Topics covered today:

1) FT definition (Page 19)

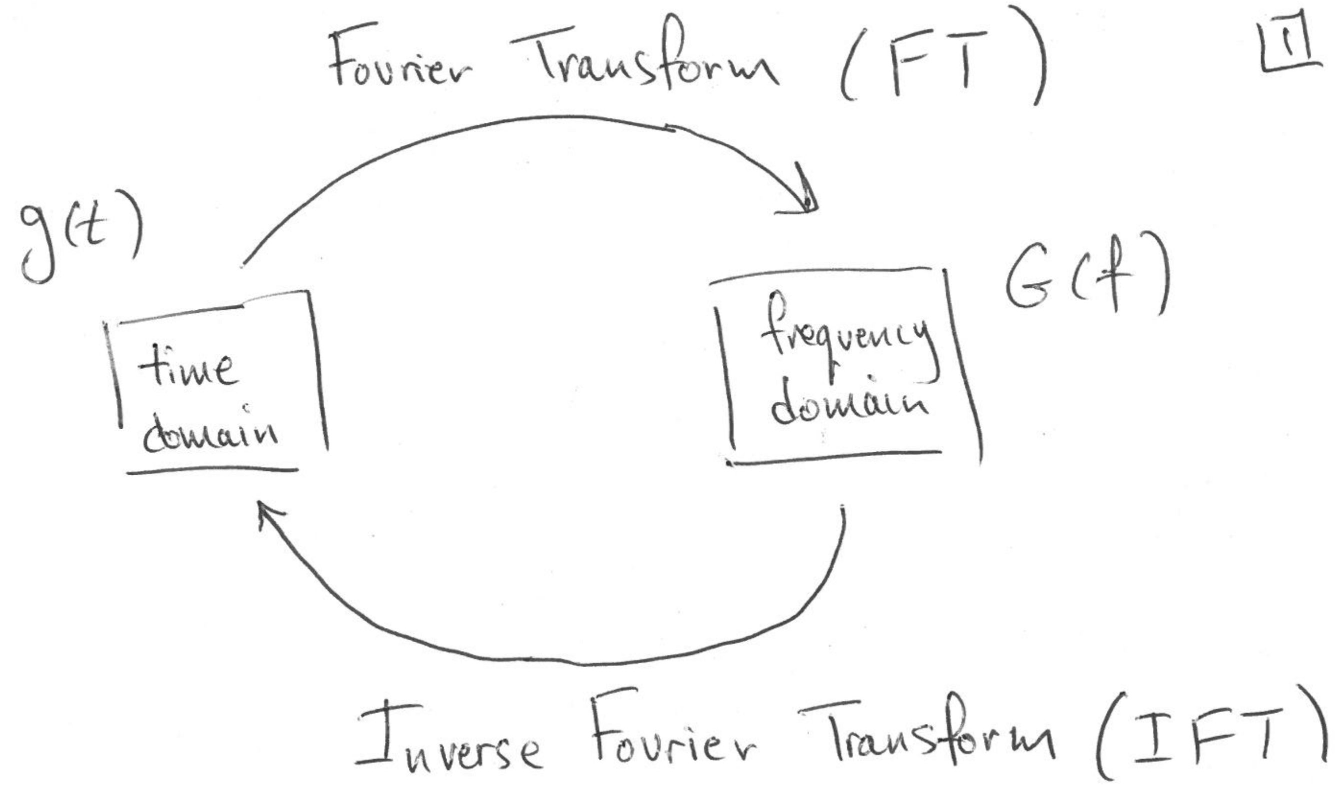
2) FT of Rectangular Pulse (Page 22)

3) FT of Exponential Pulse (Page 23)

4) FT of Combination of Exponential Pulses (Page 26)

