EE302 - Feedback Systems **Spring 2019**

Lecture 6

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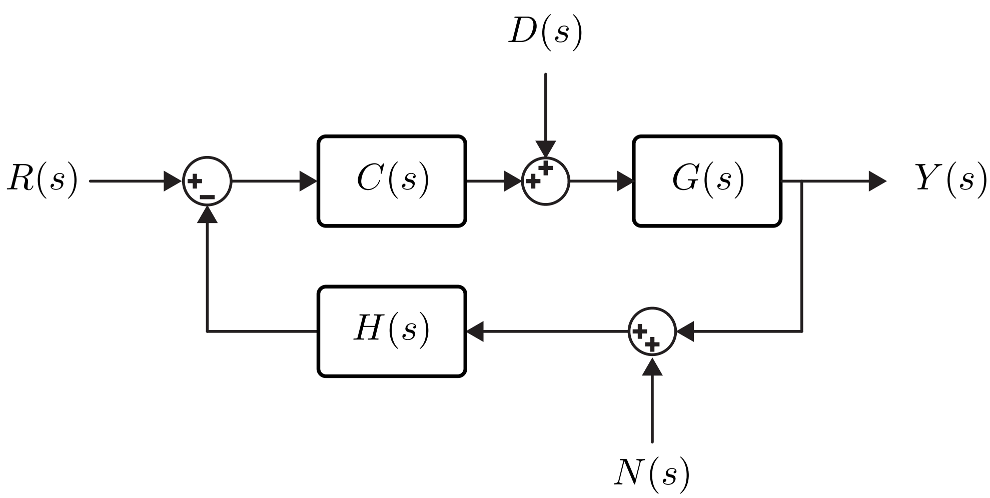
6.1 Transfer functions with multiple exogenous input

When modeling and analyzing a closed system in addition to the desired reference input signal, it is also important to model/analyze the system with unwanted disturbances and noise input signals. Let's consider the following block-diagram topology. In this closed-loop system, there exist three exogenous input signals; $r(t)$ (reference input), $d(t)$ ("disturbance" input), and $n(t)$ ("noise" input).



When modeling the response or characteristic of the system with respect to different external inputs, we assume that remaining ones are zero.

Response to $r(t)$

$$T_R(s) = \frac{Y(s)}{R(s)} = \frac{C(s)G(s)}{1 + C(s)G(s)H(s)}$$

Response to $d(t)$

$$T_D(s) = \frac{Y(s)}{D(s)} = \frac{G(s)}{1 + C(s)G(s)H(s)}$$

Response to $n(t)$

$$T_N(s) = \frac{Y(s)}{N(s)} = \frac{C(s)G(s)H(s)}{1 + C(s)G(s)H(s)}$$

If we generalize, we can write $Y(s)$ as

$$Y(s) = T_R(s)R(s) + T_D(s)D(s) + T_N(s)N(s)$$

Lets roughly analyze the desired responses under different type of inputs. Let's assume that $G(s)$ is the plant transfer function and $H(s)$ is the sensory dynamics transfer function. $C(s)$ is the transfer function of the controller.

In the ideal case, we want

- Perfect tracking of reference signal, $T_R^*(s) \approx 1$. Since it is not possible to perfectly achieve this under dynamic system constrains, we can design a "high gain" controller such that

  $$T_R(s) \approx \frac{C(s)G(s)}{C(s)G(s)H(s)} \approx \frac{1}{H(s)}$$

  If $H(s) \approx 1$, then we can have a high tracking performance from the system.

- Perfect rejection of disturbance signal, $T_R^*(s) \approx 0$. Similarly, we can design a "high gain" controller such that

  $$T_D(s) \approx \frac{G(s)}{C(s)G(s)H(s)} \approx 0$$

  It seems that the requirement on $C(s)$ is similar for good tracking and good disturbance rejection.

- Perfect rejection of noise signal, $T_N^*(s) \approx 0$. In this case, we can design a "low gain" controller (or low gain $H(s)$) such that

  $$T_N(s) \approx \frac{C(s)G(s)H(s)}{1} \approx 0$$

It seems that requirements on $C(s)$ and $H(s)$, when we consider both tracking performance, disturbance rejection, and noise rejection. This is the most well known limitation of feedback control systems. The basic idea is it is not possible to only design a controller $C(s)$ such that we reach good closed-loop performance when the system suffers from uncertainties and noises. Somehow we need to design systems such that $G(s)$, $H(s)$, and even $N(s)$ and $D(s)$ together with the designed controller achieves a good closed-loop behavior.