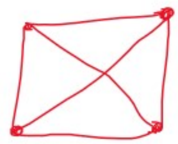
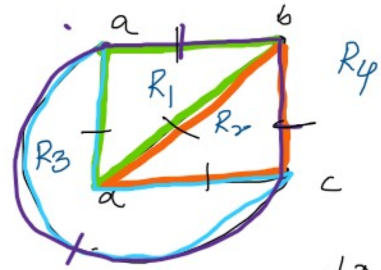
L30 18 April 2022

18 April 2022 10:07

## Planar graph :-



$K_4$




planar representation of $K_4$

**Map :-** A planar representation of the graph G.

Map divides the whole plane into different regions

**Degree of the region :-** It is denoted by $\deg(r)$

$\deg(r)$ = the length of the cycle which borders 'r'.

↳ (the no. of edges involve in the boundary)

$\deg(R_1) = 3$

$\deg(R_2) = 3$

$\deg(R_3) = 3$

$\deg(R_4) = 3$

$\overline{\phantom{xx}12\phantom{xx}}$

$$\text{sum} = 2e$$

$$12 = 2e$$

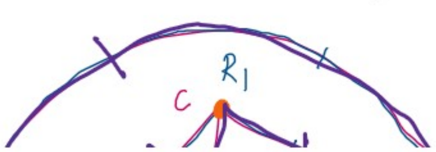
$$\boxed{e = 6}$$

**Handshaking Theorem :-**

The sum of degree of all the regions in a planar graph is equal to twice the no. of edges.

$$\boxed{\sum \deg(r) = 2e}$$

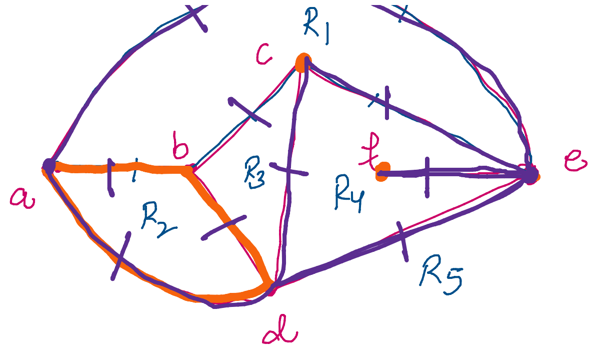

#




$\deg(R_1) = 4$

$\deg(R_2) = 3$

#



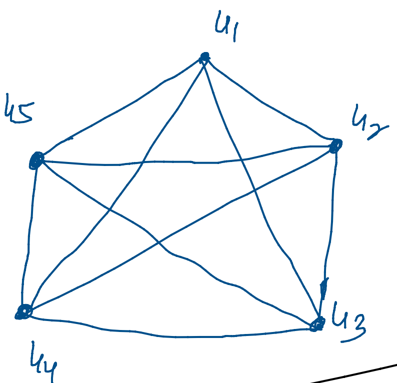

$\deg(R_1) = 4$
$\deg(R_2) = 3$
$\deg(R_3) = 3$
$\deg(R_4) = 5$ ✓
$\deg(R_5) = 3$

$\sum \deg(r) = \underline{\underline{18}}$

b c

a-b-c-a
=
b-c-a-b

$\sum \deg(r) = 2e$
$18 = 2e$
$\boxed{e = 9}$




Euler formula :-

Let G be a connected planar graph with v vertices and e edges, the planar representation divides the whole plane into 'r' regions. Then,

$$\boxed{v - e + r = 2}$$

In $K_4$
$v = 4$
$e = 6$
$r = 4$

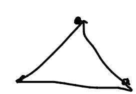
$4 - 6 + 4 = 8 - 6 = 2$

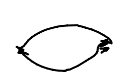
∴ it holds good for $K_4$

#

Let G be a connected planar graph with v vertices, $v \geq 3$ and e edges then.
$e \leq 3v - 6$.

Proof :- As G is a connected planar graph.
$v - e + r = 2$ ✓





As $v \geq 3$, $\deg(r) \geq 3$.

$\sum \deg(r) = 2e$

As $v \geq 3$, $\deg(s) \geq 3$.

Handshaking

$$\sum \deg(s) = 2e$$

$$2e = \sum \deg(s) \geq 3s$$

$$2e \geq 3s$$

$$\boxed{s \leq \frac{2e}{3}}$$

Euler formula

$$v - e + s = 2$$

$$s = 2 - v + e \leq \frac{2e}{3}$$

$$6 - 3v + 3e \leq 2e$$

$$3e - 2e \leq 3v - 6$$

$$\boxed{e \leq 3v - 6}$$

In $K_5$

$$v = 5$$

$$e = \frac{5(5-1)}{2} = 10$$

If $K_5$ is planar $v \geq 3$,

$$e \leq 3v - 6$$

$$10 \leq 3(5) - 6$$

$$10 \leq 15 - 6$$

$$10 \leq 9$$ X Not possible

$\Rightarrow$ $K_5$ is Non planar graph

In $K_{3,3}$

$$v = 6$$

$$e = 9$$

If $K_{3,3}$ is planar $v \geq 3$ then

$$e \leq 3v - 6$$

$$9 \leq 3(6) - 6$$

$$9 \leq 18 - 6$$

$$9 \leq 12$$

further Investigation