Design via Root Locus –

PID Controller and Compensator Design (Part 1)

Reference book: Control Systems Engineering, Third Edition, Norman S Nise

Motivations

- The RL allows us to choose the proper loop gain to meet a transient response specification. We are limited to those responses that exist along the root locus.
- If the desired transient response defined by percent overshoot and setting time etc. is not in the RL, how to speed the response to them.
- The increase in speed cannot be accomplished by a simple gain adjustment.
- One way is to replace the existing system with a system whose root locus intersects the desired design point. Expensive
- Design a controller or compensator with additional poles and zeros, so that the compensated system has a root locus that goes through the desired pole location.

System without controller

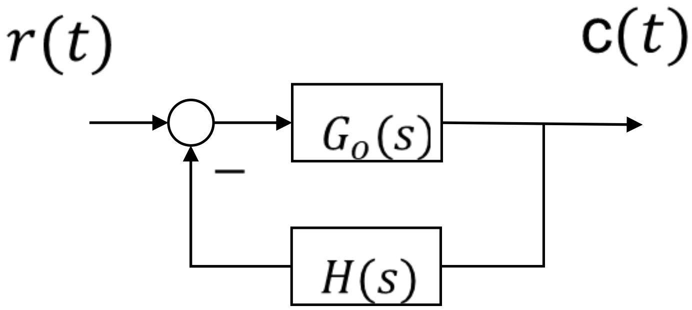
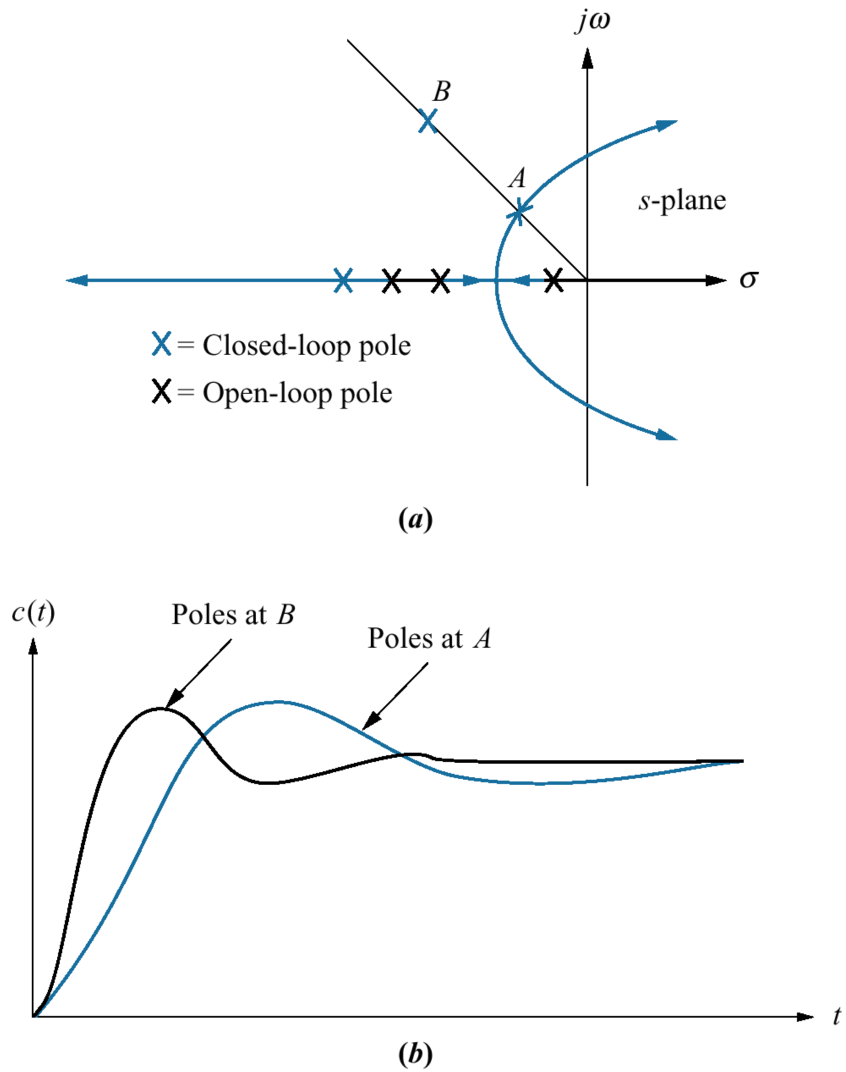


Figure 9.1
a. Sample root locus, showing possible design point via gain adjustment (*A*) and desired design point that cannot be met via simple gain adjustment (*B*);
b. responses from poles at *A* and *B*



