**1.4** Consider an infinitely thin flat plate with a 1 m chord at an angle of attack of 10° in a supersonic flow. The pressure and shear stress distributions on the upper and lower surfaces are given by $p_u = 4 \times 10^4(x-1)^2 + 5.4 \times 10^4$, $p_l = 2 \times 10^4(x-1)^2 + 1.73 \times 10^5$, $\tau_u = 288x^{-0.2}$, and $\tau_l = 731x^{-0.2}$, respectively, where $x$ is the distance from the leading edge in meters and $p$ and $\tau$ are in newtons per square meter. Calculate the normal and axial forces, the lift and drag, moments about the leading edge, and moments about the quarter chord, all per unit span. Also, calculate the location of the center of pressure.

$$N' = \int_0^c (p_l - p_u)\,dx = \int_0^1 \left(-2\times10^4(x-1)^2 + 1.19\times10^5\right)dx.$$

$$\Rightarrow N' = 1.12\times10^5\,\text{N}.$$

$$A' = \int_0^c (\tau_l + \tau_u)\,dx = \int_0^1 \left(731x^{-0.2} + 288x^{-0.2}\right)dx$$

$$\Rightarrow A' = 1274\,\text{N}.$$

$$L' = N'\cos\alpha - A'\sin\alpha = 1.12\times10^5\times\cos10^\circ - 1274\times\sin10^\circ = 1.10\times10^5\,\text{N}.$$

$$D' = N'\sin\alpha + A'\cos\alpha = 1.12\times10^5\times\sin10^\circ + 1274\times\cos\alpha$$

$$= 2.07\times10^4\,\text{N}.$$

$$M'_{LE} = \int_0^c (p_u - p_l)\,x\,dx = \int_0^1 \left(2\times10^4(x-1)^2 - 1.19\times10^5\right)x\,dx$$

$$\Rightarrow -5.78\times10^4\,\text{N}\cdot\text{m}.$$

$$M'_{c/4} = M'_{LE} + L'\left(\frac{c}{4}\right) = -5.78\times10^4 + 1.105\times10^5(0.25)$$

$$= -3.02\times10^4 \, N/m.$$

$$x_{cp} = -\frac{M'_{LE}}{N'} = -\frac{(-5.78\times10^4)}{1.12\times10^5} = 0.516\,m.$$

coefficients.

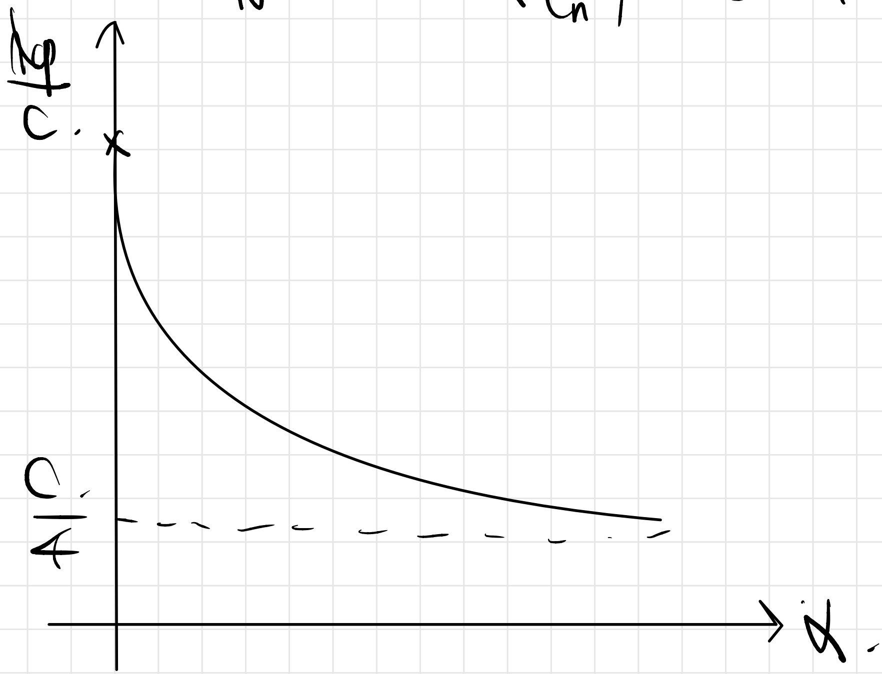
**1.6** Consider an NACA 2412 airfoil (the meaning of the number designations for standard NACA airfoil shapes is discussed in Chapter 4). The following is a tabulation of the lift, drag, and moment coefficients about the quarter chord for this airfoil, as a function of angle of attack.

