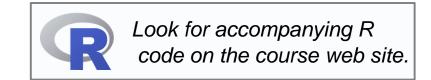
Association Analysis: Basic Concepts and Algorithms



Lecture Notes for Chapter 6

Slides by Tan, Steinbach, Kumar adapted by Michael Hahsler



Topics

- Definition
- Mining Frequent Itemsets (APRIORI)
- Concise Itemset Representation
- Alternative Methods to Find Frequent Itemsets
- Association Rule Generation
- Support Distribution
- Pattern Evaluation

Association Rule Mining

 Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

```
\{Diaper\} \rightarrow \{Beer\},\
\{Milk, Bread\} \rightarrow \{Eggs, Coke\},\
\{Beer, Bread\} \rightarrow \{Milk\},\
```

Implication means co-occurrence, not causality!



Definition: Frequent Itemset

Itemset

- A collection of one or more items
 - Example: {Milk, Bread, Diaper}
- k-itemset
 - An itemset that contains k items

Support count (σ)

- Frequency of occurrence of an itemset
- E.g. $\sigma(\{Milk, Bread, Diaper\}) = 2$

Support

- Fraction of transactions that contain an itemset
- E.g. $s(\{Milk, Bread, Diaper\})$ = $\sigma(\{Milk, Bread, Diaper\}) / |T| = 2/5$

Frequent Itemset

 An itemset whose support is greater than or equal to a *minsup* threshold

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

$$s(X) = \frac{\sigma(X)}{|T|}$$

Definition: Association Rule

Association Rule

- An implication expression of the form
 X → Y, where X and Y are itemsets
- Example: $\{Milk, Bread\} \rightarrow \{Diaper\}$

Rule Evaluation Metrics

- Support (s)
 - Fraction of transactions that contain both X and Y
- Confidence (c)
 - Measures how often items in Y appear in transactions that contain X

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example:

$$\{Milk, Bread\} \rightarrow \{Diaper\}$$

$$s = \frac{\sigma(\{Milk, Bread, Diaper\})}{|T|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\{Milk, Bread, Diaper\})}{\sigma(\{Milk, Diaper\})} = \frac{2}{3} = 0.67$$

$$c(X \rightarrow Y) = \frac{\sigma(X \cup Y)}{\sigma(X)} = \frac{s(X \cup Y)}{s(X)}$$

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Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
 - support ≥ *minsup* threshold
 - confidence ≥ minconf threshold
- Brute-force approach:
 - List all possible association rules
 - Compute the support and confidence for each rule
 - Prune rules that fail the *minsup* and *minconf* thresholds
 - ⇒ Computationally prohibitive!

Mining Association Rules

TID	Items	
1	Bread, Milk	
2	Bread, Diaper, Beer, Eggs	
3	Milk, Diaper, Beer, Coke	
4	Bread, Milk, Diaper, Beer	
5	Bread, Milk, Diaper, Coke	

Example of Rules:

```
{Milk, Diaper} \rightarrow {Beer} (s=0.4, c=0.67)
{Milk, Beer} \rightarrow {Diaper} (s=0.4, c=1.0)
{Diaper, Beer} \rightarrow {Milk} (s=0.4, c=0.67)
{Beer} \rightarrow {Milk, Diaper} (s=0.4, c=0.67)
{Diaper} \rightarrow {Milk, Beer} (s=0.4, c=0.5)
{Milk} \rightarrow {Diaper, Beer} (s=0.4, c=0.5)
```

Observations:

- All the above rules are binary partitions of the same itemset: {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

