Julia, my new friend for computing and optimization?

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E.T.
EFFICIENT TOOLS
for RESEARCH
JULIA: MY NEW FRIEND
FOR COMPUTING & OPTIMIZATION?

« Julia, my new friend for computing and optimization? »

- **Intro to the Julia programming language, for MATLAB users**
- *Date:* 14th of June 2018
- *Who:* Lilian Besson & Pierre Haessig
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Agenda for today [30 min]

1. What is Julia? [5 min]
2. Comparison with MATLAB [5 min]
3. Two examples of problems solved Julia [5 min]
4. Longer ex. on optimization with JuMP [13min]
5. Links for more information ? [2 min]

1. What is Julia ?

- **Open-source and free programming language** (MIT license)
  - Developed since 2012 (creators: MIT researchers)
  - Growing popularity worldwide, in research, data science, finance etc…
  - Multi-platform: Windows, Mac OS X, GNU/Linux...
- Designed for *performance*:
  - Interpreted *and* compiled, very efficient
  - Easy to run your code in parallel (multi-core & cluster)
- Designed to be *simple to learn and use*:
  - Easy syntax, dynamic typing (MATLAB & Python-like)

Ressources

- **Website**:
  - `JuliaLang.org` for the language
  - & `Pkg.JuliaLang.org` for packages
- Documentation : `docs.JuliaLang.org`

Comparison with MATLAB (1/3)

	Julia 😃	**MATLAB** 😢
Cost	Free ✌️	Hundreds of euros / year
License	Open-source	1 year user license (no longer after your PhD!)
Comes from	A non-profit foundation, and the community	MathWorks company
Scope	Mainly numeric	Numeric only
Performances	Very good performance	Faster than Python, slower than Julia

Comparison with MATLAB (2/3)

	Julia	MATLAB
Packaging	`Pkg` manager included. Based on `git` + GitHub, very easy to use	Toolboxes already included but 💰 have to pay if you wat more!
Editor/IDE	*Jupyter* is recommended (*Juno* is also good)	Good IDE already included
Parallel computations	Very easy, low overhead cost	Possible, high overhead

Comparison with MATLAB (3/3)

	Julia	MATLAB
Usage	Generic, worldwide 🌎	Research in academia and industry
Fame	Young but starts to be known	Old and known... In decline 📉 ?
Support?	Community[1]: StackOverflow, Forum	By MathWorks
Documentation	OK and growing, inline/online	OK, inline/online

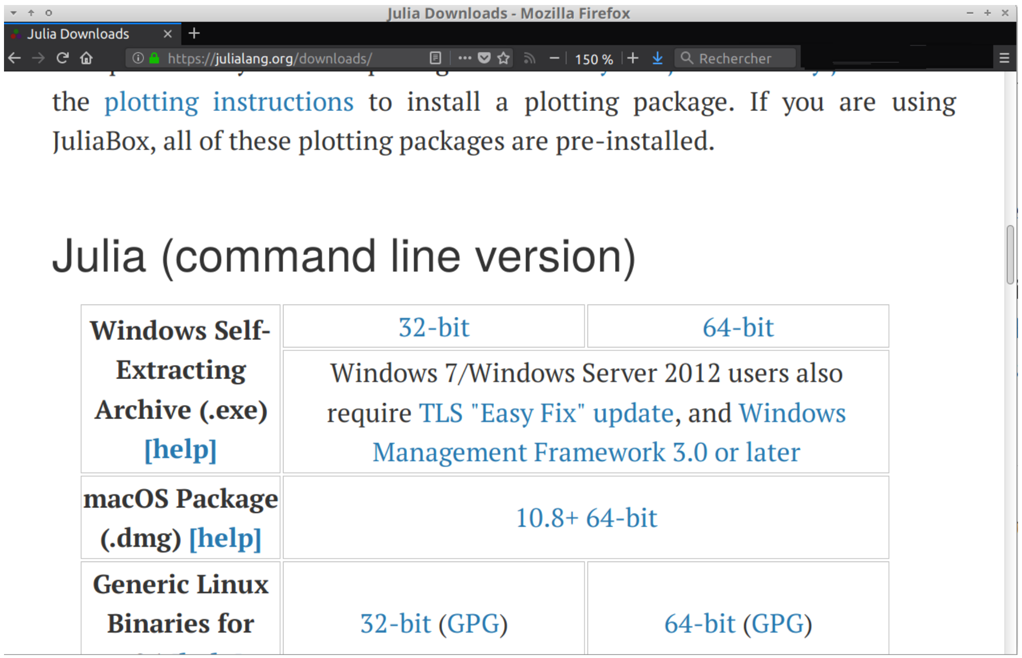
> Note[1]: Julia Computing, Inc. (founded 2015 by Julia creators) offer paid licenses (JuliaPro Enterprise) with professional support.

How to install Julia ⬇️ (1/2)

- You can try online *for free* on `JuliaBox.com`
- On Linux, Mac OS or Windows:
  - You can use the default installer 📦 from the website `JuliaLang.org/downloads`
- Takes about 4 minutes... and it's free !

> You also need Python 3 to use Jupyter ✨, I suggest to use 🐍 `Anaconda.com/download` if you don't have Python yet.

