha\hat{i} + 2\hat{j} + \hat{k}\right) = 3$

$r \cdot \left(2\hat{i} + 5\hat{j} - \alpha\hat{k}\right) = -1$

$\Rightarrow r \times a = b \times r$

$(r \times a) = -(r \times b) \Rightarrow (r \times a) + (r \times b) = 0$

$\Rightarrow r \times (a + b) = 0 \Rightarrow r = \lambda(a + b)$

$\Rightarrow\ r = \lambda\left[(1 + 2)\hat{i} + (2 - 3)\hat{j} + (-3 + 5)\hat{k}\right]$

$\Rightarrow r = \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$

$r \cdot \left(\alpha\hat{i} + 2\hat{j} + \hat{k}\right) = 3$

$\Rightarrow \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)\left(\alpha\hat{i} + 2\hat{j} + \hat{k}\right) = 3$
$\Rightarrow \lambda(3\alpha - 2 + 2) = 3$
$\Rightarrow \alpha\lambda = 1$
$r \cdot \left(2\hat{i} + 5\hat{j} - \alpha\hat{k}\right) = -1$
$\Rightarrow \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)\left(2\hat{i} + 5\hat{j} - \alpha\hat{k}\right) = -1$
$\Rightarrow \lambda(6 - 5 - 2\alpha) = -1 \Rightarrow \lambda(1 - 2\alpha) = -1$
$\Rightarrow \lambda - 2\alpha\lambda = -1 \Rightarrow \lambda - 2 = -1$
$\Rightarrow \lambda = 1$
So, $\alpha = 1$
$r = \left(3\hat{i} - \hat{j} + 2\hat{k}\right)$
$\Rightarrow |r|^2 = 9 + 1 + 4 = 14$
$\therefore \alpha + |r|^2 = 1 + 14 = 15$

Question102

Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ If $\vec{c} \cdot \left(\hat{i} + \hat{j} + 3\hat{k}\right) = 8$ then the value of $\vec{c} \cdot \left(\vec{a} \times \vec{b}\right)$ is equal to ....
[16 Mar 2021 Shift 2]

Answer: 28

Solution:

Solution:
Since, c is perpendicular to a and b.
So, $c = \lambda(a \times b)$
$a = \hat{i} + \hat{j} - \hat{k}$
$b = \hat{i} + 2\hat{j} + \hat{k}$
Now, $a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$
$= (1 + 2)\hat{i} - (1 + 1)\hat{j} + (2 - 1)\hat{k}$
$= 3\hat{i} - 2\hat{j} + \hat{k}$
$\Rightarrow c = \lambda\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$
Now, $c \cdot \left(\hat{i} + \hat{j} + 3\hat{k}\right) = 8$
$\lambda\left(3\hat{i} - 2\hat{j} + \hat{k}\right)\left(\hat{i} + \hat{j} + 3\hat{k}\right) = 8$
$\Rightarrow 4\lambda = 8$
$\therefore \lambda = 2$
So, $c = 2\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$
So, $c \cdot (a \times b) = 2\left(3\hat{i} - 2\hat{j} + \hat{k}\right)\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$
$= 2(9 + 4 + 1)$
$= 28$

Question103

If $a = \alpha\hat{j} + \beta\hat{j} + 3\hat{k}$, $b = -\beta\hat{i} - \alpha\hat{j} - \hat{k}$ and $c = \hat{i} - 2\hat{j} - \hat{k}$ such that $a \cdot b = 1$ and $b . c = -3$, then $\frac{1}{3}[(a \times b) \cdot c]$ is equal to
[17 Mar 2021 Shift 1]

Answer: 2

Solution:

Solution:
$a = \langle\alpha, \beta, 3\rangle$
$b = \langle-\beta, -\alpha, -1\rangle$
$c = \langle1, -2, -1\rangle$
$a \cdot b = 1$
$-\alpha\beta - \alpha\beta - 3 = 1$
$\alpha\beta = -2$
$b \cdot c = -3$
$-\beta + 2\alpha + 1 = -3$
$\beta - 2\alpha = 4$
$\Rightarrow \beta - 2\left(\frac{-2}{\beta}\right) = 4$
$\Rightarrow \beta^2 + 4 = 4\beta \Rightarrow \beta^2 - 4\beta + 4 = 0$
$\Rightarrow (\beta - 2)^2 = 0 \Rightarrow \beta = 2$
$\alpha\beta = -2 \Rightarrow \alpha \cdot 2 = -2$
$\Rightarrow \alpha = -1$
Hence, $\frac{1}{3}[(a \times b) \cdot c]$
$a = \langle-1, 2, 3\rangle$
$b = \langle-2, 1, -1\rangle$
$c = \langle1, -2, -1\rangle$

$$(a \times b) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ -2 & 1 & -1 \end{vmatrix} = -5\hat{i} - 7\hat{j} + 3\hat{k}$$

$(a \times b) \cdot c = \left(-5\hat{i} - 7\hat{j} + 3\hat{k}\right)\left(\hat{i} - 2\hat{j} - \hat{k}\right)$
$= -5 + 14 - 3 = 6$
$\therefore \frac{1}{3}[(a \times b) \cdot c] = \frac{1}{3} \times 6 = 2$

Question104

Let O be the origin. Let $OP = x\hat{i} + y\hat{j} - \hat{k}$ and $OQ = -\hat{i} + 2\hat{j} + 3x\hat{k}$, $x, y \in R$, $x > 0$, be such that $|PQ| = \sqrt{20}$ and the vector OP is perpendicular to OQ. If $OR = 3\hat{i} + 2\hat{j} - 7\hat{k}$, $z \in R$, is coplanar with OP and OQ, then the value of $x^2 + y^2 + z^2$ is equal to
[17 Mar 2021 Shift 2]

Options:

A. 7

B. 9

C. 2

D. 1

Answer: B

Solution:

Solution:

Given, $OP = x\hat{i} + y\hat{j} - \hat{k}$

$OQ = -\hat{i} + 2\hat{j} + 3x\hat{k}$

$OR = 3\hat{i} + z\hat{j} - 7\hat{k}$

and $|PQ| = \sqrt{20}$

Now, $|PQ| = |OQ - OP| = |OP - OQ|$

$= (x+1)\hat{i} + (y-2)\hat{j} - (1+3x)\hat{k}$

$\Rightarrow |PQ|^2 = (\sqrt{20})^2 = 20$

$\Rightarrow (x+1)^2 + (y-2)^2 + (1+3x)^2 = 20$

$$\Rightarrow (x+1)^2 + (2x-2)^2 + (1+3x)^2 = 20 \quad \left[\begin{array}{c} \because OP \perp OQ \\ \therefore OP \cdot OQ = 0 \\ \Rightarrow -x + 2y - 3x = 0 \\ \Rightarrow y = 2x \end{array}\right]$$

$\Rightarrow x^2 + 1 + 2x + 4x^2 + 4 - 8x + 1 + 9x^2 + 6x = 20$

$\Rightarrow 14x^2 + 6 = 20 \Rightarrow 14x^2 = 14$

$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1$ but x must be positive as in question conditions i.e. $x > 0$.

$\therefore x = -1$ (Rejected)

Hence, $x = 1$

$\therefore y = 2x = 2 \times 1 = 2$

Now, OP, OQ and OR are coplanar.

$\therefore [OP\ OQ\ OR] = 0$

$$\Rightarrow \begin{vmatrix} x & y & -1 \\ -1 & 2 & 3x \\ 3 & z & -7 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1 & 2 & -1 \\ -1 & 2 & 3 \\ 3 & z & -7 \end{vmatrix} = 0$$

$\Rightarrow 1(-14 - 3z) - 2(7 - 9) - 1(-z - 6) = 0 \Rightarrow z = -2$

$\therefore x^2 + y^2 + z^2 = 1 + 4 + 4 = 9$

Question105

If $\left(\vec{a} + 3\vec{b}\right)$ is perpendicular to $\left(7\vec{a} - 5\vec{b}\right)$ and $\left(\vec{a} - 4\vec{b}\right)$ is perpendicular to $\left(7\vec{a} - 2\vec{b}\right)$, then the angle between a and $\vec{b}$ (in degrees) is ________.
[25 Jul 2021 Shift 2]

Answer: 60

Solution:

$\left(\vec{a} + 3\vec{b}\right) \perp \left(7\vec{a} - 5\vec{b}\right)$

$\left(\vec{a} + 3\vec{b}\right) . \left(7\vec{a} - 5\vec{b}\right) = 0$

$7\left|\vec{a}\right|^2 - 15\left|\vec{b}\right|^2 + 16\vec{a} . \vec{b} = 0$ ......(1)

$\left(\vec{a} - 4\vec{b}\right) . \left(7\vec{a} - 2\vec{b}\right) = 0$

$7|\vec{a}|^2 + 8|\vec{b}|^2 - 30\vec{a} \cdot \vec{b} = 0$ ......(2)
from (1) & (2)
$|\vec{a}| = |\vec{b}|$
$\cos\theta = \frac{|\vec{b}|}{2|\vec{a}|}$ $\therefore \theta = 60°$

Question106

Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a} + \vec{b} + \vec{c}$. Then $36\cos^2 2\theta$ is equal to ______.
[20 Jul 2021 Shift 1]

Answer: 4

Solution:

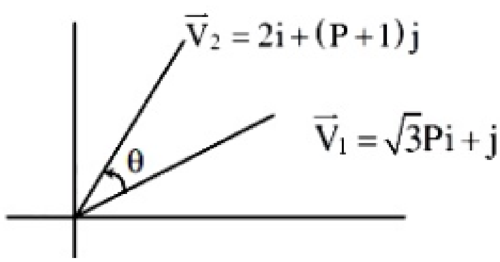
$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c}) = 3$
$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$
$\vec{a}(\vec{a} + \vec{b} + \vec{c}) = |\vec{a} + |\vec{a} + \vec{b} + \vec{c}| \cos\theta$
$\Rightarrow 1 = \sqrt{3}\cos\theta$
$\Rightarrow \cos 2\theta = -\frac{1}{3}$
$\Rightarrow 36\cos^2 2\theta = 4$

Question107

For p > 0, a vector $\vec{v}_2 = 2\hat{i} + (p + 1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1 = \sqrt{3}p\hat{i} + \hat{j}$ by an angleθ about origin in counter clockwise direction. If $\tan\theta = \frac{(\alpha\sqrt{3} - 2)}{(4\sqrt{3} + 3)}$, then the value of α is equal to ________.
[20 Jul 2021 Shift 2]

Answer: 6

Solution:



$$\left|V_1\right| = \left|V_2\right|$$
$$3P^2 + 1 = 4 + (P + 1)^2$$
$$2P^2 - 2P - 4 = 0 \Rightarrow P^2 - P - 2 = 0$$
$P = 2, -1$ (rejected)
$$\cos\theta = \frac{\vec{V}_1 . \vec{V}_2}{\left|\vec{V}_1\right|\left|\vec{V}_2\right|} = \frac{2\sqrt{3}P + (P + 1)}{\sqrt{(P + 1)^2 + 4}\sqrt{3P^2 + 1}}$$
$$\cos\theta = \frac{4\sqrt{3} + 3}{\sqrt{13}\sqrt{13}} = \frac{4\sqrt{3} + 3}{13}$$
$$\tan\theta = \frac{\sqrt{112 - 24\sqrt{3}}}{4\sqrt{3} + 3} = \frac{6\sqrt{3} - 2}{4\sqrt{3} + 3} = \frac{\alpha\sqrt{3} - 2}{4\sqrt{3} + 3}$$
$$\Rightarrow \alpha = 6$$

Question108

In a triangle ABC, if $\left|\vec{BC}\right| = 3, \left|\vec{CA}\right| = 5$ and $\left|\vec{BA}\right| = 7$, then the projection of the vector $\vec{BA}$ on $\vec{BC}$ is equal to
[20 Jul 2021 Shift 2]

Options:

A. $\frac{19}{2}$

B. $\frac{13}{2}$

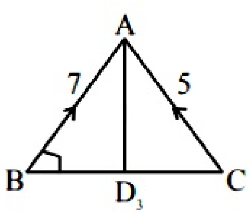
C. $\frac{11}{2}$

D. $\frac{15}{2}$

Answer: C

Solution:

Solution:



Projection of $\vec{BA}$ on $\vec{BC}$ is equal to
$$= \left|\vec{BA}\right| \cos \angle ABC$$
$$= 7\left|\frac{7^2 + 3^2 - 5^2}{2 \times 7 \times 3}\right| = \frac{11}{2}$$

Question109

Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors.
If a vector $\vec{r} = (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$, and $|\vec{r}| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to _______.
[25 Jul 2021 Shift 1]

Answer: 3

Solution:

Solution:
$\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ (Given)
$\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$

Now $(\vec{p} + \vec{q}) \times (\vec{p} - \vec{q}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$= -2\hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{r} = \pm\sqrt{3}\frac{((\vec{p} + \vec{q}) \times (\vec{p} - \vec{q}))}{|(\vec{p} + \vec{q}) \times (\vec{p} - \vec{q})|} \pm \frac{\sqrt{3}(-2\hat{i} - 2\hat{j} - 2\hat{k})}{\sqrt{2^2 + 2^2 + 2^2}}$

$\vec{r} = \pm(-\hat{i} - \hat{j} - \hat{k})$

According to question

$\vec{r} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$

So $|\alpha| = 1, |\beta| = 1, |\gamma| = 1$

$\Rightarrow |\alpha| + |\beta| + |\gamma| = 3$

Question110

If $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then $|\vec{a} \cdot \vec{b}|$ is equal to :
[25 Jul 2021 Shift 2]

Options:

A. 6

B. 4

C. 3

D. 5

Answer: A

Solution:

$|\vec{a}| = 2, |\vec{b}| = 5$
$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta = \pm 8$
$\sin\theta = \pm\frac{4}{5} \therefore \vec{a} \,.\, bv = |\vec{a}||\vec{b}| \cos\theta$
$= 10 \,.\, \left(\pm\frac{3}{5}\right) = \pm 6$
$|\vec{a} \,.\, \vec{b}| = 6$

Question111

Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Then the vector product $\left(\vec{a} + \vec{b}\right) \times \left(\left(\vec{a} \times \left(\left(\vec{a} - \vec{b}\right) \times \vec{b}\right)\right) \times \vec{b}\right)$ is equal to:
[27 Jul 2021 Shift 1]

Options:

A. $5\left(34\hat{i} - 5\hat{j} + 3\hat{k}\right)$

B. $7\left(34\hat{i} - 5\hat{j} + 3\hat{k}\right)$

C. $7\left(30\hat{i} - 5\hat{j} + 7\hat{k}\right)$

D. $5\left(30\hat{i} - 5\hat{j} + 7\hat{k}\right)$

Answer: B

Solution:

Solution:
$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$
$\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}.$
$\vec{a} + \vec{b} = 3\hat{j} + 5\hat{k};\ \vec{a} \,.\, \vec{b} = -1 + 2 + 6 = 7$
$\left(\left(\vec{a} \times \left(\left(\vec{a} - \vec{b}\right) \times \vec{b}\right)\right) \times \vec{b}\right)$
$\left(\left(\vec{a} \times \left(\vec{a} \times \vec{b} - \vec{b} \times \vec{b}\right)\right) \times \vec{b}\right)$
$\left(\vec{a} \times \left(\vec{a} \times \vec{b} - 0\right)\right) \times \vec{b}$
$\left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right) \times \vec{b}$
$\left(\left(\vec{a} \,.\, \vec{b}\right)\vec{a} - \left(\vec{a} \,.\, \vec{a}\right)\vec{b}\right) \times \vec{b}$
$\left(\vec{a} \,.\, \vec{b}\right)\vec{a} \times \vec{b} - \left(\vec{a} \,.\, \vec{a}\right)\left(\vec{b} \times \vec{b}\right)$
$\left(\vec{a} \,.\, \vec{b}\right)\left(\vec{a} \times \vec{b}\right)$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ -1 & 2 & 3 \end{vmatrix} = -\hat{i} - 5\hat{j} + 3\hat{k}$$

$\therefore 7\left(-\hat{i} - 5\hat{j} + 3\hat{k}\right)$
$\left(\vec{a} + \vec{b}\right) \times \left(7\left(-\hat{i} - 5\hat{j} + 3\hat{k}\right)\right)$
$7\left(0\hat{i} + 3\hat{j} + 5\hat{k}\right) \times \left(-\hat{i} - 5\hat{j} + 3\hat{k}\right)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 5 \\ -1 & -5 & 3 \end{vmatrix}$$

$\Rightarrow 34\hat{i} - (5)\hat{j} + \left(3\hat{k}\right)$
$\Rightarrow 34\hat{i} - 5\hat{j} + 3\hat{k}$
$\therefore 7\left(34\hat{i} - 5\hat{j} + 3\hat{k}\right)$

Question112

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is l, then the value of $3l^2$ is equal to ________.
[27 Jul 2021 Shift 1]

Answer: 2

Solution:

Solution:
$\vec{a} \times \vec{b} = \vec{c}$
Take Dot with $\vec{c}$
$\left(\vec{a} \times \vec{b}\right)\vec{c} = \left|\vec{c}\right|^2 = 2$
Projection of $\vec{b}$ or $\vec{a} \times \vec{c} = l$
$\dfrac{\left|\vec{b} \cdot \left(\vec{a} \times \vec{c}\right)\right|}{\left|\vec{a} \times \vec{c}\right|} = l$
$\therefore l = \dfrac{2}{\sqrt{6}} \Rightarrow l^2 = \dfrac{4}{6}$
$3l^2 = 2$

Question113

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} = \vec{b} \times \left(\vec{b} \times \vec{c}\right)$. If magnitudes of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are $\sqrt{2}$, 1 and 2 respectively and the angle between $\vec{b}$ and $\vec{c}$ is $\theta\left(0 < \theta < \frac{\pi}{2}\right)$, then the value of $1 + \tan\theta$ is equal to:
[27 Jul 2021 Shift 2]

Options:

A. $\sqrt{3} + 1$

B. 2

C. 1

D. $\dfrac{\sqrt{3} + 1}{\sqrt{3}}$

Answer: B

Solution:

$\vec{a} = \left(\vec{b} \cdot \vec{c}\right)\vec{b} - \left(\vec{b} \cdot \vec{b}\right)\vec{c}$
$= 1.2 \cos\theta\, \vec{b} - \vec{c}$
$\Rightarrow \vec{a} = 2\cos\theta\, \vec{b} - \vec{c}$
$\left|\vec{a}\right|^2 = (2\cos\theta)^2 + 2^2 - 2.2\cos\theta\, \vec{b} \cdot \vec{c}$
$\Rightarrow 2 = 4\cos^2\theta + 4 - 4\cos\theta \cdot 2\cos\theta$
$\Rightarrow -2 = -4\cos^2\theta$
$\Rightarrow \cos^2\theta = \frac{1}{2}$
$\Rightarrow \sec^2\theta = 2$
$\Rightarrow \tan^2\theta = 1$
$\Rightarrow \theta = \frac{\pi}{4}$
$1 + \tan\theta = 2.$

Question114

Let the vectors $(2 + a + b)\hat{i} + (a + 2b + c)\hat{j} - (b + c)\hat{k}$, $(1 + b)\hat{i} + 2b - b\hat{k}$ and $(2 + b)\hat{i} + 2b\hat{j} + (1 - b)\hat{k}$ a, b, c, ∈Rbe co-planar. Then which of the following is true? [25 Jul 2021 Shift 1]

Options:

A. $2b = a + c$

B. $3c = a + b$

C. $a = b + 2c$

D. $2a = b + c$

Answer: A

Solution:

Solution:
If the vectors are co-planar,

$$\begin{vmatrix} a+b+2 & a+2b+c & -b-c \\ b+1 & 2b & -b \\ b+2 & 2b & 1-b \end{vmatrix} = 0$$

Now $R_3 \to R_3 - R_2$, $R_1 \to R_1 - R_2$

So $$\begin{vmatrix} a+1 & a+c & -c \\ b+1 & 2b & -b \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$= (a + 1)2b - (a + c)(2b + 1) - c(-2b)$
$= 2ab + 2b - 2ab - a - 2bc - c + 2bc$
$= 2b - a - c = 0$

Question115

Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{a}| = 2$. Then which one of the following is not true?
[22 Jul 2021 Shift 2]

Options:

A. $\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c})) = \vec{0}$

B. Projection of $\vec{a}$ on $(\vec{b} \times \vec{c})$ is 2

C. $[\vec{a} \ \vec{b} \ c] + [\vec{c} \ \vec{a} \ \vec{b}] = 8$

D. $|3\vec{a} + \vec{b} - 2\vec{c}|^2 = 51$

Answer: D

Solution:

Solution:

(1) $\vec{a} \times ((\vec{b} + \vec{c}) \times (\vec{b} - \vec{c})) = \vec{0}$

$= \vec{a}(-\vec{b} \times \vec{c} + \vec{c} \times \vec{b}) = -2(\vec{a} \times (\vec{b} \times \vec{c}))$

$= -2(\vec{a} \times \vec{a})\vec{0}$

(2) Projection of $\vec{a}$ on $(\vec{b} \times \vec{c})$

$= \frac{\vec{a} . (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|} = \frac{\vec{a} . \vec{a}}{|\vec{a}|} = |\vec{a}| = 2$

(3) $[\vec{a} \ \vec{b} \ c] + [\vec{c} \ \vec{a} \ \vec{b}] = 2[\vec{a} \ \vec{b} \ c] = 2\vec{a} . (\vec{b} \times \vec{c})$

$= 2\vec{a} . \vec{a} = 2|\vec{a}|^2 = 8$

(4) $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are mutually perp vectors.

