MIT HAN LAB

Lecture 05 Quantization

Part I

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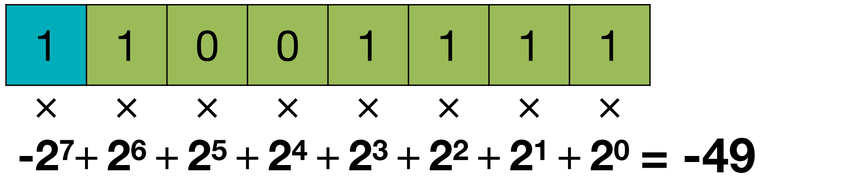
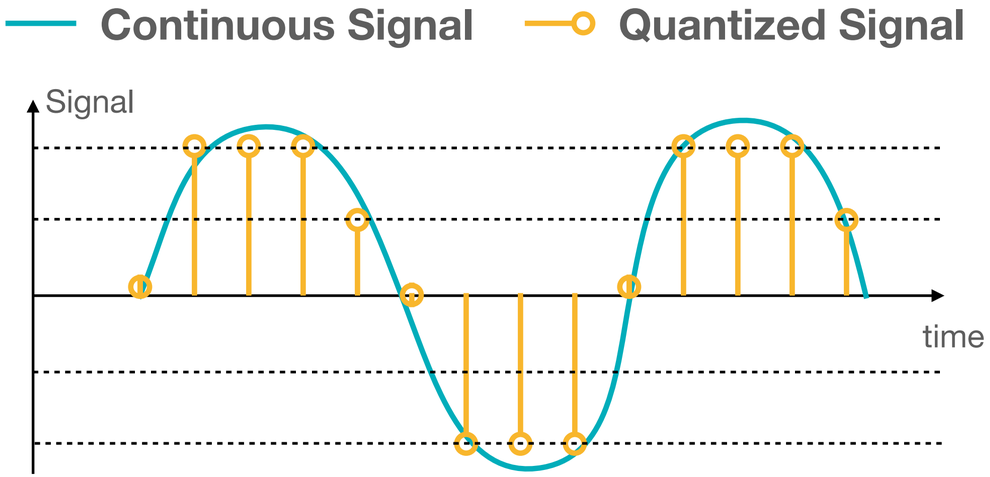
Lecture Plan

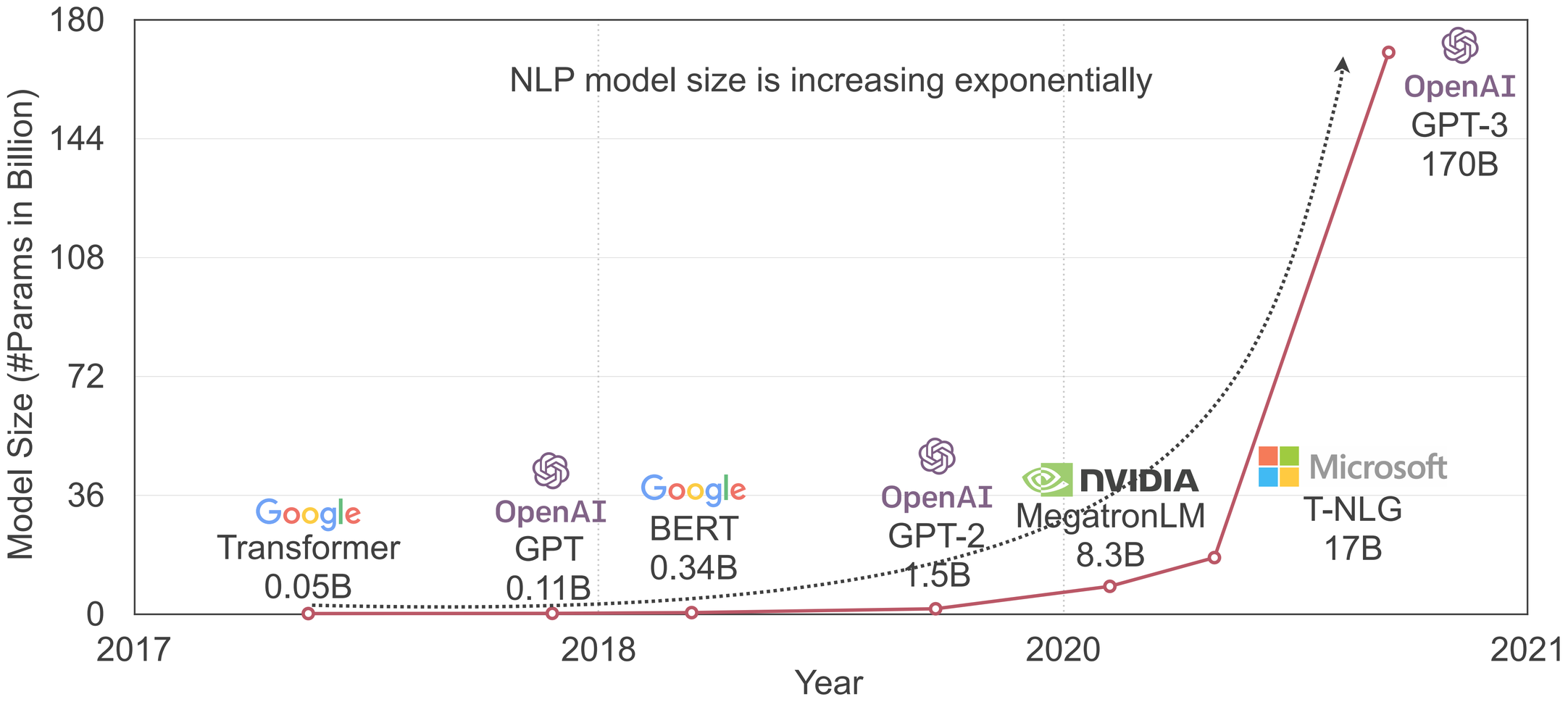
Today we will:

1. Review the numeric ***data types*** used in the modern computing systems, including integers and floating-point numbers.
2. Learn the basic concept of ***neural network quantization***
3. Learn three types of common neural network quantization:
    1. K-Means-based Quantization
    2. Linear Quantization
    3. Binary and Ternary Quantization





Today's AI is too BIG!



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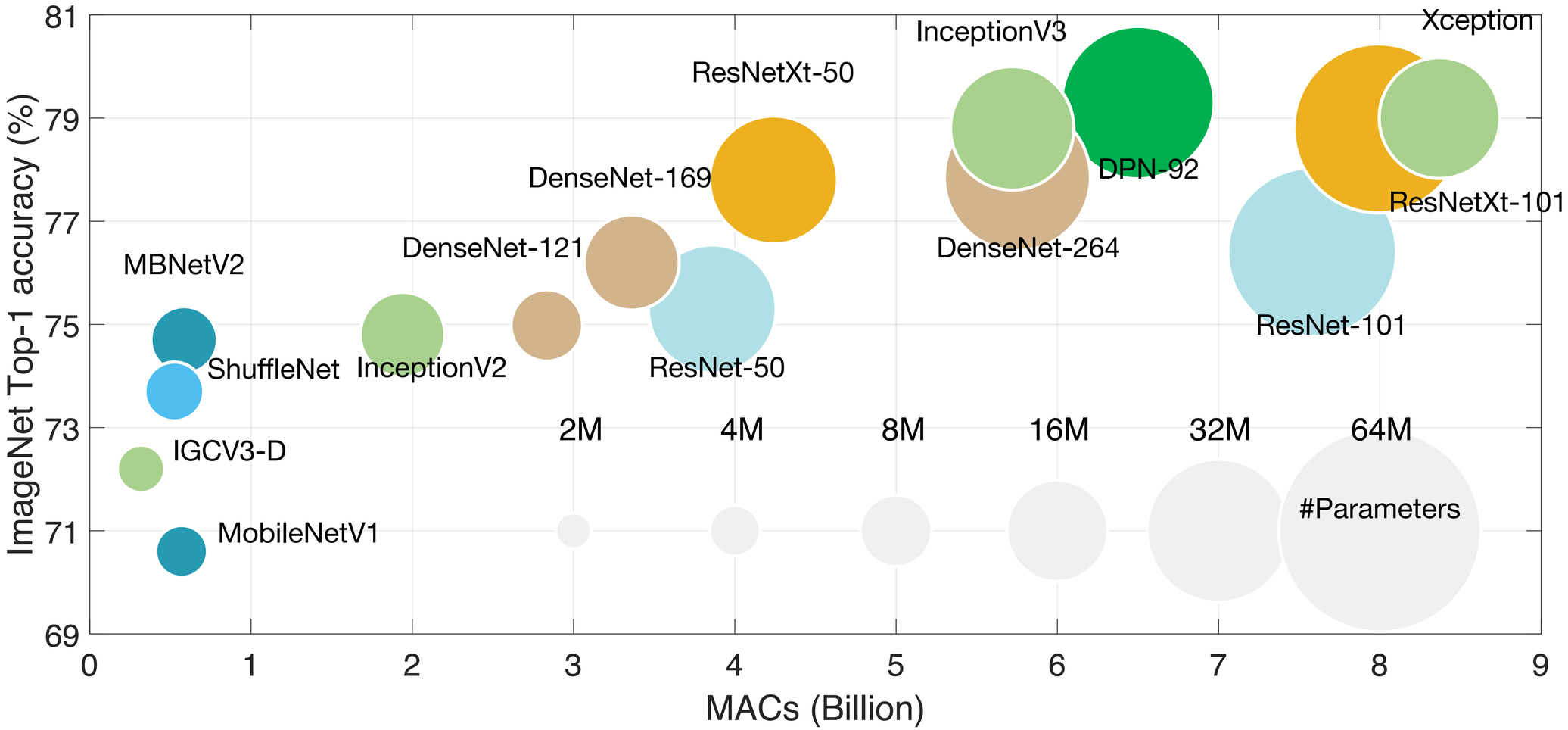


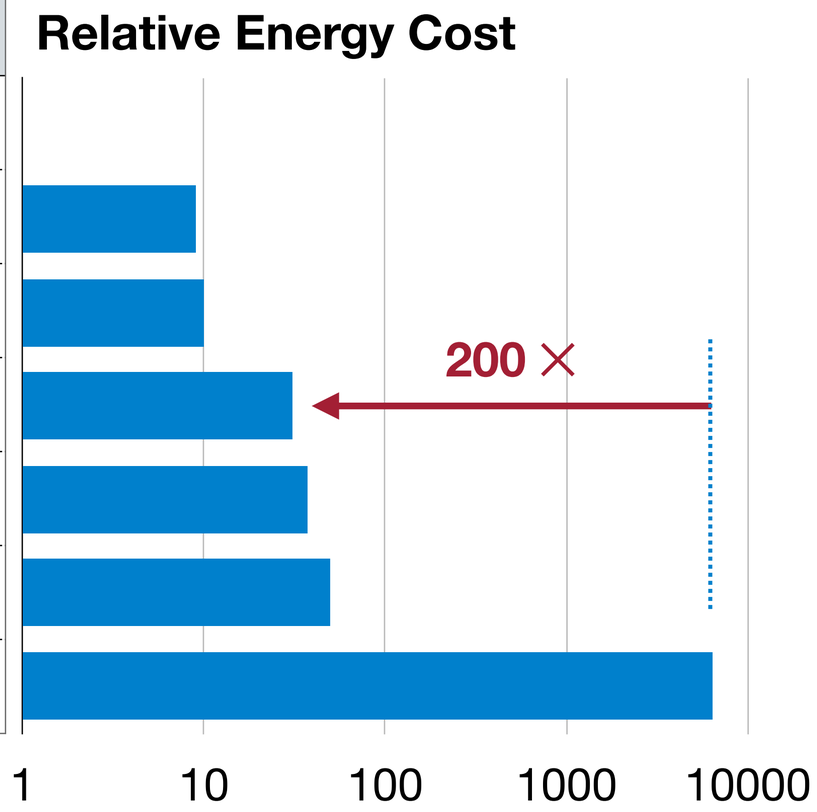
Model Compression and Hardware Acceleration for Neural Networks: A Comprehensive Survey [Deng *et al.*, IEEE 2020]

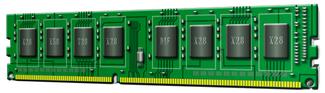
Memory is Expensive

Data Movement → More Memory Reference → More Energy

Operation	Energy [pJ]
32 bit int ADD	0.1
32 bit float ADD	0.9
32 bit Register File	1
32 bit int MULT	3.1
32 bit float MULT	3.7
32 bit SRAM Cache	5
32 bit DRAM Memory	640

Rough Energy Cost For Various Operations in 45nm 0.9V



1  = 200 ×+

This image is in the public domain

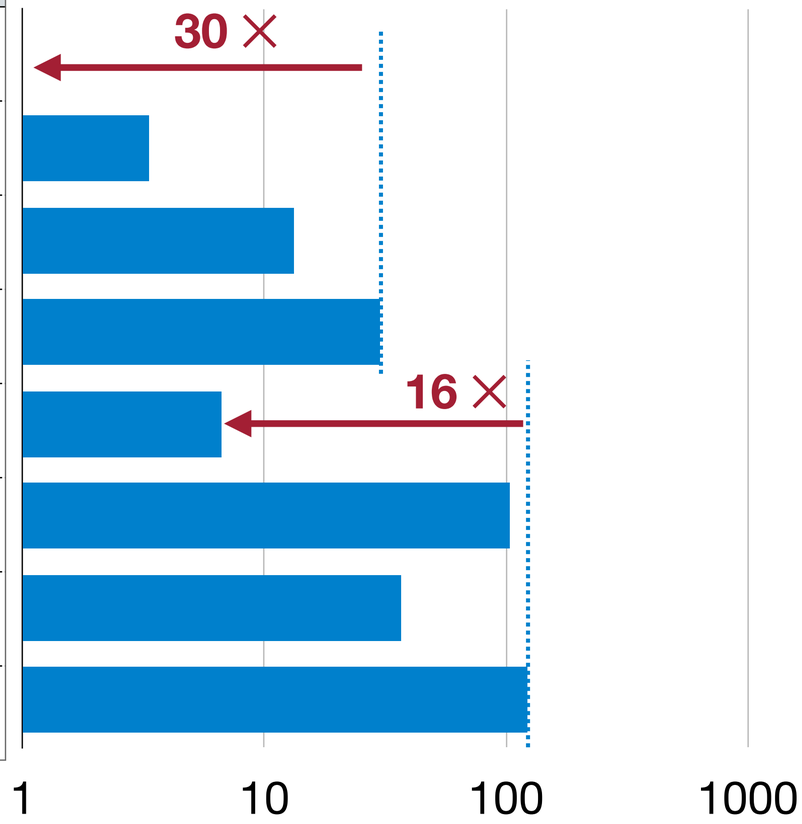
Computing's Energy Problem (and What We Can Do About it) [Horowitz, M., IEEE ISSCC 2014]

Low Bit-Width Operations are Cheap

Less Bit-Width → Less Energy

Operation	Energy [pJ]
8 bit int ADD	0.03
32 bit int ADD	0.1
16 bit float ADD	0.4
32 bit float ADD	0.9
8 bit int MULT	0.2
32 bit int MULT	3.1
16 bit float MULT	1.1
32 bit float MULT	3.7

