Frequency Response Techniques in Feedback Control Systems

https://github.com/mertankarali/Lecture-Notes/tree/master/METU-EE302/Frequency_Response

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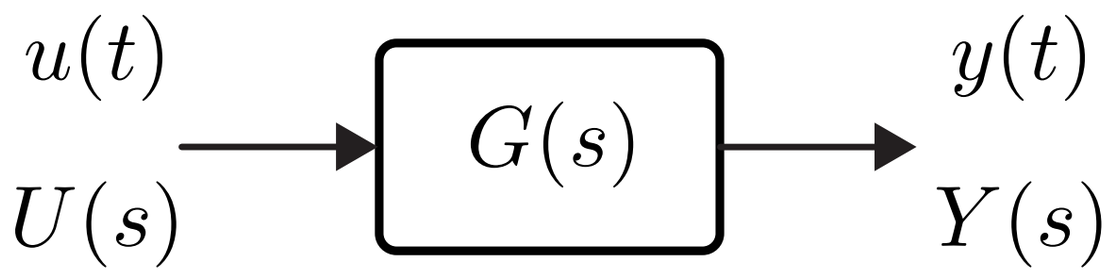
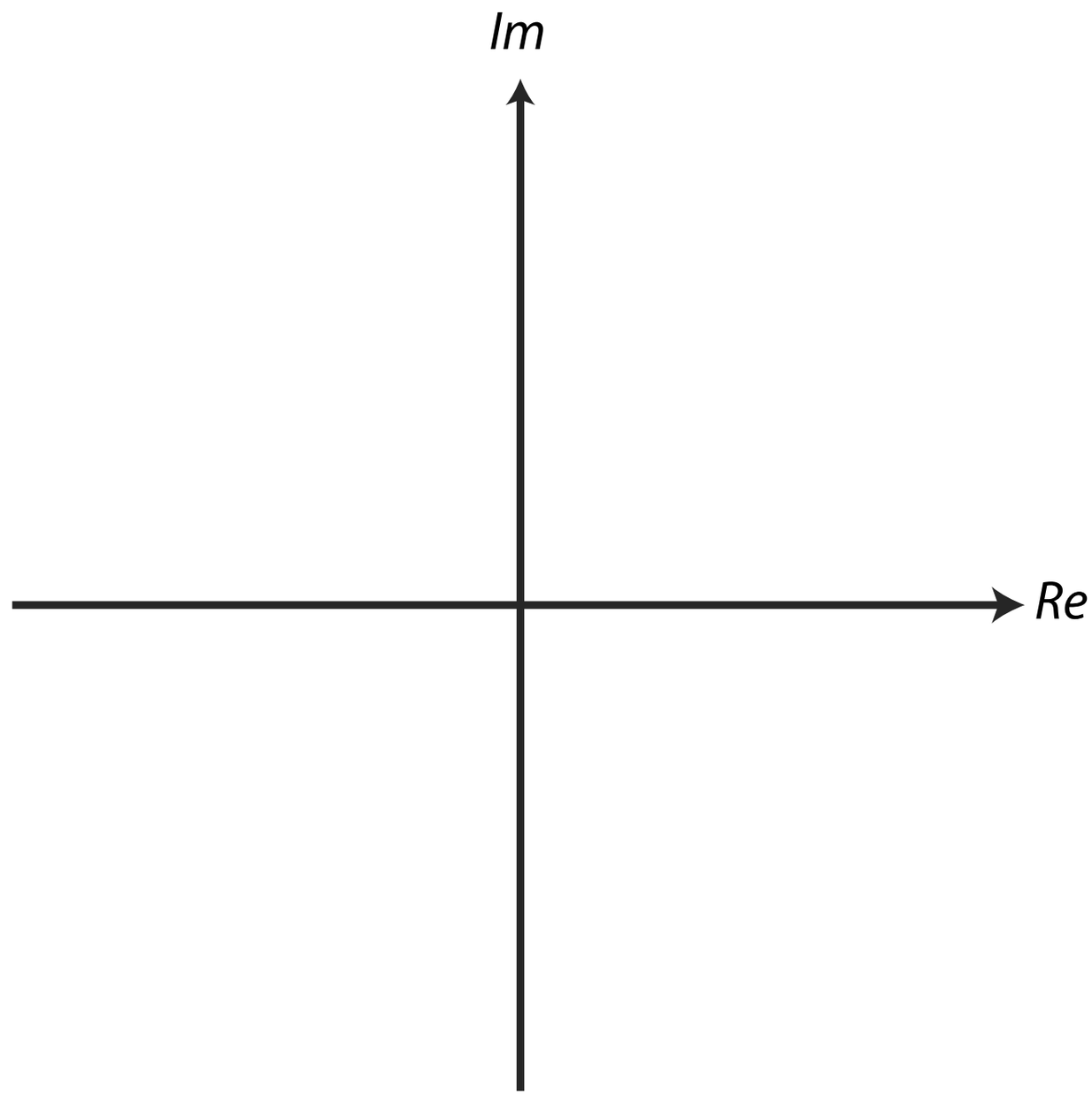
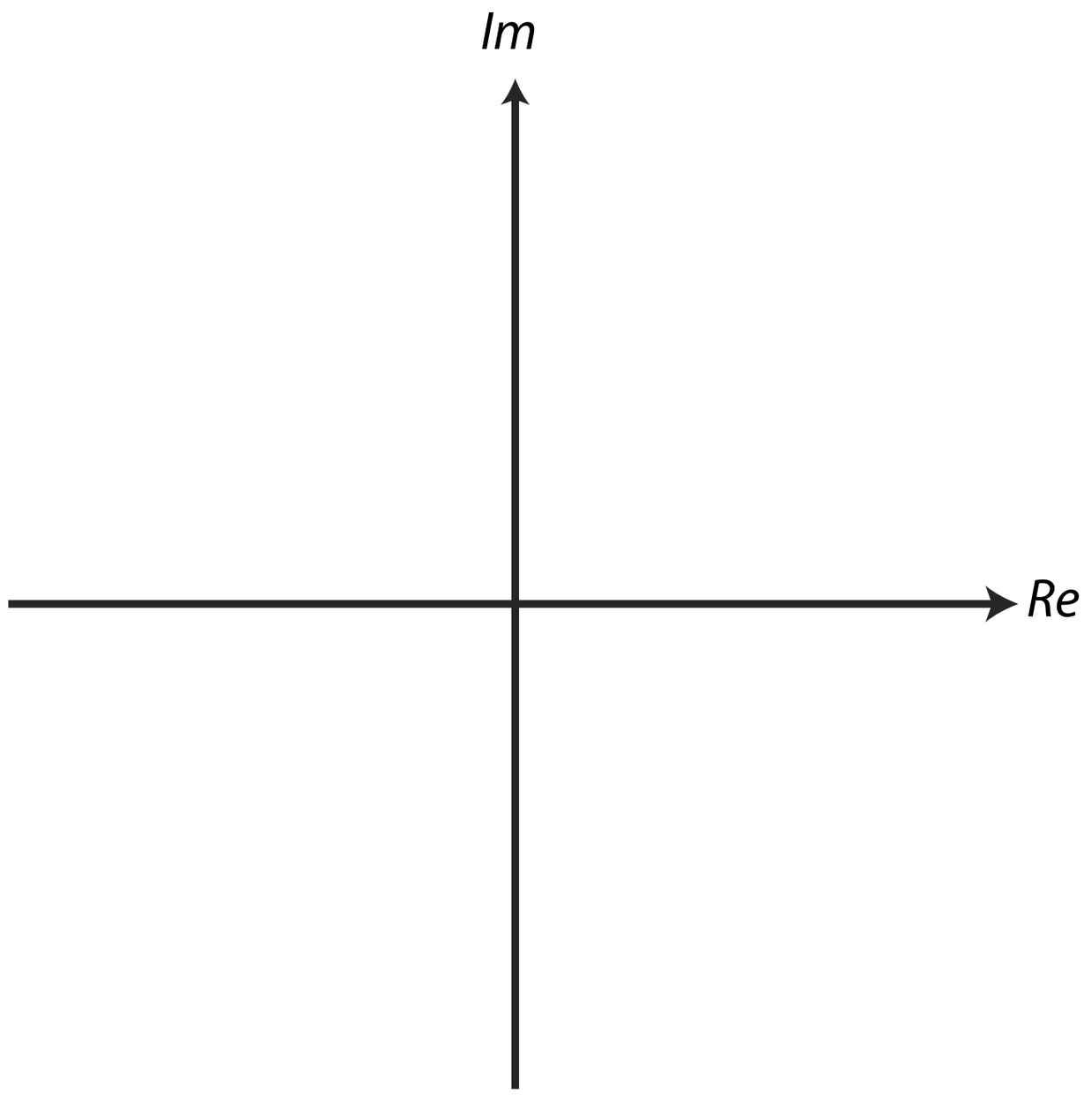
Part I - Polar Plot

