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Coverage Path Planning for Surveying Disjoint Areas

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***Abstract*— Terrain surveying using unmanned aerial vehicles (UAV) is being applied to many areas such as precision agriculture or battlefield surveillance. A core problem of the surveying task is the coverage path planning problem (CPP); it is defined as the task of determining a path for an unmanned aerial vehicle so that it observes all points of a target area. State-of-the-art planners solve the problem for a unique region. However, in some operations, for example search and rescue, it is important to plan a path that covers more than a single area. We propose a method for solving the CPP for disjoint areas. The general problem is modeled as a rural postman problem which has been demonstrated to be NP-Hard. Our solution relies on dividing the problem into two steps: optimization of the visiting order and optimization of the flight lines orientation. The method is validated through several simulations using real parameters. In addition, it is fast enough for being implemented onboard.**

I. INTRODUCTION

During the last decade, we have seen the success of unmanned aerial vehicles (UAV), best known as drones, in many tasks. Some examples are surveillance, search and rescue, mapping, precision agriculture, etc. Among those tasks, the surveying missions have been particularly successful since they allow the user to map large terrains with a relatively low cost. A core task of the surveying missions is to solve the coverage path planning (CPP); it is defined as the task of determining a path for the UAV, so that, it can scan all points of the area of interest [1]. When drones are used for surveying, it is of particular importance to plan efficient paths given that the autonomy is limited by the batteries. The CPP has been widely studied in the mobile robotics field and recently from the UAV perspective. Current approaches focus on planning optimal paths for single terrains including convex or non-convex shapes. For convex shapes, state-of-the-art methods search for the path that minimizes the number of swept lines [2]. For non-convex shapes, the methods usually divide the area into convex regions and compute paths that reduce the distance of traveling such regions [3], [4]. However, in some cases, the target area is composed of more than one terrain (disjoint areas). For instance, a UAV needs to spread fertilizer to certain areas of the crop or the UAV needs to search a car in several parking lots. We call to this problem coverage path planning for disjoint areas and we model it as a rural postman problem (RPP) which has been demonstrated NP-Hard [5]. We have set it as a new problem because in this case, the vehicle can move from one area to any of the rest, contrasting with previous problems where the vehicle can only move to the adjacent areas [6]. Moreover, we have found that the path for each area affects the cost of the global path.

We propose a method for solving the coverage path planning for disjoint areas. Due to the complexity of the problem, our solution relies on dividing the problem into two steps: optimization of the visiting order of the areas and optimization of the path for each area depending on the visiting order. In the first step, we use a genetic algorithm given that it provides accurate results in a reasonable lapse of time. For the second step, we fit a recent method that computes the path given the previous and following targets in a $O(n)$ time [7]. We have tested the method using real parameters and it is efficient enough to be implemented onboard. To the best of our knowledge, this is one of the first papers where the coverage path planning for surveying disjoint areas is addressed.

II. RELATED WORK

Recently, coverage path planning has attracted considerable attention due to its application to drone surveying. However, it is a problem that has been initially studied in the field of mobile robots. Choset and Pignon addressed the problem of covering polynomial wolds [6]. Their method, called Boustrophedon cellular decomposition (BCD), performs a line sweep decomposition of the world; such decomposition divides the world into cells. Each cell is explored in a back and forth swept and the visiting order of the cell is solved as a traveling salesman problem (TSP). Several variations of the BCD have been proposed. For example, to reduce the number of cells and waypoints [8]. However, in all the BCD approaches the movement is always done in the same direction despite of the area shape, this could lead to an inefficient path by increasing the number of turns. In this paper, we perform a comparison against the popular approach of BCD. Huang's method for coverage path planning [2] was motivated by a robot performing demining operations. Huang establishes that an optimal path should minimize the number of turns. His method proposes that the sweep direction of the back and forth pattern must be done perpendicular to the minimum width of the area represented by a polygon. However, the method does not consider the visiting order of the regions. For a further review on CPP with mobile robots the reader is referred to [1].

Coverage path planning using drones reutilizes the ideas from mobile robotics but it has included new features and restrictions, emphasizing the fact that the realization of such plans rely on the autonomy time. Li et al. [4] propose a method to cover an single area of convex or non-convex

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shape. If it is a non-convex region, their method divides it into convex cells. Then it computes the optimal orientation of the flight lines by orientating the flight lines perpendicular to the minimum span of the cell, in the same way that [2]. Finally, it optimizes the total path by selecting the best joint points (called linking points in this paper). Xu et al. [9] reuse the BCD but optimizes the path by solving a Chinese postman problem instead of a TSP.