Rough Energy Cost For Various Operations in 45nm 0.9V



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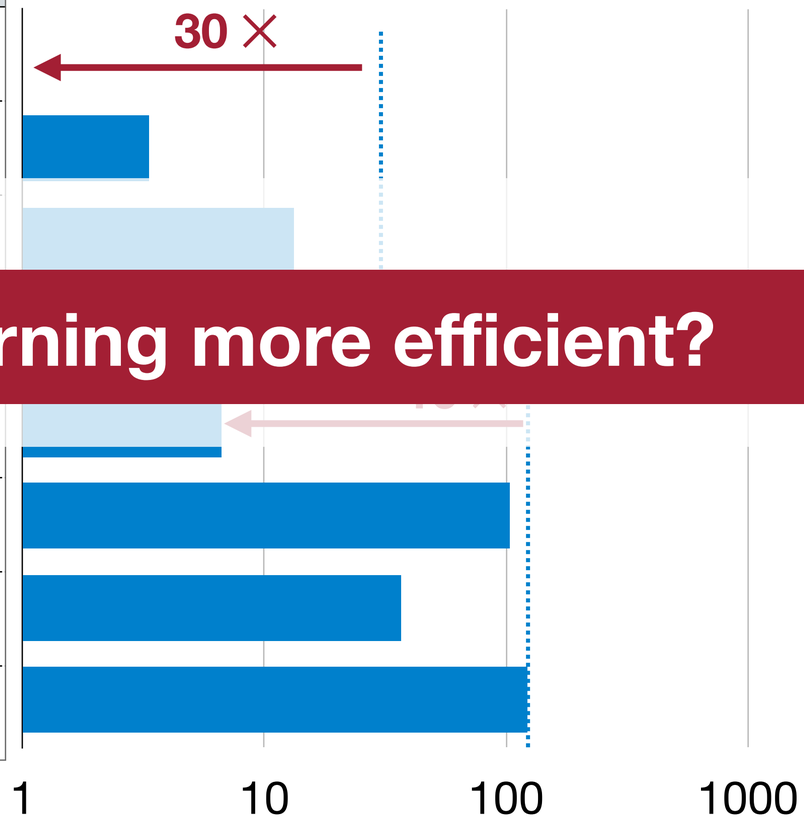
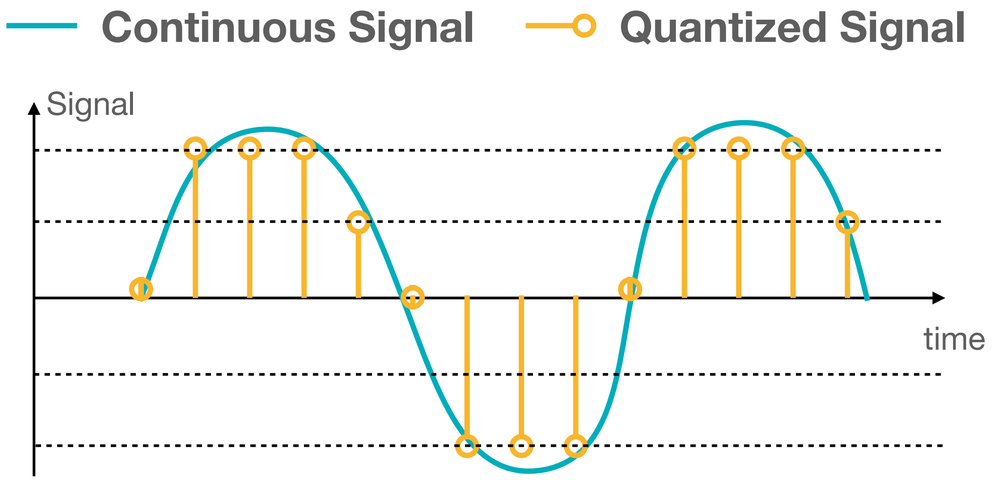


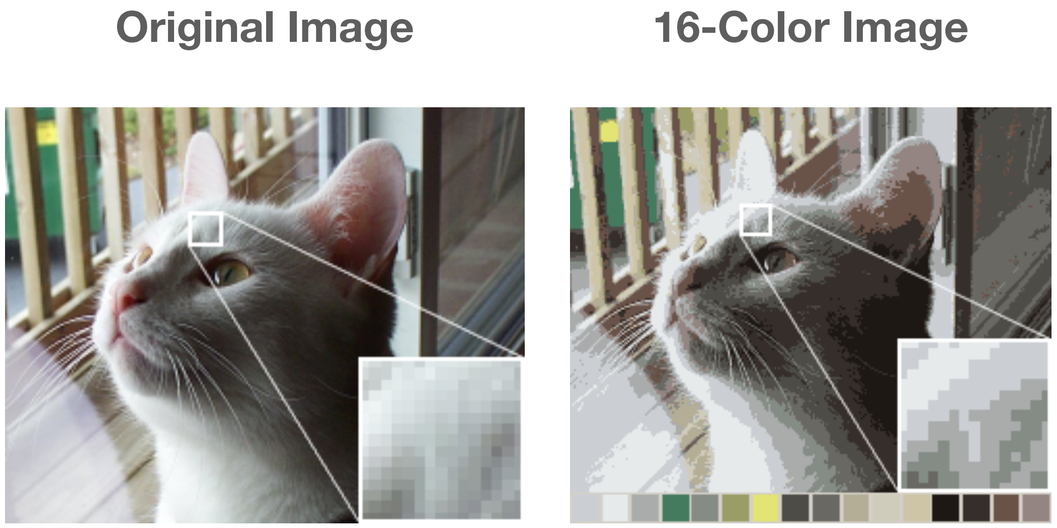
How should we make deep learning more efficient?

Computing's Energy Problem (and What We Can Do About it) [Horowitz, M., IEEE ISSCC 2014]

What is Quantization?

Quantization is the process of constraining an input from a continuous or otherwise large set of values to a discrete set.





Images are in the public domain.

Wikipedia: Quantization

Numeric Data Types

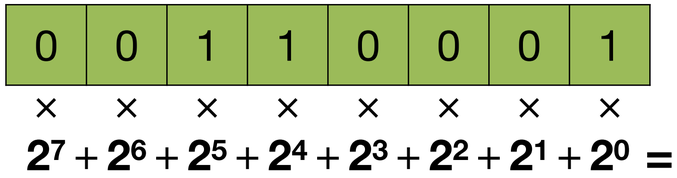
How is numeric data represented in modern computing systems?

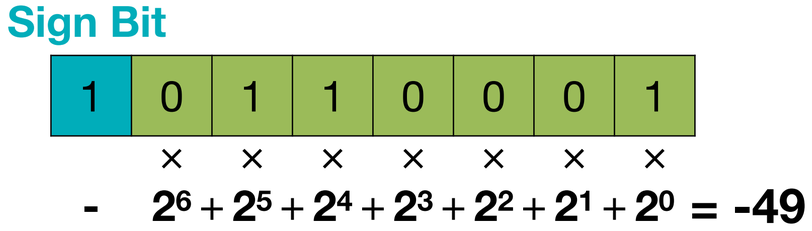
Integer

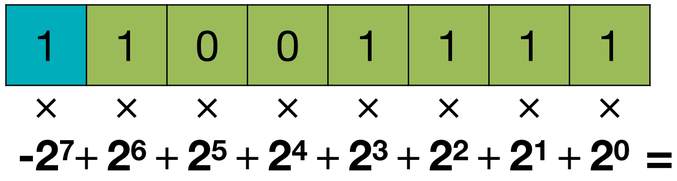
- Unsigned Integer
  - *n*-bit Range: $[0,\ 2^n - 1]$

- Signed Integer
  - Sign-Magnitude Representation
    - *n*-bit Range: $[-2^{n-1} - 1,\ 2^{n-1} - 1]$
    - Both 000…00 and 100…00 represent 0

  - Two's Complement Representation
    - *n*-bit Range: $[-2^{n-1},\ 2^{n-1} - 1]$
    - 000…00 represents 0
    - 100…00 represents -2^{n-1}







Fixed-Point Number

Integer . Fraction

“Decimal” Point

0	0	1	1	0	0	0	1
×	×	×	×	×	×	×	×

$$-2^3 + 2^2 + 2^1 + 2^0 + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} = 3.0625$$

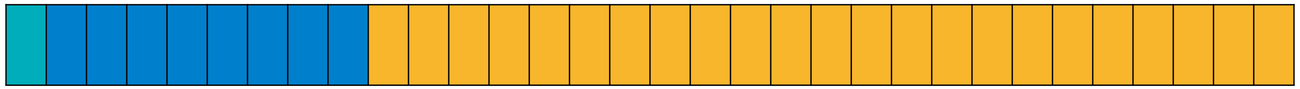
0	0	1	1	0	0	0	1
×	×	×	×	×	×	×	×

$$\left(-2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 \right) \times 2^{-4} = 49 \times 0.0625 = 3.0625$$

(using 2’s complement representation)

Floating-Point Number

Example: 32-bit floating-point number in IEEE 754

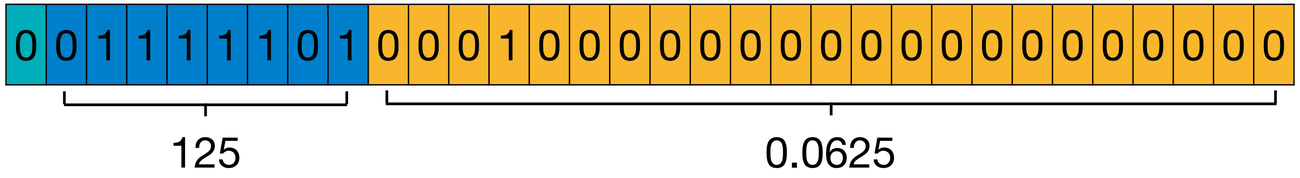


Sign **8 bit Exponent** **23 bit Fraction**

$$(-1)^{\text{sign}} \times (1 + \textbf{Fraction}) \times 2^{\textbf{Exponent}-127}$$ ← Exponent Bias = 127 = $2^{8-1}-1$

(significant / mantissa)

$$0.265625 = 1.0625 \times 2^{-2} = (1 + \underline{0.0625}) \times 2^{\underline{125}-127}$$



Floating-Point Number

Exponent Width → Range; Fraction Width → Precision

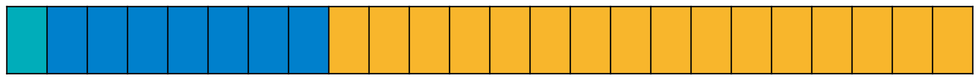
IEEE 754 Single Precision 32-bit Float (IEEE FP32)

IEEE Half Precision 16-bit Float (IEEE FP16)

Brain Float (BF16)

Nvidia TensorFloat (TF32)

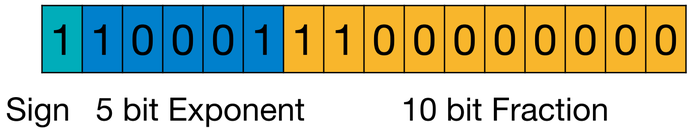
AMD 24-bit Float (AMD FP24)



Exponent (bits)	Fraction (bits)	Total (bits)
8	23	32
5	10	16
8	7	16
8	10	19
7	16	24

Numeric Data Types

- **Question**: What is the following IEEE half precision (IEEE FP16) number in decimal?



Exponent Bias = 15_{10}

  - Sign: -
  - Exponent: 10001_2 - 15_{10} = 17_{10} - 15_{10} = 2_{10}
  - Fraction: 1100000000_2 = 0.75_{10}
  - Decimal Answer = - (1 + 0.75) × 2^2 = -1.75 × 2^2 = -7.0_{10}

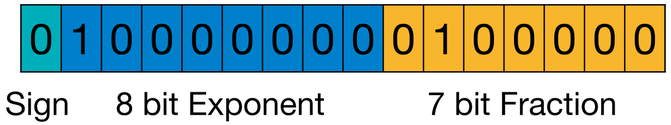
Numeric Data Types

- **Question**: What is the decimal 2.5 in Brain Float (BF16)?

$$2.5_{10} = 1.\underline{25}_{10} \times 2^{\underline{1}}$$

Exponent Bias = 127_{10}

  - Sign: +
  - Exponent Binary: $1_{10} + 127_{10} = 128_{10} = 10000000_2$
  - Fraction Binary: $0.25_{10} = 0100000_2$
  - Binary Answer



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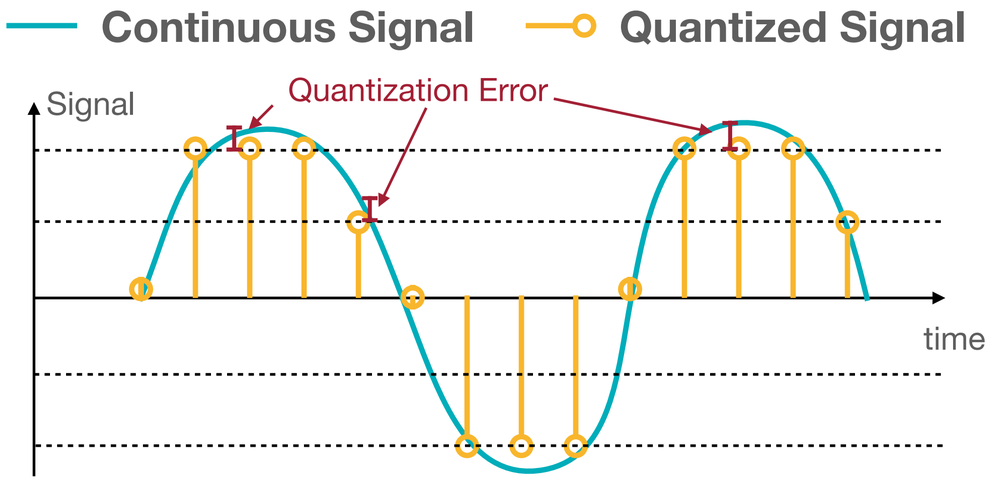
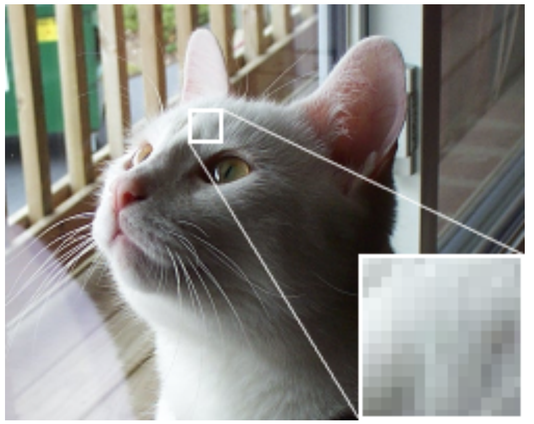
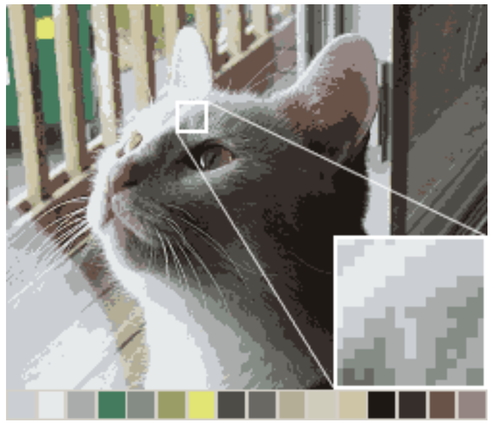


The difference between an input value and its quantized value is referred to as quantization error.

Original Image



16-Color Image

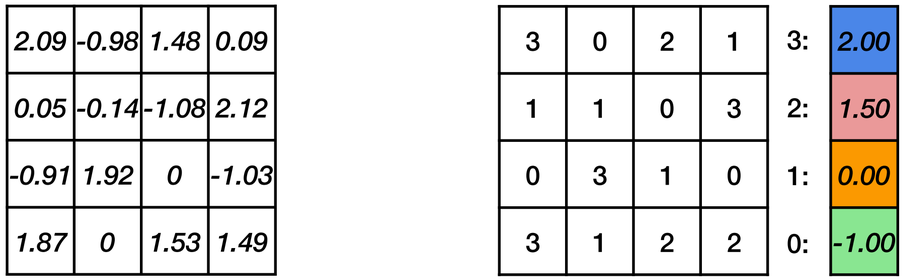


Images are in the public domain.

"Palettization"

Quantization [Wikipedia]

Neural Network Quantization: Agenda



K-Means-based Quantization

Linear Quantization

Binary/Ternary Quantization

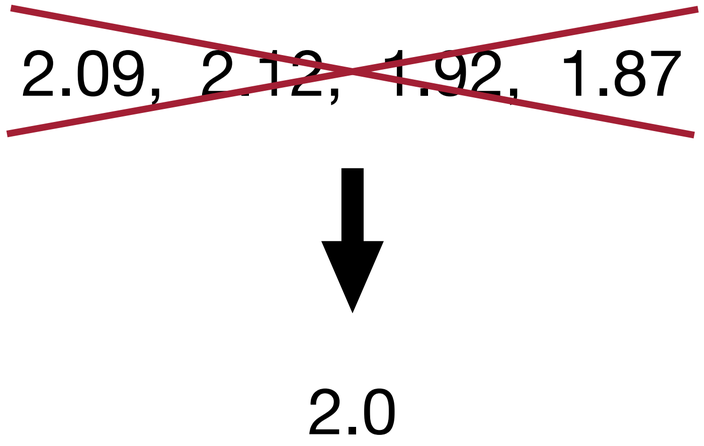
Storage	Floating-Point Weights	Integer Weights; Floating-Point Codebook
Computation	Floating-Point Arithmetic	Floating-Point Arithmetic

Neural Network Quantization

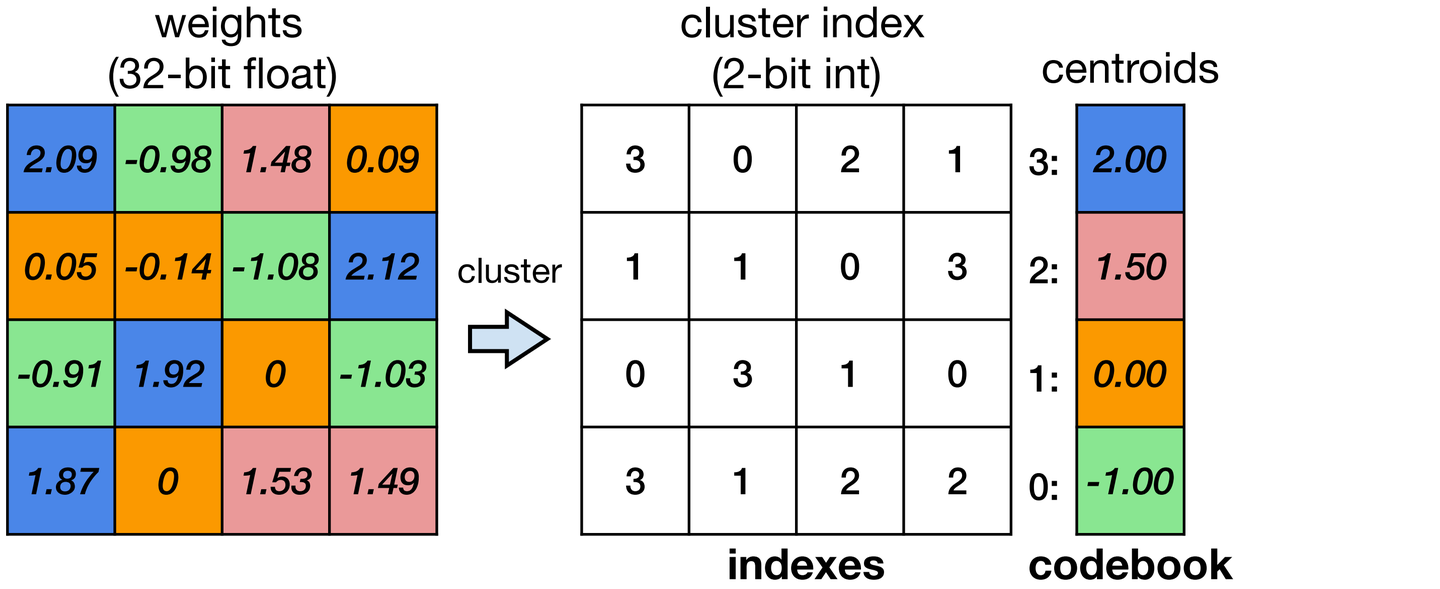
Weight Quantization

weights
(32-bit float)

2.09	*-0.98*	*1.48*	*0.09*
0.05	*-0.14*	*-1.08*	*2.12*
-0.91	*1.92*	*0*	*-1.03*
1.87	*0*	*1.53*	*1.49*



K-Means-based Weight Quantization



reconstructed weights
(32-bit float)

2.00	-1.00	1.50	0.00
0.00	0.00	-1.00	2.00
-1.00	2.00	0.00	-1.00
2.00	0.00	1.50	1.50

storage

32 bit × 16
= 512 bit = 64 B

2 bit × 16
= 32 bit = 4 B

+

32 bit × 4
= 128 bit = 16 B

= 20 B

3.2 × smaller

Assume N-bit quantization, and #parameters = $M >> 2^N$.

