EBU6503
Control Theory

Controllers

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Closed Loop Behaviour

We have seen how to determine the closed loop behaviour of a system by:

- **Analysing its closed loop transfer function and finding the output function of time**
- **Using frequency response methods (Nyquist, Nichols)**
- **Using Root Locus**

Closed Loop Behaviour

Closed loop behaviour is characterised by:

- **Damping (over-, under-, critical-)**
- **% peak overshoot**
- **Time to peak overshoot**
- **Settling time**
- **Rise time**
- **Steady-state error**

Simple Controller

But what if the closed loop behaviour is not what we would like it to be (because the user of the control system requires a specific behaviour)…..

……then we need to modify the behaviour.

We have seen from root locus that the behaviour can be modified by using a CONTROLLER that is an amplifier, and choosing a suitable gain value.

However an amplifier on its own may not give us all the desired behaviour that we need, e.g. it may give us a desired ζ but not a desired ω_d as well.

Compensators and Controllers

The compensator or controller usually (but not always) goes between the error detector and the system input:

More Advanced Controllers

Three useful controllers are:

- **Phase Lead (Advance)**
- **Phase Lag**
- **Three Term (PID)**

Phase lead and lag circuits are often called “compensators”. They usually have fixed values for a particular system.

Phase Lead/Lag Compensators

These are passive R, C circuits used to selectively alter the phase of an open loop system.

They both have one pole and one zero.

The Transfer Function is: $$G_c(s) = \frac{K(s+\alpha)}{(s+\beta)}$$

For Phase Lead $|\alpha| < |\beta|$

For Phase Lag $|\alpha| > |\beta|$

Phase Lead

A Passive RC circuit that performs this function is:

Phase Lead

The effect of adding a phase lead compensator is to:

- Make loop more stable (poles further to right)
- Increase the speed of response (reduced rise and settling times and increased bandwidth)
- Increase the phase of the forward path transfer function near the crossover frequency and so increase the relative stability (can be shown on Nichols chart)

Phase Lead

Example: Suppose we have a system

$$G_s(s) = \frac{1}{(s+1)(s+4)}$$

And controller

$$G_c(s) = \frac{K(s+5)}{(s+9)}$$

Let's look at the Root Locus…….

Phase Lag

A passive RC circuit that performs this function is:

Phase Lag

A phase lag compensator is mainly used to reduce the steady-state error, but without changing the transient response by much (the complex conjugate poles will not move much)…..

…….. In practice the compensator pole and zero will be placed near the imaginary axis……. see root locus…..

Phase Lag

Example: Suppose we have the system

$$G_s(s) = \frac{1}{(s+1)(s+4)}$$

Let's put a lag compensator with its pole and zero in the left-hand plane near the imaginary axis:

Three Term Controllers

Three Term Circuits are called "controllers" because their parameters can be varied to be used with different systems or to change the behaviour of a particular system.

The three terms are:

- **P Proportional**
- **I Integral**
- **D Derivative**

Three Term Controllers

- P This is an amplifier gain. Amplifies the error signal
- I This produces a signal proportional to the integral of the error signal
- D This produces a signal proportional to the derivative of the error signal. Also called Anticipatory Control

Consider the effects on a step input:

Proportional Term (P)

This adjusts the response of the system to a change in error signal. A high proportional term gives a large change in the output for a given change in the error.

Too large a proportional term can make the system unstable.

$$P_{out} = K_P e(t)$$

Integral Term (I)

This is used to eliminate steady-state errors.

It will tend to make the system less stable.

$$I_{out} = K_I \int_0^t e(\tau)\,d\tau$$

(A P+I controller is effectively a Phase Lag compensator with the pole at zero.)

Derivative Term (D)

This is used to damp the system response, i.e. improves stability by reducing the amplitude of the oscillations, because it predicts the system behaviour.

However, in combination with other controller terms it can make a system *less* stable if the wrong amount is used.

Also, it makes the system sensitive to noise .

It is therefore used only if necessary to achieve a desired closed loop behaviour.

$$D_{out} = K_D \frac{d}{dt} e(t)$$

Three Term Controllers

The P,I and D terms can be used in combination:

P

P+I (PI)

P+D (PD)

P+I+D (PID)

The simplest combination to meet a given specification is used, and D used only if absolutely necessary.

Three Term Controllers

The PID equations are:

Ideal parallel form

$$s(t) = K_P e(t) + K_I \int_0^t e(\tau) d\tau + K_D \frac{d}{dt} e(t)$$

Standard form

$$s(t) = K_P \left(e(t) + \frac{1}{T_I} \int_0^t e(\tau) d\tau + T_D \frac{d}{dt} e(t) \right)$$

Transfer Function

$$G_C(s) = \frac{K_D s^2 + K_P s + K_I}{s}$$

Three Term Controllers

We can illustrate the effects of adding P, PI and PD terms by using root locus to see the effects of adding the controller poles and zeroes.

The values of the controller poles and zeroes will depend on:

- **The system we wish to control, and**
- **The desired closed loop specification**

Other Control Strategies

Three term control is very useful, but the values used are a compromise and cannot provide OPTIMAL control.

So, often in practice two additional strategies are used:

- **Feedforward. This is used in conjunction with feedback control to quickly correct for disturbances to the output.**
- **Cascade. This used two PID controllers with the output of the first providing the setpoint to the second.**

FeedForward Control

Cascade Control

This is used with systems that have two very different time constants.

For example a system that controls the temperature of the water in a bath. The temperature of the water in the bath will have a long time constant, but the temperature of the heater will have a short time constant. The outer PID controller controls the temperature in the bath, the inner PID controller controls the temperature of the heater.