一、

1.√ 2.X 3.X 4.√ 5.X 6.√ 7.√ 8√

二、

1. $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ 2. $\begin{bmatrix} 0 & \frac{I}{2} \\ \frac{I}{2^{n+1}} & 0 \end{bmatrix}$ 3. (3, 4, 4)

4. 0, 26 5. $1 > a > -\frac{1}{6}$

三、解：令 $a=3,\ b=-2,\ c=-1$

$\therefore D_n = aD_{n-1} - bc\,D_{n-2}$

$\lambda^2 = 3\lambda - 2$

$\Rightarrow \lambda_1 = 1,\ \lambda_2 = 2$

$\therefore D_n = C_1 \cdot 1^n + C_2 \cdot 2^n$，代入 $D_1 = 3,\ D_2 = 7$

得 $\begin{cases} C_1 + 2C_2 = 3 \\ C_1 + 4C_2 = 7 \end{cases} \Rightarrow \begin{cases} C_1 = -1 \\ C_2 = 2 \end{cases}$

$\therefore D_n = 2^{n+1} - 1$

四、解：(1) 可以

$(A\alpha_1, A\alpha_2, A\alpha_3) = (\alpha_1, \alpha_2, \alpha_3)\begin{pmatrix} 3 & 2 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 3 \end{pmatrix}$

$\Rightarrow (\alpha_1, \alpha_2, \alpha_3) = (A\alpha_1, A\alpha_2, A\alpha_3)\begin{pmatrix} 3 & 2 & 0 \\ 0 & 3 & 2 \\ 0 & 0 & 3 \end{pmatrix}^{-1}$

$= (A\alpha_1, A\alpha_2, A\alpha_3)\begin{pmatrix} \frac{1}{3} & -\frac{2}{9} & \frac{4}{27} \\ 0 & \frac{1}{3} & -\frac{2}{9} \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$

即 $\alpha_1 = \frac{1}{3}A\alpha_1$，$\alpha_2 = -\frac{2}{9}A\alpha_1 + \frac{1}{3}A\alpha_2$，$\alpha_3 = \frac{4}{27}\alpha_1 - \frac{2}{9}A\alpha_2 + \frac{1}{3}A\alpha_3$

(2) 设存在 k_1, k_2, k_3, s.t. $k_1\alpha_1 + k_2\alpha_2 + k_3\alpha_3 = 0$

$\therefore k_1A\alpha_1 + k_2A\alpha_2 + k_3A\alpha_3 = 0 \Rightarrow 3k_1\alpha_1 + k_2(3\alpha_2 + 2\alpha_1) + k_3(3\alpha_3 + 2\alpha_2) = 0$

$\therefore (3k_1 + 2k_2)\alpha_1 + (3k_2 + 2k_3)\alpha_2 + 3k_3\alpha_3 = 0$

$\because \alpha_1, \alpha_2, \alpha_3$ 为 n维非零列向量，$\therefore k_1, k_2, k_3$ 为0.

$\therefore \alpha_1, \alpha_2, \alpha_3$ 线性无关

五：解：$\begin{pmatrix} a & 1 & 1 & 4 \\ 1 & b & 1 & 3 \\ 1 & 2b & 1 & 4 \end{pmatrix} \Rightarrow \begin{pmatrix} a & 1 & 1 & 4 \\ 0 & b & 0 & 1 \\ 1 & 0 & 1 & -2 \end{pmatrix}$ $\therefore x_2=\frac{1}{b}, b\neq 0$

$\begin{cases} x_1+x_3=-2, \\ ax_1+x_3=4-\frac{1}{b} \end{cases} \Rightarrow \begin{cases} x_1=\frac{6-\frac{1}{b}}{a-1} \\ x_3=-2+\frac{6-\frac{1}{b}}{1-a} \end{cases}$，$a\neq 1$，即

$a\neq 1, b\neq 0$ 时，有唯一解，$x=\begin{pmatrix} \frac{6-\frac{1}{b}}{a-1} \\ \frac{1}{b} \\ -2+\frac{6-\frac{1}{b}}{1-a} \end{pmatrix}$

六 (1) 解：$A\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}-3\\-3\\-3\end{pmatrix}=-3\begin{pmatrix}1\\1\\1\end{pmatrix}$ $\therefore \lambda_1=-3$，又 $A\begin{pmatrix}-1\\1\\0\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ $\therefore \lambda_2=0$

$A\begin{pmatrix}1\\1\\2\end{pmatrix}=\begin{pmatrix}2\\2\\-4\end{pmatrix}=2\begin{pmatrix}1\\1\\-2\end{pmatrix}$，$\therefore \lambda_3=2$