Configuration:
System under close loop control

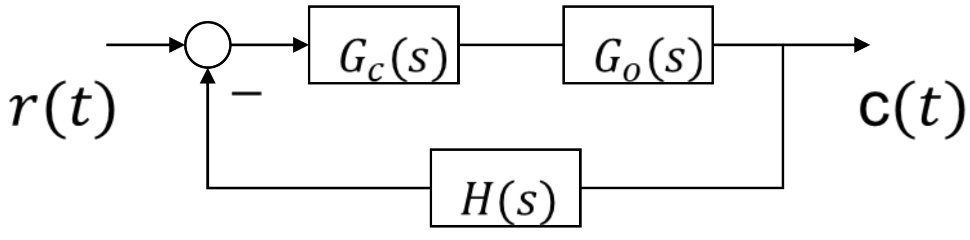
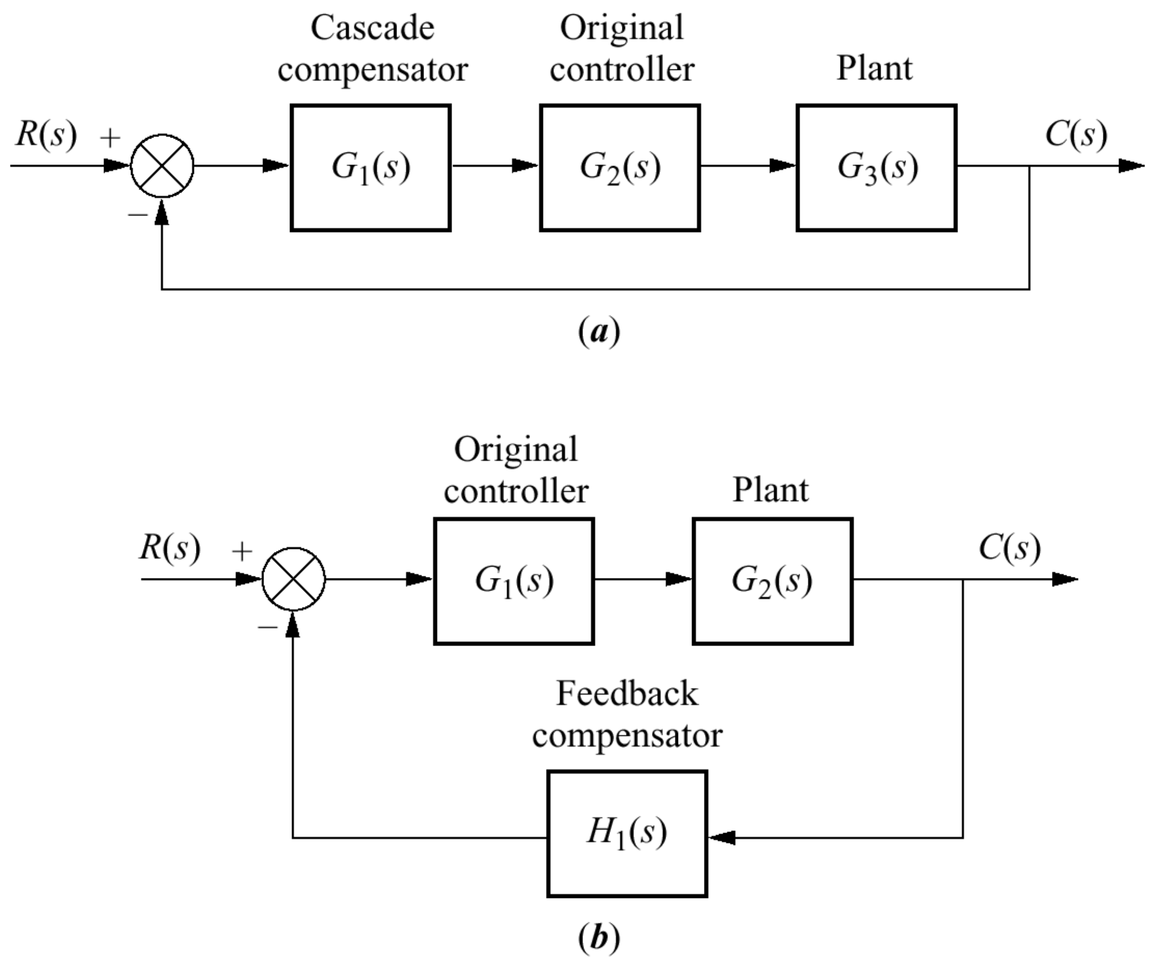


Figure 9.2

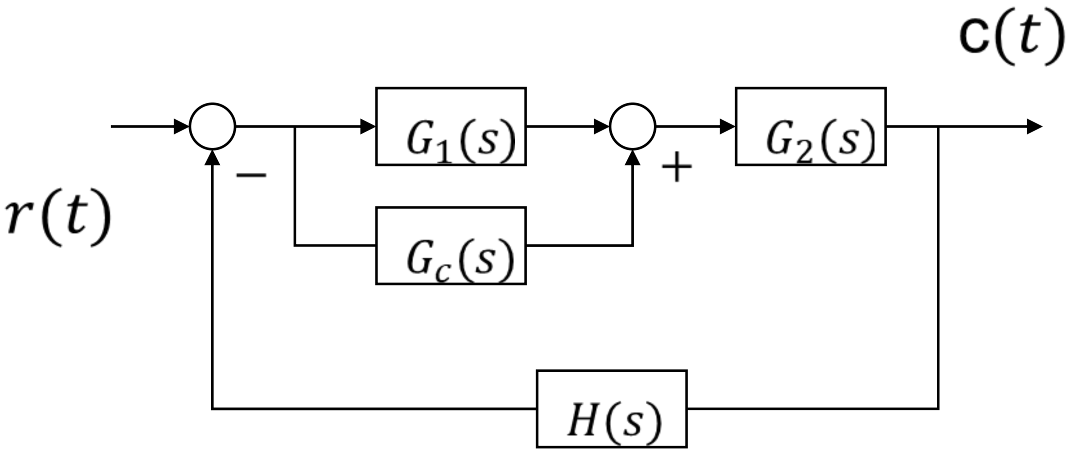
Compensation techniques:

a. cascade;

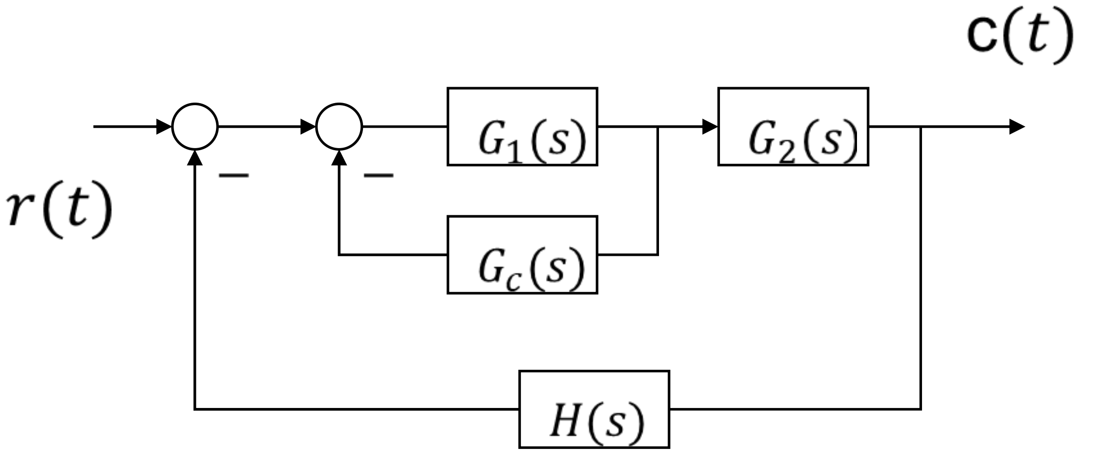
b. feedback



Feedforward control



Feedback control



Two ways of improving steady-state error via cascade compensation

- Way 1: ideal proportional plus integral (PI) compensation, reducing the error to zero (increase the type by 1)
- Way 2: compensator places the extra pole near the origin

Figure 9.3

Pole at *A* is:

a. on the root locus without compensator;

b. not on the root locus with compensator pole added;

(figure continues)

