Algebraic Identities

(1) $(a+b)^2 = a^2+b^2+2ab$

(2) $(a-b)^2 = a^2+b^2-2ab$

(3) $(a^2-b^2) = (a+b)(a-b)$

(4) $a^2+\frac{1}{a^2} = \left(a+\frac{1}{a}\right)^2-2 = \left(a-\frac{1}{a}\right)^2+2$

(5) $a^2+b^2+c^2 = (a+b+c)^2-2(ab+bc+ca)$

or, $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$

(6) $(a+b)^3 = a^3+b^3+3ab(a+b)$

(7) $(a-b)^3 = a^3-b^3-3ab(a-b)$

(8) $a^3-b^3 = (a-b)(a^2+b^2+ab)$

(9) $a^3+b^3 = (a+b)(a^2+b^2-ab)$

(10) $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

if $a+b+c=0$ then, $a^3+b^3+c^3 = 3abc$

(11) $a^2+b^2+c^2-ab-bc-ca = \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$

(12) $a^3+\frac{1}{a^3} = \left(a+\frac{1}{a}\right)^3-3\left(a+\frac{1}{a}\right)$

(13) $a^3-\frac{1}{a^3} = \left(a-\frac{1}{a}\right)^3+3\left(a-\frac{1}{a}\right)$

(14) if $x+\frac{1}{x} = K$ then $\left(x^2+\frac{1}{x^2}\right) = K^2-2$

(15) if $x+\frac{1}{x} = K$ then $\left(x^3+\frac{1}{x^3}\right) = K^3-3K$

(16) if $x-\frac{1}{x} = K$ then $\left(x^2+\frac{1}{x^2}\right) = K^2+2$

(17) if $x-\frac{1}{x} = K$ then $\left(x^3-\frac{1}{x^3}\right) = K^3+3K$

(18) if $x+\frac{1}{x} = K$ then $\left(x^5+\frac{1}{x^5}\right) = \left(x^2+\frac{1}{x^2}\right)\left(x^3+\frac{1}{x^3}\right)-\left(x+\frac{1}{x}\right)$

$= \boxed{(K^2-2)(K^3-3K)-K}$

(19) if $\left(x+\frac{1}{x}\right)=\sqrt{3}$ then (i) $x^3+\frac{1}{x^3}=0$

(ii) $x^6+1=0$

(iii) $\boxed{x^6=-1}$

(20) if $x^2+\frac{1}{x^2}=K$

(i) $x+\frac{1}{x}=\sqrt{K+2}$

(ii) $x-\frac{1}{x}=\sqrt{K-2}$

(21) $(a+b)^2-(a-b)^2=4ab$

(22) $(a+b)^2+(a-b)^2=2(a^2+b^2)$

(23) if $x+\frac{1}{x}=2$ then $\boxed{x=1}$ (always)

(24) if $x+\frac{1}{x}=-2$ then $\boxed{x=-1}$ (always)

Solutions of sheet [Algebra]

Q.1: If $x=-3$, $y=-2$ and $z=5$, then the value of $x^3+y^3+z^3$ is equal to

✓(a) 90 (b) 80 (c) 70 (d) 100

ans: here $x+y+z = -3-2+5 = 0$

We know, $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

if $a+b+c=0$ then, $a^3+b^3+c^3 = 3abc$

So, $x^3+y^3+z^3 = 3xyz = 3\times(-3)\times(-2)\times(5) = \boxed{90}$

we can also substitute values directly.

Q.2: If $(x-1)^3+x^3+(x+1)^3 = 3x(x^2-1)$, then value of x is

✓(a) 0 (b) 1 (c) 2 (d) 4

ans: Note: Solve this question by putting options. $\boxed{x=0}$

$\boxed{\text{OR}}$ expand. $[x^3-1-3x(x-1)]+x^3+[x^3+1+3x(x+1)] = 3x^3-3x$

$x^3-\cancel{1}-\cancel{3x^2}+3x+x^3+x^3+\cancel{1}+\cancel{3x^2}+3x = 3x^3-3x$

$3x^3+6x = 3x^3-3x \Rightarrow 9x=0 \Rightarrow \boxed{x=0}$

Q.3: If $x=997$, $y=998$, $z=999$, then the value of $(x^2+y^2+z^2-xy-yz-zx)$ will be

✓(a) 3 (b) 9 (c) 16 (d) 4

ans: We know, $(x^2+y^2+z^2-xy-yz-zx) = \frac{1}{2}[(x-y)^2+(y-z)^2+(z-x)^2]$

$= \frac{1}{2}[(1)^2+(1)^2+(2)^2] = \frac{6}{2} = 3$

Alternate method: Clearly, $y=x+1$, $z=x+2$ put in expression & expand.