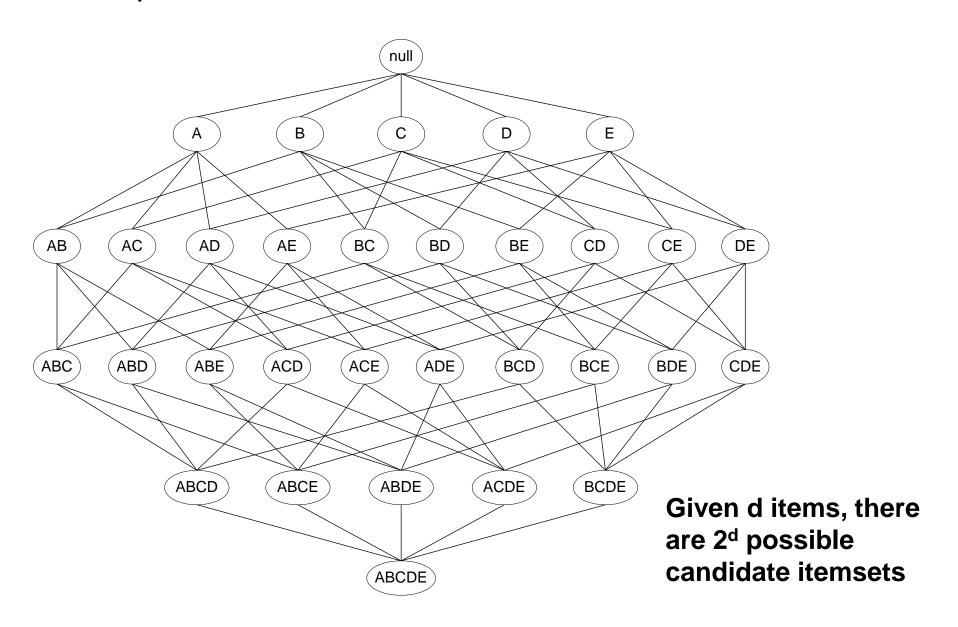
Mining Association Rules

- Two-step approach:
 - 1. Frequent Itemset Generation
 - Generate all itemsets whose support ≥ minsup

2. Rule Generation

- Generate high confidence rules from each frequent itemset,
 where each rule is a binary partitioning of a frequent itemset
- Frequent itemset generation is still computationally expensive

Frequent Itemset Generation



Reducing Number of Candidates

- Apriori principle:
 - If an itemset is frequent, then all of its subsets must also be frequent
- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

- Support of an itemset never exceeds the support of its subsets
- This is known as the anti-monotone property of support

Illustrating Apriori Principle

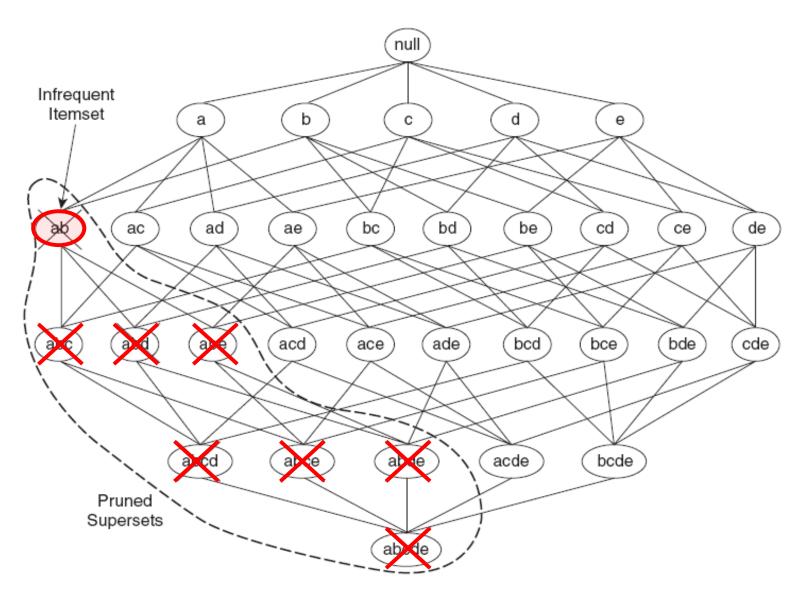


Figure 6.4. An illustration of support-based pruning. If $\{a,b\}$ is infrequent, then all supersets of $\{a,b\}$ are infrequent.

Illustrating Apriori Principle

Items (1-itemsets)

Count
4
2
4
3
4
1



Itemset	Count
{Bread,Milk}	3
{Breza,Seer}	2
{Bread,Diaper}	3
{Milk,≥eer}	2
(Milk,Diaper)	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3



Triplets (3-itemsets)

If every subset is considered,
${}^{6}C_{1} + {}^{6}C_{2} + {}^{6}C_{3} = 41$
With support-based pruning,
6 + 6 + 1 = 13

Itemset	Count
{Bread,Milk,Diaper}	3

Apriori Algorithm

•Method:

- Let k=1
- Generate frequent itemsets of length 1
- Repeat until no new frequent itemsets are identified
 - Generate length (k+1) candidate itemsets from length k frequent itemsets
 - Prune candidate itemsets containing subsets of length k that are infrequent
 - Count the support of each candidate by scanning the DB
 - Eliminate candidates that are infrequent, leaving only those that are frequent

Factors Affecting Complexity

- Choice of minimum support threshold
 - lowering support threshold results in more frequent itemsets
 - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
 - more space is needed to store support count of each item
 - if number of frequent items also increases, both computation and I/O costs may also increase
- Size of database
 - since Apriori makes multiple passes, run time of algorithm may increase with number of transactions
- Average transaction width
 - transaction width increases with denser data sets
 - This may increase max length of frequent itemsets and traversals of hash tree (number of subsets in a transaction increases with its width)

