

Design via Root Locus –

PID Controller and Lead/Lag Compensator Design (Part 2)

Reference book: Control Systems Engineering, Third Edition, Norman S Nise

Review of PI controller and Lag Compensator

Motivations

- The RL allows us to choose the proper loop gain to meet a transient response specification. We are limited to those responses that exist along the root locus.
- If the desired transient response defined by percent overshoot and setting time etc. is not in the RL, how to speed the response to them.
- The increase in speed cannot be accomplished by a simple gain adjustment.
- One way is to replace the existing system with a system whose root locus intersects the desired design point. Expensive
- Design a controller or compensator with additional poles and zeros, so that the compensated system has a root locus that goes through the desired pole location.

Two ways of improving steady-state error via cascade compensation

- Way 1: ideal proportional plus integral (PI) compensation, reducing the error to zero (increase the type by 1)
- Way 2: compensator places the extra pole near the origin

Figure 9.3

Pole at A is:

- a.** on the root locus without compensator;
 - b.** not on the root locus with compensator pole added;
- (figure continues)

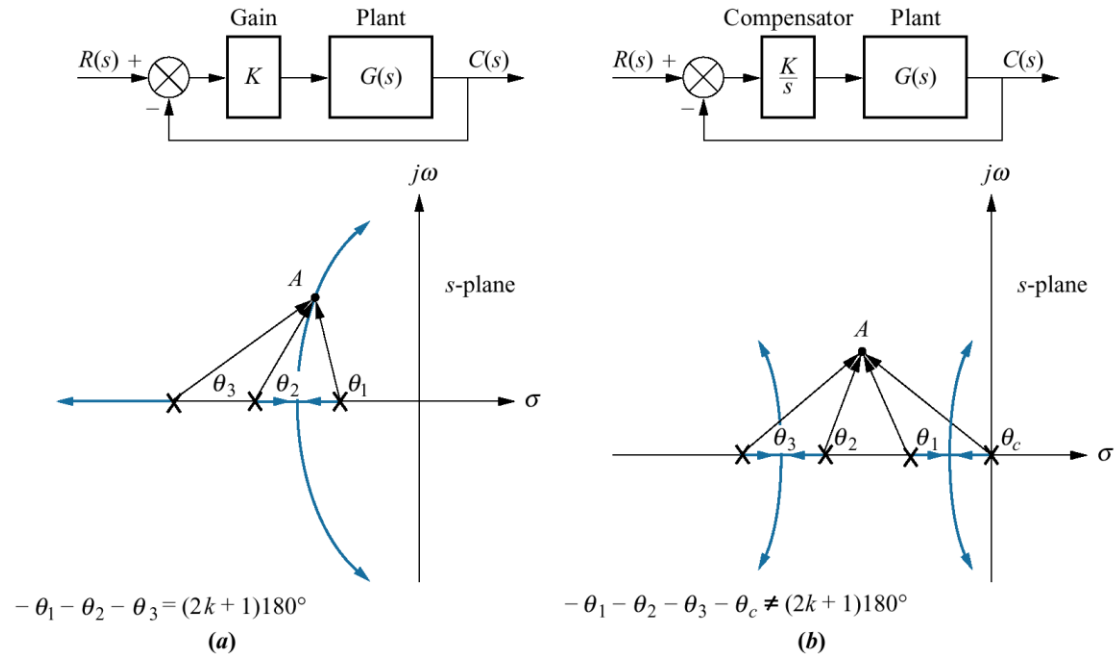
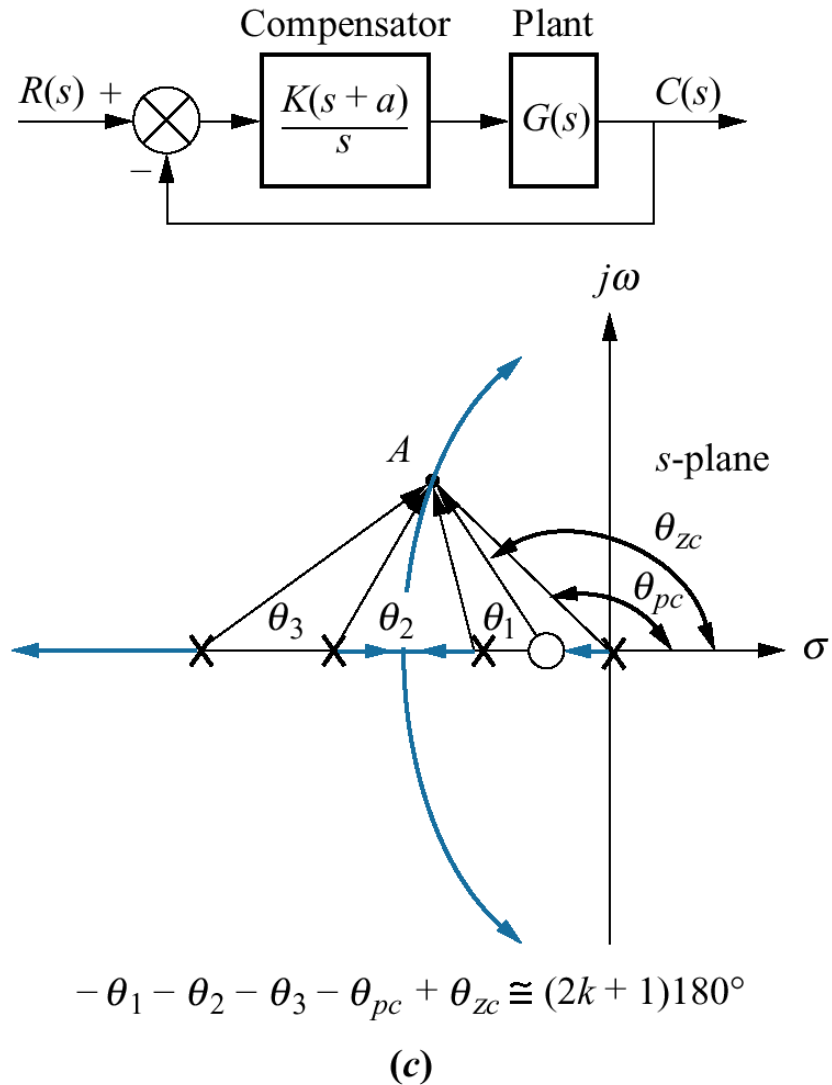


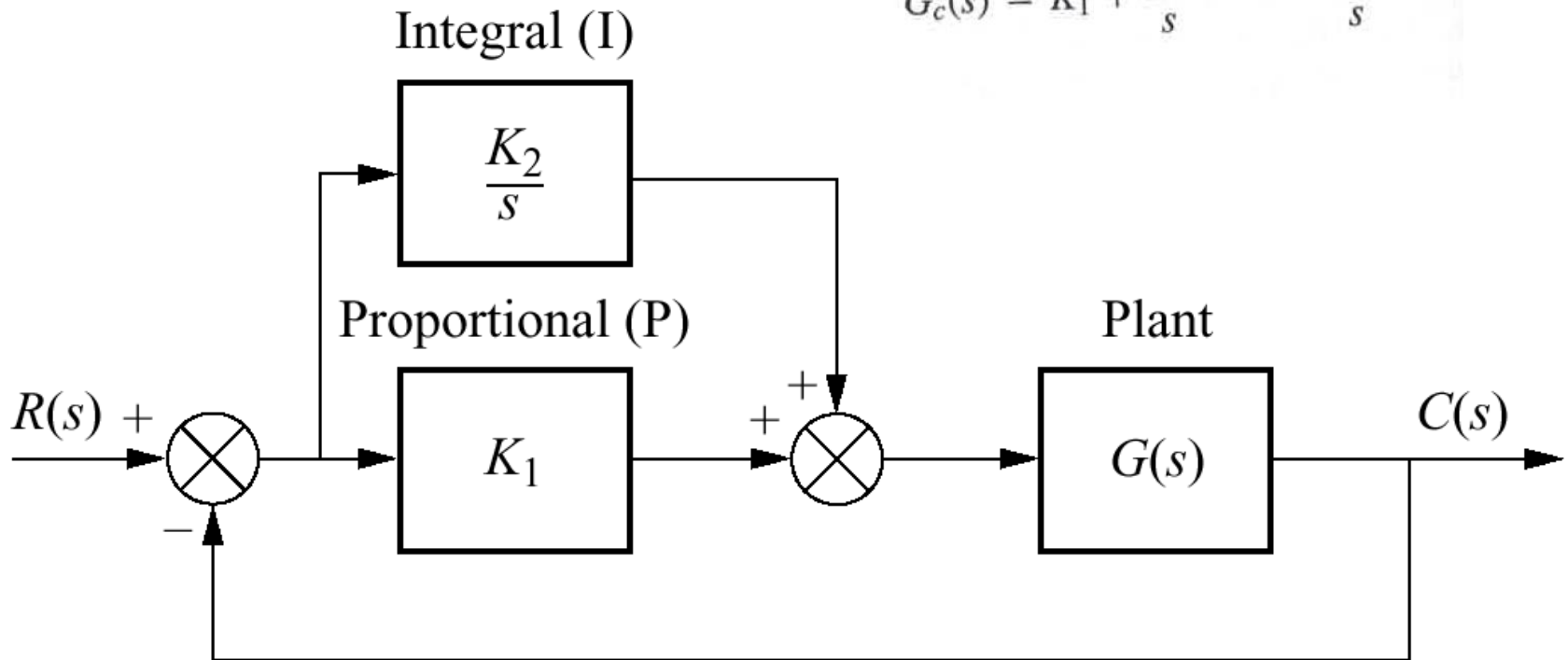
Figure 9.3 (continued)

c. approximately
on the root locus
with
compensator
pole and zero
added



PI controller (in parallel)

$$G_c(s) = K_1 + \frac{K_2}{s} = \frac{K_1 \left(s + \frac{K_2}{K_1} \right)}{s}$$



Lag Compensation

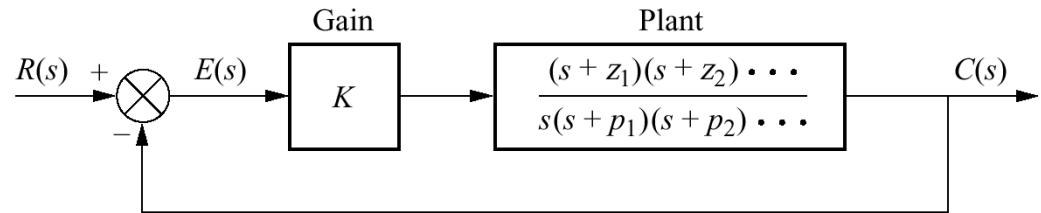
Ideal Integral compensation with its pole on the origin, requires an active integrator, called active implementation.

Lag compensation does not increase the system type, but still yield improvement in the static error constant. The idea is to place an open loop zero very near the origin pole.

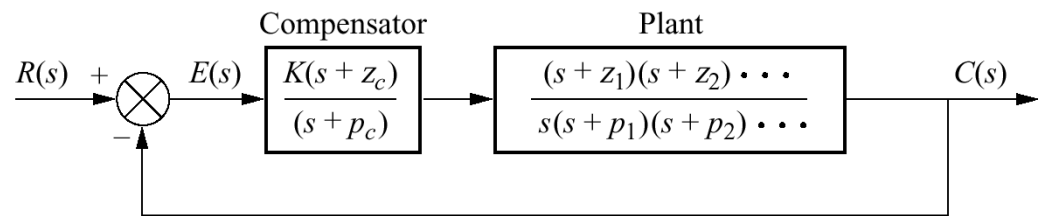
$$G_c(s) = \frac{(s + z_c)}{(s + p_c)} \quad z_c > p_c$$

Figure 9.9

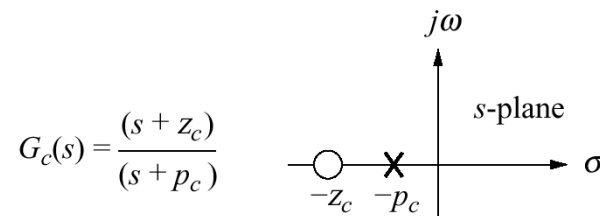
- a.** Type 1 uncompensated system;
- b.** Type 1 compensated system;
- c.** compensator pole-zero plot



(a)



(b)

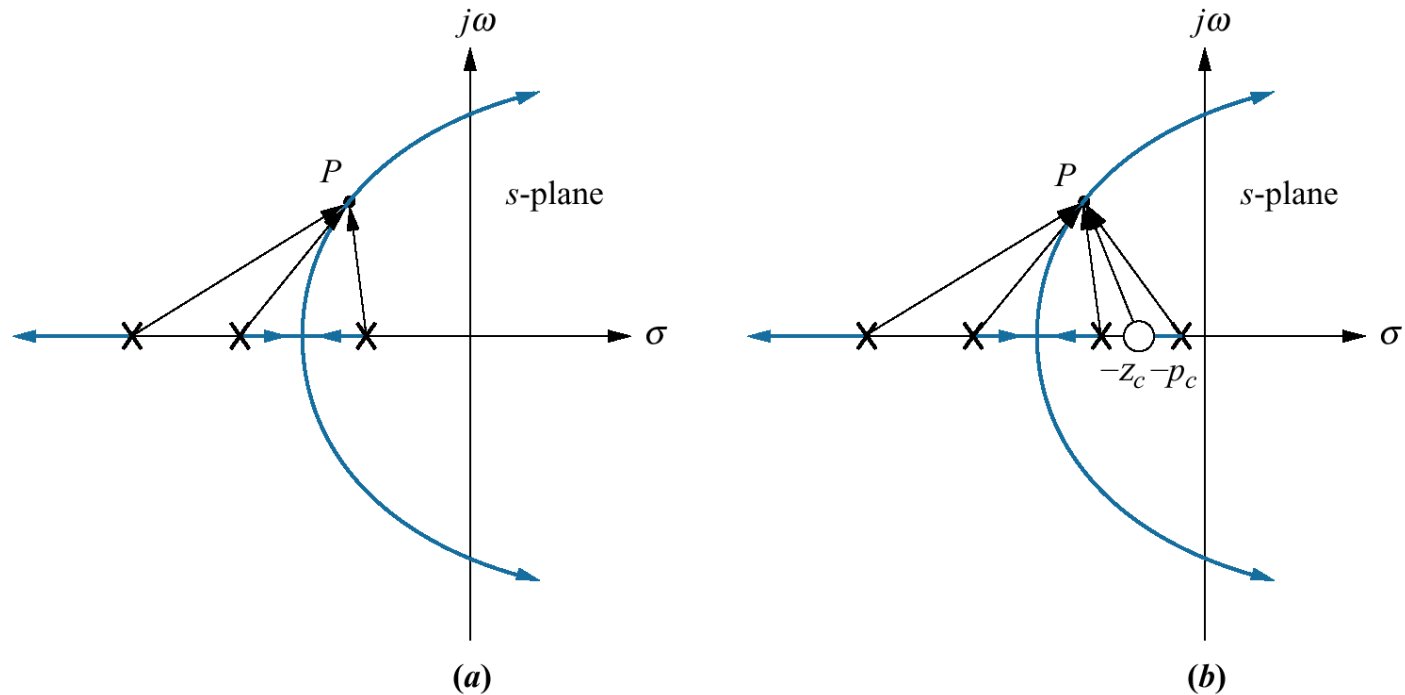


(c)

Figure 9.10

Root locus:

- a. before lag compensation;
- b. after lag compensation



Improving transient response via PD compensation

The objective is to design a response that has a desirable percent overshoot and a shorter settling time than the uncompensated system.

The transient response can be selected by choosing an appropriate closed-loop pole location on the s -plane.

Case 1: the desired point is on the RL, a simple gain adjustment is required.

Case 2: the desired point is not on the RL, the RL must be reshaped so that the compensated RL goes through the selected CL pole location.