32 bit × M
= $32M$ bit

N bit × M
= NM bit

~~32 bit × 2^N
= 2^{N+5} bit~~

$32/N$ × smaller

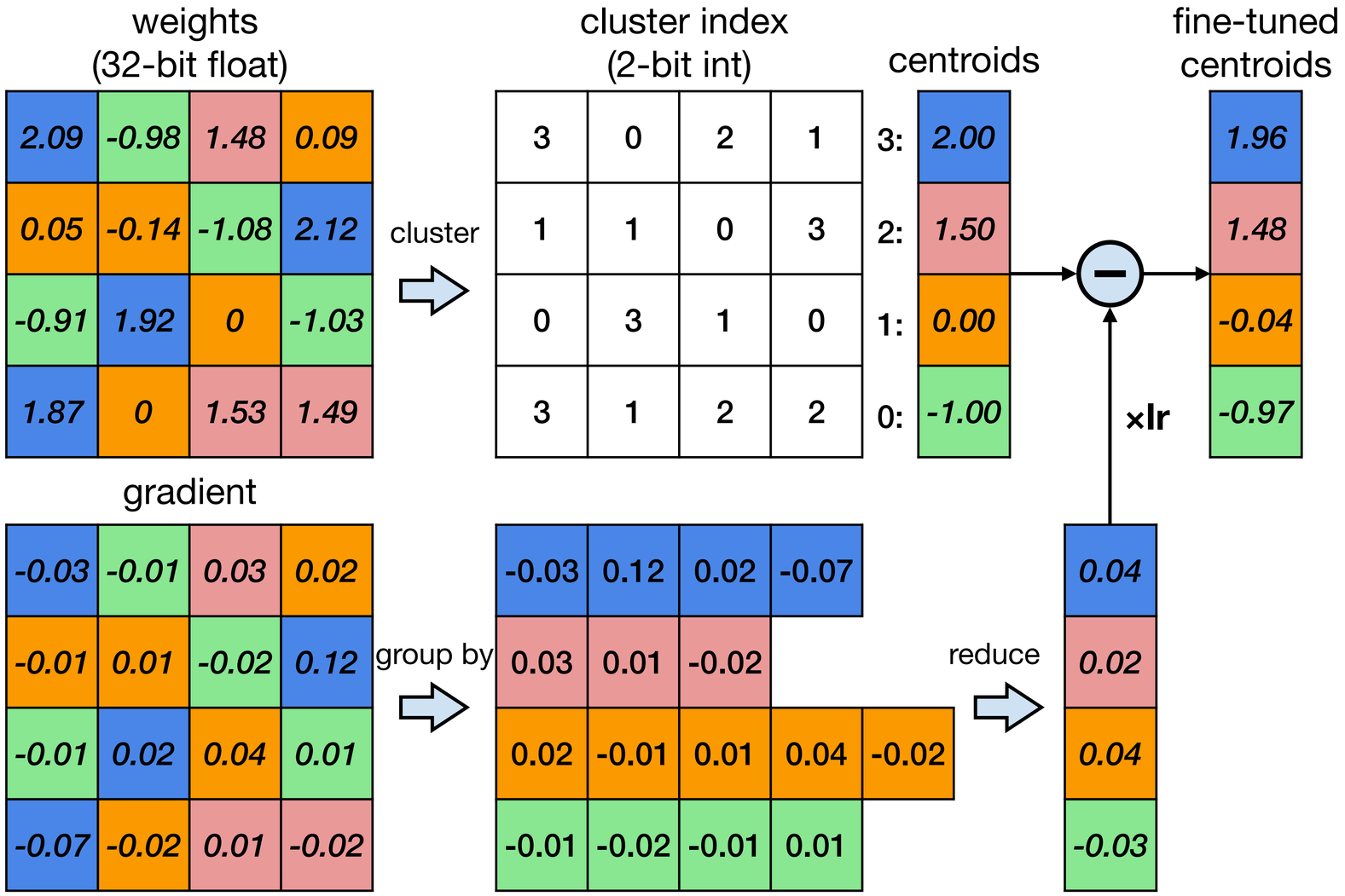
quantization error

0.09	0.02	-0.02	0.09
0.05	-0.14	-0.08	0.12
0.09	-0.08	0	-0.03
-0.13	0	0.03	-0.01

Deep Compression [Han *et al.*, ICLR 2016]

K-Means-based Weight Quantization

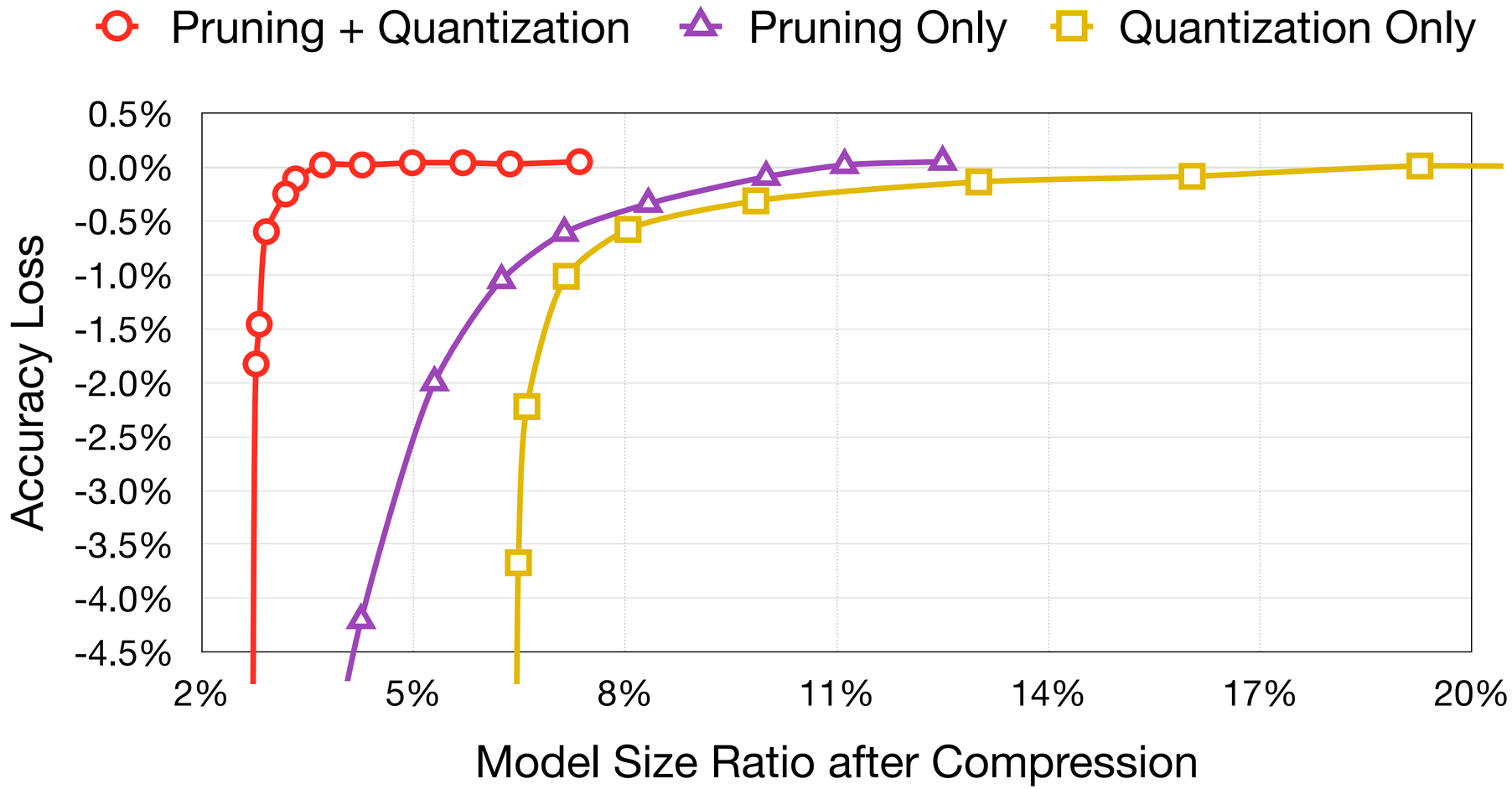
Fine-tuning Quantized Weights



Deep Compression [Han *et al.*, ICLR 2016]

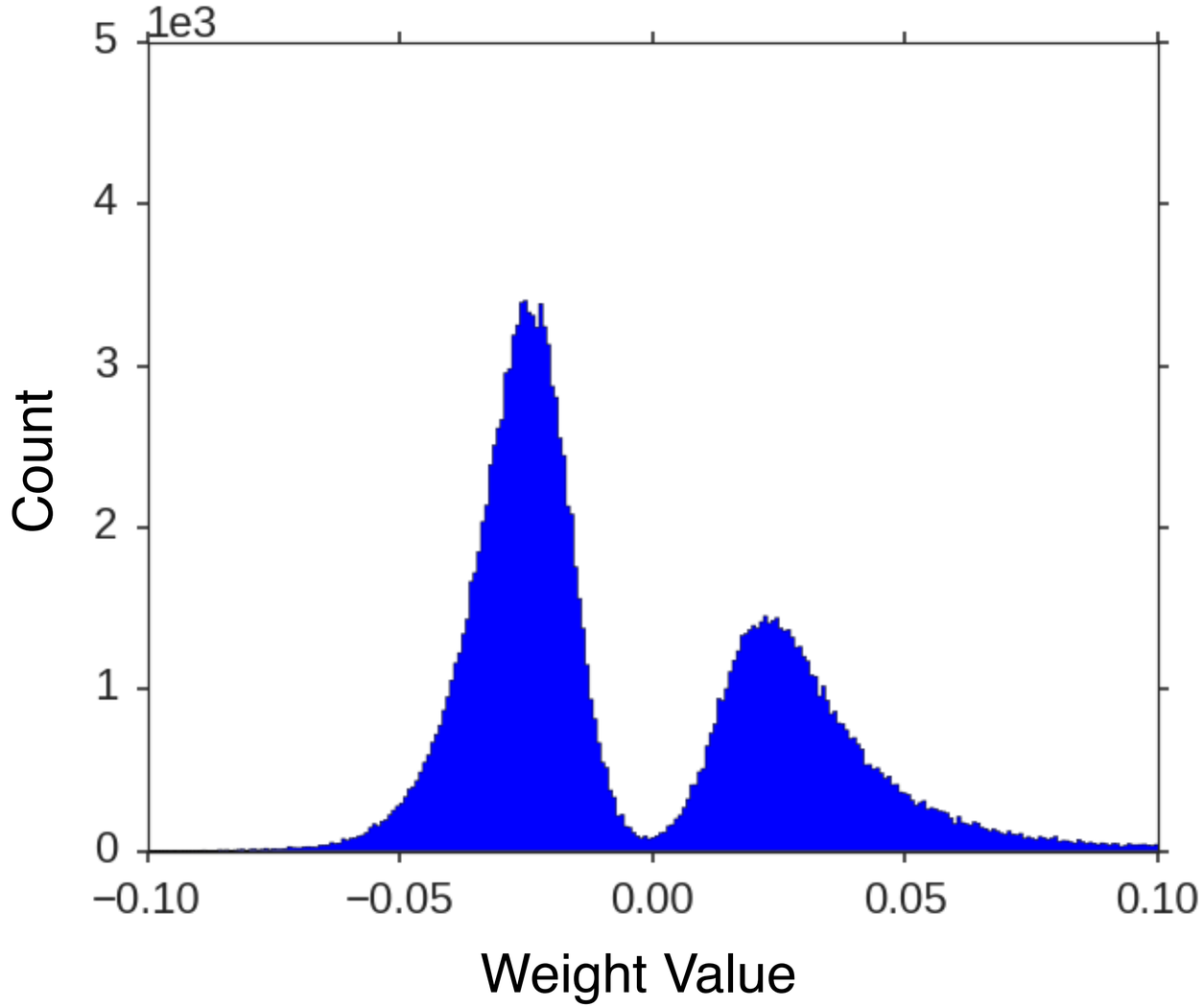
K-Means-based Weight Quantization

Accuracy vs. compression rate for AlexNet on ImageNet dataset



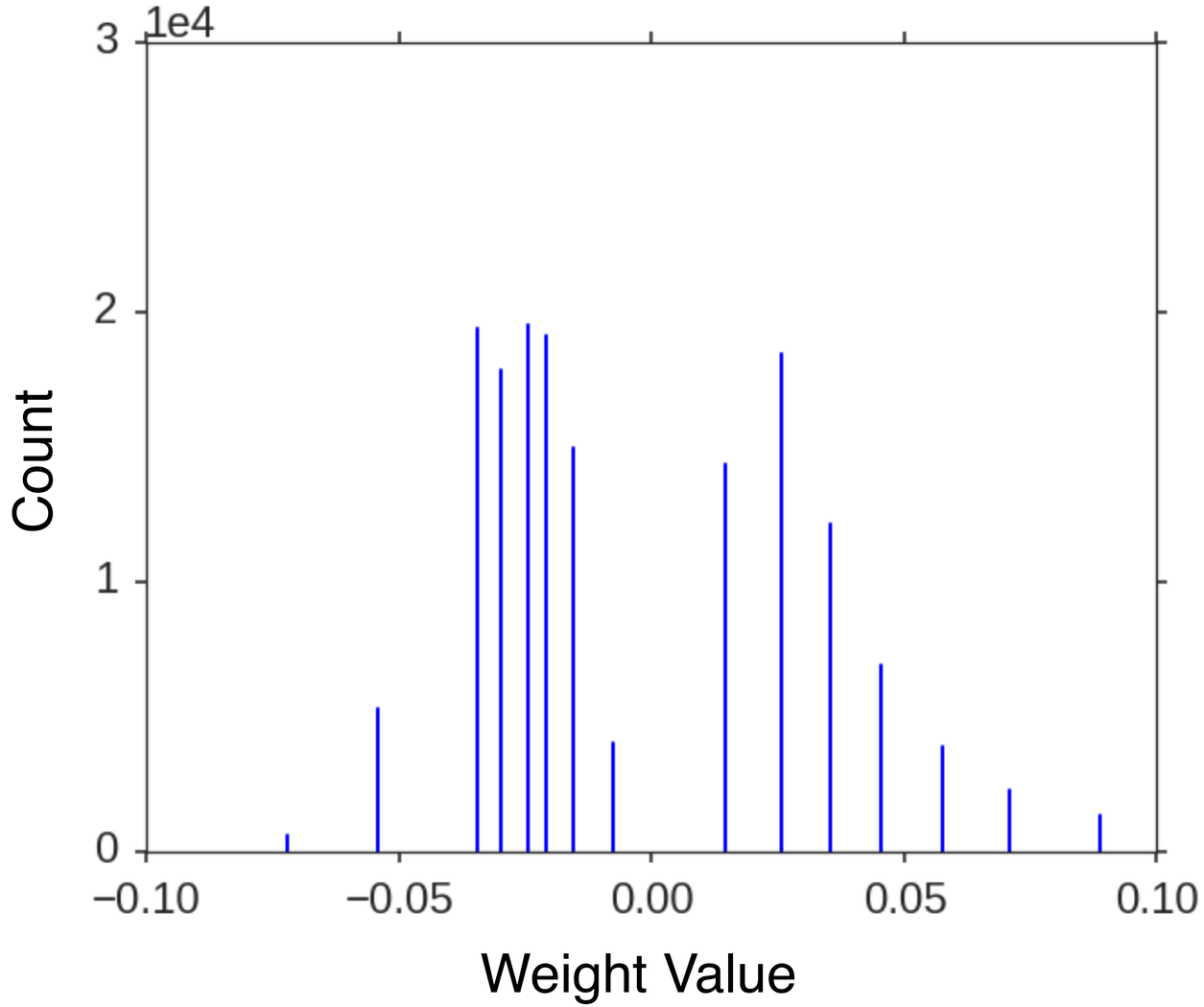
Deep Compression [Han *et al.*, ICLR 2016]