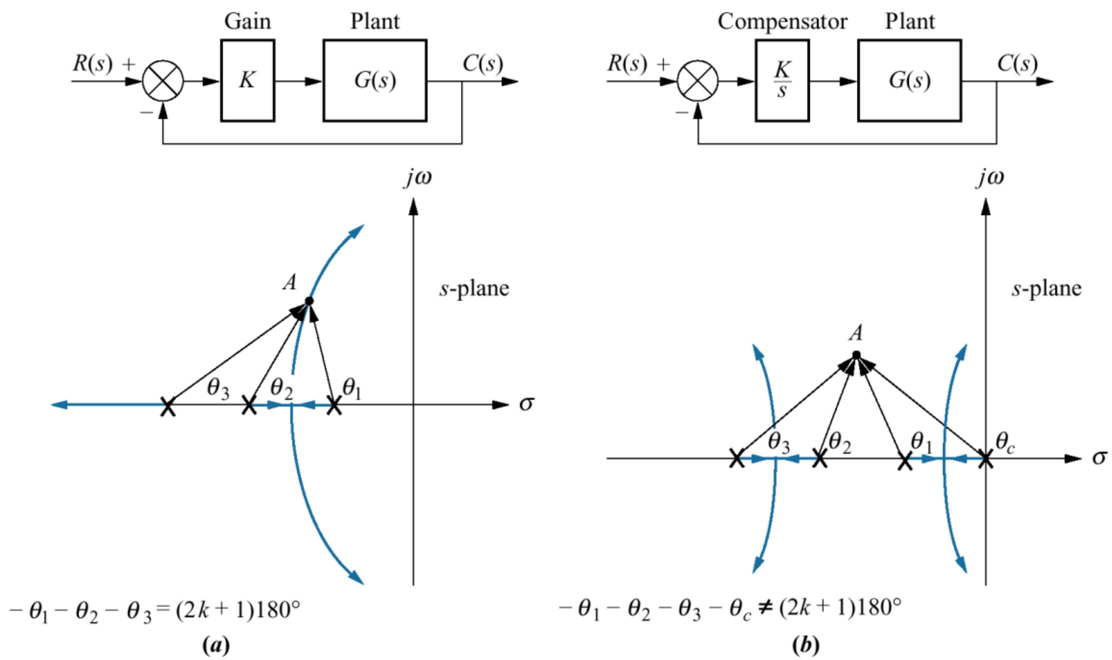
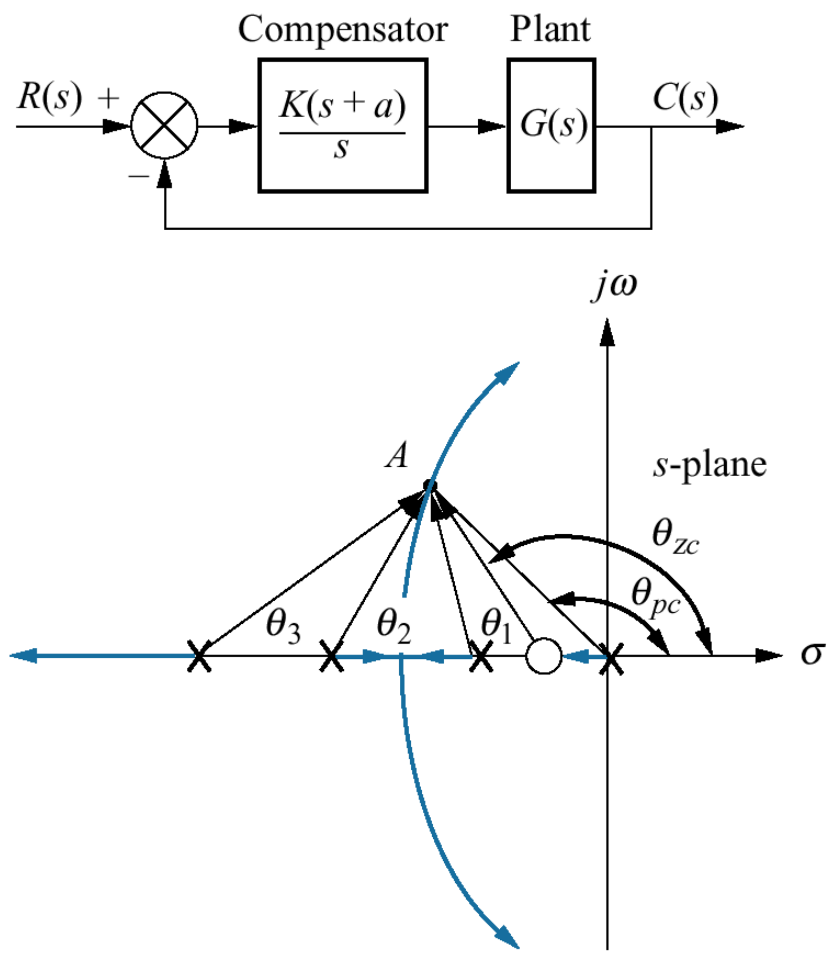


Figure 9.3 *(continued)*
c. approximately on the root locus with compensator pole and zero added

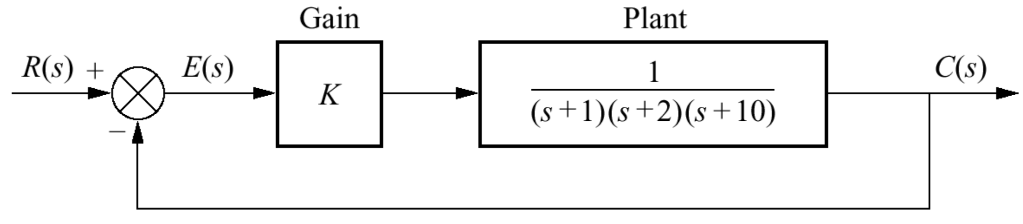


$$-\theta_1 - \theta_2 - \theta_3 - \theta_{pc} + \theta_{zc} \cong (2k+1)180^\circ$$

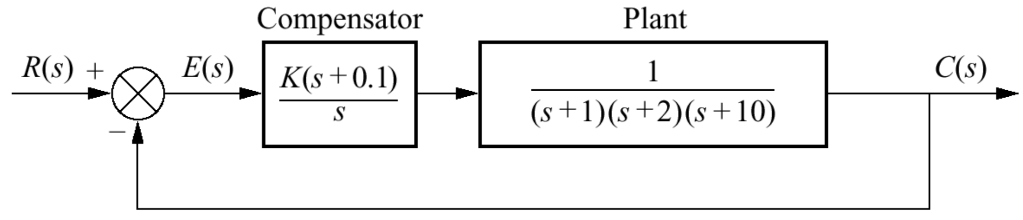
(c)

Example

Closed-loop system for Example 9.1:
a. before compensation;
b. after ideal integral compensation

