Proceeding of the 11th World Congress on Intelligent Control and Automation
Shenyang, China, June 29 - July 4 2014

A Literature Review of UAV 3D Path Planning*

Liang Yang, Juntong Qi
State Key Laboratory of Robotics
Shenyang Institute of Automation
Chinese Academy of Sciences
Shenyang 110016, China
University of Chinese Academy of Sciences
Beijing 100049, China
{yangliang1,qijt}@sia.cn

Jizhong Xiao
Department of Electrical Engineering
City University of New York,
Convent Avenue and 140th Street,
New York, USA
jxiao@ccny.cuny.edu

Xia Yong
Benxi Electric Power Supply Company
State Grid Liaoning Electric Power Company
Benxi, Liaoning Province, P.R.China
yongxias@163.com

***Abstract* –3D path planning of unmanned aerial vehicle (UAV) targets at finding an optimal and collision free path in a 3D cluttered environment while taking into account the geometric, physical and temporal constraints. Although a lot of works have been done to solve UAV 3D path planning problem, there lacks a comprehensive survey on this topic, let alone the recently published works that focus on this field. This paper analyses the most successful UAV 3D path planning algorithms that developed in recent years. This paper classifies the UAV 3D path planning methods into five categories, sampling-based algorithms, node-based algorithms, mathematical model based algorithms, Bio-inspired algorithms, and multi-fusion based algorithms. For each category a critical analysis and comparison is given. Furthermore a comprehensive applicable analysis for each kind of method is presented after considering its working mechanism and time complexity.**

Key Works – UAV 3D Path Planning, Sampling Based Algorithms, Node Based Algorithms, Multi-fusion Based Algorithms, Mathematic Model Based Algorithms.

I. INTRODUCTION

Unmanned aerial vehicles, which have the capability of vertical take-off and landing and high maneuverability, are widely used as platforms to work in various environments. When defining a mission, path planning is the crucial element of whole system. Generally speaking, path planning targets at generating a real-time global path to the target, avoiding collisions with obstacles, and optimizing a given cost function under kinodynamic constraints. The problems that path planning concerns are also what the UAV's mission must considers.

Simple 2D path planning algorithms are not able to deal with complex 3D environments, where there are quite a lot of structures constraints and uncertainties. Thus 3D path planning algorithms for UAV navigation are urgently needed nowadays, especially in complex environments such as forest, cave, and urban areas as shown in Fig.1.

Path planning in 3D environments has great potential, but unlike 2D path planning, the difficulties increase exponentially with dynamic constraints and kinematic constraints becoming much more complex. In order to plan a collision free path through the cluttered environment, a set of mathematic tools are needed to model these constraints and to store such data. From the optimization theory point of view, finding a 3D complete path is NP-hard problem, thus there exist no common solutions.

(a)Forest[25] (b)cave (c)urban area[26]
Fig.1. Examples of 3D complex environments

Over the past decades, a couple of methods are proposed to solve such problems. Algorithms implemented in 3D environments includes Visibility Graph [1] which is developed from computer science; randomly sampling search algorithms such as Rapidly-exploring Random Tree [2], and Probabilistic Roadmap [3]; optimal search algorithms like Dijsktra's algorithm [4], A* [5], and D* [6]; bio-inspired planning algorithms and etc. What must be emphasized is that this paper only pays attention to broadly applicable methods, others such as manifold based algorithm [7], which generates smooth trajectory, is ignored due to the point that it is only applicable to rigid body robots without considering aerodynamical or hydromechanical influences.

This paper discusses mainly on the efficiency of each algorithm, and suggests which one is the optimal algorithm. In 3D cluttered environments, the goal of path planning algorithms is not only to find a collision free path, but also to minimize the travel length or energy consumption. Path planning in 3D environment may face much more uncertainties, thus all these factors should be taken into consideration to get a realistic path that adaptable to different uncertainties. Local minimal and global optimal are also discussed in this paper.

This paper is organized as the following. Section 2 discusses some controversial points which need to be clarified, and gives problem definitions for further discussion. Section 3 explains the taxonomy of 3D path planning algorithms and gives a detailed analysis which includes the

*This work was supported in part by the National High Technology Research and Development Program of China under the grant 2012 AA041501, National Natural Science Foundation of China under the grant 61273025. The corresponding author is Jizhong Xiao (jxiao@ccny.cuny.edu).

978-1-4799-5825-2/14/$31.00 ©2014 IEEE

basic concept of each category and also the list of algorithms in each category. Section 4 talks about the characteristics of each kind of method, and analyzes the applicability. The last section summarizes the paper and sheds light on further research.

