EBU5375 Signals and Systems: Fourier series in Matlab

Dr Jesús Requena Carrión

Queen Mary
University of London

Background

According to Fourier Theory, a periodic CT signal $x(t)$ with period T can be expressed as a **linear combination** of **harmonically related complex exponentials**:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Where:

1. $\omega_0 = 2\pi/T$ is the fundamental frequency.
2. $k = 0, \pm 1, \pm 2 \ldots$
3. $k\omega_0$ is the k-th harmonic frequency.
4. a_k is the Fourier coefficient of the k-th harmonic frequency.

This expression is called the **synthesis equation**.

Background

Given a periodic CT signal $x(t)$, its Fourier series representation is obtained as follows:

1. Identify its period T.
2. Identify its fundamental frequency ω_0 and its harmonics $k\omega_0$.
3. Calculate the Fourier coefficient a_k by using the **analysis equation**:

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt$$

4. Express $x(t)$ as a linear combination of harmonic frequencies (synthesis equation).

Background

For instance, take the periodic square wave defined over one period T as

$$x(t) = \begin{cases} 1, & |t| < T/4 \\ 0, & T/4 < |t| < T/2 \end{cases}$$

1. Its period is by definition T
2. Its fundamental frequency $\omega_0 = 2\pi/T$, its harmonics $\omega_k = k2\pi/T$.
3. The Fourier coefficients are

$$a_k = \begin{cases} 1/2, & k = 0 \\ \frac{\sin(\pi k/2)}{k\pi}, & k \neq 0 \end{cases}$$

4. The periodic square wave can be expressed as

$$x(t) = \frac{1}{2} + \sum_{k=-\infty, k\neq 0}^{\infty} \frac{\sin(\pi k/2)}{k\pi} e^{jk\omega_0 t}$$

Objectives of the lab

In this lab, we will use Matlab to define and plot CT signals. Specifically, we will work with periodic CT signals defined as Fourier series.

We will:

1. Obtain the Fourier series of a periodic square wave with period $T = 4$.

2. Define the time vector corresponding to three periods of the periodic square wave.

3. Define and plot individual harmonic components for different harmonic frequencies $\omega_k = k\omega_0$.

4. Synthesise the periodic square wave as a Fourier series by adding harmonic components.

Step 1: Definition of the periodic CT signal

In this lab, we will work with the periodic square wave $x(t)$ with period $T = 4$ defined as:

$$x(t) = \begin{cases} 1, & |t| < 1 \\ 0, & 1 < |t| < 2 \end{cases}$$

Your job now is to

- Identify the values of its period T and its fundamental frequency ω_0.
- Draw on a piece of paper three periods of $x(t)$.
- Write down the Fourier series representation of $x(t)$, identifying the value of its Fourier coefficients and its harmonic frequencies.

Step 2: Definition of the time vector in Matlab

As you know, a time vector can be defined in Matlab by using

```
t = t_s:dt:t_e
```

where `dt` is the time difference between two consecutive time instants, and `t_s` and `t_e` are, respectivey, the first and last time instants. We will assume that the time difference is `dt=0.001`.

Your job now is to

‣ Determine the values of `t_s` and `t_e` necessary to represent 3 periods of $x(t)$ in Matlab.

Step 3: Plotting individual harmonic components

The following lines of code combine the Fourier series components corresponding to $k = 1$ and $k = -1$ into the sinusoidal signal c1. Then, c1 is plotted. Note that $1\text{i} = \sqrt{-1}$.

```
t_s = %TO BE COMPLETED BY YOU
t_e = %TO BE COMPLETED BY YOU
dt = 0.001;
t = t_s:dt:t_e; % Variable t denotes time
ek1 = (sin(pi/2)/(pi))*exp(1i*pi/2*t); % k=1 exp component
ek_1= (sin(-pi/2)/(-pi))*exp(-1i*pi/2*t); % k=-1 exp component
c1=ek1+ek_1; % k=1 sinusoidal component

figure
plot(t,C1) % plots x agains n
xlabel('t') % adds text below the X-axis
ylabel('c1(t)') % adds text beside the Y-axis
```

Your job now is to

- Enter the values of t_s and t_e and execute the lines of code.
- What is the period of the sinusoidal signal c1?

Step 3: Plotting individual harmonic components

Your job now is to

- Modify the previous lines of code to plot the sinusoidal signal `c2`, defined as the combination of the Fourier series components of $x(t)$ corresponding to $k = 2$ and $k = -2$.

- What is the period of the sinusoidal signal `c2`?

Step 3: Plotting a truncated version of $x(t)$

Given a time vector `t`, the following lines of code synthesise truncated versions of $x(t)$. The value `K` corresponds to the number of harmonic frequencies included in the synthesis equation.

```
K = %TO BE COMPLETED BY YOU

x = zeros(size(t));%define x as zero
x = 1/2; %add the component k=0 of its fourer series

for k=1:K
    ekn = (sin(k*pi/2)/(k*pi))*exp(1i*k*pi/2*t);
    ek_n= (sin(-k*pi/2)/(-k*pi))*exp(-1i*k*pi/2*t);
    x=x+ekn+ek_n;
end
figure
plot(t,x) % plots x agains n
title(['Truncated signal for K=',num2str(K)])
xlabel('t') % adds text below the X-axis
ylabel('x(t)') % adds text beside the Y-axis
```

Step 3: Plotting a truncated version of $x(t)$

Your job now is to

‣ Plot the truncated version of $x(t)$ for $K = 1, 5, 10, 50, 100$.

‣ What happens to the truncated version of $x(t)$ as K increases?