1 Nomenclature

1.1 Indices and Sets

$g \in \mathcal{G}$	Set of thermal generators.
$g \in \mathcal{G}^0_{on}$	Set of thermal generators which are initially committed (on).
$g \in \mathcal{G}^0_{off}$	Set of thermal generators which are not initially committed (off).
$w \in \mathcal{W}$	Set of renewable generators.
$t \in \mathcal{T}$	Hourly time steps: $1, \ldots, T$, $T =$ `time_periods`
$l \in \mathcal{L}_g$	Piecewise production cost intervals for thermal generator g: $1, \ldots, L_g$.
$s \in \mathcal{S}_g$	Startup categories for thermal generator g, from hottest (1) to coldest (S_g): $1, \ldots, S_g$.

1.2 System Parameters

$D(t)$	Load (demand) at time t (MW), `demand`.
$R(t)$	Spinning reserve at time t (MW), `reserves`.

1.3 Thermal Generator Parameters

CS^s_g	Startup cost in category s for generator g (\$), `startup['cost']`.
CP^l_g	Cost of operating at piecewise generation point l for generator g (MW), `piecewise_production['cost']`.
DT_g	Minimum down time for generator g (h), `time_down_minimum`.
DT^0_g	Number of time periods the unit has been off prior to the first time period for generator g, `time_down_t0`.
$\overline{P}_g$	Maximum power output for generator g (MW), `power_output_maximum`.
$\underline{P}_g$	Minimum power output for generator g (MW), `power_output_minimum`.
P^0_g	Power output for generator g (MW) in the time period prior to t=1, `power_output_t0`.
P^l_g	Power level for piecewise generation point l for generator g (MW); $P^1_g = \underline{P}_g$ and $P^{L_g}_g = \overline{P}_g$, `piecewise_production['mw']`.
RD_g	Ramp-down rate for generator g (MW/h), `ramp_down_limit`.
RU_g	Ramp-up rate for generator g (MW/h), `ramp_up_limit`.
SD_g	Shutdown capability for generator g (MW), `ramp_shutdown_limit`.
SU_g	Startup capability for generator g (MW), `ramp_startup_limit`
TS^s_g	Time offline after which the startup category s becomes active (h), `startup['lag']`.
UT_g	Minimum up time for generator g (h), `time_up_minimum`.
UT^0_g	Number of time periods the unit has been on prior to the first time period for generator g, `time_up_t0`.
U^0_g	Initial on/off status for generator g, $U^0_g = 1$ for $g \in \mathcal{G}^0_{on}$, $U^0_g = 0$ for $g \in \mathcal{G}^0_{off}$, `unit_on_t0`.
U_g	Must-run status for generator g, `must_run`.

1.4 Renewable Generator Parameters

$\overline{P}_w(t)$	Maximum renewable generation available from renewable generator w at time t (MW), `power_output_maximum`.
$\underline{P}_w(t)$	Minimum renewable generation available from renewable generator w at time t (MW), `power_output_minimum`.

1.5 Variables

$c_g(t)$	Cost of power produced above minimum for thermal generator g at time t (MW), $\in \mathbb{R}$.
$p_g(t)$	Power above minimum for thermal generator g at time t (MW), ≥ 0.
$p_w(t)$	Renewable generation used from renewable generator w at time t (MW), ≥ 0.
$r_g(t)$	Spinning reserves provided by thermal generator g at time t (MW), ≥ 0.
$u_g(t)$	Commitment status of thermal generator g at time t, $\in \{0, 1\}$.

$v_g(t)$	Startup status of thermal generator g at time t, $\in \{0,1\}$.
$w_g(t)$	Shutdown status of thermal generator g at time t, $\in \{0,1\}$.
$\delta_g^s(t)$	Startup in category s for thermal generator g at time t, $\in \{0,1\}$.
$\lambda_g^l(t)$	Fraction of power from piecewise generation point l for generator g at time t (MW), $\in [0,1]$.

2 Model Description

Below we describe the unit commitment model given by [1], with the piecewise production cost description from [2]. The unit commitment problem can then be formulated as:

$$\min \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} \left(c_g(t) + CP_g^1\, u_g(t) + \sum_{s=1}^{S_g} \left(CS_g^s \delta^s(t) \right) \right) \tag{1}$$

subject to:

$$\sum_{g\in\mathcal{G}} \left(p_g(t) + \underline{P}_g u_g(t) \right) + \sum_{w\in\mathcal{W}} p_w(t) = D(t) \qquad \forall t \in \mathcal{T} \tag{2}$$

$$\sum_{g\in\mathcal{G}} r_g(t) \geq R(t) \qquad \forall t \in \mathcal{T} \tag{3}$$