Q.4: If $a+b+c=8$, the the value of $[(a-4)^3+(b-3)^3+(c-1)^3-3(a-4)(b-3)(c-1)]$ is

(a) 2 (b) 4 (c) 1 ✓(d) 0

ans: Using, $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

here, $A=a-4$, $B=b-3$, $C=c-1$

so, $(a-4+b-3+c-1)[(a-4)^2+(b-3)^2+(c-1)^2-(a-4)(b-3)-(b-3)(c-1)-(c-1)(a-4)]$

$= (\underbrace{a+b+c}-8)[(a-4)^2+(b-3)^2+(c-1)^2-(a-4)(b-3)-(b-3)(c-1)-(c-1)(a-4)]$

$= (8-8)$ so complete product $= 0$

Q.5: If $x = \sqrt{a} + \frac{1}{\sqrt{a}}$, $y = \sqrt{a} - \frac{1}{\sqrt{a}}$ then value of $x^4 + y^4 - 2x^2y^2$ is :

✓(a) 16 (b) 20 (c) 10 (d) 5

ans: $x^4 + y^4 - 2x^2y^2 = (x^2)^2 + (y^2)^2 - 2x^2y^2 = (x^2 - y^2)^2$

$$(x^2-y^2)^2 = [(x+y)(x-y)]^2 = \left[\left(\sqrt{a}+\cancel{\tfrac{1}{\sqrt{a}}}+\sqrt{a}-\cancel{\tfrac{1}{\sqrt{a}}}\right)\left(\cancel{\sqrt{a}}+\tfrac{1}{\sqrt{a}}-\cancel{\sqrt{a}}+\tfrac{1}{\sqrt{a}}\right)\right]^2$$

$$= \left[2\cancel{\sqrt{a}} \times \frac{2}{\cancel{\sqrt{a}}}\right]^2 = \boxed{16}$$

Q.6: If $a+b+c=0$, then value of $\left[\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right]$ is

(a) 2 ✓(b) 3 (c) 4 (d) 5

ans: We know, $a^3 + b^3 + c^3 = (a+b+c)(a^2+b^2+c^2-ab-bc-ca) - 3abc$

when, $a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

$\left[\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}\right]$ taking Lcm, $\frac{(a^3+b^3+c^3)}{abc} = \frac{3a\cancel{bc}}{a\cancel{bc}} = \boxed{3}$

Q.7: If a, b, c are real and $a^3+b^3+c^3 = 3abc$ and $a+b+c \neq 0$, then the relation between a, b, c will be

(a) $a+b=c$ (b) $a+c=b$ ✓(c) $a=b=c$ (d) $b+c=a$

ans: $a^3+b^3+c^3 = 3abc$ or $a^3+b^3+c^3-3abc = 0$

Using identity, $(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = 0$

as, $a+b+c \neq 0$, so, $(a^2+b^2+c^2-ab-bc-ca) = 0$

multiply by 2 both sides, $(2a^2+2b^2+2c^2-2ab-2bc-2ca) = 0$

$a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ca = 0$

$(a-b)^2 + (b-c)^2 + (c-a)^2 = 0$ {we know $(a-b)^2, (b-c)^2, (c-a)^2 > 0$}

so, $(a-b)=0$, $(b-c)=0$, $c-a=0$

$\boxed{a=b}$, $\boxed{b=c}$, $\boxed{c=a}$ $\Rightarrow$ $\boxed{a=b=c}$

Note: We can also go by hit and trial.

