Complex Numbers and Quadratic Equations

Question1

If $S = \{z \in C : |z - i| = |z + i| = |z - 1|\}$, then, $n(S)$ is:

[27-Jan-2024 Shift 1]

Options:

A.

1

B.

0

C.

3

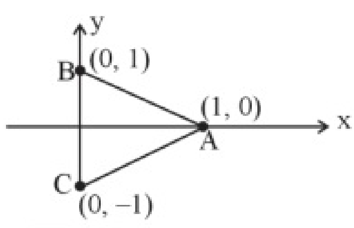
D.

2

Answer: A

Solution:

$|z - i| = |z + i| = |z - 1|$



ABC is a triangle. Hence its circum-centre will be the only point whose distance from A, B, C will be same.
So $n(S) = 1$

Question2

If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, A, B, C ≥ 0, then $5(3A - 2B - C)$ is equal to

[27-Jan-2024 Shift 1]

Answer: 5

Solution:

$x^2 + x + 1 = 0 \Rightarrow x = \omega, \omega^2 = \alpha$

Let $\alpha = \omega$

Now $(1+\alpha)^7 = -\omega^{14} = -\omega^2 = 1 + \omega$

$A = 1, B = 1, C = 0$

$\therefore 5(3A - 2B - C) = 5(3 - 2 - 0) = 5$

Question3

Let the complex numbers α and 1/α lie on the circles $|z - z_0|^2 = 4$ and $|z - z_0|^2 = 16$ respectively, where $z_0 = 1 + i$. Then, the value of $100\ |\alpha|^2$ is.___

[27-Jan-2024 Shift 2]

Answer: 20

Solution:

$|z - z_0|^2 = 4$

$\Rightarrow (\alpha - z_0)(\bar{\alpha} - \bar{z}_0) = 4$

$\Rightarrow \alpha\bar{\alpha} - \alpha\bar{z}_0 - z_0\bar{\alpha} + |z_0|^2 = 4$

$\Rightarrow |\alpha|^2 - \alpha\bar{z}_0 - z_0\bar{\alpha} = 2$ ....(1)

$|z - z_0|^2 = 16$

$\Rightarrow \left(\frac{1}{\alpha} - z_0\right)\left(\frac{1}{\alpha} - \bar{z}_0\right) = 16$

$\Rightarrow (1 - \bar{\alpha}z_0)(1 - \alpha\bar{z}_0) = 16\ |\alpha|^2$

$\Rightarrow 1 - \bar{\alpha}z_0 - \alpha\bar{z}_0 + |\alpha|^2\ |z_0|^2 = 16\ |\alpha|^2$

$\Rightarrow 1 - \bar{\alpha}z_0 - \alpha\bar{z}_0 = 14\ |\alpha|^2$ ....(2)

From (1) and (2)

$\Rightarrow 5\ |\alpha|^2 = 1$

$\Rightarrow 100\ |\alpha|^2 = 20$

Question4

If α, β are the roots of the equation, $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$, then

[27-Jan-2024 Shift 2]

Options:

A.

$2S_{12} = S_{11} + S_{10}$

B.

$S_{12} = S_{11} + S_{10}$

C.

$2S_{11} = S_{12} + S_{10}$

D.

$S_{11} = S_{10} + S_{12}$

Answer: B

Solution:

$x^2 - x - 1 = 0$

$S_n = 2023\alpha^n + 2024\beta^n$

$S_{n-1} + S_{n-2} = 2023\alpha^{n-1} + 2024\beta^{n-1} + 2023\alpha^{n-2} + 2024\beta^{n-2}$

$= 2023\alpha^{n-2}[1+\alpha] + 2024\beta^{n-2}[1+\beta]$

$= 2023\alpha^{n-2}[\alpha^2] + 2024\beta^{n-2}[\beta^2]$

$= 2023\alpha^n + 2024\beta^n$

$S_{n-1} + S_{n-2} = S_n$

Put $n = 12$

$S_{11} + S_{10} = S_{12}$

Question5

If $z = \dfrac{1}{2} - 2i$, is such that $|z+1| = \alpha z + \beta(1+i)$, $i = \sqrt{-1}$ and $\alpha, \beta \in \mathrm{R}$, then $\alpha + \beta$ is equal to

[29-Jan-2024 Shift 1]

Options:

A.

-4

B.

3

C.

2

D.

-1

Answer: B

Question6

Let α, β be the roots of the equation $x^2 - x + 2 = 0$ with $Im(\alpha) > Im(\beta)$. Then $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$ is equal to

[29-Jan-2024 Shift 1]

Answer: 13

Solution:

$$\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$$

$$= \alpha^4(\alpha - 2) + \alpha^4 - 5\alpha^2 + (\beta - 2)^2$$

$$= \alpha^5 - \alpha^4 - 5\alpha^2 + \beta^2 - 4\beta + 4$$

$$= \alpha^3(\alpha - 2) - \alpha^4 - 5\alpha^2 + \beta - 2 - 4\beta + 4$$

$$= -2\alpha^3 - 5\alpha^2 - 3\beta + 2$$

$$= -2\alpha(\alpha - 2) - 5\alpha^2 - 3\beta + 2$$

$$= -7\alpha^2 + 4\alpha - 3\beta + 2$$

$$= -7(\alpha - 2) + 4\alpha - 3\beta + 2$$

$$= -3\alpha - 3\beta + 16 = -3(1) + 16 = 13$$

Question7

Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\,(2\tan 5\pi/8)$, then (r, θ) is equal to

[29-Jan-2024 Shift 2]

Options:

A.

$$\left(2\,sec\,\frac{3\pi}{8},\ \frac{3\pi}{8}\right)$$

B.

$\left(2\ sec\ \frac{3\pi}{8},\ \frac{5\pi}{8}\right)$

C.

$\left(2\ sec\ \frac{5\pi}{8},\ \frac{3\pi}{8}\right)$

D.

$\left(2\ sec\ \frac{11\pi}{8},\ \frac{11\pi}{8}\right)$

Answer: A

Solution:

$z = 2 - i\left(2\tan\frac{5\pi}{8}\right) = x + iy(\text{ let })$

$r = \sqrt{x^2 + y^2}\ \&\ \theta = \tan^{-1}\frac{y}{x}$

$r = \sqrt{(2)^2 + \left(2\tan\frac{5\pi}{8}\right)^2}$

$= \left|2\ sec\ \frac{5\pi}{8}\right| = \left|2\ sec\left(\pi - \frac{3\pi}{8}\right)\right|$

$= 2\ sec\ \frac{3\pi}{8}$

$\&\ \theta = \tan^{-1}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right).$

$= \tan^{-1}\left(\tan^2\left(\pi - \frac{5\pi}{8}\right)\right)$

$= \frac{3\pi}{8}$

Question8

Let α,β be the roots of the equation $x^2 - \sqrt{6}x + 3 = 0$ such that Im(α) > Im(β). Let a,b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta} + \alpha^{98} = 3^n(a + ib),\ i = \sqrt{-1}.$ **Then n + a + b is equal to____**

[29-Jan-2024 Shift 2]

Answer: 49

Solution:

$$x^2 - \sqrt{6}x + 6 = 0$$

$$x = \frac{\sqrt{6} \pm i\sqrt{6}}{2} = \frac{\sqrt{6}}{2}(1 \pm i)$$

$$\alpha = \sqrt{3}\left(e^{i\frac{\pi}{4}}\right), \beta = \sqrt{3}\left(e^{-i\frac{\pi}{4}}\right)$$

$$\therefore \frac{\alpha^{99}}{\beta} + \alpha^{98} = \alpha^{98}\left(\frac{\alpha}{\beta} + 1\right)$$

$$= \frac{\alpha^{98}(\alpha + \beta)}{\beta} = 3^{49}\left(e^{i99\frac{\pi}{4}}\right) \times \sqrt{2}$$

$$= 3^{49}(-1 + i)$$

$$= 3^n(a + ib)$$

$$\therefore n = 49, a = -1, b = 1$$

$$\therefore n + a + b = 49 - 1 + 1 = 49$$

Question9

Let the set $C = \{(x, y) \mid x^2 - 2^y = 2023, x, y \in \mathbb{N}\}$.

Then $\sum_{(x, y) \in C} (x + y)$ is equal to___

[29-Jan-2024 Shift 2]

Answer: 46

Solution:

$$x^2 - 2^y = 2023$$

$$\Rightarrow x = 45, y = 1$$

$$\sum_{(x, y) \in C} (x + y) = 46.$$

Question10

If $z = x + iy, xy \neq 0$, satisfies the equation $z^2 + i\bar{z} = 0$, then $|z^2|$ is equal to :

[30-Jan-2024 Shift 1]

Options:

A.

9

B.

1

C.

4

D.

1/4

Answer: B

Solution:

$z^2 = -i\bar{z}$

$|z^2| = |i\bar{z}|$

$|z^2| = |z|$

$|z|^2 - |z| = 0$

$|z|(|z| - 1) = 0$

$|z| = 0$ (not acceptable)

$\therefore |z| = 1$

$\therefore |z|^2 = 1$

Question11

If z is a complex number, then the number of common roots of the equation $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$, is equal to :

[30-Jan-2024 Shift 2]

Options:

A.

1

B.

2

C.

0

D.

3

Answer: B

Solution:

$z^{1985}+z^{100}+1=0$ & $z^3+2z^2+2z+1=0$

$(z+1)(z^2-z+1)+2z(z+1)=0$

$(z+1)(z^2+z+1)=0$

$\Rightarrow\ z=-1,\quad z=w, w^2$

Now putting $z=-1$ not satisfy

Now put $z = w$

$\Rightarrow\ w^{1985}+w^{100}+1$

$\Rightarrow\ w^2+w+1=0$

Also, $z=w^2$

$\Rightarrow\ w^{3970}+w^{200}+1$

$\Rightarrow\ w+w^2+1=0$

Two common root

Question12

If α denotes the number of solutions of |1 − i| x = 2x and β = $\left(\frac{|z|}{\arg(z)}\right)$, where $z=\frac{\pi}{4}(1+i)^4\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i}+\frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right)$, $i=\sqrt{-1}$, then **the distance of the point (α, β) from the line 4x − 3y = 7 is____**

[31-Jan-2024 Shift 1]

Answer: 3

Solution:

$(\sqrt{2})^x=2^x\Rightarrow x=0\Rightarrow\alpha=1$

$z=\frac{\pi}{4}(1+i)^4\left[\frac{\sqrt{\pi}-\pi i-i-\sqrt{\pi}}{\pi+1}+\frac{\sqrt{\pi}-i-\pi i-\sqrt{\pi}}{1+\pi}\right]$

$=-\frac{\pi i}{2}(1+4i+6i^2+4i^3+1)$

$=2\pi i$

$\beta=\frac{2\pi}{\frac{\pi}{2}}=4$

Distance from $(1, 4)$ to $4x-3y=7$

Will be $\frac{15}{5}=3$

Question13

Let z_1 and z_2 be two complex number such that

$z_1 + z_2 = 5$ and $z_1{}^3 + z_2{}^3 = 20 + 15i$. Then $|z_1{}^4 + z_2{}^4|$ equals-

[31-Jan-2024 Shift 2]

Options:

A.

30√3

B.

75

C.

15√15

D.

25√3

Answer: B

Solution:

$z_1 + z_2 = 5$

$z_1{}^3 + z_2{}^3 = 20 + 15i$

$z_1{}^3 + z_2{}^3 = (z_1 + z_2)^3 - 3z_1z_2(z_1 + z_2)$

$z_1{}^3 + z_2{}^3 = 125 - 3z_1 \cdot z_2(5)$

$\Rightarrow 20 + 15i = 125 - 15z_1z_2$

$\Rightarrow 3z_1z_2 = 25 - 4 - 3i$

$\Rightarrow 3z_1z_2 = 21 - 3i$

$\Rightarrow z_1 \cdot z_2 = 7 - i$

$\Rightarrow (z_1 + z_2)^2 = 25$

$\Rightarrow z_1{}^2 + z_2{}^2 = 25 - 2(7 - i)$

$\Rightarrow 11 + 2i$

$(z_1{}^2 + z_2{}^2)^2 = 121 - 4 + 44i$

$\Rightarrow z_1{}^4 + z_2{}^4 + 2(7 - i)^2 = 117 + 44i$

$\Rightarrow z_1{}^4 + z_2{}^4 = 117 + 44i - 2(49 - 1 - 14i)$

$\Rightarrow |z_1{}^4 + z_2{}^4| = 75$

Question14

Let α, β ∈ N be roots of equation $x^2 - 70x + λ = 0$, where λ/2, λ/3 ∉ N.

If λ assumes the minimum possible value, then

$\frac{(\sqrt{\alpha-1}+\sqrt{\beta-1})(\lambda+35)}{|\alpha-\beta|}$ is equal to :

[30-Jan-2024 Shift 1]

Answer: 60

Solution:

$x^2-70x+\lambda=0$

$\alpha+\beta=70$

$\alpha\beta=\lambda$

$\therefore \alpha(70-\alpha)=\lambda$

Since, 2 and 3 does not divide λ

$\therefore \alpha=5, \beta=65, \lambda=325$

By putting value of α, β, λ we get the required value 60 .

Question15

The number of real solutions of the equation $x(x^2+3x|+5x-1|+6x-2|)=0$ is

[30-Jan-2024 Shift 2]

Answer: 1

Solution:

$x=0$ and $x^2+3x|+5x-1|+6x-2|=0$

Here all terms are $+ve$ except at $x=0$

So there is no value of $\mathbf{x}$

Satisfies this equation

Only solution $\mathbf{x}=0$

No of solution 1 .

Question16

Let S be the set of positive integral values of a for which
$\frac{ax^2 + 2(a+1)x + 9a + 4}{x^2 - 8x + 32} < 0, \forall x \in \mathbb{R}.$

Then, the number of elements in S is :

[31-Jan-2024 Shift 1]

Options:

A.

1

B.

0

C.

∞

D.

3

Answer: B

Solution:

$$ax^2 + 2(a+1)x + 9a + 4 < 0 \ \forall x \in R$$

$$\therefore a < 0$$

Question17

For $0 < c < b < a$, let $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements

(I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c

(II) If $\alpha \in (0, 1)$, then b may be the geometric mean of a and c

[31-Jan-2024 Shift 1]

Options:

A.

Both (I) and (II) are true

B.

Neither (I) nor (II) is true

C.

Only (II) is true

D.

Only (I) is true

Answer: A

Solution:

$f(x)=(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)$

$f(x)=a+b-2c+b+c-2a+c+a-2b=0$

$f(1)=0$

$\therefore \alpha\cdot 1=\dfrac{c+a-2b}{a+b-2c}$

$\alpha=\dfrac{c+a-2b}{a+b-2c}$

If $-1<\alpha<0$

$-1<\dfrac{c+a-2b}{a+b-2c}<0$

$b+c<2a$ and $b>\dfrac{a+c}{2}$

therefore, b cannot be G.M. between a and c.

If, $0<\alpha<1$

$0<\dfrac{c+a-2b}{a+b-2c}<1$

$b>c$ and $b<\dfrac{a+c}{2}$

Therefore, b may be the G.M. between a and c.

Question18

The number of solutions, of the equation $e^{\sin x}-2e^{-\sin x}=2$ is

[31-Jan-2024 Shift 2]

Options:

A.

2

B.

more than 2

C.

1

D.

0

Answer: D

Solution:

Take $e^{\sin x} = t(t > 0)$

$\Rightarrow t - \frac{2}{t} = 2$

$\Rightarrow \frac{t^2 - 2}{t} = 2$

$\Rightarrow t^2 - 2t - 2 = 0$

$\Rightarrow t^2 - 2t + 1 = 3$

$\Rightarrow (t - 1)^2 = 3$

$\Rightarrow t = 1 \pm \sqrt{3}$

$\Rightarrow t = 1 \pm 1.73$

$\Rightarrow t = 2.73 \text{ or } -0.73 \text{ (rejected as } t > 0)$

$\Rightarrow e^{\sin x} = 2.73$

$\Rightarrow \log_e e^{\sin x} = \log_e 2.73$

$\Rightarrow \sin x = \log_e 2.73 > 1$

So no solution.

Question19

Let a,b,c be the length of three sides of a triangle satisfying the condition $(a^2 + b^2)x^2 - 2b(a + c).x + (b^2 + c^2) = 0$. If the set of all possible values of x is the interval (α, β), then $12(\alpha^2 + \beta^2)$ is equal to____

[31-Jan-2024 Shift 2]

Answer: 36

Solution:

$$(a^2+b^2)x^2-2b(a+c)x+b^2+c^2=0$$

$$\Rightarrow a^2x^2-2abx+b^2+b^2x^2-2bcx+c^2=0$$

$$\Rightarrow (ax-b)^2+(bx-c)^2=0$$

$$\Rightarrow ax-b=0,\quad bx-c=0$$

$$\Rightarrow a+b>c \quad b+c>a \quad c+a>b$$

$$a+ax>bx \quad ax+bx>a \quad ax^2+a>ax$$

$$a+ax>ax^2 \quad ax+ax^2>a \quad x^2-x+1>0$$

$$x^2-x-1<0 \quad x^2+x-1>0 \quad \text{always true}$$

$$\frac{1-\sqrt{5}}{2}<x<\frac{1+\sqrt{5}}{2}$$

$$x<\frac{-1-\sqrt{5}}{2},\ \text{ or } x>\frac{-1+\sqrt{5}}{2}$$

$$\Rightarrow \frac{\sqrt{5}-1}{2}<x<\frac{\sqrt{5}+1}{2}$$

$$\Rightarrow \alpha=\frac{\sqrt{5}-1}{2},\beta=\frac{\sqrt{5}+1}{2}$$

$$12(\alpha^2+\beta^2)=12\left(\frac{(\sqrt{5}-1)^2+(\sqrt{5}+1)^2}{4}\right)=36$$

Question20

Let S = {z ∈ C : |z − 1| = 1 and (√2 − 1) (z + z)−i(z − z) = 2√2}. Let z_1, z_2 ∈ S be such that $|z_1|=\max\limits_{z\in S}|z|$ and $z_2=\min\limits_{z\in S}|z|$. Then $|\sqrt{2}z_1-z_2|^2$ equals :

[1-Feb-2024 Shift 1]

Options:

A.

1

B.

4

C.

3

D.

2

Answer: D

Solution:

Let $Z = x + iy$

Then $(x-1)^2 + y^2 = 1 \rightarrow (1)$

$(\sqrt{2}-1)(2x) - i(2iy) = 2\sqrt{2}$

$\Rightarrow (\sqrt{2}-1)x + y = \sqrt{2} \rightarrow (2)$

Solving (1) & (2) we get

Either $x = 1$ or $x = \dfrac{1}{2-\sqrt{2}} \rightarrow (3)$

On solving (3) with (2) we get

For $x = 1 \Rightarrow y = 1 \Rightarrow Z_2 = 1 + i$

& for

$x = \dfrac{1}{2-\sqrt{2}} \Rightarrow y = \sqrt{2} - \dfrac{1}{\sqrt{2}} \Rightarrow Z_1 = \left(1 + \dfrac{1}{\sqrt{2}}\right) + \dfrac{i}{\sqrt{2}}$

Now

$|\sqrt{2}z_1 - z_2|^2$

$= \left|\left(\dfrac{1}{\sqrt{2}} + 1\right)\sqrt{2} + i - (1+i)\right|^2$

$= (\sqrt{2})^2$

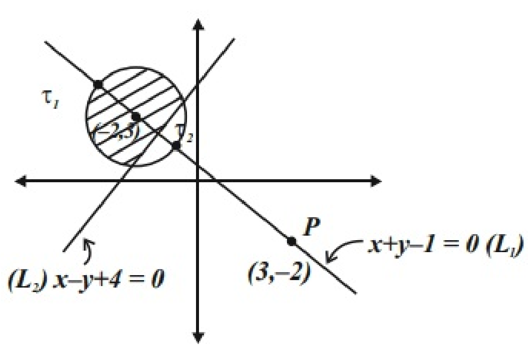
$= 2$

Question21

Let $P = \{z \in C : |z + 2 - 3i| \leq 1\}$ and $Q = \{z \in C : z(1+i) + \bar{z}(1-i) \leq -8\}$. Let in $P \cap Q$, $|z - 3 + 2i|$ be maximum and minimum at z_1 and z_2 respectively. If $|z_1|^2 + 2|z_2|^2 = \alpha + \beta\sqrt{2}$, where α, β are integers, then $\alpha + \beta$ equals_______ *[1-Feb-2024 Shift 1]*

Answer: 8

Solution:



Clearly for the shaded region z_1 is the intersection of the circle and the line passing through $P(L_1)$ and z_2 is intersection of line L_1 & L_2

Circle : $(x+2)^2+(y-3)^2=1$

$L_1 : x+y-1=0$

$L_2 : x-y+4=0$

On solving circle &L_1 we get

$z_1 : \left(-2-\frac{1}{\sqrt{2}}, 3+\frac{1}{\sqrt{2}}\right)$

On solving L_1 and z_2 is intersection of line L_1&L_2 we get $z_2 : \left(\frac{-3}{2}, \frac{5}{2}\right)$

$|z_1|^2+2|z_2|^2=14+5\sqrt{2}+17$

$=31+5\sqrt{2}$

So $\alpha=31$

$\beta=5$

$\alpha+\beta=36$

Question22

If z is a complex number such that |z| ≥ 1, then the minimum value of $\left|z+\frac{1}{2}(3+4i)\right|$ is:

[1-Feb-2024 Shift 2]

Options:

A.

5/2

B.

2

C.

3/2

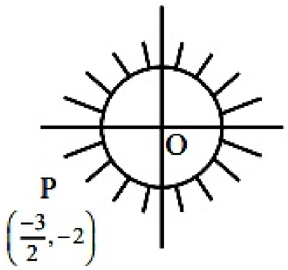
D.

None of above

Answer: D

Solution:

$|z| \geq 1$



Min. value of $\left|z + \frac{3}{2} + 2i\right|$ is actually zero.

Question23

Let $S = \{x \in R : (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10\}$

Then the number of elements in S is :

[1-Feb-2024 Shift 1]

Options:

A.

4

B.

0

C.

2

D.

1

Answer: C

Solution:

$(\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10$

Let $(\sqrt{3}+\sqrt{2})^x = t$

$t + \frac{1}{t} = 10$

$t^2 - 10t + 1 = 0$

$t = \frac{10 \pm \sqrt{100-4}}{2} = 5 \pm 2\sqrt{6}$

$(\sqrt{3}+\sqrt{2})^x = (\sqrt{3} \pm \sqrt{2})^2$

$x = 2$ or $x = -2$

Number of solutions $= 2$

Question24

Let α and β be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p,q and r be the consecutive terms of a non-constant G.P and $1/\alpha + 1/\beta + = 3/4$, then the value of $(\alpha - \beta)^2$ is :

[1-Feb-2024 Shift 2]

Options:

A.

80/9

B.

9

C.

20/3

D.

8

Answer: A

Solution:

$$px^2 + qx - r = 0$$

$$p = A,\ q = AR,\ r = AR^2$$

$$Ax^2 + ARx\ \ AR^2 = 0$$

$$x^2 + Rx - R^2 = 0$$

$$\because\ \frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$$

$$\therefore\ \frac{\alpha+\beta}{\alpha\beta} = \frac{3}{4} \Rightarrow \frac{-R}{-R^2} = \frac{3}{4} \Rightarrow R = \frac{4}{3}$$

$$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta\ = R^2 - 4(-R^2) = 5\left(\frac{16}{9}\right)$$

$$= 80/9$$

Question25

Let p, q ∈ ℝ and $(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$, $i = \sqrt{-1}$ Then $p + q + q^2$ and $p - q + q^2$ are roots of the equation.
[24-Jan-2023 Shift 1]

Options:

A. $x^2 + 4x - 1 = 0$

B. $x^2 - 4x + 1 = 0$

C. $x^2 + 4x + 1 = 0$

D. $x^2 - 4x - 1 = 0$

Answer: B

Solution:

$(1 - \sqrt{3}i)^{200} = 2^{199}(p + iq)$

$2^{200}\left(\cos\frac{\pi}{3} - i\sin\ \frac{\pi}{3}\right)^{200} = 2^{199}(p + iq)$

$2\left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = p + iq$

$p = -1,\ q = -\sqrt{3}$

$\alpha = p + q + q^2 = 2 - \sqrt{3}$

$\beta = p - q + q^2 = 2 + \sqrt{3}$

$= 4$

$= 1$

equation $x^2 - 4x + 1 = 0$

Question26

The value of $\left(\frac{1 + \sin\frac{2\pi}{9} + i\cos\frac{2\pi}{9}}{1 + \sin\ \frac{2\pi}{9} - i\cos\frac{2\pi}{9}}\right)^3$ **is**

[24-Jan-2023 Shift 2]

Options:

A. $\frac{-1}{2}(1 - i\sqrt{3})$

B. $\frac{1}{2}(1 - i\sqrt{3})$

C. $\frac{-1}{2}(\sqrt{3} - i)$

D. $\frac{1}{2}(\sqrt{3} + i)$

Answer: C

Solution:

Solution:

Let $\sin\ \frac{2\pi}{9} + i\cos\frac{2\pi}{9} = z$

$\left(\frac{1 + z}{1 + \bar{z}}\right)^3 = \left(\frac{1 + z}{1 + \ \frac{1}{z}}\right)^3 = z^3$

$\Rightarrow \left(i \left(\cos \frac{2\pi}{9} - i\sin \frac{2\pi}{9} \right) \right)^3$
$= -i \left(\cos \frac{2\pi}{3} - i\sin \frac{2\pi}{3} \right) = -i \left(\frac{-1}{2} - i \frac{\sqrt{3}}{2} \right)$
$\Rightarrow \frac{-1}{2}(\sqrt{3} - i).$

Question27

Let $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$. The set
$S = \left\{ z \in C : \left| z - z_1 \right|^2 - z - z_2 \left|^2 = z_1 - z_2 \right|^2 \right\}$
represents a
[25-Jan-2023 Shift 1]

Options:

A. straight line with sum of its intercepts on the coordinate axes equals 14

B. hyperbola with the length of the transverse axis 7

C. straight line with the sum of its intercepts on the coordinate axes equals -18

D. hyperbola with eccentricity 2

Answer: A

Solution:

Solution:
$((x-2)^2 + (y-3)^2) - ((x-3)^2 - (y-4)^2) = 1 + 1$
$\Rightarrow x + y = 7$

Question28

Let z be a complex number such that $\left| \frac{z - 2i}{z + i} \right| = 2$, $z \neq -i$. Then z lies on the circle of radius 2 and centre
[25-Jan-2023 Shift 2]

Options:

A. $(2, 0)$

B. $(0, 0)$

C. $(0, 2)$

D. $(0, -2)$

Answer: D

Solution:

Solution:

$(z-2i)(\overline{z}+2i)=4(z+i)(\overline{z}-i)$

$z\overline{z}+4+2i(z-\overline{z})=4(z\overline{z}+1+i(\overline{z}-z))$

$3z\overline{z}-6i(z-\overline{z})=0$

$x^2+y^2-2i(2iy)=0$

$x^2+y^2+4y=0$

Question29

For two non-zero complex number z_1 and z_2, if $Re(z_1z_2)=0$ and $Re(z_1+z_2)=0$, then which of the following are possible ?
(A) $Im(z_1)>0$ and $Im(z_2)>0$
(B) $Im(z_1)<0$ and $Im(z_2)>0$
(C) $Im(z_1)>0$ and $Im(z_2)<0$
(D) $Im(z_1)<0$ and $Im(z_2)<0$
Choose the correct answer from the options given below :
[29-Jan-2023 Shift 1]

Options:

A. B and D

B. B and C

C. A and B

D. A and C

Answer: B

Solution:

Solution:

$z_1=x_1+iy_1$

$z_2=x_2+y_2$

$Re(z_1z_2)=x_1x_2-y_1y_2=0$

$Re(z_1+z_2)=x_1+x_2=0$

$x_1\&x_2$ are of opposite sign

$y_1\&y_2$ are of opposite sign

Question30

Let $\alpha=8-14i$, $A=\left\{z\in C:\ \frac{\alpha z-\overline{\alpha z}}{z^2-(\overline{z})^2-112i}=1\right\}$ and
$B=\{z\in C:\ |z+3i|\ =4\}$
Then $\sum_{z\in A\cap B}(Re z-Im z)$ is equal to _______.
[29-Jan-2023 Shift 2]

Answer: 14

Solution:

$\alpha = 8 - 14i$
$z = x + iy$
$az = (8x + 14y) + i(-14x + 8y)$
$z + \bar{z} = 2x \quad z - \bar{z} = 2iy$
Set A: $\frac{2i(-14x + 8y)}{i(4xy - 112)} = 1$
$(x - 4)(y + 7) = 0$
$x = 4 \quad \text{or} \quad y = -7$
Set B: $x^2 + (y + 3)^2 = 16$
when $x = 4 \;\; y = -3$
when $y = -7 \;\; x = 0$
$\therefore A \cap B = \{4 - 3i, 0 - 7i\}$
So, $\sum_{z \in A \cap B} (\operatorname{Re} z - \operatorname{Im} z) = 4 - (-3) + (0 - (-7)) = 14$

Question31

Let $z = 1 + i$ and $z_1 = \frac{1 + i\bar{z}}{\bar{z}(1 - z) + \frac{1}{z}}$. Then $\frac{12}{\pi}$ arg(z_1) is equal to _______.

[30-Jan-2023 Shift 1]

Answer: 9

Solution:

$$z_1 = \frac{1 + i}{\bar{z}(1 - z) + \frac{1}{z}}$$

$$z_1 = \frac{1 + i(1 - i)}{(1 - i)(1 - 1 - i) + \frac{1}{1 + i}}$$

$$= \frac{1 + i - i^2}{(1 - i)(-i) + \frac{1 - i}{2}}$$

$$= \frac{2 + i}{-3i - 1} = \frac{4 + 2i}{-3i - 1}$$

$$= \frac{-(4 + 2i)(3i - 1)}{(3i)^2 - (1)^2}$$

$$\operatorname{Arg}(z_1) = \frac{3\pi}{4}$$

$$\therefore \frac{12}{\pi}\arg(z_1) = \frac{12}{\pi} \times \frac{3\pi}{4} = 9$$

Question32

For all $z \in C$ on the curve C_1: $|z| = 4$, let the locus of the point $z + \frac{1}{z}$ be the curve C_2. Then
[31-Jan-2023 Shift 1]

Options:

A. the curves C_1 and C_2 intersect at 4 points

B. the curves C_1 lies inside C_2

C. the curves C_1 and C_2 intersect at 2 points

D. the curves C_2 lies inside C_1

Answer: A

Solution:

Solution:

Let $w = z + \frac{1}{z} = 4e^{i\theta} + \frac{1}{4}e^{-i\theta}$

$\Rightarrow w = \frac{17}{4}\cos\theta + i\,\frac{15}{4}\sin\theta$

So locus of w is ellipse $\frac{x^2}{\left(\frac{17}{4}\right)^2} + \frac{y^2}{\left(\frac{15}{4}\right)^2} = 1$

Locus of z is circle $x^2 + y^2 = 16$

So intersect at 4 points

Question33

The complex number $z = \dfrac{i - 1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ is equal to:

[31-Jan-2023 Shift 2]

Options:

A. $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

B. $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$

C. $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

D. $\sqrt{2}i\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$

Answer: A

Solution:

$$Z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}} = \frac{i-1}{\frac{1}{2} + \frac{\sqrt{3}}{2}i}$$

$$= \frac{i-1}{\frac{1}{2} + \frac{\sqrt{3}}{2}i} \times \frac{\frac{1}{2} - \sqrt{\frac{3}{2}}i}{\frac{1}{2} - \sqrt{3/2}i} = \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}i$$

Apply polar form,

$$r\cos\theta = \frac{\sqrt{3}-1}{2}$$

$$r\sin\theta = \frac{\sqrt{3}+1}{2}$$

Now, $\tan\theta = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

So, $\theta = \frac{5\pi}{12}$

Question34

Let α be a root of the equation $(a-c)x^2 + (b-a)x + (c-b) = 0$ where a, b, c are distinct real numbers such that the matrix

$$\begin{bmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{bmatrix}$$

is singular. Then the value of $\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$ is [24-Jan-2023 Shift 1]

Options:

A. 6

B. 3

C. 9

D. 12

Answer: B

Solution:

Solution:

$$\Delta = 0 = \begin{vmatrix} \alpha^2 & \alpha & 1 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$$

$\Rightarrow \alpha^2(c-b) - \alpha(c-a) + (b-a) = 0$

It is singular when $\alpha = 1$

$$\frac{(a-c)^2}{(b-a)(c-b)} + \frac{(b-a)^2}{(a-c)(c-b)} + \frac{(c-b)^2}{(a-c)(b-a)}$$

$$\frac{(a-b)^3 + (b-c)^3 + (c-a)^3}{(a-b)(b-c)(c-a)}$$

$$= 3\frac{(a-b)(b-c)(c-a)}{(a-b)(b-c)(c-a)} = 3$$

Question35

Let $\lambda \in \mathbb{R}$ and let the equation E be $|x|^2 - 2|x| + |\lambda - 3| = 0$. Then the largest element in the set $S = \{x + \lambda : x$ is an integer solution of E$\}$ is [24-Jan-2023 Shift 1]

Answer: 5

Solution:

$|x|^2 - 2|x| + |\lambda - 3| = 0$
$|x|^2 - 2|x| + |\lambda - 3| - 1 = 0$
$(|x| - 1)^2 + |\lambda - 3| = 1$
At $\lambda = 3$, $x = 0$ and 2 ,
at $\lambda = 4$ or 2 , then
$x = 1$ or -1
So maximum value of $x + \lambda = 5$

Question36

The number of real solutions of the equation
$$3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0, \text{ is}$$
[24-Jan-2023 Shift 2]

Options:

A. 4

B. 0

C. 3

D. 2

Answer: B

Solution:

Solution:
$$3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$$
$$3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$$
Let $x + \frac{1}{x} = t$
$3t^2 - 2t - 1 = 0$
$3t^2 - 3t + t - 1 = 0$
$3t(t - 1) + 1(t - 1) = 0$

$(t-1)(3t+1)=0$

$t=1, -\frac{1}{3}$

$x+\frac{1}{x}=1, -\frac{1}{3} \Rightarrow$ No solution.

Question37

Let

$$\mathbf{S = \left\{ \alpha : \log_2(9^{2\alpha-4}+13) - \log_2\left(\frac{5}{2}\cdot 3^{2\alpha-4}+1\right) = 2 \right\}.}$$

Then the maximum value of β for which the equation $x^2 - 2\left(\sum_{\alpha \in s} \alpha\right)^2 x + \sum_{\alpha \in s}(\alpha+1)^2\beta = 0$ has real roots, is ______.

[25-Jan-2023 Shift 1]

Answer: 25

Solution:

$\log_2(9^{2\alpha-4}+13) - \log_2\left(\frac{5}{2}\cdot 3^{2\alpha-4}+1\right) = 2$

$\Rightarrow \frac{9^{2\alpha-4}+13}{\frac{5}{2}3^{2\alpha-4}+1} = 4$

$\sum_{\alpha \in S} \alpha = 5$ and $\sum_{\alpha \in S}(\alpha+1)^2 = 25$

$\Rightarrow x^2 - 50x + 25\beta = 0$ has real roots

$\Rightarrow \beta \le 25$

$\Rightarrow \beta_{max} = 25$

Question38

Let a ∈ R and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is ________.

[25-Jan-2023 Shift 2]

Answer: 45

Solution:

$\alpha + \beta = -60^{\frac{1}{4}}$ & $\alpha\beta = a$

Given $\alpha^4 + \beta^4 = -30$

$\Rightarrow (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = -30$

$\Rightarrow \{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2a^2 = -30$

$\Rightarrow \left\{ 60^{\frac{1}{2}} - 2a \right\}^2 - 2a^2 = -30$

$\Rightarrow 60 + 4a^2 - 4a \times 60^{\frac{1}{2}} - 2a^2 = -30$

$\Rightarrow 2a^2 - 4.60^{\frac{1}{2}}a + 90 = 0$

Product $= \dfrac{90}{2} = 45$

Question39

Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation :
[29-Jan-2023 Shift 1]

Options:

A. $7x^2 + 245x - 250 = 0$

B. $7x^2 - 245x + 250 = 0$

C. $49x^2 - 245x + 250 = 0$

D. $49x^2 + 245x + 250 = 0$

Answer: C

Solution:

Solution:

$14x^2 - 31x + 3\lambda = 0$

$\alpha + \beta = \dfrac{31}{14} \ldots (1)$ and $\alpha\beta = \dfrac{3\lambda}{14}$

$35x^2 - 53x + 4\lambda = 0$

$\alpha + \gamma = \dfrac{53}{35} \ldots (3)$ and $\alpha\gamma = \dfrac{4\lambda}{35} \ldots$

$\dfrac{(2)}{(4)} \Rightarrow \dfrac{\beta}{\gamma} = \dfrac{3 \times 35}{4 \times 14} = \dfrac{15}{8} \Rightarrow \beta = \dfrac{15}{8}\gamma$

$(1) - (3) \Rightarrow \beta - \gamma = \dfrac{31}{14} - \dfrac{53}{35} = \dfrac{155 - 106}{70} = \dfrac{7}{10}$

$\dfrac{15}{8}\gamma - \gamma = \dfrac{7}{10} \Rightarrow \gamma = \dfrac{4}{5}$

$\Rightarrow \beta = \dfrac{15}{8} \times \dfrac{4}{5} = \dfrac{3}{2}$

$\Rightarrow \alpha = \dfrac{31}{14} - \beta = \dfrac{31}{14} - \dfrac{3}{2} = \dfrac{5}{7}$

$\Rightarrow \lambda = \dfrac{14}{3}\alpha\beta = \dfrac{14}{3} \times \dfrac{5}{7} \times \dfrac{3}{2} = 5$

so, sum of roots $\dfrac{3\alpha}{\beta} + \dfrac{4\alpha}{\gamma} = \left(\dfrac{3\alpha\gamma + 4\alpha\beta}{\beta\gamma} \right)$

$= \dfrac{\left(3 \times \frac{4\lambda}{35} + 4 \times \frac{3\lambda}{14} \right)}{\beta\gamma} = \dfrac{12\lambda(14 + 35)}{14 \times 35\beta\gamma}$

$$= \frac{49 \times 12 \times 5}{490 \times \frac{3}{2} \times \frac{4}{5}} = 5$$

Product of roots

$$= \frac{3\alpha}{\beta} \times \frac{4\alpha}{\gamma} = \frac{12\alpha^2}{\beta\gamma} = \frac{12 \times \frac{25}{49}}{\frac{3}{2} \times \frac{4}{5}} = \frac{250}{49}$$

So, required equation is $x^2 - 5x + \frac{250}{49} = 0$

$\Rightarrow 49x^2 - 245x + 250 = 0$

Question40

If the value of real number a > 0 for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real roots is $\frac{3}{\sqrt{2\beta}}$ then β is equal to ________.
[30-Jan-2023 Shift 2]

Answer: 13

Solution:

Solution:
Two equations have common root

$\therefore (4a)(26a) = (-6)^2 = 36$

$\Rightarrow a^2 = \frac{9}{26} \quad \therefore a = \frac{3}{\sqrt{26}} \Rightarrow \beta = 13$

Question41

The number of real roots of the equation $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$, is:
[31-Jan-2023 Shift 1]

Options:

A. 0

B. 1

C. 3

D. 2

Answer: B

Solution:

$\sqrt{(x-1)(x-3)} + \sqrt{(x-3)(x+3)}$

$= \sqrt{4\left(x - \frac{12}{4}\right)\left(x - \frac{2}{4}\right)}$

$\Rightarrow \sqrt{x-3} = 0 \Rightarrow x = 3$ which is in domain

$\sqrt{x-1} + \sqrt{x+3} = \sqrt{4x-2}$

$2\sqrt{(x-1)(x+3)} = 2x - 4$

$x^2 + 2x - 3 = x^2 - 4x + 4$

$6x = 7$

$x = 7/6$ or

Question42

The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0, x \in R$ has: [31-Jan-2023 Shift 2]

Options:

A. two solutions and both are negative

B. no solution

C. four solutions two of which are negative

D. two solutions and only one of them is negative

Answer: A

Solution:

Solution:

$e^{4x} + 8e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0$

Let $e^x = t$

Now, $t^4 + 8t^3 + 13t^2 - 8t + 1 = 0$

Dividing equation by t^2,

$t^2 + 8t + 13 - \frac{8}{t} + \frac{1}{t^2} = 0$

$t^2 + \frac{1}{t^2} + 8\left(t - \frac{1}{t}\right) + 13 = 0$

$\left(t - \frac{1}{t}\right)^2 + 2 + 8\left(t - \frac{1}{t}\right) + 13 = 0$

Let $t - \frac{1}{t} = z$

$z^2 + 8z + 15 = 0$

$(z+3)(z+5) = 0$

$z = -3$ or $z = -5$

So, $t - \frac{1}{t} = -3$ or $t - \frac{1}{t} = -5$

$t^2 + 3t - 1 = 0$ or $t^2 + 5t - 1 = 0$

$t = \frac{-3 \pm \sqrt{13}}{2}$ or $t = \frac{-5 \pm \sqrt{29}}{2}$

as $t = e^x$ so t must be positive,

$t = \frac{\sqrt{13} - 3}{2}$ or $\frac{\sqrt{29} - 5}{2}$

So, $x = \ln\left(\frac{\sqrt{13} - 3}{2}\right)$ or $x = \ln\left(\frac{\sqrt{29} - 5}{2}\right)$

Hence two solution and both are negative.

Question43

If the center and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α, β) and γ, then $3(\alpha + \beta + \gamma)$ is equal to
[1-Feb-2023 Shift 1]

Options:

A. 11

B. 9

C. 10

D. 12

Answer: D

Solution:

Solution:

$\sqrt{(x-2)^2 + y^2} = 2\sqrt{(x-3)^2 + y^2}$

$= x^2 + y^2 - 4x + 4 = 4x^2 + 4y^2 - 24x + 36$

$= 3x^2 + 3y^2 - 20x + 32 = 0$

$= x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$

$= (\alpha, \beta) = \left(\frac{10}{3}, 0\right)$

$\gamma = \sqrt{\frac{100}{9} - \frac{32}{3}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

$3(\alpha, \beta, \gamma) = 3\left(\frac{10}{3} + \frac{2}{3}\right)$

$= 12$

Question44

Let a, b be two real numbers such that $ab < 0$. If the complex number $\frac{1+ai}{b+i}$ is of unit modulus and $a + ib$ lies on the circle $|z-1| = |2z|$, then a possible value of $\frac{1+[a]}{4b}$, where $[t]$ is greatest integer function, is :
[1-Feb-2023 Shift 2]

Options:

A. $-\frac{1}{2}$

B. -1

C. 1

D. $\frac{1}{2}$

E. 0

Answer: E

Solution:

Solution:

$\left| \frac{1+ai}{b+i} \right| = 1$

$|1+ia| = |b+i|$

$a^2 + 1 = b^2 + 1 \Rightarrow a = \pm b \Rightarrow b = -a \quad \text{as } ab < 0$

$(a+ib)$ lies on $|z-1| = |2z|$

$|a+ib-1| = 2|a+ib|$

$(a-1)^2 + b^2 = 4(a^2+b^2)$

$(a-1)^2 = a^2 = 4(2a^2)$

$1 - 2a = 6a^2 \Rightarrow 6a^2 + 2a - 1 = 0$

$a = \frac{-2 \pm \sqrt{28}}{12} = \frac{-1 \pm \sqrt{7}}{6}$

$a = \frac{\sqrt{7}-1}{6}$ and $b = \frac{1-\sqrt{7}}{6}$

$[a] = 0$

$\therefore \frac{1+[a]}{4b} = \frac{6}{4(1-\sqrt{7})} = -\left(\frac{1+\sqrt{7}}{4}\right)$

Similarly when $a = \frac{-1-\sqrt{7}}{6}$ and $b = \frac{1+\sqrt{7}}{6}$ then $[a] = -1$

$\therefore \frac{1+[a]}{4b} = \frac{1-1}{4 \times \frac{1+\sqrt{7}}{6}} = 0$

Question45

Two dice are thrown independently. Let A be the event that the number appeared on the 1^{st} die is less than the number appeared on the 2^{nd} die, B be the event that the number appeared on the 1^{st} die is even and that on the second die is odd, and C be the event that the number appeared on the 1^{st} die is odd and that on the 2^{nd} is even. Then [1-Feb-2023 Shift 2]

Options:

A. the number of favourable cases of the event $(A \cup B) \cap C$ is 6

B. A and B are mutually exclusive

C. The number of favourable cases of the events A, B and C are 15,6 and 6 respectively

D. B and C are independent

Answer: A

Solution:

Solution:

A : no. on 1^{st} die < no. on 2^{nd} die

A : no. on 1^{st} die = even & no. of 2^{nd} die = odd

C : no. on 1^{st} die = odd & no. on 2^{nd} die = even

$n(A) = 5+4+3+2+1 = 15$

$n(B) = 9$

$n(C) = 9$

$n((A \cup B) \cap C) = (A \cap C) \cup (B \cap C)$

$= (3+2+1) + 0 = 6.$

Question46

Let
$S = \{ x : x \in \mathbb{R}. \text{ and } (\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4} = 10 \}$.
Then n(S) is equal to
[1-Feb-2023 Shift 1]

Options:

A. 2

B. 4

C. 6

D. 0

Answer: B

Solution:

Solution:

Sol. Let $(\sqrt{3}+\sqrt{2})^{x^2-4} = t$

$t + \frac{1}{t} = 10$

$\Rightarrow\ t = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

$\Rightarrow\ (\sqrt{3}+\sqrt{2})^{x^2-4} = 5 + 2\sqrt{6}, 5 - 2\sqrt{6}$

$\Rightarrow\ x^2 - 4 = 2, -2 \quad \text{or} \quad x^2 = 6, 2$

$\Rightarrow\ x = \pm\sqrt{2}, \pm\sqrt{6}$

Question47

Let $a \neq b$ be two-zero real numbers. Then the number of elements in the set $X = \{ z \in C : \text{Re}(az^2 + bz) = a. \text{ and } \text{Re}(bz^2 + az) = b \}$ is equal to :
[6-Apr-2023 shift 2]

Options:

A. 0

B. 2

C. 1

D. 3

Answer: A

Solution:

Solution:

(1) Bonus

$\because z + \bar{z} = 2\,\text{Re}(z) \quad \text{If } z = x + iy$

$\Rightarrow z + \bar{z} = 2x$

$z^2 + (\bar{z})^2 = 2(x^2 - y^2)$

$(az^2 + bz) + (a\bar{z}^2 + b\bar{z}) = 2a \ldots (1)$
$(bz^2 + az) + (b\bar{z}^2 + a\bar{z}) = 2b \ldots (2)$
add (1) and (2)
$(a+b)z^2 + (a+b)z + (a+b)\bar{z}^2 + (a+b)\bar{z} = 2(a+b)$
$(a+b)[z^2 + z + (\bar{z})^2 + \bar{z}] = 2(a+b)$
sub. (1) and (2)
$(a-b)[z^2 - z + \bar{z}^2 - \bar{z}] = 2(a-b) \ldots (3)$
$z^2 + \bar{z}^2 - z - \bar{z} = 2 \ldots (4)$
Case I: If $a + b \neq 0$
From (3) & (4)
$2x + 2(x^2 - y^2) = 2 \Rightarrow x^2 - y^2 + x = 1 \ldots (5)$
$2(x^2 - y^2) - 2x = 2 \Rightarrow x^2 - y^2 - x = 1 \ldots (6)$
(5) − (6)
$2x = 0 \Rightarrow x = 0$
from (5) $y^2 = -1 \Rightarrow$ not possbible
$\therefore$ Ans $= 0$
Case II: If $a + b = 0$ then infinite number of solution.
So, the set X have infinite number of elements.

