

Private-Key Encryption and Pseudorandomness (Part II)

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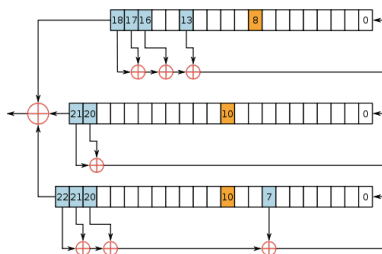
Cryptography, Autumn, 2021

- 1 Stream Ciphers And Chosen-Plaintext Attacks**
- 2 Constructing CPA-Secure Encryption Schemes**
- 3 Modes of Operation**
- 4 Security Against Chosen-Ciphertext Attacks (CCA)**

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Stream Ciphers

- **Stream cipher:** Enc. by XORing with pseudorandom stream
- **State of the art:** No standardized and popular one¹
Security is questionable, e.g., RC4 in WEP protocol in 802.11,
Linear Feedback Shift Registers (LFSRs) used in A5/1 for
GSM.

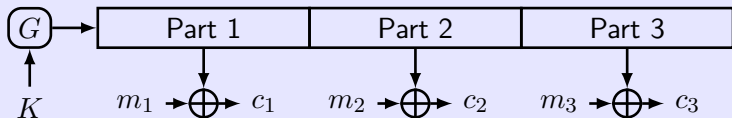


WARNING

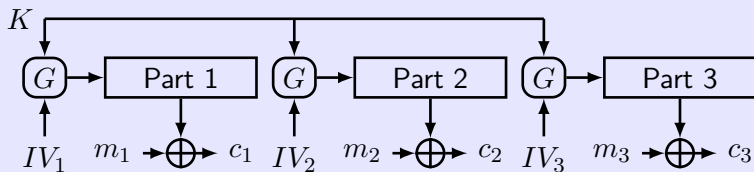
Don't use any stream cipher. If necessary, construct one from a block cipher.

¹eStream project worked on it. Salsa20/12 is a promising candidate.

Secure Multiple Encryptions Using a Stream Cipher



Synchronized Mode



Unsynchronized Mode

Initial vector IV is chosen *u.a.r* and public

Q: which mode is better in your opinion?

Keys (the IV -key pair) for multiple enc. must be independent

Attacks on 802.11b WEP

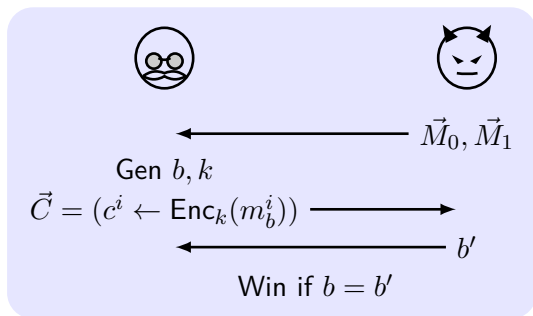
Unsynchronized mode: $\text{Enc}(m_i) := \langle IV_i, G(IV_i \| k) \oplus m_i \rangle$

- Length of IV is 24 bits, repeat IV after $2^{24} \approx 16\text{M}$ frames
- On some WiFi cards, IV resets to 0 after power cycle
- $IV_i = IV_{i-1} + 1$. For RC4, recover k after 40,000 frames

Security for Multiple Encryptions

The multiple-message eavesdropping experiment $\text{PrivK}_{\mathcal{A}, \Pi}^{\text{mult}}(n)$:

- 1 \mathcal{A} is given input 1^n , outputs $\vec{M}_0 = (m_0^1, \dots, m_0^t)$, $\vec{M}_1 = (m_1^1, \dots, m_1^t)$ with $\forall i, |m_0^i| = |m_1^i|$.
- 2 $k \leftarrow \text{Gen}(1^n)$, a random bit $b \leftarrow \{0, 1\}$ is chosen. Then $c^i \leftarrow \text{Enc}_k(m_b^i)$ and $\vec{C} = (c^1, \dots, c^t)$ is given to \mathcal{A} .
- 3 \mathcal{A} outputs b' . If $b' = b$, $\text{PrivK}_{\mathcal{A}, \Pi}^{\text{mult}} = 1$, otherwise 0.



