EE502 - Linear Systems Theory II **Spring 2023**

Lecture 12

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12.1 The Kalman Decomposition

In reachability and observability lectures, we derived two types of standards forms, specifically for unreachable systems and unobservable systems (separately). Now our goal is to propose a general standard form for a unreachable and unobservable system, based on the Kalman decomposition. The process is exactly same for bot DT and CT systems, thus we will present the decomposition for only CT systems. Let

$$\dot{x} = Ax + Bu \ , \ y \qquad = Cx + Du \ \& \ x \in \mathbb{R}^n$$

Let's assume that system is neither reachable, nor observable and

$$\text{rank}[\mathbf{R}] = r < n \ , \ \text{range}[\mathbf{R}] = \mathcal{R}$$
$$\dim[\mathcal{N}(\mathbf{O})] = \bar{o} > 0 \ , \ \bar{\mathcal{O}} = \mathcal{N}(\mathbf{O})$$

Let's consider the following similarity transformation

$$\hat{A} = T^{-1}AT \ , \ \hat{B} = T^{-1}B \ , \ \hat{C} = CT \ \& \ D = D$$

Let

$$T = \left[\begin{array}{c|c|c|c} T_{r\bar{o}} & T_{ro} & T_{\bar{r}\bar{o}} & T_{\bar{r}o} \end{array} \right]$$

Let's define sub-matrices as follows:

1. Let $\mathcal{R}\bar{\mathcal{O}} = \mathcal{R} \cap \bar{\mathcal{O}}$, i.e. $x \in \mathcal{R}\bar{\mathcal{O}} \Rightarrow x \in \mathcal{R} \ \& \ x \in \bar{\mathcal{O}}$. Columns of $T_{r\bar{o}}$ forms a basis for $\mathcal{R}\bar{\mathcal{O}}$.