Digital Signature

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Outline

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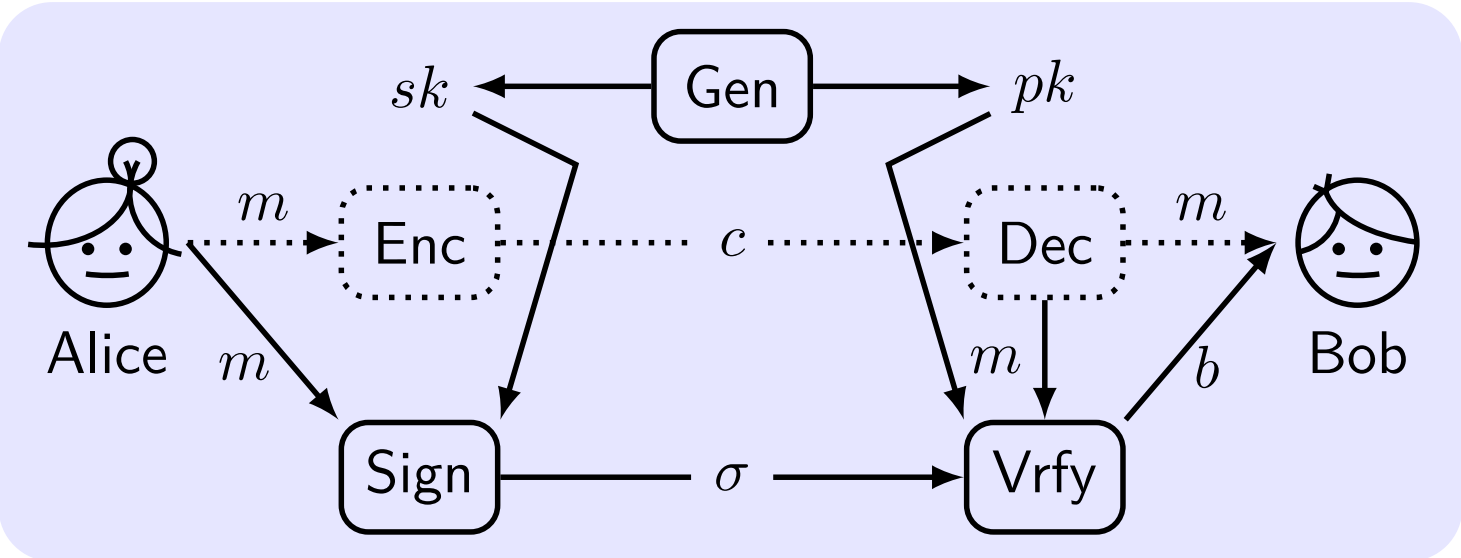
Digital Signatures – An Overview

- **Digital signature scheme** is a mathematical scheme for demonstrating the authenticity/integrity of a digital message
- Allow a **signer** S to "**sign**" a message with its own sk, anyone who knows S's pk can **verify** the authenticity/integrity
- (Comparing to MAC) digital signature is:
  - publicly verifiable
  - transferable
  - non-repudiation
  - but slow
- Q: What are the differences between digital signatures and handwritten signatures?
- Digital signature is NOT the "inverse" of public-key encryption
- Signatures are used to convey *trust* from a public key to the data which is signed

The Syntax of Digital Signature Scheme



- signature σ, a bit b means valid if $b = 1$; invalid if $b = 0$.
- **Key-generation** algorithm $(pk, sk) \leftarrow \mathsf{Gen}(1^n), |pk|, |sk| \geq n$.
- **Signing** algorithm $\sigma \leftarrow \mathsf{Sign}_{sk}(m)$.
- **Verification** algorithm $b := \mathsf{Vrfy}_{pk}(m, \sigma)$.
- **Basic correctness requirement**: $\mathsf{Vrfy}_{pk}(m, \mathsf{Sign}_{sk}(m)) = 1$.