Definition of Multi-Encryption Security

Definition 1

Π has **indistinguishable multiple encryptions in the presence of an eavesdropper** if \forall PPT \mathcal{A} , \exists negl such that

$$\Pr \left[\text{PrivK}_{\mathcal{A}, \Pi}^{\text{mult}}(n) = 1 \right] \leq \frac{1}{2} + \text{negl}(n).$$

Question:

Does any cipher we have learned so far have indistinguishable multiple encryptions in the presence of an eavesdropper?

Attack On Deterministic Multiple Encryptions

Question:

Generally, if Π 's encryption function is **deterministic**, i.e., a plaintext will be always encrypted into the same ciphertext with the same key, is Π multiple-encryption-secure?

Attack:

For the deterministic encryption, the adversary may generate $m_0^1 = m_0^2$ and $m_1^1 \neq m_1^2$, and then outputs $b' = 0$ if $c^1 = c^2$, otherwise $b' = 1$.

Chosen-Plaintext Attacks (CPA)

CPA: the adversary has the ability to obtain the encryption of plaintexts of its choice

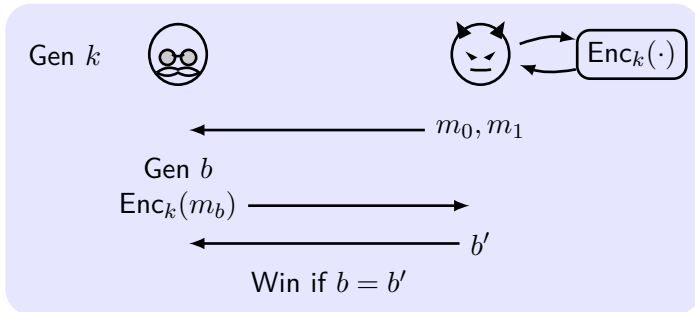
A story in WWII

- Navy cryptanalysts believe the ciphertext “AF” means “Midway island” in Japanese messages
- But the general did not believe that Midway island would be attacked
- Navy cryptanalysts sent a plaintext that the freshwater supplies at Midway island were low
- Japanese intercepted the plaintext and sent a ciphertext that “AF” was low in water
- The US forces dispatched three aircraft carriers and won

CPA Indistinguishability Experiment

The CPA indistinguishability experiment $\text{PrivK}_{\mathcal{A}, \Pi}^{\text{cpa}}(n)$:

- 1 $k \leftarrow \text{Gen}(1^n)$
- 2 \mathcal{A} is given input 1^n and **oracle access** $\mathcal{A}^{\text{Enc}_k(\cdot)}$ to $\text{Enc}_k(\cdot)$, outputs m_0, m_1 of the same length
- 3 $b \leftarrow \{0, 1\}$. Then $c \leftarrow \text{Enc}_k(m_b)$ is given to \mathcal{A}
- 4 \mathcal{A} **continues to have oracle access** to $\text{Enc}_k(\cdot)$, outputs b'
- 5 If $b' = b$, \mathcal{A} succeeded $\text{PrivK}_{\mathcal{A}, \Pi}^{\text{cpa}} = 1$, otherwise 0



Definition of CPA Security

Definition 2

Π has **indistinguishable encryptions under a CPA (CPA-secure)** if \forall PPT \mathcal{A} , \exists negl such that

$$\Pr \left[\text{PrivK}_{\mathcal{A}, \Pi}^{\text{cpa}}(n) = 1 \right] \leq \frac{1}{2} + \text{negl}(n).$$

- Q: Is any cipher we have learned so far CPA-secure? Why?

