一、求公式$(\neg p \to q) \to (q \to r)$的主合取范式和主析取范式。（10分）
真值表如下：

p	q	r	$\neg p$	$\neg p \to q$	$q \to r$	$(\neg q \to q) \to (q \to r)$
0	0	0	1	0	1	1
0	0	1	1	0	1	1
0	1	0	1	1	0	0
0	1	1	1	1	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	0	1	0	0
1	1	1	0	1	1	1

主合取范式为$(p \vee \neg q \vee r) \wedge (\neg p \vee \neg q \vee r)$
主析取范式为$(\neg p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (p \wedge \neg q \wedge \neg r) \vee (p \wedge \neg q \wedge r) \vee (p \wedge q \wedge r)$
二、用$'\downarrow'$等价表示公式$(p \to q) \to \neg r$。（10分）

$$\begin{aligned}(p \to q) \to \neg r &\iff \neg(\neg p \vee q) \vee \neg r \\ &\iff (\neg p \downarrow q) \vee \neg r \\ &\iff ((\neg p \downarrow q) \downarrow \neg r) \downarrow ((\neg p \downarrow q) \downarrow \neg r) \\ &\iff (((p \downarrow p) \downarrow q) \downarrow (r \downarrow r)) \downarrow (((p \downarrow p) \downarrow q) \downarrow (r \downarrow r))\end{aligned}$$

三、设A, B为FC中任意公式，举例说明$A \to B \vdash_{FC} \forall vA \to \forall vB$不一定成立。（5分）
根据演绎定理，原公式等价于$\vdash_{FC} (A \to B) \to (\forall vA \to \forall vB)$
构造如下解释
A, B中变元均为v，论域$D = \{0, 1\}, \bar{A}, \bar{B} : D \to \{T, F\}$
$A(0) = T, A(1) = T, B(0) = T, B(1) = F, \bar{v} = 0$
将此解释带入得

$$\begin{aligned}(\bar{A}(\bar{v}) \to \bar{B}(\bar{v})) \to (\forall v\bar{A} \to \forall v\bar{B}) &= (T \to T) \to (T \to F) \\ &= T \to F \\ &= F\end{aligned}$$

由此可得，在此解释下，该公式不成立。
四、分别用$'\uparrow'$和$'\downarrow'$等价表示公式$\neg(p \wedge \neg q) \wedge (q \wedge r)$
首先将公式化简

$$\begin{aligned}\neg(p \wedge \neg q) \wedge (q \wedge r) &\iff (\neg p \vee q) \wedge (q \wedge r) \\ &\iff (\neg p \wedge q \wedge r) \vee (q \wedge r) \\ &\iff q \wedge r\end{aligned}$$

用$'\uparrow$表示$'$

$$\begin{aligned}\neg(p \wedge \neg q) \wedge (q \wedge r) &\iff q \wedge r \\ &\iff (q \uparrow r) \uparrow (q \uparrow r)\end{aligned}$$

用‘$\downarrow'$表示

$$\begin{aligned}\neg(p \wedge \neg q) \wedge (q \wedge r) &\iff q \wedge r \\ &\iff \neg(\neg q \vee \neg r) \\ &\iff \neg q \downarrow \neg r \\ &\iff (q \downarrow q) \downarrow (r \downarrow r)\end{aligned}$$

五、在命题逻辑演算系统PC中证明：（20分）
$(1) \vdash \neg C \to (\neg B \to \neg(\neg B \to C))$
$1.(\neg B \to C) \to (\neg C \to B)$ 逆否
$2.((\neg B \to C) \to (\neg C \to B)) \to (\neg C \to ((\neg B \to C) \to B))$ 前件互换定理

