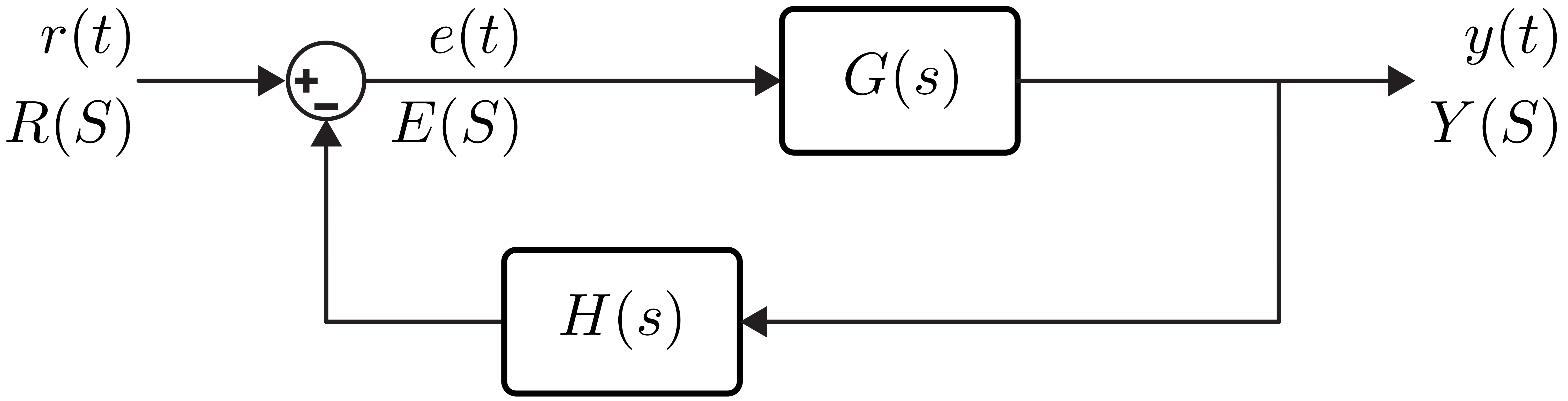
Stability Margins: Gain & Phase Margins

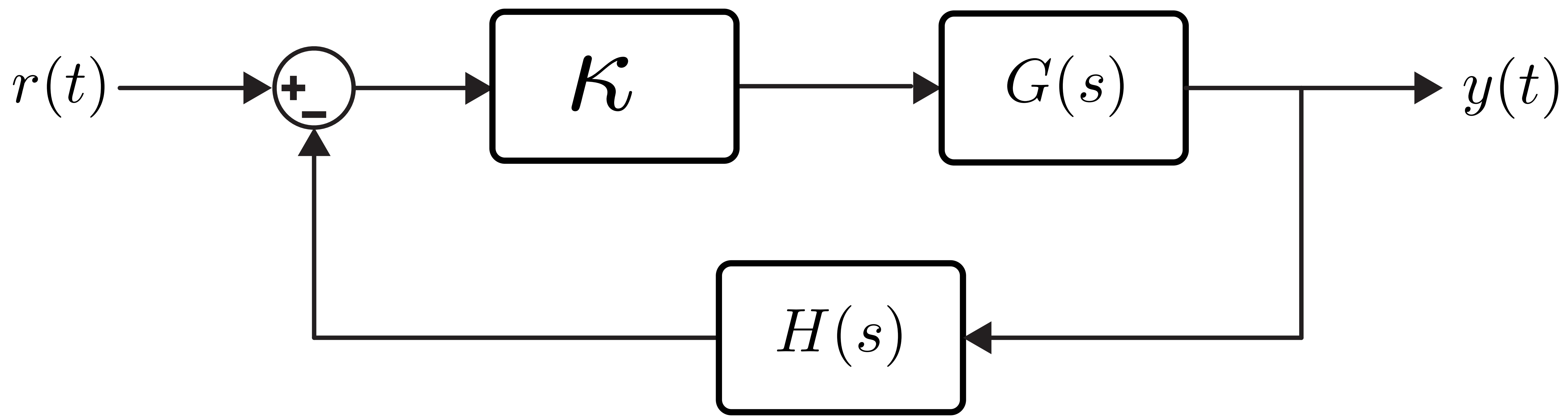
Assumptions

- Open-loop transfer function of the feedback system is a *minimum-phase* system, i.e.
  - No poles/zeros in the Open Right Half Plane
  - $\lim_{\omega\to\infty}[\frac{G_{OL}(s)}{s}]_{s=\jmath\omega} = 0$
- The feed-back system is Type $0-2$
- Phase plot of Bode diagrams of $G_{OL}(j\omega)$ crosses the -180^0 line at most once.