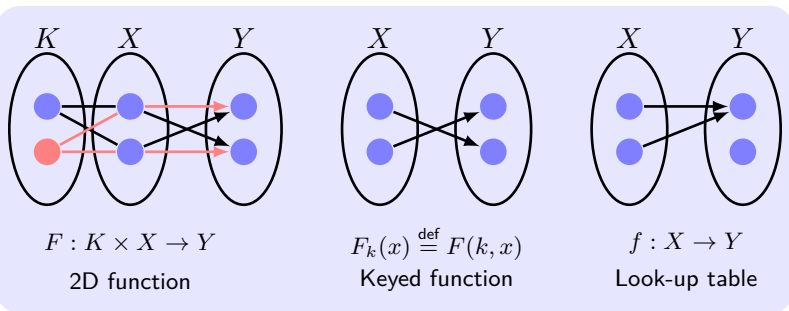
Proposition 3

*Any private-key encryption scheme that is CPA-secure also is **multiple-encryption-secure**.*

- Q: Does **multiple-encryption-security** mean CPA-security? (homework)

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Concepts on Pseudorandom Functions



- **Keyed function** $F : \{0, 1\}^* \times \{0, 1\}^* \rightarrow \{0, 1\}^*$
 $F_k : \{0, 1\}^* \rightarrow \{0, 1\}^*, F_k(x) \stackrel{\text{def}}{=} F(k, x)$
- **Look-up table** $f : \{0, 1\}^n \rightarrow \{0, 1\}^n$ with size = ? bits
- **Function family** Func_n : all functions $\{0, 1\}^n \rightarrow \{0, 1\}^n$.
 $|\text{Func}_n| = 2^{n \cdot 2^n}$
- **Length Preserving:** $\ell_{\text{key}}(n) = \ell_{\text{in}}(n) = \ell_{\text{out}}(n)$

Definition of Pseudorandom Function

Intuition: A PRF F generates a function F_k that is indistinguishable from truly random selected function f (look-up table) in Func_n .

However, the function has **exponential length**. Give D the deterministic **oracle access** $D^{\mathcal{O}}$ to the functions \mathcal{O} .

Definition 4

An efficient length-preserving, keyed function F is a **pseudorandom function (PRF)** if \forall PPT distinguishers D ,

$$\left| \Pr[D^{F_k(\cdot)}(1^n) = 1] - \Pr[D^{f(\cdot)}(1^n) = 1] \right| \leq \text{negl}(n),$$

where f is chosen *u.a.r* from Func_n .

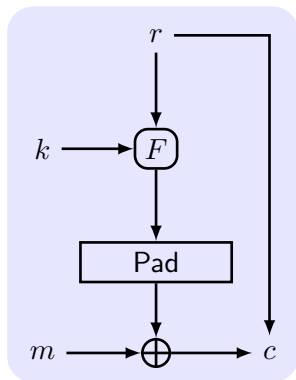
Q: Is the fixed-length OTP a PRF?

Q: Without knowing the key and the oracle access, could anyone learn something about the output from the input with a non-negligible probability?

Let $F : \{0, 1\}^n \times \{0, 1\}^n \rightarrow \{0, 1\}^n$ be a PRF. Is G a PRF?

- $G((k_1, k_2), x) = F(k_1, x) \| F(k_2, x)$
- $G(k, x) = F(k, x \oplus 1^n)$
- $G(k, x) = \begin{cases} F(k, x) & \text{when } x \neq 0^n \\ 0^n & \text{otherwise} \end{cases}$
- $G(k, x) = \begin{cases} F(k, x) & \text{when } x \neq 0^n \\ k & \text{otherwise} \end{cases}$
- $G(k, x) = F(k, x) \oplus F(k, x \oplus 1^n)$

CPA-Security from Pseudorandom Function



Construction 5

- Fresh random string r .
- $F_k(r)$: $|k| = |m| = |r| = n$.
- Gen: $k \in \{0, 1\}^n$.
- Enc: $s := F_k(r) \oplus m$,
 $c := \langle r, s \rangle$.
- Dec: $m := F_k(r) \oplus s$.