$3.\neg C \to ((\neg B \to C) \to B)$　$1,2r_{mp}$
$4.((\neg B \to C) \to B) \to (\neg B \to \neg(\neg B \to C))$　逆否
$5.\neg C \to (\neg B \to \neg(\neg B \to C))$　$3,4$三段论
$(2) \vdash ((A \to (B \to C)) \to A) \to A$
$1.\neg A \to (A \to (B \to C))$　定理3.1.3
$2.\neg(A \to (B \to C)) \to A$　1逆否
$3.A \to A$　定理3.1.1
$4.(\neg(A \to (B \to C)) \to A) \to ((A \to A) \to (((A \to (B \to C)) \to A) \to A))$　定理3.1.14
$5.((A \to (B \to C)) \to A) \to A$　$2,3,4r_{mp}$
$(3) \vdash (A \to (B \to \neg C)) \to (C \to (A \to \neg B))$
$1.\neg A \to (A \to (C \to \neg B))$　定理3.1.3
$2.(A \to (C \to \neg B)) \to (C \to (A \to \neg B))$　前件互换定理
$3.\neg A \to (C \to (A \to \neg B))$　$1,2$三段论
$4.(B \to \neg C) \to (C \to \neg B)$　逆否
$5.(C \to \neg B) \to (A \to (C \to \neg B))$　$A1$
$6.(B \to \neg C) \to (A \to (C \to \neg B))$　$4,5$三段论
$7.(B \to \neg C) \to (C \to (A \to \neg B))$　$2,6$三段论
$8.(\neg A \to (C \to (A \to \neg B))) \to (((B \to \neg C) \to (C \to (A \to \neg B))) \to ((A \to (B \to \neg C)) \to (C \to (A \to \neg B))))$　定理3.1.14

$9.(A \to (B \to \neg C)) \to (C \to (A \to \neg B))$　$3,7,8r_{mp}$
$(4)((\neg A \to B) \to C) \to D, \neg D \to \neg B, \neg A \vdash D$
$1.(B \to D) \to ((((\neg A \to B) \to C) \to D) \to ((\neg B \to ((\neg A \to B) \to C)) \to D))$　定理3.1.14
$2.(\neg D \to \neg B) \to (B \to D)$　$A3$
$3.\neg D \to \neg B$　前提
$4.B \to D$　$2,3r_{mp}$
$5.((\neg A \to B) \to C) \to D$　前提
$6.\neg A \to (A \to (B \to \neg C))$　定理3.1.3
$7.\neg A$　前提
$8.A \to (B \to \neg C)$　$6,7r_{mp}$
$9.B \to (\neg B \to C)$　定理3.1.3
$10.(A \to (B \to \neg C)) \to ((B \to (\neg B \to C)) \to ((\neg A \to B) \to (\neg B \to C)))$　定理3.1.14
$11.(\neg A \to B) \to (\neg B \to C)$　$8,9,10r_{mp}$
$12.((\neg A \to B) \to (\neg B \to C)) \to (\neg B \to ((\neg A \to B) \to C))$　前件互换定理
$13.\neg B \to ((\neg A \to B) \to C)$　$11,12r_{mp}$
$14.D$　$1,4,5,13r_{mp}$

六、在ND中证明：
$(1) \vdash (\neg A \lor B) \land (\neg B \lor C) \to (\neg A \lor C)$
只需证$(\neg A \lor B) \land (\neg B \lor C) \vdash (\neg A \lor C)$　演绎定理
$1.(\neg A \lor B) \land (\neg B \lor C) \vdash (\neg A \lor B) \land (\neg B \lor C)$　公理
$2.(\neg A \lor B) \land (\neg B \lor C) \vdash \neg A \lor B$　$1 \land$消除
$3.(\neg A \lor B) \land (\neg B \lor C); \neg A \vdash \neg A$　公理
$4.(\neg A \lor B) \land (\neg B \lor C); \neg A \vdash \neg A \lor C$　$3 \lor$引入
$5.(\neg A \lor B) \land (\neg B \lor C); B \vdash (\neg A \lor B) \land (\neg B \lor C)$　公理
$6.(\neg A \lor B) \land (\neg B \lor C); B \vdash \neg B \lor C$　$5 \land$消除
$7.(\neg A \lor B) \land (\neg B \lor C); B; \neg B \vdash B$　公理
$8.(\neg A \lor B) \land (\neg B \lor C); B; \neg B \vdash \neg B$　公理
$9.(\neg A \lor B) \land (\neg B \lor C); B; \neg B \vdash C$　$8,9\neg$消除
$10.(\neg A \lor B) \land (\neg B \lor C); B; C \vdash C$　公理
$11.(\neg A \lor B) \land (\neg B \lor C); B \vdash C$　$6,9,10 \lor$消除
$12.(\neg A \lor B) \land (\neg B \lor C); B \vdash \neg A \lor C$　$11 \lor$引入
$13.(\neg A \lor B) \land (\neg B \lor C) \vdash (\neg A \lor C)$　$2,4,12 \lor$消除
$(2) \vdash (\neg A \to \neg(A \to \neg B)) \to A$
只需证$\neg A \to \neg(A \to \neg B) \vdash A$　演绎定理
$1.\neg A \to \neg(A \to \neg B); \neg A \vdash \neg A$　公理
$2.\neg A \to \neg(A \to \neg B); \neg A \vdash \neg A \to \neg(A \to \neg B)$　公理
$3.\neg A \to \neg(A \to \neg B); \neg A \vdash \neg(A \to \neg B)$　$1,2 \to$消除
$4.\neg A \to \neg(A \to \neg B); \neg A; A \vdash \neg A$　公理
$5.\neg A \to \neg(A \to \neg B); \neg A; A \vdash A$　公理
$6.\neg A \to \neg(A \to \neg B); \neg A; A \vdash \neg B$　$4,5\neg$消除