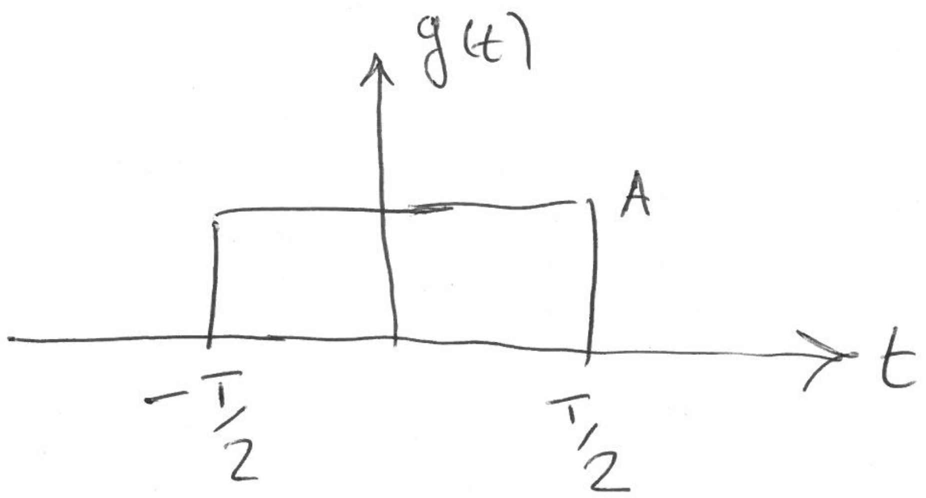
5) Time Shift Property (Page 30)

6) Frequency Shift Property (Page 30)



FT: $$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt$$

IFT: $$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{+j2\pi f t}\, dt$$



$$g(t) = A \, \mathrm{rect}\left(\frac{t}{T}\right)$$

Fourier Transform:

$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-T/2}^{T/2} A\, e^{-j2\pi f t}\, dt$$

$$= A \int_{-T/2}^{T/2} e^{-j2\pi f t}\, dt$$

$$= A \left[\frac{e^{-j2\pi f t}}{-j2\pi f} \right]_{-T/2}^{T/2}$$

$$= \frac{A}{-j2\pi f} \left[e^{-j2\pi f \frac{T}{2}} - e^{+j2\pi f \frac{T}{2}} \right]$$

$$= \frac{A}{j2\pi f} \left[e^{+j2\pi f \frac{T}{2}} - e^{-j2\pi f \frac{T}{2}} \right]$$

$$\sin\theta = \frac{1}{2j}\left[e^{j\theta} - e^{-j\theta}\right]$$

$\therefore$

$$G(f) = \frac{A}{j2\pi f}\left[2j\sin\pi f T\right]$$

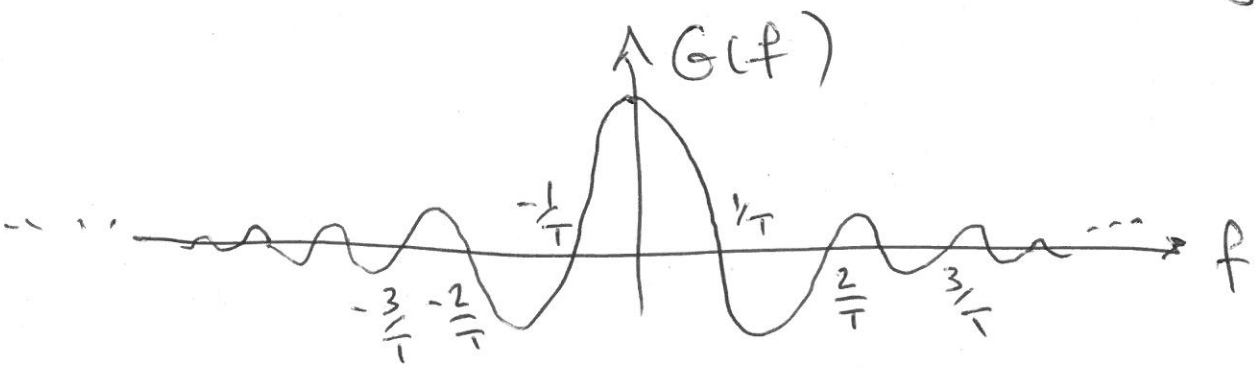
$$= \frac{A}{\pi f}\left[\sin\pi f T\right]$$

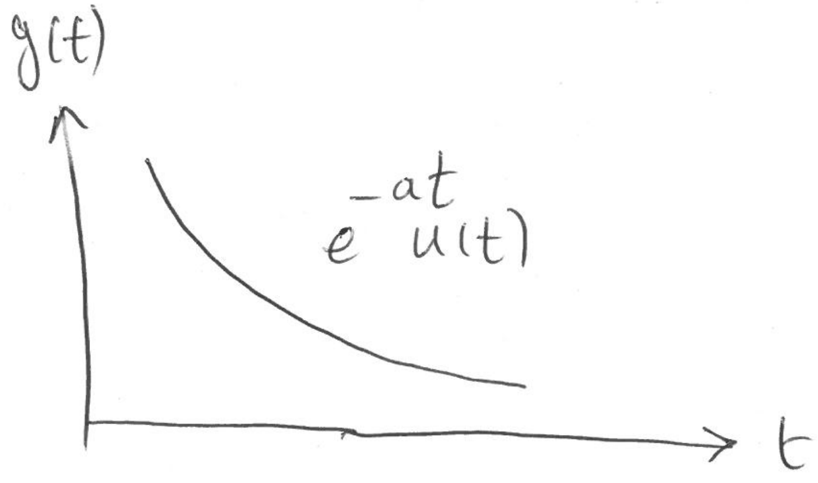
$$= AT\left[\frac{\sin\pi f T}{T\pi f}\right]$$