Theorem 6

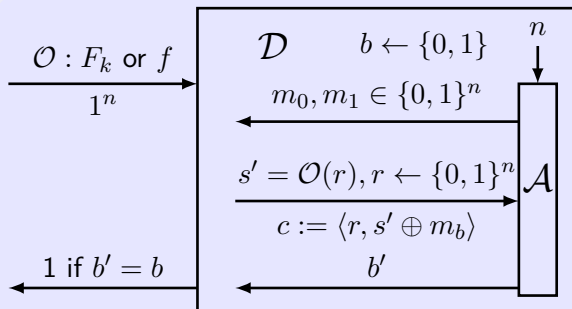
If F is a PRF, this fixed-length encryption scheme Π is CPA-secure.

Proof of CPA-Security from PRF

Idea: First, analyze the security in an idealized world where f is used in $\tilde{\Pi}$; next, claim that if Π is insecure when F_k was used then this would imply F_k is not PRF by reduction.

Proof.

Reduce D to \mathcal{A} :



Proof of CPA-Security from PRF (Cont.)

Proof.

Analyze $\Pr[\text{Break}]$, Break means $\text{PrivK}_{\mathcal{A}, \tilde{\Pi}}^{\text{cpa}}(n) = 1$:

\mathcal{A} collects $\{\langle r_i, f(r_i) \rangle\}$, $i = 1, \dots, q(n)$ with $q(n)$ queries;

The challenge $c = \langle r_c, f(r_c) \oplus m_b \rangle$.

- Repeat: $r_c \in \{r_i\}$ with probability $\frac{q(n)}{2^n}$. \mathcal{A} can know m_b .
- $\overline{\text{Repeat}}$: As OTP, $\Pr[\text{Break}] = \frac{1}{2}$

$$\begin{aligned}\Pr[\text{Break}] &= \Pr[\text{Break} \wedge \text{Repeat}] + \Pr[\text{Break} \wedge \overline{\text{Repeat}}] \\ &\leq \Pr[\text{Repeat}] + \Pr[\text{Break} | \overline{\text{Repeat}}] \\ &\leq \frac{q(n)}{2^n} + \frac{1}{2}.\end{aligned}$$

$$\Pr[D^{F_k(\cdot)}(1^n) = 1] = \Pr[\text{PrivK}_{\mathcal{A}, \Pi}^{\text{cpa}}(n) = 1] = \frac{1}{2} + \varepsilon(n).$$

$$\Pr[D^{f(\cdot)}(1^n) = 1] = \Pr[\text{PrivK}_{\mathcal{A}, \tilde{\Pi}}^{\text{cpa}}(n) = 1] = \Pr[\text{Break}] \leq \frac{1}{2} + \frac{q(n)}{2^n}.$$

$$\Pr[D^{F_k(\cdot)}(1^n) = 1] - \Pr[D^{f(\cdot)}(1^n) = 1] \geq \varepsilon(n) - \frac{q(n)}{2^n}. \quad \varepsilon(n) \text{ is negligible.} \quad \square$$

- For arbitrary-length messages, $m = m_1, \dots, m_\ell$

$$c := \langle r_1, F_k(r_1) \oplus m_1, r_2, F_k(r_2) \oplus m_2, \dots, r_\ell, F_k(r_\ell) \oplus m_\ell \rangle$$

Corollary 7

If F is a PRF, then Π is CPA-secure for arbitrary-length messages.

- **Efficiency:** $|c| = 2|m|$.

Pseudorandom Permutations

- **Bijection:** F is one-to-one and onto
- **Permutation:** A bijective function from a set to itself
- **Keyed permutation:** $\forall k, F_k(\cdot)$ is permutation
- F is a bijection $\iff F^{-1}$ is a bijection

Definition 8

An efficient, keyed permutation F is a **strong pseudorandom permutation (PRP)** if \forall PPT distinguishers D ,

$$\left| \Pr[D^{F_k(\cdot), F_k^{-1}(\cdot)}(1^n) = 1] - \Pr[D^{f(\cdot), f^{-1}(\cdot)}(1^n) = 1] \right| \leq \text{negl}(n),$$

where f is chosen *u.a.r* from the set of permutations on n -bit strings.

If F is a pseudorandom permutation then is it a PRF?

Let $X = \{0, 1\}$ (1 bit), answer the following questions.