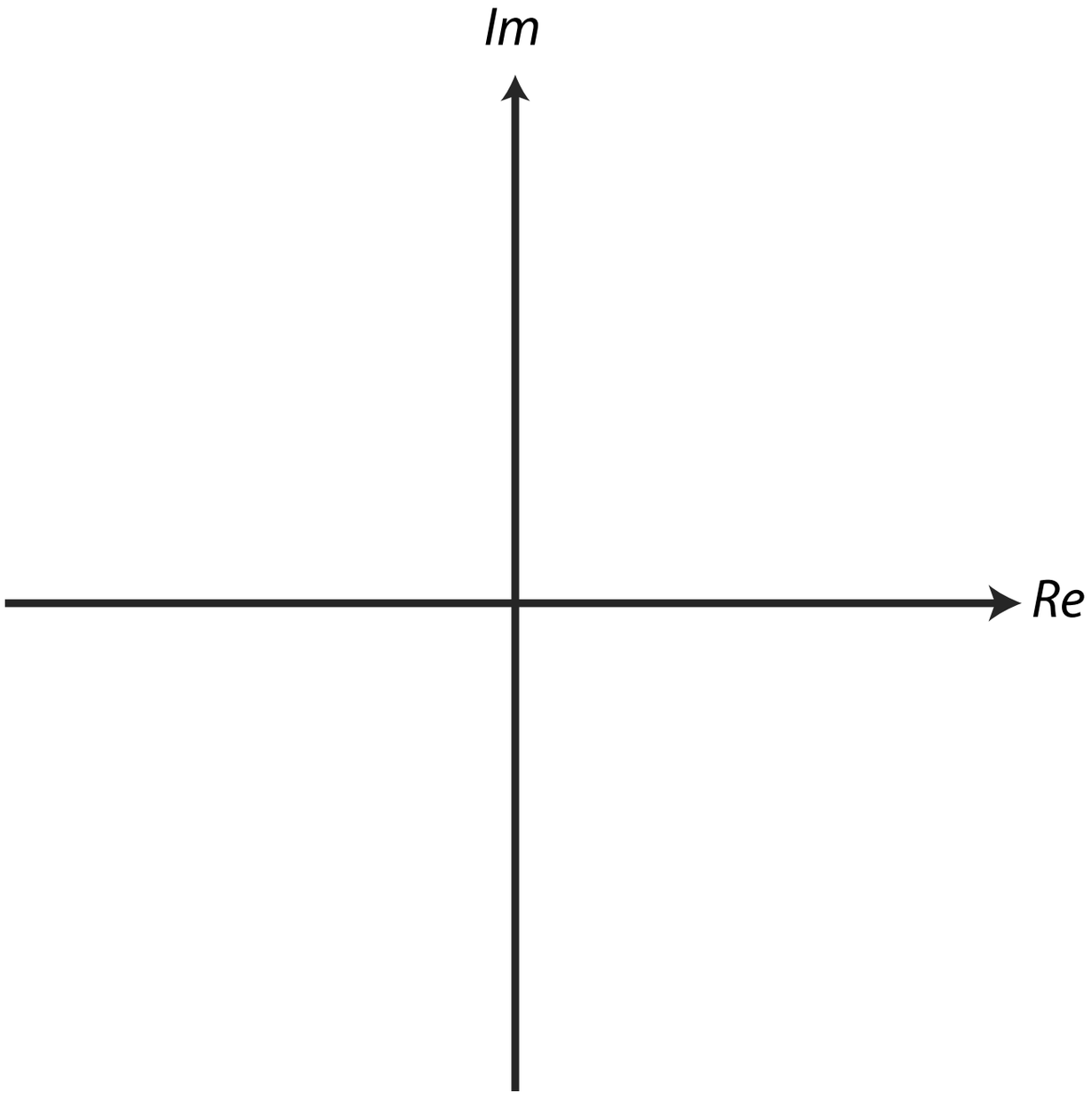

$u(t)$
$U(s)$
$G(s)$
$y(t)$
$Y(s)$

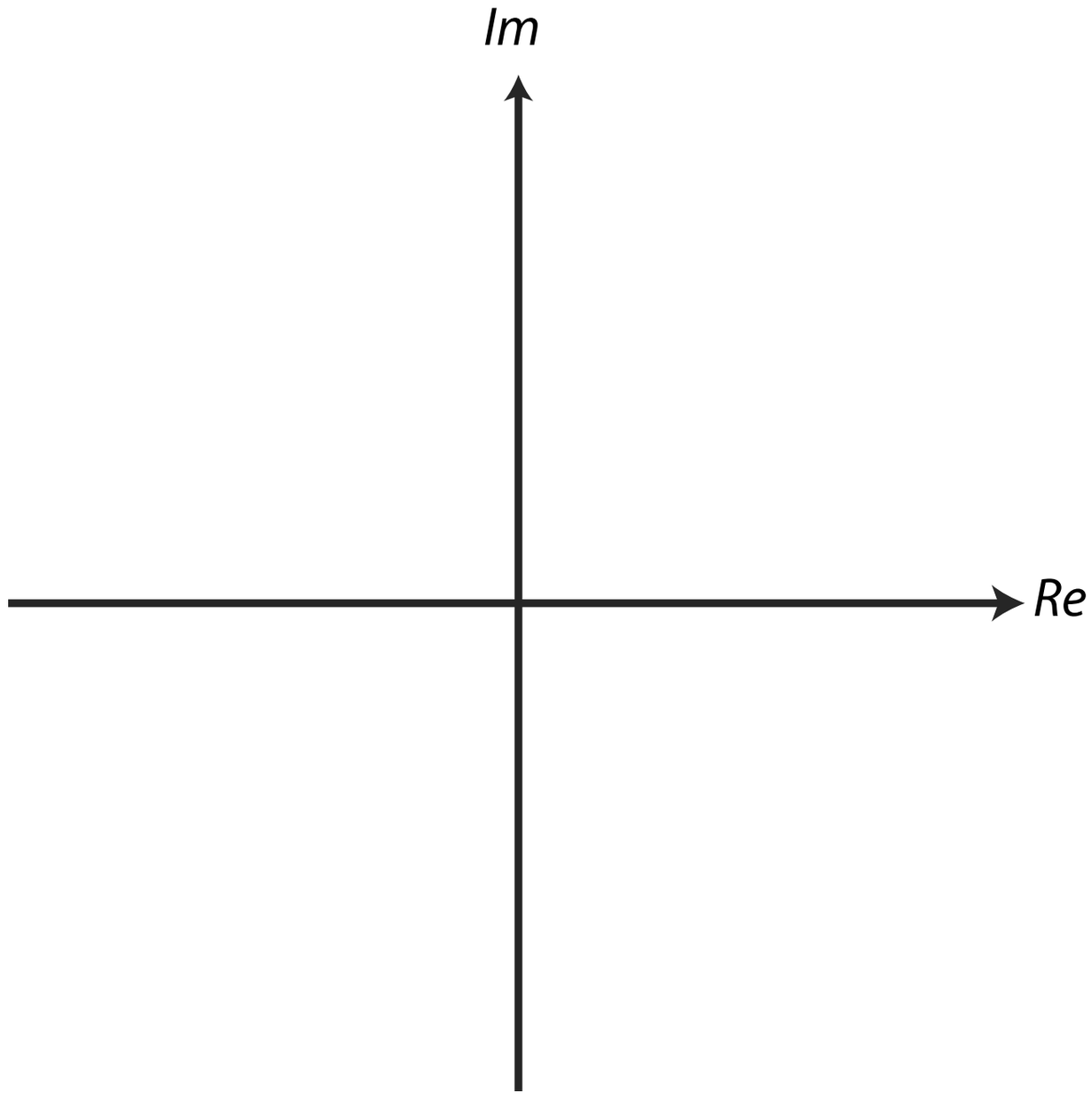
Polar Plot



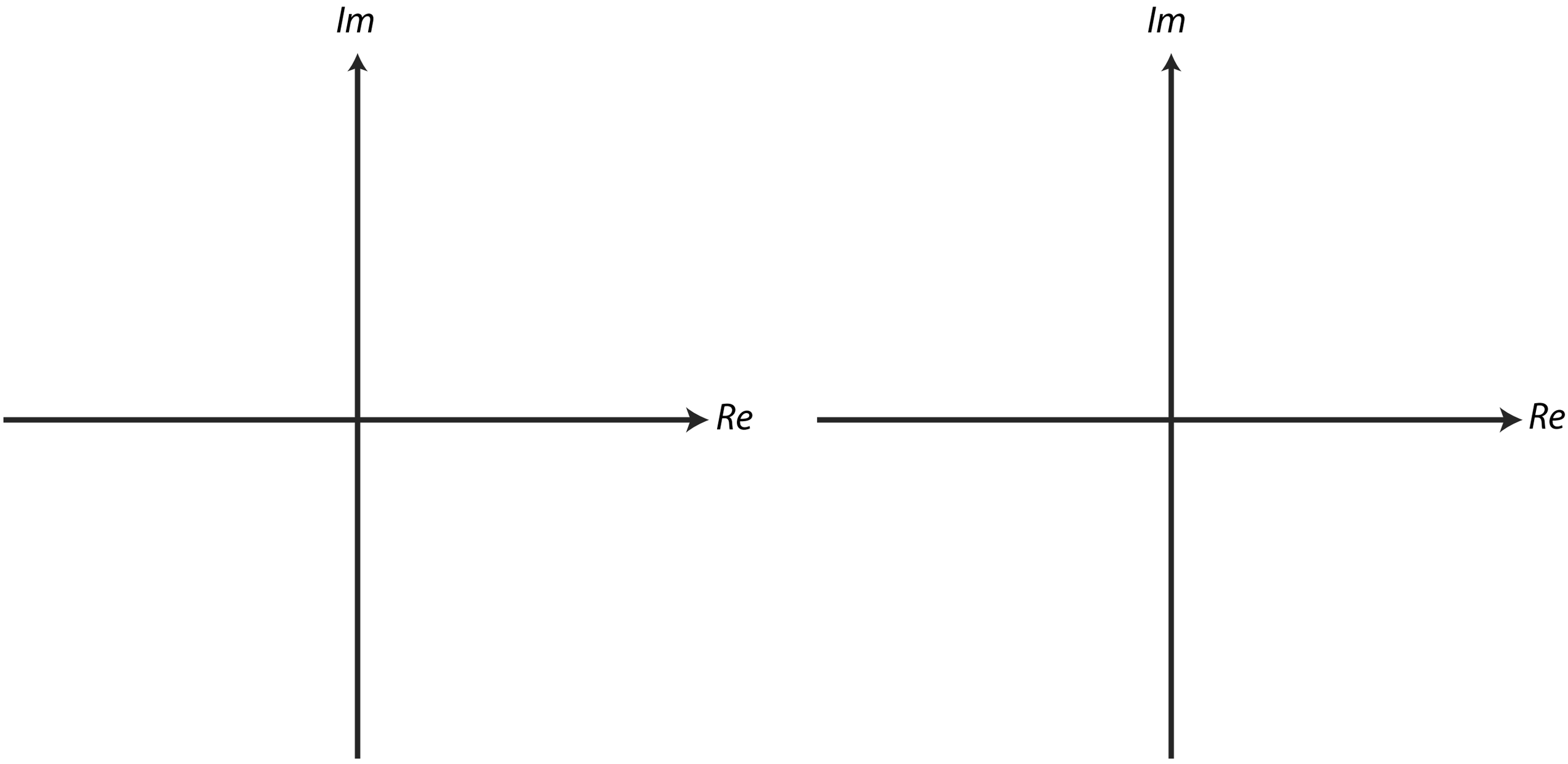


$$G_1(s) = s \quad , \quad G_2(s) = \frac{1}{s}$$

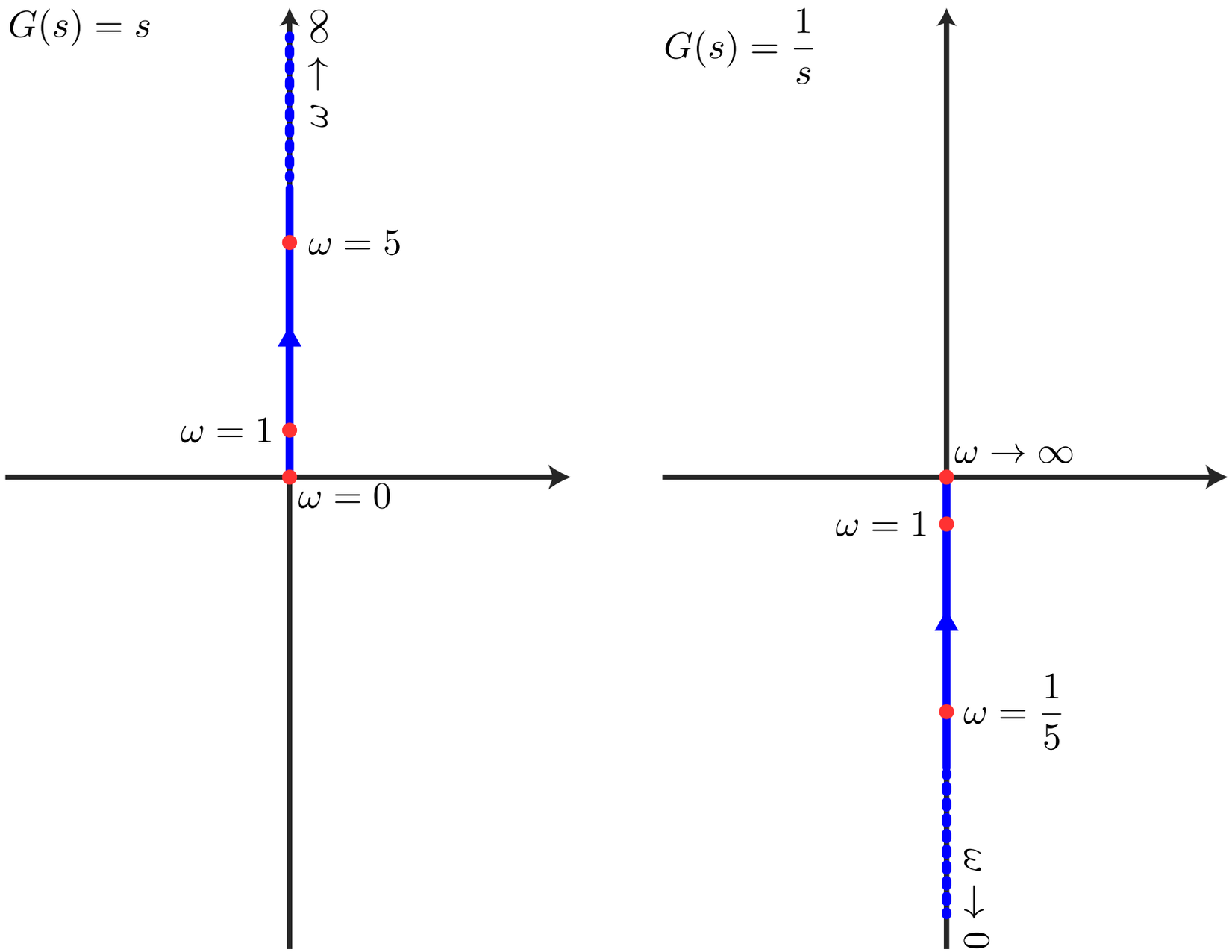




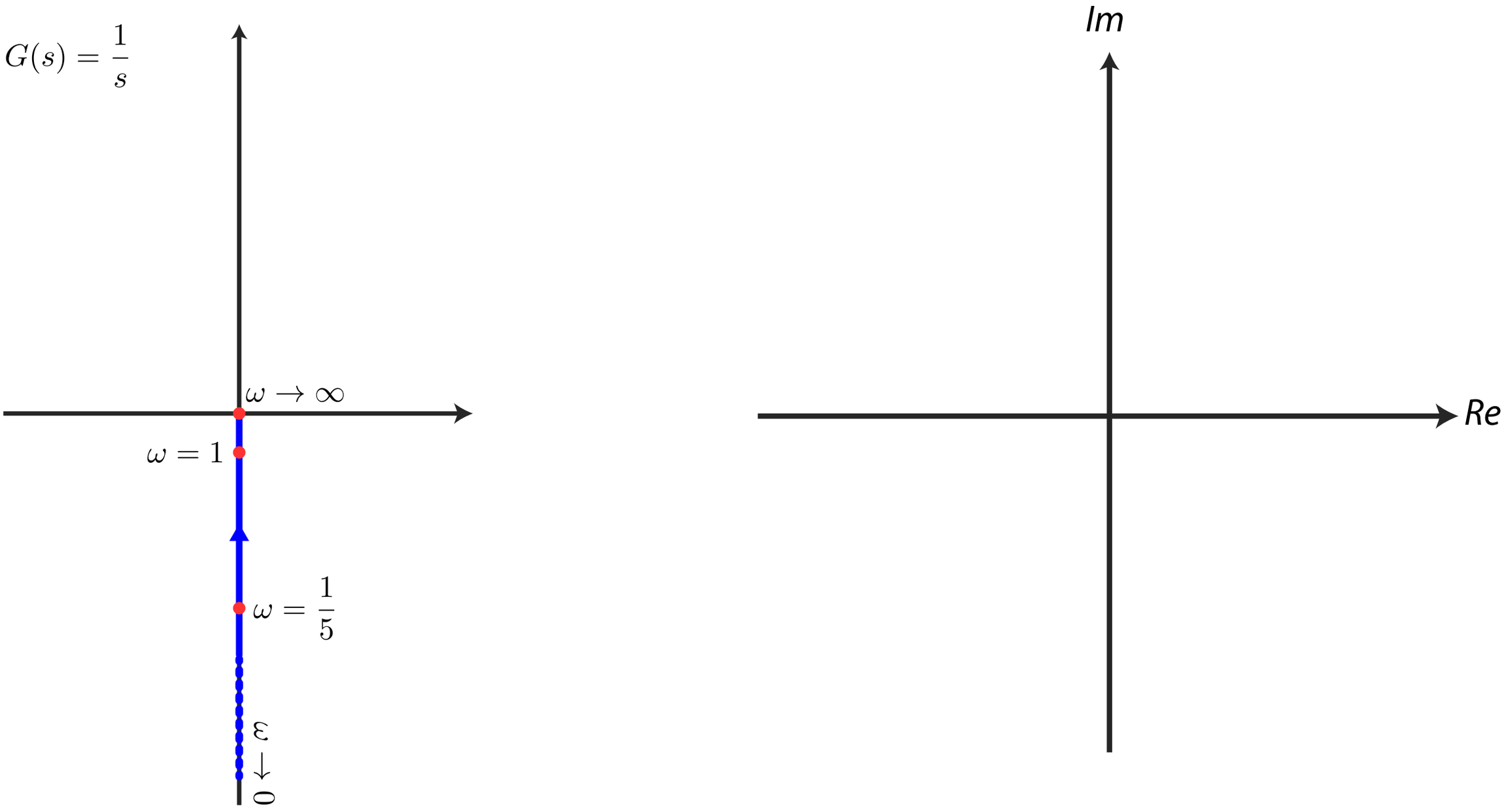
$$G_1(j\omega) = \omega\, j \quad , \quad G_2(j\omega) = \frac{-1}{\omega}\, j$$