Before Quantization: Continuous Weight



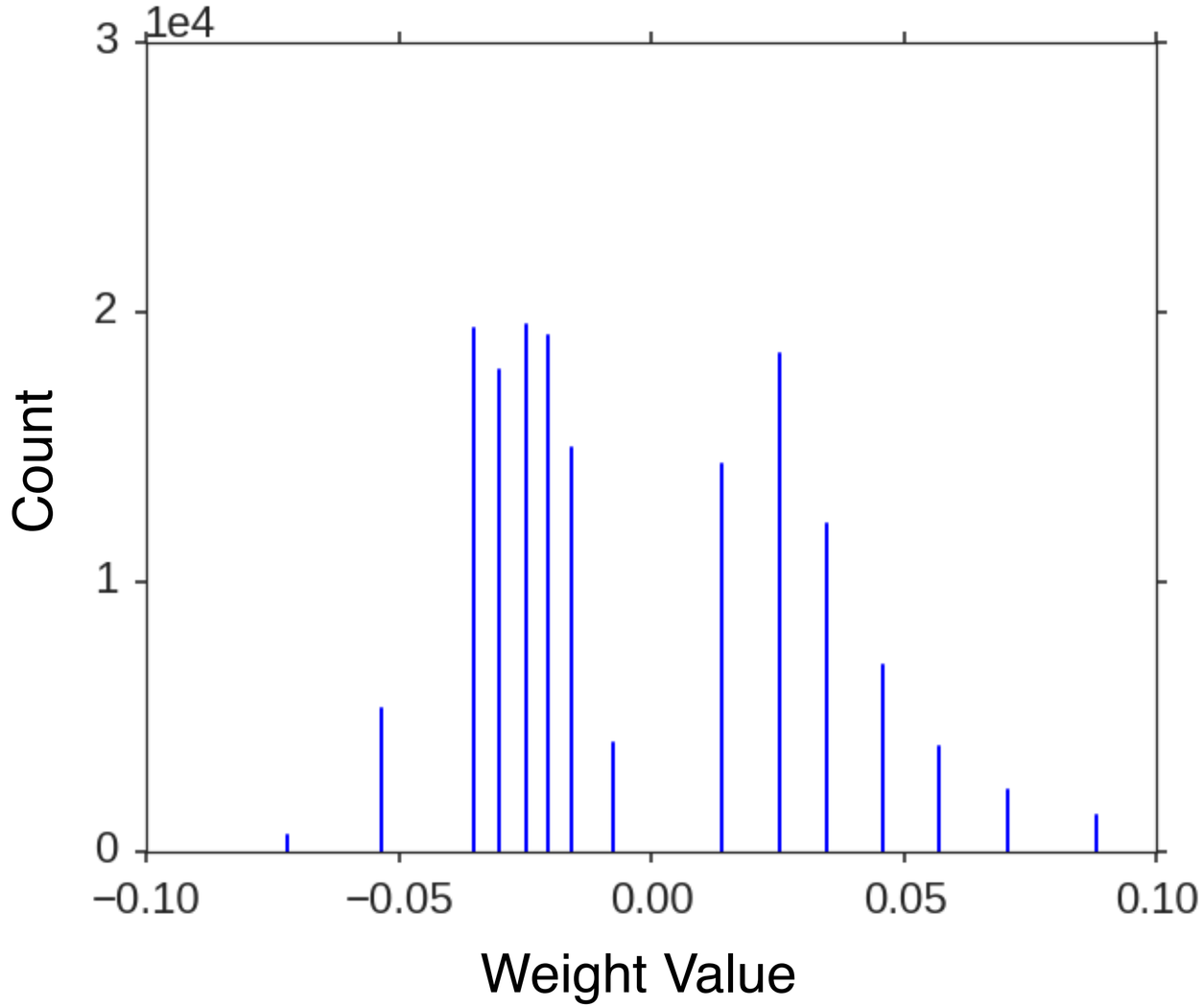
Deep Compression [Han *et al.*, ICLR 2016]

After Quantization: Discrete Weight



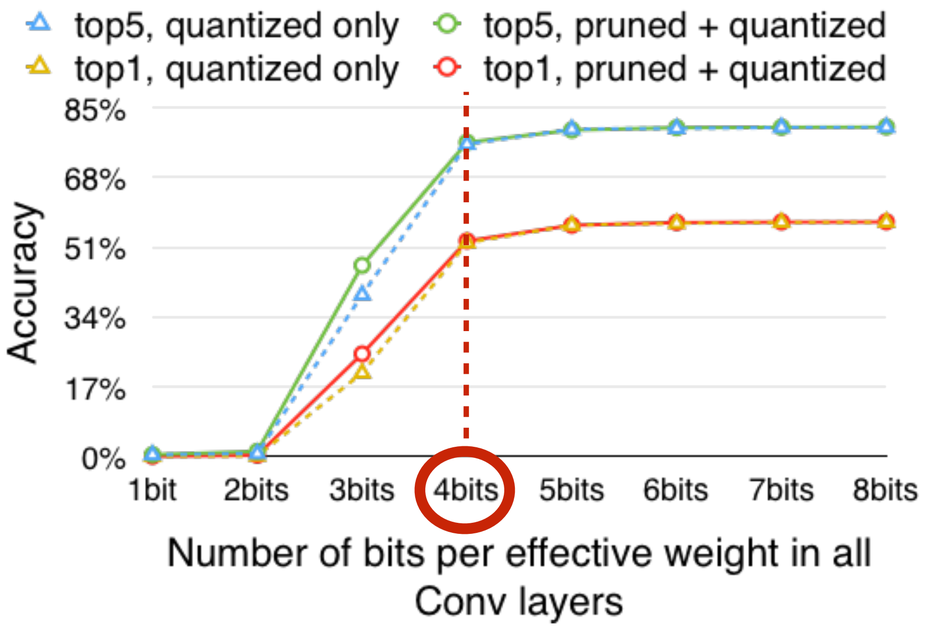
Deep Compression [Han *et al.*, ICLR 2016]

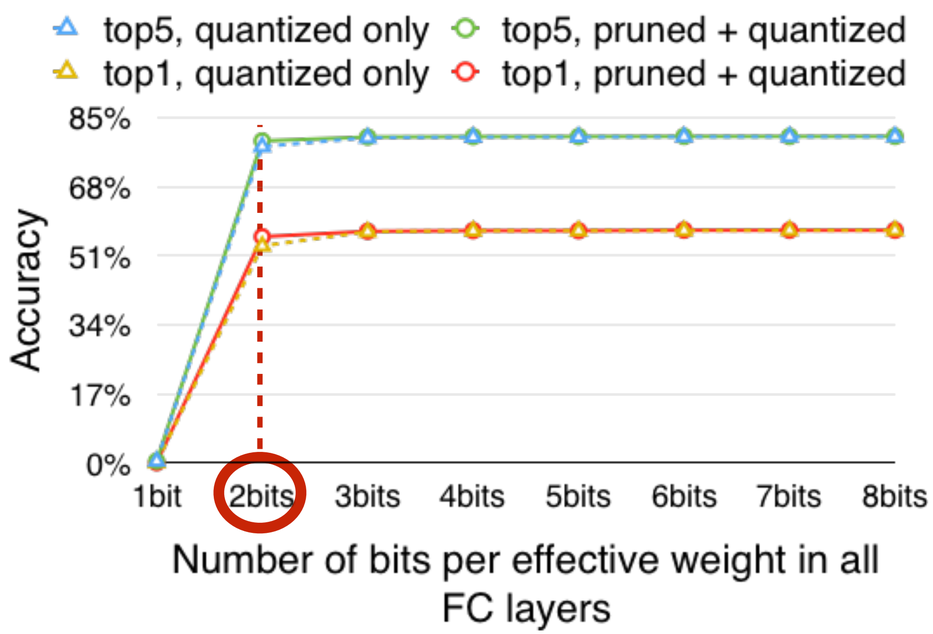
After Quantization: Discrete Weight after Training



Deep Compression [Han *et al.*, ICLR 2016]

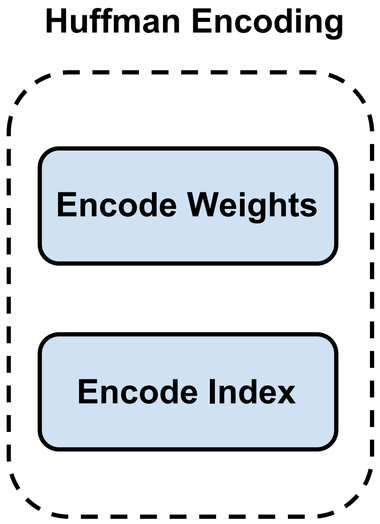
How Many Bits do We Need?

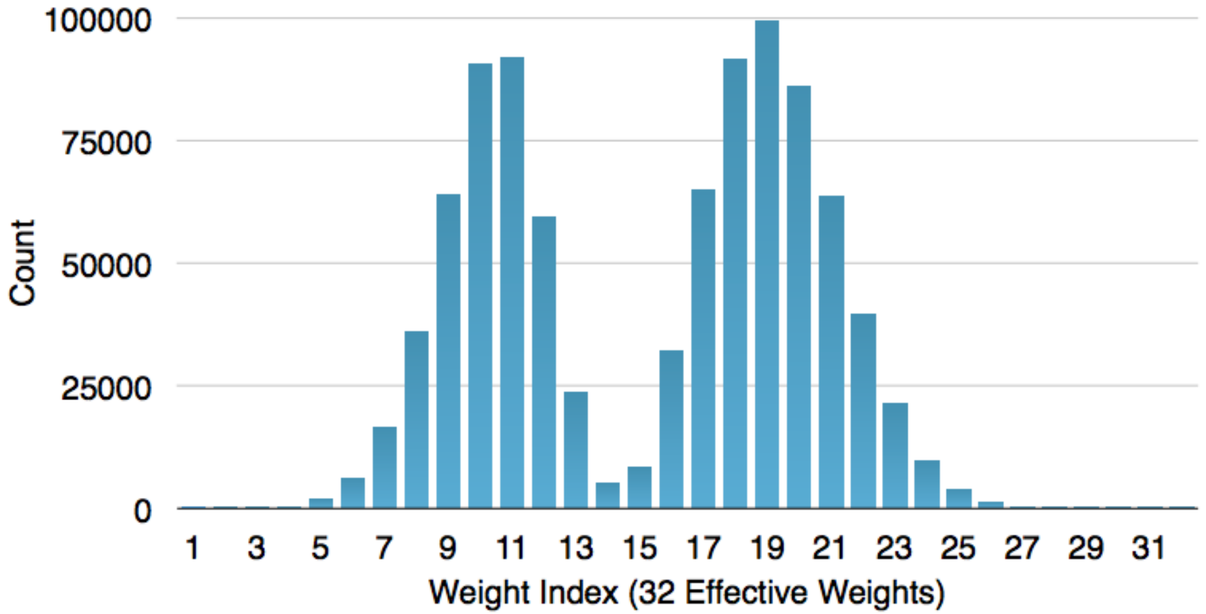




Deep Compression [Han *et al.*, ICLR 2016]

Huffman Coding

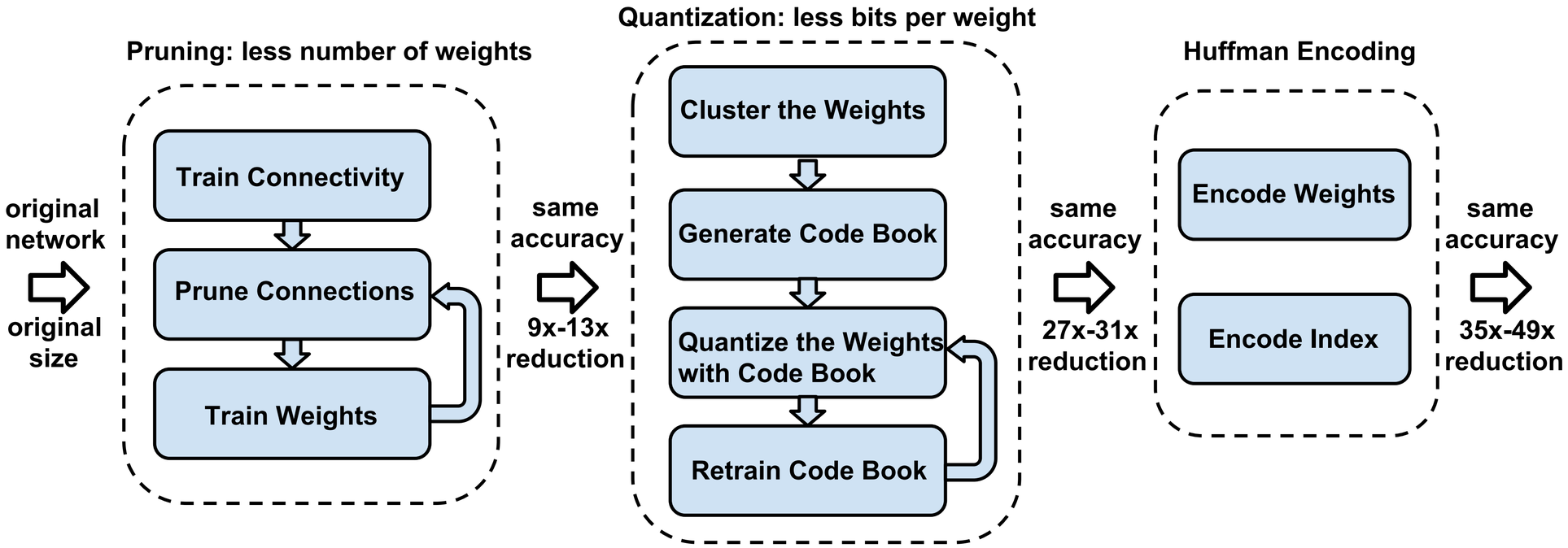




- In-frequent weights: use more bits to represent
- Frequent weights: use less bits to represent

Deep Compression [Han *et al.*, ICLR 2016]

Summary of Deep Compression



Deep Compression [Han *et al.*, ICLR 2016]

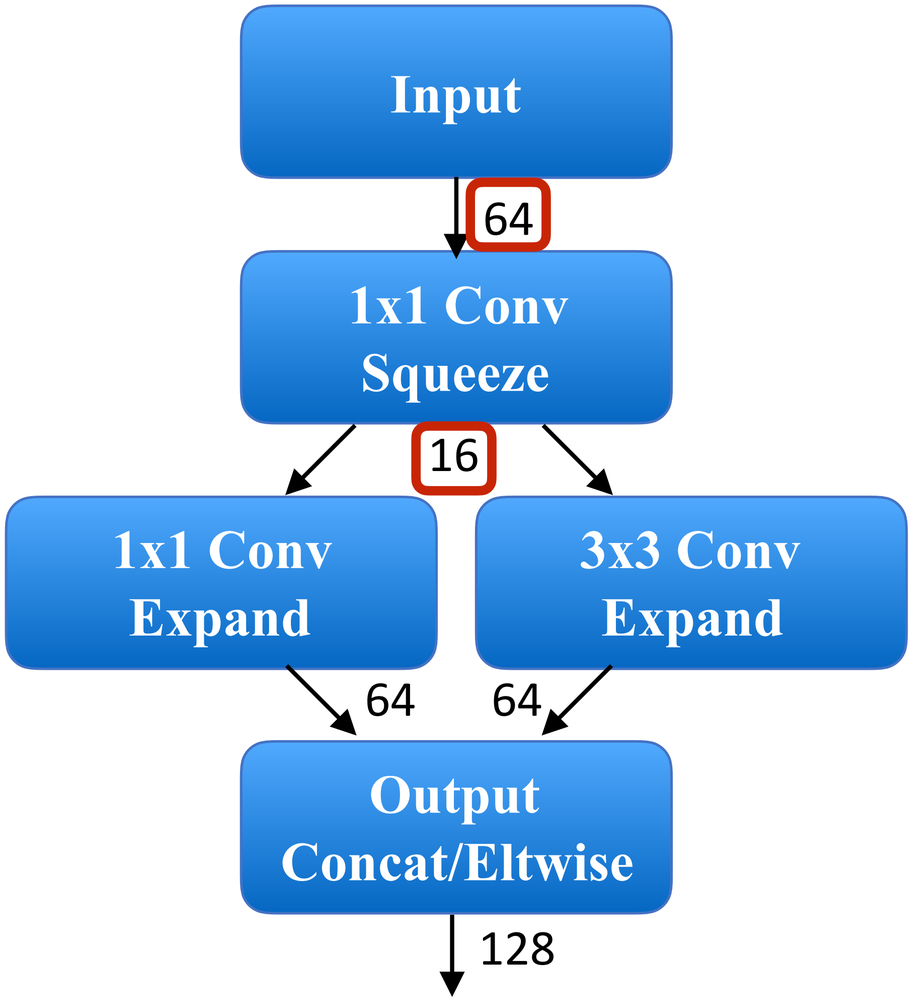
Deep Compression Results

Network	Original Size	Compressed Size	Compression Ratio	Original Accuracy	Compressed Accuracy
LeNet-300	1070KB	27KB	**40x**	98.36%	98.42%
LeNet-5	1720KB	44KB	**39x**	99.20%	99.26%
AlexNet	240MB	6.9MB	**35x**	80.27%	80.30%
VGGNet	550MB	11.3MB	**49x**	88.68%	89.09%
GoogleNet	28MB	2.8MB	**10x**	88.90%	88.92%
ResNet-18	44.6MB	4.0MB	**11x**	89.24%	89.28%

Can we make compact models to begin with?