Defining Signature Security

The signature experiment $\text{Sigforge}_{\mathcal{A},\Pi}(n)$:

1. $(pk, sk) \leftarrow \text{Gen}(1^n)$.
2. $\mathcal{A}$ is given input 1^n and oracle access to $\text{Sign}_{sk}(\cdot)$, and outputs (m, σ). $\mathcal{Q}$ is the set of queries to its oracle.
3. $\text{Sigforge}_{\mathcal{A},\Pi}(n) = 1 \iff \text{Vrfy}_{pk}(m, \sigma) = 1 \wedge m \notin \mathcal{Q}$.

Definition 1

A signature scheme Π is **existentially unforgeable under an adaptive CMA** if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that:

$$\Pr[\text{Sigforge}_{\mathcal{A},\Pi}(n) = 1] \leq \text{negl}(n).$$

Q: What's the difference on the ability of adversary between MAC and digital signature? What if an adversary is not limited to PPT?

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Insecurity of "Textbook RSA"

Construction 2

- Gen: *on input* 1^n *run* GenRSA(1^n) *to obtain* N, e, d. $pk = \langle N, e \rangle$ *and* $sk = \langle N, d \rangle$.
- Sign: *on input* sk *and* $m \in \mathbb{Z}_N^*$, $\sigma := [m^d \bmod N]$.
- Vrfy: *on input* pk *and* $m \in \mathbb{Z}_N^*$, $m \stackrel{?}{=} [\sigma^e \bmod N]$.

- **No-message attack**: choose an arbitrary $\sigma \in \mathbb{Z}_N^*$ and compute $m := [\sigma^e \bmod N]$. Output the forgery (m, σ).

$pk = \langle 15, 3 \rangle$, $\sigma = 2$, $m =? \; m^d =?$

- **Arbitrary message attack**: To forge a signature on m, choose a random m_1, set $m_2 := [m/m_1 \bmod N]$, obtain signatures σ_1, σ_2 on m_1, m_2.
  Q: $\sigma := [\_\_\_\_ \bmod N]$ is a valid signature on m.

Hashed RSA Signatures

Idea: Use hash function to break the strong algebraic relationship between the message and the signature.
RSA-FDH Signature Scheme: Random Oracle as a **Full Domain Hash (FDH)** whose image size $=$ the RSA modulus $N-1$.

- Gen: a hash function $H : \{0,1\}^* \to \mathbb{Z}_N^*$ is part of public key.
- Sign: $\sigma := [H(m)^d \bmod N]$.
- Vrfy: $\sigma^e \stackrel{?}{=} H(m) \bmod N$.
- **No-message attack**: Cannot invert $H(m) := \sigma^e \bmod N$
- **Arbitrary message attack**: $H(m/m_1)$ has no relationship with $H(m)$ and $H(m_1)$

Insecurity

There is NO known function H for which hashed RSA signatures are secure.

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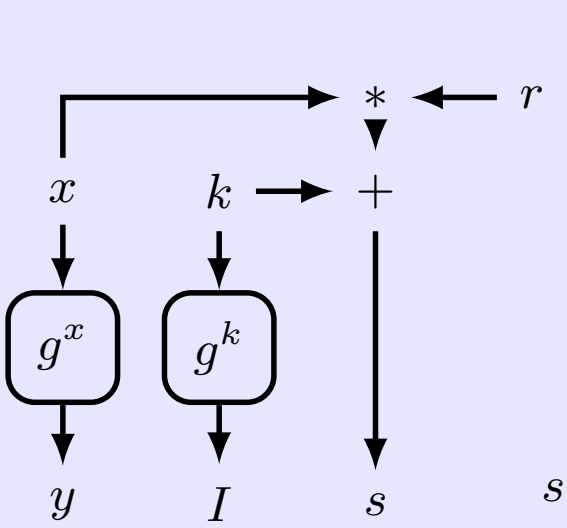
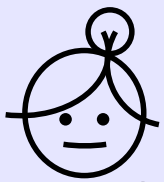
Overview of Schnorr Signature

- **Schnorr signature** (1988) is a typical instance showing the relationships among signature, identification and zero-knowledge proof.
- **Construction**: non-interactive protocol of Schnorr identification protocol, which an interactive zero-knowledge proof protocol for DLP solution.
- **Security:** proved by applying the Fiat–Shamir transformation to Schnorr identification protocol in ROM and under the assumption of DLP hardness.
- **Applications:** multisignature, threshold signature and blind signature, which are heavily used in cryptocurrency.