Q.8: Find the value of $(a^3+b^3+1-3ab)$ if $a+b+1=0$

(a) 3 ✓(b) 0 (c) -1 (d) 1

ans: $\left[\begin{array}{l} A^3+B^3+C^3 \\ -3ABC \end{array}\right] = (A+B+C)(A^2+B^2+C^2-AB-BC-CA)$

if $A+B+C=0$ then, $A^3+B^3+C^3-3ABC=0$

here $A=a$, $B=b$, $C=1$ so, $(a^3+b^3+1-3ab) = \boxed{0}$

Q.9: If $x+y+z=19$, $xyz=126$, $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{5}{7}$ and $x>0$, $y>0$, $z>0$, then the value of $x^2+y^2+z^2$ is

(a) 161 (b) 171 (c) 181 (d) 191

ans: $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{5}{7}$, $\frac{yz+xz+xy}{xyz}=\frac{5}{7}$

or, $(xy+yz+zx)=\frac{5}{7}\times(xyz)=\frac{5}{7}\times(126)=90$

$(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$

$(19)^2=x^2+y^2+z^2+(2\times 90) \Rightarrow x^2+y^2+z^2=361-180=\boxed{181}$

Q.10: If $(a-b)=4$, $(b-c)=-5$ and $(c-a)=1$, then value of $\frac{a^3+b^3+c^3-3abc}{a+b+c}$

(a) 21 (b) 20.5 (c) 42 (d) 15.5

ans: $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$$\frac{a^3+b^3+c^3-3abc}{(a+b+c)}=(a^2+b^2+c^2-ab-bc-ca)$$

$=\frac{1}{2}[2a^2+2b^2+2c^2-2ab-2bc-2ca]$

$=\frac{1}{2}[(a-b)^2+(b-c)^2+(c-a)^2]$

$=\frac{1}{2}[(4)^2+(-5)^2+(1)^2]=\frac{1}{2}\times 42=\boxed{21}$

Q.11: If $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is

(a) 1 (b) 2 (c) 3 (d) 4

ans: Given, $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$ {adding 1 to each term}

$\frac{a}{1-a}+1+\frac{b}{1-b}+1+\frac{c}{1-c}+1=1+3 \Rightarrow \frac{(\cancel{a}+1-\cancel{a})}{(1-a)}+\frac{(\cancel{b}+1-\cancel{b})}{(1-b)}+\frac{(\cancel{c}+1-\cancel{c})}{(1-c)}=4$

$\Rightarrow \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=\boxed{4}$

Q.12: If $(a-1)^2+(b+2)^2+(c+1)^2=0$, then the value of $2a-3b+7c$ is:

(a) 12 (b) 3 (c) −11 (d) 1

ans: $(a-1)^2+(b+2)^2+(c+1)^2=0$

$\Rightarrow a-1=0 \Rightarrow \boxed{a=1}$, $b+2=0 \Rightarrow \boxed{b=-2}$ $c+1=0 \Rightarrow \boxed{c=-1}$

$\Rightarrow 2a-3b+7c=(2\times 1)-3(-2)+7\times(-1)=2+6-7=\boxed{1}$

Note: Because powers are square (even) so every number is a positive no. and sum of these positive numbers can be zero only when they are zero.

Q.13: If $x^2+y^2+2x+1=0$, then value of $x^{31}+y^{35}$ is

✓(a) -1 (b) 0 (c) 1 (d) 2

ans: $x^2+2x+1+y^2=0$ or, $(x+1)^2+(y^2)=0$

$\therefore x+1=0$ and $y=0$ $\Rightarrow \boxed{x=-1}, \boxed{y=0}$

$\therefore x^{31}+y^{35} = (-1)^{31}+0 = \boxed{-1}$

Q.14: If $(3a+1)^2+(b-1)^2+(2c-3)^2=0$, then the value of $3a+b+2c$ is equal to :

✓(a) 3 (b) -1 (c) 2 (d) 5

ans: $(3a+1)^2+(b-1)^2+(2c-3)^2=0$

$\Rightarrow 3a+1=0 \Rightarrow \boxed{a=-1/3}$, $b-1=0 \Rightarrow \boxed{b=1}$, $2c-3=0 \Rightarrow \boxed{c=3/2}$

$\therefore 3a+b+2c = \cancel{3}\times\left(-\frac{1}{\cancel{3}}\right)+1+\left(\cancel{2}\times\frac{3}{\cancel{2}}\right) = -1+1+3 = \boxed{3}$

Q.15: Let $a=\sqrt{6}-\sqrt{5}$, $b=\sqrt{5}-2$, $c=2-\sqrt{3}$ then point out the correct alternative among the four alternative given below.

