T8 G1 11 Feb 2022

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Asymmetric Relation :-

A Relation R is called asymmetric if $(a,b) \in R$ implies that $(b,a) \notin R$

$A = \{1,2,3,4\}$ $R = \{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$

① Not Reflexive $\because (1,1) \notin R$

② Not Symmetric $(2,4) \in R$ but $(4,2) \notin R$

③ Not antisymmetric $\because (2,3) \in R$ and $(3,2) \in R$

④ Transitive Relation :-

A Relation R on a set A is called transitive if whenever $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ for all $a,b,c \in A$

aRb, bRc → aRc

$aRb, bRc \Rightarrow aRc$

$(a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R$

Consider the following relations on {1, 2, 3, 4}: $(1,1)$ $(3,1) \notin R_1$

$R_1 = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 1), (4, 4)\}$, → Not Transitive.

$R_2 = \{(1, 1), (1, 2), (2, 1)\}$, → $(2,1)\ (1,2) \Rightarrow (2,2) \notin R_2$ → NOT transitive.

$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$,

$R_4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$,

$R_5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$,

$R_6 = \{(3, 4)\}$. → Transitive

$(1,2)\ (2,1) \Rightarrow (1,1)$

R_2

$(1,1)\ (1,2) \Rightarrow (1,2)$ ✓

$(1,2)\ (2,1) \Rightarrow (1,1)$ ✓

$(2,1)\ (1,1) \Rightarrow (2,1)$ ✓

$(\ )\ (\ ) \Rightarrow$ [illegible] $\notin R_2$

$(2,1)\ (1,1) \Rightarrow (2,1)$

$(2,1)\ (1,2) \Rightarrow (2,2) \notin R_2$

$R_3 = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$,

$(1,1)(1,2) \Rightarrow (1,2)$

$(1,1)(1,4) \Rightarrow (1,4)$ ✓

$(1,2)(2,1) \Rightarrow (1,1)$ ✓

$(1,2)(2,2) \Rightarrow (1,2)$ ✓

$(1,4)(4,1) \Rightarrow (1,1)$ ✓

$(1,4)(4,4) \Rightarrow (1,4)$ ✓

$(2,1)(1,1) \Rightarrow (2,1)$

$(2,1)(1,2) \Rightarrow (2,2)$

$(2,1)(1,4) \Rightarrow (2,4) \notin R_3$

R_3 is not Transitive

$R_5 = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (4, 4)\}$, Transitive

$(1,1)\cancel{(1,2)} = (1,2)$ ✓ $\quad (1,2)(2,2) \Rightarrow (1,2)$ ✓ $\quad (1,3)(3,3) = (1,3) \quad (1,4)(4,4) \Rightarrow (1,4)$

$(1,1)(1,3) \Rightarrow (1,3)$ ✓ $\quad (1,2)(2,3) \Rightarrow (1,3)$ ✓ $\quad (1,3)(3,4) \Rightarrow (1,4)$

$(1,1)(1,4) \Rightarrow (1,4)$ ✓ $\quad (1,2)(2,4) \Rightarrow (1,4)$ ✓

$(2,2)(2,3) \Rightarrow (2,3) \quad (2,3)(3,3) \Rightarrow (2,3) \quad (2,4)(4,4) \Rightarrow (2,4)$

$(2,2)(2,4) = (2,4) \quad (2,3)(3,4) \Rightarrow (2,4) \quad (3,3)(3,4) \Rightarrow (3,4)$

$(3,4)(4,4) \Rightarrow (3,4)$

$R_4 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$,

$\downarrow$

Transitive

$2|4 \qquad a|b \Rightarrow b \nmid a$

$2|4 \quad 4|16 \qquad a|b$ and $b|c$

$\Rightarrow a|c$

A = the set of positive integers

$R = \{(a,b) \mid a \text{ divides } b\}$

$R = \{(a,b) \mid a \text{ divides } b\}$ → $a|c$

Reflexive, Not symmetric, antisymmetric, Transitive

No. of Reflexive Relations on a set A with 3 elements ?

$A = S$

$R_1 = \{(1,2)\ (2,3)\}$ antisymmetric

$R_2 = \{(1,2), (2,1), (2,3)\}$ → Not antisymmetric