Schnorr Identification Scheme

The prover **publicly** proves that she is the one who knows the solution x of a DLP y in a 3-round Σ-protocol.





Prover (x)

Verifier $(\mathbb{G}, q, g, y)$

$k \leftarrow \mathbb{Z}_q;\ I := g^k$ $\xrightarrow{I}$

$\xleftarrow{r}$ $r \leftarrow \mathbb{Z}_q$

$s := [rx + k \mod q]$ $\xrightarrow{s}$

$g^s \cdot y^{-r} \stackrel{?}{=} I$

Why not a simpler identification protocol?

First, the verifier generates r and sends g^r; then the prover replies with $s = g^{rx}$; last, the verifier checks $y^r \stackrel{?}{=} s$.

Proof of Schnorr Identification Scheme

Theorem 3

If the discrete-log problem is hard, then the Schnorr identification scheme is secure.

Idea: If the attacker can let $g^s \cdot y^{-r} = I$, then the attacker can compute x.

Proof.

Reduce $\mathcal{A}'$ inverting y to $\mathcal{A}$ attacking the Schnorr scheme:

1. $\mathcal{A}'$ as a verifier, answering all queries, runs $\mathcal{A}$ as a prover.
2. When $\mathcal{A}$ outputs I, $\mathcal{A}'$ chooses $r_1 \in \mathbb{Z}_q$ and send to $\mathcal{A}$, who responds with s_1.
3. $\mathcal{A}'$ runs $\mathcal{A}$ 2nd time with the same I, sends $r_2 \in \mathbb{Z}_q$ to $\mathcal{A}$ who responds with s_2.
4. If $g^{s_1} \cdot h^{-r_1} = I$ and $g^{s_2} \cdot h^{-r_2} = I$ and $r_1 \neq r_2$ then output $x = [(s_1 - s_2) \cdot (r_1 - r_2)^{-1} \mod q]$. Else, output nothing.

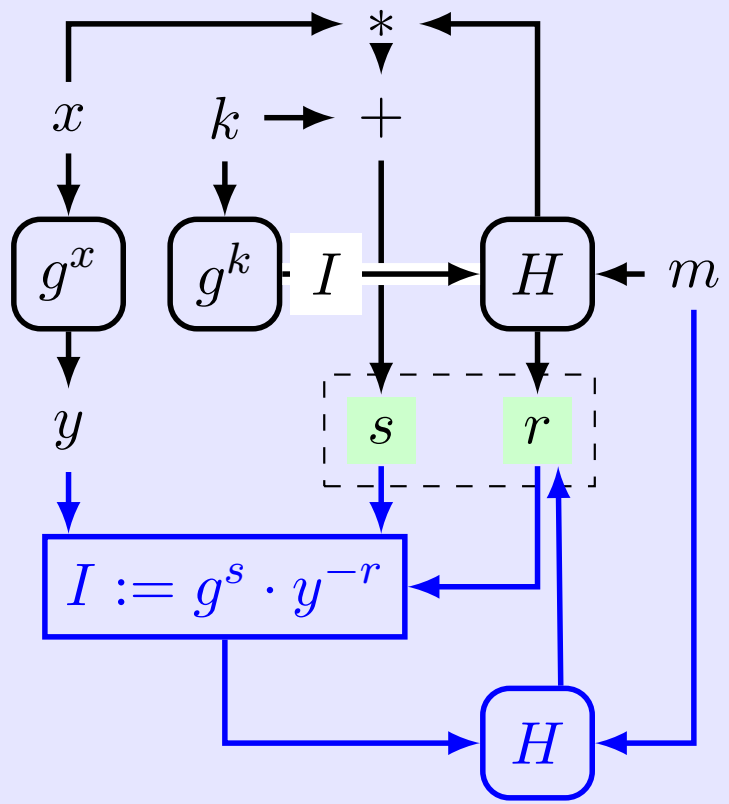
□

Schnorr Signature Scheme by Fiat-Shamir Transform

Fiat-Shamir transform: signature scheme constructed by letting the signer run the protocol by itself in ROM.