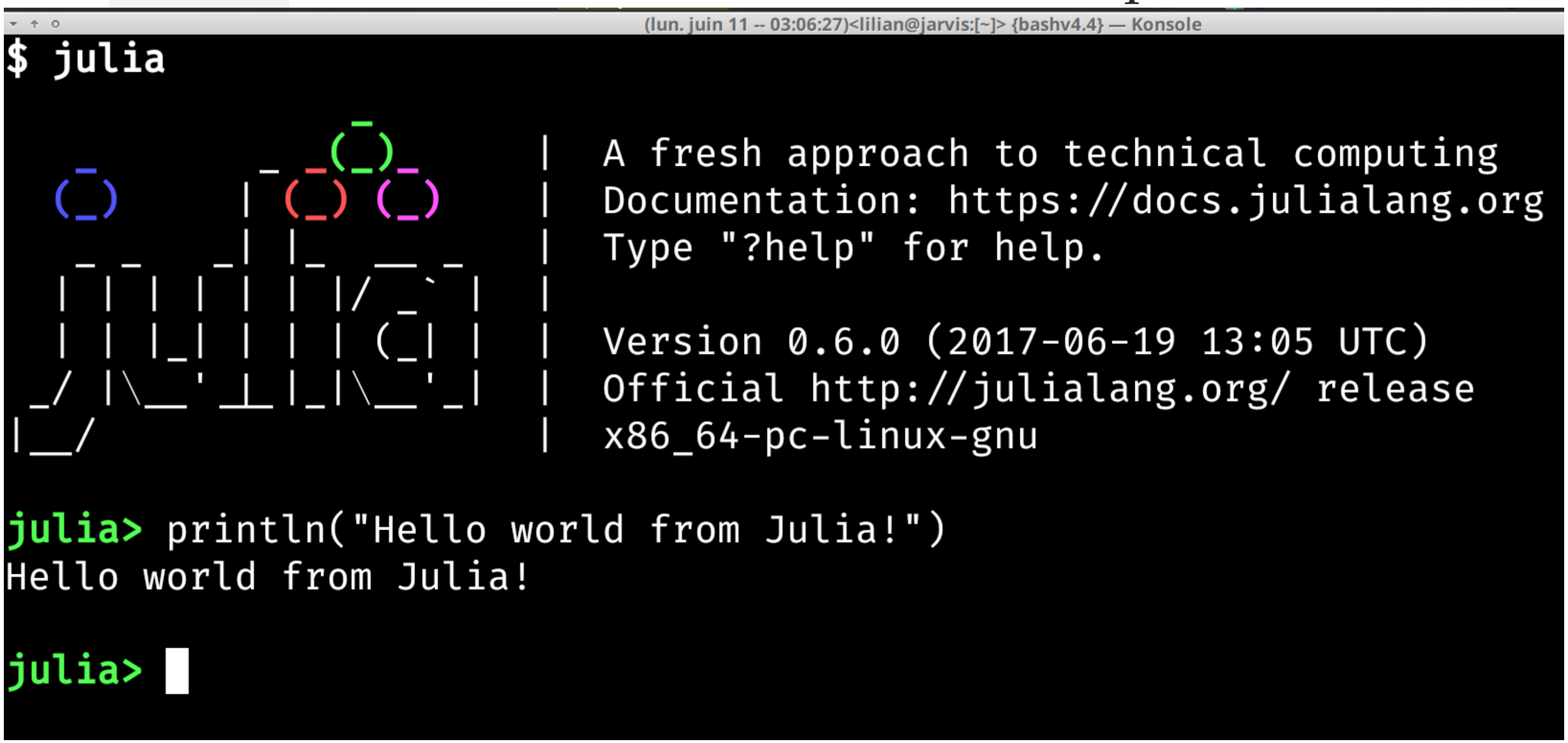
How to install Julia ⬇️ (2/2)



1. Select the binary of your platform 📦
2. Run the binary 🏃 !
3. Wait 🕐…
4. Done 👌 ! Test with `julia` in a terminal

Different tools to use Julia

- Use `julia` for the command line for short experiments

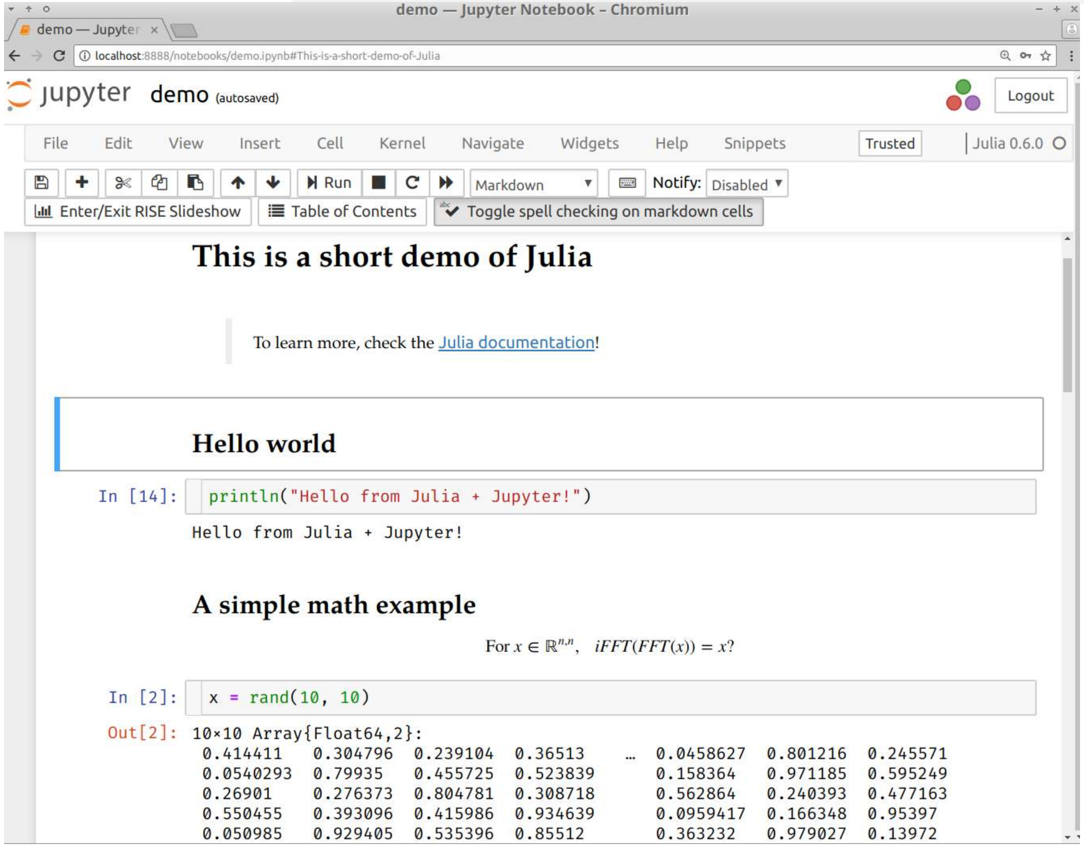


- Use the *Juno* IDE to edit large projects

> Demo time ⌚ !

