

Overview: Switching Algebra & Combinational Logic Design

- * Switching Algebra

- * Combinational Circuit Analysis & Synthesis



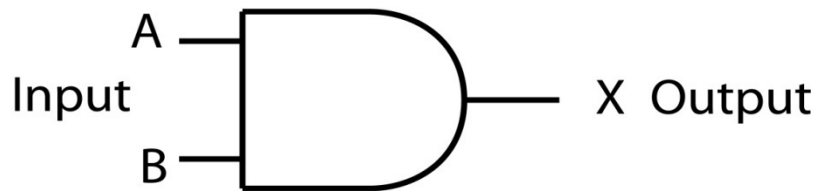
Chapters 4 & 6 – “Digital Design: Principles and Practices” book

Combinational Circuit Analysis

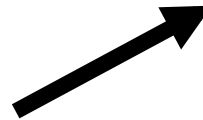
- *Analysis of combinational circuits:*
 - It requires a formal description of their logic function.
 - *Logic function description allows:*
 - *Determination of circuit behaviour* for different input combinations.
 - *Manipulation of an algebraic description* to derive different circuit structures for the logic function.
 - *Transformation of an algebraic description* into a form corresponding to an available circuit structure.

Karnaugh Maps

- *Karnaugh Map*:
 - Another approach to represent (and simplify) Boolean equations.
 - **Example** (Karnaugh map for a 2-input AND gate):



Input		Output
A	B	X
0	0	0
0	1	0
1	0	0
1	1	1



A \ B	0	1
0	0	0
1	0	1

Karnaugh Map

Karnaugh Map Simplification

- **Why use Karnaugh map simplification:**
 - Working with algebraic equations is often tedious and error-prone.
 - A **mapping technique** can be used to reduce an algebraic equation to its simplest form.
 - Simplification approaches used (consisting of grouping adjacent 1s or 0s in powers of 2) are **minterm** or **maxterm**.
 - Equations of up to 5 variables can be easily simplified by hand. However, **it becomes complicated to simplify equations with more than 5 variables.**

2-Variable Map (1/2)

- How to **build a 2-variable Karnaugh map**:
 - There will be **4 minterms** for a 2-variable equation, so use a **Karnaugh map with 4 squares** (i.e., a 2x2 table).
 - 1's and 0's on the left side and top of the map designate the values of the variable.
 - Variable **X** is **complemented** in row 0 and **uncomplemented** in row 1.
 - Variable **Y** appears **complemented** in column 0 and **uncomplemented** in column 1.

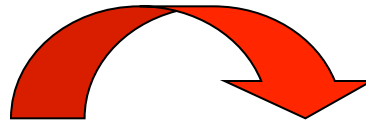
X \ Y	0	1
0	m0	m1
1	m2	m3

2-Variable Map (2/2)

- How **a 2-variable Karnaugh map works**:
 - The concept is to put a **1** in each square that has a corresponding *minterm* in the boolean equation.
 - Only four terms are possible if in *Sum of Products* form.

X \ Y	0	1
0	m0	m1
1	m2	m3

equates to




X \ Y	0	1
0	$X' \cdot Y'$	$X' \cdot Y$
1	$X \cdot Y'$	$X \cdot Y$

Example: 2-Variable Map

- Simplify the boolean equation:

$$F = X \cdot Y' + X' \cdot Y + X \cdot Y$$

- Function **F** is in *Sum of Products* form.
- Put 1's in boxes of corresponding terms and put 0's in all other boxes.



X \ Y	0	1
0	0	1
1	1	1

The Karnaugh map is a 2x2 grid. The top row (X=0) has values 0 and 1. The bottom row (X=1) has values 1 and 1. A red oval groups the two 1's in the bottom row (X=1). A green oval groups the two 1's in the right column (Y=1).

- Simplified expression for **F** is obtained by **grouping adjacent 1's** (in powers of 2) and eliminating unnecessary variables.
- Here, **there are two ways of grouping 1's**: one way is to circle the **row where X=1**; the other way is to circle the **column where Y=1**.
- Y** is eliminated from the two product terms where X=1 and **X** is eliminated from the two product terms where Y=1.

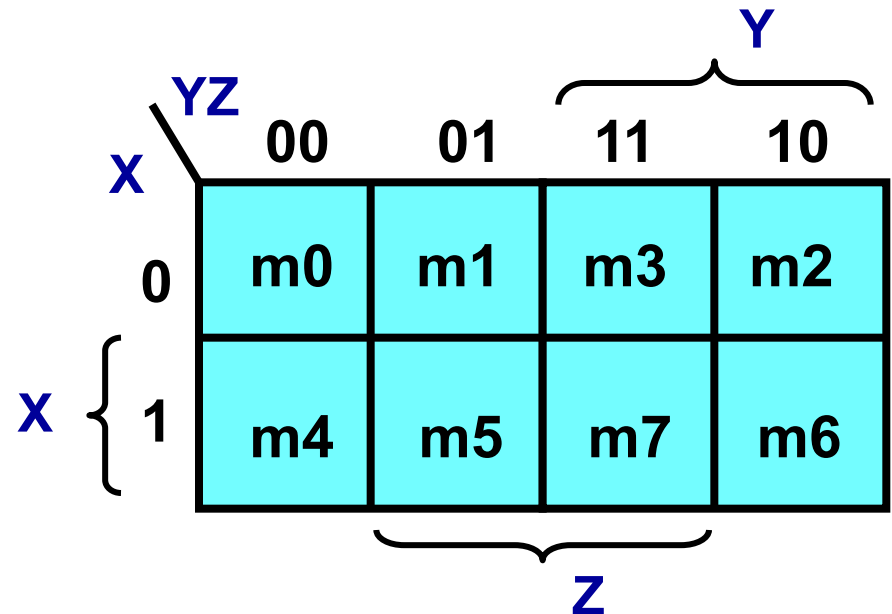
Answer: $F = X + Y$

3-Variable Map (1/2)

- How to **build a 3-variable Karnaugh map**:
 - There will be 8 *minterms* for a 3-variable equation, so use a Karnaugh map with 8 squares (usually, a 2x4 table).
 - The *minterm* numbers do not follow the normal binary counting order sequence.
 - Only 1 bit changes from one adjacent column to the next.

