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西交利物浦大学

MEC104 Experimental, Computer Skills and Sustainability: MATLAB Assignment

Due on Monday, 10th May, 2021, 18:00

Assignment Regulations

- This is an individual assignment. Every student MUST submit one soft copy of the assignment via the Learning Mall before the due date.

- A coversheet can be created in your own way, but the following information MUST be included: student ID number, full name and email address.

- In your answer sheet, all the formula, derivations, completed MATLAB scripts and functions with original highlighted text format in MATLAB editor, computational results in the command window, and plotted figures, should be part of the answers. For each question, you can use screen shots to provide 1) your coding in MATLAB editor, 2) prompts in command window, 3) results in command window, 4) results shown by plots/figures.

- There is no hard requirement on how the answer sheet must be organized. You can organize your report question by question (i.e. give one section for each question). Then, for each section, you can organize it in your own way. However, the contents and information required by each individual question MUST be provided. <u>You may follow a template on the next page to decide what information to be presented for each question.</u>

- You may refer to textbooks and lecture notes to discover approaches to problems, however, the assignment should be your own work.

- Where you do make use of other reference, please cite them in your work. Students are reminded to refer and adhere to plagiarism policy and regulations set by XJTLU. References, in IEEE style can be attached as an appendix.

- Assignments may be accepted up to 5 working days after the deadline has passed; a late penalty of 5% will be applied for each working day late without an extension being granted. Submissions over 5 working days late will not be marked. Emailed submissions will NOT be accepted without exceptional circumstances.

A Suggestion on Information to be Presented for Each Question

For each problem, for example, Problem 5:

1. Equation derivations:
1) What equation do you use in your coding?
2) Also give all the coefficients, and source terms (e.g. external force/voltage), as necessary.

2. What initial conditions, boundary conditions, time periods, domain size, etc., (computational conditions) are used? Provide schematic diagrams as necessary.

3. Main programme:
Provide the coding below, with necessary comments.

4. Functions
1) Give information on what is this function used for, and what equation is solved, related to point 1.
2) Provide the coding below, with necessary comments.

5. Results
1) Present the results required by each question, which can be numbers, data tables, figures, as appropriate.
2) Comments and analysis of the results:
    If required by a question, then you need to do this.
    If not required, you can still do this if you wish, which is great!
    If you think it is necessary to clarify your results and methods used, then please provide your comments.

6. Flow charts of your programme (if applicable).

Problem 1 Matrix Operation (20 Marks)

P1-1 (5 Marks, 1 Mark for each small question)
Type this matrix in MATLAB and use MATLAB to carry out the following instructions.

$$\boldsymbol{A} = \begin{bmatrix} 3 & 7 & -4 & 12 \\ -5 & 9 & 10 & 2 \\ 6 & 13 & 8 & 11 \\ 15 & 5 & 4 & 1 \end{bmatrix}$$

a) Create a vector **v** consisting of the elements in the second column of **A**.
b) Create a vector **w** consisting of the elements in the second row of **A**.
c) Create a 4 x 3 array **B** consisting of all elements in the second through fourth columns of **A**.
d) Create a 3 x 4 array **C** consisting of all elements in the second through fourth rows of **A**.
e) Create a 2 x 3 array **D** consisting of all elements in the first two rows and the last three columns of **A**.

P 1-2 (5 Marks, 1 Mark for each small question)
Consider the following arrays:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 4 & 100 \\ 7 & 9 & 7 \\ 3 & \pi & 42 \end{bmatrix}, \qquad \boldsymbol{B} = \ln A$$

Write MATLAB expressions to do the following.
a) Select just the second row of **B**.
b) Evaluate the sum of the second row of **B**.
c) Multiply the second column of **B** and the first column of **A** element by element.
d) Evaluate the maximum value in the vector resulting from element-by-element multiplication of the second column of **B** with the first column of **A**.
e) Use element-by-element division to divide the first row of **A** by the first three elements of the third column of **B**. Evaluate the sum of the elements of the resulting vector.