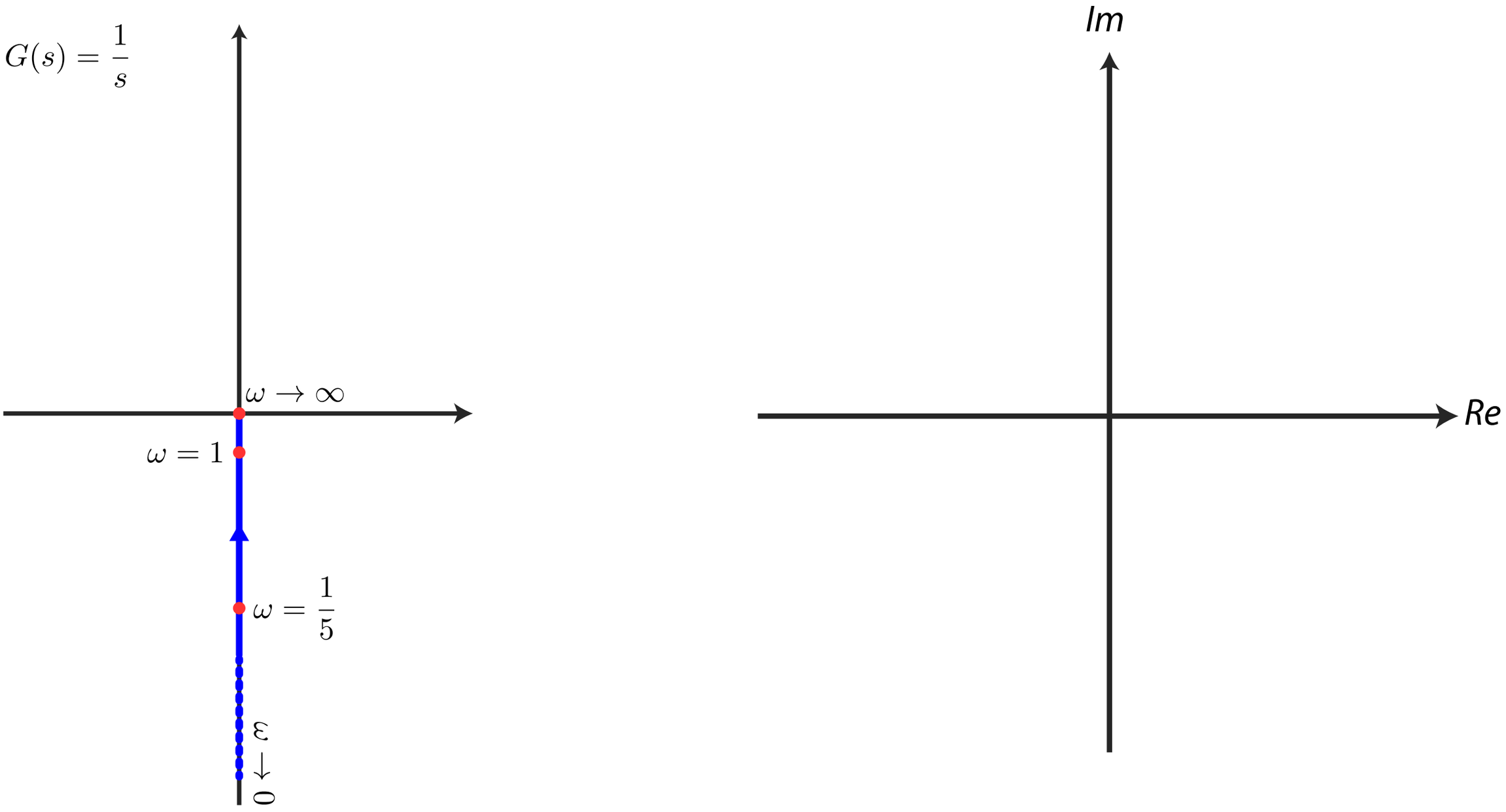
$$G_1(s) = s \quad , \quad G_2(s) = \frac{1}{s}$$



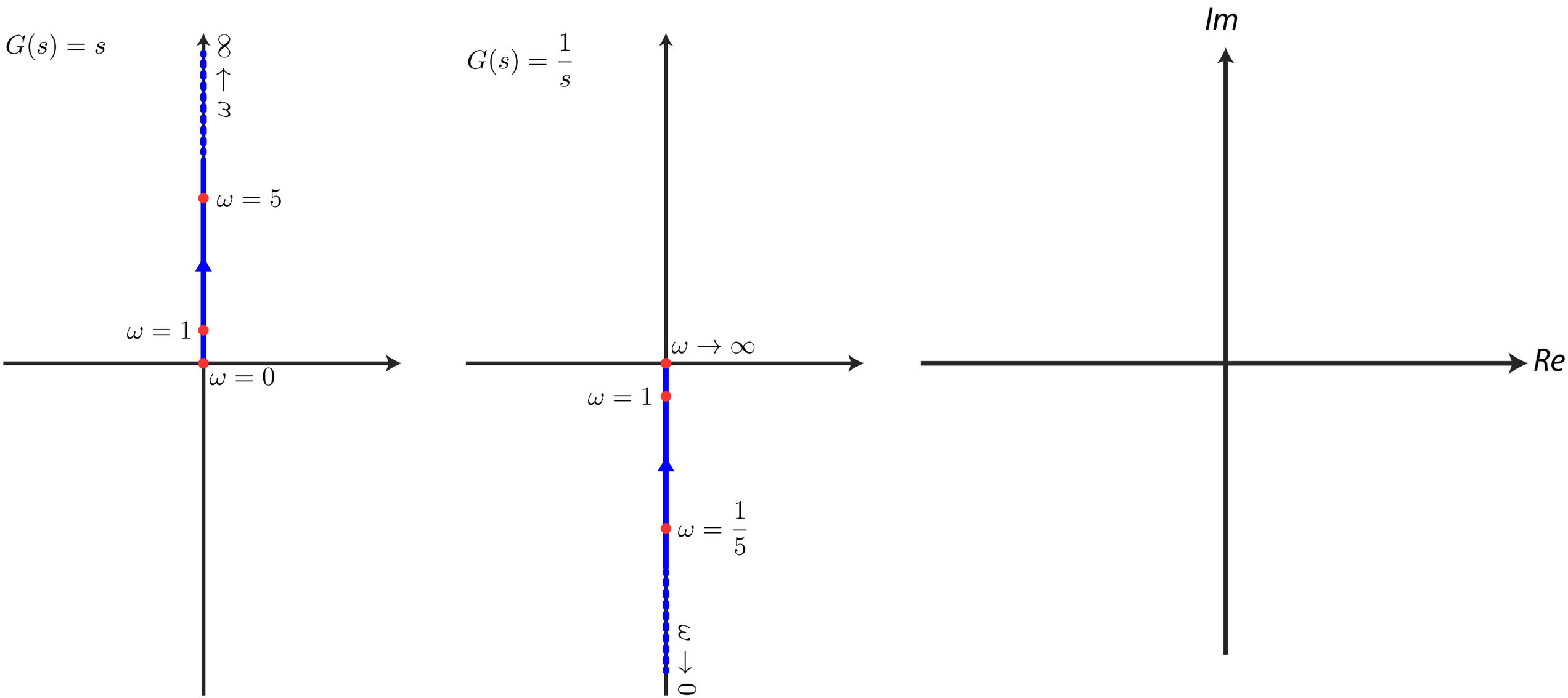
$$G_3(s) = 2 + \frac{1}{s}$$


$G(s) = \frac{1}{s}$
$\omega \to \infty$
$\omega = 1$
$\omega = \frac{1}{5}$
$\varepsilon \downarrow 0$
Im
Re

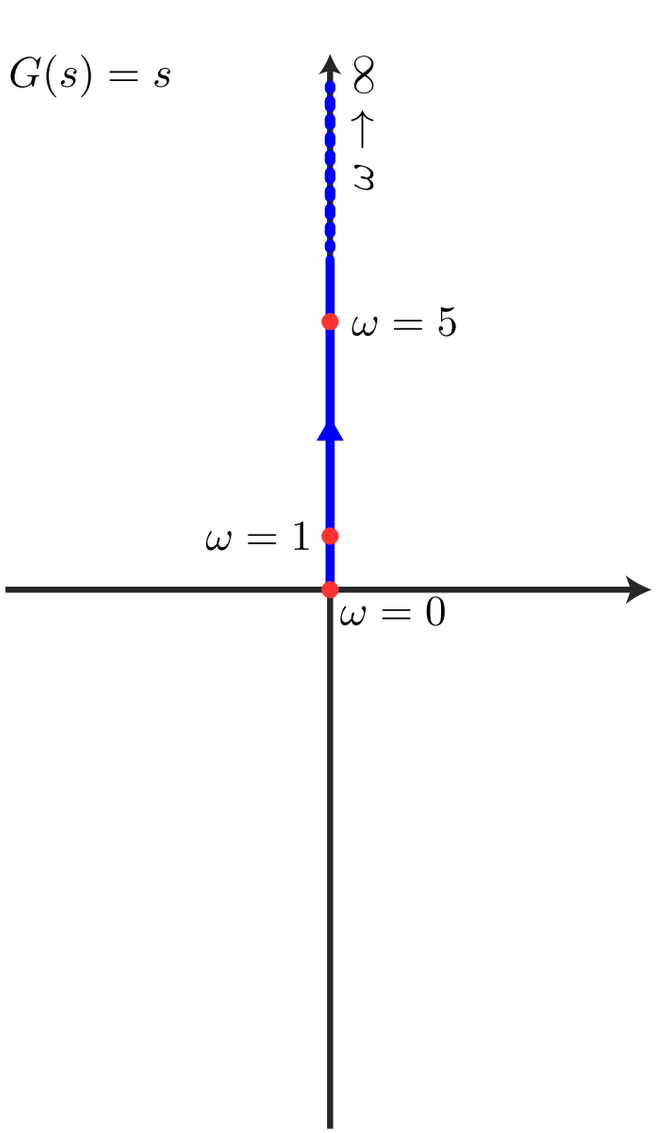
$$G_3(j\omega) = 2 - \frac{1}{\omega} j$$

