Conditions for a z-test About a Proportion:
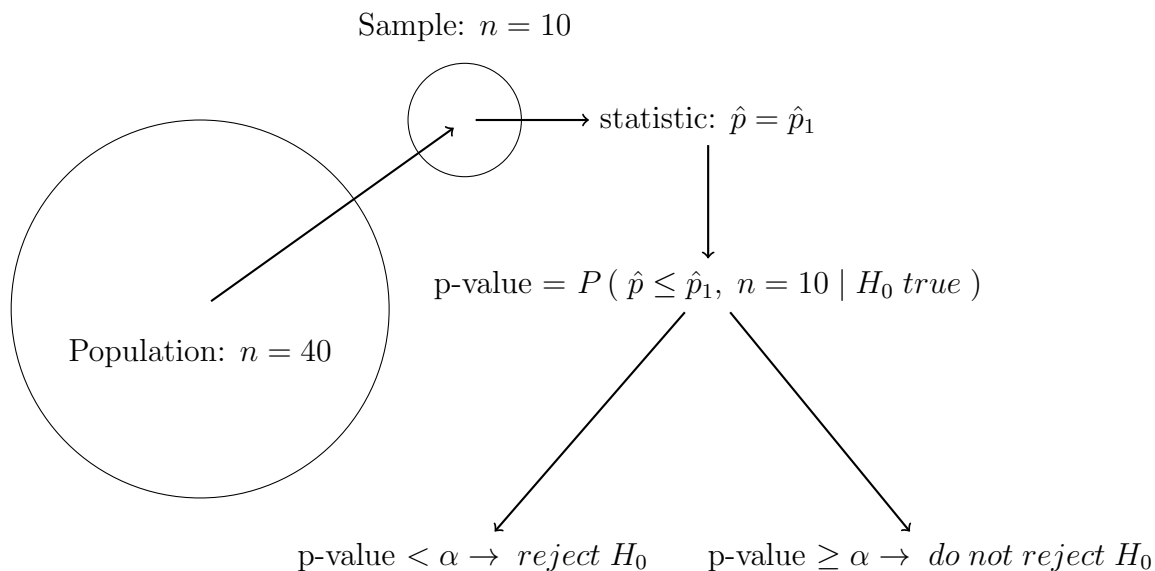
Scenario

Jules is on a team of 40 employees. Each employee gets an annual rating, the best is "exceeds expectation." Management **claimed that 10% of employees** get this rating, Jules **suspected it was actually less common**. She got an anonymous random sample of 10 ratings for employees on her team. She wants to use the sample data to test H_0: $p = 0.1$ vs. H_a: $p < 0.1$, where p is the proportion of all employees on her team who "exceeds expectation."

Hypotheses

$$\text{The null hypothesis is based on the claim itself:} \qquad H_0 : \quad p = 0.10 \qquad (1)$$
$$\text{The alternative hypothesis is based on what is suspected:} \qquad H_a : \quad p < 0.10 \qquad (2)$$

Visualize

Sample: $n = 10$

statistic: $\hat{p} = \hat{p}_1$

Population: $n = 40$

p-value $= P\,(\,\hat{p} \le \hat{p}_1,\ n = 10 \mid H_0\ true\,)$

p-value $< \alpha \rightarrow reject\ H_0$ p-value $\ge \alpha \rightarrow do\ not\ reject\ H_0$

Description

- From the population (40 employees), take a sample (10 employees), then calculate a sample statistic **(sample proportion)** and assign it to $\hat{p}_1$.

- Next calculate a p-value, remembering that a **p-value is the probability of getting a result at least as extreme, if we assume the null hypothesis is true**. Because Jules suspects that 10% are not exceeding expectations, this is the probability that the sample statistic is less than or equal to the one, that was calculated for the sample size (n = 10), given the null hypothesis is true.

- If the **p-value is less** than the predetermined significance level then the null hypothesis is rejected. If the **p-value is not less** than the significance level, then the null hypothesis can not be rejected.

Conditions Check

- **Random**: Yes. **"obtained an anonymous random sample"** is explicit in description.

- **Normal**: No. Successes and Failures must be at least equal to 10 $(np,\ n(1-p) \ge 10)$. In this case (n = 10 and p = 0.1): $(10 * 0.1 = 1\ and\ 10 * 0.9 = 9)$, so this condition is not satisfied.

- **Independence**: No. Jules is **not using replacement** and **sample size is not less than 10%** of the population. $(10/40 = 0.25)$