$\therefore |\vec{a} \times \vec{b}| = |\vec{c}| \Rightarrow |\vec{a}||\vec{b}| = |\vec{c}| \Rightarrow |\vec{b}| = \frac{|\vec{c}|}{2}$

Also, $|\vec{b} \times \vec{c}| = |a| \Rightarrow |\vec{b}||\vec{c}| = 2 \Rightarrow |\vec{c}| = 2$ & $|\vec{b}| = 1$

$|\vec{a} + \vec{b} - 2\vec{c}|^2 = (3\vec{a} + \vec{b} - 2\vec{c}) . (3\vec{a} + \vec{b} - 2\vec{c})$

$= 9|\vec{a}|^2 + |\vec{b}|^2 + 4|\vec{c}|^2$

$= (9 \times 4) + 1 + (4 \times 4)$

$= 36 + 1 + 16 = 53$

Question116

Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} + \hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} = 3\hat{i} + 2\hat{j} + 6\hat{k}$, and $|\vec{a}| = \sqrt{10}$. Then a possible value of $[\vec{a}\vec{b}\vec{c}] + [\vec{a}\vec{b}\vec{d}] + [\vec{a}\vec{c}\vec{d}]$ equal to:
[22 Jul 2021 Shift 2]

Options:

A. -42

B. -40

C. -29

D. -38

Answer: A

Solution:

Solution:

$\vec{a}=\lambda\vec{b}+\mu\vec{c}=\hat{i}(2\lambda+\mu)+\hat{j}(\lambda-\mu)+\hat{k}(\lambda+\mu)$

$\vec{a}\cdot\vec{d}=0=3(2\lambda+\mu)+2(\lambda-\mu)+6(\lambda+\mu)$

$\Rightarrow 14\lambda+7\mu=0\Rightarrow\mu=-2\lambda$

$\Rightarrow\vec{a}=(0)\hat{i}-3\lambda\hat{j}+(-\lambda)\hat{k}$

$\Rightarrow|\vec{a}|=\sqrt{10}|\lambda|=\sqrt{10}\Rightarrow|\lambda|=1$

$\Rightarrow\lambda=1$ or -1

$[\vec{a}\vec{b}\vec{c}]=0$

$[\vec{a}\vec{b}\vec{c}]+[\vec{a}\vec{b}\vec{d}]+[\vec{a}\vec{c}\vec{d}]=[\vec{a}\vec{b}+\vec{c}\vec{d}]$

$$=\begin{vmatrix}0 & -3\lambda & \lambda\\ 3 & 0 & 2\\ 3 & 2 & 6\end{vmatrix}$$

$=3\lambda(12)+\lambda(6)=42\lambda=-42$

Question117

Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|,|\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $|(\vec{a}\times\vec{b})\times\vec{c}|$ is
[20 Jul 2021 Shift 1]

Options:

A. $\frac{2}{3}$

B. 4

C. 3

D. $\frac{3}{2}$

Answer: D

Solution:

Solution:

$|\vec{a}|=3=a;\ \vec{a}\cdot\vec{c}=c$

Now $|\vec{c}-\vec{a}|=2\sqrt{2}$

$\Rightarrow c^2+a^2-2\vec{c}\cdot\vec{a}=8$

$\Rightarrow c^2+9-2(c)=8$

$\Rightarrow c^2-2c+1=0\Rightarrow c=1=|\vec{c}|$

Also, $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$
Given $(\vec{a} \times \vec{b}) = |\vec{a} \times \vec{b}||\vec{c}| \sin\frac{\pi}{6}$
$= (3)(1)(1 / 2)$
$= 3 / 2$

Question118

Let a and b be two vectors such that |2a + 3b| = |3 a + b| and the angle between a and b is 60°. If $\frac{1}{8}$ a is a unit vector, then |b| is equal to [31 Aug 2021 Shift 1]

Options:

A. 4

B. 6

C. 5

D. 8

Answer: C

Solution:

Solution:
$|2\mathbf{a} + 3\mathbf{b}| = |3\mathbf{a} + \mathbf{b}|$
$\Rightarrow |2a + 3b|^2 = |3a + b|^2$
$\Rightarrow 4|a|^2 + .9|b|^2 + 12a.b = 9|a|^2 + |b|^2 + 6a.b$
$\Rightarrow 5|a|^2 - 6a.b - 8|b|^2 = 0$
$\frac{a}{8}$ is a unit vector.
So, $|\mathbf{a}| = 8$
And,
$5.64 - 6.8|b|\left(\frac{1}{2}\right) - 8|b|^2 = 0$
$\Rightarrow |b|^2 + 3|b| - 40 = 0$
$(|b| + 8)(|b| - 5) = 0$
$|\mathbf{b}| = 5$
As, $|\mathbf{b}| = -8$ Not possible.

Question119

Let a, b andc be three vectors mutually perpendicular to each other and have same magnitude. If a vector r satisfies. a × {(r - b) × a} + b × {(r - c) × b} + c × {(r - a) × c} = 0, then r is equel to [31 Aug 2021 Shift 2]

Options:

A. $\frac{1}{3}(\mathbf{a} + \mathbf{b} + \mathbf{c})$

B. $\frac{1}{3}$ (2**a** + **b** - **c**)

C. $\frac{1}{2}$ (**a** + **b** + **c**)

D. $\frac{1}{2}$ (**a** + **b** + 2**c**)

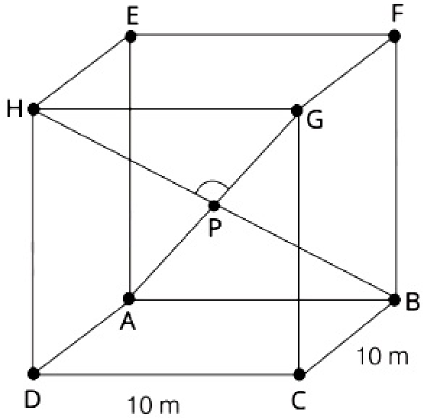
Answer: C

Solution:

Solution:
$\mathbf{a} \times [(\mathbf{r} - \mathbf{b}) \times \mathbf{a}] + \mathbf{b} \times [(\mathbf{r} - \mathbf{c}) \times \mathbf{b}] + \mathbf{c} \times [(\mathbf{r} - \mathbf{a}) \times \mathbf{c}] = 0$
$\Rightarrow \mathbf{a} . \mathbf{a} (\mathbf{r} - \mathbf{b}) - (\mathbf{a} . (\mathbf{r} - \mathbf{b})) \mathbf{a} + \mathbf{b} . \mathbf{b} (\mathbf{r} - \mathbf{c}) - (\mathbf{b} . (\mathbf{r} - \mathbf{c})) \mathbf{b} + \mathbf{c} . \mathbf{c} (\mathbf{r} - \mathbf{a}) - (\mathbf{c} . (\mathbf{r} - \mathbf{a})) \mathbf{c} = 0$
$\Rightarrow | a |^2 (r - b) - (r . a)a + | b |^2 (r - c) - (r . b)b + | c |^2 (r - a) - (r . c)c = 0$ [$\because$ **a b, c** are mutually perpendicular; $\therefore$ a . **b** = **b** . **c** = **c** . **a** = 0]
$\Rightarrow | a |^2 [3r - (a + b + c)] - [(r . a)a + (r . b)b + (r . c)c] = 0$ [$\because$ |**a**| = |**b**| = |**c**|]
$\Rightarrow | a |^2 [3r - (a + b + c) - r] = 0$
$\therefore 3\mathbf{r} - (\mathbf{a} + \mathbf{b} + \mathbf{c}) - \mathbf{r} = 0$
$\Rightarrow r = \frac{a + b + c}{2}$

Question120

A hall has a square floor of dimension 10m × 10m (see the figure) and vertical walls. If the ∠GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in m) is



[26 Aug 2021 Shift 2]

Options:

A. 5

B. $2\sqrt{10}$

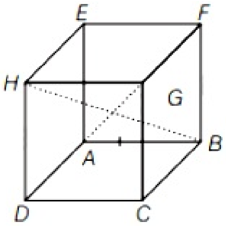
C. $5\sqrt{3}$

D. $5\sqrt{2}$

Answer: D

Solution:

Solution:
Let A = (0, 0, 0)
B = (10, 0, 0)
G = (10, 10, h)
H = (0, 10, h)



AG $= 10\hat{i} + 10\hat{j} + h\hat{k}$
BH $= -10\hat{i} + 10\hat{j} + h\hat{k}$
Since, $AG \cdot BH = |AG||BH|\cos\theta$
$(-100 + 100 + h^2) = \sqrt{h^2 + 200} \cdot \sqrt{h^2 + 200} \cdot \left(\frac{1}{5}\right)$
$\Rightarrow 5h^2 = h^2 + 200$
$\Rightarrow h^2 = 50$
$\Rightarrow h = 5\sqrt{2}$

Question121

If the projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the sum of the two vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $-\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is 1, then λ is equal to
[26 Aug 2021 Shift 2]

Answer: 5

Solution:

Let **a** $= \hat{i} + 2\hat{j} + \hat{k}$
b $= 2\hat{i} + 4\hat{j} - 5\hat{k}$
c $= -\lambda\hat{i} + 2\hat{j} + 3\hat{k}$
Now, according to the questions,
$\frac{a \cdot (b + c)}{|b + c|} = 1$
$\Rightarrow a \cdot b + a \cdot c = |b + c|$
$\Rightarrow 5 + (-\lambda + 4 + 3) = \left|(2 - \lambda)\hat{i} + 6\hat{j} - 2\hat{k}\right|$
$\Rightarrow (12 - \lambda)^2 = (2 - \lambda)^2 + 36 + 4$
$\Rightarrow \lambda^2 + 144 - 24\lambda = \lambda^2 - 4\lambda + 4 + 40$
$\Rightarrow \lambda = 5$

Question122

Let $a = \hat{i} + 5\hat{j} + \alpha\hat{k}$, $b = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $c = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $|b \times c| = 5\sqrt{3}$ and a is perpendicular to b. Then, the greatest amongst the values of $|a|^2$ is
[27 Aug 2021 Shift 1]

Answer: 90

Solution:

Solution:

Given, $a = \hat{i} + 5\hat{j} + \alpha\hat{k}$

$b = \hat{i} + 3\hat{j} + \beta\hat{k}$

and $c = -\hat{i} + 2\hat{j} - 3\hat{k}$

$\because \mathbf{a} \perp \mathbf{b}$

$\Rightarrow \mathbf{a} \cdot \mathbf{b} = 0$

$\Rightarrow (\hat{i} + 5\hat{j} + \alpha\hat{k}) \cdot (\hat{i} + 3\hat{j} + \beta\hat{k}) = 0$

$\Rightarrow 1 + 15 + \alpha\beta = 0$

or $\alpha\beta = -16$ ...(i)

Now, $b \times c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & \beta \\ -1 & 2 & -3 \end{vmatrix}$

$= \hat{i}(-9 - 2\beta) - \hat{j}(-3 + \beta) + \hat{k}(2 + 3)$

$b \times c = \hat{i}(-9 - 2\beta) + \hat{j}(3 - \beta) + 5\hat{k}$

Given, $|b \times c| = 5\sqrt{3}$

$\Rightarrow |b \times c|^2 = 75$

$\Rightarrow (-9 - 2\beta)^2 + (3 - \beta)^2 + 25 = 75$

$\Rightarrow \beta^2 + 6\beta + 8 = 0$

$\Rightarrow \beta = -2, -4$

From Eq. (i), we get

For $\beta = -2$, $\alpha = 8$

For $\beta = -4$, $\alpha = 4$

For maximum value of $|a|^2$, $\alpha = 8$

$\therefore |\alpha|^2 = 1 + 25 + 64 = 90$

Question123

If $\sum_{r=1}^{50} \tan^{-1} \frac{1}{2r^2} = p$ then the value of $\tan p$ is [26 Aug 2021 Shift 2]

Options:

A. $\frac{101}{102}$

B. $\frac{50}{51}$

C. 100

D. $\frac{51}{50}$

Answer: B

Solution:

Given, $\sum_{r=1}^{50} \tan^{-1}\frac{1}{2r^2} = p$

Now, $\Sigma\tan^{-1}\left(\frac{2}{1 + 4r^2 - 1}\right)$

$= \Sigma\tan^{-1}\left[\frac{(2r+1) - (2r-1)}{1 + (2r+1)(2r-1)}\right]$

$= \Sigma[\tan^{-1}(2r+1) - \tan^{-1}(2r-1)]$

$= (\tan^{-1}3 - \tan^{-1}1) + (\tan^{-1}5 - \tan^{-1}3) + \ldots\ldots.. + \tan^{-1}101 - \tan^{-1}99$

$= \tan^{-1}(101) - \tan^{-1}1$

$= \tan^{-1}\left(\frac{101 - 1}{1 + 101}\right) = \tan^{-1}\left(\frac{50}{51}\right)$

$\therefore \tan^{-1}\frac{50}{51} = p$

$\Rightarrow \tan p = \frac{50}{51}$

Question124

Let a = $\hat{i} + \hat{j} + \hat{k}$ and b = $\hat{j} - \hat{k}$. If c is a vector such that a × c = b and a . c = 3, then a . (b × c) is equal to [26 Aug 2021 Shift 1]

Options:

A. -2

B. -6

C. 6

D. 2

Answer: A

Solution:

Solution:

Given, **a × c = b**

a × (a × c) = a × b

(a . c)a − (a . a)c = a × b

We have, a = (1, 1, 1), b = (0, 1, −1), a . c = 3

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = -2\hat{i} + \hat{j} + \hat{k}$$

So, $3a - 3c = \left(-2\hat{i} + \hat{j} + \hat{k}\right)$

$\Rightarrow \left(3\hat{i} + 3\hat{j} + 3\hat{k}\right) - 3\bar{c} = \left(-2\hat{i} + \hat{j} + \hat{k}\right)$

$\Rightarrow 3c = \left(5\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$$\text{Now, } a \cdot (b \times c) = \left(\frac{1}{3}\right)\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 5 & 2 & 2 \end{vmatrix} = \frac{1}{3}(4 - 5 - 5) = -2$$

Question125

Let P_1, P_2, ..., P_{15} be 15 points on a circle. The number of distinct triangles formed by points $P_{\hat{i}}$, $P_{\hat{j}}$ and $P_{\hat{k}}$, such that $\hat{i} + \hat{j} + \hat{k} \neq 15$ is

[1 Sep 2021 Shift 2]

Options:

A. 12

B. 419

C. 443

D. 455

Answer: C

Solution:

Solution:

$\hat{i} + \hat{j} + \hat{k} = 15$

where, $\hat{i} = 1$, $\hat{j} + \hat{k} = 14$

$\Rightarrow (\hat{j} = 2, \hat{k} = 12), (\hat{j} = 3, \hat{k} = 11), (\hat{j} = 4, \hat{k} = 10,)$

$(\hat{j} = 5, \hat{k} = 9)(\hat{j} = 6, \hat{k} = 8)$...5 ways

$\hat{i} = 2$, $\hat{j} + \hat{k} = 13$

$\Rightarrow (\hat{j} = 3, \hat{k} = 10), \ldots, (\hat{j} = 6, \hat{k} = 7)$...4 ways

$\hat{i} = 3$, $\hat{j} + \hat{k} = 12$

$\Rightarrow (\hat{j} = 4, \hat{k} = 8), (\hat{j} = 5, \hat{k} = 7)$...2 ways

$\hat{i} = 4$, $\hat{j} + \hat{k} = 11$

$\Rightarrow (\hat{j} = 5, \hat{k} = 6)$...1 way

$\therefore$ Total = 12 ways

Then, number of possible triangles using vertices

$P_{\hat{i}}, P_{\hat{i}}, P_{\hat{k}}$ such that $\hat{i} + \hat{j} + \hat{k} \neq 15$ is

${}^{15}C_3 - 12 = 455 - 12 = 443$

Question126

A vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k} (\alpha, \beta \in R)$ lies in the plane of the vectors, $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} + 4\hat{k}$. If $\vec{a}$ bisects the angle between $\vec{b}$ and $\vec{c}$, then:

[Jan. 7, 2020 (I)]

Options:

A. $\vec{a} \cdot \hat{i} + 3 = 0$

B. $\vec{a} \cdot \hat{i} + 1 = 0$

C. $\vec{a} \cdot \hat{k} + 2 = 0$

D. $\vec{a} \cdot \hat{k} + 4 = 0$

Answer: C

Solution:

Angle bisector between $\vec{b}$ and $\vec{c}$ can be

$\vec{a} = \lambda\left(\hat{b} + \hat{c}\right)$ or $\vec{a} = \mu\left(\hat{b} - \hat{c}\right)$

If $\vec{a} = \lambda\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} + \frac{\hat{i} - \hat{j} + 4\hat{k}}{3\sqrt{2}}\right)$

$= \frac{\lambda}{3\sqrt{2}}\left[3\hat{i} + 3\hat{j} + \hat{i} - \hat{j} + 4\hat{k}\right]$

$= \frac{\lambda}{3\sqrt{2}}\left[4\hat{i} + 2\hat{j} + 4\hat{k}\right]$

Compare with $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$\frac{2\lambda}{3\sqrt{2}} = 2 \Rightarrow \lambda = 3\sqrt{2}$

$\vec{a} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

Not satisfy any option

Now consider $a = \mu\left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} - \frac{\hat{i} - \hat{j} + 4\hat{k}}{3\sqrt{2}}\right)$

$\vec{a} = \frac{\mu}{3\sqrt{2}}\left(3\hat{i} + 3\hat{j} - \hat{i} + \hat{j} - 4\hat{k}\right)$

$= \frac{\mu}{3\sqrt{2}}\left(2\hat{i} + 4\hat{j} - 4\hat{k}\right)$

Compare with $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$

$\frac{4\mu}{3\sqrt{2}} = 2 \Rightarrow \mu = \frac{3\sqrt{2}}{2}$

$\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore \vec{a} . \vec{k} + 2 = \left(\hat{i} + 2\hat{j} - 2\hat{k}\right) . \hat{k} + 2$

$= -2 + 2 = 0$

Question127

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ if $\lambda = \vec{a} . \vec{b} + \vec{b} . \vec{c} + \vec{c} . \vec{a}$ and $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$, then the ordered pair, $\left(\lambda, \vec{d}\right)$ is equal to:
[Jan. 7, 2020 (II)]

Options:

A. $\left(\frac{3}{2}, 3\vec{a} \times \vec{c}\right)$

B. $\left(-\frac{3}{2}, 3\vec{c} \times \vec{b}\right)$

C. $\left(\frac{3}{2}, 3\vec{b} \times \vec{c}\right)$

D. $\left(-\frac{3}{2}, 3\vec{a} \times \vec{b}\right)$

Answer: D

Solution:

Solution:

$\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = 0$

$3+2\left(\vec{a}\,.\,\vec{b}+\vec{b}\,.\,\vec{c}+\vec{c}\,.\,\vec{a}\right) = 0$

$\left(\vec{a}\,.\,\vec{b}+\vec{b}\,.\,\vec{c}+\vec{c}\,.\,\vec{a}\right) = \frac{-3}{2} \Rightarrow \lambda = \frac{-3}{2}$

$\vec{d} = \vec{a}\times\vec{b}+\vec{b}\times\left(-\vec{a}-\vec{b}\right)+\left(-\vec{a}-\vec{b}\right)\times a \left[\because c = -\vec{a}-\vec{b}\right]$

$= \vec{a}\times\vec{b}+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}$

$d = 3\left(\vec{a}\times\vec{b}\right)$

Question128

Let the volume of a parallelopiped whose coterminous edges are given by $\vec{u} = \hat{i}+\hat{j}+\lambda\hat{k}$, $\vec{v} = \hat{i}+\hat{j}+3\hat{k}$ and $\vec{w} = 2\hat{i}+\hat{j}+\hat{k}$ be 1cu. unit. If θ be the angle between the edges $\vec{u}$ and $\vec{w}$, then $\cos\theta$ can be: [Jan. 8, 2020 (I)]

Options:

A. $\frac{7}{6\sqrt{6}}$

B. $\frac{7}{6\sqrt{3}}$

C. $\frac{5}{7}$

D. $\frac{5}{3\sqrt{3}}$

Answer: B

Solution:

Solution:

It is given that $\vec{u} = \hat{i}+\hat{j}+\lambda\hat{k}$, $\vec{v} = \hat{i}+\hat{j}+3\hat{k}$ and $\vec{w} = 2\hat{i}+\hat{j}+\hat{k}$

Volume of $d = \left[\vec{u}\,.\,\vec{v}\,.\,\vec{w}\right]$

$$\Rightarrow \pm 1 = \begin{vmatrix} 1 & 1 & \lambda \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} \Rightarrow -\lambda+3 = \pm 1 \Rightarrow \lambda = 2 \text{ or } \lambda = 4$$

For $\lambda = 2$

$\cos\theta = \frac{2+1+2}{\sqrt{6}\sqrt{6}} = \frac{5}{6}$

For $\lambda = 4$

$\cos\theta = \frac{2+1+4}{\sqrt{6}\sqrt{18}} = \frac{7}{6\sqrt{3}}$

Question129

I et $\vec{a}$ = i − 2j + k and $\vec{b}$ = i − j + k he two vectors. If $\vec{c}$ is a vector such that

$\vec{b} \times \vec{c} = \vec{b} \times \vec{a}$ and $\vec{c} \cdot \vec{a} = 0$, then $\vec{c} \cdot \vec{b}$ is equal to:
[Jan. 8, 2020 (II)]

Options:

A. $-\frac{3}{2}$

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. -1

Answer: C

Solution:

Solution:
$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{a})$
$-(\vec{a} \cdot \vec{b})\vec{c} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$
$\Rightarrow -4\vec{c} = 6(\hat{i} - \hat{j} + \hat{k}) - 4(\hat{i} - 2\hat{j} - \hat{k})$
$\Rightarrow -4\vec{c} = 2\hat{i} - 2\hat{j} + 2\hat{k}$
$\Rightarrow \vec{c} = \frac{1}{2}(\hat{i} + \hat{j} + \hat{k})$
$\Rightarrow \vec{b} \cdot \vec{c} = -\frac{1}{2}$

Question130

The projection of the line segment joining the points (1,-1,3) and (2,-4,11) on the line joining the points (-1,2,3) and (3,-2,10) is _______. [NA Jan. 9, 2020 (I)]

Answer: 8

Solution:

Solution:
Let P(1, −1, 3), Q(2, −4, 11), R(−1, 2, 3)and S(3, −2, 10)
Then, $\overline{PQ} = \hat{i} - 3\hat{j} + 8\hat{k}$
Projection of $\overline{PQ}$ on $\overline{RS}$
$= \frac{\overline{PQ} \cdot \overline{RS}}{|\overline{RS}|} = \frac{4 + 12 + 56}{\sqrt{(4)^2 + (4)^2 + (7)^2}} = 8$

Question131

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 5$, $\vec{b}\,.\,\vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{b}$ is perpendicular to the vector $\vec{b} \times \vec{c}$, then $\left|\vec{a} \times \left(\vec{b} \times \vec{c}\right)\right|$ is equal to ______.
[NA Jan. 9, 2020 (II)]

Answer: 30

Solution:

Solution:

$\vec{b}\,.\,\vec{c} = 10 \Rightarrow |\vec{b}||\vec{c}|\cos\left(\frac{\pi}{3}\right) = 10$

$\Rightarrow 5\,.\,|\vec{c}|\,.\,\frac{1}{2} = 10 \Rightarrow |\vec{c}| = 4$

Since, is perpendicular to the vector $\vec{b} \times \vec{c}$, then

$\vec{a}\,.\left(\vec{b} \times \vec{c}\right) = 0$

Now, $\left|\vec{a} \times \left(\vec{b} \times \vec{c}\right)\right| = |\vec{a}|\left|\vec{b} \times \vec{c}\right|\sin\left(\frac{\pi}{2}\right)$

$= \sqrt{3} \times |\vec{b}||\vec{c}|\sin\frac{\pi}{3} \times 1$

Hence, $\left|\vec{a} \times \left(\vec{b} \times \vec{c}\right)\right| = 30$

Question132

Let the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ be such that $|\vec{a}| = 2$, $|\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is ________ .
[NA Sep. 05, 2020 (II)]

Answer: 6

Solution:

$\because$ Projection of $\vec{b}$ on $\vec{a}$ = Projection of $\vec{c}$ on $\vec{a}$

$\therefore \vec{a}\,.\,\vec{b} = \vec{a}\,.\,\vec{c}$

Given, $\vec{b}\,.\,\vec{c} = 0$

$\because |\vec{a} + \vec{b} - \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a}\,.\,\vec{b} - 2\vec{b}\,.\,\vec{c} - 2\vec{a}\,.\,\vec{c}$

$= 4 + 16 + 16 = 36$

$\Rightarrow |\vec{a} + \vec{b} - \vec{c}|^2 = 6$

Question133

If the vectors, $\vec{p} = (a+1)\hat{i} + a\hat{j} + a\hat{k}$, $\vec{q} = a\hat{i} + (a+1)\hat{j} + a\hat{k}$and $\vec{r} = a\hat{i} + a\hat{j} + (a+1)\hat{k}(a \in R)$ are coplanar and$3(\vec{p}.\vec{q})^2 - \lambda|\vec{r}\times\vec{q}|^2 = 0$, then the value of λ is _______.
[NA Jan. 9, 2020 (I)]

Answer: 1

Solution:

Solution:

$$\begin{vmatrix} a+1 & a & a \\ a & a+1 & a \\ a & a & a+1 \end{vmatrix} = 0$$

$\Rightarrow 3a + 1 = 0 \Rightarrow a = -\frac{1}{3}$

The given vectors

$\vec{p} = \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} - \frac{1}{3}\hat{k} = \frac{1}{3}(2\hat{i} - \hat{j} - \hat{k})$

