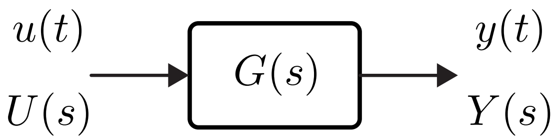
Lecture 7

Lecturer: Asst. Prof. M. Mert Ankarali

7.1 Time Domain Analysis



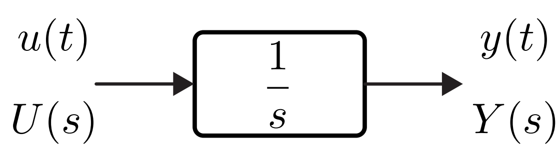
$$Y(s) = G(s)U(s)$$
$$y(t) = \mathcal{L}\{G(s)U(s)\}^{-1}$$

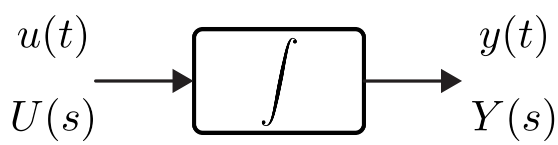
Objective:

- Calculate $y(t)$ for different $u(t)$,
- Understand the relation between the parameters and output behavior.

7.1.1 First Order Systems

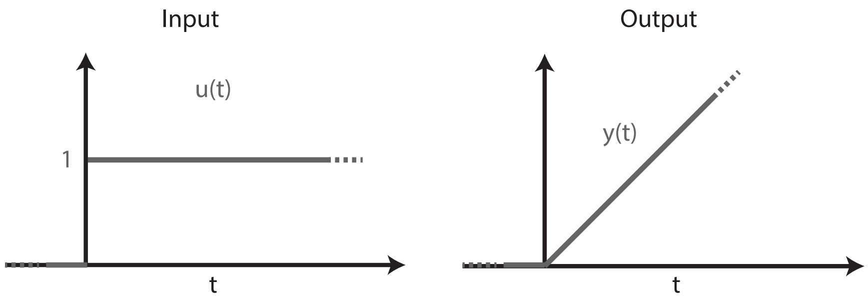
Simplest first order system is an integrator, which is also the fundamental block for higher order systems.





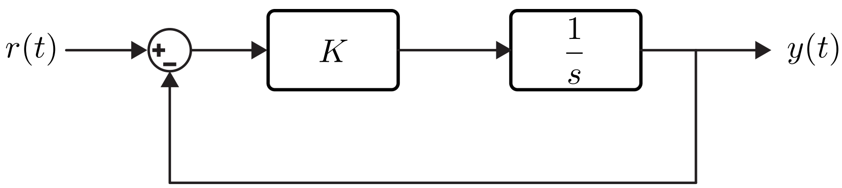
Ex 1: Compute the step-response of the integrator system, $G(s) = \frac{1}{s}$.

Solution: We assume that initial conditions are zero



$$U(s) = \frac{1}{s}$$
$$Y(s) = G(s)U(s) = \frac{1}{s^2}$$
$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}$$
$$= t\ , \text{ for } t \geq 0$$

Ex 2: Compute the step-response of the following first order system



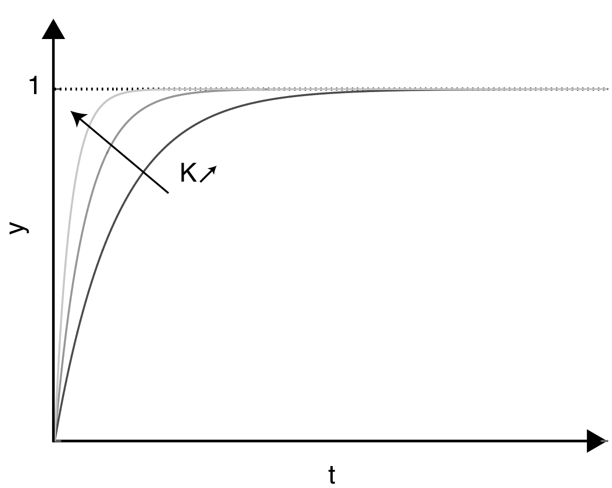
$$G(s) = \frac{K}{s+K}$$
$$Y(s) = \frac{K}{s(s+K)} = \frac{K}{s(s+K)}$$
$$= \frac{A}{s} + \frac{B}{s+K}$$