P 1-3 (5 Marks)

A mass m is suspended by three cables attached at three points B, C, and D, as shown in Figure 1. Let T_1, T_2, and T_3 be the tensions in the three cables AB, AC, and AD, respectively. If the mass m is stationary, the sum of the tension components in the x, in the y, and in the z directions must each be zero. This gives the following three equations:

$$\begin{bmatrix} \ \end{bmatrix}\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

$$\frac{T_1}{\sqrt{35}} - \frac{3T_2}{\sqrt{34}} + \frac{T_3}{\sqrt{42}} = 0$$

$$\frac{3T_1}{\sqrt{35}} - \frac{4T_3}{\sqrt{42}} = 0$$

$$\frac{5T_1}{\sqrt{35}} + \frac{5T_2}{\sqrt{34}} + \frac{5T_3}{\sqrt{42}} - mg = 0$$

$Ax = b$

$x = A^{-1}b$

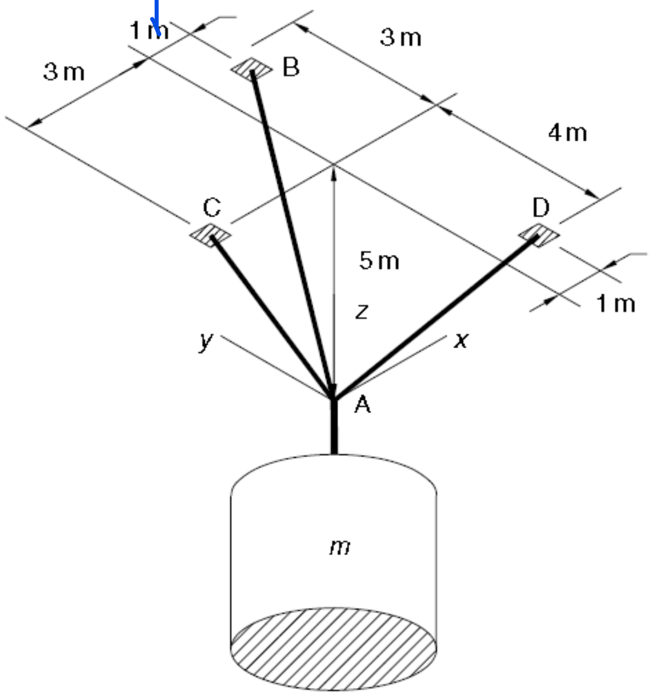


Figure 1. A mass suspended by three cables.

Determine T_1, T_2, and T_3 in terms of an unspecified value of the weight mg. (Hints: you can assume $mg = 1$, and find the values of T_1, T_2 and T_3, respectively, and the final expressions should be these values multiplied by mg.)

P 1-4 (5 Marks)

Engineers must be able to predict the rate of heat loss through a building wall to determine the heating system requirements. They do this by using the concept of thermal resistance R, which relates the heat flow rate q through a material to the temperature difference ΔT across the material: $q = \Delta T / R$. This relation is like the voltage-current relation for an electric resistor: $i = v/R$. So, the heat flow rate plays the role of electric current, and the temperature difference plays the role of the voltage difference. The SI unit for q is the watt (W), which is 1 joule/second (J/s). The wall shown in Figure 2 consists of four layers: an inner layer of plaster/lathe 10 mm thick, a layer of fiber glass insulation 125 mm thick, a layer of wood 60 mm thick, and an outer layer of brick 50 mm thick. If we assume that the inner and outer temperatures T_i and T_o have remained constant for some time, then the heat energy stored in the layers is constant, and thus the heat flow rate through each layer is the same. Applying conservation of energy gives the following equations.