Deep Compression [Han *et al.*, ICLR 2016]

SqueezeNet



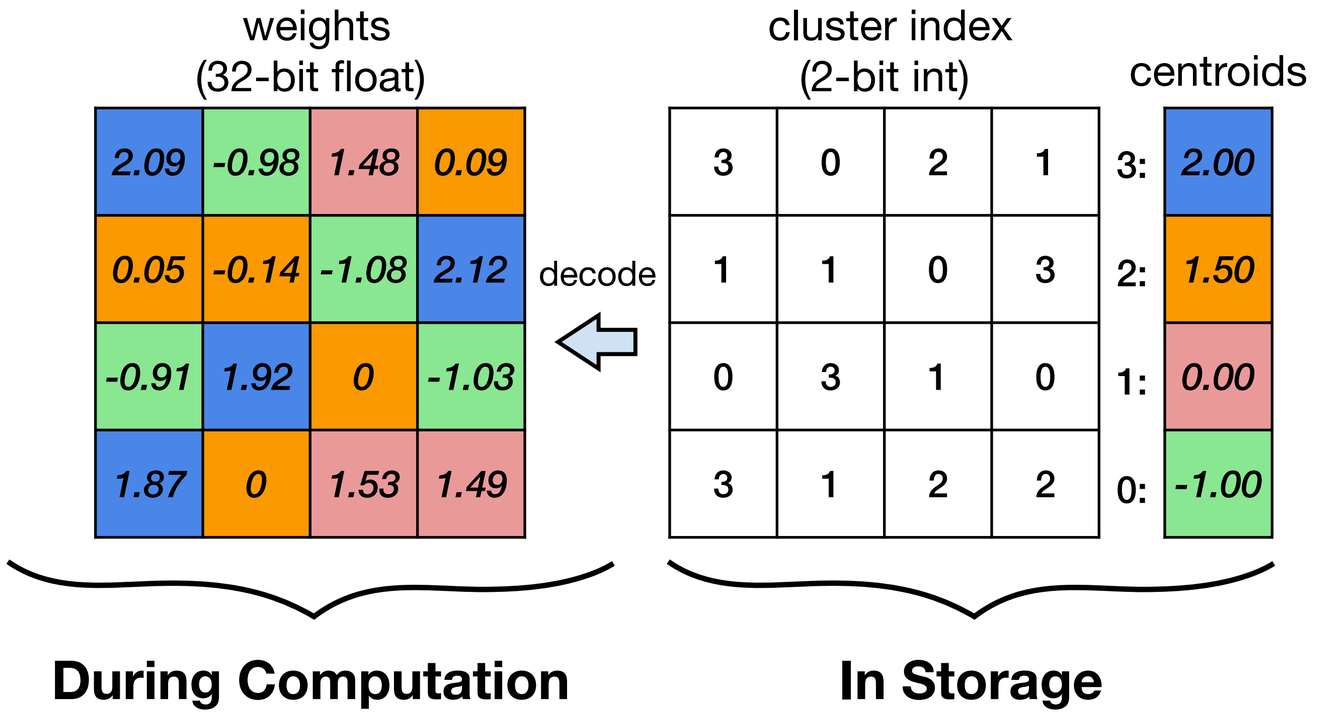
SqueezeNet: AlexNet-level accuracy with 50x fewer parameters and <0.5MB model size [Iandola *et al.*, arXiv 2016]

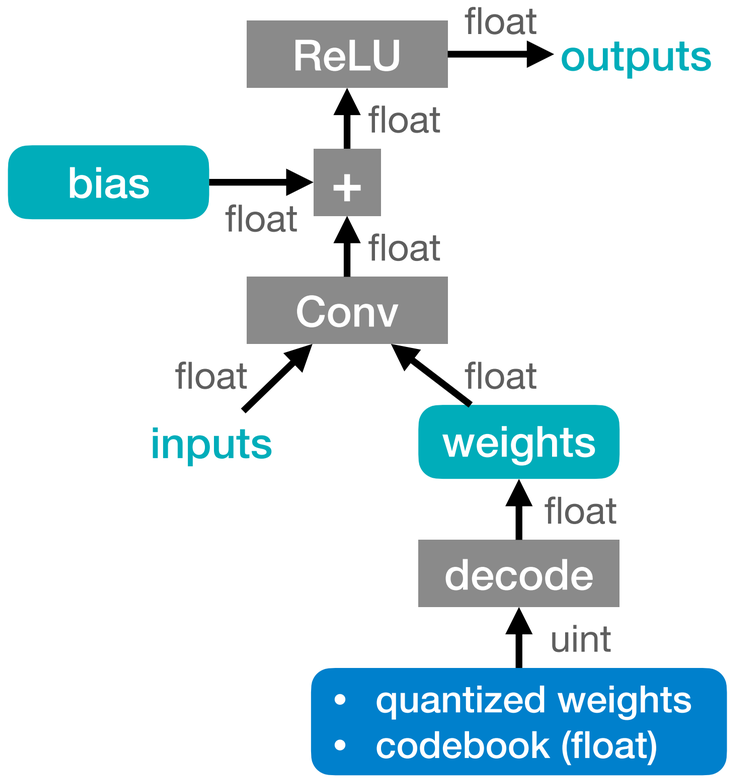
Deep Compression on SqueezeNet

Network	Approach	Size	Ratio	Top-1 Accuracy	Top-5 Accuracy
AlexNet	-	240MB	1x	57.2%	80.3%
AlexNet	SVD	48MB	5x	56.0%	79.4%
AlexNet	Deep Compression	6.9MB	35x	57.2%	80.3%
SqueezeNet	-	4.8MB	50x	57.5%	80.3%
SqueezeNet	Deep Compression	0.47MB	510x	57.5%	80.3%

SqueezeNet: AlexNet-level accuracy with 50x fewer parameters and <0.5MB model size [Iandola *et al.*, arXiv 2016]

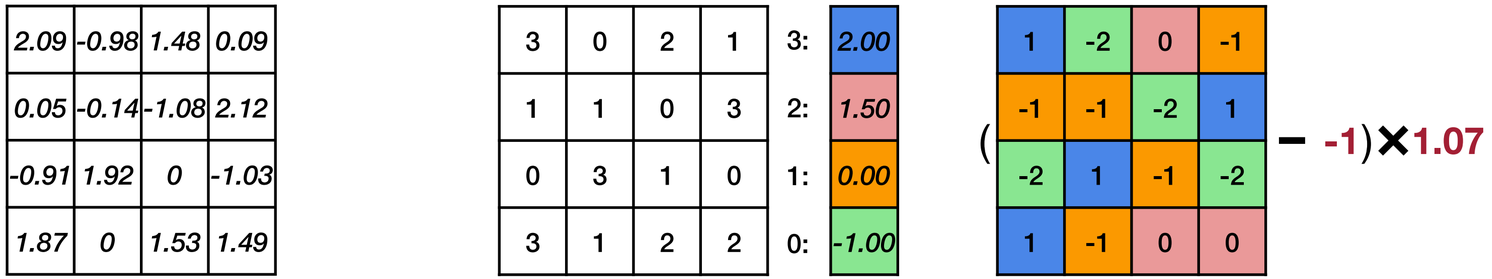
K-Means-based Weight Quantization





- The weights are decompressed using a lookup table (*i.e.*, codebook) during runtime inference.
- K-Means-based Weight Quantization only saves storage cost of a neural network model.
  - All the computation and memory access are still floating-point.

Neural Network Quantization

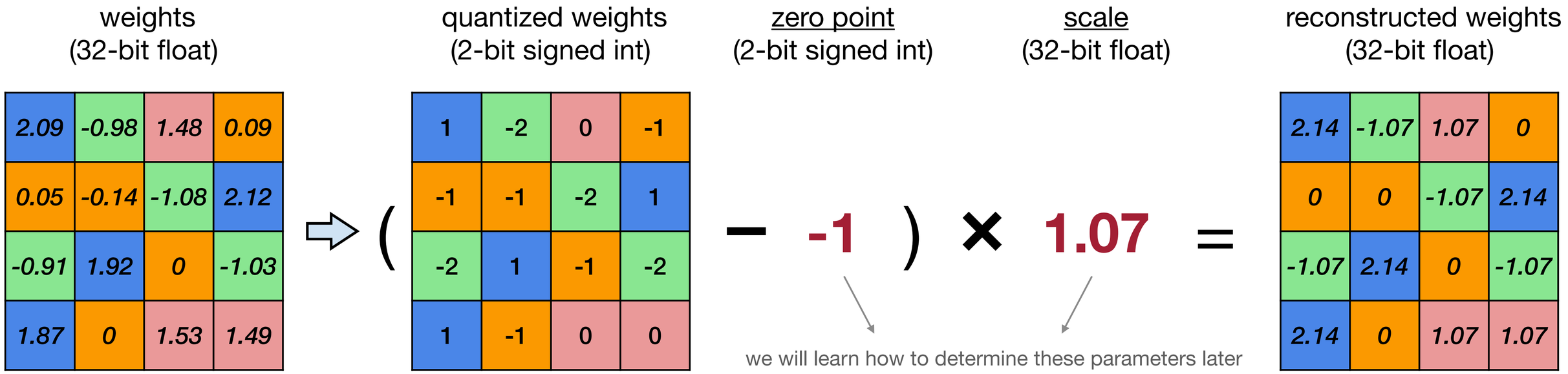


		K-Means-based Quantization	Linear Quantization	Binary/Ternary Quantization
Storage	Floating-Point Weights	Integer Weights; Floating-Point Codebook	Integer Weights	
Computation	Floating-Point Arithmetic	Floating-Point Arithmetic	Integer Arithmetic	

Linear Quantization

What is Linear Quantization?

An affine mapping of integers to real numbers



we will learn how to determine these parameters later

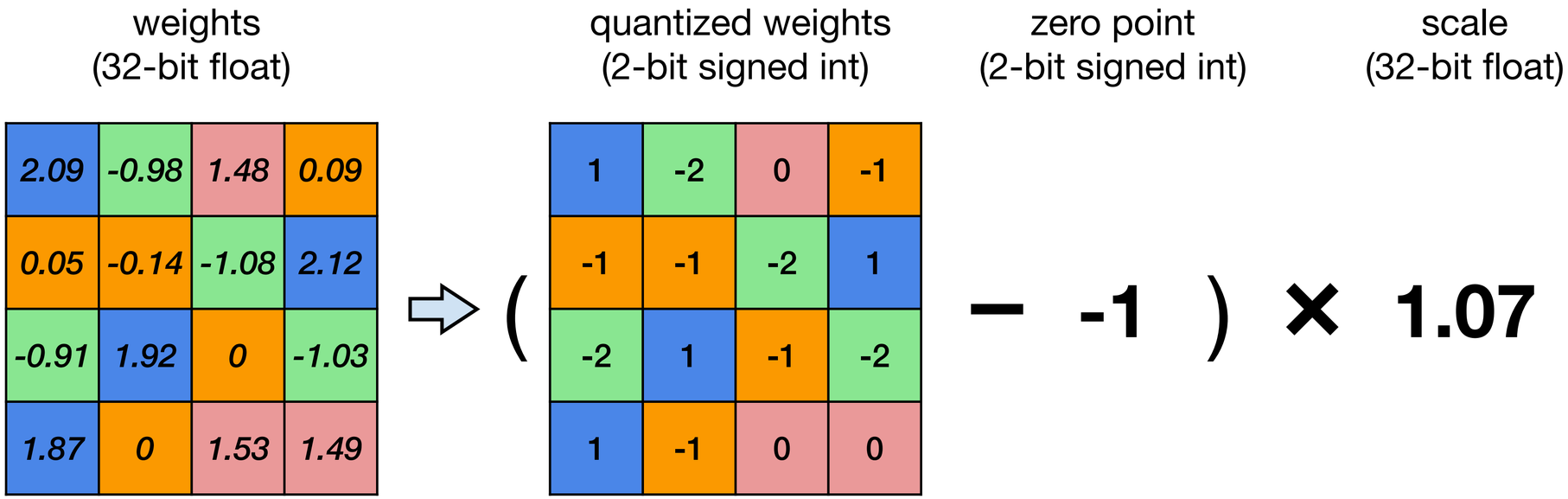
Binary	Decimal
01	1
00	0
11	-1
10	-2

quantization error

-0.05	0.09	0.41	0.09
0.05	-0.14	-0.01	-0.02
0.16	-0.22	0	0.04
-0.27	0	0.46	0.42

Linear Quantization

An affine mapping of integers to real numbers $r = S(q - Z)$



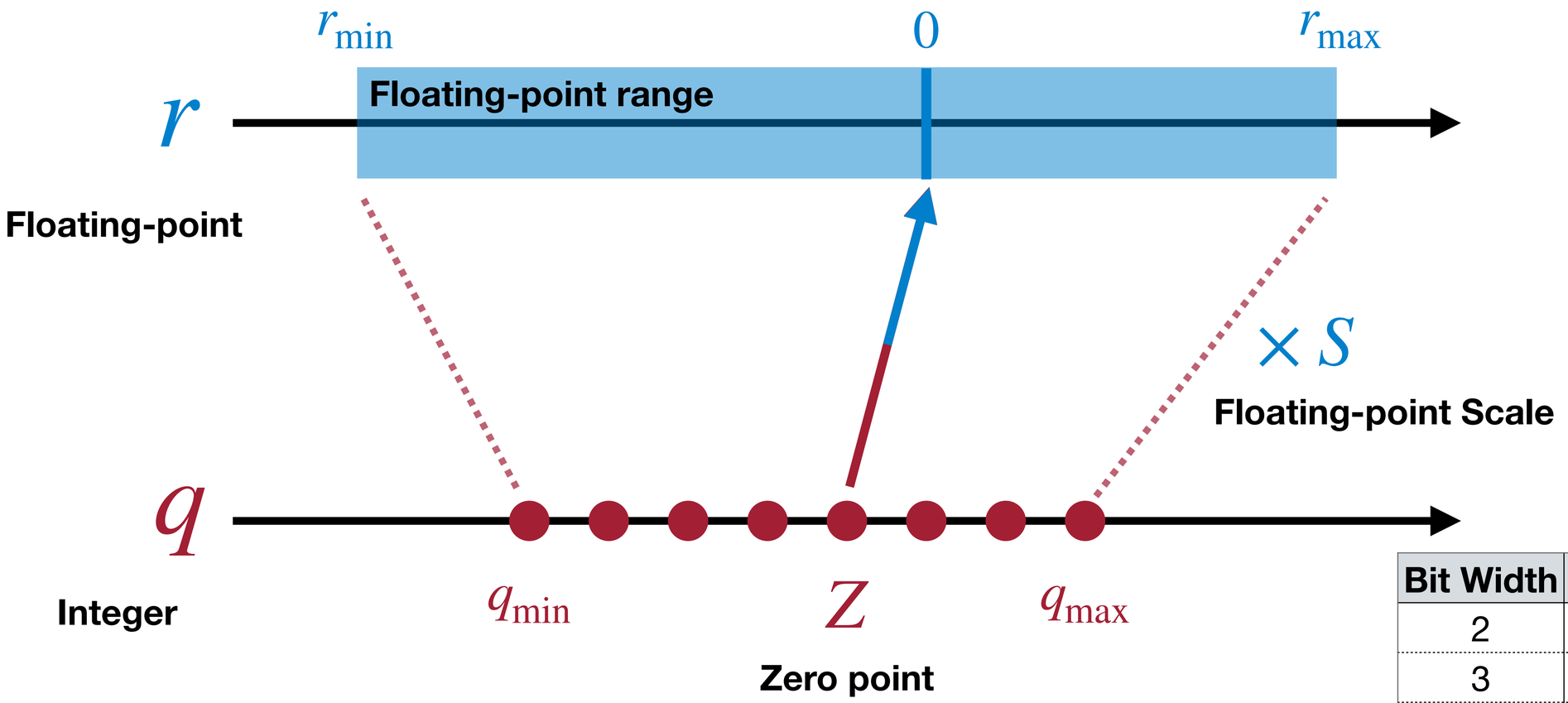
- quantization parameter
- allow real number *r=0* be exactly representable by a quantized integer Z