$\vec{q} = \frac{1}{3}(-\hat{i} + 2\hat{j} - \hat{k})$

$\vec{r} = \frac{1}{3}(-\hat{i} - \hat{j} + 2\hat{k})$

Now, $\vec{p}.\vec{q} = \frac{1}{9}(-2 - 2 + 1) = -\frac{1}{3}$

$$\vec{r}\times\vec{q} = \frac{1}{9}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix}$$

$= \frac{1}{9}(i(4-1) - j(-2-1) + k(1+2))$

$= \frac{1}{9}(3i + 3j + 3k) = \frac{i+j+k}{3}$

$|\vec{r}\times\vec{q}| = \frac{1}{3}\sqrt{3} \Rightarrow |\vec{r}\times\vec{q}|^2 = \frac{1}{3}$

$3(\vec{p}.\vec{q})^2 - \lambda|\vec{r}\times\vec{q}|^2 = 0$

$\Rightarrow 3.\frac{1}{9} - \lambda.\frac{1}{3} = 0 \Rightarrow \lambda = 1$

Question134

Let $a, b, c \in R$ be such that $a^2 + b^2 + c^2 = 1$.
If $a\cos\theta - b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right)$, where $\theta = \frac{\pi}{9}$then the angle between the vectors $a\hat{i} + b\hat{j} + c\hat{k}$ and$b\hat{i} + c\hat{j} + a\hat{k}$ is :
[Sep. 03, 2020 (II)]

Options:

A. $\frac{\pi}{2}$

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{9}$

D. 0

Answer: A

Solution:

Solution:

$a\cos\theta = b\cos\left(\theta + \frac{2\pi}{3}\right) = c\cos\left(\theta + \frac{4\pi}{3}\right) = k$

$a = \frac{k}{\cos\theta}, b = \frac{k}{\cos\left(\theta + \frac{2\pi}{3}\right)}, c = \frac{k}{\cos\left(\theta + \frac{4\pi}{3}\right)}$

$ab + bc + ca = k^2 \frac{\left[\cos\left(\theta + \frac{4\pi}{3}\right) + \cos\theta + \cos\left(\theta + \frac{2\pi}{3}\right)\right]}{\cos\left(\theta + \frac{4\pi}{3}\right) . \cos\theta . \cos\left(\theta + \frac{2\pi}{3}\right)}$

$= k^2\left[\frac{\cos\theta + 2\cos(\theta + \pi) . \cos\left(\frac{\pi}{3}\right)}{\cos\theta . \cos\left(\theta + \frac{2\pi}{3}\right) . \cos\left(\theta + \frac{4\pi}{3}\right)}\right]$

$= k^2\left[\frac{\cos\theta + 2\cos\theta\frac{1}{2}}{\cos\theta . \cos\left(\theta + \frac{2\pi}{3}\right) . \cos\left(\theta + \frac{4\pi}{3}\right)}\right] = 0$

$\cos\phi = \frac{\left(a\hat{i} + b\hat{j} + c\hat{k}\right) . \left(b\hat{i} + c\hat{j} + a\hat{k}\right)}{\sqrt{a^2 + b^2 + c^2} . \sqrt{b^2 + c^2 + a^2}}$

$= ab + bc + ca = 0$

$\phi = \frac{\pi}{2}$

Question135

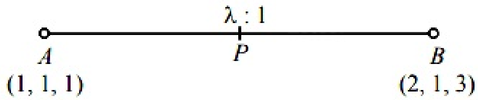
Let the position vectors of points 'A' and 'B' be $\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + 3\hat{k}$, respectively. A point 'P' divides the line segment AB internally in the ratio lambda : 1($\lambda > 0$). If O is the region and $\overrightarrow{OB} . \overrightarrow{OP} - 3\left|\overrightarrow{OA} \times \overrightarrow{OP}\right|^2 = 6$, then λ is equal to ______.
[NA Sep. 02, 2020 (II)]

Answer: 0.8

Solution:

Let position vector of A and B be $\vec{a}$ and $\vec{b}$ respectively.

$\therefore$ Position vector of P is $\overrightarrow{OP} = \frac{\lambda\vec{b} + \vec{a}}{\lambda + 1}$



Given $\overrightarrow{OB} \cdot \overrightarrow{OP} - 3\left|\overrightarrow{OA} \times \overrightarrow{OP}\right|^2 = 6$

$\Rightarrow \vec{b} \cdot \left(\frac{\lambda\vec{b} + \vec{a}}{\lambda + 1}\right) - 3\left|\vec{a} \times \frac{\lambda\vec{b} + \vec{a}}{\lambda + 1}\right|^2 = 6$

$\Rightarrow \frac{\vec{a} \cdot \vec{b} + \lambda\left|\vec{b}\right|^2}{\lambda + 1} - \frac{3\lambda^2}{(\lambda + 1)^2}\left|\vec{a} \times \vec{b}\right|^2 = 6 \left(\because \vec{a} \times \vec{b} = 2\vec{i} - \vec{j} - \vec{k} \text{ and } \vec{a} \cdot \vec{b} = 6\right)$

$\Rightarrow \frac{6 + 14\lambda}{\lambda + 1} - \frac{18\lambda^2}{(\lambda + 1)^2} = 6$

$\Rightarrow 6 + \frac{8\lambda}{\lambda + 1} - \frac{18\lambda^2}{(\lambda + 1)^2} = 6$

Let $\frac{\lambda}{\lambda + 1} = t$

$\Rightarrow 18t^2 - 8t = 0 \Rightarrow 2t(9t - 4) = 0$

$\Rightarrow t = 0, \frac{4}{9}$

$\therefore \frac{\lambda}{\lambda + 1} = \frac{4}{9} \Rightarrow \lambda = \frac{4}{5} = 0.8$

Question136

If $\vec{a}$ and $\vec{b}$ are unit vectors, then the greatest value of $\sqrt{3}\left|\vec{a} + \vec{b}\right| + \left|\vec{a} - \vec{b}\right|$ is _______.
[NA Sep. 06, 2020 (I)]

Answer: 4

Solution:

Let angle between $\vec{a}$ and $\vec{b}$ be θ.

$\left|\vec{a} + \vec{b}\right| = \sqrt{1 + 1 + 2\cos\theta} = 2\left|\cos\frac{\theta}{2}\right| \left[\because \left|\vec{a}\right| = \left|\vec{b}\right| = 1\right]$

Similarly, $\left|\vec{a} - \vec{b}\right| = 2\left|\sin\frac{\theta}{2}\right|$

So, $\sqrt{3}\left|\vec{a} + \vec{b}\right| + \left|\vec{a} - \vec{b}\right| = 2\left[\sqrt{3}\left|\cos\frac{\theta}{2}\right| + \left|\sin\frac{\theta}{2}\right|\right]$

$\because$ Maximum value of $(a\cos\theta + b\sin\theta) = \sqrt{a^2 + b^2}$

$\therefore$ Maximum value $= 2\sqrt{(\sqrt{3})^2 + (1)^2} = 4$

Question137

If $\vec{x}$ and $\vec{y}$ be two non-zero vectors such that $\left|\vec{x} + \vec{y}\right| = \left|\vec{x}\right|$ and $2\vec{x} + \lambda\vec{y}$ is perpendicular to $\vec{y}$, then the value of λ is _______.
[NA Sep. 06, 2020 (II)]

Answer: 1

Solution:

Solution:
$\because |\vec{x} + \vec{y}| = |\vec{x}|$
Squaring both sides we get
$|\vec{x}|^2 + 2\vec{x} \cdot \vec{y} + |\vec{y}|^2 = |\vec{x}|^2$
$\Rightarrow 2\vec{x} \cdot \vec{y} + \vec{y} \cdot \vec{y} = 0$ ....(i)
Also $2\vec{x} + \lambda\vec{y}$ and $\vec{y}$ are perpendicular
$\therefore 2\vec{x} \cdot \vec{y} + \lambda\vec{y} \cdot \vec{y} = 0$ ....(ii)
Comparing (i) and (ii), $\lambda = 1$

Question138

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$. Then $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$ is equal to_______.
[NA Sep. 02, 2020 (I)]

Answer: 2

Solution:

Solution:
$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$
$|\vec{a} - \vec{b}|^2 + |\vec{a} - \vec{c}|^2 = 8$
$\Rightarrow \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} = -2$
Now, $|\vec{a} + 2\vec{b}|^2 + |\vec{a} + 2\vec{c}|^2$
$= 2| = = \vec{a}|^2 + 4|\vec{b}|^2 + 4|\vec{c}|^2 + 4(\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}) = 2$

Question139

If the volume of a parallelopiped, whose coterminus edges are given by the vectors $\vec{a} = \hat{i} + \hat{j} + n\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - n\hat{k}$ and $\vec{c} = \hat{i} + n\hat{j} + 3\hat{k}$ $(n \geq 0)$, is 158cu. units, then:
[Sep. 05, 2020 (I)]

Options:

A. $\vec{a} \cdot \vec{c} = 17$

B. $\vec{b} \cdot \vec{c} = 10$

C. n = 7

D. n = 9

Answer: B

Solution:

Solution:
We know that the volume of parallelopiped

$= \left[\ \vec{a}\ \ \vec{b}\ \ \vec{c}\ \right]$

$$\begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = 158$$

$\Rightarrow (12 + n^2) - 1(6 + n) + n(2n - 4) = 158$
$\Rightarrow 3n^2 - 5n - 152 = 0$
$\Rightarrow 3n^2 - 24n + 19n - 152 = 0$
$\Rightarrow 3n(n - 8) + 19(n - 8) = 0$
$\Rightarrow n = 8$ or $n = \dfrac{-19}{3}$
$\Rightarrow n = 8 (\because n \geq 0)$
$\therefore \vec{a} = \hat{i} + \hat{j} + 8\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - 8\hat{k}$ and $\vec{c} = \hat{i} + 8\hat{j} + 3\hat{k}$
$\vec{a} \cdot \vec{c} = 1 + 8 + 24 = 33$
$\vec{b} \cdot \vec{c} = 2 + 32 - 24 = 10$

Question140

Let x_0 be the point of local maxima of $f(x) = \vec{a} \cdot \left(\vec{b} \times \vec{c}\right)$, where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$Then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ at $x = x_0$ is :
[Sep. 04, 2020 (I)]

Options:

A. -4

B. -30

C. 14

D. -22

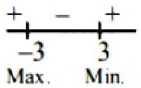
Answer: D

Solution:

Solution:
It is given that

$$f(x) = \vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix} = x^3 - 27x + 26$$

$\Rightarrow f(x) = x^3 - 27x + 26$
$\Rightarrow f'(x) = 3x^2 - 27$
For critical point $f'(x) = 0$
$\Rightarrow 3x^2 - 27 = 0 \Rightarrow x = -3, 3$



The local maxima of $f(x)$ is, $x_0 = -3$.
Then $r\bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a}$
$= -2x - 2x - 3 - 14 - 2x - x + 7x + 4 + 3x = 3x - 13$
So, value at $x = x_0$, $= \bar{a} \cdot \bar{b} + \bar{b} \cdot \bar{c} + \bar{c} \cdot \bar{a} = 3x - 13$
$= 3 \times (-3) - 13 = -22$

Question141

If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $\left|\hat{i} \times \left(\vec{a} \times \hat{i}\right)\right|^2 + \left|\hat{j} \times \left(\vec{a} \times \hat{j}\right)\right|^2 + \left|\hat{k} \times \left(\vec{a} \times \hat{k}\right)\right|^2$ is equal to ________.
[NA Sep. 04, 2020 (II)]

Answer: 18

Solution:

Solution:
$\hat{i} \times \left(\bar{a} \times \hat{i}\right) = \left(\hat{i} \cdot \hat{i}\right)\bar{a} - \left(\hat{i} \cdot \bar{a}\right)\hat{i} = \hat{j} + 2\hat{k}$
Similarly, $\hat{j} \times \left(\bar{a} \times \hat{j}\right) = 2\hat{i} + 2\hat{k}$
$\hat{k} \times \left(\hat{\bar{a}} \times \hat{k}\right) = 2\hat{i} + \hat{j}$
$\therefore \left|\hat{j} + 2\hat{k}\right|^2 + \left|2\hat{i} + 2\hat{k}\right|^2 + \left|2\hat{i} + \hat{j}\right|^2 = 5 + 8 + 5 = 18$

Question142

The sum of the distinct real values of mu, for which the vectors, $\mu\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + \mu\hat{j} + \hat{k}$, $\hat{i} + \hat{j} + \mu\hat{k}$, are co-planar,is:
[Jan. 12,2019 (I)]

Options:

A. -1

B. 0

C. 1

D. 2

Answer: A

Solution:

Solution:
∵ Three vectors $(\mu\hat{i} + \hat{j} + \hat{k})$, $(= = \hat{i} + \mu\hat{j} + \hat{k})$ and $(\hat{i} + \hat{j} + \mu\hat{k})$ are copalnar.

$$\therefore \begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$$

$\Rightarrow \mu(\mu^2 - 1) + 1 - \mu + 1 - \mu = 0$
$\Rightarrow (1 - \mu)[2 - \mu(\mu + 1)] = 0$
$\Rightarrow (1 - \mu)[\mu^2 + \mu - 2] = 0$
$\Rightarrow \mu = 1, -2$
Therefore, sum of all real values $= 1 - 2 = -1$

Question143

Let $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} + 4\hat{k}$ and $\vec{c} = 2\hat{i} + 4\hat{j} + (\lambda^2 - 1)\hat{k}$ be coplanar vectors. Then the non-zero vector $\vec{a} \times \vec{c}$ is : [Jan. 11, 2019 (I)]

Options:

A. $-10\hat{i} - 5\hat{j}$

B. $-14\hat{i} - 5\hat{j}$

C. $-14\hat{i} + 5\hat{j}$

D. $-10\hat{i} + 5\hat{j}$

Answer: D

Solution:

Solution:
$\because \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar

$$\therefore \begin{vmatrix} 1 & 2 & 4 \\ 1 & \lambda & 4 \\ 2 & 4 & (\lambda^2 - 1) \end{vmatrix} = 0$$

$\Rightarrow \lambda^3 - \lambda - 16 + 2(8 - \lambda^2 + 1) + 4(4 - 2\lambda) = 0$
$\Rightarrow \lambda^3 - 2\lambda^2 - 9\lambda + 18 = 0$
i.e., $(\lambda - 2)(\lambda - 3)(\lambda + 3) = 0$
For $\lambda = 2$, $\vec{c} = 2\hat{i} + 4\hat{j} + 3\hat{k}$

$$\therefore \vec{a} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{vmatrix} = -10\hat{i} + 5\hat{j}$$

For $\lambda = 3$ or -3, $\vec{c} = 2\vec{a} \Rightarrow \vec{a} \times c = 0$ (Rejected)

Question144

Let $\sqrt{3}\hat{i} + \hat{j}$, $\hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1 - \beta)\hat{j}$ respectively be the position vectors of the points A, B and C with respect to the origin O. If the distance of C

from the bisector of the acute angle between OA and OB is $\frac{3}{\sqrt{2}}$, then the sum of all possible values of beta is :
[Jan. 11,2019 (II)]

Options:

A. 4

B. 3

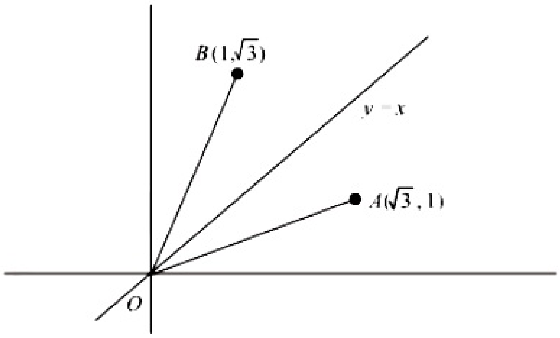
C. 2

D. 1

Answer: D

Solution:

Solution:
Since, the angle bisector of acute angle between OA and OB would be y = x



Since, the distance of C from bisector $= \frac{3}{\sqrt{2}}$

$\Rightarrow \left| \frac{\beta - (1-\beta)}{\sqrt{2}} \right| = \frac{3}{\sqrt{2}} = 2\beta = \pm 3 + 1$

$\beta = 2$ or $\beta = -1$

Hence, the sum of all possible value of $\beta = 2 + (-1) = 1$

Question145

Let $\alpha = (\lambda - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (4\lambda - 2)\vec{a} + 3\vec{b}$ be two given vectors where vectors $\vec{a}$ and $\vec{b}$ are non-collinear. The value of λ for which vectors $\vec{\alpha}$ and $\vec{\beta}$ are collinear, is:
[Jan. 10, 2019 (II)]

Options:

A. -4

B. -3

C. 4

D. 3

Answer: A

Solution:

Solution:
Let $\vec{\alpha}$ and $\vec{\beta}$ are collinear for same k
i.e., $\vec{\alpha} = k\vec{\beta}$
$(\lambda - 2)\vec{a} + \vec{b} = k\left((4\lambda - 2)\vec{a} + 3\vec{b}\right)$
$(\lambda - 2)\vec{a} + \vec{b} = k(4\lambda - 2)\vec{a} + 3k\vec{b}$
$(\lambda - 2 - k(4\lambda - 2))\vec{a} + \vec{b}(1 - 3k) = 0$
But $\vec{a}$ and $\vec{b}$ are non-collinear, then
$\lambda - 2 - k(4\lambda - 2) = 0,\ 1 - 3k = 0$
$\Rightarrow k = \frac{1}{3}$ and $\lambda - 2 - \frac{1}{3}(4\lambda - 2) = 0$
$3\lambda - 6 - 4\lambda + 2 = 0$
$\lambda = -4$

Question146

Let $\vec{a} = 2\hat{i} + \lambda_1\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}$ and $\vec{c} = 3\hat{i} + 6\hat{j} + (\lambda_3 - 1)\hat{k}$ be three vectors such that $\vec{b} = 2\vec{a}$ and $\vec{a}$ is perpendicular to $\vec{c}$ Then a possible value of $(\lambda_1, \lambda_2, \lambda_3)$ is: [Jan. 10, 2019 (I)]

Options:

A. (1,3,1)

B. $\left(-\frac{1}{2}, 4, 0\right)$

C. $\left(\frac{1}{2}, 4, -2\right)$

D. (1,5,1)

Answer: B

Solution:

Solution:
$\because \vec{b} = 2\vec{a}$
$\therefore 4\hat{i} + (3 - \lambda_2)\hat{j} + 6\hat{k}) = 4\hat{i} + 2\lambda_1\hat{j} + 6\hat{k}$
$\therefore 3 - \lambda_2 = 2\lambda_1$ ......(1)
$\because \vec{a}$ is perpendicular to $\vec{c}$
$\therefore \vec{a}\,.\,\vec{c} = 0$
$\Rightarrow 6 + 6\lambda_1 + 3(\lambda_3 - 1) = 0$
$\Rightarrow 2 + 2\lambda_1 + \lambda_3 - 1 = 0$
$\Rightarrow \lambda_3 = -2\lambda_1 - 1$ ......(2)
Since $\left(\frac{-1}{2}, 4, 0\right)$ satisfies equation (1) and (2). Hence, one of possible value of
$\lambda_1 = -\frac{1}{2}$, $\lambda_2 = 4$ and $\lambda_3 = 0$

Question147

Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection vector of $\vec{b}$ on $\vec{a}$ is $\vec{a}$.
If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$, then $\left|\vec{b}\right|$ is equal to:
[Jan. 09, 2019 (II)]

Options:

A. $\sqrt{32}$

B. 6

C. $\sqrt{22}$

D. 4

Answer: B

Solution:

Solution:

Projection of $\vec{b}$ on $\vec{a}$ = $\frac{\vec{b}\,.\,\vec{a}}{\left|\vec{a}\right|} = \frac{b_1 + b_2 + 2}{4}$

According to question $\frac{b_1 + b_2 + 2}{2} = \sqrt{1 + 1 + 2} = 2$

$\Rightarrow b_1 + b_2 = 2$ ....(1)

Since, $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$.

Hence, $\vec{a}\,.\,\vec{c} + \vec{b}\,.\,\vec{c} = 0$

$\Rightarrow 8 + 5b_1 + b_2 + 2 = 0$ ....(2)

From (1) and (2),

$b_1 = -3$, $b_2 = 5$

$\Rightarrow \vec{b} = -3\,.\,\hat{i} + 5\hat{j} + \sqrt{2}\hat{k}$

$\left|\vec{b}\right| = \sqrt{9 + 25 + 2} = 6$

Question148

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If α and β are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \frac{1}{2}\vec{b}$, then $|\alpha - \beta|$ is equal to
[Jan. 12,2019 (II)]

Options:

A. 30°

B. 90°

C. 60°

D. 45°

Answer: A

Solution:

Since, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors
$\therefore |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$
Then, $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$
$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{1}{2}\vec{b}$
$\therefore \vec{a} \cdot \vec{c} = \frac{1}{2}$ and $\vec{a} \cdot \vec{b} = 0$
$|\vec{a}||\vec{c}| \cos\beta = \frac{1}{2}$ and $|\vec{a}||\vec{b}| \cos\alpha = 0$
$\Rightarrow \beta = 60°$ and $\alpha = 90°$
Hence, $|\alpha - \beta| = 90° - 60° | = 30°$

Question149

Let $\vec{a} = i - j$, $\vec{b} = i + j + k$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 4$, then $|\vec{c}|^2$ is equal to:
[Jan 09, 2019]

Options:

A. $\frac{19}{2}$

B. 9

C. 8

D. $\frac{17}{2}$

Answer: A

Solution:

Solution:
$\because |\vec{a} \times \vec{c}|^2 = |\vec{a}|^2 |\vec{c}|^2 - (\vec{a} \cdot \vec{c})^2$
$\Rightarrow |-\vec{b}|^2 = 2|\vec{c}|^2 - 16 \Rightarrow 3 = 2|\vec{c}|^2 - 16$
$\Rightarrow |\vec{c}|^2 = \frac{19}{2}$

Question150

Let $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ be two vectors. Ifa vector perpendicular to both the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ has the magnitude 12 then one such vector is :
[April 12, 2019 (I)]

Options:

A. $4\left(2\hat{i}+2\hat{j}+2\hat{k}\right)$

B. $4\left(2\hat{i}-2\hat{j}-2\hat{k}\right)$

C. $4\left(2\hat{i}+2\hat{j}-2\hat{k}\right)$

D. $4\left(-2\hat{i}-2\hat{j}+2\hat{k}\right)$

Answer: B

Solution:

Solution:

Let vector be $\lambda\left[\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right]$

Given, $a=3\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$

$\therefore \vec{a}+\vec{b}=4\hat{i}+4\hat{j}$ and $\vec{a}-\vec{b}=2\hat{i}+4\hat{k}$

$\therefore$ vector $=\lambda\left[\left(4\hat{i}+4\hat{j}\right)\times\left(2\hat{i}+4\hat{k}\right)\right]$

$=\lambda\left[16\hat{i}-16\hat{j}-8\hat{k}\right]$ $=8\lambda\left[2\hat{i}-2\hat{j}-\hat{k}\right]$

Given that magnitude of the vector is 12.