$$= AT\,\mathrm{sinc}(fT)$$

where $\boxed{\mathrm{sinc}(x) = \frac{\sin\pi x}{\pi x}}$ page 22

zeros at $f = \frac{n}{T}$ where n is an integer





Decaying Exponential Function

$$g(t) = e^{-at} u(t)$$

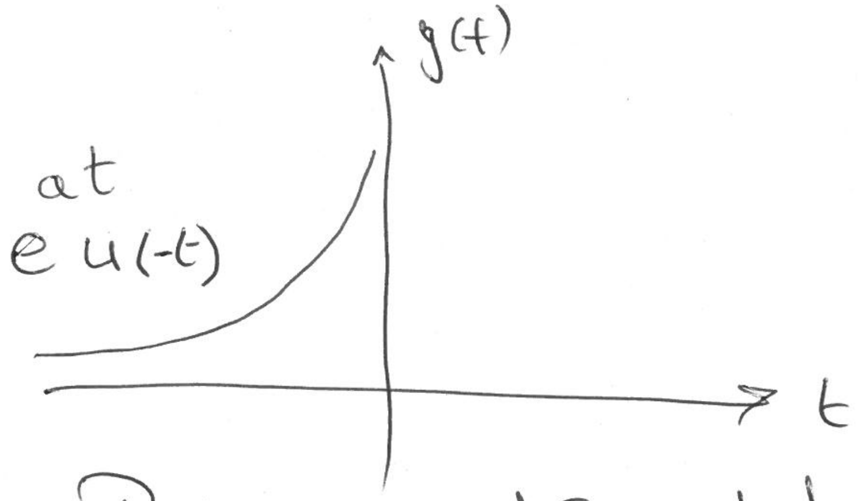
Fourier Transform:

$$G(f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{0}^{\infty} e^{-at}\, e^{-j2\pi f t}\, dt$$

$$= \left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)} \right]_{0}^{\infty}$$

$$= \frac{1}{a + j2\pi f}$$



Rising ~~Expont~~ Exponential Function

$$g(t) = e^{at} u(-t)$$

Fourier Transform:

$$G(f) = \int_{-\infty}^{0} e^{at} e^{-j2\pi f t} \, dt$$

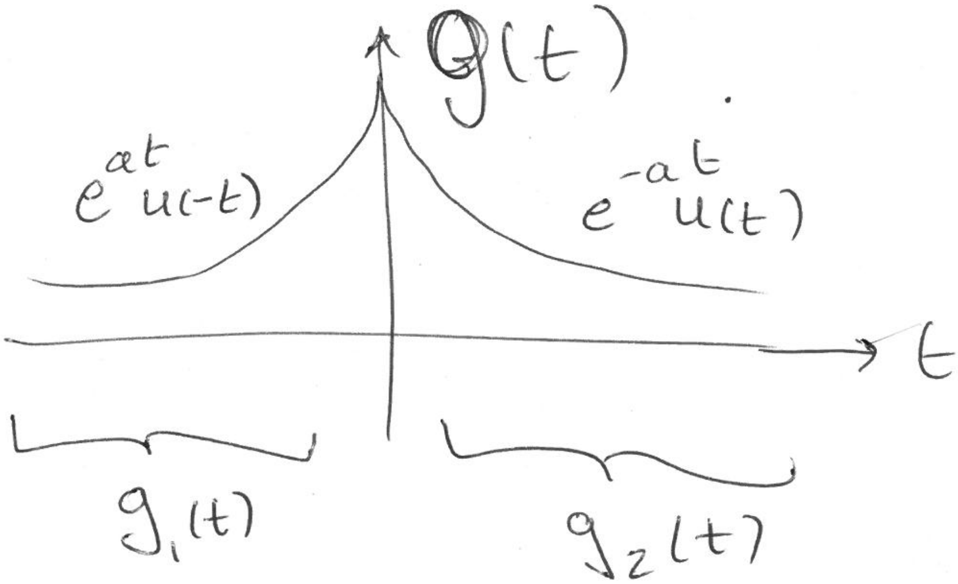
$$= \left[\frac{e^{(a-j2\pi f)t}}{a - j2\pi f} \right]_{-\infty}^{0}$$

$$= \frac{1}{a - j2\pi f}$$

$\boxed{6}$

$$g(t) = \begin{cases} e^{-at} & t > 0 \\ 1 & t = 0 \\ e^{at} & t < 0 \end{cases}$$