Sign: $k \leftarrow \mathbb{Z}_q$,
compute $I := g^k$,
$r := H(I, m)$,
$s := [rx + k \mod q]$
and output (r, s)

Vrfy: Compute I and
output 1 $\iff$ $H(I, m) \stackrel{?}{=} r$.



DSS/DSA

NIST published DSS (Digital Signature Standard) which uses Digital Signature Algorithm (DSA, a variant of ElGamal signature scheme), Elliptic Curve Digital Signature Algorithm (ECDSA), and RSA Signature Algorithm.

Construction 4

- Gen: $(\mathbb{G}, q, g) \leftarrow \mathcal{G}$. *Two hash functions* $H, F : \{0,1\}^* \to \mathbb{Z}_q$. $x \leftarrow \mathbb{Z}_q$ *and* $y := g^x$.
  $pk = \langle \mathbb{G}, q, g, y, H, F \rangle$. $sk = \langle \mathbb{G}, q, g, x, H, F \rangle$.
- Sign: $k \leftarrow \mathbb{Z}_q^*$ *and* $r := F(g^k)$, $s := (H(m) + xr) \cdot k^{-1}$. *Output* (r, s).
- Vrfy: *Output* $1 \iff r \stackrel{?}{=} F(g^{H(m) \cdot s^{-1}} y^{r \cdot s^{-1}})$.

Q: Is the verification correct?

Security of DSS/DSA

Insecurity

Security of DSS relies on the hardness of discrete log problem. But NO proof of security for DSS based on discrete log assumption.

The entropy, secrecy and uniqueness of k is critical.

- Case I: If k is predictable, then x leaks, since $s := [(H(m) + xr) \cdot k^{-1} \bmod q]$, and only x is unknown;
- Case II: If the same k is ever used to generate two different signatures under the same x, then both k and x leaks under two signatures.
  Q: how?
  This attack has been used to learn the private key of the Sony PlayStation (PS3) in 2010.

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One-Time Signature

One-Time Signature (OTS): Under a weaker attack scenario, sign only one message with one secret.
The OTS experiment $\text{Sigforge}^{\text{1-time}}_{\mathcal{A},\Pi}(n)$:

1. $(pk, sk) \leftarrow \text{Gen}(1^n)$.
2. $\mathcal{A}$ is given input 1^n and a single query m' to $\text{Sign}_{sk}(\cdot)$, and outputs (m, σ), $m \neq m'$.
3. $\text{Sigforge}^{\text{1-time}}_{\mathcal{A},\Pi}(n) = 1 \iff \text{Vrfy}_{pk}(m, \sigma) = 1$.

Definition 5

A signature scheme Π is **existentially unforgeable under a single-message attack** if $\forall$ PPT $\mathcal{A}$, $\exists$ negl such that:

$$\Pr[\text{Sigforge}^{\text{1-time}}_{\mathcal{A},\Pi}(n) = 1] \leq \text{negl}(n).$$

Lamport's OTS (1979)

Idea: OTS from OWF; one mapping per bit.

Construction 6

f is a one-way function.

- Gen*: on input 1^n, for $i \in \{1, \ldots, \ell\}$:*
  1. *choose random $x_{i,0}, x_{i,1} \leftarrow \{0,1\}^n$.*
  2. *compute $y_{i,0} := f(x_{i,0})$ and $y_{i,1} := f(x_{i,1})$.*

$$pk = \begin{pmatrix} y_{1,0} & y_{2,0} & \cdots & y_{\ell,0} \\ y_{1,1} & y_{2,1} & \cdots & y_{\ell,1} \end{pmatrix} \quad sk = \begin{pmatrix} x_{1,0} & x_{2,0} & \cdots & x_{\ell,0} \\ x_{1,1} & x_{2,1} & \cdots & x_{\ell,1} \end{pmatrix}.$$

- Sign*: $m = m_1 \cdots m_\ell$, output $\sigma = (x_{1,m_1}, \ldots, x_{\ell,m_\ell})$.*
- Vrfy*: $\sigma = (x_1, \ldots, x_\ell)$, output $1 \iff f(x_i) = y_{i,m_i}$, for all i.*

Theorem 7

If f is OWF, Π is OTS for messages of length polynomial ℓ.