$\therefore 12=8|\lambda|\sqrt{4+4+1}\Rightarrow|\lambda|=\frac{1}{2}$

$\therefore$ required vector is $\pm 4\left(2\hat{i}-2\hat{j}-2\hat{k}\right)$

Question151

If the volume of parallelopiped formed by the vectors $\hat{i}+\lambda\hat{j}+\hat{k}$, $\hat{j}+\lambda\hat{k}$ and $\lambda\hat{i}+\hat{k}$ is minimum, then λ is equalto:
[April 12, 2019 (I)]

Options:

A. $-\frac{1}{\sqrt{3}}$

B. $\frac{1}{\sqrt{3}}$

C. $\sqrt{3}$

D. $-\sqrt{3}$

Answer: B

Solution:

Solution:

Volume of the parallelepiped is,

$V=|{|\table 1 ,\λ, 1 ; 0 , 1 , \λ;\λ,0 ,1 }||

 ={ |1(1)+\λ(\λ^{2})+1(-\λ)|}$

$=|\lambda^{3}-\lambda+1|$

Let $f(x)=x^{3}-x+1$

On differentiating, $f'(x)=3x^{2}-1$

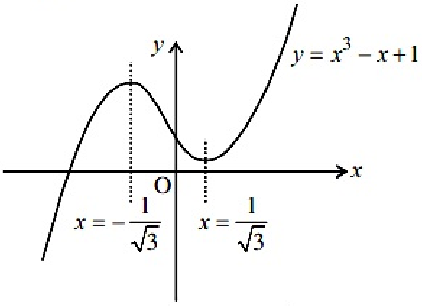
Now, $f'(x) = 0$

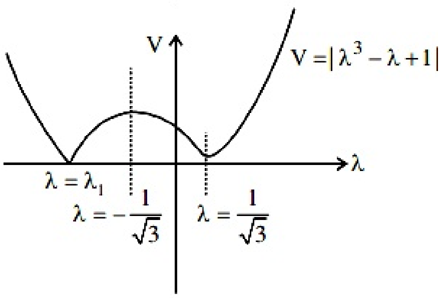
$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$

and $f''(x) = 6x$

Since, $f''\left(\frac{1}{\sqrt{3}}\right) > 0$

$\therefore x = \frac{1}{\sqrt{3}}$ is point of local minima.





For $\lambda = \lambda_1$, volume of parallelopiped is zero.

$\therefore$ vectors are coplanar.

Question152

Let $\alpha \in R$ and the three vectors $\vec{a} = \alpha\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \alpha\hat{k}$ and $\vec{c} = \alpha\hat{i} - 2\hat{j} + 3\hat{k}$. Then the set $S = (\alpha : \vec{a}, \vec{b}$ and $\vec{c}$ are coplanar) [April 12, 2019 (II)]

Options:

A. is singleton

B. is empty

C. contains exactly two positive numbers

D. contains exactly two numbers only one of which is positive

Answer: B

Solution:

Solution:

Let, three vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar,

then $[\vec{a}, \vec{b}, \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} \alpha & 1 & 3 \\ 2 & 1 & -\alpha \\ \alpha & -2 & 3 \end{vmatrix} = 0 \Rightarrow \alpha^2 + 6 = 0$$

$\because$ no real value of 'α' exist.

$\therefore$ set S is an empty set.

Question153

If a unit vector $\vec{a}$ makes angles $\pi / 3$ with $\hat{i}$, $\pi / 4$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of is:
[April 09, 2019 (II)]

Options:

A. $\frac{5\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{5\pi}{12}$

D. $\frac{2\pi}{3}$

Answer: D

Solution:

Solution:

Let $\cos\alpha$, $\cos\beta$, $\cos\gamma$ be direction cosines of a.

$\therefore \cos\alpha = \cos\frac{\pi}{3}$, $\cos\beta = \cos\frac{\pi}{4}$ and $\cos\gamma = \cos\theta$

$\Rightarrow \cos^2\frac{\pi}{3} + \cos^2\frac{\pi}{4} + \cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{4}$

$\Rightarrow \cos\theta = \pm\frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

Question154

Let $\vec{\alpha} = 3\hat{i} + \hat{j}$ and $\vec{\beta} = 2\hat{i} - \hat{j} + 3\hat{k}$. If $\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$, then $\vec{\beta}_1 \times \vec{\beta}_2$ is equal to:
[April 09, 2019 (I)]

Options:

A. $-3\hat{i} + 9\hat{j} + 5\hat{k}$

B. $3\hat{i} - 9\hat{j} - 5\hat{k}$

C. $\frac{1}{2}\left(-3\hat{i} + 9\hat{j} + 5\hat{k}\right)$

D. $\frac{1}{2}\left(3\hat{i} - 9\hat{j} + 5\hat{k}\right)$

Answer: C

Solution:

Solution:

$\vec{\beta} = \vec{\beta}_1 - \vec{\beta}_2$ ....(1)

Since, $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

$\therefore \vec{\beta}_2 . \vec{\alpha} = 0$

Since, $\vec{\beta}_1$ is parallel to $\vec{a}$.

then $\vec{\beta}_1 = \lambda\vec{\alpha}$ (say)

$\vec{a} . \vec{\beta} = \vec{a} . \vec{\beta}_1 - \alpha . \vec{\beta}_2$

$\Rightarrow 5 = \lambda\alpha^2 \Rightarrow 5 = \lambda \times 10 \left(\because \left|\vec{\alpha}\right| = \sqrt{10}\right)$

$\Rightarrow \lambda = \frac{1}{2} \therefore \vec{B}_1 = \frac{\alpha}{2}$

$\therefore \vec{\beta}_1 = \frac{\vec{\alpha}}{2}$

Cross product with $\vec{B}_1$ in equation (1)

$\Rightarrow \vec{\beta} \times \vec{B}_1 = -\vec{B}_2 \times \vec{B}_1$

$\Rightarrow \vec{\beta} \times \vec{B}_1 = \vec{B}_1 \times \vec{B}_2 \Rightarrow \vec{\beta}_1 \times \vec{\beta}_2 = \frac{\vec{\beta} \times \vec{\alpha}}{2}$

$$\Rightarrow \vec{B}_1 \times \vec{B}_2 = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 3 & 1 & 0 \end{vmatrix}$$

$= \frac{1}{2}\left[-3\hat{i} - \hat{j}(-9) + \hat{k}(5)\right] = \frac{1}{2}\left[-3\hat{i} + 9\hat{j} + 5\hat{k}\right]$

Question155

The magnitude of the projection of the vector $2\hat{i} + 3\hat{j} + \hat{k}$ on the vector perpendicular to the plane containing the vectors $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, is :
[April 08, 2019 (I)]

Options:

A. $\frac{\sqrt{3}}{2}$

B. $\sqrt{6}$

C. $3\sqrt{6}$

D. $\sqrt{\frac{3}{2}}$

Answer: D

Solution:

Solution:

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ vector perpendicular to $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b}$

$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$

Now, projection of vector $\vec{c} = 2\hat{i} + 3 = = \hat{j} + \hat{k}$ on $\vec{a} \times \vec{b}$ is $= \left| \frac{\vec{c} \cdot (\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|} \right| = \left| \frac{2 - 6 + 1}{\sqrt{6}} \right| = \frac{3}{\sqrt{6}} = \frac{\sqrt{3}}{2}$

Question156

Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x.
Then $|\vec{a} \times \vec{b}| = r$ is possible if :
[April 08, 2019 (II)]

Options:

A. $\sqrt{\frac{3}{2}} < r \le 3\sqrt{\frac{3}{2}}$

B. $r \ge 5\sqrt{\frac{3}{2}}$

C. $0 < r \le \sqrt{\frac{3}{2}}$

D. $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$

Answer: B

Solution:

Solution:

Given, $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

Now, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & x \\ 1 & -1 & 1 \end{vmatrix}$

$= (2 + x)\hat{i} + (x - 3)\hat{j} - 5\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{(2 + x)^2 + (x - 3)^2 + (-5)^2} = r$

$\Rightarrow r = \sqrt{4 + x^2 + 4x + x^2 + 9 - 6x + 25}$

$= \sqrt{2x^2 - 2x + 38} = \sqrt{2\left(x^2 - x + \frac{1}{4}\right) + 38 - \frac{1}{2}}$

$= \sqrt{2\left(x - \frac{1}{2}\right)^2 + \frac{75}{2}} \Rightarrow r \ge \sqrt{\frac{75}{2}} \Rightarrow r \ge 5\sqrt{\frac{3}{2}}$

Question157

Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u} \cdot \vec{b} - 24$, then $|\vec{u}|^2$ is equal to:

[2018]

Options:

A. 315

B. 256

C. 84

D. 336

Answer: D

Solution:

Solution:

$\because \vec{u}, \vec{a}$ & $\vec{b}$ are coplanar

$\therefore \vec{u} = \lambda(\vec{a} \times \vec{b}) \times \vec{a} = \lambda\{\vec{a}^2 . \vec{b} - (\vec{a} . \vec{b})\vec{a}\}$

$= \lambda\{-4\hat{i} + 8\hat{j} + 16\hat{k}\} = \lambda'\{-\hat{i} + 2\hat{j} + 4\hat{k}\}$

Also, $\vec{u} . \vec{b} = 24 \Rightarrow \lambda' = 4$

$\therefore \vec{u} = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$\Rightarrow |\vec{u}|^2 = 336$

Question158

If the position vectors of the vertices A, B and C of a ΔABC are respectively $4\hat{i} + 7\hat{j} + 8\hat{k}$, $2\hat{i} + 3\hat{j} + 4\hat{k}$ and $2\hat{i} + 5\hat{j} + 7\hat{k}$, then the position vector of the point, where the bisector of ∠A meets BC is [Online April 15, 2018]

Options:

A. $\frac{1}{2}(4\hat{i} + 8\hat{j} + 11\hat{k})$

B. $\frac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$

C. $\frac{1}{4}(8\hat{i} + 14\hat{j} + 9\hat{k})$

D. $\frac{1}{3}(6\hat{i} + 11\hat{j} + 15\hat{k})$

Answer: B

Solution:

Solution:

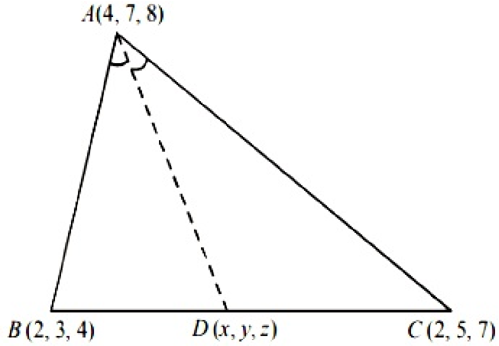
Suppose angular bisector of A meets BC at D(x, y, z)

Using angular bisector theorem,

$\frac{AB}{AC} = \frac{BD}{DC}$

$\frac{BD}{DC} = \frac{\sqrt{(4-2)^2 + (7-3)^2 + (8-4)^2}}{\sqrt{(4-2)^2 + (7-5)^2 + (8-7)^2}}$

$= \frac{\sqrt{2^2 + 4^2 + 4^2}}{\sqrt{2^2 + 2^2 + 1^2}} = \frac{6}{3} = 2$



So, D(x, y, z) ≡ $\left(\frac{(2)(2) + (1)(2)}{2 + 1}, \frac{(2)(5) + (1)(3)}{2 + 1}\right.$
$\left.\frac{(2)(7) + (1)(4)}{2 + 1}\right)$

$D(x, y, z) \equiv \left(\frac{6}{3}, \frac{13}{3}, \frac{18}{3}\right)$

Therefore, position vector of point $P = \frac{1}{3}\left(6\hat{i} + 13\hat{j} + 18\hat{k}\right)$

Question159

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$ and a vector $\vec{b}$ be such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} . \vec{b} = 3$. Then $\left|\vec{b}\right|$ equals?
[Online April 16, 2018]

Options:

A. $\sqrt{\frac{11}{3}}$

B. $\frac{\sqrt{11}}{3}$

C. $\frac{11}{\sqrt{3}}$

D. $\frac{11}{3}$

Answer: A

Solution:

Solution:

$\because \vec{a} = \hat{i} + \hat{j} + \hat{k} \Rightarrow \left|\vec{a}\right| = \sqrt{3}$

& $\vec{c} = \hat{j} - \hat{k} \Rightarrow \left|\vec{c}\right|\sqrt{2}$

Now, $\vec{a} \times \vec{b} = \vec{c}$ (Given)

$\Rightarrow \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta = \left|\vec{c}\right|$

$\Rightarrow \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta = \sqrt{2}$ ....(i)

Also $\vec{a} . \vec{b} = 3 \Rightarrow \left|\vec{a}\right|\left|\vec{b}\right|\cos\theta = 3$ ....(ii)

Dividing [i] by [ii], we get

$\tan\theta = \frac{\sqrt{2}}{3} \therefore \sin\theta = \frac{\sqrt{2}}{\sqrt{11}}$
Substituting value of $\sin\theta$ in [i] we get
$\sqrt{3}|\vec{b}|\frac{\sqrt{2}}{\sqrt{11}} = \sqrt{2}$
$|\vec{b}| = \frac{\sqrt{11}}{\sqrt{3}}$

Question160

If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are unit vectors such that $\vec{a} + 2\vec{b} + 2\vec{c} = \vec{0}$, then $|\vec{a} \times \vec{c}|$ is equal to
[Online April 15, 2018]

Options:

A. $\frac{1}{4}$

B. $\frac{\sqrt{15}}{4}$

C. $\frac{15}{16}$

D. $\frac{\sqrt{15}}{16}$

Answer: B

Solution:

Solution:
$\because \vec{a} + 2\vec{b} + 2\vec{c} = \vec{0}$ [Given]
$\Rightarrow \vec{a} + 2\vec{c} = -2\vec{b} \Rightarrow (\vec{a} + 2\vec{c}) . (\vec{a} + 2\vec{c}) = (-2\vec{b})(-2\vec{b})$
$\Rightarrow \vec{a} . \vec{a} + 4\vec{c} . \vec{c} + 4\vec{a} . \vec{c} = 4\vec{b} . \vec{b} \Rightarrow 1 + 4 + 4\vec{a} . \vec{c} = 4$
$\Rightarrow \vec{a} . \vec{c} = \frac{-1}{4}$
$\because |\vec{a} . \vec{c}|^2 + |\vec{a} \times \vec{c}|^2 = 1$ ($\vec{a}$ is unit vector)
$\Rightarrow \frac{1}{16} + |\vec{a} \times \vec{c}|^2 = 1$
$\Rightarrow |\vec{a} \times \vec{c}|^2 = \frac{15}{16} \Rightarrow |\vec{a} \times \vec{c}| = \frac{\sqrt{15}}{4}$

Question161

Let $\vec{a} = 2\hat{i} + \hat{j} - 2k$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ and the angle between $\vec{c}$and $\vec{a} \times \vec{b}$ be 30°. Then $\vec{a} . \vec{c}$ is equal to :
[2017]

Options:

A. $\frac{1}{8}$

B. $\frac{25}{8}$

C. 2

D. 5

Answer: C

Solution:

Solution:

Given : $\vec{a} = 2\hat{i} + \hat{j} - 2k, \vec{b} = \hat{i} + \hat{j}$

$\Rightarrow |\vec{a}| = 3$

$\therefore \vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{2^2 + 2^2 + 1^2} = 3$

We have $(\vec{a} \times \vec{b}) \times \vec{c} = |\vec{a} \times \vec{b}||\vec{c}| \sin 30\, n$

$\Rightarrow |(\vec{a} \times \vec{b}) \times \vec{c}| = 3|\vec{c}| . \frac{1}{2} \Rightarrow 3 = 3|\vec{c}| . \frac{1}{2}$

$\therefore |\vec{c}| = 2$

Now $|\vec{c} - \vec{a}| = 3$

On squaring, we get

$\Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} . \vec{c} = 9 \Rightarrow 4 + 9 - 2\vec{a} . \vec{c} = 9$

$\Rightarrow \vec{a} . \vec{c} = 2 [\because \vec{c} . \vec{a} = \vec{a} . \vec{c}]$

Question162

If the vector $\vec{b} = 3\hat{j} + 4\hat{k}$ is written as the sum of a vector $\vec{b}_1$, parallel to $\vec{a} = \hat{i} + \hat{j}$ and a vector $\vec{b}_2$, perpendicular to $\vec{a}$, then $\vec{b}_1 \times \vec{b}_2$ is equal to: [Online April 9, 2017]

Options:

A. $-3\hat{i} + 3\hat{j} - 9\hat{k}$

B. $6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$

C. $-6\hat{i} + 6\hat{j} - \frac{9}{2}\hat{k}$

D. $3\hat{i} - 3\hat{j} + 9\hat{k}$

Answer: B

Solution:

Solution:

$$\vec{b}_1 = \frac{(\vec{b}_1 . \vec{a})\hat{a}}{1} = \left\{ \frac{(3\hat{j} + 4\hat{k}) . (\hat{i} + \hat{j})}{\sqrt{2}} \right\} \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}} \right)$$

$= \frac{3(\hat{i}+\hat{j})}{\sqrt{2}\times\sqrt{2}} = \frac{3(\hat{i}+\hat{j})}{2}$

$\vec{b}_1 + \vec{b}_2 = \vec{b}$

$\Rightarrow \vec{b}_2 = \vec{b} - \vec{b}_1 = (3\hat{j}+4\hat{k}) - \frac{3}{2}(\hat{i}+\hat{j})$

$\Rightarrow \vec{b}_2 = -\frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} + 4\hat{k}$

$\& \ \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{2} & \frac{3}{2} & 0 \\ -\frac{3}{2} & \frac{3}{2} & 4 \end{vmatrix}$

$\Rightarrow \vec{b}_1 \times \vec{b}_2 = \hat{i}(6) - \hat{j}(6) + \hat{k}\left(-\frac{9}{4}+\frac{9}{4}\right)$

$\Rightarrow 6\hat{i} - 6\hat{j} + \frac{9}{2}\hat{k}$

Question163

The area (in sq. units) of the parallelogram whose diagonals are along the vectors $8\hat{i} - 6\hat{j}$ and $3\hat{i} + 4\hat{j} - 12\hat{k}$, is :
[Online April 8, 2017]

Options:

A. 26

B. 65

C. 20

D. 52

Answer: B

Solution:

Solution:

Let; $d_1 = 8\hat{i} - 6\hat{j} + 0\hat{k}$ & $d_2 = 3\hat{i} + 4\hat{j} - 12\hat{k} \therefore d_1 \times d_2 | = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -6 & 0 \\ 3 & 4 & -12 \end{vmatrix} = \left|72\hat{i} - (-96)\hat{j} + 50\hat{k}\right|$

$\Rightarrow d_1 \times d_2 | = \sqrt{16900} = 130$

$\therefore$ Area of parallelogram $= \frac{1}{2}\left|d_1 \times d_2\right| = \frac{1}{2} \times 130 = 65$

Question164

Let ABC be a triangle whose circumcentre is at P. If the position vectors A, B, C and P are $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{4}$ respectively, then the position vector of the orthocentre of this triangle, is :

[Online April 10, 2016]

Options:

A. $-\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

B. $\vec{a}+\vec{b}+c$

C. $\left(\frac{\vec{a}+\vec{b}+\vec{c}}{2}\right)$

D. $\vec{0}$

Answer: C

Solution:

Solution:

Position vector of centriod $\vec{G}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$

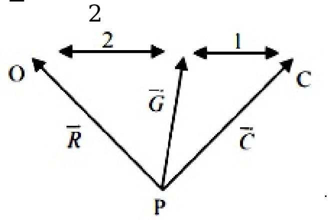
Position vector of circum centre $\vec{C}=\frac{\vec{a}+\vec{b}+\vec{c}}{4}$

$\vec{G}=\frac{2\vec{C}+\vec{r}}{3}$

$3\vec{G}=2\vec{C}+\vec{r}$

$r=3\vec{G}-2\vec{C}=\left(\vec{a}+\vec{b}+\vec{c}\right)-2\left(\frac{\vec{a}+\vec{b}+\vec{c}}{4}\right)$

$=\frac{\vec{a}+\vec{b}+\vec{c}}{2}$



Question165

In a triangle ABC, right angled at the vertex A, if the position vectors of A, B and C are respectively $3\hat{i}+\hat{j}-\hat{k}$, $-\hat{i}+3\hat{j}+p\hat{k}$ and $5\hat{i}+a\hat{j}-4\hat{k}$, then the point(pq) lies on a line:
[Online April 9, 2016]

Options:

A. making an obtuse angle with the positive direction of x -axis

B. parallel to x -axis

C. parallel to y-axis

D. making an acute angle with the positive direction of x -axis

Answer: D

Solution:

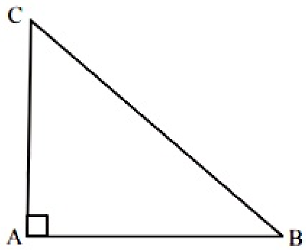
Solution:

$\overline{AB} = -4\hat{i} + 2\hat{j} + (p+1)\hat{k}$

$\overline{AC} = \underline{2\hat{i}} + (q-1)\hat{j} - 3\hat{k}$

$\overline{AB} \perp \overline{AC}$

$\Rightarrow \overline{AB} \cdot \overline{AC} = 0$



$-8 + 2(q-1) - 3(p+1) = 0$

$3p - 2q + 13 = 0$

(p, q) lies on $3x - 2y + 13 = 0$

slope $= \frac{3}{2}$

$\therefore$ Acute angle with x -axis

Question166

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is:
[2016]

Options:

A. $\frac{2\pi}{3}$

B. $\frac{5\pi}{6}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{2}$

Answer: B

Solution:

Solution:

$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(b + c)$

$\Rightarrow (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$

On comparing both sides

$\vec{a} \cdot \vec{b} = -\frac{\sqrt{3}}{2} \Rightarrow \cos\theta = -\frac{\sqrt{3}}{2}$ [$\because \vec{a}$ and $\vec{b}$ are unit vectors]

where θ is the angle between $\vec{a}$ and $\vec{b}$

$\theta = \frac{5\pi}{6}$

Question167

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is :
[2015]

Options:

A. $\frac{2}{3}$

B. $\frac{-2\sqrt{3}}{3}$

C. $\frac{2\sqrt{2}}{3}$

D. $\frac{-\sqrt{2}}{3}$

Answer: C

Solution:

Solution:

$\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow -\vec{c} \times \left(\vec{a} \times \vec{b}\right) = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow -\left(\vec{c}\,.\,\vec{b}\right)\vec{a} + \left(\vec{c}\,.\,\vec{a}\right)\vec{b} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\Rightarrow -\left|\vec{b}\right|\left|\vec{c}\right|\cos\theta\,\vec{a} + \left(\vec{c}\,.\,\vec{a}\right)\vec{b} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

$\because \vec{a}, \vec{b}, \vec{c}$ are non collinear, the above equation is possible only when

$-\cos\theta = \frac{1}{3}$ and $\vec{c}\,.\,\vec{a} = 0$

$\Rightarrow \cos\theta = -\frac{1}{3}$

$\Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}; \theta \in II$ quad

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Question168

In a parallelogram ABD, $\left|\vec{AB}\right| = a$, $\left|\vec{AD}\right| = b$ and $\left|\vec{AC}\right| = c$, then $\vec{DA}\,.\,\vec{AB}$ has the value :
[Online April 11, 2015]

Options:

A. $\frac{1}{2}(a^2 + b^2 + c^2)$

B. $\frac{1}{2}(a^2 - b^2 + c^2)$

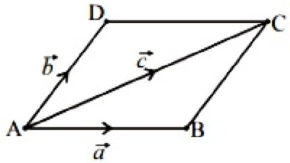
C. $\frac{1}{2}(a^2 + b^2 - c^2)$

D. $\frac{1}{3}(b^2 + c^2 - a^2)$

Answer: C

Solution:

Solution:
Let $|\overline{AB}| = a$, $|\overline{AD}| = b$ and $|\overline{AC}| = c$
We have $\overline{AB} + \overline{AD} = \overline{AC}$