Way 1: Proportional-plus-derivative (PD) controller

$$G_c(s) = s + z_c$$

Way 2: Lead compensator

$$G_c(s) = \frac{(s + z_c)}{(s + p_c)} \quad z_c < p_c$$

Figure 9.15

Using ideal
derivative
compensation:

compensation:

a. uncompensated;

b. compensator
zero at -2 ;

(figure continues)

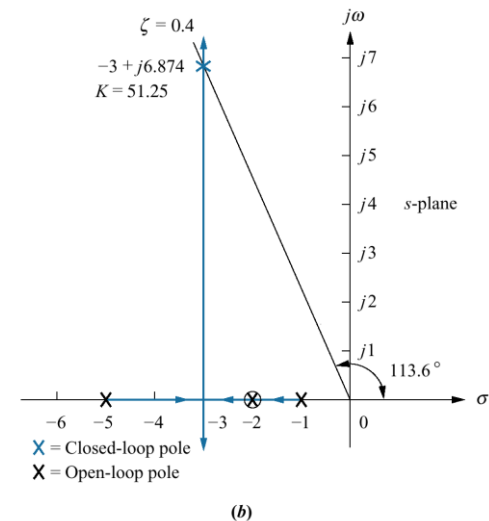
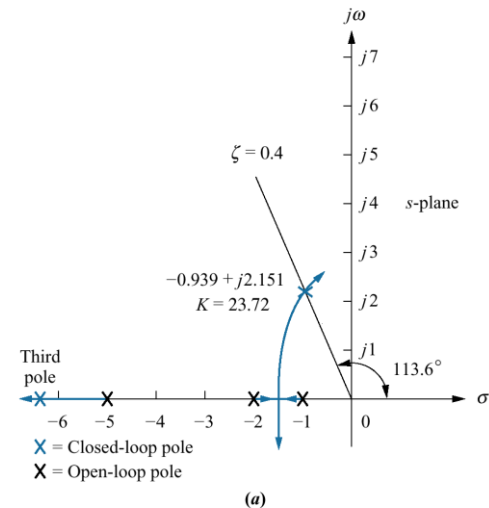


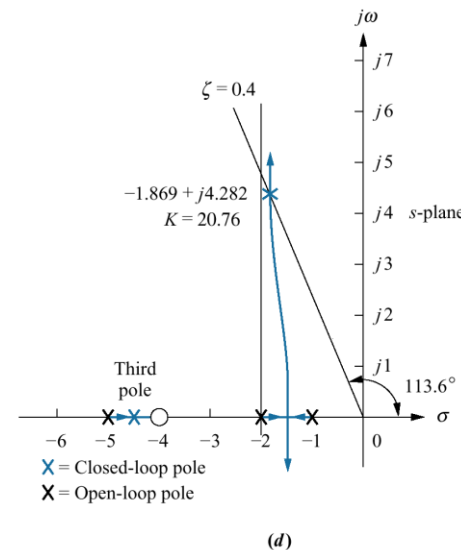
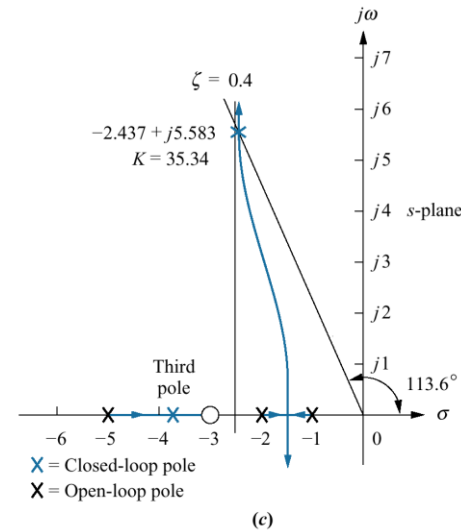
Figure 9.15*(continued)***c.** compensator
zero at -3 ;**d.** compensator
zero at -4 

Figure 9.16
Uncompensated system and ideal derivative compensation solutions from Table 9.2

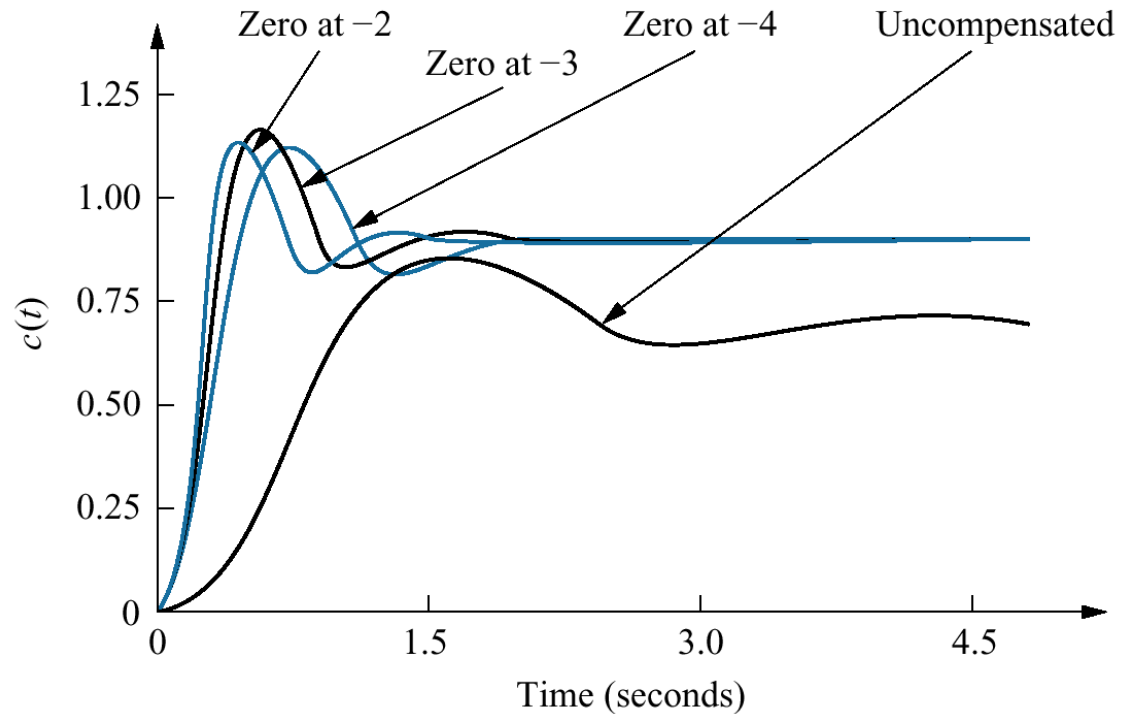
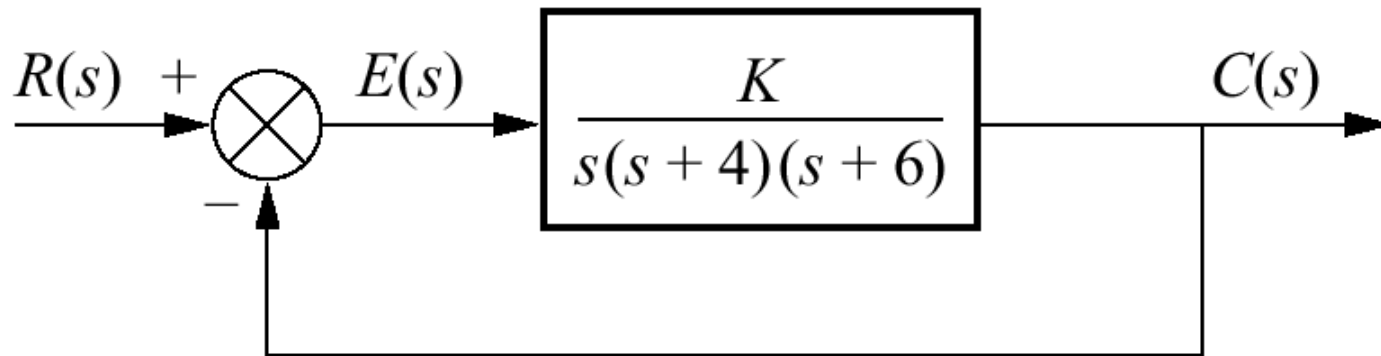


Table 9.2
Predicted characteristics for the
systems of Figure 9.15

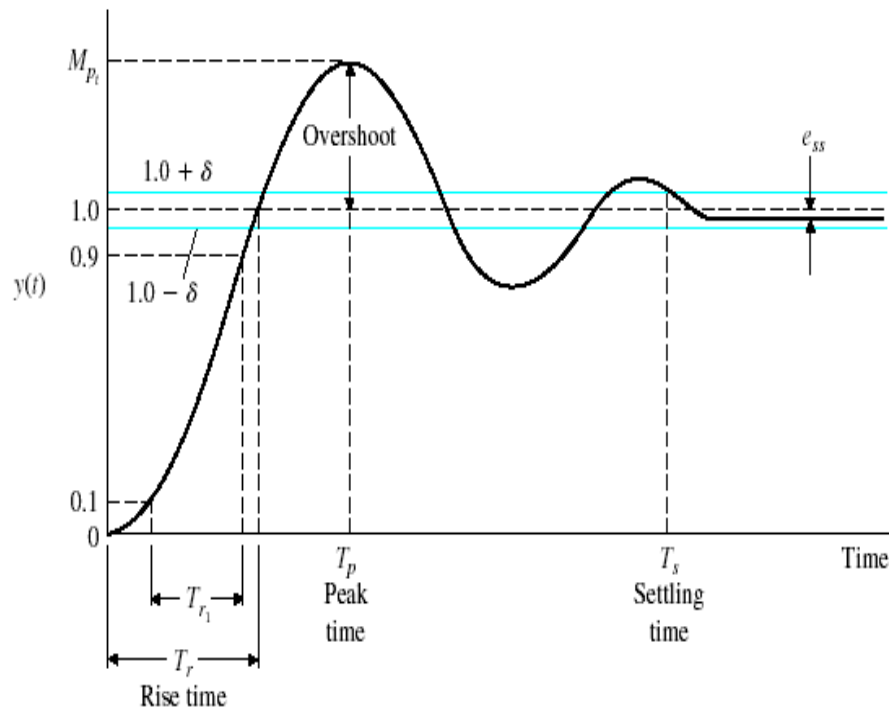
	Uncompensated	Compensation b	Compensation c	Compensation d
	K	$K(s + 2)$	$K(s + 3)$	$K(s + 4)$
Plant and compensator	$(s + 1)(s + 2)(s + 5)$	$(s + 1)(s + 2)(s + 5)$	$(s + 1)(s + 2)(s + 5)$	$(s + 1)(s + 2)(s + 5)$
Dom. poles	$-0.939 \pm j2.151$	$-3 \pm j6.874$	$-2.437 \pm j5.583$	$-1.869 \pm j4.282$
K	23.72	51.25	35.34	20.76
ζ	0.4	0.4	0.4	0.4
ω_n	2.347	7.5	6.091	4.673
%OS	25.38	25.38	25.38	25.38
T_s	4.26	1.33	1.64	2.14
T_p	1.46	0.46	0.56	0.733
K_p	2.372	10.25	10.6	8.304
$e(\infty)$	0.297	0.089	0.086	0.107
Third pole	-6.123	None	-3.127	-4.262
Zero	None	None	-3	-4
Comments	Second-order approx. OK	Pure second-order	Second-order approx. OK	Second-order approx. OK

PD controller design example

Given the system, design an PD controller to yield a 16% overshoot, with a threefold reduction in settling time



Performance of a second-order system



PO=percent overshoot

Pv= peak value

Fv = final value

$$PO = \frac{M_{pv} - fv}{fv} \cdot 100$$

$$T_s = \frac{4}{\zeta \cdot \omega_n}$$

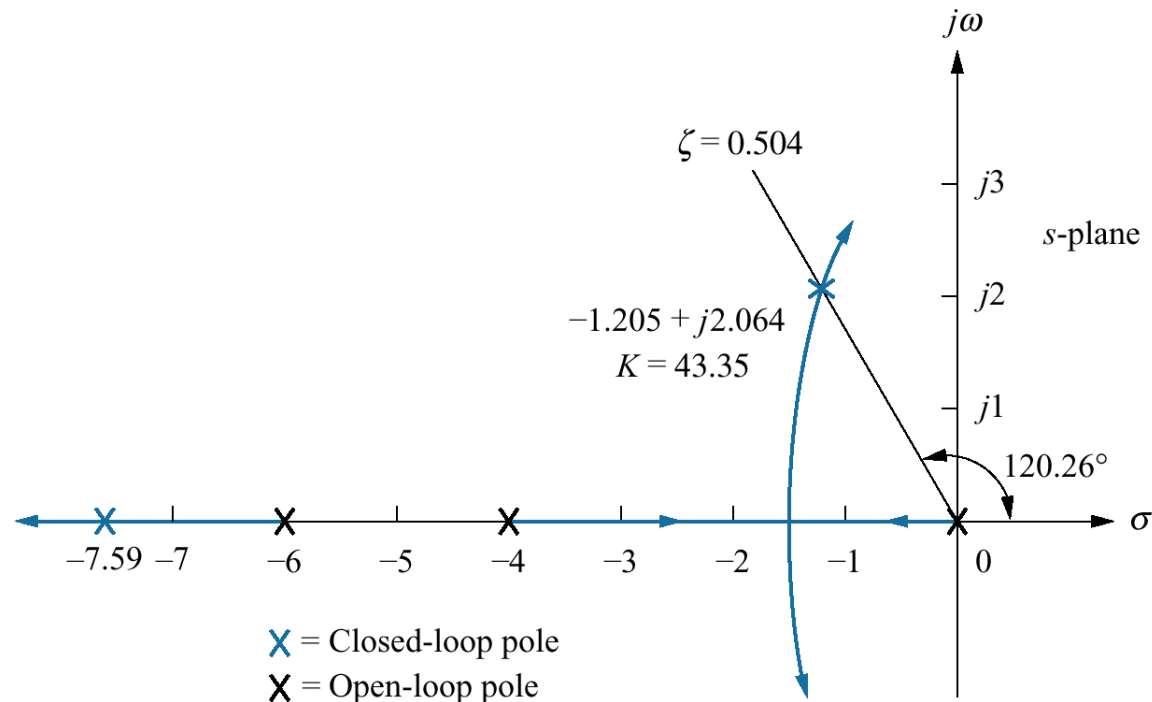
$$T_p = \frac{\pi}{\omega_n \cdot \sqrt{1 - \zeta^2}}$$

$$M_{pv} = 1 + e^{-\zeta \cdot \frac{\pi}{\sqrt{1 - \zeta^2}}}$$

$$PO = 100 \cdot e^{-\zeta \cdot \frac{\pi}{\sqrt{1 - \zeta^2}}}$$

Figure 9.18

Root locus for uncompensated system shown in Figure 9.17



Overshoot 16% is equivalent to damping rate 0.504
Settling time T_s

$$T_s = \frac{4}{\zeta\omega_n} = \frac{4}{1.205} = 3.320$$

The desired settling time $T_s = 3.320/3 = 1.107$

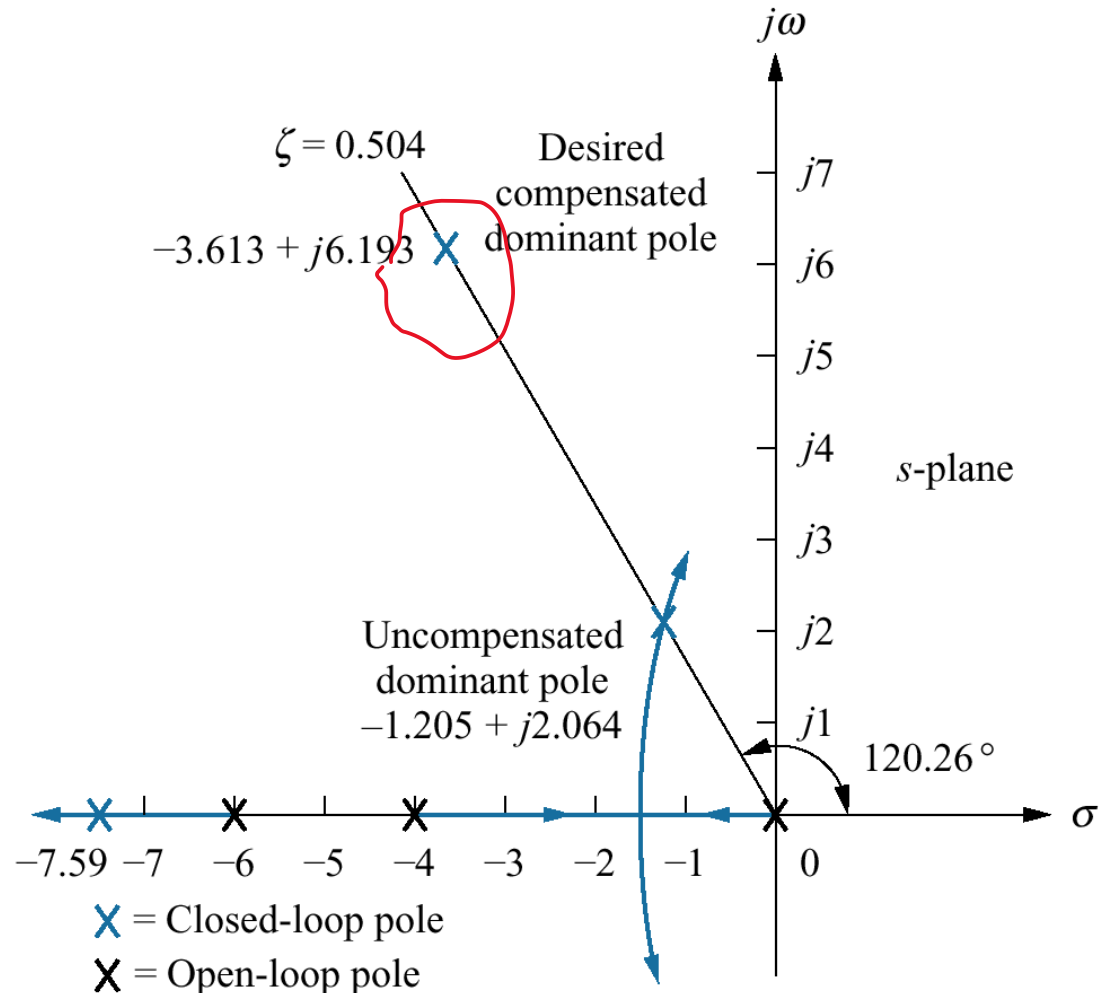
$$\sigma = \frac{4}{T_s} = \frac{4}{1.107} = 3.613$$

the real part will be -3.613

The imaginary part = 6.193

The desired compensated point is $-3.613 + j6.193$

Figure 9.19
Compensated dominant pole superimposed over the uncompensated root locus for Example 9.3