- 1 What are the functions in the permutation over X ?
- 2 $K = \{0, 1\}$, what is the simplest permutation $F(k, x)$ over X ?
- 3 Is your F a secure PRP?
- 4 Is your F a secure PRF?
- 5 What if $X = \{0, 1\}^{128}$ and $K = \{0, 1\}^{128}$?
- 6 Could you give a (or another) PRP over $X = \{0, 1\}^{128}$?

Proposition 9

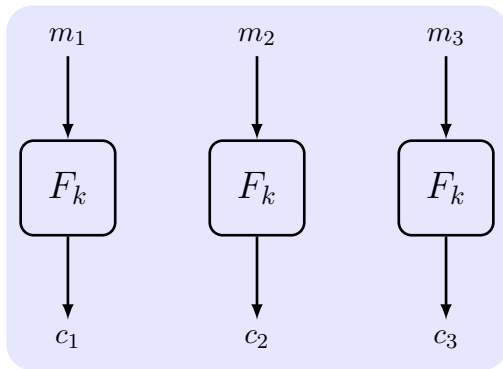
IF F is a PRP and additionally $\ell_{in}(n) \geq n$, then F is also a PRF.

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Modes of Operation:

- A way of encrypting arbitrary-length messages using a PRP or PRF
- A way of constructing a PRG from a PRP or PRF

Electronic Code Book (ECB) Mode

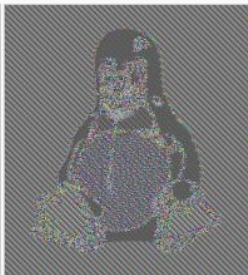


- Q: is it indistinguishable in the presence of an eavesdropper?
- Q: can F be any PRF?

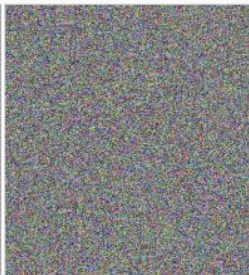
Attack on ECB mode



Original image

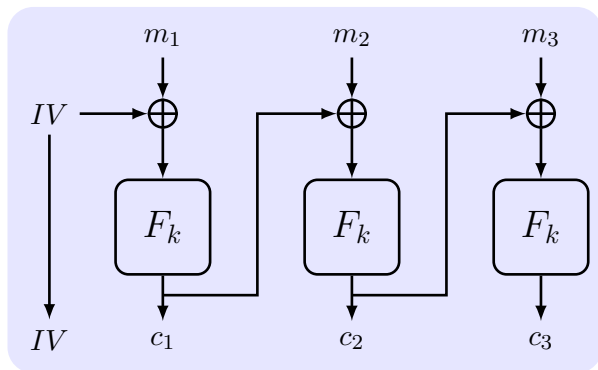


Encrypted using ECB mode



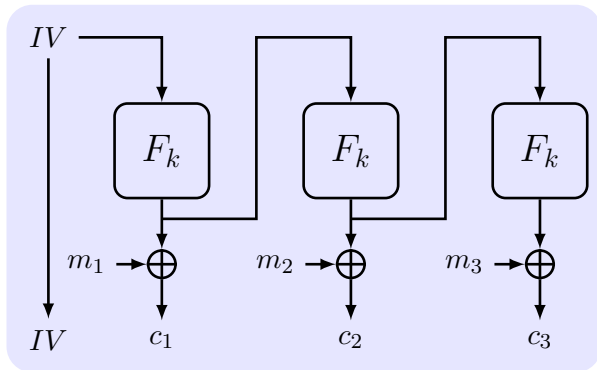
Modes other than ECB result in pseudo-randomness

Cipher Block Chaining (CBC) Mode



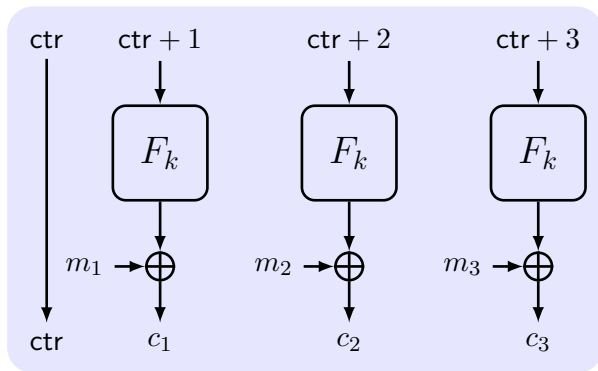
- IV : initial vector, a fresh random string.
- Q: is it CPA-secure? what if IV is always 0?
- Q: is the encryption parallelizable, i.e., outputting c_2 before getting c_1 ?
- Q: can F be any PRF?