On squaring both the side, we get
$|\overline{AB}|^2 + |\overline{AD}|^2 + 2\overline{AB} \cdot \overline{AD} = |\overline{AC}|^2$
$\Rightarrow a^2 + b^2 + 2\overline{AB} \cdot (-\overline{DA}) = c^2$
$\Rightarrow 2\overline{AB} \cdot \overline{DA} = a^2 + b^2 - c^2$
$\Rightarrow \overline{DA} \cdot \overline{AB} = \frac{1}{2}(a^2 + b^2 - c^2)$

Question169

Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$. If $\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})$, then $2|\vec{c}|$ is equal to: [Online April 10, 2015]

Options:

A. $\sqrt{55}$

B. $\sqrt{37}$

C. $\sqrt{51}$

D. $\sqrt{43}$

Answer: A

Solution:

Solution:
$|\vec{a} + \vec{b}| = \sqrt{3}$
angle between $\vec{a}$ and $\vec{b}$ is 60°.
$\vec{a} \times \vec{b}$ is $\perp^r$ to plane containing $\vec{a}$ and $\vec{b}$
$\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})$
$\vec{c} = \sqrt{|\vec{a}|^2 + 4|\vec{b}|^2 + 2.2|\vec{a}|^2 \cos 60° \, \vec{n}_1} + 3|\vec{a}||\vec{b}| \sin 60° \, \vec{n}_2 + 3|\vec{a}||\vec{b}| \sin 60° \cdot \vec{n}_2$
$\vec{n}_1 \perp^r \vec{n}_2$
$|\vec{c}|^2 = (1 + 4 + 2) + 9 \times \frac{3}{4} \Rightarrow |\vec{c}|^2 = 7 + 27 / 4 = 55 / 4$

$2|\vec{c}| = \sqrt{55}$

Question170

If $\hat{x}$, $\hat{y}$ and $\hat{z}$ are three unit vectors in three-dimensional space, then the minimum value of $|\hat{x}+\hat{y}|^2 + |\hat{y}+\hat{z}|^2 + |\hat{z}+\hat{x}|^2$
[Online April 12, 2014]

Options:

A. $\frac{3}{2}$

B. 3

C. $3\sqrt{3}$

D. 6

Answer: B

Solution:

Solution:
$(\hat{x}+\hat{y}+\hat{z})^2 \geq 0$
$\Rightarrow 3 + 2\sum \hat{x}\cdot\hat{y} \geq 0$
$\Rightarrow 2\sum \hat{x}\cdot\hat{y} \geq -3$
Now, $|\hat{x}+\hat{y}|^2 + |\hat{y}+\hat{z}|^2 + |\hat{z}+\hat{x}|^2$
$= 6 + 2\sum x\cdot\hat{y} \geq 6 + (-3)$
$\Rightarrow |\hat{x}+\hat{y}|^2 + |\hat{y}+\hat{z}|^2 + |\hat{z}+\hat{x}|^2 \geq 3$

Question171

If $|\vec{a}| = 2, |\vec{b}| = 3$ and $|2\vec{a} - \vec{b}| = 5$, then $|2\vec{a} + \vec{b}|$ equals:
[Online April 9, 2014]

Options:

A. 17

B. 7

C. 5

D. 1

Answer: C

Solution:

Given $\left|2\vec{a}-\vec{b}\right| = 5$

$\sqrt{(2|\vec{a}|)^2+|\vec{b}|^2 - 2\times 2\vec{a}|\vec{b}|\cos\theta} = 5$

Putting values of $|\vec{a}|$ and $|\vec{b}|$, we get

$(2\times 2)^2 + (3)^2 - 24\cos\theta = 25$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

$\left|2\vec{a}+\vec{b}\right| = \sqrt{16+9+24\cos\theta} = \sqrt{25} = 5$

Question172

If $\left[\ \vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\ \right] = \lambda\left[\vec{a}, \vec{b}, \vec{c}\right]^2$ then λ is equal to [2014]

Options:

A. 0

B. 1

C. 2

D. 3

Answer: B

Solution:

Solution:

$L.H.S = \left[(\vec{a}\times\vec{b}).\left[(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})\right]\right]$

$= (\vec{a}\times\vec{b}).\left[(\vec{b}\times\vec{c}.\vec{a})\vec{c} - (\vec{b}\times\vec{c}\vec{c})\vec{a}\right]$

$= (\vec{a}\times\vec{b}).\vec{b}\vec{c}\vec{a}]\vec{c}]$ $[\because b\times c.c = 0]$

$= [\vec{a}\vec{b}\vec{c}].\vec{a}\times\vec{b}.\vec{c}) = [\vec{a}\vec{b}\vec{c}]^2$

$= [\vec{a}\times\vec{b}\vec{b}\times\vec{c}\vec{c}\times\vec{a}]] = [\vec{a}\vec{b}\vec{c}]^2$

So $\lambda = 1$

Question173

If $\vec{x} = 3\hat{i} - 6\hat{j} - \hat{k}$, $\vec{y} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{z} = 3\hat{i} - 4\hat{j} - 12\hat{k}$ then the magnitude of the projection of $\vec{x}\times\vec{y}$ on $\vec{z}$ is: [Online April 19, 2014]

Options:

A. 12

B. 15

C. 14

D. 13

Answer: C

Solution:

Solution:

Let $\vec{x} = 3\hat{i} - 6\hat{j} - \hat{k}$, $\vec{y} = \hat{i} + 4\hat{j} - 3\hat{k}$ and $\vec{z} = 3\hat{i} - 4\hat{j} - 12\hat{k}$

Now, $\vec{x} \times \vec{y} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -6 & -1 \\ 1 & 4 & -3 \end{vmatrix} = 22\hat{i} + 8\hat{j} + 18\hat{k}$

Projection of $\hat{x} \times \hat{y}$ on $\hat{z} = \frac{(\hat{x} + \hat{y}).(z)}{|\hat{z}|}$

$= \frac{22(3) + 8(-4) + 18(-12)}{\sqrt{9 + 16 + 144}} = \frac{-182}{13} = -14$

Now, magnitude of proection = 14.

Question174

If $|\vec{c}|^2 = 60$ and $\vec{c} \times (\hat{i} + 2\hat{j} + 5\hat{k}) = \vec{0}$, then a value of $c.(-7\hat{i} + 2\hat{j} + 3\hat{k})$ is [Online April 11, 2014]

Options:

A. $4\sqrt{2}$

B. 12

C. 24

D. $12\sqrt{2}$

Answer: D

Solution:

Let, $\vec{c} = a\hat{i} + b\hat{j} + c\hat{k}$

Given, $\vec{c} \times (\hat{i} + 2\hat{j} + 5\hat{k}) = \vec{0}$

$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ 1 & 2 & 5 \end{vmatrix} = \vec{0}$

$\Rightarrow (5b - 2c)\hat{i} - (5a - c)\hat{j} + (2a - b)\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

Comparingboth sides, we get

$5b - 2c = 0; 5a - c = 0; 2a - b = 0$

or $5b = 2c; 5a = c; 2a = b$

Also given $|\vec{c}|^2 = 60 \Rightarrow a^2 + b^2 + c^2 = 60$

Putting the value of b and c in above eqn., we get

$a^2 + (2a)^2 + (5a)^2 = 60$

$\Rightarrow a^2 + 4a^2 + 25a^2 = 60$ $ \⇒ 30 a^{2}=60

$$a^{2}=2$

$a = \pm\sqrt{2}; b = 2\sqrt{2}; c = 5\sqrt{2}$

Now, $\vec{c} = a\hat{i} + b\hat{j} + c\hat{k}$

$\therefore \vec{c} = \sqrt{2}\hat{i} + 2\sqrt{2}\hat{j} + 5\sqrt{2}\hat{k}$

Value of $\vec{c}\cdot\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$ is
$\left(\sqrt{2}\hat{i}+2\sqrt{2}\hat{j}+5\sqrt{2}\hat{k}\right)\cdot\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$
$=-7\sqrt{2}+4\sqrt{2}+15\sqrt{2}=12\sqrt{2}$

Question175

If the vectors $\vec{AB}=3\hat{i}+4\hat{k}$ and $\vec{AC}=5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is [2013]

Options:

A. $\sqrt{18}$

B. $\sqrt{72}$

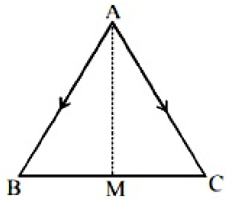
C. $\sqrt{33}$

D. $\sqrt{45}$

Answer: C

Solution:

Solution:
(c) We have, $\vec{AB}+\vec{BC}+\vec{CA}=0\Rightarrow\vec{BC}=\vec{AC}-\vec{AB}$
Let M be mid-point of BC
Now, $\vec{BM}=\frac{\vec{AC}-\vec{AB}}{2}\left(\text{because}\,\vec{BM}=\frac{\vec{BC}}{2}\right)$



Also, we have
$\vec{AB}+\vec{BM}+\vec{MA}=0$
$\Rightarrow\vec{AB}+\frac{\vec{AC}-\vec{AB}}{2}=\vec{AM}$
$\Rightarrow1\vec{AM}=\frac{\vec{AB}+\vec{AC}}{2}=4\hat{i}-\hat{j}+4\hat{k}$
$\Rightarrow\left|\vec{AM}\right|=\sqrt{33}$

Question176

If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, then the value of α for which the vectors $\vec{u}=(\alpha-2)\vec{a}+\vec{b}$ and $\vec{v}=(2+3\alpha)\vec{a}-3\vec{b}$ are collinear is: [Online April 23, 2013]

Options:

A. $\frac{3}{2}$

B. $\frac{2}{3}$

C. $-\frac{3}{2}$

D. $-\frac{2}{3}$

Answer: B

Solution:

Solution:

Since, $\vec{u}$ and $\vec{v}$ are collinear, therefore $\overrightarrow{ku}+\vec{v}=0$

$\Rightarrow[k(\alpha-2)+2+3\alpha]\vec{a}+(k-3)\vec{b}=0$ ....(i)

Since $\vec{a}$ and $\vec{b}$ are non-collinear, then for some constant m and n,

$m\vec{a}+n\vec{b}=0 \Rightarrow m=0, n=0$

Hence from equation (i)

$k-3=0 \Rightarrow k=3$

And $k(\alpha-2)+2+3\alpha=0$

$\Rightarrow 3(\alpha-2)+2+3\alpha=0 \Rightarrow \alpha=\frac{2}{3}$

Question177

If $\hat{a}$, $\hat{b}$ and $\hat{c}$ are unit vectors satisfying $\hat{a}-\sqrt{3}\hat{b}+\hat{c}=\vec{0}$,then the angle between the vectors $\hat{a}$ and $\hat{c}$ is : [Online April 22, 2013]

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: B

Solution:

Solution:

Let angle between $\hat{a}$ and $\hat{c}$ be θ.

Now, $\hat{a}-\sqrt{3}\hat{b}+\hat{c}=\vec{0}$

$\Rightarrow(\hat{a}+\hat{c})=\sqrt{3}\hat{b}$

$\Rightarrow(\hat{a}+\hat{c}).(\hat{a}+\hat{c})=3(\hat{b}.\hat{b})$

$\Rightarrow\hat{a}.\hat{a}+\hat{a}.\hat{c}+\hat{c}.\hat{a}+\hat{c}.\hat{c}=3\times 1$

$\Rightarrow 1+2\cos\theta+1=3$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

Question178

Let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector of the type $\vec{b} + \lambda\vec{c}$ for some scalar λ,whose projection on $\vec{a}$ is of magnitude $\sqrt{\frac{2}{3}}$ is :

[Online April 9, 2013]

Options:

A. $2\hat{i} + \hat{j} + 5\hat{k}$

B. $2\hat{i} + 3\hat{j} - 3\hat{k}$

C. $2\hat{i} - \hat{j} + 5\hat{k}$

D. $2\hat{i} + 3\hat{j} + 3\hat{k}$

Answer: B

Solution:

Solution:

Let $\vec{d} = \vec{b} + \lambda\vec{c}$

$\therefore \vec{d} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + \hat{j} - 2\hat{k})$

$= (1 + \lambda)\hat{i} + (2 + \lambda)\hat{j} - (1 + 2\lambda)\hat{k}$

If θ be the angle between $\vec{d}$ and $\vec{a}$, then projection of $\vec{d}$ or $(\vec{b} + \lambda\vec{c})$ on $\vec{a}$

$= |\vec{d}|\cos\theta = |\vec{d}| \left| \left(\frac{\vec{d}\,.\,\vec{a}}{|\vec{d}||\vec{a}|} \right) \right| = \frac{\vec{d}\,.\,\vec{a}}{|\vec{a}|}$

$= \frac{2(\lambda + 1) - (\lambda + 2) - (2\lambda + 1)}{\sqrt{4 + 1 + 1}} = \frac{-\lambda - 1}{\sqrt{6}}$

But projection of $\vec{d}$ on $\vec{a} = \sqrt{\frac{2}{3}}$

$\therefore -\frac{\lambda + 1}{\sqrt{6}} = \sqrt{\frac{2}{3}} \Rightarrow \frac{\lambda^2 + 2\lambda + 1}{6} = \frac{2}{3}$

$\Rightarrow \lambda^2 + 2\lambda - 3 = 0 \Rightarrow \lambda^2 + 3\lambda - \lambda - 3 = 0$

$\Rightarrow \lambda(\lambda + 3) - 1(\lambda + 3) = 0, \Rightarrow \lambda = 1, -3$

when $\lambda = 1$, then $\vec{b} + \lambda\vec{c} = 2\hat{i} + 3\hat{j} - 3\hat{k}$

when $\lambda = -3$, then $\vec{b} + \lambda\vec{c} = -2\hat{i} - \hat{j} + 5\hat{k}$

Question179

Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a} \bullet \vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 30°, then $\left|(\vec{a} \times \vec{b}) \times \vec{c}\right|$ equals:

[Online April 25, 2013]

Options:

A. $\frac{1}{2}$

B. $\frac{3\sqrt{3}}{2}$

C. 3

D. $\frac{3}{2}$

Answer: D

Solution:

Solution:

$\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = \hat{i} + \hat{j}$

$\Rightarrow |\vec{a}| = 3$

and $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{4+4+1} = 3$

Now, $|\vec{c} - \vec{a}| = 2\sqrt{2} \Rightarrow |\vec{c} - \vec{a}|^2 = 8$

$\Rightarrow |\vec{c} - \vec{a}|.(\vec{c} - \vec{a}) = 8$

$\Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c}\,.\,\vec{a} = 8$

$\Rightarrow |\vec{c}|^2 + 9 - 2|\vec{c}| = 8$

$\Rightarrow (|\vec{c}| - 1)^2 = 0 \Rightarrow |\vec{c}| = 1$

$\therefore |(\vec{a} \times \vec{b}) \times \vec{c}| = |\vec{a} \times \vec{b}||\vec{c}| \sin 30° = 3 \times 1 \times \frac{1}{2} = \frac{3}{2}$

Question180

The vector $(\hat{i} \times \vec{a}\vec{b})\hat{i} + (\hat{j} \times \vec{a}\,.\,\vec{b})\hat{j} + (\hat{k} \times \vec{a}\,.\,\vec{b})\hat{k}$ is equal to :
[Online April 9, 2013]

Options:

A. $\vec{b} \times \vec{a}$

B. $\vec{a}$

C. $\vec{a} \times \vec{b}$

D. $\vec{b}$

Answer: C

Solution:

$\left(\hat{i} \times \vec{a}\vec{b}\right)\hat{i} + \left(\hat{j} \times \vec{a} . \vec{b}\right)\hat{j} + \left(\hat{k} \times \vec{a} . \vec{b}\right)\hat{k}$
$\left(\hat{i} . \vec{a} \times \vec{b}\right)\hat{i} + \left(\hat{j} . \vec{a} \times \vec{b}\right)\hat{j} + \left(\hat{k} . \vec{a} \times \vec{b}\right)\hat{k}$ $\left(\because \vec{a} \times \vec{b} . \vec{c} = \vec{a} . \vec{b} \times \vec{c}\right)$
$\left(\vec{a} \times \vec{b}\right)\hat{i} + \left(\vec{a} \times \vec{b}\right)\hat{j} + \left(\vec{a} \times \vec{b}\right)\hat{k} = \vec{a} \times \vec{b}$

Question181

Statement 1: If the points (1, 2, 2), (2, 1, 2) and (2, 2, z) and (1, 1, 1) are coplanar, then z = 2.
Statement 2: If the 4 points P, Q, R and S are coplanar, then the volume of the tetrahedron PQRS is 0.
[Online May 12, 2012]

Options:

A. Statement 1 is false,, Statement 2 is true.

B. Statement 1 is true, Statement 2 is false.

C. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.

D. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

Answer: A

Solution:

Statement - 1
Points (1, 2, 2), (2, 1, 2), (2, 2, z) and (1, 1, 1) are coplanar then z = 2 which is false.

$$\because \begin{vmatrix} 1 & -1 & 0 \\ 1 & 0 & z-2 \\ 0 & -1 & -1 \end{vmatrix} = 0$$

$\Rightarrow 1(z-2) + 1(-1) = 0 \Rightarrow z = 3$
Statement -2 is the true statement.

Question182

If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = r\hat{i} + \hat{j} + (2r-1)\hat{k}$ are three vectors such that $\vec{c}$ is parallel to the plane of $\vec{a}$ and $\vec{b}$, then r is equal to
[Online May 19, 2012]

Options:

A. 1

B. -1

C. 0

D. 2

Answer: C

Solution:

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, and $\vec{c} = r\hat{i} + \hat{j} + (2r-1)\hat{k}$
Since, $\vec{c}$ is parallel to the plane of $\vec{a}$ and $\vec{b}$ therefore,
$\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.

$$\therefore \begin{vmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ r & 1 & 2r-1 \end{vmatrix} = 0$$

$\Rightarrow 1(6r - 3 + 1) + 2(4r - 2 + r) + 3(2 - 3r) = 0$
$\Rightarrow 6r - 2 + 10r - 4 + 6 - 9r = 0$
$\Rightarrow r = 0$

Question183

Let ABCD be a parallelogram such that $\vec{AB} = \vec{q}$, $\vec{AD} = \vec{p}$and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincide with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by : [2012]

Options:

A. $\vec{r} = 3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

B. $\vec{r} = -\vec{q} + \frac{(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

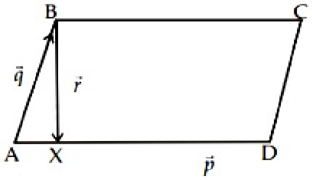
C. $\vec{r} = \vec{q} - \frac{(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

D. $\vec{r} = -3\vec{q} - \frac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

Answer: B

Solution:

Solution:
Let ABCD be a parallelogram such that $\overline{AB} = \vec{q}$, $\overline{AD} = \vec{p}$ and $\angle BAD$ be an acute angle.
We have



$$\overline{AX} = \left(\frac{\vec{p}\cdot\vec{q}}{|\vec{p}|}\right)\left(\frac{\vec{p}}{|\vec{p}|}\right) = \frac{\vec{p}\cdot\vec{q}}{|\vec{p}|^2}\vec{p}$$

From triangle law

Let $\vec{r} = \overline{BX} = \overline{BA} + \overline{AX} = -\vec{q} + \frac{\vec{p}\cdot\vec{q}}{|\vec{p}|^2}\vec{p}$

Question184

Let $\vec{a}$ and $\vec{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other,then the angle between $\hat{a}$ and $\hat{b}$ is:
[2012]

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

Solution:
Given that $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ and $|\hat{a}| = |\hat{b}| = 1$
Since $\vec{c}$ and $\vec{d}$ are perpendicular to each other
$\therefore \vec{c} \cdot \vec{d} = 0$
$\Rightarrow (\hat{a} + 2\hat{b}) \cdot (5\hat{a} - 4\hat{b}) = 0$
$\Rightarrow 5 + 6\hat{a} \cdot \hat{b} - 8 = 0$
$\Rightarrow \hat{a} \cdot \hat{b} = \frac{1}{2} \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

Question185

If $a + b + c = 0$, $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$, then the angle between $\vec{a}$ and $\vec{b}$ is
[Online May 19, 2012]

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. $\frac{\pi}{2}$

Answer: A

Solution:

Solution:
Let $a + b + c = 0 \Rightarrow (a + b) = -c$
$\Rightarrow (a + b)^2 = c^2$
$\Rightarrow a^2 + b^2 + 2a \cdot b = c^2$
$\Rightarrow 9 + 25 + 2.3.5 \cos\theta = 49$
$\left(\because |\vec{a}| = 3, |\vec{b}| = 5 \text{ and } |\vec{c}| = 7\right)$
$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

Question186

A unit vector which is perpendicular to the vector $2\hat{i} - \hat{j} + 2\hat{k}$ and is coplanar with the vectors $\hat{i} + \hat{j} - \hat{k}$ and $2\hat{i} + 2\hat{j} - \hat{k}$ is [Online May 12, 2012]

Options:

A. $\frac{2\hat{j} + \hat{k}}{\sqrt{5}}$

B. $\frac{3\hat{i} + 2\hat{j} - 2\hat{k}}{\sqrt{17}}$

C. $\frac{3\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{17}}$

D. $\frac{2\hat{i} + 2\hat{j} - 1\hat{k}}{3}$

Answer: D

Solution:

Let $x\hat{i} + y\hat{j} + z\hat{k}$ be the required unit vector.
Since $\hat{a}$ is perpendicular to $\left(2\hat{i} - \hat{j} + 2\hat{k}\right)$.
$\therefore 2x - y + 2z = 0$ ....(i)
Since vector $x\hat{i} + y\hat{j} + z\hat{k}$ is coplanar with the vector $\hat{i} + \hat{j} - \hat{k}$ and $2\hat{i} + 2\hat{j} - \hat{k}$
$\therefore x\hat{i} + y\hat{j} + z\hat{k} = p\left(\hat{i} + \hat{j} - \hat{k}\right) + q\left(2\hat{i} + 2\hat{j} - \hat{k}\right)$
where p and q are some scalars.
$\Rightarrow x\hat{i} + y\hat{j} + z\hat{k} = (p + 2q)\hat{i} + (p + 2q)\hat{j} - (p + q)\hat{k}$
$\Rightarrow x = p + 2q, y = p + 2q, z = -p - q$
Now from equation (i),
$2p + 4q - p - 2q - 2p - 2q = 0$
$\Rightarrow -p = 0 \Rightarrow p = 0$
$\therefore x = 2q, y = 2q, z = -q$
Since vector $x\hat{i} + y\hat{j} + z\hat{k}$ is a unit vector, therefore
$\left|x\hat{i} + y\hat{j} + z\hat{k}\right| = 1$
$\Rightarrow \sqrt{x^2 + y^2 + z^2} = 1$
$\Rightarrow x^2 + y^2 + z^2 = 1$
$\Rightarrow 4q^2 + 4q^2 + q^2 = 1$
$\Rightarrow 9q^2 = 1 \Rightarrow q = \pm\frac{1}{3}$

When $q = \frac{1}{3}$, then $x = \frac{2}{3}, y = \frac{2}{3}, z = -\frac{1}{3}$

When $q = -\frac{1}{3}$, then $x = -\frac{2}{3}, y = -\frac{2}{3}, z = \frac{1}{3}$

Here required unit vector is $\frac{2\hat{i} + 2\hat{j} - 1\hat{k}}{3}$

or $-\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{1}{3}\hat{k}$

Question187

ABCD is parallelogram. The position vectors of A and C are respectively, $3\hat{i} + 3\hat{j} + 5\hat{k}$ and $\hat{i} - 5\hat{j} - 5\hat{k}$. If M is the midpoint of the diagonal DB, then the magnitude of the projection of $\vec{OM}$ on $\vec{OC}$, where O is the origin, is
[Online May 7, 2012]

Options:

A. $7\sqrt{51}$

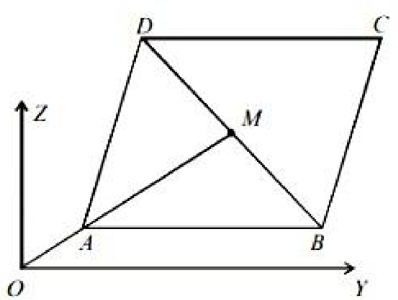
B. $\frac{7}{\sqrt{50}}$

C. $7\sqrt{50}$

D. $\frac{7}{\sqrt{51}}$

Answer: D

Solution:



In a parallelogram, diagonals bisect each other. So, mid point of DB is also the mid-point of AC.