The imaginary part of

$$\omega_d = 3.613 \tan(180^\circ - 120.26^\circ) = 6.193$$

Design the location

All the angles to the design point of all the poles and zeros of the compensated system plus the angle contribution by the compensator zero should be 180°

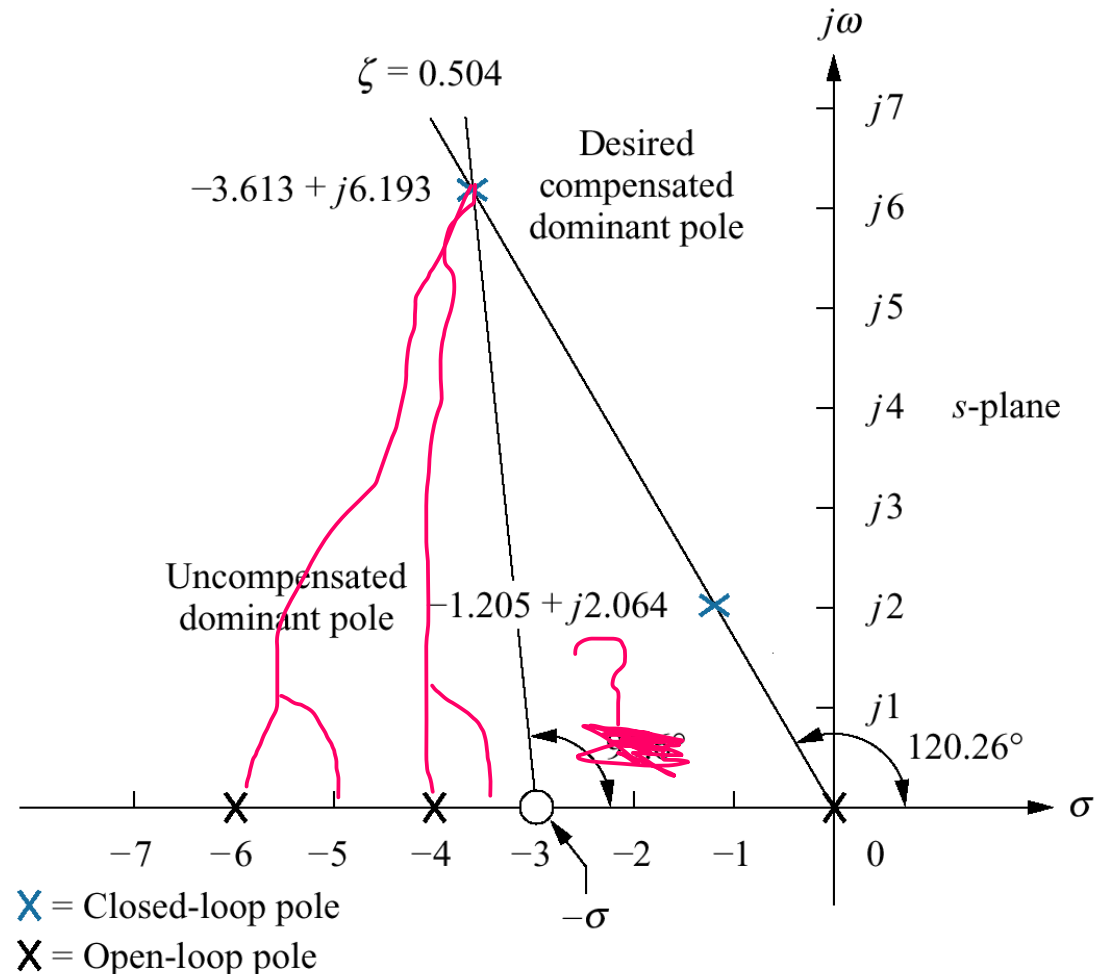


Figure 9.20

Evaluating the location of the compensating zero for Example 9.3

$$\frac{6.193}{3.613 - \sigma} = \tan(180^\circ - 95.6^\circ)$$

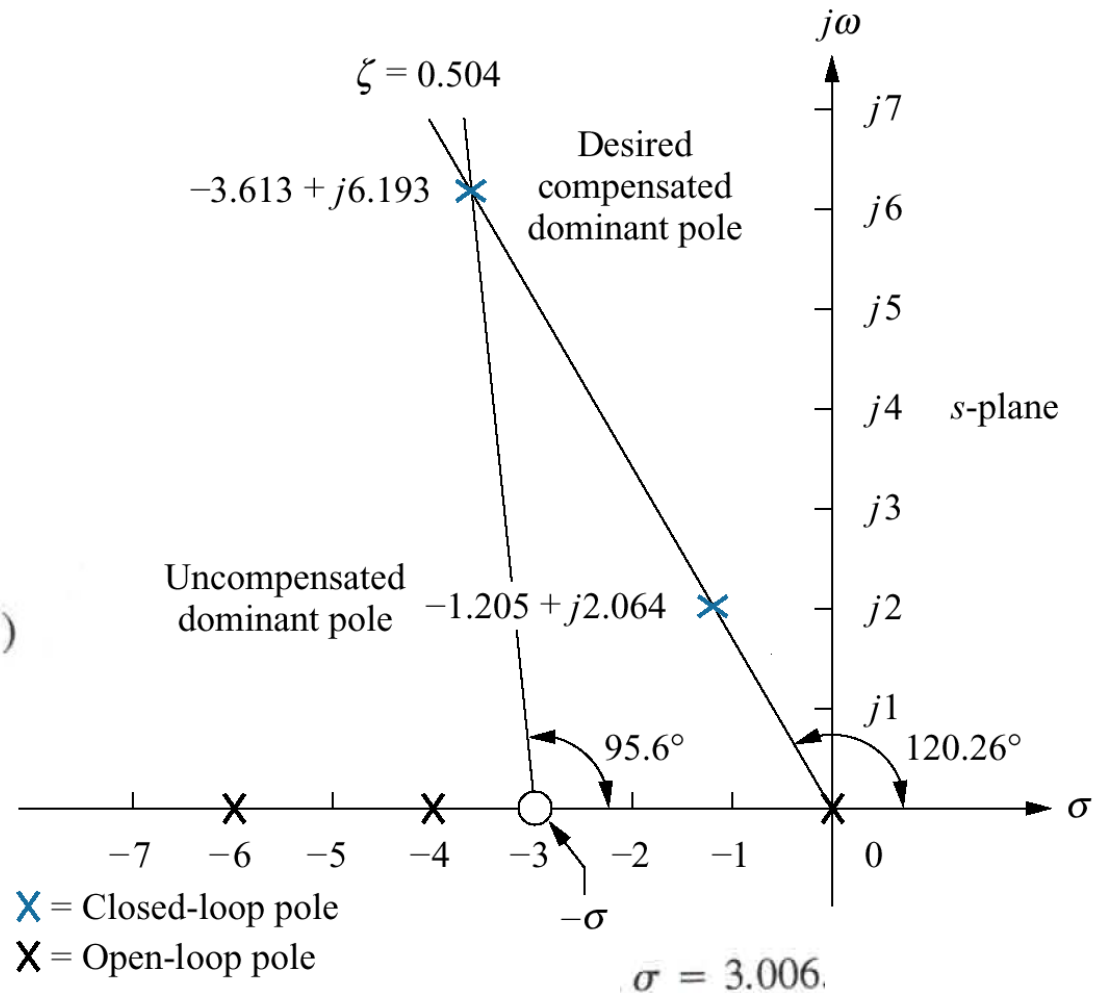


Figure 9.21
Root locus for
the
compensated
system of
Example 9.3

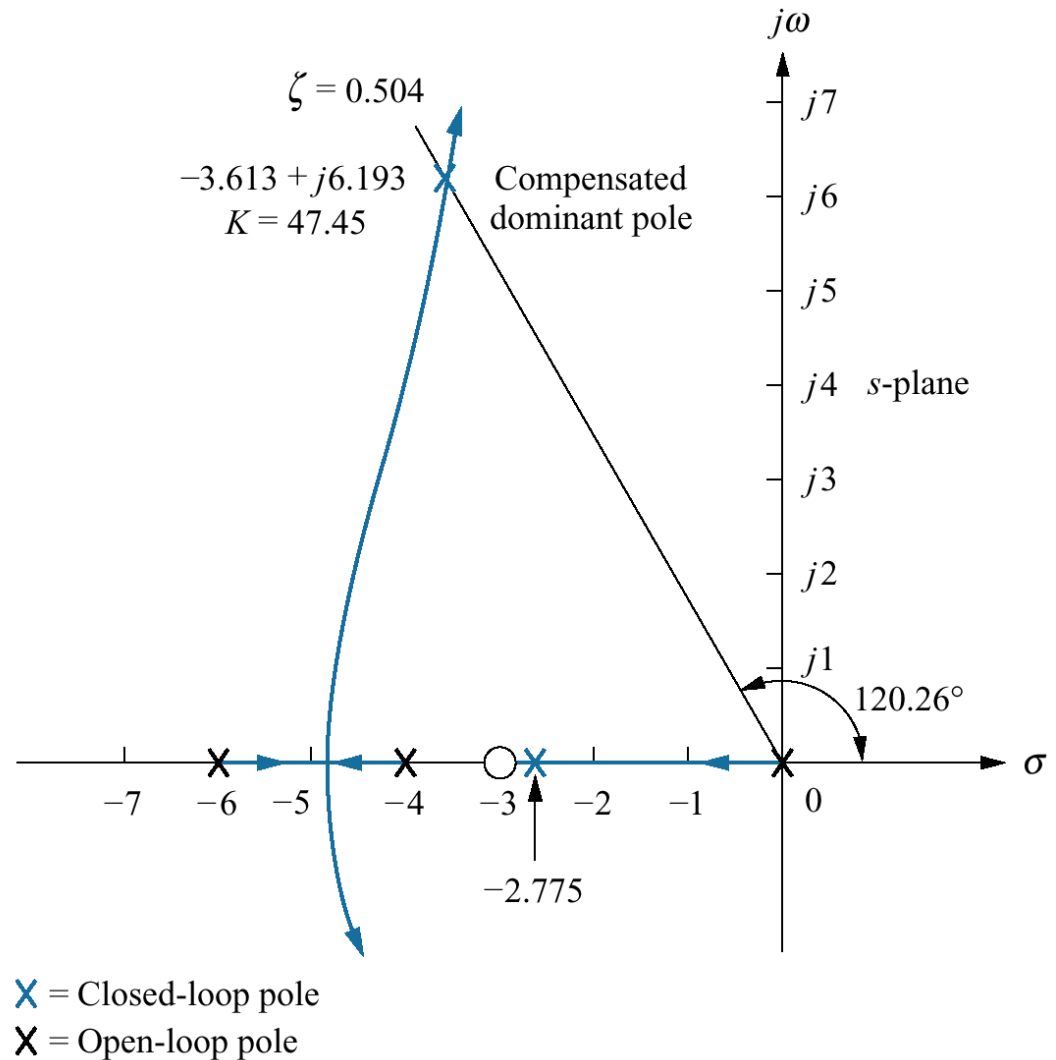


Figure 9.22
Uncompensated
and
compensated
system step
responses of
Example 9.3

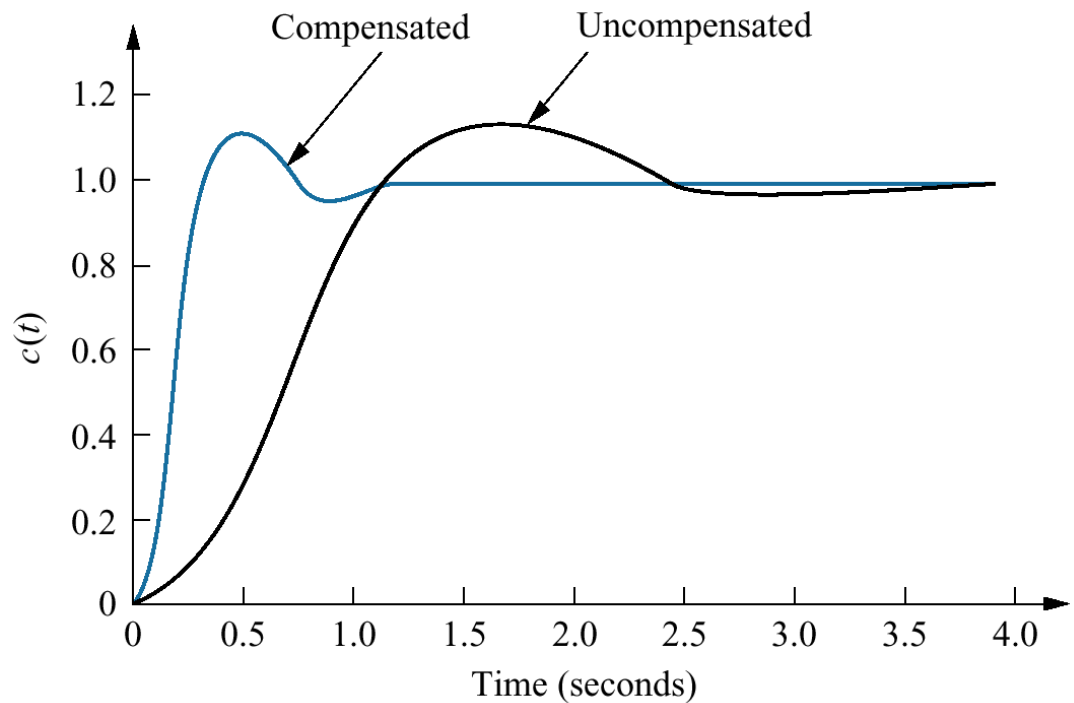


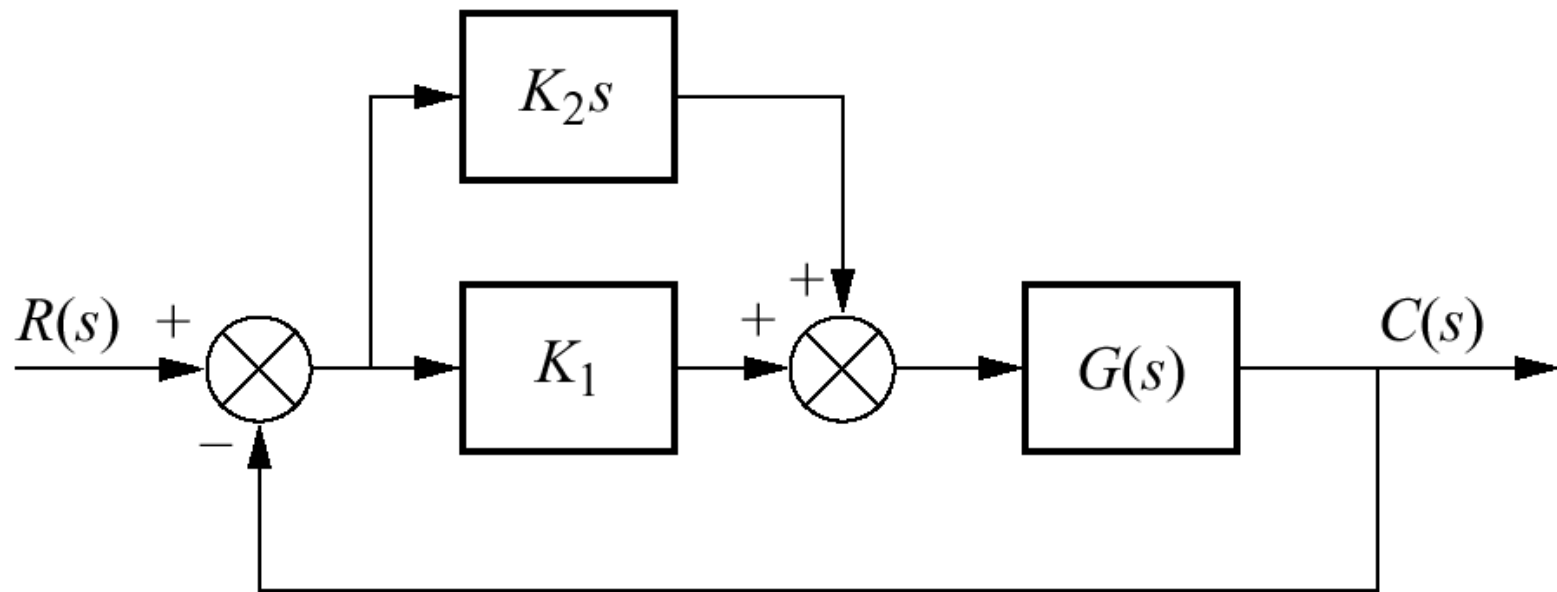
Table 9.3

Uncompensated and compensated system characteristics for Example 9.3

	Uncompensated	Simulation	Compensated	Simulation
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$		$\frac{K(s+3.006)}{s(s+4)(s+6)}$	
Dominant poles	$-1.205 \pm j2.064$		$-3.613 \pm j6.193$	
K	43.35		47.45	
ζ	0.504		0.504	
ω_n	2.39		7.17	
%OS	16	14.8	16	11.8
T_s	3.320	3.6	1.107	1.2
T_p	1.522	1.7	0.507	0.5
K_v	1.806		5.94	
$e(\infty)$	0.554		0.168	
Third pole	-7.591		-2.775	
Zero	None		-3.006	
Comments	Second-order approx. OK		Pole-zero not canceling	

Figure 9.23
PD controller

$$G_c(s) = K_2s + K_1 = K_2 \left(s + \frac{K_1}{K_2} \right)$$



Lead compensation

Ideal derivative compensator needs an active network to implement, i.e. an additional power supplier, and noise due to differentiation is big.