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Maximal Frequent Itemset

An itemset is maximal frequent if none of its immediate supersets

is frequent

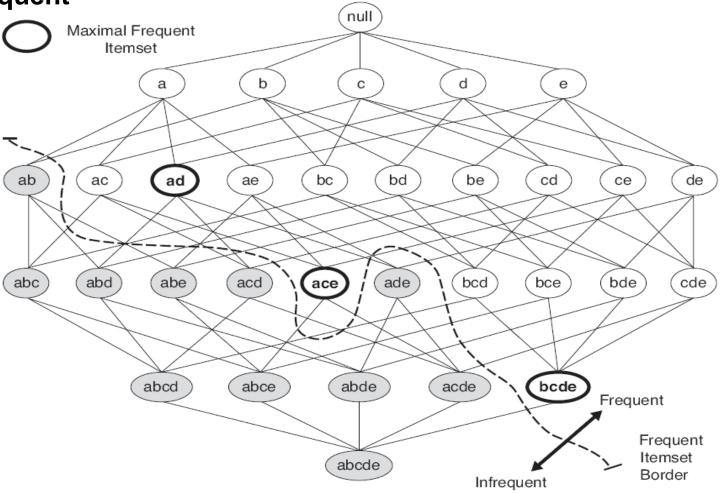


Figure 6.16. Maximal frequent itemset.

Closed Itemset

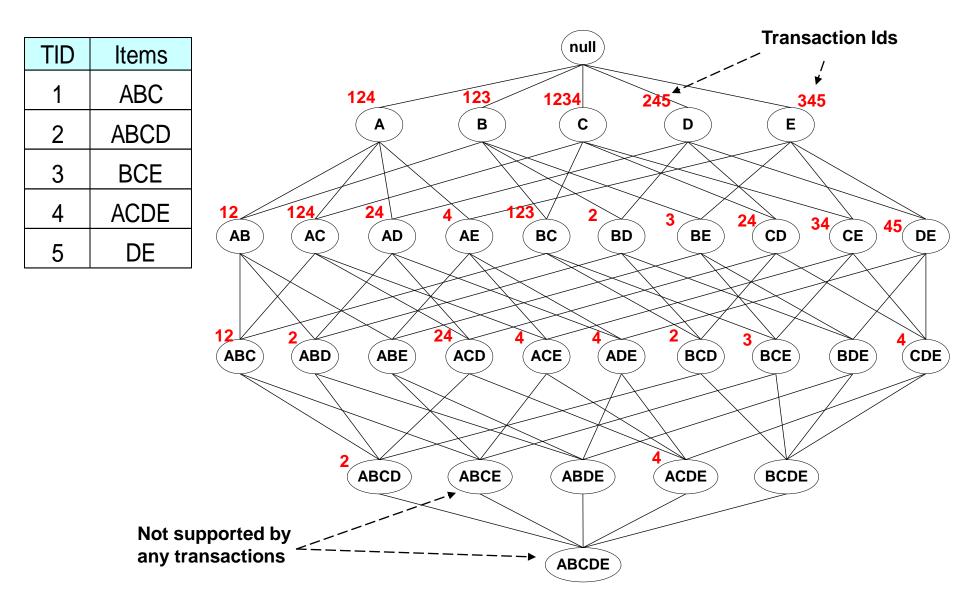
 An itemset is closed if none of its immediate supersets has the same support as the itemset (can only have smaller support -> see APRIORI principle)