| $\alpha$ (degrees) | $c_l$ | $c_d$ | $c_{m,c/4}$ |
|---|---|---|---|
| −2.0 | 0.05 | 0.006 | −0.042 |
| 0 | 0.25 | 0.006 | −0.040 |
| 2.0 | 0.44 | 0.006 | −0.038 |
| 4.0 | 0.64 | 0.007 | −0.036 |
| 6.0 | 0.85 | 0.0075 | −0.036 |
| 8.0 | 1.08 | 0.0092 | −0.036 |
| 10.0 | 1.26 | 0.0115 | −0.034 |
| 12.0 | 1.43 | 0.0150 | −0.030 |
| 14.0 | 1.56 | 0.0186 | −0.025 |

From this table, plot on graph paper the variation of $x_{cp}/c$ as a function of $\alpha$.

$C_n = C_l \times \cos\alpha + C_d \times \sin\alpha.$

$$x_{cp} = \frac{c}{4} - \frac{M'_{c/4}}{N'} = \frac{c}{4} - c \times \left(\frac{C_{m(\frac{c}{4})}}{C_n}\right) \Rightarrow \frac{x_{cp}}{c} = \frac{1}{4} - \left(\frac{C_m}{C_n}\right).$$



**1.9** Consider two different flows over geometrically similar airfoil shapes, one airfoil being twice the size of the other. The flow over the smaller airfoil has freestream properties given by $T_\infty = 200$ K, $\rho_\infty = 1.23$ kg/m$^3$, and $V_\infty = 100$ m/s. The flow over the larger airfoil is described by $T_\infty = 800$ K, $\rho_\infty = 1.739$ kg/m$^3$, and $V_\infty = 200$ m/s. Assume that both $\mu$ and $a$ are proportional to $T^{1/2}$. Are the two flows dynamically similar?

$\mu \propto T^{\frac{1}{2}} \quad a \propto T^{\frac{1}{2}}$

$$\frac{M_1}{M_2} = \frac{V_1/a_1}{V_2/a_2} = \frac{V_1}{V_2} \times \sqrt{\frac{T_2}{T_1}} = \frac{100}{200} \times \sqrt{\frac{800}{200}} = 1.$$

$$\frac{Re_1}{Re_2} = \frac{\rho_1 V_1 c_1/\mu_1}{\rho_2 V_2 c_2/\mu_2} = \frac{\rho_1 V_1 c_1}{\rho_2 V_2 c_2} \times \sqrt{\frac{T_2}{T_1}} = \left(\frac{1.23}{1.739}\right)\left(\frac{100}{200}\right)\left(\frac{1}{2}\right)\sqrt{\frac{800}{200}}$$

$= 0.354 \ (\neq 1)$. $\therefore$ Not dynamically similar.

**1.11** A U-tube mercury manometer is used to measure the pressure at a point on the wing of a wind-tunnel model. One side of the manometer is connected to the model, and the other side is open to the atmosphere. Atmospheric pressure and the density of liquid mercury are $1.01 \times 10^5$ N/m$^2$ and $1.36 \times 10^4$ kg/m$^3$, respectively. When the displacement of the two columns of mercury is 20 cm, with the high column on the model side, what is the pressure on the wing?

$$P_b = P_a - \rho g \Delta h = 1.01\times10^5 - (1.36)(10^4)(9.8)(0.2)$$

$$P_b = 7.43 \times 10^4 \text{ N/m}^2$$

column on the model side, what is the pressure on the wing?

**1.12** The German Zeppelins of World War I were dirigibles with the following typical characteristics: volume = 15,000 m$^3$ and maximum diameter = 14.0 m. Consider a Zeppelin flying at a velocity of 30 m/s at a standard altitude of 1000 m (look up the corresponding density in Appendix D). The Zeppelin is at a small angle of attack such that its lift coefficient is 0.05 (based on the maximum cross-sectional area). The Zeppelin is flying in straight-and-level flight with no acceleration. Calculate the total weight of the Zeppelin.

$$B = \rho \cdot g \cdot V = (15000)(1.1117)(9.8) = 1.634 \times 10^5 \text{ N}$$

$$q_\infty = \frac{1}{2}\rho V^2 = \frac{1}{2}(1.1117)(30)^2 = 500 \text{ N/m}^2$$

$$S = \frac{\pi d^2}{4} = \frac{\pi (14)^2}{4} = 153.9 \text{ m}^2$$

$$L = q_\infty S C_L = (500)(153.9)(0.05) = 3847 \text{ N}$$

$$W = 1.634 \times 10^5 + 3847 = 1.67 \times 10^5 \text{ N}$$

$$W = 1.67 \times 10^5 \text{ N}$$