EE302 - Feedback Systems **Spring 2019**

Lecture 6

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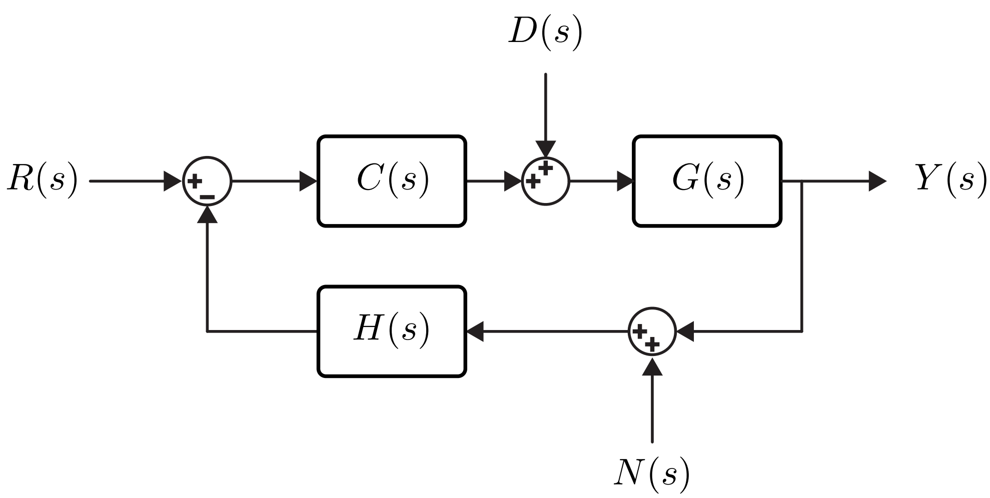
6.1 Transfer functions with multiple exogenous input

When modeling and analyzing a closed system in addition to the desired reference input signal, it is also important to model/analyze the system with unwanted disturbances and noise input signals. Let's consider the following block-diagram topology. In this closed-loop system, there exist three exogenous input signals; $r(t)$ (reference input), $d(t)$ ("disturbance" input), and $n(t)$ ("noise" input).



When modeling the response or characteristic of the system with respect to different external inputs, we assume that remaining ones are zero.

Response to $r(t)$

$$T_R(s) = \frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)H(s)}$$

Response to $d(t)$

$$T_D(s) = \frac{Y(s)}{D(s)} = \frac{G(s)}{1 + C(s)G(s)H(s)}$$

Response to $n(t)$

$$T_N(s) = \frac{Y(s)}{N(s)} = \frac{C(s)G(s)H(s)}{1 + C(s)G(s)H(s)}$$

If we generalize, we can write $Y(s)$ as

$$Y(s) = T_R(s)R(s) + T_D(s)D(s) + T_N(s)N(s)$$

Lets roughly analyze the desired responses under different type of inputs. Let's assume that $G(s)$ is the plant transfer function and $H(s)$ is the sensory dynamics transfer function. $C(s)$ is the transfer function of the controller.

In the ideal case, we want

- Perfect tracking of reference signal, $T_R^*(s) \approx 1$. Since it is not possible to perfectly achieve this under dynamic system constrains, we can design a "high gain" controller such that

  $$T_R(s) \approx \frac{C(s)G(s)}{C(s)G(s)H(s)} \approx \frac{1}{H(s)}$$

  If $H(s) \approx 1$, then we can have a high tracking performance from the system.

- Perfect rejection of disturbance signal, $T_R^*(s) \approx 0$. Similarly, we can design a "high gain" controller such that

  $$T_D(s) \approx \frac{G(s)}{C(s)G(s)H(s)} \approx 0$$

  It seems that the requirement on $C(s)$ is similar for good tracking and good disturbance rejection.

- Perfect rejection of noise signal, $T_N^*(s) \approx 0$. In this case, we can design a "low gain" controller (or low gain $H(s)$) such that

  $$T_N(s) \approx \frac{C(s)G(s)H(s)}{1} \approx 0$$

It seems that requirements on $C(s)$ and $H(s)$ start conflicting when we consider both tracking performance, disturbance rejection, and noise rejection. This paradox is the most well-known limitation of feedback control systems. The basic idea is that one can not only concentrate on designing a controller $C(s)$ such that we reach excellent closed-loop tracking performance when the system suffers from uncertainties and noises. Somehow we need to design $G(s)$, $H(s)$, and even $N(s)$ and $D(s)$ together such that the whole system achieves a "good" closed-loop behavior.