Double Exponential Function
(Symmetric)



$$g(t) = g_1(t) + g_2(t)$$

$\therefore$

$$G(f) = G_1(f) + G_2(f)$$

Superposition property

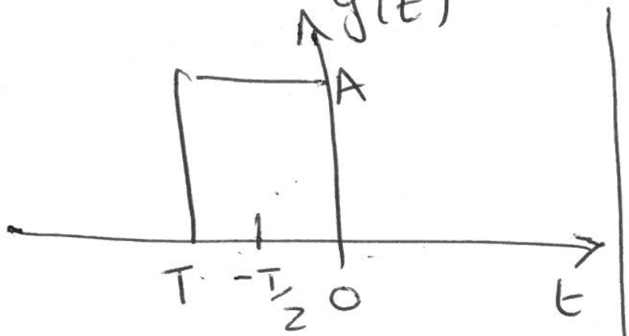
$$= \frac{1}{a - j2\pi f} + \frac{1}{a + j2\pi f}$$

$$= \frac{2a}{a^2 + (2\pi f)^2}$$

Time Shift Property

$$g(t) \rightleftharpoons G(f)$$

$$g(t-t_0) \rightleftharpoons G(f)e^{-j2\pi f t_0}$$



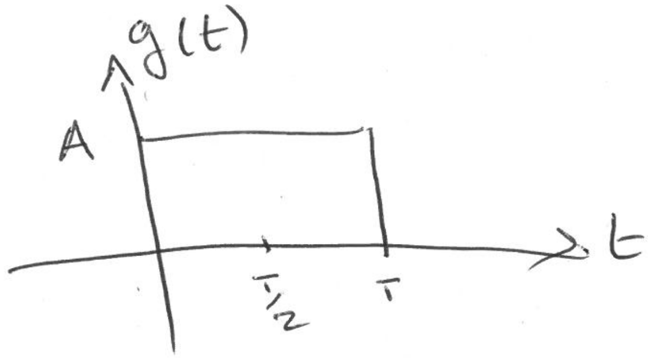
$$A\,\text{rect}(t/T) \rightleftharpoons AT\,\text{sinc}(fT)$$

what is $g(t)$?

$$g(t) = A\,\text{rect}\left(\frac{t+T/2}{T}\right)$$

$\therefore$

$$G(f) = AT\,\text{sinc}(fT)e^{j2\pi f\frac{T}{2}}$$



$$A\,\text{rect}(t/T) \rightleftharpoons AT\,\text{sinc}(fT)$$

what is $g(t)$?