$20 - T_1 \quad T_1 - T_2 \quad T_2 - T_3 \quad T_3 - T_0$

$$q = \frac{1}{R_1}(T_i - T_1) = \frac{1}{R_2}(T_1 - T_2) = \frac{1}{R_3}(T_2 - T_3) = \frac{1}{R_4}(T_3 - T_o)$$

The thermal resistance of a solid material is given by $R = D/k$, where D is the material thickness and k is the material's thermal conductivity. For the given materials, the resistances for a wall area of 1 m^2 are $R_1 = 0.036$, $R_2 = 4.01$, $R_3 = 0.408$, and $R_4 = 0.038$ K/W. Suppose that $T_i = 20$ °C and $T_o = -10$ °C. Find the other three temperatures and the heat loss rate q, in watts. Also, compute the heat loss rate if the wall's area is 10 m^2.

$\frac{T_1}{0.036}$

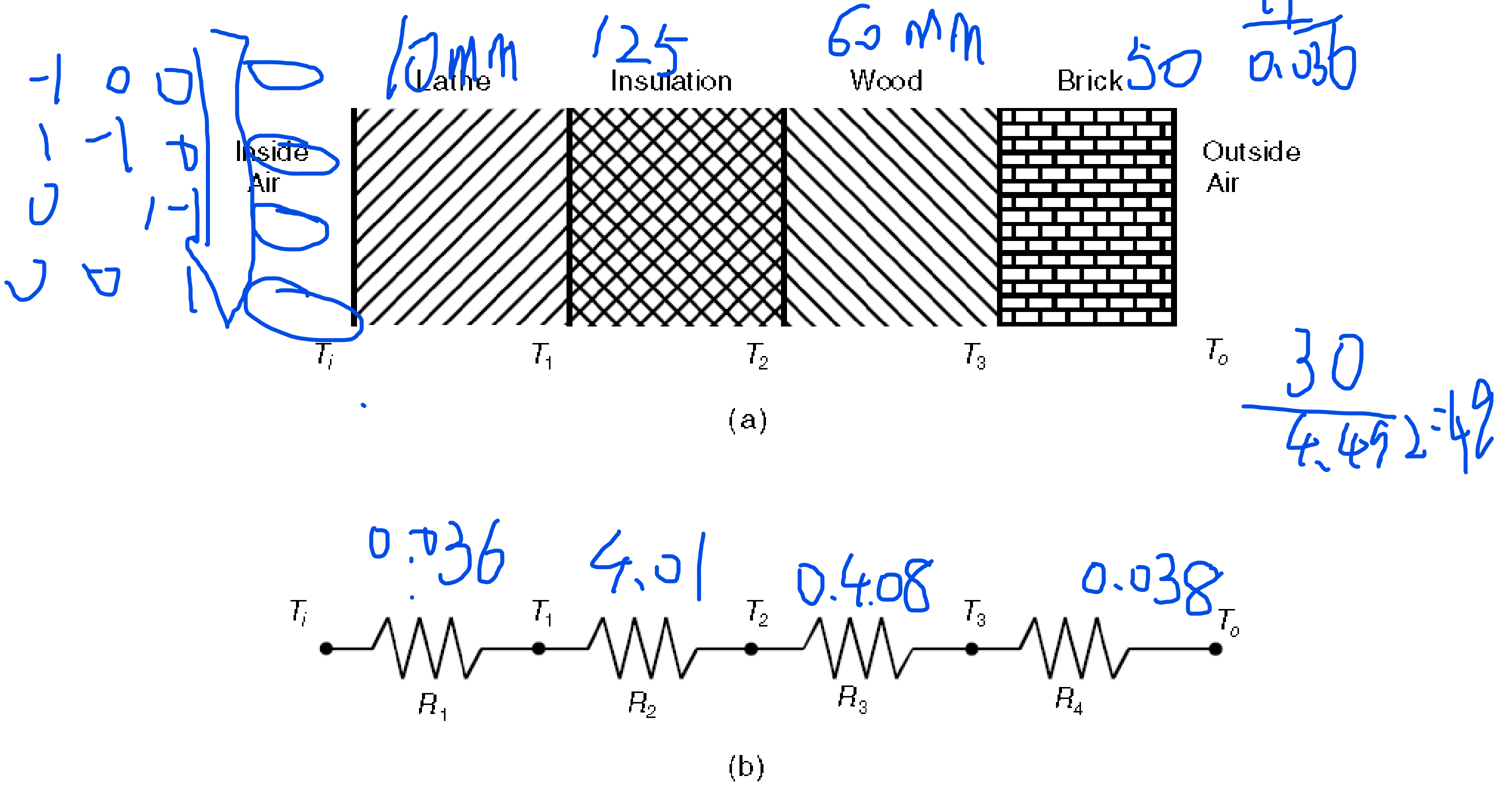


Figure 2. The schematic for P1-4.