Different tools to use Julia

- Use **J**u*pyter* notebooks to write or share your experiments (examples: `github.com/Naereen/notebooks`)



> Demo time ⌚ !

📦 How to install modules in Julia ?

- Installing is **easy** !

```
julia> Pkd.add("IJulia")  # installs IJulia
```

- Updating also!

```
julia> Pkg.update()
```

🔍 How to find the module you need ?

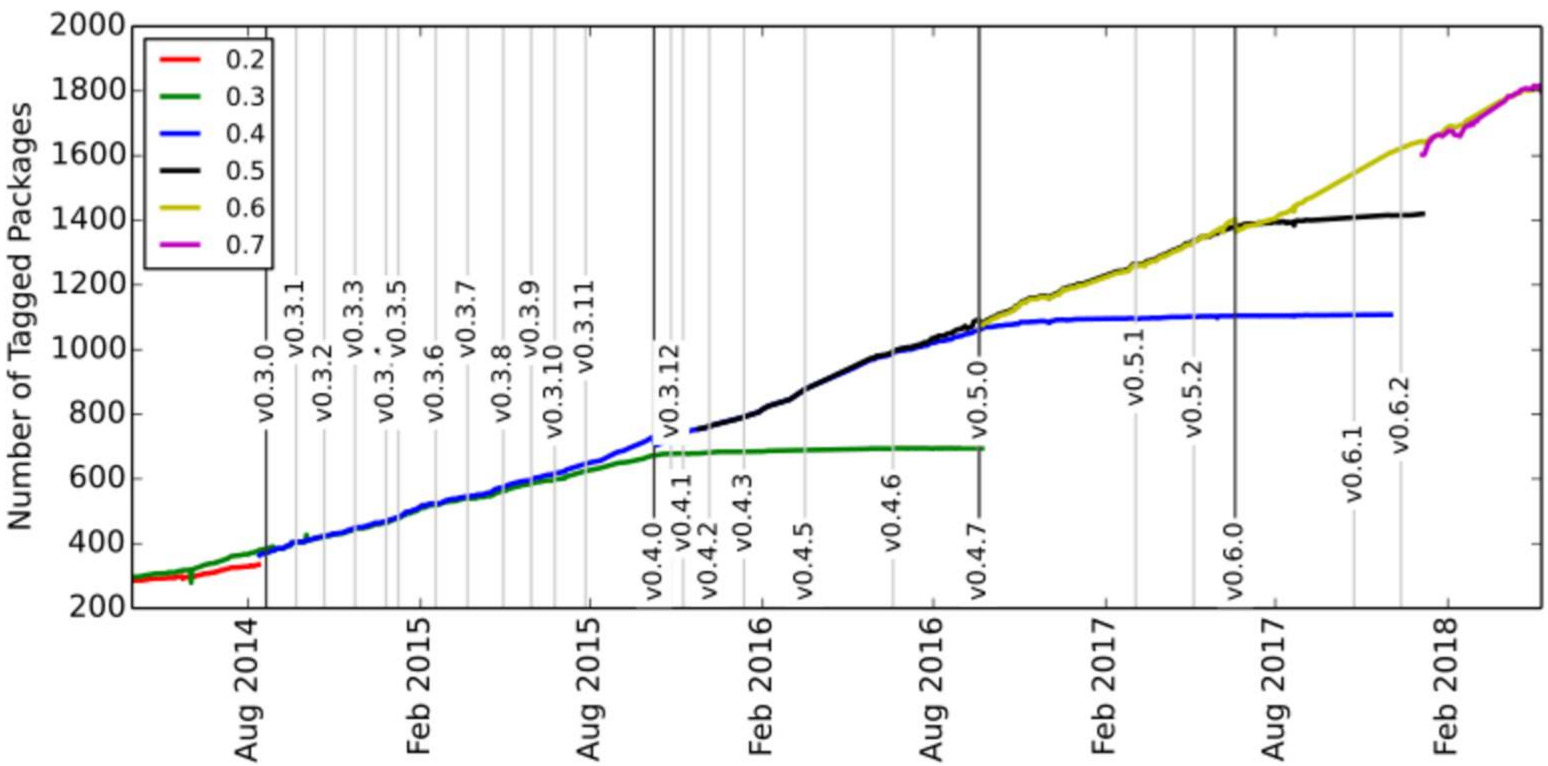
- First… ask your colleagues 😄 !
- Complete list on `Pkg.JuliaLang.org`

📦 Overview of famous Julia modules

- Plotting:
  - `Winston.jl` for easy plotting like MATLAB
  - `PyPlot.jl` interface to Matplotlib (Python)
- The JuliaDiffEq collection for **differential equations**
- The JuliaOpt collection for **optimization**
- The JuliaStats collection for **statistics**
- And many more!

> Find more specific packages on `GitHub.com/svaksha/Julia.jl`

Many packages, and a quickly growing community



> Julia is still in development, in version v0.6 but version 1.0 is planned soon!

