EE502 - Linear Systems Theory II **Spring 2023**

Lecture 15

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15.1 Poles & Zeros of MIMO Systems

15.1.1 Poles & Zeros of SISO Systems

Let $G(s)$ (or $G(z)$ in DT case) and $\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$ are the transfer function and a *minimal* state-space representation of a SISO LTI system.

p_0 is a pole of the system if

- $\lim_{s \to p_0} G(s) = \infty$
- p_0 is an eigenvalue of A

whereas z_0 is a pole of the system if

- $\lim_{s \to z_0} G(s) = 0$
- steady-state part of the zero state response to $u(t) = e^{z_0 t}$

$$y_{ss}(t) = C(z_0 I - A)^{-1} B e^{z_0 t} = 0$$

15.1.2 Poles of MISO Systems

Unlike MIMO zeros, definition and derivation of MIMO poles is much more straightforward

Let $G(s)$ (or $G(z)$ in DT case) and $\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right)$ are the transfer function matrix and a *minimal* state-space representation of a MIMO LTI system.

p_0 is a pole of the system if

- $\exists (i,j)$ s.t. $\lim_{s \to p_0} G_{ij}(s) = \infty$
- $\lim_{s \to p_0} ||G(s)|| = \infty$
- p_0 is an eigenvalue of A

in other words in the context of transfer function matrix p_0 is a pole of the system if it is a pole of any entry of $G(s)$. Understanding and derivation of the multiplicities of a pole based on transfer function matrix is a little bit tricky and not very intuitive for the context and scope of the class. Thus, we can simply state that we can find the algebraic and geometric multiplicity of a pole based on Jordan decomposition of A provided that state-space representation is minimal.

Ex 15.1

$$G(s) = \begin{bmatrix} \frac{s+1}{(s+2)^2} & 0 & 0 \\ 0 & \frac{s}{(s+1)(s+2)} & 0 \end{bmatrix}$$

We can clearly see that $p_1 = -2$ and $p_2 = -2$ are the poles of the system. Note that $z = -1$ also a zero of the firs entry of the transfer function matrix (and indeed it is a zero of the system). This states that a MIMO system can have a pole and a zero at the same location.