Output Feedback (OFB) Mode



- Q: is it CPA-secure?
- Q: is the encryption parallelizable?
- Q: can F be any PRF?

Counter (CTR) Mode



- ctr is an IV
- Q: is it CPA-secure?
- Q: is the encryption parallelizable?
- Q: can F be any PRF?

CTR Mode Is CPA-secure

Theorem 10

If F is a PRF, then randomized CTR mode is CPA-secure.

Proof.

The message length and the number of query are $q(n)$.

Overlap: the sequence for the challenge overlaps the sequences for the queries from the adversary.

ctr^* : ctr in the challenge. ctr_i : ctr in the queries, $i = 1, \dots, q(n)$.

Overlap: $\text{ctr}_i - q(n) < \text{ctr}^* < \text{ctr}_i + q(n)$.

$$\Pr[\text{Overlap}] \leq \frac{2q(n) - 1}{2^n} \cdot q(n)$$



Proof of CPA-secure CTR Mode (Cont.)

Proof.

See proof of theorem 6. (1) Analyze Break : $\text{PrivK}_{\mathcal{A}, \tilde{\Pi}}^{\text{cpa}}(n) = 1$.

$$\begin{aligned}\Pr[\text{Break}] &= \Pr[\text{Break} \wedge \text{Overlap}] + \Pr[\text{Break} \wedge \overline{\text{Overlap}}] \\ &\leq \Pr[\text{Overlap}] + \Pr[\text{Break} | \overline{\text{Overlap}}] \\ &\leq \frac{2q(n)^2}{2^n} + \frac{1}{2}.\end{aligned}$$

(2) Reduce D to \mathcal{A}

$$\Pr[D^{f(\cdot)}(1^n) = 1] = \Pr[\text{PrivK}_{\mathcal{A}, \tilde{\Pi}}^{\text{cpa}}(n) = 1] \leq \frac{2q(n)^2}{2^n} + \frac{1}{2}$$

$$\Pr[D^{F_k(\cdot)}(1^n) = 1] = \Pr[\text{PrivK}_{\mathcal{A}, \Pi}^{\text{cpa}}(n) = 1] \leq \frac{1}{2} + \varepsilon(n)$$

If F is PRP, $\varepsilon(n)$ is negligible.



IV Should Not Be Predictable

If *IV* is predictable, then CBC/OFB/CTR mode is not CPA-secure.

Q: Why? (homework)

Bug in SSL/TLS 1.0

IV for record $\#i$ is last CT block of record $\#(i - 1)$.

API in OpenSSL

```
void AES_cbc_encrypt (  
    const unsigned char *in,  
    unsigned char        *out,  
    size_t                length,  
    const AES_KEY         *key,  
    unsigned char        *ivec,    User supplies IV  
    AES_ENCRYPT or AES_DECRYPT);
```


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Security Against CCA

The CCA indistinguishability experiment $\text{PrivK}_{\mathcal{A}, \Pi}^{\text{cca}}(n)$:

- 1 $k \leftarrow \text{Gen}(1^n)$.
- 2 \mathcal{A} is given input 1^n and oracle access $\mathcal{A}^{\text{Enc}_k(\cdot)}$ and $\mathcal{A}^{\text{Dec}_k(\cdot)}$, outputs m_0, m_1 of the same length.
- 3 $b \leftarrow \{0, 1\}$. $c \leftarrow \text{Enc}_k(m_b)$ is given to \mathcal{A} .
- 4 \mathcal{A} continues to have oracle access **except for c** , outputs b' .
- 5 If $b' = b$, \mathcal{A} succeeded $\text{PrivK}_{\mathcal{A}, \Pi}^{\text{cca}} = 1$, otherwise 0.

