2.4 一元多项式的表示和相加

- 一元多项式$P_n(x)=a_0+a_1x+a_2x^2+\ldots+a_nx^n$
  - 由n+1个系数唯一确定。在计算机中可用一个线性表$P_n=(a_0, a_1, a_2,\ldots, a_n)$来表示，按照指数升序排列系数，定义线性表
- 例1：一元多项式： $P_4(x)=5+3x+12x^2+23x^4$
  - 对应的**线性表:** $P_4=(5, 3, 12, 0, 23)$
  - 即： $P_4(x)=5+3x+12x^2+0x^3+23x^4$
- 例2：一元多项式： $P_5(x)=11+3x+12x^3+17x^5$
  - 对应的**线性表:** $P_5=(11, 3, 0, 12, 0, 17)$
  - 即： $P_5(x)=11+3x+0x^2+12x^3+0x^4+17x^5$

2.4 一元多项式的表示和相加

- 两个多项式相加$P_n(x)+ Q_m(x)$:

> $P_n(x)$ 对应的线性表：$(a_0, a_1, a_2,\ldots, a_n)$

> $Q_m(x)$对应的线性表：$(b_0, b_1, b_2,\ldots, b_m)$

> 假设$n \le m$，则和多项式的线性表：

$$(a_0+b_0, a_1+b_1, a_2+b_2,\ldots, a_n+b_n, b_{n+1}, \ldots, b_m)$$

- 相加算法实现：首先确定表示一元多项式的线性表的存储方式-----**顺序存储结构，链式存储结构**均可；其次研究不同存储方式下的相加算法

线性表采用顺序存储结构存放

- 实现方式：数组
- 问题：求两个一元多项式的**和**

$$R_m(x) = P_n(x) + Q_m(x): n \le m。$$

分析：一元多项式$P_n(x)$和$P_m(x)$分别用一维数组**A[ ]**和**B[ ]**表示；其和用一维数组 **R[ ]**表示。则有

(1) **R[i]=A[i]+B[i], i≤n；**

(2) **R[i]=B[i], n<i≤m。**

问题？

- $P_{2001}(x)=12+10x^{120}+23x^{2001}$——线性表有2002个数据元素，其中1999个为0
- $S_{2000}(x)=1+2x^{2000}$——线性表有2001个数据元素，其中1999个为0
- 一元多项式的指数很高且相邻的指数相差很大时，宜只存放系数非零项的系数和相应的指数，否则浪费存储，但系数非零项**按指数升序排列**
- $S_{2000}(x)=1+2x^{2000}$只存放指数为0和2000两项即可，即：$S_{2001}=((1,0),(2, 2000))$
- $P_{2001}(x)=12+10x^{120}+23x^{2001}$只存放指数为0,120和2001三项即可，$P_{2001}=((12,0),(10,120),(23, 2001))$

指数相差很大的多项式

- 只存放系数非零项，顺序存储结构：每个数组元素存一个非零项——系数(coef)和指数(exp)
- $P_{2001}(x)=12+10x^{120}+23x^{2001}$

	coef	exp
elem[0]	12	0
elem[1]	10	120
elem[2]	23	2001
elem[3]		
elem[4]		

指数相差很大的多项式

- define MAXSIZE 100
- typedef struct

  { int coef,exp} Elemtype;
- typedef struct

  {Elemtype elem[MAXSIZE];

  int length;

  } SeqPoly;
- SeqPoly p;

相加

- $P_{2001}(x)=12+10x^{120}+23x^{2001}$
- $Q_{3000}(x)=-8x^{10}+45x^{120}-23x^{2001}+7x^{3000}$
- $P_{2001}=((12,0),(10,120),(23,2001))$
- $Q_{3000}=((-8,10),(45,120),(-23,2001),(7,3000))$

	coef	exp
P.elem[0]	12	0
P.elem[1]	10	120
P.elem[2]	23	2001
P.elem[3]		
P.elem[4]		

	coef	exp
Q.elem[0]	-8	10
Q.elem[1]	45	120
Q.elem[2]	-23	2001
Q.elem[3]	7	3000
Q.elem[4]		

相加

- $P_{2001}(x)=12+10x^{120}+23x^{2001}$
- $Q_{3000}(x)=-8x^{10}+45x^{120}-23x^{2001}+7x^{3000}$
- $P_{2001}=((12,0),(10,120),(23,2001))$
- $Q_{3000}=((-8,10),(45,120),(-23,2001),(7,3000))$

	coef	exp
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P.elem[2]	23	2001
P.elem[3]		
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	coef	exp
Q.elem[0]	-8	10
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	coef	exp
P.elem[0]	12	0
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P.elem[3]	23	2001
P.elem[4]		

	coef	exp
Q.elem[0]	-8	10
Q.elem[1]	45	120
Q.elem[2]	-23	2001
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Q.elem[4]		

相加

- $P_{2001}(x)=12+10x^{120}+23x^{2001}$
- $Q_{3000}(x)=-8x^{10}+45x^{120}-23x^{2001}+7x^{3000}$
- $P_{2001}=((12,0),(10,120),(23,2001))$
- $Q_{3000}=((-8,10),(45,120),(-23,2001),(7,3000))$

	coef	exp
P.elem[0]	12	0
P.elem[1]	10	120
P.elem[2]	10	120
P.elem[3]	23	2001
P.elem[4]		

	coef	exp
Q.elem[0]	-8	10
Q.elem[1]	45	120
Q.elem[2]	-23	2001
Q.elem[3]	7	3000
Q.elem[4]		

相加

- $P_{2001}(x)=12+10x^{120}+23x^{2001}$
- $Q_{3000}(x)=-8x^{10}+45x^{120}-23x^{2001}+7x^{3000}$
- $P_{2001}=((12,0),(10,120),(23,2001))$
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	coef	exp
P.elem[0]	12	0
P.elem[1]	-8	10
P.elem[2]	10	120
P.elem[3]	23	2001
P.elem[4]		

	coef	exp
Q.elem[0]	-8	10
Q.elem[1]	45	120
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相加

- $P_{2001}(x)=12+10x^{120}+23x^{2001}$
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- $P_{2001}=((12,0),(10,120),(23,2001))$
- $Q_{3000}=((-8,10),(45,120),(-23,2001),(7,3000))$

	coef	exp
P.elem[0]	12	0
P.elem[1]	-8	10
P.elem[2]	10	120
P.elem[3]	23	2001
P.elem[4]		

	coef	exp
Q.elem[0]	-8	10
Q.elem[1]	45	120
Q.elem[2]	-23	2001
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相加

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- $Q_{3000}(x)=-8x^{10}+45x^{120}-23x^{2001}+7x^{3000}$
- $P_{2001}=((12,0),(10,120),(23,2001))$
- $Q_{3000}=((-8,10),(45,120),(-23,2001),(7,3000))$

	coef	exp
P.elem[0]	12	0
P.elem[1]	-8	10
P.elem[2]	55	120
P.elem[3]	23	2001
P.elem[4]		

	coef	exp
Q.elem[0]	-8	10
Q.elem[1]	45	120
Q.elem[2]	-23	2001
Q.elem[3]	7	3000
Q.elem[4]		

相加

- $P_{2001}(x)=12+10x^{120}+23x^{2001}$
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- $P_{2001}=((12,0),(10,120),(23,2001))$
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	coef	exp
P.elem[0]	12	0
P.elem[1]	-8	10
P.elem[2]	55	120
P.elem[3]	23	2001
P.elem[4]		

	coef	exp
Q.elem[0]	-8	10
Q.elem[1]	45	120
Q.elem[2]	-23	2001
Q.elem[3]	7	3000
Q.elem[4]		

相加

- $P_{2001}(x)=12+10x^{120}+23x^{2001}$
- $Q_{3000}(x)=-8x^{10}+45x^{120} -23x^{2001}+7x^{3000}$
- P_{2001}=((12,0),(10,120),(23,2001))
- Q_{3000}=((-8,10),(45,120),(-23,2001), (7,3000))

	coef	exp
P.elem[0]	12	0
P.elem[1]	-8	10
P.elem[2]	55	120
P.elem[3]	23	2001
P.elem[4]		

	coef	exp
Q.elem[0]	-8	10
Q.elem[1]	45	120
Q.elem[2]	-23	2001
Q.elem[3]	7	3000
Q.elem[4]		

和: ((12,0), (-8,10) ,(55,120) , (7,3000))

顺序存储结构插入、删除需要移动数据，以多项式相加运算为例，若采用数组存放，相加运算的结果要保存在元多项式被加数中，则可能需要插入、删除，要移动数据----

建议采用链表

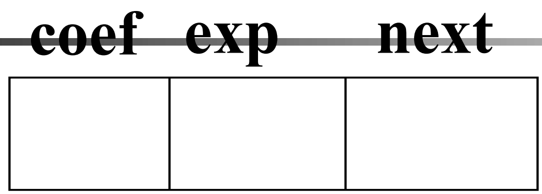
相加

- $P_{2001}(x)=12+10x^{120}+23x^{2001}$
- $Q_{3000}(x)=-8x^{10}+45x^{120}-23x^{2001}+7x^{3000}$
- P_{2001}=((12,0),(10,120),(23,2001))
- Q_{3000}=((-8,10),(45,120),(-23,2001), (7,3000))

	coef	exp
P.elem[0]	12	0
P.elem[1]	-8	10
P.elem[2]	55	120
P.elem[3]	7	3000
P.elem[4]		

	coef	exp
Q.elem[0]	-8	10
Q.elem[1]	45	120
Q.elem[2]	-23	2001
Q.elem[3]	7	3000
Q.elem[4]		

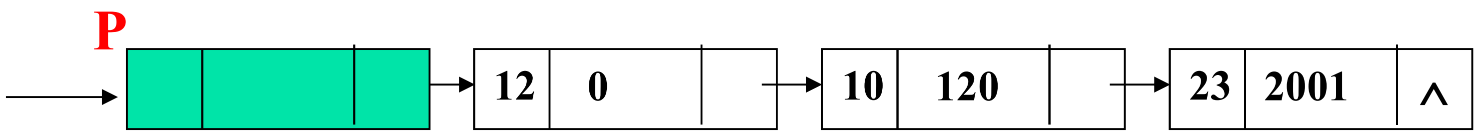
链式存储结构：将多项式的每一系数非零项拉成单链表。表中每个结点由三个域组成：系数域（coef）、指数域（exp）、指针域（next）