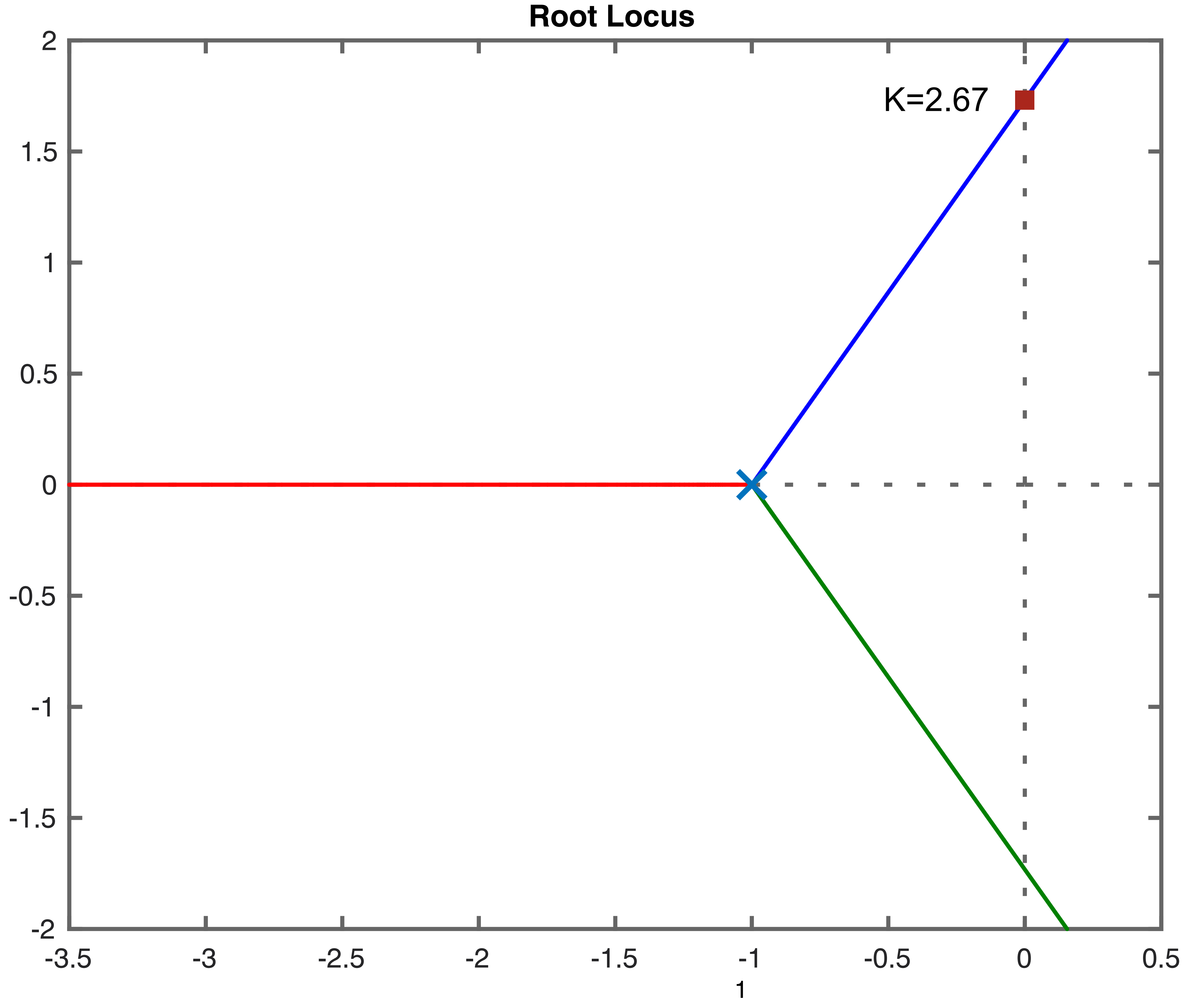
Gain Margin, g_m

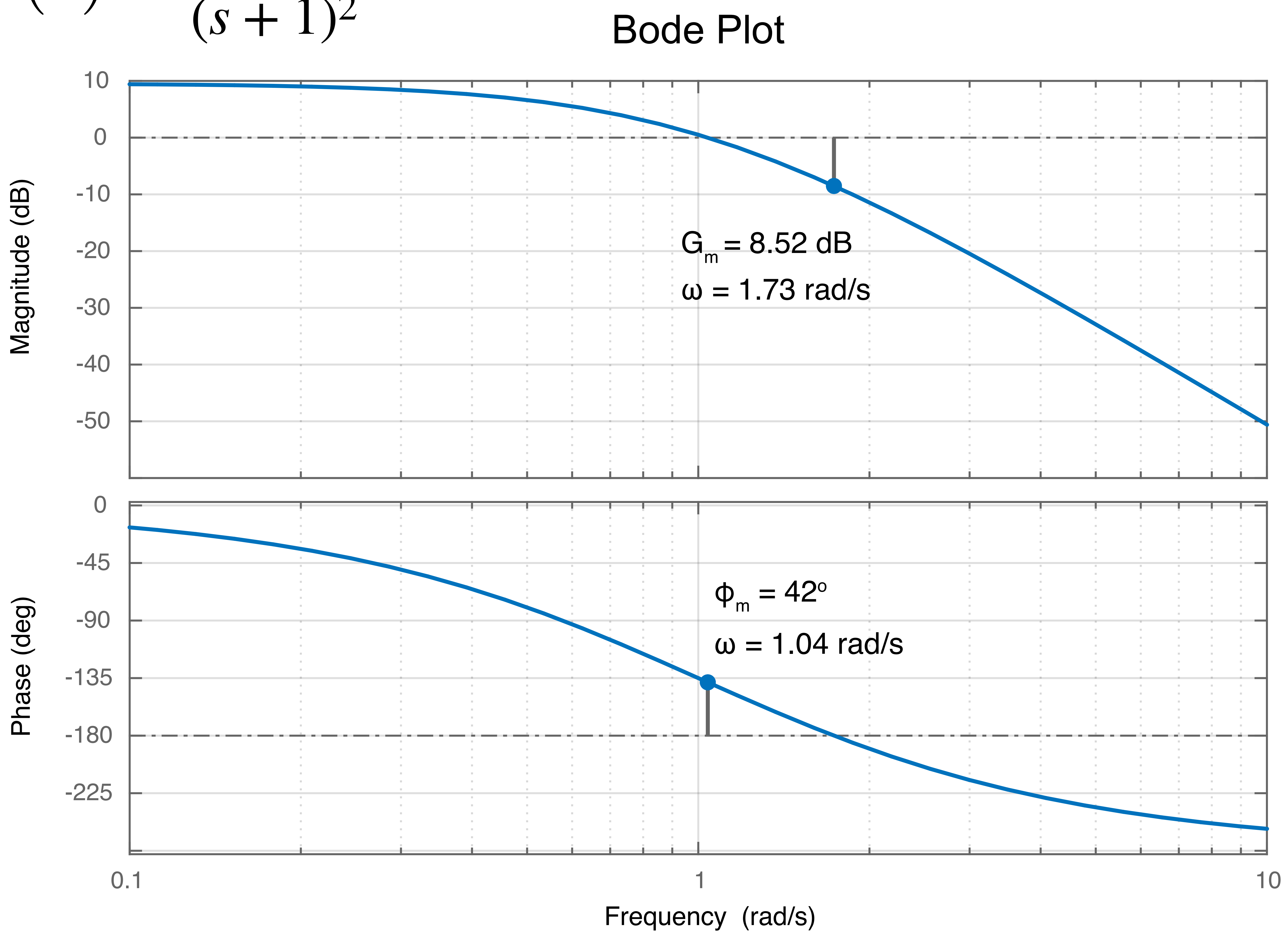
g_m : the change in open-loop gain required to reach the "stability limit"