2. Main differences in syntax between Julia and MATLAB

> Ref: `CheatSheets.QuanteCon.org`

2. Main differences in syntax between Julia and MATLAB

> Ref: `CheatSheets.QuanteCon.org`

	Julia	MATLAB
File ext.	`.jl`	`.m`
Comment	`# blabla...`	`% blabla...`
Indexing	`a[1]` to `a[end]`	`a(1)` to `a(end)`
Slicing	`a[1:100]` (view)	`a(1:100)` (⚠️ copy)
Operations	Linear algebra by default	Linear algebra by default
Block	Use `end` to close all blocks	Use `endif` `endfor` etc

	Julia	MATLAB
Help	`?func`	`help func`
And	`a & b`	`a && b`
Or	`a | b`	`a || b`
Datatype	`Array` of *any* type	multi-dim doubles array
Array	`[1 2; 3 4]`	`[1 2; 3 4]`
Size	`size(a)`	`size(a)`
Nb Dim	`ndims(a)`	`ndims(a)`
Last	`a[end]`	`a(end)`

	Julia	MATLAB
Tranpose	`a.'`	`a.'`
Conj. transpose	`a'`	`a'`
Matrix x	`a * b`	`a * b`
Element-wise x	`a .* b`	`a .* b`
Element-wise /	`a ./ b`	`a ./ b`
Element-wise ^	`a ^ 3`	`a .^ 3`
Zeros	`zeros(2, 3, 5)`	`zeros(2, 3, 5)`
Ones	`ones(2, 3, 5)`	`ones(2, 3, 5)`
Identity	`eye(10)`	`eye(10)`
Range	`range(0, 100, 2)` or `1:2:100`	`1:2:100`

	Julia	MATLAB
Maximum	`max(a)`	`max(max(a))` ?
Random matrix	`rand(3, 4)`	`rand(3, 4)`
L^2 **Norm**	`norm(v)`	`norm(v)`
Inverse	`inv(a)`	`inv(a)`
Solve syst.	`a \ b`	`a \ b`
Eigen vals	`V, D = eig(a)`	`[V,D]=eig(a)`
FFT/IFFT	`fft(a)`, `ifft(a)`	`fft(a)`, `ifft(a)`

> Very close to MATLAB for linear algebra!

3. Scientific problems solved with Julia

> Just to give examples of syntax and modules

1. 1D numerical integration and plot
2. Solving a 2^{nd} order Ordinary Differential Equation

3.1. 1D numerical integration and plot

> Exercise: evaluate and plot this function on $[-1, 1]$:
>
> $$\mathrm{Ei}(x) := \int_{-x}^{\infty} \frac{\mathrm{e}^u}{u} \,\mathrm{d}u$$

How to?

Use packages and everything is easy!

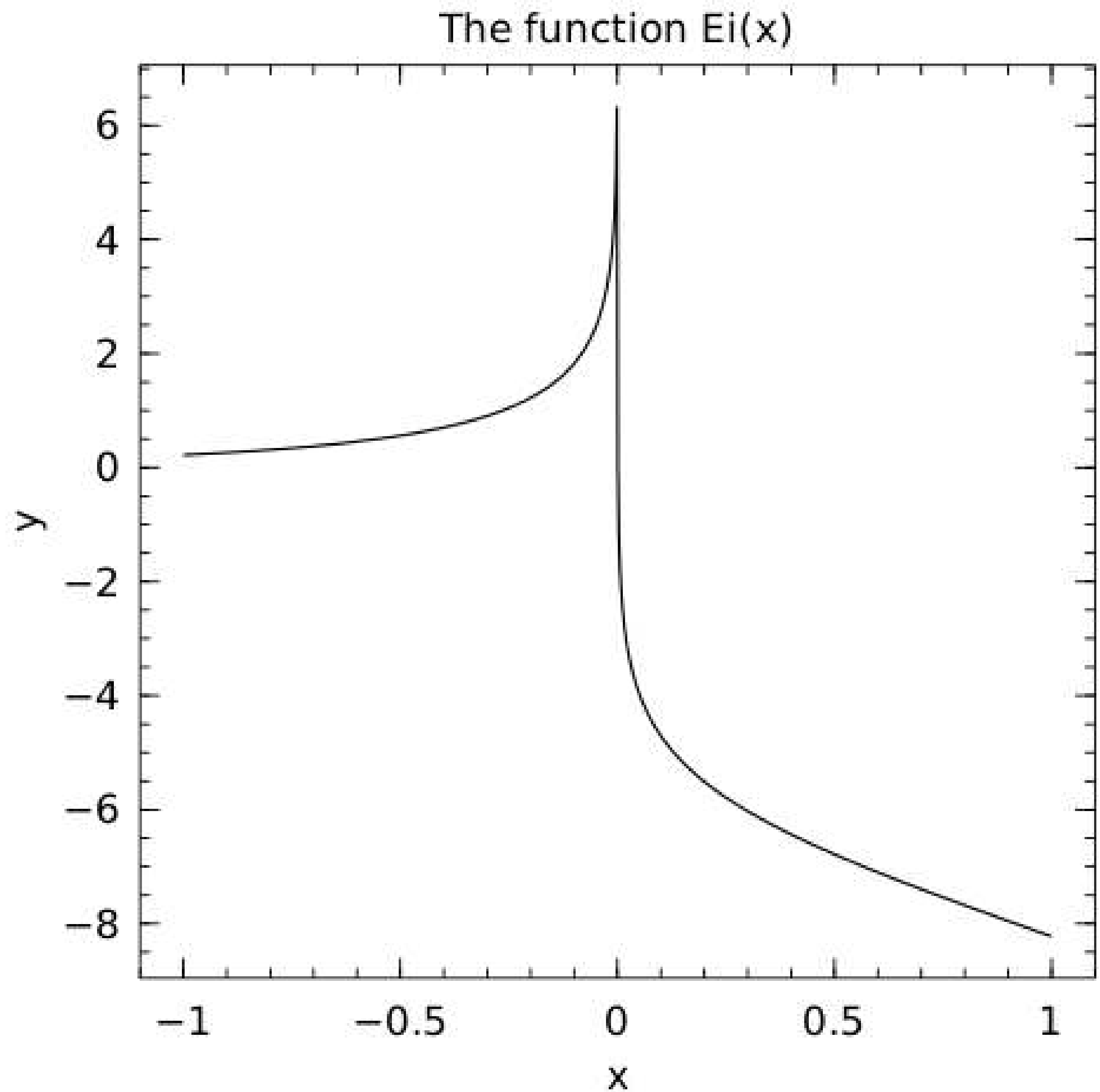
- `QuadGK.jl` for integration
- `Winston.jl` for 2D plotting

```julia
using QuadGK

function Ei(x, minfloat=1e-3, maxfloat=100)
    f = t -> exp(-t) / t  # inline function
    if x > 0
        return quadgk(f, -x, -minfloat)[1]
             + quadgk(f, minfloat, maxfloat)[1]
    else
        return quadgk(f, -x, maxfloat)[1]
    end
end

X = linspace(-1, 1, 1000)  # 1000 points
Y = [ Ei(x) for x in X ]   # Python-like syntax!

using Winston
plot(X, Y)
title("The function Ei(x)")
xlabel("x"); ylabel("y")
savefig("figures/Ei_integral.png")
```