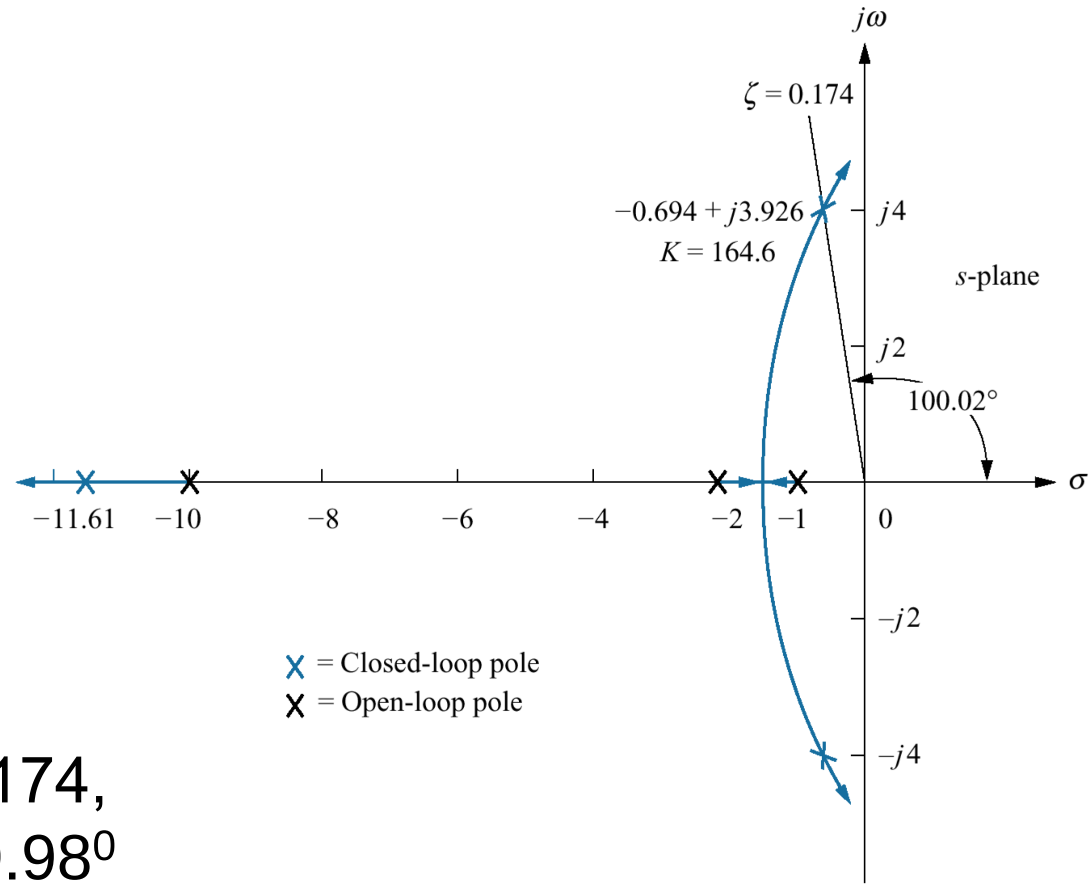


(a)



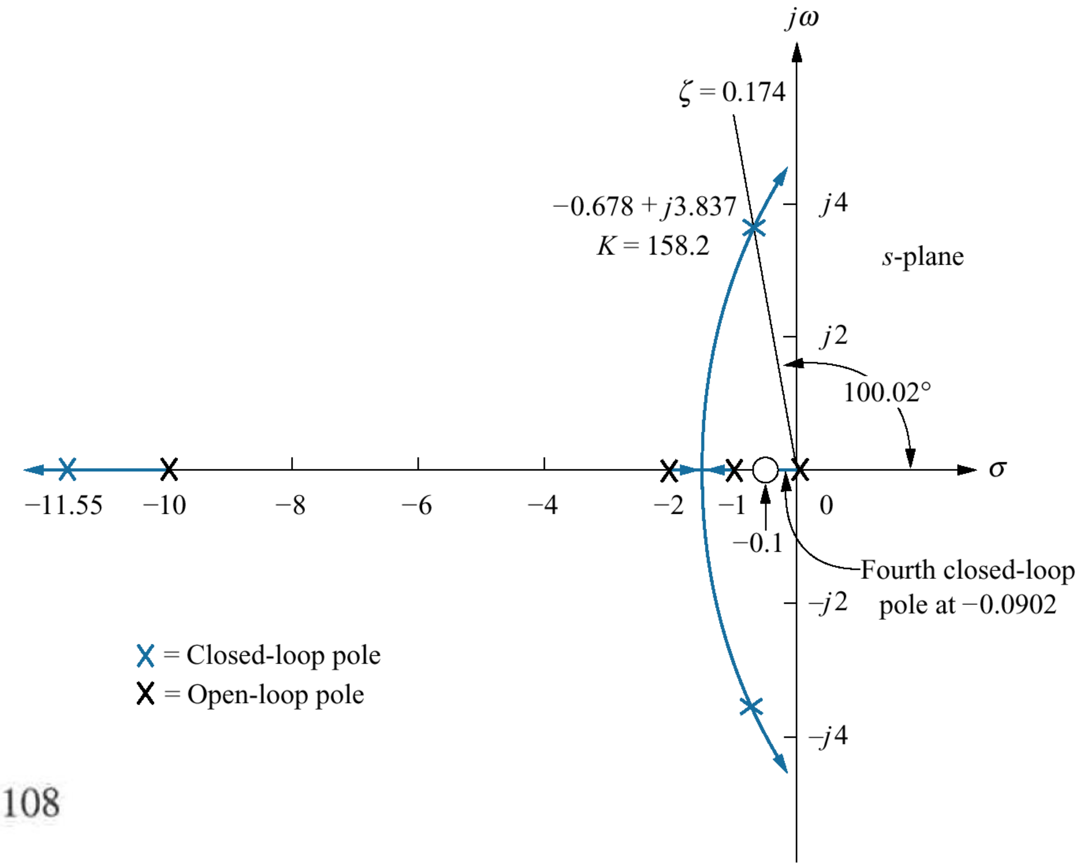
(b)

Figure 9.5
Root locus for uncompensated system of Figure 9.4(*a*)



Damping rate $\zeta=0.174$, $\cos\Theta=0.174$, $\Theta=79.98^0$

Figure 9.6
Root locus for compensated system of Figure 9.4(b)



$$e(\infty) = \frac{1}{1+K_p} = \frac{1}{1+8.23} = 0.108$$

Table 5.5 Summary of Steady-State Errors

Number of Integrations in $G_c(s)G(s)$, Type Number	Input Step, $r(t) = A$, $R(s) = A/s$	 Ramp, At, A/s^2	 Parabola, $At^2/2$, A/s^3
0	$e_{ss} = \dfrac{A}{1 + K_p}$	Infinite	Infinite
1	$e_{ss} = 0$	$\dfrac{A}{K_v}$	Infinite
2	$e_{ss} = 0$	0	$\dfrac{A}{K_a}$