结点结构示意图

```
typedef struct node{
      int   coef;//系数
      int    exp ;//指数
      struct node *next;}PNode,  *Poly;
```

$$P_{2001}(x)=12+10x^{120}+23x^{2001}$$



链表表示适合于经常增减非零项。

多项式相加的运算规则

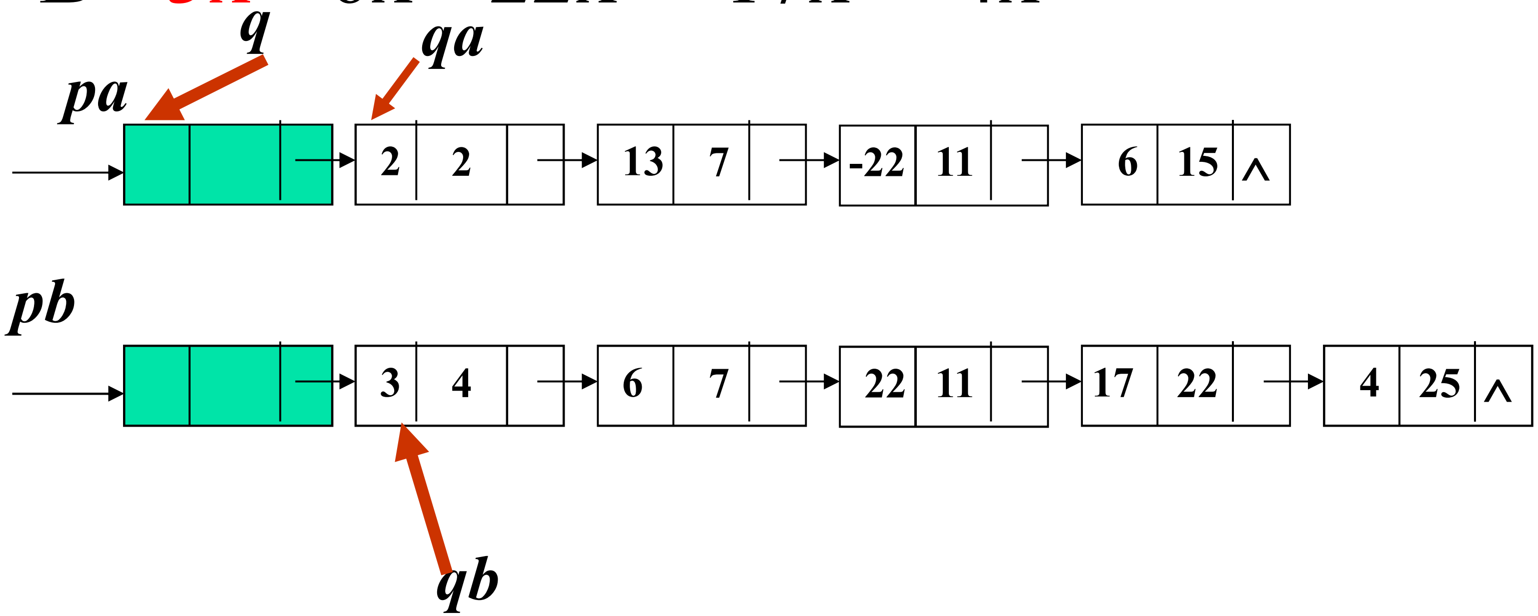
- 两个多项式中所有指数相等的项对应系数相加，若和不为零，则构成“和多项式”的一项；所有指数不同的项均复抄到和“多项式”中。
- **问题：** A=A+B

➢ 设A、B采用链式存储结构存放，头指针分别为pa、pb。qa、qb分别指向A、B多项式的当前搜索结点。q指向qa的直接前趋结点。

➢ 相加后只保留和多项式

多项式相加的运算规则

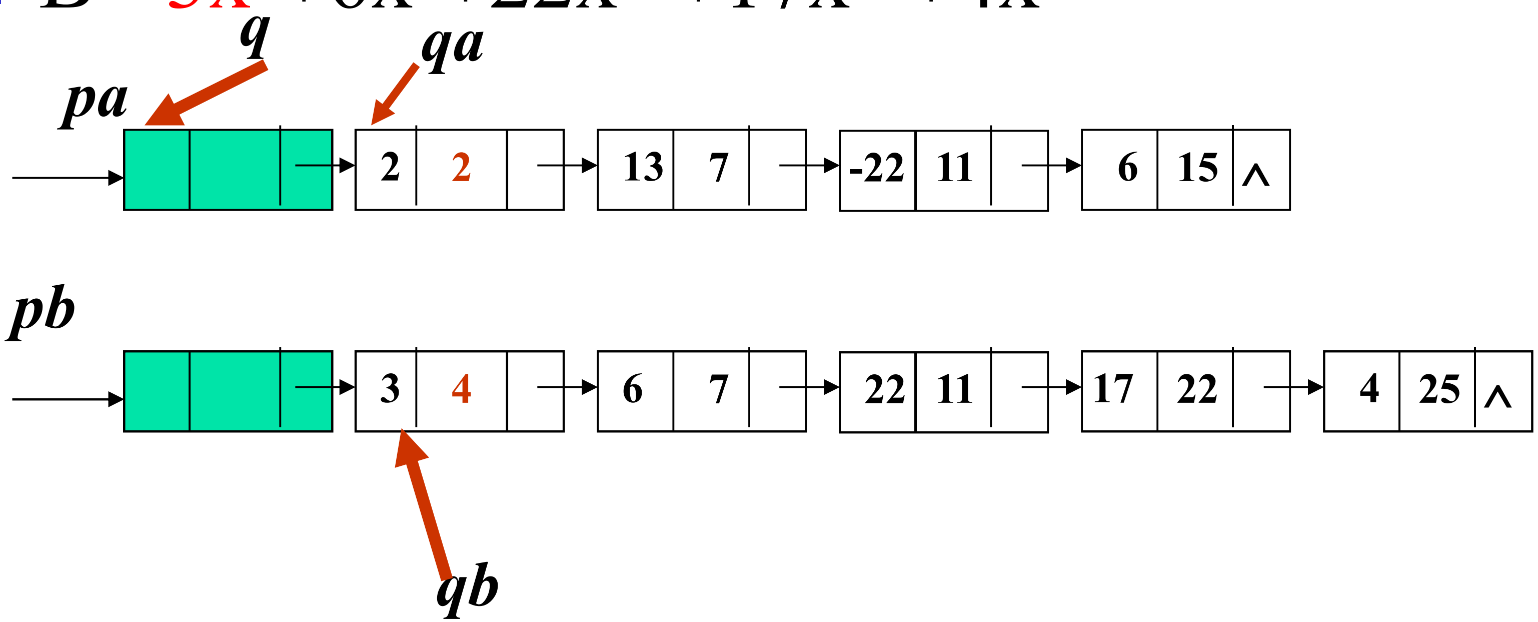
- $A=2x^2+13x^7-22x^{11}+6x^{15}$
- $B=3x^4+6x^7+22x^{11}+17x^{22}+4x^{25}$



qa=pa->next;q=pa;qb=pb->next;

多项式相加的运算规则

- $A=2x^2+13x^7-22x^{11}+6x^{15}$
- $B=3x^4+6x^7+22x^{11}+17x^{22}+4x^{25}$

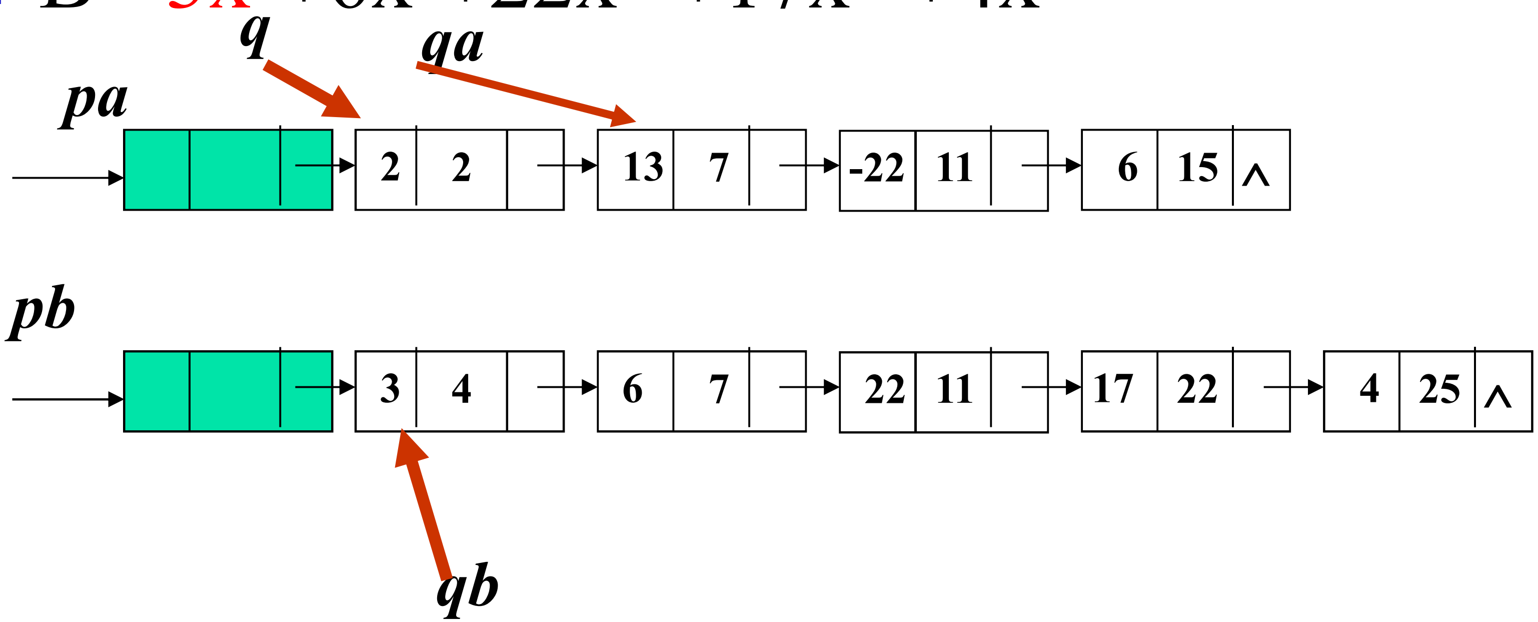


被加数qa->exp和加数qb->exp进行比较

被加数qa->exp<加数qb->exp, $2x^2$留在和多项式里

$A=2x^2$

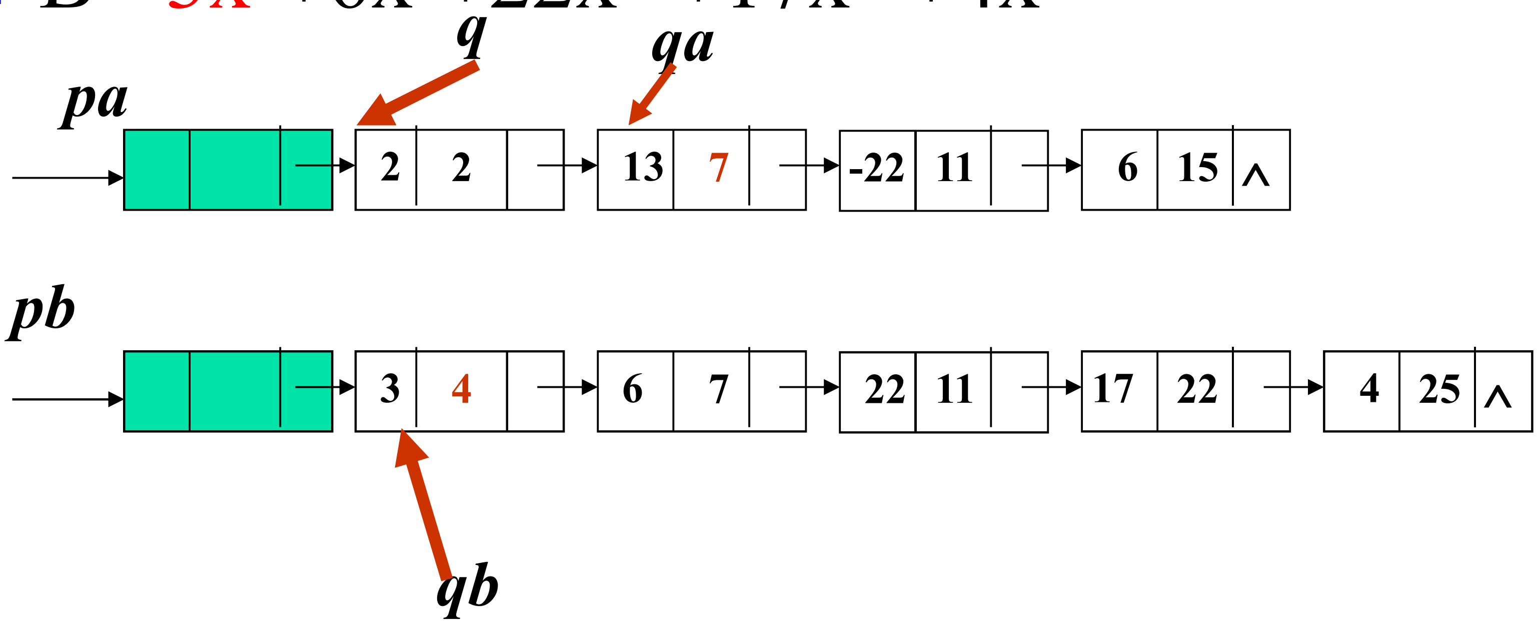
- $A=2x^2+13x^7-22x^{11}+6x^{15}$
- $B=3x^4+6x^7+22x^{11}+17x^{22}+4x^{25}$



