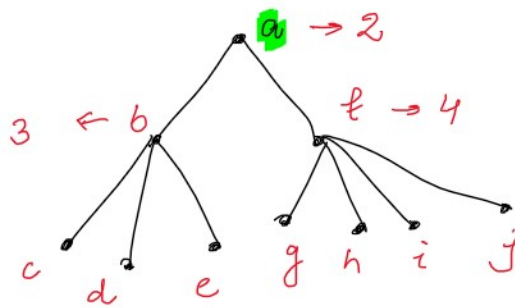
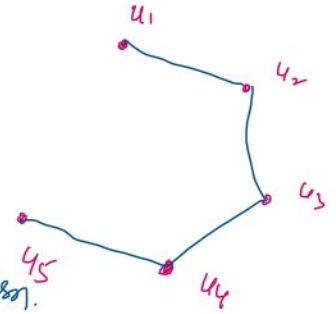


# A tree with  $n$  vertices has  $(n-1)$  edges.

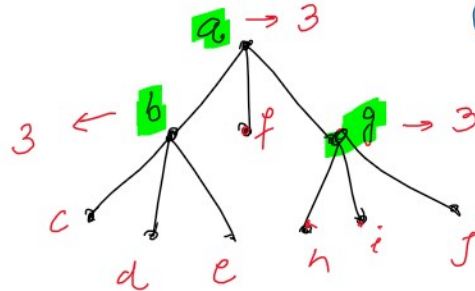
# m-ary tree :-

A tree is s.t.b m-ary tree if every internal vertex has no more than  $m$ -children.



4-ary tree

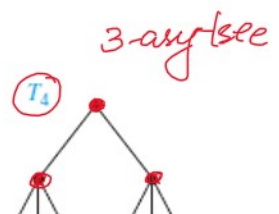
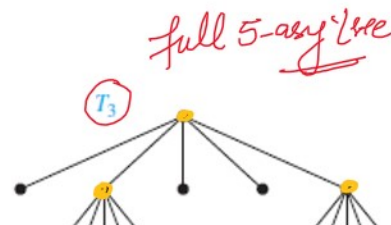
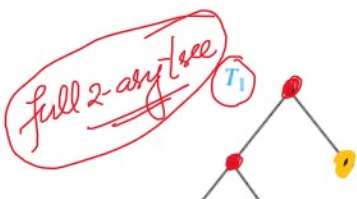
# full m-ary tree :- A tree is s.t.b full m-ary tree if every internal vertex has exactly  $m$ -children.

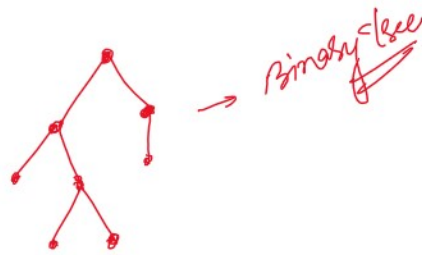
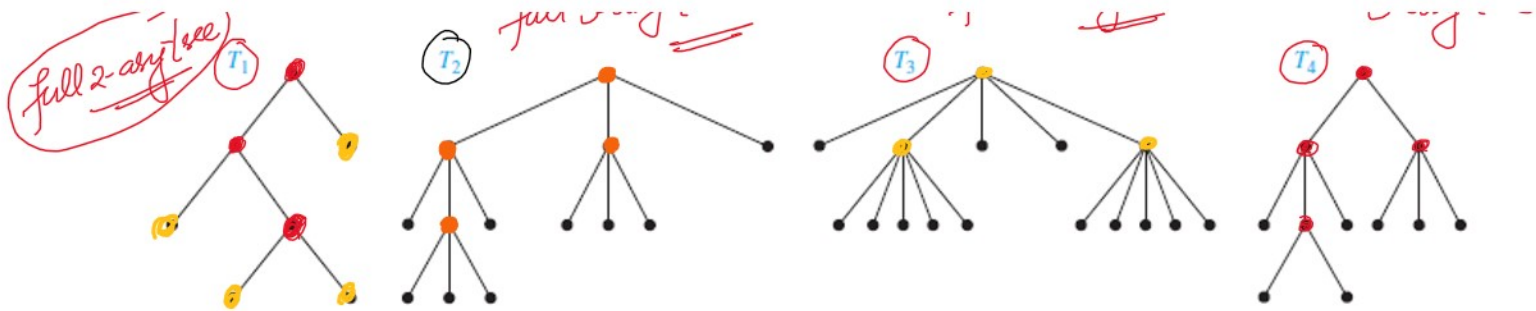


full 3-ary tree

Binary tree :- A m-ary tree is called binary tree if  $m=2$

2-ary tree





# A full  $m$ -ary tree with  $i$  internal vertices contains  $mi+1$  vertices.

Total no. of children =  $mi$

$n \rightarrow$  total no. of vertices

$i \rightarrow$  internal vertices

$l \rightarrow$  no. of leaves

$n = i + l$

$n = mi + 1$   $\rightarrow$  full  $m$ -ary tree

# A full  $m$ -ary tree with  $i$  internal vertices contains  $mi+1$  leaves.

$$\begin{aligned} n &= mi + 1 \\ n &= i + l \end{aligned}$$

$$\begin{aligned} mi + 1 &= i + l \\ l &= mi + 1 - i = (m-1)i + 1 \end{aligned}$$