(a) $b<a<c$ (b) $a<c<b$ (c) $b<c<a$ ✓(d) $a<b<c$

ans: Rationalize all terms, $a=\frac{(\sqrt{6}-\sqrt{5})\times(\sqrt{6}+\sqrt{5})}{(\sqrt{6}+\sqrt{5})} = \frac{1}{(\sqrt{6}+\sqrt{5})}$

$b=\frac{(\sqrt{5}-2)\times(\sqrt{5}+2)}{(\sqrt{5}+2)} = \frac{1}{(\sqrt{5}+2)}$ and $c=\frac{(2-\sqrt{3})(2+\sqrt{3})}{(2+\sqrt{3})} = \frac{1}{(2+\sqrt{3})}$

Now the term whose denominator is largest is the smallest term.

So, $\boxed{a<b<c}$

Q.16: If $\frac{x}{a}=\frac{1}{a}-\frac{a}{x}$, then the value of $x-x^2$ is :

(a) $-a$ (b) $\frac{1}{a}$ (c) $-\frac{1}{a}$ ✓(d) a^2

ans: $\frac{x}{a}=\frac{1}{a}-\frac{a}{x} \Rightarrow \frac{x}{a}-\frac{a}{x}=\frac{1}{a} \Rightarrow \frac{x^2-a^2}{\cancel{a}x}=\frac{1}{\cancel{a}}$

$x^2-a^2=x \Rightarrow \boxed{x^2-x=a^2}$

Q.17: If $a^2+b^2+c^2+3=2(a+b+c)$ then the value of $(a+b+c)$ is :

(a) 2 ✓(b) 3 (c) 4 (d) 5

ans: $a^2+b^2+c^2+3 = 2a+2b+2c$

$(a^2-2a+1)+(b^2-2b+1)+(c^2-2c+1)=0$

or, $(a-1)^2+(b-1)^2+(c-1)^2=0$, Now $a-1=0 \Rightarrow \boxed{a=1}$

$\boxed{b=1}$, $\boxed{c=1}$ So, $a+b+c = \boxed{3}$

Q.18: If $x-y = \frac{x+y}{7} = \frac{xy}{4}$, the numerical value of xy is:

✓(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

ans: $x-y = \frac{x+y}{7} = \frac{xy}{4}$

$x-y = \frac{xy}{4}$ also, $x+y = \frac{7xy}{4}$

$\frac{1}{y} - \frac{1}{x} = \frac{1}{4}$ —— ① or, $\frac{1}{y} + \frac{1}{x} = \frac{7}{4}$ —— ②

eq. ① + ② $\frac{2}{y} = \frac{8}{4} \Rightarrow \boxed{y=1}$ eq. ② − ① $\frac{2}{x} = \frac{6}{4} \Rightarrow \boxed{x = \frac{4}{3}}$

$\Rightarrow xy = 1 \times \frac{4}{3} = \boxed{\frac{4}{3}}$

Q.19: If $a+b+c=0$, then the value of $\left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right)\left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$ is

(a) 8 (b) −3 ✓(c) 9 (d) 0

ans: $\left(\frac{a+b}{c} + \frac{b+c}{a} + \frac{c+a}{b}\right)\left(\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}\right)$

$\Rightarrow \left[\frac{-c}{c} + \frac{-a}{a} + \frac{-b}{b}\right]\left[\frac{a}{-a} + \frac{b}{-b} + \frac{c}{-c}\right] = (-3) \times (-3) = \boxed{9}$

Q.20: If a, b, c are non-zero, $a + \frac{1}{b} = 1$ and $b + \frac{1}{c} = 1$, then value of abc

✓(a) −1 (b) 3 (c) −3 (d) 1

ans: $a + \frac{1}{b} = 1 \Rightarrow \boxed{ab+1 = b}$, also $b + \frac{1}{c} = 1 \Rightarrow b = 1 - \frac{1}{c}$

$ab+1 = 1 - \frac{1}{c} \Rightarrow ab = -\frac{1}{c} \Rightarrow abc = \boxed{-1}$ ans

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Q.21: If $a^2+b^2 = 5ab$, then the value of $\left(\frac{a^2}{b^2} + \frac{b^2}{a^2}\right)$ is:

(a) 32 (b) 16 ✓(c) 23 (d) −23

ans: $a^2 + b^2 = 5ab$

or, $\frac{a^2}{ab} + \frac{b^2}{ab} = 5 \Rightarrow \left(\frac{a}{b} + \frac{b}{a}\right) = 5$

so, $\left(\frac{a^2}{b^2} + \frac{b^2}{a^2}\right) = (5)^2 - 2 = 25-2 = \boxed{23}$