Mid-point of M $= 2\hat{i} - \hat{j}$

Direction ratio of OC = $(1, -5, -5)$

Direction ratio of OM $= (2, -1, 0)$

Angle θ between OM and OC is given by

$$\cos\theta = \frac{(1 \times 2) + (-5)(-1) + (-5)(0)}{\sqrt{2^2 + (-1)^2}\sqrt{(1)^2 + (-5)^2 + (-5)^2}}$$

$$= \frac{2+5}{\sqrt{5}\sqrt{51}} = \frac{7}{\sqrt{5}\sqrt{51}}$$

Projection of $\vec{OM}$ on $\vec{OC}$ is given by

$$|OM| \,.\, \cos\theta = \sqrt{5} \times \frac{7}{\sqrt{5} \times \sqrt{51}} = \frac{7}{\sqrt{51}}$$

Question188

Statement 1: The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ lie in the same plane if and only if $\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = 0$
Statement 2: The vectors $\vec{u}$ and $\vec{v}$ are perpendicular if and only if $\vec{u} \cdot \vec{v} = 0$ where $\vec{u} \times \vec{v}$ is a vector perpendicular to the plane of $\vec{u}$ and $\vec{v}$.
[Online May 26, 2012]

Options:

A. Statement 1 is false, Statement 2 is true.

B. Statement 1 is true, Statement 2 is true, Statement 2 is correct explanation for Statement 1.

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is true, Statement 2 is true, , Statement 2 is not a correct explanation for Statement 1.

Answer: C

Solution:

Solution:
Statement - 1
The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ lie in the same plane.
$\Rightarrow \vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar.
We know, the necessary and sufficient conditions for three vectors to be coplanar is that $\left[\vec{a}\vec{b}\vec{c}\right] = 0$
i.e. $\vec{a} \cdot \left(\vec{b} \times \vec{c}\right) = 0$
Hence, statement- 1 is true.

Question189

If $\vec{u} = \hat{j} + 4\hat{k}$, $\vec{v} = \hat{i} + 3\hat{k}$ and $\vec{w} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$ are vectors in 3 - dimensional space, then the maximum possible value of $\left|\vec{u} \times \vec{v} \cdot \vec{w}\right|$ is
[Online May 12, 2012]

Options:

A. $\sqrt{3}$

B. 5

C. $\sqrt{14}$

D. 7

Answer: B

Solution:

Let $\vec{u} = \hat{j} + 4\hat{k}$, $\vec{v} = \hat{i} + 3\hat{k}$ and $\vec{w} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$

Now, $\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 4 \\ 1 & 0 & -3 \end{vmatrix} = \hat{i}(-3) - \hat{j}(-4) + \hat{k}(-1)$

$= -3\hat{i} + 4\hat{j} - \hat{k}$

Now, $(\vec{u} \times \vec{v}) \cdot \vec{w} = (-3\hat{i} + 4\hat{j} - \hat{k}) \cdot (\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$

$= -3\cos\theta + 4\sin\theta$

Now, maximum possible value of $|-3\cos\theta + 4\sin\theta| = \sqrt{(-3)^2 + (4)^2} = \sqrt{25} = 5$

Question190

If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + (2\lambda - 1)\hat{k}$ are coplanar vectors, then λ is equal to [Online May 7, 2012]

Options:

A. 0

B. -1

C. 2

D. 1

Answer: A

Solution:

Solution:

Since $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + (2\lambda - 1)\hat{k}$ are coplanar

therefore $[\vec{a}\,\vec{b}\,\vec{c}] = 0$

i.e., $\begin{vmatrix} 1 & 2 & \lambda \\ -2 & 3 & 1 \\ 3 & -1 & 2\lambda - 1 \end{vmatrix} = 0$

$\Rightarrow 1(6\lambda - 2) - 2(-4\lambda - 1) + \lambda(-7) = 0$

$\Rightarrow (6\lambda - 2) + 8\lambda + 2 + 2 + 2\lambda - 9\lambda = 0$

$\Rightarrow 7\lambda = 0 \Rightarrow \lambda = 0$

Question191

Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non-zero vectors which are pairwise non-collinear. If $\vec{a} + 3\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 2\vec{c}$ is collinear with $\vec{a}$, then $\vec{a} + 3\vec{b} + 6\vec{c}$ is : [2011RS]

Options:

A. $\vec{a}$

B. $\vec{c}$

C. $\vec{0}$

D. $\vec{a}+\vec{c}$

Answer: C

Solution:

As per question
$\vec{a}+3\vec{b}=\lambda\vec{c}$ ....(i)
$\vec{b}+2\vec{c}=\mu\vec{a}$ ....(ii)
On solving equations (i) and (ii)
$(1+3\mu)\vec{a}-(\lambda+6)\vec{c}=0$
As $\vec{a}$ and $\vec{c}$ are non collinear,
$\therefore 1+3\mu=0$ and $\lambda+6=0$
From(i) $\vec{a}+3\vec{b}+6\vec{c}=\vec{0}$

Question192

If the $p\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+q\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+r\hat{k}$ $(p\neq q\neq r\neq 1)$ vector are coplanar, then the value of $pqr-(p+q+r)$ is [2011RS]

Options:

A. 2

B. 0

C. -1

D. -2

Answer: D

Solution:

Solution:
The given vectors are coplanar then

$$\begin{vmatrix} p & 1 & 1 \\ 1 & q & 1 \\ 1 & 1 & r \end{vmatrix}=0$$

$\Rightarrow p(qr-1)+1(1-r)+1(1-q)=0$
$\Rightarrow pqr-p+1-r+1-q=0$
$\Rightarrow pqr-(p+q+r)=-2$

Question193

The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors

satisfying $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 0$ Then the vector $\vec{d}$ is equal to
[2011]

Options:

A. $\vec{c} + \left(\frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} \right) \vec{b}$

B. $\vec{b} + \left(\frac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} \right) \vec{c}$

C. $\vec{c} - \left(\frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} \right) \vec{b}$

D. $\vec{b} - \left(\frac{\vec{b} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} \right) \vec{c}$

Answer: C

Solution:

Solution:

Given that $\vec{a} \cdot \vec{d} \neq 0$, $\vec{a} \cdot \vec{d} = 0$

Now, $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$

$\Rightarrow \vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{a} \times \left(\vec{b} \times \vec{d}\right)$

$\Rightarrow \left(\vec{a} \cdot \vec{c}\right)\vec{b} - \left(\vec{a} \cdot \vec{b}\right)\vec{c} = \left(a \cdot \vec{d}\right)\vec{b} - \left(\vec{a} \cdot \vec{b}\right)\vec{d}$

$\Rightarrow \left(\vec{a} \cdot \vec{b}\right)\vec{d} = -\left(\vec{a} \cdot \vec{c}\right)\vec{b} + \left(\vec{a}\vec{b}\right)\vec{c}$

$\vec{d} = \vec{c} - \left(\frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} \right) \vec{b}$

Question194

If $\vec{a} = \frac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and $\vec{b} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $\left(2\vec{a} - \vec{b}\right)\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} + 2\vec{b}\right)\right]$ is
[2011]

Options:

A. -3

B. 5

C. 3

D. -5

Answer: D

Solution:

Solution:
$\left(2\vec{a}-\vec{b}\right)\left(\left(\vec{a}\times\vec{b}\right)\times\left(\vec{a}+2\vec{b}\right)\right)$
$=\left(2\vec{a}-\vec{b}\right).\left(\left(\vec{a}\times\vec{b}\right)\times\vec{a}+2\left(\vec{a}\times\vec{b}\right)\times\vec{b}\right)$
$=\left(2\vec{a}-\vec{b}\right)\left(\left(\vec{a}.a\right)\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{a}+2\left(\vec{a}.\vec{b}\right)\vec{b}-2\left(\vec{b}.b\right)\vec{a}\right)$
$=\left(2\vec{a}-\vec{b}\right)\left(\vec{b}-0+0-2\vec{a}\right)$
From given values we get
$\vec{a}.\vec{b}=0$ and $\vec{b}.\vec{b}=1$
$=-4\vec{a}.\vec{a}-\vec{b}.\vec{b}=-5$

Question195

If the vectors $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}+\hat{k}$ and $\vec{c}=\lambda\hat{i}+\hat{j}+\mu\hat{k}$ are mutually orthogonal, then $(\lambda,\mu)=$ [2010]

Options:

A. (2,-3)

B. (-2,3)

C. (3,-2)

D. (-3,2)

Answer: D

Solution:

Solution:
Given that, $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually orthogonal
$\therefore \vec{a}.\vec{b}=0,\ \vec{b}.\vec{c}=0,\ \vec{c}.\vec{a}=0$
$\Rightarrow 2\lambda+4+\mu=0$ ......(i)
$\lambda-1+2\mu=0$ ....(ii)
On solving (i) and (ii), we get $\lambda=-3,\ \mu=2$

Question196

Let $\vec{a}=\hat{j}-\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a}\times\vec{b}+\vec{c}=\vec{0}$ and $\vec{a}.\vec{b}=3$ is [2010]

Options:

A. $2\hat{i}-\hat{j}+2\hat{k}$

B. $\hat{i}-\hat{j}-2\hat{k}$

C. $\hat{i}+\hat{j}-2\hat{k}$

D. $-\hat{i}+\hat{j}-2\hat{k}$

Answer: D

Solution:

Solution:
Given that
$\vec{c}=\vec{b}\times\vec{a}\Rightarrow\vec{b}.\vec{c}=\vec{b}.\left(\vec{b}\times\vec{a}\right)\Rightarrow\vec{b}.\vec{c}=0$
$\Rightarrow\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right).\left(\hat{i}-\hat{j}-\hat{k}\right)=0$
where $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$
$b_1-b_2-b_3=0$ ......(i)
and $\vec{a}.\vec{b}=3\Rightarrow\left(\hat{j}-\hat{k}\right).\left(b_1\hat{i}+b_2\hat{j}+b_3\hat{k}\right)=3$
$\Rightarrow b_2-b_3=3$
From equation (i)
$b_1=b_2+b_3=(3+b_3)+b_3=3+2b_3$
$\vec{b}=(3+2b_3)\hat{i}+(3+b_3)\hat{j}+b_3\hat{k}$
From the option given, it is clear that b_3 equal to either 2 or -2
If $b_3=2$ then $\vec{b}=7\hat{i}+5\hat{j}+2\hat{k}$ which is not possible If
$b_3=-2$, then $\vec{b}=-\hat{i}+\hat{j}-2\hat{k}$

Question197

If $\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors and p, q are real numbers, then the equality $\left[3\vec{u}\,p\vec{v}\,p\vec{\omega}\right]-\left[p\vec{v}\vec{\omega}\,q\vec{u}\right]-\left[2\vec{\omega}\,q\vec{v}\,q\vec{u}\right]=0$holds for : [2009]

Options:

A. exactly two values of (p, q)

B. more than two but not all values of (p, q)

C. all values of (p, q)

D. exactly one value of (p, q)

Answer: D

Solution:

Solution:
$\vec{u}, \vec{v}, \vec{w}$ are non-coplanar vectors
$\therefore\left[\vec{u}, \vec{v}, \vec{w}\right]\neq 0$
Now, $\left[3\vec{u}, p\vec{v}, p\vec{w}\right]-\left[p\vec{v}, p\vec{w}, q\vec{u}\right]-\left[2\vec{w}, q\vec{v}, q\vec{u}\right]=0$
$\Rightarrow 3p^2\left[\vec{u}, \vec{v}, \vec{w}\right]-pq\left[\vec{v}, \vec{w}, \vec{u}\right]-2q^2\left[\vec{w}, \vec{v}, \vec{u}\right]=0$
$\Rightarrow 3p^2\left[\vec{u}, \vec{v}, \vec{w}\right]-pq\left[\vec{u}, \vec{v}, \vec{w}\right]-2q^2\left[\vec{u}, \vec{v}, \vec{w}\right]=0$
$\Rightarrow(3p^2-pq+2q^2)\left[\vec{u}, \vec{v}, \vec{w}\right]=0$
$\Rightarrow 3p^2-pq+2q^2=0$
$\left(\because\left[\vec{u}, \vec{v}, \vec{w}\right]=0\right)$
$\Rightarrow 2p^2+p^2-pq+\frac{q^2}{4}+\frac{7q^2}{4}=0$

$\Rightarrow 2p^2 + \left(p - \frac{q}{2}\right)^2 + \frac{7}{4}q^2 = 0$
$\Rightarrow p = 0, q = 0, p = q / 2$
This is possible only when $p = 0, q = 0$
$\therefore$ There is exactly one value of (p, q)

Question198

The vector $\vec{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}$ lies in the plane of the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{j} + \hat{k}$ and bisects the angle between $\vec{b}$ and $\vec{c}$. Then which one of the following gives possible values of α and β ? [2008]

Options:

A. $\alpha = 2, \beta = 2$

B. $\alpha = 1, \beta = 2$

C. $\alpha = 2, \beta = 1$

D. $\alpha = 1, \beta = 1$

Answer: D

Solution:

Solution:
$\because \vec{a}$ lies in the plane of $\vec{b}$ and $\vec{c}$
$\therefore \vec{a} = \vec{b} + \lambda\vec{c}$
$\Rightarrow \alpha\hat{i} + 2\hat{j} + \beta\hat{k} = \hat{i} + \hat{j} + \lambda\left(\hat{j} + \hat{k}\right)$
$\Rightarrow \alpha = 1, 2 = 1 + \lambda, \beta = \lambda$
$\Rightarrow \alpha = 1, \beta = 1$

Question199

The non-zero vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{a} = 8\vec{b}$and $\vec{c} = -7\vec{b}$. Then the angle between $\vec{a}$ and $\vec{c}$ is [2008]

Options:

A. 0

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. pi

Answer: D

Solution:

Solution:
Clearly $\vec{a} = -\frac{8}{7}\vec{c}$
$\Rightarrow \vec{a} \parallel \vec{c}$ and are opposite in direction
$\therefore$ Angle between $\vec{a}$ and $\vec{c}$ is π

Question200

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals [2007]

Options:

A. -4

B. -2

C. 0

D. 1 .

Answer: B

Solution:

Solution:
Given $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$.
Given that $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar
$\therefore \left[\vec{a}\vec{b}\vec{c}\right] = 0$
i.e. $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & (x-2) & -1 \end{vmatrix} = 0$
$\Rightarrow 1[1 - 2(x-2)] - 1[-1 - 2x] + 1[x - 2 + x] = 0$
$\Rightarrow 1 - 2x + 4 + 1 + 2x + 2x - 2 = 0$
$\Rightarrow 2x = -4 \Rightarrow x = -2$

Question201

If $\hat{u}$ and $\hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u} \times 3\hat{v}$ is a unit vector for [2007]

Options:

A. no value of θ

B. exactly one value of θ

C. exactly two values of θ

D. more than two values of θ

Answer: B

Solution:

Solution:
Given that $|2\hat{u} \times 3\hat{v}| = 1$ and θ is acute angle between $\hat{u}$ and $\hat{v}, |\hat{u}| = 1, |\hat{v}| = 1$
$\Rightarrow |2\hat{u} \times 3\hat{v}| = 6|\hat{u}||\hat{v}||\sin\theta| = 1$
$\Rightarrow 6 \mid \sin\theta \Big| = 1 \Rightarrow \sin\theta = \frac{1}{6}$
Hence, there is exactly one value of θ for which $2\hat{u} \times 3\hat{v}$ is a unit vector.

Question202

A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. The angle of projection is
[2007]

Options:

A. $\tan^{-1}\frac{bc}{a(c-a)}$

B. $\tan^{-1}\frac{bc}{a}$

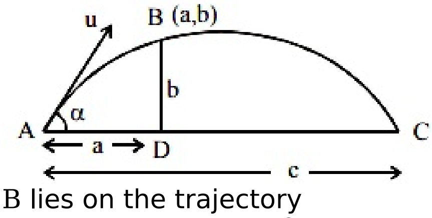
C. $\tan^{-1}\frac{b}{ac}$

D. 45°.

Answer: A

Solution:

Let B be the top of the wall whose coordinates will be (a, b). Range (R) = c



B lies on the trajectory

$\therefore y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$

$\Rightarrow b = a\tan\alpha - \frac{1}{2}g\frac{a^2}{u^2\cos^2\alpha}$

$\Rightarrow b = a\tan\alpha\left[1 - \frac{ga}{2u^2\cos^2\alpha\tan\alpha}\right]$

$= a\tan\alpha\left[1 - \frac{a}{\frac{2u^2}{g}\cos^2\alpha \cdot \frac{\sin\alpha}{\cos\alpha}}\right]$

$$= a\tan\alpha\left[1 - \frac{a}{\frac{u^2 . 2\sin\alpha\cos\alpha}{g}}\right]$$

$$= a\tan\alpha\left[1 - \frac{a}{\frac{u^2\sin 2\alpha}{g}}\right]$$

$$= a\tan\alpha\left[1 - \frac{a}{R}\right] \quad \left(\because R = \frac{u^2\sin^2\alpha}{g}\right)$$

$$\Rightarrow b = a\tan\alpha\left[1 - \frac{a}{c}\right]$$

$$\Rightarrow b = a\tan\alpha . \left(\frac{c-a}{c}\right)$$

$$\Rightarrow \tan\alpha = \frac{bc}{a(c-a)}$$

The angle of projection,

$$\alpha = \tan^{-1}\frac{bc}{a(c-a)}$$

Question203

A body weighing 13 kg is suspended by two strings 5m and 12m long, their other ends being fastened to the extremities of a rod 13m long. If the rod be so held that the body hangs immediately below the middle point, then tensions in the strings are [2007]

Options:

A. 5 kg and 12 kg

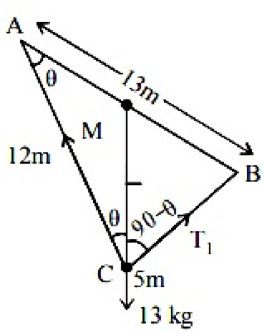
B. 5 kg and 13 kg

C. 12 kg and 13 kg

D. 5 kg and 5 k

Answer: A

Solution:



In ΔABC

$\because 13^2 = 5^2 + 12^2 \Rightarrow AB^2 = AC^2 + BC^2$

$\Rightarrow \angle ACB = 90°$

M is mid point of the hypotenuse AB, therefore $MA = MB = MC$

$\Rightarrow \angle A = \angle ACM = \theta$

Applying Lami's theorem at C, we get

$$\frac{T_1}{\sin(180-\theta)} = \frac{T_2}{\sin(90+\theta)} = \frac{13kg}{\sin 90°}$$

$\Rightarrow T_1 = 13\sin\theta$ and $T_2 = 13\cos\theta$

$\Rightarrow T_1 = 13 \times \frac{5}{13}$ and $T_2 = 13 \times \frac{12}{13}$
$\Rightarrow T_1 = 5kg$ and $T_2 = 12kg$

Question204

The resultant of two forces Pn and 3n is a force of 7n. If thedirection of 3n force were reversed, the resultant would be $\sqrt{19}$ n. The value of P is [2007]

Options:

A. 3 n

B. 4 n

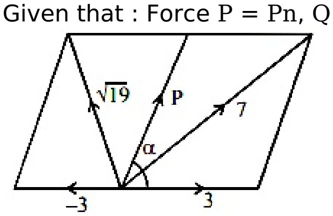
C. 5 n

D. 6 n.

Answer: C

Solution:

Solution:
Given that : Force P = Pn, Q = 3n, resultant R = 7n & P′ = Pn, Q′ = (−3)n, R′ = $\sqrt{19}$n



We know that $R^2 = P^2 + Q^2 + 2PQ\cos\alpha$
$\Rightarrow (7)^2 = P^2 + (3)^2 + 2 \times P \times 3\cos\alpha$
$\Rightarrow 49 = P^2 + 9 + 6P\cos\alpha$
$\Rightarrow 40 = P^2 + 6P\cos\alpha$ .....(i)
and $(\sqrt{19})^2 = P^2 + (-3)^2 + 2P \times -3\cos\alpha$
$\Rightarrow 19 = P^2 + 9 - 6P\cos\alpha$
$\Rightarrow 10 = P^2 - 6P\cos\alpha$ .....(ii)
Adding (i) and(ii) $50 = 2P^2$
$\Rightarrow P^2 = 25 \Rightarrow P = 5n$

Question205

ABC is a triangle, right angled at A. The resultant of the forces acting along $\overline{AB}$, $\overline{BC}$ with magnitudes $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively is the force along $\overline{AD}$, where D is the foot of the perpendicular from A onto BC. The magnitude of the resultant is [2006]

Options:

A. $\frac{AB^2 + AC^2}{(AB)^2(AC)^2}$

B. $\frac{(AB)(AC)}{AB + AC}$

C. $\frac{1}{AB} + \frac{1}{AC}$

D. $\frac{1}{AD}$

Answer: D

Solution:

Solution:

If we consider unit vectors $\hat{i}$ and $\hat{j}$ in the direction AB and AC respectively and its magnitude $\frac{1}{AB}$ and $\frac{1}{AC}$ respectively, then as per question, forces along AB and AC respectively are

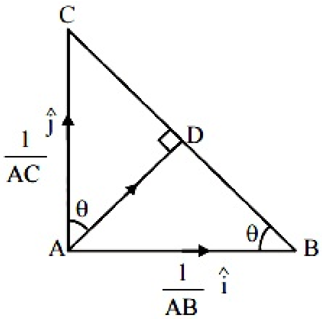
$\left(\frac{1}{AB}\right)\hat{i}$ and $\left(\frac{1}{AC}\right)\hat{j}$

∴ Their resultant along AD = $\left(\frac{1}{AB}\right)\hat{i} + \left(\frac{1}{AC}\right)\hat{j}$

∴ Magnitude of resultant is

$= \sqrt{\left(\frac{1}{AB}\right)^2 + \left(\frac{1}{AC}\right)^2} = \sqrt{\frac{AC^2 + AB^2}{AB^2 + AC^2}}$ $[\because AC^2 + AB^2 = BC^2]$

$= \frac{BC}{AB \, . \, AC}$



$\because \Delta ABC \sim \Delta DBA$

$\Rightarrow \frac{BC}{AB} = \frac{AC}{AD} \Rightarrow \frac{BC}{AB \times AC} = \frac{1}{AD}$

∴ The required magnitude of resultant becomes $\frac{1}{AD}$

Question206

The values of a, for which points A, B, C with position vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $a\hat{i} - 3\hat{j} + \hat{k}$ respectively are the vertices of a right angled triangle with C = $\frac{\pi}{2}$ are [2006]

Options:

A. 2 and 1

B. -2 and -1

C. -2 and 1

D. 2 and -1

Answer: A

Solution:

