T10 G1 18 Feb 2022

18 February 2022 11:22

Def:- The elements a and b of a poset (S, R) are called Comparable if either aRb or bRa.

$(Z^+, |)$ poset

$2|4$

2 and 4 are comparable

but

3 and 5 are not comparable → incomparable

$3 \nmid 5$ and $5 \nmid 3$

If neither aRb nor bRa then a and b are called incomparable elements

Def:- If (S, R) is a poset and every two elements of the set are comparable to each other. then S is called Linearly ordered set or totally ordered set.

\# $S = \{2, 4, 8, 16, 32\}, \ |$ → divisibility

$2|4$ $\quad 4|8$ $\quad 8|16$

$2|8$ $\quad 4|16$ $\quad 8|32$

$2|16$ $\quad 4|32$ $\quad 16|32$

$2|32$

$2R4R8R16R32$

$2|4|8|16|32$

(2) $(Z, \leq)$ is a poset.

$\ldots\ldots -3 \leq -2 \leq -1 \leq 0 \leq 1 \leq 2 \leq 3 \leq 4$

A totally ordered set is also called as chain

(3) $S = \{6, 16, 8, 2, 12\}, \ |$

③ $S = \{6, 16, 8, 2, 12\}, \mid$

$6 \nmid 8$ and $8 \nmid 6$ $\qquad$ $8 \nmid 12$ and $12 \nmid 8$

6 and 8 → incomparable elements

8 and 12, 12 and 16

Not a Linearly ordered set.

\# Which of these pairs of elements are comparable in the poset $(Z^+, \mid)$

ⓐ 5, 15 $\qquad$ ⓑ 6, 9 $\qquad$ ⓒ 8, 16 $\qquad$ ⓓ 7, 7

$5 \mid 15$ $\qquad$ $6 \nmid 9$ and $9 \nmid 6$ $\qquad$ $8 \mid 16$ $\qquad$ $7 \mid 7$

Hasse Diagram

$A = \{1, 2, 3, 4\}, \leq$

$1 \leq 1$ $\qquad$ $2 \leq 3$ $\qquad$ $3 \leq 3$ $\qquad$ ~~$4 \leq 4$~~

$1 \leq 2$ $\qquad$ $2 \leq 4$ $\qquad$ ~~$3 \leq 4$~~

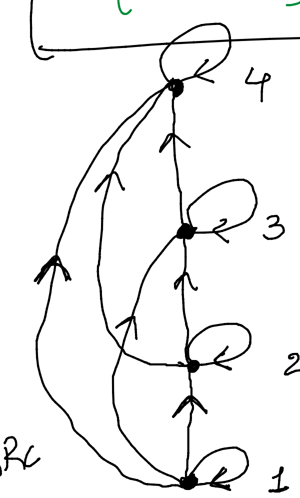
$1 \leq 3$ $\qquad$ $2 \leq 2$

$1 \leq 4$

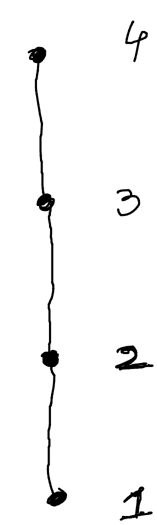
① Reflexive

② antisymmetric
$aRb \not\Rightarrow bRa$

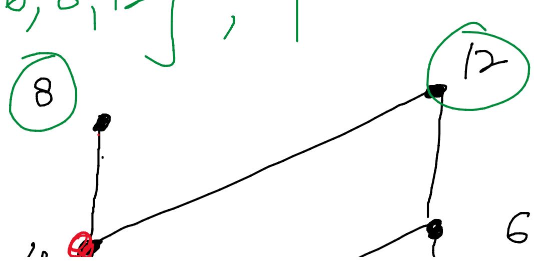
③ Transitive $aRb, bRc \rightarrow aRc$



Hasse Diagram



$A = \{1, 2, 3, 4, 6, 8, 12\}, \mid$



$1|2 \quad 2|4$

$\Rightarrow 1|4 \quad \underline{\underline{4|12}}$

$\underline{\underline{1|12}}$