Algebra

1. $(a+b)^2 = a^2+b^2+2ab$
2. $(a-b)^2 = a^2+b^2-2ab$
3. $(a^2-b^2) = (a+b)(a-b)$
4. $a^2+\frac{1}{a^2} = \left(a+\frac{1}{a}\right)^2-2 = \left(a-\frac{1}{a}\right)^2+2$
5. $a^2+b^2+c^2 = (a+b+c)^2-2(ab+bc+ca)$
6. $(a+b)^3 = a^3+b^3+3ab(a+b)$
7. $a^3-b^3 = (a-b)(a^2+b^2+ab)$
8. $(a-b)^3 = a^3-b^3-3ab(a-b)$
9. $a^3+b^3 = (a+b)(a^2+b^2-ab)$
10. $a^3+\frac{1}{a^3} = \left(a+\frac{1}{a}\right)^3-3\left(a+\frac{1}{a}\right)$
11. $a^3-\frac{1}{a^3} = \left(a-\frac{1}{a}\right)^3+3\left(a-\frac{1}{a}\right)$
12. $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

If $a+b+c=0$, then

$a^3+b^3+c^3 = 3abc$

14. if $x + \frac{1}{x} = K$

then $\left[x^2 + \frac{1}{x^2} = K^2 - 2\right]$

15. if $x + \frac{1}{x} = K$

then $\left[x^3 + \frac{1}{x^3} = K^3 - 3K\right]$

16. if $x + \frac{1}{x} = \sqrt{3}$

(i) then $x^3 + \frac{1}{x^3} = 0$

(ii) $x^6 + 1 = 0$

(iii) $x^6 = -1$

17. if $x^2 + \frac{1}{x^2} = K$

(i) $x + \frac{1}{x} = \sqrt{K+2}$

(ii) $x - \frac{1}{x} = \sqrt{K-2}$

18. if $x - \frac{1}{x} = K$

then $\left[x^2 + \frac{1}{x^2} = K^2 + 2\right]$

19. if $x - \frac{1}{x} = K$

$\left[x^3 - \frac{1}{x^3} = K^3 + 3K\right]$

20. if $x + \frac{1}{x} = K$

$x^5 + \frac{1}{x^5} = \left(x^2 + \frac{1}{x^2}\right)\left(x^3 + \frac{1}{x^3}\right) - \left(x + \frac{1}{x}\right)$

$= (K^2 - 2)(K^3 - 3K) - K$

21. $(a+b)^2 - (a-b)^2 = 4ab$

22. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2$

23. if $x + \frac{1}{x} = 2$

then $x = 1$ (always)

24. if $x + \frac{1}{x} = -2$

then $x = -1$ (always)