Torres et al. [3] propose a method based on the same coverage optimality for a polygon defined by Huang [2] but optimizes the path between target regions. A drawback of their method is that the calculation of the optimal orientation has a quadratic complexity, given that for each edge of the polygon, it has to compute the distance to each vertex. Balampanis et al. [10] propose a method for coverage with multiple UAVs which is based on the region growing algorithm. In [11], Balampanis proposes a spiral like pattern to cover the areas, however such pattern requires additional instruments such as a gimbal to maintain the camera pointing to the ground. For covering complex polygons, in [12] and [13], the polygon is decomposed into subfields and the sum of their widths (altitudes) is minimized. An alternative approach for complex polygons is the constriction and decomposition method [14] where the set of cells is created with a trade off between the length of the path and the difficult of executing it under kinematics constraints.

Recent works have addressed the problem of minimizing the energy wasted or they have analyzed the effect of the wind on the survey. Di Franco et al. [15] compute the optimal velocity for the flight, however they do not consider the turns and they only address a single area. The method of Modares et al. [16] computes a path that minimizes the total energy for a fleet of drones. The authors solve the problem by formulating a vehicle routing problem. Modares's method divides the space into cells and the drone can only travel between adjacent cells, in the same way that [17], unlike them, in our problem the drone could flight from one area to any other so that the number of connections for each cell (area) is combinatorially increased. Coombes et al. [18] propose a method to estimate the time that the UAV needs to complete a boustrophedon path under wind conditions.

III. Background

Each terrain is represented by a polygon, m. We assume that the polygons are convex and they do not intersect between them. In case that a polygon is not convex, it can be converted to convex by calculating its convex hull or it can be split into convex parts.

The surveying mission is performed by a multirotor vehicle which carries a camera that is pointing to the ground. We assume that the terrains are plain, therefore, the UAV maintains a constant altitude. The configuration of the UAV is defined by $q = (x, y, \theta)$, where x and y are coordinates over the north-east-down (NED) frame and θ is the orientation with respect to the east axis. In addition, we assume that the UAV has two motion primitives: translation in straight line at speed v and rotation in site at heading rate ω.

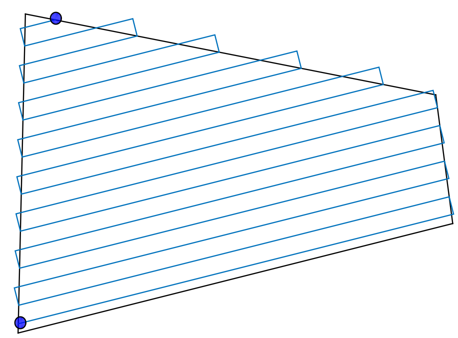


Fig. 1. Example of a back and forth pattern for surveying a convex polygon.

A. Back and Forth Pattern

A survey task consist on flying over the surface of all target polygons. A coverage path is a 2D spatial curve parametrized by its length, $P(s) = [x_c(s), y_c(s)]^T$. When the UAV follows the coverage path, the UAV can take all the required photos while guarantees a minimum overlap between images. For each polygon, the coverage path has the shape of a back and forth pattern (BFP) [1]. We use the BFP because it allows us to maintain an overlap between images in order to register the information, maintains the camera pointing to the ground at straight segments and reduces the number of turns of the aircraft [2]. When a UAV follows a BFP, it flights in straight segments, called flight lines, until it reaches the frontier of the polygon, then it takes another flight line to return. For example, movements from south to north and from north to south. The BFP ends when the polygon is completely swept. See Fig. 1. The distance between flight lines is calculated accordingly to the camera specifications and survey needs [15]. The BFP can be resumed into a series of "waypoints" where the aircraft changes its orientation while in straight segments the UAV keeps a constant speed. Therefore, a BFP is resumed as $P = \{p_0, \ldots, p_l\}$. The pattern could be pointing to any direction, namely, the flight lines point to a certain angle with respect to the north. Formally, let α be the counter-clock wise rotation angle of the flight lines with respect to the north axis. Let P_m^α define a path that covers a polygon m with flight orientation equal to α. The waypoints of the path are defined as $P_m^\alpha = \{p_0^{m,\alpha}, \ldots, p_l^{m,\alpha}\}$. See Fig. 2 where each of the polygons is covered by a path with a particular orientation. The point from where the UAV takes off and lands is called home, p^h. The points of the BFP where UAV enters or exits the polygon are called linking points; they match with the initial and last waypoints, $p_0^{m,\alpha}$ and $p_l^{m,\alpha}$. The cost between two consecutive waypoints, $p_i = [x_i, y_i]^T$ and $p_{i+1} = [x_{i+1}, y_{i+1}]^T$, is defined by the time that it takes to orientate the UAV plus the time to reach the second point, see equation (1).