II. PRELIMINARY MATERIAL

This section presents some preliminary material such as the definition of path planning and what is the different between path planning and trajectory planning. The distinction is never addressed explicitly by the majority of the path planning literature.

A. Problem Statement

Unmanned aerial vehicles (UAVs), which have the ability to fly autonomously without human's help, are widely used. When facing with complex situations in outdoor or indoor environments, they need a path planner to determine where they should go to fulfil their mission. It is important to define what is path planning. Based on the works in [8,9,10,11], we here give a clear definition of 3D path planning of UAVs.

In this paper UAVs are assumed to fly in a three-dimension (R^3) space, called the workspace w . The work space will often have obstacles, let wo_i be the ith obstacle. The free workspace without obstacles is the overall area represented by $w_{free} = w \setminus \cup_i wo_i$. The initial point x_{init} is an element in w_{free} , and the goal region x_{goal} is also an element in w_{free} . Thus a path planning problem is defined by a triplet $(x_{init}, x_{goal}, w_{free})$, and the following definitions are given:

Definition 1-Path Planning: Given a function $\delta : [0,T] \to R^3$ of bounded variation, where $\delta(0) = x_{init}$ and $\delta(T) = x_{goal}$. If there exist a process Φ which can guarantee $\delta(\tau) \in w_{free}$, for all $\tau \in [0,T]$, then Φ is called Path Planning.

Definition 2-Optimal Path Planning: Given a path planning problem $(x_{init}, x_{goal}, w_{free})$ and a cost function $c : \sum \to R \geq 0$ ($\sum$ denote the set of all paths), if a process fulfil definition 1 to find a path δ', and $c(\delta')$=min{ $c(\delta)$, δ is the set of all feasible path}, then δ' is the optimal path and Φ' is optimal path planning.

B. Path Planning and Trajectory Planning

Path planning and Trajectory planning are two distinct problems in robotics, but they are intimately related. There exist several works [9, 12] that explicitly differentiate the two concepts. Synthesizing all the corresponding knowledge together, this paper supports the following definition for further discussion.

Definition 3 – Trajectory planning usually refers to the problem of taking the solution from a robot path planning algorithm and determining how to move along the path. The **path** can be either a continuous curve or discrete line segments in w_{free} that connecting the start node x_{init} to the goal end node x_{goal} . When the path is parameterized by time t, then it is a **trajectory**, which can be described mathematically as a twice-differentiable polynomial, i.e., the velocities and accelerations can be computed by taking the first and second derivatives with respect to time. Trajectory planning is to find smooth and continuous trajectory segments to move along the path.

III. UAV 3D PATH PLANNING ALGORITHM TAXONOMY

A lot of UAV 3D path planning algorithms have been proposed in recent years. This paper filtrates a large number of outstanding papers and books, some of them are listed in the reference and others are not (due to space limitations). Some of the representative algorithms are Rapidly-exploring Random Trees (RRT), Probabilistic Roadmaps(PRM), Artificial Potential Field [13], and etc. Fig. 2 shows the taxonomy of current approaches of 3D path planning algorithms.

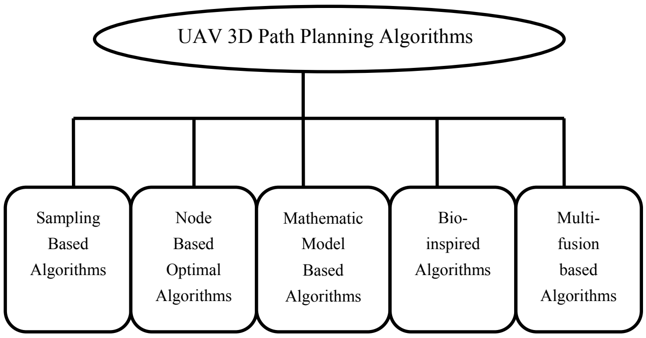

Fig.2. 3D path planning algorithm taxonomy

This paper divides all UAV 3D path planning algorithms into five categories and each category contains a various of methods which conform to certain characteristics. The following sub-sections show the detailed methods of each category, and this paper discusses the characteristics of each category.

A. Sampling Based Algorithms

This method needs some pre-known information of the workspace where the robots operate. It usually sample the environment as a set of nodes, or other forms, then map the environment or just search randomly to find an optimal path.

This paper classifies the sampling based algorithms as two sub-categories, **Passive** and **Active**. Active means algorithm like Rapidly-exploring Random Trees can form a skeleton to the goal all by its own processing procedure. Passive means algorithms like Probabilistic Roadmaps (PRM) really can generate a road net map from start node to the goal, but there exist a set of paths, thus a combination of search algorithms is needed to fulfil the task. According to this, this paper sorts out a set of algorithms which cannot independently generate a single path, and classifies them as passive.

