L6 1 Feb 2022

01 February 2022 11:06

Predicates & Quantifiers :→

we know that

"Every computer connected to the university network is functioning properly." → ?

"MATH3 is functioning properly." → ?

where MATH3 is one of the computers connected to the university network.

" x is greater than 3 "

the variable which is the subject of the statement

predicate is the property that subject can have.

$P(x) :\rightarrow$ x is greater than 3

propositional function P at x

once a value is assigned to x then P(x) becomes a proposition.

$P(2) :\rightarrow$ 2 is greater than 3.

False

. Let $P(x)$ denote the statement "$x \leq 4$." What are these truth values?

a) $P(0)$ **b)** $P(4)$ **c)** $P(6)$

a) $P(0)$: $0 \leq 4$ True

b) $4 \leq 4$ True

c) $6 \leq 4$ False

Consider the statement " "

Consider the statement

$Q(x,y)$ denotes the statement "$x = y+3$"

$Q(1,4) \Rightarrow 1 = 4+3$

$\Rightarrow 1 = 7$ False

(x, y)

$Q(3,0) \Rightarrow 3 = 0+3$

$\Rightarrow 3 = 3$ True

Similarly, we can let $R(x, y, z)$ denote the statement "$x + y = z$."

What are the truth values of the propositions $R(1, 2, 3)$ and $R(0, 0, 1)$?

$R(1,2,3)$:- $1+2=3$

$\Rightarrow 3 = 3$ True

$R(0,0,1)$:- $0+0=1$

$\Rightarrow 0 = 1$ False

Let $P(x)$ be the statement "the word x contains the letter a." What are these truth values?

a) $P(\text{orange})$ True b) $P(\text{lemon})$ – False

c) $P(\text{true})$ → False d) $P(\text{false})$ → True

{is greater than 3}

Quantifiers :→

The *universal quantification* of $P(x)$ is the statement

"$P(x)$ for all values of x in the domain."

The notation $\forall x P(x)$ denotes the universal quantification of $P(x)$. Here $\forall$ is called the **universal quantifier.** We read $\forall x P(x)$ as "for all $x P(x)$" or "for every $x P(x)$." An element for which $P(x)$ is false is called a **counterexample** of $\forall x P(x)$.

$\forall x P(x)$ as for all x $P(x)$

Let $P(x)$ be the statement "$x + 1 > x$." What is the truth value of the quantification $\forall x P(x)$, where the domain consists of all real numbers?

$P(x)$ "$x+1 > x$"

$\forall x P(x) \rightarrow$ True

$P(4)$ $4+1>4$

$5>4$

True

Let $Q(x)$ be the statement "$x < 2$." What is the truth value of the quantification $\forall x Q(x)$, where the domain consists of all real numbers?

$\forall x Q(x)$ $\forall x, x < 2$

$\downarrow$ False

$\boxed{Q(3) \text{ is false}}$

statement	True	False
$\forall x P(x)$	$P(x)$ is true for every x	There is one x for which $P(x)$ is false.

Statement	True	False
$\forall x\, P(x)$	$P(x)$ is true for every x	There is one x for which $P(x)$ is false.