Proceedings of the 36th
Conference on Decision & Control
San Diego, California USA • December 1997

A Full Block S-Procedure with Applications

Carsten W. Scherer[1]
Mechanical Engineering Systems and Control Group
Delft University of Technology
Mekelweg 2
2628 CD Delft, The Netherlands
c.scherer@wbmt.tudelft.nl

Abstract
In this paper we provide a general result that allows to equivalently translate robust performance analysis specifications characterized through a single quadratic Lyapunov function into the corresponding analysis test with multipliers. Just as an illustration we apply the technique to robust quadratic and robust generalized H_2 performance, and we comment on the wide range of its applicability. Finally, we reveal how this technique allows to approach LPV problems in which the control input and measurement output matrix are parameter dependent. The latter is made possible by letting the parameter enter the LPV controller via a kernel representation that generalizes the more conventional LFT structure.

1 A Full Block S-Procedure

Suppose $\mathcal{S}$ is a subspace of $\mathbb{R}^n$, $T \in \mathbb{R}^{l\times n}$ is a full row rank matrix, and $\mathbf{\Delta} \subset \mathbb{R}^{k\times l}$ is a compact set of matrices of full row rank. Define the family of subspaces

$$\mathcal{S}_\Delta := \mathcal{S} \cap \ker(\Delta T) = \{x \in \mathcal{S} : Tx \in \ker(\Delta)\}$$

indexed by $\Delta \in \mathbf{\Delta}$.

In the terminology of the behavioral approach, $\mathcal{S}$ is the system, T picks the interconnection variables that are constrained by the uncertainties, the elements of $\Delta \in \mathbf{\Delta}$ define kernel representations of the possible uncertainties, and $\mathcal{S}_\Delta$ is the perturbed system.

Suppose $N \in \mathbb{R}^{n\times n}$ is a fixed symmetric matrix. The goal is to render the implicit negativity condition

$$\forall \Delta \in \mathbf{\Delta}: \quad N < 0 \text{ on } \mathcal{S}_\Delta$$

explicit. We want to relate this property, under certain technical hypotheses, to the existence of a multiplier P that satisfies

$$N + T^T P T < 0 \text{ on } \mathcal{S} \quad \text{and} \quad P > 0 \text{ on } \ker(\Delta)$$

for all uncertainties $\Delta \in \mathbf{\Delta}$.

The required technical condition will be related to a certain well-posedness property; here it amounts to the complementarity of the subspace $\mathcal{S}_\Delta$ to a fixed subspace $\mathcal{S}_0 \subset \mathcal{S}$ that is sufficiently large. Moreover, the quadratic form N is supposed to be nonnegative on this subspace; in the applications we have in mind, this is a property on the performance index under consideration that is, interestingly enough, indispensable in reducing the underlying controller design problem to an LMI problem. To be precise, we require

$$\dim(\mathcal{S}_0) \geq k \quad \text{and} \quad N \geq 0 \text{ on } \mathcal{S}_0.$$

Theorem 1 *The condition*

$$\forall \Delta \in \mathbf{\Delta}: \ \mathcal{S}_\Delta \cap \mathcal{S}_0 = \{0\}, \ N < 0 \ \textit{on} \ \mathcal{S}_\Delta$$

holds iff there exists a matrix P that satisfies

$$\forall \Delta \in \mathbf{\Delta}: \begin{cases} N + T^T P T < 0 \ \textit{on } \mathcal{S} \\ P > 0 \ \textit{on } \ker(\Delta) \end{cases}$$

2 Application to Robust Performance Problems

Consider the first order image representation

$$\begin{pmatrix} \dot{x} \\ z_u \\ z_p \end{pmatrix} = \begin{pmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix} \begin{pmatrix} x \\ w_1 \\ w_2 \end{pmatrix} \tag{1}$$

of a system (with A Hurwitz) in L_2. We can assume w.l.o.g. that the third block column of the matrix has full column rank.

Here, w_1 and w_2 are latent variables; z_p are the variables on which we impose the performance specification, and z_u are the interconnection variables to let the parameters enter the system.

[1]I would like to thank Ted Iwasaki (Tokyo Institute of Technology) for helpful discussions.

