TABLE A6.2 *Fourier-Transform Pairs*

Time Function	*Fourier Transform*
$\text{rect}\left(\frac{t}{T}\right)$	$T \text{ sinc } (fT)$
$\text{sinc } (2Wt)$	$\frac{1}{2W}\text{rect}\left(\frac{f}{2W}\right)$
$\exp(-at)u(t), \quad a > 0$	$\frac{1}{a + j2\pi f}$
$\exp(-a\lvert t\rvert), \quad a > 0$	$\frac{2a}{a^2 + (2\pi f)^2}$
$\exp(-\pi t^2)$	$\exp(-\pi f^2)$
$\begin{cases} 1 - \frac{\lvert t\rvert}{T}, & \lvert t\rvert < T \\ 0, & \lvert t\rvert \geq T \end{cases}$	$T \text{ sinc}^2(fT)$
$\delta(t)$	1
1	$\delta(f)$
$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$
$\exp(j2\pi f_c t)$	$\delta(f - f_c)$
$\cos(2\pi f_c t)$	$\frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$
$\sin(2\pi f_c t)$	$\frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$
$\text{sgn}(t)$	$\frac{1}{j\pi f}$
$\frac{1}{\pi t}$	$-j \text{ sgn}(f)$
$u(t)$	$\frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
$\sum_{i=-\infty}^{\infty} \delta(t - iT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_0}\right)$

Notes: $u(t)$ = unit step function
$\delta(t)$ = Dirac delta function
$\text{rect}(t)$ = rectangular function
$\text{sgn}(t)$ = signum function
$\text{sinc}(t)$ = sinc function

TABLE A6.3 *Hilbert-Transform Pairs*[a]

Time Function	*Hilbert Transform*
$m(t) \cos(2\pi f_c t)$	$m(t) \sin(2\pi f_c t)$
$m(t) \sin(2\pi f_c t)$	$-m(t) \cos(2\pi f_c t)$
$\cos(2\pi f_c t)$	$\sin(2\pi f_c t)$
$\sin(2\pi f_c t)$	$-\cos(2\pi f_c t)$
$\delta(t)$	$\frac{1}{\pi t}$
$\frac{1}{t}$	$-\pi\delta(t)$

[a]In the first two pairs, it is assumed that $m(t)$ is band limited to the interval $-W \leq f \leq W$, where $W < f_c$.

TABLE A6.4 *Trigonometric Identities*

$$\exp(\pm j\theta) = \cos\theta \pm j\sin\theta$$
$$\cos\theta = \tfrac{1}{2}[\exp(j\theta) + \exp(-j\theta)]$$
$$\sin\theta = \frac{1}{2j}[\exp(j\theta) - \exp(-j\theta)]$$
$$\sin^2\theta + \cos^2\theta = 1$$
$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$
$$\cos^2\theta = \tfrac{1}{2}[1 + \cos(2\theta)]$$
$$\sin^2\theta = \tfrac{1}{2}[1 - \cos(2\theta)]$$
$$2\sin\theta\cos\theta = \sin(2\theta)$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$
$$\sin\alpha\sin\beta = \tfrac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$
$$\cos\alpha\cos\beta = \tfrac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$
$$\sin\alpha\cos\beta = \tfrac{1}{2}[\sin(\alpha - \beta) + \sin(\alpha + \beta)]$$

TABLE A6.5 *Series Expansions*

Taylor series	$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$
where	$f^{(n)}(a) = \frac{d^n f(x)}{dx^n}\big\vert_{x=a}$
MacLaurin series	$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$
where	$f^{(n)}(0) = \frac{d^n f(x)}{dx^n}\big\vert_{x=0}$
Binomial series	$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots, \quad \vert nx\vert < 1$
Exponential series	$\exp x = 1 + x + \frac{1}{2!}x^2 + \cdots$
Logarithmic series	$\ln(1+x) = x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \cdots$
Trigonometric series	$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots$
	$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots$
	$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$
	$\sin^{-1}x = x + \frac{1}{6}x^3 + \frac{3}{40}x^5 + \cdots$
	$\tan^{-1}x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \cdots, \quad \vert x\vert < 1$
	$\mathrm{sinc}\, x = 1 - \frac{1}{3!}(\pi x)^2 + \frac{1}{5!}(\pi x)^4 - \cdots$