Problem 2 (20 Marks)

P5-1 (10 Marks, 7 Marks for programming and 3 Marks for displaying results)

The equation of motion for a pendulum whose base is accelerating horizontally with an acceleration *a(t)* is

$y'' = a(t) \cos y - 9 \sin y$

$$L\ddot{\theta} + g \sin\theta = a(t) \cos\theta$$

Suppose that $g = 9.81\, m/s^2$, $L = 1\, m$, and $\dot{\theta}(0) = 0$. Plot $\theta(t)$ for $0 \le t \le 10\, s$ for the following three cases:

a) The acceleration is constant: $a = 5\, m/s^2$, and $\theta(0) = 0.5\, rad$.

b) The acceleration is constant: $a = 5\, m/s^2$, and $\theta(0) = 3\, rad$.

c) The acceleration is linear with time: $a = 0.5t\, m/s^2$, and $\theta(0) = 3\, rad$.

$y'' = 5 \cos y - 9.81 \sin y$

P5-2 (10 Marks, 7 Marks for programming and 3 Marks for displaying results)

The following equation describes the motion of a certain mass connected to a spring, with no friction

$10 \sin t$

$$3\ddot{y} + 75y = f(t)$$

where $f(t)$ is an applied force. Suppose the applied force is sinusoidal with a frequency of $\omega\, rad/s$ and an amplitude of 10 N: $f(t) = 10 \sin(\omega t)$.

Suppose that the initial conditions are $y(0) = \dot{y}(0) = 0$. Plot $y(t)$ for $0 \le t \le 20\, s$. Do this for the following three cases. Compare the results of each case:

a) $\omega = 1\, rad/s$

b) $\omega = 5\, rad/s$

c) $\omega = 10\, rad/s$

Problem 3 (15 Marks, 5 Marks for a) and 10 Marks for b))

The equations for an armature-controlled dc motor (Figure 3) are the following. The Motor's current is i and its rotational velocity is ω.

0.003 0.8 0.05

$$L\frac{di}{dt} = -Ri - K_e\omega + v(t) \quad 20$$

J, = J

0.05

$$J\frac{d\omega}{dt} = K_T i - \cancel{c\omega}$$

8×10⁻⁵

where L, R, and J are the motor's inductance, resistance, and inertia; K_T and K_e are the torque constant and back emf constant; c is a viscous damping constant; and $v(t)$ is the applied voltage.

Use the values $R = 0.8\,\Omega$, $L = 0.003\,H$, $K_T = 0.05\,N\cdot m/A$, $K_e = 0.05\,V\cdot s/rad$, $c = 0$ and $J = 8\times10^{-5}\,kg\cdot m^2$.

V I t

a) Suppose the applied voltage is 20 V. Plot the motor's speed and current versus time. Choose the simulation time period large enough to show the motor's speed becoming constant.

b) Suppose the applied voltage is trapezoidal as given below.

$$v(t) = \begin{cases} 400t & 0 \le t < 0.05 \\ 20 & 0.05 \le t \le 0.2 \\ -400(t-0.2)+20 & 0.2 < t \le 0.25 \\ 0 & t > 0.25 \end{cases}$$

0.2

Plot the motor's speed versus time for $0 \le t \le 0.3\ s$. Also plot the applied voltage versus time. How well does the motor speed follow a trapezoidal profile?

For both a) and b), the initial conditions are $i(0) = 0$ and $\omega(0) = 0$. You can use either MATLAB programming (read Tutorial 3 for solving ODEs) or Simulink (read Tutorial 6.)