$$\angle[G_{OL}(j\omega_{pc})] = \pm - 180^0 \quad \Rightarrow \quad g_m = \frac{1}{|G_{OL}(j\omega_{pc})|} \quad \text{or} \quad G_m = -20\log_{10}|G_{OL}(j\omega_{pc})|$$

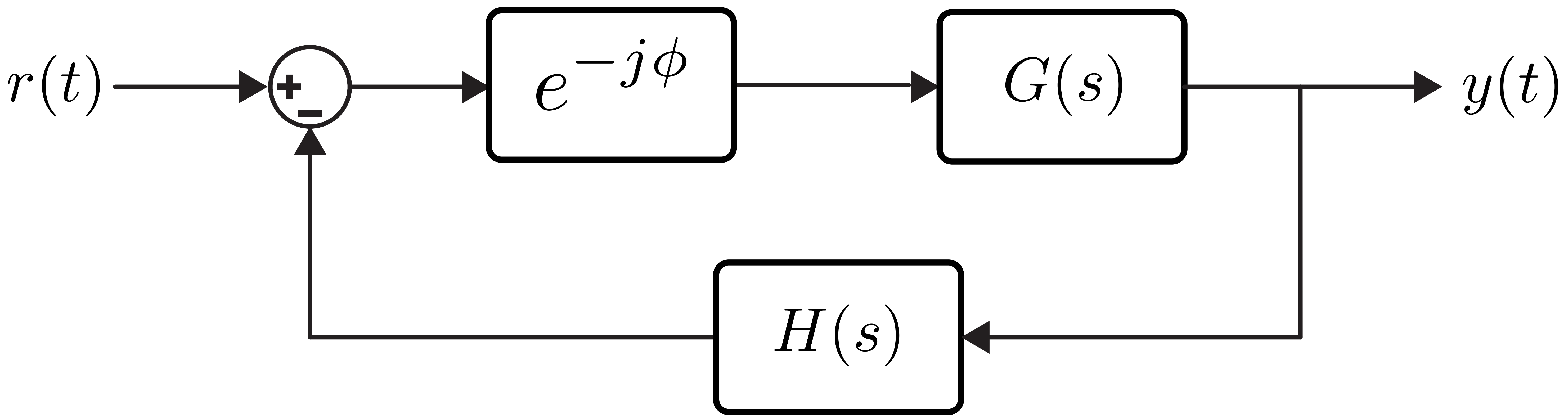
$$G(s) = \frac{3}{(s+1)^2}$$


Root Locus
K=2.67
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1


Bode Plot
Magnitude (dB)
10
0
-10
-20
-30
-40
-50
G_m = 8.52 dB
ω = 1.73 rad/s
Phase (deg)
0
-45
-90
-135
-180
-225
ϕ_m = 42°
ω = 1.04 rad/s
0.1
1
10
Frequency (rad/s)

