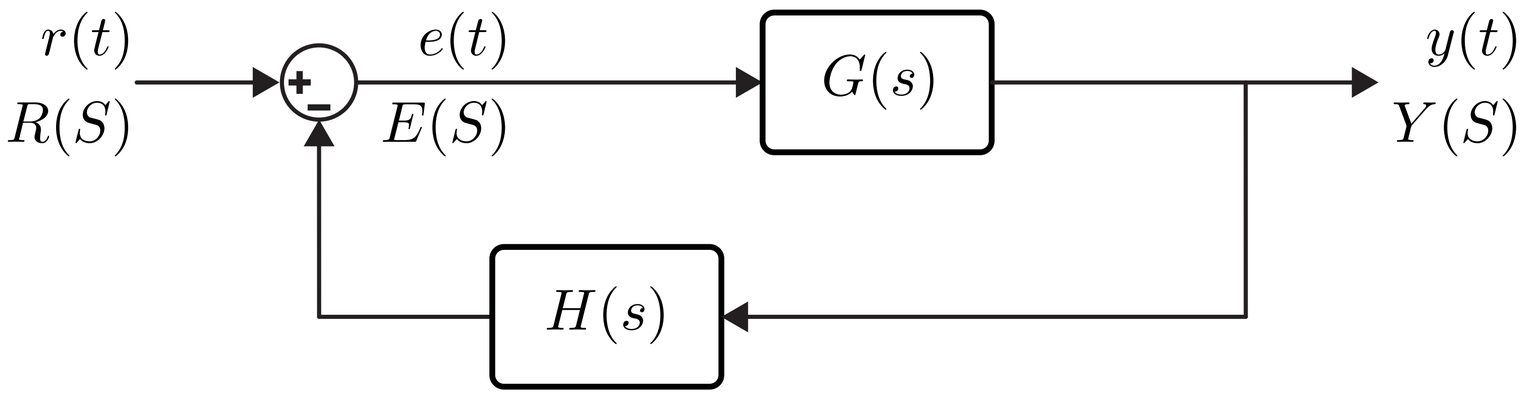
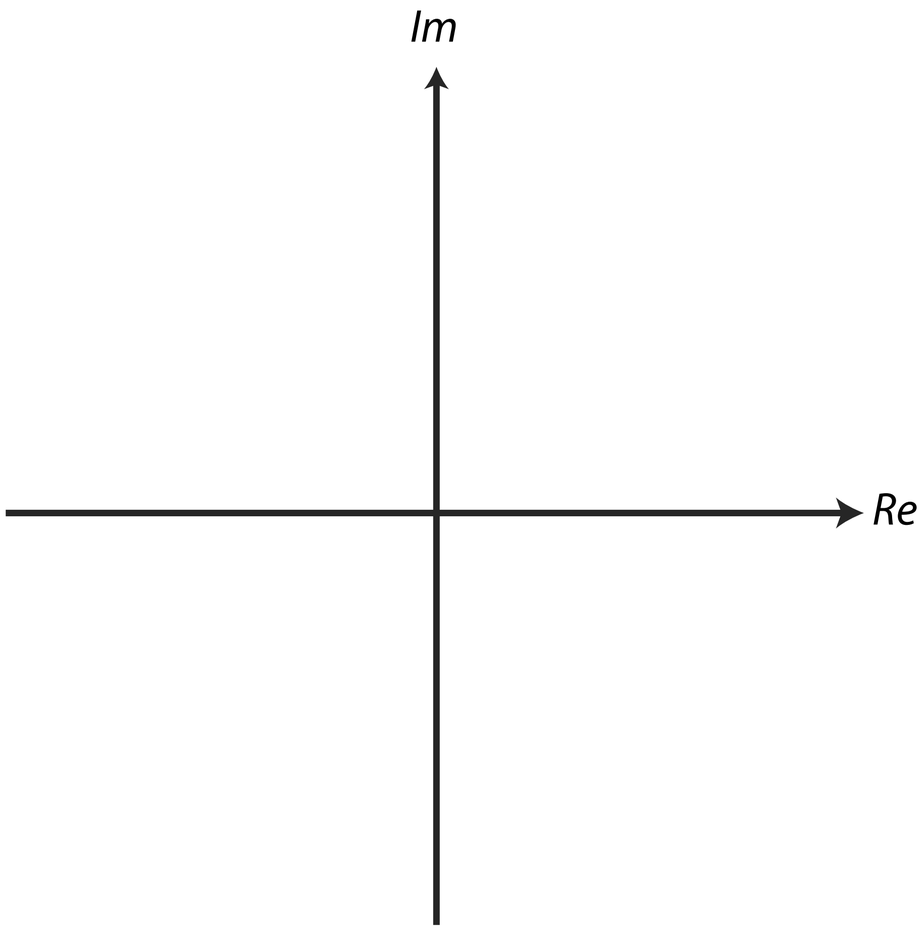
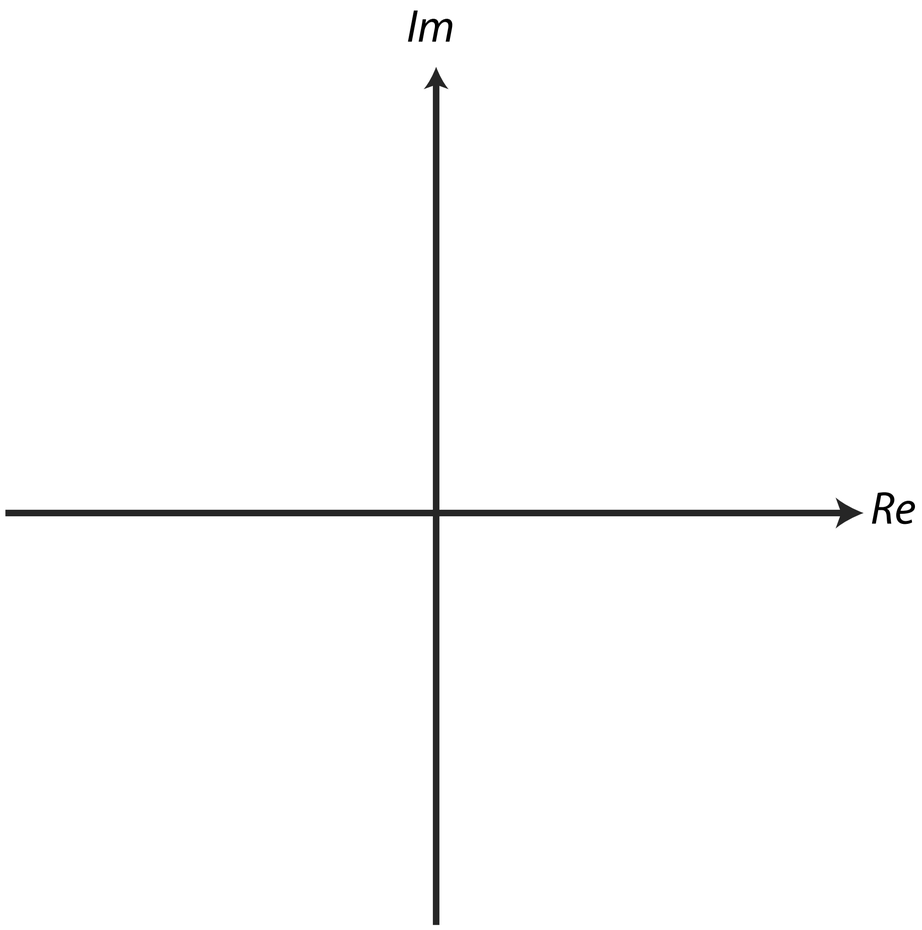
Stability Margins: Gain & Phase Margins