Example of Lamport's OTS

Signing $m = 011$

$$sk = \begin{pmatrix} x_{1,0} & x_{2,0} & x_{3,0} \\ x_{1,1} & x_{2,1} & x_{3,1} \end{pmatrix} \implies \sigma = \_\_\_\_$$

$\sigma = (x_1, x_2, x_3)$:

$$pk = \begin{pmatrix} y_{1,0} & y_{2,0} & y_{3,0} \\ y_{1,1} & y_{2,1} & y_{3,1} \end{pmatrix} \implies \begin{array}{l} f(x_1) \stackrel{?}{=} \_\_\_\_ \\ f(x_2) \stackrel{?}{=} \_\_\_\_ \\ f(x_3) \stackrel{?}{=} \_\_\_\_ \end{array}$$

Proof of Lamport's OTS Security

Idea: If $m \neq m'$, then $\exists i^*, m_{i*} = b^* \neq m'_{i*}$. So to forge a signature on m can invert a single y_{i^*,b^*} at least.

Proof.

Reduce $\mathcal{I}$ inverting y to $\mathcal{A}$ attacking Π:

1. Construct pk: Choose $i^* \leftarrow \{1, \ldots, \ell\}$ and $b^* \leftarrow \{0, 1\}$, set $y_{i^*,b^*} := y$. For $i \neq i^*$, $y_{i,b} := f(x_{i,b})$.
2. $\mathcal{A}$ queries m': If $m'_{i_*} = b^*$, stop. Otherwise, return $\sigma = (x_{1,m'_1}, \ldots, x_{\ell,m'_\ell})$.
3. When $\mathcal{A}$ outputs (m, σ), $\sigma = (x_1, \ldots, x_\ell)$, if $\mathcal{A}$ output a forgery at (i^*, b^*): $\mathsf{Vrfy}_{pk}(m, \sigma) = 1$ and $m_{i^*} = b^* \neq m'_{i^*}$, then output x_{i^*,b^*}.

$$\Pr[\mathcal{I} \text{ succeeds}] \geq \frac{1}{2\ell} \Pr[\mathcal{A} \text{ succeeds}]$$

Stateful Signature Scheme

Idea: OTS by signing with **"new" key** derived from **"old" state**.

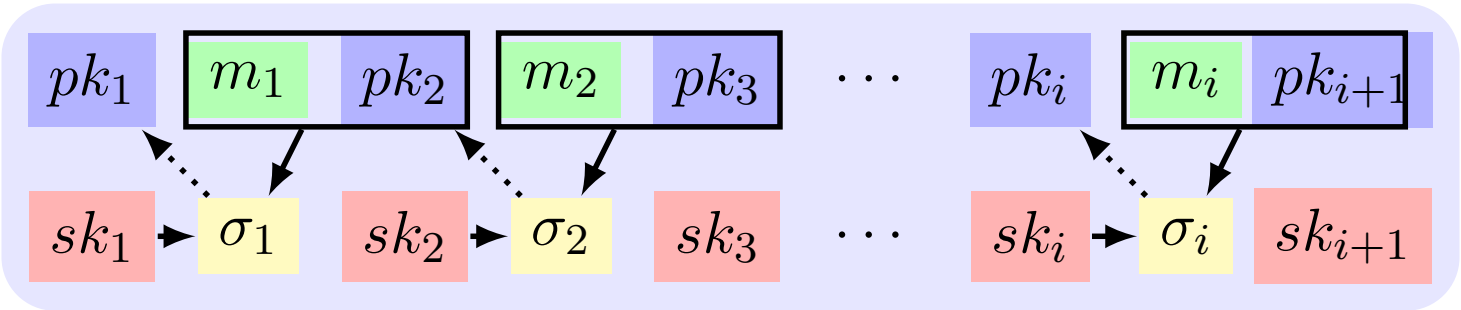
Definition 8 (Stateful signature scheme)

- **Key-generation** algorithm $(pk, sk, s_0) \leftarrow \mathsf{Gen}(1^n)$. s_0 is **initial state**.
- **Signing** algorithm $(\sigma, s_i) \leftarrow \mathsf{Sign}_{sk, s_{i-1}}(m)$.
- **Verification** algorithm $b := \mathsf{Vrfy}_{pk}(m, \sigma)$.

