L20 22 Mar 2022

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A graph G has 21 edges, 3 vertices of degree 4 and all other vertices are of degree 3. Find the no. of vertices in G.

$e = 21$

$n = ?$

Handshaking

$$\sum \deg(v) = 2e$$

$$3(4) + (n-3)\cdot 3 = 2e$$

$$12 + 3n - 9 = 2(21)$$

$$3n = 42 - 3$$

$$n = \frac{39}{3} = 13$$

$$\sum \deg(v) = 2e$$

$$3\cdot(4) + x(3) = 2(21)$$

$$12 + 3x = 42$$

$$3x = 42 - 12$$

$$3x = 30$$

$$x = 10$$

Total $= 10 + 3 = 13$

odd vertices

A vertex is s.t.b odd vertex if its degree is odd no.

Even vertices

A vertex is s.t.b even vertex if its degree is an even no.

The no. of odd vertices in a graph is always even.

Let G be the graph

By Handshaking Lemma

$$\sum \deg(v) = 2e$$

$$\underbrace{\sum_{v \in V_{odd}} \deg(v)} + \underbrace{\sum_{v \in V_{Even}} \deg(v)} = 2e$$

$1, 3, 5 = 9$ (odd)

$1, 1, 3, 3, 5$

$$\sum_{v \in V_{odd}} \deg(v) = \underbrace{2e}_{\text{Even}} - \underbrace{\sum_{v \in V_{even}} \deg(v)}_{\text{Even}}$$

$$\sum_{v \in V_{odd}} \deg(v) = \text{Even}$$

$\Rightarrow$ The no. of odd degree vertices are even in no.

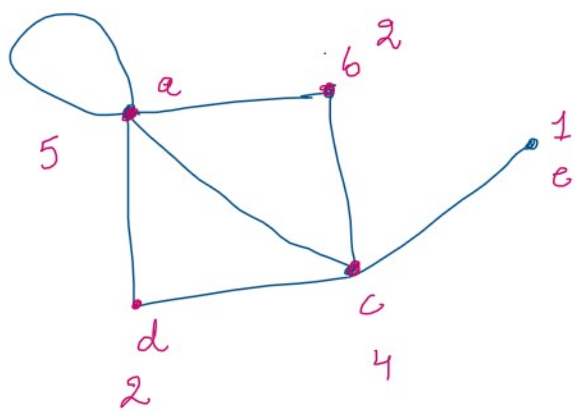
⇒ The no. of odd degree vertices are even.

The possible no. of vertices in a graph with degree 7 and all other vertices with degree 2 is.

→ odd no.

(a) 3. (b) 4. (c) 5 (d) 1

Degree Sequence :- is the sequence of degree of all the vertices of the graph with non increasing order



5, 4, 2, 2, 1

A sequence $d_1, d_2, d_3, \ldots d_n$ is called graphic if it is the degree sequence of a simple graph.

5, 4, 3, 2, 1, 0

Sum of degree of all vertice

$5+4+3+2+1+0$
$= 15 = 2e$

Not possible

$e = 15/2$

6, 5, 4, 3, 2, 1 Not possible

6, 5, 4, 3, 2, 1 Not possible

odd in no.

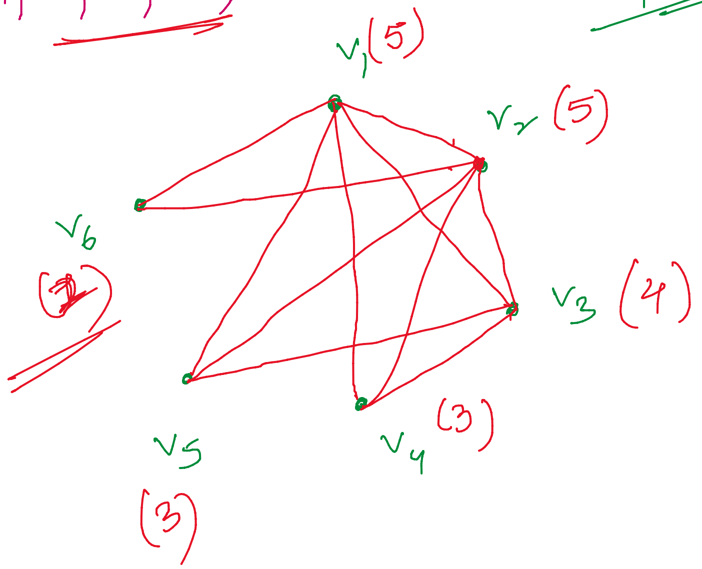
v_1 v_2 v_3 v_4 v_5 v_6

5, 5, 4, 3, 2, 1,

Not possible

sum = 20

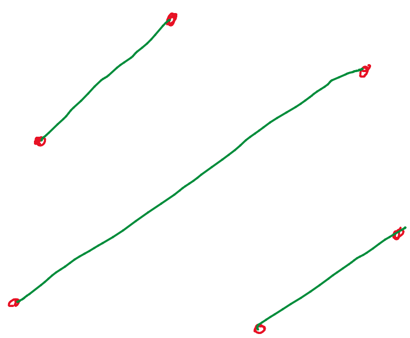
e = 10



1, 1, 1, 1, 1, 1

sum = 6

e = 3



3, 3, 3, 3, 2