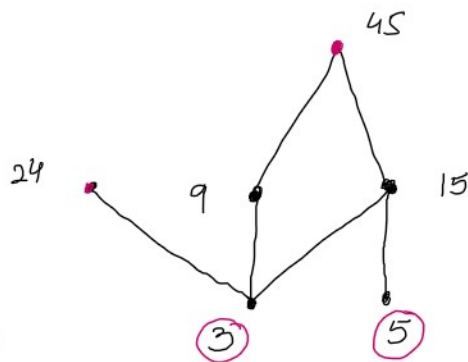


Answer these questions for the poset  $(\{3, 5, 9, 15, 24, 45\}, |)$ .

- a) Find the maximal elements.  $\rightarrow 24, 45$
- b) Find the minimal elements.  $\rightarrow 3, 5$
- c) Is there a greatest element?  $No$
- d) Is there a least element?  $\rightarrow No$
- e) Find all upper bounds of  $\{3, 5\}$ .  $\rightarrow 15, 45$
- f) Find the least upper bound of  $\{3, 5\}$ , if it exists.  $\rightarrow 15$
- g) Find all lower bounds of  $\{15, 45\}$ .  $\rightarrow 15, 5, 3$
- h) Find the greatest lower bound of  $\{15, 45\}$ , if it exists.  $\rightarrow 15$



$$\begin{array}{r} 15 \mid 15 \\ 15 \mid 45 \end{array}$$

Least upper bound and greatest lower bound  $\rightarrow$   
 $(l.u.b.)$   $(g.l.b.)$

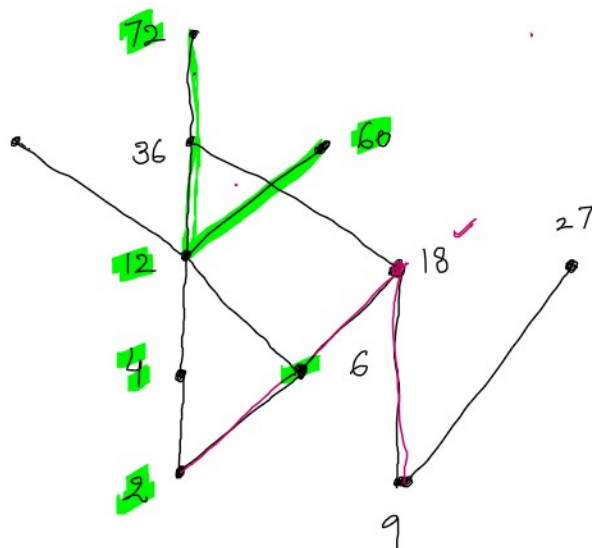
Let  $A$  be the subset of the poset  $(S, R)$

$x$  is the least upper bound of  $A$  if  $a R x$  whenever  $a \in A$  and  $x R z$  whenever  $z$  is an upper bound of  $A$ .

$y$  is the greatest lower bound of  $A$  if  $y R a$  whenever  $a \in A$  and  $z R y$  whenever  $z$  is an upper bound of  $A$ .

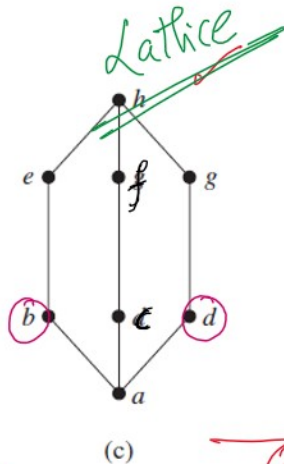
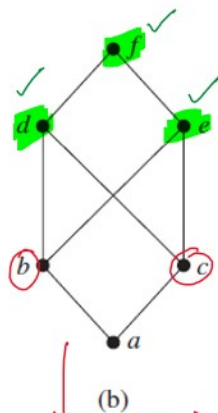
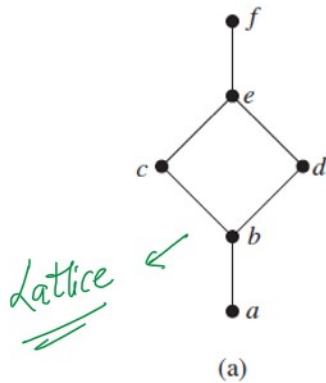
Answer these questions for the poset  $(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |)$ .

- a) Find the maximal elements.  $72, 48, 60, 27$
- b) Find the minimal elements.  $2, 9$
- c) Is there a greatest element?  $No$
- d) Is there a least element?  $No$
- e) Find all upper bounds of  $\{2, 9\}$ .  $18, 36, 72$
- f) Find the least upper bound of  $\{2, 9\}$ , if it exists.  $18$
- g) Find all lower bounds of  $\{60, 72\}$ .  $2, 4, 6, 12$
- h) Find the greatest lower bound of  $\{60, 72\}$ , if it exists.  $\rightarrow 12$



Lattice  $\Rightarrow$

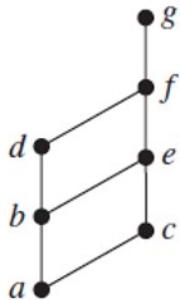
A partially ordered set in which every pair of elements has both a least upper bound and the greatest lower bound is called a Lattice.



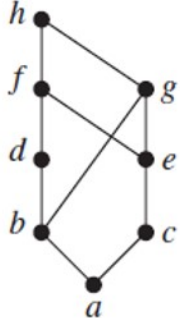
⑥  
upper bound of  
b and c  
 $= \{d, e, f\}$   
least upper bound  
 $= \text{None}$

⑦

a)



b)



c)