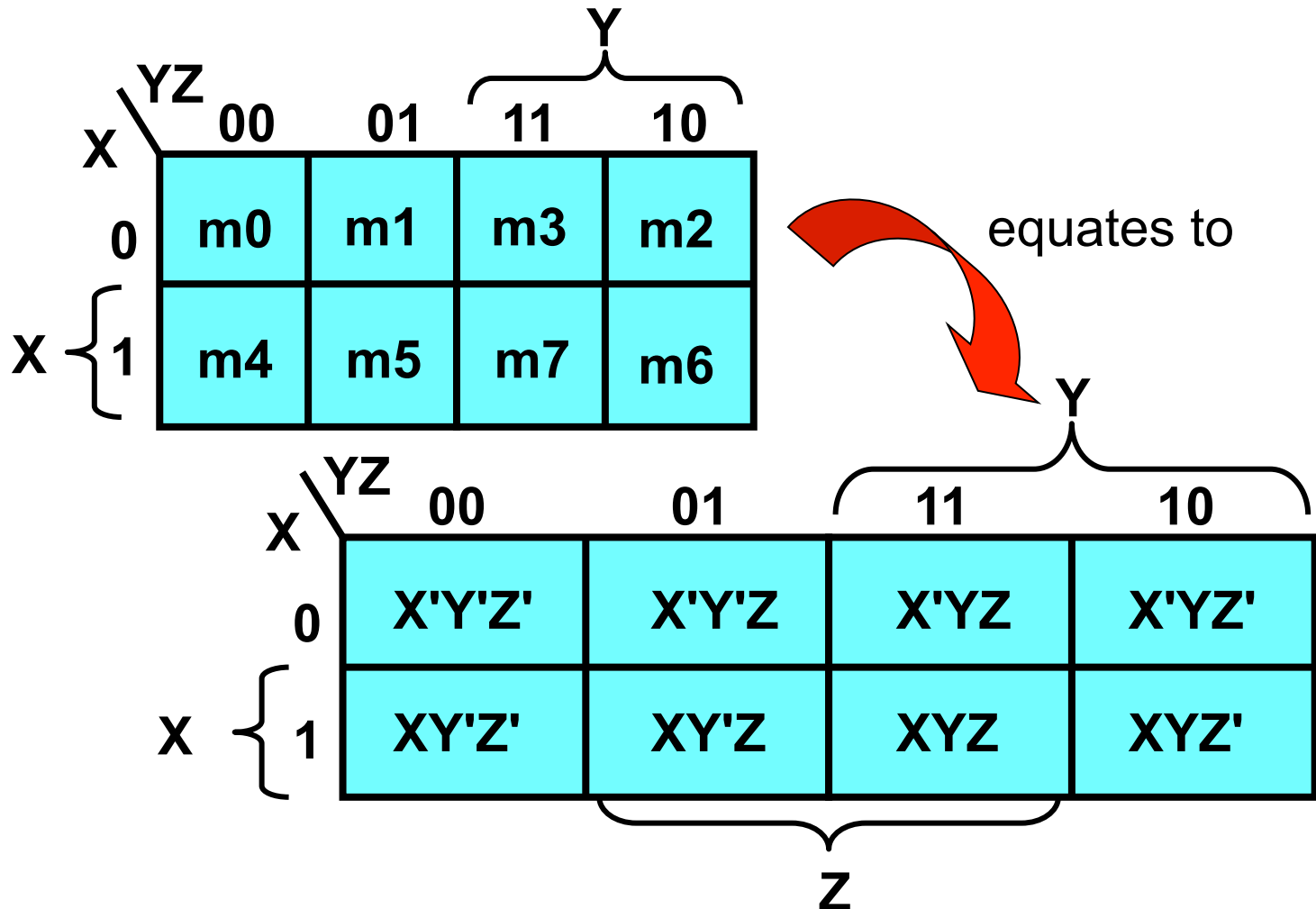


What does this remind you of?



3-Variable Map (2/2)

- How a **3-variable Karnaugh map** works:



Example 1: 3-Variable Map

Simplify: $F(X, Y, Z) = \Sigma m(1, 2, 5, 6) = 001, 010, 101, 110$

Note: Circle 1's in horizontal or vertical groups of 1, 2, 4 or 8 only (*powers of 2*)!

$$m1 + m5 = X'Y'Z + XY'Z$$

$$= (X' + X)(Y'Z)$$

$\therefore X$ is redundant

$$m2 + m6 = X'YZ' + XYZ'$$

$$= (X' + X)(YZ')$$

$\therefore X$ is redundant

	YZ	00	01	11	10
X	0	0	1	0	1
X	1	0	1	0	1

ANSWER: $F = Y'Z + YZ'$

Example 2: 3-Variable Map

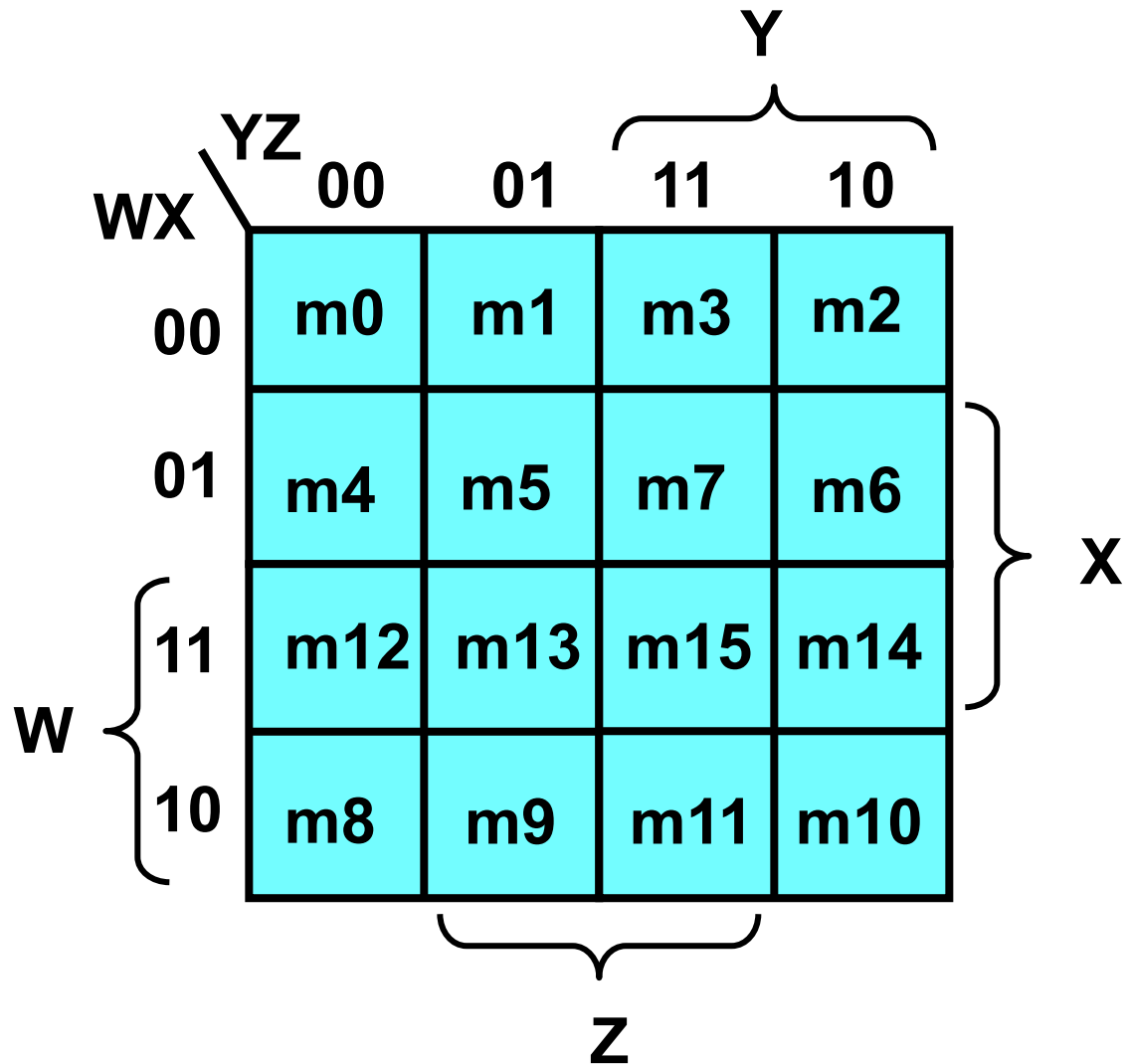
To be completed in class ...

Simplify, using a Karnaugh map:

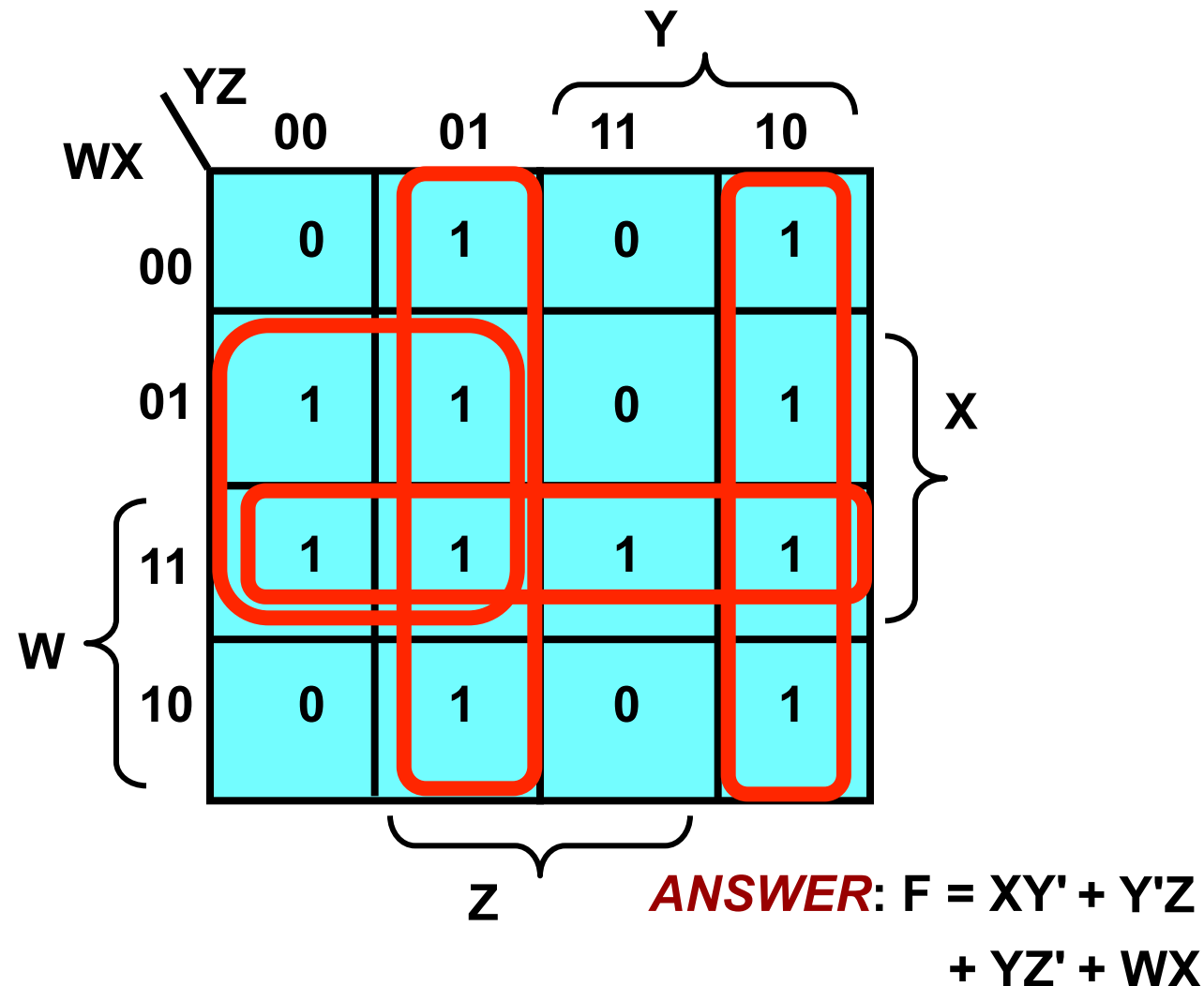
$$F(X, Y, Z) = \sum m(3, 4, 5, 6, 7)$$

X \ YZ	YZ			
	00	01	11	10
0				
1				

4-Variable Map



Example 1: 4-Variable Map



W	X	Y	Z	F	Truth Table
0	0	0	0	0	
0	0	0	1	1	
0	0	1	0	1	
0	0	1	1	0	
0	1	0	0	1	
0	1	0	1	1	
0	1	1	0	1	
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	1	
1	0	1	0	1	
1	0	1	1	0	
1	1	0	0	1	
1	1	0	1	1	
1	1	1	0	1	
1	1	1	1	1	

Example 2: 4-Variable Map

- This is an **example of a Karnaugh map with all possible groupings** (except circles with a single value):

		YZ			
		00	01	11	10
WX	00	1	1	0	0
	01	1	1	0	0
	11	0	1	1	0
	10	0	0	1	1

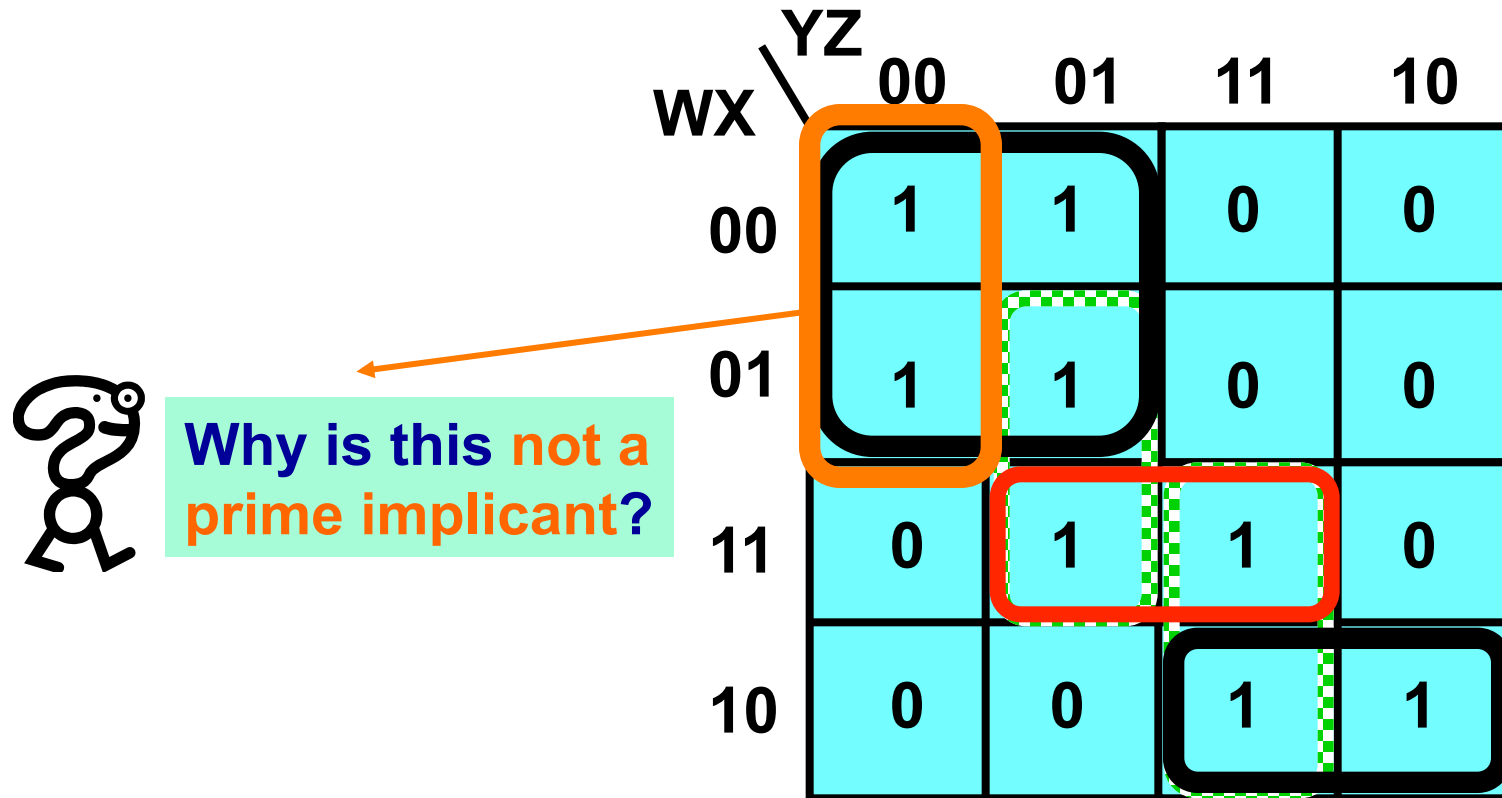
Prime Implicants

- **Challenge when using K-maps:** To select the right groups.
 - **IF** the number of groups is not minimised **AND**
 - the size of each group is not maximised **THEN**,
 - Resulting expression will still be equivalent to the original one.
 - BUT**
 - Resulting expression will not be a **minimal sum of products** (or MSP).

- Good **approach to finding an actual MSP:**
 - Find all of the largest possible groupings of 1's (these are called the **prime implicants**); but only in powers of 2!
 - The final MSP will contain a subset of these prime implicants.

