

# Delta function Properties

$$1) \int_{-\infty}^{\infty} \delta(t) dt = 1$$

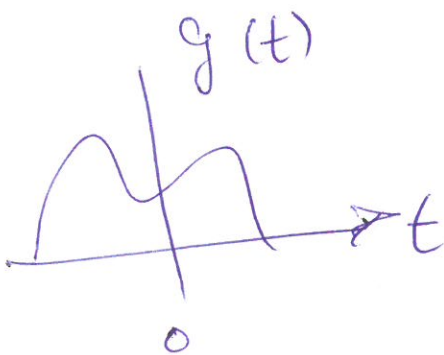
$$\int_{-\infty}^{\infty} \delta(t-t_0) dt = 1$$

$$2) \int_{-\infty}^{\infty} g(t) \delta(t) dt = g(0)$$

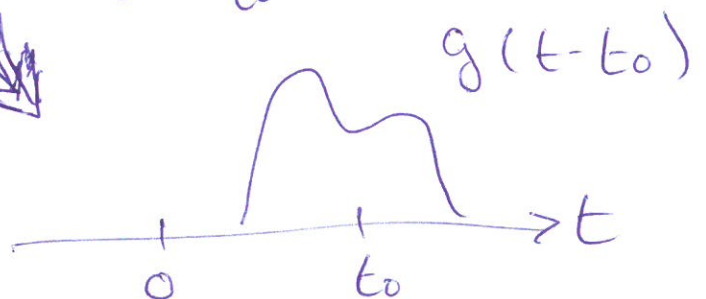
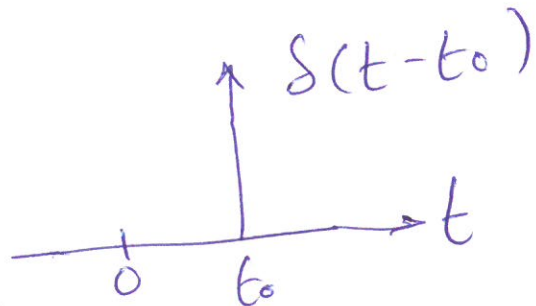
$$\int_{-\infty}^{\infty} g(t) \delta(t-t_0) dt = g(t_0)$$

$$3) g(t) * \delta(t) = g(t)$$

$$g(t) * \delta(t-t_0) = g(t-t_0)$$



\*



$$4) \delta(t) \Rightarrow 1$$

$$5) 1 \Leftrightarrow \delta(f)$$

} duality property

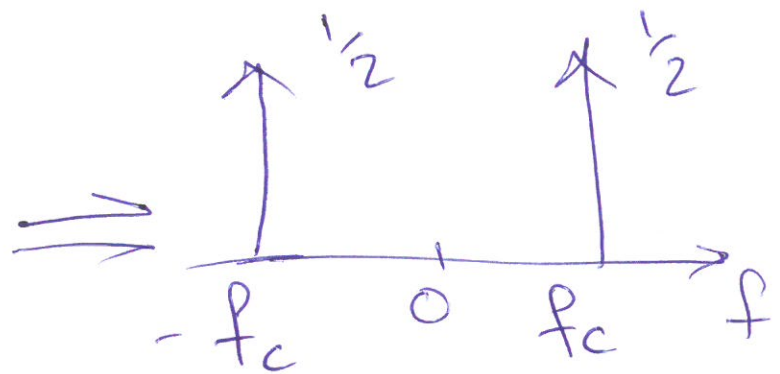
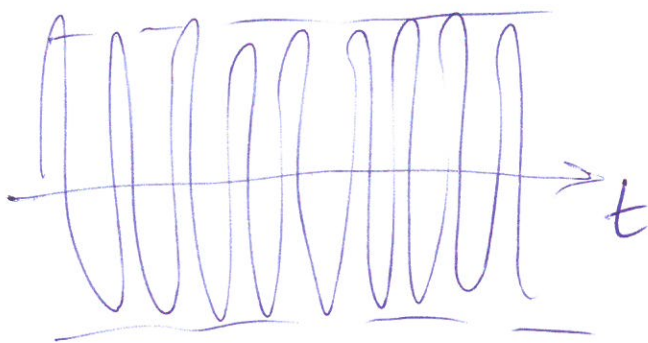
$$6) 1 \cdot e^{j2\pi f_c t} \Leftrightarrow \delta(f - f_c)$$

↳ frequency shift property

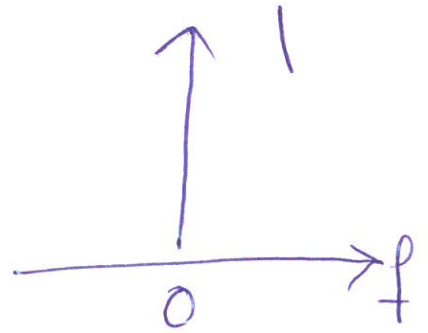
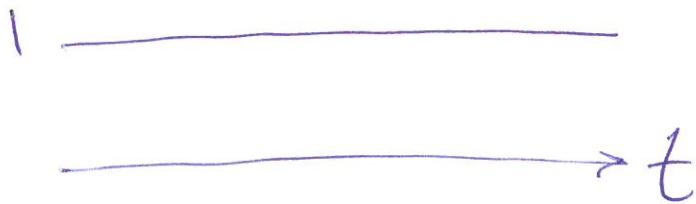
$$7) \cos(2\pi f_c t) = \frac{1}{2} [e^{j2\pi f_c t} + e^{-j2\pi f_c t}]$$

∴

$$\cos(2\pi f_c t) \Leftrightarrow \frac{1}{2} [\delta(f - f_c) + \delta(f + f_c)]$$



dc (i.e. frequency = 0)



1



$\delta(f)$

$$8) \quad g_1(t) \cdot g_2(t) \iff G_1(f) * G_2(f)$$

convolution property

example:

$$\text{rect}\left(\frac{t}{T}\right) \cos(2\pi f_0 t) \iff ?$$

— earlier:

$$\frac{1}{2} \text{rect}\left(\frac{t}{T}\right) \left[ e^{j2\pi f_0 t} + e^{-j2\pi f_0 t} \right] \iff$$

$$\frac{T}{2} \text{sinc}((f - f_0)T)$$

$$+ \frac{T}{2} \text{sinc}((f + f_0)T)$$

Using convolution property

$$g_1(t)g_2(t) \iff G_1(f) * G_2(f)$$

$$\text{rect}\left(\frac{t}{T}\right) \cos(2\pi f_0 t) \iff$$

$$T \text{sinc}(fT) *$$

$$\frac{1}{2} [\delta(f-f_0) + \delta(f+f_0)]$$

$$= \frac{T}{2} \text{sinc}((f-f_0)T) + \frac{T}{2} \text{sinc}((f+f_0)T)$$