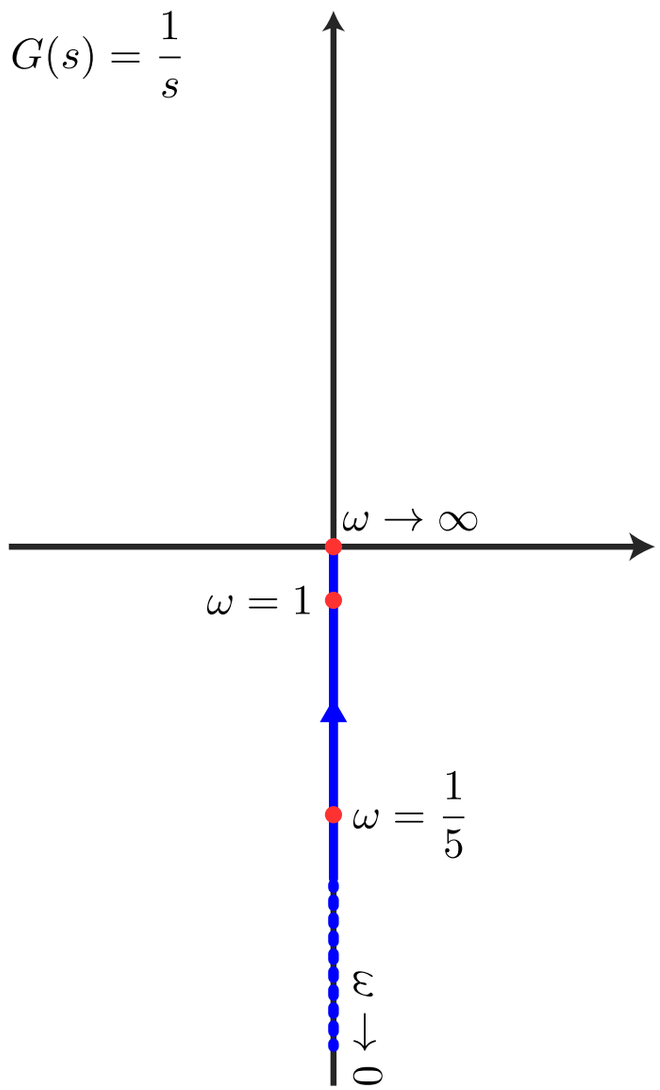


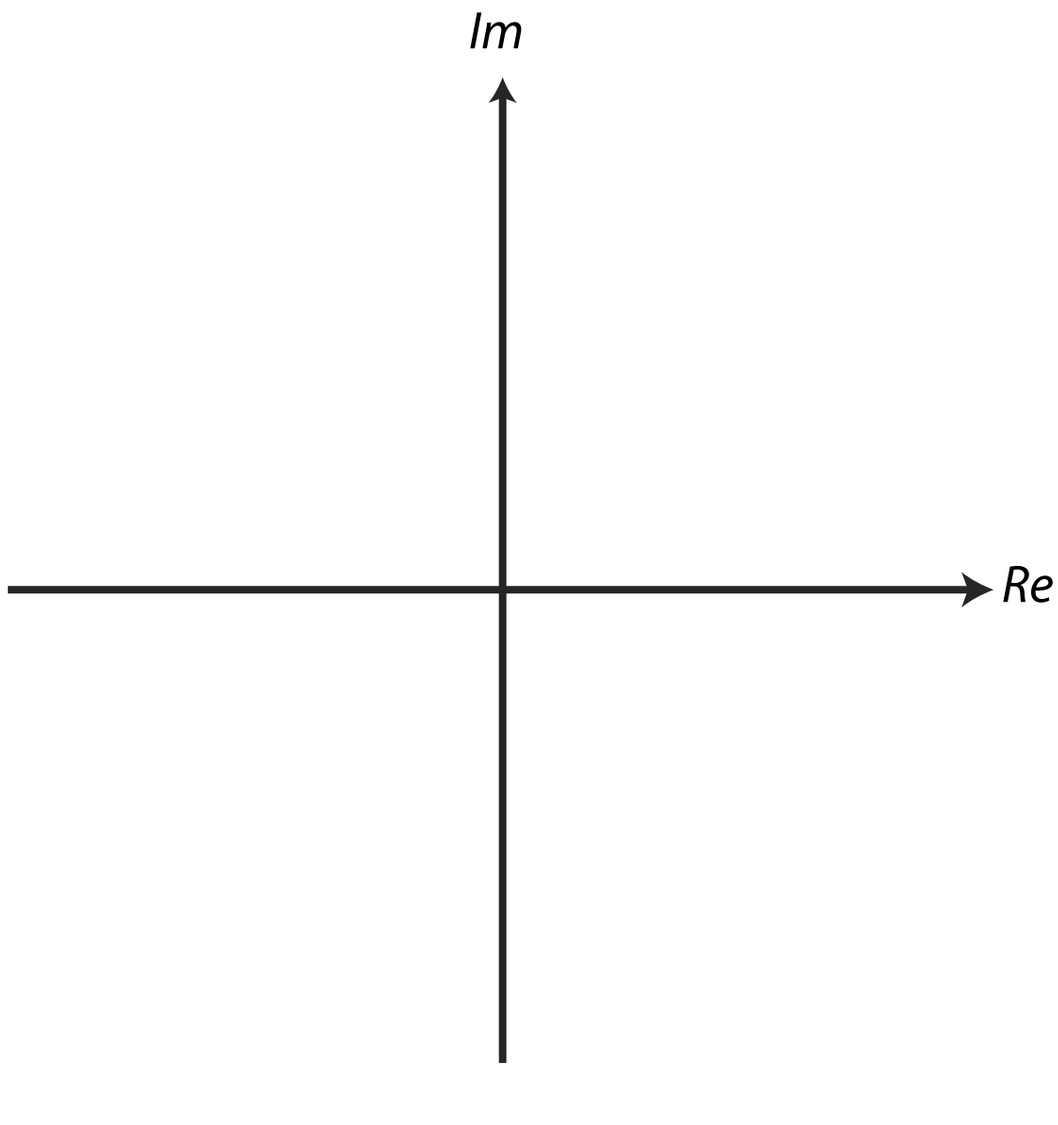
$$G_4(s) = s + \frac{1}{s} + 2$$



$$G_4(j\omega) = \left[\frac{\omega^2 - 1}{\omega}\right] j + 2$$







$G(s) = s$

$\omega = 5$

$\omega = 1$

$\omega = 0$

$G(s) = \frac{1}{s}$

$\omega \to \infty$

$\omega = 1$

$\omega = \frac{1}{5}$

$\varepsilon \downarrow 0$

$G(s) = 2 + \frac{1}{s}$

$\omega \to \infty$

$\omega = 1$

$\omega = \frac{1}{5}$

$\varepsilon \downarrow 0$

$G(s) = 2 + s + \frac{1}{s}$

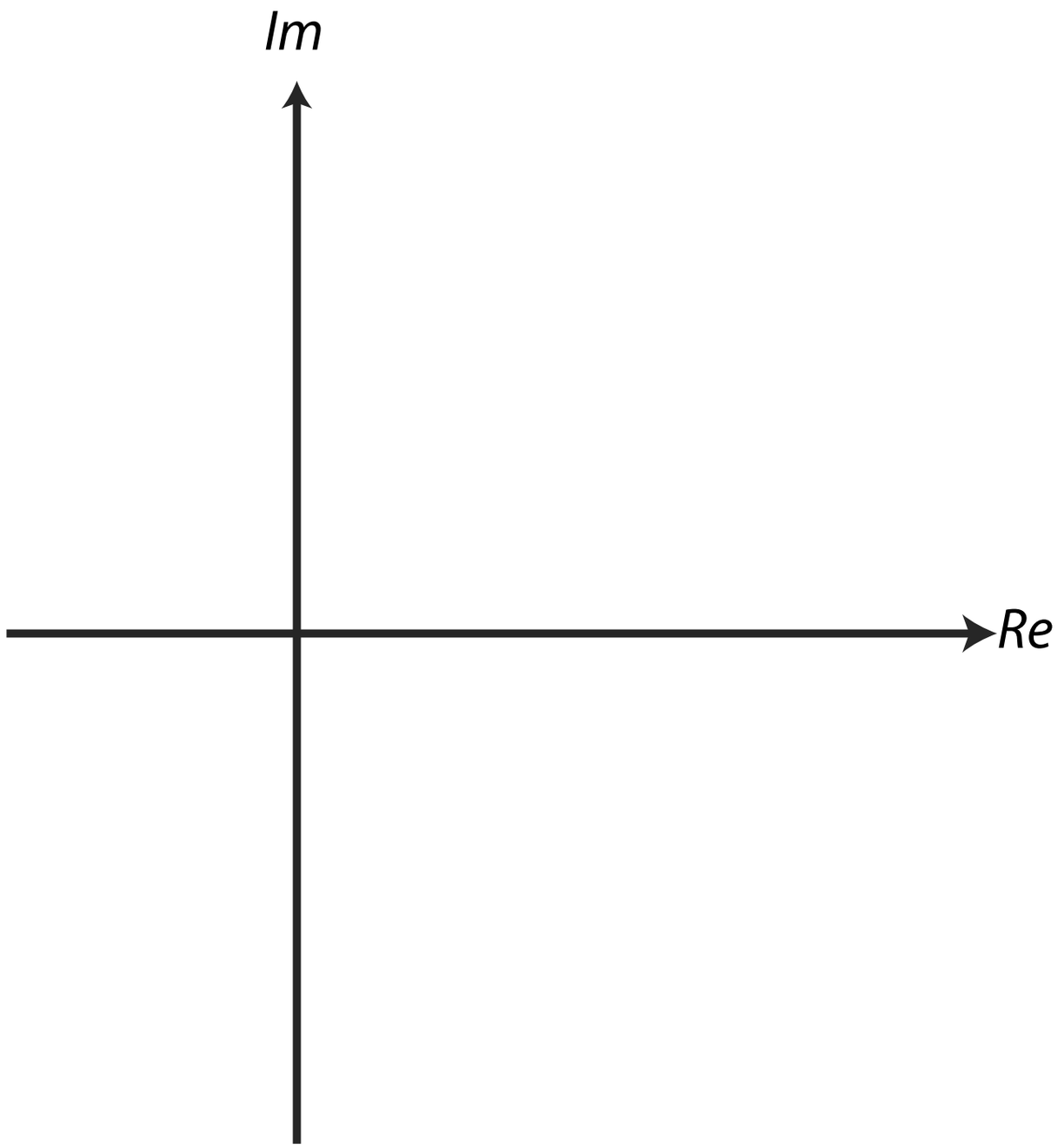
$\omega = 5$

$\omega = 1$

$\omega = \frac{1}{5}$

$\varepsilon \downarrow 0$

$$G_1(s) = \frac{1}{s+1}$$

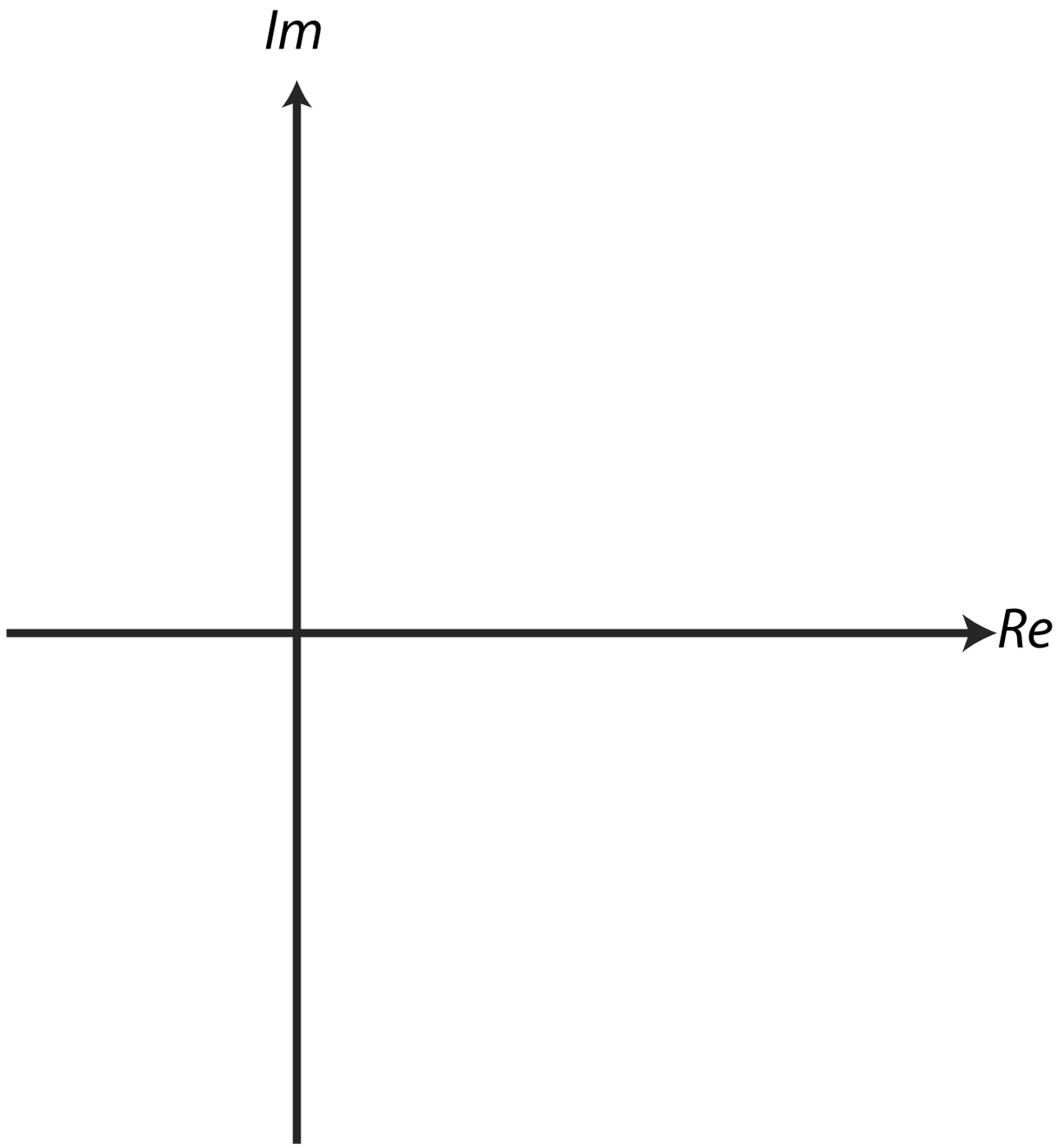


$$G_1(s) = \frac{1}{s+1}$$

$$G_1(j\omega) = \frac{1}{j\omega+1} = \frac{1-j\omega}{\omega^2+1} = \frac{1}{\omega^2+1} - \frac{\omega}{\omega^2+1}j$$

