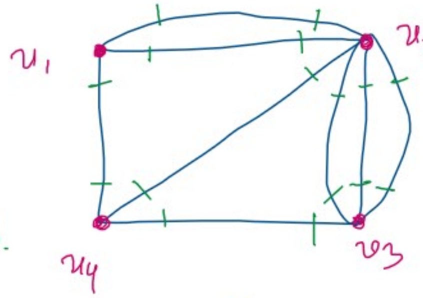
L19 21 Mar 2022

21 March 2022 10:06

Degree of a vertex :→ $\deg(v)$ = the no. of edges involve that vertex v.

$\deg(v_1) = 3$



$\deg(v_2) = 6$

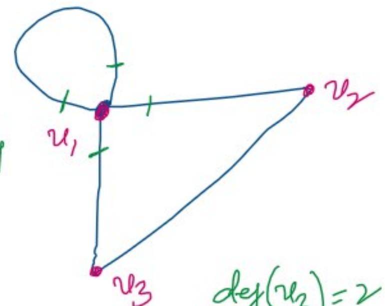
$\deg(v_3) = 4$

$\deg(v_4) = 3.$

imp

[loop is counted twice for calculating the degree]

$\deg(v_1) = 4$



$\deg(v_2) = 2$

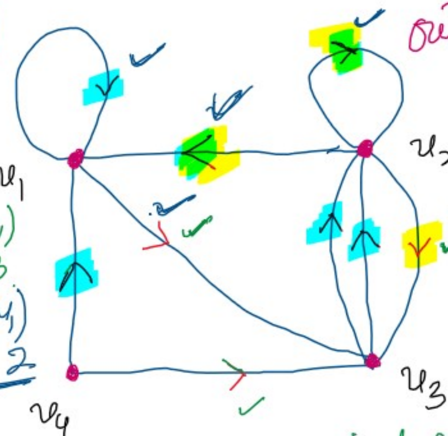
$\deg(v_3) = 2$

indegree(v) or $\deg^-(v)$

↓

the no. of edges whose direction is towards 'v'.

outdegree(v) or $\deg^+(v)$



$\text{indeg}(v_1) = 3$

$\text{outdeg}(v_1) = 2$

$\text{indeg}(v_2) = 3$

$\text{outdeg}(v_2) = 3$

$\text{indeg}(v_3) = 3.$

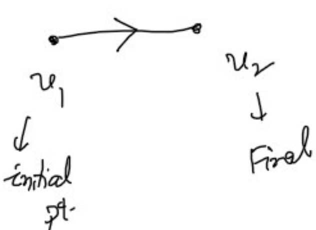
$\text{outdeg}(v_3) = 2$

$\text{indeg}(v_4) = 0$

$\text{outdeg}(v_4) = 2$

outdegree(v) or $\deg^+(v)$

the no. of edges which are directed away from v



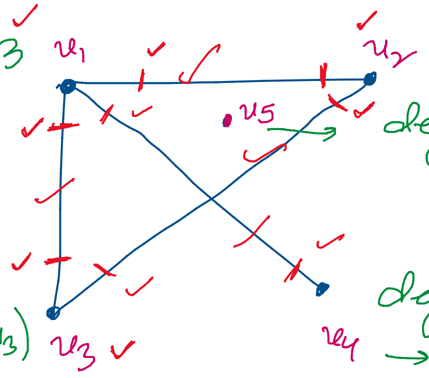
$$\boxed{\deg(v) = \text{indegree}(v) + \text{outdegree}(v)}$$

Isolated vertex :→

A vertex with degree zero is called isolated vertex.

isolated vertex.

Pendent vertex :→ A vertex with degree 1 is called pendent vertex.



$deg(v_1) = 3$

$deg(v_2) = 2$

$deg(v_5) = 0$ → isolated vertex

$deg(v_3) = 2$

$deg(v_4) = 1$

v_4 → pendent vertex

$Sum = 3 + 2 + 2 + 1 = 8$

$8 = 2e$

$e = 4$

Handshaking Theorem :→

The sum of degree of all the vertices is equal to twice the no. of edges.

$$\boxed{\sum deg(v) = 2e}$$

Draw a graph with 5 vertices each with degree 3. ?

Not possible

v_1, v_2, v_3, v_4, v_5

$\downarrow$ 3, 3, 3, 3, 3

sum of degree of all vertices

$= 5(3) = 15$

$15 = 2e$

$\Rightarrow e = 15/2$?

The maximum degree of a vertex in a simple graph with n vertex is $= n-1$

→ No multiple edges, no loops

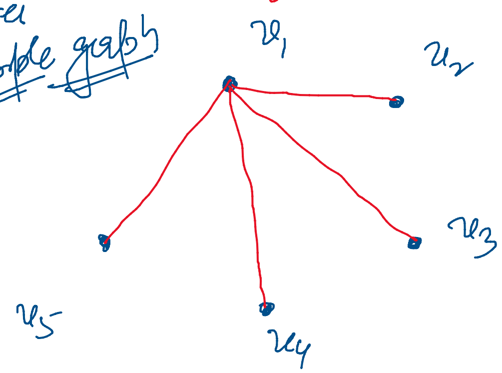
Max. $deg(v_1) = 4$

$= n-1$

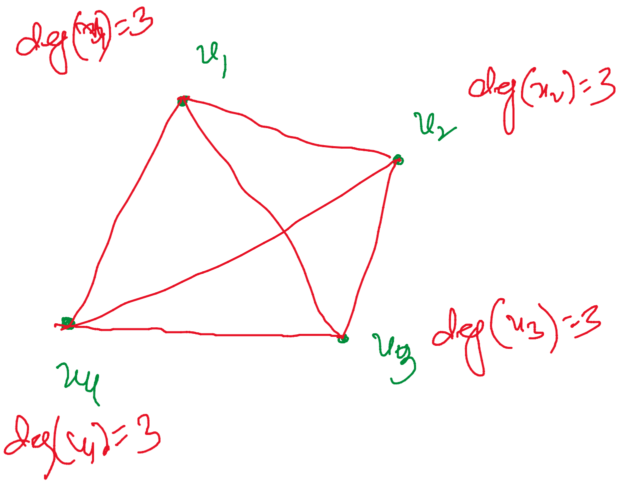
No multiple edges, no loops

5 vertices in a simple graph

Max. $\deg(v_1) = 4$



The max. no. of edges in a simple graph with n vertices.



$\frac{4(4-1)}{2} = 6$

Handshaking Lemma

Sum of degree $= 2e_{max.}$

$(n-1) + (n-1) + (n-1) + \cdots n$ vertices $=$

$2e_{max.}$

$n(n-1) = 2e_{max.}$

$$\Rightarrow \boxed{e_{max.} = \frac{n(n-1)}{2}}$$