q=qa;qa=qa->next

$A=2x^2$

- $A=2x^2+13x^7-22x^{11}+6x^{15}$
- $B=3x^4+6x^7+22x^{11}+17x^{22}+4x^{25}$

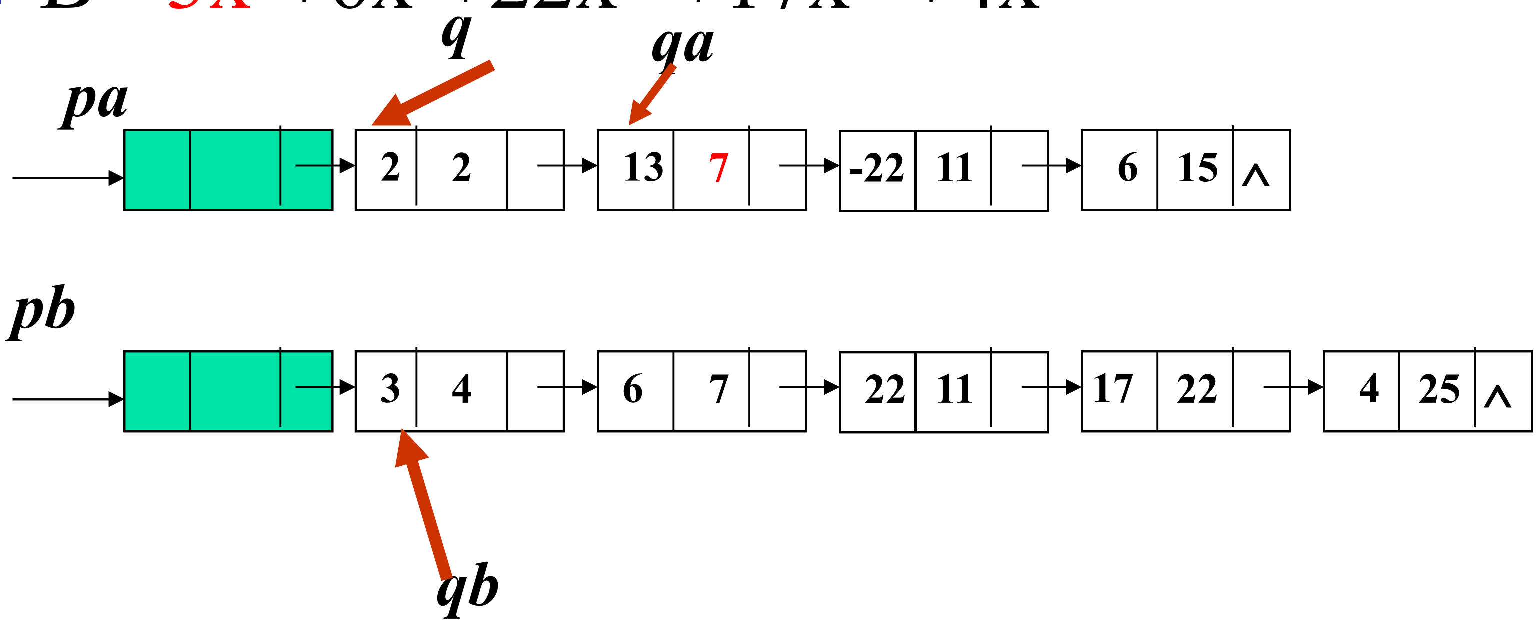


被加数qa->exp和加数qb->exp进行比较

被加数qa->exp>加数qb->exp, $3x^4$留在和多项式里

$A=2x^2+3x^4$

- $A=2x^2+13x^7-22x^{11}+6x^{15}$
- $B=3x^4+6x^7+22x^{11}+17x^{22}+4x^{25}$



被加数qa->exp和加数qb->exp进行比较

被加数qa->exp>加数qb->exp, $3x^4$留在和多项式里

$A=2x^2+3x^4$

- $A=2x^2+13x^7-22x^{11}+6x^{15}$
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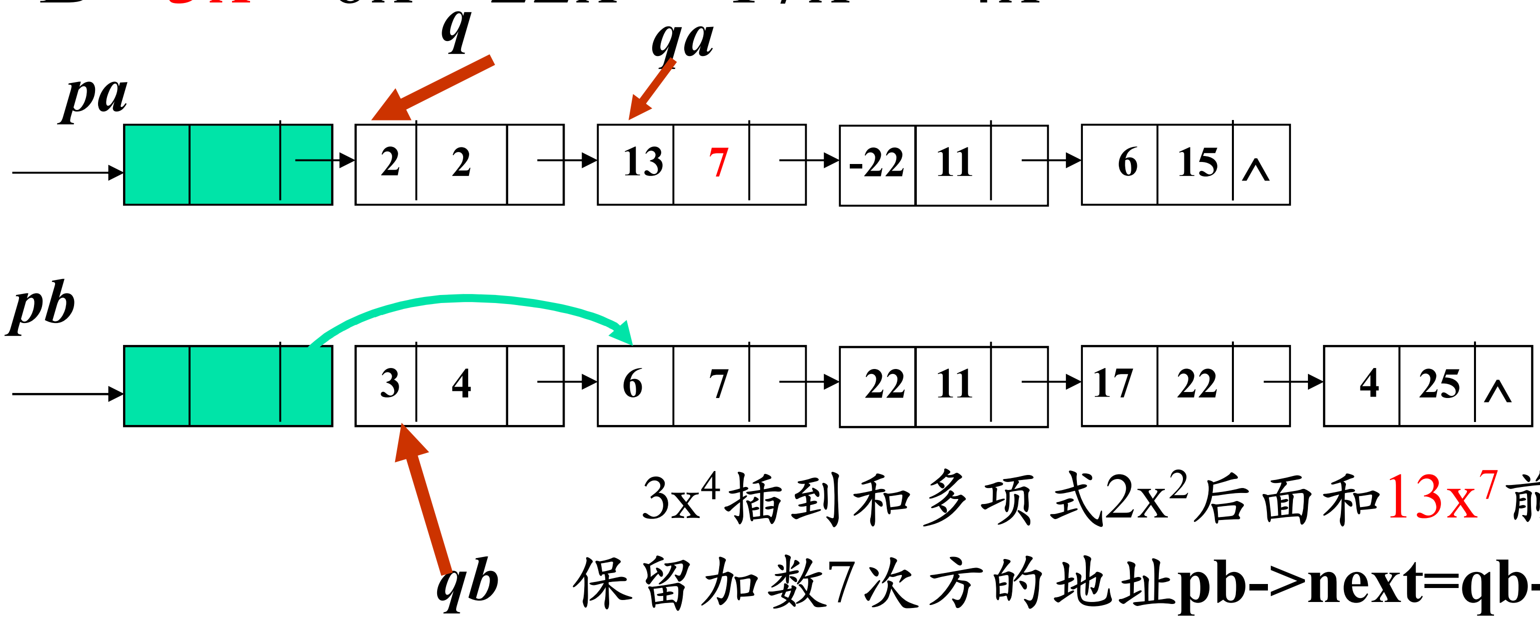
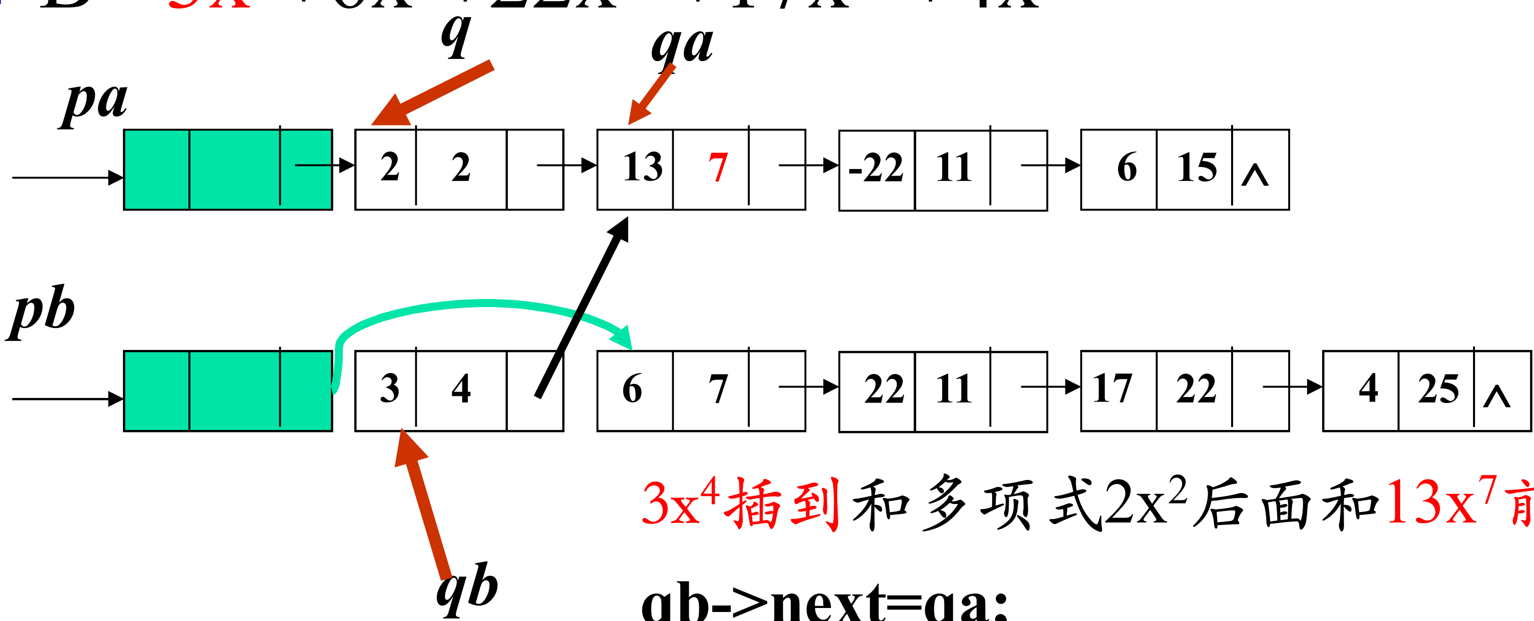