Definition 11

Π has **indistinguishable encryptions under a CCA (CCA-secure)** if \forall PPT \mathcal{A} , \exists negl such that

$$\Pr \left[\text{PrivK}_{\mathcal{A}, \Pi}^{\text{cca}}(n) = 1 \right] \leq \frac{1}{2} + \text{negl}(n).$$

Understanding CCA-security

- In real world, the adversary might conduct CCA by influencing what gets decrypted
 - If the communication is not authenticated, then an adversary may send certain ciphertexts on behalf of the honest party
- CCA-security implies “**non-malleability**”
- None of the above scheme is CCA-secure

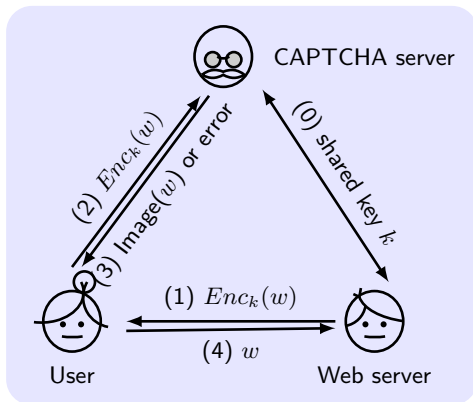
CCA against Construction 5

\mathcal{A} gives m_0, m_1 and gets $c = \langle r, F_k(r) \oplus m_b \rangle$, and then queries c' which is the same with c except that a single bit is flipped. The $m' = c' \oplus F_k(r)$ should be the same with m_b **except ____?**

Q: Show that the above modes (CBC, OFB and CTR) are also not CCA-secure. (homework)

Padding-Oracle Attacks: Real-world Case

CAPTCHA server will return an error when deciphering the CT of a CAPTCHA text received from a user.

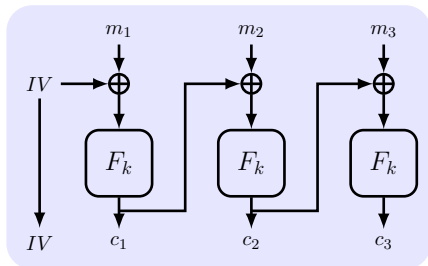


Padding-Oracle Attacks

PKCS #5 Padding: append b bytes of b to the message in order to make the total length a multiple of the block length (append a dummy block if needed). The decryption server will return a **Bad Padding Error** for incorrect padding.

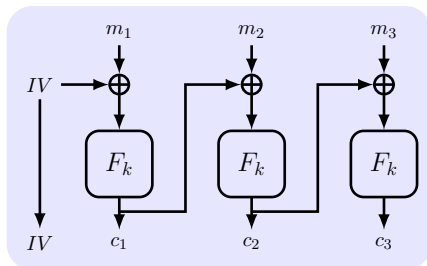
Padding-Oracle Attacks:

- In a one-block CBC, by modifying the 1st byte of IV , attacker can learn whether m is NULL. If yes, error will occur.



- append $\{b\}^b$ as a dummy block if m is NULL
- change the 1st byte of IV from x to y , get decrypted block $(x \oplus y \oplus b) || \{b\}^{b-1}$, and trigger an error

Padding-Oracle Attacks (Cont.)



- If no error, then learn whether m is 1 byte by modifying the 2nd byte of IV and so on (changing the ciphertext)
- Once learn the length of m , learn the last byte of m (s) by modifying the one before the last block in the ciphertext
- $m_{last} = \dots s || \{b\}^b$, $c_{last-1} = \dots t || \{\cdot\}^b$
- modify c_{last-1} to $c'_{last-1} = \dots u || (\{\cdot\}^b \oplus \{b\}^b \oplus \{b+1\}^b)$
- Q: If no padding error, then $s = ?$

- Asymptotic approach, proof of reduction, indistinguishable
- PRG, PRF, PRP, stream cipher, block cipher
- Security/construction against eavesdropping/CPA
- EBC, CBC, OFB, CTR
- CCA, padding-oracle attack