Q.22: For what value(s) of K the expression $P + \frac{1}{4}\sqrt{P} + K^2$ is a perfect square

(a) $\pm\frac{1}{3}$ (b) $\pm\frac{1}{4}$ ✓(c) $\pm\frac{1}{8}$ (d) $\pm\frac{1}{2}$

ans: $(\sqrt{P})^2 \boxed{\pm 2\sqrt{P} \times K} + K^2$ compare middle term, $+2\cancel{\sqrt{P}} \times K = \frac{1}{4}\cancel{\sqrt{P}} \Rightarrow \boxed{K = \frac{1}{8}}$

again, $(K) \times (-2\sqrt{P}) = \frac{1}{4}\sqrt{P} \Rightarrow K \times (-2\cancel{\sqrt{P}}) = \frac{1}{4}\cancel{\sqrt{P}} \Rightarrow \boxed{K = -\frac{1}{8}}$

Q.23: If $a+b=1$, $c+d=1$ and $a-b=\frac{d}{c}$, then the value of c^2-d^2 is

(a) $\frac{a}{b}$ ✓(b) $\frac{b}{a}$ (c) 1 (d) -1

ans: $c^2-d^2 = (c+d)(c-d)$
$= 1 \times (c-d)$

for finding $(c-d)$; $a-b = \frac{d}{c}$

$\frac{1}{a-b} = \frac{c}{d} = \frac{a+b}{a-b}$

(applying componendo & div.) $\frac{c+d}{c-d} = \frac{a+\cancel{b}+a-\cancel{b}}{(a+b)-(a-b)}$

$\Rightarrow \frac{1}{c-d} = \frac{2a}{2b} = \frac{a}{b} \Rightarrow \boxed{c-d = \frac{b}{a}}$

Q.24: If $a^4+b^4 = a^2b^2$, then (a^6+b^6) equals

✓(a) 0 (b) 1 (c) a^2+b^2 (d) $a^2b^4+a^4b^2$

ans: $a^4+b^4 = a^2b^2 \Rightarrow a^4+b^4-a^2b^2 = 0$ ——— (1)

also, $a^6+b^6 = (a^2)^3+(b^2)^3 = (a^2+b^2)(a^4+b^4-a^2b^2)$

$= (a^2+b^2) \times 0$ {from eq. 1}

$= \boxed{0}$

Q.25: Find the value of $x^{18}+x^{12}+x^6+1$ if $x+\frac{1}{x} = \sqrt{3}$

✓(a) 0 (b) 1 (c) 2 (d) 3

ans: if $x+\frac{1}{x} = \sqrt{3}$ then $x^3+\frac{1}{x^3} = (\sqrt{3})^3 - 3\sqrt{3} = 3\sqrt{3}-3\sqrt{3} = 0$

$x^3+\frac{1}{x^3} = 0 \Rightarrow x^6+1 = 0 \Rightarrow \boxed{x^6 = -1}$

so, $x^{18}+x^{12}+x^6+1 = (x^6)^3+(x^6)^2+x^6+1 = (-1)^3+(-1)^2+(-1)+1$

$= -\cancel{1}+\cancel{1}-\cancel{1}+\cancel{1} = \boxed{0}$

Q.26: If $x+\frac{1}{4x} = \frac{3}{2}$, find the value of $8x^3+\frac{1}{8x^3}$

✓(a) 18 (b) 36 (c) 24 (d) 16

ans: $x+\frac{1}{4x} = \frac{3}{2}$, so, $2x+\frac{1}{2x} = 2\times\frac{3}{2} \Rightarrow 2x+\frac{1}{2x} = 3$

$8x^3+\frac{1}{8x^3} = (3)^3-(3\times 3) = 27-9 = \boxed{18}$

Q.27: If $\frac{1}{x+y} = \frac{1}{x}+\frac{1}{y}$ $(x\neq 0, x\neq y)$, then the value of x^3-y^3 is

✓(a) 0 (b) 1 (c) -1 (d) 2

ans: $\frac{1}{x+y} = \frac{1}{x}+\frac{1}{y} \Rightarrow \frac{1}{x+y} = \frac{x+y}{xy} \Rightarrow (x+y)^2 = xy \Rightarrow x^2+y^2+2xy = xy$