$3x^4$插到和多项式$2x^2$后面和$13x^7$前面

保留加数7次方的地址**pb->next=qb->next;**

$A=2x^2+3x^4$

- $A=2x^2+13x^7-22x^{11}+6x^{15}$
- $B=3x^4+6x^7+22x^{11}+17x^{22}+4x^{25}$

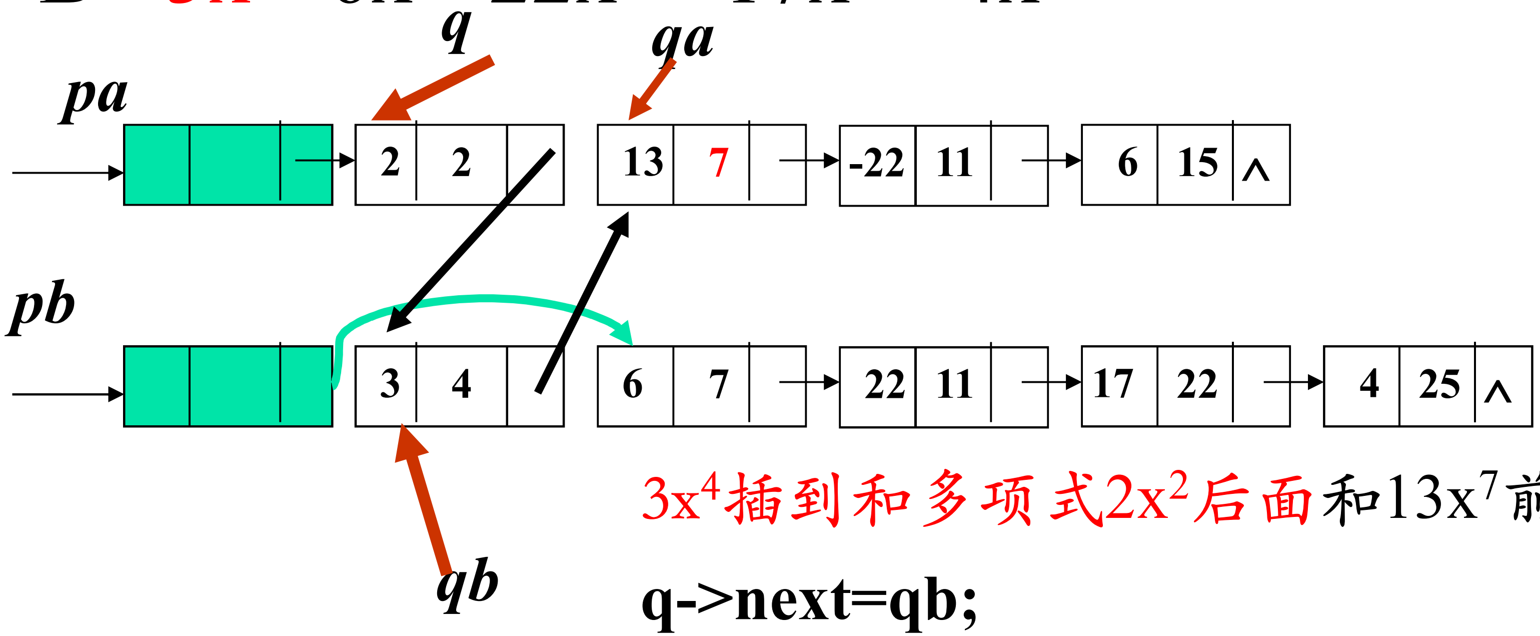


$3x^4$插到和多项式$2x^2$后面和$13x^7$前面

qb->next=qa;

$A=2x^2+3x^4$

- $A=2x^2+13x^7-22x^{11}+6x^{15}$
- $B=3x^4+6x^7+22x^{11}+17x^{22}+4x^{25}$

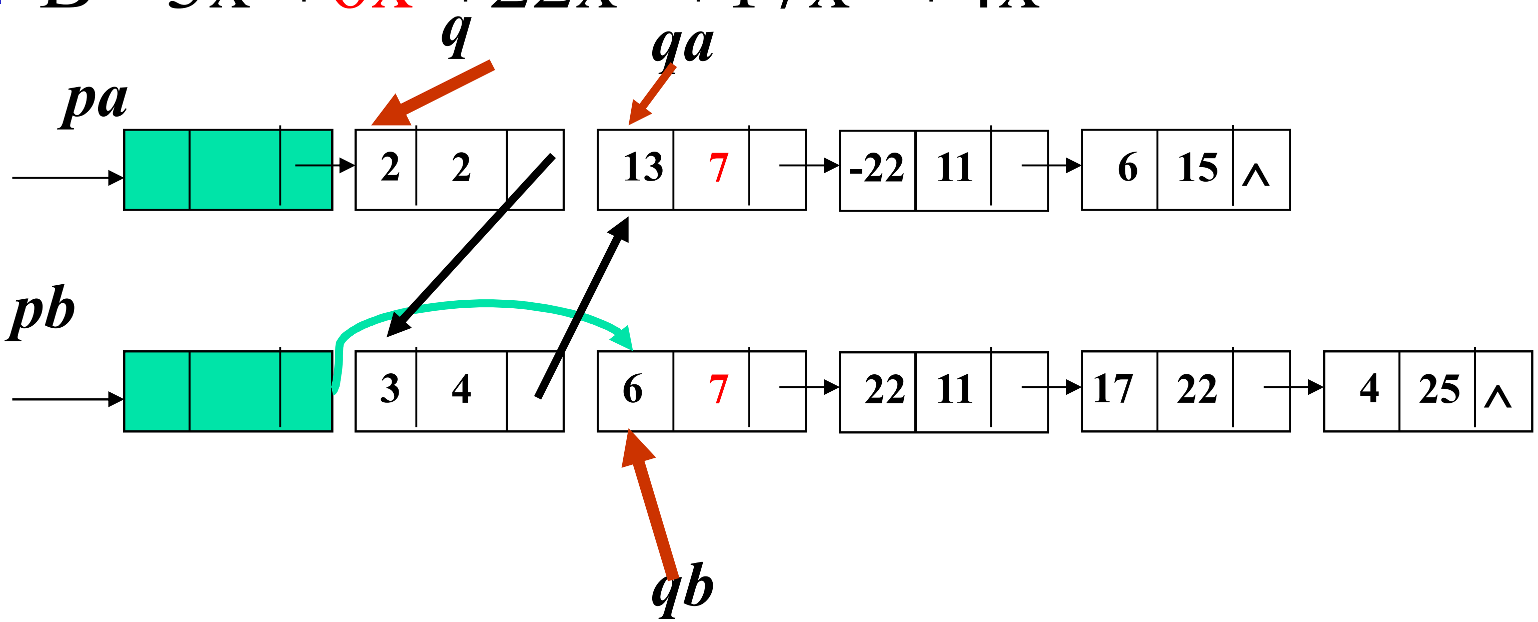


$3x^4$插到和多项式$2x^2$后面和$13x^7$前面

q->next=qb;

$A=2x^2+3x^4$

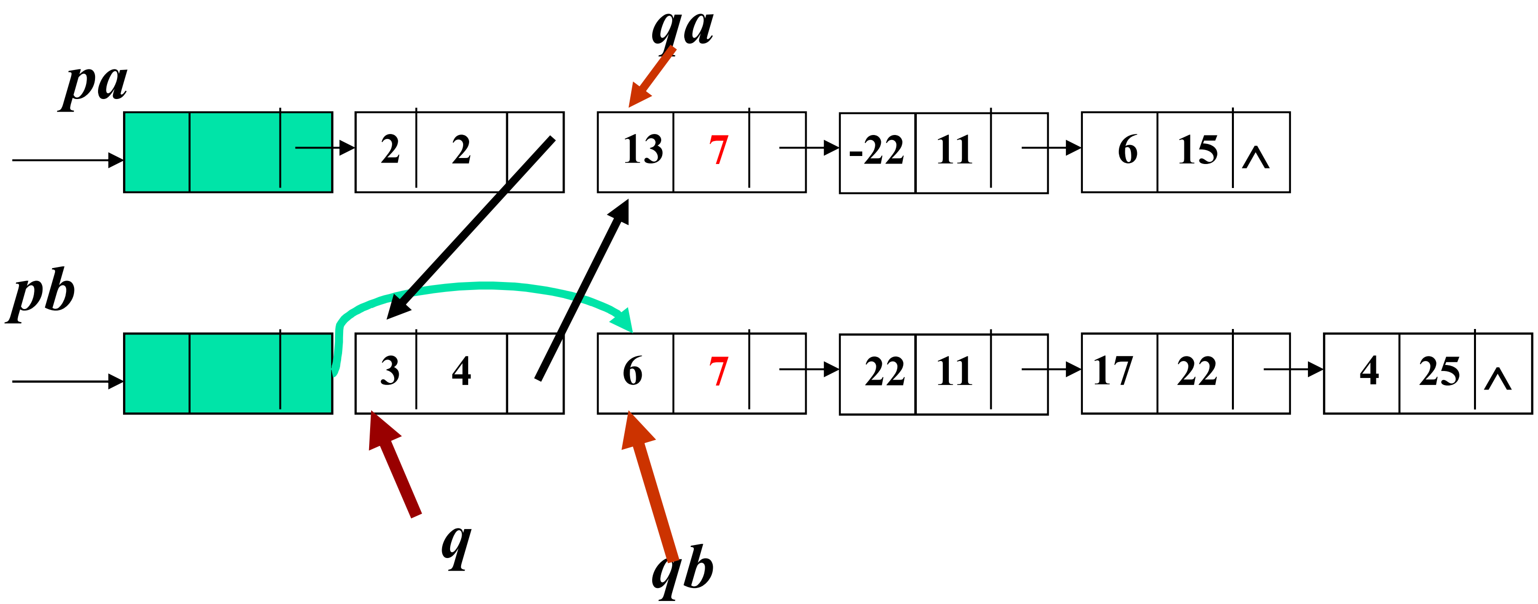
- $A=2x^2+13x^7-22x^{11}+6x^{15}$
- $B=3x^4+6x^7+22x^{11}+17x^{22}+4x^{25}$



下一次加数查看$6x^7$ **qb= pb->next;**

$A=2x^2+3x^4$

- $A=2x^2+13x^7-22x^{11}+6x^{15}$
- $B=3x^4+6x^7+22x^{11}+17x^{22}+4x^{25}$



$A=2x^2+3x^4$

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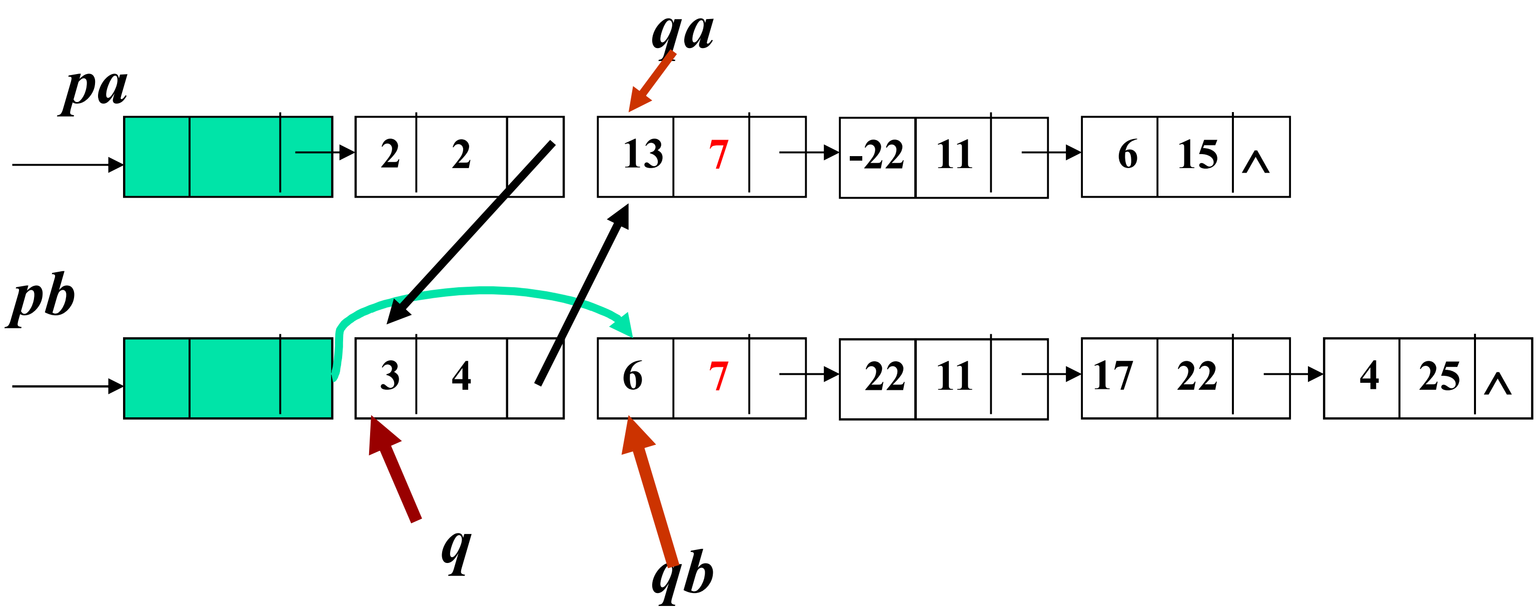