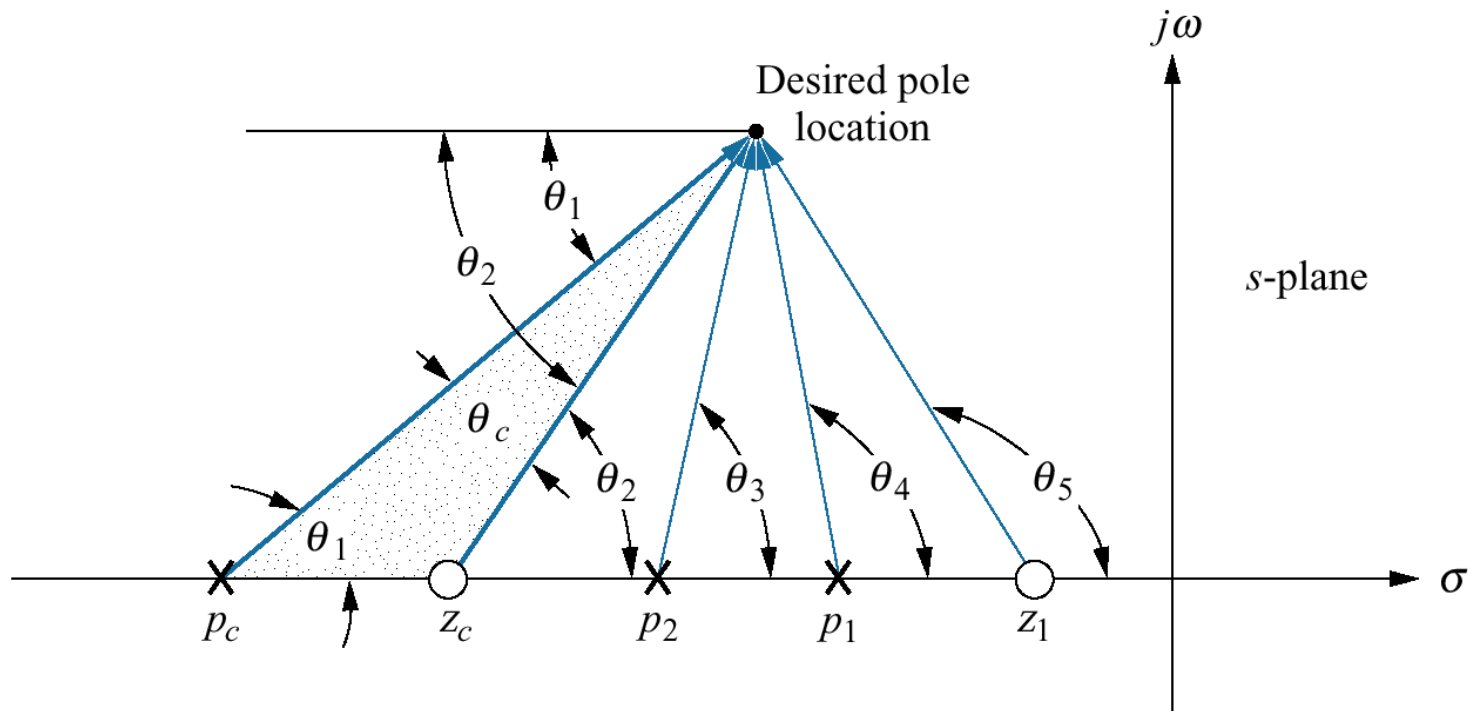
Lead compensator

$$G_c(s) = \frac{(s + z_c)}{(s + p_c)} \quad z_c < p_c$$

Figure 9.24

Geometry of lead compensation

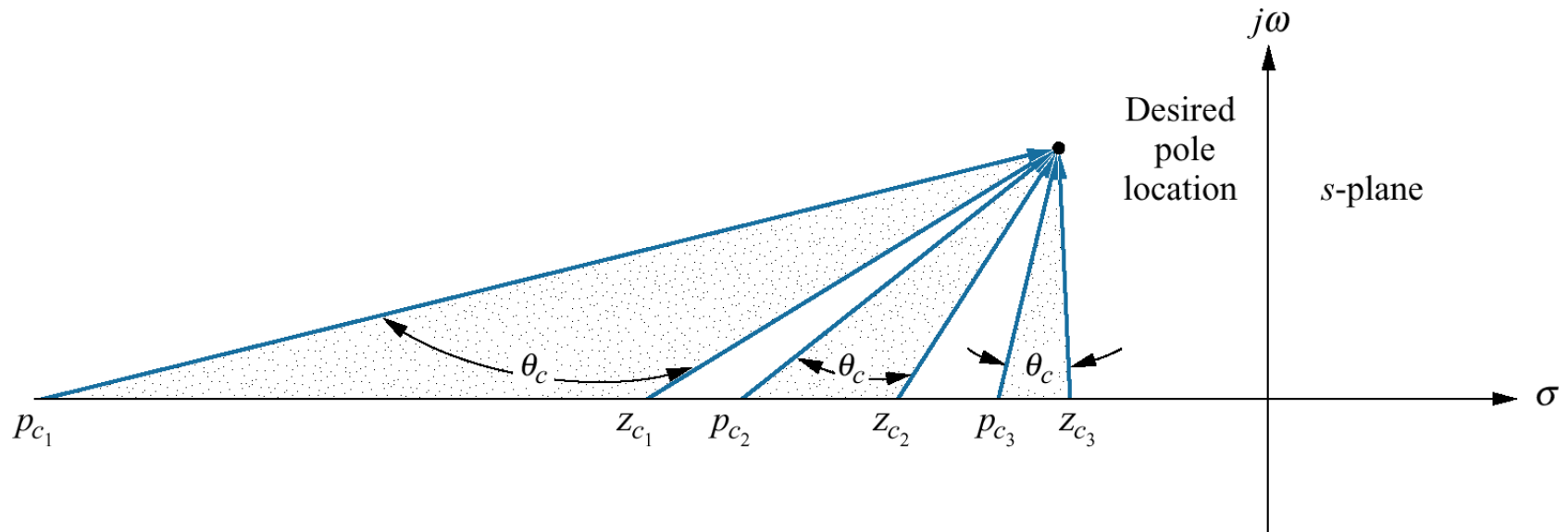
$$\theta_2 - \theta_1 - \theta_3 - \theta_4 + \theta_5 = (2k + 1)180^\circ$$



Θ_c is the angle contribution made by the compensator

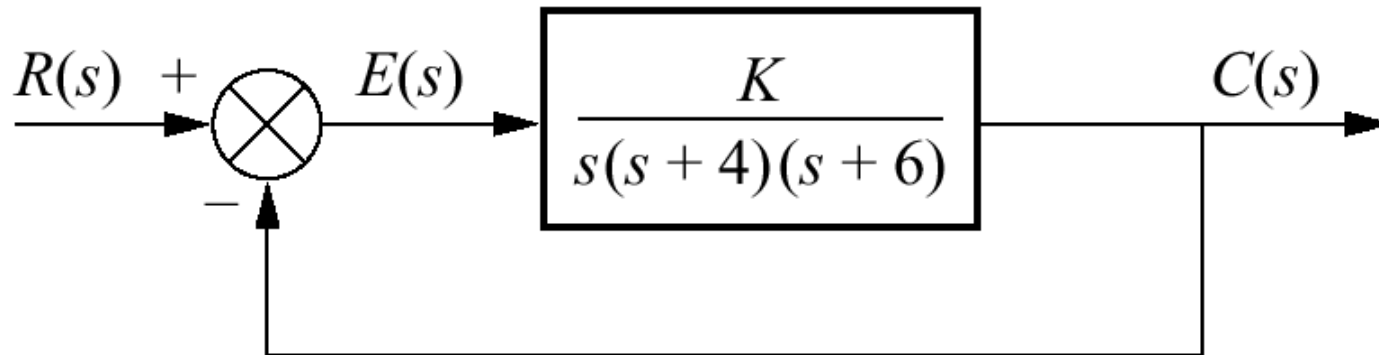
Figure 9.25

Three of the infinite possible lead compensator solutions



Example of lead compensator design

Design three lead compensators for the system that will reduce the settling time by a factor of 2 while maintaining 30% overshoot. Compare the system characteristics among the three designs.



Overshoot 30% is equivalent to damping rate 0.358
Settling time T_s

$$T_s = \frac{4}{\zeta\omega_n} = 4/1.007=3.972$$

The desired settling time $T_s=3.972/2=1.986$

$$\sigma = \frac{4}{T_s} = 4/1.986=2.014$$

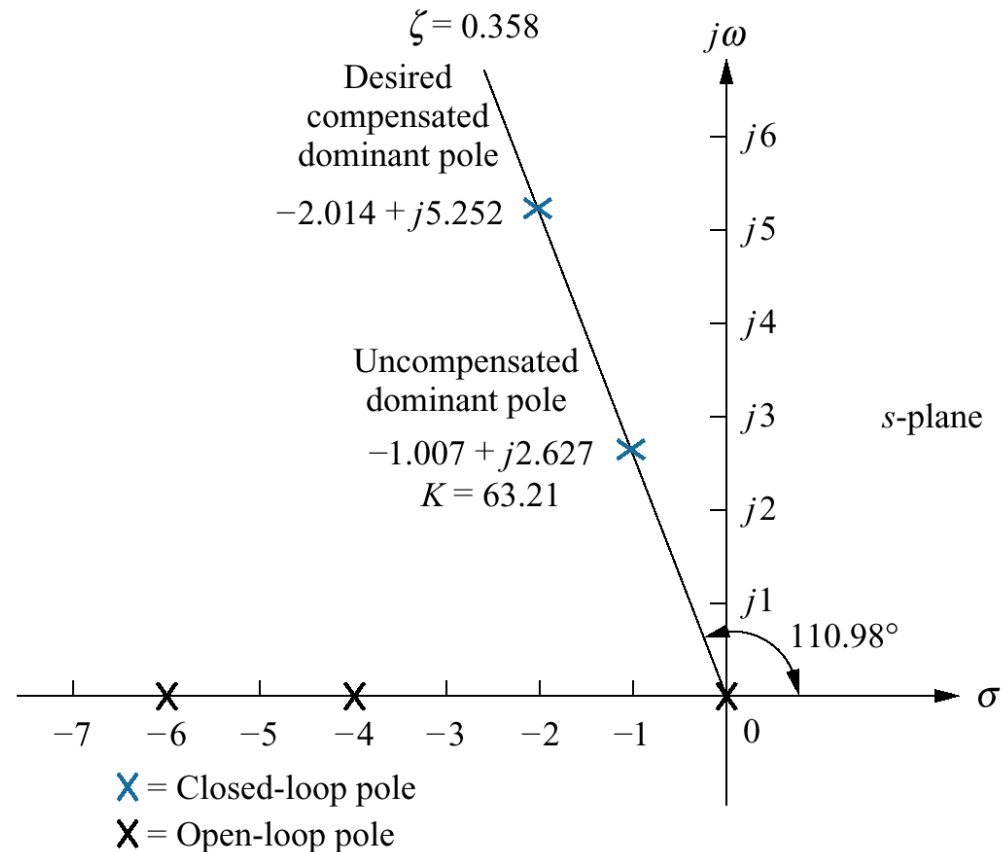
the real part will be -2.014

The imaginary part = $-2.014\tan(110.98)=5.252$

The desired compensated point is $-2.014+j5.252$

Figure 9.26

Lead compensator design, showing evaluation of uncompensated and compensated dominant poles for Example 9.4



Let arbitrarily assume the compensator zero $z_c = -5$

$$G_c(s) = \frac{(s + z_c)}{(s + p_c)}$$

In order to find the location of the pole p_c , we need to find out the angle contribution made by the compensator pole

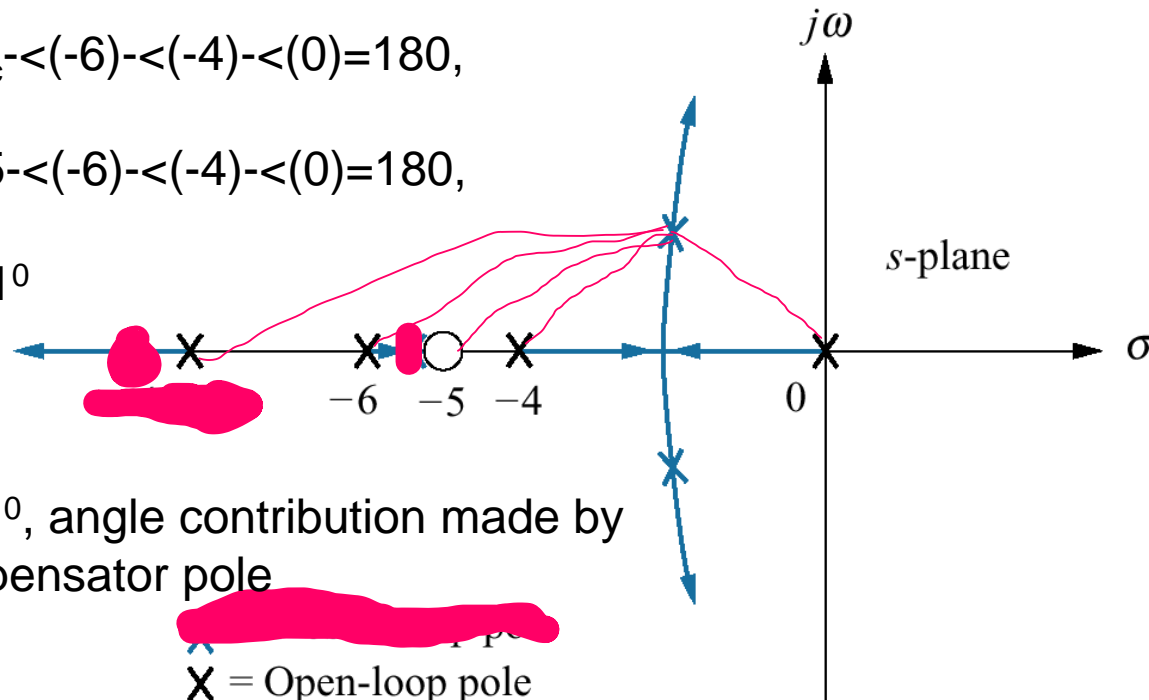
$$-\angle p_c + \angle z_c - \angle(-6) - \angle(-4) - \angle(0) = 180^\circ,$$

$$-\angle p_c + \angle(-5) - \angle(-6) - \angle(-4) - \angle(0) = 180^\circ,$$

$$\angle p_c = 7.31^\circ$$

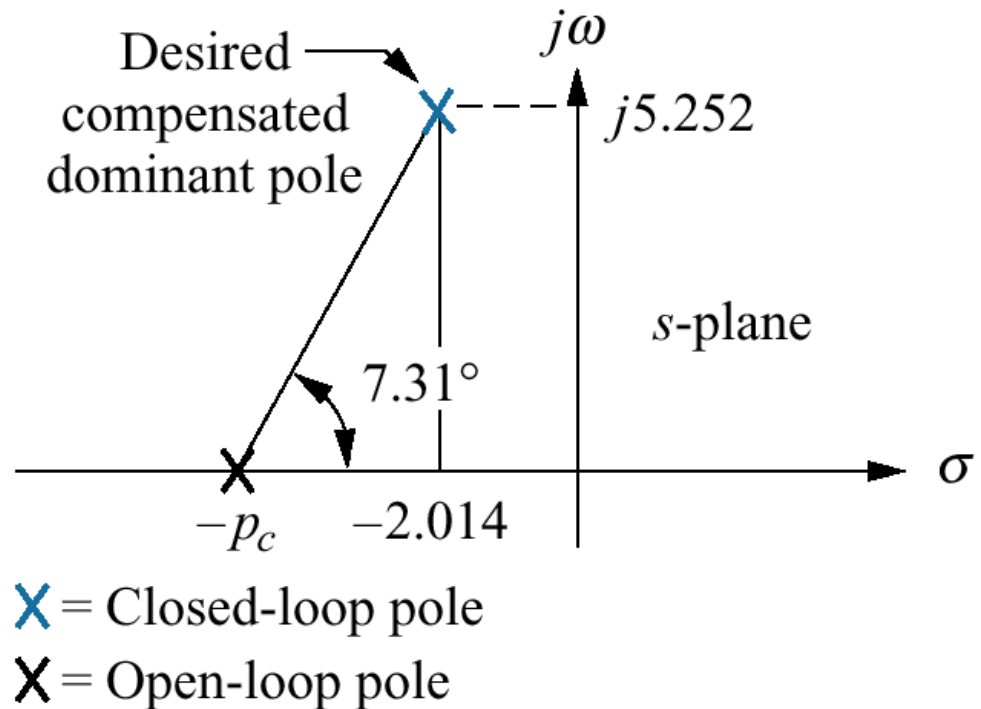
$\angle p_c = 7.31^\circ$, angle contribution made by the compensator pole

\times = Open-loop pole



Note: This figure is not drawn to scale.