The function Ei(x)
6
4
2
0
−2
−4
−6
−8
y
−1
−0.5
0
0.5
1
x

3.2. Solving a 2^{nd} order ODE

> Goal: solve and plot the differential equation of a pendulum:
>
> $$\theta''(t) + b\,\theta'(t) + c\,\sin(\theta(t)) = 0$$
>
> For $b = 1/4, c = 5, \theta(0) = \pi - 0.1, \theta'(0) = 0, t \in [0, 10]$

How to?

Use packages!

- `DifferentialEquations.jl` function for ODE integration
- `Winston.jl` for 2D plotting

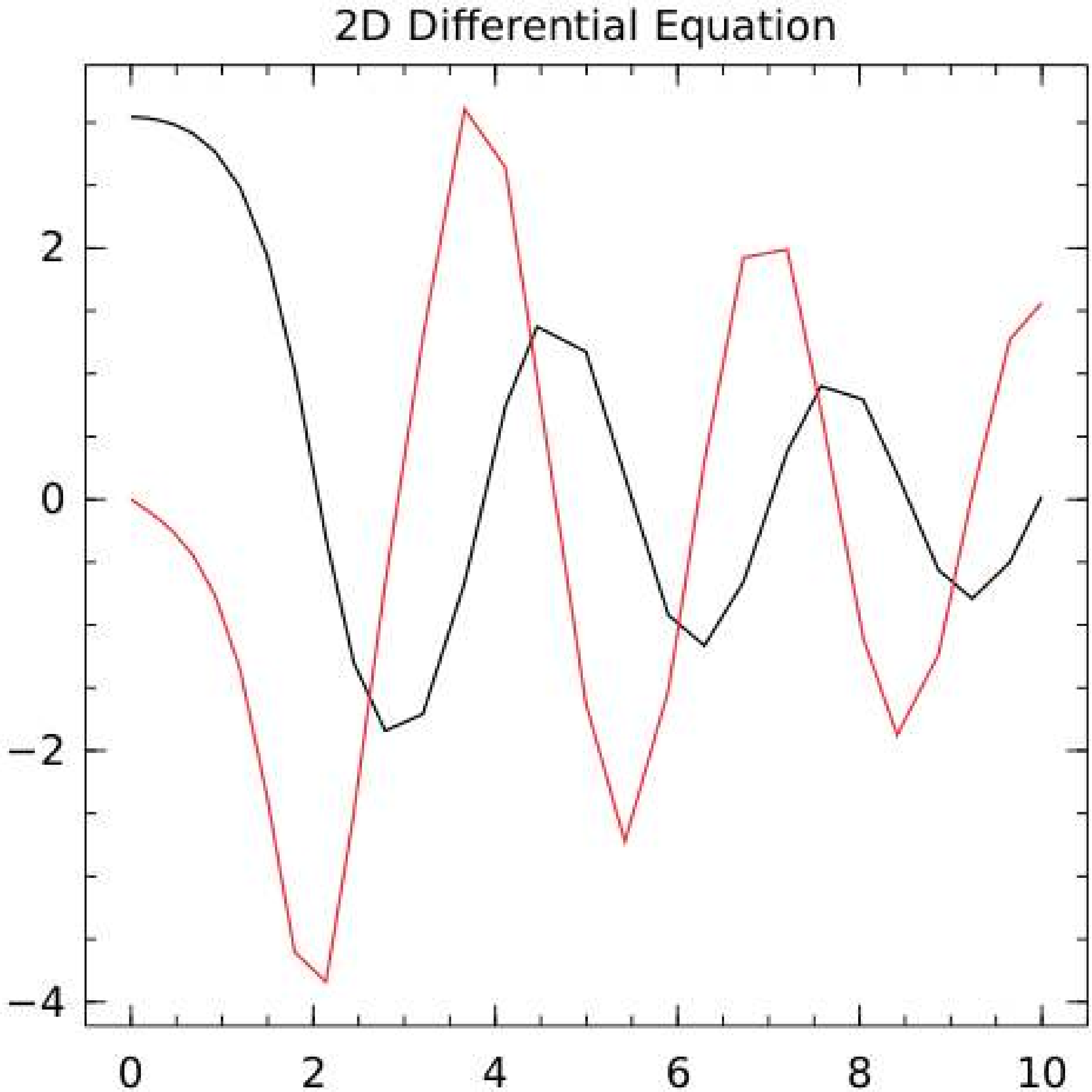
```julia
using DifferentialEquations

b, c = 0.25, 5.0
y0 = [pi - 0.1, 0]

# macro magic!
pend2 = @ode_def Pendulum begin
  dθ = ω  # ← yes, this is UTF8, θ and ω in text
  dω = (-b * ω) - (c * sin(θ))
end

prob = ODEProblem(pend, y0, (0.0, 10.0))
sol = solve(prob)              # ↑ solve on interval [0,10]
t, y = sol.t, hcat(sol.u...)'

using Winston
plot(t, y[:, 1], t, y[:, 2])
title("2D Differential Equation")
savefig("figures/Pendulum_solution.png")
```