Error Constant

k_p is called *position error constant* and is given by

$$k_p = \lim_{s \to 0} G(s)$$

k_v is called *velocity error constant* and is given by

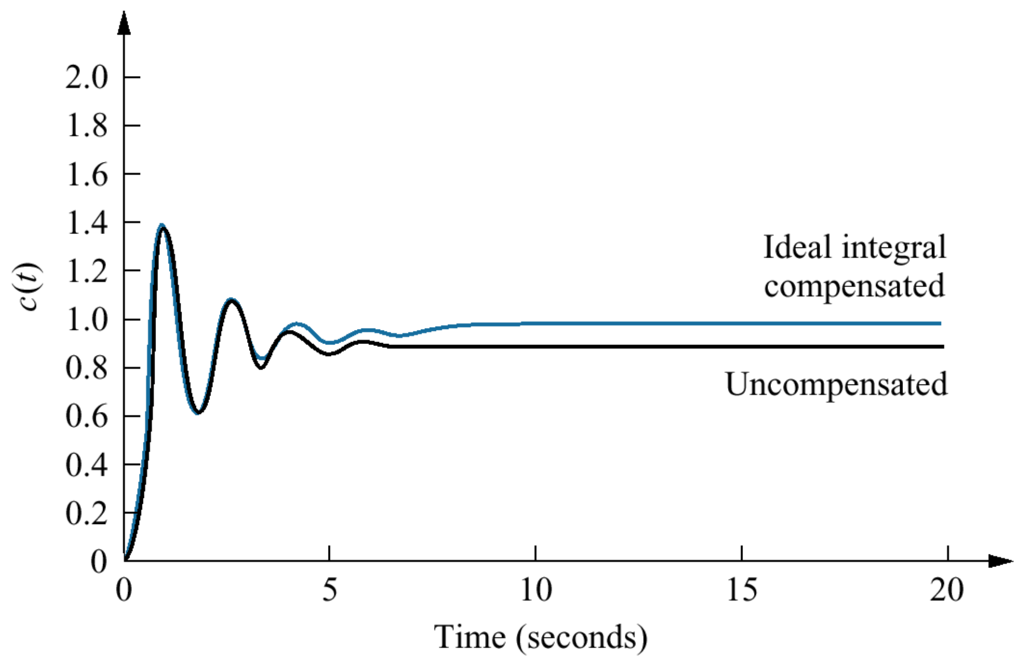
$$k_v = \lim_{s \to 0} sG(s)$$

k_a is called *acceleration error constant* and is given by

$$k_a = \lim_{s \to 0} s^2 G(s)$$

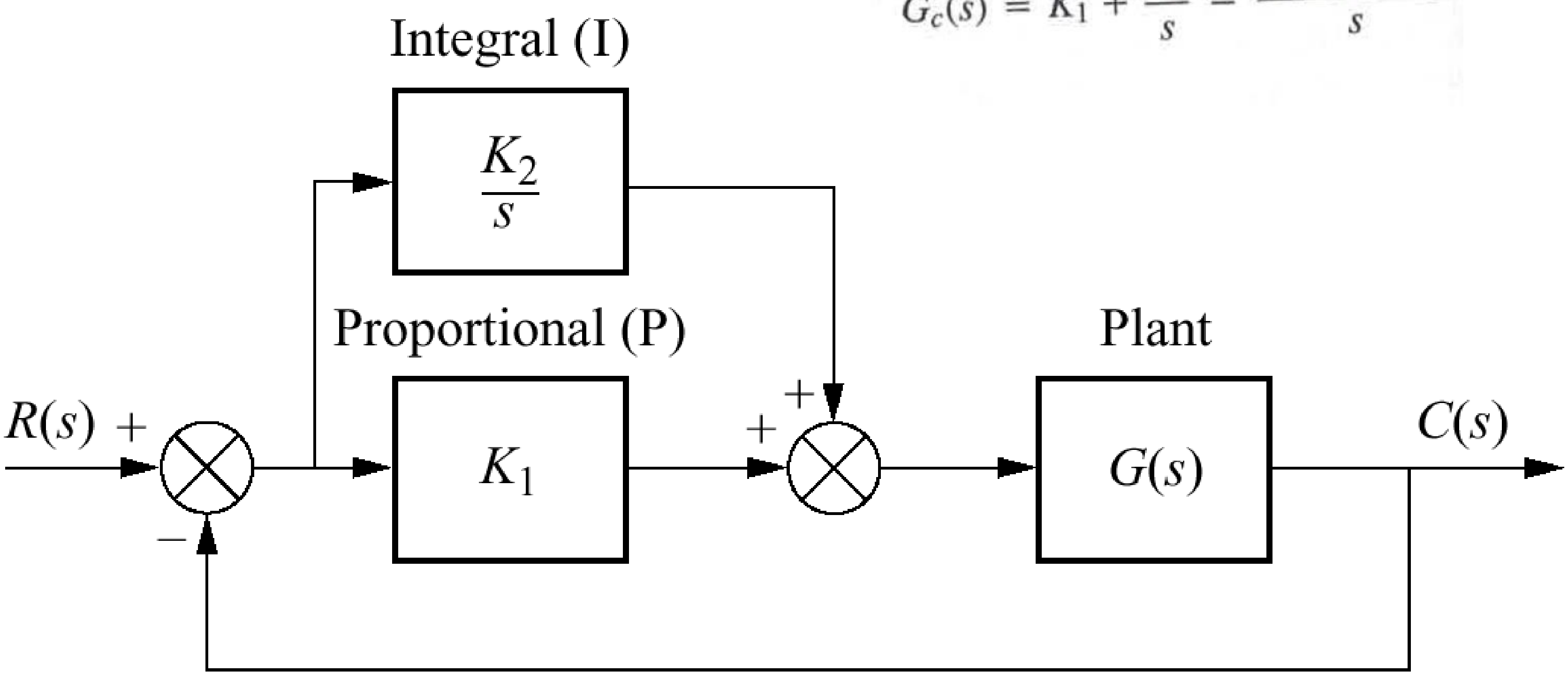
Figure 9.7

Ideal integral compensated system response and the uncompensated system response of Example 9.1



PI controller （in parallel)

$$G_c(s) = K_1 + \frac{K_2}{s} = \frac{K_1\left(s + \frac{K_2}{K_1}\right)}{s}$$



Lag Compensation

Ideal Integral compensation with its pole on the origin, requires an active integrator, called active implementation.