- quantization parameter

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Linear Quantization

An affine mapping of integers to real numbers $r = S(q - Z)$

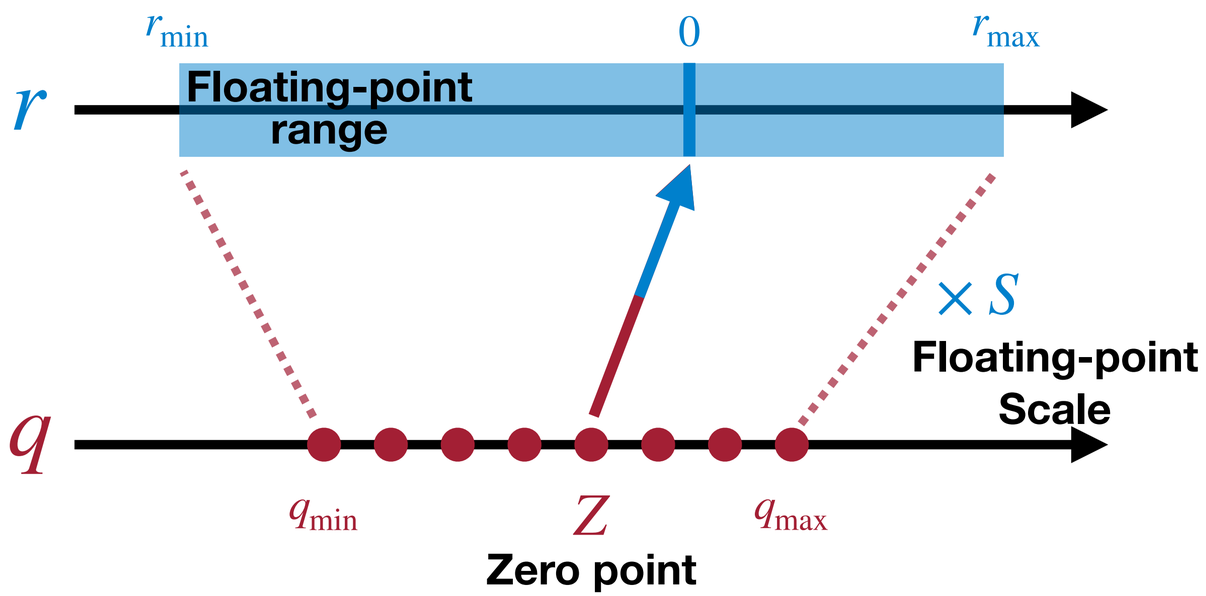


Bit Width	q_{min}	q_{max}
2	-2	1
3	-4	3
4	-8	7
N	-2^{N-1}	$2^{N-1}-1$

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Scale of Linear Quantization

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



$$r_{max} = S\left(q_{max} - Z\right)$$

$$r_{min} = S\left(q_{min} - Z\right)$$

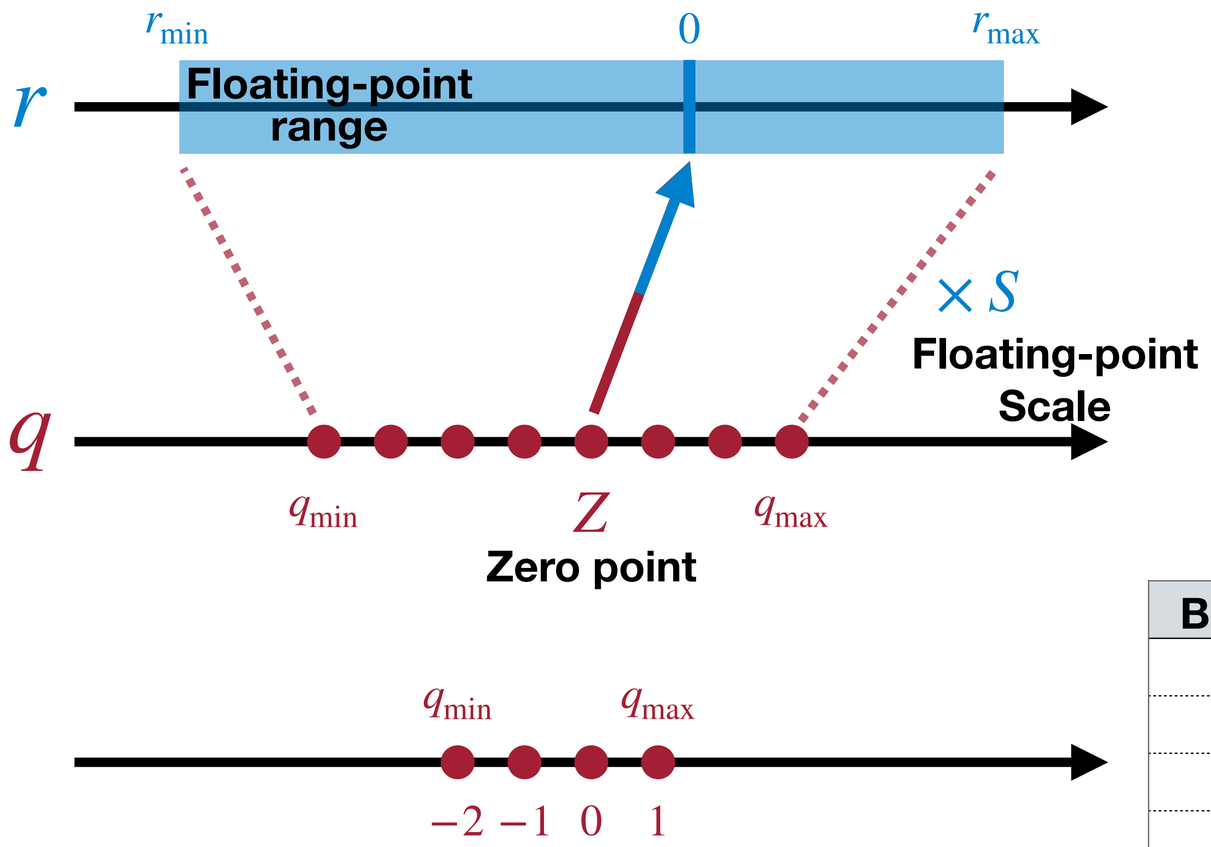
$$\downarrow$$

$$r_{max} - r_{min} = S\left(q_{max} - q_{min}\right)$$

$$S = \frac{r_{max} - r_{min}}{q_{max} - q_{min}}$$

Scale of Linear Quantization

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



2.09	*-0.98*	*1.48*	*0.09*
0.05	*-0.14*	*-1.08*	*2.12*
-0.91	*1.92*	*0*	*-1.03*
1.87	*0*	*1.53*	*1.49*

Binary	Decimal
01	1
00	0
11	-1
10	-2

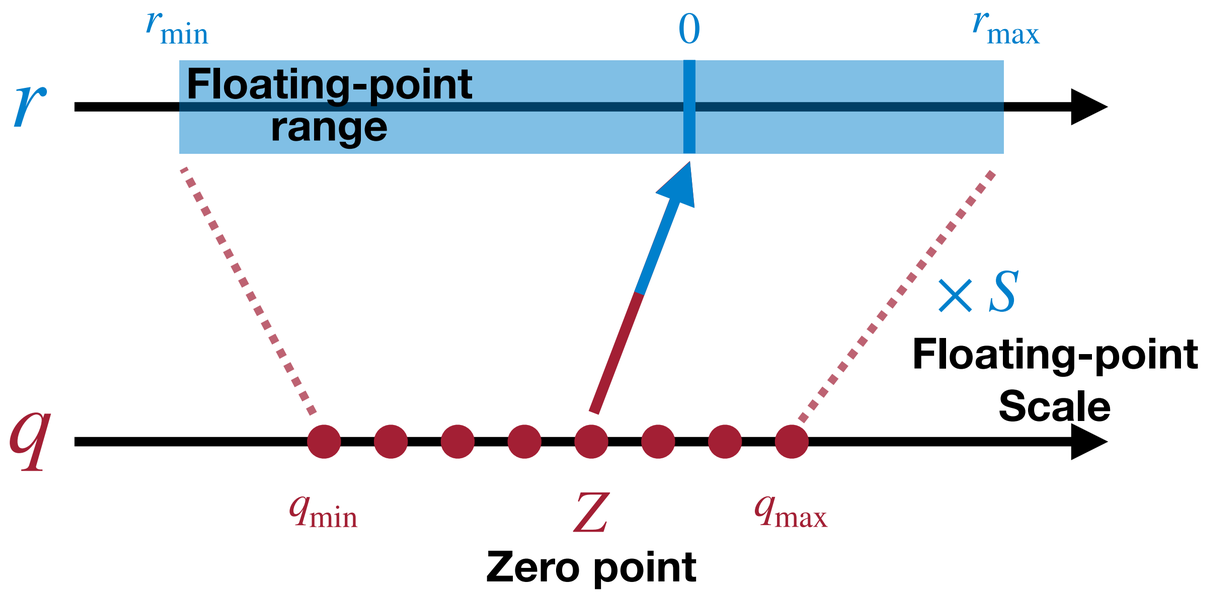
$$S = \frac{r_{max} - r_{min}}{q_{max} - q_{min}}$$

$$= \frac{2.12 - (-1.08)}{1 - (-2)}$$

$$= 1.07$$

Zero Point of Linear Quantization

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



$$r_{min} = S\left(q_{min} - Z\right)$$

$$\downarrow$$

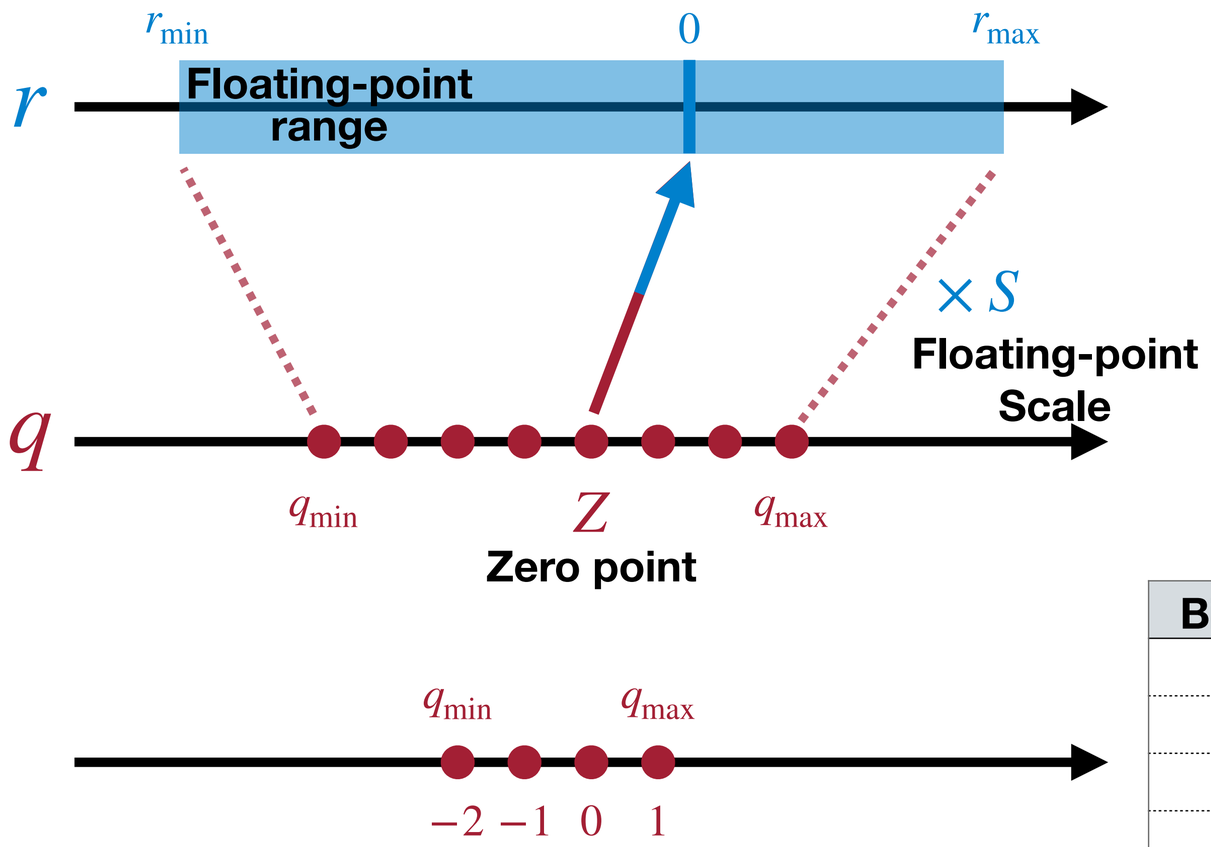
$$Z = q_{min} - \frac{r_{min}}{S}$$

$$\downarrow$$

$$Z = \text{round}\left(q_{min} - \frac{r_{min}}{S}\right)$$

Zero Point of Linear Quantization

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$



2.09	*-0.98*	*1.48*	*0.09*
0.05	*-0.14*	*-1.08*	*2.12*
-0.91	*1.92*	*0*	*-1.03*
1.87	*0*	*1.53*	*1.49*

Binary	Decimal
01	1
00	0
11	-1
10	-2

$$Z = q_{min} - \frac{r_{min}}{S}$$

$$= \text{round}(-2 - \frac{-1.08}{1.07})$$

$$= -1$$

Linear Quantized Matrix Multiplication

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following matrix multiplication.

$$\mathbf{Y} = \mathbf{W}\mathbf{X}$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}\left(\mathbf{q}_{\mathbf{W}} - Z_{\mathbf{W}}\right) \cdot S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\right)$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}\left(\mathbf{q}_{\mathbf{W}} - Z_{\mathbf{W}}\right)\left(\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\right) + Z_{\mathbf{Y}}$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}} + Z_{\mathbf{W}}Z_{\mathbf{X}}\right) + Z_{\mathbf{Y}}$$

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Linear Quantized Matrix Multiplication

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$$\mathbf{Y} = \mathbf{W}\mathbf{X}$$

$$\mathbf{q_Y} = \frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}} \left(\mathbf{q_W q_X} - Z_\mathbf{W} \mathbf{q_X} - Z_\mathbf{X} \mathbf{q_W} + Z_\mathbf{W} Z_\mathbf{X} \right) + Z_\mathbf{Y}$$

Precompute

Rescale to *N*-bit Integer

***N*-bit Integer Multiplication**
32-bit Integer Addition/Subtraction

***N*-bit Integer Addition**

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Linear Quantized Matrix Multiplication

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- Consider the following matrix multiplication.

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$$\mathbf{q}_\mathbf{Y} = \frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}} \left(\mathbf{q}_\mathbf{W}\mathbf{q}_\mathbf{X} - Z_\mathbf{W}\mathbf{q}_\mathbf{X} - Z_\mathbf{X}\mathbf{q}_\mathbf{W} + Z_\mathbf{W}Z_\mathbf{X} \right) + Z_\mathbf{Y}$$

- Empirically, the scale $\frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}}$ is always in the interval (0, 1).