$$g(t) = A\,\text{rect}\left(\frac{t-T/2}{T}\right)$$

$\therefore$

$$G(f) = AT\,\text{sinc}(fT)e^{-j2\pi f\frac{T}{2}}$$

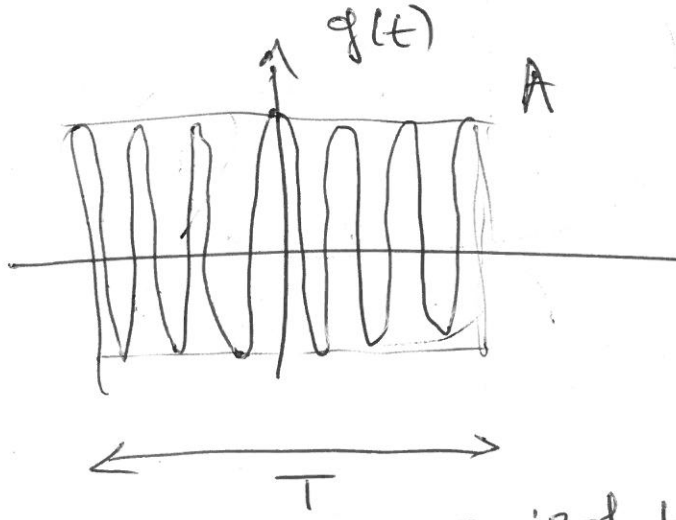
Frequency Shift Property

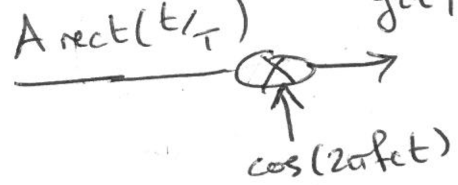
$$g(t) \rightleftharpoons G(f)$$

$$g(t)e^{+j2\pi f_c t} \rightleftharpoons G(f-f_c)$$

Find the Fourier Transform of

$$g(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)\cos(2\pi f_c t)$$





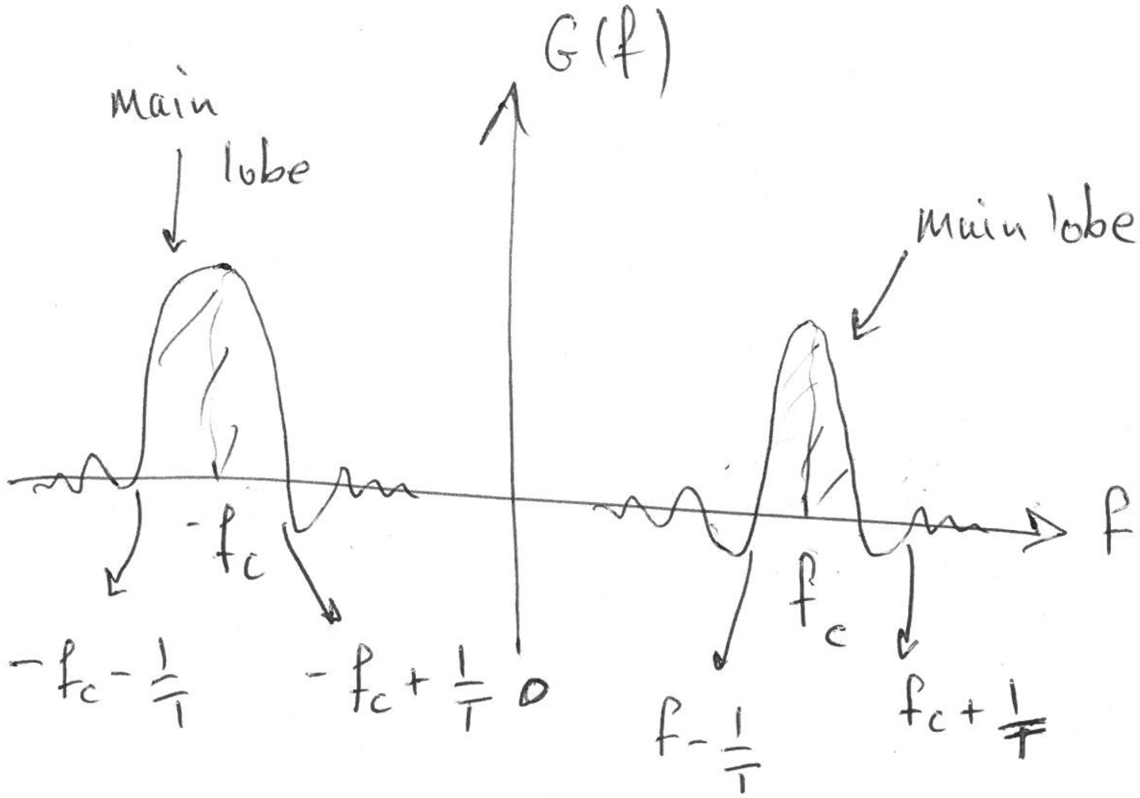
since $\cos 2\pi f_c t = \frac{1}{2}\left[e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right]$

rewrite $g(t)$:

$$g(t) = \frac{A}{2}\,\mathrm{rect}\left(t/T\right)e^{j2\pi f_c t} + \frac{A}{2}\,\mathrm{rect}\left(t/T\right)e^{-j2\pi f_c t}$$

$\therefore$

$$G(f) = \frac{AT}{2}\,\mathrm{sinc}\left((f-f_c)T\right) + \frac{A}{2}T\,\mathrm{sinc}\left((f+f_c)T\right)$$



G(f)
main lobe
main lobe
f
$-f_c$
f_c
$-f_c - \frac{1}{T}$
$-f_c + \frac{1}{T}$
0
$f - \frac{1}{T}$
$f_c + \frac{1}{T}$