We consider the linear parameter-varying (LPV) systems obtained as follows: they are parametrized by all continuous curves

$$\Delta : [0, \infty) \to \mathbf{\Delta}$$

with a given set of values

$$\mathbf{\Delta} \subset \mathbb{R}^{k \times l}$$

that captures both the size and the structure of the parameters. We assume that $\mathbf{\Delta}$ is compact and consists of full row rank matrices only. These parameter curves enter (1) via a kernel representation as

$$\Delta(t) z_u(t) = 0. \qquad (2)$$

We will clarify below that this generalizes the more standard LFT structure.

As a first property, we intend to characterize that the representation of LPV systems is *well-posed*:

$$\Delta D_{11} \text{ is nonsingular for every } \Delta \in \mathbf{\Delta}. \qquad (3)$$

In the case of well-posedness (3), we observe that the LPV system admits the alternative representation

$$\begin{pmatrix} \dot{x} \\ z_p \end{pmatrix} = \begin{pmatrix} A(\Delta(t)) & B(\Delta(t)) \\ C(\Delta(t)) & D(\Delta(t)) \end{pmatrix} \begin{pmatrix} x \\ w_2 \end{pmatrix} \qquad (4)$$

where

$$\begin{pmatrix} A(\Delta) & B(\Delta) \\ C(\Delta) & D(\Delta) \end{pmatrix} = \begin{pmatrix} A & B_2 \\ C_2 & D_{22} \end{pmatrix} + \\ + \begin{pmatrix} B_1 \\ D_{21} \end{pmatrix} (\Delta D_{11})^{-1} \Delta \begin{pmatrix} C_1 & D_{12} \end{pmatrix}.$$

Given the performance index P_p, the second goal is to guarantee uniform (in the uncertainty) robust exponential stability, and robust quadratic performance:

$$\int_0^\infty z_p(t)^T P_p z_p(t)\, dt \leq 0 \qquad (5)$$

holds for any trajectory of any of the LPV systems with $x(0) = 0$. Let us include the following technical hypotheses:

$$D_{11} \text{ has } k \text{ columns}, \quad D_{21}^T P_p D_{21} \geq 0.$$

The first property is obviously necessary for well-posedness; the second property holds for the standard H_∞ or positive real index and many others.

It is well-known and elementary to show that robust exponential stability and robust quadratic performance is guaranteed by the existence of some $X > 0$ such that

$$\forall \Delta \in \mathbf{\Delta} : * \begin{pmatrix} 0 & X & 0 \\ X & 0 & 0 \\ 0 & 0 & P_p \end{pmatrix} \begin{pmatrix} I & 0 \\ A(\Delta) & B(\Delta) \\ C(\Delta) & D(\Delta) \end{pmatrix} < 0. \quad (6)$$

(Note that throughout this paper we will employ the abbreviation $*PM$ for $M^T PM$.)

We will use Theorem 1 to equivalently reformulate this characterization as

$$* \begin{pmatrix} 0 & X & 0 & 0 \\ X & 0 & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P_p \end{pmatrix} \begin{pmatrix} I & 0 & 0 \\ A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix} < 0, \qquad (7)$$

where P is a *multiplier* that satisfies

$$\forall \Delta \in \mathbf{\Delta} : \quad P > 0 \text{ on } \ker(\Delta). \qquad (8)$$

Theorem 2 *Well-posedness (3) and (6) hold iff there exists a P with (7) and (8).*

Proof. We just apply Theorem 1 to

$$N = \begin{pmatrix} 0 & X & 0 & 0 \\ X & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_p \end{pmatrix}, \ T = \begin{pmatrix} 0 & 0 & I & 0 \end{pmatrix},$$

$$\mathcal{S} = \text{im} \begin{pmatrix} I & 0 & 0 \\ A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix}, \ \mathcal{S}_0 = \text{im} \begin{pmatrix} 0 \\ B_2 \\ D_{12} \\ D_{22} \end{pmatrix}.$$

■

Before commenting on this result, let us look at the robust generalized H_2 problem that offers an interesting additional insight into the solution of robust mixed problems.