Suppose that someone starts a chain letter. Each person who receives the letter is asked to send it on to four other people. Some people do this, but others do not send any letters. How many people have seen the letter, including the first person, if no one receives more than one letter and if the chain letter ends after there have been 100 people who read it but did not send it out? How many people sent out the letter?

$\rightarrow n = ?$

$\rightarrow i = ?$

$\rightarrow n = ?$

and if the chain letter ends after there have been 100 people who read it but did not send it out?  
How many people sent out the letter?  $\rightarrow i = ?$

full 4-ary tree

$$m=4$$

$$l=100$$

$$\begin{aligned} n &= i + l \\ n &= 100 + 33 \\ n &= 133 \end{aligned}$$

$$\begin{aligned} n &= mi + 1 \\ n &= i + l \end{aligned}$$

$$\begin{aligned} mi + 1 &= i + l \\ 4i + 1 &= i + 100 \\ 4i - i &= 99 \end{aligned}$$

$$3i = 99 \Rightarrow i = 33$$

How many leaves does a full 3-ary tree with 100 vertices have?

$$\begin{aligned} m &= 3 \\ n &= mi + 1 \\ 100 &= 3i + 1 \\ 99 &= 3i \\ i &= 33 \end{aligned}$$

$$n = 100$$

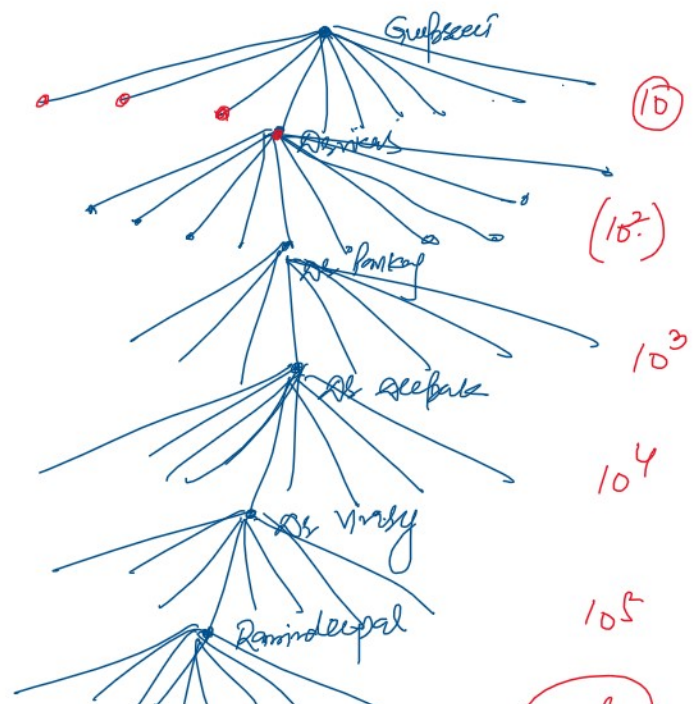
$$l = ?$$

$$\begin{aligned} n &= i + l \\ 100 &= 33 + l \end{aligned}$$

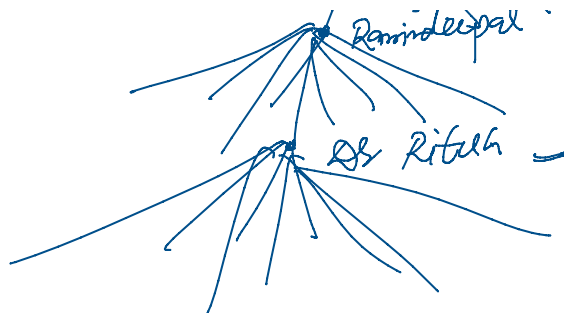
$$l = 100 - 33 = 67$$

A chain letter starts with a person sending a letter out to 10 others. Each person is asked to send the letter out to 10 others, and each letter contains a list of the previous six people in the chain. Unless there are fewer than six names in the list, each person sends one dollar to the first person in this list, removes the name of this person from the list, moves up each of the other five names one position, and inserts his or her name at the end of this list. If no person breaks the chain and no one receives more than one letter, how much money will a person in the chain ultimately receive?

$$\begin{aligned} 10^6 \\ 10,000,000 \end{aligned}$$



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