Question48

For α, β, z ∈ C and λ > 1, if $\sqrt{\lambda - 1}$ is the radius of the circle $|z-\alpha|^2 + |z-\beta|^2 = 2\lambda$, then $|\alpha - \beta|$ is equal to _______ :
[6-Apr-2023 shift 2]

Answer: 2

Solution:

$|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$
$z_1 = \alpha, z_2 = \beta$
$|\alpha - \beta|^2 = 2\lambda$
$|\alpha - \beta| = \sqrt{2\lambda}$
$2r = \sqrt{2\lambda}$
$2\sqrt{\lambda - 1} = \sqrt{2\lambda}$
$\Rightarrow 4(\lambda - 1) = 2\lambda$
$\lambda = 2$
$|\alpha - \beta| = 2$

Question49

If for $z = \alpha + i\beta$, $|z + 2| = z + 4(1 + i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation
[8-Apr-2023 shift 1]

Options:

A. $x^2 + 3x - 4 = 0$

B. $x^2 + 7x + 12 = 0$

C. $x^2 + x - 12 = 0$

D. $x^2 + 2x - 3 = 0$

Answer: B

Solution:

Solution:

$| z + 2 | = | \alpha + i\beta + 2 |$

$= \alpha + i\beta + 4 + 4i$

$\sqrt{(\alpha + 2)^2 + \beta^2} = (\alpha + 4) + i(\beta + 4) \quad \beta + 4 = 0$

$(\alpha + 2)^2 + 16 = (\alpha + 4)^2$

$\alpha^2 + 4 + 4\alpha + 16 = \alpha^2 + 16 + 8\alpha$

$4 = 4\alpha$

$\alpha = 1$

$\alpha = 1, \beta = -4$

$\alpha + \beta = -3, \alpha\beta = -4$

Sum of roots $= -7$

Product of roots $= 12$

$x^2 + 7x + 12 = 0$

Question50

Let A = $\left\{ \theta \in (0, 2\pi) : \frac{1 + 2i\sin\theta}{1 - i\sin\theta} \right.$. is purely imaginary $\}$. Then the sum of the elements in A is.
[8-Apr-2023 shift 2]

Options:

A. π

B. 3π

C. 4π

D. 2π

Answer: C

Solution:

Solution:

$z = \frac{1 + 2i\sin\theta}{1 - i\sin\theta} \times \frac{1 + i\sin\theta}{1 + i\sin\theta}$

$z = \frac{1 - 2\sin^2\theta + i(3\sin\theta)}{1 + \sin^2\theta}$

$\mathrm{Re}(z) = 0$

$\frac{1 - 2\sin^2\theta}{1 + \sin^2\theta} = 0$

$\sin\theta = \frac{\pm 1}{\sqrt{2}}$

$A = \left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$

sum $= 4\pi$(Option 3)

Question51

Let the complex number z = x+ iy be such that $\frac{2z-3i}{2z+i}$ is purely imaginary. If $x+y^2=0$, then y^4+y^2-y is equal to :
[10-Apr-2023 shift 1]

Options:

A. $\frac{3}{2}$

B. $\frac{2}{3}$

C. $\frac{4}{3}$

D. $\frac{3}{4}$

Answer: D

Solution:

Solution:

$z = x + iy$

$\frac{(2z-3i)}{2z+i} =$ purely imaginary

Means $\operatorname{Re}\left(\frac{2z-3i}{2z+i}\right) = 0$

$\Rightarrow \frac{(2z-3i)}{(2z+i)} = \frac{2(x+iy)-3i}{2(x+iy)+i}$

$= \frac{2x+2yi-3i}{2x+i2y+i}$

$= \frac{2x+i(2y-3)}{2x+i(2y+1)} \times \frac{2x-i(2y+1)}{2x-i(2y+1)}$

$\operatorname{Re}\left[\frac{2z-3i}{2z+i}\right] = \frac{4x^2+(2y-3)(2y+1)}{4x^2+(2y+1)^2} = 0$

$\Rightarrow 4x^2+(2y-3)(2y+1) = 0$

$\Rightarrow 4x^2+[4y^2+2y-6y-3] = 0$

$\because x+y^2 = 0 \Rightarrow x = -y^2$

$\Rightarrow 4(-y^2)^2+4y^2-4y-3 = 0$

$\Rightarrow 4y^4+4y^2-4y = 3$

$\Rightarrow y^4+y^2-y = \frac{3}{4}$

Therefore, correct answer is option (4).

Question52

Let S = $\left\{ z = x+iy : \frac{2z-3i}{4z+2i}\right.$. is a real number $\}$. Then which of the following is NOT correct?
[10-Apr-2023 shift 2]

Options:

A. $y \in \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$

B. $(x, y) = \left(0, -\frac{1}{2}\right)$

C. $x = 0$

D. $y + x^2 + y^2 \neq -\frac{1}{4}$

Answer: B

Solution:

Solution:

$\frac{2z - 3i}{4z + 2i} \in R$

$\frac{2(x + iy) - 3i}{4(x + iy) + 2i} = \frac{2x + (2y - 3)i}{4x + (4y + 2)i} \times \frac{4x - (4y + 2)i}{4x - (4y + 2)i}$

$4x(2y - 3) - 2x(4y + 2) = 0$

$x = 0 \;\; y \neq -\frac{1}{2}$

Ans. $= 2$

Question53

Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to :

[11-Apr-2023 shift 1]

Options:

A. $\pi - \tan^{-1}\frac{8}{9}$

B. $-\pi + \tan^{-1}\frac{8}{9}$

C. $\pi - \tan^{-1}\frac{33}{5}$

D. $-\pi + \tan^{-1}\frac{33}{5}$

Answer: A

Solution:

Solution:

$W_1 = z_i i = (5 + 4i)i = -4 + 5i \ldots$ (i)

$W_1 = z_2(-i) = (3 + 5i)(-i) = 5 - 3i \ldots$ (2)

$W_1 - W_2 = -9 + 8i$

Principal argument $= \pi - \tan^{-1}\left(\frac{8}{9}\right)$

Question54

For $a \in C$, let $A = \{z \in C : Re(a + \overline{z}) > Im(\overline{a} + z)\}$ and $B = \{z \in C : Re(a + \overline{z}) < Im(\overline{a} + z)\}$. The among the two statements:
(S1) : If Re(a), Im(a) > 0, then the set A contains all the real numbers
(S2) : If Re(a), Im(a) < 0, then the set B contains all the real numbers,
[11-Apr-2023 shift 2]

Options:

A. only (S1) is true

B. both are false

C. only (S2) is true

D. both are true

Answer: B

Solution:

Let $a = x_1 + iy_1 z = x + iy$
$\text{Now}\, Re(a + \overline{z}) > Im(\overline{a} + z)$
$\therefore x_1 + x > -y_1 + y$
$x_1 = 2, y_1 = 10, x = -12, y = 0$
Given inequality is not valid for these values.
S1 is false.
$\text{Now}\ \ Re(a + \overline{z}) < Im(\overline{a} + z)$
$x_1 + x < -y_1 + y$
$x_1 = -2, y_1 = -10, x = 12, y = 0$
Given inequality is not valid for these values. S2 is false.

Question55

Let $S = \left\{ z \in C - \{i, 2i\} : \frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in R \right\}$. If $\alpha - \frac{13}{11}i \in S, a \in R - \{0\}$, then $242\alpha^2$ is equal to _______.
[11-Apr-2023 shift 2]

Answer: 1680

Solution:

Solution:

$$\left(\frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \right) \in R$$

$$\Rightarrow 1 + \frac{(11iz - 13)}{(z^2 - 3iz - 2)} \in R$$

Put $Z = \alpha - \frac{13}{11}i$

$\Rightarrow (z^2 - 3iz - 2)$ is imaginary

Put $z = x + iy$

$\Rightarrow (x^2 - y^2 + 2xyi - 3ix + 3y - 2) \in$ Imaginary

$\Rightarrow Re(x^2 - y^2 + 3y - 2 + (2xy - 3x)i) = 0$

$\Rightarrow x^2 - y^2 + 3y - 2 = 0$

$x^2 = y^2 - 3y + 2$

$x^2 = (y-1)(y-2) \therefore z = \alpha - \frac{13}{11}i$

Put $x = \alpha, y = \frac{-13}{11}$

$\alpha^2 = \left(\frac{-13}{11} - 11\right)\left(\frac{-13}{11} - 2\right)$

$\alpha^2 = \frac{(24 \times 35)}{121}$

$242\alpha^2 = 48 \times 35 = 1680$

Question56

Let C be the circle in the complex plane with centre $z_0 = \frac{1}{2}(1 + 3i)$ and radius $r = 1$. Let $z_1 = 1 + i$ and the complex number z_2 be outside the circle C such that $|z_1 - z_0||z_2 - z_0| = 1$. If $z_0 \cdot z_1$ and z_2 are collinear, then the smaller value of $|z_2|^2$ is equal to

[12-Apr-2023 shift 1]

Options:

A. $\frac{7}{2}$

B. $\frac{13}{2}$

C. $\frac{5}{2}$

D. $\frac{3}{2}$

Answer: C

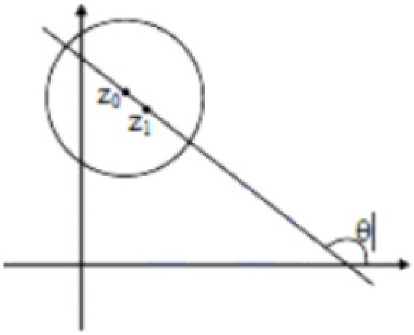
Solution:

Solution:

$|z_1 - z_0| = \left|\frac{1-i}{2}\right| = \frac{1}{2}$

$\Rightarrow |z_2 - z_0| = \sqrt{2}$: centre $\left(\frac{1}{2}, \frac{3}{2}\right)$

$z_0\left(\frac{1}{2}, \frac{3}{2}\right)$ and $z_1(1, 1)$



$\tan\theta = -1 \Rightarrow \theta = 135^\circ$
$z_2\left(\frac{1}{2} + \sqrt{2}\cos 135^\circ,\ \frac{3}{2} + \sqrt{2}\sin 135^\circ\right)$
or
$\left(\frac{1}{2} - \sqrt{2}\cos 135^\circ,\ \frac{3}{2} - \sqrt{2}\sin 135^\circ\right)$
$\Rightarrow z_2\left(-\frac{1}{2}, \frac{5}{2}\right)$ or $z_2\left(\frac{3}{2}, \frac{1}{2}\right)$
$\Rightarrow |z_3|^2 = \frac{26}{4}, \frac{5}{2}$
$\Rightarrow |z_2|_{min}{}^2 = \frac{5}{2}$

Question57

Let $S = \{z \in C : \bar{z} = i(z^2 + Re(\bar{z}))\}$. Then $\sum\limits_{z \in S} |z|^2$ is equal to [13-Apr-2023 shift 2]

Options:

A. 4

B. $\frac{7}{2}$

C. 3

D. $\frac{5}{2}$

Answer: A

Solution:

Let $z = x + iy$
$\bar{z} = i(z^2 + Re(\bar{z}))$
$\Rightarrow x - iy = i(x^2 - y^2 + 2ixy + x)$
$\Rightarrow x - iy = -2xy + i(x^2 - y^2 + x)$
$x + 2xy = 0$ and $x^2 - y^2 + x + y = 0$
$x(1 + 2y) = 0$ and $x^2 - y^2 + x + y = 0$
If $x = 0$ then $-y^2 + y = 0$
$\Rightarrow y = 1, 0$
If $y = \frac{-1}{2}$ then $x^2 - \frac{1}{4} + x - \frac{1}{2} = 0$
$\Rightarrow x = -\frac{3}{2}, \frac{1}{2}$
$= \left\{0 + i0, 0 + i, -\frac{3}{2} - \frac{1}{2}i, \frac{1}{2} - \frac{1}{2}i\right\}$
$\sum\limits_{Z \in S\,|\,Z|}^{2\,=\,0\,+\,1\,+} \frac{9}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 4$

Question58

If the set $\left\{ \operatorname{Re}\left(\frac{z - \bar{z} + z\bar{z}}{2 - 3z + 5z} \right) : z \in C, \operatorname{Re}(z) = 3 \right\}$ is equal to the interval $(\alpha, \beta]$, then $24(\beta - \alpha)$ is equal to
[15-Apr-2023 shift 1]

Options:

A. 36

B. 27

C. 30

D. 42

Answer: C

Solution:

Solution:

Let $z_1 = \left(\frac{z - \bar{z} + z\bar{z}}{2 - 3z + 5\bar{z}} \right)$

Let $z = 3 + iy$

$\bar{z} = 3 - iy$

$$z_1 = \frac{2iy + (9 + y^2)}{2 - 3(3 + iy) + 5(3 - iy)}$$

$$= \frac{9 + y^2 + i(2y)}{8 - 8iy}$$

$$= \frac{(9 + y^2) + i(2y)}{8(1 - iy)}$$

$$\operatorname{Re}(z_1) = \frac{(9 + y^2) - 2y^2}{8(1 + y^2)}$$

$$= \frac{9 - y^2}{8(1 + y^2)}$$

$$= \frac{1}{8}\left[\frac{10 - (1 + y^2)}{(1 + y^2)} \right]$$

$$= \frac{1}{8}\left[\frac{10}{(1 + y^2)} - 1 \right]$$

$1 + y^2 \in [1, \infty]$

$\frac{1}{1 + y^2} \in (0, 1]$

$\frac{10}{1 + y^2} \in (0, 10]$

$\frac{10}{1 + y^2} - 1 \in (-1, 9]$

$\operatorname{Re}(z_1) \in \left(\frac{-1}{8}, \frac{9}{8} \right]$

$\alpha = \frac{-1}{8}, \beta = \frac{9}{8}$

$24(\beta - \alpha) = 24\left(\frac{9}{8} + \frac{1}{8} \right) = 30$

Question59

The sum of all the roots of the equation $|x^2 - 8x + 15| - 2x + 7 = 0$ is : [6-Apr-2023 shift 1]

Options:

A. $11 - \sqrt{3}$

B. $9 - \sqrt{3}$

C. $9 + \sqrt{3}$

D. $11 + \sqrt{3}$

Answer: C

Solution:

Solution:

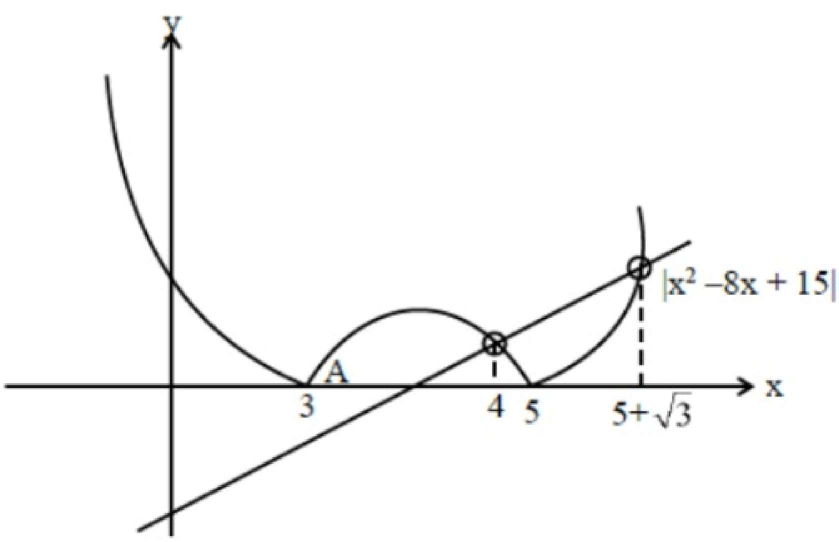
$|x^2 - 8x + 15| = 2x - 7$

$x^2 - 8x + 15 = 2x - 7$ & $x^2 - 8x + 15 = 7 - 2x$

$x^2 - 10x + 22 = 0$ & $x^2 - 6x + 8 = 0$

$x_1 = 5 + \sqrt{3}$ $x_2 = 5 - \sqrt{3}$ (reject) $x_3 = 4$ $x_4 = 2$ (reject)

Sum of of roots is $= 5 + \sqrt{3} + 4 = 9 + \sqrt{3}$



Question60

Let α, β, γ, be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to
[8-Apr-2023 shift 1]

Options:

A. $\frac{155}{8}$

B. 21

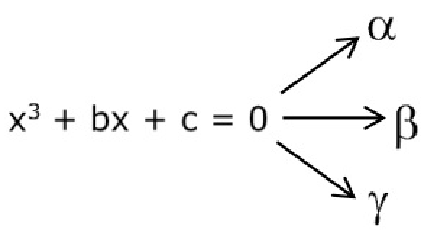
C. 19

D. $\frac{169}{8}$

Answer: C

Solution:

Solution:



$\beta\gamma = 1$
$\alpha = -1$
Put $\alpha = -1$
$-1 - b + c = 0$
$c - b = 1$
also
$\alpha \cdot \beta \cdot \gamma = -c$
$-1 = -c \Rightarrow c = 1$
$\therefore b = 0$
$x^3 + 1 = 0$
$\alpha = -1, \beta = -w, \gamma = -w^2$
$\therefore b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$
$0 + 2 + 3 + 6 + 8 = 19$

Question61

Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5\,|\,x + 2\,| - 4 = 0$ respectively, where [x] denotes the greatest integer leq x. Then $m^2 + mn + n^2$ is equal to
[8-Apr-2023 shift 2]

Answer: 9

Solution:

$x^2 - 12x + [x] + 31 = 0$
$\{x\} = x^2 - 11x + 31$
$0 \le x^2 - 11x + 31 < 1$
$x^2 - 11x + 30n < 0$
$x \in (5, 6)$
so $[x] = 5$
$x^2 - 12x + 5 + 31 = 0$
$x^2 - 12x + 36 = 0$
$x = 6$ but $x \in (5, 6)$
so $x \in \varphi$
$m = 0$

Now

$$x^2 - 5|x+2| - 4 = 0$$

$x \ge -2$	$x < -2$
$x^2 - 5x - 14 = 0$	$x^2 + 5x + 6 = 0$
$(x-7)(x+2) = 0$	$(x+3)(x+2) = 0$
$x = 7, -2$	$x = -3, -2$

$x = \{7, -2, -3\}$
$n = 3$
$m^2 + mn + n^2 = n^2 = 9$

Question62

If a and b are the roots of the equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is equal to ________.
[11-Apr-2023 shift 1]

Answer: 51

Solution:

By newton's theorem

$$S_{n+2} - 7S_{n+1} - S_n = 0$$
$$S_{21} - 7S_{20} - S_{19} = 0$$
$$S_{20} - 7S_{19} - S_{18} = 0$$
$$S_{19} - 7S_{18} - S_{17} = 0$$
$$\frac{S_{21} + S_{17}}{S_{19}} = \frac{S_{21} + (S_{19} - 7S_{18})}{S_{19}}$$
$$= \frac{50S_{19} + (S_{21} - 7S_{20})}{S_{19}}$$
$$= 51 \cdot \frac{S_{19}}{S_{19}} = 51$$

Question63

The number of points where the curve

$f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1, x \in R$ cuts x-axis, is equal to ______
[11-Apr-2023 shift 2]

Answer: 2

Solution:

Let $e^{2x} = t$

$\Rightarrow t^4 - t^3 - 3t^2 - t + 1 = 0$

$\Rightarrow t_2 + \frac{1}{t_2} - \left(t + \frac{1}{t}\right) - 3 = 0$

$\Rightarrow \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 5 = 0$

$\Rightarrow t + \frac{1}{t} = \frac{1 + \sqrt{21}}{2}$

Two real values of t.

Question64

Let α, β be the roots of the quadratic equation $x^2 + \sqrt{6}x + 3 = 0$. Then $\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$ is equal to
[12-Apr-2023 shift 1]

Options:

A. 9

B. 729

C. 72

D. 81

Answer: D

Solution:

Solution:

$\alpha, \beta = \frac{-\sqrt{6} \pm \sqrt{6 - 12}}{2} = \frac{-\sqrt{6} \pm \sqrt{6}i}{2}$

$= \sqrt{3e^{\pm \frac{3\pi i}{4}}}$

Required expression

$$\frac{(\sqrt{3})^{23}\left(2\cos\frac{69\pi}{4}\right) + (\sqrt{3})^{14}\left(2\cos\frac{42\pi}{4}\right)}{(\sqrt{3})^{15}\left(2\cos\frac{45\pi}{4}\right) + (\sqrt{3})^{10}\left(2\cos\frac{30\pi}{4}\right)}$$

$(\sqrt{3})^8 = 81$

Question65

Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$, Then $\alpha^{14} + \beta^{14}$ is equal to
[13-Apr-2023 shift 2]

Options:

A. $-128\sqrt{2}$

B. $-64\sqrt{2}$

C. -128

D. -64

Answer: C

Solution:

Solution:

$x^2 - \sqrt{2}x + 2 = 0$

$x = \dfrac{\sqrt{2} \pm \sqrt{-6}}{2}$

$= \sqrt{2}\left(\dfrac{1 \pm i\sqrt{3}}{2}\right)$

$= -\sqrt{2}\omega, -\sqrt{2}\omega^2$

$\Rightarrow \alpha - \sqrt{2}\omega, \beta = -\sqrt{2}\omega^2$

$\alpha^{14} + \beta^{14} = 2^7(\omega^{14} + \omega^{28}) = 2^7(\omega^2 + \omega) = -128$

Question66

The number of real roots of the equation $x|x| - 5|x + 2| + 6 = 0$, is
[15-Apr-2023 shift 1]

Options:

A. 5

B. 6

C. 4

D. 3

Answer: D

Solution:

Solution:

$x|x| - 5|x + 2| + 6 = 0$

$C - 1 : -x \in [0, \infty]$

$x^2 - 5x - 4 = 0$

$x = \dfrac{5 \pm \sqrt{25 + 16}}{2} = \dfrac{5 + \sqrt{41}}{2}$

$x = \frac{5 \pm \sqrt{41}}{2}$
$C - 2 : - : -x \in [-2, 0)$
$-x^2 - 5x - 4 = 0$
$x^2 + 5x + 4 = 0$
$x = -1, -4$
$x = -1$
$C - 3 : x \in [-\infty, -2)$
$-x^2 + 5x + 16 = 0$
$x^2 - 5x - 16 = 0$
$x = \frac{5 \pm \sqrt{25 + 64}}{2}$
$x = \frac{5 \pm \sqrt{89}}{2}$
$x = \frac{5 - \sqrt{89}}{2}$

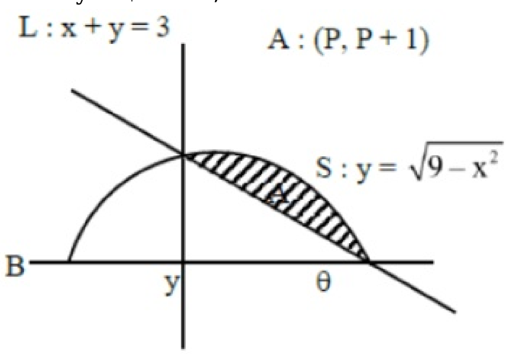
Question67

Let the point (p, p + 1) lie inside the region $E = \left\{ (x, y) : 3 - x \le y \le \sqrt{9 - x^2}, 0 \le x \le 3 \right\}$. If the set of all values of p is the interval (a, b), then $b^2 + b - a^2$ is equal to _______.
[6-Apr-2023 shift 1]

Answer: 3

Solution:

Solution:
$3 - x \le y \le \sqrt{9 - x^2}; 0 \le x \le 3$



$L(A) > 0 \Rightarrow P + P + 1 - 3 > 0 \Rightarrow P > 1 \ldots (1)$
$S(A) < 0 \Rightarrow P + 1 - \sqrt{9 - P^2} < 0$
$\Rightarrow P + 1 < \sqrt{9 - P^2}$
$\Rightarrow P + 2P + 1 < 9 - P^2$
$\Rightarrow 2P^2 + 2P - 8 < 0$
$\Rightarrow P^2 + P - 4 < 0$
$\Rightarrow P \in \left(\frac{-1 - \sqrt{17}}{2}, \frac{-1 + \sqrt{17}}{2} \right) \ldots (2)$
$(1) \cap (2) P \in \left(1, \frac{\sqrt{17} - 1}{2} \right) \equiv (a, b)$
$b^2 + b - a^2 = 4 - 1 = 3$

Question68

Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_c c}$. Then $6a + 5bc$ is equal to ______.
[10-Apr-2023 shift 1]

Answer: 8

Solution:

Solution:
$(2a)^{\ln a} = (bc)^{\ln b}$ $2a > 0, bc > 0$
$\ln a(\ln 2 + \ln a) = \ln b(\ln b + \ln c)$
$\ln 2 \cdot \ln b = \ln c \cdot \ln a$
$\ln 2 = \alpha, \ln a = x, \ln b = y, \ln c = z$
$\alpha y = xz$
$x(\alpha + x) = y(y + z)$
$\alpha = \frac{xz}{y}$
$x\left(\frac{xz}{y} + x\right) = y(y + z)$
$x^2(z + y) = y^2(y + z)$
$y + z = 0$ or $x^2 = y^2 \Rightarrow x = -y$
$bc = 1$ or $ab = 1$
$bc = 1$ or $ab = 1$

(1) if $bc = 1 \Rightarrow (2a)^{\ln a} = 1$ $\nearrow a = 1$, $\searrow a = 1/2$

$(a, b, c) = \left(\frac{1}{2}, \lambda, \frac{1}{\lambda}\right), \lambda \neq 1, 2, \frac{1}{2}$
then
$6a + 5bc = 3 + 5 = 8$
(II) $(a, b, c) = \left(\lambda, \frac{1}{\lambda}, \frac{1}{2}\right), \lambda \neq 1, 2, \frac{1}{2}$
In this situation infinite answer are possible
So, Bonus.

Question69

The number of integral solutions x of $\log_{\left(x + \frac{7}{2}\right)}\left(\frac{x - 7}{2x - 3}\right)^2 \geq 0$ is :

[11-Apr-2023 shift 1]

Options:

A. 5

B. 7

C. 8

D. 6

Answer: D

Solution:

Solution:

$\log_{x+\frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2 \geq 0$

Feasible region: $x+\frac{7}{2} > 0 \Rightarrow x > -\frac{7}{2}$

And $x+\frac{7}{2} \neq 1 \Rightarrow x \neq \frac{-5}{2}$

Taking intersection: $x \in \left(\frac{-7}{2}, \infty\right) - \left\{-\frac{5}{2}, \frac{3}{2}, 7\right\}$

Now $\log_a b \geq 0$ if $a > 1$ and $b \geq 1$

$a \in (0, 1)$ and $b \in (0, 1)$

$C - I;\ x+\frac{7}{2} > 1$ and $\left(\frac{x-7}{2x-3}\right)^2 \geq 1$

$x > -\frac{5}{2};\ (2x-3)^2-(x-7)^2 \leq 0$

$(2x-3+x-7)(2x-3-x+7) \leq 0$

$(3x-10)(x+4) \leq 0$

$x \in \left[-4, \frac{10}{3}\right]$

Intersection: $x \in \left(\frac{-5}{2}, \frac{10}{3}\right]$

$C - \Pi : x+\frac{7}{2} \in (0, 1)$ and $\left(\frac{x-7}{2x-3}\right)^2 \in (0, 1)$

$0 < x+\frac{7}{2} < 1;\ \left(\frac{x-7}{2x-3}\right)^2 < 1$

$-\frac{7}{2} < x < \frac{-5}{2};\ (x-7)^2 < (2x-3)^2$

$x \in (-\infty, -4) \cup \left(\frac{10}{3}, \infty\right)$

No common values of x.

Hence intersection with feasible region

We get $x \in \left(\frac{-5}{2}, \frac{10}{3}\right] - \left\{\frac{3}{2}\right\}$

Integral value of x are $\{-2, -1, 0, 1, 2, 3\}$

No. of integral values $= 6$

Question70

If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2+\lambda x-1=0$ is 15 , then $6(\alpha^3+\beta^3)^2$ is equal to: [24-Jun-2022-Shift-1]

Options:

A. 18

B. 24

C. 36

D. 96

Answer: B

Solution:

$3x^2 + \lambda x - 1 = 0$

Given, two roots are α and β.

$\therefore$ Sum of roots $= \alpha + \beta = \dfrac{-\lambda}{3}$

And product of roots $= \alpha\beta = \dfrac{-1}{3}$

Given that,

Sum of square of reciprocal of roots α and β is 15 .

$\therefore \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} = 15$

$\Rightarrow \dfrac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = 15$

$\Rightarrow \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = 15$

$\Rightarrow \dfrac{\dfrac{\lambda^2}{9} + 2 \times \dfrac{1}{3}}{\dfrac{1}{9}} = 15$

$\Rightarrow \dfrac{\dfrac{\lambda^2 + 6}{9}}{\dfrac{1}{9}} = 15$

$\Rightarrow \lambda^2 + 6 = 15$

$\Rightarrow \lambda^2 = 9$

Now, $6(\alpha^3 + \beta^3)^2$

$= 6\{(\alpha+\beta)(\alpha^2 + \beta^2 - \alpha\beta)\}^2$

$= 6(\alpha+\beta)^2[(\alpha+\beta)^2 - 2\alpha\beta - \alpha\beta]^2$

$= 6\left(\dfrac{-\lambda}{3}\right)^2\left[\left(\dfrac{-\lambda}{3}\right)^2 - 3\cdot\dfrac{-1}{3}\right]^2$

$= 6 \times \dfrac{\lambda^2}{9} \times \left[\dfrac{\lambda^2}{9} + 1\right]$

$= 6 \times \dfrac{9}{9} \times \left[\dfrac{9}{9} + 1\right]^2$

$= 6 \times (2)^2$

$= 6 \times 4 = 24$

Question71

Let S = { z ∈ C: | z − 3 | ≤1 and z(4 + 3i) + $\bar{z}$(4 − 3i) ≤ 24 }. If α + iβ is the point in S which is closest to 4i, then 25(α + β) is equal to____
[24-Jun-2022-Shift-2]

Answer: 80

Solution:

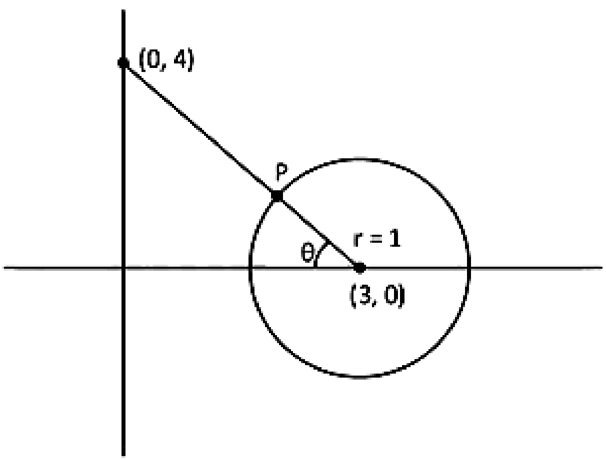
Here $|z-3| < 1$

$\Rightarrow (x-3)^2 + y^2 < 1$

and $z = (4+3i) + \bar{z}(4-3i) \leq 24$

$\Rightarrow 4x - 3y \leq 12$

$\tan\theta = \dfrac{4}{3}$



∴ Coordinate of $P = (3 - \cos\theta, \sin\theta)$

$= \left(3 - \dfrac{3}{5}, \dfrac{4}{5}\right)$

$\therefore \alpha + i\beta = \dfrac{12}{5} + \dfrac{4}{5}i$

$\therefore 25(\alpha + \beta) = 80$

Question72

Let a circle C in complex plane pass through the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then arg(z) is equal to:
[25-Jun-2022-Shift-1]

Options:

A. $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \pi$

B. $\tan^{-1}\left(\frac{24}{7}\right) - \pi$

C. $\tan^{-1}(3) - \pi$

D. $\tan^{-1}\left(\frac{3}{4}\right) - \pi$

Answer: B

Solution:

Solution:

$z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$

Clearly, $C \equiv x^2 + y^2 = 25$

Let $z(x, y)$

$\Rightarrow \left(\frac{y-4}{x-3}\right)\left(\frac{2}{-4}\right) = -1$

$\Rightarrow y = 2x - 2 \equiv L$

$\therefore$ z is intersection of C&L

$\Rightarrow z \equiv \left(\frac{-7}{5}, \frac{-24}{5}\right)$

$\therefore \text{Arg}(z) = -\pi + \tan^{-1}\left(\frac{24}{7}\right)$

Question73

Let z_1 and z_2 be two complex numbers such that $\overline{z_1} = i\overline{z_2}$ and

$\arg\left(\frac{z_1}{z_2}\right) = \pi$. Then

[25-Jun-2022-Shift-2]

Options:

A. $\arg z_2 = \frac{\pi}{4}$

B. $\arg z_2 = -\frac{3\pi}{4}$

C. $\arg z_1 = \frac{\pi}{4}$

D. $\arg z_1 = -\frac{3\pi}{4}$

Answer: C

Solution:

Solution:

$\because \frac{z_1}{z_2} = -i \Rightarrow z_1 = -iz_2$

$\Rightarrow \arg(z_1) = -\frac{\pi}{2} + \arg(z_2) \ldots\ldots$ (i)

Also $\arg(z_1) - \arg(\bar{z}_2) = \pi$

$\Rightarrow \arg(z_1) + \arg(z_2) = \pi \ldots.$ (ii)

From (i) and (ii), we get $\arg(z_1) = \frac{\pi}{4}$ and $\arg(z_2) = \frac{3\pi}{4}$

Question74

Let $A = \left\{ z \in C : \left| \frac{z+1}{z-1} \right| < 1 \right\}$ and $B = \left\{ z \in C : \arg\left(\frac{z-1}{z+1} \right) = \frac{2\pi}{3} \right\}$. Then $A \cap B$ is :

[26-Jun-2022-Shift-1]

Options:

A. a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

B. a portion of a circle centred at $\left(0, -\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

C. an empty

D. a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

Answer: B

Solution:

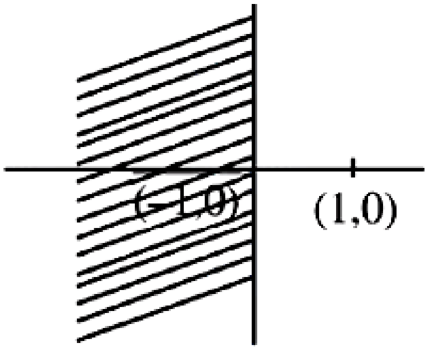
Set A

$\Rightarrow \left|\frac{z+1}{z-1}\right| < 1$

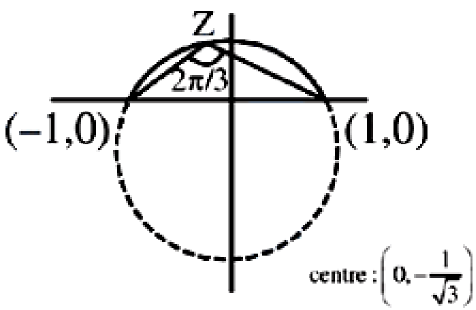
$\Rightarrow |z+1| < |z-1|$

$\Rightarrow (x+1)^2 + y^2 < (x-1)^2 + y^2$

$\Rightarrow x < 0$



Set B



$\Rightarrow \arg\left(\frac{z-1}{z+1}\right) = \frac{2\pi}{3}$

$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right) = \frac{2\pi}{3}$

$\Rightarrow x^2 + y^2 + \frac{2y}{\sqrt{3}} - 1 = 0$

$A \cap B$

$\Rightarrow$ Centre $\left(0, -\frac{1}{\sqrt{3}}\right)$

Question75

If $z^2 + z + 1 = 0$, $z \in C$, then $\left|\sum_{n=1}^{15}\left(z^n + (-1)^n \frac{1}{z^n}\right)^2\right|$ is equal to____
[26-Jun-2022-Shift-2]

Answer: 2

Solution:

$\because z^2 + z + 1 = 0$

$\Rightarrow \omega$ or ω^2

$$\because \left| \sum_{n=1}^{15} \left(z^n + (-1)^n \frac{1}{z^n} \right)^2 \right|$$

$$= \left| \sum_{n=1}^{15} z^{2n} + \sum_{n=1}^{15} z^{-2n} + 2 \cdot \sum_{n=1}^{15} (-1)^n \right|$$

$= | 0 + 0 - 2 |$

$= 2$

Question76

The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z} = iz^2$ is :
[27-Jun-2022-Shift-1]

Options:

A. $\frac{3\sqrt{3}}{4}$

B. $\frac{3\sqrt{3}}{2}$

C. $\frac{3}{2}$

D. $\frac{3}{4}$

Answer: A

Solution:

$\bar{z} = iz^2$
Let $z = x + iy$
$x - iy = i(x^2 - y^2 + 2xiy)$
$x - iy = i(x^2 - y^2) - 2xy$
$\therefore x = -2yx \text{ or } x^2 - y^2 = -y$
$x = 0 \text{ or } y = -\frac{1}{2}$
Case - I
$x = 0$
$-y^2 = -y$
$y = 0, 1$
Case - II
$y = -\frac{1}{2}$
$\Rightarrow x^2 - \frac{1}{4} = \frac{1}{2} \Rightarrow x = \pm \frac{\sqrt{3}}{2}$
$x = \left\{ 0, i, \frac{\sqrt{3}}{2} - \frac{i}{2}, \frac{-\sqrt{3}}{2} - \frac{i}{2} \right\}$

$$\text{Area of polygon } = \frac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ \frac{\sqrt{3}}{2} & \frac{-1}{2} & 1 \\ \frac{-\sqrt{3}}{2} & \frac{-1}{2} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left|-\sqrt{3} - \frac{\sqrt{3}}{2}\right| = \frac{3\sqrt{3}}{4}$$

Question77

The number of points of intersection of $|z-(4+3i)| = 2$ and $|z| + |z-4| = 6, z \in C$, is [27-Jun-2022-Shift-2]

Options:

A. 0

B. 1

C. 2

D. 3

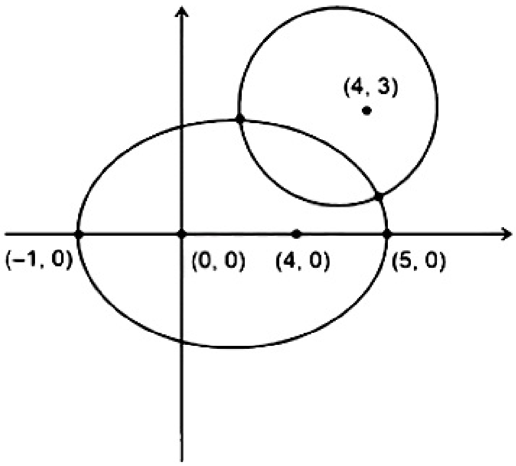
Answer: C

Solution:

Solution:

$C_1 : |z-(4+3i)| = 2$ and $C_2 : |z| + |z-4| = 6, z \in C$

C_1 represents a circle with centre $(4, 3)$ and radius 2 and C_2 represents a ellipse with focii at $(0, 0)$ and $(4, 0)$ and length of major axis $= 6$, and length of semi-major axis $2\sqrt{5}$ and $(4, 2)$ lies inside the both C_1 and C_2 and $(4, 3)$ lies outside the C_2



$\therefore$ number of intersection points $= 2$

Question78

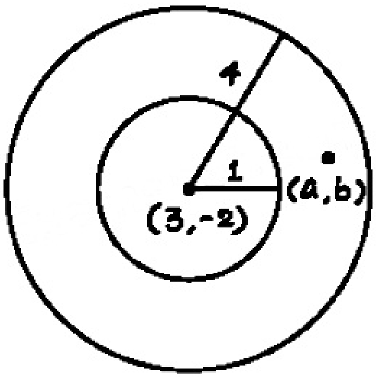
The number of elements in the set $\{z = a + ib \in C : a, b \in Z$ and $1 < |z-3+2i| < 4\}$ is___

[28-Jun-2022-Shift-1]

Answer: 40

Solution:

Solution:
$1 < |Z - 3 + 2i| < 4$



$1 < (a-3)^2 + (b+2)^2 < 16$
$(0, \pm2), (\pm2, 0), (\pm1, \pm2), (\pm2, \pm1)$
$(\pm2, \pm3), (3\pm, \pm2), (\pm1, \pm1), (2\pm, \pm2)$
$(\pm3, 0), (0, \pm3), (\pm3 \pm 1), (\pm1, \pm3)$
Total 40 points

Question79

Sum of squares of modulus of all the complex numbers z satisfying $\bar{z} = iz^2 + z^2 - z$ is equal to_____
[28-Jun-2022-Shift-2]

Answer: 2

Solution:

Let $z = x + iy$
So $2x = (1 + i)(x^2 - y^2 + 2xyi)$
$\Rightarrow 2x = x^2 - y^2 - 2xy$
(i) and
$x^2 - y^2 + 2xy = 0$
From (i) and (ii) we get
$x = 0 \text{ or } y = -\frac{1}{2}$
When $x = 0$ we get $y = 0$
When $y = -\frac{1}{2}$ we get $x^2 - x - \frac{1}{4} = 0$
$\Rightarrow x = \frac{-1 \pm \sqrt{2}}{2}$

So there will be total 3 possible values of z, which are 0, $\left(\frac{-1+\sqrt{2}}{2}\right)-\frac{1}{2}i$ and $\left(\frac{-1-\sqrt{2}}{2}\right)-\frac{1}{2}i$

Sum of squares of modulus

$= 0+\left(\frac{\sqrt{2}-1}{2}\right)^2+\frac{1}{4}+\left(\frac{\sqrt{2}+1}{2}\right)^2=+\frac{1}{4}$

$= 2$

Question80

Let α and β be the roots of the equation $x^2+(2i-1)=0$. Then, the value of $|\alpha^8+\beta^8|$ is equal to:
[29-Jun-2022-Shift-1]

Options:

A. 50

B. 250

C. 1250

D. 1500

Answer: A

Solution:

Solution:
Given equation,
$x^2+(2i-1)=0$
$\Rightarrow x^2=1-2i$
Let α and β are the two roots of the equation.
As, we know roots of a equation satisfy the equation so
$\alpha^2=1-2i$
and $\beta^2=1-2i$
$\therefore \alpha^2=\beta^2=1-2i$
$\therefore \left|\alpha^2\right| = \sqrt{1^2+(-2)^2}=\sqrt{15}$

Now, $\left|\alpha^8+\beta^8\right|$

$\left|\alpha^8+\alpha^8\right|$

$= 2\left|\alpha^8\right|$

$= 2\left|\alpha^2\right|^4$

$= 2(\sqrt{5})^4$
$= 2\times 25$
$= 50$

Question81

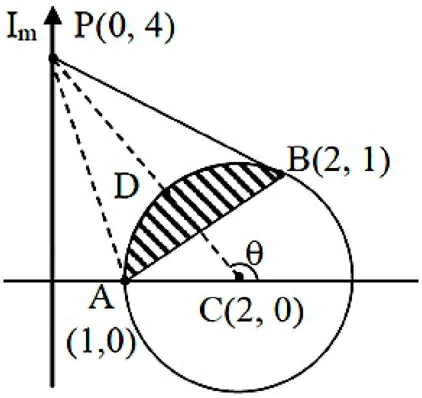
Let $S=\{z\in C: |z-2|\le 1, z(1+i)+\bar{z}(1-i)\le 2\}$. Let $|z-4i|$ attains minimum and maximum values, respectively, at $z_1\in S$ and $z_2\in S$. If $5(z_1|^2+z_2|^2)=\alpha+\beta\sqrt{5}$, where α and β are integers, then the value of

α + β is equal to_____
[29-Jun-2022-Shift-1]

Answer: 26

Solution:

Solution:
$|z-2| \le 1$



$(x-2)^2 + y^2 \le 1$
&
$z(1+i) + \bar{z}(1-i) \le 2$
Put $z = x + iy$
$\therefore x - y \le 1 \ldots\ldots$ (2)
$PA = \sqrt{17}$, $PB = \sqrt{13}$
Maximum is PA & Minimum is PD
Let $D(2+\cos\theta, 0+\sin\theta)$
$\therefore m_{cp} = \tan\theta = -2$
$\cos\theta = -\frac{1}{\sqrt{5}}$, $\sin\theta = \frac{2}{\sqrt{5}}$
$\therefore D\left(2 - \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$
$\Rightarrow z_1 = \left(2 - \frac{1}{\sqrt{5}}\right) + \frac{2i}{\sqrt{5}}$
$\left|z_1\right| = \frac{25-4\sqrt{5}}{5\&} z_2 = 1$
$\therefore \left|z_2\right|^2 = 1$
$\therefore 5\left(\left|z_1\right|^2 + \left|z_2\right|^2\right) = 30 - 4\sqrt{5}$
$\therefore \alpha = 30$
$\beta = -4$
$\therefore \alpha + \beta = 26$

Question82

Let arg(z) represent the principal argument of the complex number z. Then, $|z| = 3$ and $\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$ intersect
[29-Jun-2022-Shift-2]

Options:

A. exactly at one point.

B. exactly at two points.

C. nowhere.

D. at infinitely many points.

Answer: C

Solution:

Solution:

Let $z = x + iy$

$\therefore |z| = \sqrt{x^2 + y^2}$

Given, $|z| = 3$

$\therefore \sqrt{x^2 + y^2} = 3$

$\Rightarrow x^2 + y^2 = 9 = 3^2$

This represent a circle with center at (0, 0) and radius = 3

Now, given

$\arg(z - 1) - \arg(z + 1) = \frac{\pi}{4}$

$\Rightarrow \arg(x + iy - 1) - \arg(x + iy + 1) = \frac{\pi}{4}$

$\Rightarrow \arg(x - 1 + iy) - \arg(x + 1 + iy) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{y}{x-1} - \frac{y}{x+1}}{1 + \frac{y}{x-1} \times \frac{y}{x+1}}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{xy + y - xy + y}{x^2 - 1}}{\frac{x^2 - 1 + y^2}{x^2 - 1}}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{xy + y - xy + y}{x^2 - 1 + y^2}\right) = \frac{\pi}{4}$

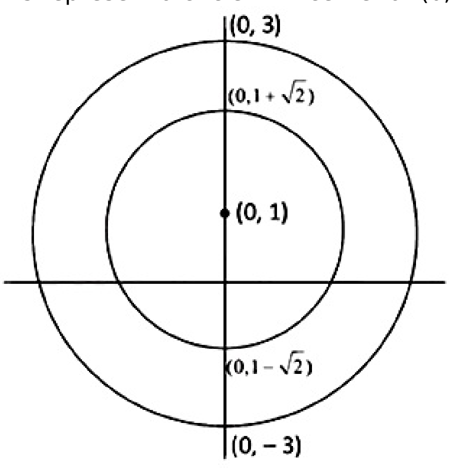
$\Rightarrow \frac{2y}{x^2 - 1 + y^2} = \tan\left(\frac{\pi}{4}\right)$

$\Rightarrow 2y = x^2 + y^2 - 1$

$\Rightarrow x^2 + y^2 - 2y - 1 = 0$

$\Rightarrow x^2 + (y - 1)^2 = (\sqrt{2})^2$

This represent a circle with center at (0, 1) and radius $\sqrt{2}$.