Suppose T_2 and T_∞ are two matrices of full row rank whose number of columns equals the size of z_p. Then we intend to characterize that

$$||T_\infty z_p||_\infty \leq ||T_2 z_p||_2 \qquad (9)$$

holds for the whole family of systems (4) with $x(0) = 0$. If we have $T_2 z_p = w_2$, this property amounts to the gain of the mapping $L_2 \ni w_2 \to T_2 z_p \in L_\infty$ defined by (4) with $x(0) = 0$ being robustly not larger than one. This gain has been called generalized H_2-norm [15].

Let us assume that

$$T_\infty D(\Delta) = 0$$

what is indeed required to ensure $||T_\infty z_p||_\infty < \infty$ for all $w_2 \in L_2$ in (4). Then the following result is very easy to prove: If there exists an $X > 0$ such that, for all $\Delta \in \mathbf{\Delta}$,

$$* \begin{pmatrix} 0 & X & 0 \\ X & 0 & 0 \\ 0 & 0 & -T_2^T T_2 \end{pmatrix} \begin{pmatrix} I & 0 \\ A(\Delta) & B(\Delta) \\ C(\Delta) & D(\Delta) \end{pmatrix} < 0, \qquad (10)$$

$$\begin{pmatrix} I \\ C(\Delta) \end{pmatrix}^T \begin{pmatrix} -X & 0 \\ 0 & T_\infty^T T_\infty \end{pmatrix} \begin{pmatrix} I \\ C(\Delta) \end{pmatrix} < 0 \qquad (11)$$

then (4) is robustly exponentially stable and (9) holds for any system trajectory.

We end up with two inequalities in the parameter Δ. Therefore, we have to apply Theorem 1 to each of these inequalities individually, what leads to two independent multipliers to equivalently reformulate this test.

Theorem 3 *Suppose $T_2 D_{21} = 0$. Then well-posedness (3) and (10), (11) hold iff there exist multipliers P_1 and P_2 that both satisfy (8) and*

$$* \begin{pmatrix} 0 & X & 0 & 0 \\ X & 0 & 0 & 0 \\ 0 & 0 & P_1 & 0 \\ 0 & 0 & 0 & -T_2^T T_2 \end{pmatrix} \begin{pmatrix} I & 0 & 0 \\ A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{pmatrix} < 0$$

as well as

$$* \begin{pmatrix} -X & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & T_\infty^T T_\infty \end{pmatrix} \begin{pmatrix} I & 0 \\ C_1 & D_{11} \\ C_2 & D_{21} \end{pmatrix} < 0.$$

Remarks.

- The equivalences of these robust performance characterizations seem not to have appeared in the literature. They extend [14, 10] to robust performance problems for general LFT uncertainty descriptions. Comparable robust performance specifications with multipliers that are only indirectly described have been provided in [19, 16].
- It is an important structural insight that, in Theorem 2, the combined multiplier $\begin{pmatrix} P & 0 \\ 0 & P_p \end{pmatrix}$ for performance and parameter can be taken block-diagonal.
- If the parameter has a block-diagonal structure, the channel-wise application of the standard S-procedure leads from (6) to (7) with a block-diagonal scaling P. It is know that this step introduces conservatism. Using multipliers which are full and whose structure is not explicitly specified at the outset leads to a reformulation without conservatism. Therefore, we call the technique presented here a *full-block S-procedure.*
- An important aspect is the ease to proceed from (6) to (7) in a formal manner, just by referring to Theorem 1. Moreover, the derivation is not only straightforward, but leads to simple formulas that favorably compare with their sometimes pretty intricate counterparts in the literature.
- There are numerous further applications of the full block S-procedure that are currently under investigation. As most prominent ones, we mention that one can straightforwardly extend general robust mixed problems as proposed in [8, 12] to full block scalings what reduces conservatism; see also [19]. Moreover, the techniques apply to analysis problems with parameter dependent Lyapunov functions along the lines of [3].