Solution:
$\overrightarrow{CA} = (2-a)\hat{i} + 2\hat{j}$; $\overrightarrow{CB} = (1-a)\hat{i} - 6\hat{k}$ $\left[\because \overrightarrow{CA} \perp \overrightarrow{CB}\right]$
$\therefore \overrightarrow{CA} \cdot \overrightarrow{CB} = 0 \Rightarrow (2-a)(1-a) = 0$
$\Rightarrow a = 2, 1$

Question207

If $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \vec{a} \times \left(\vec{b} \times \vec{c}\right)$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three vectors such that $\vec{a}\vec{b} \neq 0$, $\vec{b} \cdot \vec{c} \neq 0$ then $\vec{a}$ and $\vec{c}$ are
[2006]

Options:

A. inclined at an angle of $\frac{\pi}{3}$ between them

B. inclined at an angle of $\frac{\pi}{6}$ between them

C. perpendicular

D. parallel

Answer: D

Solution:

Solution:
$(\bar{a} \times \bar{b}) \times \bar{c} = \bar{a} \times (\bar{b} \times \bar{c})$, $\overline{ab} \neq 0$, $\bar{b} \cdot \bar{c} \neq 0$
$\Rightarrow (\bar{a} \cdot \bar{c}) \cdot \bar{b} - (\bar{b} \cdot \bar{c})\bar{a} = (\bar{a} \cdot \bar{c}) \cdot \bar{b} - (\bar{a} \cdot \bar{b}) \cdot \bar{c}$
$\Rightarrow (\bar{a} \cdot \bar{b}) \cdot \bar{c} = (\bar{b} \cdot \bar{c})\bar{a} \Rightarrow \bar{a} \parallel \bar{c}$

Question208

A body falling from rest under gravity passes a certain point P. It was at a distance of 400 m from P, 4s prior to passing through P. If $g = 10m / s^2$, then the height above the point P from where the body began to fall is
[2006]

Options:

A. 720 m

B. 900 m

C. 320 m

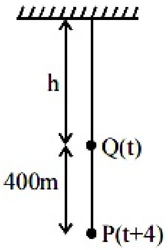
D. 680 m

Answer: A

Solution:

Solution:

We know that $h = \frac{1}{2}gt^2$ and $h + 400 = \frac{1}{2}g(t+4)^2$



Subtracting, we get $400 = 8g + 4gt$

$\Rightarrow t = 8sec$

$\therefore h = \frac{1}{2} \times 10 \times 64 = 320m$

$\therefore$ Required height $= 320 + 400 = 720m$

Question209

A particle has two velocities of equal magnitude inclined to each other at an angle θ . If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Then θ is [2006]

Options:

A. 90°

B. 120°

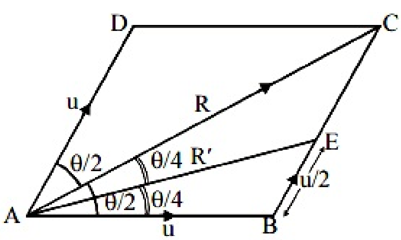
C. 45°

D. 60°

Answer: B

Solution:

Let two velocities u and u at an angle θ to each other the resultant is given by



$R^2 = u^2 + u^2 + 2u^2 \cos\theta = 2u^2(1 + \cos\theta)$

$\Rightarrow R^2 = 4u^2 \cos^2\theta / 2$ or $R = 2u\cos\frac{\theta}{2}$

Now in second case, the new resultant AE (i.e., R') bisects $\angle CAB$, therefore using angle bisector theorem in ΔABC, we get

$\frac{AB}{AC} = \frac{BE}{EC} \Rightarrow \frac{u}{R} = \frac{u/2}{u/2} \Rightarrow R = u$

$\Rightarrow 2u\cos\frac{\theta}{2} = u$
$\Rightarrow \cos\frac{\theta}{2} = \frac{1}{2} = \cos 60° \Rightarrow \frac{\theta}{2} = 60°$
or $\theta = 120°$

Question210

If C is the mid point of AB and P is any point outside AB, then [2005]

Options:

A. $\overline{PA} + \overline{PB} = 2\overline{PC}$

B. $\overline{PA} + \overline{PB} = \overline{PC}$

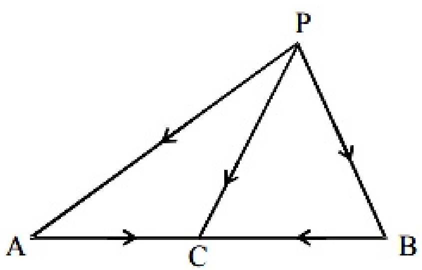
C. $\overline{PA} + \overline{PB} + 2\overline{PC} = \overline{0}$

D. $\overline{PA} + \overline{PB} + \overline{PC} = \overline{0}$

Answer: A

Solution:

$\overline{PA} + \overline{AP} = 0$ and $\overline{PC} + \overline{CP} = 0$
$\Rightarrow \overline{PA} + \overline{AC} + \overline{CP} = 0$........(i)
Similarly, $\overline{PB} + \overline{BC} + \overline{CP} = 0$ ......(ii)
Adding eqn. (i) and (ii), we get
$\overline{PA} + \overline{PB} + \overline{AC} + \overline{BC} + 2\overline{CP} = 0$
Since $\overline{AC} = -\overline{BC}$ & $\overline{CP} = -\overline{PC}$
$\Rightarrow \overline{PA} + \overline{PB} - 2\overline{PC} = 0$



Question211

Let a, b and c be distinct non- negative numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ lie in a plane,then c is [2005]

Options:

A. the Geometric Mean of a and b

B. the Arithmetic Mean of a and b

C. equal to zero

D. the Harmonic Mean of a and b

Answer: A

Solution:

Vector $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0 \Rightarrow c^2 = ab \Rightarrow c = \sqrt{ab}$$

$\therefore c$ is G.M. of a and b.

Question212

Let $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$. Then $[\vec{a}, \vec{b}, \vec{c}]$ depends on [2005]

Options:

A. only y

B. only x

C. both x and y

D. neither x nor y

Answer: D

Solution:

Solution:

Given that $\vec{a} = \hat{i} - \hat{k}$, $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$ and $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$

$$\therefore [\vec{a}\vec{b}\vec{c}] = \vec{a}\,.\,(\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$

$= 1[1 + x - y - x + x^2] - [x^2 - y]$

$= 1 - y + x^2 - x^2 + y = 1$

Hence $[\vec{a}\vec{b}\vec{c}]$ is independent of x and y both.

Question213

If $\vec{a}, \vec{b}, \vec{c}$ are non coplanar vectors and λ is a real number then $\left[\lambda(\vec{a} + \vec{b}) \;\; \lambda^2\vec{b} \;\; \lambda\vec{c}\right] = \left[\vec{a} \;\; \vec{b} + \vec{c} \;\; \vec{b}\right]$ for [2005]

Options:

A. exactly one value of λ

B. no value of λ

C. exactly three values of λ

D. exactly two values of λ

Answer: B

Solution:

Solution:

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ Given that $\left[\ \lambda(\vec{a}+\vec{b})\quad \lambda^2\vec{b}\quad \lambda\vec{c}\ \right] = \left[\ \vec{a}\quad \vec{b}+\vec{c}\quad \vec{b}\ \right]$

$$\begin{vmatrix} \lambda(a_1+b_1) & \lambda(a_2+b_2) & \lambda(a_3+b_3) \\ \lambda^2 b_1 & \lambda^2 b_2 & \lambda^2 b_3 \\ \lambda c_1 & \lambda c_2 & \lambda c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$\Rightarrow \lambda^4 \begin{vmatrix} a_1+b_1 & a_2+b_2 & a_3+b_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1+c_1 & b_2+c_2 & b_3+c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$R_1 \to R_1 - R_2$ in Ist det.

and $R_2 \to R_2 - R_3$ in 2 nd det.

$$\Rightarrow \lambda^4 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$\Rightarrow \lambda^4 = -1$

Hence λ has no real values.

Question214

For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to [2005]

Options:

A. $3\vec{a}^2$

B. $\vec{a}^2$

C. $2\vec{a}^2$

D. $4\vec{a}^2$

Answer: C

Solution:

Let $\vec{a} = x\vec{i} + y\vec{j} + z\vec{k}$
$\vec{a} \times \vec{i} = z\vec{j} - y\vec{k} \Rightarrow |\vec{a} \times i|^2 = y^2 + z^2$
Similarly, $|\vec{a} \times \vec{j}|^2 = x^2 + z^2$ and $|\vec{a} \times \vec{k}|^2 = x^2 + y^2$
Adding all above equation
$\Rightarrow (\vec{a} \times \vec{i})^2 + (\vec{a} \times \vec{j})^2 + (\vec{a} \times \vec{k})^2$
$= 2(x^2 + y^2 + z^2) = 2|\vec{a}|^2$

Question215

The resultant R of two forces acting on a particle is at right angles to one of them and its magnitude is one third of the other force. The ratio of larger force to smaller one is:
[2005]

Options:

A. 2: 1

B. $3 : \sqrt{2}$

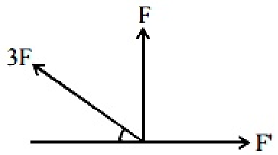
C. 3: 2

D. $3 : 2\sqrt{2}$

Answer: D

Solution:

Solution:



According to question $F' = 3F \cos\theta$ and $F = 3F \sin\theta$
$\Rightarrow F' = 2\sqrt{2}F$
$\Rightarrow F : F' : : 3 : 2\sqrt{2}$

Question216

A and B are two like parallel forces. A couple of moment H lies in the plane of A and B and is contained with them. The resultant of A and B after combining is displaced through a distance
[2005]

Options:

A. $\frac{2H}{A - B}$

B. $\frac{H}{A + B}$

C. $\frac{H}{2(A+B)}$

D. $\frac{H}{A-B}$

Answer: B

Solution:

Solution:
Let A and B be displaced by a distance x then Change in moment of $(A+B)=$ applied moments
$\Rightarrow (A+B)\times x = H \Rightarrow x = \frac{H}{A+B}$

Question217

A particle is projected from a point O with velocity u at an angle of 60° with the horizontal. When it is moving in a direction at right angles to its direction at O, its velocity then is given by [2005]

Options:

A. $\frac{u}{3}$

B. $\frac{u}{2}$

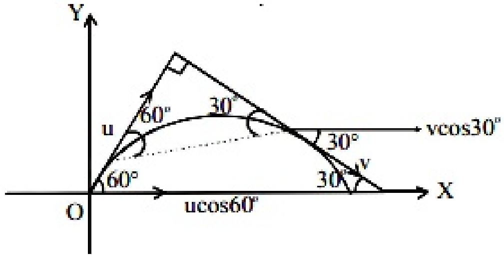
C. $\frac{2u}{3}$

D. $\frac{u}{\sqrt{3}}$

Answer: D

Solution:

As per question $u\cos 60° = v\cos 30°$
(as horizontal component of velocity remains the same)
$\Rightarrow u\,.\,\frac{1}{2} = v\,.\,\frac{\sqrt{3}}{2}$ or $v = \frac{1}{\sqrt{3}}u$



Question218

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-coplanar vectors and lambda is a real number,then the

vectors $\vec{a} + 2\vec{b} + 3\vec{c}$, $\lambda\vec{b} + 4\vec{c}$ and $(2\lambda - 1)\vec{c}$ are non coplanar for
[2004]

Options:

A. no value of λ

B. all except one value of λ

C. all except two values of λ

D. all values of λ

Answer: C

Solution:

Solution:

If vectors $\vec{a} + 2\vec{b} + 3\vec{c}$, $\lambda\vec{b} + 4\vec{c}$ and $(2\lambda - 1)\vec{c}$ are coplanar then $\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda - 1 \end{vmatrix} = 0$

$\Rightarrow \lambda(2\lambda - 1) = 0 \Rightarrow \lambda = 0$ or $\frac{1}{2}$

$\therefore$ Forces are noncoplanar for all λ, except $\lambda = 0, \frac{1}{2}$

Question219

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of these are collinear. If the vector $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$(λ being some non-zero scalar) then $\vec{a} + 2\vec{b} + 6\vec{c}$ equals
[2004]

Options:

A. 0

B. $\lambda\vec{b}$

C. $\lambda\vec{c}$

D. $\lambda\vec{a}$

Answer: C

Solution:

Given that $\vec{a} + 2\vec{b}$ is collinear with $\vec{c}$ and $\vec{b} + 3\vec{c}$ is collinear with $\vec{a}$

Let $\vec{a} + 2\vec{b} = t\vec{c}$ and $\vec{b} + 3\vec{c} = s\vec{a}$, where t and s are scalars

$\therefore \vec{a} + 2\vec{b} + 6\vec{c} = t\vec{c} + 6\vec{c}$

$= (t + 6)\vec{c}\left[\text{ using } \vec{a} + 2\vec{b} = t\vec{c}\right]$

$= \lambda\vec{c}$, where $\lambda = t + 6$

Question220

Let $\vec{u}$, $\vec{v}$, $\vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$. If the projection $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\vec{v}$, $\vec{w}$ are perpendicular to each other then $|\vec{u} - \vec{v} + \vec{w}|$ equals
[2004]

Options:

A. 14

B. $\sqrt{7}$

C. $\sqrt{14}$

D. 2

Answer: C

Solution:

Solution:

Projection of $\vec{v}$ along $\vec{u}$ = $\frac{\vec{v} \cdot \vec{u}}{|\vec{u}|} = \vec{v} \cdot \vec{u}$

projection of $\vec{w}$ along $\vec{u}$ = $\frac{\vec{w} \cdot \vec{u}}{|\vec{u}|} = \vec{w}\vec{u}$

Given $\vec{v} \cdot \vec{u} = \vec{w} \cdot \vec{u}$ ....(1)

Also , $\vec{v} \cdot \vec{w} = 0 [\because \vec{v} \perp \vec{w}]$ ....(2)

Now $|\vec{u} - \vec{v} + \vec{w}|^2$

$= |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 - 2\vec{u} \cdot \vec{v} - 2\vec{v} \cdot \vec{w} + 2\vec{u} \cdot \vec{w}$

$= 1 + 4 + 9 + 0$[From(1) and (2)] $= 14$

$\therefore |\vec{u} - \vec{v} + \vec{w}| = \sqrt{14}$

Question221

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be non-zero vectors such that $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3} |\vec{b}| |\vec{c}| \vec{a}$. If θ is the acute angle between the vectors $\vec{b}$ and $\vec{c}$, then $\sin\theta$ equals
[2004]

Options:

A. $\frac{2\sqrt{2}}{3}$

B. $\frac{\sqrt{2}}{3}$

C. $\frac{2}{3}$

D. $\frac{1}{3}$

Answer: A

Solution:

Solution:

Given that $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

Clearly $\vec{a}$ and $\vec{b}$ are non collinear

$\Rightarrow\left(\vec{a} . \vec{c}\right)\vec{b} - \left(\vec{b} . \vec{c}\right)\vec{a} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|\vec{a}$

Comparing both side.

$\therefore\vec{a} . \vec{c} = 0$ and $-\vec{b} . \vec{c} = \frac{1}{3}\left|\vec{b}\right|\left|\vec{c}\right|$

$\Rightarrow\cos\theta = \frac{-1}{3}$

$\therefore\sin\theta = \sqrt{1 - \frac{1}{9}} = \frac{2\sqrt{2}}{3}$

[θ is acute angle between $\vec{b}$ and $\vec{c}$]

Question222

If t_1 and t_2 are the times of flight of two particles having the same initial velocity u and range R on the horizontal,then $t_1^{\,2} + t_2^{\,2}$ is equal to [2004]

Options:

A. 1

B. $4u^2 / g^2$

C. $u^2 / 2g$

D. u^2 / g

Answer: B

Solution:

Solution:

For same horizontal range the angles of projectionmust be α and $\frac{\pi}{2} - \alpha$

$\therefore t_1 = \frac{2u\sin\alpha}{g}$ .... (i)

$t_2 = \frac{2u\sin\left(\frac{\pi}{2} - \alpha\right)}{g} = \frac{2u\cos\alpha}{g}$ .... (ii)

Squaring and adding eqn. (i) and (ii),

$\therefore t_1^{\,2} + t_2^{\,2} = \frac{4u^2}{g^2}$

Question223

A velocity $\frac{1}{4}$m / s is resolved into two components alongOA and OB making angles 30° and 45° respectively with the given velocity. Then the component along OB is
[2004]

Options:

A. $\frac{1}{8}(\sqrt{6}-\sqrt{2})$m / s

B. $\frac{1}{4}(\sqrt{3}-1)$m / s

C. $\frac{1}{4}$m / s

D. $\frac{1}{8}$m / s

Answer: A

Solution:

Solution:
Given v = $\frac{1}{4}$m / s, component along OB

$$= \frac{v\sin 30°}{\sin(45° + 30°)} = \frac{14 \times \frac{1}{2}}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{\sqrt{6}-\sqrt{2}}{8}$$

Question224

A paticle moves towards east from a point A to a point B at the rate of 4 km/h and then towards north from B to C at the rate of 5km/hr. If AB = 12 km and BC = 5 km, then its average speed for its journey from A to C and resultant average velocity direct from A to C are respectively
[2004]

Options:

A. $\frac{13}{9}$km / h and $\frac{17}{9}$km / h

B. $\frac{13}{4}$km / h and $\frac{17}{4}$km / h

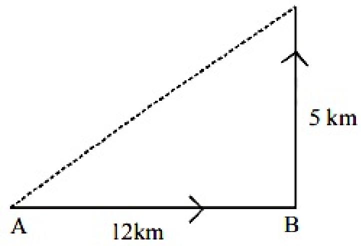
C. $\frac{17}{9}$km / h and $\frac{13}{9}$km / h

D. $\frac{17}{4}$km / h and $\frac{13}{4}$km / h

Answer: D

Solution:

Time taken by the particle in complete journey $T = \frac{12}{4} + \frac{5}{5} = 4\mathrm{hr}$



$\therefore$Average speed $= \frac{12+5}{4} = \frac{17}{4}$

Average velocity $= \sqrt{\frac{12^2+5^2}{4}} = \frac{13}{4}$

Question225

Three forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ acting along IA, IB and IC, where I is the incentre of a ΔABC are in equilibrium. Then $\vec{P} : \vec{Q} : \vec{R}$ is [2004]

Options:

A. $\operatorname{cosec}\frac{A}{2} : \operatorname{cosec}\frac{B}{2} : \operatorname{cosec}\frac{C}{2}$

B. $\sin\frac{A}{2} : \sin\frac{B}{2} : \sin\frac{C}{2}$

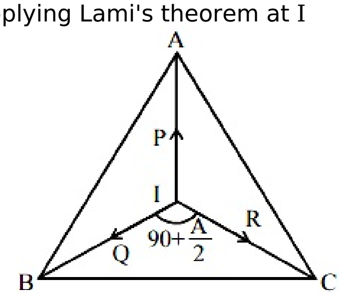
C. $\sec\frac{A}{2} : \sec\frac{B}{2} : \sec\frac{C}{2}$

D. $\cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$

Answer: D

Solution:

Solution:
Let I is incentre of ΔABC.
$\therefore IA, IB, IC$ are bisectors of the angles A, B and C.
Now $\angle BIC = 180 - \frac{B}{2} - \frac{C}{2} = 90° + \frac{A}{2}$etc
Applying Lami's theorem at I



$$\frac{P}{\sin\left(90° + \frac{A}{2}\right)} = \frac{Q}{\sin\left(90° + \frac{B}{2}\right)} = \frac{R}{\sin\left(90° + \frac{C}{2}\right)}$$

$\Rightarrow P : Q : R = \cos\frac{A}{2} : \cos\frac{B}{2} : \cos\frac{C}{2}$

Question226

In a right angle ΔABC, $\angle A = 90°$ and sides a, b, c are respectively, 5cm, 4cm and 3cm. If a force $\vec{F}$ has moments 0,9 and 16 in N cm. units respectively about vertices A, B and C, then magnitude of $\vec{F}$ is
[2004]

Options:

A. 9

B. 4

C. 5

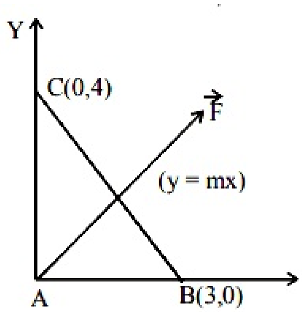
D. 3

Answer: C

Solution:

Since, the moment about A is zero, hence $\vec{F}$ passes through A. Taking A as origin. Let the line of action of force $\vec{F}$ be $y = mx$. (see figure)

Moment about B = $\frac{3m}{\sqrt{1+m^2}}\left|\vec{F}\right| = 9$ ....(1)



Moment about C = $\frac{4}{\sqrt{1+m^2}}\left|\vec{F}\right| = 16.$

Dividing (1) by (2), we get

$m = \frac{3}{4} \Rightarrow \left|\vec{F}\right| = 5N$

Question227

With two forces acting at point, the maximum affect is obtained when their resultant is 4N. If they act at right angles, then their resultant is 3N. Then the forces are
[2004]

Options:

A. $\left(2 + \frac{1}{2}\sqrt{3}\right)N$ and $\left(2 - \frac{1}{2}\sqrt{3}\right)N$

B. $(2+\sqrt{3})\mathrm{N}$ and $(2-\sqrt{3})\mathrm{N}$

C. $\left(2+\frac{1}{2}\sqrt{2}\right)\mathrm{N}$ and $\left(2-\frac{1}{2}\sqrt{2}\right)\mathrm{N}$

D. $(2+\sqrt{2})\mathrm{N}$ and $(2-\sqrt{2})\mathrm{N}$

Answer: C

Solution:

Solution:
Let forces be P and Q. then $P+Q=4$ ....(1)
and $P^2+Q^2=3^2$ ....(2)
Solving eqns. (1) and (2), we get the forces
$\left(2+\frac{1}{2}\sqrt{2}\right)\mathrm{N}$ and $\left(2-\frac{1}{2}\sqrt{2}\right)\mathrm{N}$

Question228

A particle is acted upon by constant forces $4\hat{i}+\hat{j}-3\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ which displace it from a point $\hat{i}+2\hat{j}+3\hat{k}$ to the point $5\hat{i}+4\hat{j}+\hat{k}$. The work done in standard units by the forces is given by [2004]

Options:

A. 15

B. 30

C. 25

D. 40

Answer: D

Solution:

Resultant of forces
$\vec{F}=4\hat{i}+\hat{j}-3\hat{k}+3\hat{i}+\hat{j}-\hat{k}$
Displacement
$\vec{d}=5\hat{i}+4\hat{j}+\hat{k}-\left(\hat{i}+2\hat{j}+3\hat{k}\right)=4\hat{i}+2\hat{j}-2\hat{k}$
$\therefore$ Work done $=\vec{F}\cdot\vec{d}=28+4+8=40$

Question229

Consider points A, B, C and D with position vectors $7\hat{i}-4\hat{j}+7\hat{k}$, $\hat{i}-6\hat{j}+10\hat{k}$, $-\hat{i}-3\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+5\hat{k}$ respectively. Then ABCD is a

[2003]

Options:

A. parallelogram but not a rhombus

B. square

C. rhombus

D. None

Answer: D

Solution:

Solution:
Given that $A = (7, -4, 7)$, $B = (1, -6, 10)$, $C = (-1, -3, 4)$
and $D = (5, -1, 5)$
$\therefore AB = \sqrt{(7-1)^2 + (-4+6)^2 + (7-10)^2}$
$= \sqrt{36 + 4 + 9} = 7$
Similarly, $BC = 7$, $CD = \sqrt{41}$, $DA = \sqrt{17}$
∴ None of the options is satisfied.