From diagram you can see both the circles do not cut anywhere.

Question83

The sum of all the real roots of the equation $(e^{2x} - 4)(6e^{2x} - 5e^{x} + 1) = 0$ is
[24-Jun-2022-Shift-2]

Options:

A. $\log_e 3$

B. $-\log_e 3$

C. $\log_e 6$

D. $-\log_e 6$

Answer: B

Solution:

Solution:

$(e^{2x} - 4)(6e^{2x} - 5e^{x} + 1) = 0$

Let $e^{x} = t$

$\therefore (t^{2} - 4)(6t^{2} - 5t + 1) = 0$

$\Rightarrow (t^{2} - 4)(2t - 1)(3t - 1) = 0$

$\therefore t = 2, -2, \frac{1}{2}, \frac{1}{3}$

$\therefore e^{x} = 2 \Rightarrow x = \ln 2$

$e^{x} = -2$(not possible)

$e^{x} = \frac{1}{2} \Rightarrow x = -\ln 2$

$e^{x} = \frac{1}{3} \Rightarrow x = -\ln 3$

$\therefore$ Sum of all real roots

$= \ln 2 - \ln 2 - \ln 3$

$= -\ln 3$

Question84

For a natural number n, let $\alpha_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8}$ is___
[25-Jun-2022-Shift-1]

Answer: 4

Solution:

Solution:

$\alpha_n = 19^n - 12^n$

Let equation of roots 12&19 i.e.

$x^2 - 31x + 228 = 0$

$\Rightarrow (31 - x) = \frac{228}{x}$ (where x can be 19 or 12)

$\therefore \frac{31\alpha_9 - \alpha_{10}}{57\alpha_8} = \frac{31(19^9 - 12^9) - (19^{10} - 12^{10})}{57(19^8 - 12^8)}$

$= \frac{19^9(31 - 19) - 12^9(31 - 12)}{57(19^8 - 12^8)}$

$= \frac{228(19^8 - 12^8)}{57(19^8 - 12^8)} = 4$

Question85

Let a, b ∈ R be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to :
[25-Jun-2022-Shift-2]

Options:

A. 37

B. 58

C. 68

D. 92

Answer: B

Solution:

Solution:
$ax^2 - 2bx + 15 = 0$ has repeated root so $b^2 = 15a$ and $\alpha = \frac{15}{b}$
$\because \alpha$ is a root of $x^2 - 2bx + 21 = 0$
So $\frac{225}{b^2} = 9 \Rightarrow b^2 = 25$
Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 4b^2 - 42 = 100 - 42 = 58$

Question86

The sum of the cubes of all the roots of the equation

$x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ is ____
[26-Jun-2022-Shift-1]

Answer: 36

Solution:

Solution:

$x^4 - 3x^3 - x^2 - x^2 + 3x + 1 = 0$

$(x^2 - 1)(x^2 - 3x - 1) = 0$

Let the root of $x^2 - 3x - 1 = 0$ be α and β and other two roots of given equation are 1 and -1

So sum of cubes of roots

$= 1^3 + (-1)^3 + \alpha^3 + \beta^3$

$= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= (3)^3 - 3(-1)(3)$

$= 36$

Question87

If the sum of all the roots of the equation $e^{2x} - 11e^{x} - 45e^{-x} + \frac{81}{2} = 0$ is $\log_e p$, then p is equal to____
[27-Jun-2022-Shift-1]

Answer: 45

Solution:

Let $e^x = t$ then equation reduces to
$t^2 - 11t - \frac{45}{t} + \frac{81}{2} = 0$
$\Rightarrow 2t^3 - 22t^2 + 81t - 45 = 0 \ldots..$ (i)
if roots of $e^{2xt} - 11e^x - 45e^{-x} + \frac{81}{2} = 0$ are α, β, γ then roots of (i) will be $e^{\alpha_1}e^{\alpha_2}e^{\alpha 3}$ using product of roots
$e^{\alpha_1 + \alpha_2 + \alpha_3} = 45$
$\Rightarrow \alpha_1 + \alpha_2 + \alpha_3 = \ln 45 \Rightarrow p = 45$

Question88

Let α, β be the roots of the equation $x^2 - 4\lambda x + 5 = 0$ and α, γ be the roots of the equation $x^2 - (3\sqrt{2} + 2\sqrt{3})x + 7 + 3\lambda\sqrt{3} = 0$, $\lambda > 0$. If $\beta + \gamma = 3\sqrt{2}$, then $(\alpha + 2\beta + \gamma)^2$ is equal to___
[27-Jun-2022-Shift-2]

Answer: 98

Solution:

$\because \alpha, \beta$ are roots of $x^2 - 4\lambda x + 5 = 0$
$\therefore \alpha + \beta = 4\lambda$ and $\alpha\beta = 5$
Also, α, γ are roots of
$x^2 - (3\sqrt{2} + 2\sqrt{3})x + 7 + 3\sqrt{3}\lambda = 0, \lambda > 0$
$\therefore \alpha + \gamma = 3\sqrt{2} + 2\sqrt{3}, \alpha\gamma = 7 + 3\sqrt{3}\lambda$
$\because \alpha$ is common root
$\therefore \alpha^2 - 4\lambda\alpha + 5 = 0$
and $\alpha^2 - (3\sqrt{2} + 2\sqrt{3})\alpha + 7 + 3\sqrt{3}\lambda = 0$
From (i) - (ii) : we get $\alpha = \dfrac{2 + 3\sqrt{3}\lambda}{3\sqrt{2} + 2\sqrt{3} - 4\lambda}$
$\because \beta + \gamma = 3\sqrt{2}$
$\therefore 4\lambda + 3\sqrt{2} + 2\sqrt{3} - 2\alpha = 3\sqrt{2}$
$\Rightarrow 3\sqrt{2} = 4\lambda + 3\sqrt{2} + 2\sqrt{3} - \dfrac{4 + 6\sqrt{3}\lambda}{3\sqrt{2} + 2\sqrt{3} - 4\lambda}$
$\Rightarrow 8\lambda^2 + 3(\sqrt{3} + 2\sqrt{2})\lambda - 4 - 3\sqrt{6} = 0$
$\therefore \lambda = \dfrac{6\sqrt{2} - 3\sqrt{2} \pm \sqrt{9(11 - 4\sqrt{6}) + 32(4 + 3\sqrt{6})}}{16}$
$\therefore \lambda = \sqrt{2}$
$\therefore (\alpha + 2\beta + \gamma)^2 = (\alpha + \beta + \beta + \gamma)^2$
$= (4\sqrt{2} + 3\sqrt{2})^2$
$= (7\sqrt{2})^2 = 98$

Question89

The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is_____
[28-Jun-2022-Shift-1]

Answer: 2

Solution:

Dividing by e^{2x}
$e^{2x} + 4e^x - 58 + 4e^{-x} + e^{-2x} = 0$
$\Rightarrow (e^x + e^{-x})^2 + 4(e^x + e^{-x}) - 60 = 0$

Let $e^x + e^{-x} = t \in [2, \infty)$
$\Rightarrow t^2 + 4t - 60 = 0$
$\Rightarrow t = 6$ is only possible solution
$e^x + e^{-x} = 6 \Rightarrow e^{2x} - 6e^x + 1 = 0$
Let $e^x = p$
$p^2 - 6p + 1 = 0$
$\Rightarrow p = \frac{3+\sqrt{5}}{2}$ or $\frac{3-\sqrt{5}}{2}$
So $x = \ln\left(\frac{3+\sqrt{5}}{2}\right)$ or $\ln\left(\frac{3-\sqrt{5}}{2}\right)$

Question90

Let $f(x)$ be a quadratic polynomial such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is -1, then the sum of the roots of $f(x) = 0$ is equal to :
[28-Jun-2022-Shift-2]

Options:

A. $\frac{11}{3}$

B. $\frac{7}{3}$

C. $\frac{13}{3}$

D. $\frac{14}{3}$

Answer: A

Solution:

Solution:
$\because x = -1$ be the roots of $f(x) = 0$
$\therefore$ Let $f(x) = A(x+1)(x-1)$...... (i)
Now, $f(-2) + f(3) = 0$
$\Rightarrow A[-1(-2-b) + 4(3-b)] = 0$
$b = \frac{14}{3}$
$\therefore$ Second root of $f(x) = 0$ will be $\frac{14}{3}$
$\therefore$ Sum of roots $= \frac{14}{3} - 1 = \frac{11}{3}$

Question91

Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then, the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to :
[29-Jun-2022-Shift-2]

Options:

A. 1

B. α

C. 1 + α

D. 1 + 2α

Answer: A

Solution:

Given, α is a root of the equation $1 + x^2 + x^4 = 0$
$\therefore \alpha$ will satisfy the equation.
$\therefore 1 + \alpha^2 + \alpha^4 = 0$
$$\alpha^2 = \frac{-1 \pm \sqrt{1-4}}{2}$$
$$= \frac{-1 \pm \sqrt{3}i}{2}$$
$\therefore \alpha^2 = \omega ar \omega^2$
Now,
$\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$
$= \alpha \cdot (\alpha^2)^{505} + (\alpha^2)^{1011} - \alpha \cdot (\alpha^2)^{1516}$
$= \alpha(\omega)^{505} + (\omega)^{1011} - \alpha \cdot (\omega)^{1516}$
$= \alpha \cdot (\omega^3)^{168} \cdot \omega + (\omega^3)^{337} - \alpha \cdot (\omega^3)^{505} \cdot \omega$
$= \alpha\omega + 1 - \alpha\omega$
$= 1$

Question92

Let x, y > 0. If $x^3y^2 = 2^{15}$, then the least value of 3x + 2y is [24-Jun-2022-Shift-2]

Options:

A. 30

B. 32

C. 36

D. 40

Answer: D

Solution:

$x, y > 0$ and $x^3 y^2 = 2^{15}$

Now, $3x + 2y = (x + x + x) + (y + y)$

So, by $A \cdot M \geq$ G.M inequality

$$\frac{3x + 2y}{5} \geq \sqrt[5]{x^3 \cdot y^2}$$

$$\therefore 3x + 2y \geq 5 \sqrt[5]{2^{15}} \geq 40$$

$\therefore$ Least value of $3x + 4y = 40$

Question93

Let p and q be two real numbers such that p + q = 3 and $p^4 + q^4 = 369$. Then $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2}$ is equal to____
[26-Jun-2022-Shift-2]

Answer: 4

Solution:

Solution:

$\because p + q = 3 \ldots\ldots$ (i)

and $p^4 + q^4 = 369 \ldots\ldots$ (ii)

$\{(p+q)^2 - 2pq\}^2 - 2p^2q^2 = 369$

or $(9 - 2pq)^2 - 2(pq)^2 = 369$

or $(pq)^2 - 18pq - 144 = 0$

$\therefore pq = -6$ or 24

But $pq = 24$ is not possible

$\therefore pq = -6$

Hence, $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2} = \left(\frac{pq}{p+q}\right)^2 = (-2)^2 = 4$

Question94

If α, β, γ, δ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to :
[25-Jul-2022-Shift-1]

Options:

A. -4

B. -1

C. 1

D. 4

Answer: B

Solution:

Solution:
When, $x^5 = 1$
then $x^5 - 1 = 0$
$\Rightarrow (x - 1)(x^4 + x^3 + x^2 + x + 1) = 0$
Given, $x^4 + x^3 + x^2 + x + 1 = 0$ has roots α, β, γ and 8 .
$\therefore$ Roots of $x^5 - 1 = 0$ are $1, \alpha, \beta, \gamma$ and 8
We know, Sum of p^{th} power of n^{th} roots of unity $= 0$. (If p is not multiple of n) or n (If p is multiple of n)
$\therefore$ Here, Sum of p^{th} power of n^{th} roots of unity
Here, $p = 2021$, which is not multiple of 5 .
$\therefore 1^{2021} + \alpha^{2021} + \beta^{2021} + \gamma^{2021} + 8^{2021} = 0$
$\Rightarrow \alpha^{2021} + \beta^{2021} + \gamma^{2021} + 8^{2021} = -1$

Question95

For $n \in N$, let $S_n = \left\{ z \in C : |z - 3 + 2i| = \frac{n}{4} \right\}$ and
$T_n = \left\{ z \in C : |z - 2 + 3i| = \frac{1}{n} \right\}$.
Then the number of elements in the set $\{n \in N : S_n \cap T_n = \varphi\}$ is :
[25-Jul-2022-Shift-1]

Options:

A. 0

B. 2

C. 3

D. 4

Answer: D

Solution:

Solution:
$S_n = \left\{ z \in C : |z - 3 + 2i| = \frac{n}{4} \right\}$ represents a circle with centre $C_1(3, -2)$ and radius $r_1 = \frac{n}{4}$
Similarly T_n represents circle with centre $C_2(2, -3)$ and radius $r_2 = \frac{1}{n}$
As $S_n \cap T_n = \varphi$
$C_1C_2 > r_1 + r_2$ OR $C_1C_2 < |r_1 - r_2|$
$\sqrt{2} > \frac{n}{4} + \frac{1}{n}$ OR $\sqrt{2} < \left| \frac{n}{4} - \frac{1}{n} \right|$
$n = 1, 2, 3, 4$ n may take infinite values

Question96

For $z \in C$ if the minimum value of $(|z - 3\sqrt{2}| + |z - p\sqrt{2}i|)$ is $5\sqrt{2}$, then a value Question: of p is
[25-Jul-2022-Shift-2]

Options:

A. 3

B. $\frac{7}{2}$

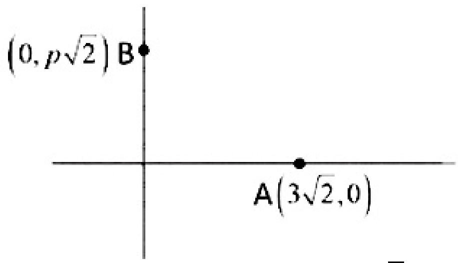
C. 4

D. $\frac{9}{2}$

Answer: C

Solution:

Solution:



It is sum of distance of z from $(3\sqrt{2}, 0)$ and $(0, p\sqrt{2})$ For minimising, z should lie on AB and $AB = 5\sqrt{2}$
$(AB)^2 = 18 + 2p^2$
$p = \pm 4$

Question97

Let 0 be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $Re(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true?
[26-Jul-2022-Shift-1]

Options:

A. $\arg z_2 = \pi - \tan^{-1} 3$

B. $\arg(z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$

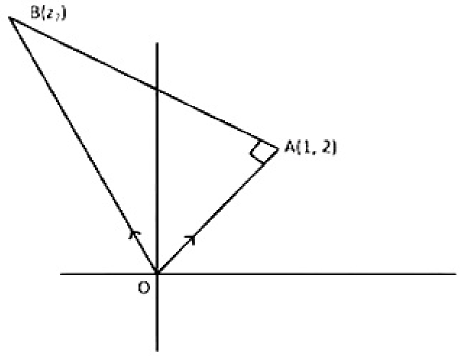
C. $z_2 | = \sqrt{10}$

D. $2z_1 - z_2 | = 5$

Answer: D

Solution:

Solution:



$$\frac{z_2 - 0}{(1 + 2i) - 0} = \frac{|OB|}{|OA|} e^{\frac{i\pi}{4}}$$

$$\Rightarrow \frac{z_2}{1 + 2i} = \sqrt{2} e^{\frac{i\pi}{4}}$$

OR $z_2 = (1 + 2i)(1 + i)$

$= -1 + 3i$

$\arg z_2 = \pi - \tan^{-1} 3$

$|z_2| = \sqrt{10}$

$z_1 - 2z_2 = (1 + 2i) + 2 - 6i = 3 - 4i$

$\arg(z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$

$|2z_1 - z_2| = | 2 + 4i + 1 - 3i | = | 3 + i | = \sqrt{10}$

Question98

If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z - i| - | z + 5i | = 0$, then [26-Jul-2022-Shift-2]

Options:

A. $x + 2y - 4 = 0$

B. $x^2 + y - 4 = 0$

C. $x + 2y + 4 = 0$

D. $x^2 - y + 3 = 0$

Answer: C

Solution:

Solution:

$|z - i| = | z + 5i|$

So, z lies on $\perp^r$ bisector of $(0, 1)$ and $(0, -5)$

i.e., line $y = -2$

as $|z| = 2$

$\Rightarrow z = -2i$

$x = 0$ and $y = -2$

so, $x + 2y + 4 = 0$

Question99

Let the minimum value v_0 of $v = |z|^2 + |z-3|^2 + |z-6i|^2, z \in C$ is attained at $z = z_0$. Then $\left|2z_0^2 - \overline{z}_0^3 + 3\right|^2 + v_0^2$ is equal to
[27-Jul-2022-Shift-1]

Options:

A. 1000

B. 1024

C. 1105

D. 1196

Answer: A

Solution:

Solution:
Let $z = x + iy$
$v = x^2 + y^2 + (x-3)^2 + y^2 + x^2 + (y-6)^2$
$= (3x^2 - 6x + 9) + (3y^2 - 12y + 36)$
$= 3(x^2 + y^2 - 2x - 4y + 15)$
$= 3[(x-1)^2 + (y-2)^2 + 10]$
v_{min} at $z = 1 + 2i = z_0$ and $v_0 = 30$
so $|2(1+2i)^2 - (1-2i)^3 + 3|^2 + 900$
$= |2(-3+4i) - (1 - 8i^3 - 6i(1-2i) + .3|^2 + 900.$
$= |-6 + 8i - (1 + 8i - 6i - 12) + 3|^2 + 900$
$= |8 + 6i|^2 + 900$

$= |8 + 6i|^2 + 900$
$= 1000$

Question100

Let $S = \{z \in C : z^2 + \overline{z} = 0\}$. Then $\sum_{z \in S} (Re(z) + Im(z))$ is equal to ________.
[27-Jul-2022-Shift-1]

Answer: 0

Solution:

$\because z^2 + \overline{z} = 0$
Let $z = x + iy$
$\therefore x^2 - y^2 + 2ixy + x - iy = 0$
$(x^2 - y^2 + x) + i(2xy - y) = 0$
$\therefore x^2 + y^2 = 0$ and $(2x-1)y = 0$
if $x = +\frac{1}{2}$ then $y = \pm\frac{\sqrt{3}}{2}$
And if $y = 0$ then $x = 0, -1$

$\therefore z = 0 + 0i, -1 + 0i, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$

$\therefore \sum (R_e(z) + m(z)) = 0$

Question101

Let S be the set of all (α, β), π < α, β < 2π, for which the complex number $\frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely imaginary and $\frac{1 + i\cos\beta}{1 - 2i\cos\beta}$ is purely real. Let $Z_{\alpha\beta} = \sin 2\alpha + i\cos 2\beta, (\alpha, \beta) \in S$. Then $\sum_{(\alpha, \beta) \in S} \left(iZ_{\alpha\beta} + \frac{1}{iZ_{\alpha\beta}} \right)$ is equal to :

[27-Jul-2022-Shift-2]

Options:

A. 3

B. 3i

C. 1

D. 2 − i

Answer: C

Solution:

Solution:

$\because \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha}$ is purely imaginary

$\therefore \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha} + \frac{1 + i\sin\alpha}{1 - 2i\sin\alpha} = 0$

$\Rightarrow 1 - 2\sin^2\alpha = 0$

$\therefore \alpha = \frac{5\pi}{4}, \frac{7\pi}{4}$

and $\frac{1 + i\cos\beta}{1 - 2i\cos\beta}$ is purely real

$\frac{1 + i\cos\beta}{1 - 2i\cos\beta} - \frac{1 - i\cos\beta}{1 + 2i\cos\beta} = 0$

$\Rightarrow \cos\beta = 0$

$\therefore \beta = \frac{3\pi}{2}$

$\therefore S = \left\{ \left(\frac{5\pi}{2}, \frac{3\pi}{2} \right), \left(\frac{7\pi}{4}, \frac{3\pi}{2} \right) \right\}$

$Z_{\alpha\beta} = 1 - i$ and $Z_{\alpha\beta} = -1 - i$

$\therefore \sum_{(\alpha, \beta) \in S} \left(iZ_{\alpha\beta} + \frac{1}{iZ_{\alpha\beta}} \right) = i(-2i) + \frac{1}{i}\left[\frac{1}{1 + i} + \frac{1}{-1 + i} \right]$

$= 2 + \frac{1}{i} \frac{2i}{-2} = 1$

Question102

Let $S_1 = \left\{ z_1 \in C : z_1 - 3 \right| = \frac{1}{2} \right\}$ and $S_2 = \{z_2 \in C : z_2 - |z_2 + 1|| = z_2 + |z_2 - 1||\}$. Then, for $z_1 \in S_1$ and

$z_2 \in S_2$, **the least value of** $z_2 - z_1|$ **is :**
[28-Jul-2022-Shift-1]

Options:

A. 0

B. $\frac{1}{2}$

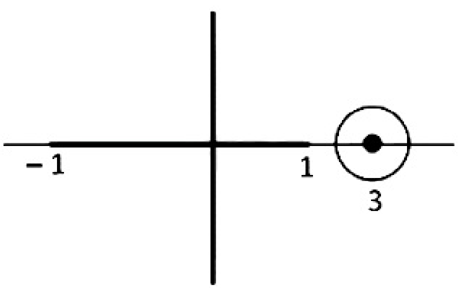
C. $\frac{3}{2}$

D. $\frac{5}{2}$

Answer: C

Solution:

Solution:
$\because |Z_2 + |Z_2 - 1||^2 = |Z_2 - |Z_2 + 1||^2$
$\Rightarrow (Z_2 + |Z_2 - 1|)(\overline{Z}_2 + |Z_2 - 1|) = (Z_2 - |Z_2 + 1|)(\overline{Z}_2 - |Z_2 + 1|)$
$\Rightarrow Z_2(|Z_2 - 1| + |Z_2 + 1|) + \overline{Z}_2(|Z_2 - 1| + |Z_2 + 1|) = |Z_2 + 1|^2 - |Z_2 - 1|^2$
$\Rightarrow (Z_2 + \overline{Z}_2)(|Z_2 + 1| + |Z_2 - 1|) = 2(Z_2 + \overline{Z}_2)$
$\Rightarrow$ Either $Z_2 + \overline{Z}_2 = 0$ or $|Z_2 + 1| + |Z_2 - 1| = 2$
So, Z_2 lies on imaginary axis or on real axis within $[-1, 1]$
Also $|Z_1 - 3| = \frac{1}{2} \Rightarrow Z_1$ lies on the circle having center 3 and radius $\frac{1}{2}$.



Clearly $|Z_1 - Z_2|_{\min} = \frac{3}{2}$

Question103

Let $z = a + ib$, $b \neq 0$ **be complex numbers satisfying** $z^2 = \overline{z} \cdot 2^{1-|z|}$. **Then the least value of** $n \in N$ **, such that** $z^n = (z+1)^n$, **is equal to ________.**
[28-Jul-2022-Shift-2]

Answer: 6

Solution:

Solution:
$\because z^2 = \overline{z} \cdot 2^{1-|z|}$...... (1)

$\Rightarrow |z|^2 = |\bar{z}| \cdot 2^{1-|z|}$
$\Rightarrow |z| = 2^{1-|z|}$
$\because b \neq 0 \Rightarrow |z| \neq 0$
$\therefore |z| = 1 \ldots\ldots (2)$
$\because z = a + ib$ then $\sqrt{a^2 + b^2} = 1 \ldots\ldots (3)$
Now again from equation (1), equation (2), equation (3) we get :
$a^2 - b^2 + i2ab = (a - ib)2^0$
$\therefore a^2 - b^2 = a$ and $2ab = -b$
$\therefore a = -\frac{1}{2}$ and $b = \pm\frac{\sqrt{3}}{2}$
$z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ or $z = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$
$z^n = (z+1)^n \Rightarrow \left(\frac{z+1}{z}\right)^n = 1$
$\left(1 + \frac{1}{z}\right)^n = 1$
$\left(\frac{1 + \sqrt{3}i}{2}\right) = 1$, then minimum value of n is 6 .

Question104

If z = 2 + 3i, then $z^5 + (\bar{z})^5$ is equal to :
[29-Jul-2022-Shift-1]

Options:

A. 244

B. 224

C. 245

D. 265

Answer: A

Solution:

Solution:
$z = (2 + 3i)$
$\Rightarrow z^5 = (2 + 3i)((2 + 3i)^2)^2$
$= (2 + 3i)(-5 + 12i)^2$
$= (2 + 3i)(-119 - 120i)$
$= -238 - 240i - 357i + 360$
$= 122 - 597i$
$\bar{z}^5 = 122 + 597i$
$z^5 + \bar{z}^5 = 244$

Question105

If $z \neq 0$ be a complex number such that $\left|z - \frac{1}{z}\right| = 2$, then the maximum value of |z| is:
[29-Jul-2022-Shift-2]

Options:

A. $\sqrt{2}$

B. 1

C. $\sqrt{2}-1$

D. $\sqrt{2}+1$

Answer: D

Solution:

Solution:
We know,

$||z_1| - |z_2|| \le |z_1 + z_2| \le |z_1| + |z_2|$

$\therefore ||z| - \frac{1}{|z|}| \le |z - \frac{1}{z}|$

$\Rightarrow ||z| - \frac{1}{|z|}| \le 2 \left[\text{ Given } |z - \frac{1}{z}| = 2 \right]$

$\Rightarrow \left| \frac{|z|^2-1}{|z|} \right| \le 2$

$\Rightarrow -2 \le \frac{|z|^2-1}{|z|} \le 2$

$\therefore \frac{|z|^2-1}{|z|} \le 2$

$\Rightarrow |z|^2 - 1 \le 2|z|$

$\Rightarrow |z|^2 - 2|z| - 1 \le 0$

$\Rightarrow |z|^2 - 2|z| + 1 - 2 \le 0$

$\Rightarrow (|z|-1)^2 - 2 \le 0$

$\Rightarrow -\sqrt{2} \le |z| - 1 \le \sqrt{2}$

$\Rightarrow 1-\sqrt{2} \le |z| \le 1+\sqrt{2} \ldots\ldots (1)$

or

$-2 \le \frac{|z|^2-1}{|z|}$

$\Rightarrow |z|^2 - 1 \le -2|z|$

$\Rightarrow |z|^2 + 2|z| - 1 \le 0$

$\Rightarrow |z|^2 + 2|z| + 1 - 2 \le 0$

$\Rightarrow (|z|+1)^2 - 2 \le 0$

$\Rightarrow -\sqrt{2} \le |z| + 1 \le +\sqrt{2}$

$\Rightarrow -\sqrt{2} - 1 \le |z| \le \sqrt{2} - 1 \ldots\ldots (2)$

From (1) and (2) we get,

Maximum value of $|z| = \sqrt{2}+1$ and minimum value of $|z| = -\sqrt{2}-1$

Question106

Let $S = \{z = x + iy : |z - 1 + i| \ge |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of x, for which $u = 2x + iy \in S$ for some $y \in \mathbb{R}$, is [29-Jul-2022-Shift-2]

Options:

A. $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$

B. $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

C. $\left(-\sqrt{2},\ \frac{1}{2}\right]$

D. $\left(-\frac{1}{\sqrt{2}},\ \frac{1}{2\sqrt{2}}\right]$

Answer: B

Solution:

Question107

If the numbers appeared on the two throws of a fair six faced die are α and β, then the probability that $x^2 + \alpha x + \beta > 0$, for all $x \in R$, is : [25-Jul-2022-Shift-1]

Options:

A. $\frac{17}{36}$

B. $\frac{4}{9}$

C. $\frac{1}{2}$

D. $\frac{19}{36}$

Answer: A

Solution:

Solution:
For $x^2 + \alpha x + \beta > 0\ \forall x \in R$ to hold, we should have $\alpha^2 - 4\beta < 0$
If $\alpha = 1$, β can be 1, 2, 3, 4, 5, 6 i.e., 6 choices
If $\alpha = 2$, β can be 2, 3, 4, 5, 6 i.e., 5 choices
If $\alpha = 3$, β can be 3, 4, 5, 6 i.e., 4 choices
If $\alpha = 4$, β can be 5 or 6 i.e., 2 choices
If $\alpha = 6$, No possible value for β i.e., 0 choices
Hence total favourable outcomes
$= 6 + 5 + 4 + 2 + 0 + 0$
$= 17$
Total possible choices for α and β $= 6 \times 6 = 36$
Required probability $= \frac{17}{36}$

Question108

Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is

equal to
[25-Jul-2022-Shift-1]

Answer: 38

Solution:

Solution:

$\because$ Roots of $2ax^2 - 8ax + 1 = 0$ are $\frac{1}{p}$ and $\frac{1}{r}$ and roots of $6bx^2 + 12bx + 1 = 0$ are $\frac{1}{q}$ and $\frac{1}{s}$

Let $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ as $\alpha - 3\beta, \alpha - \beta, \alpha + \beta, \alpha + 3\beta$

So sum of roots $2\alpha - 2\beta = 4$ and $2\alpha + 2\beta = -2$

Clearly $\alpha = \frac{1}{2}$ and $\beta = -\frac{3}{2}$

Now product of roots, $\frac{1}{p} \cdot \frac{1}{r} = \frac{1}{2a} = -5 \Rightarrow \frac{1}{a} = -10$

and $\frac{1}{q} \cdot \frac{1}{x} = \frac{1}{6b} = -8 \Rightarrow \frac{1}{b} = -48$

So, $\frac{1}{a} - \frac{1}{b} = 38$

Question109

If for some p, q, r $\in$ R, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2 + r^2}{p^2}$ is equal to ________.
[26-Jul-2022-Shift-1]

Answer: 272

Solution:

$$(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array}$$

$\therefore \alpha + \beta > 0$ and $\alpha\beta > 0$

Also, it has a common root with $x^2 + 2x - 8 = 0$

$\therefore$ The common root between above two equations is 4 .

$\Rightarrow 16(p^2 + q^2) - 8q(p + r) + q^2 + r^2 = 0$

$\Rightarrow (16p^2 - 8pq + q^2) + (16q^2 - 8qr + r^2) = 0$

$\Rightarrow (4p - q)^2 + (4q - r)^2 = 0$

$\Rightarrow q = 4p$ and $r = 16p$

$\therefore \frac{q^2 + r^2}{p^2} = \frac{16p^2 + 256p^2}{p^2} = 272$

Question110

The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is ____________.
[26-Jul-2022-Shift-1]

Answer: 3

Solution:

Solution:
$x^8 - x^7 - x^6 + x^5 + 3x^4 - 4x^3 - 2x^2 + 4x - 1 = 0$
$\Rightarrow x^7(x-1) - x^5(x-1) + 3x^3(x-1) - x(x^2-1) + 2x(1-x) + (x-1) = 0$
$\Rightarrow (x-1)(x^7 - x^5 + 3x^3 - x(x+1) - 2x + 1) = 0$
$\Rightarrow (x-1)(x^7 - x^5 + 3x^3 - x^2 - 3x + 1) = 0$
$\Rightarrow (x-1)(x^5(x^2-1) + 3x(x^2-1) - 1(x^2-1)) = 0$
$\Rightarrow (x-1)(x^2-1)(x^5 + 3x - 1) = 0$
$\therefore x = \pm 1$ are roots of above equation and $x^5 + 3x - 1$ is a monotonic term hence vanishs at exactly one value of x other than 1 or -1.
$\therefore$3 real roots.

Question111

The minimum value of the sum of the squares of the roots of $x^2 + (3-a)x + 1 = 2a$ is:
[26-Jul-2022-Shift-2]

Options:

A. 4

B. 5

C. 6

D. 8

Answer: C

Solution:

Solution:

$$x^2 + (3-a)x + 1 = 2a \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$\alpha + \beta = a - 3,\ \alpha\beta = 1 - 2a$
$\Rightarrow \alpha^2 + \beta^2 = (a-3)^2 - 2(1-2a)$
$= a^2 - 6a + 9 - 2 + 4a$
$= a^2 - 2a + 7$
$= (a-1)^2 + 6$

So, $\alpha^2 + \beta^2 \geq 6$

Question112

Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of $(a + b - c)$ is __________.
[26-Jul-2022-Shift-2]

Options:

A. 12

B. 13

C. 14

D. 16

Answer: A

Solution:

Solution:
Abscissae of PQ are roots of $x^2 - 4x - 6 = 0$
Ordinates of PQ are roots of $y^2 + 2y - 7 = 0$
and PQ is diameter
⇒ Equation of circle is
$x^2 + y^2 - 4x + 2y - 13 = 0$
But, given $x^2 + y^2 + 2ax + 2by + c = 0$
By comparison $a = -2, b = 1, c = -13$
$\Rightarrow a + b - c = -2 + 1 + 13 = 12$

Question113

If α, β are the roots of the equation
$$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right) + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$$
then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is :
[27-Jul-2022-Shift-2]

Options:

A. $3x^2 - 20x - 12 = 0$

B. $3x^2 - 10x - 4 = 0$

C. $3x^2 - 10x + 2 = 0$

D. $3x^2 - 20x + 16 = 0$

Answer: B

Solution:

Solution:

$3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}} = 3^{\sqrt{\log_3 5}} - \left(3^{\log_3 5}\right)^{\sqrt{\log_5 3}}$

$3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} = 5^{(\log_5 3)^{\frac{2}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} = 0$

Note: In the given equation ' x ' is missing.

So

$x^2 - 5x + 3(-1) = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

$\alpha + \beta + \frac{1}{\alpha} + \frac{1}{\beta} = (\alpha + \beta) + \frac{\alpha + \beta}{\alpha\beta}$

$= 5 - \frac{5}{3} = \frac{10}{3}$

$\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = 2 + \alpha\beta + \frac{1}{\alpha\beta} = 2 - 3 - \frac{1}{3} = \frac{-4}{3}$

So Equation must be option (B).

Question114

The sum of all real values of x for which $\frac{3x^2 - 9x + 17}{x^2 + 3x + 10} - \frac{5x^2 - 7x + 19}{3x^2 + 5x + 12}$ is equal to ________.

[28-Jul-2022-Shift-1]

Answer: 6

Solution:

Solution:

$\frac{3x^2 - 9x + 17}{x^2 + 3x + 10} = \frac{5x^2 - 7x + 19}{3x^2 + 5x + 12}$

$\Rightarrow \frac{3x^2 - 9x + 17}{5x^2 - 7x + 19} = \frac{x^2 + 3x + 10}{3x^2 + 5x + 12}$

$\frac{-2x^2 - 2x - 2}{5x^2 - 7x + 19} = \frac{-2x^2 - 2x - 2}{3x^2 + 5x + 12}$

Either $x^2 + x + 1 = 0$ or No real roots $\Rightarrow 5x^2 - 7x + 19 = 3x^2 + 5x + 12$

$2x^2 - 12x + 7 = 0$

sum of roots $= 6$

Question115

Let α, β be the roots of the equation $x^2 - \sqrt{2}x + \sqrt{6} = 0$ and $\frac{1}{\alpha^2} + 1, \frac{1}{\beta^2} + 1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are :

[28-Jul-2022-Shift-2]

Options:

A. non-real complex numbers

B. real and both negative

C. real and both positive

D. real and exactly one of them is positive

Answer: B

Solution:

Solution:

$\alpha + \beta = \sqrt{2}, \alpha\beta = \sqrt{6}$

$\frac{1}{\alpha^2} + 1 + \frac{1}{\beta^2} + 1 = 2 + \frac{\alpha^2 + \beta^2}{6}$

$= 2 + \frac{2 - 2\sqrt{6}}{6} = -a$

$\left(\frac{1}{\alpha^2} + 1\right)\left(\frac{1}{\beta^2} + 1\right) = 1 + \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\alpha^2\beta^2}$

$= \frac{7}{6} + \frac{2 - 2\sqrt{6}}{6} = b$

$\Rightarrow a + b = \frac{-5}{6}$

So, equation is $x^2 + \frac{17x}{6} + \frac{7}{6} = 0$

OR $6x^2 + 17x + 7 = 0$

Both roots of equation are - ve and distinct

Question116

Let $f(x) = ax^2 + bx + c$ be such that $f(1) = 3$, $f(-2) = \lambda$ and $f(3) = 4$. If $f(0) + f(1) + f(-2) + f(3) = 14$, then λ is equal to : [28-Jul-2022-Shift-2]

Options:

A. -4

B. $\frac{13}{2}$

C. $\frac{23}{2}$

D. 4

Answer: D

Solution:

Solution:

$f(1) = a + b + c = 3 \ldots\ldots$ (i)

$f(3) = 9a + 3b + c = 4 \ldots.$ (ii)

$f(0) + f(1) + f(-2) + f(3) = 14$

OR $c + 3 + (4a - 2b + c) + 4 = 14$

OR $4a - 2b + 2c = 7 \ldots.$ (iii)

From (i) and (ii) $8a + 2b = 1 \ldots.$ (iv)

From (iii) $-(2) \times$ (i)

$\Rightarrow 2a - 4b = 1 \ldots\ldots (v)$
From (iv) and (v) $a = \frac{1}{6}$, $b = \frac{-1}{6}$ and $c = 3$
$f(-2) = 4a - 2b + c$
$= \frac{4}{6} + \frac{2}{6} + 3 = 4$

Question117

Let $\alpha, \beta (\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n, n \in N$, then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}{}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to [29-Jul-2022-Shift-2]

Options:

Answer: 16

Solution:

Solution:
α and β are the roots of the quadratic equation $x^2 - x - 4 = 0$.
$\therefore \alpha$ and β are satisfy the given equation.
$\alpha^2 - \alpha - 4 = 0$
$\Rightarrow \alpha^{n+1} - \alpha^n - 4\alpha^{n-1} = 0 \ldots\ldots (i)$
and $\beta^2 - \beta - 4 = 0$
$\Rightarrow \beta^{n+1} - \beta^n - 4\beta^{n-1} = 0 \quad \cdots\cdots (2)$Substituting (2) from (1), we get,
$(\alpha^{n+1} - \beta^{n+1}) - (\alpha^n - \beta^n) - 4(\alpha^{n-1} - \beta^{n-1}) = 0$
$\Rightarrow P_{n+1} - P_n - 4P_{n-1} = 0$
$\Rightarrow P_{n+1} = P_n + 4P_{n-1}$
$\Rightarrow P_{n+1} - P_n = 4P_{n-1}$
For $n = 14$, $P_{15} - P_{14} = 4P_{13}$
For $n = 15$, $P_{16} - P_{15} = 4P_{14}$
Now, $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}{}^2 + P_{14}P_{15}}{P_{13}P_{14}}$
$= \frac{P_{16}(P_{15} - P_{14}) - P_{15}(P_{15} - P_{14})}{P_{13}P_{14}}$
$= \frac{(P_{15} - P_{14})(P_{16} - P_{15})}{P_{13}P_{14}}$
$= \frac{(4P_{13})(4P_{14})}{P_{13}P_{14}}$
$= 16$

Question118

Let $S = \left\{ x \in [-6, 3] - \{-2, 2\} : \frac{|x+3| - 1}{|x| - 2} \geq 0 \right\}$ and $T = \{x \in \mathbb{Z} : x^2 - 7|x| + 9 \leq 0\}$ Then the number of elements in $S \cap T$ is : [28-Jul-2022-Shift-2]

Options:

A. 7

B. 5

C. 4

D. 3

Answer: D

Solution:

Solution:
$|x^2| - 7 \mid x \mid + 9 \le 0$
$\Rightarrow \mid x \Big| \in \left[\frac{7-\sqrt{13}}{2}, \frac{7+\sqrt{13}}{2} \right]$
As $x \in Z$
So, x can be $\pm 2, \pm 3, \pm 4, \pm 5$
Out of these values of x,
$x = 3, -4, -5$
satisfy S as well
$n(S \cap T) = 3$

Question119

Let $i = \sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$ and $n = [|k|]$ be the greatest integral part of $|k|$. Then, $\sum_{j=0}^{n+5} (j+5)^2 - \sum_{j=0}^{n+5} (j+5)$ is equal to
[2021, 24 Feb. Shift-II]

Answer: 310

Solution:

Solution:
Given, $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}} = k$
$\because \; -1 + i\sqrt{3} = 2e^{i2\pi/3}$
$1 + i\sqrt{3} = 2e^{i\pi/3}$
$1 - i = \sqrt{2}e^{-i\pi/4}$
$1 + i = \sqrt{2}e^{i\pi/4}$
Now, $\frac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{(\sqrt{2}e^{-i\pi/4})^{24}} + \frac{(2e^{i\pi/3})^{21}}{(\sqrt{2}e^{i\pi/4})^{24}}$
$= \frac{2^{21} \cdot e^{i14\pi}}{2^{12} \cdot e^{-i6\pi}} + \frac{2^{21} \cdot e^{i7\pi}}{2^{12} \cdot e^{i6\pi}}$
$= 2^9 \cdot e^{i20\pi} + 2^9 \cdot e^{i\pi}$
$= 2^9(1) + 2^9(-1)$
$\Rightarrow 2^9 - 2^9 = 0 = k$ (given)
$\therefore \; n = [|k|] = [101] = 0$
Now, $\sum_{j=0}^{5} (j+5)^2 - \sum_{j=0}^{5} (j+5) \; [\because n = 0]$
$= [5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2]$
$-[5 + 6 + 7 + 8 + 9 + 10]$
$= [\,(1^2 + 2^2 + 3^2 + \ldots + 10^2) -$
$(1^2 + 2^2 + \ldots + 4^2)\,] - [\,(1 + 2 + 3 + \ldots + 10)$

$-(1+2+3+4)]$

$= \left[\frac{10\times11\times21}{6} - \frac{4\times5\times9}{6}\right] - \left[\frac{10\times11}{2} - \frac{4\times5}{2}\right]$

$= (385-30)-(55-10)$

$= 385-45 = 310$

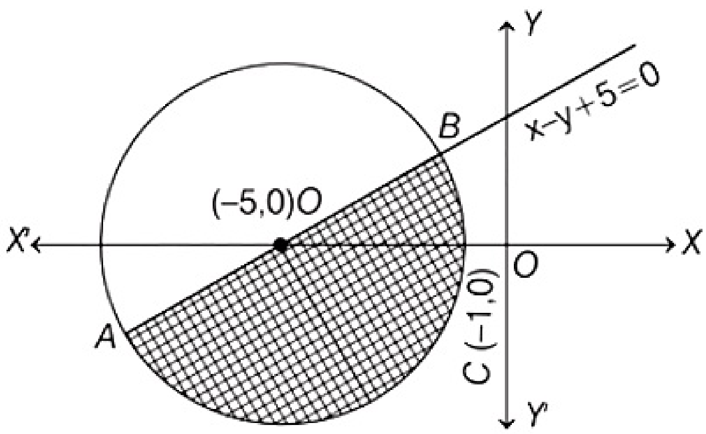
Question120

Let z be those complex numbers which satisfy $|z+5| \le 4$ and $z(1+i)+\overline{z}(1-i) \ge -10$, $i=\sqrt{-1}$. If the maximum value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $(\alpha+\beta)$ is
[2021, 26 Feb. Shift-II]

Answer: 48

Solution:

Solution:
Given, $|z+5| \le 4$, which is equation of circle.
$|z+5| \le 4$
$\Rightarrow (x+5)^2+y^2 \le 16$
and $z(1+i)+\overline{z}(1-i) \ge -10$
$\Rightarrow (z+\overline{z})+i(z-\overline{z}) \ge -10$
$\Rightarrow x-y+5 \ge 0$
From Eqs. (i) and (ii), region bounded by inequalities are



Now, $|z+1|^2 = |z-(-1)|^2$
Maximum value of $|z+1|^2$ will be equal to $(AC)^2$.
Now, $(x+5)^2+y^2 = 16$
and $x-y+5=0$
Given, $y = \pm2\sqrt{2}$
and $x = \pm2\sqrt{2}-5$
∴ Coordinates are
c$A(-2\sqrt{2}-5, -2\sqrt{2})$
$B(2\sqrt{2}-5, 2\sqrt{2})$
$C(-1, 0)$
Then,
$AC^2 = (2\sqrt{2}+4)^2+(2\sqrt{2})^2$
$= 32+16\sqrt{2}$
Given, that maximum value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$
$\Rightarrow \alpha+\beta\sqrt{2} = 32+16\sqrt{2}$
$\Rightarrow \alpha=32, \beta=16$
$\therefore \alpha+\beta = 32+16 = 48$

Question121

Let the lines $(2-i)z=(2+i)\overline{z}$ and $(2+i)z+(i-2)\overline{z}-4i=0$, (here $i^2=-1$) be normal to a circle C. If the line $iz+\overline{z}+1+i=0$ is tangent to this circle C, then its radius is
[2021, 25 Feb. Shift-1]

Options:

A. $\frac{3}{\sqrt{2}}$

B. $\frac{1}{2\sqrt{2}}$

C. $3\sqrt{2}$

D. $\frac{3}{2\sqrt{2}}$

Answer: D

Solution:

Solution:
Given, $(2-i)z=(2+i)\overline{z}$
Let $z=x+iy$, then $\overline{z}=x-iy$
$\Rightarrow(2-i)(x+iy)=(2+i)(x-iy)$
$\Rightarrow 2x-ix+2iy+y=2x+ix-2iy+y$
$\Rightarrow 2ix-4iy=0$
$\therefore$ Equation of line $L_1 \Rightarrow x-2y=0$ $\cdots\cdots$ (i)
Also, $(2+i)z+(i-2)\overline{z}-4i=0$
$\Rightarrow(2+i)(x+iy)+(i-2)(x-iy)-4i=0$
$\Rightarrow 2x+ix+2iy-y+ix-2x+y$
$+2iy-4i=0$
$\Rightarrow$ $2ix+4iy-4i=0$
$\therefore$ Equation of line $L_2 \Rightarrow x+2y-2=0$... (ii)
From Eqs. (i) and (ii),
$4y=2$ or $y=1/2$ and $x=1$
Hence, centre $=(1,1/2)$
Equation of third line
$L_3 \Rightarrow iz+\overline{z}+1+i=0$
$\Rightarrow$ $i(x+iy)+(x-iy)+1+i=0$
$\Rightarrow$ $ix-y+x-iy+1+i=0$
$\Rightarrow$ $(x-y+1)+i(x-y+1)=0$
$\therefore$ Radius = Distance of point $(1,1/2)$ to the line $x-y+1=0$

$$\therefore\ r=\frac{\left|1-\frac{1}{2}+1\right|}{\sqrt{1^2+1^2}}=\frac{3}{2\sqrt{2}}$$

Question122

Let α and β be two real numbers, such that $\alpha+\beta=1$ and $\alpha\beta=-1$. Let $p_n=(\alpha)^n+(\beta)^n$, $p_{n-1}=11$ and $p_{n+1}=29$, for some integer $n\geq 1$. Then, the value of $p_n{}^2$ is

[2021, 26 Feb. Shift-III]

Answer: 324

Solution:

Given that, $\alpha + \beta = 1$, $\alpha\beta = -1$
Let α, β be roots of quadratic equation, then the quadratic equation be
$x^2 - x - 1 = 0$
Now, $\alpha^2 - \alpha - 1 = 0$
$\Rightarrow \alpha^2 = \alpha + 1 \quad \cdots\cdots (i)$
Similarly, $\beta^2 = \beta + 1 \quad \cdots\cdots (ii)$
Multiply α^{n-1} in Eq. (i), we get
$\alpha^{n+1} = \alpha^n + \alpha^{n-1} \quad \cdots\cdots (iii)$
Multiply β^{n-1} in Eq. (ii), we get
$\beta^{n+1} = \beta^n + \beta^{n-1} \quad \cdots\cdots (iv)$
Add Eqs. (iii) and (iv), we get
$\alpha^{n+1} + \beta^{n+1} = (\alpha^n + \beta^n) + (\alpha^{n-1} + \beta^{n-1})$
$p_{n+1} = p_n + p_{n-1}$
$29 = p_n + 11$
$\Rightarrow P_n = 18$
$p_n^{\ 2} = (18)^2 = 324$

Question123

The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is
[2021, 26 Feb. Shift-1]

Answer: 1

Solution:

$\log_4(x-1) = \log_2(x-3)$ (given)
$\Rightarrow \log_2 2(x-1) = \log_2(x-3)$
Using property of logarithm,
$\log_b c^a = \dfrac{1}{c}\log_b a$
$\Rightarrow \dfrac{1}{2}\log_2(x-1) = \log_2(x-3)$
$\Rightarrow \log_2(x-1) = 2\log_2(x-3)$
$\Rightarrow \log_2(x-1) = \log_2(x-3)^2$
On comparing, $x - 1 = (x-3)^2$
or $x - 1 = x^2 + 9 - 6x$
$\Rightarrow x^2 - 7x + 10 = 0$
$\Rightarrow x^2 - 5x - 2x + 10 = 0$
$\Rightarrow (x-5)(x-2) = 0$
$\Rightarrow x = 2, 5$
$x = 2$(rejected) as $x > 1$
$\therefore x = 5$ is only solution i.e. number of solution is 1.