3 Application to LPV Control

For the discussion of LPV control we concentrate on the quadratic performance specification with index P_p that is, in addition, non-singular. In contrast to robust control, in LPV control it is assumed that the parameter curve is on-line measurable. These design problems can be approached either by directly using the analysis test (6) [2, 4, 11] or by proceeding with the multiplier version (7) [13, 1, 5, 9, 18, 16].

The former suffers from the disadvantage that the matrices defining the control input and the measured output are not allowed to depend on the parameter. In the latter, usually a restricted class of structured scalings is employed. One of the main motivations for the full block S-procedure is to overcome these restrictions in LPV control.

Since we need dualization, the parameter dependent system is assumed to admit a slightly more special description as

$$\begin{pmatrix} \dot{x} \\ w_u \\ z_u \\ w_p \\ z_p \\ y \end{pmatrix} = \begin{pmatrix} A & B_1 & B_2 & B \\ 0 & I & 0 & 0 \\ C_1 & D_{11} & D_{12} & E_1 \\ 0 & 0 & I & 0 \\ C_2 & D_{21} & D_{22} & E_2 \\ C & F_1 & F_2 & 0 \end{pmatrix} \begin{pmatrix} x \\ w_u \\ w_p \\ u \end{pmatrix} \qquad (12)$$

with parameters entering as

$$\Delta(t) \begin{pmatrix} w_u(t) \\ z_u(t) \end{pmatrix} = 0. \qquad (13)$$

As usual, u denotes the control input variable and y the measured output variable.

We assume that the controller is described as

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c \begin{pmatrix} y \\ w_c \end{pmatrix} \\ \begin{pmatrix} u \\ z_c \end{pmatrix} &= C_c x_c + D_c \begin{pmatrix} y \\ w_c \end{pmatrix} \end{aligned} \tag{14}$$

and a specific parameter curve $\Delta(.)$ enters as

$$\Delta_c(\Delta(t)) \begin{pmatrix} w_c(t) \\ z_c(t) \end{pmatrix} = 0 \tag{15}$$

for a to be constructed scheduling function $\Delta_c : \mathbf{\Delta} \to \mathbb{R}^{k_c \times l_c}$.

The description of the controlled system is obtained by interconnecting the LTI systems (12), (14) to get

$$\begin{pmatrix} \dot{x} \\ w_u \\ z_u \\ w_c \\ z_c \\ w_p \\ z_p \end{pmatrix} = \begin{pmatrix} \mathcal{A} & \mathcal{B}_1 & \mathcal{B}_2 & \mathcal{B}_3 \\ 0 & I & 0 & 0 \\ \mathcal{C}_1 & \mathcal{D}_{11} & \mathcal{D}_{12} & \mathcal{D}_{13} \\ 0 & 0 & I & 0 \\ \mathcal{C}_2 & \mathcal{D}_{21} & \mathcal{D}_{22} & \mathcal{D}_{23} \\ 0 & 0 & 0 & I \\ \mathcal{C}_3 & \mathcal{D}_{31} & \mathcal{D}_{32} & \mathcal{D}_{33} \end{pmatrix} \begin{pmatrix} x \\ w_u \\ w_c \\ w_p \end{pmatrix} \tag{16}$$

and letting the parameters enter via (13), (15).

The LPV problem now reads as follows: Find an LTI controller (14) and a scheduling function Δ_c such that the controlled system (16), (13), (15) is robustly exponentially stable and robustly satisfies the performance specification (5).

Robust stability and robust performance is characterized through Theorem 2 by employing multipliers P that satisfy

$$\begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} > 0 \text{ on } \ker \begin{pmatrix} \Delta & 0 \\ 0 & \Delta_c(\Delta) \end{pmatrix} \tag{17}$$

for all $\Delta \in \mathbf{\Delta}$. Note that one can dualize this test [10, 16] to arrive at a formulation with the dual performance index $\tilde{P}_p$ and the dual multipliers $\tilde{P}$ that fulfill

$$\begin{pmatrix} \tilde{P}_{11} & \tilde{P}_{12} \\ \tilde{P}_{21} & \tilde{P}_{22} \end{pmatrix} < 0 \text{ on } \operatorname{im} \begin{pmatrix} \Delta^T & 0 \\ 0 & \Delta_c(\Delta)^T \end{pmatrix}.$$

The duality coupling for the performance index and the multipliers is $\tilde{P}_p = P_p^{-1}$ and $\tilde{P} = P^{-1}$ respectively.