$A=2x^2+3x^4+19x^7$

- $A=2x^2+13x^7-22x^{11}+6x^{15}$
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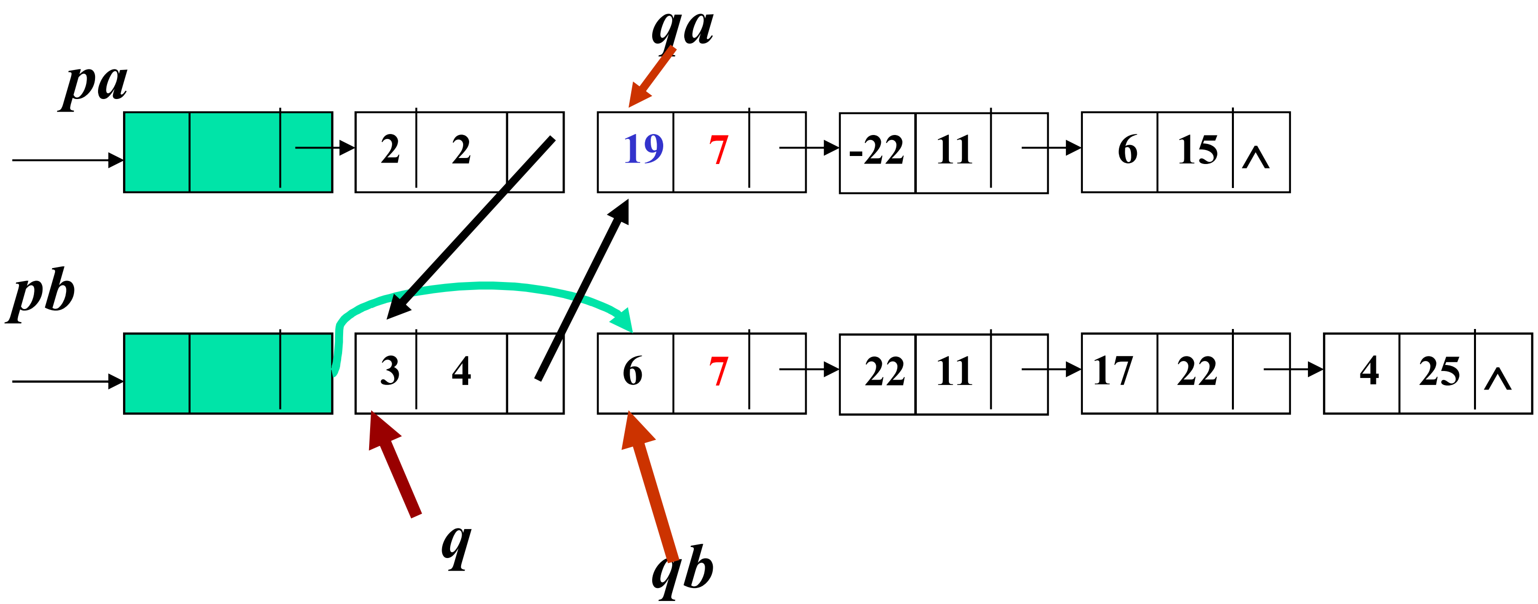

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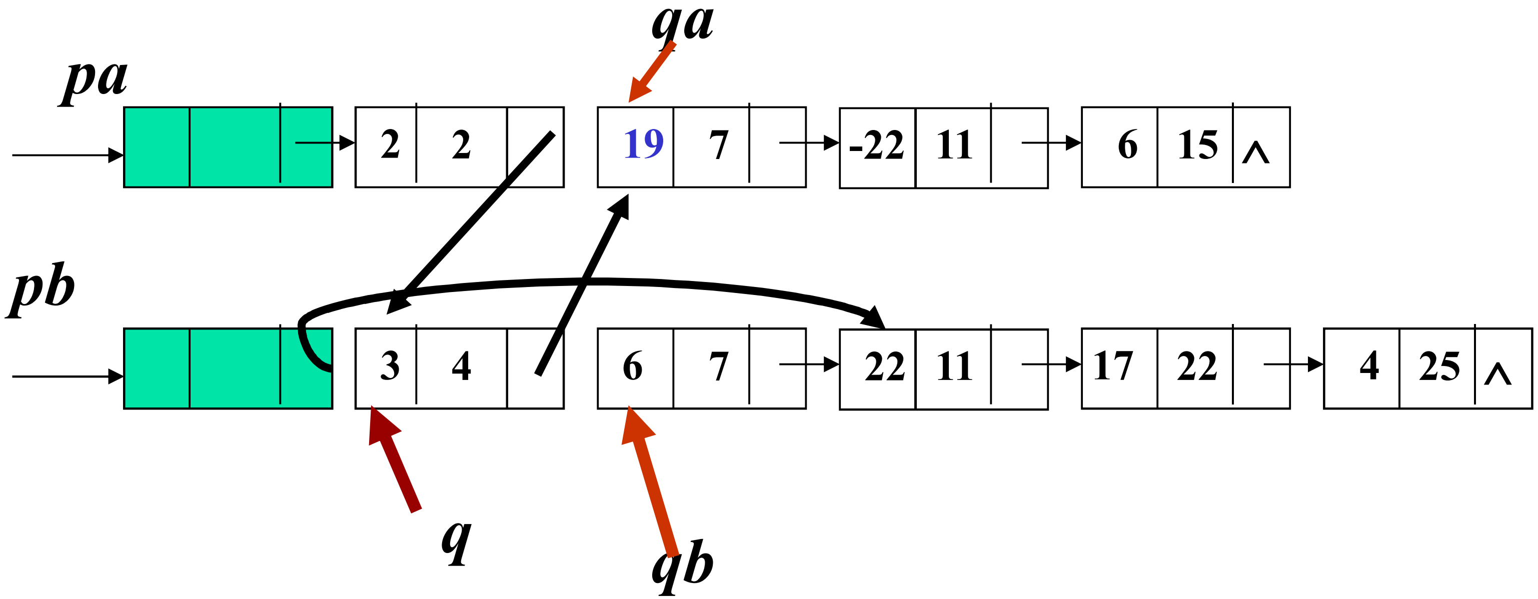

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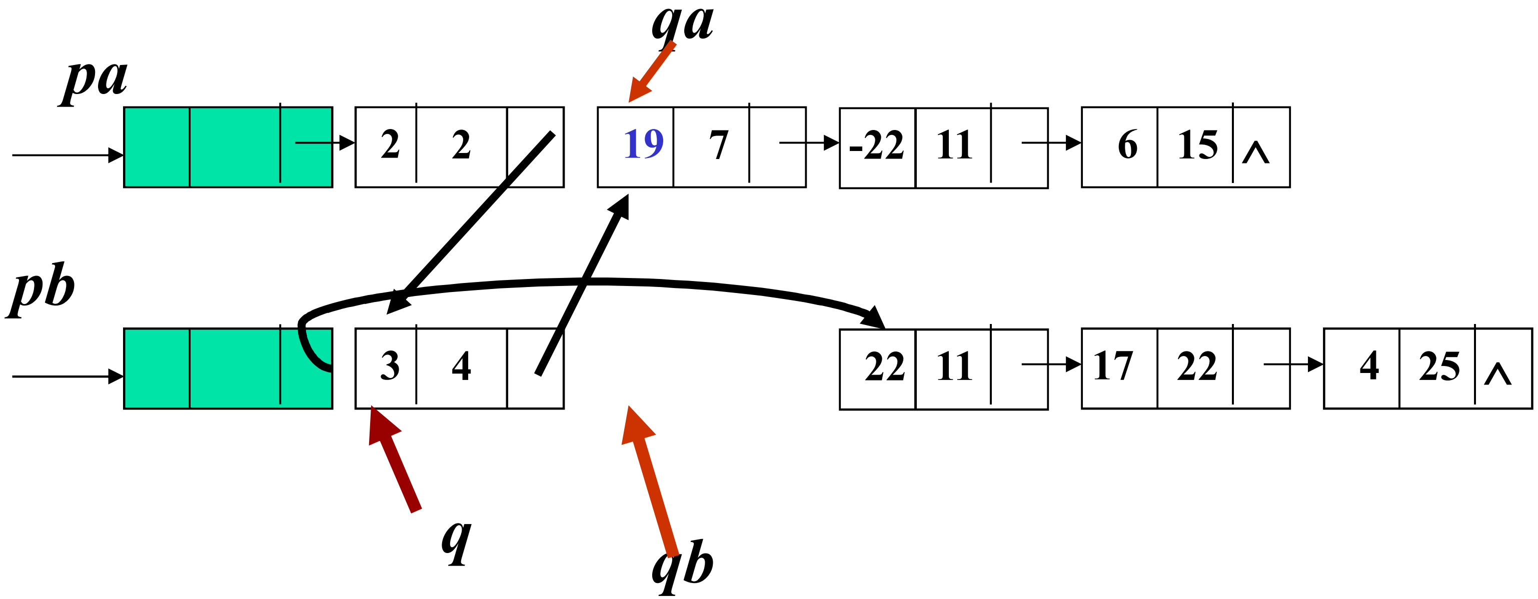

$A=2x^2+3x^4+19x^7$

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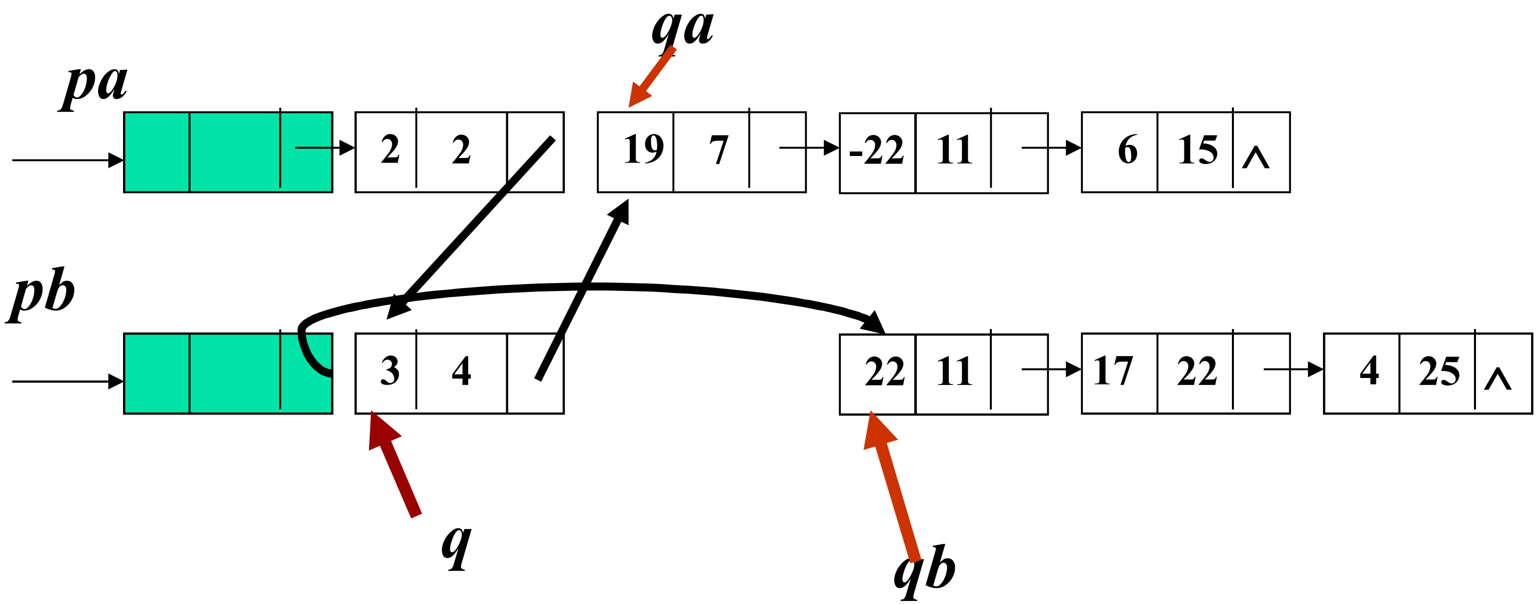

