EE502 - Linear Systems Theory II

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Lecture 16

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16.1 Minimality of Interconnected Systems

In this section we shall examine the conditions under which minimality is lost when minimal subsystems are interconnected in various configurations,

16.2 Series - Cascade Connection

Consider the following system structure where two sub systems, with transfer functions $G_1(s)$ and $G_2(s)$ and associated minimal representations $\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}$ and $\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix}$, connected in series/cascade configuration.

$$\begin{array}{c|c}
u = u_1 \\
\hline
\begin{pmatrix} A_1 & B_1 \\
\hline
C_1 & D_1 \end{array}
\end{array}
\begin{array}{c|c}
y_1 = u_2 \\
\hline
\begin{pmatrix} A_2 & B_2 \\
\hline
C_2 & D_2 \end{array}
\end{array}$$

The transfer function of the connection is simply equal to $G(s) = G_2(s)G_1(s)$. Let x_1 and x_2 state-variables of the sub-systems, then natural choice of the state variable for the series connection is $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Under this definition the state-space representation for the whole system can be found as

$$A = \begin{bmatrix} A_1 & 0 \\ B_2C_1 & A_2 \end{bmatrix}, B = \begin{bmatrix} B_1 \\ B_2D_1 \end{bmatrix}, C = \begin{bmatrix} D_2C_1 & C_2 \end{bmatrix}, D = \begin{bmatrix} D_2D_2 \end{bmatrix}$$

Let's analyze the observability of the connection via PBH test.