Example: K-Map with Prime Implicants

- Here is an **example of a Karnaugh map with *prime implicants*** (all of the marked groups are prime implicants):

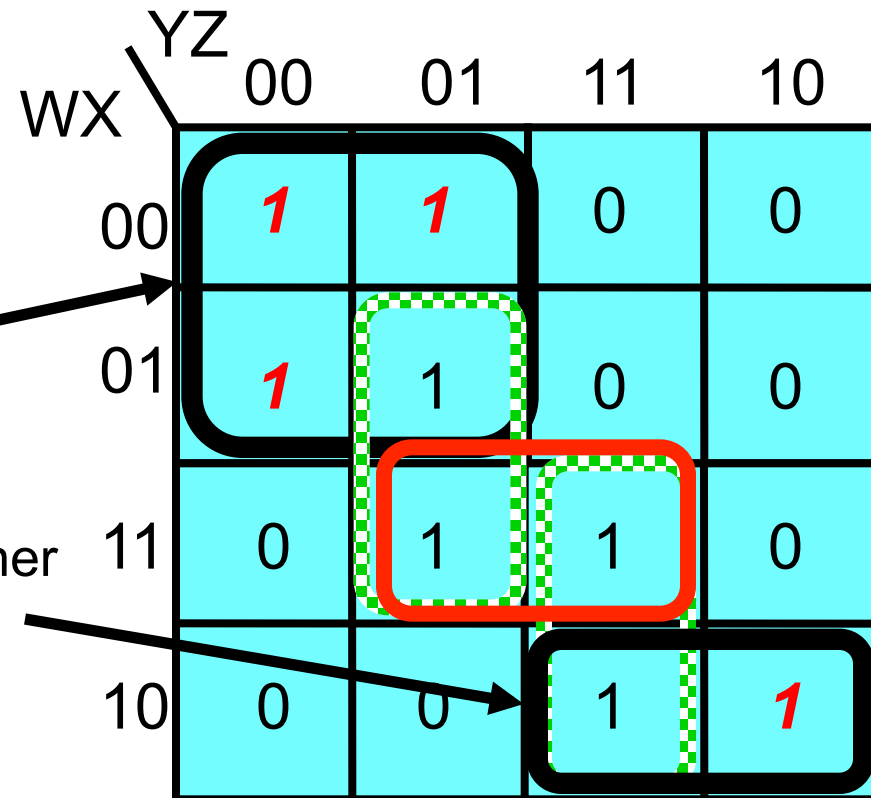


Essential Prime Implicants (EPIs)

- If any group contains a **minterm** that isn't also covered by another overlapping group, then that is an **EPI**.
- EPIs** appear in the MSP, since they contain **minterms** that no other groups include.
- Example:**

EPI: because no other group covers **m0**, **m1**, and **m4**.

EPI: because no other group covers **m10**.



Another Example : Covering the other *Minterms*

- Example from previous slide:
 - Pick as few other prime implicants as necessary, to ensure that all the *minterms* are covered.
 - After choosing the **green** rectangles in the example, there are just two *minterms* left to be covered: *m13* and *m15*.
 - These are both included in the **red** implicant, **WXZ** .
 - The resulting equation is,

$$F = W'Y' + WXZ + WX'Y$$

YZ WX \		00	01	11	10
		00	01	11	10
00	1	1	0	0	
01	1	1	0	0	
11	0	1	1	0	
10	0	0	1	1	

Relationships: F , F' , Σ , Π

Karnaugh Map for F

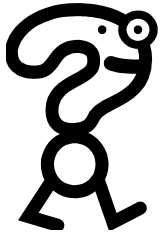
Karnaugh Map for F'

Minimal Sum of Products for F	Minimal Product of Sums for F	Minimal Sum of Products for F'	Minimal Product of Sums for F'
What to do: Loop 1's and read as <i>minterms</i> .	What to do: Loop 0's and read as <i>maxterms</i> (or complement SOP for F).	What to do: Loop 1's.	What to do: Loop 0's and read as <i>maxterms</i> (or complement SOP for F).

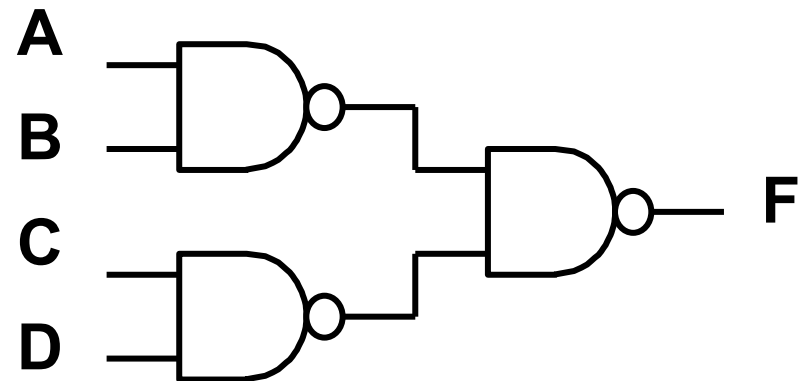
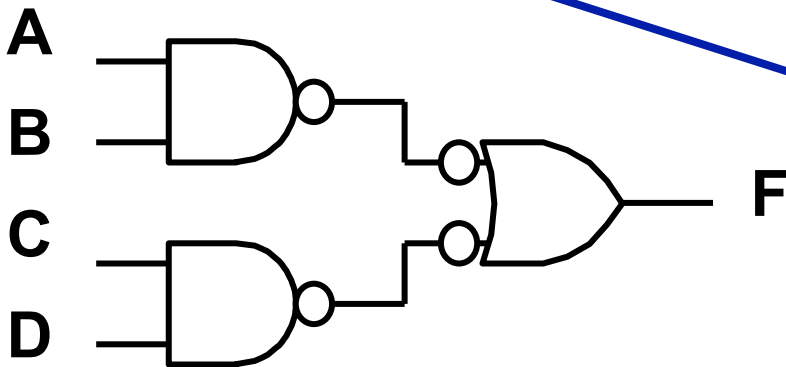
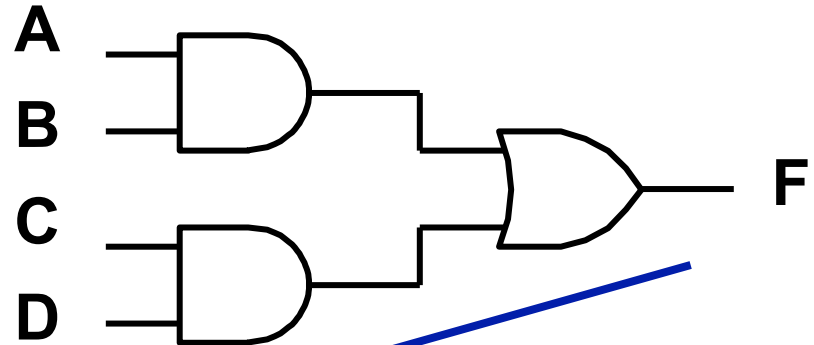


Converting to Negative Logic

- Three different (but *equivalent*) circuits that implement the equation **$F = AB + CD$** :



What are the equations describing the two bottom diagrams?



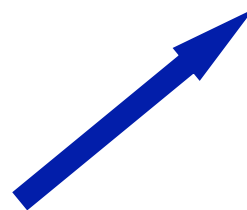
Example: SOP of F (1/2)

- Find the *minimal sum of products* and draw the *NAND gate* implementation for:
$$F = X'Y'Z + X'YZ' + X'YZ + XY'Z' + XY'Z$$

- What to do:**

- ✓ First look for the largest possible group.
- ✓ If there are still 1's to cover, keep circling until all are covered.
- ✓ Read off essential prime implicants necessary for minimal cover.

X \ YZ	YZ			
	00	01	11	10
0	0	1	1	1
1	1	1	0	0



$$F = XY' + X'Y + Y'Z$$

$$F = F'' = (XY' + X'Y + Y'Z)''$$

$$F = ((XY')'(X'Y)'(Y'Z)')'$$

Example: SOP of F (2/2) *To be completed in class ...*

- Draw the *NAND gate* implementation of: $F = XY' + X'Y + Y'Z$
 - ✓ 1. Draw the normal circuit:
 - ✓ 2. Apply DeMorgan's theorems graphically:
 - ✓ 3. Convert the 3-inverted input gate to *NAND*:

Example: SOP of F'

To be completed in class ...

- Find the *minimal sum of products* for F' where,

$$F = X'Y'Z + X'YZ' + X'YZ + XY'Z' + XY'Z$$

$$\therefore F' = (X+Y+Z')(X+Y'+Z)(X+Y'+Z')(X'+Y+Z)(X'+Y+Z')$$

- What to do:**

✓ Map F'. ✓ Circle the 1's.