The synthesis inequalities for the LPV problem at hand are obtained along standard lines [13, 1, 5, 9, 8, 16]: Start with the primal and dual analysis inequalities for the controller system which involve the Lyapunov matrices $\mathcal{X}$ and $\mathcal{X}^{-1}$. Then eliminate the controller parameters for which one requires to compute basis matrices Φ and Ψ of

$$\ker \begin{pmatrix} B^T & E_1^T & E_2^T \end{pmatrix}, \ \ker \begin{pmatrix} C & F_1 & F_2 \end{pmatrix}.$$

Due to the particular structure, the resulting two inequalities simplify considerably; one ends up with the LMIs

$$* \left(\begin{array}{cc|cc|cc} 0 & X & 0 & 0 & 0 & 0 \\ X & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & Q & S & 0 & 0 \\ 0 & 0 & S^T & R & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & Q_p & S_p \\ 0 & 0 & 0 & 0 & S_p^T & R_p \end{array}\right) \left(\begin{array}{ccc} I & 0 & 0 \\ A & B_1 & B_2 \\ \hline 0 & I & 0 \\ C_1 & D_{11} & D_{12} \\ \hline 0 & 0 & I \\ C_2 & D_{21} & D_{22} \end{array}\right) \Psi < 0 \tag{18}$$

$$* \left(\begin{array}{cc|cc|cc} 0 & Y & 0 & 0 & 0 & 0 \\ Y & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & \tilde{Q} & \tilde{S} & 0 & 0 \\ 0 & 0 & \tilde{S}^T & \tilde{R} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & \tilde{Q}_p & \tilde{S}_p \\ 0 & 0 & 0 & 0 & \tilde{S}_p^T & \tilde{R}_p \end{array}\right) \left(\begin{array}{ccc} A^T & C_1^T & C_2^T \\ -I & 0 & 0 \\ \hline B_1^T & D_{11}^T & D_{21}^T \\ 0 & -I & 0 \\ \hline B_2^T & D_{12}^T & D_{22}^T \\ 0 & 0 & -I \end{array}\right) \Phi > 0 \tag{19}$$

in the symmetric matrices X, Y coupled as

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} > 0 \tag{20}$$

and in

$$P_{11} = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}, \ \tilde{P}_{11} = \begin{pmatrix} \tilde{Q} & \tilde{S} \\ \tilde{S}^T & \tilde{R} \end{pmatrix}$$

which satisfy

$$\forall \Delta \in \mathbf{\Delta} : \begin{cases} P_{11} > 0 \text{ on } \ker(\Delta) \\ \tilde{P}_{11} < 0 \text{ on } \operatorname{im}(\Delta^T) \end{cases}. \tag{21}$$

We observe that other parts of the multipliers simply drop out and do not occur in this result. In this way one proves the necessity part of the following theorem.

Theorem 4 *There exists an LPV controller (14), (15) for (12), (13) such that the controlled system satisfies the condition for robust performance in Theorem 2 if and only if there exist X, Y, P_{11}, $\tilde{P}_{11}$ that fulfill (18)-(20) and (21).*

As an important novel aspect, we make no assumption on the multipliers. Indeed, this causes the main difficulties in proving the reverse direction by constructing a suitable LPV controller.

Let us briefly describe how to construct such a controller. For that purpose suppose that X, Y, P_{11}, $\tilde{P}_{11}$ satisfy the synthesis conditions.

The most difficult step in the construction is covered by the following theorem.

Theorem 5 *Suppose the matrices P_{11}, $\tilde{P}_{11}$ fulfill (21). Then there exists a continuous function $\Delta_c(\Delta)$ and an extension P_{12}, P_{21}, P_{22} such that*

$$P = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \text{ satisfies } P^{-1} = \begin{pmatrix} \tilde{P}_{11} & * \\ * & * \end{pmatrix}$$

and such that (17) holds for all $\Delta \in \mathbf{\Delta}$.

