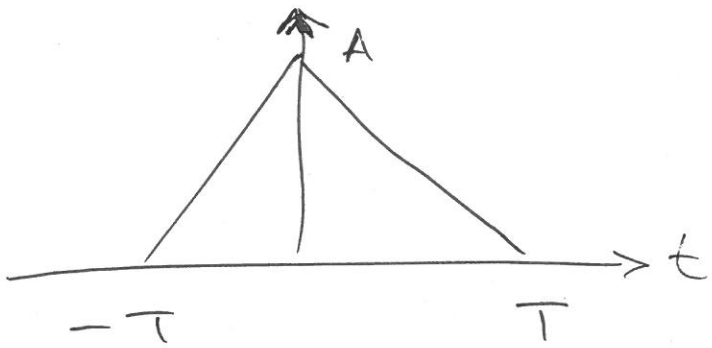
Fourier Transform Tutorial

(a) $G(f) = 16\,\text{sinc}^2(4(f-10^6)) + 16\,\text{sinc}^2(4(f+10^6))$

we know that

$$A\,\text{tri}(t/T) \rightleftharpoons AT\,\text{sinc}^2(fT)$$



$AT = 16, \quad T = 4 \quad \therefore A = 4$

$$g(t) = 4\,\text{tri}\left(\tfrac{t}{4}\right)e^{j2\pi 10^6 t} + 4\,\text{tri}\left(\tfrac{t}{4}\right)e^{-j2\pi 10^6 t}$$

$$= 8\,\text{tri}\left(\tfrac{t}{4}\right)\cos(2\pi 10^6 t)$$

b) $G(f) = 12\,\text{sinc}(4f)\sin(4\pi f)$

$$= \frac{6}{j}\left[\text{sinc}(4f)e^{j2\pi(2)f} - \text{sinc}(4f)e^{-j2\pi(2)f}\right]$$

↳ time shift property

Note that

$$A\,\text{rect}(t/T) \rightleftharpoons AT\,\text{sinc}(fT)$$

therefore

$$\frac{1}{4}\,\text{rect}\left(\frac{t}{4}\right) \rightleftharpoons \text{sinc}(4f)$$

$$\therefore g(t) = \frac{6}{j}\left[\frac{1}{4}\,\text{rect}\left(\frac{t+2}{4}\right) - \frac{1}{4}\,\text{rect}\left(\frac{t-2}{4}\right)\right]$$

$$= \frac{3}{2j}\,\text{rect}\left(\frac{t+2}{4}\right) - \frac{3}{2j}\,\text{rect}\left(\frac{t-2}{4}\right)$$

(c)

(i) $g(t) = e^{-t}\sin(2\pi f_0 t)\,u(t)$

$$= \frac{e^{-t}}{2j}\left[e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}\right]u(t)$$

↳ frequency shift property

also we know

$$e^{-t}u(t) \rightleftharpoons \frac{1}{1+j2\pi f}$$

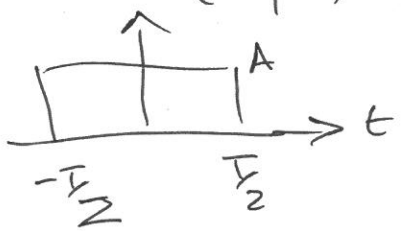
$\therefore$

$$G(f) = \frac{1}{2j}\left\{\frac{1}{1+j2\pi(f-f_0)} - \frac{1}{1+j2\pi(f+f_0)}\right\}$$

(ii) $g(t) = 8\,\text{rect}\left(\frac{t}{4}\right)\cos(2\pi 10^6 t)$

we know

$$A\,\text{rect}\left(\frac{t}{T}\right) \rightleftharpoons AT\,\text{sinc}(fT)$$



frequency shift property

$\therefore$

$$g(t) = 8\,\text{rect}\left(\frac{t}{4}\right)\left(\frac{e^{j2\pi 10^6 t} + e^{-j2\pi 10^6 t}}{2}\right)$$

$\therefore$

$$G(f) = 16\,\text{sinc}((f-10^6)4) + 16\,\text{sinc}((f+10^6)4)$$

d)

(i) $10\,tri\left(2t-\frac{1}{2}\right)$

$= 10\,tri\left(\frac{4t-1}{2}\right)$

$= 10\,tri\left(\frac{4(t-1/4)}{2}\right)$

$= 10\,tri\left(2(t-1/4)\right)$ → time shift property

$\therefore$

$G(f) = 5\,sinc^2\left(\frac{f}{2}\right) e^{-j2\pi f/4}$

(ii) $g(t) = 8\,tri\left(\frac{t}{2}\right)\cos(2\pi 10^6 t)$

$= \frac{8}{2}\,tri\left(\frac{t}{2}\right)\left\{ e^{j2\pi 10^6 t} + e^{-j2\pi 10^6 t} \right\}$

$\therefore$

$G(f) = 8\,sinc^2\left((f-10^6)2\right) + 8\,sinc^2\left((f+10^6)2\right)$