$A=2x^2+3x^4+19x^7$

- $A=2x^2+13x^7-22x^{11}+6x^{15}$
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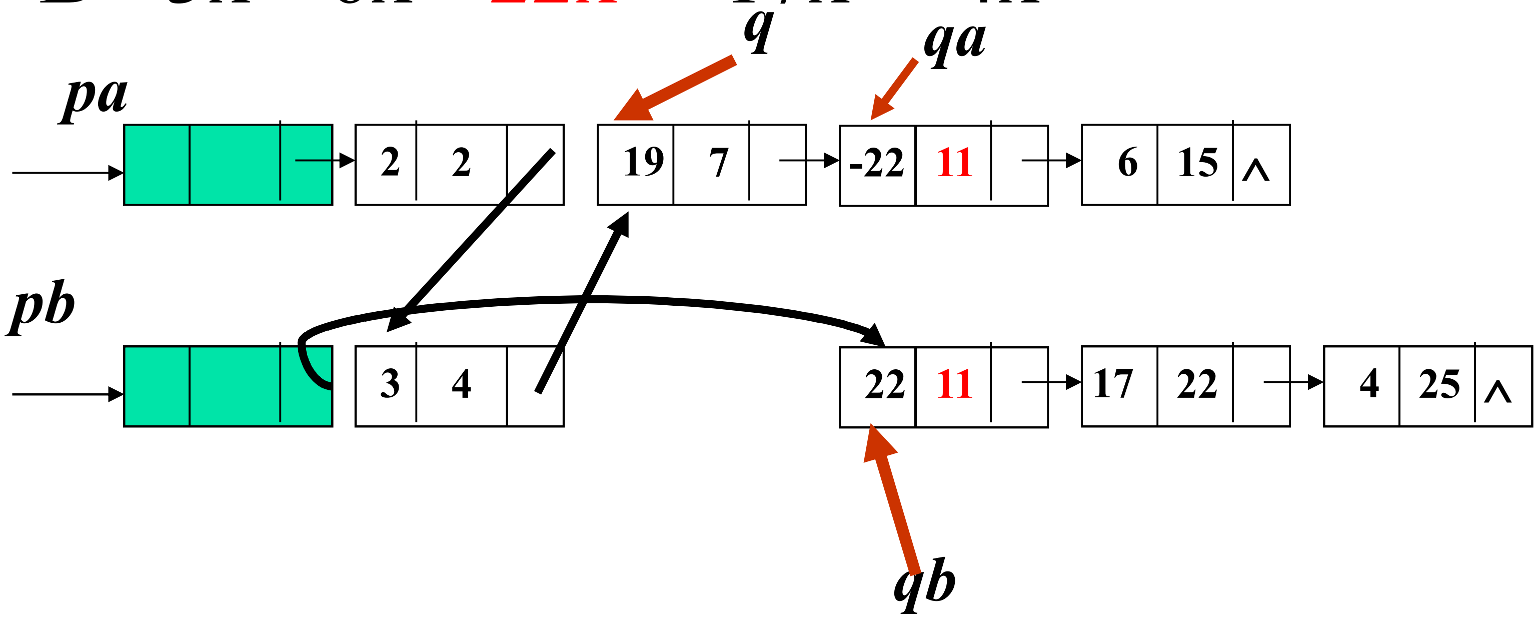

$A=2x^2+3x^4+19x^7$

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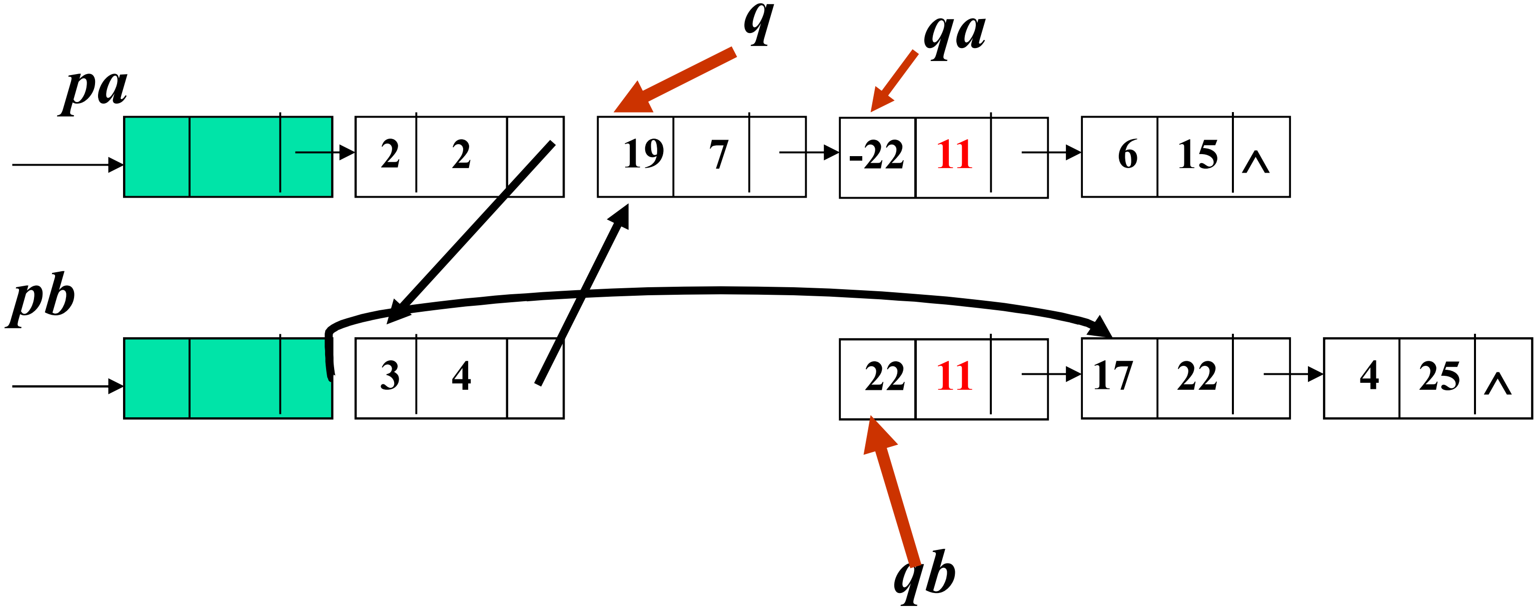

$A=2x^2+3x^4+19x^7$

- $A=2x^2+13x^7-22x^{11}+6x^{15}$
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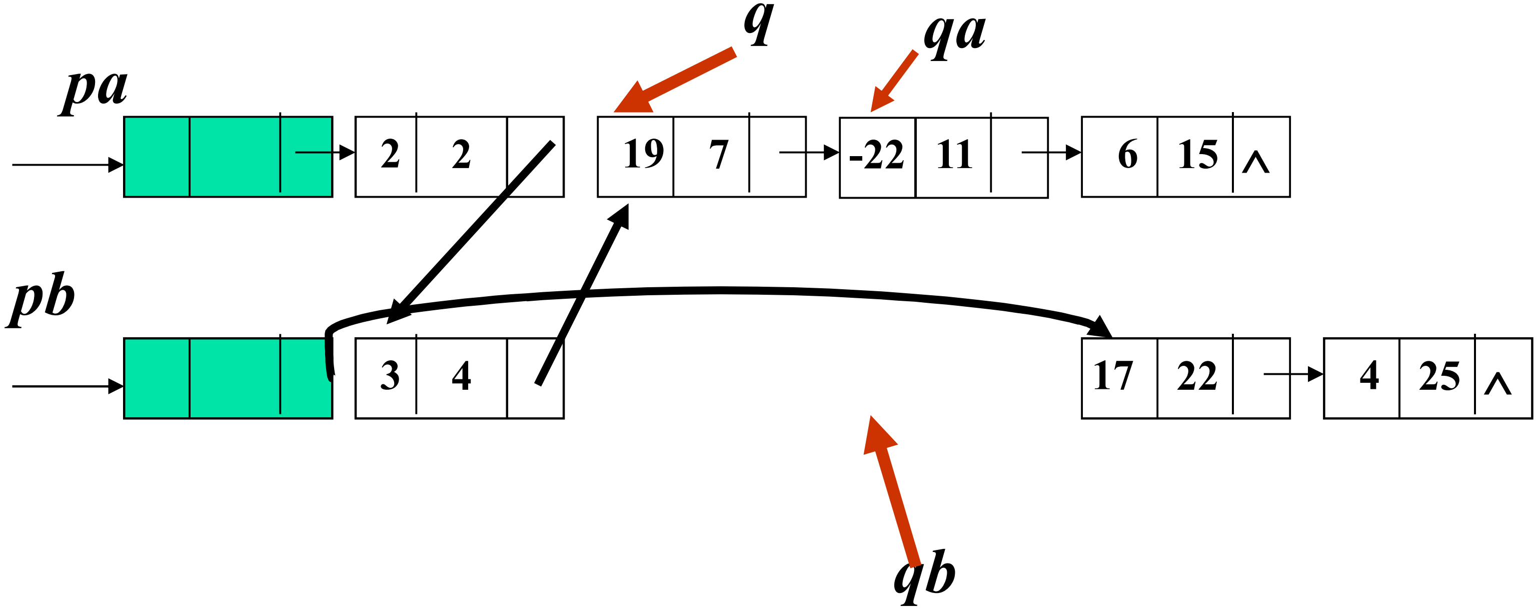
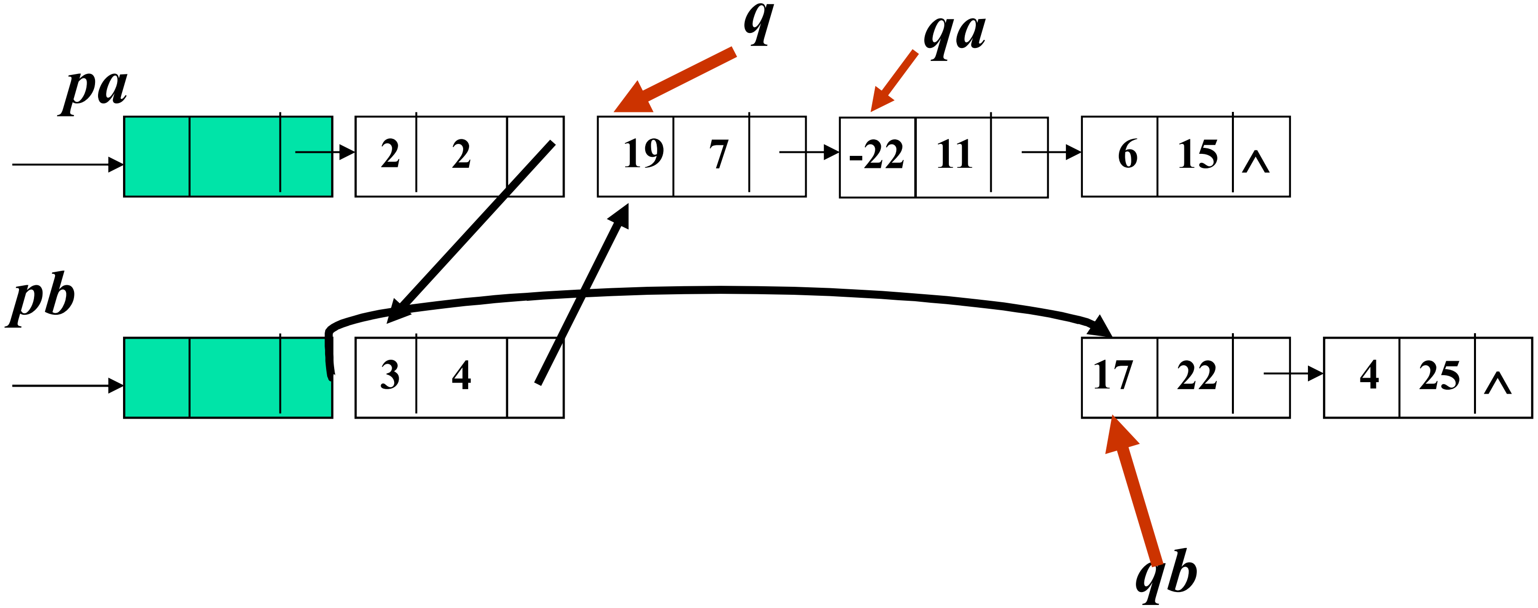


