

Bounded Lattice :-

A lattice  $L$  is p.t.b bounded if  $L$  has a least element and a greatest element.

① e.g. Non Negative integers,  $<$

$$(\mathbb{Z}^+, <)$$

$$\underbrace{0 < 1 < 2 < 3 < 4 < 5 < 6 < 7 \dots}_{\text{Least elem.}}$$

②

$$A = \{a, b\}$$

$$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

$$\emptyset \subseteq \emptyset$$

$$\emptyset \subseteq \{a\}$$

$$\emptyset \subseteq \{b\}$$

$$\emptyset \subseteq \{a, b\}$$

Bounded Lattice

$$\emptyset \subseteq \{a, b\}$$

$$\emptyset \rightarrow \text{least elem}$$

$$\{a\} \subseteq \{a, b\}$$

$$\{a, b\} \rightarrow \text{Greatest elem.}$$

$$\{b\} \subseteq \{a, b\}$$

$$\{a\} \subseteq \{a, b\}$$

Distributive Lattice :-

$$(a, b) \quad \begin{matrix} \text{join} \\ a \vee b \end{matrix}$$

Least upper bound of  
a and b

meet .

$$a \wedge b$$

Greatest lower bound of  
a and b

A lattice is p.t.b distributive if it satisfies distributive laws

$$\text{i.e. } ① x \wedge (y \vee z) = \underline{(x \wedge y) \vee (x \wedge z)}$$

$$② x \vee (y \wedge z) = \underline{(x \vee y) \wedge (x \vee z)}$$

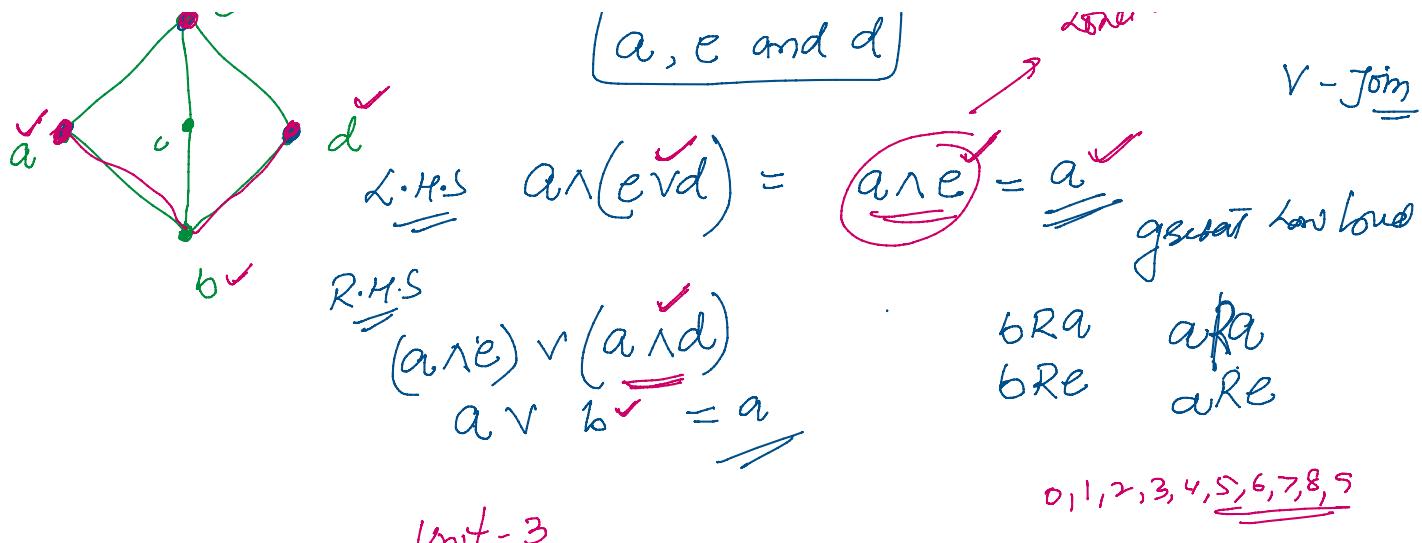
$$\begin{array}{l} e \wedge x \\ e \wedge y \\ \hline e \wedge z \end{array}$$



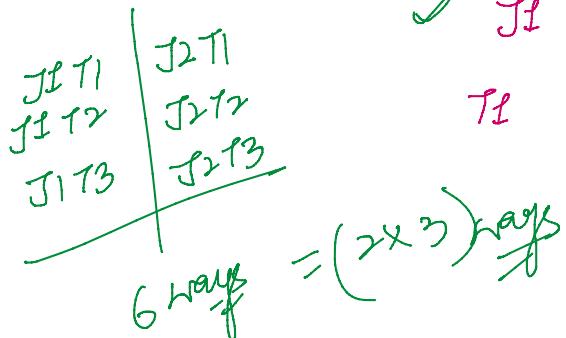
$$a, b \text{ and } c$$

$$\text{Join} = a \vee b$$

$$V-Join$$



Fundamental principle of counting :-



"If an event can occur in  $m$  different ways, following which another event can occur in  $n$  different ways, then the total number of occurrence of the events in the given order is  $m \times n$ ."

How many 3-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 if the digits can be repeated?

1, 2, 3, 4, 5, 6

$$6 \times 6 \times 3 = \underline{\underline{108}}$$

How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming that

- (i) repetition of the digits is allowed?
- (ii) repetition of the digits is not allowed?

$$(i) \quad \begin{array}{c} X \ X \ X \\ \downarrow \quad \downarrow \quad \downarrow \\ 5 \times 5 \times 5 = \underline{\underline{125}} \end{array}$$

$$(ii) \quad \begin{array}{c} X \ X \ X \\ \downarrow \quad \downarrow \quad | \\ 5 \times 4 \times 3 = \underline{\underline{60}} \end{array}$$

A coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

$$\begin{array}{c} X \ X \ X \\ \downarrow \quad \downarrow \quad \downarrow \\ T \quad H \quad T \quad H \quad T \quad H \\ | \quad | \quad | \quad | \quad | \quad | \\ 2 \times 2 \times 2 = \underline{\underline{8}} \end{array}$$

**Example 4** Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.

flags →	2	3	4	5
	$\begin{array}{c} X \ X \\ \downarrow \quad \downarrow \\ 5 \times 4 \end{array}$	$\begin{array}{c} X \ X \ X \\ \downarrow \quad \downarrow \quad \downarrow \\ 5 \times 4 \times 3 \end{array}$	$\begin{array}{c} X \ X \ X \ X \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 5 \times 4 \times 3 \times 2 \end{array}$	$\begin{array}{c} X \ X \ X \ X \ X \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 5 \times 4 \times 3 \times 2 \times 1 \end{array}$

$$20 + 60 + 120 + 120$$

$$= \underline{\underline{320}}$$