(2) $B=A^{2022}\begin{pmatrix}0\\2\\1\end{pmatrix}=[PD^{2022}P^{-1}]\begin{pmatrix}0\\2\\1\end{pmatrix}=P\begin{pmatrix}\lambda_1^{2022} & & \\ & \lambda_2^{2022} & \\ & & \lambda_3^{2022}\end{pmatrix}P^{T}\begin{pmatrix}0\\2\\1\end{pmatrix}$，

$\lambda_1, \lambda_2, \lambda_3$ 对应特征向量分别为 $\begin{pmatrix}1\\1\\1\end{pmatrix}$、$\begin{pmatrix}-1\\1\\0\end{pmatrix}$、$\begin{pmatrix}1\\1\\-2\end{pmatrix}$，$P=\begin{pmatrix}1 & -1 & 1\\ 1 & 1 & 1\\ 1 & 0 & -2\end{pmatrix}$

$P^{-1}=\frac{P^*}{|P|}=-\frac{1}{6}\begin{pmatrix}-2 & 3 & -1\\ -2 & -3 & -1\\ -2 & 0 & 2\end{pmatrix}^{T}=\begin{pmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ -\frac{1}{2} & \frac{1}{2} & 0\\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{3}\end{pmatrix}$

$\therefore B=\begin{pmatrix}1 & -1 & 1\\ 1 & 1 & 1\\ 1 & 0 & -2\end{pmatrix}\begin{pmatrix}(-3)^{2022} & & \\ & 0 & \\ & & 2^{2022}\end{pmatrix}\begin{pmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ -\frac{1}{2} & \frac{1}{2} & 0\\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{3}\end{pmatrix}=\begin{pmatrix}3^{2021}+\frac{2^{2021}}{3} & 3^{2021}+\frac{2^{2021}}{3} & 3^{2021}-\frac{2^{2022}}{3}\\ 3^{2021}+\frac{2^{2021}}{3} & 3^{2021}+\frac{2^{2021}}{3} & 3^{2021}-\frac{2^{2022}}{3}\\ 3^{2021}-\frac{2^{2022}}{3} & 3^{2021}-\frac{2^{2022}}{3} & 3^{2021}+\frac{2^{2023}}{3}\end{pmatrix}$

七、解: $A=\begin{pmatrix} a & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, $|\lambda E-A|=\begin{vmatrix} \lambda-a & -1 & 0 \\ -1 & \lambda-2 & 0 \\ 0 & 0 & \lambda-3 \end{vmatrix}=(\lambda-3)[(\lambda-a)(\lambda-2)-1]$

$\lambda_1=b$, $\lambda_2=3$, $\lambda_3=1$ $\quad\therefore \begin{cases}(b-a)(b-2)-1=0 \\ (1-a)(1-2)-1=0\end{cases} \Rightarrow \begin{cases} b=3\text{或}b=1(\text{舍}) \\ a=2 \end{cases}$

$\therefore \lambda_3=1$时, $\xi_3=\begin{pmatrix}-1\\1\\0\end{pmatrix}$, $\lambda_1=3$, $\lambda_2=3$时, $\xi_1=\begin{pmatrix}1\\1\\0\end{pmatrix}$, $\xi_2=\begin{pmatrix}0\\0\\1\end{pmatrix}$

$\therefore C=\begin{pmatrix} 1 & 0 & -1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

八. (1) 令$\alpha\beta^T=B_{(n\times n)}$, $\alpha,\beta\in R^n$ $\quad\therefore 0\le r(B)\le \min\{r(\alpha), r(\beta^T)\}$

当α,β为非零列向量时, $r(B)=1$, 当α,β到有一个为零向量时 $r(B)=0$.

(2) 证: A,B为n阶实矩阵,

当 $r(B)=0$ 时, 即$B=0$ $|A+s\alpha\beta^T|=|A|=a+0\cdot s$, 对$\forall s\in R$成立

当 $r(B)=1$时, B有一个不为0的特征值λ_1, 和$n-1$个为0的特征值$\lambda_2,\cdots\lambda_n$

$\therefore$ B可相似对角化为 $\begin{pmatrix}\lambda_1 & & & \\ & 0 & & \\ & & 0 & \\ & & & \ddots\ 0\end{pmatrix}$

$\therefore |A+s\alpha\beta^T|=\begin{pmatrix} a_{11}+s\lambda_1 & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn}\end{pmatrix}=(a_{11}+s\lambda_1)A_{11}+a_{21}A_{21}+\cdots+a_{n1}A_{n1}$

$=|A|+s\lambda_1 A_{11}$

$=a+bs$, 对$\forall s\in R$成立

证毕.