A simple stateful signature scheme for OTS:
Generate (pk_i, sk_i) independently, set $pk := (pk_1, \ldots, pk_\ell)$ and $sk := (sk_1, \ldots, sk_\ell)$.
Start from the state 1, sign the s-th message with sk_s, verify with pk_s, and update the state to $s + 1$.
Weakness: the upper bound ℓ must be fixed in advance.

"Chain-Based" Signatures

Idea: generate keys "on-the-fly" and sign the key chain.



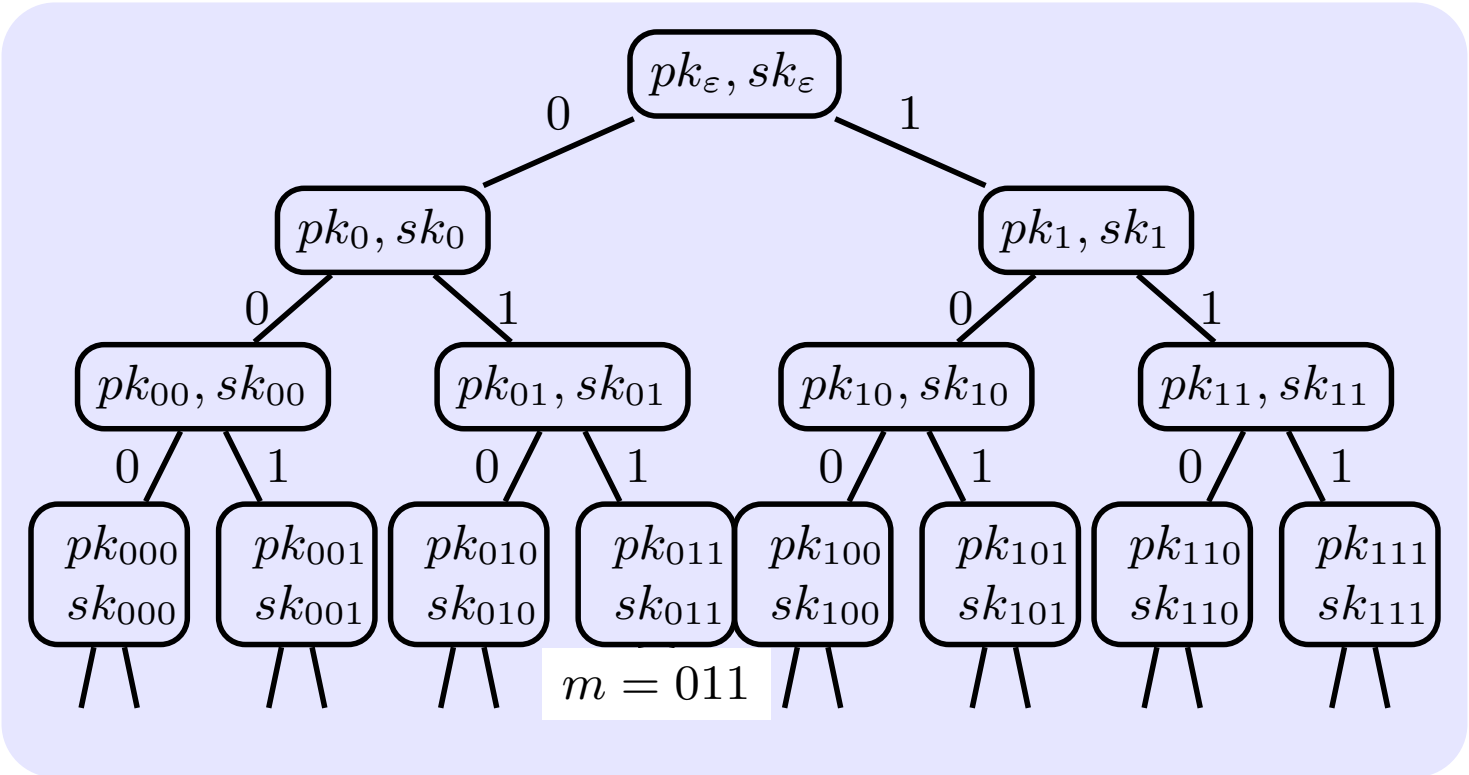
Use a single public key pk_1, sign each m_i and pk_{i+1} with sk_i:

$$\sigma_i \leftarrow \mathsf{Sign}_{sk_i}(m_i \| pk_{i+1}),$$

output $\langle pk_{i+1}, \sigma_i \rangle$, and verify σ_i with pk_i.
The signature is $(pk_{i+1}, \sigma_i, \{m_j, pk_{j+1}, \sigma_j\}_{j=1}^{i-1})$.
Weakness: stateful, not efficient, revealing all previous messages.

"Tree-Based" Signatures

Idea: generate a chain of keys for each message and sign the chain.



- root is ε (empty string), leaf is a message m, and internal nodes (pk_w, sk_w), where w is the prefix of m.
- each node pk_w "certifies" its children $pk_{w0} \| pk_{w1}$ or w.

A Stateless Solution

Idea: use deterministic randomness to emulate the state of tree.

Use PRF F and two keys k, k' (secrets) to generate pk_w, sk_w:

1. compute $r_w := F_k(w)$.
2. compute $(pk_w, sk_w) := \mathsf{Gen}(1^n; r_w)$, using r_w as random coins.

k' is used to generate r'_w that is used to compute σ_w.

Lemma 9

If OWF exist, then $\exists$ OTS (for messages of arbitrary length).

Theorem 10

If OWF exists, then $\exists$ (stateless) secure signature scheme.

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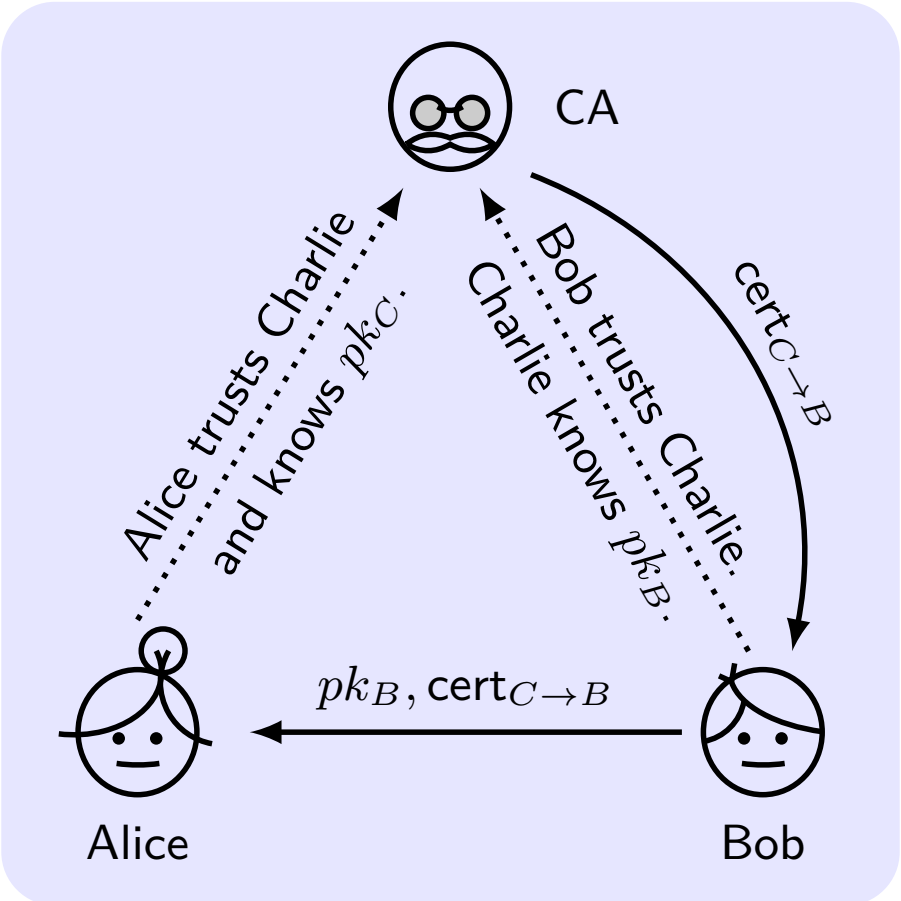
Certificates



