L16 02 Mar 2022

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Combination :→

n (objects)

↓

r (at a times)

$$nC_r = C(n,r) = \frac{n!}{r!(n-r)!}$$

$$nC_r = \frac{1}{r!}\, nP_r \Rightarrow \boxed{nP_r = r!\, nC_r}$$

$$nC_n = \frac{n!}{n!\,(n-n)!} = \frac{n!}{n!\,0!} = 1$$

$$nC_0 = \frac{n!}{0!\,(n-0)!} = \frac{n!}{1\cdot n!} = 1$$

$$nC_{n-r} = \frac{n!}{(n-r)!\,(n-(n-r))!} = \frac{n!}{(n-r)!\,r!} = nC_r$$

$$\boxed{nC_r = nC_{n-r}}$$

nC_0 nC_1 nC_2 nC_3 --- nC_{n-1} nC_n

$15C_7 = 15C_8$

Example 18 A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

2 Men 3 Women = 5 persons = n

(a) $5C_3 = \frac{5!}{3!\,2!} = \frac{5\times4\times3!}{3!\times2} = 10$

↓ $3 = r$

(b) 5 → 3 ; 2 Men → 1 Men ; 3 Women → 2 Women

3 ↓ — 1 Men ↓ — 2 Women ↓

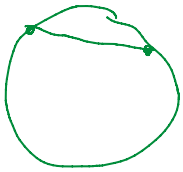
$2C_1 \times 3C_2 = ?$

How many chords can be drawn through 21 points on a circle?

$n = 21$

$r = 2$

$21C_2 = \frac{21!}{2!(21-2)!} = ?$



In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

17 players → 12 + 5 Bowlers

↓ 11 players

↓ 7 players + 4 Bowlers

$= {}^{12}C_7 \times {}^{5}C_4 = \frac{12!}{7!\,5!} \times \frac{5!}{4!\,1!}$

$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7!}{7! \times 4 \times 3 \times 2 \times 1} =$

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In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

9 Course = 7 Course + 2 Compulsory

↓ 3 Course ↓ 2

$7C_3 = \frac{7!}{3!\,4!} = ?$

Example 21 A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girl ? (ii) at least one boy and one girl ? (iii) at least 3 girls ?

4 girls and 7 boys = 11 persons

i) No Girl

$${}^7C_5 = \frac{7!}{5!\,2!}$$

ii) at least one boy and one girl

1B 4G + 2B 3G + 3B 2G + 4B 1G

${}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

iii) at least 3 girls

3G 2B + 4G 1B

${}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$ =

Example 22 (a) Find the number of words with or without meaning which can be made using all the letters of the word AGAIN. If (b) these words are written as in a dictionary, what will be the 50th word?

(a) AGAIN

$$\frac{5!}{2!} = \frac{5\times4\times3\times2!}{2!} = 60$$

(b) A XXXX — GIAN

$4! = 24$

G XXXX — AAIN

$$\frac{4!}{2!} = \frac{24}{2} = 12$$

I XXXX — AAGN

$$\frac{4!}{2} = \frac{24}{2} = 12$$

$24+12+12 = 48$

50th word

NAAIG

49th NAAGI

50th word NAAIG

Example 24 In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

$5! \times {}^{6}P_{3}$