Lag compensation does not increase the system type, but still yield improvement in the static error constant. The idea is to place an open loop zero very near the origin pole.

$$G_c(s) = \frac{(s + z_c)}{(s + p_c)} \qquad z_c > p_c$$

Figure 9.9

a. Type 1 uncompensated system;
b. Type 1 compensated system;
c. compensator pole-zero plot

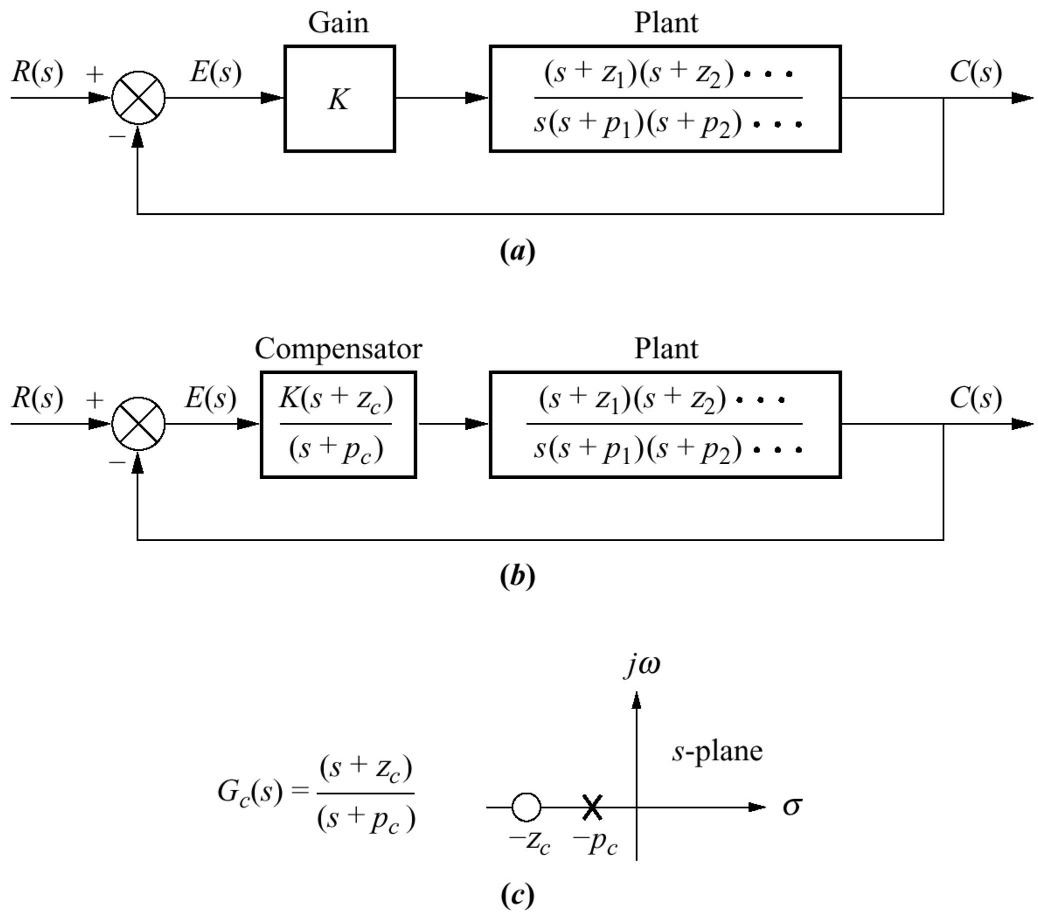
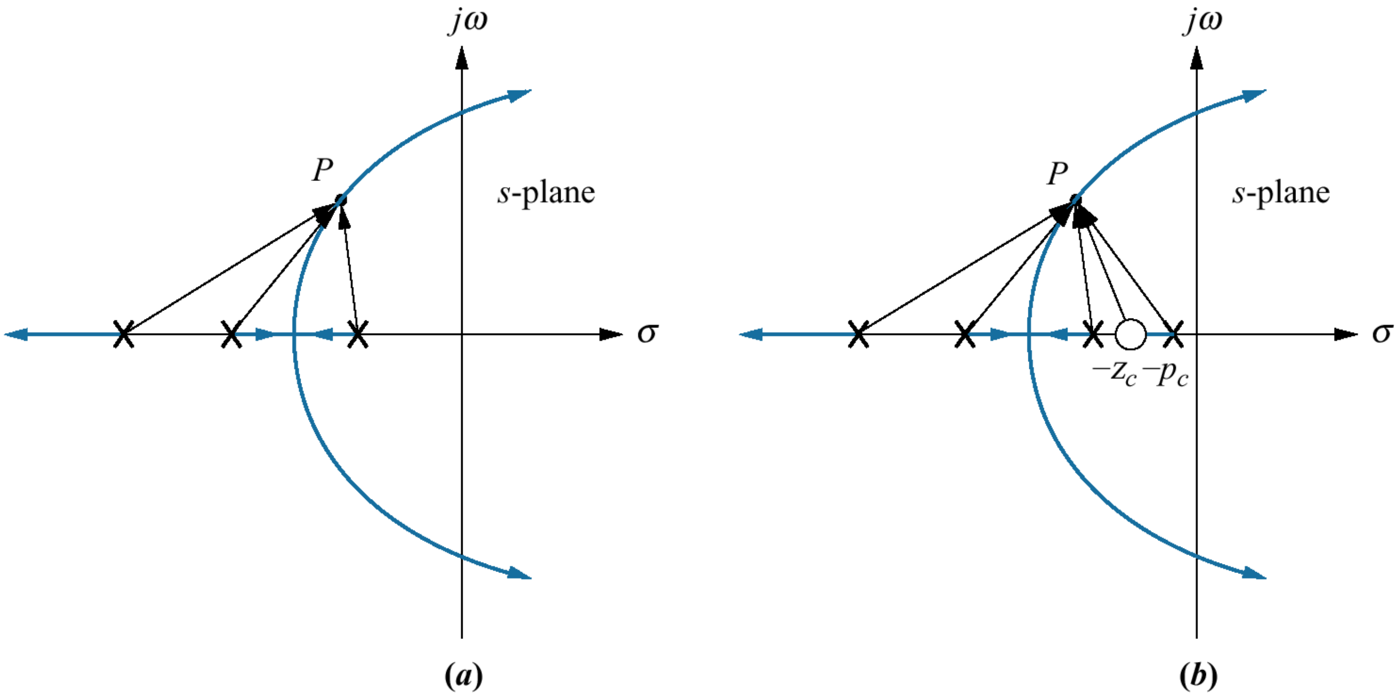


Figure 9.10
Root locus:
a. before lag compensation;
b. after lag compensation



Compensation Effect:

On the transient response – the compensated point P is still at the approximately the same location in the RL.

On the required gain K – virtually the same

On the steady-state error – the improvement in the compensated system's k (k_p, k_v ,k_a) is the ratio of magnitude of z_c to the p_c. The steady-state error will improve the static error constant by a factor z_c/p_c