$A=2x^2+3x^4+19x^7$

- $A=2x^2+13x^7-22x^{11}+6x^{15}$
- $B=3x^4+6x^7+22x^{11}+17x^{22}+4x^{25}$



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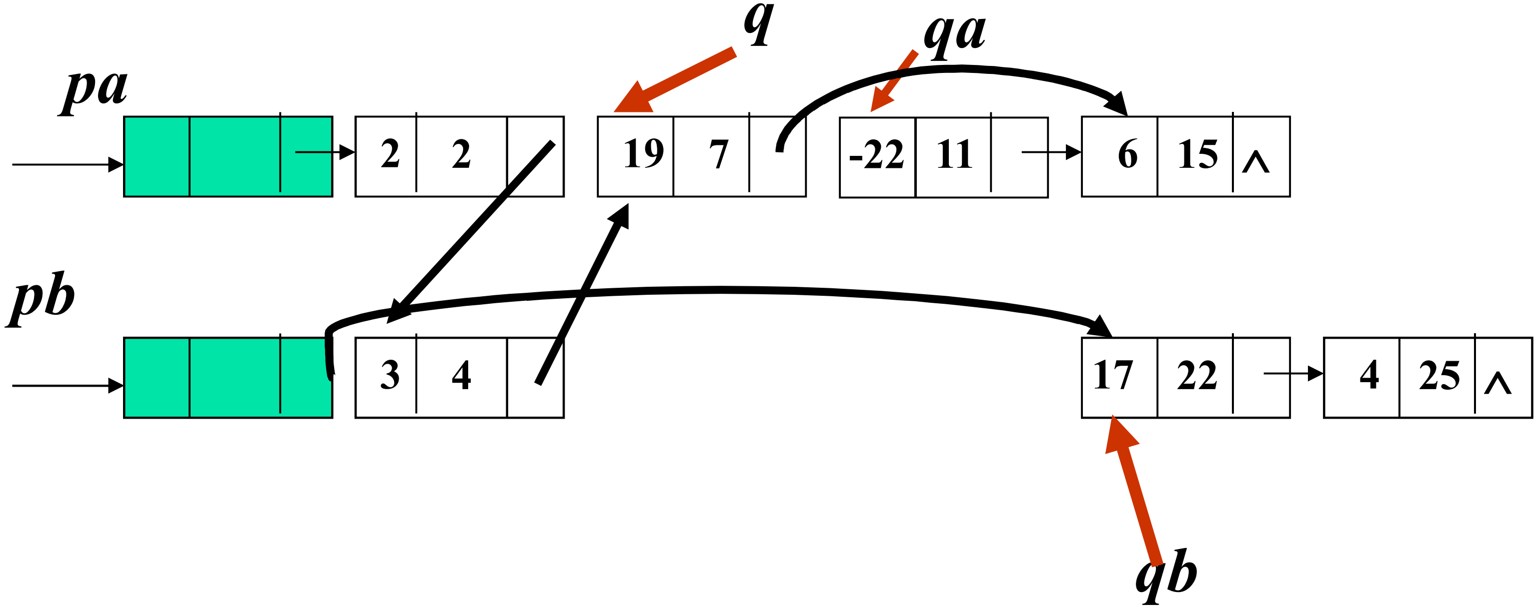

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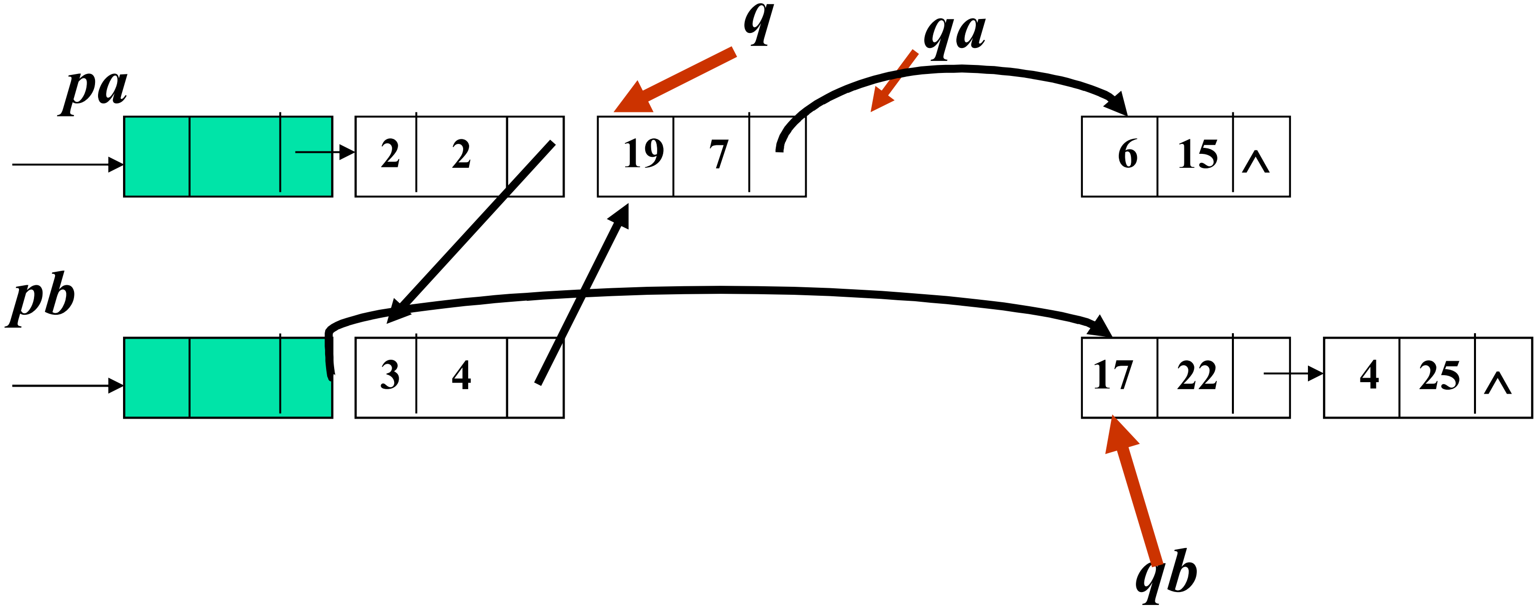

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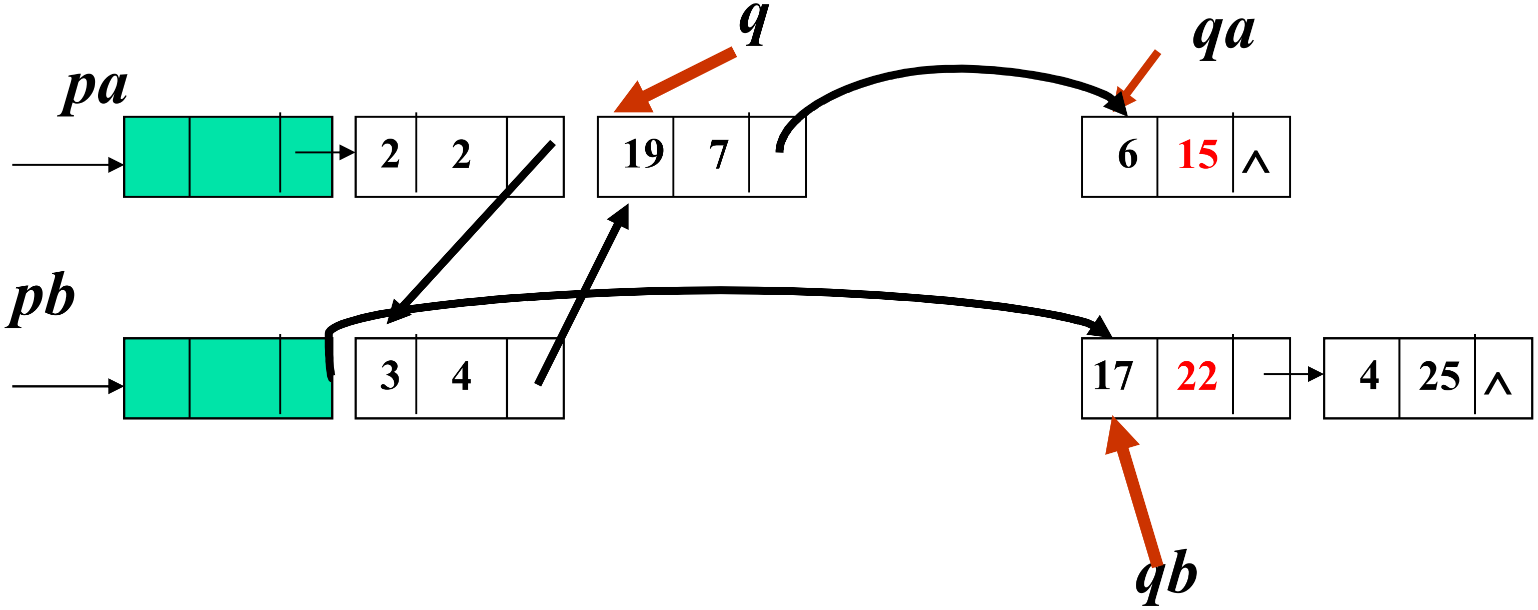

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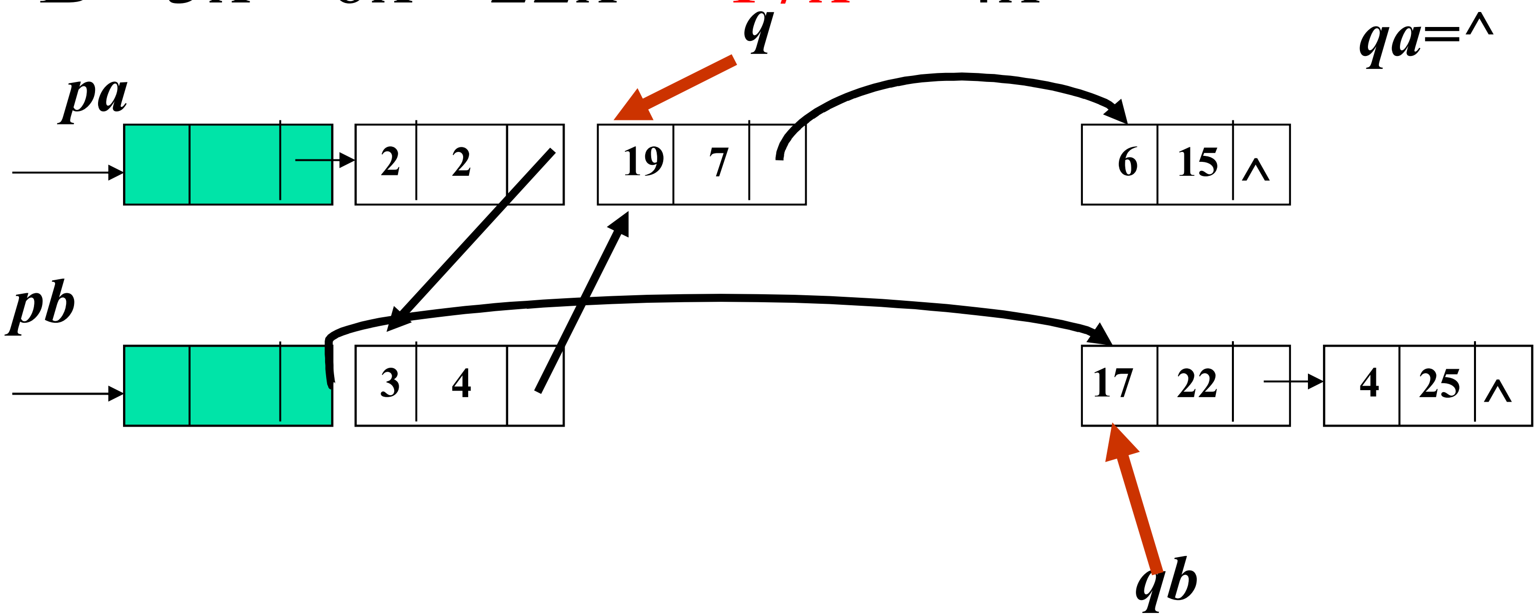