$$c(p_i, p_{i+1}) = \theta_\Delta/\omega + ||p_{i+1} - p_i||/v \tag{1}$$

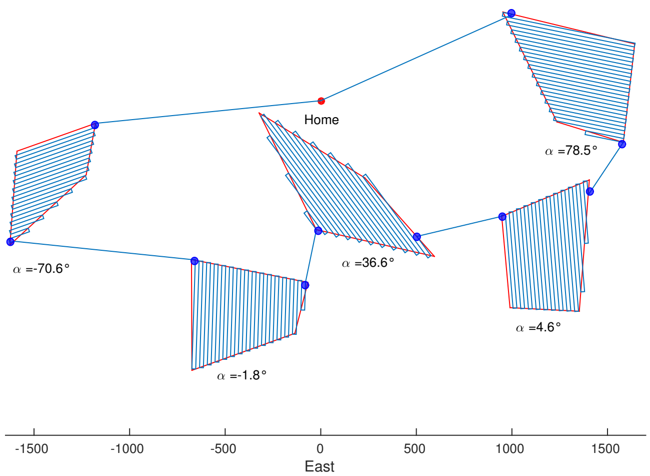


Fig. 2. Example of a coverage path (drawn in blue) for disjoint areas (drawn in red). It starts from the home point (red point), pass though the polygons following a back and forth pattern and finally returns to home. The angles denote the orientation of the flight lines with respect to the north. Linking points are drawn in blue. Units are in meters.

The cost of a path is the sum of the time for reaching all the waypoints of the path, see equation (2).

$$c(P_m^\alpha) = \sum_{i=0}^{l-1} c(p_i, p_{i+1}) \tag{2}$$

IV. Coverage Path Planning for Disjoint Areas Problem

In this section, we analyze the problem of coverage path planning for disjoint areas. The problem is stated as follows: Given a set of n convex polygons, $M = \{m_1, m_2 \dots m_n\}$, compute a coverage path that surveys all the polygons in the shortest time. If the motion primitives are restricted to straight lines and rotations in site, the problem is restricted to compute the series of waypoints, ρ, that surveys all the polygons in the shortest time. ρ implicitly contains the visiting order of the polygons and the search pattern for each of them, $P_{m_i}^{\alpha_i}$.

Assuming that each polygon is covered by a back and forth search pattern, we model the problem as an undirected Rural Postman Problem (RPP) where the cost for each edge depends on the orientations of the BFPs. The RPP is a variation of the Chinese postman problem and it is defined on a undirected graph $G = (V, E)$, where V is a list of vertices and E is a list of edges. Each edge, e, has an associated cost $c(e)$. Unlike the Chinese postman problem, the RPP defines a subset of edges, $E_R \in E$, that must be part of the path. Then, the problem is to determine a least cost tour traversing each edge of E_R at least once [5].

In this problem, the set V is composed by the home point and the linking points. Namely,

$$V = p^h \cup \{p_j^{m_i,\alpha_i} | i = 1 \dots n, j = 0_i \vee j = l_i\} \tag{3}$$

where p^h is the home point and $p_j^{m_i,\alpha_i}$ is a linking point of the BFP with orientation α_i that covers the polygon m_i.

Based on the vertices we form a complete graph. So, V has $2n+1$ vertices and E has $n(2n+1)$ edges. The especial subset of edges, E_R, is defined by the edges that connect two linking points of the same polygon, see equation (4).

$$E_R = \{(p_a^{m_b,\alpha_b}, p_c^{m_d,\alpha_d}) | b = d\} \tag{4}$$

On the other hand, edges that connect linking points of different polygons do not belong to E_R.