TID	Items
1	{A,B}
2	$\{B,C,D\}$
3	$\{A,B,C,D\}$
4	$\{A,B,D\}$
5	$\{A,B,C,D\}$

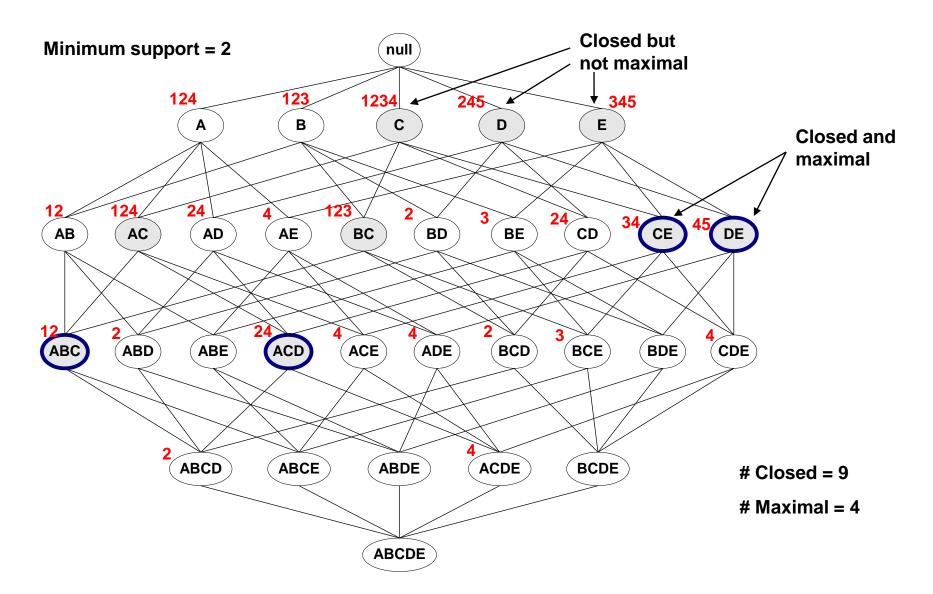
Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
$\{A,B,D\}$	3
$\{A,C,D\}$	2
{B,C,D}	3
{A,B,C,D}	2

Maximal vs Closed Itemsets



Maximal vs Closed Frequent Itemsets



Maximal vs Closed Itemsets

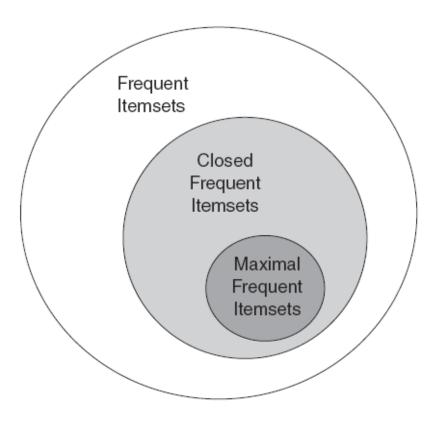


Figure 6.18. Relationships among frequent, maximal frequent, and closed frequent itemsets.

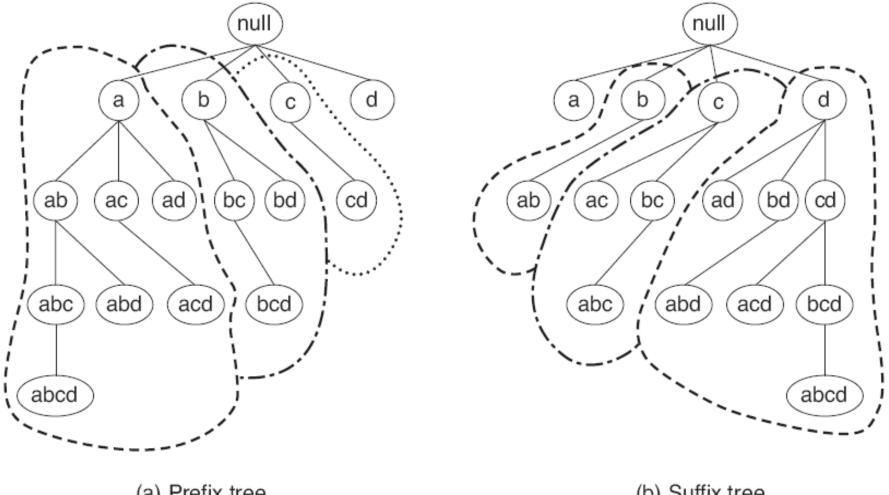


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Alternative Methods for Frequent Itemset Generation

- Traversal of Itemset Lattice
 - Equivalent Classes



(a) Prefix tree.

(b) Suffix tree.

Alternative Methods for Frequent Itemset Generation

•Representation of Database: horizontal vs vertical data layout

Horizontal Data Layout

TID	Items
1	a,b,e
2	b,c,d
3	c,e
4	a,c,d
5	a,b,c,d
6	a,e
7	a,b
8	a,b,c
9	a,c,d
10	b

Vertical Data Layout

а	b	С	d	е
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

Figure 6.23. Horizontal and vertical data format.