2D Differential Equation
2
0
−2
−4
0
2
4
6
8
10

Examples

1. **Iterative computation**: signal filtering
2. **Optimization**: robust regression on RADAR data

Ex. 1: Iterative computation

Objective:

- show the efficiency of Julia's Just-in-Time (JIT) compilation
- but also its fragility…

Note: you can find companion notebooks on GitHub

Iterative computation: signal filtering

The classical saying:

> *« Vectorized code often runs much faster than the corresponding code containing loops. »* (cf. MATLAB doc)

does not hold for Julia, because of its **Just-in-Time compiler**.

Example of a computation that cannot be vectorized

Smoothing of a signal $\{u_k\}_{k\in\mathbb{N}}$:

$$y_k = a y_{k-1} + (1-a) u_k, \quad k \in\mathbb{N}^+$$

Parameter a tunes the smoothing (none: $a = 0$, strong $a \to 1^-$).

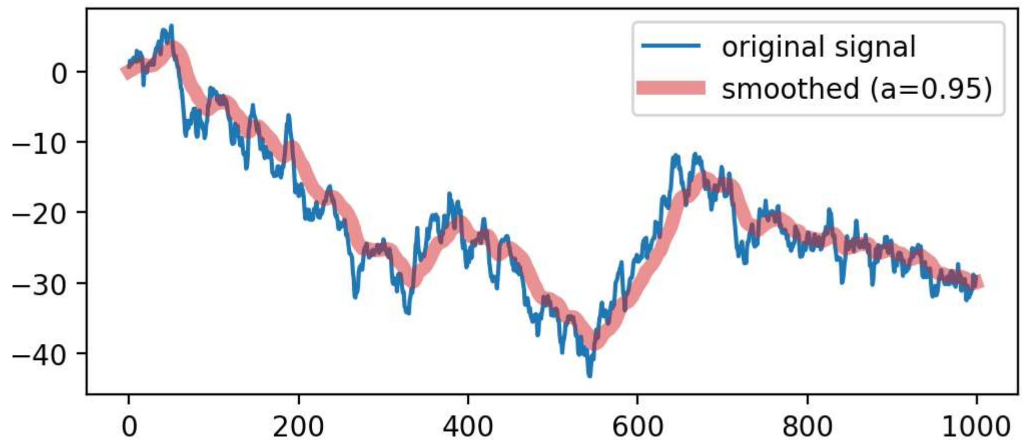
💥 Iteration (`for` loop) **cannot** be avoided.

Signal filtering in Julia 👌

```julia
function smooth(u, a)
    y = zeros(u)

    y[1] = (1-a)*u[1]
    for k=2:length(u)  # this loop is NOT slow!
        y[k] = a*y[k-1] + (1-a)*u[k]
    end

    return y
end
```



Performance of the signal filter

Implementation	Time for $10\,\mathrm{Mpts}$	Notes
Julia ✨	$50 - 70\,\mathrm{ms}$	**Fast! Easy!** 👌
Matlab 2017b*	$90 - 100\,\mathrm{ms}$	💵
Octave 4.2	$88\,000\,\mathrm{ms}$	**slow!!** 🐌🐌🐌
Scilab	$7\,800\,\mathrm{ms}$	**slow!!** 🐌🐌
Python native	$4\,400\,\mathrm{ms}$	**slow!** 🐌
SciPy's `lfilter`	$70\,\mathrm{ms}$	many lines of C
Python + `@numba.jit`	$50\,\mathrm{ms}$	since 2012

*on a VM which adds an overhead of 20% to 40%,
(so that 70 ms → 100 ms)

(Matlab and Python implementations)

```
function y = smooth(u, a)
    y = zeros(length(u),1);
    y(1) = (1-a)*u(1);
    for k=2:length(u)
        y(k) = a*y(k-1) + (1-a)*u(k);
    end
end
```

Python (optionally with Numba's `jit` decorator)

```
@numba.jit  # <- factor ×100 speed-up!
def smooth_jit(u, a):
    y = np.zeros_like(u)
    y[0] = (1-a)*u[0]
    for k in range(1, len(u)):
        y[k] = a*y[k-1] + (1-a)*u[k]
    return y
```

Conclusion on the performance

For this simple iterative computation:

- Julia performs very well, much better than native Python
- but it's possible to get the same with fresh Python tools (Numba)
- more realistic examples are needed

Fragility of Julia's JIT Compilation 💥

The efficiency of the compiled code relies on **type inference**.

```julia
function smooth1(u, a)
    y = 0
    for k=1:length(u)
        y = a*y + (1-a)*u[k]
    end
    return y
end
```

```julia
function smooth2(u, a)
    y = 0.0   # <- difference is here!
    for k=1:length(u)
        y = a*y + (1-a)*u[k]
    end
    return y
end
```

An order of magnitude difference 🐌 vs 🏃

```
julia> @time smooth1(u, 0.9);
  0.212018 seconds (30.00 M allocations: 457.764 MiB ...)
```

```
julia> @time smooth2(u, 0.9);
  0.024883 seconds (5 allocations: 176 bytes)
```

Fortunately, Julia gives a good diagnosis tool 🛠️

```
julia> @code_warntype smooth1(u, 0.9);
...  # ↓ we spot a detail
y::Union{Float64, Int64}
...
```

`y` is either `Float64` or `Int64` when it should be just `Float64`.