Question124

Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is
[2021, 25 Feb. Shift-II]

Options:

A. 4

B. 3

C. 2

D. 1

Answer: C

Solution:

Solution:
We have, $x^2 - 6x - 2 = 0$
Given, α and β are roots of above quadratic equation, then
$\alpha^2 - 6\alpha - 2 = 0$
$\beta^2 - 6\beta - 2 = 0$
Also, given $a_n = \alpha^n - \beta^n$, then

$$\frac{a_{10} - 2a_8}{3a_9}$$

$$= \frac{(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^{10} - 2\alpha^8 - \beta^{10} + 2\beta^8}{3(\alpha^9 - \beta^9)} = \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{3(\alpha^9 - \beta^9)}$$

[from Eqs. (i) and (ii) $\alpha^2 - 2 = 6\alpha$, $\beta^2 - 2 = 6\beta$]

$$= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{6\alpha^9 - 6\beta^9}{3(\alpha^9 - \beta^9)} = \frac{6(\alpha^9 - \beta^9)}{3(\alpha^9 - \beta^9)}$$

$= 2$

Question125

If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to
[2021, 25 Feb. Shift-II]

Options:

A. 3

B. -3

C. 7

D. -7

Answer: D

Solution:

Solution:
Given, root of $z^2 + \alpha z + \beta = 0$ is $1 - 2i$.
Since, it is quadratic equation and one root is complex in nature, its another root is complex conjugate.
$\therefore$ Two roots are $1 - 2i$ and $1 + 2i$.
Now, sum of roots $= -\dfrac{\alpha}{1} = -\alpha$
$= (1 - 2i) + (1 + 2i) = 2$
Gives, $\alpha = -2$
Product of roots $= \dfrac{\beta}{1} = \beta$
$= (1 - 2i)(1 + 2i) = 1 + 4 = 5$
Gives, $\beta = 5$
$\therefore\ \alpha - \beta = -2 - 5 = -7$

Question126

The integer ' k ', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is
[2021, 25 Feb. Shift-1]

Options:

A. 3

B. 2

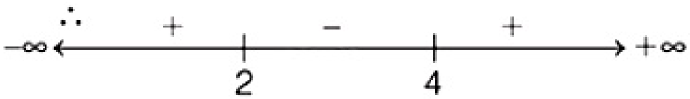
C. 0

D. 4

Answer: A

Solution:

Given, $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$
Here, $a > 0$
$\therefore D < 0$
$\Rightarrow\ [2(3k - 1)]^2 - 4(8k^2 - 7) < 0$
$\Rightarrow\ 4(9k^2 + 1 - 6k) - 4(8k^2 - 7) < 0$
$\Rightarrow\ k^2 - 6k + 8 < 0$
$\Rightarrow\ (k - 4)(k - 2) < 0$
$\therefore$



$k \in (2, 4)$
$\therefore$ Required integer, $k = 3$

Question127

The sum of 162th power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is
[2021, 26 Feb. Shift-1]

Answer: 3

Solution:

Given, $x^3 - 2x^2 + 2x - 1 = 0$
i.e. $(x^3 - 1) - (2x^2 - 2x) = 0$
$\Rightarrow (x - 1)(x^2 + x + 1) - 2x(x - 1) = 0$
$\Rightarrow (x - 1)(x^2 + x + 1 - 2x) = 0$
$\Rightarrow (x - 1)(x^2 - x + 1) = 0$
$\therefore x = 1$ and $x = \dfrac{-(-1) \pm \sqrt{1 - 4}}{2} = \dfrac{1 \pm \sqrt{3}i}{2}$
$\therefore$ Roots are $1, -\omega_1 - \omega^2$.
Then, sum of 162^{th} power of the roots
$= (1)^{162} + (-\omega)^{162} + (-\omega^2)^{162}$
$= 1 + \omega^{162} + \omega^{324}$
$= 1 + (\omega^3)^{54} + (\omega^3)^{108}$
$= 1 + (1)^{54} + (1)^{108}$ $[\because \omega^3 = 1]$
$= 1 + 1 + 1 = 3$

Question128

Let a, b, c be in an arithmetic progression. Let the centroid of the triangle with vertices (a, c), (2, b) and (a, b) be $\left(\frac{10}{3}, \frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is
[2021, 24 Feb. Shift-II]

Options:

A. $\frac{71}{256}$

B. $\frac{69}{256}$

C. $-\frac{69}{256}$

D. $-\frac{71}{256}$

Answer: D

Solution:

Given, a, b, c are in AP.
(a, c), (2, b), (a, b) are vertices of triangle.

Centroid $= \left(\frac{10}{3}, \frac{7}{3}\right)$

α and β are the roots of equation $ax^2 + bx + 1 = 0$
$\because$a, b, c are in AP.
$\therefore\ 2b = a + c$

Centroid $= \left(\frac{a+2+a}{3}, \frac{c+b+b}{3}\right)$

$= \left(\frac{2a+2}{3}, \frac{c+2b}{3}\right) = \left(\frac{10}{3}, \frac{7}{3}\right)$

$\Rightarrow\ \frac{2a+2}{3} = \frac{10}{3}$ and $\frac{c+2b}{3} = \frac{7}{3}$

$\Rightarrow\ a = 4$
$\Rightarrow\ c + a + c = 7\ [\because 2b = a + c]$
$\Rightarrow\ 2c = 7 - 4\ [\because a = 4]$
$c = 3 / 2$

Also, $2b = a + c = 4 + \frac{3}{2}$

$\Rightarrow\ b = 11 / 4$
Now, α and β are roots of $ax^2 + bx + 1 = 0$

$\therefore\ \alpha + \beta = \frac{-b}{a} = \frac{-11/4}{4}$

$\Rightarrow\ \alpha + \beta = \frac{-11}{16}$

$\Rightarrow\ \alpha\beta = \frac{1}{a} = \frac{1}{4}$

$\Rightarrow\ \alpha\beta = \frac{1}{4}$

Now, $\alpha^2 \& + \beta^2 - \alpha\beta$

$= (\alpha + \beta)^2 - 3\alpha\beta$

$= \left(\frac{-11}{16}\right)^2 - 3 \times \frac{1}{4}$

$= \frac{121 - 192}{256} = \frac{-71}{256}$

Question129

The number of the real roots of the equation $(x+1)^2 + |\, x - 5 \,| = \frac{27}{4}$ is [2021,24 Feb. Shift-II]

Answer: 2

Solution:

Given, equation $(x+1)^2 + |\,(x-5)\,| = \frac{27}{4}$

Case I For $x \geq 5$

$l\,l\ \Rightarrow (x+1)^2 + (x-5) = \frac{27}{4}$

$\Rightarrow\ x^2 + 3x - 4 = \frac{27}{4}$

$\Rightarrow 4x^2 + 12x - 43 = 0$

$\therefore\ x\ = \frac{-12 \pm \sqrt{144 + 688}}{8}$

$= \frac{-12 \pm \sqrt{832}}{8} = \frac{-12 \pm 28.8}{8}$

$x = \frac{-3 \pm 7.2}{8}$

$x = \frac{-3 + 7.2}{8}, \frac{-3 - 7.2}{8}$

Both the values are less than 5.

∴ No solution from here.

Case II $x < 5$

$\Rightarrow (x + 1)^2 + (x - 5) = \frac{27}{4}$

$\Rightarrow x^2 + x - 6 - \frac{27}{4} = 0$

$\Rightarrow 4x^2 + 4x - 3 = 0$

$\Rightarrow x = \frac{-4 \pm \sqrt{16 + 48}}{8}$

$= \frac{-4 \pm 8}{8}$

$\Rightarrow x = \frac{-12}{8}, \frac{4}{8}$, both are less than 5.

∴ These values must be the solution. Hence, here 2 real roots are possible.

Question130

If the least and the largest real values of α, for which the equation $z + \alpha | z - 1 | + 2i = 0$ $(z \in C$ and $i = \sqrt{-1})$ has a solution, are p and q respectively, then $4(p^2 + q^2)$ is equal to
[2021,24 Feb. Shift-I]

Answer: 10

Solution:

Given, $\alpha_{\text{least}} = p$

$\alpha_{\max} = q$

Equation given is $z + \alpha | z - 1 | + 2i = 0$;

$z \in C$ and $i = \sqrt{-1}$

Let $z = x + iy$

Then, $z + \alpha | z - 1 | + 2i = 0$

$\Rightarrow x + iy + \alpha \sqrt{(x - 1)^2 + y^2} + 2i = 0$

$\Rightarrow \left(x + \alpha\sqrt{(x-1)^2 + y^2}\right) + i(y+2) = 0$
$\therefore y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2 + y^2} = 0$
$y = -2$ and $x^2 = \alpha^2(x^2 + 1 - 2x + y^2)$
$x^2 = \alpha^2(x^2 - 2x + 5)$ $(\because y = -2)$
$\Rightarrow \alpha^2 = \dfrac{x^2}{x^2 - 2x + 5}$
$\therefore \alpha^2 \in \left[0, \dfrac{5}{4}\right]$
$\therefore \alpha \in \left[-\dfrac{\sqrt{5}}{2}, \dfrac{\sqrt{5}}{2}\right]$
Now, $4(p^2 + q^2) = 4[(\alpha_{least})^2 + (\alpha_{max})^2]$
$= 4\left[\left(-\dfrac{\sqrt{5}}{2}\right)^2 + \left(\dfrac{\sqrt{5}}{2}\right)^2\right]$
$= 4 \times \left[\dfrac{5}{4} + \dfrac{5}{4}\right] = 10$

Question131

Let p and q be two positive numbers such that p + q = 2 and $p^4 + q^4 = 272$. Then p and q are roots of the equation: [24-Feb-2021 Shift 1]

Options:

A. $x^2 - 2x + 2 = 0$

B. $x^2 - 2x + 8 = 0$

C. $x^2 - 2x + 136 = 0$

D. $x^2 - 2x + 16 = 0$

Answer: D

Solution:

Solution:
We have
$(p^2 + q^2)^2 - 2p^2q^2 = 272$
$((p+q)^2 - 2pq)^2 - 2p^2q^2 = 272$
$(4 - 2pq)^2 - 2p^2q^2 = 272$
$16 - 16pq + 2p^2q^2 = 272$
$(pq)^2 - 8pq - 128 = 0$
$pq = \dfrac{8 \pm 24}{2} = 16, -8$
$\therefore pq = 16$ $(\because p, q > 0)$

$\therefore$ Required equation :
$x^2 - (2)x + 16 = 0$

Question132

If the equation $a|z|^2 + \overline{\overline{\alpha} z + \alpha\overline{z}} + d = 0$ represents a circle, wherea,d are real constants, then which of the following condition is correct? [2021, 18 March Shift-I]

Options:

A. $|\alpha|^2 - ad \neq 0$

B. $|\alpha|^2 - ad > 0$ and $a \in R - \{0\}$

C. $|\alpha|^2 - ad \geq 0$ and $a \in R$

D. $\alpha = 0$, a, d $\in R^+$

Answer: B

Solution:

Solution:
Given, $a|z|^2 + \overline{\overline{\alpha}z + \alpha\overline{z}} + d = 0$
$\Rightarrow a|z|^2 + \alpha\overline{z} + \overline{\alpha}z + d = 0$ ...(i)
Putting $z = x + iy$ and $\alpha = p + iq$ in Eq. (i),
we get
$a(x^2 + y^2) + (p + iq)(x - iy) + (p - iq)$
$\Rightarrow (x + iy) + d = 0$
$a(x^2 + y^2) + px + qy - ipy + iqx + px + qy \quad - iqx + ipy + d = 0$
$\Rightarrow a(x^2 + y^2) + 2px + 2qy + d = 0$
$\Rightarrow x^2 + y^2 + \left(\frac{2p}{a}\right)x + \left(\frac{2q}{a}\right)y + \frac{d}{a} = 0$ be a cricle
If $a \neq 0$ and $r^2 = \left(\frac{p^2}{a^2} + \frac{q^2}{a^2} - \frac{d}{a}\right) > 0$If $a \neq 0$ and $r^2 = \left(\frac{p^2}{a^2} + \frac{q^2}{a^2} - \frac{d}{a}\right) > 0$
$\Rightarrow p^2 + q^2 - ad > 0$
$\Rightarrow |\alpha|^2 - ad > 0$
and $a \in R - \{0\}$

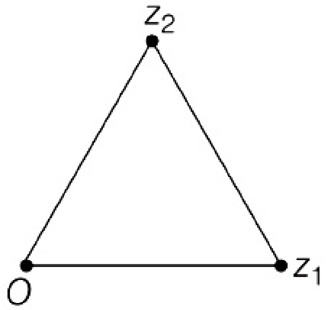
Question133

Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of $|a|$ is [2021, 18 March Shift-I]

Answer: 6

Solution:

Solution:
Given, z_1, z_2 are the roots of
$z^2 + az + 12 = 0$
$\therefore z_1 + z_2 = \frac{-a}{1} = -a$
and $z_1 z_2 = \frac{12}{1} = 12$
Now, z_1, z_2 and origin forms an equilateral triangle.



l c $\therefore$ $z_1^2 + z_2^2 + 0^2 = z_1 z_2 + 0 + 0$
$\Rightarrow z_1^2 + z_1^2 = z_1 z_2$
$\Rightarrow z_1^2 + z_2^2 + 2z_1z_2 = z_1z_2 + 2z_1z_2$
$\Rightarrow (z_1 + z_2)^2 = 3z_1z_2$
$\Rightarrow (-a)^2 = 3 \times (12)$
$\Rightarrow a^2 = 36 \Rightarrow |a|^2 = 36$
$\Rightarrow |a| = \pm 6$
But $|a| \geq 0$
$\therefore |a| = 6$

Question134

Let a complex number be $w = 1 - \sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w, is equal to
[2021, 18 March Shift-III]

Options:

A. 4

B. $\frac{1}{2}$

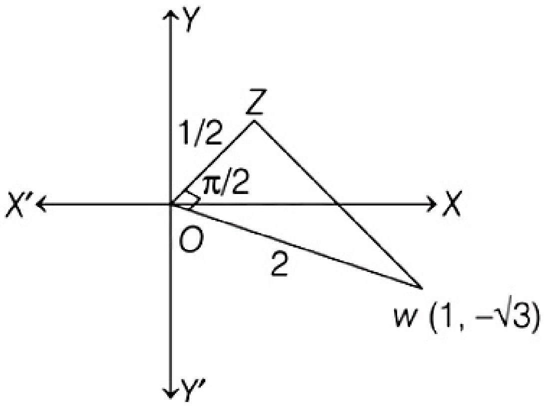
C. $\frac{1}{4}$

D. 2

Answer: B

Solution:

Solution:
Given, $w = 1 - \sqrt{3}i$
$\Rightarrow |w| = \sqrt{(1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = 2$
and $|zw| = 1 \Rightarrow |z||w| = 1$
$\Rightarrow |z| = \frac{1}{|w|} = \frac{1}{2}$



∴ Area of Δ = $\frac{1}{2} \times \frac{1}{2} \times 2 = \frac{1}{2}$

Question135

Let S_1, S_2 and S_3 be three sets defined as
$S_1 = \{z \in C : |z - 1| \le \sqrt{2}\}$
$S_2 = \{z \in C : Re[(1 - i)z] \ge 1\}$
$S_3 = \{z \in C : Im(z) \le 1\}$
Then, the set $S_1 \cap S_2 \cap S_3$
[2021, 17 March Shift-II]

Options:

A. is a singleton

B. has exactly two elements

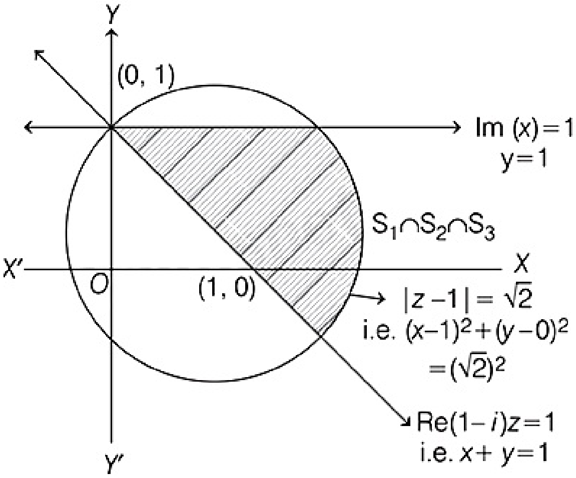
C. has infinitely many elements

D. has exactly three elements

Answer: C

Solution:

Solution:
For $|z - 1| \le \sqrt{2}$, . . . (i)
z lies on and inside the circle of radius $\sqrt{2}$ units and centre $(1, 0)$.
For S_2 , let $z = x + iy$
Now $(1 - i)(z) = (1 - i)(x + iy)$
$= x + iy - ix + y = (x + y) + i(y - x)$
$\therefore Re[(1 - i)z] = (x + y)$, which is greater than or equal to one.
i.e., $x + y \ge 1 \quad \cdots\cdots (ii)$
Also, for S_3
Let $z = x + iy$
$\therefore I_m(z) = y$, which is less than or equal to
one.
i.e., $y \le 1 \quad \cdots\cdots (iii)$
Concept Draw the graph of Eqs. (i), (ii) and (iii) and then select the common region bounded by Eqs. (i), (ii) and (iii) for $S_1 \cap S_2 \cap S_3$.



$\therefore S_1 \cap S_2 \cap S_3$ has infinitely many elements.

Question136

The area of the triangle with vertices A(z), B(iz) and C(z + iz) is [2021, 17 March Shift-I]

Options:

A. 1

B. $\frac{1}{2}|z|^2$

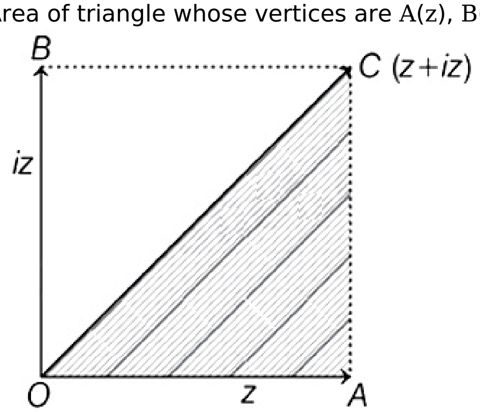
C. $\frac{1}{2}$

D. $\frac{1}{2}|z+iz|^2$

Answer: B

Solution:

Solution:
Area of triangle whose vertices are A(z), B(iz), C(z + iz)



Area of the triangle

$= \frac{1}{2}|z|\,|iz| = \frac{1}{2}|z|^2$

Question137

The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\ldots\infty}}}}$ is

[2021, 17 March Shift-I]

Options:

A. $2+\frac{2}{5}\sqrt{30}$

B. $2+\frac{4}{\sqrt{5}}\sqrt{30}$

C. $4+\frac{4}{\sqrt{5}}\sqrt{30}$

D. $5+\frac{2}{5}\sqrt{30}$

Answer: A

Solution:

Solution:

Let $x=4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\ldots\infty}}}}$

$x=4+\cfrac{1}{5+\cfrac{1}{x}}$

$\Rightarrow (x-4)(5x+1)=x$

$\Rightarrow 5x^2-19x-4=x$

$\Rightarrow 5x^2-20x-4=0$

$\Rightarrow x=\frac{20\pm\sqrt{400+80}}{10}$

$\Rightarrow x=\frac{20\pm\sqrt{480}}{10}$

$\Rightarrow x=2\pm\sqrt{\frac{480}{100}}$

$=2\pm\frac{2}{5}\sqrt{30}$

$\because$ x nless 0

So, $x=2+\frac{2}{5}\sqrt{30}$

Question138

The number of elements in the set $\{x\in R:(|x|-3)\,|\,x+4\,|\;=6\}$ is equal to
[2021, 16 March Shift-1]

Options:

A. 3

B. 2

C. 4

D. 1

Answer: B

Solution:

Solution:
Given, set $= \{x \in R : (|x| - 3) \mid x + 4 \mid = 6\}$
As, we already know

$$|x| = \begin{cases} x_1 & x \geq 0 \\ -x_1 & x < 0. \end{cases} \text{ and}$$

$$|x+4| = \begin{cases} x+4 & x \geq -4 \\ -(x+4) & x < -4. \end{cases}$$

Case I
$x < -4$
$r(-x-3)(-x-4) = 6$
$(x+3)(x+4) = 6$
$x^2 + 7x + 12 = 6$
$x^2 + 7x + 6 = 0$
$(x+6)(x+1) = 0$
$x = -6 \text{ or } x = -1$
We will reject $x = -1$ as, $-1 > -4$
$\therefore$ When $x < -4$, $x = -6$ is the solution.
Case II
$-4 \leq x < 0$
$(-x-3)(x+4) = 6$
$\Rightarrow -(x+3)(x+4) = 6$
$\Rightarrow -(x^2 + 7x + 12) = 6$
$\Rightarrow x^2 + 7x + 18 = 0$
As, the discriminant of this quadratic
equation is $D = 7^2 - 4 \cdot 18 = 49 - 72 = -23$
$\because D = -23$ and $D < 0$
So, no real roots and as per the question,
x$\in$ R.
No solution when $-4 \leq x < 0$.

Case III
$x \geq 0$
$(|x| - 3) \mid x + 4 \mid = 6$
$\Rightarrow (x-3)(x+4) = 6$
$\Rightarrow x^2 + x - 12 = 6$
$\Rightarrow x^2 + x - 18 = 0$
$x = \dfrac{-1 \pm \sqrt{1+72}}{2} = \dfrac{-1 \pm \sqrt{73}}{2}$
We will reject $x = \dfrac{-1-\sqrt{73}}{2}$ as $\dfrac{-1-\sqrt{73}}{2} < 0$ and here, $x \geq 0$.
So, $x = \dfrac{-1+\sqrt{73}}{2}$, when $x \geq 0$.
$\therefore x = -6$ and $x = \dfrac{-1+\sqrt{73}}{2}$
are the two solutions which belong to the set.
Hence, number of solutions $= 2$

Question139

Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients, such that $\int_0^1 P(x)dx = 1$ and $P(x)$ leaves remainder 5 when it is divided by $(x - 2)$. Then, the value of $g(b + c)$ is
[2021, 16 March Shift-II]

Options:

A. 9

B. 15

C. 7

D. 11

Answer: C

Solution:

Solution:
$P(x) = x^2 + bx + c$
$\Rightarrow \int_0^1 (x^2 + bx + c)dx = 1$
$\Rightarrow \left[\frac{x^3}{3} + \frac{bx^2}{2} + cx\right]_0^1 = 1$
$\Rightarrow \frac{1}{3} + \frac{b}{2} + c = 1$
$\Rightarrow b + 2c = 4/3 \quad \cdots\cdots (i)$
And, $P(x) = (x - 2) \cdot Q(x) + 5$
When, $x = 2$
$P(2) = 5$
$4 + 2b + c = 5$
$c = 1 - 2b \quad \cdots\cdots (ii)$
Putting $c = 1 - 2b$ in Eq. (i),
$b + 2(1 - 2b) = 4/3$
$\Rightarrow -3b + 2 = 4/3$
$\Rightarrow b = 2/9$
$\therefore c = 1 - 4/9 = 5/9$
$9(b + c) = 9\left(\frac{2}{9} + \frac{5}{9}\right) = 7$

Question140

Let z and w be two complex numbers, such that $w = z\bar{z} - 2z + 2$, $\left|\frac{z + i}{z - 3i}\right| = 1$ and Re(w) has minimum value. Then, the minimum value of $n \in N$ for which w^n is real, is equal to
[2021, 16 March Shift-1]

Answer: 4

Solution:

Solution:

Given, $w = z\bar{z} - 2z + z$

$\left| \frac{z+i}{z-3i} \right| = 1$

$\Rightarrow \ |z + i| = |z - 3i|$

Let $z = x + iy$

$\Rightarrow |x + i(y + 1)| = |x + i(y - 3)|$

$\Rightarrow x^2 + (y+1)^2 = x^2 + (y-3)^2$

$\Rightarrow 2y + 1 = -6y + 9$

$\therefore y = 1$

Now, $w = z\bar{z} - 2z + 2$

$w = |z|^2 - 2z + 2$

$\Rightarrow w = x^2 + y^2 - 2(x + iy) + 2$

$\Rightarrow w = (x^2 + y^2 - 2x + 2) + i(-2y)$

$\Rightarrow w = (x^2 + 1 - 2x + 2) + i(-2)$

$w = (x-1)^2 + 2 - 2i$

Re(w) has minimum value.

So, $(x-1)^2 + 2$ is minimum when $x = 1$

$\therefore w = 2 - 2i$

$= 2(1 - i)$

$= 2\sqrt{2}\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)$

$w = 2\sqrt{2}e^{-i\pi/4}$

Now, $w^n = (2\sqrt{2})^n e^{\frac{-in\pi}{4}}$

$= (2\sqrt{2})^n \left[\cos\left(\frac{n\pi}{4} \right) - i\sin\left(\frac{n\pi}{4} \right) \right]$

This has to be zero for w^n to be real.

So, $\sin\left(\frac{n\pi}{4} \right) = 0$

$\Rightarrow \ \frac{n\pi}{4} = 0, \pi, 2\pi, 3\pi \ldots$

$\Rightarrow n = 0, 4, 8, 12 \ldots$

The minimum value of n is $4 (n \in N)$.

Question141

The least value of |z|, where z is a complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{\left||z|+1\right|} log_{e^2} \right) \geq log_{\sqrt{2}}|5\sqrt{7} + 9i|,$$

$i = \sqrt{-1}$, is equal to:

[2021, 16 March Shift-II]

Options:

A. 3

B. $\sqrt{5}$

C. 2

D. 8

Answer: A

Solution:

Solution:

$\exp\left[\frac{(|z|+3)(|z|-1)}{(|z|+1)}\times\log_e 2\right]\geq\log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|$

$\exp\left[\frac{(|z|+3)(|z|-1)}{(|z|+1)}\times\log_e 2\right]\geq\log_{\sqrt{2}}16$

$\Rightarrow\ \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|$

$\Rightarrow\ \frac{(|z|+3)(|z|-1)}{(|z|+1)}\geq 3$

$\Rightarrow\ |\ z2^3$

$\Rightarrow\ |\ z\ |+1\ )$

$\Rightarrow\ (|z|-3)(|z|+2)\geq 0$

$\Rightarrow\ |\ z\ |\ =3$

Question142

If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x)=f(x^3)+xg(x^3)$ is divisible by x^2+x+1, then $P(1)$ is equal to [2021, 18 March Shift-II]

Answer: 0

Solution:

Solution:
Method (1)
Given , $P(x)=f(x^3)+xg(x^3)\ \cdots\cdots(i)$
$\therefore\ P(1)=f(1)+g(1)\ \cdots\cdots(ii)$
Given, $P(x)$ is divisible by (x^2+x+1).
$\therefore\ P(x)=Q(x).(x^2+x+1)$
As, we know that ω and ω^2 are non-real cube roots of unity and this is also root of $x^2+x+1=0$
$\therefore\ P(\omega)=P(\omega^2)=0$
As, we know that ω and ω^2 are non-real cube roots of unity and this is also root of $x^2+x+1=0$
$\therefore\ P(\omega)=P(\omega^2)=0\ \ldots\ (iii)$
From Eq. (i),
$P(\omega)=f(\omega^3)+\omega[g(\omega)^3]=0$[from Eq. (iii)]
$\Rightarrow f(1)+\omega g(1)\ =0\ \ldots\ (iv)$
and $P(\omega^2)\ =0$ [from Eq. (iii)]
$\Rightarrow f(\omega^6)+\omega^2\cdot g(\omega^6)=0$
$\Rightarrow\ f(1)+\omega^2 g(1)=0\ \cdots\cdots(v)$
Now, adding Eqs. (iv) and (v), we get
$2f(1)+(\omega+\omega^2)g(1)=0$
$\Rightarrow\ 2f(1)-1g(1)=0\ (\because 1+\omega+\omega^2=0)$
$\Rightarrow\ 2f(1)=g(1)\ \ldots$ (vi)
Subtracting Eq. (iv) from Eq. (v), we get
$0+(\omega-\omega^2)g(1)=0$
$\Rightarrow\ g(1)=0$
$f(1)=\frac{g(1)}{2}=\frac{0}{2}$ [from Eq. (vi)]
$\Rightarrow\ f(1)=0$
From Eq. (ii), $P(1)=f(1)+g(1)=0+0=0$
Method (2)
$\because\ P(\omega)=0$
$\Rightarrow\ f(1)+\omega g(1)=0$
$\Rightarrow\ f(1)+\left(-\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)g(1)=0$

$\Rightarrow \left(f(1) - \frac{g(1)}{2}\right) + i\left(\frac{\sqrt{3}}{2}g(1)\right) = 0$

On comparing real and imaginary parts from both sides, we have

$f(1) - \frac{g(1)}{2} = 0, \quad \frac{\sqrt{3}}{2}g(1) = 0$

$\Rightarrow f(1) = \frac{g(1)}{2}, \quad \Rightarrow g(1) = 0$

$\therefore f(1) = \frac{0}{2} = 0$

$\therefore P(1) = f(1) + g(1) = 0 + 0 = 0$

Question143

The value of $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots \infty}}}}$ is equal to
[2021, 18 March shift-I]

Options:

A. $1.5 + \sqrt{3}$

B. $2 + \sqrt{3}$

C. $3 + 2\sqrt{3}$

D. $4 + \sqrt{3}$

Answer: A

Solution:

Solution:

Let $x = 3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots \infty}}}}$

So, $x = 3 + \cfrac{1}{4 + \cfrac{1}{x}} = 3 + \cfrac{1}{\cfrac{4x+1}{x}} = 3 + \frac{x}{4x+1}$

$\Rightarrow (x - 3) = \frac{x}{4x+1}$

$\Rightarrow (4x+1)(x-3) = x$

$\Rightarrow 4x^2 - 12x - 3 = 0$

$\Rightarrow x = \frac{3 \pm 2\sqrt{3}}{2}$

$\Rightarrow x = \frac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}$

But from above, $x > 0$

$\therefore$ Only positive value of x is accepted

$\therefore x = 1.5 + \sqrt{3}$

Question144

Let C be the set of all complex numbers. Let
$S_1 = \{z \in C \mid |z - 3 - 2i|^2 = 8\}$,
$S_2 = \{z \in C \mid Re(z) \geq 5\}$ and
$S_3 = \{z \in C \mid |z - \bar{z}| \geq 8$.
Then, the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to
[2021, 27 July Shift-1]

Options:

A. 1

B. 0

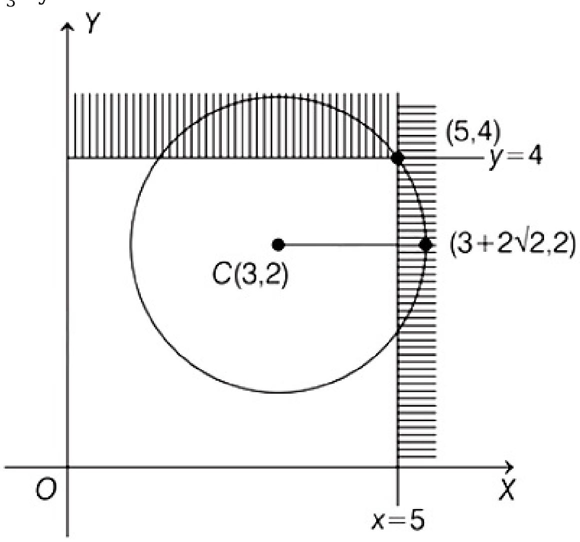
C. 2

D. Infinite

Answer: A

Solution:

Solution:
$S_1 : |z - 3 - 2i|^2 = 8$
$\Rightarrow \ |(x + iy) - (3 + 2i)|^2 = 8$
$\Rightarrow \ |(x - 3) + i(y - 2)|^2 = 8$
$\Rightarrow (x - 3)^2 + (y - 2)^2 = 8$
$S_2 : Re(z) \geq 5$
$x \geq 5$
$S_3 : |z - \bar{z}| \geq 8$
$|(x + iy) - (x - iy)| \geq 8$
$\Rightarrow 2y \geq 8$
$\Rightarrow y \geq 4$
$S_1 : (x - 3)^2 + (y - 2)^2 = 8$
$S_2 : x \geq 5$
$S_3 : y \geq 4$



Circle passes through (5, 4) as shown in the figure.
⇒ There is exactly one point (5, 4) in $S_1 \cap S_2 \cap S_3$.

Question145

The point P(a, b) undergoes the following three transformations successively
(A) Reflection about the line y = x.
(B) Translation through 2 units along the positive direction of X-axis.
(C) Rotation through angle $\frac{\pi}{4}$
about the origin in the anti-clockwise direction.
If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$,
then the value of 2a + b is equal to
[2021, 27 July Shift-II]

Options:

A. 13

B. 9

C. 5

D. 7

Answer: B

Solution:

Solution:
The image of P(a, b) along y = x is Q(b, a). Translating it 2 units along the positive direction of X -axis, it becomes R(b + 2, a). Then, rotation through $\frac{\pi}{4}$ about the origin in the anticlockwise direction, the final position of the point P is $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$.
Now, applying rotational theorem,

$$-\frac{1}{\sqrt{2}}+\frac{7}{\sqrt{2}}i=[(b+2)+ai].\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$

$$\Rightarrow -\frac{1}{\sqrt{2}}+\frac{7}{\sqrt{2}}i=\left(\frac{b+2}{\sqrt{2}}-\frac{a}{\sqrt{2}}\right)+i\left(\frac{b+2}{\sqrt{2}}+\frac{a}{\sqrt{2}}\right)$$

$$\Rightarrow -\frac{1}{\sqrt{2}}+\frac{7}{\sqrt{2}}i=\left(\frac{b-a+2}{\sqrt{2}}\right)+i\left(\frac{a+b+2}{\sqrt{2}}\right)$$

l l So, $\frac{b-a+2}{\sqrt{2}}=-\frac{1}{\sqrt{2}}$

$\Rightarrow b-a=-3 \quad \cdots\cdots(i)$

and $\frac{a+b+2}{\sqrt{2}}=\frac{7}{\sqrt{2}}$

$\Rightarrow a+b=5 \quad \cdots\cdots(ii)$

Adding Eqs. (i) and (ii),
$2b=2\Rightarrow b=1$
Substitute the value of b in Eq. (ii), a = 4
Now, $2a+b=2\times4+1=9$

Question146

Let C be the set of all complex numbers.
Let $S_1=\{z\in C: |z-2|\le 1\}$ and $S_2=\{z\in C: z(1+i)+\bar{z}(1-i)\ge 4\}$.
Then, the maximum value of $z-\frac{5}{2}|^2$ for $z\in S_1\cap S_2$ is equal to

[2021, 27 July Shift-II]

Options:

A. $\frac{3+2\sqrt{2}}{4}$

B. $\frac{5+2\sqrt{2}}{2}$

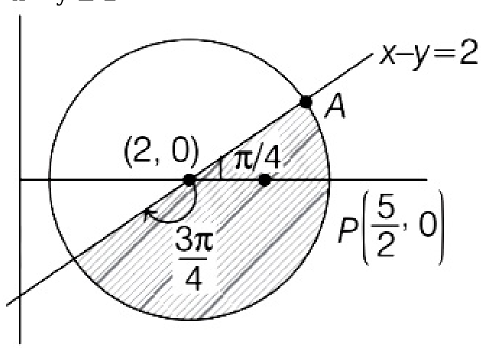
C. $\frac{3+2\sqrt{2}}{2}$

D. $\frac{5+2\sqrt{2}}{4}$

Answer: D

Solution:

Solution:
Let $S_1 = \{z \in C : |z-2| \leq 1\}$
and $S_2 = \{z \in C : z(1+i) + \bar{z}(1-i) \geq 4\}$
Now $|z-2| \leq 1$
Let $z = x + iy$
$\Rightarrow |x + iy - 2| \leq 1$
$\Rightarrow (x-2)^2 + y^2 \leq 1$
Also, $z(1+i) + \bar{z}(1-i) \geq 4$
$\Rightarrow (x+iy)(1+i) + (x-iy)(1-i) \geq 4$
$\Rightarrow 2x - 2y \geq 4$
$\Rightarrow x - y \geq 2$



Let point on circle be $A(2+\cos\theta, \sin\theta)$,
$\theta \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$

$(AP)^2 = \left(2 + \cos\theta - \frac{5}{2}\right)^2 + \sin^2\theta$

$\Rightarrow (AP)^2 = \cos^2\theta + \frac{1}{4} - \cos\theta + \sin^2\theta$

$\Rightarrow (AP)^2 = \frac{5}{4} - \cos\theta$

For $(AP)^2$ to be maximum, $\theta = -\frac{3\pi}{4}$

$\Rightarrow (AP)^2 = \frac{5}{4} + \frac{1}{\sqrt{2}}$

$\Rightarrow (AP)^2 = \frac{5+2\sqrt{2}}{4}$

Question147

Let α, β be two roots of the equation $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$. Then,

$\alpha^8 + \beta^8$ is equal to
[2021, 27 July Shift-l]

Options:

A. 10

B. 50

C. 100

D. 160

Answer: B

Solution:

Solution:

$x^2 + (20)^{\frac{1}{4}} \cdot x + (5)^{\frac{1}{2}} = 0$.
roots α & β.

$\alpha + \beta = -(20)^{\frac{1}{4}}$

$\alpha\beta = (5)^{\frac{1}{2}}$.
$\alpha^8 + \beta^8 = (\alpha^4)^2 + (\beta^4)^2$
$= (\alpha^4 - \beta^4)^2 + 2(\alpha\beta)^4$. $\cdots\cdots$ (i)
$\Rightarrow (\alpha + \beta)^2 = (\alpha^2 + \beta^2) + 2\alpha\beta$.

$\Rightarrow (20)^{\frac{1}{2}} = (\alpha^2 + \beta^2) + 2 \cdot 5^{\frac{1}{2}}$

$\Rightarrow 2 \cdot (5)^{\frac{1}{2}} = (\alpha^2 + \beta^2) + 2 \cdot 5^{\frac{1}{2}}$
$\Rightarrow 0 = (\alpha^2 + \beta^2)$
From eqn (1)
$\alpha^8 + p^8 = ((\alpha^2 + p^2) \cdot (\alpha^2 - \beta^2))^2 + 2 \cdot (5)^{1/2}$
$= 0 + 2 \times 5^2$
$= 2 \times 25$
$= 50$ (Ans)

Question148

The number of real roots of the equation $e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$ is equal to ....
[2021, 27 July Shift-II]

Answer: 2

Solution:

Given equation,
$e^{4x} - e^{3x} - 4e^{2x} - e^x + 1 = 0$
Let $e^x = t > 0$
$t^4 - t^3 - 4t^2 - t + 1 = 0$

$\Rightarrow\ t^2 - t - 4 - \frac{1}{t} + \frac{1}{t^2} = 0$

$\Rightarrow t^2 + \frac{1}{t^2} + 2 - \left(t + \frac{1}{t}\right) - 6 = 0$

$\Rightarrow\ \left(t + \frac{1}{t}\right)^2 - \left(t + \frac{1}{t}\right) - 6 = 0$

Let $\alpha = t + \frac{1}{t} \geq 2$

l c $\Rightarrow \alpha^2 - \alpha - 6 = 0$
$\Rightarrow \alpha^2 - 3\alpha + 2\alpha - 6 = 0$
$\Rightarrow \alpha(\alpha - 3) + 2(\alpha - 3) = 0$
$\Rightarrow (\alpha - 3)(\alpha + 2) = 0$
$\Rightarrow \alpha = 3$ or $\alpha = -2$ (not possible)
$\Rightarrow t + \frac{1}{t} = 3$
$\Rightarrow t^2 - 3t + 1 = 0$
$\therefore$ The number of real roots $= 2$

Question149

The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ is [2021, 25 July Shift-1]

Options:

A. 2

B. 4

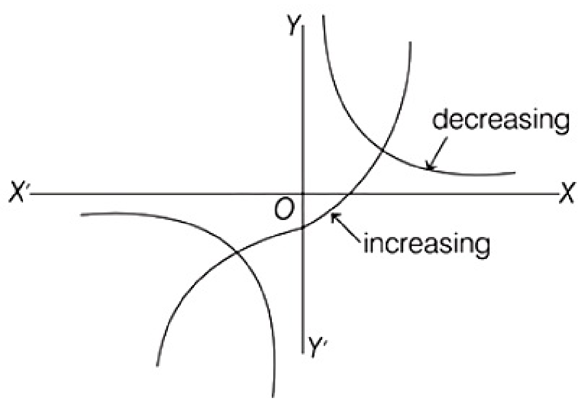
C. 6

D. 1

Answer: A

Solution:

Solution:
$e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$
$\Rightarrow (e^{3x} - 1)^2 - e^x(e^{3x} - 1) = 12e^{2x}$
$\Rightarrow (e^{3x} - 1)(e^{3x} - e^x - 1) = 12e^{2x}$
$\Rightarrow (e^{3x} - 1)(e^x - e^{-x} - e^{-2x}) = 12$
$\Rightarrow e^x - e^{-x} - e^{-2x} = \frac{12}{e^{3x} - 1}$



Hence, the number of real roots is 2.

