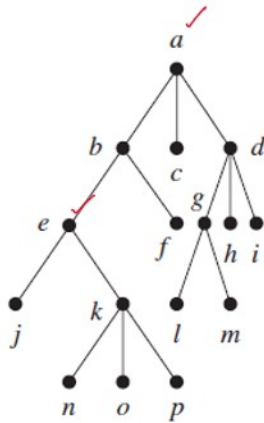


Ordered rooted tree:- is a rooted tree in which childrens of all the internal vertices are in some order.

## Traversal Algorithms

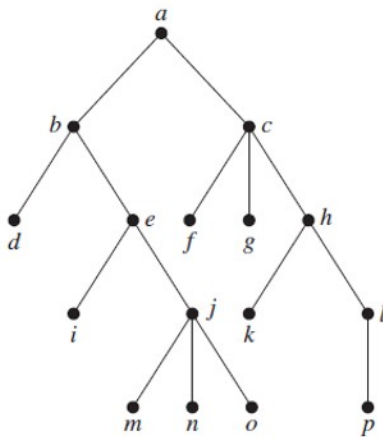
Procedures for systematically visiting every vertex of an ordered rooted tree are called **traversal algorithms**. We will describe three of the most commonly used such algorithms, preorder traversal, inorder traversal, and postorder traversal. Each of these algorithms can be defined recursively. We first define preorder traversal.



a b e j k n o p f c d g l  
m h i

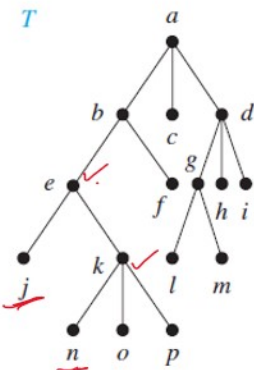
Preorder traversal: Visit root,  
visit subtrees left to right

Preorder



a b d e i j m n o c f g h k l p

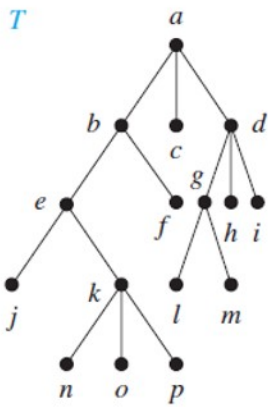
T



j e n k o p b f a c l g m d h i

Inorder traversal: Visit leftmost  
subtree, visit root, visit other  
subtrees left to right

T



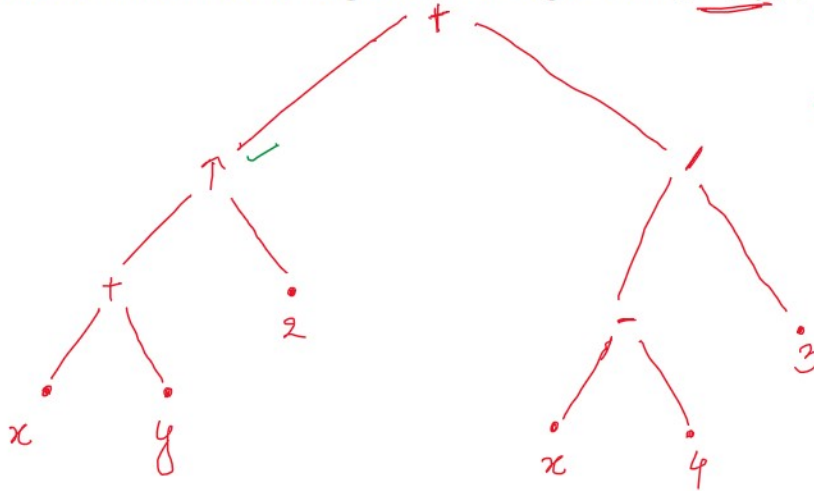
j n o p k e f b c l m g h i d a

Postorder traversal: Visit subtrees left to right, visit root

## Infix, Prefix, and Postfix Notation

We can represent complicated expressions, such as compound propositions, combinations of sets, and arithmetic expressions using ordered rooted trees. For instance, consider the representation of an arithmetic expression involving the operators  $+$  (addition),  $-$  (subtraction),  $*$  (multiplication),  $/$  (division), and  $\uparrow$  (exponentiation). We will use parentheses to indicate the order of the operations. An ordered rooted tree can be used to represent such expressions, where the internal vertices represent operations, and the leaves represent the variables or numbers. Each operation operates on its left and right subtrees (in that order).