$\Rightarrow \boxed{x^2+y^2-xy=0}$ $x^3-y^3 = (x-y)(x^2+y^2-xy) = (x-y)\times 0 = \boxed{0}$

Q.28: If $xy(x+y) = 1$, then the value of $\frac{1}{x^3y^3} - x^3 - y^3$ is :

(a) 0 (b) 1 ✓(c) 3 (d) -2

ans: $xy(x+y) = 1 \Rightarrow x+y = \frac{1}{xy}$ cube both side, $(x+y)^3 = \frac{1}{(xy)^3}$

$\Rightarrow x^3+y^3+3xy(x+y) = \frac{1}{x^3y^3} \Rightarrow \frac{1}{x^3y^3} - x^3 - y^3 = 3\cancel{xy} \times \frac{1}{\cancel{xy}} = \boxed{3}$

Q.29: If $x^4 + \frac{1}{x^4} = 119$ and $x > 1$, then the value of $x^3 - \frac{1}{x^3}$ is

(a) 54 (B) 18 (C) 72 ✓(d) 36

ans: $x^4 + \frac{1}{x^4} = 119 \Rightarrow x^2 + \frac{1}{x^2} = \sqrt{119+2} = 11$

$x^2 + \frac{1}{x^2} = 11 \Rightarrow x - \frac{1}{x} = \sqrt{11-2} = \sqrt{9} \Rightarrow x - \frac{1}{x} = \pm 3$

$x^3 - \frac{1}{x^3} = (3)^3 + (3\times3) = 27+9 = \boxed{36}$

Q.30: If $x+y = z$, then the expression $x^3+y^3-z^3+3xyz$

✓(a) 0 (b) $3xyz$ (c) $-3xyz$ (d) z^3

ans: $x+y = z \Rightarrow x+y+(-z) = 0$ If $a+b+c=0$

then $a^3+b^3+c^3-3abc = 0 \Rightarrow x^3+y^3+(-z)^3 - 3xy(-z) = 0$

$\Rightarrow \boxed{x^3+y^3-z^3+3xyz = 0}$

Q.31: If $a + \frac{1}{a} = \sqrt{3}$, then the value of $a^6 - \frac{1}{a^6} + 2$ will be :

(a) 1 ✓(b) 2 (c) $3\sqrt{3}$ (d) 5

ans: We know, when $a + \frac{1}{a} = \sqrt{3} \Rightarrow \boxed{a^6 = -1}$

$\Rightarrow a^6 - \frac{1}{a^6} + 2 = -1 - \frac{1}{(-1)} + 2 = -\cancel{1} + \cancel{1} + 2 = \boxed{2}$

Q.32: If $x^2+1 = 2x$, then the value of $\frac{\left(x^4 + \frac{1}{x^2}\right)}{(x^2 - 3x + 1)}$ is

(a) 0 (b) 1 (c) 2 (d) -2

ans: $x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow \boxed{x = 1}$

So, $\frac{\left(x^4 + \frac{1}{x^2}\right)}{(x^2-3x+1)} = \frac{(1+1)}{(1-3+1)} = \frac{2}{-1} = \boxed{-2}$

Q.33: If $\frac{x}{x^2-2x+1} = \frac{1}{3}$, then the value of $x^3 + \frac{1}{x^3}$ is :

(a) 81 ✓(b) 110 (c) 125 (d) 27

ans: $\frac{x}{\cancel{x}\left(x - 2 + \frac{1}{x}\right)} = \frac{1}{3} \Rightarrow x + \frac{1}{x} = 5 \Rightarrow x^3 + \frac{1}{x^3} = (5)^3 - (3\times5) = 125 - 15$

$= \boxed{110}$

Q.34: If $x+\frac{1}{x+1}=1$, then $(x+1)^5+\frac{1}{(x+1)^5}$ equals

(a) 1 (b) 2 (c) 4 (d) 8

ans: $(x+1)+\frac{1}{(x+1)}=2$ (adding 1 both sides)

$(x+1)^2+\frac{1}{(x+1)^2}=(2)^2-2=\boxed{2}$, and $(x+1)^3+\frac{1}{(x+1)^3}=(2)^3-3(2)=\boxed{2}$

$\left[(x+1)^2+\frac{1}{(x+1)^2}\right]\times\left[(x+1)^3+\frac{1}{(x+1)^3}\right]=4$

$\Rightarrow (x+1)^5+\frac{1}{(x+1)}+(x+1)+\frac{1}{(x+1)^5}=4 \Rightarrow (x+1)^5+\frac{1}{(x+1)^5}+1+1=4$

$\Rightarrow (x+1)^5+\frac{1}{(x+1)^5}=\boxed{2}$

Note: We can also find value of x and then substitute.

