Permutations and Combinations:

0.1 Permutations

Description: This is the template for a subsection.

Quote: "Here is an example quote"

Intuition: Or findings, explanations, etc.

$$X' = \{d, e, f, \dots z\} \qquad Y' \neq \{1, 2, 3\}$$

0.2 Combinations

Description: Combinations are all or part of a set of objects. They are not dependent on order like permutations. Combinations can be calculated with or without repetition.

Question: How many different 5-card hands can be made from a standard deck of 52 cards?

Permutations: First, 52 cards could be drawn: $\underline{52}\ \dots$, then 51 cards: $\underline{52}\ \underline{51}\ \dots$, and so on: $\underline{52}\ \underline{51}\ \underline{50}\ \underline{49}\ \underline{48}$. Multiplying this factorial will return **ALL** the possible permutations and **DUPLICATE** results:

$$52 * 51 * 50 * 49 * 48 \quad = \quad \text{Permutations} = 311,875,200 \quad \Rightarrow \quad \textbf{A, K, Q, J, 10} \neq \textbf{10, J, Q, K, A} \tag{1}$$

How many ways to arrange five cards? First, 5 cards could be drawn: $\underline{5}\ \dots$, then 4 cards: $\underline{5}\ \underline{4}\ \dots$, until 1 card: $\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$. Multiplying this factorial will return **ALL** the possible ways to arrange 5 cards.

$$5 * 4 * 3 * 2 * 1 \quad = \quad \text{Permutations} = 120 \tag{2}$$

How many unique 5-card hands? Dividing all the possible ways to order 5 cards in a deck, by all the possible ways arrange 5 cards returns the result. **The number of permutations divided by the number of ways to arrange one hand.**

$$\frac{\text{Possible ways to order 5 cards in a deck (permutations)}}{\text{Possible ways to arrange 5 cards (combinations)}} = \frac{311,875,200}{120} = 2,598,960 \tag{3}$$

Formulas: Relating the above intuition to the formulas and notations. There are many ways to do this. Verbally saying things like: "n choose k" or "n factorial over k factorial, times n minus k factorial". Also it can be written or denoted in a myriad of ways including:

$$\frac{52 * 51 * 50 * 49 * 48}{5 * 4 * 3 * 2 * 1} = \frac{52!}{5!47!} = \frac{52!}{5!(52-5)!} = \frac{n!}{k!(n-k)!} \tag{4}$$

Note: In the above example the 47! is implicitly cancelled out.

$$\binom{n}{k} = C_k^n = {}_nC_k = {}^nC_k = C\,(n, k) = \binom{52}{5} \tag{5}$$

$$\binom{52}{5} = C_5^{52} = {}_{52}C_5 = {}^{52}C_5 = C\,(52, 5) = \binom{n}{k} \tag{6}$$