$$|G_1(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

$$\angle[G_1(j\omega)] = \arctan(-\omega)$$

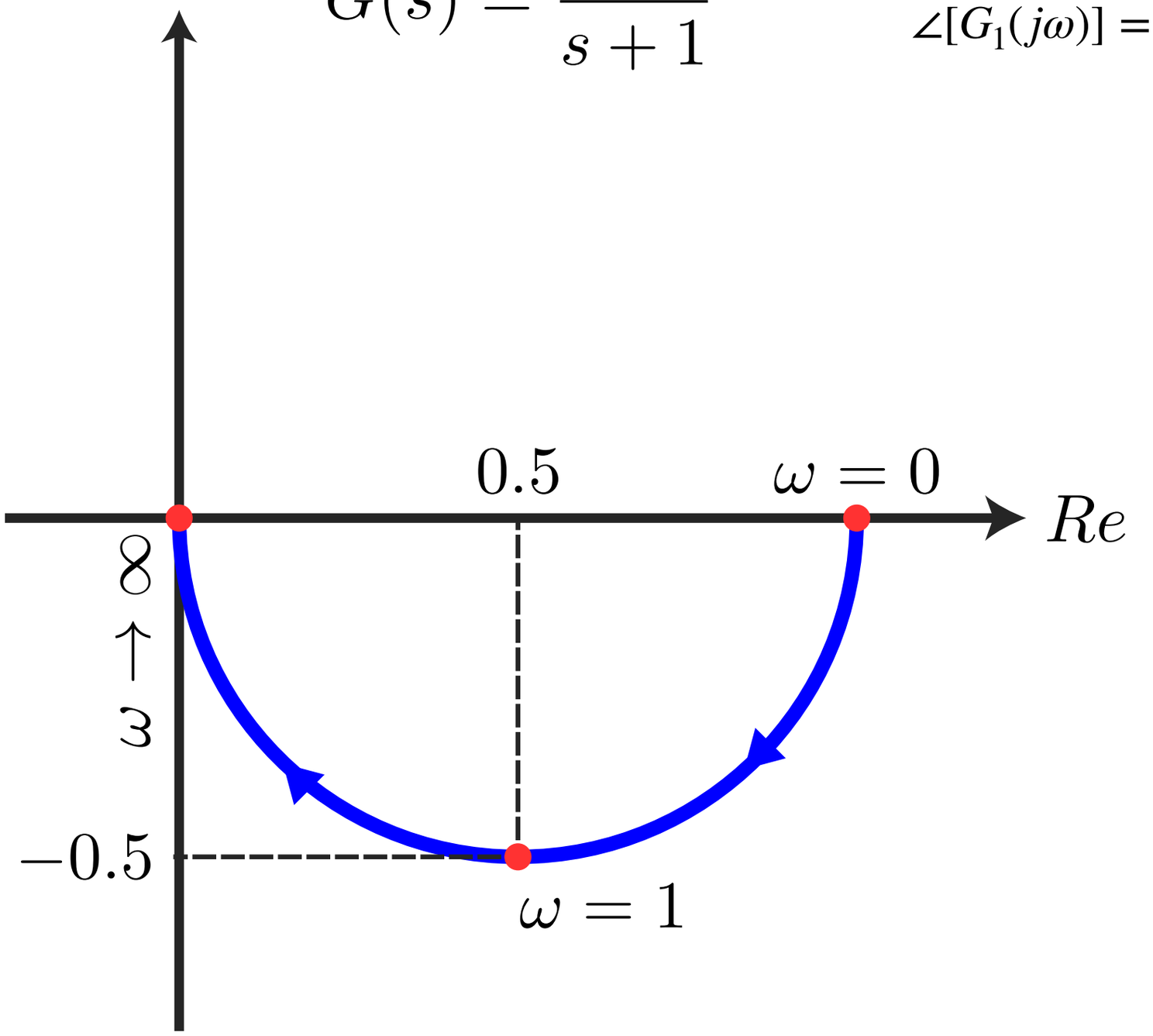


$$G_1(j\omega) = \frac{1}{j\omega + 1} = \frac{1 - j\omega}{\omega^2 + 1} = \frac{1}{\omega^2 + 1} - \frac{\omega}{\omega^2 + 1} j$$

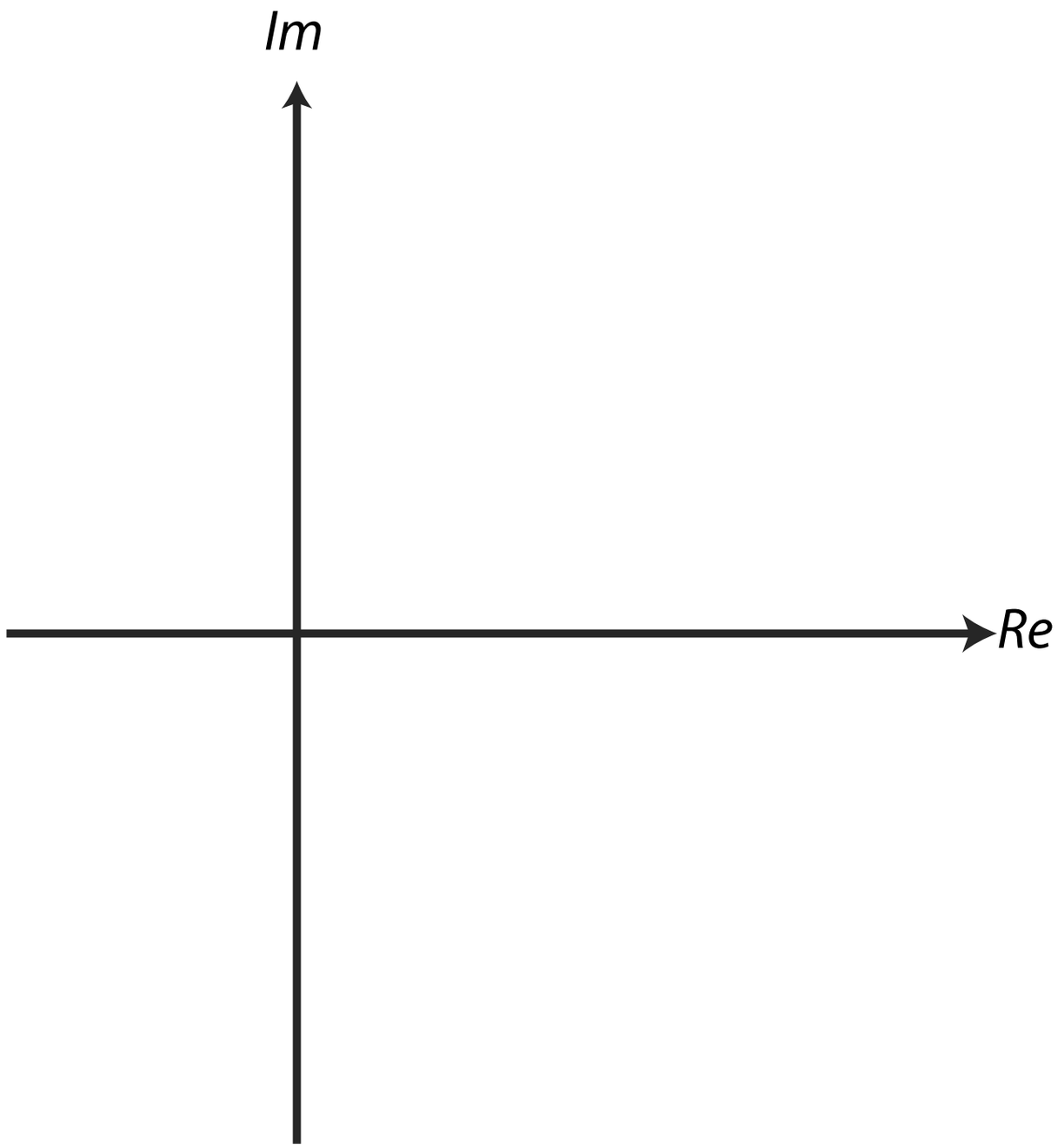
$$|G_1(j\omega)| = \frac{1}{\sqrt{1 + \omega^2}}$$