The cost for an edge, $e = (p_a^{m_b,\alpha_b}, p_c^{m_d,\alpha_d})$, that does not belong to E_R is computed as the time that takes to the UAV to move directly from one vertex to another. Namely:

$$c(e \notin E_R) = c(p_a^{m_b,\alpha_b}, p_c^{m_d,\alpha_d}) \tag{5}$$

where $c()$ is computed accordingly to equation (1). If the edge belongs to E_R, $e = (p_a^{m_b,\alpha_b}, p_c^{m_b,\alpha_b})$, then the cost is computed as the cost of the BFP that covers the associated polygon. Please, see equation (6).

$$c(e \in E_R) = c(P_{m_b}^{\alpha_b}) \tag{6}$$

Using the previous modeling, the computed tour must pass through each polygon and only some connections between polygons will be added. However, it is important to notice that the costs depends on the orientations of the back and forth patterns, this increments the dimensionality of the problem because for each particular set of orientations the RPP changes. In the next section, we present an strategy to deal with the problem.

V. Two Steps Path Planning Strategy

The RPP was proved to be NP-Hard [19], therefore, to find an optimal solution for a large number of polygons is intractable. We propose a two steps strategy that is based on the particular characteristics of the problem. This strategy allows us to find a near optimal path in a reasonable computation time. In the fist step of the strategy, we plan a visiting order for the polygons. In a second step, we compute the BFP for each polygon.

The reason for solving the visiting order first is because we have noticed that in most of the cases the edges that connect polygons, $E \setminus E_R$, do not vary significantly with respect to the orientation of each BFP, such orientations are denoted by $A = \{\alpha_1, \dots, \alpha_m\}$. To compute the visiting order we convert the graph into a reduced graph, $G' = \{V', E'\}$. The set of new vertices, V', is composed by the home point plus a new vertex for each polygon: $V' = \{v_i | i = 0, 1, \dots n\}$ where $v_o = p^h$ and $v_i | i > 0$ represents the polygon m_i. The set of new edges E' connects all the vertices in V'. Then we compute the visiting order by solving a Traveling Salesman

Problem (TSP). An issue with this formulation is that the cost of each edge is not known in advance since it depends on the orientations A. Our proposition is to approximate the cost by using the distance between the centroids of the polygons. To solve the TSP problem, we have implemented a Genetic Algorithm (GA). It uses a chromosome that defines a possible solution for the TSP on the graph G'. We have implemented a GA since it has provided good results, shown adequate computation time and it can be parallelized [20]. An alternative to the GA is to use the Lin-Kernigham Heuristic [21] which starts from one solution and iteratively improves it by applying certain transformations. A comparison of both heuristics for this particular context is left for future work.

Once we have found the visiting order, we sort the set of polygons M and the set of vertices V' respectively. Then, we proceed to the second step were we compute the BFP for each polygon. In this step, we use the planner proposed in [7] that computes, in $O(n)$ time, the path that covers a convex polygonal area given the starting and ending points. Given that the method is based on the rotating calipers algorithm we will call it rotating calipers planner (RCP). The RCP fits in this context given that we already have approximations of the previous and next points. Therefore, for each polygon m_i we send to the RCP the polygon m_i, the previous estimated point v_{i-1} and the next estimated point v_{i+1}. Note that, $v_{n+1} = v_0$ given that the UAV returns to home. Then, for each query the RCP returns the orientation α_i, and the path $P_{m_i}^{\alpha_i}$.

Finally, the full path is:

$$\rho = \left\{ p^h, \bigcup_i P_{m_i}^{\alpha_i}, p^h \right\} \tag{7}$$

VI. Experiments

We have implemented the method in MatLab and we have performed several simulations comparing the performance of the two steps path planning method versus two alternative approaches. The objective of the experiment is to compare the computation time and quality of the solutions. The compared methods are the following:

- Boustrophedon cellular decomposition (BCD) [6]. Each polygon is considered a cell. A back and forth path is computed for each polygon. Then a TSP is formulated and solved. We use the centroids of the polygons as vertices of the graph. Since the UAV can flight to any polygon the adjacency matrix connects all the vertices.
- Two steps path planner (TSPP). The method proposed in this paper.
- Optimization using genetic algorithms (GA). An alternative method to the TSPP. In this approach, first we obtain the visiting order for the polygons, we formulate and solve the TSP problem in the same way that the TSPP. Then, we obtain the orientation angles for each polygon. Unlike the TSPP, we formulate a second optimization problem where the objective function is the cost of the total path. The parameters to find are the orientation angles $A = \{\alpha_1 \dots \alpha_n\}$. To solve the optimization we implement another standard genetic algorithm which iterates and finds the best combination of parameters.