This results allows to find extended scalings and a suitable scheduling function. If we observe that (18)-(20) are nothing but the synthesis inequalities for the quadratic performance problem with index $\begin{pmatrix} P & 0 \\ 0 & P_p \end{pmatrix}$, the construction of the LTI part of the controller can be obtained by a standard nominal design procedure.

Remark. Note that the synthesis inequalities for designing a robust controller (w_c, z_c are absent) are given by (18)-(20), (21) including the duality coupling

$$\tilde{P}_{11} = P_{11}^{-1}.$$

This relation renders these conditions, as well-known, non-convex in the variables P_{11} and $\tilde{P}_{11}$.

Under specific structural hypotheses on the multipliers (as discussed below), this procedure has been followed in [16] and is worked out in full detail in [17]. A full discussion of the novel procedure including all the proofs will be available in a forthcoming paper.

Note that the multipliers in (21) are described by infinitely many linear matrix inequalities. They can be reduced to finitely many inequalities by gridding the parameter space $\mathbf{\Delta}$. Instead, however, we propose to constrain the scalings, possibly involving conservatism, such that one can exploit convexity in order to reduce the test to finitely many LMIs that are amenable to standard software.

Let us illustrate this technique and the benefit of the presented approach over existing ones by briefly resorting to the standard LFT description of uncertain systems; in that case the parameters enter as

$$w_u(t) = \delta(t) z_u(t) \tag{22}$$

where

$$\delta = \begin{pmatrix} \delta_1 I_1 & & 0 \\ & \ddots & \\ 0 & & \delta_m I_m \end{pmatrix}$$

with the $\dim(I_j)$ times repeated scalar parameters δ_j varying in $[-1, 1]$. Note that (22) can be written as

$$\begin{pmatrix} I & -\delta(t) \end{pmatrix} \begin{pmatrix} w_u(t) \\ z_u(t) \end{pmatrix} = 0$$

such that it nicely fits in our more general scenario. If we recall that

$$\ker \begin{pmatrix} I & -\delta \end{pmatrix} = \operatorname{im} \begin{pmatrix} \delta \\ I \end{pmatrix},$$

the constraints (21) on the scalings hence read as

$$\begin{pmatrix} \delta \\ I \end{pmatrix}^T \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} \delta \\ I \end{pmatrix} > 0 \tag{23}$$

and

$$\begin{pmatrix} I \\ -\delta^T \end{pmatrix}^T \begin{pmatrix} \tilde{Q} & \tilde{S} \\ \tilde{S}^T & \tilde{R} \end{pmatrix} \begin{pmatrix} I \\ -\delta^T \end{pmatrix} < 0 \tag{24}$$

for all $\delta_j \in [-1, 1]$. (Note that δ happens to be symmetric; since the generalization to a possibly non-symmetric structure δ is straightforward, we neglect this extra property.)

Let us now denote the extreme points of the set of all δ by δ^j. If we impose the strong constraint $Q < 0$ and $\tilde{R} > 0$ on parts of the multipliers, we infer that (23), (24) hold for all δ iff they hold for all extreme points δ^j. This is the situation considered in our previous work [16]. However, it is simple to relax the strong negativity/positivity condition by referring to a partial convexity argument. Indeed, if partitioning

$$Q = \begin{pmatrix} Q_1 & & * \\ & \ddots & \\ * & & Q_m \end{pmatrix}, \quad \tilde{R} = \begin{pmatrix} \tilde{R}_1 & & * \\ & \ddots & \\ * & & \tilde{R}_m \end{pmatrix}$$

according to δ, δ^T, we can work with the relaxed negativity/positivity constraint

$$Q_j < 0, \ \tilde{R}_j > 0 \tag{25}$$

imposed on the diagonal blocks only. This implies that the left-hand sides of (23), (24) are partially concave, convex functions of δ respectively. Hence one can again conclude that (23), (24) are satisfied for all δ iff they are are fulfilled for the extreme points δ^j.

The set of synthesis inequalities then consists of (18)-(20), (23)-(24) for the extreme points, and (25), such that the feasibility test amounts to a standard LMI problem.

In the talk we will provide an example which reveals that this relaxation can considerably reduce the conservatism.