$$\angle[G_1(j\omega)] = \arctan(-\omega)$$

$$G(s) = \frac{1}{s + 1}$$



$$G_2(s) = \frac{s}{s+1}$$

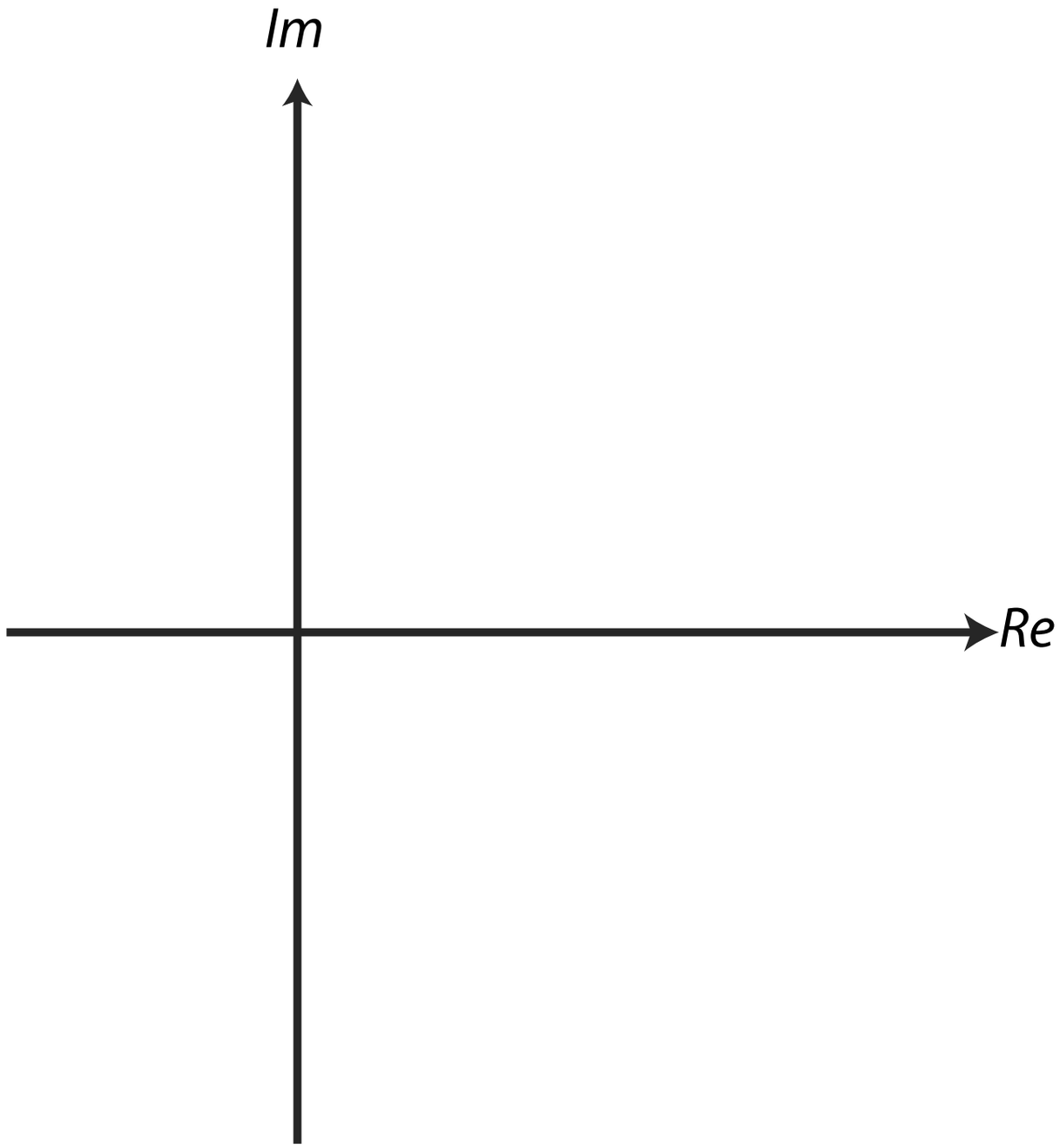


$$G_2(s) = \frac{s}{s+1}$$

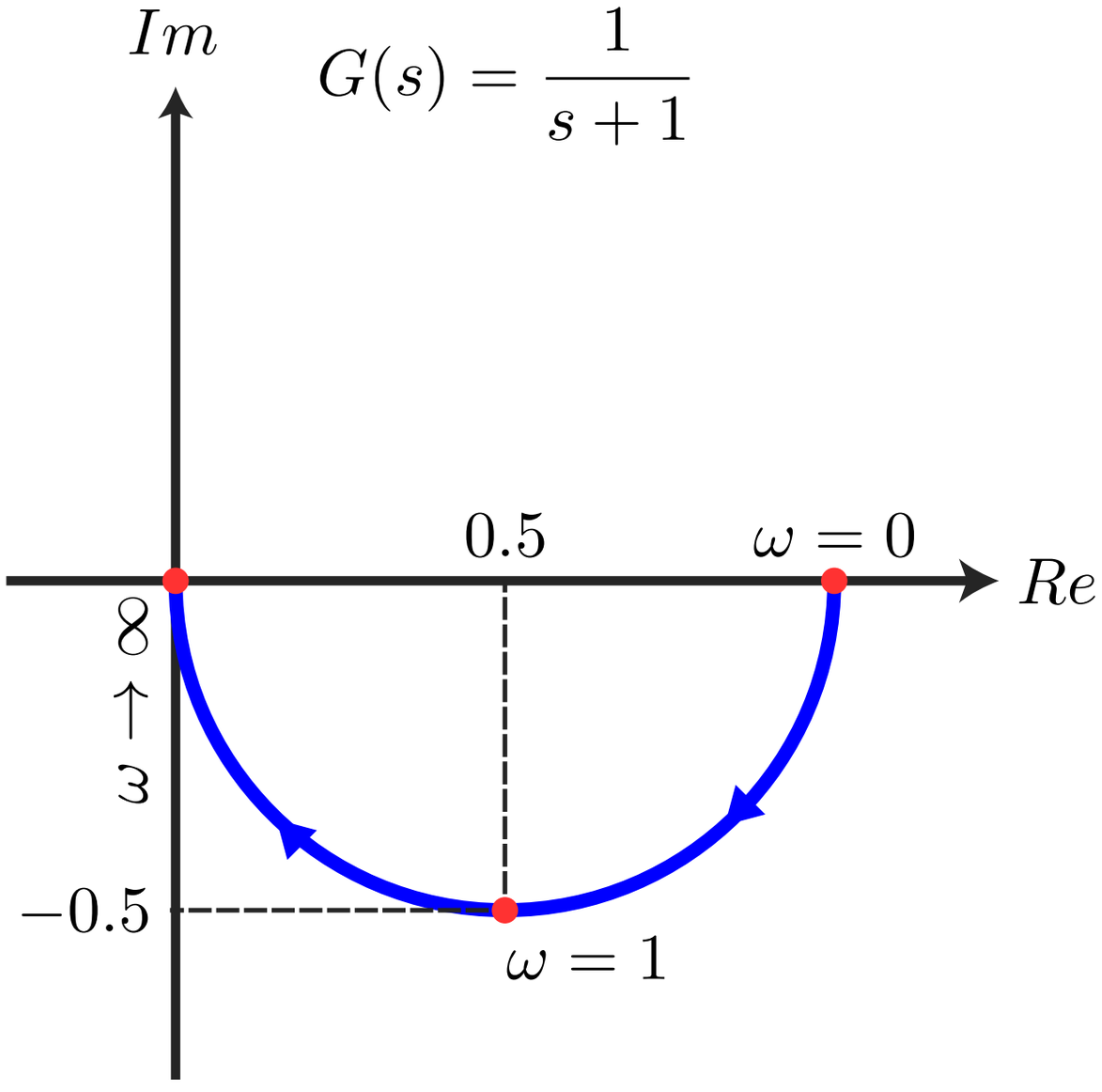
$$G_2(j\omega) = \frac{j\omega}{j\omega+1} = \frac{j\omega+\omega^2}{\omega^2+1} = \frac{\omega^2}{\omega^2+1} + \frac{\omega}{\omega^2+1}j$$

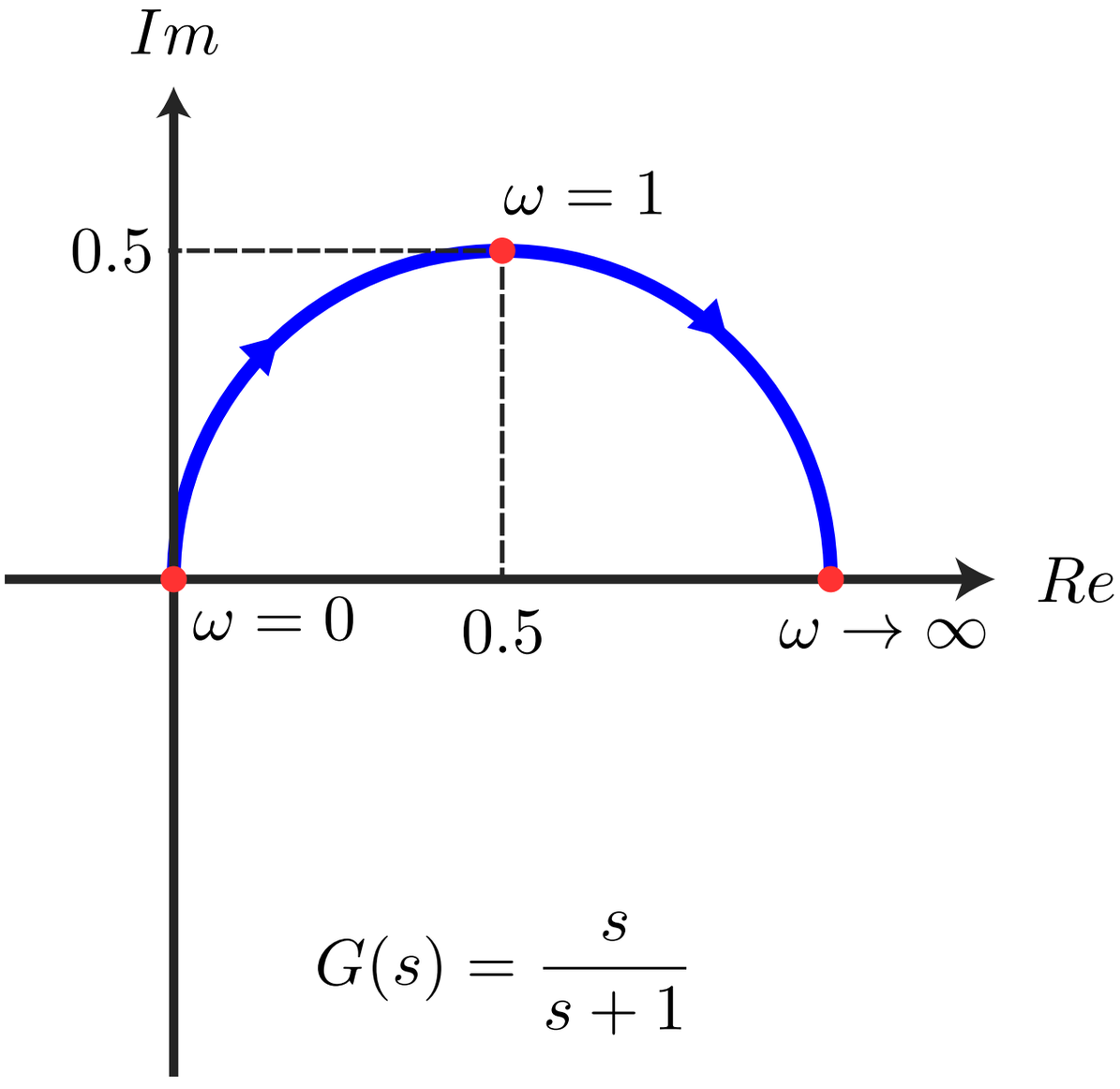
$$|G_2(j\omega)| = \sqrt{\frac{\omega^2}{1+\omega^2}}$$

$$\angle[G_2(j\omega)] = \arctan(1/\omega)$$



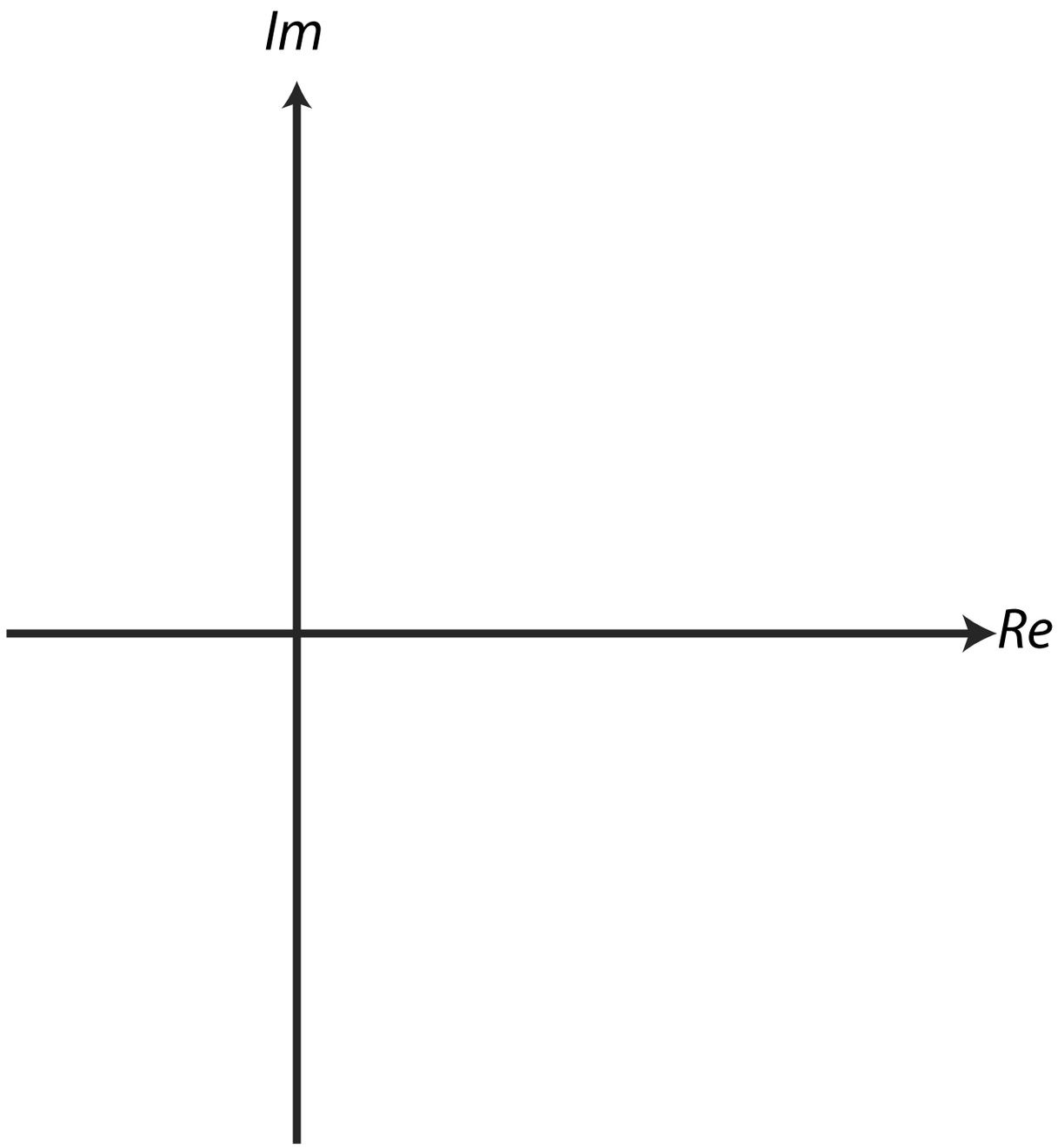
$$G(s) = \frac{1}{s+1}$$





$$G(s) = \frac{s}{s+1}$$

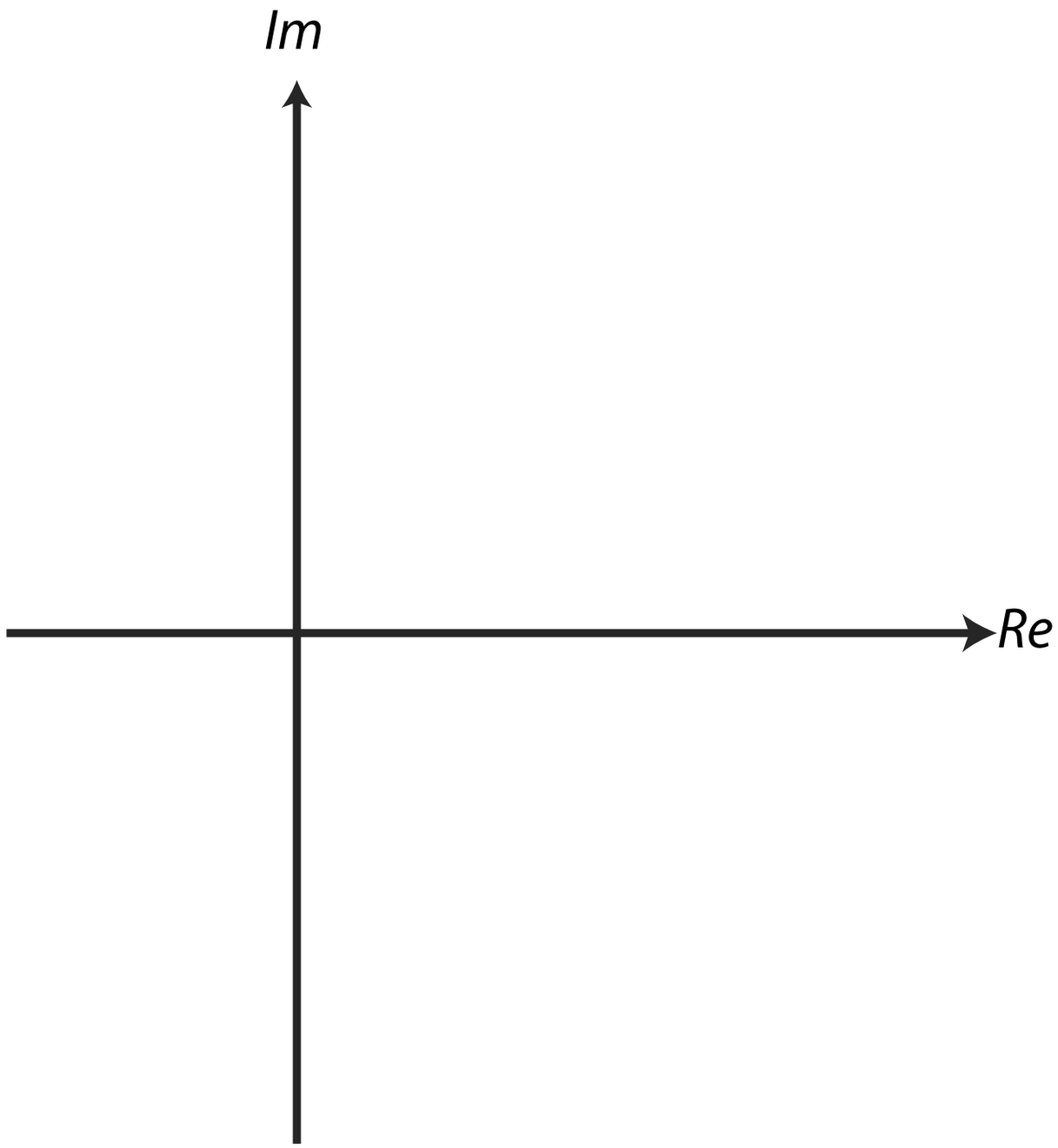
$$G(s) = \frac{1}{(s+1)^2}$$



$$G(s) = \frac{1}{(s+1)^2}$$

$$G(j\omega) = \frac{1}{(j\omega+1)^2} = \frac{(-j\omega+1)^2}{(\omega^2+1)^2}$$

$$= \left[\left(1-\omega^2\right) + j(-2\omega)\right] \frac{1}{(\omega^2+1)^2}$$