✓ Read off essential prime implicants as *minterms*.



F is as in previous example.

X \ YZ	00	01	11	10
	0	1	1	0
0				
1				

Summary: Karnaugh Maps (1/2)

- Karnaugh maps are an *alternative* to Switching Algebra for *simplifying expressions*:
 - The result is an MSP (or MPS) that leads to a minimal 2-level circuit.
 - It is easy to handle *don't-care conditions*.
 - Karnaugh maps are really only good for manual simplification of fairly small expressions.



We'll look at these
in a few slides.

Summary: Karnaugh Maps (2/2)

- Things to keep in mind:
 - Remember the correct order of *minterms* on the K-map.
 - When grouping, it is possible to wrap around all sides of the K-map, and the groups can overlap.
 - Make as few groups (of adjacent 1s or 0s) as possible, but make each of them as large as possible (*only in powers of 2* i.e., 1, 2, 4, 8, ...).
 - There may be more than one valid solution!!

Map Manipulation

- Often a truth table will be generated that has several combinations that the designer doesn't care about.
- Expressions can be simplified by using DON'T CARE cases:
 - Sometimes it is possible to eliminate essential prime implicants.
- Example:
 - Consider the following *incompletely specified function*:
$$\left\{ \begin{array}{l} F(W,X,Y,Z) = \Sigma m(1, 3, 7, 11, 15) \\ d(W,X,Y,Z) = \Sigma m(0, 2, 5) \rightarrow \text{These are DON'T CARE cases.} \end{array} \right.$$

4-Variable Karnaugh Map

W	X	Y	Z	F
0	0	0	0	X
0	0	0	1	1
0	0	1	0	X
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

$$F(W,X,Y,Z) = \sum m(1, 3, 7, 11, 15)$$

$$d(W,X,Y,Z) = \sum m(0, 2, 5)$$

WX \ YZ		00	01	11	10
		00	01	11	10
00		X	1	1	X
01		0	X	1	0
11		0	0	1	0
10		0	0	1	0

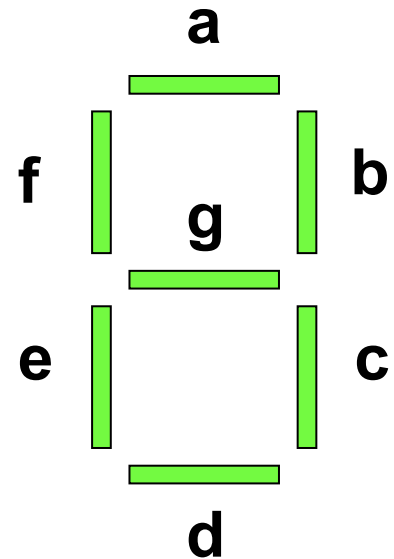
Answer:

$$F = YZ + W'X'$$

Example (1/9): Design Procedure

- **Problem Specification:**

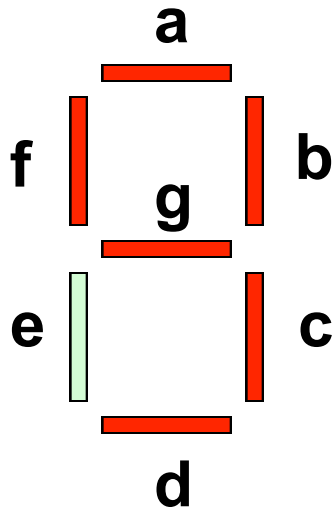
- Design a *code converter* to decode from BCD to a 7-segment display.
- The combinational circuit must accept a BCD digit *and* generate the appropriate outputs to display the digit in a 7-segment format as shown here.



Example (2/9): BCD and I/O

- *BCD numbers are 4 digits long*, so
 - e.g., $1_{10} = 0001_2$ and $10_{10} = 00010000_2$.
- **Inputs:**
 - We only have to output decimal digits 0-9 on the 7-segment display, so *only 4 inputs are needed!*
- **Outputs:**
 - Need *one for each segment a-g*.
 - Output for each function *a-g* should be:
 - *1* if that segment should be *on*.
 - *0* if it should be *off*.

Example (3/9): BCD 7-Segment Truth Table

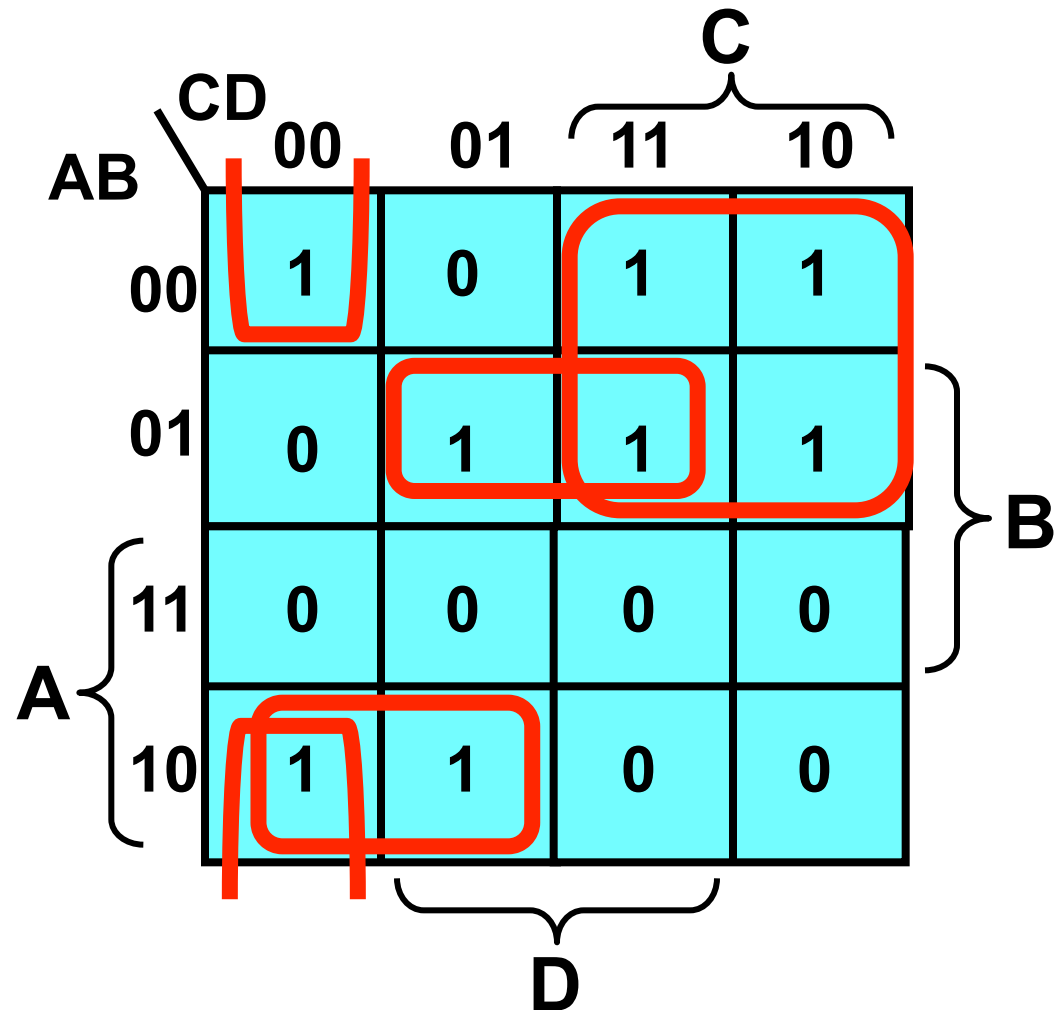


	A	B	C	D	a	b	c	d	e	f	g
0 ⇒	0	0	0	0	1	1	1	1	1	1	0
1 ⇒	0	0	0	1	0	1	1	0	0	0	0
2 ⇒	0	0	1	0	1	1	0	1	1	0	1
3 ⇒	0	0	1	1	1	1	1	1	0	0	1
4 ⇒	0	1	0	0	0	1	1	0	0	1	1
5 ⇒	0	1	0	1	1	0	1	1	0	1	1
6 ⇒	0	1	1	0	1	0	1	1	1	1	1
7 ⇒	0	1	1	1	1	1	1	0	0	0	0
8 ⇒	1	0	0	0	1	1	1	1	1	1	1
9 ⇒	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	0	0	0	0	0	0	0
	1	0	1	1	0	0	0	0	0	0	0
	1	1	0	0	0	0	0	0	0	0	0
	1	1	0	1	0	0	0	0	0	0	0
	1	1	1	0	0	0	0	0	0	0	0
	1	1	1	1	0	0	0	0	0	0	0