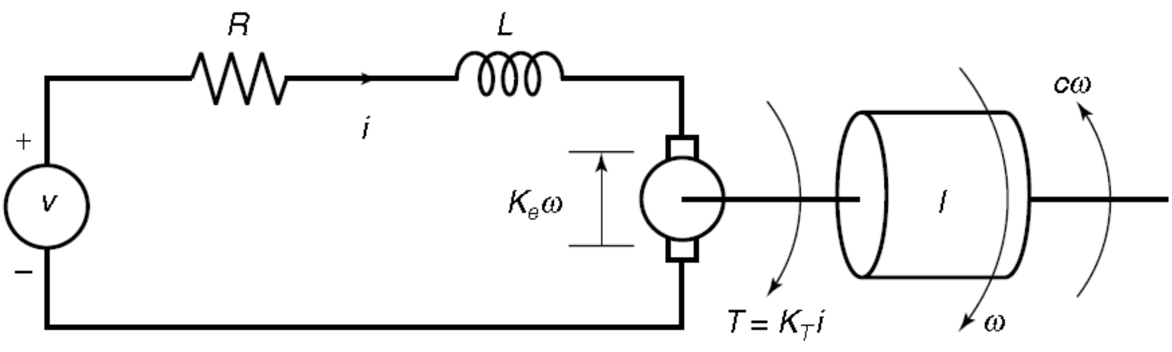


Figure 3. The armature-controlled dc motor of Problem 3.

Problem 4 (15 Marks)

Consider a hydradulic system as shown in Figure 4. The model is this system is given below which solves for the height of the fluid, h, in the tank:

$$\rho A \frac{dh}{dt} = q + \frac{1}{R_l} SSR(p_l - p) - \frac{1}{R_r} SSR(p - p_r)$$

where

ρ: the fluid mass density
A: the bottom area of the tank
q: the mass flow rate of the flow source
p: the bottom pressure in the tank
p_l: supply pressure from the left-hand side tube
p_r: supply pressure from the right-hand side tube
p_a: atmospheric pressure
R_l: flow resistance from the left-hand side tube
R_r: flow resistance from the right-hand side tube

and

$$SSR(\Delta p) = \begin{cases} \sqrt{\Delta p} & if\ \Delta p > 0 \\ -\sqrt{\Delta p} & if\ \Delta p < 0 \end{cases}$$

with Δp the pressure difference across the flow resistances.

The mass flow rates out of left and right outlets are, respectively:

$$q_{lo} = \frac{\rho g h}{R_l}$$
$$q_{ro} = \frac{\rho g h}{R_r}$$

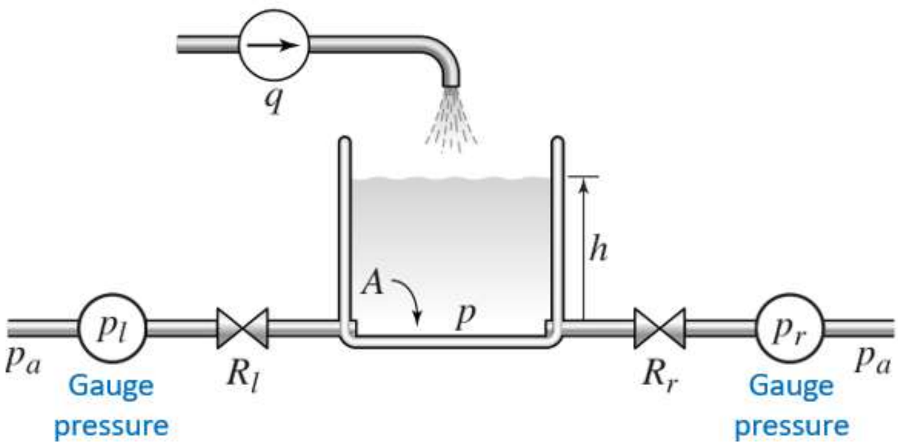


Figure 4. A hydraulic system with a flow source and two pumps

Now, we have a hydraulic flow system as shown in Figure 5. The parameters are: $A_1 = 3\,ft^2$, $A_2 = 5\,ft^2$, $R_1 = 30\,ft^{-1} \cdot sec^{-1}$, $R_2 = 40\,ft^{-1} \cdot sec^{-1}$, $\rho = 1.94\,slug/ft^3$, $q_{mi} = 0.5\,slug/sec$. The initial fluid heights in the two tanks are, respectively, $h_1(0) = 2\,ft$ and $h_2(0) = 5\,ft$.

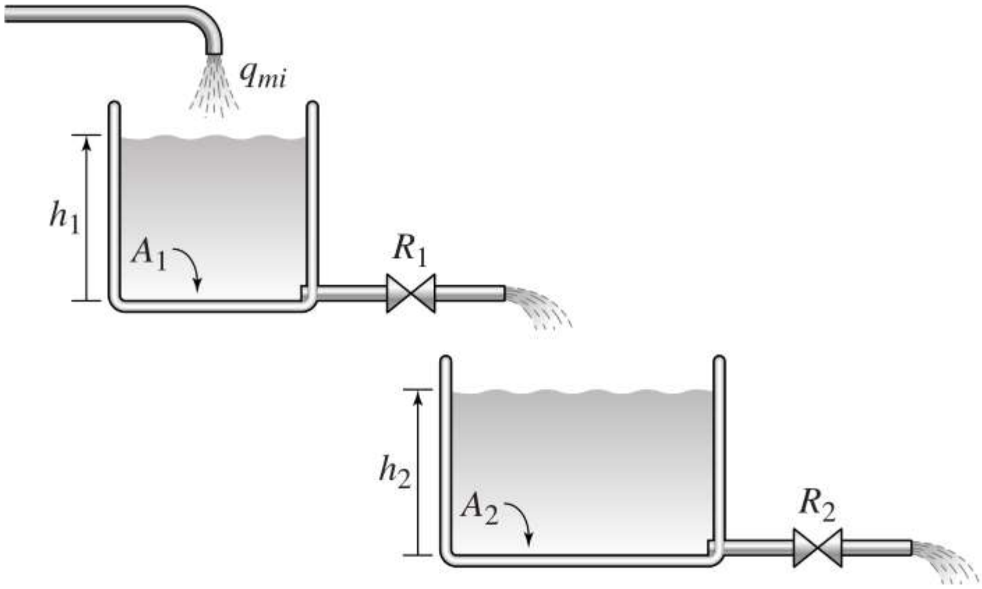


Figure 5. The hydraulic flow system for Problem 4.

Question: Obtain the plots of $h_1(t)$ and $h_2(t)$.

Note: It is recommended to use Simulink to solve this question, by referring to a similar example in Tutorial 6. If you cannot access MATLAB, for which you have to use Octave, then you can use direct programming, by referring to examples of solving ordinary differential equations in Tutorial 3.

Problem 5 (30 Marks, with a) 10 Marks, b) 8 Marks, c) 7 Marks, d) 5 Marks)

Consider a 2-dimensional rectangular area (Figure 5), without existence of internal electrical changes, and given the potentials at the 4 boundaries (i.e. the boundary conditions), and also considering the uniform medium inside the area, the distribution of electrical potential, φ, is governed by the Laplace equation:

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \tag{5-1}$$

The boundary conditions are given by: $\varphi|_{x=0} = \varphi_1$, $\varphi|_{x=a} = \varphi_2$, $\varphi|_{y=0} = \varphi_3$, $\varphi|_{y=b} = \varphi_4$. Normally, it is difficult to solve this equation analytically, especially for all non-zero boundary conditions. Thus, many numerical methods are proposed to solve these cases, for example, the finite different method (FDM), which can solve the Laplace equation using an iterative method, and based on the following algebraic expression:

$$\varphi_{i,j}^{n+1} = \varphi_{i,j}^{n} + \frac{\omega}{4}\left(\varphi_{i,j+1}^{n} + \varphi_{i,j-1}^{n+1} + \varphi_{i-1,j}^{n+1} + \varphi_{i+1,j}^{n} - 4\varphi_{i,j}^{n}\right) \tag{5-2}$$