Assume the uncompensated system shown in Figure 9.9(*a*). The static error constant, K_{v_O}, for the system is

$$K_{v_O} = \frac{K z_1 z_2 \cdots}{p_1 p_2 \cdots} \tag{9.3}$$

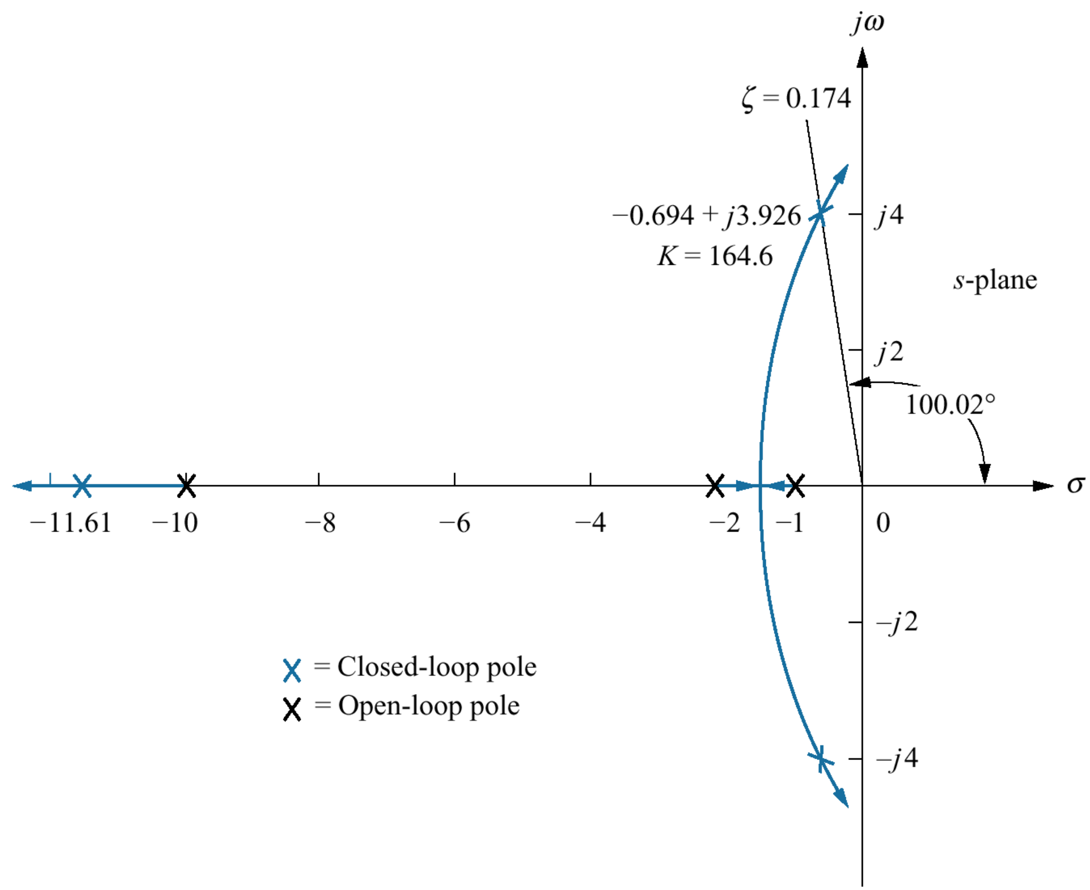
Assuming the lag compensator shown in Figure 9.9(*b*) and (*c*), the new static error constant is

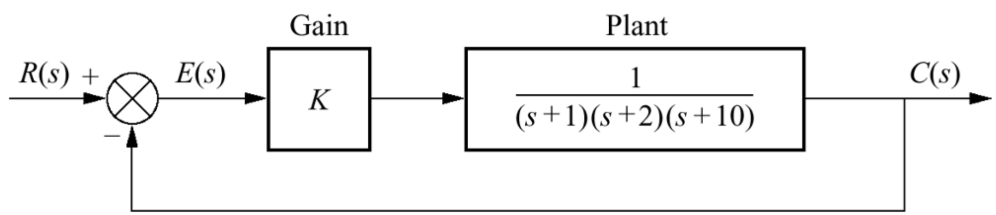
$$K_{v_N} = \frac{(K z_1 z_2 \cdots)(z_c)}{(p_1 p_2 \cdots)(p_c)} \tag{9.4}$$

$$K_{v_N} = K_{v_O}\frac{z_c}{p_c} > K_{v_O}$$

Lag compensator design

Compensate the system to improve the steady-state error (0.108) by a factor of 10 if the system is operating with a damping ratio of 0.174





Solution The uncompensated system error from Example 9.1 was 0.108 with $K_p = 8.23$. A tenfold improvement means a steady-state error of

$$e(\infty) = \frac{0.108}{10} = 0.0108 \tag{9.6}$$

Since

$$e(\infty) = \frac{1}{1 + K_p} = 0.0108 \tag{9.7}$$

rearranging and solving for the required K_p yields

$$K_p = \frac{1 - e(\infty)}{e(\infty)} = \frac{1 - 0.0108}{0.0108} = 91.59 \tag{9.8}$$

The improvement in K_p from the uncompensated system to the compensated system is the required ratio of the compensator zero to the compensator pole, or

$$\frac{z_c}{p_c} = \frac{K_{p_N}}{K_{p_O}} = \frac{91.59}{8.23} = 11.13 \tag{9.9}$$