$7.\neg A \to \neg(A \to \neg B); \neg A \vdash A \to \neg B$　6演绎定理

$8.\neg A \to \neg(A \to \neg B) \vdash A$　$3,7\neg$引入

七、在FC中证明：（20分）

(1) $\vdash (\exists x P(x) \to \forall x Q(x)) \to \forall x(P(x) \to Q(x))$

只需证$\exists x P(x) \to \forall x Q(x) \vdash \forall x(P(x) \to Q(x))$　演绎定理

$1.P(x) \to \exists x P(x)$　定理5.2.2　前提

$2.\forall x Q(x) \to Q(x)$　定理5.2.1

$3.\exists x P(x) \to \forall x Q(x) \vdash \exists x P(x) \to \forall x Q(x)$　前提

$4.\exists x P(x) \to \forall x Q(x) \vdash P(x) \to Q(x)$　$1,3,2$传递

$5.\exists x P(x) \to \forall x Q(x) \vdash \forall x(P(x) \to Q(x))$　4定理5.2.5

(2) $\forall x(P(x) \to \neg(Q(y) \to \neg R(x))) \vdash \exists x P(x) \to Q(y)$

只需证$\forall x(P(x) \to \neg(Q(y) \to \neg R(x))), \exists x P(x) \vdash Q(y)$　演绎定理

$1.\forall x(P(x) \to \neg(Q(y) \to \neg R(x))), \exists x P(x) \vdash \exists x P(x)$

$2.\forall x(P(x) \to \neg(Q(y) \to \neg R(x))), \exists x P(x); P(x) \vdash P(x)$　前提

$3.\forall x(P(x) \to \neg(Q(y) \to \neg R(x))), \exists x P(x); P(x) \vdash \forall x(P(x) \to \neg(Q(y) \to \neg R(x)))$　前提

$4.\forall x(P(x) \to \neg(Q(y) \to \neg R(x))) \to (P(x) \to \neg(Q(y) \to \neg R(x)))$　定理5.2.1

$5.\forall x(P(x) \to \neg(Q(y) \to \neg R(x))), \exists x P(x); P(x) \vdash P(x) \to \neg(Q(y) \to \neg R(x))$　$3,4r_{mp}$

$6.\forall x(P(x) \to \neg(Q(y) \to \neg R(x))), \exists x P(x); P(x) \vdash \neg(Q(y) \to \neg R(x))$　$2,5r_{mp}$

$7.\neg Q(y) \to (Q(y) \to \neg R(x))$　定理3.1.3

$8.\neg(Q(y) \to \neg R(x)) \to Q(y)$　7逆否

$9.\forall x(P(x) \to \neg(Q(y) \to \neg R(x))), \exists x P(x); P(x) \vdash Q(y)$　$6,8r_{mp}$