We can compute A and B as

$$A = \lim_{s\to 0}\left[sY(s)\right] = \frac{K}{K} = 1$$
$$B = \lim_{s\to -K}\left[(s+K)Y(s)\right] = \frac{K}{-K} = -1$$

Then, we can compute $y(t)$ as

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{s+K}\right\}$$
$$= \left[1 - e^{-Kt}\right]\ , \text{ for } t \geq 0$$



Proportional Controller

$$\lim_{t\to\infty} y(t) = 1$$
$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty}(y(t) - u(t)) = 0$$

Zero steady-state error $\forall K > 0$
"Convergence speed" $\nearrow$ as $K \nearrow$

Ex 2: Find the unit-ramp response for the same system

$$
\begin{aligned}
Y(s) &= \frac{K}{s^2(s+K)} \\
&= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+K}
\end{aligned}
$$

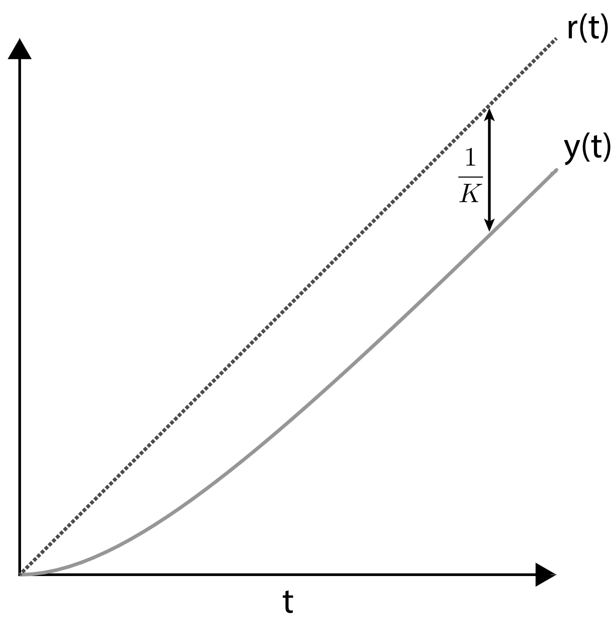
A, B, and C can be computed as

$$
\begin{aligned}
C &= \lim_{s \to -K} \left[(s+K)Y(s)\right] = \frac{1}{K} \\
B &= \lim_{s \to 0} \left[s^2 Y(s)\right] = 1 \\
A &= \lim_{s \to 0} \frac{d}{ds}\left[s^2 Y(s)\right] = \lim_{s \to 0} \frac{d}{ds}\left[\frac{K}{(s+K)}\right] = \lim_{s \to 0}\left[\frac{-K}{(s+K)^2}\right] = \frac{-1}{K}
\end{aligned}
$$

Then, we can compute $y(t)$ as

$$
\begin{aligned}
y(t) &= \frac{-1}{K} + t + \frac{1}{K}e^{-Kt}\ , \text{ for } t \geq 0 \\
y(t) &= t + \frac{-1}{K}\left[1 - \frac{1}{K}e^{-Kt}\right]\ , \text{ for } t \geq 0
\end{aligned}
$$

Note that $r(t) = t\ ,$ for $t \geq 0$.



Proportional Controller

$$e(t) = r(t) - y(t) = \frac{1}{K}\left[1 - \frac{1}{K}e^{-Kt}\right]$$

$$\lim_{t \to \infty} e(t) = \frac{1}{K}$$

Non-zero steady-state error
Steady-state error $\searrow$ as $K \nearrow$