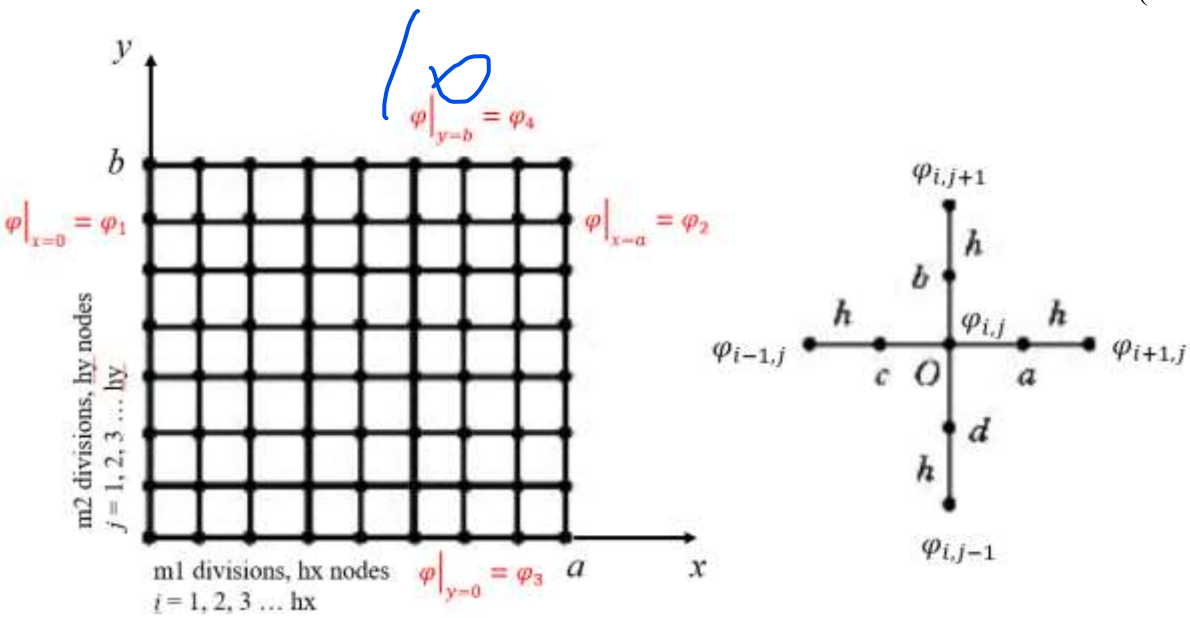
where n is iterative number, i and j are, respective, the node number in x and y directions. Solution of this algebraic equation is based on dividing the area in to a large number of small areas (called meshing). With the mesh size of h for each small area (Figure 5), the area are divided into m1 (=a/h) divisions in x direction and m2 (=b/h) in y direction, and thus hx = m1+1 (with i = 1, 2, 3 … hx) and hy = m2+1 (with j = 1, 2, 3 … hy) node numbers in x and y directions, respectively. The smaller the value of h, the more accurate solution can be obtained.

The solution procedure aims to solve the value of electrical potential on every node of the divided area, i.e. $\varphi_{i,j}$, which is given below:

1) Specify the boundary conditions, i.e. $\varphi|_{x=0} = \varphi_{1,j} = \varphi_1$, $\varphi|_{x=a} = \varphi_{hx,j} = \varphi_2$, $\varphi|_{y=0} = \varphi_{i,1} = \varphi_3$, $\varphi|_{y=b} = \varphi_{i,hy} = \varphi_4$.
2) Provide the initial values of $\varphi_{i,j}$ on every node of the divided area.
3) Use Eq. 5-2 to solve $\varphi_{i,j}$ iteratively, following the fixed order of nodes: from left to right (x from 0 to a, with i from 1 to hx) and then from bottom to top (y from 0 to b, with j from 1 to hy).
4) For each iteration step, i.e. n+1, check the difference between solution from that of the previous step, and stop the solution procedure until the difference is smaller than a pre-set tolerance (e.g. Tol = 10^{-5}). This means the solution procedure should complete when $\left|\varphi_{i,j}^{n+1} - \varphi_{i,j}^{n}\right| \leq Tol.$
5) Display the results.