Figure 9.27
s-plane picture
used to calculate
the location of
the compensator
pole for
Example 9.4



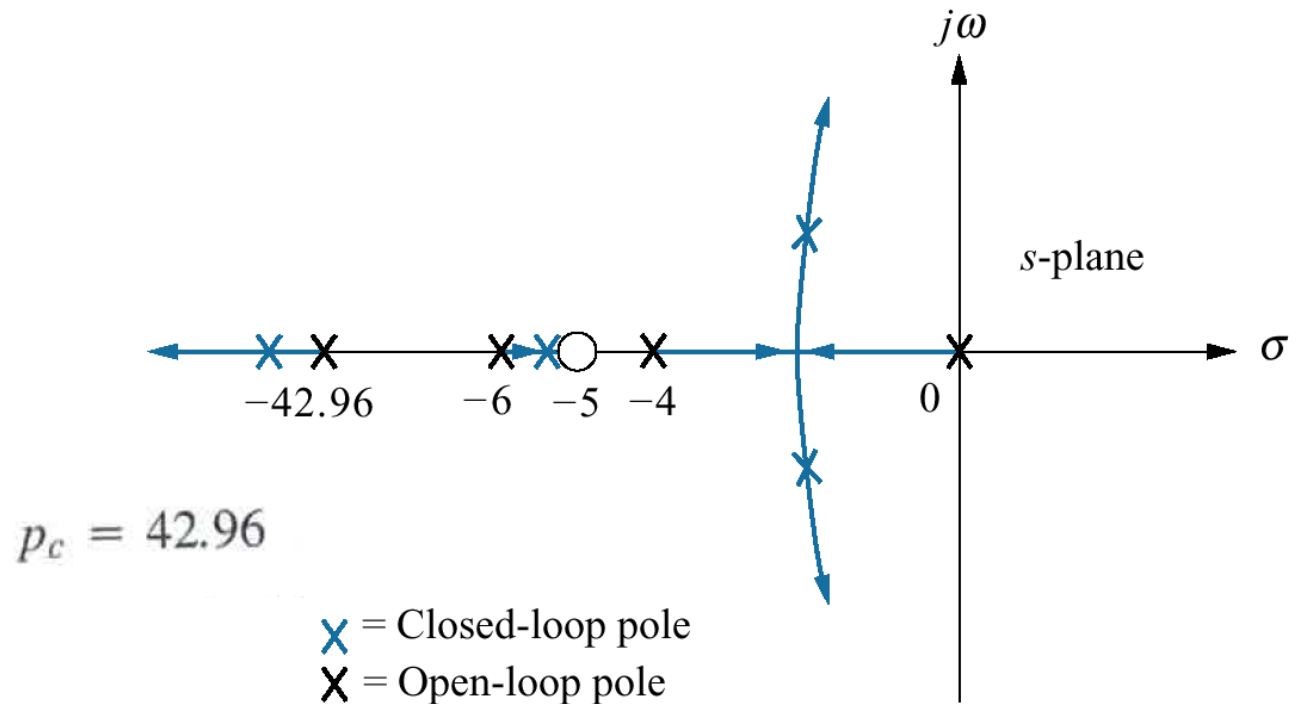
Note: This figure is not drawn to scale.

$$\frac{5.252}{p_c - 2.014} = \tan 7.31^\circ$$

$$p_c = 42.96$$

Figure 9.28

Compensated system root locus



Note: This figure is not drawn to scale.

Figure 9.29
Uncompensated system and lead compensation responses for Example 9.4

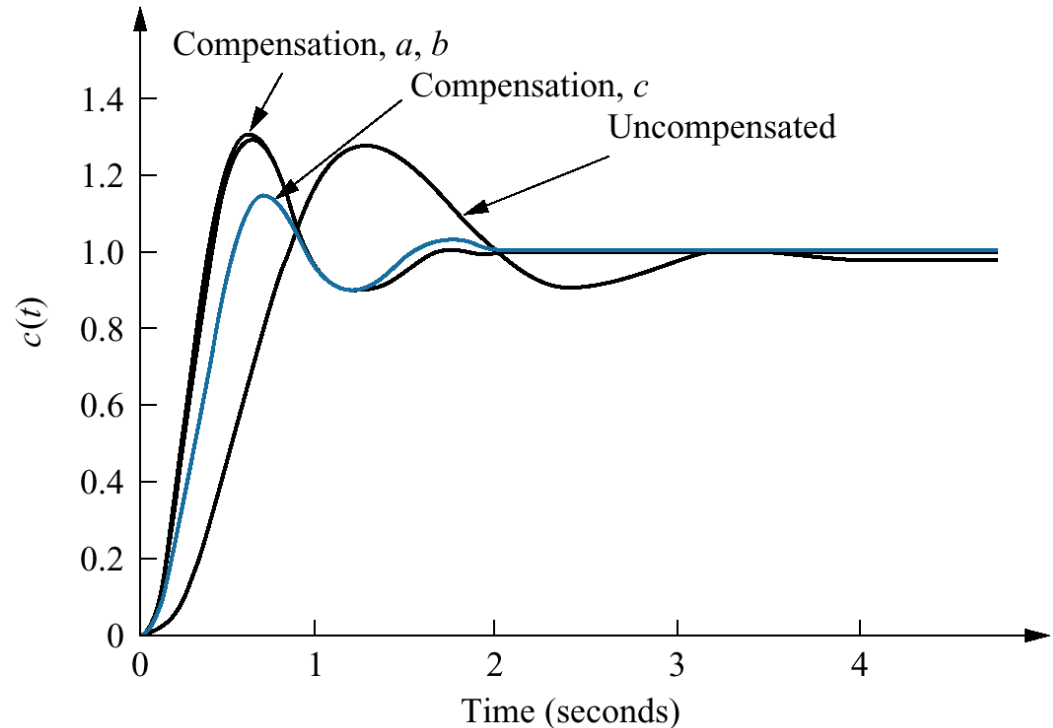


Table 9.4

Comparison of lead compensation designs for Example 9.4

	Uncompensated	Compensation a	Compensation b	Compensation c
Plant and compensator	$\frac{K}{s(s+4)(s+6)}$	$\frac{K(s+5)}{s(s+4)(s+6)(s+42.96)}$	$\frac{K(s+4)}{s(s+4)(s+6)(s+20.09)}$	$\frac{K(s+2)}{s(s+4)(s+6)(s+8.971)}$
Dominant poles	$-1.007 \pm j2.627$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$	$-2.014 \pm j5.252$
K	63.21	1423	698.1	345.6
ζ	0.358	0.358	0.358	0.358
ω_n	2.813	5.625	5.625	5.625
%OS	30 (28)	30 (30.7)	30 (28.2)	30 (14.5)
T_s	3.972 (4)	1.986 (2)	1.986 (2)	1.986 (1.7)
T_p	1.196 (1.3)	0.598 (0.6)	0.598 (0.6)	0.598 (0.7)
K_v	2.634	6.9	5.791	3.21
$e(\infty)$	0.380	0.145	0.173	0.312
Other poles	-7.986	-43.8, -5.134	-22.06	-13.3, -1.642
Zero	None	-5	None	-2
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK	No pole-zero cancellation

Note: Simulation results are shown in parentheses.

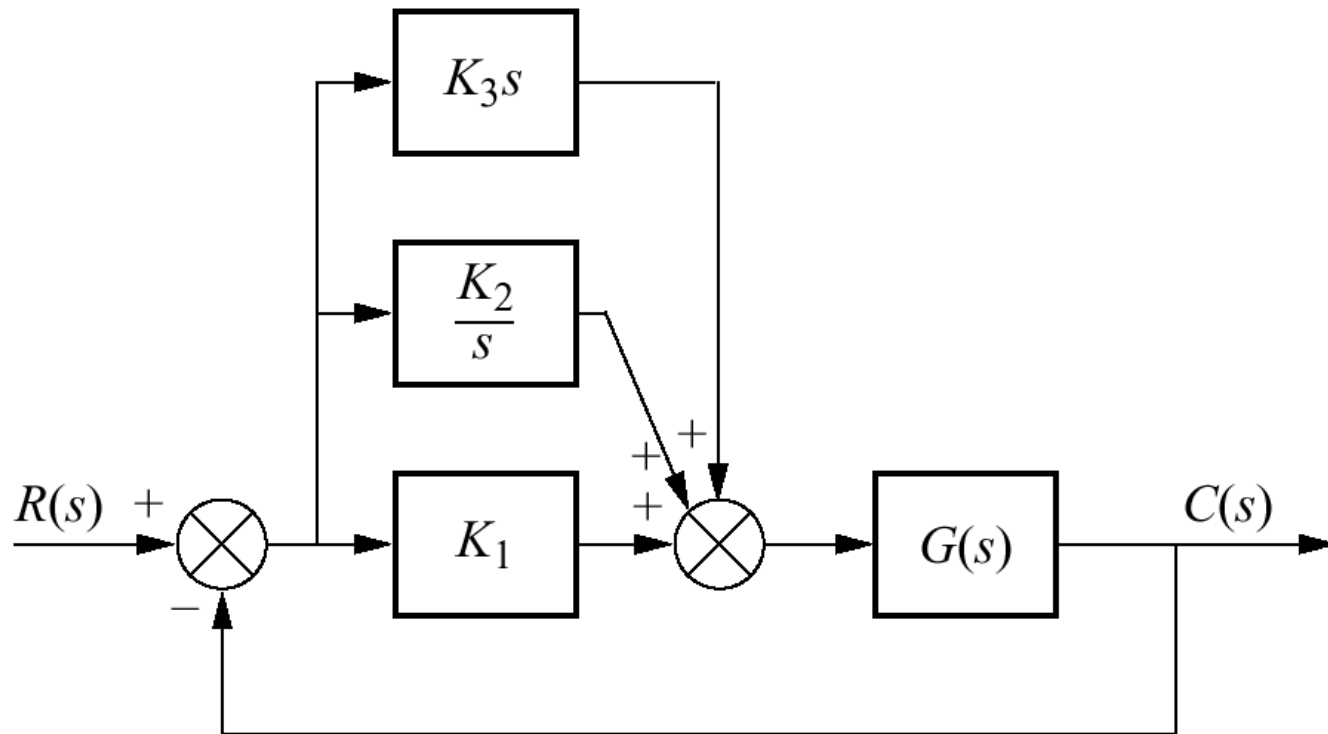
Improving steady-state error and transient response

Design an active PD controller followed by an active PI controller, the resulting compensator is a PID (proportional-plus-integral-plus-derivative) controller.

Design a passive lead compensator and then design a passive lag compensator, the resulting compensator is a lag-lead compensator.

PID controller

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3s = \frac{K_1s + K_2 + K_3s^2}{s} = \frac{K_3 \left(s^2 + \frac{K_1}{K_3}s + \frac{K_2}{K_3} \right)}{s}$$



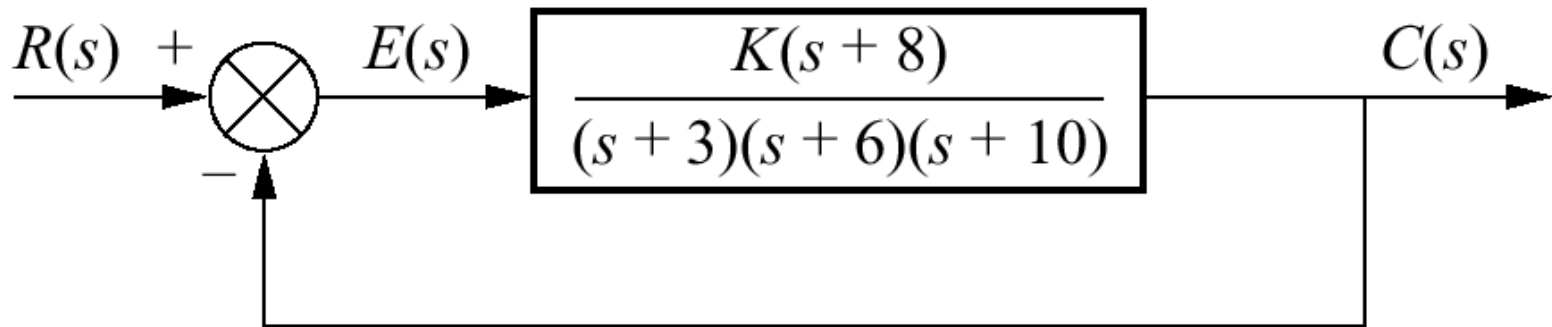
Design steps of PID controller:

1. Evaluate the performance of the uncompensated system to determine how much improvement in transient response is required.
2. Design the PD controller to meet the transient response specification
3. Simulate the system to be sure all requirements have been met.
4. Re-design if the simulation shows the requirements have not been met.
5. Design the PI controller to yield the required steady-state error
6. Determine the gain, k_1 , k_2 , k_3
7. Simulate the system to be sure all requirements have been met.
8. Redesign if the simulation shows the requirements have not been met.

Figure 9.31

Uncompensated feedback control system for Example 9.5

Problem: design a PID controller so that the system can operate with a peak time that is two-thirds that of the uncompensated system at 20% overshoot and with zero steady-state error for a step input.