Question150

If α, β are roots of the equation. $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2}\right)$ is equal to [2021, 25 July Shift-1]

Answer: 1

Solution:

$x^2 + 5\sqrt{2}x + 10 = 0$

$P_n = \alpha^n - \beta^n$

$$\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} = \frac{P_{17}(5\sqrt{2}P_{19} + P_{20})}{P_{18}(5\sqrt{2}P_{18} + P_{19})}$$

$\Rightarrow x^{18}(x^2 + 5\sqrt{2}x + 16) = 0$

$\Rightarrow x^{20} + 5\sqrt{2}x^{19} + x^{18} = 0$

$(\alpha^{20} - \beta^{20}) + 5\sqrt{2}(\alpha^{19} - \beta^{19}) + (\alpha^{18} - \beta^{18}) = 0$

$P_{20} + 5\sqrt{2}P_{19} + P_{18} = 0$

Similarly,

$P_{19} + 5\sqrt{2}P_{18} + P_{17} = 0$

So, $\frac{P_{17}(5\sqrt{2}P_{19} + P_{20})}{P_{18}(5\sqrt{2}P_{18} + P_{19})} = \frac{P_{17}(-P_{18})}{P_{18}(-P_{17})} = 1$

Question151

The number of real solutions of the equation $x^2 - |x| - 12 = 0$ is [2021, 25 July Shift-II]

Options:

A. 2

B. 3

C. 1

D. 4

Answer: A

Solution:

Solution:
Given equation,
$cx^2 - |x| - 12 = 0$
$\Rightarrow \quad |x^2| - |x| - 12 = 0$
$\Rightarrow |x|^2 - 4|x| + 3|x| - 12 = 0$
$\Rightarrow (|x| - 4)(|x| + 3) = 0$
So $\quad |x| - 4 = 0$ or $|x| + 3 = 0$
$|x| = 4$ or $|x| = -3$ (not possible)
$x = \pm 4$
Hence, the number of real solutions $= 2$

Question152

Let [x] denote the greatest integer less than or equal to x. Then, the values of $x \in R$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval
[2021, 22 July Shift-II]

Options:

A. $\left[0, \frac{1}{e}\right)$

B. $[\log_e 2, \log_e 3)$

C. $[1, e)$

D. $[0, \log_e 2)$

Answer: D

Solution:

Solution:
$[e^x]^2 + [e^x + 1] - 3 = 0$
$\Rightarrow [e^x]^2 + [e^x] + 1 - 3 = 0$
$\Rightarrow [e^x]^2 + [e^x] - 2 = 0$
$\Rightarrow ([e^x] - 1)([e^x] + 2) = 0$
$[e^x] = 1$ or $[e^x] = -2$
Not possible as $e^x > 0$.
$\Rightarrow [e^x] = 1$
$\Rightarrow 1 \le e^x < 2$
$\Rightarrow 0 \le x < \log_e 2$

Question153

If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of
$\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to
[2021, 20 July Shift-1]

Options:

A. 56×3^{25}

B. 56×3^{24}

C. 52×3^{24}

D. 28×3^{25}

Answer: C

Solution:

Solution:

$x^2 + 3^{\frac{1}{4}}x + 3^{\frac{1}{2}} = 0$

$\therefore\ x = \dfrac{-3^{1/4} \pm \sqrt{3^{1/2} - 4 \cdot 3^{1/2}}}{2}$

$= \dfrac{3^{1/4}(-1 \pm \sqrt{3}i)}{2}$

$= 3^{1/4}\left(\dfrac{-1+\sqrt{3}i}{2}\right) \text{or} 3^{1/4}\left(\dfrac{-1-\sqrt{3}i}{2}\right)$

$= 3^{1/4}\omega \text{ or } 3^{1/4}\omega^2$

$= 3^{1/4}\omega$ or $3^{1/4}\omega^2$

Now, $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$

$= \alpha^{108} - \alpha^{96} + \beta^{108} - \beta^{96}$

$= (\alpha^{108} + \beta^{108}) - (\alpha^{96} + \beta^{96})$

$= \{(3^{1/4}\omega)^{108} + (3^{1/4}\omega^2)^{108}\}$

$- \{(3^{1/4}\omega)^{96} + (3^{1/4}\omega^2)^{96}\}$

$= 3^{27}(\omega^{108} + \omega^{216}) - 3^{24}(\omega^{96} + \omega^{192})$

$= 3^{27}(2) - 3^{24}(2) = 3^{24}(54) - 3^{24}(2)$

$= 3^{24}(52) = 52 \times 3^{24}$

Question154

The number of solutions of the equation $\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x+1)^2 - 4 = 0$ $x > 0$, is
[2021, 20 July Shift-II]

Answer: 1

Solution:

$\log_{(x+1)}(2x^2 + 7x + 5)$
$+\log_{(2x+5)}(x+1)^2 - 4 = 0$
$= \log_{(x+1)}\{(2x+5)(x+1)\}$
$+2\log_{(2x+5)}(x+1) - 4 = 0$

$= \log_{(x+1)}(2x+5) + \log_{(x+1)}(x+1)$
$+2\log_{(2x+5)}(x+1) - 4 = 0$
$= \log_{(x+1)}(2x+5) + 2\log_{(2x+5)}(x+1) - 3 = 0$
$[\because \log_a a = 1]$

$$= \log_{(x+1)}(2x+5) + 2\frac{\log_{(x+1)}(x+1)}{\log_{(x+1)}(2x+5)} = 3$$

Let $\log_{(x+1)}(2x+5) = t$

$t + \frac{2}{t} = 3 \Rightarrow t^2 - 3t + 2 = 0$
$(t-1))t-2) = 0$
$\Rightarrow t = 1, t = 2$
$\Rightarrow \log_{(x+1)}(2x+5) = 1$ and
$\log_{(x+1)}(2x+5) = 2$
$2x+5 = (x+1)$
and $2x+5 = (x+1)^2$
$x = -4$
and $2x+5 = x^2+1+2x$
i.e., $x^2 = 4$
$\Rightarrow x = +2, -2$

Given, $x > 0$
$x = -4, x = -2$ are discarde(d)
$\therefore x = 2$ is only solution.

Question155

If the real part of the complex number $z = \frac{3+2i\cos\theta}{1-3i\cos\theta}$, $\theta \in \left(0, \frac{\pi}{2}\right)$ is zero, then the value of $\sin^2 3\theta + \cos^2\theta$ is equal to
[2021, 27 July Shift-11]

Answer: 1

Solution:

Solution:
We have,

$$l\,z = \frac{3+2i\cos\theta}{1-3i\cos\theta} = \frac{3+2i\cos\theta}{1-3i\cos\theta} \times \frac{1+3i\cos\theta}{1+3i\cos\theta}$$

$$= \frac{(3-6\cos^2\theta) + i(9\cos\theta + 2\cos\theta)}{1+9\cos^2\theta}$$

$$z = \frac{(3-6\cos^2\theta) + (11\cos\theta)i}{1+9\cos^2\theta}$$

Given, Re(z) =0

$$\Rightarrow \frac{3-6\cos^2\theta}{1+9\cos^2\theta} = 0$$

$\Rightarrow 3 - 6\cos^2\theta = 0$

$$\Rightarrow \cos^2\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = \frac{\pi}{4}\left\{\theta \in \left(0, \frac{\pi}{2}\right)\right\}$$

$$\Rightarrow \quad = \frac{1}{2} + \frac{1}{2} = 1$$

Hence, $\sin^2 3\theta + \cos^2\theta$

$$= \sin^2\frac{3\pi}{4} + \cos^2\frac{\pi}{4} = \frac{1}{2} + \frac{1}{2} = 1$$

Question156

Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then, the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to
[2021, 22 July Shift-11]

Options:

A. 1

B. $\frac{4}{3}$

C. $\frac{3}{2}$

D. 2

Answer: B

Solution:

Solution:
$z^2 + 3z = 0$
$z = x + iy$
$\Rightarrow (x^2 - y^2) + i(2xy) + 3(x - iy) = 0$
$\Rightarrow (x^2 - y^2 + 3x) + i(2xy - 3y) = 0$
$\begin{cases} x^2 - y^2 + 3x = 0 \\ y(2x - 3) = 0. \end{cases}$
$y = 0$ or $x = \frac{3}{2}$
If $y = 0$,
$\Rightarrow x(x + 3) = 0$
$\Rightarrow x = 0, -3$
$\Rightarrow$ So, $(0, 0)$ and $(-3, 0)$ are solutions, when
$y = 0$.
When $x = \frac{3}{2}$, $\frac{9}{4} - y^2 + \frac{9}{2} = 0 \Rightarrow y^2 = \frac{27}{4}$
$\Rightarrow y = \pm \frac{3\sqrt{3}}{2}$
$\therefore \left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)$ and $\left(\frac{3}{2}, -\frac{3\sqrt{3}}{2}\right)$
There are 4 solutions.
$\sum_{k=0}^{\infty} \left(\frac{1}{n^k}\right) = 1 + \frac{1}{4} + \frac{1}{4^2} + \dots . \infty$
$= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$

Question157

If the real part of the complex number $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta \in (0, \pi)$, then the value of the integral

$\int_0^{\theta} \sin x \, d x$ is equal to
[2021, 22 July Shift-II]

Options:

A. 1

B. 2

C. -1

D. 0

Answer: A

Solution:

Solution:
Let $z = (1 - \cos\theta + 2i\sin\theta)^{-1}$

$\Rightarrow z = \dfrac{1}{1 - \cos\theta + 2i\sin\theta}$

$= \dfrac{1}{1 - \cos\theta + 2i\sin\theta} \times \dfrac{1 - \cos\theta - 2i\sin\theta}{1 - \cos\theta - 2i\sin\theta}$

$= \dfrac{(1 - \cos\theta) - 2i\sin\theta}{(1 - \cos\theta)^2 - (2i\sin\theta)^2}$

$= \dfrac{2\sin^2\dfrac{\theta}{2} - 4i\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}}{4\sin^4\dfrac{\theta}{2} + 16\sin^2\dfrac{\theta}{2}\cos^2\dfrac{\theta}{2}}$

$= \dfrac{2\sin\dfrac{\theta}{2}\left(\sin\dfrac{\theta}{2} - 2i\cos\dfrac{\theta}{2}\right)}{4\sin^2\dfrac{\theta}{2}\left(\sin^2\dfrac{\theta}{2} + 4\cos^2\dfrac{\theta}{2}\right)}$

$= \dfrac{\sin\dfrac{\theta}{2} - 2i\cos\dfrac{\theta}{2}}{2\sin\dfrac{\theta}{2}\left(\sin^2\dfrac{\theta}{2} + 4\cos^2\dfrac{\theta}{2}\right)}$

Now, $\mathrm{Re}(z) = \dfrac{\sin\dfrac{\theta}{2}}{2\sin\dfrac{\theta}{2}\left(\sin^2\dfrac{\theta}{2} + 4\cos^2\dfrac{\theta}{2}\right)}$

$= \dfrac{1}{2\left(1 + 3\cos^2\dfrac{\theta}{2}\right)}$

Given, $\mathrm{Re}(z) = \dfrac{1}{5}$

$\Rightarrow \dfrac{1}{2\left(1 + 3\cos^2\dfrac{\theta}{2}\right)} = \dfrac{1}{5}$

$\Rightarrow 1 + 3\cos^2\dfrac{\theta}{2} = \dfrac{5}{2} \Rightarrow \cos^2\dfrac{\theta}{2} = \dfrac{1}{2}$

$\Rightarrow \cos\dfrac{\theta}{2} = \pm\dfrac{1}{\sqrt{2}}$

$\therefore \dfrac{\theta}{2} = n\pi \pm \dfrac{\pi}{4}$

$\therefore \theta = 2n\pi \pm \dfrac{\pi}{2}$

Given, range is $\theta \in (0, \pi)$.

$\therefore \theta = \dfrac{\pi}{2}$

Now, $\int_0^{\theta} \sin x \, d x = \int_0^{\frac{\pi}{2}} \sin x \, d x$

$\int_0^{\frac{\pi}{2}} \sin x \, d x = -\cos x\,]_0^{\pi/2}$

$= -\left(\cos\frac{\pi}{2} - \cos 0\right)$
$= -(0 - 1) = 1$

Question158

If z and ω are two complex numbers such that |zω| = 1 and arg (z) − arg(ω) = $\frac{3\pi}{2}$, then arg $\left(\frac{1 - 2\bar{z}\omega}{1 + 3\bar{z}\omega}\right)$ is
(Here, arg(z) denotes the principal argument of complex number z)
[2021, 20 July Shift-1]

Options:

A. $\frac{\pi}{4}$

B. $-\frac{3\pi}{4}$

C. $-\frac{\pi}{4}$

D. $\frac{3\pi}{4}$

Answer: B

Solution:

Solution:
$|zW| = 1, \arg(z) - \arg(w) = \frac{3\pi}{2}$
Let $z = re^{i\theta}$
$w = \frac{1}{r}e^{i\left(\theta - \frac{3\pi}{2}\right)} \Rightarrow \bar{z} = re^{-i\theta}$
$w\bar{z} = \frac{1}{r}e^{i\left(\theta - \frac{3\pi}{2}\right)} \cdot re^{-i\theta}$
$\Rightarrow w\bar{z} = e^{i\left(\theta - \frac{3\pi}{2} - \theta\right)} = e^{-i\frac{3\pi}{2}}$
$\Rightarrow w\bar{z} = \cos\left(\frac{-3\pi}{2}\right) + i\sin\left(\frac{-3\pi}{2}\right)$
$\Rightarrow w\bar{z} = 0 + i$
$\Rightarrow w\bar{Z} = i$
$\left(\frac{1 - 2w\bar{z}}{1 + 3w\bar{z}}\right) = \left(\frac{1 - 2i}{1 + 3i} \times \frac{1 - 3i}{1 - 3i}\right)$
$= \frac{1 - 2i - 3i + 6i^2}{10} = \frac{-5 - 5i}{10}$
$\therefore \arg = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4}$

Question159

Let Z_1 and Z_2 be two complex numbers such that $\arg(Z_1 - Z_2) = \frac{\pi}{4}$ and Z_1, Z_2 satisfy the equation |Z − 3| = Re(Z). Then, the imaginary part of

$Z_1 + Z_2$ is equal to
[2021, 27 Aug. Shift-11]

Answer: 6

Solution:

Solution:
Let $Z_1 = a_1 + ib_1$, $Z_2 = a_2 + ib_2$
$Z_1 - Z_2 = (a_1 - a_2) + i(b_1 - b_2)$
$\arg(Z_1 - Z_2) = \frac{\pi}{4} \Rightarrow \tan^{-1}\left(\frac{b_1 - b_2}{a_1 - a_2}\right) = \frac{\pi}{4}$
$ll \Rightarrow b_1 - b_2 = a_1 - a_2$
Also, $|Z_1 - 3| = \mathrm{Re}(Z_1)$
$\Rightarrow (a_1 - 3)^2 + b_1^2 = a_1^2$
and $|Z_2 - 3| = \mathrm{Re}(Z_2)$
$\Rightarrow (a_2 - 3)^2 + b_2^2 = a_2^2$
$\Rightarrow (a_1 - 3)^2 - (a_2 - 3)^2 + b_1^2 - b_2^2$
$= a_1^2 - a_2^2$
$\Rightarrow (a_1 - a_2)(a_1 + a_2 - 6) + (b_1 - b_2)(b_1 + b_2)$
$= (a_1 - a_2)(a_1 + a_2)$
$\Rightarrow a_1 + a_2 - 6 + b_1 + b_2 = a_1 + a_2$
$\Rightarrow b_1 + b_2 = 6$
$\Rightarrow lm(Z_1 + Z_2) = 6$
[using Eq. (i).]

Question160

The sum of the roots of the equation $x + 1 - 2\log_2(3 + 2^x)$ $+ 2\log_4(10 - 2^{-x}) = 0$ is
[2021, 31 Aug. Shift-II]

Options:

A. $\log_2 14$

B. $\log_2 11$

C. $\log_2 12$

D. $\log_2 13$

Answer: B

Solution:

Solution:
$x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$

$\Rightarrow$

$x + 1 - 2\log_2(3 + 2^x) + \log_2\left(\frac{10 \cdot 2^x - 1}{2^x}\right) = 0$

$\Rightarrow x + 1 - 2\log_2(3 + 2^x) + \log_2(10 \cdot 2^x - 1)$

$\Rightarrow 1 + \log_2\left(\frac{10 \cdot 2^x - 1}{(3 + 2^x)^2}\right) = 0$

$\Rightarrow \frac{10 \cdot 2^x - 1}{9 + (2^x)^2 + 6 \cdot 2^x} = \frac{1}{2}$

$\Rightarrow (2^x)^2 - 14 \cdot 2^x + 11 = 0$

Let $2^x = y$

$\Rightarrow y^2 - 14y + 11 = 0$

Let $2^x = y$

$\Rightarrow y^2 - 14y + 11 = 0$

$y = \frac{14 \pm \sqrt{152}}{2} = 7 \pm \frac{\sqrt{152}}{2}$

$y_1 = 7 + \frac{\sqrt{152}}{2},$

$y_2 = 7 - \frac{\sqrt{152}}{2}$

$\Rightarrow 2^{x_1} = 7 + \frac{\sqrt{152}}{2},$

$2^{x_2} = 7 - \frac{\sqrt{152}}{2}$

$\Rightarrow x_1 = \log_2\left(7 + \frac{\sqrt{152}}{2}\right)$

$x_2 = \log_2\left(7 - \frac{\sqrt{152}}{2}\right)$

$\therefore$ Sum of roots $= x_1 + x_2$

$= \log_2\left(49 - \frac{152}{4}\right) = \log_2 11$

Question161

The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is
[2021, 27 Aug. Shift-I]

Answer: 4

Solution:

Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 4 = 0$

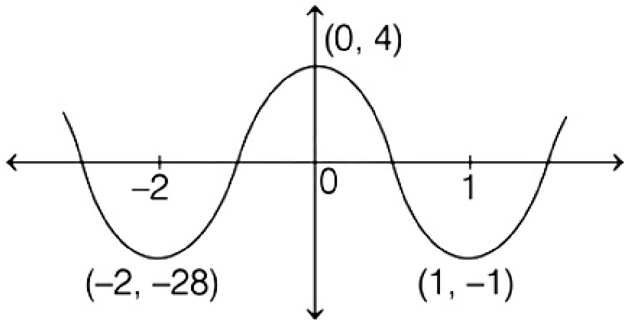
Differentiating w.r.t. x_1

$f'(x) = 12x^3 + 12x^2 - 24x = 0$

$\Rightarrow 12x(x^2 + x - 2) = 0$

$\Rightarrow x(x + 2)(x - 1) = 0$

Critical point $x = 0, 1, -2$



Graph of $y = f(x)$
Number of real roots $= 4$

Question162

The set of all values of $k > -1$, for which the equation $(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$ has real roots, is
[2021, 27 Aug. Shift-II]

Options:

A. $\left(1, \frac{5}{2}\right]$

B. $[2, 3)$

C. $\left[-\frac{1}{2}, 1\right)$

D. $\left(\frac{1}{2}, \frac{3}{2}\right] - \{1\}$

Answer: A

Solution:

Solution:
Given,
$(3x^2 + 4x + 3)^2 - (k + 1)(3x^2 + 4x + 3)$
$(3x^2 + 4x + 2) + k(3x^2 + 4x + 2)^2 = 0$
Let $y = 3x^2 + 4x + 2$
Then, given equation becomes
$(y + 1)^2 - (k + 1)y(y + 1) + ky^2 = 0$
$\Rightarrow y^2 + 2y + 1 - ky^2 - ky - y^2 - y + ky^2 = 0$
$\Rightarrow y + 1 - ky = 0$
$\Rightarrow y(1 - k) = -1$
$\Rightarrow y = \frac{1}{k - 1}$
$\Rightarrow 3x^2 + 4x + 2 - \frac{1}{k - 1} = 0$
For real roots, $D \geq 0$
$\Rightarrow 16 - 4 \cdot 3 \cdot \left(2 - \frac{1}{k - 1}\right) \geq 0$
$\Rightarrow -8 + \frac{12}{k - 1} \geq 0 \Rightarrow \frac{3}{k - 1} \geq 2$
$\Rightarrow \frac{3 - 2k + 2}{k - 1} \geq 0 \Rightarrow \frac{2k - 5}{k - 1} \leq 0$

$\Rightarrow\ k \in \left(1,\ \frac{5}{2}\right]\ [\because k \neq 1]$

Question163

The sum of all integral values of k (k ≠ 0) for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is
[2021,26 Aug. Shift-I]

Answer: 66

Solution:

Solution:

$\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k} \Rightarrow x \in R - \{1, 2\}$

$k(2x - 4 - x + 1) = 2(x^2 - 3x + 2)$

$k(x - 3) = 2(x^2 - 3x + 2)$

$2x^2 - (6 + k)x + 3k + 4 = 0$

For no real roots $b^2 - 4ac < 0$

$\therefore (k + 6)^2 - 8 \cdot (3k + 4) < 0$

$\Rightarrow\quad k^2 - 12k - 4 < 0$

$\Rightarrow\ (k - 6)^2 - 32 < 0$

$\Rightarrow\quad (k - 6)^2 < 32$

$\Rightarrow\ -4\sqrt{2} < k - 6 < 4\sqrt{2}$

$\Rightarrow\ 6 - 4\sqrt{2} < k < 6 + 4\sqrt{2}$

Integral $k \in \{1, 2, 3, 4, \ldots .11\}$

Sum $= 66$

Question164

If $(\sqrt{3} + i)^{100} = 2^{99}(p + iq)$, then p and q are roots of the equation
[2021, 26 Aug. Shift-III]

Options:

A. $x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0$

B. $x^2 + (\sqrt{3} + 1)x + \sqrt{3} = 0$

C. $x^2 + (\sqrt{3} - 1)x - \sqrt{3} = 0$

D. $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

Answer: A

Solution:

Solution:
$(\sqrt{3}+i)^{100} = 2^{99}(p+iq)$

$2^{100} e^{i100\frac{\pi}{6}} = 2^{99}(p+iq)$

$\Rightarrow\ 2e^{i\frac{2\pi}{3}} = p+iq$

$\Rightarrow 2\left[\cos\left(\pi-\frac{\pi}{3}\right)+i\sin\left(\pi-\frac{\pi}{3}\right)\right] = p+iq$

$\Rightarrow\ (-1+i\sqrt{3}) = p+iq$

$\Rightarrow\ p=-1$ and $q=\sqrt{3}$

Equation whose roots are -1 and $\sqrt{3}i$ is

$\Rightarrow\ (x+1)(x-\sqrt{3})=0$

$x^2-(\sqrt{3}-1)x-\sqrt{3}=0$

Question165

Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$ and α and γ are the roots of equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to
[2021,26 Aug. Shift-II]

Answer: 18

Solution:

Solution:
We have, α is common root of the equations $x^2-x+2\lambda=0$ and $3x^2-10x+27\lambda=0$.
Now, common root of these equations is $(3\alpha^2-10\alpha+27\lambda)-(3\alpha^2-3\alpha+6\lambda)=0 \Rightarrow\ -7\alpha+21\lambda=0$

$\Rightarrow\ \alpha=3\lambda$

Again, α is root of $x^2-x+2\lambda=0$

$\therefore\ \alpha^2-\alpha+2\lambda=0$

$\Rightarrow\ (3\lambda)^2-3\lambda+2\lambda=0$

$\Rightarrow\ 9\lambda^2-\lambda=0$

$\Rightarrow\ \lambda(9\lambda-1)=0$

$\Rightarrow\ \lambda=0,\ \frac{1}{9}$

$\Rightarrow\ \lambda=\frac{1}{9}\ [\because \lambda \neq 0]$

$\therefore\ \alpha=3\lambda=3\times\frac{1}{9}=\frac{1}{3}$

Again, α and β are roots of the equation
$x^2-x+2\lambda=0$

l l $\therefore\ \alpha+\beta=\frac{-(-1)}{1}=1$

$\Rightarrow\beta=1-\alpha=1-\frac{1}{3}=\frac{2}{3}$

And α and γ are the roots of the equation $3x^2-10x+27\lambda=0$

$\therefore\ \alpha+\gamma=\frac{-(-10)}{3}=\frac{10}{3}$

$\Rightarrow\ \gamma=\frac{10}{3}-\alpha=\frac{10}{3}-\frac{1}{3}=\frac{9}{3}=3$

$\therefore\ \frac{\beta\gamma}{\lambda}=\frac{\left(\frac{2}{3}\right)\times(3)}{\left(\frac{1}{9}\right)}=18$

Question166

If S = $\left\{ z \in C : \frac{z-i}{z+2i} \in R \right\}$, then
[2021, 27 Aug. Shift-1]

Options:

A. S contains exactly two elements.

B. S contains only one element.

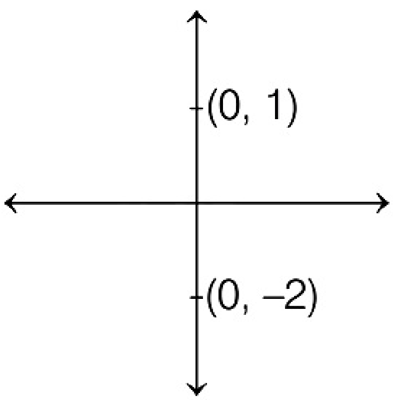
C. S is a circle in the complex plane.

D. S is a straight line in the complex plane.

Answer: D

Solution:

Solution:
Given, $\frac{z-i}{z+2i} \in R$



$\Rightarrow \arg\left(\frac{z-i}{z+2i}\right) = 0$ or π
$\Rightarrow i, -2i, z$ are collinear.
$\Rightarrow$S is a straight line in the complex plane.

Question167

Let $z = \frac{1 - i\sqrt{3}}{2}$ and $i = \sqrt{-1}$. Then the value of $21 + \left(z + \frac{1}{z}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \left(z^3 + \frac{1}{z^3}\right)^3 + \ldots + \left(z^{21} + \frac{1}{z^{21}}\right)^3$ is
[2021, 26 Aug. Shift-1]

Answer: 13

Solution:

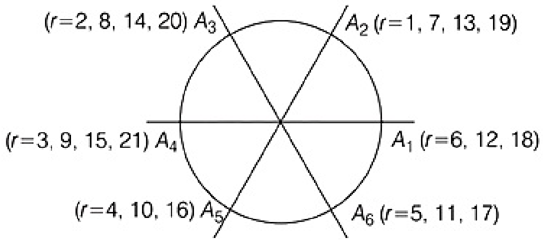
Solution:

$z = \frac{1}{2} - \frac{\sqrt{3}}{2}i = e^{\frac{-i\pi}{3}}$

Again, $z^r + \frac{1}{z^r} = z^r + \bar{z}^r = 2\mathrm{Re}(z^r)$

$[\because |z^r| = 1] \quad = 2\cos\left(\frac{r\pi}{3}\right)$

$21 + \sum_{r=1}^{21}\left(z^r + \frac{1}{z^r}\right)^3 = 21 + \sum_{r=1}^{21} 8\cos^3\left(\frac{\pi r}{3}\right)$



Now, all the diametric ends will cancel out each other. Only a single value at A_L will remain which is -1.

So, $21 + 8(-1) = 13$

Question168

The equation $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$ represents a circle with [2021, 26 Aug. Shift-I]

Options:

A. centre at $(0, -1)$ and radius $\sqrt{2}$

B. centre at $(0, 1)$ and radius $\sqrt{2}$

C. centre at $(0, 0)$ and radius $\sqrt{2}$

D. centre at $(0, 1)$ and radius 2

Answer: B

Solution:

Solution:

We have, $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4} \Rightarrow \arg(z-1) - \arg(z+1) = \frac{\pi}{4}$

Let $z = x + iy$

$\arg[(x-1) + iy] - \arg[(x+1) + iy] = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right) = \frac{\pi}{4}$

$\Rightarrow \left(\frac{\frac{y}{x-1} - \frac{y}{x+1}}{1 + \frac{y}{x-1}\cdot\frac{y}{x+1}}\right) = \tan\frac{\pi}{4}$

$\Rightarrow \frac{y(x+1) - y(x-1)}{(x^2-1) + y^2} = 1$

$\Rightarrow 2y = x^2 + y^2 - 1$

$\Rightarrow x^2 + y^2 - 2y - 1 = 0$
$\Rightarrow x^2 + (y - 1)^2 = 2$
$\Rightarrow x^2 + (y - 1)^2 = (\sqrt{2})^2$
Which is a circle with Centre $(0, 1)$ and Radius $= \sqrt{2}$ units

----

Question169

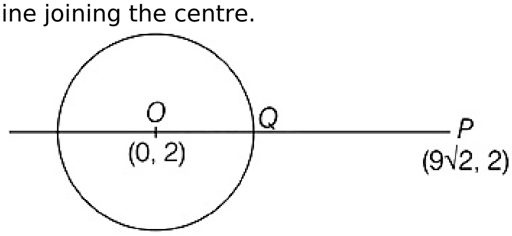
A point z moves in the complex plane such that arg $\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $|z - 9\sqrt{2} - 2i|^2$ equal to
[2021, 31 Aug. Shift-I]

Answer: 98

Solution:

Solution:
$\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$
If
$z = x + iy$
$\arg[(x-2) + iy] - \arg[(x+2) + iy] = \frac{\pi}{4}$
$\Rightarrow \tan^{-1}\left(\frac{y}{x-2}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$
$\Rightarrow \dfrac{\frac{y}{x-2} - \frac{y}{x+2}}{1 + \frac{y}{x-2} \cdot \frac{y}{x+2}} = \tan\left(\frac{\pi}{4}\right)$

$l \Rightarrow \frac{xy + 2y - xy + 2y}{x^2 + y^2 - 4} = 1$
$\Rightarrow 4y = x^2 + y^2 - 4$
$\Rightarrow x^2 + y^2 - 4y - 4 = 0$
z is a circle.
Centre $= (0, 2)$, Radius $= (2\sqrt{2})$
$|z - 9\sqrt{2} - 2i|^2$ is the distance of $(9\sqrt{2}, 2)$ from any point on circle. Distance will be minimum when $(9\sqrt{2}, 2)$ will lie on the line joining the centre.



$PQ = OP - OQ$
$= 9\sqrt{2} - 2\sqrt{2} = 7\sqrt{2}$
$PO^2 = (7\sqrt{2})^2 = 98$

Question170

If z is a complex number such that $\frac{z-i}{z-1}$ is purely imaginary, then the minimum value of $|z - (3 + 3i)|$ is

[2021, 31 Aug. Shift-II]

Options:

A. $2\sqrt{2}-1$

B. $6\sqrt{2}$

C. $3\sqrt{2}$

D. $2\sqrt{2}$

Answer: D

Solution:

Solution:
Let $z = x + iy$

$c\ \frac{z-i}{z-1} = \frac{x+i(y-1)}{(x-1)+iy} \times \frac{(x-1)-iy}{(x-1)-iy}$

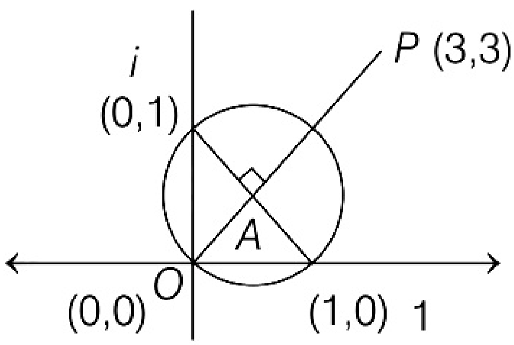
$= \frac{x(x-1)+y(y-1)}{(x-1)^2+y^2} + i\left[\frac{(x-1)(y-1)-xy}{(x-1)^2+y^2}\right]$

As $\frac{z-i}{z-1}$ is purely imaginary,

$x^2+y^2-x-y = 0$

$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2 = 0$

This is a circle with centre $\left(\frac{1}{2}, \frac{1}{2}\right)$, radius $= \frac{1}{2}$ which passes through origin as shown in the figure.



Minimum $|z-(3+3i)| = OP - OA$

$\sqrt{(3-0)^2+(3-0)^2} - \sqrt{\left(\frac{1}{2}-0\right)^2+\left(\frac{1}{2}-0\right)^2}$

$= 3\sqrt{2}-\sqrt{2}$

$= 2\sqrt{2}$

Question171

The least positive integer n such that $\frac{(2i)^n}{(1-i)^{n-2}}$, $i = \sqrt{-1}$, is a positive integer, is
[2021, 26 Aug. Shift-11]

Answer: 6

Solution:

Solution:
We have,
$$\frac{(2i)^n}{(1-i)^{n-2}} = \frac{(2i)^n}{(1-i)^n(1-i)^{-2}}$$
$$= \left(\frac{2i}{1-i}\right)^n(1-i)^2$$
$$= \left[\frac{2i(1+i)}{(1-i)(1+i)}\right]^n(1+i^2-2i)$$
$$= \left(\frac{2i-2}{2}\right)^n(1-1-2i)$$
$$= (i-1)^n(-2i)$$

If $n = 1$, $(i-1)(-2i) = -2i^2 + 2i = 2 + 2i$
If $n = 2$, $-2i(i-1)^2 = -2i(-2i) = -4$
If $n = 4$, $-2i(i-1)^4 = -2i(-2i)(-2i) = 8i$
If $n = 6$, $-2i(i-1)^6 = -2i(-2i)(-2i)(-2i) = 16$
So, least value of n for which given complex is positive is 6 .

Question172

The numbers of pairs (a, b) of real numbers, such that whenever α is a root of the equation $x^2 + ax + b = 0$, $\alpha^2 - 2$ is also a root of this equation, is
[2021, 01 Sep. Shift-II]

Options:

A. 6

B. 2

C. 4

D. 8

Answer: A

Solution:

Solution:
Given equation $x^2 + ax + b = 0$
It has two roots (not necessarily real α and β)
$\Rightarrow$ Either $\alpha = \beta$ or $\alpha \neq \beta$
1. If $\alpha = \beta \Rightarrow \alpha = \alpha^2 - 2 \Rightarrow \alpha = -1, 2$
When $\alpha = -1$, then $(a, b) = (2, 1)$
When $\alpha = 2$, then $(a, b) = (-4, 4)$
II. If $\alpha \neq \beta$, then
(a) $\alpha = \alpha^2 - 2$ and $\beta = \beta^2 - 2$
Here, $(\alpha, \beta) = (2, -1)$ or $(-1, 2)$
Hence $(a, b) = (-\alpha - \beta, \alpha\beta) = (-1, -2)$
(b) $\alpha = \beta^2 - 2$ and $\beta = \alpha^2 - 2$
Then $\alpha - \beta = \beta^2 - \alpha^2 = (\beta - \alpha)(\beta + \alpha)$
$\because \alpha \neq \beta$
$\Rightarrow \alpha + \beta = \beta^2 + \alpha^2 - 4$
or $\alpha + \beta = (\alpha + \beta)^2 - 2\alpha\beta - 4$
$\Rightarrow -1 = 1 - 2\alpha\beta - 4 \Rightarrow \alpha\beta = -1$
$\Rightarrow (a, b) = (-\alpha - \beta, \alpha\beta) = (1, -1)$

(c) $\alpha = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta \Rightarrow \alpha = -\beta$
Thus, $\alpha = 2, \beta = -2$
or $\alpha = -1, \beta = 1$
$\therefore$ $(a, b) = (0, -4)$ and $(0, -1)$
(d) $\beta = \alpha^2 - 2 = \beta^2 - 2$ and $\alpha \neq \beta$(as in (c))
$\Rightarrow$ We get 6 pairs of (a, b)
They are $(2, 1), (-4, 4), (-1, -2), (1, -1)$,
$(0, -4)$, and $(0, -1)$.

Question173

If $\frac{3+i\sin\theta}{4-i\cos\theta}$, $\theta \in [0,2\pi]$, is a real number, then an argument of $\sin\theta + i\cos\theta$ is:
[Jan. 7, 2020 (II)]

Options:

A. $\pi - \tan^{-1}\left(\frac{4}{3}\right)$

B. $\pi - \tan^{-1}\left(\frac{3}{4}\right)$

C. $-\tan^{-1}\left(\frac{3}{4}\right)$

D. $\tan^{-1}\left(\frac{4}{3}\right)$

Answer: A

Solution:

Solution:
Let $z = \frac{3+i\sin\theta}{4-i\cos\theta}$, after rationalising
$z = \frac{(3+i\sin\theta)}{(4-i\cos\theta)} \times \frac{(4+i\cos\theta)}{(4+i\cos\theta)}$
As z is purely real
$\Rightarrow 3\cos\theta + 4\sin\theta = 0 \Rightarrow \tan\theta = -\frac{3}{4}$
$\arg(\sin\theta + i\cos\theta) = \pi + \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right)$
$= \pi + \tan^{-1}\left(-\frac{4}{3}\right) = \pi - \tan^{-1}\left(\frac{4}{3}\right)$

Question174

Let z be a complex number such that $\left|\frac{Z-i}{Z+2i}\right| = 1$ and $|Z| = \frac{5}{2}$. Then the value of $|Z+3i|$ is:
[Jan. 9, 2020 (I)]

Options:

A. $\sqrt{10}$

B. $\frac{7}{2}$

C. $\frac{15}{4}$

D. $2\sqrt{3}$

Answer: B

Solution:

Solution:
Let $z = x + iy$
Then, $\left|\frac{z - i}{z + 2i}\right| = 1 \Rightarrow x^2 + (y-1)^2$
$= x^2 + (y+2)^2 \Rightarrow -2y + 1 = 4y + 4$
$\Rightarrow 6y = -3 \Rightarrow y = -\frac{1}{2}$
$\because \; |z| = \frac{5}{2} \Rightarrow x^2 + y^2 = \frac{25}{4}$
$\Rightarrow x^2 = \frac{24}{4} = 6$
$\therefore \; z = x + iy \;\; \Rightarrow z = \pm\sqrt{6} - \frac{i}{2}$
$|z + 3i| = \sqrt{6 + \frac{25}{4}} = \sqrt{\frac{49}{4}}$
$\Rightarrow \; |z + 3i| = \frac{7}{2}$

Question175

If z be a complex number satisfying $|Re(z)| + |Im(Z)| = 4$, then $|Z|$ cannot be:
[Jan. 9, 2020 (II)]

Options:

A. $\sqrt{\frac{17}{2}}$

B. $\sqrt{10}$

C. $\sqrt{7}$

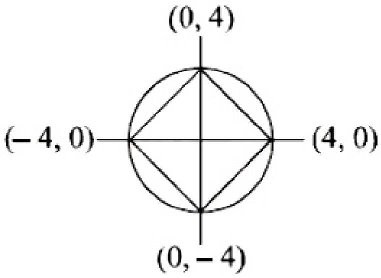
D. $\sqrt{8}$

Answer: C

Solution:

Solution:
$z = x + iy \; |x| + |y| = 4$
$|z| = \sqrt{x^2 + y^2}$
Minimum value of
$|z| = 2\sqrt{2}$
Maximum value of
$|z| = 4 \; |z| \in [\sqrt{8}, \sqrt{16}]$

So, |z| can't be $\sqrt{7}$.



Question176

Let $\alpha = \frac{-1+i\sqrt{3}}{2}$. If $a = (1+\alpha)\sum_{k=0}^{100}\alpha^{2k}$ and $b = \sum_{k=0}^{100}\alpha^{3k}$, then a and b are the roots of the quadratic equation:
[Jan. 8, 2020 (II)]

Options:

A. $x^2 + 101x + 100 = 0$

B. $x^2 - 102x + 101 = 0$

C. $x^2 - 101x + 100 = 0$

D. $x^2 + 102x + 101 = 0$

Answer: B

Solution:

Solution:
Let $\alpha = \omega$, $b = 1 + \omega^3 + \omega^6 + \ldots. = 101$
$a = (1+\omega)(1+\omega^2+\omega^4+\ldots..\omega^{198}+\omega^{200})$
$= (1+\omega)\dfrac{(1-(\omega^2)^{101})}{1-\omega^2} = \dfrac{(\omega+1)(\omega^{202}-1)}{(\omega^2-1)}$
$\Rightarrow a = \dfrac{(1+\omega)(1-\omega)}{1-\omega^2} = 1$
Required equation $= x^2 - (101+1)x + (101)\times 1 = 0$
$\Rightarrow x^2 - 102x + 101 = 0$

Question177

If $\operatorname{Re}\left(\frac{z-1}{2z+i}\right) = 1$, where $z = x + iy$, then the point (x, y) lies on a :
[Jan. 7, 2020 (I)]

Options:

A. circle whose centre is at $\left(-\frac{1}{2}, -\frac{3}{2}\right)$.

B. straight line whose slope is $-\frac{2}{3}$.

C. straight line whose slope is $\frac{3}{2}$.

D. circle whose diameter is $\frac{\sqrt{5}}{2}$.

Answer: D

Solution:

Solution:

$\because z = x + iy$

$\left(\frac{z-1}{2z+i}\right) = \frac{(x-1)+iy}{2(x+iy)+i}$

$= \frac{(x-1)+iy}{2x+(2y+1)i} \times \frac{2x-(2y+1)i}{2x-(2y+1)i}$

$\text{Re}\left(\frac{z+1}{2z+i}\right) = \frac{2x(x-1)+y(2y+1)}{(2x)^2+(2y+1)^2} = 1$

$\Rightarrow \left(x+\frac{1}{2}\right)^2 + \left(y+\frac{3}{4}\right)^2 = \left(\frac{\sqrt{5}}{4}\right)^2$

Question178

The number of real roots of the equation, $e^{4x} + e^{3x} - 4e^{2x} + e^{x} + 1 = 0$ is: [Jan. 9, 2020 (I)]

Options:

A. 1

B. 3

C. 2

D. 4

Answer: A

Solution:

Solution:

Let $e^x = t \in (0, \infty)$

Given equation

$t^4 + t^3 - 4t^2 + t + 1 = 0$

$\Rightarrow t^2 + t - 4 + \frac{1}{t} + \frac{1}{t^2} = 0$

$\Rightarrow \left(t^2 + \frac{1}{t^2}\right) + \left(t + \frac{1}{t}\right) - 4 = 0$

Let $t + \frac{1}{t} = y$

$(y^2 - 2) + y - 4 = 0 \quad \Rightarrow \quad y^2 + y - 6 = 0$

$y^2 + y - 6 = 0 \quad \Rightarrow \quad y = -3, 2$

$\Rightarrow \; y = 2 \quad \Rightarrow t + \frac{1}{t} = 2$

$\Rightarrow \; e^x + e^{-x} = 2$

$x = 0$, is the only solution of the equation

Hence, there only one solution of the given equation.