Alternative Algorithms

FP-growth

- Use a compressed representation of the database using an FP-tree
- Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets

ECLAT

- Store transaction id-lists (vertical data layout).
- Performs fast tid-list intersection (bit-wise XOR) to count itemset frequencies

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Rule Generation

•Given a frequent itemset L, find all non-empty subsets $X=f \subset L$ and Y=L-f such that $X \to Y$ satisfies the minimum confidence requirement

$$c(X \rightarrow Y) = \frac{\sigma(X \cup Y)}{\sigma(X)}$$

- If {A,B,C,D} is a frequent itemset, candidate rules:

ABC
$$\rightarrow$$
D, ABD \rightarrow C, ACD \rightarrow B, BCD \rightarrow A, A \rightarrow BCD, B \rightarrow ACD, C \rightarrow ABD, D \rightarrow ABC AB \rightarrow CD, AC \rightarrow BD, AD \rightarrow BC, BC \rightarrow AD, BD \rightarrow AC, CD \rightarrow AB,

If |L| = k, then there are $2^k - 2$ candidate association rules (ignoring $L \to \emptyset$ and $\emptyset \to L$)

Rule Generation

- •How to efficiently generate rules from frequent itemsets?
 - In general, confidence does not have an anti-monotone property

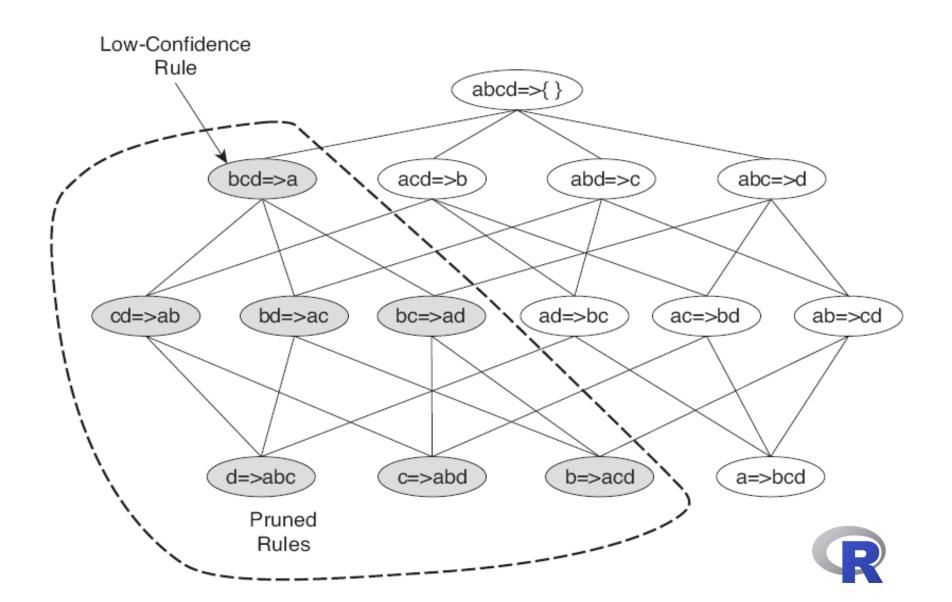
 $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$

- But confidence of rules generated from the same itemset has an anti-monotone property
- e.g., L = {A,B,C,D}:

$$c(ABC \rightarrow D) \ge c(AB \rightarrow CD) \ge c(A \rightarrow BCD)$$

 Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

Rule Generation for Apriori Algorithm



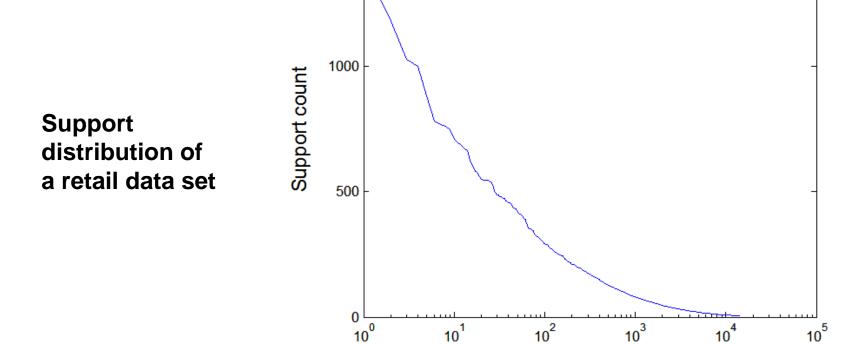
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Effect of Support Distribution

1500

Many real data sets have skewed support distribution



Sorted items

Effect of Support Distribution

- How to set the appropriate minsup threshold?
 - If *minsup* is set too high, we could miss itemsets involving interesting rare items (e.g., expensive products)
 - If minsup is set too low, it is computationally expensive and the number of itemsets is very large
- Using a single minimum support threshold may not be effective