Step 1: Evaluate the performance of the uncompensated system

a) 20% overshoot is equivalent to damping rate $\xi=0.456$

b) Dominant pole is $-5.415 + (-)j10.57$ with $k=121.5$

c) Peak time $T_p=0.297$

$$T_p = \frac{\pi}{\omega_n \cdot \sqrt{1 - \zeta^2}}$$

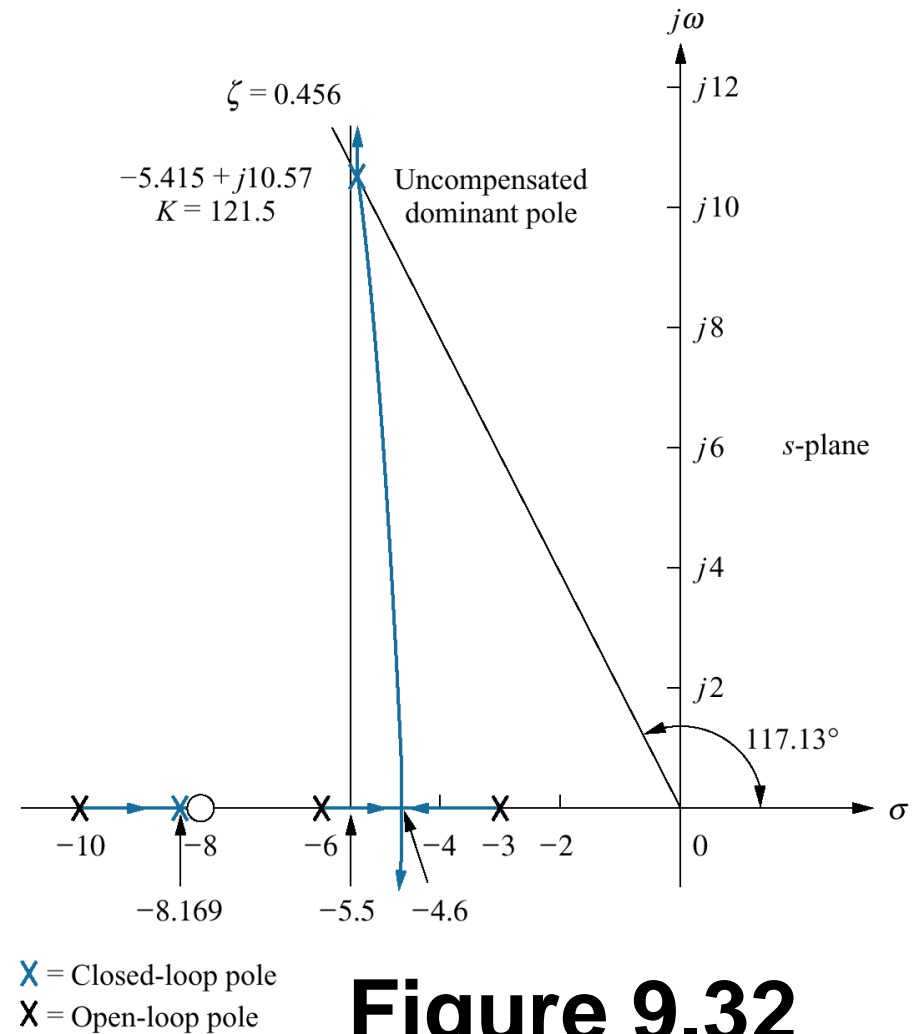


Figure 9.32

Root locus for the uncompensated system of Example 9.5

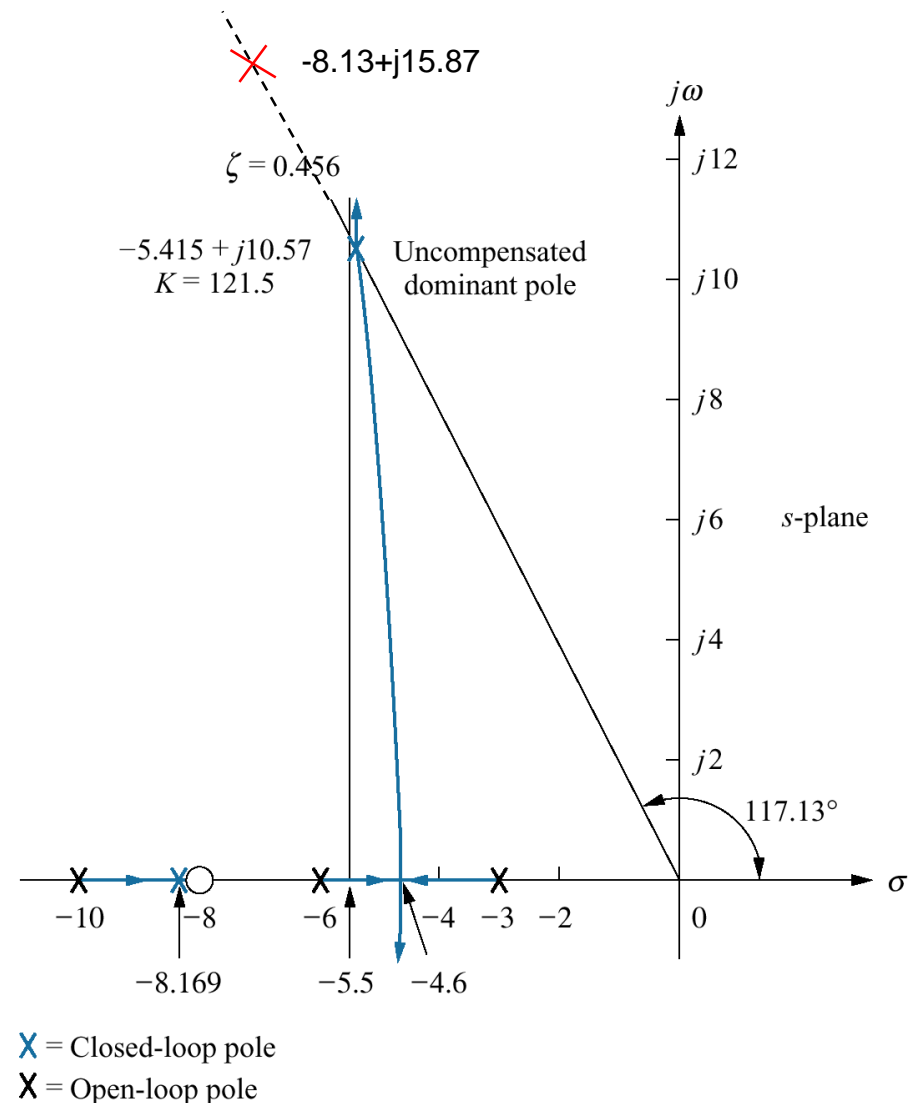
Step 2: to compensate the system to reduce the peak time to 2/3 of the uncompensated system.

The imaginary part of the compensated dominant pole is

$$\omega_d = \frac{\pi}{T_p} = \frac{\pi}{(2/3)(0.297)} = 15.87$$

The real part of the compensated dominant pole is

$$\sigma = \frac{\omega_d}{\tan 117.13^\circ} = -8.13$$



Step 2: Design a PD controller.

$$G_{PD}(s) = s + z_c$$

The sum of the angles from the uncompensated system's poles and zeros to the desired dominant pole is -198.37° .

The contribution required from the compensator is $180^\circ - 198.37^\circ = 18.37^\circ$

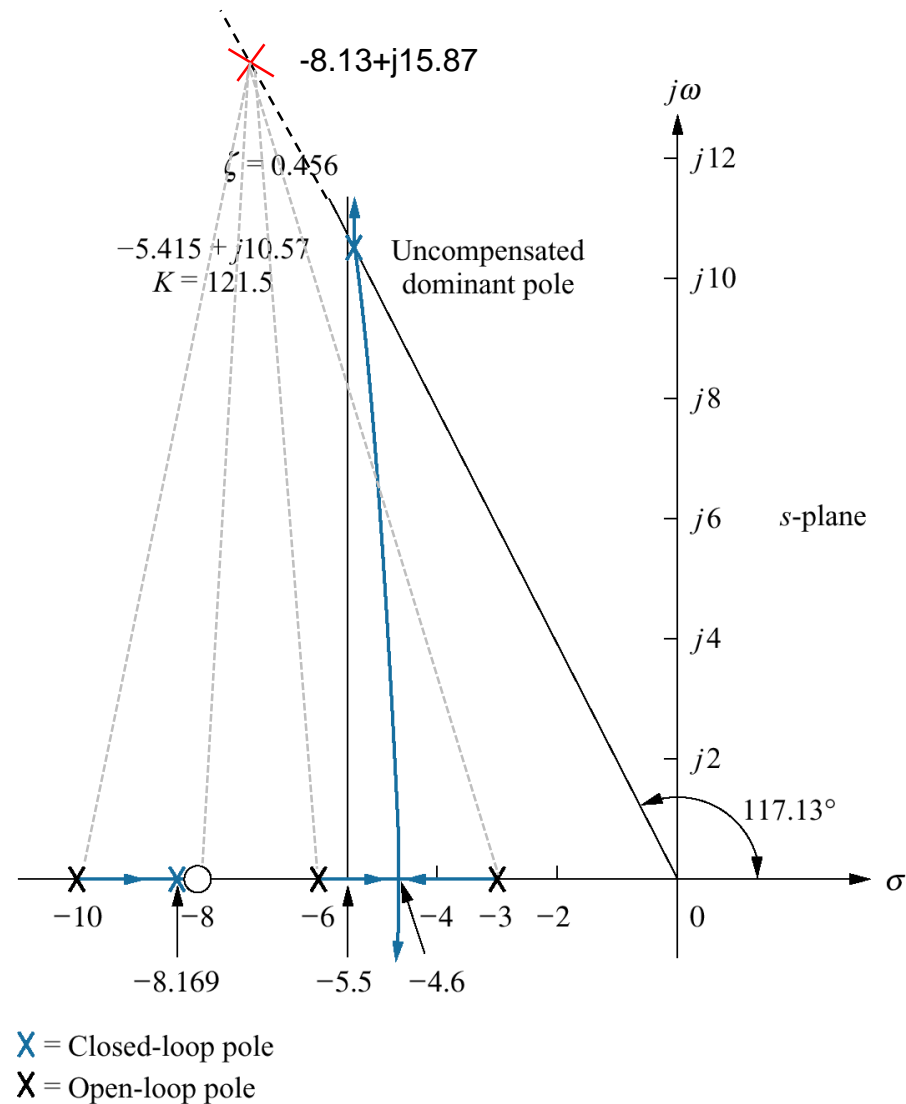
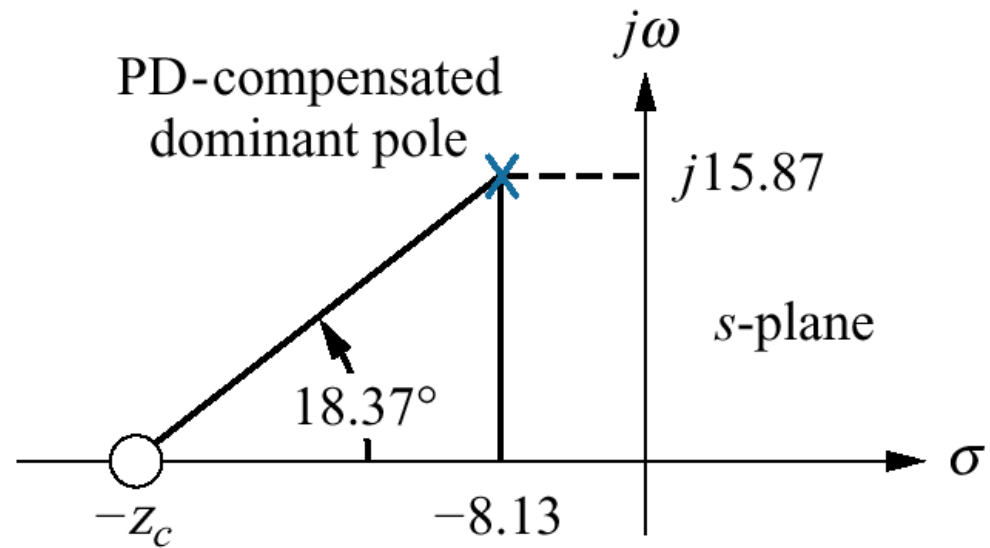


Figure 9.33

Calculating the PD compensator zero for Example 9.5

$$\frac{15.87}{z_c - 8.13} = \tan 18.37^\circ$$

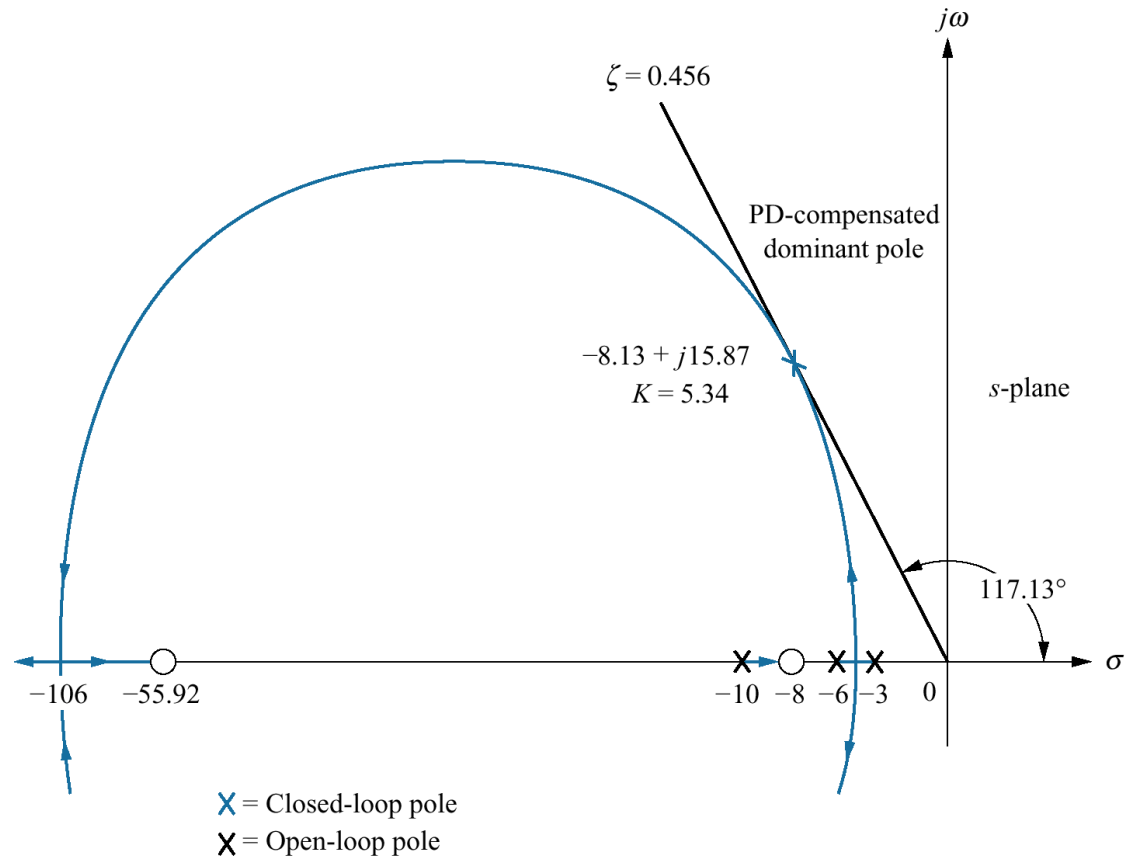
$z_c = 8.13$,
PD controller is
 $G_{PD}(s) = s + 8.13$



X = Closed-loop pole

Note: This figure is not drawn to scale.

Figure 9.34
Root locus for
PD-compensated
system of
Example 9.5



Note: This figure is not drawn to scale.

Step 3 and 4: simulate the PD-controlled system. We see the reduction in peak time and the improvement in steady-state error.

Step 5: Add a PI controller to reduce the steady-state error to zero for a step input.

Any ideal integral compensator zero will work, as long as the zero is placed close to the origin.

$$G_{PI}(s) = \frac{s + 0.5}{s}$$

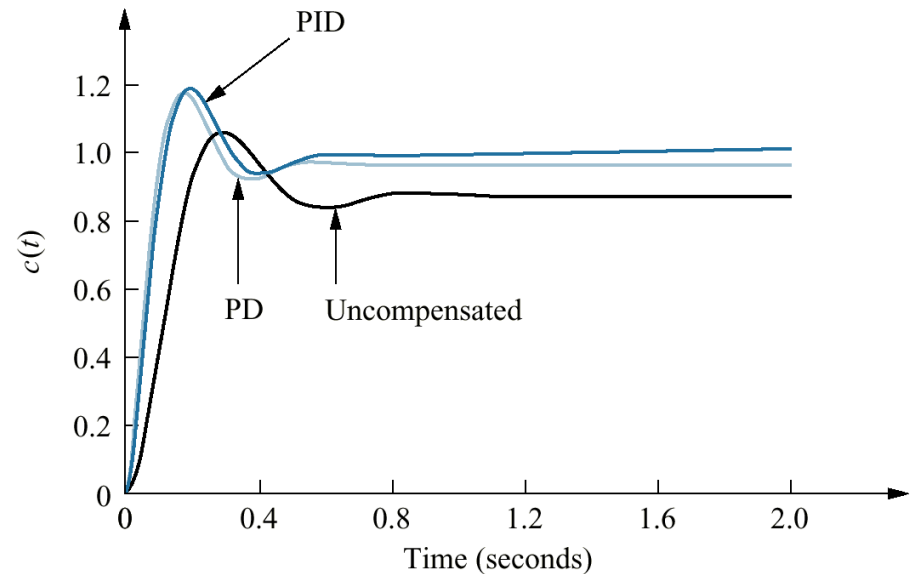
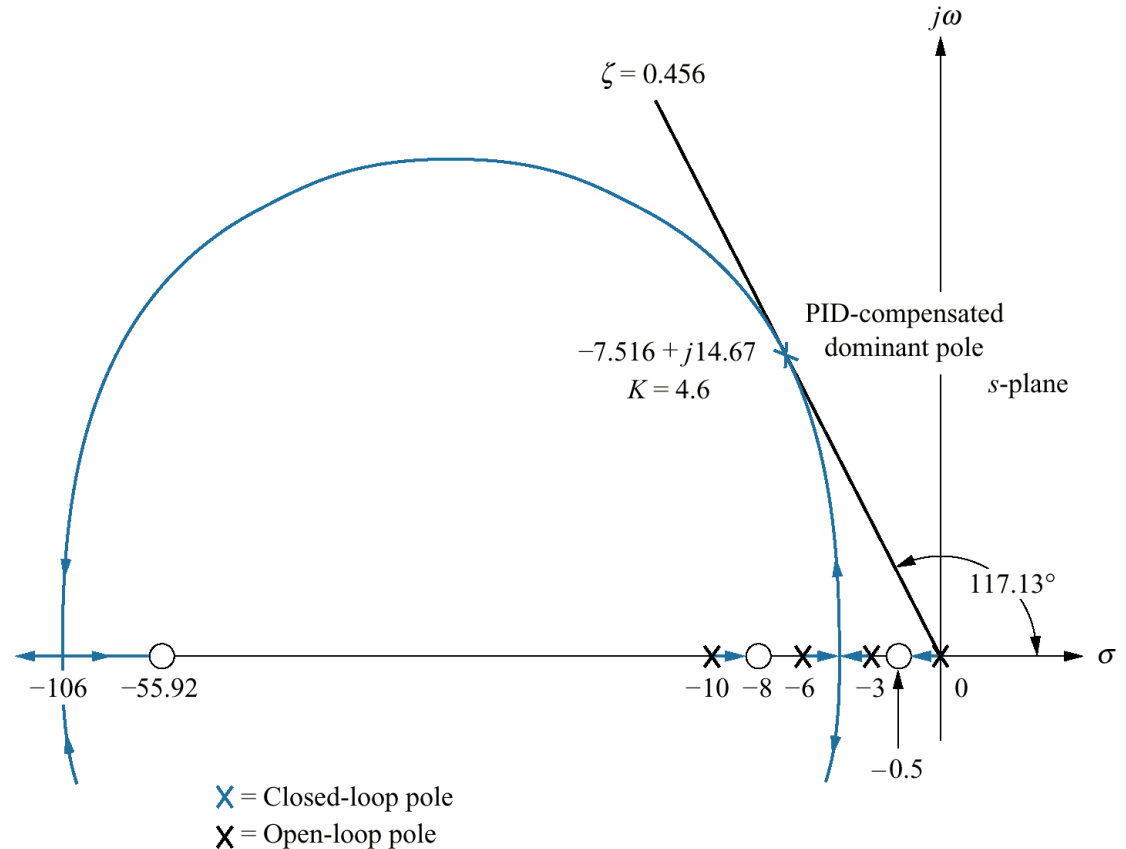


Figure 9.36
Root locus for
PID-
compensated
system
of Example 9.5



Note: This figure is not drawn to scale.