Certificates $\text{cert}_{C\to B} \stackrel{\text{def}}{=} \text{Sign}_{sk_C}(\text{'Bob's key is } pk_B\text{'})$.

How Alice learn CA's key? How CA learn Bob's key?

Public-Key Infrastructure (PKI)

- **A single CA**: is trusted by everybody.
  - Strength: simple
  - Weakness: single-point-of-failure
- **Multiple CAs**: are trusted by everybody.
  - Strength: robust
  - Weakness: cannikin law
- **Delegation and certificate chains**: The trust is transitive.
  - Strength: ease the burden on the root CA.
  - Weakness: difficult for management, cannikin law.
- **"Web of trust"**: No central points of trust, e.g., PGP.
  - Strength: robust, work at "grass-roots" level.
  - Weakness: difficult to manage/give a guarantee on trust.

Invalidating Certificates

- **Expiration**: include an *expiry date* in the certificate.

$$\text{cert}_{C \to B} \stackrel{\text{def}}{=} \text{Sign}_{sk_C}(\text{'bob's key is } pk_B\text{'}, \text{ date}).$$

- **Revocation**: explicitly revoke the certificate.

$$\text{cert}_{C \to B} \stackrel{\text{def}}{=} \text{Sign}_{sk_C}(\text{'bob's key is } pk_B\text{'}, \#\#\#).$$

"###" represents the serial number of this certificate.
Cumulated Revocation: CA generates *certificate revocation list* (CRL) containing the serial numbers of all revoked certificates, signs CRL with the current date.

Exclusive Ownership

Exclusive Ownership: Given any signature generated by a public key pk, no adversary can find $pk' \neq pk$ such that the signature can be verified with pk'.

Duplicate Signature Key Selection Attack

- Does the validity of a signature with Bob's public key imply that Bob produced the signature with his private key?
- No. For example, the signature σ of m is generated by Alice with RSA. Bob's key pair is $(e = 1, d = 1)$ and $N = \sigma - m$. Then the signature is also verified with Bob's public key. $\sigma^e \bmod N = \sigma \bmod (\sigma - m) = m$.
- This attack could be used to cheat [Let's Encrypt] in the ownership of domain name. [1]
- Defence: Check the public key before verification.

[1] https://www.ietf.org/mail-archive/web/acme/current/msg00484.html

Signcryption

Signcryption: In a group, each has two pairs of keys: (ek, dk) for enc, and (vk, sk) for sig. And all public keys are distributed to everyone. A sender S and a receiver R should do to secure both privacy (no other learns m except S and R) and authenticity (R is sure about the message is sent from S).

Which signcryption scheme is secure?

- Enc-then-Auth: send $\left\langle S, c \leftarrow \mathsf{Enc}_{ek_R}(m), \mathsf{Sign}_{sk_S}(c) \right\rangle$
- Auth-then-Enc I: $\sigma \leftarrow \mathsf{Sign}_{sk_S}(m)$, send $\langle S, \mathsf{Enc}_{ek_R}(m \| \sigma) \rangle$
- Auth-then-Enc II: $\sigma \leftarrow \mathsf{Sign}_{sk_S}(m \| R)$, send $\langle S, \mathsf{Enc}_{ek_R}(S \| m \| \sigma) \rangle$
- Any other method?

Summary

- Digital signature provides publicly verifiable authentication and integrity.
- Signagures is related to something only someone knows, which can be publicly verified.
- Signatures are used to convey *trust* from a public key to the data which is signed.