Cause: initialization `y=0` vs. `y=0.0`!

Ex. 2: Optimization in Julia

Objective: demonstrate **JuMP**, a Modeling Language for Optimization in Julia.

- Some researchers migrate to Julia just for this!
- I use JuMP for **my research** (energy management)

Note: you can find companion notebooks on GitHub

Optimization problem example

Example problem: identifying the sea clutter in Weather Radar data.

- is a **robust regression** problem
  - $\hookrightarrow$ is an optimization problem!

An « IETR-colored » example, inspired by:

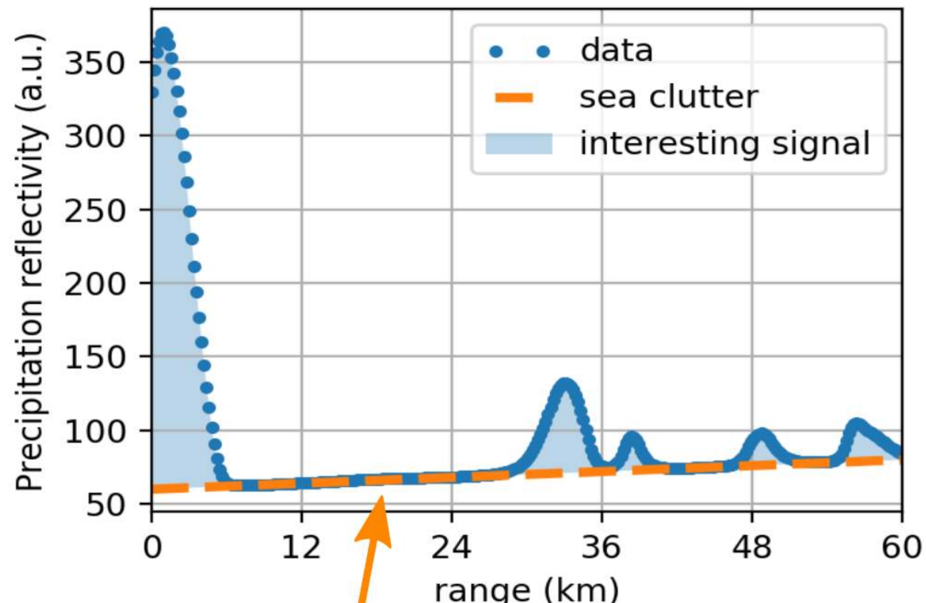
- Radar data+photo: P.-J. Trombe *et al.*, « Weather radars – the new eyes for offshore wind farms?,» *Wind Energy*, 2014.
- Regression methods: S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004. (Example 6.2).

Weather radar: the problem of sea clutter

Offshore wind farm (Horns Rev, Denmark)

Weather radar: X-band (9.41 GHz), 60 km range

Reflection data, for one azimuth



Objective: **fit this trend** (to substract it)

Given n data points (x_i, y_i), fit a linear trend:

$$\hat{y} = a.x + b$$

An **optimization problem** with two parameters: a (slope), b (intercept)

Regression as an optimization problem

The parameters for the trend (a, b) should minimize a criterion J which penalizes the residuals $r_i = y_i - \hat{y} = y_i - a.x + b$:

$$J(a, b) = \sum_i \phi(r_i)$$

where ϕ is the *penaly function*, to be chosen:

- $\phi(r) = r^2$: quadratic deviation $\longrightarrow$ least squares regression
- $\phi(r) = |r|$: absolute value deviation
- $\phi(r) = h(r)$: Huber loss
- ...

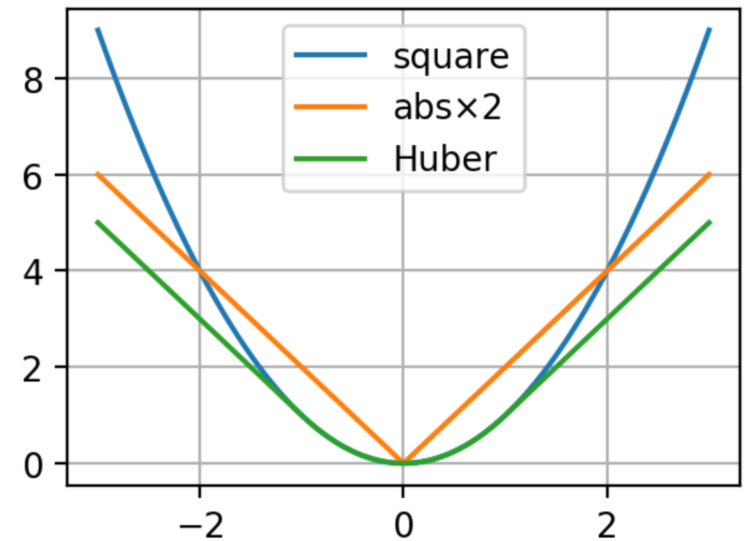
🔧 Choice of penalty function

The choice of the loss function influences:

- the optimization result (fit quality)
  - *e.g.*, in the presence of outliers
- the properties of optimization problem: convexity, smoothness

Properties of each function

- quadratic: convex, smooth, heavy weight for strong deviations
- absolute value: convex, not smooth
- Huber: a mix of the two



🛠 How to solve the regression problem?