The numerical iteration (i.e. Eq. 5-2) needs to define a relaxation factor which can be estimated by the following equation

$$\omega = \frac{2}{1 + \sqrt{1 - \left[\frac{\cos\left(\frac{\pi}{m1}\right) + \cos\left(\frac{\pi}{m2}\right)}{2}\right]^2}} \tag{5-3}$$



(a) The computational domain

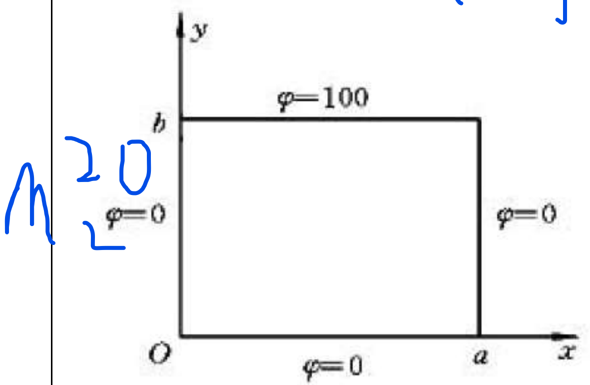
(b) Node (i, j) and its surrounding nodes

Figure 6. The 2D rectangular area of Problem 5, which shows the mesh and the nodes.

Now, write a programme to use FDM to solve the electrical potential distribution of the following two cases.

Case 1

$\varphi_{ij} = 1$



Use: a = 16 cm; b = 10 cm; h = 0.5 cm
Boundary conditions are as shown.
Use the initial condition of 1 V for electrical potential at all nodes.

$m_1 = 32$

$m_2 = 20$

node $hx = 33$

node $hy = 21$

Case 2

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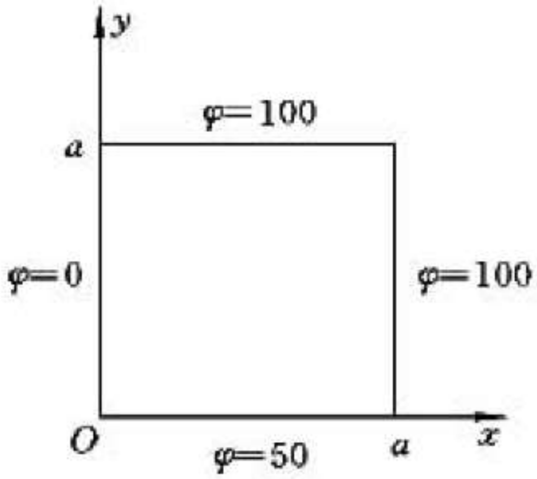
<td>
</td>
<td>Use: a = 10 cm; b = a = 10 cm; h = 0.5 cm
Boundary conditions are as shown.
Use the initial condition of 1 V for electrical potential at all nodes.</td>
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Requirements:

a) Write a function file for the solver of the Laplace equation defined by Eqs. 5-2 and 5-3. The function output should be the matrix containing the values of φ in all nodes of the area. It is also required this function should display the final iteration number (n) for the solution procedure.

b) Define the problem from manually input relevant values from the command window, including: (1) Dimensions of the computational area (a, b); (2) Mesh size h; (b) 4 boundary conditions. (Hints: You can use "input" command to do this, e.g. try this command: a=input('Input a number:').)

c) Display the results, use "mesh" and "contour" commands to plot the results, respectively, and thus 2 figures are required.

d) Draw the flow charts of your programme, including a flow chart for the main programme and those for all functions.