For each sub-category, passive contains the elements like 3D voronoi [14], Rapidly-exploring Random Graph [15], PRM, K-PRM [11], S-PRM [10], Visibility Graphs, Corridor Map[16], and etc; Active includes the elements such as RRT, Dynamic Domain RRT(DDRRT) [17], RRT-Star(RRT*) [18], Artificial Potential Field, and etc.

This paper regards the similar algorithms, such as RRT, RRT*, DDRRT, and RRG, as a series. Thus four series can be synthesized, and they are RRT series, Artificial Potential Field series, Voronoi series and PRM series. For each series, the basic version cannot generate optimal path globally.

Yang [2] propounded a collision free path using RRT, but RRT has no re-planning procedure and optimizing capability. Thus improved version such as RRT*, DDRRT, and RRG are proposed to solve this. Xiao [3] proposed an improved PRM and combined it with A* to solve the problem that PRM cannot generate an optimal path by itself. Voronoi is a map construction method that cannot lead to generate an optimal path, Liu [38] solved this problem by integrating with node based algorithms. Artificial potential field method has the characteristic of low computational complexity, but it is easy to be trapped into local minima, Sigurd [13] combined voronoi with navigation function to achieve globally collision free path.

B. Node Based Optimal Algorithms

It can be quite obviously comprehended from the words that Node Based Optimal Algorithms generate path based on a set of nodes. The above section divides sampling based Algorithms into two categories, one is passive like PRM which cannot pick up a way out by itself, thus a complementary of search algorithm is urgently needed in order to find an optimal path. Analysing from the working mechanism, Node Based Optimal algorithms share the same property that they search through a set of nodes on a graph or a map, where pre-information sensing and processing procedures are already executed.

Algorithms like Dijkstra's algorithm, A*, Lifelong Planning A*(LPA) [19], Theta* [5], Lazy Theta* [20], Dynamic A* (D*), D*-Lite [21], Harmony Search [22] and etc are belong to node based optimal algorithms. This kind of algorithm is a special form of dynamic programming. When a map or graph is already constructed, they first define a cost function, and then search each nodes and arcs to find a cost minimum path.

Musliman [4] showed that Dijkstra's algorithms can find a the shortest way of a certain graph. To reduce the total number of states of Dijkstra's algorithm, Filippis [5] introduced a heuristic estimation of the cost to achieve a faster convergence but time saving method. Dynamic A*(D*) , which first introduced by Stentz [6], is a sensor based algorithms that changes its edge's weights to form a temporal map. D* is now widely used due to the fact that it can guarantees stability. Harmony Search (HS) which first proposed by [22], this method is proposed to solve the weakness of linear programming. HS can replan the path to find an optimal path with fast convergence, [23] have implemented in UAV to obtain aerial coverage optimization.

C. Mathematic Model Based Algorithms

Mathematic Model based Algorithms include Linear Programming, Optimal Control, and etc. Although these algorithms really can be classified as sampling based algorithms, but after taking its planning mechanisms and computational complexity into consideration, this paper sorts them out. These methods model the environment as well as the body, considering the kinematic and dynamic constraints and then bound the cost function with all the inequalities or equations to achieve an optimal solution.

Mathematic model based algorithms are classified as sub-parts, Linear Programming, and Optimal Control [24]. And Linear Programming contains Mixed-Integer Linear Programming [25], Binary Linear Programming[26], Non-linear Programming [27,28] and etc. This kind of algorithm considers almost all the factors, and then a cost function is defined to judge the current selection until an optimal path is found. A basic mathematic model based algorithm planning problem depicts in Fig.3.

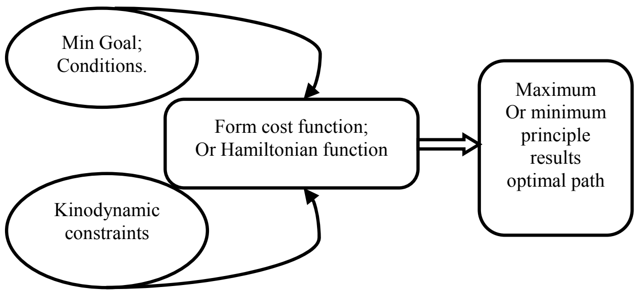

Fig. 3. Procedure of Basic Mathematic model based problem

Miller [24] dealt with path planning problem in optimal control framework, which combines cost criterion and Hamiltonnian function to form a boundary value problem (BVP), and resulted a realistic optimal path globally. Mixed-Integer Linear Programming (MILP) was first introduced to solve receding horizon optimization problem [41] for unmanned aerial vehicles, but an increase in the number of variables would increase the number of evaluations of the recursive functions which causes a heavy computational burden. Binary Linear Programming [26] can be regarded as a simplified version of MILP, but it ignores so much information to form a skeleton like road map.