Topics

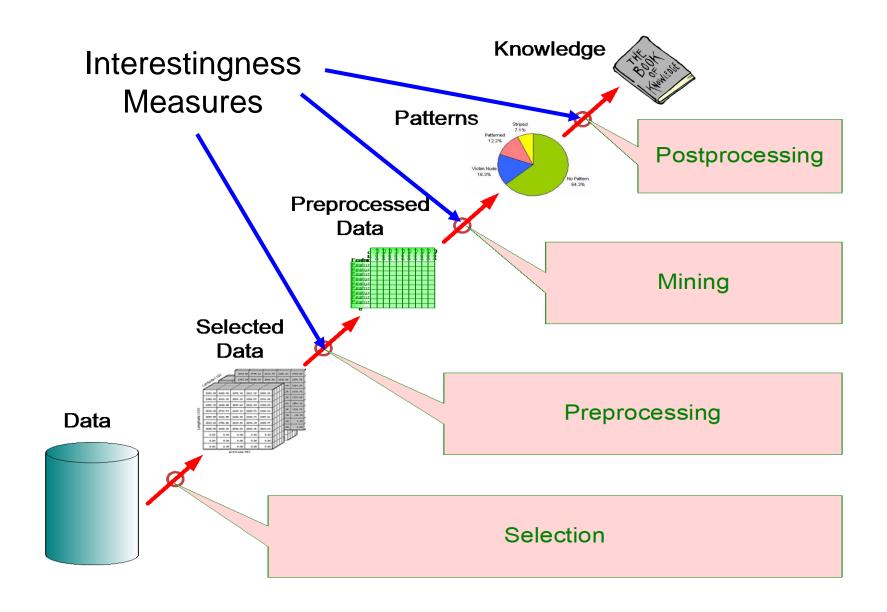
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Pattern Evaluation

- Association rule algorithms tend to produce too many rules. Many of them are
 - uninteresting or
 - redundant

- Interestingness measures can be used to prune/rank the derived patterns
- A rule $\{A,B,C\} \rightarrow \{D\}$ can be considered **redundant** if $\{A,B\}$ $\rightarrow \{D\}$ has the same or higher confidence.

Application of Interestingness Measure



Computing Interestingness Measure

•Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

	Y	\overline{Y}	
X	f ₁₁	f ₁₀	f ₁₊
\overline{X}	f ₀₁	f ₀₀	f _{o+}
	f ₊₁	f ₊₀	ΙΤΙ

f₁₁: support of X and Y

f₁₀: support of X and not Y

f₀₁: support of not X and Y

f₀₀: support of not X and not Y

error

Used to define various measures

e.g., support, confidence, lift, Gini, J-measure, etc.

$$\sup(\{X,Y\}) = \frac{f_{11}}{|T|} \text{ estimates } P(X,Y)$$

$$conf(X \to Y) = \frac{f_{11}}{f_{1+}} \text{ estimates } P(Y \mid X)$$

Drawback of Confidence

	Coffee	Coffee	
Tea	15	5	20
Tea	75	5	80
	90	10	100

Association Rule: Tea → Coffee

Support = P(Coffee, Tea) =
$$15/100 = 0.15$$

Confidence= P(Coffee | Tea) = $15/20 = 0.75$
but P(Coffee) = $90/100 = 0.9$

- ⇒ Although confidence is high, rule is misleading
- \Rightarrow P($\overline{\text{Coffee}}$ |Tea) = 75/80 = **0.9375**

Statistical Independence

- Population of 1000 students
 - 600 students know how to swim (S)
 - 700 students know how to bike (B)
 - 450 students know how to swim and bike (S,B)
 - P(S,B) = 450/1000 = 0.45 (observed joint prob.)
 - $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$ (expected under indep.)
 - $P(S,B) = P(S) \times P(B) => Statistical independence$
 - $P(S,B) > P(S) \times P(B) => Positively correlated$
 - $P(S,B) < P(S) \times P(B) => Negatively correlated$

Statistical-based Measures

 Measures that take statistical dependence into account for rule: $X \to Y$

Lift = Interest =
$$\frac{P(Y|X)}{P(Y)} = \frac{P(X,Y)}{P(X)P(Y)}$$

Deviation from independence $PS = P(X,Y) - P(X)P(Y)$

independence

$$\Phi = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1 - P(X)]P(Y)[1 - P(Y)]}}$$
Correlation

Example: Lift/Interest

	Coffee	Coffee	
Tea	15	5	20
Tea	75	5	80
	90	10	100

Association Rule: Tea \rightarrow Coffee

Conf(Tea
$$\rightarrow$$
 Coffee)= P(Coffee|Tea) = P(Coffee,Tea)/P(Tea)
= .15/.2 = 0.75
but P(Coffee) = 0.9
 \Rightarrow Lift(Tea \rightarrow Coffee) = P(Coffee,Tee)/(P(Coffee)P(Tee))
= .15/(.9 x .2) = **0.8333**