What is the ordered rooted tree that represents the expression  $((x + y) \uparrow 2) + ((x - 4)/3)$ ?



$+ \uparrow + x y 2 / - x 4 3$

What is the value of the prefix expression  $+ - * 2 3 5 / \uparrow 2 3 4$ ?

$+ - * 2 3 5 / \uparrow 2 3 4$

$2 \uparrow 3$   
 $2^3 = 8$

$+ - * 5 2 5 / 8 11$

$$\begin{array}{rcl}
 + & - & * \ 2 \ 3 \ 5 \ / \ 8 \ 4 \quad \leftarrow \\
 & & \quad \quad \quad 2^2 = 8 \\
 & & \quad \quad \quad 8/4 = 2 \\
 + & - & * \ 2 \ 3 \ 5 \ 2 \quad \leftarrow \\
 & & \quad \quad \quad 2 * 3 = 6 \\
 + & - & 6 \ 5 \ 2 \quad \leftarrow \\
 & & \quad \quad \quad 6 - 5 = 1 \\
 + & 1 \ 2 \quad \leftarrow \\
 & \quad \quad 1 + 2 = \underline{\underline{3}}
 \end{array}$$

3 ✓

What is the value of the postfix expression  $7 \ 2 \ 3 \ * \ - \ 4 \ \uparrow \ 9 \ 3 \ / \ +$ ?

$$\begin{array}{rcl}
 & & \quad \quad \quad 2 * 3 = 6 \\
 7 \ 6 \ - \ 4 \ \uparrow \ 9 \ 3 \ / \ + \\
 \quad \quad \quad 7 - 6 = 1 \\
 1 \ 4 \ \uparrow \ 9 \ 3 \ / \ + \\
 \quad \quad \quad 1^4 = 1 \\
 \quad \quad \quad \quad \quad \quad 9 \ 3 \ / \ + \\
 \quad \quad \quad \quad \quad \quad 9 \ 3 = 3 \\
 1 \ 3 \ + \\
 \quad \quad \quad 1 + 3 = \underline{\underline{4}}
 \end{array}$$

$$1+3=4$$

What is the value of each of these prefix expressions?

a)  $- * 2 / 8 4 3$

b)  $\uparrow - * 3 3 * 4 2 5$

~~c)  $+ - \uparrow 3 2 \uparrow 2 3 / 6 - 4 2$~~

d)  $* + 3 + 3 \uparrow 3 + 3 3 3$

$+ - \uparrow 3 2 \uparrow 2 3 / 6 - 4 2$

$+ - \uparrow 3 2 \uparrow 2 3 / 6 2$   
 $4-2=2$   
 $6/2=3$

$+ - \uparrow 3 2 \uparrow 2 3 3$   
 $2^3=8$

$+ - \uparrow 3 2 8 3$   
 $3^2=9$

$+ - 9 8 3$   
 $9-8=1$

$+ 1 3$   
 $1+3=4$

What is the value of each of these postfix expressions?

a)  $5 2 1 - - 3 1 4 + + *$

b)  $9 3 / 5 + 7 2 - *$

~~c)  $3 2 * 2 \uparrow 5 3 - 8 4 / * -$~~

$6 2 \uparrow 5 3 - 8 4 / * -$

$36 5 3 - 8 4 / * -$

$36 2 8 4 / * -$

$$36 \quad (2 \quad 2 \quad *) \quad -$$

$$36 \quad 4 \quad - \quad = \quad 36 - 4 = \underline{32}$$