D. Bio-inspired Algorithms

Bio-inspired Algorithms originate from mimicking biological behaviour to deal with problems. This path planning method leaves out the process of constructing complex environment models, and proposes a strong searching method to converge to the goal stably.

This paper divides the bio-inspired algorithms into Evolutionary Algorithm (EA) [29] and Neural Network(NN) [30] algorithm due to the fact that they analyze at different levels. And to the terms of evolutionary algorithms, it contains genetic algorithm [31], memetic algorithm[32], particle swarm optimization [33], ant colony optimization[34], and shuffled frog leaping algorithm[35].

Evolutionary Algorithm starts by selecting randomly feasible solutions as the first generation. Then takes the environment, robot's capacity, goal, and other constraints into consideration, the planner evaluates the fitness of each individual. The next step, a set of individuals is selected as parents for the next generations according to their fitness. The last step is a mutation and crossover step, and stops the process when a pre-set value is achieved. The best fitness individual is decoded as the optimal path, as Fig.4 shows.

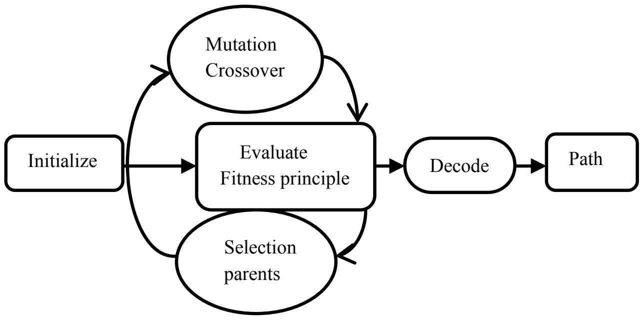

Fig. 4.Evolution process for path planning

Neural network approach aims to generate a dynamic landscape for the neural activities, as well as artificial potential field method, the unsearched areas globally attract the robot in the entire space. Always a shunting equation is defined to guarantees that the positive UAV's activity can propagate to all the free unexplored space, but negative activity only stays locally.

Evolutionary algorithm [29] was propounded to solve the problem that traditional Linear Programming often fail to solve NP-hard problems that with large number of variables. But because of the crossover operator is chose partly random, this kind of algorithms sometimes suffer the problem of premature convergence. Kroumov [30] implemented in robots to achieve global optimal path with the help of neural network algorithm. But when implemented in 3D environment, the neighbour neurons will explode to 26, thus implement it on-line seem to be unrealistic.

D. Multi-fusion Based Algorithms

Current 3D path planning algorithms tend to integrate with other algorithms, or combine one by one, aim to plan an optimal path (length, or time, or energy, or threat optimal). Say artificial potential field algorithms, without navigation function or other methods' combination they usually tend to drop into local minima. Probabilistic Roadmaps also cannot generate an optimal single path by itself. Thus this paper classifies this kind of algorithms, which generated by combining several algorithms together to achieve global optimal path, as Multi-fusion Based Algorithms.

Multi-fusion based algorithms manage with problems that a single algorithm proposed cannot achieve an optimal result individually. Xiao [3] used 3D grid to represent the environment and 3D PRM to form a roadmap in obstacle free space, at last A* node based optimal algorithm is combined to achieve an optimal path. Masehian [36] introduced a visibility graph, Voronoi diagram, and potential field(VVP) integrated algorithm, investigating the extension of VVP algorithm to 3D space shows effective tradeoff between the shortest and safest path. Scholer[39] combined visibility graph and Dijkstra's algorithm(or Geodesics) to find an optimal solution for path planning problem in 3D. Various of methods can be classified as this kind, but here cannot list one by one.

Based on the principle of each algorithm, this paper classifies these multi-fusion based algorithms into two categories: (a) A kind of path planning algorithms which is formed by integrating several path planning algorithms, in order to work simultaneously to find an optimal path. This paper calls these 'Integration of Algorithms'. (b) A kind of algorithms which is made up of several path planning algorithms. And it works in the typical form that when one algorithm completes its part, another one then works immediately. This paper calls these 'Algorithms ranking'. Table.1 illustrates several typical algorithms of each sub-category.