Note: Lift < 1, therefore Coffee and Tea are negatively associated

	#	Measure	Formula
There are lots of	1	ϕ -coefficient	$\frac{P(A,B) - P(A)P(B)}{\sqrt{P(A)P(B)(1 - P(A))(1 - P(B))}}$
measures proposed	2	Goodman-Kruskal's (λ)	$\frac{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}{\sum_{j} \max_{k} P(A_{j}, B_{k}) + \sum_{k} \max_{j} P(A_{j}, B_{k}) - \max_{j} P(A_{j}) - \max_{k} P(B_{k})}{2 - \max_{j} P(A_{j}) - \max_{k} P(B_{k})}$
in the literature	3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
	4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{\overline{B}} = \frac{\alpha-1}{\alpha-1}$
	5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB}) + P(A,B)P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})} + \sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha} - 1}{\sqrt{\alpha} + 1}$
Some measures are good for certain	6	Kappa (κ)	$\frac{P(A,B)P(AB)+\sqrt{P(A,B)P(A,B)}}{P(A,B)+P(\overline{A},B)-P(A)P(B)-P(\overline{A})P(\overline{B})}$ $1-P(A)P(B)-P(\overline{A})P(\overline{B})$ $\sum_{i}\sum_{j}P(A_{i},B_{j})\log\frac{P(A_{i},B_{j})}{P(A_{i})P(B_{j})}$
applications, but not	7	Mutual Information (M)	$\overline{\min(-\sum_{i} P(A_i) \log P(A_i), -\sum_{j} P(B_j) \log P(B_j))}$
for others	8	J-Measure (J)	$\max\left(P(A,B)\log(rac{P(B A)}{P(B)}) + P(A\overline{B})\log(rac{P(\overline{B} A)}{P(\overline{B})}), ight.$
			$P(A,B)\log(rac{P(A B)}{P(A)}) + P(\overline{A}B)\log(rac{P(\overline{A} B)}{P(A)})$
	9	Gini index (G)	$= \max \left(P(A)[P(B A)^2 + P(\overline{B} A)^2] + P(\overline{A})[P(B \overline{A})^2 + P(\overline{B} \overline{A})^2] \right)$
What criteria should			$-P(B)^2-P(\overline{B})^2,$
we use to determine			$P(B)[P(A B)^{2} + P(\overline{A} B)^{2}] + P(\overline{B})[P(A \overline{B})^{2} + P(\overline{A} \overline{B})^{2}]$
whether a measure			$-P(A)^2-P(\overline{A})^2$
is good or bad?	10	Support (s)	P(A,B)
•	11	Confidence (c)	$\max(P(B A), P(A B))$
	12	Laplace (L)	$\max\left(rac{NP(A,B)+1}{NP(A)+2},rac{NP(A,B)+1}{NP(B)+2} ight)$
What about Apriori-	13	Conviction (V)	$\max\left(rac{P(A)P(\overline{B})}{P(A\overline{B})},rac{P(B)P(\overline{A})}{P(B\overline{A})} ight)$
style support based	14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
pruning? How does	15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
it affect these	16	Piatetsky-Shapiro's (PS)	P(A,B) - P(A)P(B)
measures?	17	Certainty factor (F)	$\max\left(rac{P(B A)-P(B)}{1-P(B)},rac{P(A B)-P(A)}{1-P(A)} ight)$
	18	Added Value (AV)	$\max(P(B A) - P(B), P(A B) - P(A))$
	19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
	20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
	21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B A)-P(B),P(A B)-P(A))$

Comparing Different Measures

10 examples of contingency tables:

Example	f ₁₁	f ₁₀	f ₀₁	f ₀₀		
E1	8123	83	424	1370		
E2	8330	2	622	1046		
E3	9481	94	127	298		
E4	3954	3080	5	2961		
E5	2886	1363	1320	4431		
E6	1500	2000	500	6000		
E7	4000	2000	1000	3000		
E8	4000	2000	2000	2000		
E9	1720	7121	5	1154		
E10	61	2483	4	7452		