$$\sum_{t=1}^{\min\{UT_g - UT_g^0, T\}} (u_g(t) - 1) = 0 \qquad \forall g \in \mathcal{G}_{on}^0 \tag{4}$$

$$\sum_{t=1}^{\min\{DT_g - DT_g^0, T\}} u_g(t) = 0 \qquad \forall g \in \mathcal{G}_{off}^0 \tag{5}$$

$$u_g(1) - U_g^0 = v_g(1) - w_g(1) \qquad \forall g \in \mathcal{G} \tag{6}$$

$$\sum_{s=1}^{S_g - 1} \sum_{t=\max\{1, TS_g^{s+1} - DT_g^0 + 1\}}^{\min\{TS_g^{s+1} - 1, T\}} \delta_g^s(t) = 0 \qquad \forall g \in \mathcal{G} \tag{7}$$

$$p_g(1) + r_g(1) - U_g^0(P_g^0 - \underline{P}_g) \leq RU_g \qquad \forall g \in \mathcal{G} \tag{8}$$

$$U_g^0(P_g^0 - \underline{P}_g) - p_g(1) \leq RD_g \qquad \forall g \in \mathcal{G} \tag{9}$$

$$U_g^0(P_g^0 - \underline{P}_g) \leq (\overline{P}_g - \underline{P}_g)U_g^0 - \max\{(\overline{P}_g - SD_g), 0\} w_g(1) \qquad \forall g \in \mathcal{G} \tag{10}$$

$$u_g(t) \geq U_g \qquad \forall t \in \mathcal{T},\ \forall g \in \mathcal{G} \tag{11}$$

$$u_g(t) - u_g(t-1) = v_g(t) - w_g(t) \qquad \forall t \in \mathcal{T} \setminus \{1\},\ \forall g \in \mathcal{G} \tag{12}$$

$$\sum_{i=t-\min\{UT_g, T\}+1}^{t} v_g(i) \leq u_g(t) \qquad \forall t \in \{\min\{UT_g, T\} \ldots, T\},\ \forall g \in \mathcal{G} \tag{13}$$

$$\sum_{i=t-\min\{DT_g. T\}+1}^{t} w_g(i) \leq 1 - u_g(t) \qquad \forall t \in \{\min\{DT_g, T\}, \ldots, T\},\ \forall g \in \mathcal{G} \tag{14}$$

$$\delta_g^s(t) \leq \sum_{i=TS_g^s}^{TS_g^{s+1}-1} w_g(t-i) \qquad \forall t \in \{TS_g^{s+1}, \ldots, T\},\ \forall s \in \mathcal{S}_g \setminus \{S_g\},\ \forall g \in \mathcal{G} \tag{15}$$

$$v_g(t) = \sum_{s=1}^{S_g} \delta_g^s(t) \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \tag{16}$$

$$p_g(t) + r_g(t) \leq (\overline{P}_g - \underline{P}_g)u_g(t) - \max\{(\overline{P}_g - SU_g), 0\}v_g(t) \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \tag{17}$$

$$p_g(t) + r_g(t) \leq (\overline{P}_g - \underline{P}_g)u_g(t) - \max\{(\overline{P}_g - SD_g), 0\}w_g(t+1) \qquad \forall t \in \mathcal{T} \setminus \{T\}, \forall g \in \mathcal{G} \tag{18}$$

$$p_g(t) + r_g(t) - p_g(t-1) \leq RU_g \qquad \forall t \in \mathcal{T} \setminus \{1\}, \forall g \in \mathcal{G} \tag{19}$$

$$p_g(t-1) - p_g(t) \leq RD_g \qquad \forall t \in \mathcal{T} \setminus \{1\}, \forall g \in \mathcal{G} \tag{20}$$

$$p_g(t) = \sum_{l \in \mathcal{L}_g} (P_g^l - P_g^1)\lambda_g^l(t) \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \tag{21}$$

$$c_g(t) = \sum_{l \in \mathcal{L}_g} (CP_g^l - CP_g^1)\lambda_g^l(t) \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \tag{22}$$

$$u_g(t) = \sum_{l \in \mathcal{L}_g} \lambda_g^l(t) \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G} \tag{23}$$

$$\underline{P}_w(t) \leq p_w(t) \leq \overline{P}_w(t) \qquad \forall t \in \mathcal{T}, \forall w \in \mathcal{W} \tag{24}$$

Note that in constraints (4), (5), and (7), we use the convention that empty sums are 0.

References

[1] Morales-España, G., Latorre, J. M., and Ramos, A. Tight and compact MILP formulation for the thermal unit commitment problem. *IEEE Transactions on Power Systems 28*, 4 (2013), 4897–4908.

[2] Sridhar, S., Linderoth, J., and Luedtke, J. Locally ideal formulations for piecewise linear functions with indicator variables. *Operations Research Letters 41*, 6 (2013), 627–632.