TABLE I
TYPICAL ALGORITHMS OF MULTI-FUSION BASED ALGORITHMS

Name of sub-category	Typical algorithms of each sub-category
Integration of Algorithms	visibility graph, Voronoi diagram, and potential field(VVP) algorithms; Voronoi potential field algorithms; Neural network potential field algorithms.
Algorithms ranking	Voronoi Node based optimal algorithms; PRM Node based optimal algorithms; GIS-MCDA algorithms[40]; visibility graph Node based optimal algorithms [38]; visibility graph Geodesics algorithm[39].

IV. ANALYSIS AND APPLICABILITY

Section 3 states the taxonomy of UAV 3D path planning algorithms, and a concise summarization of each category will be given below.

Sampling based algorithms first sample the environment as a set of nodes, then connect the nodes by a 'near' procedure, or depth-first search procedure, or etc. At last this kind of methods begins to search to achieve an optimal path. This kind of method is easy to implement and tend to have a simple structure. Thus they are appropriate for static and on-line planning conditions.

Node based algorithms deal with nodes, and neglect the typical way of forming a map. They commonly deal with node information which can transform distance between nodes into calculation weight, and search directly to find an optimal path globally. This kind of method can be combined with other methods to achieve global optimal, and they are appropriate to work on-line.

Mathematic model based algorithms aim to describing the whole workspace in a typical optimal control form. It describes almost all the constraints, which are dynamic and kinematic, in mathematical form and bound the cost function tightly. Although almost always load a heavy computational burden on computer, this kind of method now performs well enough with the improvement of computer technology. These algorithms are appropriate to work off-line.

Bio-inspired algorithms are heuristic methods, and can excellently deal with complex unstructured constraints as well as other NP problem. This kind of algorithms optimizes the path by mutation, but mutation process requires a long iteration time. Thus this kind of algorithms can only work off-line.

Multi-fusion based algorithms integrate several algorithms' advantages together to achieve global optimal and cost minimum. Often consider time saving and information completely simultaneously, and sometimes several simple relative method combine to form a rather well perform method, thus appropriate to implement on-line.

Based on the above analysis, Table.2 gives an intuitional knowledge of each category's processing time complexity, static (S) or dynamic (D) environment, and real-time applicability.

TABLE II
PROPERTIES OF EACH KIND OF METHODS

Method	Elements of each method	Time Complexity	S/D Environment	Real-time
Sampling Based Algorithms	Voronoi [14], RRT [15], PRM[3], K-PRM [11], S-PRM [10], Visibility Graphs [38], Corridor Map[16], DDRRT [17], RRT* [18]	$0(n\log n) \le T \le 0(n^2)$	S and D	On-line
Node Based Algorithms	Dijkstra's Algorithms [4], A* [3], D*[6], LPA [19], Theta* [5], Lazy Theta* [20], D*-Lite [21], Harmony Search [22]	$0(m\log n) \le T \le 0(n^2)$	S and D	On-line
Mathematic Model Based Algorithms	Optimal Control [22], Mixed-Integer Linear Programming [23], Binary Linear Programming[24], Non-linear Programming [27,28]	Depending on the polynomial equation	S and D	Off-line
Bio-inspired Algorithms	NN [30], genetic algorithm [31], memetic algorithm[32], particle swarm optimization [33], ant colony optimization[34], shuffled frog leaping algorithm[35].	$T \ge 0(n^2)$	S	Off-line
Multi-fusion Based Algorithms	VVP [36], PRM Node based optimal algorithms [3], GIS-MCDA algorithms [40], visibility graph Node based optimal algorithms [38], visibility graph Geodesics algorithm[39]	$0(n\log n) \le T$	Depending on the algorithm	On-line

V. CONCLUSION

In order to support a comprehensive review of 3D path planning of unmanned aerial vehicle (UAV), this paper analyzes a large number of current path planning algorithms successfully implemented in 3D cluttered environments for unmanned aerial vehicles. After analyzing many different approaches of UAV 3D path planning, this paper classifies all the approaches into five categories: Sampling based algorithms, Node based optimal algorithms; Mathematic model based algorithms, Bio-inspired algorithms, and Multi-fusion based algorithms. What should be emphasized is that this paper first names a special kind of path planning algorithms, that is, multi-fusion based algorithms.

This paper first lists the algorithms in each category that synthesized among almost all the works reported during recent years, and then supports a critical discussion of each algorithm. A detailed analysis on time complexity and applicability of methods in each category is given in section 4. Based on the idea of integration we will implement further works mainly on algorithms integration in order to produce a more efficiency method.

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