Arbitrarily selecting

$$p_c = 0.01 \tag{9.10}$$

we use Eq. (9.9) and find

$$z_c = 11.13 p_c \approx 0.111 \tag{9.11}$$

Figure 9.11
Compensated system for Example 9.2

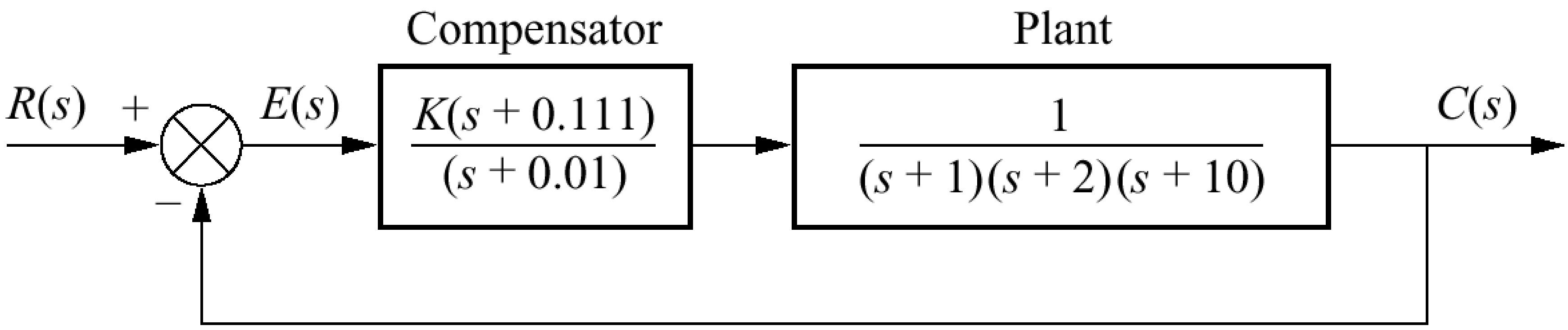
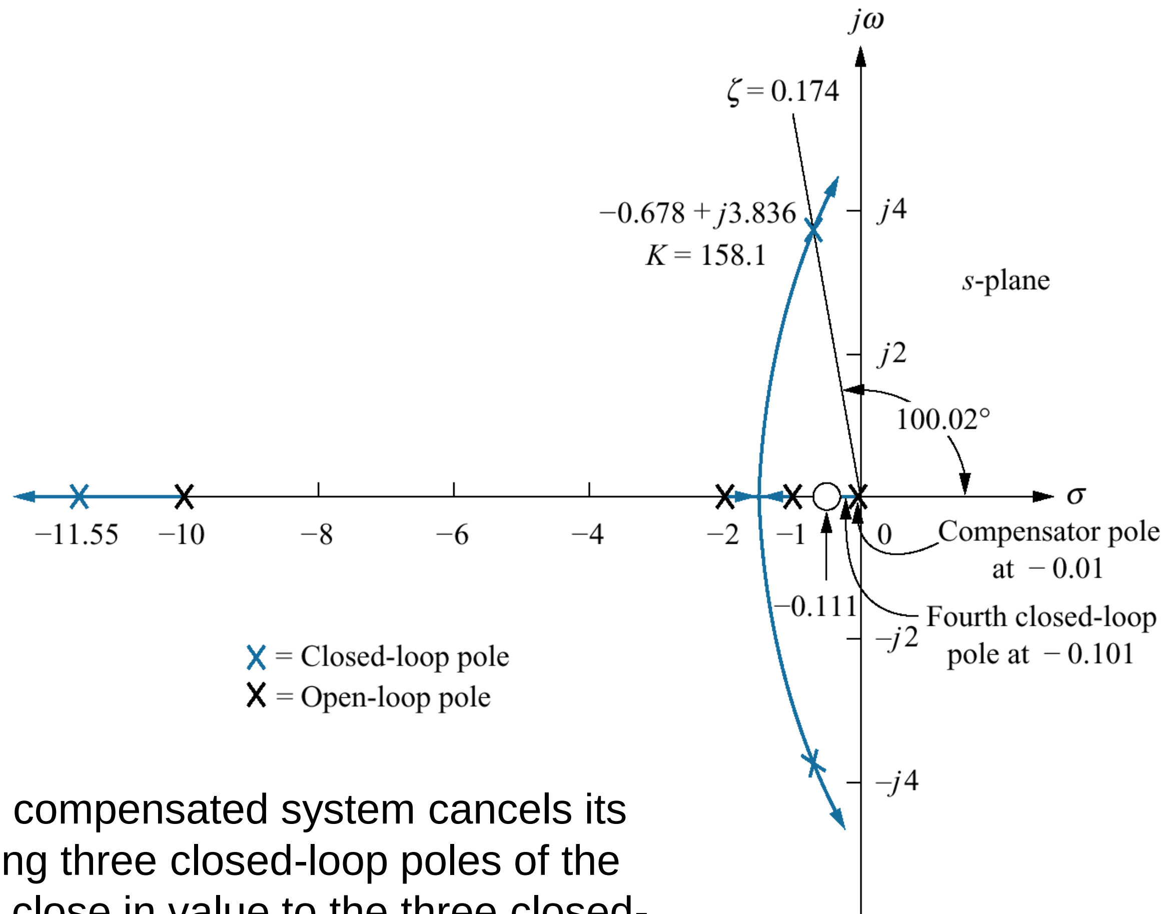


Figure 9.12
Root locus for compensated system of Figure 9.11



The fourth pole (-0.01) of the compensated system cancels its zero. This leaves the remaining three closed-loop poles of the uncompensated system very close in value to the three closed-loop poles of the uncompendated system. The transient response of the both system is approximately the same, but the stead-state error is 9.818 times less.

Table 9.1
Predicted characteristics of uncompensated and lag-compensated systems for Example 9.2

Parameter	Uncompensated	Lag-compensated
Plant and compensator	$\dfrac{K}{(s+1)(s+2)(s+10)}$	$\dfrac{K(s+0.111)}{(s+1)(s+2)(s+10)(s+0.01)}$
K	164.6	158.1
K_p	8.23	87.75
$e(\infty)$	0.108	0.011
Dominant second-order poles	$-0.694 \pm j3.926$	$-0.678 \pm j3.836$
Third pole	-11.61	-11.55
Fourth pole	None	-0.101
Zero	None	-0.111

Figure 9.13

Step responses of uncompensated and lag-compensated systems for Example 9.2

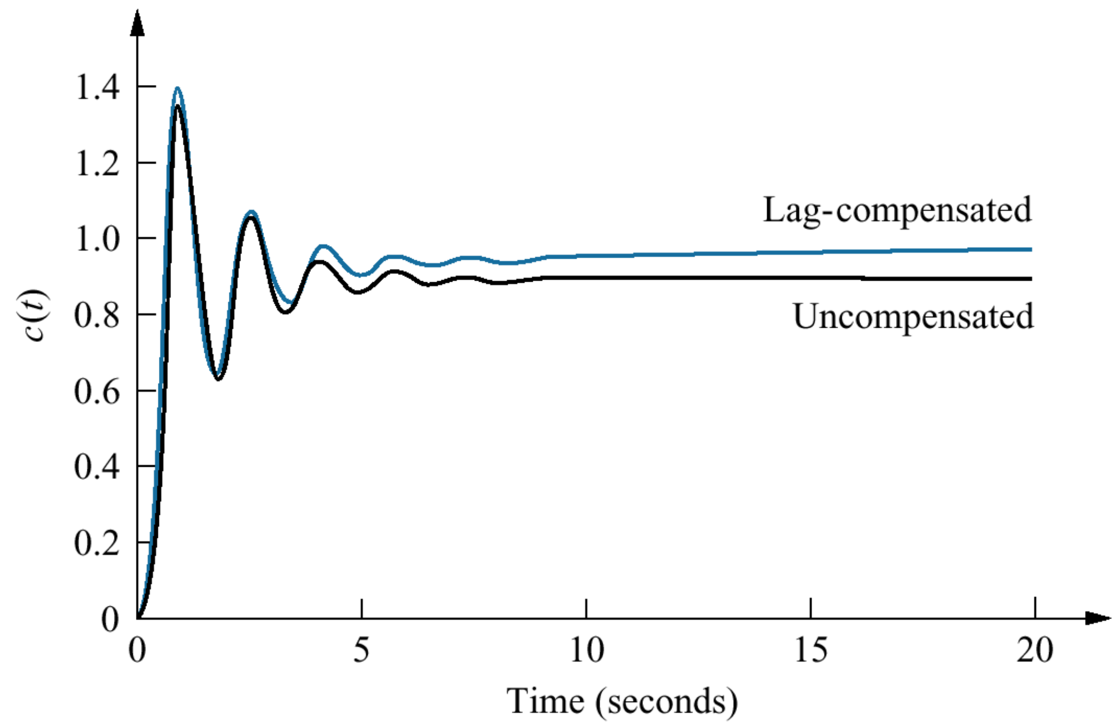


Figure 9.14

Step responses of the system for Example 9.2 using different lag compensators