Example (4/9): Karnaugh Map for 'a'

$$a = B'C'D' + AB'C' + A'BD + A'C$$

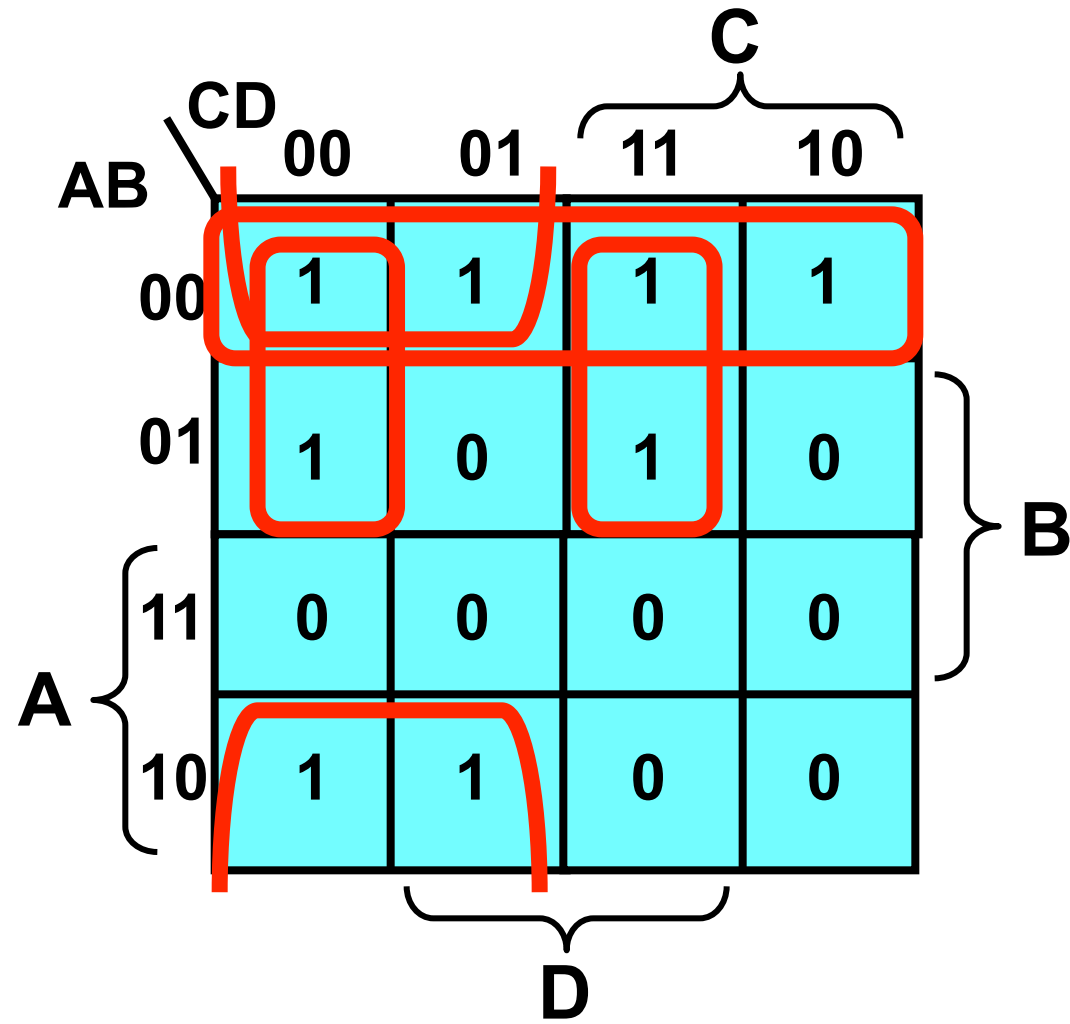
A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0



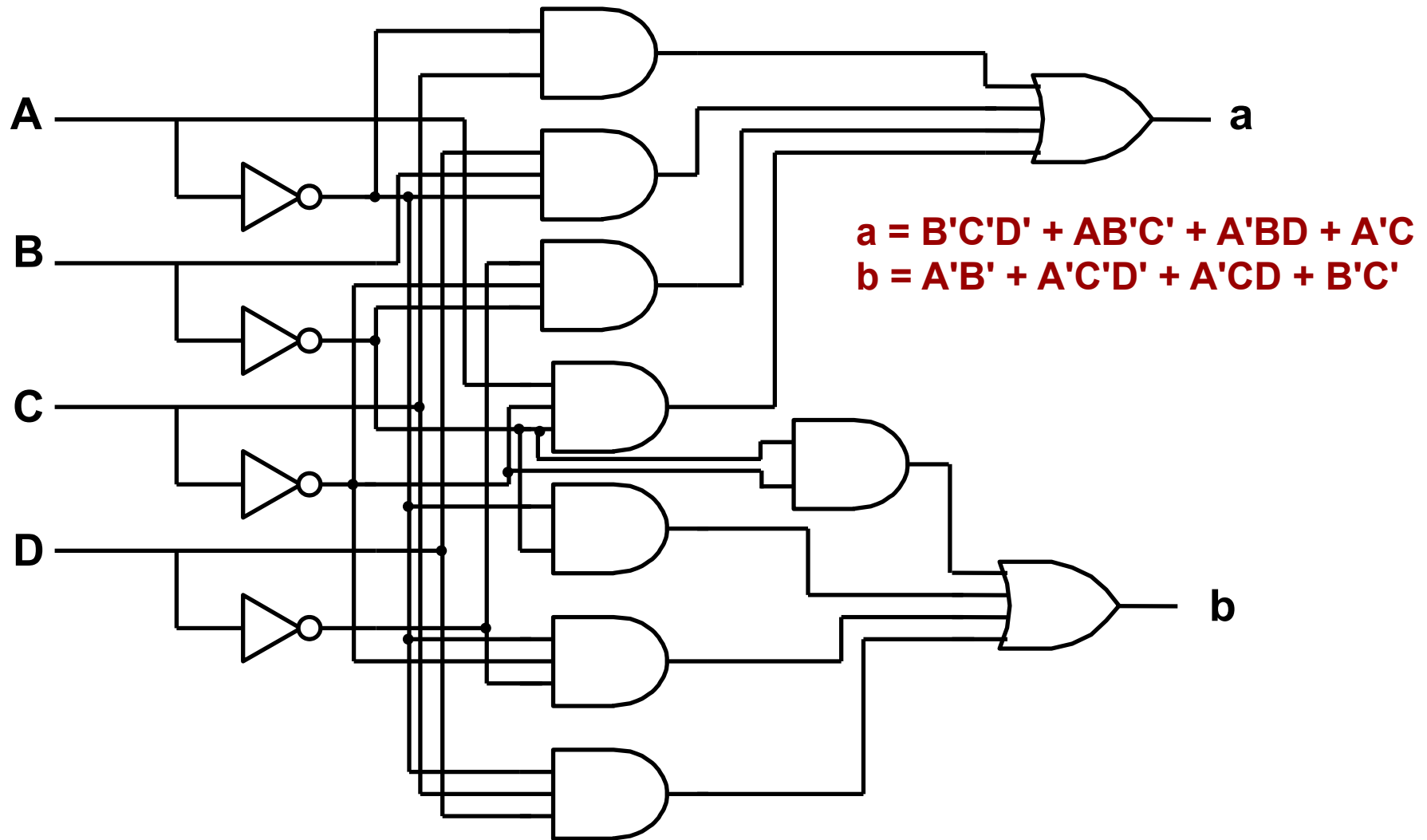
Example (5/9): Karnaugh Map for 'b'

$$b = A'B' + A'C'D' + A'CD + B'C'$$

A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0



Example (6/9): BCD 7-Segment Decoder (for *a* & *b*)



Example (7/9): Karnaugh Map for 'c'

c =

A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0

		C			
		CD			
		00	01	11	10
A	AB	00	01	11	10
	00	1	1	1	0
	01	1	1	1	1
	11	0	0	0	0
	10	1	1	0	0

Example (8/9): Karnaugh Map for 'd'

d =

		C			
		00	01	11	10
A	AB	00	01	11	10
	00	1	0	1	1
	01	0	1	0	1
	11	0	0	0	0
	10	1	1	0	0
		D			

A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0

Example (9/9): BCD 7-Segment Decoder (for c & d)

To be completed in class ...

