Switching Algebra Extra exercises

1. Use switching-algebra theorems to simplify each of the following logic functions:

a. F = WXYZ(WXYZ’ + WX’YZ + W’XYZ + WXY’Z)

b. F = AB + ABC’D + ABDE’ + A’BC’E + A’B’C’E

c. F = MRP + QO’R’ + MN + ONM + QPMO’

d. F = (V + Y + Z)(V’ + W + X’)(V’ + X + Y’)(V + X’)

Answer:

a.
F = W·X·Y·Z·(W·X·Y·Z’ + W·X’·Y·Z + W’·X·Y·Z + W·X·Y’·Z)
= W·X·Y·Z·W·X·Y·Z’ + W·X·Y·Z·W·X’·Y·Z
+ W·X·Y·Z·W’·X·Y·Z + W·X·Y·Z·W·X·Y’·Z (T8)
= 0 + 0 + 0 + 0 (T6’, T5’, T2’)
= 0 (A4’)

b.
F = A·B + A·B·C’·D + A·B·D·E’ + A’·B·C’·E + A’·B’·C’·E
= A·B + A·B·D·E’ + A’·B·C’·E + A’·B’·C’·E (T9)
= A·B + A’·B·C’·E + A’·B’·C’·E (T9)
= A·B + A’·C’·E (T10)

c.
F = M·R·P + Q·O’·R’ + M·N + Q·P·M·O’ + O·N·M
= M·R·P + Q·O’·R’ + Q·P·M·O’ + M·N + O·N·M (T6)
= M·R·P + Q·O’·R’ + Q·P·M·O’ + M·N (T9)
= R·(M·P) + R’·(Q·O’) + (M·P)·(Q·O’) + M·N (T6’,T7’)
= R·(M·P) + R’·(Q·O’) + M·N (T11)
= R·M·P + R’·Q·O’ + M·N (T7’)

d.
F = (V+Y+Z)(V’+W+X’)(V’+X+Y’)(V+X’)
=(V+Y+Z)(V+X’)(V’+W+X’)(V’+X+Y’)
=(V+X’Y+X’Z)(V’+(W+X’)(X+Y’)) [T8’]
=(V+X’Y+X’Z)(V’+WX+WY’+X’X+X’Y’)
=(V+X’Y+X’Z)(V’+WX+X’Y’+WY’)
=(V+X’Y+X’Z)(V’+WX+X’Y’) [Consensus]
=VV’+V(WX+X’Y’)+V’(X’Y+X’Z)+X’(Y+Z)(WX+X’Y’)
=VWX+VX’Y’+V’X’Y+V’X’Z+X’Y’(Y+Z)
=VWX+VX’Y’+V’X’Y+V’X’Z+X’Y’Z
=VWX+VX’Y’+X’(V’Y+V’Z+Y’Z)
=VWX+VX’Y’+V’X’Y+X’Y’Z [Consensus]

2. Write the truth table for each of the following logic functions:

a. F = X'Y + X'Y'Z

b. F = AB' + B'C + CD' + CA'

c. F = (A' + B'CD)(B' + C' + DE')

d. F = (((A + B')' + C)' + D)'

Answer:

a.

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

b.

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

c.

A	B	C	D	E	F
0	0	0	0	0	1
0	0	0	0	1	1
0	0	0	1	0	1
0	0	0	1	1	1
0	0	1	0	0	1
0	0	1	0	1	1
0	0	1	1	0	1
0	0	1	1	1	1
0	1	0	0	0	1
0	1	0	0	1	1
0	1	0	1	0	1
0	1	0	1	1	1
0	1	1	0	0	0
0	1	1	0	1	0
0	1	1	1	0	1
0	1	1	1	1	0
1	0	0	0	0	0
1	0	0	0	1	0
1	0	0	1	0	0
1	0	0	1	1	0
1	0	1	0	0	0
1	0	1	0	1	0
1	0	1	1	0	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	0	0	1	0
1	1	0	1	0	0
1	1	0	1	1	0
1	1	1	0	0	0
1	1	1	0	1	0
1	1	1	1	0	0
1	1	1	1	1	0

d.

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

3. Write the canonical sum and product for each of the following logic functions:

a. $F = \sum_{X, Y} (1,2)$

b. $F = \prod_{A,B,C} (1, 2, 4)$

c. $F = \sum_{A,B,C,D} (1, 2, 5, 6)$

d. $F = X' + YZ$

Answer:

a. $F = X'\cdot Y + Y'\cdot X = (X + Y)\cdot(X' + Y') = \sum m(1, 2) = \prod M(0, 3)$

b. $F = \prod_M (1, 2, 4) = \sum m(0, 3, 5, 6, 7)$

c. $F = \sum_m (1, 2, 5, 6) = \prod_M (0, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15)$

d. $F = X'(Y+Y')(Z+Z') + (X+X')YZ = X'(YZ + YZ' + Y'Z + Y'Z') + XYZ + X'YZ$

$= X'YZ + X'YZ' + X'Y'Z + X'Y'Z' + XYZ$

$= \sum_m (011, 010, 001, 000, 111) = \sum_m (0, 1, 2, 3, 7) = \prod_M (4, 5, 6)$