Question179

The least positive value of ' a ' for which the equation, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots is ________ [Jan. 8, 2020 (I)]

Answer: 8

Solution:

Solution:

Since, $2x^2 + (a - 10)x + \frac{33}{2} = 2a$ has real roots,

$\therefore\ D \geq 0$

$\Rightarrow\ (a - 10)^2 - 4(2)\left(\frac{33}{2} - 2a\right) \geq 0$

$\Rightarrow\ (a - 10)^2 - 4(33 - 4a) \geq 0$

$\Rightarrow\ a^2 - 4a - 32 \geq 0$

$\Rightarrow\ (a - 8)(a + 4) \geq 0$

$\Rightarrow\ a \leq -4 \cup a \geq 8$

$\Rightarrow\ a \in (-\infty, -4] \cup [8, \infty)$

Question180

If the equation, $x^2 + bx + 45 = 0 (b \in R)$ has conjugate complex roots and they satisfy $|z + 1| = 2\sqrt{10}$, then: [Jan. 8, 2020 (I)]

Options:

A. $b^2 - b = 30$

B. $b^2 + b = 72$

C. $b^2 - b = 42$

D. $b^2 + b = 12$

Answer: A

Solution:

Solution:

Let $z = \alpha \pm i\beta$ be the complex roots of the equation

So, sum of roots $= 2\alpha = -b$ and

Product of roots $= \alpha^2 + \beta^2 = 45$

$(\alpha + 1)^2 + \beta^2 = 40$

Given, $|z + 1| = 2\sqrt{10}$

$\Rightarrow\ (\alpha + 1)^2 - \alpha^2 = -5\ [\because \beta^2 = 45 - \alpha^2]$

$\Rightarrow\ 2\alpha + 1 = -5\ \ \Rightarrow\ \ 2\alpha = -6$

Hence, $b = 6$ and $b^2 - b = 30$

Question181

Let α and β be the roots of the equation $x^2 - x - 1 = 0$. If $p_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true?
[Jan. 7, 2020 (II)]

Options:

A. $p_3 = p_5 - p_4$

B. $P_5 = 11$

C. $(p_1 + p_2 + p_3 + p_4 + p_5) = 26$

D. $p_5 = p_2 \cdot p_3$

Answer: D

Solution:

Solution:
$\alpha^5 = 5\alpha + 3$
$\beta^5 = 5\beta + 3$
$p_5 = 5(\alpha + \beta) + 6 = 5(1) + 6$
$\left[\because \text{ from } x^2 - x - 1 = 0, \alpha + \beta = \frac{-b}{a} = 1\right]$
$p_5 = 11$ and $p_5 = \alpha^2 + \beta^2 = \alpha + 1 + \beta + 1$
$p_2 = 3$ and $p_3 = \alpha^3 + \beta^3 = 2\alpha + 1 + 2\beta + 1$
$= 2(1) + 2 = 4$
$p_2 \times p_3 = 12$ and $p_5 = 11 \Rightarrow p_5 \neq p_2 \times p_3$

Question182

Let α and $\overline{\beta}$ be two real roots of the equation $(k+1)\tan^2 x - \sqrt{2} \,.\, \lambda \tan x = (1 - k)$, where $k(\neq -1)$ and λ are real numbers. If $\tan^2(\alpha + \beta) = 50$, then a value of λ is:
[Jan. 7, 2020 (I)]

Options:

A. $10\sqrt{2}$

B. 10

C. 5

D. $5\sqrt{2}$

Answer: B

Solution:

Solution:
$(k+1)\tan^2 x - \sqrt{2}\lambda \tan x + (k-1) = 0$
$\tan\alpha + \tan\beta = \dfrac{\sqrt{2}\lambda}{k+1}$ [Sum of roots]
$\tan\alpha \cdot \tan\beta = \dfrac{k-1}{k+1}$ [Product of roots]
$\therefore\ \tan(\alpha+\beta) = \dfrac{\frac{\sqrt{2}\lambda}{k+1}}{1 - \frac{k-1}{k+1}} = \dfrac{\sqrt{2\lambda}}{2} = \dfrac{\lambda}{\sqrt{2}}$
$\tan^2(\alpha+\beta) = \dfrac{\lambda^2}{2} = 50$
$\lambda = 10$

Question183

Let a, b $\in$ R, a $\neq$ 0 be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to : [Jan. 9, 2020 (II)]

Options:

A. 25

B. 26

C. 28

D. 24

Answer: A

Solution:

Solution:
$ax^2 - 2bx + 5 = 0$
If α and α are roots of equations, then sum of roots
$2\alpha = \dfrac{2b}{a} \Rightarrow \alpha = \dfrac{b}{a}$
and product of roots $= \alpha^2 = \dfrac{5}{a} \Rightarrow \dfrac{b^2}{a^2} = \dfrac{5}{a}$
$\Rightarrow\ b^2 = 5a\ \ (a \neq 0)\ .\ .\ .(i)$
For $x^2 - 2bx - 10 = 0$
$\alpha + \beta = 2b\ .\ .\ .(ii)$
and $\alpha\beta = -10\ .\ .\ .(iii)$
$\alpha = \dfrac{b}{a}$ is also root of $x^2 - 2bx - 10 = 0$
$\Rightarrow\ b^2 - 2ab^2 - 10a^2 = 0$
By eqn. (i) $\Rightarrow 5a - 10a^2 - 10a^2 = 0$
$\Rightarrow\ 20a^2 = 5a$
$\Rightarrow\ a = \dfrac{1}{4}$ and $b^2 = \dfrac{5}{4}$
$\alpha^2 = 20$ and $\beta^2 = 5$
Now, $\alpha^2 + \beta^2 = 5 + 20 = 25$

Question184

If the four complex numbers z, $\bar{z}$, $\bar{z} - 2\mathrm{Re}(\bar{z})$ and $z - 2\mathrm{Re}(z)$ represent the vertices of a square of side 4 units in the Argand plane, then $|z|$ is equal to :
[Sep. 05, 2020 (I)]

Options:

A. $4\sqrt{2}$

B. 4

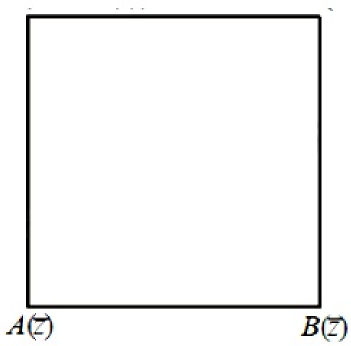
C. $2\sqrt{2}$

D. 2

Answer: C

Solution:

Solution:
$D(z - 2\mathrm{Re}(z))$ $C(\bar{z} - 2\mathrm{Re}(\bar{z}))$



Let $z = x + iy$
$\because$ Length of side of square $= 4$ units
Then, $|z - \bar{z}| = 4 \Rightarrow |2iy| = 4 \Rightarrow |y| = 2$
Also, $|z - (z - 2\mathrm{Re}(z))| = 4$
$\Rightarrow |2\mathrm{Re}(z)| = 4 \Rightarrow |2x| = 4 \Rightarrow |x| = 2$
$\therefore |z| = \sqrt{x^2 + y^2} = \sqrt{4 + 4} = 2\sqrt{2}$

Question185

The value of $\left(\frac{-1 + i\sqrt{3}}{1 - i}\right)^{30}$ is:
[Sep. 05, 2020 (II)]

Options:

A. -2^{15}

B. $2^{15}i$

C. $-2^{15}i$

D. 6^{5}

Answer: C

Solution:

Solution:

$\because -1+\sqrt{3}i = 2\cdot e^{\frac{2\pi}{3}i}$ and $1-i = \sqrt{2}\cdot e^{-\frac{i\pi}{4}}$

$\therefore \left(\frac{-1+\sqrt{3}i}{1-i}\right)^{30} = \left(\sqrt{2}e^{\left(\frac{2\pi}{3}+\frac{\pi}{4}\right)i}\right)^{30}$

$= 2^{15}\cdot e^{-\frac{\pi}{2}i} = -2^{15}\cdot i.$

Question186

If $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$, (n,m ∈ N), then the greatest common divisor of the least values of m and n is________.
[Sep. 03, 2020 (I)]

Answer: 4

Solution:

Solution:

Given that $\left(\frac{1+i}{1-i}\right)^{m/2} = \left(\frac{1+i}{i-1}\right)^{n/3} = 1$

$\Rightarrow \left(\frac{(1+i)^2}{2}\right)^{m/2} = \left(\frac{(1+i)^2}{-2}\right)^{n/3} = 1$

$\Rightarrow i^{m/2} = (-i)^{n/3} = 1$

m(least) = 8, n(least) = 12

GCD(8, 12) = 4

Question187

If Z_1, Z_2 are complex numbers such that

$\mathrm{Re}(z_1) = |Z_1 - 1|$, $\mathrm{Re}(Z_2) = |Z_2 - 1|$ and $\arg(Z_1 - Z_2) = \frac{\pi}{6}$, then

$\mathrm{Im}(Z_1 + Z_2)$ is equal to:

[Sep. 03, 2020 (II)]

Options:

A. $\frac{2}{\sqrt{3}}$

B. $2\sqrt{3}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{1}{\sqrt{3}}$

Answer: B

Solution:

Solution:
Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$
$\because |z_1 - 1| = \mathrm{Re}(z_1)$
$\Rightarrow (x_1 - 1)^2 + y_1{}^2 = x_1{}^2$
$\Rightarrow y_1{}^2 - 2x_1 + 1| = 0$ ....(i)
$|z_2 - 1| = \mathrm{Re}(z_2) \Rightarrow (x_2 - 1)^2 + y_2{}^2 = x_2{}^2$
$\Rightarrow y_2{}^2 - 2x_2 + 1 = 0$ ....(ii)
From eqn. (i) - (ii)
$y_1{}^2 - y_2{}^2 - 2(x_1 - x_2) = 0$
$\Rightarrow y_1 + y_2 = 2\left(\frac{x_1 - x_2}{y_1 - y_2}\right)$ ....(iii)
$\because \arg(z_1 - z_2) = \frac{\pi}{6}$
$\Rightarrow \tan^{-1}\left(\frac{y_1 - y_2}{x_1 - x_2}\right) = \frac{\pi}{6}$
$\Rightarrow \frac{y_1 - y_2}{x_1 - x_2} = \frac{1}{\sqrt{3}}$
$\Rightarrow \frac{2}{y_1 + y_2} = \frac{1}{\sqrt{3}}$ $\left[\text{From, } \frac{y_1 - y_2}{x_1 - x_2} = \frac{2}{y_1 + y_2}\right]$
$\therefore y_1 + y_2 = 2\sqrt{3} \Rightarrow \mathrm{l\,m}(z_1 + z_2) = 2\sqrt{3}$

Question188

Let $z = x + iy$ be a non-zero complex number such that $z^2 = i\,|z|^2$, where $i = \sqrt{-1}$, then z lies on the:
[Sep. 06, 2020 (II)]

Options:

A. line, $y = -x$

B. imaginary axis

C. line, $y = x$

D. real axis

Answer: C

Solution:

Solution:
Let $z = x + iy$
$\because z^2 = i\,|z|^2$
$\therefore x^2 - y^2 + 2ixy = i(x^2 + y^2)$
$\Rightarrow x^2 - y^2 = 0$ and $2xy = x^2 + y^2$
$\Rightarrow (x - y)(x + y) = 0$ and $(x - y)^2 = 0$
$\Rightarrow x = y$

Question189

If a and b are real numbers such that $(2+\alpha)^4 = a + b\alpha$, where $\alpha = \frac{-1+i\sqrt{3}}{2}$, then a + b is equal to :

[Sep. 04, 2020 (II)]

Options:

A. 9

B. 24

C. 33

D. 57

Answer: A

Solution:

Solution:

Given that, $\alpha = \frac{-1+\sqrt{3}i}{2} = \omega$

$\therefore (2+\omega)^4 = a + b\omega \Rightarrow (4+\omega^2+4\omega)^2 = a + b\omega$

$\Rightarrow (\omega^2 + 4(1+\omega))^2 = a + b\omega$

$\Rightarrow (\omega^2 - 4\omega^2)^2 = a + b\omega$

$[\because 1+\omega = -\omega^2]$

$\Rightarrow (-3\omega^2)^2 = a + b\omega \Rightarrow 9\omega^4 = a + b\omega$

$\Rightarrow 9\omega = a + b\omega\ (\because \omega^3 = 1)$

On comparing ,$a = 0$, $b = 9$

$\Rightarrow a + b = 0 + 9 = 9$

Question190

The value of $\left(\frac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ **is :**

[Sep. 02, 2020 (I)]

Options:

A. $\frac{1}{2}(1-i\sqrt{3})$

B. $\frac{1}{2}(\sqrt{3}-i)$

C. $-\frac{1}{2}(\sqrt{3}-i)$

D. $-\frac{1}{2}(1-i\sqrt{3})$

Answer: C

Solution:

Solution:

$$\left(\frac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3$$

$$=\left(\frac{2\cos^2\frac{5\pi}{36}+i2\sin\frac{5\pi}{36}\cdot\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-i2\sin\frac{5\pi}{36}\cdot\cos\frac{5\pi}{36}}\right)^3$$

$$=\left(\frac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3=\left(\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}\right)^6$$

$$=\cos\left(6\times\frac{5\pi}{36}\right)+i\sin\left(6\times\frac{5\pi}{36}\right)=\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}$$

$$=-\frac{\sqrt{3}}{2}+i\frac{1}{2}=-\frac{1}{2}(\sqrt{3}-i)$$

Question191

The imaginary part of $(3+2\sqrt{-54})^{1/2}-(3-2\sqrt{-54})^{1/2}$ can be: [Sep. 02, 2020 (II)]

Options:

A. $-\sqrt{6}$

B. $-2\sqrt{6}$

C. 6

D. $\sqrt{6}$

Answer: B

Solution:

Solution:

$3+2\sqrt{-54}=3+6\sqrt{6}i$

Let $\sqrt{3+6\sqrt{6}i}=a+ib$

$\Rightarrow a^2-b^2=3$ and $ab=3\sqrt{6}$

$\Rightarrow a^2+b^2=\sqrt{(a^2-b^2)^2+4a^2b^2}=15$

So, $a=\pm3$ and $b=\pm\sqrt{6}$

$\sqrt{3+6\sqrt{6}i}=\pm(3+\sqrt{6}i)$

Similarly, $\sqrt{3-6\sqrt{6}i}=\pm(3-\sqrt{6}i)$

$\mathrm{Im}\left(\sqrt{3+6\sqrt{6}i}-\sqrt{3-6\sqrt{6}i}\right)=\pm2\sqrt{6}$

Question192

If α and β be two roots of the equation $x^2-64x+256=0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is:

[Sep. 06, 2020 (I)]

Options:

A. 2

B. 3

C. 1

D. 4

Answer: A

Solution:

Solution:

$\because \alpha + \beta = 64, \alpha\beta = 256$

$\frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}} = \frac{\alpha + \beta}{(\alpha\beta)^{5/8}} = \frac{64}{(2^8)^{5/8}} = \frac{64}{32} = 2$

Question193

If α and β are the roots of the equation 2x(2x + 1) = 1, then β is equal to:

[Sep. 06, 2020 (II)]

Options:

A. $2\alpha(\alpha + 1)$

B. $-2\alpha(\alpha + 1)$

C. $2\alpha(\alpha - 1)$

D. $2\alpha^2$

Answer: B

Solution:

Solution:

Let α and β be the roots of the given quadratic equation,

$2x^2 + 2x - 1 = 0$

Then, $\alpha + \beta = -\frac{1}{2} \Rightarrow -1 = 2\alpha + 2\beta$

and $4\alpha^2 + 2\alpha - 1 = 0$ [$\because \alpha$ is root of eq. (i)]

$\Rightarrow 4\alpha^2 + 2\alpha + 2\alpha + 2\beta = 0 \Rightarrow \beta = -2\alpha(\alpha + 1)$

Question194

The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$, is:
[Sep. 05, 2020 (I)]

Options:

A. $\frac{5}{9}$

B. $\frac{25}{81}$

C. $\frac{5}{27}$

D. $\frac{25}{9}$

Answer: B

Solution:

Solution:
Let $|x| = y$ then
$9y^2 - 18y + 5 = 0$
$\Rightarrow 9y^2 - 15y - 3y + 5 = 0$
$\Rightarrow (3y - 1)(3y - 5) = 0$
$\Rightarrow y = \frac{1}{3}$ or $\frac{5}{3} \Rightarrow \left|x\right| = \frac{1}{3}$ or $\frac{5}{3}$
Roots are $\pm\frac{1}{3}$ and $\pm\frac{5}{3}$
Product $= \frac{25}{81}$

Question195

If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$ the the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to :
[Sep. 05, 2020 (II)]

Options:

A. $\frac{27}{32}$

B. $\frac{1}{24}$

C. $\frac{3}{8}$

D. $\frac{27}{16}$

Answer: D

Solution:

Solution:
Let α and β be the roots of the quadratic equation $7x^2 - 3x - 2 = 0$

$\therefore \alpha + \beta = \frac{3}{7}, \alpha\beta = \frac{-2}{7}$

Now, $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$

$= \frac{\alpha - \alpha\beta(\alpha+\beta) + \beta}{1 - (\alpha^2 + \beta^2) + (\alpha\beta)^2}$

$= \frac{(\alpha+\beta) - \alpha\beta(\alpha+\beta)}{1 - (\alpha+\beta)^2 + 2\alpha\beta + (\alpha\beta)^2}$

$= \frac{\frac{3}{7} + \frac{2}{7} \times \frac{3}{7}}{1 - \frac{9}{49} + 2 \times \frac{-2}{7} + \frac{4}{49}} = \frac{27}{16}$

Question196

Let u = $\frac{2z+i}{z-ki}$, z = x + iy and k > 0. If the curve represented by Re(u) + I m(u) = 1 intersects the y -axis at the points P and Q where PQ = 5, then the value of k is :
[Sep. 04, 2020 (I)]

Options:

A. 3 / 2

B. 1 / 2

C. 4

D. 2

Answer: D

Solution:

Solution:

$u = \frac{2(x+iy)+i}{(x+iy)-ki} = \frac{2x + i(2y+1)}{x + i(y-k)}$

Real part of u = $Re(u) = \frac{2x^2 + (y-K)(2y+1)}{x^2 + (y-K)^2}$

Imaginary part of u

$= Im(u) = \frac{-2x(y-K) + x(2y+1)}{x^2 + (y-K)^2}$

$\because Re(u) + Im(u) = 1$

$\Rightarrow 2x^2 + 2y^2 - 2Ky + y - K - 2xy + 2Kx + 2xy + x$

$= x^2 + y^2 + K^2 - 2Ky$

Since, the curve intersect at y -axis

$\therefore x = 0$

$\Rightarrow y^2 + y - K(K+1) = 0$

Let y_1 and y_2 are roots of equations if $x = 0$

$\because y_1 + y_2 = -1$

$y_1 y_2 = -(K^2 + K)$

$\therefore (y_1 - y_2)^2 = (1 + 4K^2 + 4K)$

Given $PQ = 5 \Rightarrow |y_1 - y_2| = 5$

$\Rightarrow 4K^2 + 4K - 24 = 0 \Rightarrow K = 2$ or -3

as $K > 0, \therefore K = 2$

Question197

Let $\lambda \neq 0$ be in R. If α and β are roots of the equation, $x^2 - x + 2\lambda = 0$ and α and γ are the roots of the equation, $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to
[Sep. 04, 2020 (II)]

Options:

A. 27

B. 18

C. 9

D. 36

Answer: B

Solution:

Solution:
Since α is common root of $x^2 - x + 2\lambda = 0$ and $3x^2 - 10x + 27\lambda = 0$
$\therefore 3\alpha^2 - 10\alpha + 27\lambda = 0$ ....(i)
$3\alpha^2 - 3\alpha + 6\lambda = 0$ ....(ii)
$\therefore$ On subtract, we get $\alpha = 3\lambda$
Now, $\alpha\beta = 2\lambda \Rightarrow 3\lambda \cdot \beta = 2\lambda \Rightarrow \beta = \frac{2}{3}$
$\Rightarrow \alpha + \beta = 1 \Rightarrow 3\lambda + \frac{2}{3} = 1 \Rightarrow \lambda = \frac{1}{9}$ and
$\alpha\gamma = 9\lambda \Rightarrow 3\lambda \cdot \gamma = 9\lambda \Rightarrow \gamma = 3$
$\therefore \frac{\beta\gamma}{\lambda} = 18$

Question198

If α and β are the roots of the equation $x^2 + px + 2 = 0$ and $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the roots of the equation $2x^2 + 2qx + 1 = 0$ then $\left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$ is equal to
[Sep. 03, 2020 (I)]

Options:

A. $\frac{9}{4}(9 + q^2)$

B. $\frac{9}{4}(9 - q^2)$

C. $\frac{9}{4}(9 + p^2)$

D. $\frac{9}{4}(9 - p^2)$

Answer: D

Solution:

Solution:

$\alpha \cdot \beta = 2$ and $\alpha + \beta = -p$ also $\frac{1}{\alpha} + \frac{1}{\beta} = -q$

$\Rightarrow p = 2q$

$Now \left(\alpha - \frac{1}{\alpha}\right)\left(\beta - \frac{1}{\beta}\right)\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$

$= \left[\alpha\beta + \frac{1}{\alpha\beta} - \frac{\alpha}{\beta} - \frac{\beta}{\alpha}\right]\left[\alpha\beta + \frac{1}{\alpha\beta} + 2\right]$

$= \frac{9}{2}\left[\frac{5}{2} - \frac{\alpha^2 + \beta^2}{2}\right] = \frac{9}{4}[5 - (p^2 - 4)]$

$= \frac{9}{4}(9 - p^2)$ $[\because \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta]$

Question199

The set of all real values of λ for which the quadratic equations, $(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$ always have exactly one root in the interval (0,1) is :
[Sep. 03, 2020 (II)]

Options:

A. (0,2)

B. (2,4]

C. (1,3]

D. (-3,-1)

Answer: C

Solution:

Solution:
The given quadratic equation is
$(\lambda^2 + 1)x^2 - 4\lambda x + 2 = 0$
$\because$ One root is in the interval (0,1)
$\therefore f(0)f(1) \leq 0$
$\Rightarrow 2(\lambda^2 + 1 - 4\lambda + 2) \leq 0$
$\Rightarrow 2(\lambda^2 - 4\lambda + 3) \leq 0$
$(\lambda - 1)(\lambda - 3) \leq 0 \Rightarrow \lambda \in [1, 3]$
But at $\lambda = 1$, both roots are 1 so $\lambda \neq 1$
$\therefore \lambda \in (1, 3]$

Question200

Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, n = 1, 2, 3, ..., then :
[Sep. 02, 2020 (I)]

Options:

A. $6S_6 + 5S_5 = 2S_4$

B. $6S_6 + 5S_5 + 2S_4 = 0$

C. $5S_6 + 6S_5 = 2S_4$

D. $5S_6 + 6S_5 + 2S_4 = 0$

Answer: C

Solution:

Solution:
Since, α and β are the roots of the equation
$5x^2 + 6x - 2 = 0$
Then, $5\alpha^2 + 6\alpha - 2 = 0$, $5\beta^2 + 6\beta - 2 = 0$
$5\alpha^2 + 6\alpha = 2$
$5S_6 + 6S_5 = 5(\alpha^6 + \beta^6) + 6(\alpha^5 + \beta^5)$
$= (5\alpha^4 + 6\alpha^5) + (5\beta^6 + 6\beta^5)$
$= \alpha^4(5\alpha^2 + 6\alpha) + \beta^4(5\beta^2 + 6\beta)$
$= 2(\alpha^4 + \beta^4) = 2S_4$

Question201

If λ be the ratio of the roots of the quadratic equation in x, $3m^2x^2 + m(m-4)x + 2 = 0$, then the least value of m for which $\lambda + \frac{1}{\lambda} = 1$, is:
[Jan. 12, 2019 (I)]

Options:

A. $2 - \sqrt{3}$

B. $4 - 3\sqrt{2}$

C. $-2 + \sqrt{2}$

D. $4 - 2\sqrt{3}$

Answer: B

Solution:

Solution:
Let roots of the quadratic equation are α, β.
Given, $\lambda = \frac{\alpha}{\beta}$ and $\lambda + \frac{1}{\lambda} = 1 \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1$
$\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = 1 \ldots (i)$
The quadratic equation is, $3m^2x^2 + m(m-4)x + 2 = 0$
$\therefore \alpha + \beta = \frac{m(4-m)}{3m^2} = \frac{4-m}{3m}$ and $\alpha\beta = \frac{2}{3m^2}$
Put these values in eq (1)
$$\frac{\left(\frac{4-m}{3m}\right)^2}{\frac{2}{3m^2}} = 3$$
$\Rightarrow (m-4)^2 = 18 \Rightarrow m = 4 \pm \sqrt{18}$

Therefore, least value is
$4-\sqrt{18}=4-3\sqrt{2}$

Question202

If one real root of the quadratic equation $81x^2+kx+256=0$ is cube of the other root, then a value of k is :
[Jan. 11, 2019 (I)]

Options:

A. -81

B. 100

C. 144

D. -300

Answer: D

Solution:

Solution:
Let α and β be the roots of the equation,
$81x^2+kx+256=0$
Given $(\alpha)^{\frac{1}{3}}=\beta \Rightarrow \alpha=\beta^3$
$\because$ Product of the roots $=\frac{256}{81}$
$\therefore\ (\alpha)(\beta)=\frac{256}{81}$
$\Rightarrow\ \beta^4=\left(\frac{4}{3}\right)^4 \Rightarrow \beta=\frac{4}{3} \Rightarrow \alpha=\frac{64}{27}$
Sum of the roots $=-\frac{k}{81}$
$\therefore\ \alpha+\beta=-\frac{k}{81} \Rightarrow \frac{4}{3}+\frac{64}{27}=-\frac{k}{81}$
$\Rightarrow\ k=-300$

Question203

Consider the quadratic equation $(c-5)x^2-2cx+(c-4)=0$ $c\neq 5$. Let S be the set of all integral values of c for which one root of the equation lies in the interval (0,2) and its other root lies in the interval (2,3) . Then the number of elements in S is:
[Jan. 10, 2019 (I)]

Options:

A. 18

B. 12

C. 10

D. 11

Answer: D

Solution:

Solution:
Consider the quadratic equation
$(c-5)x^2 - 2cx + (c-4) = 0$
Now, $f(0), f(3) > 0$ and $f(0) . f(2) < 0$
$\Rightarrow (c-4)(4c-49) > 0$ and $(c-4)(c-24) < 0$
$\Rightarrow c \in (-\infty, 4) \cup \left(\frac{49}{4}, \infty\right)$ and $c \in (4, 24)$
$\Rightarrow c \in \left(\frac{49}{4}, 24\right)$
Integral values in the interval $\left(\frac{49}{4}, 24\right)$ are 13, 14, ..., 23
$\therefore$ S = {13, 14, ..., 23}

Question204

The value of λ such that sum of the squares of the roots of the quadratic equation, $x^2 + (3-\lambda)x + 2 = \lambda$ has the least value is: [Jan. 10, 2019 (II)]

Options:

A. $\frac{15}{8}$

B. 1

C. $\frac{4}{9}$

D. 2

Answer: D

Solution:

Solution:
The given quadratic equation is
$x^2 + (3-\lambda)x + 2 = \lambda$
Sum of roots $= \alpha + \beta = \lambda - 3$
Product of roots $= \alpha\beta = 2 - \lambda$
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= (\lambda - 3)^2 - 2(2 - \lambda)$
$= \lambda^2 - 4\lambda + 5 = (\lambda - 2)^2 + 1$
For least $(\alpha^2 + \beta^2) \lambda = 2$

Question205

Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to:

[Jan. 9, 2019 (I)]

Options:

A. -256

B. 512

C. -512

D. 256

Answer: A

Solution:

Solution:
Consider the equation
$x^2 + 2x + 2 = 0$
$x = \dfrac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$
Let $\alpha = -1 + i, \beta = -1 - i$
$\alpha^{15} + \beta^{15} = (-1+i)^{15} + (-1-i)^{15}$
$= \left(\sqrt{2}e^{i\frac{3\pi}{4}}\right)^{15} + \left(\sqrt{2}e^{-i\frac{3\pi}{4}}\right)^{15}$
$= (\sqrt{2})^{15}\left[e^{\frac{i45\pi}{4}} + e^{\frac{-i45\pi}{4}}\right]$
$= (\sqrt{2})^{15} \cdot 2\cos\dfrac{45\pi}{4} = (\sqrt{2})^{15} \cdot 2\cos\dfrac{3\pi}{4}$
$= \dfrac{-2}{\sqrt{2}}(\sqrt{2})^{15}$
$= -2(\sqrt{2})^{14} = -256$

Question206

The number of all possible positive integral values of α for which the roots of the quadratic equation, $6x^2 - 11x + \alpha = 0$ are rational numbers is:
[Jan. 09, 2019 (II)]

Options:

A. 3

B. 2

C. 4

D. 5

Answer: A

Solution:

Solution:
The roots of $6x^2 - 11x + \alpha = 0$ are rational numbers.
$\therefore$ Discriminant D must be perfect square number.
$D = (-11)^2 - 4 \cdot 6 \cdot \alpha$

$= 121 - 24\alpha$ must be a perfect square
Hence, possible values for α are
$\alpha = 3, 4, 5$
$\therefore$3 positive integral values are possible.

Question207

If both the roots of the quadratic equation $x^2 - mx + 4 = 0$ are real and distinct and they lie in the interval [1, 5], then m lies in the interval: [Jan. 09, 2019 (II)]

Options:

A. (-5,-4)

B. (4,5)

C. (5,6)

D. (3,4)

Answer: B

Solution:

Solution:
Given quadratic equation is: $x^2 - mx + 4 = 0$
Both the roots are real and distinct.
So, discriminant $B^2 - 4AC > 0$
$\therefore\ m^2 - 4 \cdot 1 \cdot 4 > 0$
$\therefore\ (m-4)(m+4) > 0$
$\therefore\ m \in (-\infty, -4) \cup (4, \infty)$ ....(i)
Since, both roots lies in [1,5]
$\therefore\ -\frac{-m}{2} \in (1, 5)$
$\Rightarrow\ m \in (2, 10)$
And $1 \cdot (1 - m + 4) > 0 \Rightarrow m < 5$
$\therefore\ m \in (-\infty, 5)\ \ldots$ (iii)
And $1 \cdot (25 - 5m + 4) > 0 \Rightarrow m < \frac{29}{5}$
$\therefore\ m \in \left(-\infty\ \frac{29}{5}\right)\ \ldots$ (iv)
From (i), (ii), (iii) and (iv), $m \in (4, 5)$

Question208

Let z_0 be a root of the quadratic equation, $x^2 + x + 1 = 0$. If $z = 3 + 6iz_0^{81} - 3iz_0^{93}$, then arg z is equal to: [Jan. 09, 2019 (II)]

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. 0

Answer: A

Solution:

Solution:
$\because z_0$ is a root of quadratic equation
$x^2 + x + 1 = 0$
$\therefore\ z_0 = \omega$ or $\omega^2 \Rightarrow {z_0}^3 = 1$
$\therefore\ z = 3 + 6i{z_0}^{81} - 3i{z_0}^{93}$
$= 3 + 6i((z_0)^3)^{27} - 3i((z_0)^3)^{31}$
$= 3 + 6i - 3i$
$= 3 + 3i$
$\therefore\ \arg(z) \tan^{-1}\left(\frac{3}{3}\right) = \frac{\pi}{4}$

Question209

If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to:
[Jan. 10, 2019 (I)]

Options:

A. $\frac{3}{4}$

B. $\frac{5}{4}$

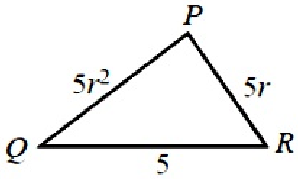
C. $\frac{7}{4}$

D. $\frac{3}{2}$

Answer: C

Solution:

Solution:



ΔPQR is possible if
$5 + 5r > 5r^2$
$\Rightarrow\ 1 + r > r^2$
$\Rightarrow\ r^2 - r - 1 < 0$
$\Rightarrow \left(r - \frac{1}{2} + \frac{\sqrt{5}}{2}\right)\left(r - \frac{1}{2} - \frac{\sqrt{5}}{2}\right) < 0$
$\Rightarrow r \in \left(\frac{-\sqrt{5}+1}{2}, \frac{\sqrt{5}+1}{2}\right)$

$\because \ \frac{7}{4} \notin \left(\frac{-\sqrt{5}+1}{2}, \frac{\sqrt{5}+1}{2} \right) \therefore r \neq \frac{7}{4}$

Question210

If $\frac{z-\alpha}{z+\alpha}(\alpha \in R)$ is a purely imaginary number and $|z| = 2$, then a value of α is:
[Jan. 12, 2019 (I)]

Options:

A. 2

B. 1

C. $\frac{1}{2}$

D. $\sqrt{2}$

Answer: A

Solution:

Solution:

Let $t = \frac{z-\alpha}{z+\alpha}$

$\because$ t is purely imaginary number.

$\therefore t + \bar{t} = 0$

$\Rightarrow \frac{z-\alpha}{z+\alpha} + \frac{\bar{z}-\alpha}{\bar{z}+\alpha} = 0$

$\Rightarrow (z-\alpha)(\bar{z}+\alpha) + (\bar{z}-\alpha)(z+\alpha) = 0$

$\Rightarrow z\bar{z} - \alpha^2 + z\bar{z} - \alpha^2 = 0$

$\Rightarrow z\bar{z} - \alpha^2 = 0$

$\Rightarrow |z|^2 - \alpha^2 = 0$

$\Rightarrow \alpha^2 = 4$

$\Rightarrow \alpha = \pm 2$

Question211

Let z_1 and z_2 be two complex numbers satisfying $|z_1| = 9$ and $|z_2| - |3| - |4i|| = |4$. Then the minimum value of $|z_1 - z_2|$ is :
[Jan. 12, 2019 (II)]

Options:

A. 0

B. $\sqrt{2}$

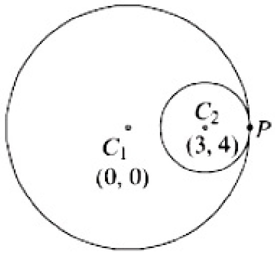
C. 1

D. 2

Answer: A

Solution:

Solution:
$|z_1| = 9, |z_2 - 3 - 4i| = 4$
z_1 lies on a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$
z_2 lies on a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$
So, minimum value of $|z_1 - z_2|$ is zero at point of contact (i.e. A)



Question212

Let z be a complex number such that $|z| + z = 3 + i$ (where $i = \sqrt{-1}$) Then $|z|$ is equal to : [Jan. 11, 2019 (II)]

Options:

A. $\frac{\sqrt{34}}{3}$

B. $\frac{5}{3}$

C. $\frac{\sqrt{41}}{4}$

D. $\frac{5}{4}$

Answer: B

Solution:

Solution:
Since, $|z| + z = 3 + i$
Let $z = a + ib$, then
$|z| + z = 3 + i \Rightarrow \sqrt{a^2 + b^2} + a + ib = 3 + i$
Compare real and imaginary coefficients on both sides
$b = 1, \sqrt{a^2 + b^2} + a = 3$
$\sqrt{a^2 + 1} = 3 - a$
$a^2 + 1 = a^2 + 9 - 6a$
$6a = 8 \Rightarrow a = \frac{4}{3}$
Then
$|z| = \sqrt{\left(\frac{4}{3}\right)^2 + 1} = \sqrt{\frac{16}{9} + 1} = \frac{5}{3}$

Question213

Let z_1 and z_2 be any two non-zero complex numbers such that $3z_1| = 4z_2|$. If $z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1}$ then
[Jan. 10 2019 (II)]

Options:

A. $Re(z) = 0$

B. $|z| = \sqrt{\frac{5}{2}}$

C. $|z| = \frac{1}{2}\sqrt{\frac{17}{2}}$

D. $I\,m(z) = 0$

E. None of Above

Answer: E

Solution:

Solution:
(none)

Let $z_1 = r_1e^{i\theta}$ and $z_2 = r_2e^{i\varphi}$

$3\,|\,z_1\,| = 4\,|\,z_2\,| \Rightarrow 3r_1 = 4r_2$

$z = \frac{3z_1}{2z_2} + \frac{2z_2}{3z_1} = \frac{3r_1}{2r_2}e^{i(\theta-\varphi)} + \frac{2}{3}\frac{r_2}{r_1}e^{i(\varphi-\theta)}$

$= \frac{3}{2}\times\frac{4}{3}(\cos(\theta-\varphi) + i\sin(\theta-\varphi)) +$
$\frac{2}{3}\times\frac{3}{4}[\cos(\theta-\varphi) - i\sin(\theta-\varphi)]$

$z = \left(2 + \frac{1}{2}\right)\cos(\theta-\varphi) + i\left(2 - \frac{1}{2}\right)\sin(\theta-\varphi)$

$\therefore\ |z| = \sqrt{\frac{25}{4}\cos^2(\theta-\varphi) + \frac{9}{4}\sin^2(\theta-\varphi)}$

$= \sqrt{\frac{16}{4}\cos^2(\theta-\varphi) + \frac{9}{4}} \Rightarrow \frac{3}{2} \le |z| \le \frac{5}{2}$

Question214

Let $A = \left\{\theta \in \left(-\frac{\pi}{2}, \pi\right) : \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary }\right\}$.
Then the sum of the elements in A is:
[Jan. 9 2019 (I)]

Options:

A. $\frac{5\pi}{6}$

B. π

C. $\frac{3\pi}{4}$

D. $\frac{2\pi}{3}$

Answer: D

Solution:

Solution:

Suppose $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$

Since, z is purely imaginary, then $z + \bar{z} = 0$

$\Rightarrow \frac{3+2i\sin\theta}{1-2i\sin\theta} + \frac{3-2i\sin\theta}{1+2i\sin\theta} = 0$

$\Rightarrow \frac{(3+2i\sin\theta)(1+2i\sin\theta)+(3-2i\sin\theta)(1-2i\sin\theta)}{1+4\sin^2\theta}$

$= 0$

$\Rightarrow \sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

Now, the sum of elements in $A = -\frac{\pi}{3} + \frac{\pi}{3} + \frac{2\pi}{3} = \frac{2\pi}{3}$

Question215

Let $\left(-2-\frac{1}{3}i\right)^3 = \frac{x+iy}{27}(i = \sqrt{-1})$, where x and y are real numbers then y − x equals:
[Jan. 11, 2019 (I)]

Options:

A. 91

B. -85

C. 85

D. -91

Answer: A

Solution:

Solution:

$-(6+i)^3 = x + iy$

$\Rightarrow -[216 + i^3 + 18i(6+i)] = x + iy$

$\Rightarrow -[216 - i + 108i - 18] = x + iy$

$\Rightarrow -216 + i - 108i + 18 = x + iy$

$\Rightarrow -198 - 107i = x + iy$

$\Rightarrow x = -198, y = -107$

$\Rightarrow y - x = -107 + 198 = 91$

Question216

Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If R(z) and I (z) respectively denote the real and imaginary parts of z, then:
[Jan. 10, 2019 (II)]

Options:

A. $I(z) = 0$

B. $R(z) > 0$ and $I(z) > 0$

C. $R(z) < 0$ and $I(z) > 0$

D. $R(z) = -(c)$

Answer: A

Solution:

Solution:

$z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$

$= \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^5 + \left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^5$

$= \left(e^{i\frac{\pi}{6}}\right)^5 + \left(e^{-i\frac{\pi}{6}}\right)^5 = 2\cos\frac{\pi}{6} = \sqrt{3}$

$\Rightarrow I(z) = 0, \mathrm{Re}(z) = \sqrt{3}$

Question217

Let $z \in C$ with $Im(Z) = 10$ and it satisfies $\frac{2z - n}{2z + n} = 2i - 1$ for some natural number n.Then:
[April 12, 2019 (II)]

Options:

A. $n = 20$ and $\mathrm{Re}(z) = -10$

B. $n = 40$ and $\mathrm{Re}(z) = 10$

C. $n = 40$ and $\mathrm{Re}(z) = -10$

D. $n = 20$ and $\mathrm{Re}(z) = 10$

Answer: C

Solution:

Solution:

Let $\mathrm{Re}(z) = x$ i.e., $z = x + 10i$
$2z - n = (2i - 1)(2z + n)$
$(2x - n) + 20i = (2i - 1)((2x + n) + 20i)$
On comparing real and imaginary parts,
$-(2x + n) - 40 = 2x - n$ and $20 = 4x + 2n - 20$
$\Rightarrow 4x = -40$ and $40 = -40 + 2n$
$\Rightarrow x = -10$ and $n = 40$

Hence, $\mathrm{Re}(z) = -10$

Question218

The equation $|Z - i| = |Z - 1|$, $i = \sqrt{-1}$, represents:
[April 12, 2019 (I)]

Options:

A. a circle of radius $\frac{1}{2}$

B. the line through the origin with slope 1.

C. a circle of radius 1.

D. the line through the origin with slope -1.

Answer: B

Solution:

Solution:
Given equation is, $|z-1| = |z-i|$
$\Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2$ [Here $, z = x + iy$]
$\Rightarrow 1 - 2x = 1 - 2y \Rightarrow x - y = 0$
Hence, locus is straight line with slope 1.

Question219

if $a > 0$ and $Z = \frac{(1+i)^2}{a-i}$, has magnitude $\sqrt{\frac{2}{5}}$, then $\bar{z}$ is equal to:

[April 10, 2019 (I)]

Options:

A. $-\frac{1}{2} - \frac{3}{5}i$

B. $-\frac{3}{5} - \frac{1}{5}i$

C. $\frac{1}{5} - \frac{3}{5}i$

D. $-\frac{1}{5} + \frac{3}{5}i$

Answer: A

Solution:

Solution:
$z = \frac{(1+i)^2}{a-i} \times \frac{a+i}{a+i}$

$z = \frac{(1-1+2i)(a+i)}{a^2+1} = \frac{2ai-2}{a^2+1}$

$|z| = \sqrt{\left(\frac{-2}{a^2+1}\right)^2 + \left(\frac{2a}{a^2+1}\right)^2} = \sqrt{\frac{4+4a^2}{(a^2+1)^2}}$

$\Rightarrow |z| = \sqrt{\frac{4(1+a^2)}{(1+a^2)^2}} = \frac{2}{\sqrt{1+a^2}}$ ....(i)

Since, it is given that $|z| = \sqrt{\frac{2}{5}}$

Then, from equation (i),

$\sqrt{\frac{2}{5}} = \frac{2}{\sqrt{1+a^2}}$

Now, square on both side; we get

$\Rightarrow 1 + a^2 = 10 \Rightarrow a = \pm 3$

Since, it is given that $a > 0 \Rightarrow a = 3$ Then, $z = \frac{(1+i)^2}{a-i} = \frac{1+i^2+2i}{3-i} = \frac{2i}{3-i}$

$= \frac{2i(3+i)}{10} = \frac{-1+3i}{5}$

Hence, $\bar{z} = \frac{-1}{5} - \frac{3}{5}i$

Question220

If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$, then:
[April 10, 2019 (II)]

Options:

A. $\bar{z}\omega = i$

B. $z\bar{\omega} = \frac{-1+i}{\sqrt{2}}$

C. $\bar{z}\omega = -i$

D. $z\bar{\omega} = \frac{1-i}{\sqrt{2}}$

Answer: C

Solution:

Solution:

Given $|z\omega| = 1 \ldots$ (i)

and $\arg\left(\frac{z}{\omega}\right) = \frac{\pi}{2}$

$\therefore \frac{z}{\omega} + \frac{\bar{z}}{\bar{\omega}} = 0 \ \left[\because \mathrm{Re}\left(\frac{z}{\omega}\right) = 0\right]$

$\Rightarrow z\bar{\omega} = -\overline{z\omega}$

from equation (i), $z\bar{z}\omega\bar{\omega} = 1$ [using $z\bar{z} = z|^2$]

$(\bar{z}\omega)^2 = -1 \Rightarrow \bar{z}\omega = \pm i$

from equation (ii), $-\arg(\bar{z}) - \arg\omega = \frac{\pi}{2} - \arg(\bar{z}\omega) = \frac{-\pi}{2}$

Hence, $\bar{z}w = -i$

Question221

Let $z \in C$ be such that $|z| < 1$. If $\omega = \frac{5+3z}{5(1-z)}$, then :
[April 09, 2019 (II)]

Options:

A. $5\mathrm{Re}(\omega) > 4$

B. $4\mathrm{Im}(\omega) > 5$

C. $5\mathrm{Re}(\omega) > 1$

D. $5\mathrm{Im}(\omega) < 1$

Answer: C

Solution:

Solution:

$\omega = \frac{5+3z}{5-5z} \Rightarrow 5\omega - 5\omega z = 5 + 3z$

$\Rightarrow 5\omega - 5 = z(3+5\omega) \Rightarrow z = \frac{5(\omega-1)}{3+5\omega}$

$\because |z| < 1, \therefore 5|\omega - 1| < |3 + 5\omega|$

$\Rightarrow 25(\omega\overline{\omega} - \omega - \overline{\omega} + 1) < 9 + 25\omega\overline{\omega} + 15\omega + 15\overline{\omega}$

$(\because |z|^2 = z\overline{z})$

$\Rightarrow 16 < 40\omega + 40\overline{\omega} \Rightarrow \omega + \overline{\omega} > \frac{2}{5} \Rightarrow 2\mathrm{Re}(\omega) > \frac{2}{5}$

$\Rightarrow \mathrm{Re}(\omega) > \frac{1}{5}$

Question222

If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}(i = \sqrt{-1})$, then $(1 + iz + z^5 + iz^8)^9$ is equal to:
[April 08, 2019 (II)]

Options:

A. 0

B. 1

C. $(-1 + 2i)^9$

D. -1

Answer: D

Solution:

Solution:

$\left(\frac{\sqrt{3}}{2} + \frac{i}{2} = -i\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -i\omega\right.$

where ω is imaginary cube root of unity.