$10.\forall x(P(x) \to \neg(Q(y) \to \neg R(x))), \exists x P(x) \vdash Q(y)$　$1,9$存在消除

八、只要是计算机系的本科生或者研究生，就一定学过C语言和$Java$语言。

如果是学过C语言或者$C++$语言的学生，那么就一定会编程。

因此只要是计算机系的本科生，就会编程。

将上面三句话分别用谓词公式表示出来，并在FC中证明其推理的正确性。（15分）

设x是全体学生，

$P(x)$表示x是计算机系的本科生，$Q(x)$表示x是计算机系的研究生，

$R(x)$表示x学过C语言，$S(x)$表示x学过$Java$语言，

$T(x)$表示x学过$C++$语言，$U(x)$表示x会编程

第一句：$\forall x((P(x) \wedge \neg Q(x)) \vee (\neg P(x) \wedge Q(x)) \to R(x) \wedge S(x))$

第二句：$\forall x(R(x) \vee T(x) \to U(x))$

第三句：$\forall x((P(x) \wedge \neg Q(x)) \vee (\neg P(x) \wedge Q(x)) \to R(x) \wedge S(x)), \forall x(R(x) \vee T(x) \to U(x)) \vdash \forall x(P(x) \wedge \neg Q(x) \to U(x))$

设公式集$\Gamma = \{\forall x((P(x) \wedge \neg Q(x)) \vee (\neg P(x) \wedge Q(x)) \to R(x) \wedge S(x)), \forall x(R(x) \vee T(x) \to U(x))\}$

$1.\Gamma; P(x) \wedge \neg Q(x) \vdash P(x) \wedge \neg Q(x)$　前提

$2.P(x) \wedge \neg Q(x) \to (P(x) \wedge \neg Q(x)) \vee (\neg P(x) \wedge Q(x))$　定理3.1.15

$3.\Gamma; P(x) \wedge \neg Q(x) \vdash (P(x) \wedge \neg Q(x)) \vee (\neg P(x) \wedge Q(x))$　$1,2r_{mp}$

$4.\Gamma; P(x) \wedge \neg Q(x) \vdash \forall x((P(x) \wedge \neg Q(x)) \vee (\neg P(x) \wedge Q(x)) \to R(x) \wedge S(x))$　前提

$5.\forall x((P(x) \wedge \neg Q(x)) \vee (\neg P(x) \wedge Q(x)) \to R(x) \wedge S(x)) \to ((P(x) \wedge \neg Q(x)) \vee (\neg P(x) \wedge Q(x)) \to R(x) \wedge S(x))$　定理5.2.1

$6.\Gamma; P(x) \wedge \neg Q(x) \vdash (P(x) \wedge \neg Q(x)) \vee (\neg P(x) \wedge Q(x)) \to R(x) \wedge S(x)$　$4,5r_{mp}$

$7.\Gamma; P(x) \wedge \neg Q(x) \vdash R(x) \wedge S(x)$　$3,6r_{mp}$

$8.R(x) \wedge S(x) \to R(x)$　定理3.1.16

$9.\Gamma; P(x) \wedge \neg Q(x) \vdash R(x)$　$7,8r_{mp}$

$10.R(x) \to R(x) \vee T(x)$　定理3.1.15

$11.\Gamma; P(x) \wedge \neg Q(x) \vdash R(x) \vee T(x)$　$9,10r_{mp}$

$12.\Gamma; P(x) \wedge \neg Q(x) \vdash \forall x(R(x) \vee T(x) \to U(x))$　前提

$13.\forall x(R(x) \vee T(x) \to U(x)) \to (R(x) \vee T(x) \to U(x))$　定理5.2.1

$14.\Gamma; P(x) \wedge \neg Q(x) \vdash R(x) \vee T(x) \to U(x)$　$12,13r_{mp}$

$15.\Gamma; P(x) \wedge \neg Q(x) \vdash U(x)$　$11,14r_{mp}$

$16.\Gamma \vdash P(x) \wedge \neg Q(x) \to U(x)$　15演绎定理

$17.\Gamma \vdash \forall x(P(x) \wedge \neg Q(x) \to U(x))$　16定理5.2.5

综上，以上推理正确