Step 6: Determine the gains K1, K2, K3

$$\begin{aligned} G_{\text{PID}}(s) &= \frac{K(s + 55.92)(s + 0.5)}{s} = \frac{4.6(s + 55.92)(s + 0.5)}{s} \\ &= \frac{4.6(s^2 + 56.42s + 27.96)}{s} \end{aligned}$$

K1 =259.5, K2=128.6, K3=4.6

Steps 7 and 8:

Simulate the result and redesign if necessary

Figure 9.35

Step responses for uncompensated, PD-compensated, and PID-compensated systems of Example 9.5

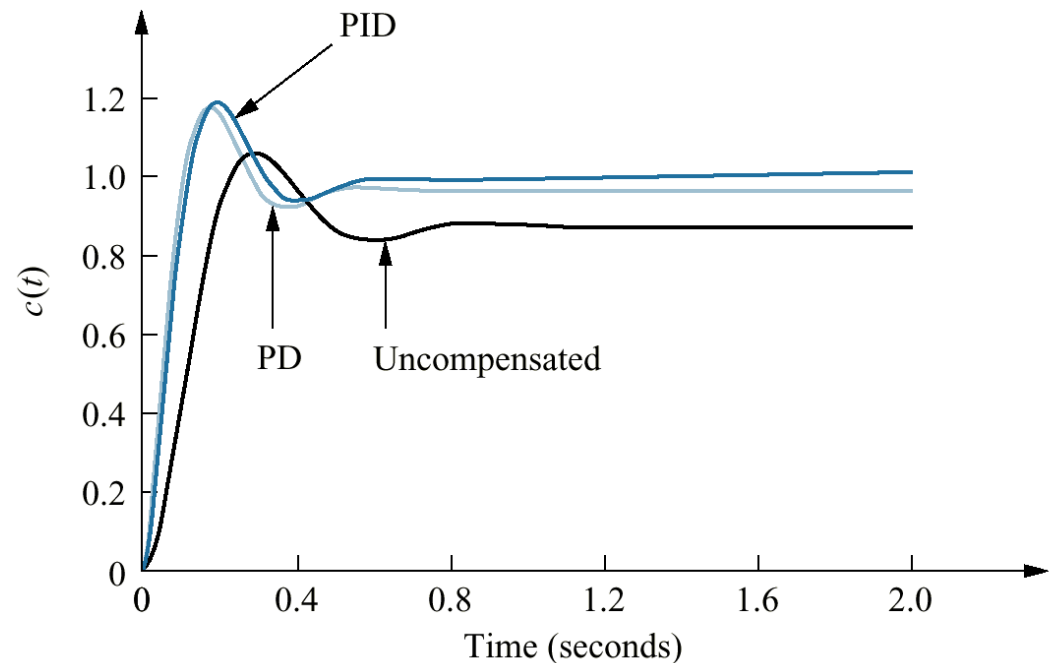


Table 9.5

Predicted characteristics of uncompensated, PD-, and PID- compensated systems of Example 9.5

	Uncompensated	PD-compensated	PID-compensated
Plant and compensator	$\frac{K(s+8)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)}{(s+3)(s+6)(s+10)}$	$\frac{K(s+8)(s+55.92)(s+0.5)}{(s+3)(s+6)(s+10)s}$
Dominant poles	$-5.415 \pm j10.57$	$-8.13 \pm j15.87$	$-7.516 \pm j14.67$
K	121.5	5.34	4.6
ζ	0.456	0.456	0.456
ω_n	11.88	17.83	16.49
%OS	20	20	20
T_s	0.739	0.492	0.532
T_p	0.297	0.198	0.214
K_p	5.4	13.27	∞
$e(\infty)$	0.156	0.070	0
Other poles	-8.169	-8.079	-8.099, -0.468
Zeros	-8	-8, -55.92	-8, -55.92, -0.5
Comments	Second-order approx. OK	Second-order approx. OK	Zero at -55.92 and -0.5 not canceled

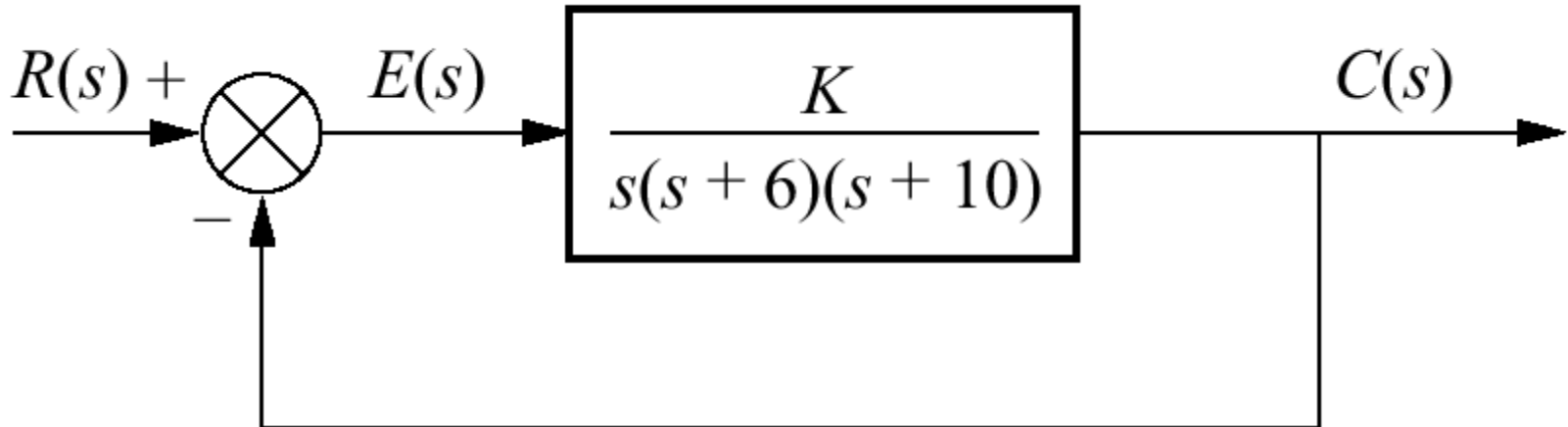
Lag-lead compensator design

Design steps:

1. Evaluate the performance of the uncompensated system to determine how much improvement in transient response is required.
2. Design the lead compensator to meet the transient response specification
3. Simulate the system to be sure all requirements have been met.
4. Re-design if the simulation shows the requirements have not been met.
5. Evaluate the steady-state error performance for the lead-compensated system to determine how much improvement in steady-state error is required.
6. Design the lag compensator to yield the required steady-state error
7. Simulate the system to be sure all requirements have been met.
8. Redesign if the simulation shows the requirements have not been met.

Example Uncompensated system for Example 9.6

Design a lag-lead compensator so that the system will operate with 20% overshoot, and a twofold reduction in settling time, and a tenfold improvement in steady-state error for a ramp input.

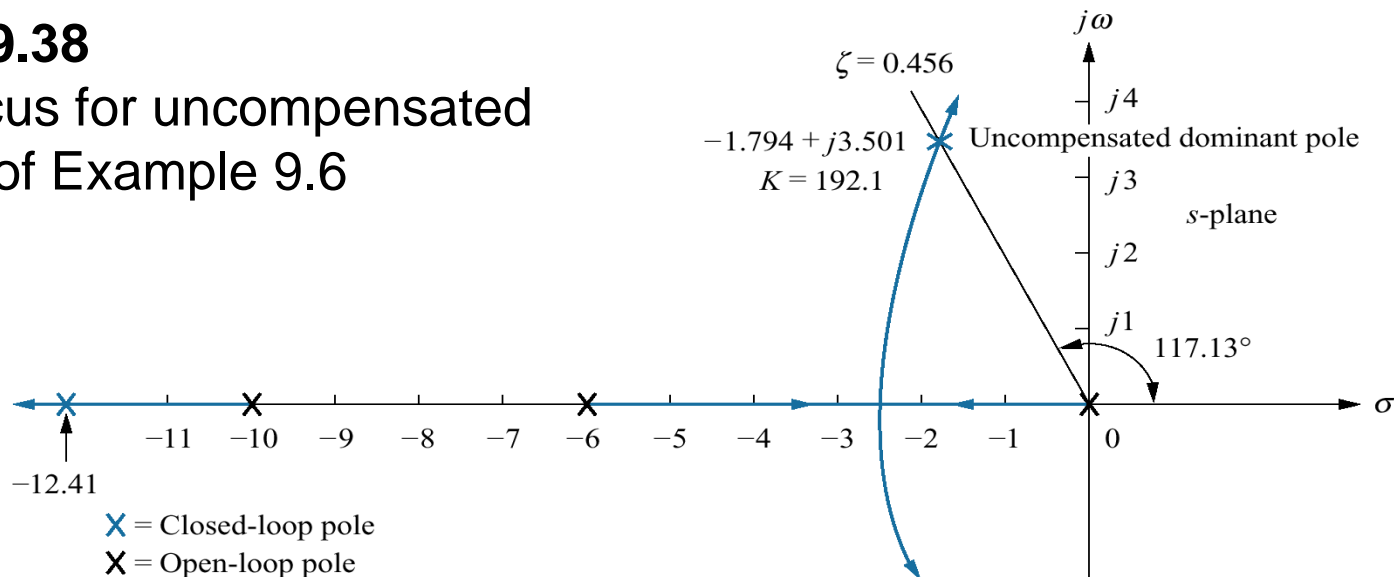


Step 1: Evaluate the performance of the uncompensated system.

a) 20% overshoot is equivalent to damping rate $\xi=0.456$, $\theta=117.13^\circ$

b) Dominant pole is $-1.794 + (-)j3.501$ with $k=192.1$

Figure 9.38
Root locus for uncompensated
system of Example 9.6



Step 2: Design a lead compensator to realize a twofold reduction in settling time.

$$T_s = \frac{4}{\zeta \cdot \omega_n}$$

The real part of the desired dominant pole

$$-\xi\omega_n = -2 \times 1.794 = -3.588$$

The imaginary part of the desired dominant pole

$$\omega_d = \xi\omega_n \tan 117.13^\circ = 7.003$$

Arbitrarily select $z_c = -6$, the compensator zero which is coincident with the open-loop pole.

To find the location of the compensator pole, sum the angles to the design point from the uncompensated system's poles and zeros and the compensator zero (-6) and get -164.65° . The angle contribution expected from the compensator pole is -15.35°

Figure 9.38

Root locus for
uncompensated
system of
Example 9.6

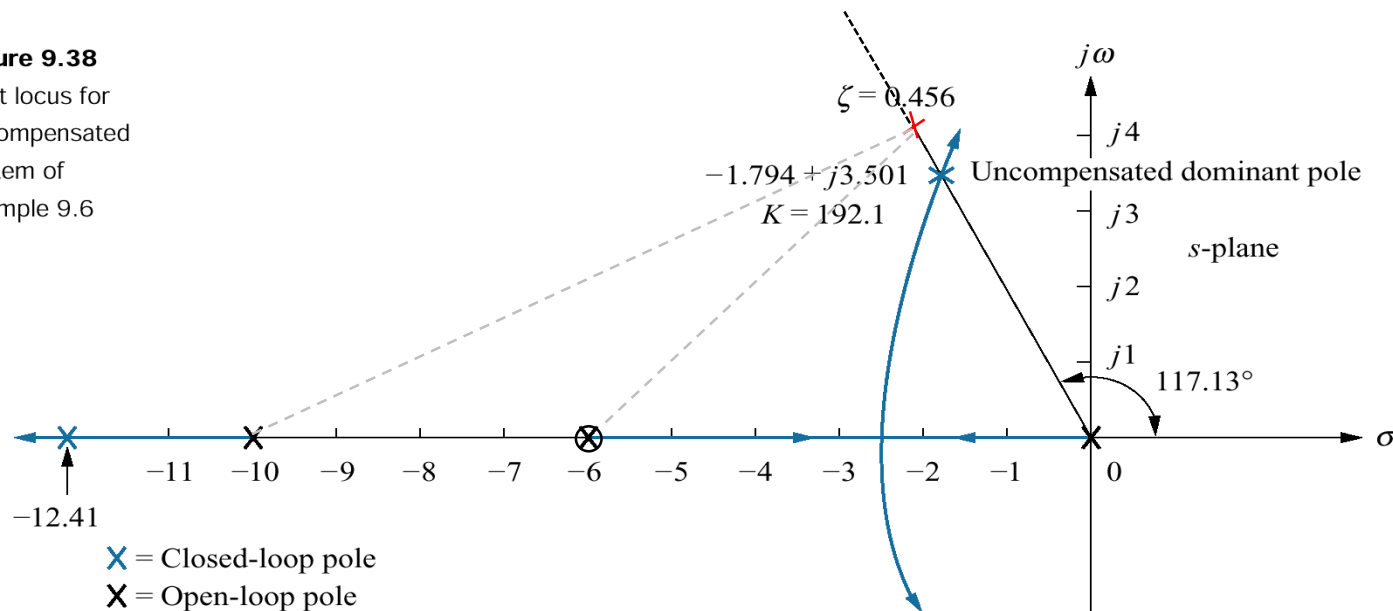


Figure 9.39

Evaluating the
compensator pole for
Example 9.6

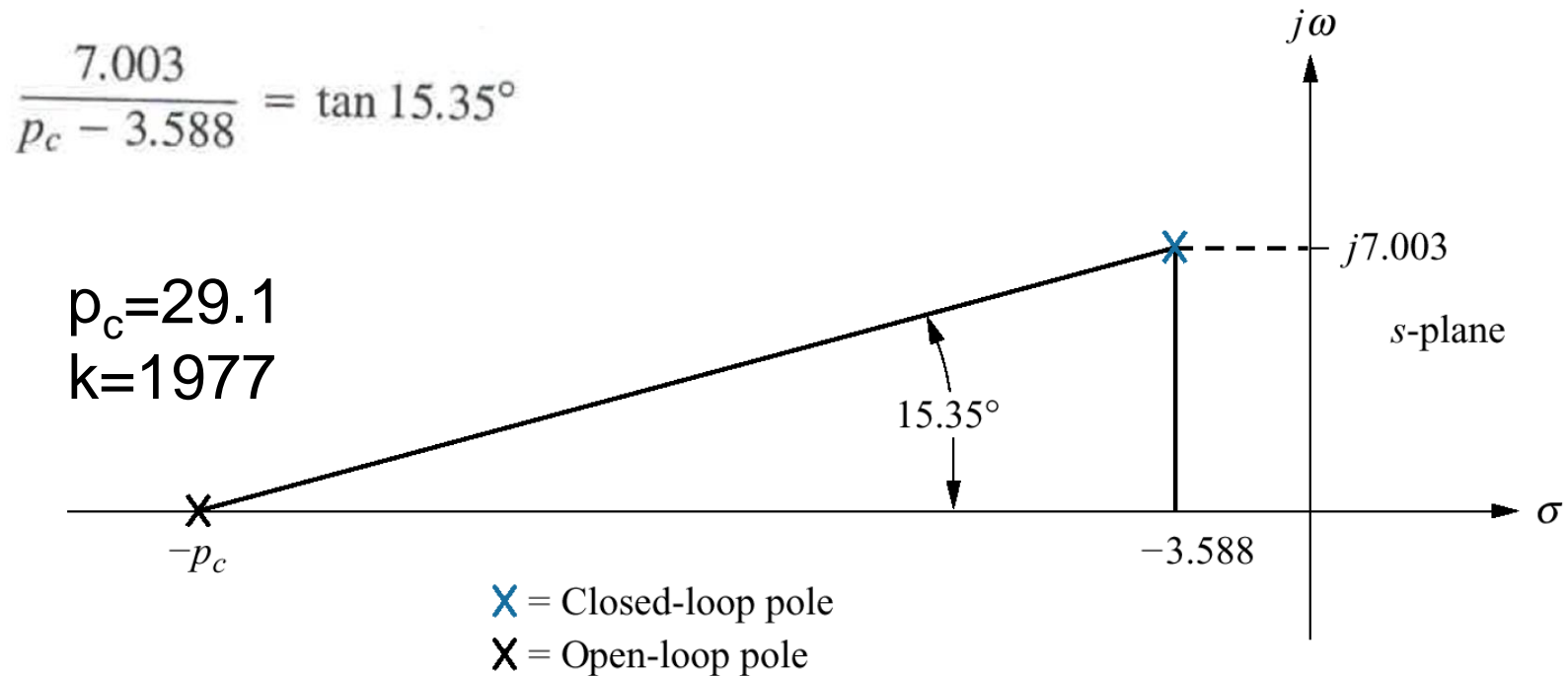


Table 9.6

Predicted characteristics of uncompensated, lead-compensated, and lag-lead-compensated systems of Example 9.6

	Uncompensated	Lead-compensated	Lag-lead-compensated
Plant and compensator	$\frac{K}{s(s+6)(s+10)}$	$\frac{K}{s(s+10)(s+29.1)}$	$\frac{K(s+0.04713)}{s(s+10)(s+29.1)(s+0.01)}$
Dominant poles	$-1.794 \pm j3.501$	$-3.588 \pm j7.003$	$-3.574 \pm j6.976$
K	192.1	1977	1971
ζ	0.456	0.456	0.456
ω_n	3.934	7.869	7.838
%OS	20	20	20
T_s	2.230	1.115	1.119
T_p	0.897	0.449	0.450
K_v	3.202	6.794	31.92
$e(\infty)$	0.312	0.147	0.0313
Third pole	-12.41	-31.92	-31.91, -0.0474
Zero	None	None	-0.04713
Comments	Second-order approx. OK	Second-order approx. OK	Second-order approx. OK

Steps 3 and 4: simulate the design and the result of the lead compensator is satisfactory.