Rankings of contingency tables using various measures:

support & confidence

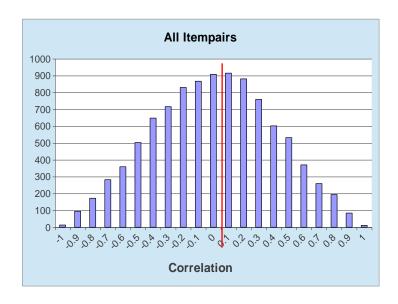
						_		_	-		_			_							
#	φ	λ	α	Q	Y	κ	M	J	G	s	c	L	V	I	IS	PS	\boldsymbol{F}	AV	S	<i>چ</i>	K
E1	1	1	3	3	3	1	2	2	1	3	5	5	4	6	2	2	4	6	1	2	5
E2	2	2	1	1	1	2	1	3	2	2	1	1	1	8	3	5	1	8	2	3	6
E3	3	3	4	4	4	3	3	8	7	1	4	4	6	10	1	8	6	10	3	1	10
E4	4	7	2	2	2	5	4	1	3	6	2	2	2	4	4	1	2	3	4	5	1
E5	5	4	8	8	8	4	7	5	4	7	9	9	9	3	6	3	9	4	5	6	3
E6	6	6	7	7	7	7	6	4	6	9	8	8	7	2	8	6	7	2	7	8	2
E7	7	5	9	9	9	6	8	6	5	4	7	7	8	5	5	4	8	5	6	4	4
E8	8	9	10	10	10	8	10	10	8	4	10	10	10	9	7	7	10	9	8	7	9
E9	9	9	5	5	5	9	9	7	9	8	3	3	3	7	9	9	3	7	9	9	8
E10	10	8	6	6	6	10	5	9	10	10	6	6	5	1	10	10	5	1	10	۵	7
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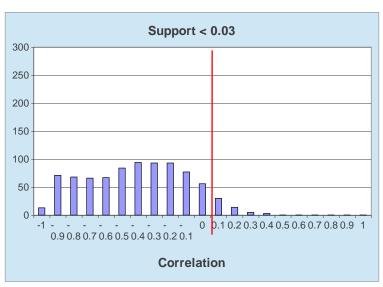
Support-based Pruning

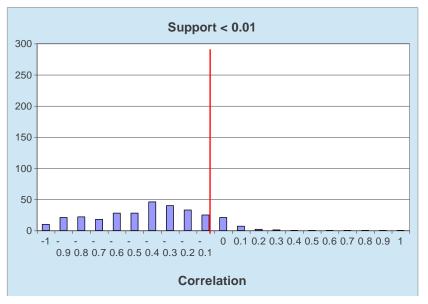
 Most of the association rule mining algorithms use support measure to prune rules and itemsets

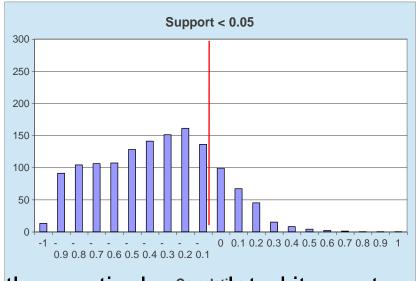
- Study effect of support pruning on correlation of itemsets
 - Generate 10,000 random contingency tables
 - Compute support and pairwise correlation for each table
 - Apply support-based pruning and examine the tables that are removed

Effect of Support-based Pruning









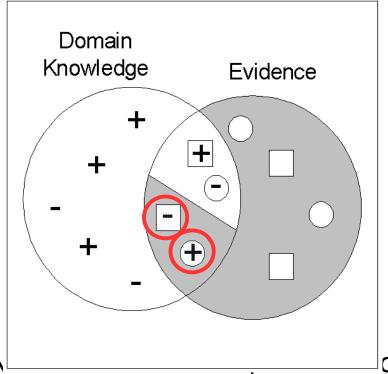
Support-based pruning eliminates mostly negatively correlated itemsets

Subjective Interestingness Measure

- Objective measure:
 - Rank patterns based on statistics computed from data
 - e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).
- Subjective measure:
 - Rank patterns according to user's interpretation
 - A pattern is subjectively interesting if it contradicts the expectation of a user (Silberschatz & Tuzhilin)
 - A pattern is subjectively interesting if it is actionable (Silberschatz & Tuzhilin)

Interestingness via Unexpectedness

Need to model expectation of users (domain knowledge)



- + Pattern expected to be frequent
- Pattern expected to be infrequent
- Pattern found to be frequent
- Pattern found to be infrequent
- **+** Expected Patterns
- Unexpected Patterns

of users with evidence from data

(i.e., extracted patterns)

Applications for Association Rules

- Market Basket Analysis
 Marketing & Retail. E.g., frequent itemsets give information about "other customer who bought this item also bought X"
- Exploratory Data Analysis
 Find correlation in very large (= many transactions),
 high-dimensional (= many items) data
- Intrusion Detection
 Rules with low support but very high lift
- Build Rule-based Classifiers
 Class association rules (CARs)