Q.35: If $\frac{1}{a}-\frac{1}{b}=\frac{1}{a-b}$, then the value of a^3+b^3 is;

(a) 0 (b) -1 (c) 1 (d) 2

ans: $\frac{1}{a}-\frac{1}{b}=\frac{1}{a-b} \Rightarrow \frac{b-a}{ab}=\frac{1}{a-b} \Rightarrow -\frac{(a-b)}{ab}=\frac{1}{(a-b)} \Rightarrow -(a-b)^2=ab$

$(a-b)^2=-ab \Rightarrow a^2+b^2-2ab+ab=0 \Rightarrow \boxed{a^2+b^2-ab=0}$

$a^3+b^3=(a+b)(a^2+b^2-ab)=(a+b)\times(0)=\boxed{0}$

Q.36: If $a+b+c=6$, $a^2+b^2+c^2=14$ and $a^3+b^3+c^3=36$, then the value of abc is:

(a) 3 (b) 6 (c) 9 (d) 12

ans: $(a+b+c)^2=a^2+b^2+c^2+2(ab+bc+ca)$

$(6)^2=14+2(ab+bc+ca) \Rightarrow \boxed{ab+bc+ca=11}$

Now, $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$36-3abc=6\times(14-11) \Rightarrow 3abc=18 \Rightarrow \boxed{abc=6}$

Q.37: If $x=y=333$ and $z=334$, then the value of $x^3+y^3+z^3-3xyz$ is:

(a) 0 (b) 667 (c) 1000 (d) 2334

ans: $x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$

$=\frac{1}{2}(333+333+334)\left[0+(-1)^2+(1)^2\right]=\boxed{1000}$

Q.38: If $p-2q=4$, then the value of $p^3-8q^3-24pq-64$ is:

(a) 2 (b) 0 (c) 3 (d) -1

ans: Cubing, $(p-2q)^3=(4)^3 \Rightarrow p^3-8q^3-3\times p\times 2q(p-2q) \Rightarrow p^3-8q^3-6p^2q+12pq^2=64$

$p^3-8q^3\ 6pq(p-2q)=64 \Rightarrow p^3-8q^3-6pq(4)-64=0$

$\Rightarrow \boxed{p^3-8q^3-24pq-64=0}$

Q.39: If $x+\frac{1}{x}=2$ and x is real, then the value of $x^{17}+\frac{1}{x^{19}}$ is

(a) 1 (b) 0 ✓(c) 2 (d) −2

ans: $x+\frac{1}{x}=2$ (assume $x=1$, so, $1+1=2$)

$x^{17}+\frac{1}{x^{19}} = (1)^{17}+\frac{1}{(1)^{19}} = 1+1 = \boxed{2}$

Q.40: Find the value of $x^3-6x^2+12x-13$ if $x=\sqrt[3]{5}+2$

(a) −1 (b) 1 (c) 2 ✓(d) 0

ans: $x=\sqrt[3]{5}+2$ or, $x-2=\sqrt[3]{5}$ cubing both sides

$(x-2)^3=(\sqrt[3]{5})^3 \Rightarrow x^3-2^3-3\times x\times 2(x-2)=5 \Rightarrow x^3-6x^2+12x-8-5=0$

$\Rightarrow \boxed{x^3-6x^2+12x-13=0}$

Q.41: find the value of $\sqrt{(x^2+y^2+z)(x+y-3z)}+\sqrt[3]{xy^3z^2}$ when $x=1, y=-3, z=-1$

(a) 1 ✓(b) 0 (c) −1 (d) 1/2

ans: $\sqrt{[(1)^2+(-3)^2+(-1)][(1-\cancel{3}+\cancel{3})]}+[1\times(-3)^3\times(-1)^2]^{1/3}$

$=\sqrt{(1+9-1)(1)}+[(-3)^3]^{1/3} = 3-3 = \boxed{0}$

Q.42: If a, b, c be all positive integers, then the least positive value of $a^3+b^3+c^3-3abc$ is :