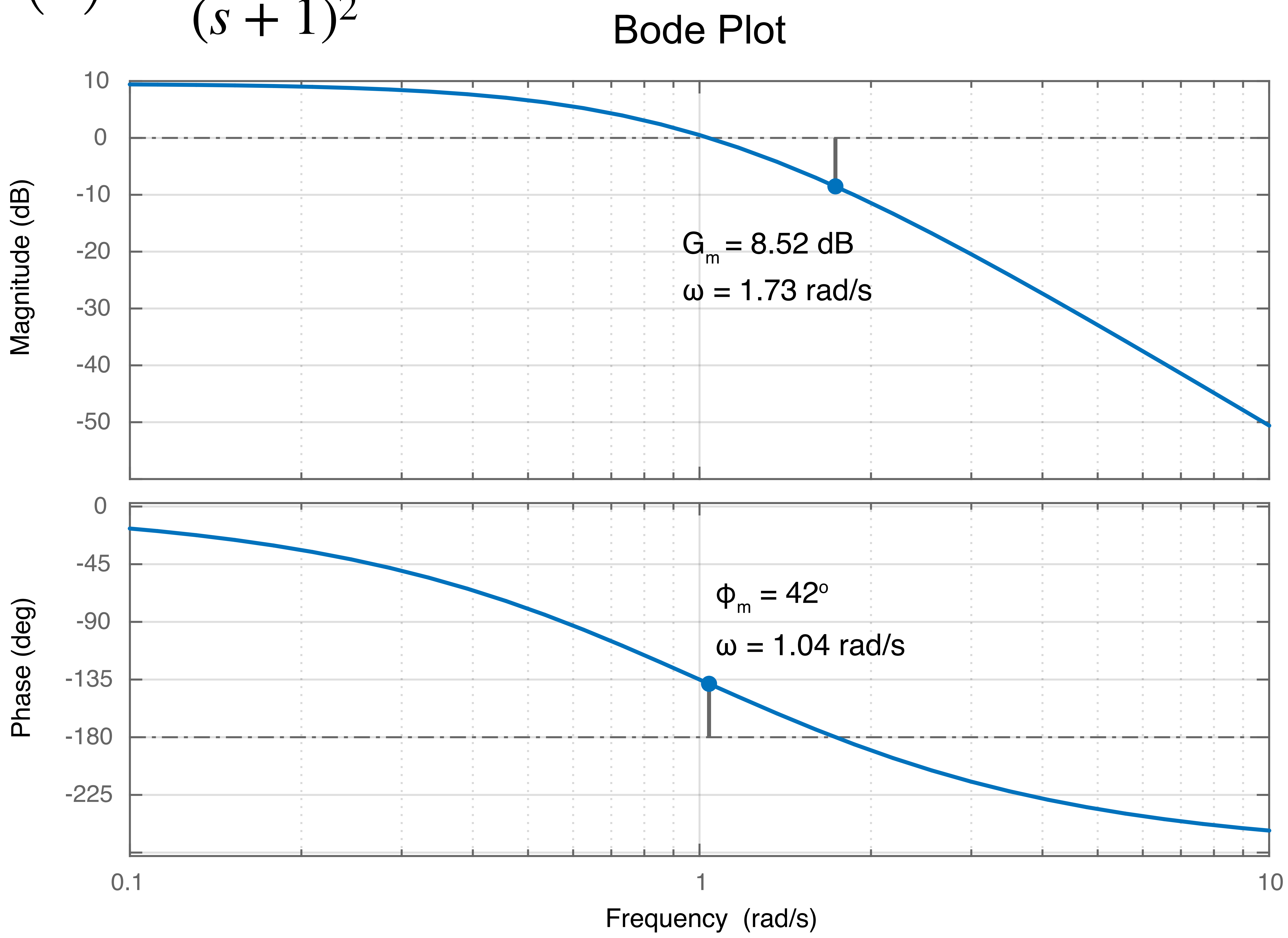
Phase Margin, ϕ_m

ϕ_m : the amount of “phase lag” required to reach the stability limit



$$|G_{OL}(j\omega_{gc})| = 1 \quad \text{or} \quad M_{dB}\{G_{OL}(j\omega_{gc})\} = 0 \text{ dB} \quad \Rightarrow \quad \phi_m = \pi + \angle G_{OL}(j\omega_{gc})$$

$$G(s) = \frac{3}{(s+1)^2}$$


Bode Plot
Magnitude (dB)
10
0
-10
-20
-30
-40
-50
Gm = 8.52 dB
ω = 1.73 rad/s
Phase (deg)
0
-45
-90
-135
-180
-225
Φm = 42°
ω = 1.04 rad/s
0.1
1
10
Frequency (rad/s)

Closed-Loop System is BIBO Stable $\Longleftrightarrow G_m > 0 \ \& \ \phi_m > 0$