$$G(s) = \frac{1}{(s+1)^2}$$

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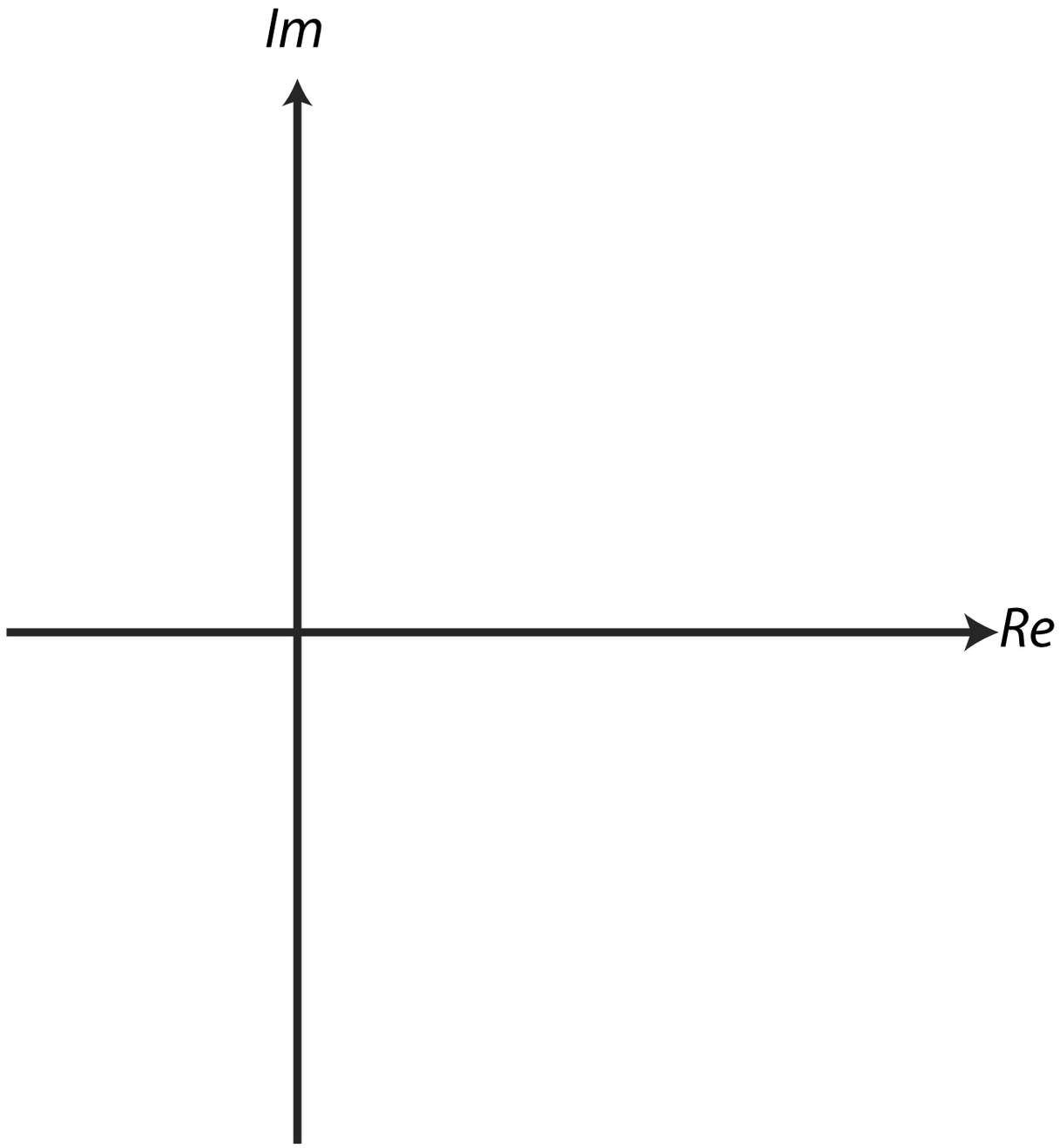
$$= \left[\left(1-\omega^2\right) + j(-2\omega)\right] \frac{1}{(\omega^2+1)^2}$$

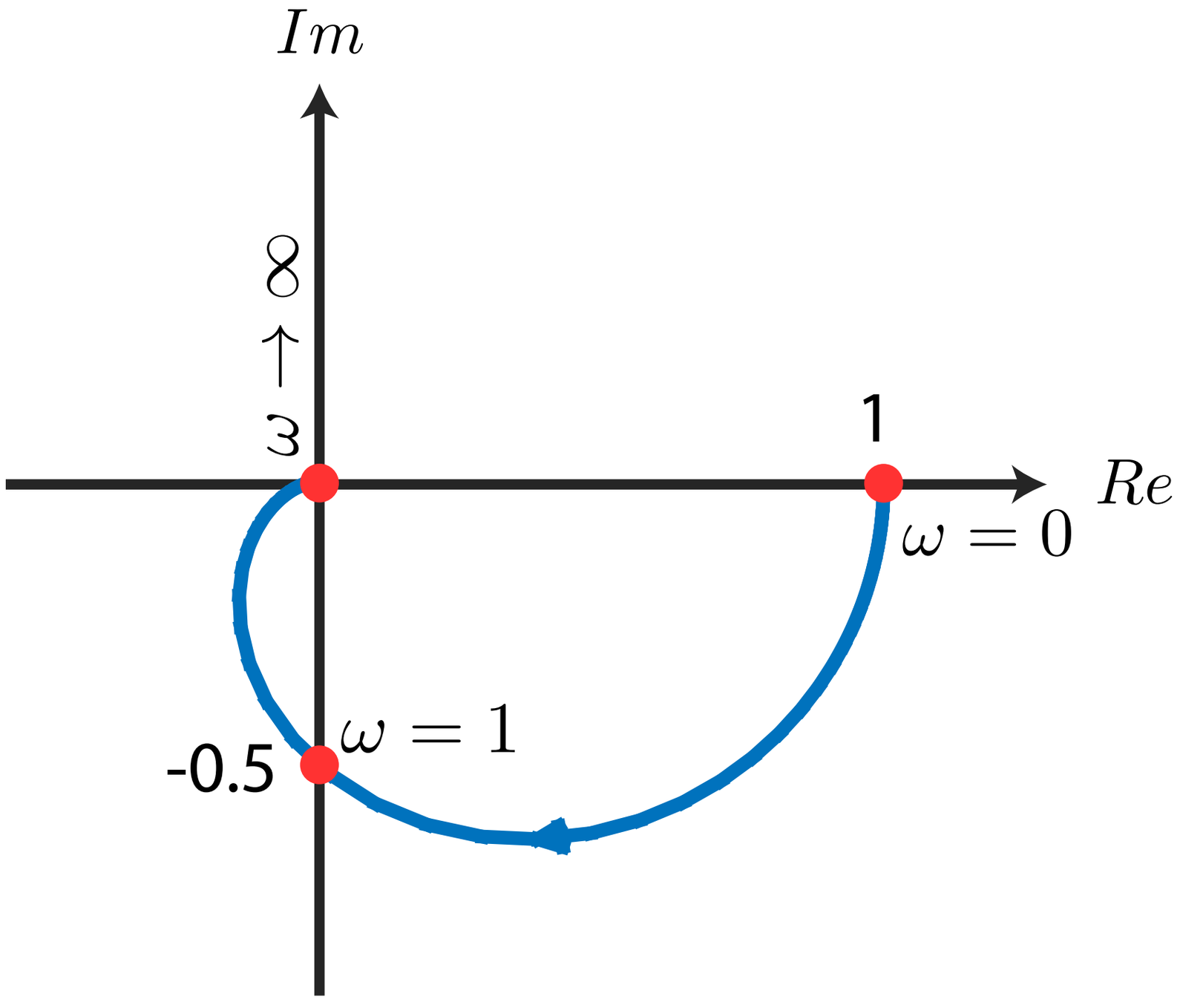


$$\omega \to 0 \;\Rightarrow G(j\omega) = 1$$

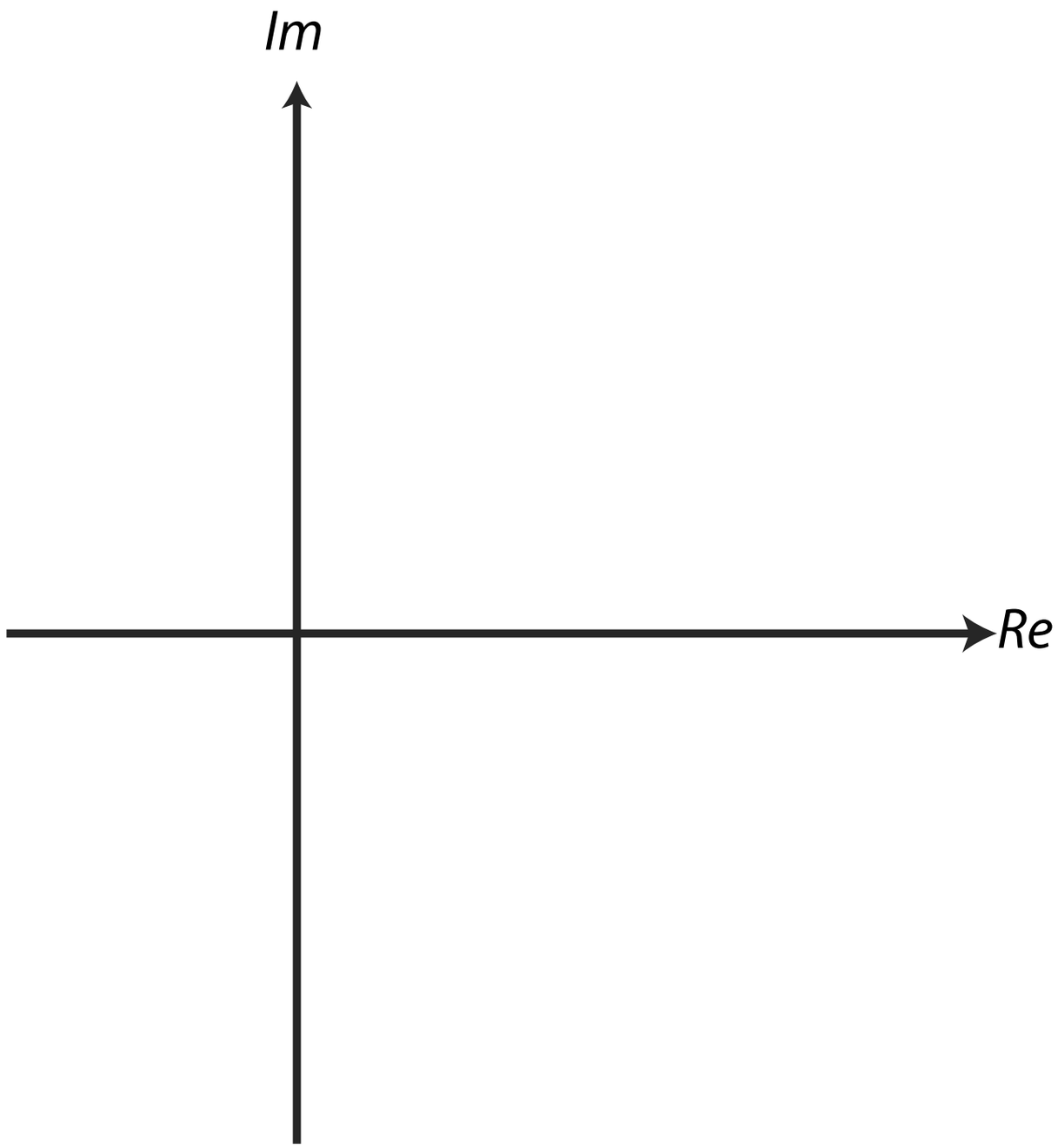
$$\omega \to 1 \;\Rightarrow G(j\omega) = -0.5j$$