(a) 1 (b) 2 (c) 4 (d) 3

ans: according to options, minimum values $a=1$, $b=1$, $c=2$

$(1)^3+(1)^3+(2)^3-3(1)(1)(2) = (2+8)-(3\times2) = 10-6 = 4$

$\Rightarrow$ least positive value of $a^3+b^3+c^3-3abc = \boxed{4}$

Q.43: If $x=6+\frac{1}{x}$, then the value of $x^4+\frac{1}{x^4}$ is

(a) 1448 ✓(b) 1442 (c) 1444 (d) 1446

ans: $x-\frac{1}{x}=6 \Rightarrow x^2+\frac{1}{x^2}=(6)^2+2=\boxed{38}$ $x^4+\frac{1}{x^4}=(38)^2-2=\boxed{1442}$

Q.44: If $x^2-3x+1=0$, then the value of $\frac{x^6+x^4+x^2+1}{x^3}$ will be

(a) 18 (b) 15 ✓(c) 21 (d) 30

ans: $x^2+1=3x \Rightarrow x+\frac{1}{x}=3$ —— (1) Given, $\frac{x^6+x^4+x^2+1}{x^3}$

$\Rightarrow \frac{x^6}{x^3}+\frac{x^4}{x^3}+\frac{x^2}{x^3}+\frac{1}{x^3} = x^3+x+\frac{1}{x}+\frac{1}{x^3} = (x+\frac{1}{x})+(x^3+\frac{1}{x^3})$

$= 3+[(3)^3-(3\times3)] = 3+18$

$= \boxed{21}$

Q.45: If $x^2+y^2+1=2x$, then the value of x^3+y^3 is

(a) 2 (b) 0 (c) -1 (d) 1 ✓

ans: $x^2-2x+1+y^2=0 \Rightarrow (x-1)^2+y^2=0 \Rightarrow x-1=0 \Rightarrow \boxed{x=1}\ \boxed{y=0}$

$\Rightarrow x^3+y^3=(1)^3+(0)^3=\boxed{1}$

Q.46: If $p=99$, then the value of $p(p^2+3p+3)$ is

(a) 10000000 (b) 999000 (c) 999999 ✓ (d) 990000

ans: $p(p^2+3p+3) = p^3+3p^2+3p+1-1 = (p+1)^3-1$

$= (99+1)^3-1 = \boxed{999999}$

Q.47: If $x=997$, $y=998$, and $z=999$, then the value of $x^2+y^2+z^2-xy-yz-zx$ is:

(a) 0 (b) 1 (c) -1 (d) 3

ans: $x^2+y^2+z^2-xy-yz-zx = \frac{1}{2}[(x-y)^2+(y-z)^2+(z-x)^2]$

$= \frac{1}{2}[(-1)^2+(-1)^2+(2)^2] = \frac{1}{2}\times 6 = \boxed{3}$

Q.48: If $p^3+3p^2+3p=7$, then the value of p^2+2p is

(a) 4 (b) 3 ✓ (c) 5 (d) 6

ans: $p^3+3p^2+3p+1=7+1 \Rightarrow (p+1)^3=8=(2)^3 \Rightarrow p+1=2 \Rightarrow \boxed{p=1}$

$\Rightarrow p^2+2p=(1)^2+(2\times1) = \boxed{3}$

Q.49: If $t^2-4t+1=0$, then the value of $t^3+\frac{1}{t^3}$ is

(a) 44 (b) 48 (c) 52 ✓ (d) 64

ans: $t^2+1=4t \Rightarrow \frac{t^2}{t}+\frac{1}{t}=4 \Rightarrow t+\frac{1}{t}=4$

$\left(t^3+\frac{1}{t^3}\right)=(4)^3-(3\times4) = 64-12 = \boxed{52}$

Q.50: The expression x^4-2x^2+K will be a perfect square when the value of K is

(a) 2 (b) 1 ✓ (c) -1 (d) -2

ans: We know, $(a-b)^2=a^2-2ab+b^2$

given expression x^4-2x^2+K

Comparing both equations

$a^2=x^4 \Rightarrow \boxed{a=x^2},\ \boxed{K=b^2}$

$-\cancel{2}ab=-\cancel{2}x^2 \Rightarrow ab=x^2 \Rightarrow x^2\times b=x^2 \Rightarrow \boxed{b=1}$

$K=b^2=1^2=1$ So, for $\boxed{K=1}$ expression will be perfect square.