Now, $(1 + iz + z^5 + iz^8)^9$

$= (1 + \omega - i\omega^2 + i\omega^2)^9 = (1+\omega)^9$

$= (-\omega^2)^9 = -\omega^{18} = -1 \ (\because 1 + \omega + \omega^2 = 0)$

Question223

If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta$ $-2\sin\theta = 00$, $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})(\alpha-\beta)^{24}}$ is equal to :
[April 10, 2019 (I)]

Options:

A. $\frac{2^{12}}{(\sin\theta-4)^{12}}$

B. $\frac{2^{12}}{(\sin\theta+8)^{12}}$

C. $\frac{2^{12}}{(\sin\theta-8)^{6}}$

D. $\frac{2^{6}}{(\sin\theta+8)^{12}}$

Answer: B

Solution:

Solution:
Given equation is,
$x^2 + x\sin\theta - 2\sin\theta = 0$
$\alpha+\beta = -\sin\theta$ and $\alpha\beta = -2\sin\theta$
$\frac{(\alpha^{12}+\beta^{12})\alpha^{12}\beta^{12}}{(\alpha^{12}+\beta^{12})(\alpha-\beta)^{24}} = \frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}}$
$\therefore \ |\alpha-\beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{\sin^2\theta + 8\sin\theta}$
$\therefore \frac{(\alpha\beta)^{12}}{(\alpha-\beta)^{24}} = \frac{(2\sin\theta)^{12}}{\sin^{12}\theta(\sin\theta+8)^{12}} = \frac{2^{12}}{(\sin\theta+8)^{12}}$

Question224

The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ is:
[April 10, 2019 (II)]

Options:

A. 3

B. 2

C. 4

D. 1

Answer: D

Solution:

Solution:
Let $2^x - 1 = t$
$5 + | t | = (t+1)(t-1) \Rightarrow | t | = t^2 - 6$
When $t > 0$, $t^2 - t - 6 = 0 \Rightarrow t = 3$ or -2
$t = -2$(rejected)
When $t < 0$, $t^2 + t - 6 = 0 \Rightarrow t = -3$ or 2 (both rejected)
$\therefore 2^x - 1 = 3 \Rightarrow 2^x = 4 \Rightarrow x = 2$

Question225

Let p, q ∈ R. If $2 - \sqrt{3}$ is a root of the quadratic equation, $x^2 + px + q = 0$, then:
[April 10, 2019 (II)]

Options:

A. $p^2 - 4q + 12 = 0$

B. $q^2 - 4p - 16 = 0$

C. $q^2 + 4p + 14 = 0$

D. $p^2 - 4q - 12 = 0$

Answer: D

Solution:

Solution:
Since $2 - \sqrt{3}$ is a root of the quadratic equation
$x^2 + px + q = 0$
$\therefore 2 + \sqrt{3}$ is the other root
$\Rightarrow x^2 + px + q = [x - (2 - \sqrt{3})][x - (2 + \sqrt{3})]$
$= x^2 - (2 + \sqrt{3})x - (2 - \sqrt{3})x + (2^2 - (\sqrt{3})^2)$
$= x^2 - 4x + 1$
Now, by comparing $p = -4$, $q = 1$
$\Rightarrow p^2 - 4q - 12 = 16 - 4 - 12 = 0$

Question226

If m is chosen in the quadratic equation $(m^2 + 1)x^2 - 3x + (m^2 + 1)^2 = 0$ such that the sum of its roots is greatest, then the absolute difference of the cubes of its roots is:
[April 09, 2019 (II)]

Options:

A. $10\sqrt{5}$

B. $8\sqrt{3}$

C. $8\sqrt{5}$

D. $4\sqrt{3}$

Answer: C

Solution:

Solution:

Sum of roots $= \dfrac{3}{m^2+1}$

$\because$ sum of roots is greatest. $\therefore m = 0$

Hence equation becomes $x^2 - 3x + 1 = 0$

Now, $\alpha + \beta = 3$, $\alpha\beta = 1 \Rightarrow |-\alpha - \beta| = \sqrt{5}$

$|\alpha^3 - \beta^3| = |(\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta)| = \sqrt{5}(9 - 1) = 8\sqrt{5}$

Question227

The sum of the solutions of the equation $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$, $(x > 0)$ is equal to: [April 8, 2019 (I)]

Options:

A. 9

B. 12

C. 4

D. 10

Answer: D

Solution:

Solution:

Let $\sqrt{x} = a$

$\therefore$ given equation will become:

$|a - 2| + a(a - 4) + 2 = 0$

$\Rightarrow |a - 2| + a^2 - 4a + 4 - 2 = 0$

$\Rightarrow |a - 2| + (a - 2)^2 - 2 = 0$

Let $|a - 2| = y$(Clearly $y \ge 0$)

$\Rightarrow y + y^2 - 2 = 0$

$\Rightarrow y = 1$ or -2 (rejected) $\Rightarrow |a - 2| = 1 \Rightarrow a = 1, 3$

When $\sqrt{x} = 1 \Rightarrow x = 1$

When $\sqrt{x} = 3 \Rightarrow x = 9$

Hence, the required sum of solutions of the equation

$= 10$

Question228

If α and β be the roots of the equation $x^2 - 2x + 2 = 0$, then the least value of n for which $\left(\frac{\alpha}{\beta}\right)^n = 1$ is: [April 8, 2019 (I)]

Options:

A. 2

B. 5

C. 4

D. 3

Answer: C

Solution:

Solution:
The given quadratic equation is $x^2 - 2x + 2 = 0$
Then, the roots of the this equation are $\frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$
Now, $\frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = i$
or $\frac{\alpha}{\beta} = \frac{1-i}{1+i} = \frac{(1-i)^2}{1-i^2} = -i$ So , $\frac{\alpha}{\beta} = \pm i$
N ow, $\left(\frac{\alpha}{\beta}\right)^n = 1 \Rightarrow (\pm i)^n = 1$
$\Rightarrow$ n must be a multiple of 4
Hence, the required least value of $n = 4$

Question229

The set of all $\alpha \in R$, for which $w = \frac{1+(1-8\alpha)z}{1-z}$ is a purely imaginary number, for all $z \in C$ satisfying $|z| = 1$ and $\text{Re}z \neq 1$, is [Online April 15, 2018]

Options:

A. $\{0\}$

B. an empty set

C. $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$

D. equal to R

Answer: A

Solution:

Solution:
$\because |z| = 1 \& \text{Re}z \neq 1$
Suppose $z = x + iy \Rightarrow x^2 + y^2 = 1 \ldots\ldots$ (i)
Now, $w = \frac{1+(1-8\alpha)z}{1-z}$
$\Rightarrow w = \frac{1+(1-8\alpha)(x+iy)}{1-(x+iy)}$
$\Rightarrow w = \frac{1+(1-8\alpha)(x+iy))((1-x)+ iy)}{1-(x+iy))((1-x)+ iy)}$
$\Rightarrow$ $w = \frac{[(1+x(1-8\alpha)(1-x)-(1-8\alpha)y^2]}{(1-x)^2+y^2}$

$$+ i\frac{[(1+x(1-8\alpha))y-(1-8\alpha)y(1-x)]}{(1-x)^2+y^2}$$

As, w is purely imaginary. So,

$$\text{Re}w = \frac{[(1+x(1-8\alpha))(1-x)-(1-8\alpha)y^2]}{(1-x)^2+y^2} = 0$$

$\Rightarrow (1-x)+x(1-8\alpha)(1-x) = (1-8\alpha)y^2$

$\Rightarrow (1-x)+x(1-8\alpha)-x^2(1-8\alpha) = (1-8x)y^2$

$\Rightarrow (1-x)+x(1-8\alpha) = 1-8\alpha$[From (i), $x^2+y^2 = 1$]

$\Rightarrow 1-8\alpha = 1 \Rightarrow \alpha = 0$

$\therefore \alpha \in \{0\}$

Question230

The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$, is

[Online April 16, 2018]

Options:

A. 2

B. 6

C. 5

D. 3

Answer: D

Solution:

Solution:

Let $l = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$

$\therefore l = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right) \times \left(\frac{1+i\sqrt{3}}{1+i\sqrt{3}}\right)$

$= \left(\frac{-2+i2\sqrt{3}}{4}\right) = \left(\frac{1-i\sqrt{3}}{-2}\right)$

Also, $l = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right) \times \left(\frac{1-i\sqrt{3}}{1-i\sqrt{3}}\right)$

$= \left(\frac{4}{-2-i2\sqrt{3}}\right) = \left(\frac{-2}{1+i\sqrt{3}}\right)$

Now, $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^3 = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right) \times \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right) \times \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$

$= \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right) \times \left(\frac{-2}{1+i\sqrt{3}}\right) \times \left(\frac{1-i\sqrt{3}}{-2}\right) = 1$

$\therefore$ least positive integer n is 3 .

Question231

Let p, q and r be real numbers ($p \neq q, r \neq 0$), such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to.

[Online April 16, 2018]

Options:

A. $p^2 + q^2 + r^2$

B. $p^2 + q^2$

C. $2(p^2 + q^2)$

D. $\frac{p^2 + q^2}{2}$

Answer: B

Solution:

Solution:

$\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$

$\frac{x+p+x+q}{(x+p)(x+q)} = \frac{1}{r}$

$(2x + p + q)r = x^2 + px + qx + pq$

$x^2 + (p + q - 2r)x + pq - pr - qr = 0$

Let α and β be the roots.

$\therefore \alpha + \beta = -(p + q - 2r)$ ....(i)

$\& \alpha\beta = pq - pr - qr$ ....(ii)

$\because \alpha = -\beta$ (given)

∴ in eq. (1), we get

$\Rightarrow -(p + q - 2r) = 0$ ....(iii)

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-(p + q - 2r))^2 - 2(pq - pr - qr)\ldots.$ (from (i) and (ii))

$= p^2 + q^2 + 4r^2 + 2pq - 4pr - 4qr - 2pq + 2pr + 2qr$

$= p^2 + q^2 + 4r^2 - 2pr - 2qr$

$= p^2 + q^2 + 2r(2r - p - q)$ … (from (iii))

$= p^2 + q^2 + 0$

$= p^2 + q^2$

Question232

If an angle A of a ΔABC satisfies $5\cos A + 3 = 0$, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are. [Online April 16, 2018]

Options:

A. $\sin A, \sec A$

B. $\sec A, \tan A$

C. $\tan A, \cos A$

D. $\sec A, \cot A$

Answer: B

Solution:

Solution:

Here, $9x^2 + 27x + 20 = 0$

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \dfrac{-27 \pm \sqrt{27^2 - 4 \times 9 \times 20}}{2 \times 9}$

$\Rightarrow x = -\dfrac{4}{3}, -\dfrac{5}{3}$

Given, $\cos A = -\dfrac{3}{5}$

$\therefore \sec A = \dfrac{1}{\cos A} = -\dfrac{5}{3}$

Here, A is an obtuse angle.

$\therefore \tan A = -\sqrt{\sec^2 A - 1} = -\dfrac{4}{3}$

Hence, roots of the equation are $\sec A$ and $\tan A$

Question233

If $\tan A$ and $\tan B$ are the roots of the quadratic equation, $3x^2 - 10x - 25 = 0$ then the value of $3\sin^2(A + B) - 10\sin(A + B) \cdot \cos(A + B) - 25\cos^2(A + B)$ is [Online April 15, 2018]

Options:

A. 25

B. -25

C. -10

D. 10

Answer: B

Solution:

Solution:

As $\tan A$ and $\tan B$ are the roots of $3x^2 - 10x - 25 = 0$,

So, $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{\frac{10}{3}}{1 + \frac{25}{3}} = \dfrac{10/3}{28/3} = \dfrac{5}{14}$

Now, $\cos 2(A + B) = -1 + 2\cos^2(A + B)$

$= \dfrac{1 - \tan^2(A + B)}{1 + \tan^2(A + B)} \Rightarrow \cos^2(A + B) = \dfrac{196}{221}$

$\therefore 3\sin^2(A + B) - 10\sin(A + B)\cos(A + B) - 25\cos^2(A + B)$

$= \cos^2(A + B)[3\tan^2(A + B) - 10\tan(A + B) - 25]$

$= \dfrac{75 - 700 - 4900}{196} \times \dfrac{196}{221} = -\dfrac{5525}{196} \times \dfrac{196}{221} = -25$

Question234

If $f(x)$ is a quadratic expression such that $f\ + f\ = 0$ and -1 is a root of $f(x) = 0$, then the other root of $f(x) = 0$ is [Online April 15, 2018]

Options:

A. $-\frac{5}{8}$

B. $-\frac{8}{5}$

C. $\frac{5}{8}$

D. $\frac{8}{5}$

Answer: D

Solution:

Solution:
If a and -1 are the roots of the polynomial, then we get
$f(x) = x^2 + (1 - a)x - a$
$\therefore\ f(1) = 2 - 2a$
and $f(2) = 6 - 3a$
As, $f(1) + f(2) = 0 \Rightarrow 2 - 2a + 6 - 3a = 0 \Rightarrow a = \frac{8}{5}$
Therefore, the other root is $\frac{8}{5}$

Question235

If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to : [2018]

Options:

A. 0

B. 1

C. 2

D. -1

Answer: B

Solution:

Solution:
α, β are roots of $x^2 - x + 1 = 0$
$\therefore\ \alpha = -\omega$ and $\beta = -\omega^2$
where ω is cube root of unity
$\therefore\ \alpha^{101} + \beta^{107} = (-\omega)^{101} + (-\omega)^{107}$
$= -[\omega^2 + \omega] = -[-1] = 1$

Question236

If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2-\lambda)x + (10-\lambda) = 0$ is minimum, then the magnitude of the difference of the roots of this equation is
[Online April 15, 2018]

Options:

A. 20

B. $2\sqrt{5}$

C. $2\sqrt{7}$

D. $4\sqrt{2}$

Answer: B

Solution:

Solution:
Let, the roots of the equation, $x^2 + (2-\lambda)x + (10-\lambda) = 0$ are α and β
Also roots of the given equation are
$\frac{\lambda - 2 \pm \sqrt{4 - 4\lambda + \lambda^2 - 40 + 4\lambda}}{2} = \frac{\lambda - 2 \pm \sqrt{\lambda^2 - 36}}{2}$
The magnitude of the difference of the roots is $|\sqrt{\lambda^2 - 36}$
So, $\alpha^3 + \beta^3 = \frac{(\lambda-2)^3}{4} + \frac{3(\lambda-2)(\lambda^2-36)}{4}$
$= \frac{(\lambda-2)(4\lambda^2 - 4\lambda - 104)}{4} = (\lambda-2)(\lambda^2 - \lambda - 26) = f(\lambda)$
As $f(\lambda)$ attains its minimum value at $\lambda = 4$
Therefore, the magintude of the difference of the roots is
$|i\sqrt{20}| = 2\sqrt{5}$

Question237

If $|z - 3 + 2i| \leq 4$ then the difference between the greatest value and the least value of $|z|$ is
[Online April 15, 2018]

Options:

A. $\sqrt{13}$

B. $2\sqrt{13}$

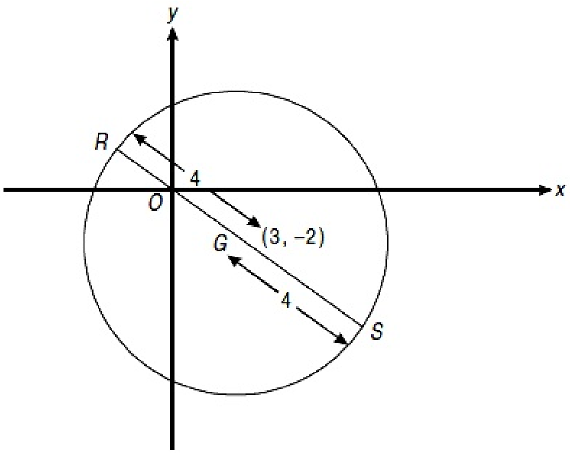
C. 8

D. $4 + \sqrt{13}$

Answer: B

Solution:

Solution:
$|z - (3 - 2i)| \leq 4$ represents a circle whose centre is (3,-2) and radius $= 4$
$|z| = |z - 0|$ represents the distance of point z ' from origin (0,0)



Suppose RS is the normal of the circle passing through origin ' O ' and G is its center (3,-2) .
Here, OR is the least distance and OS is the greatest distance
$OR = RG - OG$ and $OS = OG + GS$
As, $RG = GS = 4$
$OG = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$
From (i), $OR = 4 - \sqrt{13}$ and $OS = 4 + \sqrt{13}$
So, required difference $= (4 + \sqrt{13}) - (4 - \sqrt{13})$
$= \sqrt{13} + \sqrt{13} = 2\sqrt{13}$

Question238

If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + \ldots\ldots + (x + \overline{n-1})(x+n) = 10n$ has two consecutive integral solutions, then n is equal to: [2017]

Options:

A. 11

B. 12

C. 9

D. 10

Answer: A

Solution:

Solution:

We have, $\sum_{r=1}^{n} (x + r - 1)(x + r) = 10n$

$\sum_{r=1}^{n} (x^2 + xr + (r-1)x + r^2 - r) = 10n$

$\Rightarrow \sum_{r=1}^{n} (x^2 + (2r-1)x + r(r-1)) = 10n$

$\Rightarrow nx^2 + \{1 + 3 + 5 + \ldots. + (2n-1)\}x$
$+ \{1.2 + 2.3 + \ldots. + (n-1)n\} = 10n$

$\Rightarrow nx^2 + n^2x + \dfrac{(n-1)n(n+1)}{3} = 10n$

$\Rightarrow x^2 + nx + \dfrac{n^2 - 31}{3} = 0$

Let α and $\alpha + 1$ be its two solutions
($\because$ it has two consequtive integral solutions)

$\Rightarrow\ \alpha + (\alpha + 1) = -n$

$\Rightarrow\ \alpha = \dfrac{-n-1}{2}$ ....(i)

Also $\alpha(\alpha + 1) = \dfrac{n^2 - 31}{3}$ ....(ii)

Putting value of (i) in (ii), we get

$-\left(\dfrac{n+1}{2}\right)\left(\dfrac{1-n}{2}\right) = \dfrac{n^2-31}{3}$

$\Rightarrow n^2 = 121 \Rightarrow n = 11$

Question239

The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is: [Online April 9, 2017]

Options:

A. 16

B. 14

C. -4

D. -5

Answer: C

Solution:

Solution:

$(x-1)(x^2 + 5x - 50) = 0$

$\Rightarrow\ (x-1)(x+10)(x-5) = 0$

$\Rightarrow\ x = 1, 5, -10$

Sum $= -4$

Question240

Let p(x) be a quadratic polynomial such that $p(0) = 1$. If p(x) leaves remainder 4 when divided by $x - 1$ and it leaves remainder 6 when divided by $x + 1$; then [Online April 8, 2017]

Options:

A. $p(b) = 11$

B. $p(b) = 19$

C. $p(-2) = 19$

D. $p(-2) = 11$

Answer: C

Solution:

Solution:
Let $p(x) = ax^2 + bx + c$
$\because p(0) = 1 \Rightarrow c = 1$
Also, $p(1) = 4$ & $p(-1) = 6$
$\Rightarrow a + b + 1 = 4$ & $a - b + 1 = 6$
$\Rightarrow a + b = 3$ & $a - b = 5$
$\Rightarrow a = 4$ & $b = -1$
$p(x) = 4x^2 - x + 1$
$p(b) = 16 - 2 + 1 = 15$
$p(-2) = 16 + 2 + 1 = 19$

Question241

A value of θ for which $\frac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$ is purely imaginary, is:
[2016]

Options:

A. $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

B. $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{6}$

Answer: B

Solution:

Solution:
Rationalizing the given expression $\frac{(2 + 3i\sin\theta)(1 + 2i\sin\theta)}{1 + 4\sin^2\theta}$
For the given expression to be purely imaginary, real part of the above expression should be equal to zero.
$\Rightarrow \frac{2 - 6\sin^2\theta}{1 + 4\sin^2\theta} = 0 \quad \Rightarrow \sin^2\theta = \frac{1}{3}$
$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{3}} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Question242

The point represented by 2 + i in the Argand plane moves 1 unit eastwards, then 2 units northwards and finally from there $2\sqrt{2}$ units in the south-westwards direction. Then its new position in the Argand plane is at the point represented by :
[Online April 9, 2016]

Options:

A. $1 + i$

B. $2 + 2i$

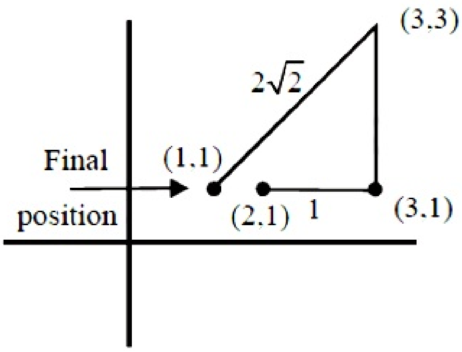
C. $-2 - 2i$

D. $-1 - i$

Answer: A

Solution:

Solution:



So new position is at the point $1 + i$

Question243

The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is: [2016]

Options:

A. 6

B. 5

C. 3

D. -4

Answer: C

Solution:

$(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$
Case I
$x^2 - 5x + 5 = 1$ and $x^2 + 4x - 60$ can be any real number $\Rightarrow x = 1, 4$
Case II
$x^2 - 5x + 5 = -1$ and $x^2 + 4x - 60$ has to be an even number
$\Rightarrow x = 2, 3$
where 3 is rejected because for $x = 3$,
$x^2 + 4x - 60$ is odd
Case III
$x^2 - 5x + 5$ can be any real number and

$x^2 + 4x - 60 = 0$
$\Rightarrow x = -10, 6$
$\Rightarrow$ Sum of all values of x
$= -10 + 6 + 2 + 1 + 4 = 3$

Question244

If x is a solution of the equation, $\sqrt{2x+1} - \sqrt{2x-1} = 1$, $\left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2 - 1}$ is equal to:
[Online April 10, 2016]

Options:

A. $\frac{3}{4}$

B. $\frac{1}{2}$

C. $2\sqrt{2}$

D. 2

Answer: A

Solution:

Solution:
$\sqrt{2x+1} - \sqrt{2x-1} = 1$
$\Rightarrow\ 2x + 1 + 2x - 1 - 2\sqrt{4x^2 - 1} = 1$
$\Rightarrow\ 4x - 1 = 2\sqrt{4x^2 - 1}$
$\Rightarrow\ 16x^2 - 8x + 1 = 16x^2 - 4$
$\Rightarrow\ 8x = 5$
$\Rightarrow\ x = \frac{5}{8}$ which satisfies equation (i)
So, $\sqrt{4x^2 - 1} = \frac{3}{4}$

Question245

If the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have a common root different from -1, then $|b|$ is equal to :
[Online April 9, 2016]

Options:

A. 2

B. 3

C. $\sqrt{3}$

D. $\sqrt{2}$

Answer: C

Solution:

Solution:
$x^2 + bx - 1 = 0$ common root
$x^2 + x + b = 0$
$x = \dfrac{b+1}{b-1}$
Put $x = \dfrac{b+1}{b-1}$ in equation
$\left(\dfrac{b+1}{b-1}\right)^2 + \left(\dfrac{b+1}{b-1}\right) + b = 0$
Put $x = \dfrac{b+1}{b-1}$ in equation
$\left(\dfrac{b+1}{b-1}\right)^2 + \left(\dfrac{b+1}{b-1}\right) + b = 0$
$(b+1)^2 + (b+1)(b-1) + b(b-1)^2 = 0$
$b^2 + 1 + 2b + b^2 - 1 + b(b^2 - 2b + 1) = 0$
$2b^2 + 2b + b^3 - 2b^2 + b = 0$
$b^3 + 3b = 0$
$b(b^2 + 3) = 0$
$b^2 = -3$
$b = \pm\sqrt{3}i$
$|b| = \sqrt{3}$

Question246

If z is a non-real complex number, then the minimum value of $\frac{\operatorname{l\,m}z^5}{(\operatorname{l\,m}z)^5}$ is : [Online April 11, 2015]

Options:

A. -1

B. -4

C. -2

D. -5

Answer: B

Solution:

Solution:
Let $z = re^{i\theta}$
Consider $\dfrac{\operatorname{I\,m}z^5}{(\operatorname{I\,m}z)^5} = \dfrac{r^5(\sin 5\theta)}{r^5(\sin\theta)^5}$
$(\because e^{i\theta} = \cos\theta + i\sin\theta)$
$= \dfrac{\sin 5\theta}{\sin^5\theta} = \dfrac{16\sin^5\theta - 20\sin^3\theta + 5\sin\theta}{\sin^5\theta}$
$= \dfrac{16\sin^5\theta}{\sin^5\theta} - \dfrac{20\sin^3\theta}{\sin^5\theta} + \dfrac{5\sin\theta}{\sin^5\theta}$
$= 5\operatorname{cosec}^4\theta - 20\operatorname{cosec}^2\theta + 16$
minimum value of $\dfrac{\operatorname{I\,m}z^5}{(\operatorname{I\,m}z)^5}$ is -4

Question247

A complex number z is said to be unimodular if |z| = 1.
Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a:
[2015]

Options:

A. circle of radius 2.

B. circle of radius $\sqrt{2}$.

C. straight line parallel to x -axis

D. straight line parallel to y-axis.

Answer: A

Solution:

Solution:

$\left| \frac{z_1 - 2z_2}{2 - z_1\bar{z}_2} \right| = 1$

$\Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$

$\Rightarrow (z_1 - 2z_2)(z_1 - 2z_2) = (2 - z_1\bar{z}_2)\overline{(2 - z_1\bar{z}_2)}$

$\Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$

$\Rightarrow (z_1\bar{z}_1) - 2z_1\bar{z}_2 - 2\bar{z}_1 z_2 + 4z_2\bar{z}_2$

$= 4 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2 + z_1\bar{z}_1 z_2\bar{z}_2$

$\Rightarrow |z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2 |z_2|^2$

$\Rightarrow |z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2 |z_2|^2 = 0$

$(|z_1|^2 - 4)(1 - |z_2|^2) = 0$

$\because |z_2| \neq 1$

$\therefore |z_1|^2 = 4$

$\Rightarrow |z_1| = 2$

$\Rightarrow$ Point z_1 lies on circle of radius 2

Question248

Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to:
[2015]

Options:

A. 3

B. -3

C. 6

D. -6

Answer: A

Solution:

Solution:

$\alpha, \beta = \dfrac{6 \pm \sqrt{36+8}}{2} = 3 \pm \sqrt{11}$

$\alpha = 3 + \sqrt{11}, \beta = 3 - \sqrt{11}$

$\therefore a_n = (3+\sqrt{11})^n - (3-\sqrt{11})^n \dfrac{a_{10} - 2a_8}{2a_9}$

$= \dfrac{(3+\sqrt{11})^{10} - (3-\sqrt{11})^{10} - 2(3+\sqrt{11})^8 + 2(3-\sqrt{11})^8}{2[(3+\sqrt{11})^9 - (3-\sqrt{11})^9]}$

$= \dfrac{(3+\sqrt{11})^8[(3+\sqrt{11})^2 - 2] + (3-\sqrt{11})^8[2 - (3-\sqrt{11})^2]}{2[(3+\sqrt{11})^9 - (3-\sqrt{11})^9]}$

$= \dfrac{(3+\sqrt{11})^8(9 + 11 + 6\sqrt{11} - 2) + (3-\sqrt{11})^8(2 - 9 - 11 + 6\sqrt{11})}{2[(3+\sqrt{11})^9 - (3-\sqrt{11})^9]}$

$= \dfrac{6(3+\sqrt{11})^9 - 6(3-\sqrt{11})^9}{2[(3+\sqrt{11})^9 - (3-\sqrt{11})^9]} = \dfrac{6}{2} = 3$

Question249

If the two roots of the equation, $(a-1)(x^4 + x^2 + 1) + (a+1)(x^2 + x + 1)^2 = 0$ are real and distinct, then the set of all values of 'a' is
[Online April 11, 2015]
:

Options:

A. $\left(0, \frac{1}{2}\right)$

B. $\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$

C. $\left(-\frac{1}{2}, 0\right)$

D. $(-\infty, -2) \cup (2, \infty)$

Answer: B

Solution:

Solution:

$(a-1)(x^4 + x^2 + 1) + (a+1)(x^2 + x + 1)^2 = 0$

$\Rightarrow (a-1)(x^2 + x + 1)(x^2 - x + 1) + (a+1)(x^2 + x + 1)^2 = 0$

$\Rightarrow (x^2 + x + 1)[(a-1)(x^2 - x + 1) + (a+1)(x^2 + x + 1)] = 0$

$\Rightarrow (x^2 + x + 1)(ax^2 + x + a) = 0$

For roots to be distinct and real, $a \neq 0$ and $1 - 4a^2 > 0$

$\Rightarrow a \neq 0$ and $a^2 < \dfrac{1}{4}$

$\Rightarrow a \in \left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right)$

Question250

If $2 + 3i$ is one of the roots of the equation $2x^3 - 9x^2 + kx - 13 = 0$, $k \in R$, then the real root of this equation:
[Online April 10, 2015]

Options:

A. exists and is equal to $-\frac{1}{2}$.

B. exists and is equal to $\frac{1}{2}$

C. exists and is equal to 1 .

D. does not exist.

Answer: B

Solution:

Solution:
$\alpha = 2 + 3i; \beta = 2 - 3i, \gamma = ?$
$\alpha\beta\gamma = \frac{13}{2}\left[\text{ since product of roots } = \frac{d}{a}\right]$
$\Rightarrow (4 + 9)\gamma = \frac{13}{2} \Rightarrow \gamma = \frac{1}{2}$

Question251

If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$:
[2014]

Options:

A. is strictly greater than $\frac{5}{2}$

B. is strictly greater than $\frac{3}{2}$ but less than $\frac{5}{2}$

C. is equal to $\frac{5}{2}$

D. lie in the interval (1,2)

Answer: D

Solution:

Solution:
We know minimum value of $|Z_1 + Z_2|$ is

$|Z_1| - |Z_2|$. Thus minimum value of $\left|Z + \frac{1}{2}\right|$ is $|Z| - \frac{1}{2}|$

$\leq \left|Z + \frac{1}{2}\right| \leq \left|Z\right| + \frac{1}{2}$

Since, $|Z| \geq 2$ therefore

$2 - \frac{1}{2} < \left|Z + \frac{1}{2}\right| < 2 + \frac{1}{2}$

$\Rightarrow \frac{3}{2} < \left|Z + \frac{1}{2}\right| < \frac{5}{2}$

Question252

For all complex numbers z of the form $1 + i\alpha$, $\alpha \in R$, if $z^2 = x + iy$, then [Online April 19, 2014]

Options:

A. $y^2 - 4x + 2 = 0$

B. $y^2 + 4x - 4 = 0$

C. $y^2 - 4x - 4 = 0$

D. $y^2 + 4x + 2 = 0$

Answer: B

Solution:

Solution:

Let $z = 1 + i\alpha$, $\alpha \in R$

$z^2 = (1 + i\alpha)(1 + i\alpha)$

$x + iy = (1 + 2i\alpha - \alpha^2)$

On comparing real and imaginary parts, we get

$x = 1 - \alpha^2$, $y = 2\alpha$

Now, consider option (b), which is

$y^2 + 4x - 4 = 0$

LHS : $y^2 + 4x - 4 = (2\alpha)^2 + 4(1 - \alpha^2) - 4$

$= 4\alpha^2 + 4 - 4\alpha^2 - 4$

$= 0 = R.H.S$

Hence, $y^2 + 4x - 4 = 0$

Question253

Let $z \neq -i$ be any complex number such that $\frac{z - i}{z + i}$ is a purely imaginary number. Then $z + \frac{1}{z}$ is: [Online April 12, 2014]

Options:

A. zero

B. anynon-zero real number other than 1 .

C. any non-zero real number.

D. a purely imaginary number.

Answer: C

Solution:

Solution:
Let $z = x + iy$

$\frac{z - i}{z + i}$ is purely imaginary means its real part is zero.

$$\frac{x + iy - i}{x + iy + i} = \frac{x + i(y - 1)}{x + i(y + 1)} \times \frac{x - i(y + 1)}{x - i(y + 1)}$$

$$= \frac{x^2 - 2ix(y + 1) + xi(y - 1) + y^2 - 1}{x^2 + (y + 1)^2}$$

$$= \frac{x^2 + y^2 - 1}{x^2 + (y + 1)^2} - \frac{2xi}{x^2 + (y + 1)^2}$$

for pure imaginary, we have

$$\frac{x^2 + y^2 - 1}{x^2 + (y + 1)^2} = 0$$

$\Rightarrow x^2 + y^2 = 1$

$\Rightarrow (x + iy)(x - iy) = 1$

$\Rightarrow x + iy = \frac{1}{x - iy} = z$

and $\frac{1}{z} = x - iy$

$z + \frac{1}{z} = (x + iy) + (x - iy) = 2x$

$\left(z + \frac{1}{z}\right)$ is any non-zero real number

Question254

If z_1, z_2 and z_3, z_4 are 2 pairs of complex conjugate numbers, then $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$ equals: [Online April 11, 2014]

Options:

A. 0

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{2}$

D. π

Answer: A

Solution:

Solution:

Consider $\arg\left(\frac{z_1}{z_4}\right) + \arg\left(\frac{z_2}{z_3}\right)$

$= \arg(z_1) - \arg(z_4) + \arg(z_2) - \arg(z_3)$
$= (\arg(z_1) + \arg(z_2)) - (\arg(z_3) + \arg(z_4))$

given $\left(\begin{array}{l} z_2 = \bar{z}_1 \\ z_4 = \bar{z}_3 \end{array} \right)$

$= (\arg(z_1) + \arg(\bar{z}_1)) - (\arg(z_3) + \arg(\bar{z}_3))$

$\left\{ \begin{array}{l} \text{also } (\arg(\bar{z}_1) = -\arg(z_1)) \\ \arg(\bar{z}_3) = -\arg(z_3) \end{array} \right\}$

$= (\arg(z_1) - \arg(z_1)) - (\arg(z_3) - \arg(z_3))$
$= 0 - 0 = 0$

Question255

Let $w(\mathrm{Im}\, w \neq 0)$ be a complex number. Then the set of all complex number z satisfying the equation $w - \overline{wz} = k(1 - z)$, for some real number k, is
[Online April 9, 2014]

Options:

A. $\{z : |z| = 1\}$

B. $\{z : z = \bar{z}\}$

C. $\{z : z \neq 1\}$

D. $\{z : |z| = 1, z \neq 1\}$

Answer: D

Solution:

Solution:
Consider the equation
$w - \bar{w}z = k(1 - z), k \in R$
Clearly $z \neq 1$ and $\frac{w - \bar{w}z}{1 - z}$ is purely real
$\therefore \overline{\left(\frac{w - \bar{w}z}{1 - z}\right)} = \frac{w - \bar{w}z}{1 - z}$
$\Rightarrow \frac{\bar{w} - w\bar{z}}{1 - \bar{z}} = \frac{w - \bar{w}z}{1 - z}$
$\Rightarrow \bar{w} - \bar{w}z - w\bar{z} + wz\bar{z} = w - w\bar{z} - \bar{w}z + \bar{w}z\bar{z}$
$\Rightarrow \bar{w} + w|z|^2 = w + \bar{w}|z|^2$
$\Rightarrow (w - \bar{w})(|z|^2) = w - \bar{w}$
$\Rightarrow |z|^2 = 1 \ (\because \mathrm{Im}\, w \neq 0)$
$\Rightarrow |z| = 1$ and $z \neq 1$
$\therefore$ The required set is $\{z : |z| = 1, z \neq 1\}$

Question256

If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\leq x$) has no integral solution, then all possible values of a lie in the interval:
[2014]

Options:

A. (-2,-1)

B. $(-\infty, -2) \cup (2, \infty)$

C. (-1,0)∪(0, 1)

D. (1,2)

Answer: C

Solution:

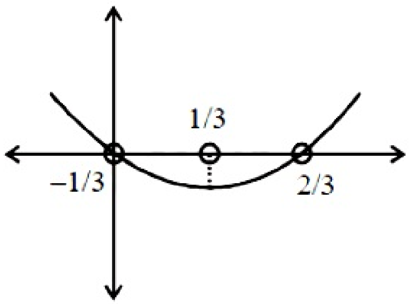
Solution:

Consider $-3(x-[x])^2 + 2[x-[x]) + a^2 = 0$

$\Rightarrow\ 3\{x\}^2 - 2\{x\} - a^2 = 0\ \ (\because x - [x] = \{x\})$

$\Rightarrow\ 3\left(\{x\}^2 - \frac{2}{3}\{x\}\right) = a^2, a \neq 0$

$\Rightarrow\ a^2 = 3\{x\}\left(\{x\} - \frac{2}{3}\right)$



Now, $\{x\} \in (0, 1)$ and $\frac{-2}{3} \leq a^2 < 1$ (by graph)

Since, x is not an integer

$\therefore\ a \in (-1, 1) - \{0\}$

$\Rightarrow a \in (-1, 0) \cup (0, 1)$

Question257

The equation $\sqrt{3x^2 + x + 5} = x - 3$, where x is real, has; [Online April 19, 2014]

Options:

A. no solution

B. exactly one solution

C. exactly two solution

D. exactly four solution

Answer: A

Solution:

Solution:

Consider $\sqrt{3x^2 + x + 5} = x - 3$

Squaring both the sides, we get
$3x^2 + x + 5 = (x - 3)^2$
$\Rightarrow 3x^2 + x + 5 = x^2 + 9 - 6x$
$\Rightarrow 2x^2 + 7x - 4 = 0$
$\Rightarrow 2x^2 + 8x - x - 4 = 0$
$\Rightarrow 2x(x + 4) - 1(x + 4) = 0$
$\Rightarrow x = \frac{1}{2}$ or $x = -4$
For $x = \frac{1}{2}$ and $x = -4$
L.H.S. ≠ R.H.S. of equation, $\sqrt{3x^2 + x + 5} = x - 3$
Also, for every $x \in R$, $LH\,S \neq RH\,S$ of the given equation.
∴ Given equation has no solution.

Question258

The sum of the roots of the equation, $x^2 + |\,2x - 3\,| - 4 = 0$, is: [Online April 12, 2014]

Options:

A. 2

B. -2

C. $\sqrt{2}$

D. $-\sqrt{2}$

Answer: C

Solution:

Solution:
$x^2 + |\,2x - 3\,| - 4 = 0$

$$|2x - 3| = \begin{cases} (2x - 3) & \text{if} \quad x > \frac{3}{2} \\ -(2x - 3) & \text{if} \quad x < \frac{3}{2}. \end{cases}$$

for $x > \frac{3}{2}$, $x^2 + 2x - 3 - 4 = 0$
$x^2 + 2x - 7 = 0$
$x = \frac{-2 \pm \sqrt{4 + 28}}{2} = \frac{-2 \pm 4\sqrt{2}}{2} = -1 \pm 2\sqrt{2}$
Here $x = 2\sqrt{2} - 1$ $\left\{ 2\sqrt{2} - 1 < \frac{3}{2} \right\}$
for $x < \frac{3}{2}$
$x^2 - 2x + 3 - 4 = 0$
$\Rightarrow x^2 - 2x - 1 = 0$
$\Rightarrow x = \frac{2 \pm \sqrt{4 + 4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$
Here $x = 1 - \sqrt{2}$ $\left\{ (1 - \sqrt{2}) < \frac{3}{2} \right\}$
Sum of roots : $(2\sqrt{2} - 1) + (1 - \sqrt{2}) = \sqrt{2}$

Question259

If α and β are roots of the equation, $x^2 - 4\sqrt{2}kx + 2e^{4\ln k} - 1 = 0$ for some k, and $\alpha^2 + \beta^2 = 66$, then $\alpha^3 + \beta^3$ is equal to:
[Online April 11, 2014]

Options:

A. $248\sqrt{2}$

B. $280\sqrt{2}$

C. $-32\sqrt{2}$

D. $-280\sqrt{2}$

Answer: D

Solution:

Solution:
$x^2 - 4\sqrt{2}kx + 2e^{4\ln k} - 1 = 0$
or, $x^2 - 4\sqrt{2}kx + 2k^4 - 1 = 0$
$\alpha + \beta = 4\sqrt{2}k$ and $\alpha \cdot \beta = 2k^4 - 1$
Squaring both sides, we get
$(\alpha + \beta)^2 = (4\sqrt{2}k)^2 \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 32k^2$
$66 + 2\alpha\beta = 32k^2$
$66 + 2(2k^4 - 1) = 32k^2$
$66 + 4k^4 - 2 = 32k^2 \Rightarrow 4k^4 - 32k^2 + 64 = 0$
or, $k^4 - 8k^2 + 16 = 0 \Rightarrow (k^2)^2 - 8k^2 + 16 = 0$
$\Rightarrow (k^2 - 4)(k^2 - 4) = 0 \Rightarrow k^2 = 4, k^2 = 4$
$\Rightarrow k = \pm 2$
Now, $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$
$\therefore \alpha^3 + \beta^3 = (4\sqrt{2}k)[66 - (2k^4 - 1)]$
Putting $k = -2$, ($k = +2$ cannot be taken because it does not satisfy the above equation)
$\therefore \alpha^3 + \beta^3 = (4\sqrt{2}(-2))[66 - 2(-2)^4 - 1]$
$\alpha^3 + \beta^3 = (-8\sqrt{2})(66 - 32 + 1) = (-8\sqrt{2})$
$\therefore \alpha^3 + \beta^3 = -280\sqrt{2}$

Question260

If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation, $ax^2 + bx + 1 = 0 (a \neq 0, a, b, \in R)$, then the equation, $x(x + b^3) + (a^3 - 3abx) = 0$ as roots :
[Online April 9, 2014]

Options:

A. $\alpha^{3/2}$ and $\beta^{3/2}$

B. $\alpha\beta^{1/2}$ and $\alpha^{1/2}\beta$

C. $\sqrt{\alpha\beta}$ and $\alpha\beta$

D. $\alpha^{-\frac{3}{2}}$ and β^{-3} ?

Answer: A

Solution:

Solution:

Let $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ be the roots of $ax^2 + bx + 1 = 0$

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \left(\frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}}\right) = -\frac{b}{a}$

$\frac{1}{\sqrt{\alpha}\sqrt{\beta}} = \frac{1}{a} \Rightarrow a = \sqrt{\alpha\beta}$

$b = -(\sqrt{\alpha} + \sqrt{\beta})$

$x(x + b^3) + (a^3 - 3abx) = 0$

$\Rightarrow\ x^2 + (b^3 - 3ab)x + a^3 = 0$

Putting values of a and b, we get

$x^2 + [(-\sqrt{\alpha} + \sqrt{\beta})^3 + 3(\sqrt{\alpha\beta})(\sqrt{\alpha} + \sqrt{\beta})] + (\alpha\beta)^{3/2} = 0$

$\Rightarrow x^2 - [\alpha^{3/2} + \beta^{3/2} + 3\sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta}) - 3\sqrt{\alpha\beta}(\sqrt{\alpha} + \sqrt{\beta})]x + (\alpha\beta)^{3/2} = 0$

$\Rightarrow\ x^2 - (\alpha^{3/2} + \beta^{3/2})x + \alpha^{3/2}\beta^{3/2} = 0$

Roots of this equation are $\alpha^{3/2}, \beta^{3/2}$

Question261

If non-zero real numbers b and c are such that $\min f(x) > \max g(x)$, where $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2 (x \in R)$ then $\left|\frac{c}{b}\right|$ lies in the interval:
[Online April 19, 2014]

Options:

A. $\left(0, \frac{1}{2}\right)$

B. $\left[\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

C. $\left[\frac{1}{\sqrt{2}}, \sqrt{2}\right]$

D. $(\sqrt{2}, \infty)$

Answer: D

Solution:

Solution:

We have

$f(x) = x^2 + 2bx + 2c^2$

and $g(x) = -x^2 - 2cx + b^2, (x \in R)$

$\Rightarrow f(x) = (x + b)^2 + 2c^2 - b^2$

and $g(x) = -(x + c)^2 + b^2 + c^2$

Now, $f_{min} = 2c^2 - b^2$ and $g_{max} = b^2 + c^2$

Given : $\min f(x) > \max g(x)$

$\Rightarrow 2c^2 - b^2 > b^2 + c^2$

$\Rightarrow c^2 > 2b^2$

$\Rightarrow |c| > |b|\sqrt{2}$

$\Rightarrow \frac{|c|}{|b|} > \sqrt{2} \Rightarrow \left|\frac{c}{b}\right| > \sqrt{2}$

$\Rightarrow \left| \frac{c}{b} \right| \in (\sqrt{2}, \infty)$

Question262

If equations $ax^2 + bx + c = 0 (a, b, c \in R, a \neq 0)$ and $2x^2 + 3x + 4 = 0$ have a common root, then a : b : c equals:
[Online April 9, 2014]

Options:

A. 1: 2: 3

B. 2: 3: 4

C. 4: 3: 2

D. 3: 2: 1

Answer: B

Solution:

Solution:
Let α, β be the common roots of both the equations.
For first equation $ax^2 + bx + c = 0$
we have
$\alpha + \beta = \frac{-b}{a}$ . . .(i)
$\alpha \cdot \beta = \frac{c}{a}$ . . .(ii)
For second equation $2x^2 + 3x + 4 = 0$
we have
$\alpha + \beta = \frac{-3}{2}$ . . .(iii)
$\alpha \cdot \beta = \frac{2}{1}$. . . (iv)
Now, from (i) & (iii) & from (ii) & (iv)
$\frac{-b}{a} = \frac{-3}{2} \quad \frac{c}{a} = \frac{2}{1}$
$\frac{b}{a} = \frac{3/2}{1}$
Therefore on comparing we get $a = 1, b = \frac{3}{2}$ & $c = 2$
putting these values in first equation, we get
$x^2 + \frac{3}{2}x + 2 = 0$ or $2x^2 + 3x + 4 = 0$
from this, we get $a = 2, b = 3; c = 4$
or $a : b : c = 2 : 3 : 4$

Question263

If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+z}\right)$ equals:
[2013]

Options:

A. $-\theta$

B. $\frac{\pi}{2}-\theta$

C. θ

D. $\pi-\theta$

Answer: C

Solution:

Solution:
Given $|z| = 1, \arg z = \theta$
$\Rightarrow \bar{z} = \frac{1}{z}$
$\therefore \arg\left(\frac{1+z}{1+\bar{z}}\right) = \arg\left(\frac{1+z}{1+\frac{1}{z}}\right) = \arg(z) = \theta$

Question264

Let z satisfy $|z| = 1$ and $z = 1-\bar{z}$.
Statement 1 : z is a real number.
Statement 2: Principal argument of z is $\frac{\pi}{3}$
[Online April 25, 2013]

Options:

A. Statement 1 is true Statement 2 is true; Statement 2 is a correct explanation for Statement 1
.

B. Statement 1 is false; Statement 2 is true

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is true; Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.

Answer: B

Solution:

Solution:
Let $z = x + iy, \bar{z} = x - iy$
Now, $z = 1 - \bar{z}$
$\Rightarrow x + iy = 1 - (x - iy)$
$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$
Now, $|z| = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow y^2 = 1 - x^2$
$\Rightarrow y = \pm\frac{\sqrt{3}}{2}$
N ow, $\tan\theta = \frac{y}{x}$ (θ is the argument)

$= \frac{\sqrt{3}}{2} \div \frac{1}{2}$ (+ ve since only principal argument)
$= \sqrt{3}$
$\Rightarrow \theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$
Hence, z is not a real number
So, statement- 1 is false and 2 is true.