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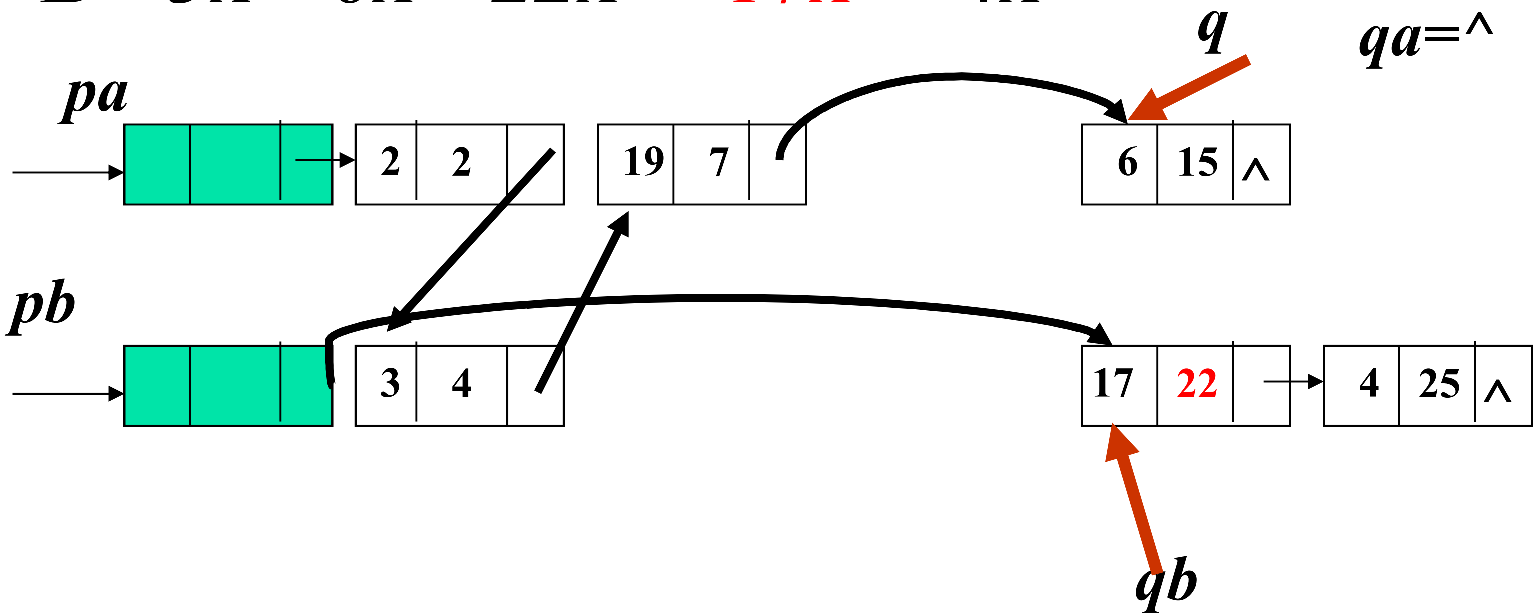

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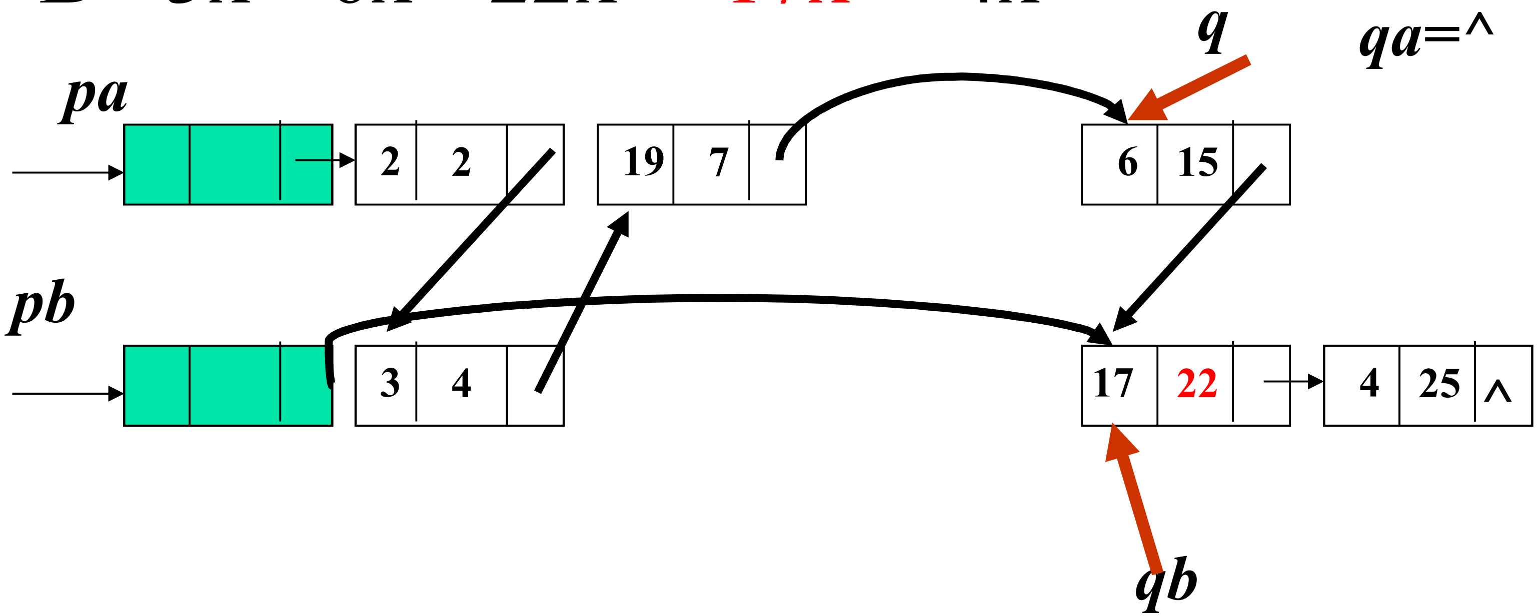

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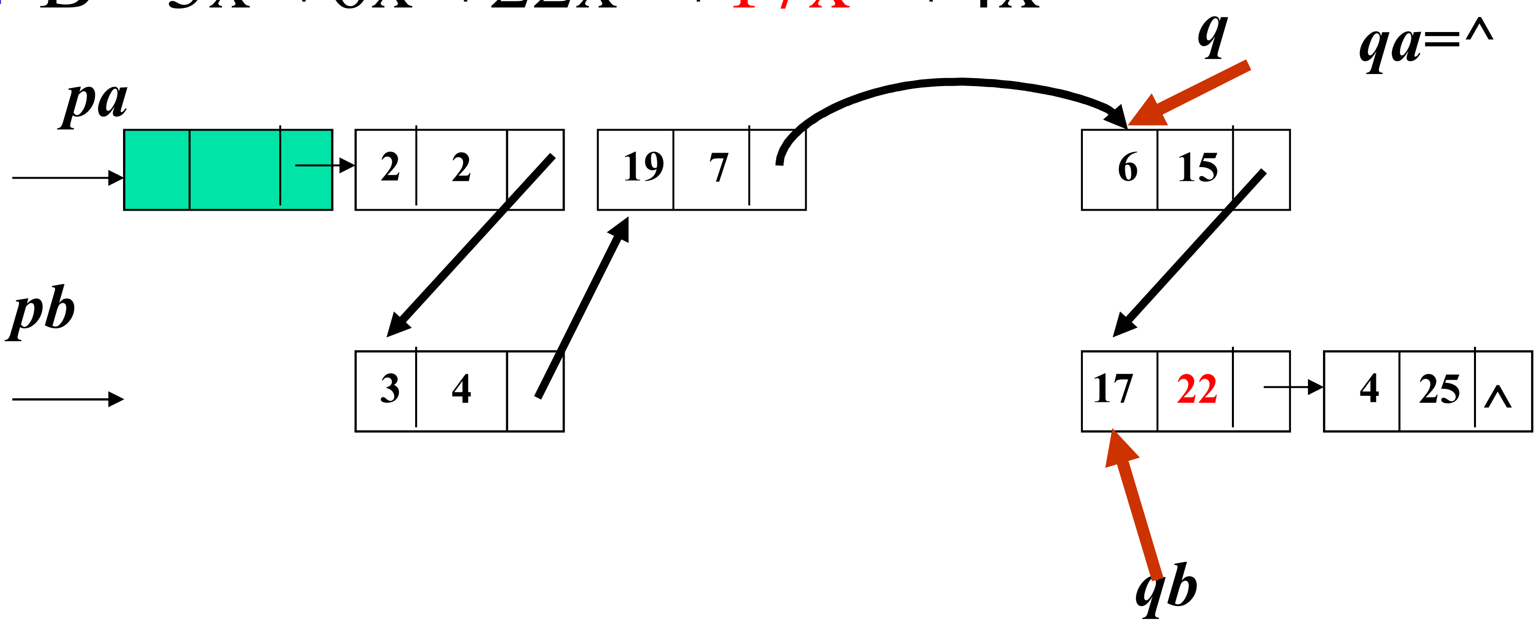

多项式相加的运算规则

- 如果qa->exp=qb->exp:则求其系数之和sum=qa->coef+qb->coef。如果sum不为零，修改qa结点的系数qa->coef=sum，
  qa、qb指针后移,将后移前qb指向的结点归还；否则qa、qb指针后移，将后移前qa、qb指向的结点归还。
- 如果qa->exp>qb->exp：则把qb结点插在qa结点之前，qb指针在原链表上后移。
- 如果qa->exp<qb->exp:则qa指针后移。

- 多项式相乘：利用多项式相加可实现多项式相乘，因为乘法运算可分解为加法运算。

```
Void add(poly &pa,poly &pb)
{
   poly qa,qb,q;
   qa=pa->next;q=pa;qb=pb->next;
    while(qa&&qb)
    if(qa->exp<qb->exp){q=qa;qa=qa->next}
    else if(qa->exp==qb->exp)
           { sum= qa->coef+qb->coef;
              pb->next=qb->next;
              free(qb);qb=pb->next;
             if(sum==0){ q->next=qa->next;
                             free(qa);qa=q->next; }
             else{qa->coef=sum;q=qa;qa=qa->next;}
            }
     else{     pb->next=qb->next;
                qb->next=qa;q->next=qb;
               q=qb;qb=pb->next;}
     if(qb)q->next=qb;
     free(pb);
}
```