k	BCD	GA	TSPP
5	2,374 s	2,280 s	**2,246** s
10	4,458 s	4,370 s	**4,271** s
20	7,666 s	7,426 s	**7,174** s
10	12,831 s	12,800 s	**11,956** s
80	30,194 s	31,210 s	**28,623** s

TABLE I

Cost of the path computed by each method. The best cost for each set of polygons is remarked.

k	BCD	GA	TSPP
5	0.095 s	49.79 s	**0.084** s
10	**0.046** s	107.0 s	0.089 s
20	**0.092** s	425.1 s	0.169 s
40	**0.258** s	2,004.5 s	0.376 s
80	**0.631** s	8,744.2 s	0.962 s

TABLE II

Computation time of each planner using different amounts of polygons

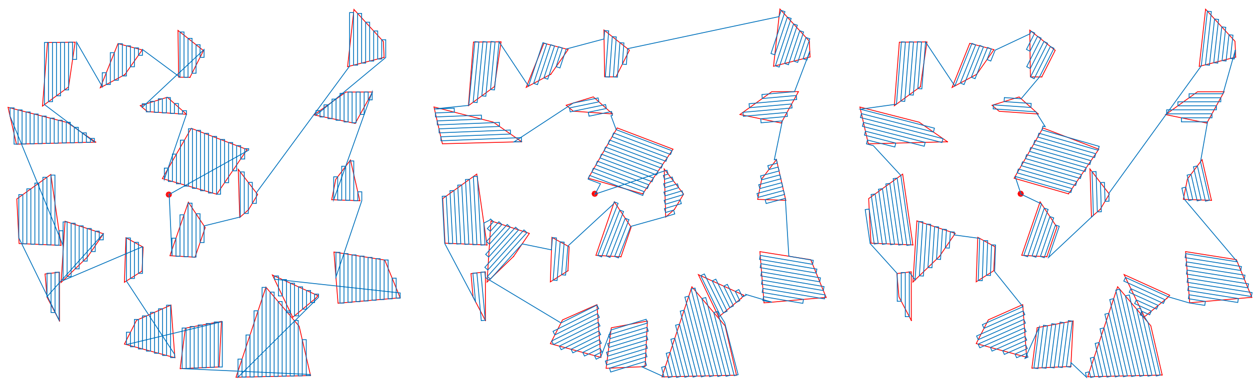
Each method returns a full path, ρ, that covers the polygons. The cost of the full path is computed as the time that the UAV takes to complete it, see equation (8).

$$cost(\rho) = \sum_{i=1,q-1} c(\rho(i), \rho(i+1)) \tag{8}$$

where q is the number of waypoints of ρ and c is calculated according to equation 1. The rotational and translational speeds were fixed to 0.785 radians/s and 7 m/s respectively. The methods were run in a intel core i7 machine with 2GB of RAM.

The experiment was carried out as follows: A set of k random convex polygons is generated and it is tested with each method. The values for k were 5, 10, 20, 40, 80. The parameters for the TSP standard genetic algorithm are: population equal to 100, number of iterations given to the GA equal to 100, 100, 100, 500, 1000 respectively (Note that the number of iterations for the cases of 40 and 80 polygons were increased in order to deal with the big amount of polygons). The results are summarized in tables I and II.

The results show that the proposed TSPP strategy provides the best paths with a competitive computation time. See in table I that the costs of the TSPP paths are the smallest for all the tested scenarios. The average saving with respect to the BCD approach is 5.6% and with respect to the GA approach it is 4.4%. This saving is more relevant for scenarios where the coverage must be done as fast as possible, for example, in search and rescue operations. On the other hand, see in table II that the computation time for the TSPP strategy it is almost the same that the BCD (in all cases it is less than a second) and it is quite smaller than the GA approach. Based on this computation times, an onboard implementation of the TSPP is feasible.