Question230

If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2)$,$(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product abc equals
[2003]

Options:

A. 0

B. 2

C. -1

D. 1

Answer: C

Solution:

Solution:

Given $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow (1 + abc) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

Given that $(1, a, a^2)$, $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar

$$\therefore \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \text{ (given condition)}$$

$\therefore 1 + abc = 0 \Rightarrow abc = -1$

Question231

The vectors $\vec{AB} = 3\hat{i} + 4\hat{k}$ & $\vec{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is [2003]

Options:

A. $\sqrt{288}$

B. $\sqrt{18}$

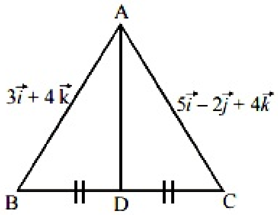
C. $\sqrt{72}$

D. $\sqrt{33}$

Answer: D

Solution:

Solution:



Given that AD is median of ΔABC.

$\therefore \overline{AD} = \dfrac{(3+5)\hat{i} + (0-2)\hat{j} + (4+4)\hat{k}}{2} = 4i - j + 4k$

$|\overline{AD}| = \sqrt{16 + 16 + 1} = \sqrt{33}$

Question232

$\vec{a}, \vec{b}, \vec{c}$ are 3 vectors, such that $\vec{a} + \vec{b} + \vec{c} = 0, |\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is equal to

[2003]

Options:

A. 1

B. 0

C. -7

D. 7

Answer: C

Solution:

Given that $\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \left(\vec{a} + \vec{b} + \vec{c}\right) . \left(\vec{a} + \vec{b} + \vec{c}\right) = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\left(\vec{a} . \vec{b} + \vec{b} . \vec{c} + \vec{c} . \vec{a}\right) = 0$

$1 + 4 + 9 + 2\left(\vec{a} . \vec{b} + \vec{b} . \vec{c} + \vec{c} . \vec{a}\right) = 0$

$\vec{a} . \vec{b} + \vec{b} . \vec{c} + \vec{c} . \vec{a} = \dfrac{-1 - 4 - 9}{2} = -7$

Question233

If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non- coplanar vectors, then $\left(\vec{u} + \vec{v} - \vec{w}\right) . \left(\vec{u} - \vec{v}\right) \times \left(\vec{v} - \vec{w}\right)$ equals
[2003]

Options:

A. $3\vec{u} . \vec{v} \times \vec{w}$

B. 0

C. $\vec{u} . \left(\vec{v} \times \vec{w}\right)$

D. $\vec{u} . \vec{w} \times \vec{v}$

Answer: C

Solution:

$\left(\vec{u} + \vec{v} - \vec{w}\right) . \left(\vec{u} - \vec{v}\right) \times \left(\vec{v} - \vec{w}\right)$

$= \left(\vec{u} + \vec{v} - \vec{w}\right) . \left(\vec{u} \times \vec{v} - \vec{u} \times \vec{w} - \vec{v} \times \vec{v} + \vec{v} \times \vec{w}\right)$

$= \left(\vec{u} + \vec{v} - \vec{w}\right) . \left(\vec{u} \times \vec{v} - \vec{u} \times \vec{w} + \vec{v} \times \vec{w}\right) \left[\because \vec{v} \times \vec{v} = 0\right]$

$= \vec{u} . \left(\vec{u} \times \vec{v}\right) - \vec{u} . \left(\vec{u} \times \vec{w}\right) + \vec{u} . \left(\vec{v} \times \vec{w}\right) + \vec{v} . \left(\vec{u} \times \vec{v}\right) - \vec{v} . \left(\vec{u} \times \vec{w}\right) + \vec{v} . \left(\vec{v} \times \vec{w}\right) - \vec{w} . \left(\vec{u} \times \vec{v}\right)$
$+ \vec{w} . \left(\vec{u} \times \vec{w}\right) - \vec{w} . \left(\vec{v} \times \vec{w}\right)$

We know that $\left[\vec{a}, \vec{a}, \vec{b}\right] = 0$

$= \vec{u} . \left(\vec{v} \times \vec{w}\right) - \vec{v} . \left(\vec{u} \times \vec{w}\right) - \vec{w} . \left(\vec{u} \times \vec{v}\right)$

$= \left[\vec{u}\vec{v}\vec{w})\right] + \left[\vec{v}\vec{w}\vec{u})\right] - \left[\vec{w}\vec{u}\vec{v}\right] = \vec{u} . \left(\vec{v} \times \vec{w}\right)$

Question234

A tetrahedron has vertices at O(0, 0, 0), A(1, 2, 1)B(2, 1, 3)and C(−1, 1, 2). Then the angle between the faces OAB and ABC will be [2003]

Options:

A. 90°

B. $\cos^{-1}\left(\frac{19}{35}\right)$

C. $\cos^{-1}\left(\frac{17}{31}\right)$

D. 30°

Answer: B

Solution:

Solution:
Normal vector of the face OAB

$$= \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 5\hat{i} - \hat{j} - 3\hat{k}$$

Normal vector of the face ABC

$$= \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ -2 & -1 & 1 \end{vmatrix} = \hat{i} - 5\hat{j} - 3k$$

Angle between the faces = angle between their normals

$\cos\theta = \left|\frac{5+5+9}{\sqrt{35}\sqrt{35}}\right| = \frac{19}{35}$ or $\theta = \cos^{-1}\left(\frac{19}{35}\right)$

Question235

Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \hat{n} = 0$, then $|\vec{w} \cdot \hat{n}|$ is equal to [2003]

Options:

A. 3

B. 0

C. 1

D. 2

Answer: A

Solution:

Solution:
Given that $\vec{u} \cdot \hat{n} = 0$ and $\vec{v} \cdot \vec{n} = 0$
$\Rightarrow \vec{n}$ is perpendicular both $\vec{u}$ and $\vec{v}$,
$\therefore \hat{n} = \frac{\vec{u} \times \vec{v}}{|\vec{u}||\vec{v}|}$

$$\hat{n} = \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}}{\sqrt{2} \times \sqrt{2}} = \frac{-2\hat{k}}{2} = -k$$

$\left|\vec{\omega} \cdot \hat{n}\right| = |(i + 2j + 3k) \cdot (-k)| = |-3| = 3$

Question236

Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in [2003]

Options:

A. H.P

B. A.G..P

C. A.P

D. G..P.

Answer: A

Solution:

Solution:
Let β be the inclination of the plane to the horizontal and u be the velocity of projection of the projectile
We have $R_1 = \frac{u^2}{g(1 + \sin\beta)}$ and $R_2 = \frac{u^2}{g(1 - \sin\beta)}$
Adding above equations
$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2g}{u^2}$ or $\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R}\left[\because R = \frac{u^2}{g}\right]$
$\therefore R_1$, R, R_2 are in H.P.

Question237

Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{u}$ and the other from rest with uniform acceleration $\vec{f}$. Let α be the angle between their directions of motion. The relative velocity of the second particle w.r.t. the first is

least after a time
[2003]

Options:

A. $\frac{u \cos \alpha}{f}$

B. $\frac{u \sin \alpha}{f}$

C. $\frac{f \cos \alpha}{u}$

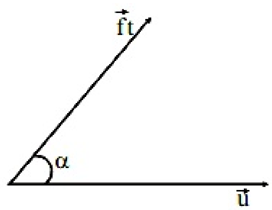
D. $u \sin \alpha$

Answer: A

Solution:

Solution:

Let the two velocities be $\vec{v}_1 = u\hat{i}$ and $\vec{v}_2 = (f t \cos \alpha)\hat{i} + (f t \sin \alpha)\hat{j}$



∴ Relative velocity of second with respect to first

$\vec{v} = \vec{v}_2 - \vec{v}_1 = (f t \cos \alpha - u)\hat{i} + f t \sin \alpha\,\hat{j}$

$\Rightarrow |\vec{v}|^2 = (f t \cos \alpha - u)^2 + (f t \sin \alpha)^2$

$= f^2 t^2 + u^2 - 2uf t \cos \alpha$

For $|\vec{v}|$ to be min and max. we should have

$\frac{d\ |v|^2}{d\,t} = 0 \Rightarrow 2f^2 t - 2uf \cos \alpha = 0$

$\Rightarrow t = \frac{u \cos \alpha}{f}$

Also $\frac{d^2\,|v|^2}{d\,t^2} = 2f^2 = +ve$

∴ $|v|^2$ and hence $|v|$ is least at the time $\frac{u \cos \alpha}{f}$

Question238

Two stones are projected from the top of a cliff h metres high, with the same speed u, so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected horizontally and the other is projected at an angle θ to the horizontal then tan θ equals
[2003]

Options:

A. $u\sqrt{\frac{2}{gh}}$

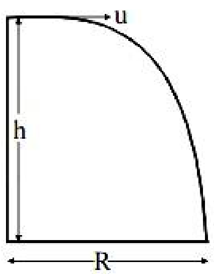
B. $\sqrt{\frac{2u}{gh}}$

C. $2g\sqrt{\frac{u}{h}}$

D. $2h\sqrt{\frac{u}{g}}$

Answer: A

Solution:



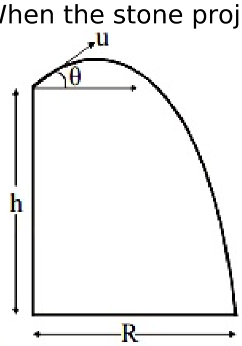
Given that the stone projected horizontally. For horizontal motion,
Distance = speed × time ⇒ $R = ut$
and for vertical motion

$h = 0 \times t + \frac{1}{2}gt^2$

$\Rightarrow t = \sqrt{\frac{2h}{g}}$

∴ We get $R = u\sqrt{\frac{2h}{g}}$ ... ...(1)

When the stone projected at an angle θ, for horizontal and vertical motions, we have



$R = u\cos\theta \times t$ ....(2)

and $h = -u\sin\theta \times t + \frac{1}{2}gt^2$....(3)

From eqns. (1) and (2) we get

$u\sqrt{\frac{2h}{g}} = u\cos\theta \times t$

$\Rightarrow t = \frac{1}{\cos\theta}\sqrt{\frac{2h}{g}}$

Putting the value of t in eq (3) we get

$h = -\frac{u\sin\theta}{\cos\theta}\sqrt{\frac{2h}{g}} + \frac{1}{2}g\left[\frac{2h}{g\cos^2\theta}\right]$

$h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\sec^2\theta$

$h = -u\sqrt{\frac{2h}{g}}\tan\theta + h\tan^2\theta + h$

$\tan^2\theta - u\sqrt{\frac{2}{hg}}\tan\theta = 0; \therefore \tan\theta = u\sqrt{\frac{2}{hg}}$

Question239

A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by
[2003]

Options:

A. $\sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$

B. $2s\left(\frac{1}{f}+\frac{1}{r}\right)$

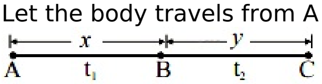
C. $\frac{\sqrt{2s}}{\frac{1}{f}+\frac{1}{r}}$

D. $\sqrt{2s(f+r)}$

Answer: A

Solution:

Solution:
Let the body travels from A to B with constant acceleration t and from B to C with constant retardation r.



If $AB = x$, $BC = y$, time taken from A to B $= t_1$ and time taken from B to C $= t_2$, then $s = x + y$ and $t = t_1 + t_2$
For the motion from A to B
$v^2 = u^2 + 2fs \Rightarrow v^2 = 2fx (\because u = 0)$
$\Rightarrow x = \frac{v^2}{2f}$ ....(1)
and $v = u + ft \Rightarrow v = ft_1$
$\Rightarrow t_1 = \frac{v}{f}$ ....(2)
For the motion from B to C
$v^2 = u^2 + 2fs$
$\Rightarrow 0 = v^2 - 2ry \Rightarrow y = \frac{v^2}{2r}$ ....(3)
and $v = u + ft \Rightarrow 0 = v - rt_2$
$\Rightarrow t_2 = \frac{v}{r}$
Adding equations (1) and (3), we get
$x + y = \frac{v^2}{2}\left[\frac{1}{f}+\frac{1}{r}\right] = s$
Adding equations (2) and (4), we get
$t_1 + t_2 = v\left[\frac{1}{f}+\frac{1}{r}\right] = t$
$\frac{t^2}{2s} = \frac{v^2\left[\frac{1}{f}+\frac{1}{r}\right]^2}{2\times\frac{v^2}{2}\left(\frac{1}{f}+\frac{1}{r}\right)} = \frac{1}{f}+\frac{1}{r}$
$t = \sqrt{2s\left(\frac{1}{f}+\frac{1}{r}\right)}$

Question240

The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed,then $\vec{R}$ is again doubled. Then $P^2 : Q^2 : R^2$ is [2003]

Options:

A. 2: 3: 1

B. 3: 1: 1

C. 2: 3: 2

D. 1: 2: 3 .

Answer: C

Solution:

Solution:
$R^2 = P^2 + Q^2 + 2PQ\cos\theta$ .... (1)
When $\vec{Q}$ and $\vec{R}$ are doubled
$4R^2 = P^2 + 4Q^2 + 4PQ\cos\theta$ .... (2)
When $\vec{Q}$ is reversed and $\vec{R}$ is doubled
$4R^2 = P^2 + Q^2 - 2PQ\cos\theta$ .... (3)
Adding(1) and (3), $5R^2 = 2P^2 + 2Q^2$
$\Rightarrow 2P^2 + 2Q^2 - 5R^2 = 0$ .... (4)
Applying (3) × 2 + (2), $12R^2 = 3P^2 + 6Q^2$
$\Rightarrow 3P^2 + 6Q^2 - 12R^2 = 0$....(5)
From (4) and (5) $\frac{P^2}{-24+30} = \frac{Q^2}{24-15} = \frac{R^2}{12-6}$
$\frac{P^2}{6} = \frac{Q^2}{9} = \frac{R^2}{6}$ or $P^2 : Q^2 : R^2 = 2 : 3 : 2$

Question241

**A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle the moment of the couple thus formed is $\vec{H}$. If instead, the force $\vec{P}$ are turned through an angle α, then the moment of couple becomes
[2003]**

Options:

A. $\vec{H}\sin\alpha - \vec{G}\cos\alpha$

B. $\vec{G}\sin\alpha - \vec{H}\cos\alpha$

C. $\vec{H}\sin\alpha + \vec{G}\cos\alpha$

D. $\vec{G}\sin\alpha + \vec{H}\cos\alpha$

Answer: C

Solution:

Solution:

We know that $\vec{G} = \vec{r} \times \vec{p}$; $|\vec{G}| = |\vec{r}||\vec{p}|\sin\theta$

$|\vec{H}| = |\vec{r}||\vec{p}|\cos\theta$ [$\because \sin(90° + \theta) = \cos\theta$]

$G = |\vec{r}||\vec{p}|\sin\theta$ .....(1)

$H = |\vec{r}||\vec{p}|\cos\theta$.....(2)

$x = |\vec{r}||\vec{p}|\sin(\theta + \alpha)$.....(3)

From(1), (2) & (3), $x = \vec{G}\cos\alpha + \vec{H}\sin\alpha$

Question242

A particle acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} - 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The total work done by the forces is [2003]

Options:

A. 50 units

B. 20 units

C. 30 units

D. 40 units.

Answer: D

Solution:

Solution:

$\vec{F} = \vec{F}_1 + \vec{F}_2 = 7i + 2j - 4k$

$\vec{d}$ = Positon Vector of $\vec{B}$ − Position Vector of $\vec{A}$

$= 4i + 2j - 2k$

$W = \vec{F} \cdot \vec{d} = 28 + 4 + 8 = 40$ unit

Question243

If $|\vec{a}| = 5, |\vec{b}| = 4, |\vec{c}| = 3$ thus what will be the value of $|\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}|$, given that $\vec{a} + \vec{b} + \vec{c} = 0$ [2002]

Options:

A. 25

B. 50

C. -25

D. -50

Answer: A

Solution:

Given that $\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow\left|\vec{a}+\vec{b}+\vec{c}\right|^2=0$

$\Rightarrow|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0\Rightarrow 25+16+9+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=0$

$\Rightarrow\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)=-25$

$\therefore\left|\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right|=25$

Question244

If sdaa $\vec{a}$, $\vec{b}$, $\vec{c}$ are vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $\left|\vec{a}\right|=7,\left|\vec{b}\right|=5,\left|\vec{c}\right|=3$ then angle between vector $\vec{b}$ and $\vec{c}$ is [2002]

Options:

A. 60°

B. 30°

C. 45°

D. 90°

Answer: A

Solution:

Given that $\vec{a}+\vec{b}+\vec{c}=0\Rightarrow\vec{b}+\vec{c}=-\vec{a}$

$\Rightarrow|\vec{b}+\vec{c}|^2=|\vec{a}|^2=5^2+3^2+2\vec{b}.\vec{c}=7^2$

$\Rightarrow 2\left|\vec{b}\right|\left|\vec{c}\right|\cos\theta=49-34=15;$

$\Rightarrow 2\times 5\times 3\cos\theta=15;$

$\Rightarrow\cos\theta=1/2;\Rightarrow\theta=\frac{\pi}{3}=60°$

Question245

If $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$ then $\vec{a}+\vec{b}+\vec{c}=$ [2002]

Options:

A. abc

B. -1

C. 0

D. 2

Answer: C

Solution:

Let $\vec{a}+\vec{b}+\vec{c}=\vec{r}$ Then $\vec{a}\times\left(\vec{a}+\vec{b}+\vec{c}\right)=\vec{a}\times\vec{r}$
$\Rightarrow 0+\vec{a}\times\vec{b}+\vec{a}\times\vec{c}=\vec{a}\times\vec{r}$
$\Rightarrow \vec{a}\times\vec{b}-\vec{c}\times\vec{a}=\vec{a}\times\vec{r}\Rightarrow\vec{a}\times\vec{r}=\vec{0}\ \left[\because\vec{a}\times\vec{b}=\vec{c}\times\vec{a}\right]$
Similarly $\vec{b}\times\vec{r}=\vec{0}$ & $\vec{c}\times\vec{r}=\vec{0}$
Above three conditions can be hold if and only if $\vec{r}=\vec{0}$

Question246

$\vec{a}=3\hat{i}-5\hat{j}$ and $\vec{b}=6\hat{i}+3\hat{j}$ are two vectors and $\vec{c}$ is a vector such that $\vec{c}=\vec{a}\times\vec{b}$ then $\left|\vec{a}\right|:\left|\vec{b}\right|:\left|\vec{c}\right|$
[2002]

Options:

A. $\sqrt{34}:\sqrt{45}:\sqrt{39}$

B. $\sqrt{34}:\sqrt{45}:39$

C. 34: 39: 45

D. 39: 35: 34

Answer: B

Solution:

Solution:

We have $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-5&0\\6&3&0\end{vmatrix}=39\hat{k}=\vec{c}$
Also $\left|\vec{a}\right|=\sqrt{34},\left|\vec{b}\right|=\sqrt{45},\left|\vec{c}\right|=39;$
$\therefore\left|\vec{a}\right|:\left|\vec{b}\right|:\left|\vec{c}\right|=\sqrt{34}:\sqrt{45}:39$

Question247

If the vectors $\vec{c}$, $\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\vec{b}=\hat{j}$ are such that $\vec{a}$, $\vec{c}$ and $\vec{b}$ form a right handed system then $\vec{c}$ is:
[2002]

Options:

A. $z\hat{i} - x\hat{k}$

B. $\vec{0}$

C. $y\hat{j}$

D. $-z\hat{i} + x\hat{k}$

Answer: A

Solution:

Solution:

Given that $\vec{a}, \vec{c}, \vec{b}$ form a right handed system,

$$\therefore \vec{c} = \vec{b} \times \vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 0 & 1 & 0 \\ x & y & z \end{vmatrix} = z\hat{i} - x\hat{k}$$

Question248

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are vectors such that $[\vec{a}\vec{b}\vec{c}] = 4$ then $[\vec{a} \times \vec{b} \;\; \vec{b} \times \vec{c} \;\; \vec{c} \times \vec{a}] =$ [2002]

Options:

A. 16

B. 64

C. 4

D. 8

Answer: A

Solution:

Solution:

$[\vec{a} \times \vec{b} \;\; \vec{b} \times \vec{c} \;\; \vec{c} \times \vec{a}]$

$(\vec{a} \times \vec{b}) . \{(\vec{b} \times \vec{c}) \times (\vec{c} \times \vec{a})\} \because \vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} . \vec{c})\vec{b} - (\vec{a} . \vec{b})\vec{c}$

$= (\vec{a} \times \vec{b}) . \{(\vec{m} . \vec{a})\vec{c} - (\vec{m} . \vec{c})\vec{a}\}$ (where $\vec{m} = \vec{b} \times \vec{c}$)

$= \{(\vec{a} \times \vec{b}) . \vec{c}\} . \{\vec{a} . (\vec{b} \times \vec{c})\} = [\vec{a}\vec{b}\vec{c}]^2 = 4^2 = 16$

Question249

If $|\vec{a}| = 4, |\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\pi / 6$then $(\vec{a} \times \vec{b})^2$ is equal to

[2002]

Options:

A. 48

B. 16

C. $\vec{a}$

D. None of these

Answer: B

Solution:

Since, $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\frac{\pi}{6} = 4 \times 2 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}.$

We know that,

$(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$

$\Rightarrow (\vec{a} \times \vec{b})^2 + 48 = 16 \times 4$

$\Rightarrow (\vec{a} \times \vec{b})^2 = 16$

Question250

A bead of weight w can slide on smooth circular wire in a vertical plane. The bead is attached by a light thread to the highest point of the wire and in equilibrium, the thread is taut and make an angle θ with the vertical then tension of the thread and reaction of the wire on the bead are
[2002]

Options:

A. $T = w\cos\theta \quad R = w\tan\theta$

B. $T = 2w\cos\theta \quad R = w$

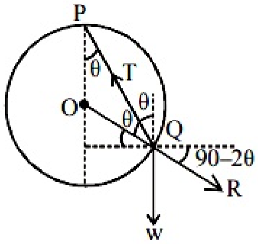
C. $T = w \quad R = w\sin\theta$

D. $T = w\sin\theta \quad R = w\cot\theta$

Answer: B

Solution:

From figure angle $TQW = 180 - \theta$; $\angle RQW = 2\theta$;
$\angle RQT = 180 - \theta$



Applying Lami's theorem at Q.

$\frac{T}{\sin 2\theta} = \frac{R}{\sin(180-\theta)} = \frac{W}{\sin(180-\theta)}$

$\Rightarrow R = W$ and $T = 2W\cos\theta$

Question251

The sum of two forces is 18 N and resultant whose direction is at right angles to the smaller force is 12 N. The magnitude of the two forces are [2002]

Options:

A. 13, 5

B. 12, 6

C. 14, 4

D. 11, 7

Answer: A

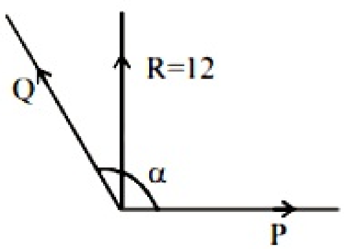
Solution:

Given that $P + Q = 18$ ....(1)

We know that

$P^2 + Q^2 + 2PQ\cos\alpha = 144$ ......(2)

$\tan 90° = \frac{Q\sin\alpha}{P + Q\cos\alpha}$



$\Rightarrow P + Q\cos\alpha = 0$ ......(3)

From (2) and (3),

$Q^2 - P^2 = 144 \Rightarrow (Q-P)(Q+P) = 144$

$Q - P = \frac{144}{18} = 8$

From (1), On solving, we get $Q = 13$, $P = 5$