$$\omega \to \infty \;\Rightarrow |G(j\omega)| \to 0 \quad \& \quad \angle[G(j\omega)] \to -\pi$$

$$G(s) = \frac{1}{(s+1)^2}$$



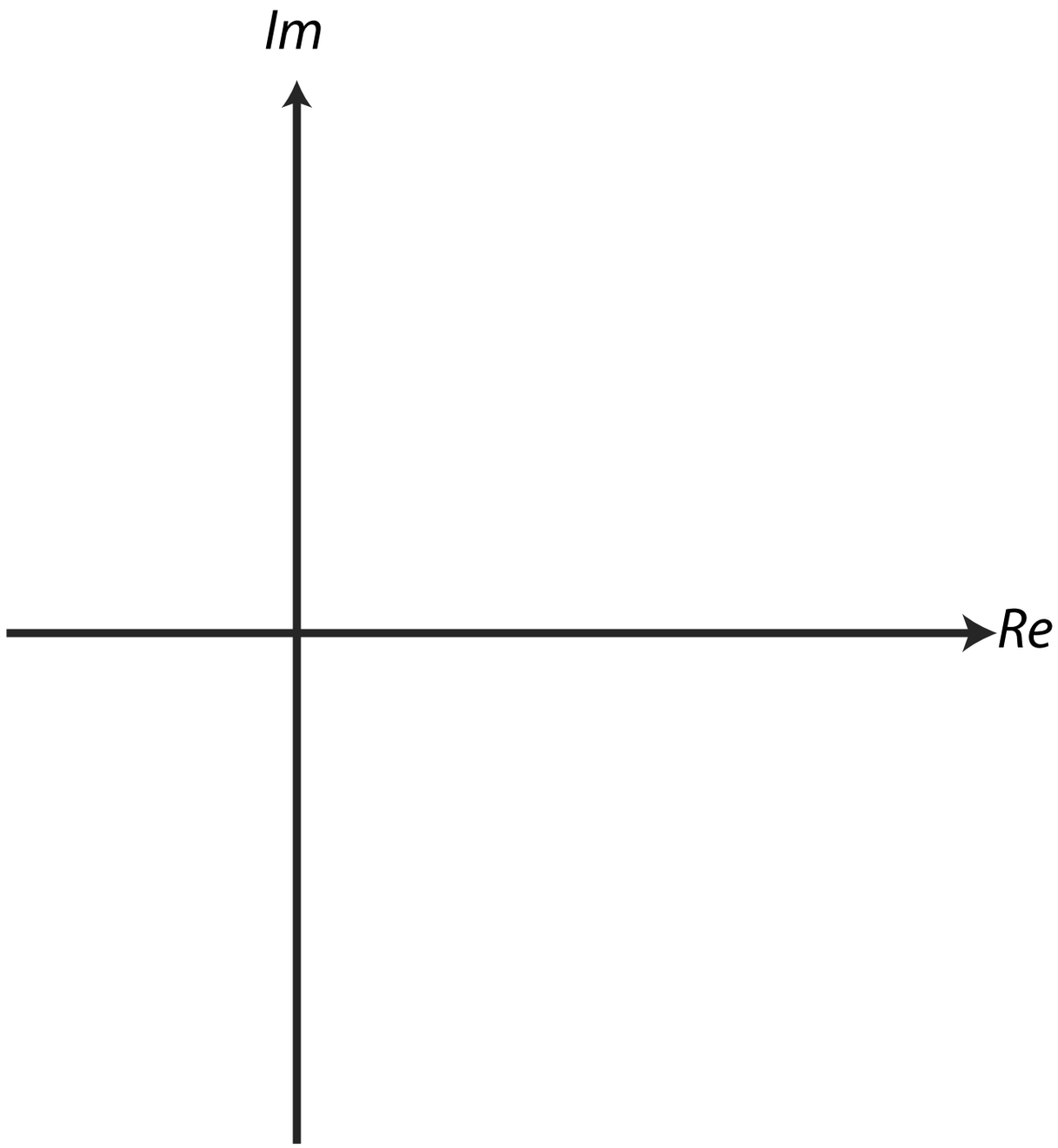
$$G(s) = \frac{1}{(s+1)^3}$$



$$G(s) = \frac{1}{(s+1)^3}$$

$$G(j\omega) = \frac{1}{(j\omega+1)^3} = \frac{(-j\omega+1)^3}{(\omega^2+1)^3}$$

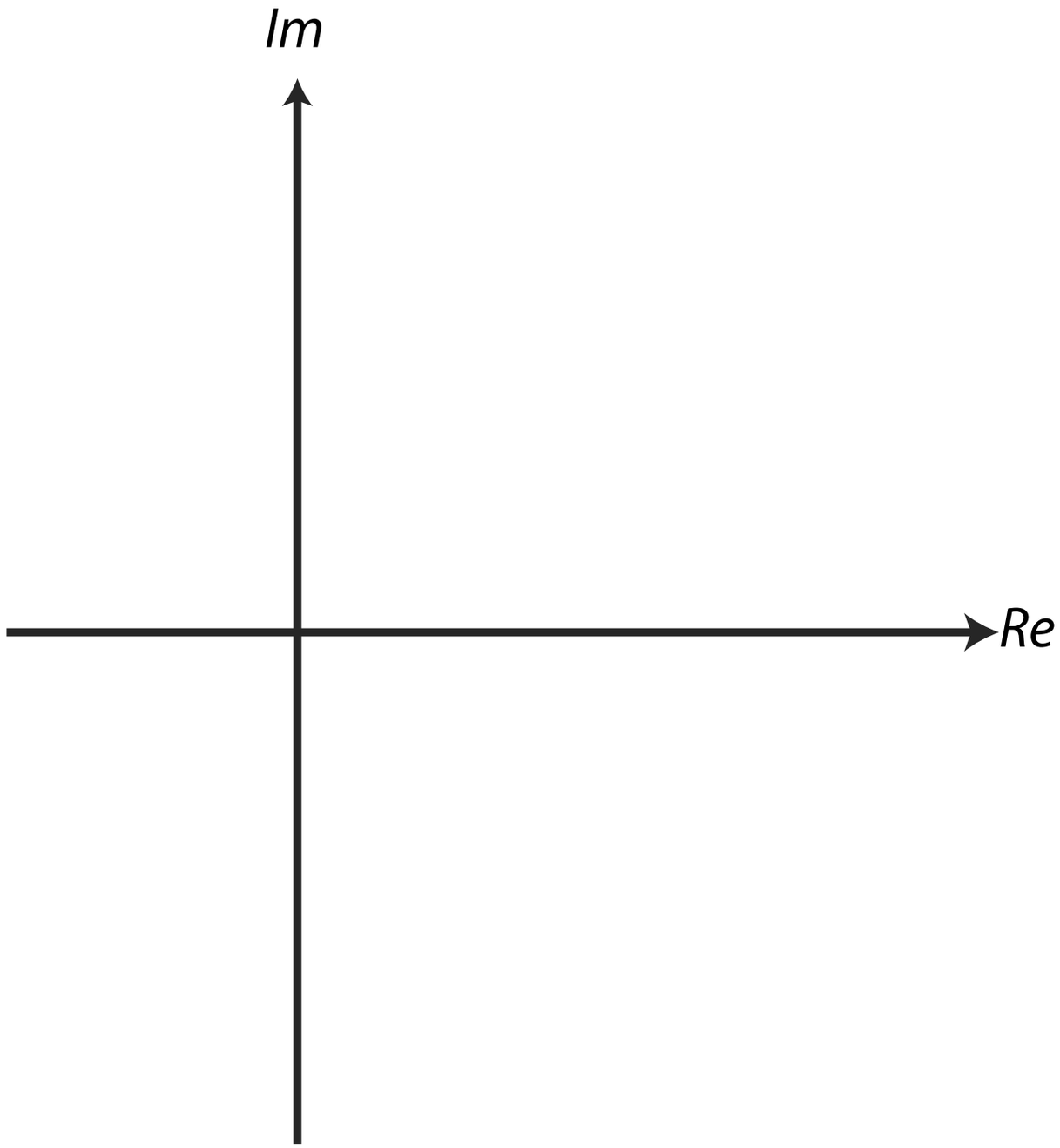
$$= \left[\left(1-3\omega^2\right) + j(\omega^3 - 3\omega)\right] \frac{1}{(\omega^2+1)^3}$$



$$G(s) = \frac{1}{(s+1)^3}$$

$$G(j\omega) = \frac{1}{(j\omega+1)^3} = \frac{(-j\omega+1)^3}{(\omega^2+1)^3}$$

$$= \left[\left(1-3\omega^2\right) + j(\omega^3 - 3\omega)\right] \frac{1}{(\omega^2+1)^3}$$



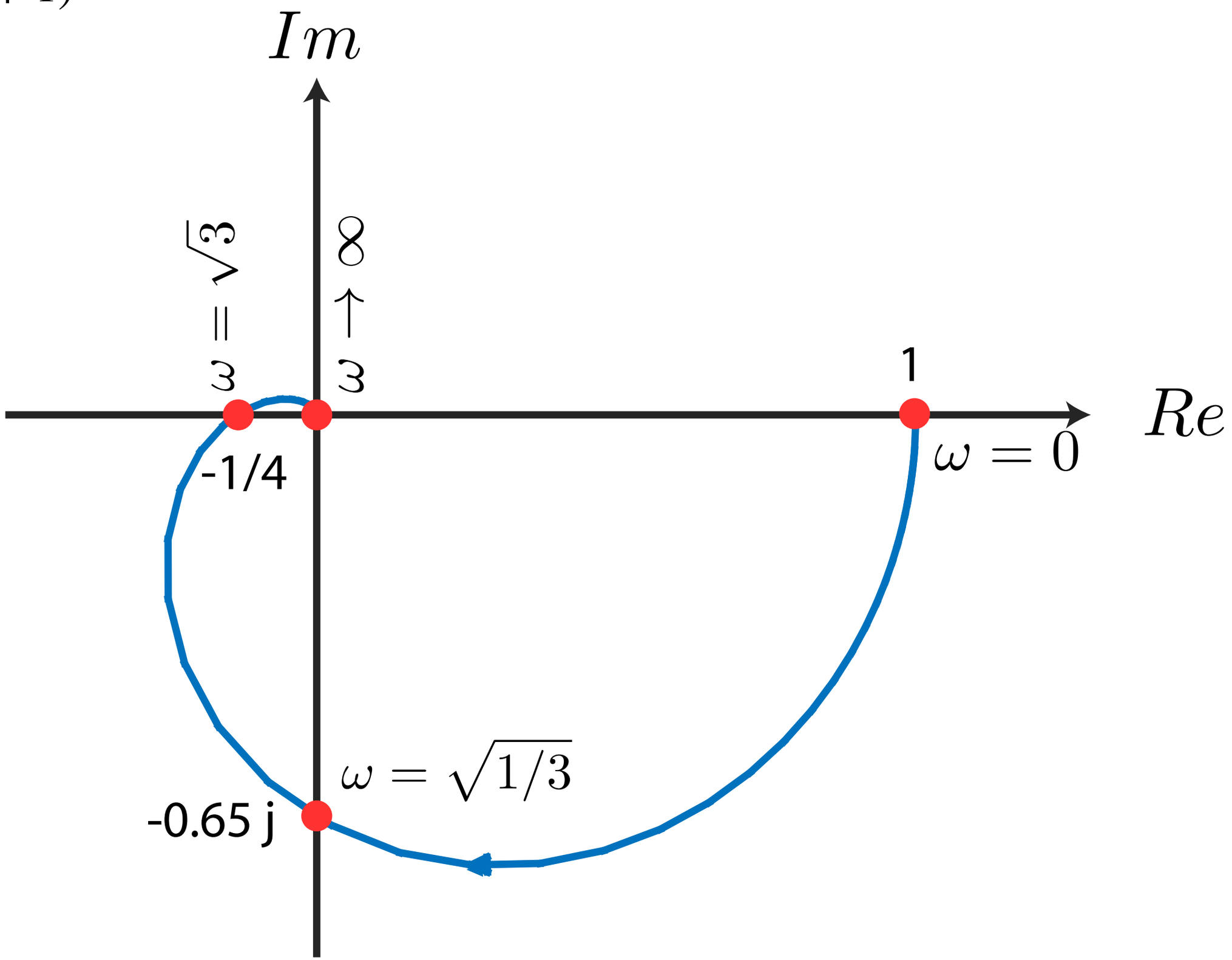
$$\omega \to 0 \;\Rightarrow G(j\omega) = 1$$

$$\omega \to \sqrt{1/3} \;\Rightarrow G(j\omega) = -0.65j$$

$$\omega \to \sqrt{3} \;\Rightarrow G(j\omega) = -1/4$$

$$\omega \to \infty \;\Rightarrow |G(j\omega)| \to 0 \quad \& \quad \angle[G(j\omega)] \to \pi/2$$

$$G(s) = \frac{1}{(s+1)^3}$$


Im
Re
$-\infty \rightarrow -3$
$-3 = \sqrt{3}$
1
$\omega = 0$
-1/4
$\omega = \sqrt{1/3}$
-0.65 j