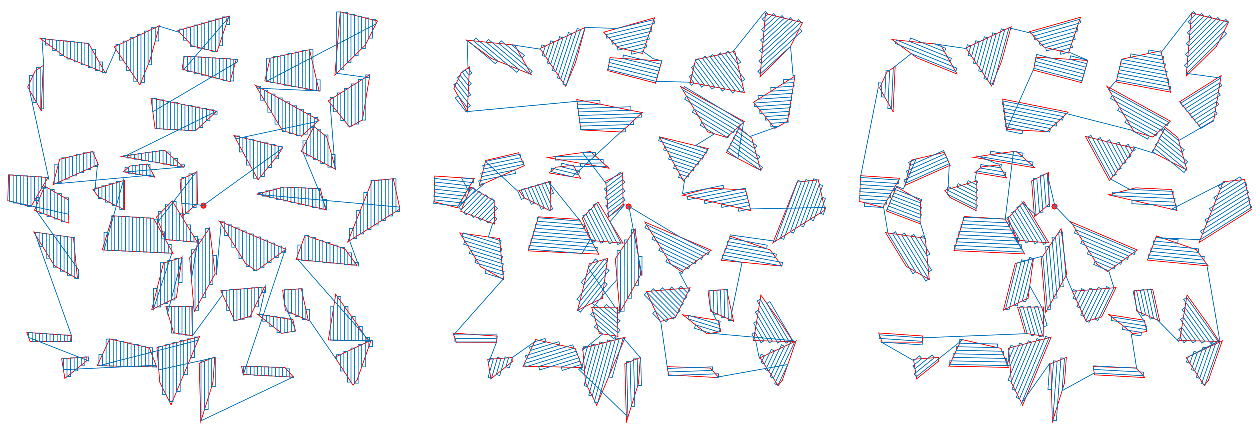


(a) Boustrophedon cellular decomposition (BCD).

(b) Genetic algorithm (GA).

(c) Two steps path planning (TSPP).

Fig. 3. Comparison of the paths computed by each method for 20 disjoint areas.

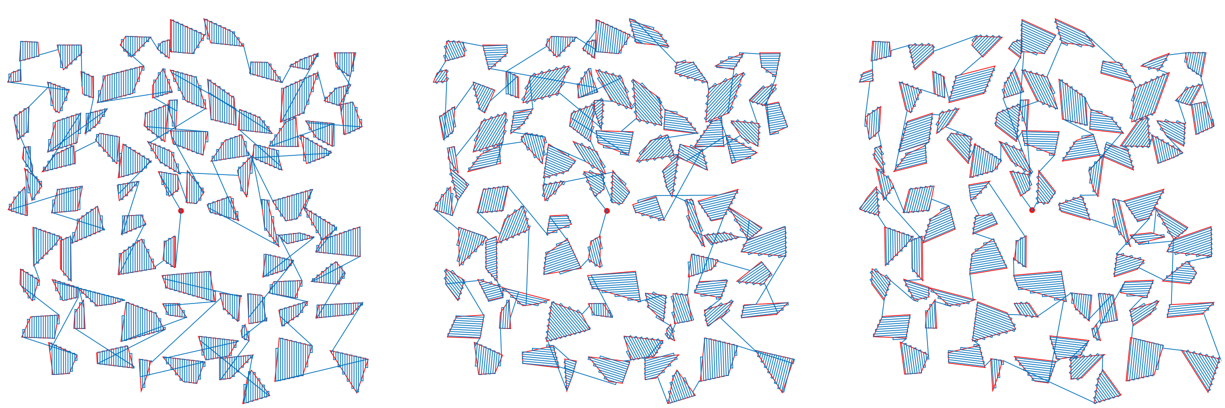


(a) Boustrophedon cellular decomposition (BCD).

(b) Genetic algorithm (GA).

(c) Two steps path planning (TSPP).

Fig. 4. Comparison of the paths computed by each method for 40 disjoint areas.



(a) Boustrophedon cellular decomposition (BCD).

(b) Genetic algorithm (GA).

(c) Two steps path planning (TSPP).

Fig. 5. Comparison of the paths computed by each method for 80 disjoint areas.

Figures 3, 4 and 5 show examples of the paths computed using each method. We can observe that the BCD approach computes the worst paths given that the orientation of the back and forth patterns are fixed. The GA provides better paths, however, we can still notice some straight sections over already covered areas. The TSPP provides the best paths of the three methods. The orientation of the flight lines provides clear paths reducing the distance between polygons and reducing the flights over already covered areas.

VII. Conclusions

We have presented a method for computing the path for a UAV that covers a set of disjoint areas. The method decomposes the problem into optimization of the visiting order and computation of the back and forth pattern for each area. The simulations show that the proposed method provides the best paths with respect to the compared methods and it is fast enough to be implemented onboard. To the best of our knowledge, this is one of the first coverage path planners that specifically deals with disjoint areas. In a future work, we will investigate alternative optimization heuristics for the TSP problem, in addition, we will test the method on the search and rescue task using unmanned aerial vehicles.

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