Fixed-point Multiplication

$$\frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}} = 2^{-n} M_0, \quad \text{where} \quad M_0 \in [0.5, 1)$$

Bit Shift

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Linear Quantized Matrix Multiplication

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following matrix multiplication.

$$\mathbf{Y} = \mathbf{W}\mathbf{X}$$

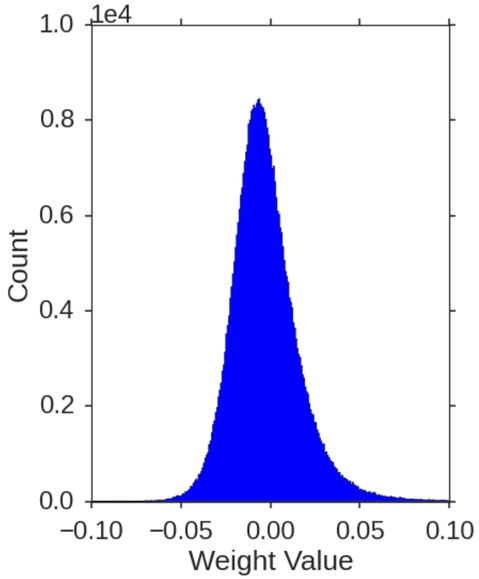
$$\mathbf{q_Y} = \boxed{\frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}}} \Big(\boxed{\mathbf{q_W q_X} - Z_\mathbf{W}\mathbf{q_X} - Z_\mathbf{X}\mathbf{q_W} + Z_\mathbf{W} Z_\mathbf{X}} \Big) + \boxed{Z_\mathbf{Y}}$$

Precompute

Rescale to *N*-bit Integer

***N*-bit Integer Multiplication**
32-bit Integer Addition/Subtraction

***N*-bit Integer Addition**

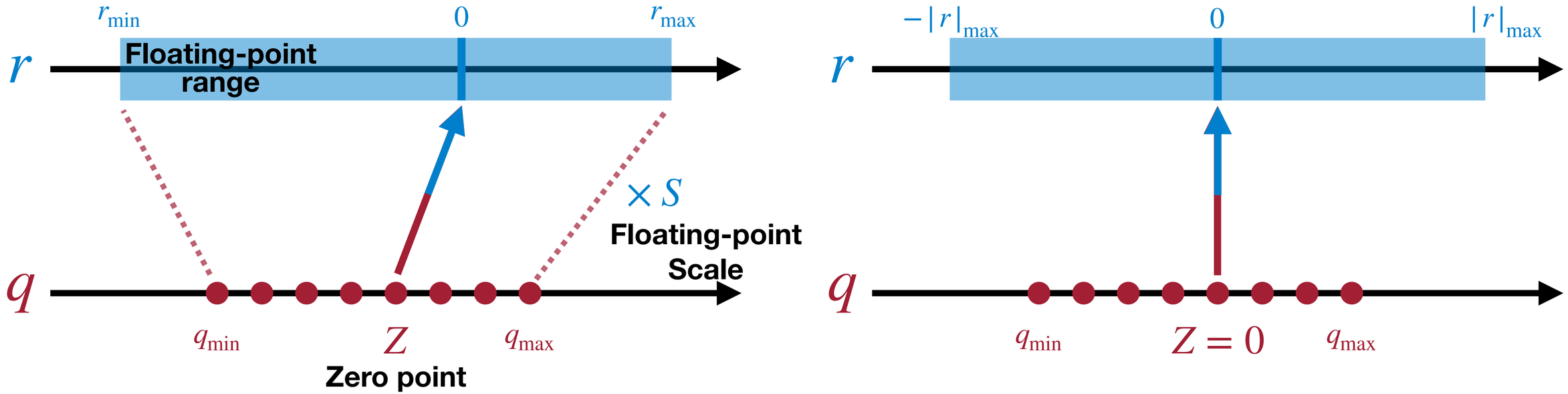


$$Z_\mathbf{W} = 0?$$

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Symmetric Linear Quantization

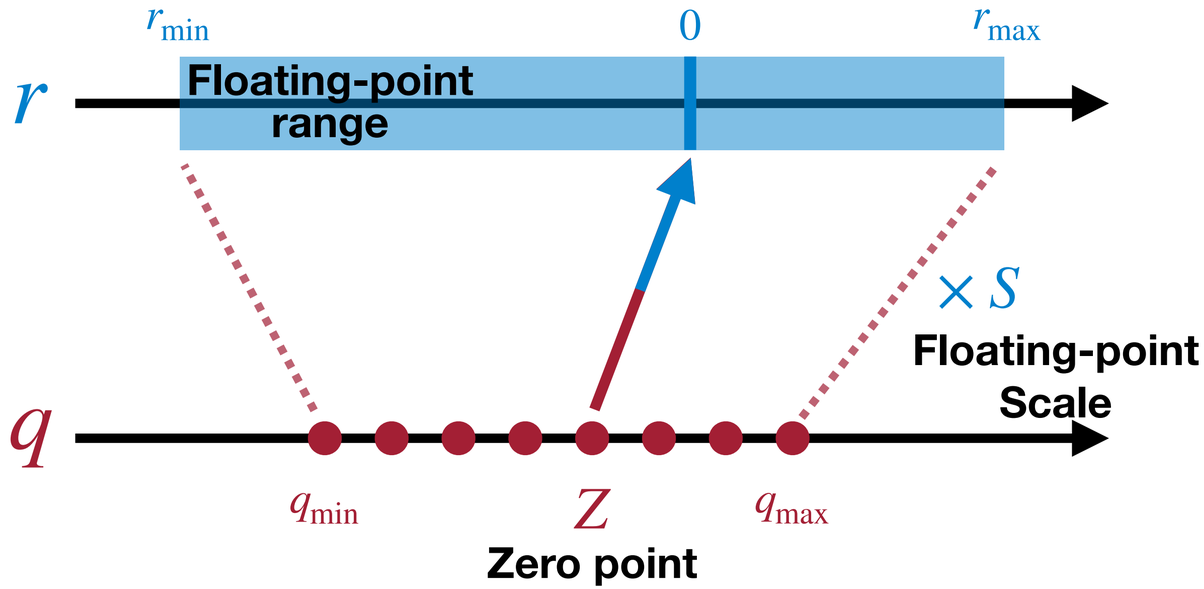
Zero point $Z = 0$ and Symmetric floating-point range

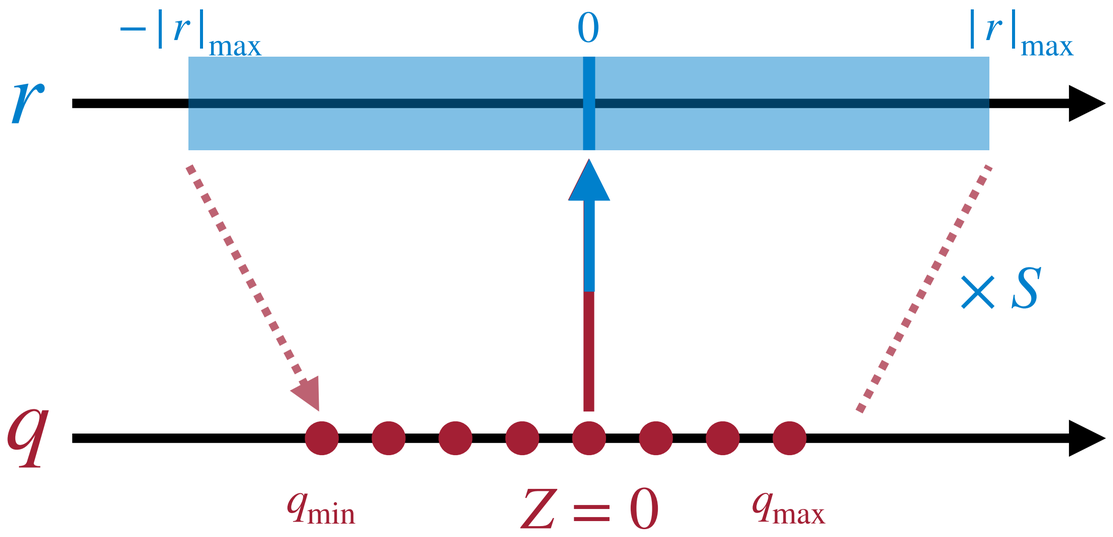


Bit Width	q_{min}	q_{max}
2	-2	1
3	-4	3
4	-8	7
N	-2^{N-1}	$2^{N-1}-1$

Symmetric Linear Quantization

Full range mode





$$S = \frac{r_{\max} - r_{\min}}{q_{\max} - q_{\min}}$$

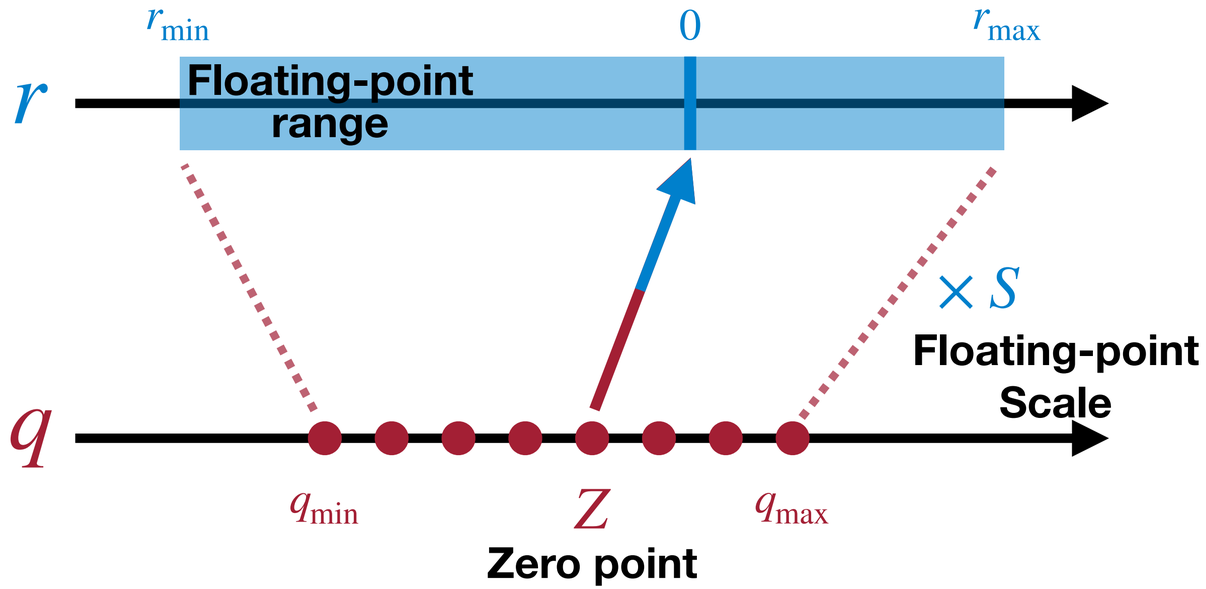
$$S = \frac{r_{\min}}{q_{\min} - Z} = \frac{-|r|_{\max}}{q_{\min}} = \frac{|r|_{\max}}{2^{N-1}}$$

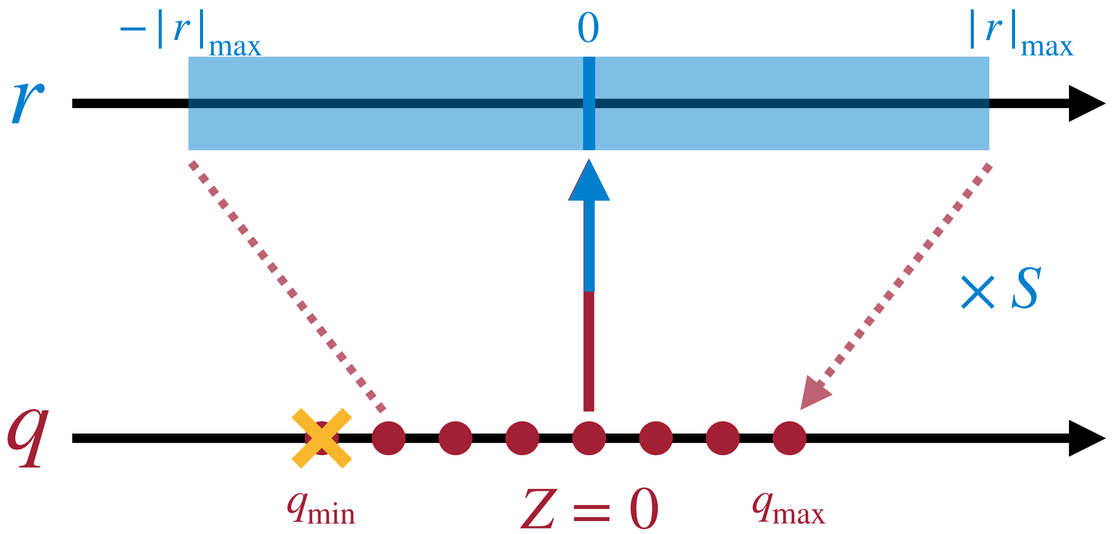
Bit Width	q_{min}	q_{max}
2	-2	1
3	-4	3
4	-8	7
N	-2^{N-1}	$2^{N-1}-1$

- use full range of quantized integers
- example: PyTorch's native quantization, ONNX

Symmetric Linear Quantization

Restricted range mode





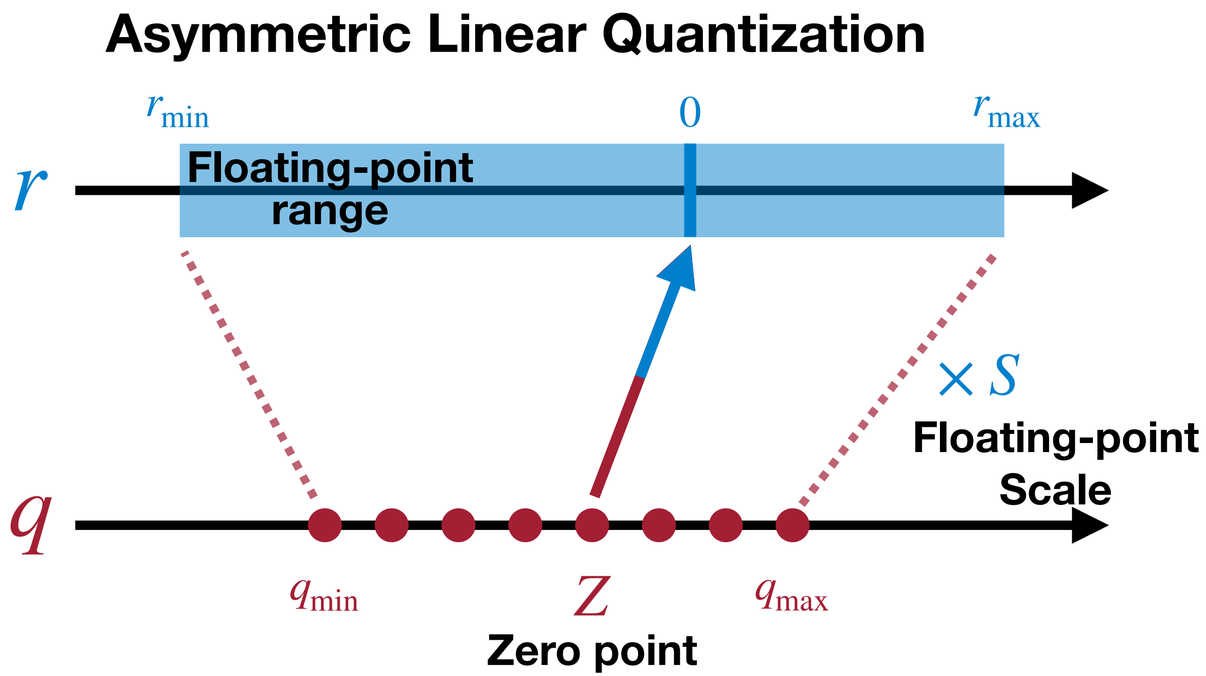
$$S = \frac{r_{max} - r_{min}}{q_{max} - q_{min}}$$

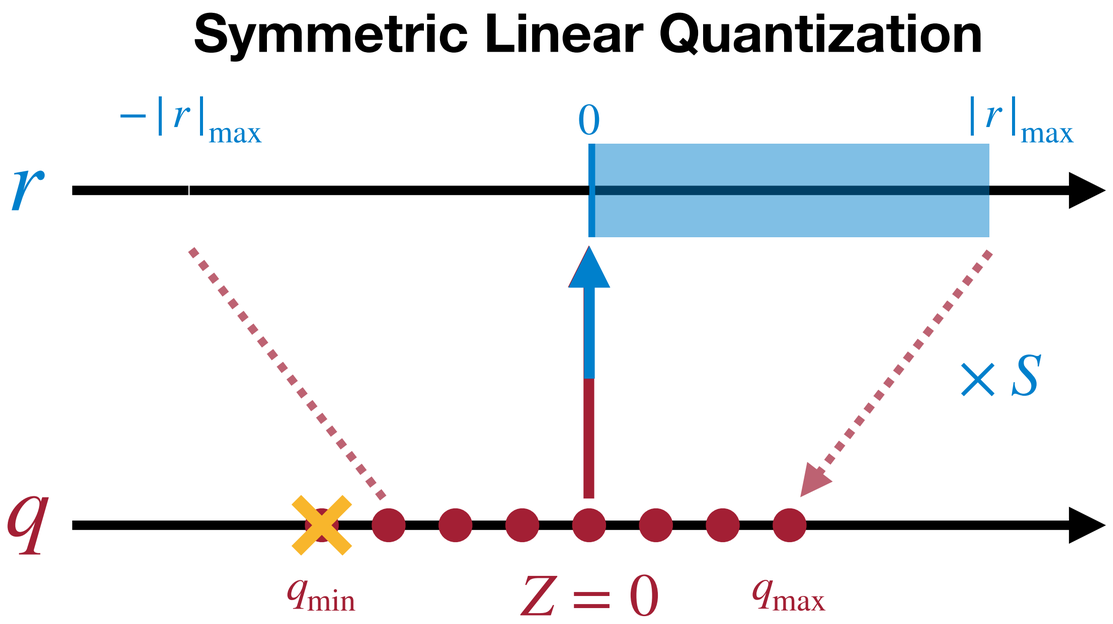
$$S = \frac{r_{max}}{q_{max} - Z} = \frac{|r|_{max}}{q_{max}} = \frac{|r|_{max}}{2^{N-1} - 1}$$

Bit Width	q_{min}	q_{max}
2	-2	1
3	-4	3
4	-8	7
N	-2^{N-1}	$2^{N-1}-1$

- example: TensorFlow, NVIDIA TensorRT, Intel DNNL

Asymmetric *vs.* Symmetric





- The quantized range is fully used.
- The implementation is more complex, and zero points require additional logic in hardware.