Option 1: a big bag of tools

A specific package for each type of regression:

- « least square toolbox » ($\longrightarrow$ MultivariateStats.jl)
- « least absolute value toolbox » ($\longrightarrow$ quantile regression)
- « Huber toolbox » (*i.e.*, robust regression $\longrightarrow$??)
- ...

Option 2: the « One Tool »

$\Longrightarrow$ a **Modeling Language for Optimization**

- more **freedom to explore variants** of the problem

Modeling Languages for Optimization

*Purpose: make it easy to **specify** and **solve** optimization problems without expert knowledge.*

JuMP: optimization modeling in Julia

- The JuMP package offers a domain-specific modeling language for mathematical optimization.

JuMP **interfaces with many optimization solvers**: open-source (Ipopt, GLPK, Clp, ECOS...) and commercial (CPLEX, Gurobi, MOSEK...).

- Other Modeling Languages for Optimization:
  - Standalone software: AMPL, GAMS
  - Matlab: YALMIP (previous seminar), CVX
  - Python: Pyomo, PuLP, CVXPy

> Claim: JuMP is **fast**, thanks to Julia's metaprogramming capabilities (generation of Julia code within Julia code).

📈 Regression with JuMP — common part

- Given `x` and `y` the 300 data points:

```
m = Model(solver = ECOSSolver())

@variable(m, a)
@variable(m, b)

res = a*x .- y + b
```

`res` (« residuals ») is an Array of 300 elements of type `JuMP.GenericAffExpr{Float64,JuMP.Variable}`, *i.e.*, a semi-symbolic affine expression.

- Now, we need to specify the penalty on those residuals.

Regression choice: least squares regression

$$\min \sum_i r_i^2$$

Reformulated as a Second-Order Cone Program (SOCP):

$$\min j, \quad \text{such that } \|r\|_2 \leq j$$

```
@variable(m, j)
@constraint(m, norm(res) <= j)
@objective(m, Min, j)
```

(SOCP problem $\Longrightarrow$ ECOS solver)

Regression choice: least absolute deviation

$$\min \sum_i |r_i|$$

Reformulated as a Linear Program (LP)

$$\min \sum_i t_i, \quad \text{such that } -t_i \leq r_i \leq t_i$$

```
@variable(m, t[1:n])
@constraint(m, res .<= t)
@constraint(m, res .>= -t)
@objective(m, Min, sum(t))
```

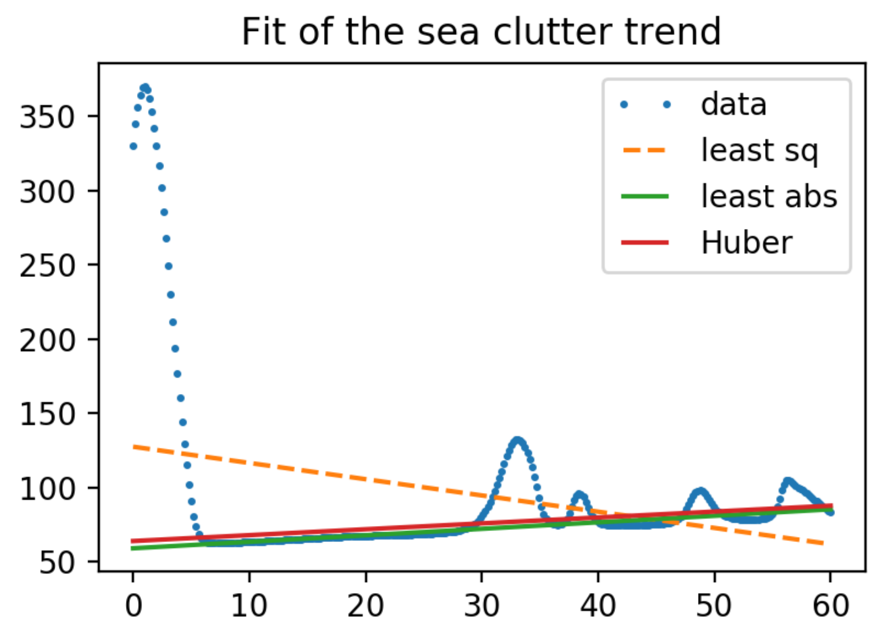
Solve! ⚙

```
julia> solve(m)
[solver blabla... ⌛ ]
:Optimal  # hopefully
```

```
julia> getvalue(a), getvalue(b)
(-1.094, 127.52)  # for least squares
```

Observations:

- least abs. val., Huber ✅
- least squares ❎



JuMP: summary

A modeling language for optimization, *within Julia*:

- gives access to all classical optimization solvers
- very fast (claim)
- gives freedom to explore many variations of an optimization problem (fast prototyping)

More on optimization with Julia:

- JuliaOpt: host organization of JuMP
- Optim.jl: implementation of classics in Julia (*e.g.*, Nelder-Mead)
- JuliaDiff: Automatic Differentiation to compute gradients, thanks to Julia's strong capability for code introspection

Conclusion (1/2)

Sum-up

- I hope you got a good introduction to Julia 👌
- It's not hard to migrate from MATLAB to Julia
- Good start: `docs.JuliaLang.org/en/stable/manual/getting-started`
- Julia is fast!
- Free and open source!
- Can be very efficient for some applications!

Conclusion (2/2)

> *Thanks for joining* 👏 *!*

Your mission, if you accept it... 💥

1. 👶 *Padawan level:* Train yourself a little bit on Julia
   ↪ `JuliaBox.com` ? Or install it on your laptop!
   And read introduction in the Julia manual!
2. 👩‍🎓 *Jedi level:* Try to solve a numerical system, from your research or teaching, **in Julia instead of MATLAB**
3. ⚔️ *Master level:* From now on, try to use open-source & free tools for your research (Julia, Python and others)… 🤑

Thank you !!