$c =$

$d =$

Example (1/2): Parity Generators

- **Problem Specification:** Build a circuit that will generate the appropriate even parity bit for the 3-bit input.
 - *Inputs:* X = 1st bit, Y = 2nd bit; Z = 3rd bit
 - *Outputs:* F = parity bit



Remember: In a 4-bit number, there will be *even parity*, if there is an even number of 1's.



What is the requirement for having *odd parity*?

Example (2/2): 3-Bit Even Parity Generator

To be completed in class ...

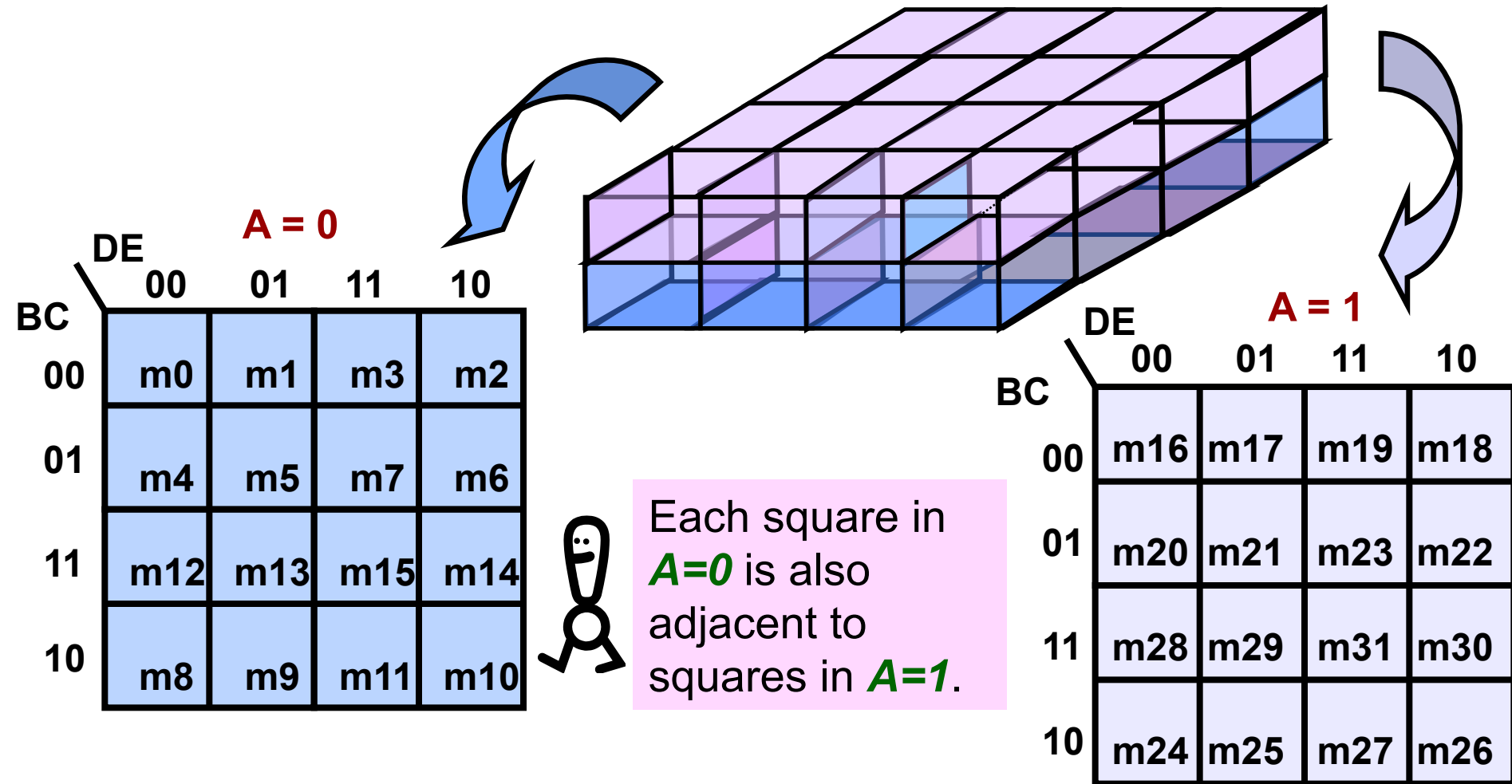
Truth Table

X	Y	Z	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	



X \ YZ	00	01	11	10
0				
1				

5-Variable Karnaugh Map



Example (1/2): 5-Variable Karnaugh Map

$$F(A,B,C,D,E) = \sum m(0, 10, 11, 14, 15, 16, 20, 24, 26, 27, 28, 30, 31)$$

$$= 00000, 01010, 01011, 01110, 01111, 10000, 10100, 11000, 11010, 11011, 11100, 11110, 11111$$

A = 0

		DE			
		00	01	11	10
BC	00	1	0	0	0
	01	0	0	0	0
	11	0	0	1	1
	10	0	0	1	1

A = 1

		DE			
		00	01	11	10
BC	00	1	0	0	0
	01	1	0	0	0
	11	1	0	1	1
	10	1	0	1	1

Example (2/2): 5-Variable Karnaugh Map

Next step: Grouping ... Start with largest possible – in this map it's 32, down to 1. Don't circle smaller groups if they don't include uncovered 1's!

Cross-map (i.e., $A=0$ and $A=1$) so no A ; but every other variable is the same, so $B'C'D'E'$.

Only in map $A=1$; B and C change, so $AD'E'$.

A = 0

DE \ BC	00	01	11	10
00	1	0	0	0
01	0	0	0	0
11	0	0	1	1
10	0	0	1	1

A = 1

DE \ BC	00	01	11	10
00	1	0	0	0
01	1	0	0	0
11	1	0	1	1
10	1	0	1	1

Remember the cross-map adjacency!

$$F = B'C'D'E' + AD'E' + BD$$

Cross-map (i.e., $A=0$ and $A=1$) so no A ; in both maps C and E change, so BD .

General Procedure: Designing Combinational Circuits

1. **Specification**: Write specification for the circuit if not already available.
 - Specify/label input(s) and output(s).
2. **Formulation**: Derive *Truth Table* or *initial Boolean equations* defining the relationships between inputs and outputs, if not in the specification.
3. **Optimisation**: Minimise the design using *Switching Algebra, Karnaugh Map, software*.
 - Draw logic diagram for the resulting circuit using AND/ OR/ NOT gates.
4. **Technology Mapping**: Map the logic diagram to the implementation technology selected (e.g., map into NANDs).
5. **Verification**: Verify the correctness of the final design *manually* or *using simulation*.

