Mathematical Tables 477

TABLE A6.4 Trigonometric Identities

$$\exp(\pm j\theta) = \cos \theta \pm j \sin \theta$$

$$\cos \theta = \frac{1}{2} [\exp(j\theta) + \exp(-j\theta)]$$

$$\sin \theta = \frac{1}{2j} [\exp(j\theta) - \exp(-j\theta)]$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\cos^2 \theta - \sin^2 \theta = \cos(2\theta)$$

$$\cos^2 \theta = \frac{1}{2} [1 + \cos(2\theta)]$$

$$\sin^2 \theta = \frac{1}{2} [1 - \cos(2\theta)]$$

$$2 \sin \theta \cos \theta = \sin(2\theta)$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

TABLE A6.5 Series Expansions

Taylor series
$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \dots$$
 where
$$f^{(n)}(a) = \frac{d^n f(x)}{dx^n}|_{x=a}$$
 MacLaurin series
$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \dots + \frac{f^{(n)}(0)}{n!}x^n + \dots$$
 where
$$f^{(n)}(0) = \frac{d^n f(x)}{dx^n}|_{x=0}$$
 Binomial series
$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{nx}{2!}x^2 + \dots$$
 Exponential series
$$\ln(1+x) = x - \frac{1}{2!}x^2 + \frac{1}{3!}x^3 - \dots$$
 Trigonometric series
$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots$$
 Trigonometric series
$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \dots$$

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots$$

$$\sin^{-1}x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \dots$$

$$\tan^{-1}x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \dots, \quad |x| < 1$$

$$\sin x = 1 - \frac{1}{3!}(\pi x)^2 + \frac{1}{5!}(\pi x)^4 - \dots$$