Question265

Let $a = Im\left(\frac{1+z^2}{2iz}\right)$, where z is any non-zero complex number. The set $A = \{a : |z| = 1 \text{ and } z \neq \pm 1\}$ is equal to:
[Online April 23, 2013]

Options:

A. (-1,1)

B. [-1,1]

C. [0,1)

D. (-1,0]

Answer: A

Solution:

Solution:
Let $z = x + iy \Rightarrow z^2 = x^2 - y^2 + 2ixy$
Now, $\frac{1+z^2}{2iz} = \frac{1 + x^2 - y^2 + 2ixy}{2i(x+iy)} = \frac{(x^2 - y^2 + 1) + 2ixy}{2ix - 2y}$
$= \frac{(x^2 - y^2 + 1) + 2ixy}{-2y + 2ix} \times \frac{-2y - 2ix}{-2y - 2ix}$
$= \frac{y(x^2 + y^2 - 1) + x(x^2 + y^2 + 1)i}{2(x^2 + y^2)}$
$a = \frac{x(x^2 + y^2 + 1)}{2(x^2 + y^2)}$
Since, $|z| = 1 \Rightarrow \sqrt{x^2 + y^2} = 1 \Rightarrow x^2 + y^2 = 1$
$\therefore a = \frac{x(1+1)}{2 \times 1} = x$
Also $z \neq 1 \Rightarrow x + iy \neq 1$
$\therefore A = (-1, 1)$

Question266

If $Z_1 \neq 0$ and Z_2 be two complex numbers such that $\frac{Z_2}{Z_1}$ is a purely imaginary number, then $\left|\frac{2Z_1 + 3Z_2}{2Z_1 - 3Z_2}\right|$ is equal to:
[Online April 9, 2013]

Options:

A. 2

B. 5

C. 3

D. 1

Answer: D

Solution:

Solution:

Let $z_1 = 1 + i$ and $z_2 = 1 - i$

$$\frac{z_2}{z_1} = \frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = -i$$

$$\frac{2z_1 + 3z_2}{2z_1 - 3z_2} = \frac{2 + 3\left(\frac{z_2}{z_1}\right)}{2 - 3\left(\frac{z_2}{z_1}\right)} = \frac{2-3i}{2+3i}$$

$$\left|\frac{2z_1 + 3z_2}{2z_1 - 3z_2}\right| = \left|\frac{2-3i}{2+3i}\right| = \left|\frac{2-3i}{2+3i}\right| \quad \left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$$

$$= \frac{\sqrt{4+9}}{\sqrt{4+9}} = 1$$

Question267

If p and q are non-zero real numbers and $\alpha^3 + \beta^3 = -p$, $\alpha\beta = q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta}$, $\frac{\beta^2}{\alpha}$ is: [Online April 25, 2013]

Options:

A. $px^2 - qx + p^2 = 0$

B. $qx^2 + px + q^2 = 0$

C. $px^2 + qx + p^2 = 0$

D. $qx^2 - px + q^2 = 0$

Answer: B

Solution:

Solution:

Given $\alpha^3 + \beta^3 = -p$ and $\alpha\beta = q$

Let $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ be the root of required quadratic equation.

So, $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{\alpha^3 + \beta^3}{\alpha\beta} = \frac{-p}{q}$

and $\frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = \alpha\beta = q$

Hence, required quadratic equation is

$x^2 - \left(\frac{-p}{q}\right)x + q = 0$

$\Rightarrow x^2 + \frac{p}{q}x + q = 0 \Rightarrow qx^2 + px + q^2 = 0$

Question268

If α and β are roots of the equation $x^2 + px + \frac{3p}{4} = 0$ such that $|\alpha - \beta| = \sqrt{10}$, then p belongs to the set :
[Online April 22, 2013]

Options:

A. $\{2, -5\}$

B. $\{-3, 2\}$

C. $\{-2, 5\}$

D. $\{3, -5\}$

Answer: C

Solution:

Solution:
Given quadratic eqn. is
$x^2 + px + \frac{3p}{4} = 0$
So, $\alpha + \beta = -p$, $\alpha\beta = \frac{3p}{4}$
Now, given $|\alpha - \beta| = \sqrt{10}$
$\Rightarrow \alpha - \beta = \pm\sqrt{10}$
$\Rightarrow (\alpha - \beta)^2 = 10 \Rightarrow \alpha^2 + \beta^2 - 2\alpha\beta = 10$
$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 10$
$\Rightarrow p^2 - 4 \times \frac{3p}{4} = 10 \Rightarrow p^2 - 3p - 10 = 0$
$\Rightarrow p = -2, 5 \Rightarrow p \in \{-2, 5\}$

Question269

If a complex number z statisfies the equation $z + \sqrt{2}\ | z + 1 |\ + i = 0$, then $|z|$ is equal to :
[Online April 22, 2013]

Options:

A. 2

B. $\sqrt{3}$

C. $\sqrt{5}$

D. 1

Answer: C

Solution:

Solution:
Given equation is
$z + \sqrt{2} \mid z + 1 \mid + i = 0$
put $z = x + iy$ in the given equation.
$(x + iy) + \sqrt{2} \mid x + iy + 1 \mid + i = 0$
$\Rightarrow x + iy + \sqrt{2}\left[\sqrt{(x+1)^2 + y^2}\right] + i = 0$
Now, equating real and imaginary part, we get
$x + \sqrt{2}\sqrt{(x+1)^2 + y^2} = 0$ and
$y + 1 = 0 \Rightarrow y = -1$
$\Rightarrow x + \sqrt{2}\sqrt{(x+1)^2 + (-1)^2} = 0 \quad (\because y = -1)$
$\Rightarrow \sqrt{2}\sqrt{(x+1)^2 + 1} = -x$
$\Rightarrow 2[(x+1)^2 + 1] = x^2$
$\Rightarrow x^2 + 4x + 4 = 0$
$\Rightarrow x = -2$
Thus, $z = -2 + i(-1) \Rightarrow \mid z \mid = \sqrt{5}$

Question270

If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$ a, b, c ∈ R, have a common root, then a : b : c is [2013]

Options:

A. 1: 2: 3

B. 3: 2: 1

C. 1: 3: 2

D. 3: 1: 2

Answer: A

Solution:

Solution:
Given equations are
$x^2 + 2x + 3 = 0 \ \ldots$ (i)
$ax^2 + bx + c = 0 \ \ldots$ (ii)
Roots of equation (i) are imaginary roots in order pair.
According to the question (ii) will also have both roots same as (i).
Thus $\frac{a}{1} = \frac{b}{2} = \frac{c}{3} = \lambda$(say)
$\Rightarrow \ a = \lambda, b = 2\lambda, c = 3\lambda$
Hence, required ratio is 1: 2: 3

Question271

The least integral value α of x such that $\frac{x-5}{x^2+5x-14} > 0$ satisfies:
[Online April 23, 2013]

Options:

A. $\alpha^2 + 3\alpha - 4 = 0$

B. $\alpha^2 - 5\alpha + 4 = 0$

C. $\alpha^2 - 7\alpha + 6 = 0$

D. $\alpha^2 + 5\alpha - 6 = 0$

Answer: A

Solution:

Solution:
$\frac{x-5}{x^2+5x-14} > 0 \Rightarrow x^2 + 5x - 14 < x - 5$
$\Rightarrow x^2 + 4x - 9 < 0$
$\Rightarrow \alpha = -5, -4, -3, -2, -1, 0, 1$
$\alpha = -5$ does not satisfy any of the options
$\alpha = -4$ satisfy the option (a) $\alpha^2 + 3\alpha - 4 = 0$

Question272

The values of ' a ' for which one root of the equation $x^2 - (a+1)x + a^2 + a - 8 = 0$ exceeds 2 and the other is lesser than 2 , are given by :
[Online April 9, 2013]

Options:

A. $3 < a < 10$

B. $a \geq 10$

C. $-2 < a < 3$

D. $a \leq -2$

Answer: C

Solution:

Solution:
$x^2 - (a+1)x + a^2 + a - 8 = 0$
Since roots are different, therefore $D > 0$
$\Rightarrow (a+1)^2 - 4(a^2 + a - 8) > 0$
$\Rightarrow (a-3)(3a+1) < 0$
There are two cases arises
Case I. $a - 3 > 0$ and $3a + 1 < 0$
$\Rightarrow a > 3$ and $a < -\frac{11}{3}$
Hence, no solution in this case
Case II :$a - 3 < 0$ and $3a + 11 > 0$

$\Rightarrow a < 3$ and $a > -\frac{11}{3}$
$\therefore -\frac{11}{3} < a < 3 \Rightarrow -2 < a < 3$

Question273

$|z_1 + z_2|^2 + |z_1 - z_2|^2$ is equal to
[Online May 26, 2012]

Options:

A. $2(|z_1| + |z_2|)$

B. $2(|z_1|^2 + |z_2|^2)$

C. $|z_1||z_2|$

D. $|z_1|^2 + |z_2|^2$

Answer: B

Solution:

Solution:
$|z_1 + z_2|^2 + |z_1 - z_2|^2$
$= |z_1|^2 + |z_2|^2 + 2|z_1||z_2| + |z_1|^2 + |z_2|^2 - 2|z_1||z_2|$
$= 2|z_1|^2 + 2|z_2|^2 = 2[|z_1|^2 + |z_2|^2]$

Question274

Let Z and W be complex numbers such that $|Z| = |W|$, and arg Z denotes the principal argument of Z.
Statement 1: If $\arg Z + \arg W = \pi$, then $Z = -\overline{W}$
Statement 2: $|Z| = |W|$, implies $\arg Z - \arg \overline{W} = \pi$
[Online May 19, 2012]

Options:

A. Statement 1 is true, Statement 2 is false.

B. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1 .

C. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1 .

D. Statement 1 is false, Statement 2 is true.

Answer: A

Solution:

Solution:
Let $|Z| = |W| = r \Rightarrow Z = re^{i\theta}, W = re^{i\varphi}$
where $\theta + \varphi = \pi$
$\therefore \overline{W} = re^{-i\varphi}$
Now, $Z = re^{i(\pi - \varphi)} = re^{i\pi} \times e^{-i\varphi} = -re^{-i\varphi}$
$= -\overline{W}$
Thus, statement- 1 is true but statement- 2 is false.

Question275

Let Z_1 and Z_2 be any two complex number. Statement 1: $|Z_1 - Z_2| \geq |Z_1| - |Z_2|$
Statement 2: $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$
[Online May 7, 2012]

Options:

A. Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation of Statement 1.

B. Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation of Statement 1.

C. Statement 1 is true, Statement 2 is false.

D. Statement 1 is false, Statement 2 is true.

Answer: B

Solution:

Solution:
Statement -1 and 2 both are true. It is fundamental property. But Statement -2 is not correct explanation for Statement -1.

Question276

If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies:
[2012]

Options:

A. either on the real axis or on a circle passing through the origin.

B. on a circle with centre at the origin

C. either on the real axis or on a circle not passing through the origin.

D. on the imaginary axis.

Answer: A

Solution:

Solution:

$\frac{z^2}{z-1} = \frac{\bar{z}^2}{\bar{z}-1}\left[\because \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}\right]$

$\Rightarrow z\bar{z}z - z^2 = z \cdot \bar{z} \cdot \bar{z} - \bar{z}^2$

$\Rightarrow |z|^2 \cdot z - z^2 = |z|^2 \cdot \bar{z} - \bar{z}^2$

$\Rightarrow |z|^2 (z - \bar{z}) - (z - \bar{z})(z + \bar{z}) = 0$

$\Rightarrow (z - \bar{z})(|z|^2 - (z + \bar{z})) = 0$

Either $z - \bar{z} = 0$ or $|z|^2 - (z + \bar{z}) = 0$

Either $z = \bar{z} \Rightarrow$ real axis

or $|z|^2 = z + \bar{z} \Rightarrow z\bar{z} - z - \bar{z} = 0$

represents a circle passing through origin.

Question277

Let p, q, r ∈ R and r > p > 0. If the quadratic equation $px^2 + qx + r = 0$ has two complex roots α and β, then |α|+ |β| is [Online May 19, 2012]

Options:

A. equal tol

B. less than 2 but not equal to 1

C. greater than 2

D. equal to 2

Answer: C

Solution:

Solution:

Given quadratic equation is $px^2 + qx + r = 0$

$D = q^2 - 4pr$

Since α and β are two complex root

$\therefore \beta = \bar{\alpha} \Rightarrow |\beta| = |\bar{\alpha}| \Rightarrow |\beta| = |\alpha| \quad (\because |\bar{\alpha}| = |\alpha|)$

Consider

$|\alpha| + |\beta| = |\alpha| + |\alpha| \quad (\because |\beta| = |\alpha|)$

$= 2|\alpha| > 2.1 = 2 \; (\because |\alpha| > 1)$

Hence, $|\alpha| + |\beta|$ is greater than 2

Question278

If the sum of the square of the roots of the equation $x^2 - (\sin\alpha - 2)x - (1 + \sin\alpha) = 0$ is least, then α is equal to [Online May 12, 2012]

Options:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

Answer: D

Solution:

Solution:
Given equation is
$x^2 - (\sin\alpha - 2)x - (1 + \sin\alpha) = 0$
Let x_1 and x_2 be two roots of quadratic equation
$\therefore x_1 + x_2 = \sin\alpha - 2$ and $x_1 x_2 = -(1 + \sin\alpha)$
$(x_1 + x_2)^2 = (\sin\alpha - 2)^2 = \sin^2\alpha + 4 - 4\sin\alpha$
$\Rightarrow\ {x_1}^2 + {x_2}^2 = \sin^2\alpha + 4 - 4\sin\alpha - 2x_1x_2$
$= \sin^2\alpha + 4 - 4\sin\alpha + 2(1 + \sin\alpha)$
$= \sin^2\alpha - 2\sin\alpha + 6$
Now, By putting
$\alpha = \frac{\pi}{6}, \alpha = \frac{\pi}{4}, \alpha = \frac{\pi}{3}$ and $\alpha = \frac{\pi}{2}$ in (i) one by one
We get least value of ${x_1}^2 + {x_2}^2$ at $\frac{\pi}{2}$
Hence, $\alpha = \frac{\pi}{2}$

Question279

The value of k for which the equation $(k - 2)x^2 + 8x + k + 4 = 0$ has both roots real, distinct and negative is [Online May 7, 2012]

Options:

A. 6

B. 3

C. 4

D. 1

Answer: B

Solution:

Solution:
$(k - 2)x^2 + 8x + k + 4 = 0$
If real roots then,
$8^2 - 4(k - 2)(k + 4) > 0$
$\Rightarrow\ k^2 + 2k - 8 < 16$
$\Rightarrow k^2 + 6k - 4k - 24 < 0$
$\Rightarrow (k + 6)(k - 4) < 0$
$\Rightarrow\ -6 < k < 4$
If both roots are negative
then $\alpha\beta$ is + ve

$\Rightarrow \quad \frac{k+4}{k-2} > 0 \Rightarrow k > -4$
Also, $\frac{k-2}{k+4} > 0 \Rightarrow k > 2$
Roots are real so $-6 < k < 4$
So, 6 and 4 are not correct.
Since, $k > 2$, so 1 is also not correct value of k.
$\therefore k = 3$

Question280

If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals [2011]

Options:

A. (1,1)

B. (1,0)

C. (-1,1)

D. (0,1)

Answer: A

Solution:

Solution:
$(1+\omega)^7 = A + B\omega$
$(-\omega^2)^7 = A + B\omega (\because \omega^{14} = \omega^{12} \cdot \omega^2 = \omega^2)$
$-\omega^2 = A + B\omega$
$1 + \omega = A + B\omega$
$\Rightarrow A = 1, B = 1$

Question281

Let for $a \neq a_1 \neq 0$ $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(b)$ is : [2011 RS]

Options:

A. 3

B. 9

C. 6

D. 18

Answer: D

Solution:

Solution:
$p(x) = 0$
$\Rightarrow\ f(x) = g(x)$
$\Rightarrow\ ax^2 + bx + c = a_1x^2 + b_1x + c_1$
$\Rightarrow\ (a - a_1)x^2 + (b - b_1)x + (c - c_1) = 0$
It has only one solution, $x = -1$
$\Rightarrow\ b - b_1 = a - a_1 + c - c_1\ \ldots(i)$
Sum of roots $\dfrac{-(b - b_1)}{(a - a_1)} = -1 - 1$
$\Rightarrow\ \dfrac{b - b_1}{2(a - a_1)} = 1$ ....(ii)
$\Rightarrow\ b - b_1 = 2(a - a_1)$
Now $p(-2) = 2$
$\Rightarrow f(-2) - g(-2) = 2$
$\Rightarrow\ 4a - 2b + c - 4a_1 + 2b_1 - c_1 = 2$
$\Rightarrow 4(a - a_1) - 2(b - b_1) + (c - c_1) = 2\ldots(iii)$
From equations, (i), (ii) and (iii)
$a - a_1 = c - c_1 = \dfrac{1}{2}(b - b_1) = 2$
Now, $p(2) = f(2) - g(2)$
$= 4(a - a_1) + 2(b - b_1) + (c - c_1)$
$= 8 + 8 + 2 = 18$

Question282

Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4,3) . Rahul made a mistake in writing down coefficient of x to get roots (3,2) . The correct roots of equation are : [2011 RS]

Options:

A. 6,1

B. 4,3

C. -6,-1

D. -4,-3

Answer: A

Solution:

Solution:
Let the correct equation be
$ax^2 + bx + c = 0$
Now, Sachin's equation
$ax^2 + bx + c' = 0$
Given that, roots found by Sachin's are 4 and 3
$\Rightarrow -\dfrac{b}{a} = 7$ ....(i)
Rahul's equation, $ax^2 + b'x + c = 0$
Given that roots found by Rahul's are 3 and 2

$\Rightarrow \frac{c}{a} = 6$ ....(ii)
From (i) and (ii), roots of the correct equation
$x^2 - 7x + 6 = 0$ are 6 and 1

Question283

Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Rez = 1 then it is necessary that :
[2011]

Options:

A. $\beta \in (-1, 0)$

B. $|\beta| = 1$

C. $\beta \in (1, \infty)$

D. $\beta \in (0, 1)$

Answer: C

Solution:

Solution:
Since both the roots of given quadratic equation lie in the line $\mathrm{Rez} = 1$ i.e., $x = 1$, hence real part of both the roots are 1
Let both roots be $1 + i\alpha$ and $1 - i\alpha$
Product of the roots, $1 + \alpha^2 = \beta$
$\because \alpha^2 + 1 \geq 1$
$\therefore \beta \geq 1 \Rightarrow \because \beta \in (1, \infty)$

Question284

The number of complex numbers z such that $|z - 1| = | z + 1 | = | z - i|$ equals
[2010]

Options:

A. 1

B. 2

C. ∞

D. 0

Answer: A

Solution:

Solution:
Let $z = x + iy$

$|z-1| = |z+1| \Rightarrow (x-1)^2 + y^2 = (x+1)^2 + y^2$
$\Rightarrow x = 0 \Rightarrow$ Re $z = 0$
$|z-1| = |z-i| \Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2$
$\Rightarrow x = y$
$|z+1| = |z-i| \Rightarrow (x+1)^2 + y^2 = x^2 + (y-1)^2$
$\Rightarrow x = -y$
Only (0,0) will satisfy all conditions.
$\Rightarrow$ Number of complex number $z = 1$

Question285

If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2000} =$ [2010]

Options:

A. -1

B. 1

C. 2

D. -2

Answer: B

Solution:

Solution:

$x^2 - x + 1 = 0 \Rightarrow x = \dfrac{1 \pm \sqrt{1-4}}{2}$

$x = \dfrac{1 \pm \sqrt{3}i}{2}$

$\alpha = \dfrac{1}{2} + i\dfrac{\sqrt{3}}{2} = -\omega^2$

$\beta = \dfrac{1}{2} - \dfrac{i\sqrt{3}}{2} = -\omega$

$\alpha^{2009} + \beta^{2009} = (-\omega^2)^{2009} + (-\omega)^{2009}$

$= -\omega^2 - \omega = 1$

Question286

If the roots of the equation $bx^2 + cx + a = 0$ be imaginary, then for all real values of x, the expression $3b^2x^2 + 6bcx + 2c^2$ is : [2009]

Options:

A. less than $4ab$

B. greater than $-4ab$

C. less than $-4ab$

D. greater than $4ab$

Answer: B

Solution:

Solution:
Given that roots of the equation
$bx^2 + cx + a = 0$ are imaginary
$\therefore\ c^2 - 4ab < 0$
Let $y = 3b^2x^2 + 6bcx + 2c^2$
$\Rightarrow 3b^2x^2 + 6bcx + 2c^2 - y = 0$
As x is real, $D \geq 0$
$\Rightarrow 36b^2c^2 - 12b^2(2c^2 - y) \geq 0$
$\Rightarrow 12b^2(3c^2 - 2c^2 + y) \geq 0 [\because b^2 \geq 0]$
$\Rightarrow\ c^2 + y \geq 0 \quad \Rightarrow \quad y \geq -c^2$
But from eqn. (i), $c^2 < 4ab$ or $-c^2 > -4ab$
$\therefore$ we get $y \geq -c^2 > -4ab$
$\Rightarrow\ y > -4ab$

Question287

If $z - \frac{4}{z} \Big| = 2$, then the maximum value of $|z|$ is equal to :
[2009]

Options:

A. $\sqrt{5} + 1$

B. 2

C. $2 + \sqrt{2}$

D. $\sqrt{3} + 1$

Answer: A

Solution:

Solution:
Given that $\left|z - \frac{4}{z}\right| = 2$
$|z| = \left|z - \frac{4}{z} + \frac{4}{z}\right| \leq \left|z - \frac{4}{z}\right| + \frac{4}{|z|}$
$\Rightarrow |\ z \Big| \leq 2 + \frac{4}{|z|}$
$\Rightarrow |\ z\ |^2 - 2\ |\ z\ | - 4 \leq 0$
$\Rightarrow \left(|z| - \frac{2 + \sqrt{20}}{2}\right)\left(|z| - \frac{2 - \sqrt{20}}{2}\right) \leq 0$
$\Rightarrow (|z| - (1 + \sqrt{5}))(|z| - (1 - \sqrt{5})) \leq 0$
$\frac{+}{-\infty} \ - \ \frac{+}{\infty}$
$(1 - \sqrt{5})(1 + \sqrt{5})$
$\Rightarrow\ (-\sqrt{5} + 1) \leq |\ z\ | \leq (\sqrt{5} + 1)$
$\Rightarrow\ \ |\ z\ |\ \max = \sqrt{5} + 1$

Question288

The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is [2009]

Options:

A. 1

B. 4

C. 3

D. 2

Answer: D

Solution:

Solution:
Let the roots of equation $x^2 - 6x + a = 0$ be α and 4 β and that of the equation $x^2 - cx + 6 = 0$ be α and 3β.Then
$\alpha + 4\beta = 6 \ \ldots\ (i)\ \ 4\alpha\beta = a\ldots\ (ii)$
and $\alpha + 3\beta = c\ldots$ (iii) $3\alpha\beta = 6\ldots$ (iv)
$\Rightarrow\ a = 8$(from (ii) and (iv))
∴ The equation becomes $x^2 - 6x + 8 = 0$
$\Rightarrow\ (x - 2)(x - 4) = 0$
⇒ roots are 2 and 4
$\Rightarrow \alpha = 2, \beta = 1$ ∴ Common root is 2

Question289

The conjugate of a complex number is $\frac{1}{i - 1}$ then that complex number is [2008]

Options:

A. $\frac{-1}{i - 1}$

B. $\frac{1}{i + 1}$

C. $\frac{-1}{i + 1}$

D. $\frac{1}{i - 1}$

Answer: C

Solution:

Solution:

$\left(\frac{1}{i-1}\right) = \frac{1}{(i-1)} = \frac{1}{-i-1} = \frac{-1}{i+1}$

Question290

If $|z + 4| \le 3$, then the maximum value of $|z + 1|$ is [2007]

Options:

A. 6

B. 0

C. 4

D. 10

Answer: A

Solution:

Solution:
$|z+1| = |z+4-3| \le |z+4| + |-3| \le |3| + |-3|$
$\Rightarrow |z+1| \le 6 \Rightarrow |z+1|_{max} = 6$

Question291

If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$, then the set of possible values of a is [2007]

Options:

A. $(3, \infty)$

B. $(-\infty, -3)$

C. (-3,3)

D. $(-3, \infty)$

Answer: C

Solution:

Solution:
Let α and β are roots of the equation
$x^2 + ax + 1 = 0$
$\alpha + \beta = -a$ and $\alpha\beta = 1$
Given that $|\alpha - \beta| < \sqrt{5}$
$\Rightarrow \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} < \sqrt{5}$
$(\because (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta)$

$\Rightarrow\sqrt{a^2-4}<\sqrt{5}\Rightarrow a^2-4<5$
$\Rightarrow a^2-9<0\Rightarrow a^2<9\Rightarrow -3<a<3$
$\Rightarrow a\in(-3,3)$

Question292

All the values of m for which both roots of the equation $x^2-2mx+m^2-1=0$ are greater than -2 but less than 4 lie in the interval
[2006]

Options:

A. $-2<m<0$

B. $m>3$

C. $-1<m<3$

D. $1<m<4$

Answer: C

Solution:

Solution:
Given equation is $x^2-2mx+m^2-1=0$
$\Rightarrow(x-m)^2-1=0$
$\Rightarrow(x-m+1)(x-m-1)=0$
$\Rightarrow x=m-1,m+1$
$m-1>-2$ and $m+1<4$
$\Rightarrow m>-1$ and $m<3\Rightarrow -1<m<3$

Question293

If the roots of the quadratic equation $x^2+px+q=0$ are $\tan 30^\circ$ and $\tan 15^\circ$ respectively, then the value of $2+q-p$ is
[2006]

Options:

A. 2

B. 3

C. 0

D. 1

Answer: B

Solution:

Solution:
Given that $x^2 + px + q = 0$
Sum of roots $= \tan 30^\circ + \tan 15^\circ = -p$
Product of roots $= \tan 30^\circ \cdot \tan 15^\circ = q$
$\tan 45^\circ = \dfrac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \cdot \tan 15^\circ} \Rightarrow \dfrac{-p}{1-q} = 1$
$\Rightarrow -p = 1 - q \Rightarrow q - p = 1$
$\therefore 2 + q - p = 3$

Question294

If $z^2 + z + 1 = 0$, where z is complex number, then the value of $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \ldots\ldots\ldots + \left(z^6 + \frac{1}{z^6}\right)^2$ is [2006]

Options:

A. 18

B. 54

C. 6

D. 12

Answer: D

Solution:

Solution:
$z^2 + z + 1 = 0 \Rightarrow z = \omega$ or ω^2
So, $z + \dfrac{1}{z} = \omega + \omega^2 = -1$
$\left[\because \dfrac{1}{z} = \omega^2 \text{ and } 1 + \omega + \omega^2 = 0\right]$
$z^2 + \dfrac{1}{z^2} = \omega^2 + \omega = -1$
$[\because \omega^3 = 1]$
$z^3 + \dfrac{1}{z^3} = \omega^3 + \omega^3 = 2$
$z^4 + \dfrac{1}{z^4} = -1,\ z^5 + \dfrac{1}{z^5} = -1$
and $z^6 + \dfrac{1}{z^6} = 2$
$\therefore$ The given sum $= 1 + 1 + 4 + 1 + 1 + 4 = 12$

Question295

If x is real, the maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is [2006]

Options:

A. $\frac{1}{4}$

B. 41

C. 1

D. $\frac{17}{7}$

Answer: B

Solution:

Solution:

$y = \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$

$3x^2(y - 1) + 9x(y - 1) + 7y - 17 = 0$

$D \geq 0$ $\because$ x is real

$81(y - 1)^2 - 4 \times 3(y - 1)(7y - 17) \geq 0$

$\Rightarrow (y - 1)(y - 41) \leq 0 \Rightarrow 1 \leq y \leq 41$

$\therefore$ Max value of y is 41

Question296

If $\omega = \frac{z}{z - \frac{1}{3}i}$ and $|\omega| = 1$, then z lies on

[2005]

Options:

A. an ellipse

B. a circle

C. a straight line

D. a parabola

Answer: C

Solution:

Solution:

Given that $w = \frac{z}{z - \frac{1}{3}i}$

$\Rightarrow |w| = \frac{|z|}{\left|z - \frac{1}{3}i\right|} = 1 \quad \left[\because \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\right]$

$\Rightarrow |z| = \left|z - \frac{1}{3}i\right|$

$\Rightarrow$ distance of z from origin and point $\left(0, \frac{1}{3}\right)$ is same hence z lies on bisector of the line joining points (0,0) and (0, 1 / 3)

Hence z lies on a straight line.

Question297

If z_1 and z_2 are two non- zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to
[2005]

Options:

A. $\frac{\pi}{2}$

B. $-\pi$

C. 0

D. $\frac{-\pi}{2}$

Answer: C

Solution:

Solution:
$|z_1 + z_2| = |z_1| + |z_2| \Rightarrow z_1$ and z_2 are collinear and are to the same side of origin; hence $\arg z_1 - \arg z_2 = 0$.

Question298

If the cube roots of unity are 1, ω, ω^2 then the roots of the equation $(x - 1)^3 + 8 = 0$, are
[2005]

Options:

A. $-1, -1 + 2\omega, -1 - 2\omega^2$

B. -1,-1,-1

C. $-1, 1 - 2\omega, 1 - 2\omega^2$

D. $-1, 1 + 2\omega, 1 + 2\omega^2$

Answer: C

Solution:

Solution:
$\because (x - 1)^3 + 8 = 0 \Rightarrow (x - 1) = (-2)(1)^{1/3}$
$\Rightarrow x - 1 = -2$ or -2ω or $-2\omega^2$
or $x = -1$ or $1 - 2\omega$ or $1 - 2\omega^2$

Question299

In a triangle PQR, $\angle R = \frac{\pi}{2}$. If $\tan\left(\frac{P}{2}\right)$ and $-\tan\left(\frac{Q}{2}\right)$ are the roots of

$ax^2 + bx + c = 0$, $a \neq 0$ then
[2005]

Options:

A. $a = b + c$

B. $c = a + b$

C. $b = c$

D. $b = a + c$

Answer: B

Solution:

Solution:

$\tan\left(\frac{P}{2}\right)$, $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$

$\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right) = -\frac{b}{a}$

$\tan\left(\frac{P}{2}\right) \cdot \tan\left(\frac{Q}{2}\right) = \frac{c}{a}$

$\dfrac{\tan\left(\frac{P}{2}\right) + \tan\left(\frac{Q}{2}\right)}{1 - \tan\left(\frac{P}{2}\right)\tan\left(\frac{Q}{2}\right)} = \tan\left(\frac{P}{2} + \frac{Q}{2}\right) = 1$

$\left[\because P + Q = \frac{\pi}{2}\right]$

$\Rightarrow \dfrac{-\frac{b}{a}}{1 - \frac{c}{a}} = 1 \Rightarrow -\frac{b}{a} = \frac{a - c}{a}$

$\Rightarrow -b = a - c \Rightarrow c = a + b$

Question300

If the roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals
[2005]

Options:

A. -2

B. 3

C. 2

D. 1

Answer: D

Solution:

Solution:

Let α, $\alpha + 1$ be roots

Then $\alpha + \alpha + 1 = b =$ sum of roots
$\alpha(\alpha + 1) = c =$ product of roots
$\therefore\ b^2 - 4c = (2\alpha + 1)^2 - 4\alpha(\alpha + 1) = 1$

Question301

If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval [2005]

Options:

A. (5,6]

B. $(6, \infty)$

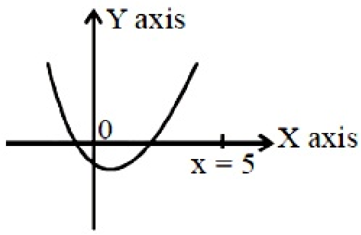
C. $(-\infty, 4)$

D. [4,5]

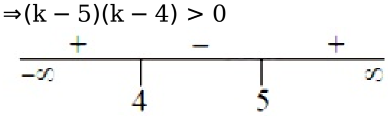
Answer: C

Solution:

Solution:
Given that both roots of quadratic equation are less than 5 then (i)



Discriminant ≥0
$4k^2 - 4(k^2 + k - 5) \geq 0$
$4k^2 - 4k^2 - 4k + 20 \geq 0$
$4k \leq 20 \Rightarrow k \leq 5$
(ii) $p(5) > 0$
$\Rightarrow f(5) > 0; 25 - 10k + k^2 + k - 5 > 0$
$\Rightarrow k^2 - 9k + 20 > 0$
$\Rightarrow k(k - 4) - 5(k - 4) > 0$
$\Rightarrow (k - 5)(k - 4) > 0$



(iii) $\dfrac{\text{Sum of roots}}{2} < 5$
$\Rightarrow -\dfrac{b}{2a} = \dfrac{2k}{2} < 5$
$\Rightarrow k < 5$
The intersection of (i), (ii) & (iii) gives
$k \in (-\infty, 4)$

Question302

The value of a for which the sum of the squares of the roots of the

equation $x^2 - (a-2)x - a - 1 = 0$ assume the least value is [2005]

Options:

A. 1

B. 0

C. 3

D. 2

Answer: A

Solution:

Solution:
Given equation is $x^2 - (a-2)x - a - 1 = 0$
$\Rightarrow \alpha + \beta = a - 2$; $\alpha\beta = -(a+1)$
$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= a^2 - 2a + 6 = (a-1)^2 + 5$
For min. value of $\alpha^2 + \beta^2$, $a - 1 = 0$
$\Rightarrow a = 1$

Question303

If $z = x - iy$ and $z^{\frac{1}{3}} = p + iq$, then $\left(\frac{x}{p} + \frac{y}{q}\right) / (p^2 + q^2)$ is equal to [2004]

Options:

A. -2

B. -1

C. 2

D. 1

Answer: A

Solution:

Solution:
Given that $z^{\frac{1}{3}} = p + iq$
$\Rightarrow z = p^3 + (iq)^3 + 3p(iq)(p + iq)$
$\Rightarrow x - iy = p^3 - 3pq^2 + i(3p^2q - q^3)$
Comparing both side, we get
$\therefore x = p^3 - 3pq^2 \Rightarrow \frac{x}{p} = p^2 - 3q^2$ .... (i)
and $y = q^3 - 3p^2q \Rightarrow \frac{y}{q} = q^2 - 3p^2$......... . (ii)
Adding (i) and (ii), we get
$\therefore \frac{x}{p} + \frac{y}{q} = -2p^2 - 2q^2 \quad \therefore \left(\frac{x}{p} + \frac{y}{q}\right) / (p^2 + q^2) = -2$

Question304

Let z and w be complex numbers such that $\bar{z} + i\overline{w} = 0$ and $\arg zw = \pi$. Then arg z equals [2004]

Options:

A. $\frac{5\pi}{4}$

B. $\frac{\pi}{2}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{4}$

Answer: C

Solution:

Solution:
Given that $\arg zw = \pi$
$\Rightarrow \arg z + \arg w = \pi$
$\bar{z} + i\overline{w} = 0 \Rightarrow \bar{z} = -i\overline{w}$
Replace i by $-i$, we get
$\therefore z = iw \Rightarrow \arg z = \frac{\pi}{2} + \arg w$
$\Rightarrow \arg z = \frac{\pi}{2} + \pi - \arg z$ (from (i))
$\arg z = \frac{3\pi}{4}$

Question305

If $|z^2 - 1| = |z|^2 + 1$, then z lies on [2004]

Options:

A. an ellipse

B. the imaginary axis

C. a circle

D. the real axis

Answer: B

Solution:

Solution:

Given that $|z^2 - 1| = |z|^2 + 1 \Rightarrow |z^2 - 1|^2 = (z\bar{z} + 1)^2$
$[\because |z|^2 = z\bar{z}]$
$\Rightarrow (z^2 - 1)(\bar{z}^2 - 1) = (\bar{z} + 1)^2 (\because \overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2)$
$\Rightarrow z^2\bar{z}^2 - z^2 - \bar{z}^2 + 1 = z^2\bar{z}^2 + 2z\bar{z} + 1$
$\Rightarrow z^2 + 2z\bar{z} + \bar{z}^2 = 0$
$\Rightarrow (z + \bar{z})^2 = 0 \Rightarrow z = -\bar{z}$
$\Rightarrow z$ is purely imaginary

Question306

If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of ˚q′ is [2004]

Options:

A. 4

B. 12

C. 3

D. $\frac{49}{4}$

Answer: D

Solution:

Solution:
Given that 4 is a root of $x^2 + px + 12 = 0$
$\Rightarrow 16 + 4p + 12 = 0 \Rightarrow p = -7$
Now, the equation $x^2 + px + q = 0$ has equal roots.
$\therefore D = 0$
$\Rightarrow p^2 - 4q = 0 \Rightarrow q = \frac{p^2}{4} = \frac{49}{4}$

Question307

If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$ then its root are [2004]

Options:

A. -1,2

B. -1,1

C. 0,-1

D. 0,1

Answer: C

Solution:

Solution:
Let the second root be α.
Then $\alpha + (1 - p) = -p \Rightarrow \alpha = -1$
Also $\alpha . (1 - p) = 1 - p$
$\Rightarrow (\alpha - 1)(1 - p) = 0 \Rightarrow p = 1 [\because \alpha = -1]$
$\therefore$ Roots are $\alpha = -1$ and $1 - p = 0$

Question308

If $\left(\frac{1+i}{1-i}\right)^x = 1$ then
[2003]

Options:

A. $x = 2n + 1$, where n is any positive integer

B. $x = 4n$, where n is any positive integer

C. $x = 2n$, where n is any positive integer

D. $x = 4n + 1$, where n is any positive integer.

Answer: B

Solution:

Solution:
Given that
$\left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow \left[\frac{(1+i)^2}{1-i^2}\right]^x = 1$
$\left(\frac{1+i^2+2i}{1+1}\right)^x = 1 \Rightarrow (i)^x = 1; \quad \therefore x = 4n; \quad n \in I^+$

Question309

If z and ω are two non-zero complex numbers such that $|z\omega| = 1$ and $\mathrm{Arg}(z) - \mathrm{Arg}(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to
[2003]

Options:

A. -1

B. 1

C. $-i$

D. i

Answer: A

Solution:

Solution:
$|\overline{z}\omega| = |\overline{z}||\omega| = |z||\omega| = |z\omega| = 1 [\because |\overline{z}| = |z|]$
$\operatorname{Arg}(\overline{z}\omega) = \arg(\overline{z}) + \arg(\omega)$
$= -\arg(z) + \arg\omega = -\frac{\pi}{2}$
$[\because \arg(\overline{z}) = -\arg(z)]$
$\therefore \overline{z}\omega = -1$

Question310

The number of real solutions of the equation $x^2 - 3|x| + 2 = 0$ is [2003]

Options:

A. 3

B. 2

C. 4

D. 1

Answer: C

Solution:

Solution:
Given that $x^2 - 3|x| + 2 = 0 \Rightarrow |x|^2 - 3|x| + 2 = 0$
$\Rightarrow (|x| - 2)(|x| - 1) = 0$
$\Rightarrow |x| = 1, 2 \Rightarrow x = \pm 1, \pm 2$
$\therefore$ No. of solution $= 4$

Question311

The value of ' a ' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is [2003]

Options:

A. $-\frac{1}{3}$

B. $\frac{2}{3}$

C. $-\frac{2}{3}$

D. $\frac{1}{3}$

Answer: B

Solution:

Solution:
Let one roots of given equation be α
$\therefore$ Second roots be 2α then
$\alpha + 2\alpha = 3\alpha = \frac{1-3a}{a^2-5a+3}$
$\Rightarrow \alpha = \frac{1-3a}{3(a^2-5a+3)}$ ....(i)
and $\alpha.2\alpha = 2\alpha^2 = \frac{2}{a^2-5a+3}$
$\therefore 2\left[\frac{1}{9}\frac{(1-3a)^2}{(a^2-5a+3)^2}\right] = \frac{2}{a^2-5a+3}$
[from (i)]
$\frac{(1-3a)^2}{(a^2-5a+3)} = 9$
$\Rightarrow 9a^2 - 6a + 1 = 9a^2 - 45a + 27$
$\Rightarrow 39a = 26 \Rightarrow a = \frac{2}{3}$

Question312

Let Z_1 and Z_2 be two roots of the equation $Z^2 + aZ + b = 0$, Z being complex. Further, assume that the origin, Z_1 and Z_2 form an equilateral triangle. Then
[2003]

Options:

A. $a^2 = 4b$

B. $a^2 = b$

C. $a^2 = 2b$

D. $a^2 = 3b$

Answer: D

Solution:

Solution:
Given that $Z^2 + aZ + b = 0$;
$Z_1 + Z_2 = -a$ & $Z_1Z_2 = b$
0, Z_1, Z_2 form an equilateral triangle $\therefore 0^2 + Z_1^2 + Z_2^2 = 0 . Z_1 + Z_1 \cdot Z_2 + Z_2 \cdot 0$
(for an equilateral triangle,
$Z_1^2 + Z_2^2 + Z_3^2 = Z_1Z_2 + Z_2Z_3 + Z_3Z_1$)
$\Rightarrow Z_1^2 + Z_2^2 = Z_1Z_2$
$\Rightarrow (Z_1 + Z_2)^2 = 3Z_1Z_2$
$\therefore a^2 = 3b$

Question313

If $|z-4| < |z-2|$, its solution is given by
[2002]

Options:

A. $Re(z) > 0$

B. $Re(z) < 0$

C. $Re(z) > 3$

D. $Re(z) > 2$

Answer: C

Solution:

Solution:
Given that $|z-4| < |z-2$
Let $z = x + iy$
$\Rightarrow |(x-4) + iy)| < |(x-2) + iy|$
$\Rightarrow (x-4)^2 + y^2 < (x-2)^2 + y^2$
$\Rightarrow x^2 - 8x + 16 < x^2 - 4x + 4 \Rightarrow 12 < 4x$
$\Rightarrow x > 3 \Rightarrow Re(z) > 3$

Question314

z and w are two non zero complex numbers such that $|z| = |w|$ and $Arg z + Arg w = \pi$ then z equals
[2002]

Options:

A. $\overline{\omega}$

B. $-\overline{\omega}$

C. ω

D. $-\omega$

Answer: B

Solution:

Solution:
Let $|z| = |\omega| = r$
$\therefore z = re^{i\theta}, \omega = re^{i\varphi}$ where $\theta + \varphi = \pi$
$\therefore z = re^{i(\pi - \varphi)} = re^{i\pi} \cdot e^{-i\varphi = -re^{-i\varphi}} = -\overline{\omega}$
$[\because e^{i\pi} = -1$ and $\overline{\omega} = re^{-i\varphi}]$

Question315

The locus of the centre of a circle which touches the circle $|z - z_1| = a$

and $|z - z_2| = b$ externally (z, z_1 & z_2 are complex numbers) will be [2002]

Options:

A. an ellipse

B. a hyperbola

C. a circle

D. none of these

Answer: B

Solution:

Solution:
Let the circle be $|z - z_0| = r$. Then according to given conditions $|z_0 - z_1| = r + a$ ... (i)
$|z_0 - z_2| = r + b$ ....(ii)
Subtract (ii) from (i)
we get $|z_0 - z_1| - | z_0 - z_2 | = a - b$.
∴ Locus of centre z_0 is $|z - z_1| - | z - z_2|$
$= a - b$, which represents a hyperbola.

Question316

If p and q are the roots of the equation $x^2 + px + q = 0$, then [2002]

Options:

A. $p = 1, q = -2$

B. $p = 0, q = 1$

C. $p = -2, q = 0$

D. $p = -2, q = 1$

Answer: A

Solution:

Solution:
$p + q = -p \Rightarrow q = 2p$
and $pq = q \Rightarrow q(p - 1) = 0$
$\Rightarrow q = 0$ or $p = 1$
If $q = 0$, then $p = 0$
or $p = 1$, then $q = -2$.

Question317

Product of real roots of the equation $t^2x^2 + | x | + 9 = 0$

[2002]

Options:

A. is always positive

B. is always negative

C. does not exist

D. none of these

Answer: A

Solution:

Solution:
Product of real roots $= \frac{c}{a} = \frac{9}{t^2} > 0, \forall t \in R$
$\therefore$ Product of real roots is always positive.

Question318

Difference between the corresponding roots of $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ is same and $a \neq b$, then
[2002]

Options:

A. $a + b + 4 = 0$

B. $a + b - 4 = 0$

C. $a - b - 4 = 0$

D. $a - b + 4 = 0$

Answer: A

Solution:

Solution:
Let α and β are roots of the equation $x^2 + ax + b = 0$ and γ and δ be the roots of the equation $x^2 + bx + a = 0$ respectively
$\therefore \alpha + \beta = -a, \alpha\beta = b$ and $\gamma + \delta = -b, \gamma\delta = a$
Given $|\alpha - \beta| = |\gamma - \delta| \Rightarrow (\alpha - \beta)^2 = (\gamma - \delta)^2$
$\Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = (\gamma + \delta)^2 - 4\gamma\delta$
$\Rightarrow a^2 - 4b = b^2 - 4a$
$\Rightarrow (a^2 - b^2) + 4(a - b) = 0$
$\Rightarrow a + b + 4 = 0 \ (\because a \neq b)$

Question319

If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$ then the equation having α / β and β / α as its roots is

[2002]

Options:

A. $3x^2 - 19x + 3 = 0$

B. $3x^2 + 19x - 3 = 0$

C. $3x^2 - 19x - 3 = 0$

D. $x^2 - 5x + 3 = 0.$

Answer: A

Solution:

Given that $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$;
$\Rightarrow \alpha$ & β are roots of equation, $x^2 = 5x - 3$
or $x^2 - 5x + 3 = 0$
$\therefore \alpha + \beta = 5$ and $\alpha\beta = 3$
Thus, the equation whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is

$$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha\beta}{\alpha\beta} = 0$$

$$\Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0$$

or $3x^2 - 19x + 3 = 0$