4 Conclusion

We have given a general full block S-procedure in order to rewrite robust performance tests formulated in terms of a constant Lyapunov matrix into the corresponding multiplier test without conservatism. As an application, we have given a full solution to the corresponding LPV synthesis problem where the multipliers are in no way restricted. This has been made possible by proposing a novel scheme to schedule the LPV controller; the parameters are viewed to define a kernel representation of a static system that is interconnected with the LTI part of the controller.

References

[1] P. Apkarian and P. Gahinet, "A convex characterization of gain-scheduled $\mathcal{H}_\infty$ controllers," *IEEE Trans. Automat. Control*, vol. 40, pp. 853-864, 1995.

[2] P. Apkarian, P. Gahinet, and G. Becker, "Self-scheduled $\mathcal{H}_\infty$ control of linear parameter-varying systems," *Proc. Amer. Contr. Conf.*, pp. 856-860, 1994.

[3] P. Apkarian, E. Feron, and P. Gahinet, "A parameter-dependent Lyapunov approach to robust control with real parametric uncertainty", *Proc. European Contr. Conf.* 1995.

[4] G. Becker, A. Packard, D. Philbrick, and G. Balas, "Control of parametrically-dependent linear systems: A single quadratic Lyapunov approach," *Proc. Amer. Contr. Conf., San Francisco, CA*, pp. 2795-2799, 1993.

[5] G. Becker, A. Packard, "Robust performance of linear parametrically varying systems using parametrically dependent linear feedback," *Systems Control Lett.*, vol. 23, pp. 205-215, 1994.

[6] S.P. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishan, *Linear Matrix Inequalities in Systems and Control Theory.* SIAM Studies in Applied Mathematics 15, SIAM, Philadelphia 1994.

[7] P. Gahinet, A. Nemirovskii, A.J. Laub, and M. Chilali, "The LMI control toolbox," *Proc. 33rd IEEE Conf. Decision Contr.*, pp. 2038-2041, 1994.

[8] L. El Ghaoui and J.P. Folcher, "Multiobjective robust control of LTI systems subject to unstructured perturbations," *Systems Control Lett.*, vol. 28, pp. 23-30, 1996.

[9] A. Helmersson, *Methods for Robust Gain-Scheduling,* Ph.D. Thesis, Linköping University, Sweden, 1995.

[10] T. Iwasaki, S. Hara and T. Asai, "Well-posedness theorem: A classification of LMI/BMI-reducible robust control problems," preprint, 1995.

[11] I.E. Köse, F. Jabbari, W.E. Schmittendorf, "A direct characterization of L_2-gain controllers for LPV systems," *Proc. IEEE Conf. Decision Contr.*, pp. 3990-3995, 1996.

[12] I. Masubuchi, A. Ohara, N. Suda, "Robust multiobjective controller design via convex optimization," *Proc. IEEE Conf. Decision Contr.*, pp. 263-264, 1996.

[13] A. Packard, "Gain-scheduling via linear fractional transformations," *Systems Control Lett.*, vol. 22, pp. 79-92, 1994.

[14] A. Rantzer and A. Megretski, "System analysis via integral quadratic constraints," *Proc. IEEE Conf. Decision Contr.*, pp. 3062-3067, 1994.

[15] M.A. Rotea, "Generalized $\mathcal{H}_2$ control," *Automatica*, vol. 29, pp. 373-385, 1993.

[16] C.W. Scherer, "Robust Generalized H_2 Control for Uncertain and LPV Systems with General Scalings," *Proc. IEEE Conf. Decision Contr.*, pp. 3970-3975, 1996.

[17] C.W. Scherer, "Robust mixed control and LPV control with full block scalings," preprint, 1997.

[18] G. Scorletti, L. El Ghaoui, "Improved linear matrix inequality conditions for gain scheduling," *Proc. IEEE Conf. Decision Contr.*, pp. 3626-3631, 1995.

[19] H. Tokunaga, T. Iwasaki, S. Hara, "Multiobjective robust control with transient specifications," *Proc. IEEE Conf. Decision Contr.*, pp. 3482-3483, 1996.