Step 5: Design the lag compensator to improve the steady-state error
Open-loop transfer function of the uncompensated system:

$$G(s) = \frac{192.1}{s(s + 6)(s + 10)}$$

$$K_v = 192.1 / (6 \times 10) = 3.201$$

Open-loop transfer of the lead compensated system

$$G_{LC}(s) = \frac{1977}{s(s + 10)(s + 29.1)}$$

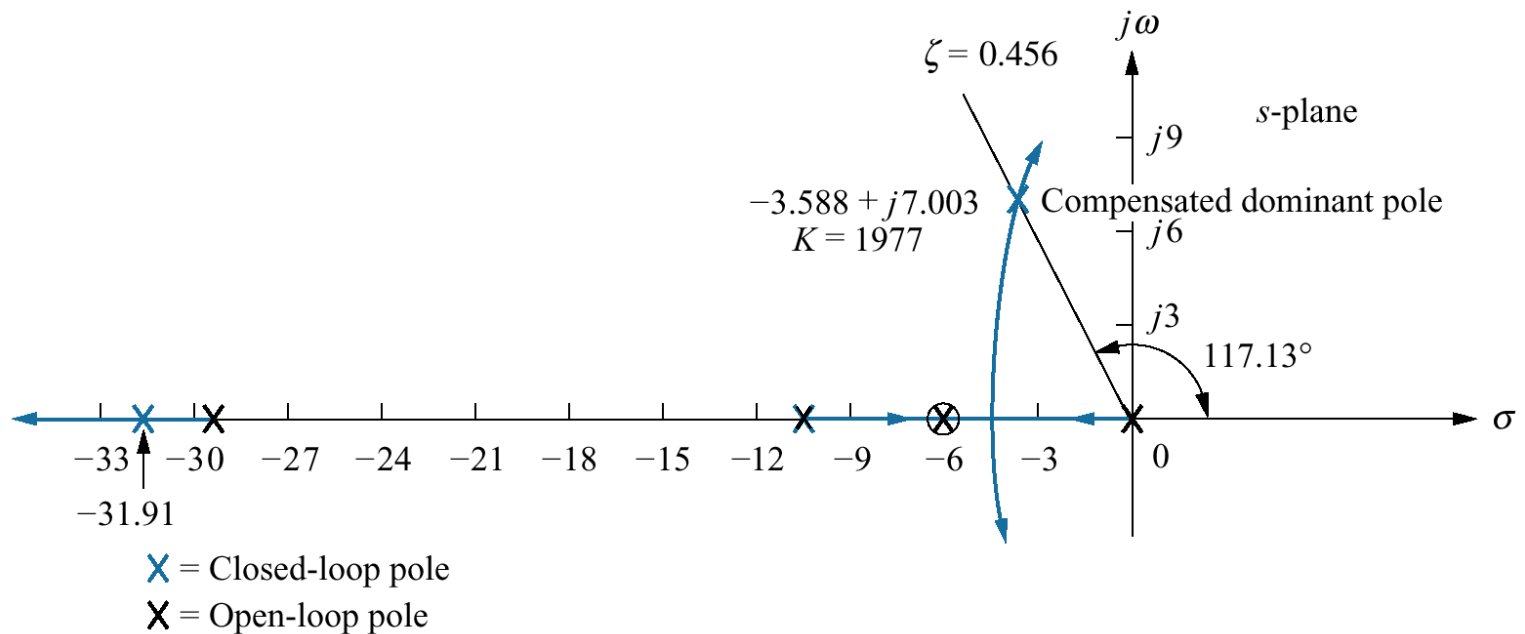
$$K_v = 6.794$$

The lag-lead compensator must provide a tenfold improvement

The lag compensator must improve the steady-state error by the factor $10 / 6.792 / 3.201 = 4.713$

Figure 9.40

Root locus for lead-compensated system of Example 9.6



Step 6: Design the lag compensator, arbitrarily choose the lag compensator pole at $p_c=0.01$,

$$K_{vN} = K_{vO} \frac{z_c}{p_c} > K_{vO}$$

$$z_c = (K_{vN}/K_{vO}) \times p_c = 4.713 \times 0.01 = 0.04713$$

Lag compensator:
$$G_{\text{lag}}(s) = \frac{(s + 0.04713)}{(s + 0.01)}$$

Lag-lead compensated open-loop transfer function

$$G_{\text{LLC}}(s) = \frac{K(s + 0.04713)}{s(s + 10)(s + 29.1)(s + 0.01)}$$

$$K_v = 1971 \times 0.04713 / (10 \times 29.1 \times 0.01) = 319.22$$

$$e_{ss} = 1/K_v = 1/319.22 = 0.00313$$

Figure 9.41

Root locus for lag-lead-compensated system of Example 9.6

$$G_{LLC}(s) = \frac{K(s + 0.04713)}{s(s + 10)(s + 29.1)(s + 0.01)}$$

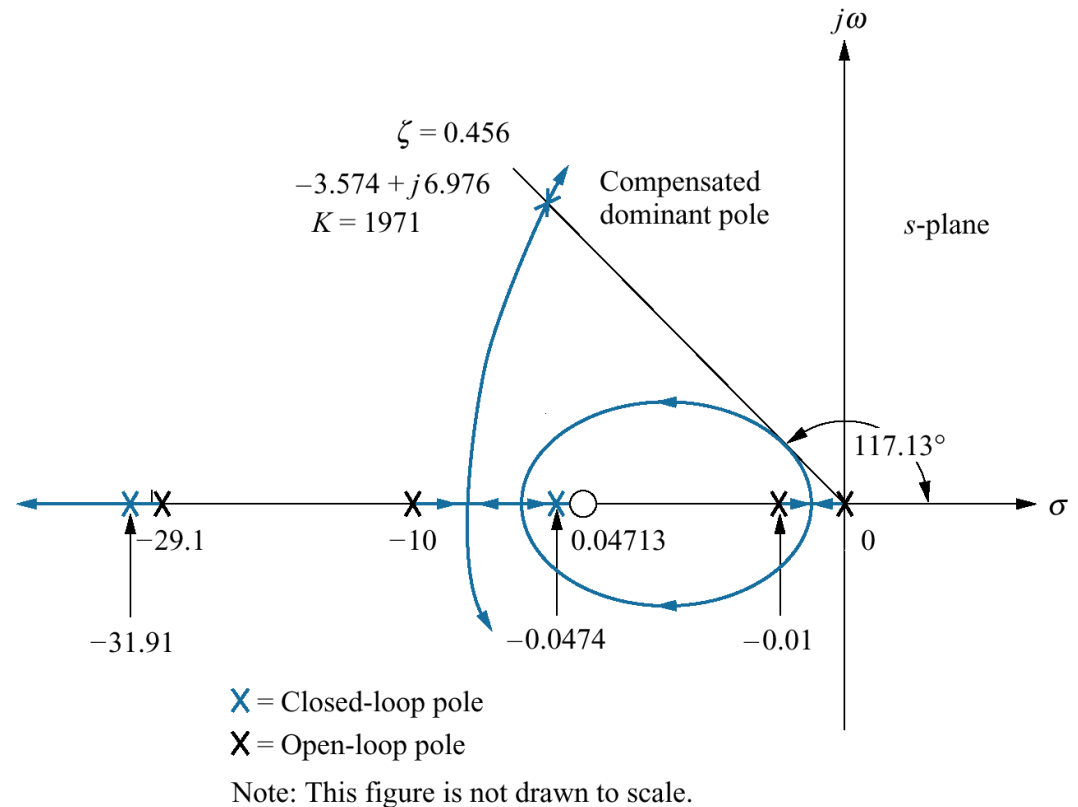


Figure 9.42
Improvement
in step
response for
lag-lead-
compensated
system of
Example 9.6

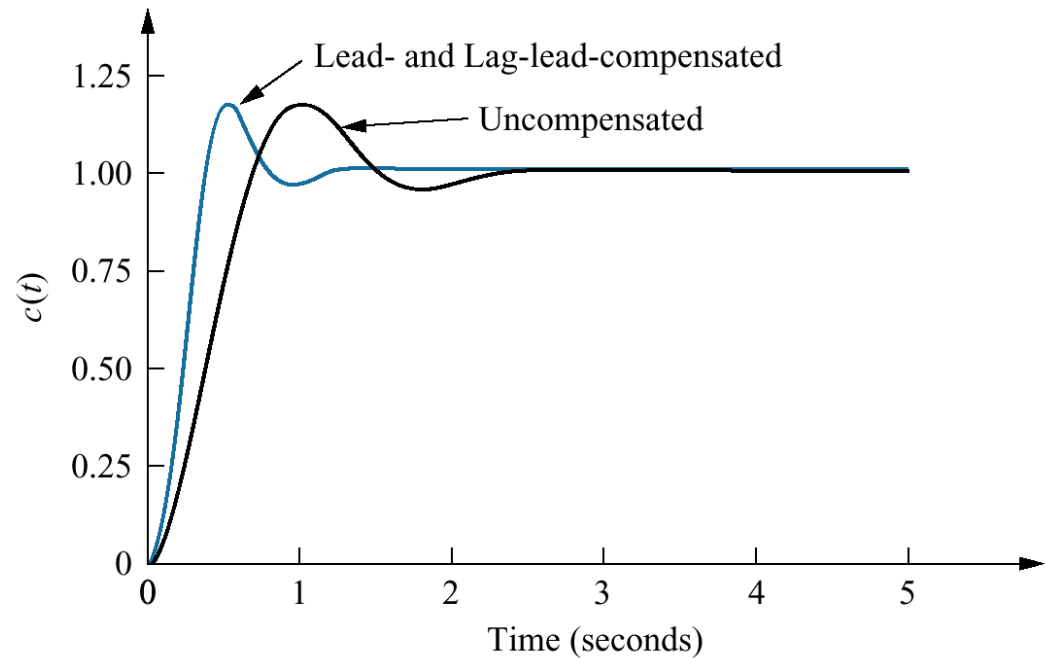
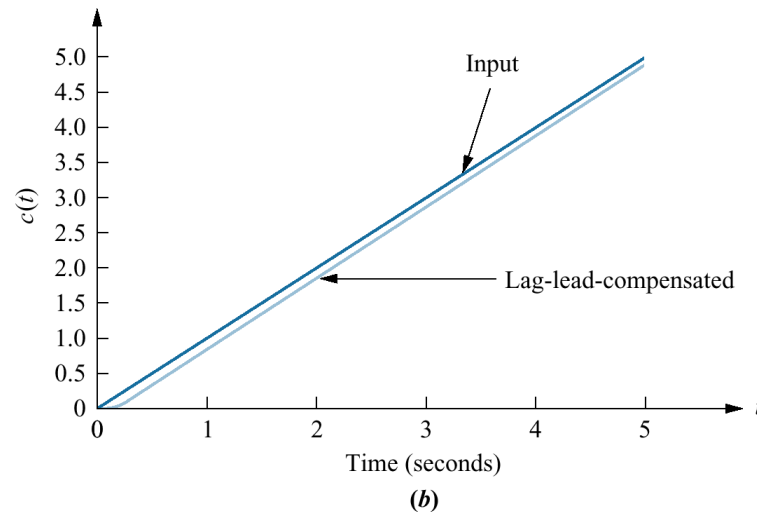
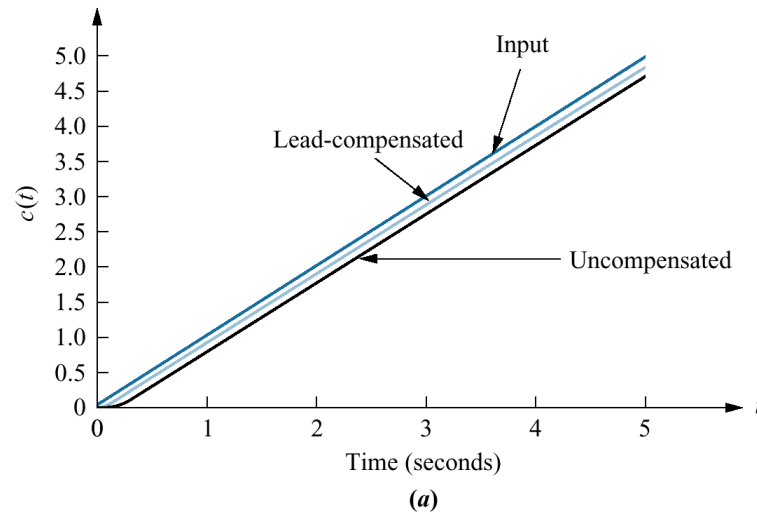


Figure 9.43
Improvement in
ramp response
error for the
system of
Example 9.6:
a. lead-
compensated;
b. lag-lead-
compensated



Summary

Lead/Lag Compensators

They both have one pole and one zero

$$G_c(s) = \frac{(s + z_c)}{(s + p_c)}$$

Lead compensator $z_c < p_c$

Lag compensator $z_c > p_c$

Lead compensator $a > 1$

$$G_c(s) = \frac{1 + aTs}{1 + Ts}$$

Lag compensator $a < 1$

Effect of the Lead/Lag compensator

Lead compensator:

- 1) Make loop more stable
- 2) Increase the speed of response

Lag compensator:

- 1) Reduce the steady-state error without changing the transient response by much

PID Controller

Three terms are:

P Proportional $P_{out} = K_P e(t)$

I Integral $I_{out} = K_I \int_0^t e(\tau) d\tau$

D Derivative $D_{out} = K_D \frac{d}{dt} e(t)$

P, I and D terms can be used in combination:

P control

PI control

PD control

PID control

Effect of the PID Controller

PD controller:

- 1) Improve the transient response performance

PI controller:

- 1) Reduce the steady-state error to zero

PID controller

PI controller to meet a given specification is used, and D used only if absolutely necessary.

PID Controller

Ideal parallel form:

$$s(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{d}{dt} e(t)$$

Standard form:

$$s(t) = K_P \left(e(t) + \frac{1}{T_I} \int_0^t e(\tau) d\tau + T_D \frac{d}{dt} e(t) \right)$$

Transfer function:

$$G_C(s) = \frac{K_D s^2 + K_P s + K_I}{s}$$

Table 9.7

Types of cascade compensators 1

Function	Compensator	Transfer function	Characteristics
Improve steady-state error	PI	$K \frac{s + z_c}{s}$	<ol style="list-style-type: none"> 1. Increases system type 2. Error becomes zero 3. Zero at $-z_c$ is small and negative 4. Active circuits required to implement
Improve steady-state error	Lag	$K \frac{s + z_c}{s + p_c}$	<ol style="list-style-type: none"> 1. Error improved but not driven to zero 2. Pole at $-p_c$ is small and negative 3. Zero at $-z_c$ is close and to the left of pole at $-p_c$ 4. Active circuits not required to implement
Improve transient response	PD	$K(s + z_c)$	<ol style="list-style-type: none"> 1. Zero at $-z_c$ is selected to put design point on root locus 2. Active circuits required to implement 3. Can cause noise and saturation; implement with rate feedback or with a pole (lead)
Improve transient response	Lead	$K \frac{s + z_c}{s + p_c}$	<ol style="list-style-type: none"> 1. Zero at $-z_c$ and pole at $-p_c$ are selected to put design point on root locus 2. Pole at $-p_c$ is more negative than zero at $-z_c$ 3. Active circuits not required to implement

Table 9.7**Types of cascade compensators 2**

Function	Compensator	Transfer function	Characteristics
Improve steady-state error and transient response	PID	$K \frac{(s + z_{\text{lag}})(s + z_{\text{lead}})}{s}$	<ol style="list-style-type: none"> 1. Lag zero at $-z_{\text{lag}}$ and pole at origin improve steady-state error 2. Lead zero at $-z_{\text{lead}}$ improves transient response 3. Lag zero at $-z_{\text{lag}}$ is close and to the left of the origin 4. Lead zero at $-z_{\text{lead}}$ is selected to put design point on root locus 5. Active circuits required to implement 6. Can cause noise and saturation; implement with rate feedback or with an additional pole
Improve steady-state error and transient response	Lag-lead	$K \frac{(s + z_{\text{lag}})(s + z_{\text{lead}})}{(s + p_{\text{lag}})(s + p_{\text{lead}})}$	<ol style="list-style-type: none"> 1. Lag pole at $-p_{\text{lag}}$ and lag zero at $-z_{\text{lag}}$ are used to improve steady-state error 2. Lead pole at $-p_{\text{lead}}$ and lead zero at $-z_{\text{lead}}$ are used to improve transient response 3. Lag pole at $-p_{\text{lag}}$ is small and negative 4. Lag zero at $-z_{\text{lag}}$ is close and to the left of lag pole at $-p_{\text{lag}}$ 5. Lead zero at $-z_{\text{lead}}$ and lead pole at $-p_{\text{lead}}$ are selected to put design point on root locus 6. Lead pole at $-p_{\text{lead}}$ is more negative than lead zero at $-z_{\text{lead}}$ 7. Active circuits not required to implement