- The quantized range will be wasted for biased float range.
  - Activation tensor is non-negative after ReLU, and thus symmetric quantization will lose 1 bit effectively.
- The implementation is much simpler.

Neural Network Distiller

Linear Quantized Matrix Multiplication

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following matrix multiplication.

$$\mathbf{Y} = \mathbf{W}\mathbf{X}$$

$$\mathbf{q_Y} = \frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}} \left(\mathbf{q_W q_X} - Z_\mathbf{W} \mathbf{q_X} - Z_\mathbf{X} \mathbf{q_W} + Z_\mathbf{W} Z_\mathbf{X} \right) + Z_\mathbf{Y}$$

Precompute

Rescale to *N*-bit Integer

***N*-bit Integer Multiplication**
32-bit Integer Addition/Subtraction

***N*-bit Integer Addition**

$$\mathbf{q_Y} = \frac{S_\mathbf{W} S_\mathbf{X}}{S_\mathbf{Y}} \left(\mathbf{q_W q_X} - Z_\mathbf{X} \mathbf{q_W} \right) + Z_\mathbf{Y}$$

$Z_\mathbf{W} = 0$

Linear Quantized Fully-Connected Layer

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- So far, we ignore bias. Now we consider the following fully-connected layer with bias.

$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mathbf{b}$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}\left(\mathbf{q}_{\mathbf{W}} - Z_{\mathbf{W}}\right) \cdot S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\right) + S_{\mathbf{b}}\left(\mathbf{q}_{\mathbf{b}} - Z_{\mathbf{b}}\right)$$

$$\downarrow \quad Z_{\mathbf{W}} = 0$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = \underline{S_{\mathbf{W}}S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}}\right)} + \underline{S_{\mathbf{b}}\left(\mathbf{q}_{\mathbf{b}} - Z_{\mathbf{b}}\right)}$$

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$$\downarrow \quad Z_{\mathbf{W}} = 0$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}}\right) + S_{\mathbf{b}}\left(\mathbf{q}_{\mathbf{b}} - Z_{\mathbf{b}}\right)$$

$$\downarrow \quad Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}}S_{\mathbf{X}}$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}}S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}} + \mathbf{q}_{\mathbf{b}}\right)$$

Linear Quantized Fully-Connected Layer

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- So far, we ignore bias. Now we consider the following fully-connected layer with bias.

$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mathbf{b}$$

$$Z_{\mathbf{W}} = 0 \downarrow Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}} S_{\mathbf{X}}$$

$$S_{\mathbf{Y}}\left(\mathbf{q}_{\mathbf{Y}} - Z_{\mathbf{Y}}\right) = S_{\mathbf{W}} S_{\mathbf{X}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}} + \mathbf{q}_{\mathbf{b}}\right)$$

$$\downarrow$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} + \underbrace{\mathbf{q}_{\mathbf{b}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}}}_{\text{Precompute}}\right) + Z_{\mathbf{Y}}$$

$$\downarrow \quad \mathbf{q}_{bias} = \mathbf{q}_{\mathbf{b}} - Z_{\mathbf{X}}\mathbf{q}_{\mathbf{W}}$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}}\left(\mathbf{q}_{\mathbf{W}}\mathbf{q}_{\mathbf{X}} + \mathbf{q}_{bias}\right) + Z_{\mathbf{Y}}$$

Linear Quantized Fully-Connected Layer

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

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$$\mathbf{Y} = \mathbf{W}\mathbf{X} + \mathbf{b}$$

$$Z_{\mathbf{W}} = 0$$

$$Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}} S_{\mathbf{X}}$$

$$\mathbf{q}_{bias} = \mathbf{q}_{\mathbf{b}} - Z_{\mathbf{X}} \mathbf{q}_{\mathbf{W}}$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}} \left(\mathbf{q}_{\mathbf{W}} \mathbf{q}_{\mathbf{X}} + \mathbf{q}_{bias} \right) + Z_{\mathbf{Y}}$$

Rescale to *N*-bit Int | **_N_-bit Int Mult. 32-bit Int Add.** | **_N_-bit Int Add**

Note: both $\mathbf{q}_{\mathbf{b}}$ and $\mathbf{q}_{bias}$ are 32 bits.

Linear Quantized Convolution Layer

Linear Quantization is an affine mapping of integers to real numbers $r = S(q - Z)$

- Consider the following convolution layer.

$$\mathbf{Y} = \text{Conv}(\mathbf{W}, \mathbf{X}) + \mathbf{b}$$

$$Z_{\mathbf{W}} = 0$$

$$Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}} S_{\mathbf{X}}$$

$$\mathbf{q}_{bias} = \mathbf{q}_{\mathbf{b}} - \text{Conv}(\mathbf{q}_{\mathbf{W}}, Z_{\mathbf{X}})$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}} \left(\text{Conv}(\mathbf{q}_{\mathbf{W}}, \mathbf{q}_{\mathbf{X}}) + \mathbf{q}_{bias} \right) + Z_{\mathbf{Y}}$$

Rescale to *N*-bit Int

***N*-bit Int Mult. 32-bit Int Add.**

***N*-bit Int Add**

Note: both $\mathbf{q}_{\mathbf{b}}$ and $\mathbf{q}_{bias}$ are 32 bits.

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$$\mathbf{Y} = \text{Conv}(\mathbf{W}, \mathbf{X}) + \mathbf{b}$$

$$Z_{\mathbf{W}} = 0$$

$$Z_{\mathbf{b}} = 0, \quad S_{\mathbf{b}} = S_{\mathbf{W}} S_{\mathbf{X}}$$

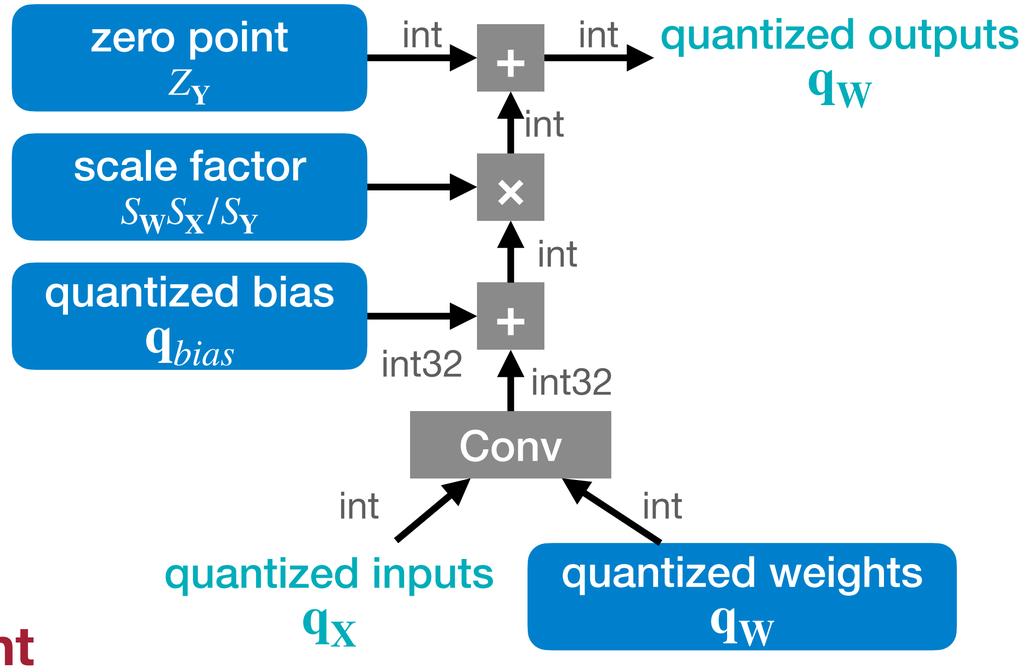
$$\mathbf{q}_{bias} = \mathbf{q}_{\mathbf{b}} - \text{Conv}(\mathbf{q}_{\mathbf{W}}, Z_{\mathbf{X}})$$

$$\mathbf{q}_{\mathbf{Y}} = \frac{S_{\mathbf{W}} S_{\mathbf{X}}}{S_{\mathbf{Y}}} \left(\text{Conv}(\mathbf{q}_{\mathbf{W}}, \mathbf{q}_{\mathbf{X}}) + \mathbf{q}_{bias} \right) + Z_{\mathbf{Y}}$$

Rescale to *N*-bit Int

***N*-bit Int Mult. 32-bit Int Add.**

***N*-bit Int Add**

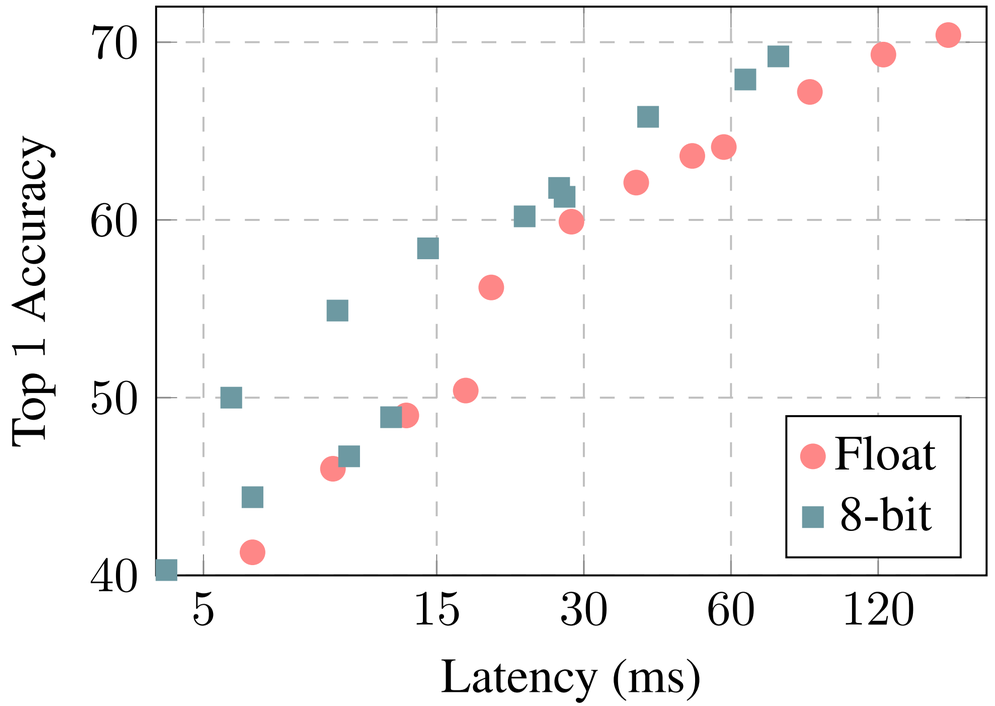


Note: both $\mathbf{q}_{\mathbf{b}}$ and $\mathbf{q}_{bias}$ are 32 bits.

INT8 Linear Quantization

An affine mapping of integers to real numbers $r = S(q - Z)$

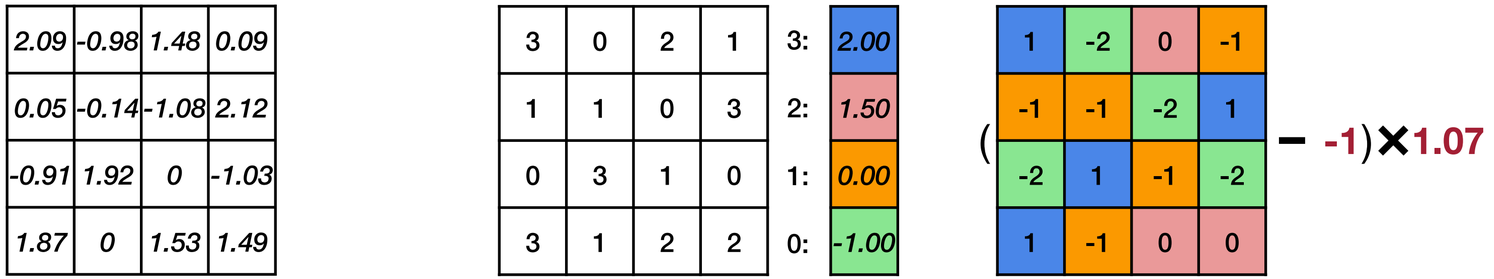
Neural Network	ResNet-50	Inception-V3
Floating-point Accuracy	76.4%	78.4%
8-bit Integer-quantized Acurracy	74.9%	75.4%



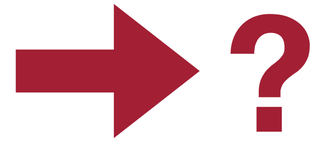
Latency-vs-accuracy tradeoff of float vs. integer-only MobileNets on ImageNet using Snapdragon 835 big cores.

Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob *et al.*, CVPR 2018]

Neural Network Quantization



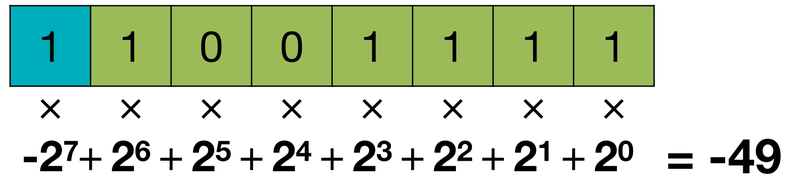
		K-Means-based Quantization	**Linear Quantization**
Storage	Floating-Point Weights	Integer Weights; Floating-Point Codebook	Integer Weights
Computation	Floating-Point Arithmetic	Floating-Point Arithmetic	Integer Arithmetic



Summary of Today's Lecture

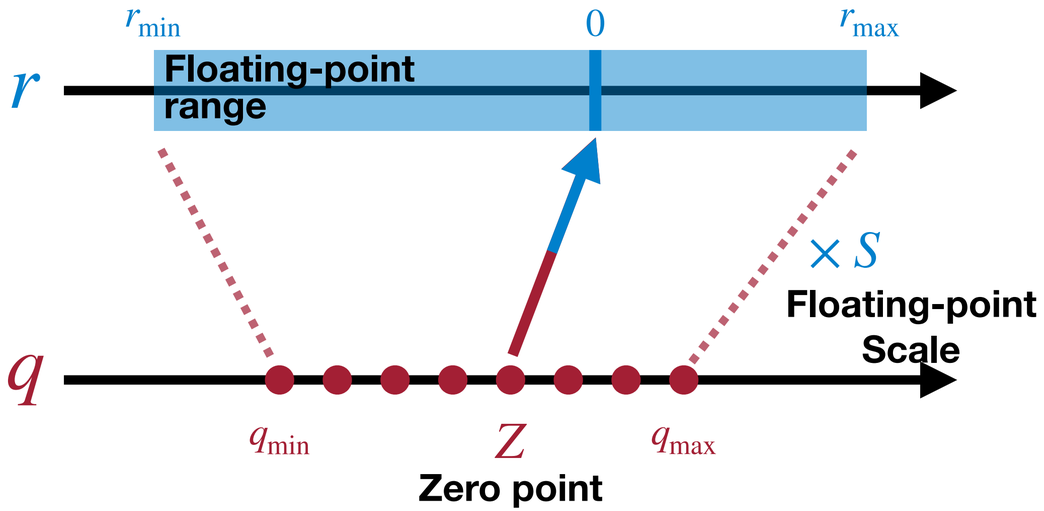
Today, we reviewed and learned

- the numeric data types used in the modern computing systems, including integers and floating-point numbers.



- the basic concept of **neural network quantization**: *converting the weights and activations of neural networks into a limited discrete set of numbers.*

- three types of common neural network quantization:
  - K-Means-based Quantization
  - Linear Quantization



References

1. Model Compression and Hardware Acceleration for Neural Networks: A Comprehensive Survey [Deng et al., IEEE 2020]
2. Computing's Energy Problem (and What We Can Do About it) [Horowitz, M., IEEE ISSCC 2014]
3. Deep Compression [Han et al., ICLR 2016]
4. Neural Network Distiller: https://intellabs.github.io/distiller/algo_quantization.html
5. Quantization and Training of Neural Networks for Efficient Integer-Arithmetic-Only Inference [Jacob et al., CVPR 2018]
6. BinaryConnect: Training Deep Neural Networks with Binary Weights during Propagations [Courbariaux et al., NeurIPS 2015]
7. Binarized Neural Networks: Training Deep Neural Networks with Weights and Activations Constrained to +1 or –1. [Courbariaux et al., Arxiv 2016]
8. XNOR-Net: ImageNet Classification using Binary Convolutional Neural Networks [Rastegari et al., ECCV 2016]
9. Ternary Weight Networks [Li et al., Arxiv 2016]
10. Trained Ternary Quantization [Zhu et al., ICLR 2017]