Practical Considerations:

- Cost of gates (Number)
- Maximum allowed delay
- Fan-in/Fan-out

Example: Circuit Design (1/2)

- *Question*: Design a circuit that has a 3-bit input and a single output (F) specified as follows:
 - $F = 0$, when the input is less than $(5)_{10}$
 - $F = 1$, otherwise

Step 1: Specification

- Label the inputs (3 bits) as X , Y , Z : X is the most significant bit, Z is the least significant bit.
- The output (1 bit) is F :
 - $F = 1 \rightarrow (101)_2, (110)_2, (111)_2$
 - $F = 0 \rightarrow$ other inputs

Example: Circuit Design (2/2)

Step 2: Formulation

- Obtain the **Truth Table** 

Step 3: Optimisation

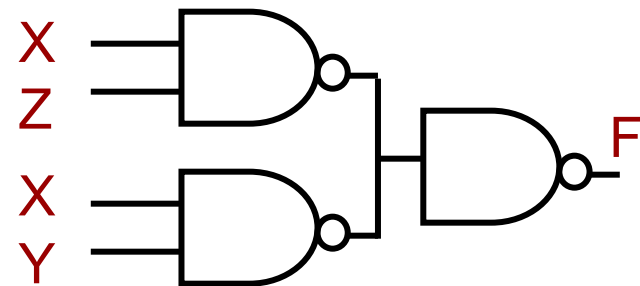
X \ YZ				
	00	01	11	10
0	0	0	0	0
1	0	1	1	1

$$F = XZ + XY$$

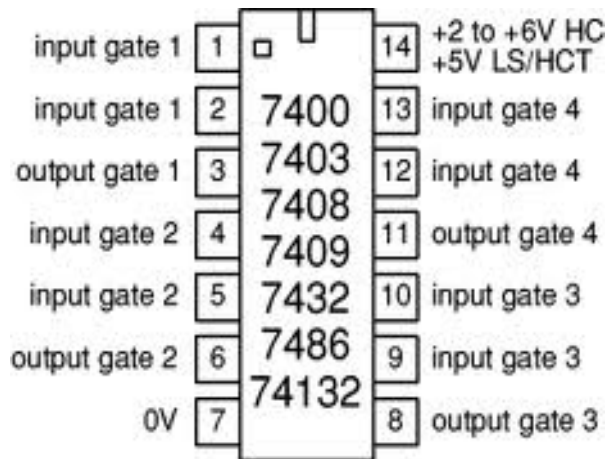
X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Step 4: Optimisation (e.g., NAND implementation)

- $F = F'' = (XZ + XY)'' = ((XZ)' \cdot (XY)')'$,
applying *DeMorgan's theorem*



Standard Pin Layout for Common Logic Gates



Quad 2-input gates

7400 quad 2-input NAND

7403 quad 2-input NAND with open collector outputs

7408 quad 2-input AND

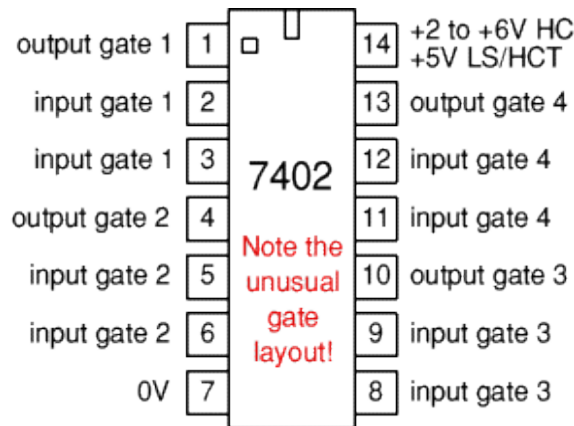
7409 quad 2-input AND with open collector outputs

7432 quad 2-input OR

7486 quad 2-input XOR

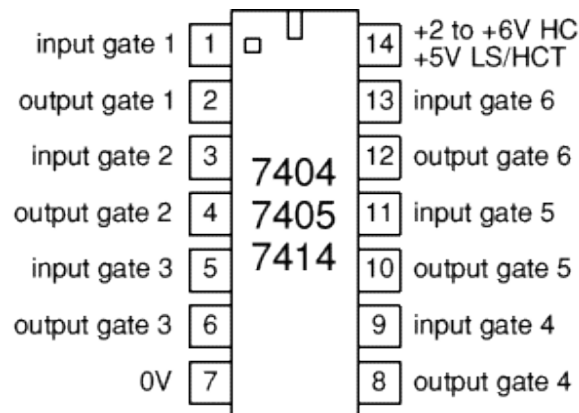
74132 quad 2-input NAND with Schmitt trigger inputs

7402 quad 2-input NOR

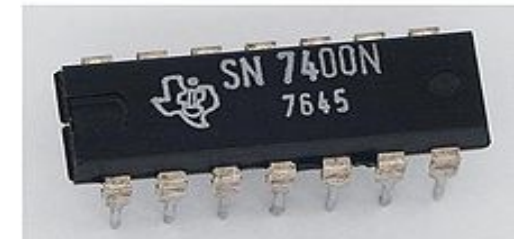
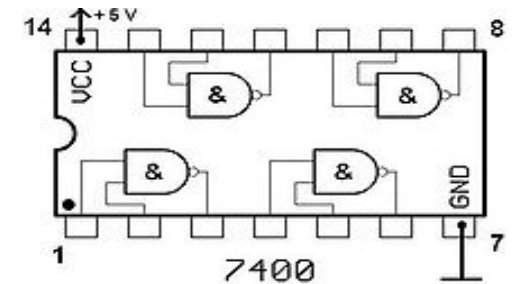


Hex NOT gates / Inverters

7404 hex NOT



NAND Gates in DIP Packages



Standard Pin Layout for Common Logic Gates

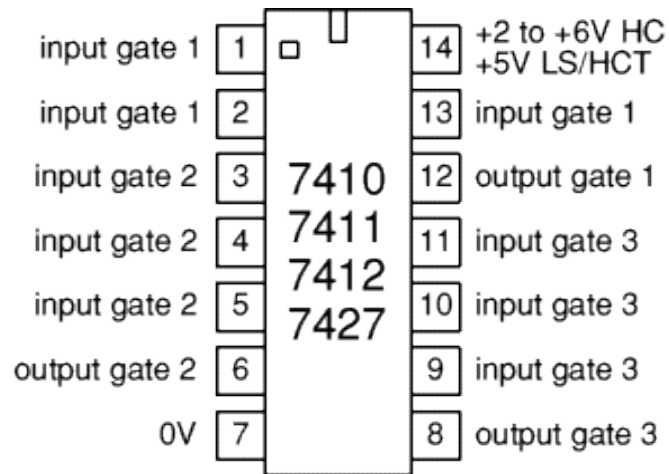
Triple 3-input gates

7410 triple 3-input NAND

7411 triple 3-input AND

7412 triple 3-input NAND with open collector outputs

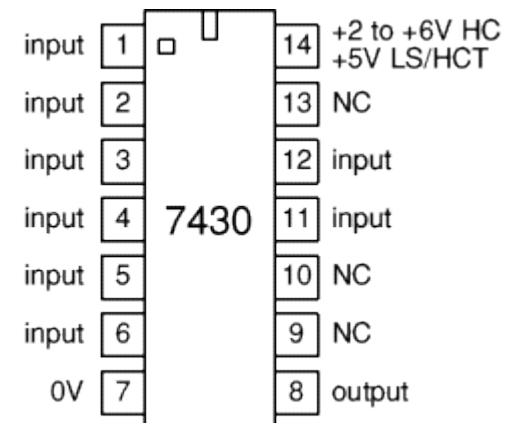
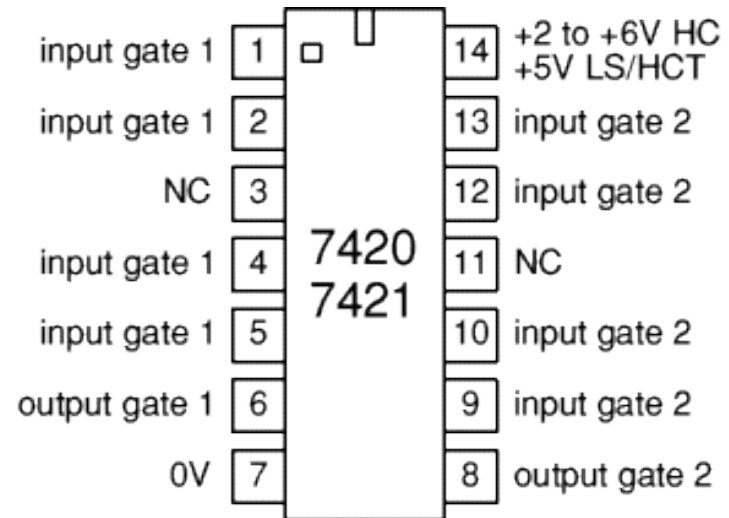
7427 triple 3-input NOR



Dual 4-input gates

7420 dual 4-input NAND

7421 dual 4-input AND



7430 8-input NAND gate