https://github.com/mertankarali/Lecture-Notes/tree/master/METU-EE302/Lecture%2017

Lecture #17




Im
Re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Assumptions

- $G_{OL}(s)$ is a *minimum-phase* system, i.e.
  - No poles/zeros in the Open Right Half Plane
  - $\lim_{\omega \to \infty} [\frac{G_{OL}(s)}{s}]_{s=\jmath\omega} = 0$
- The feed-back system is Type $0 - 2$
- Polar plot of $G_{OL}(j\omega)$ crosses the negative real-axis at most once.

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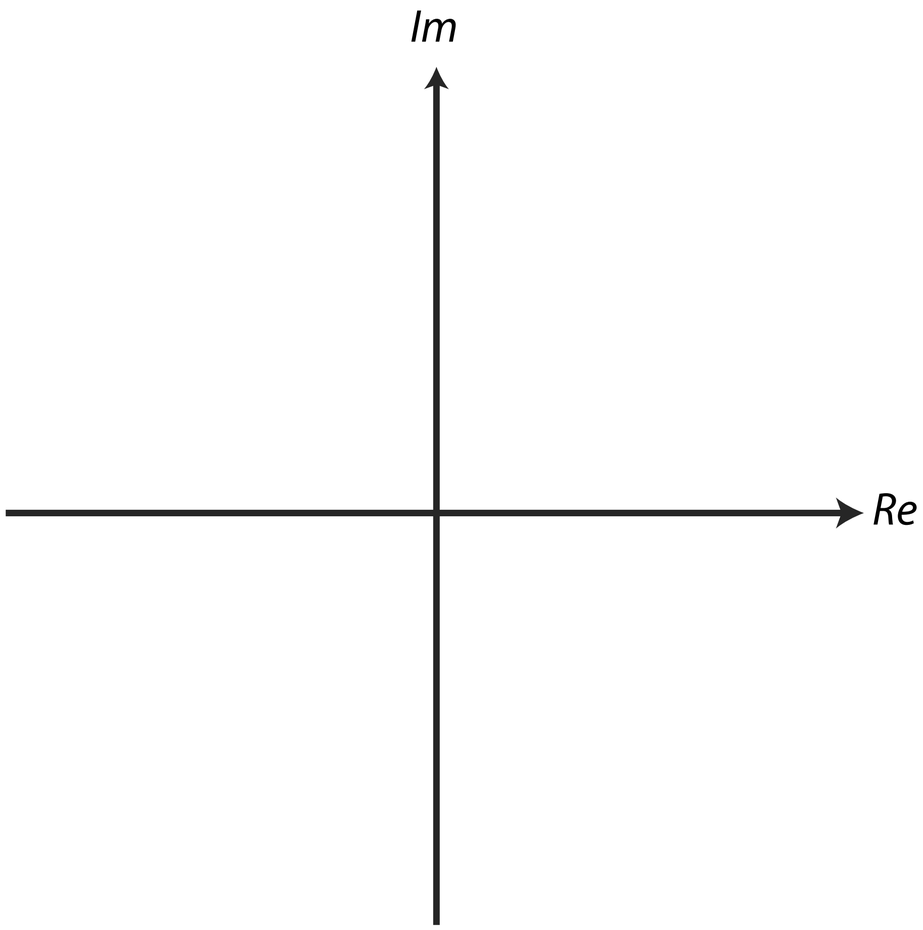
Assumptions $\Rightarrow$ Gain & Phase Margin from Polar Plot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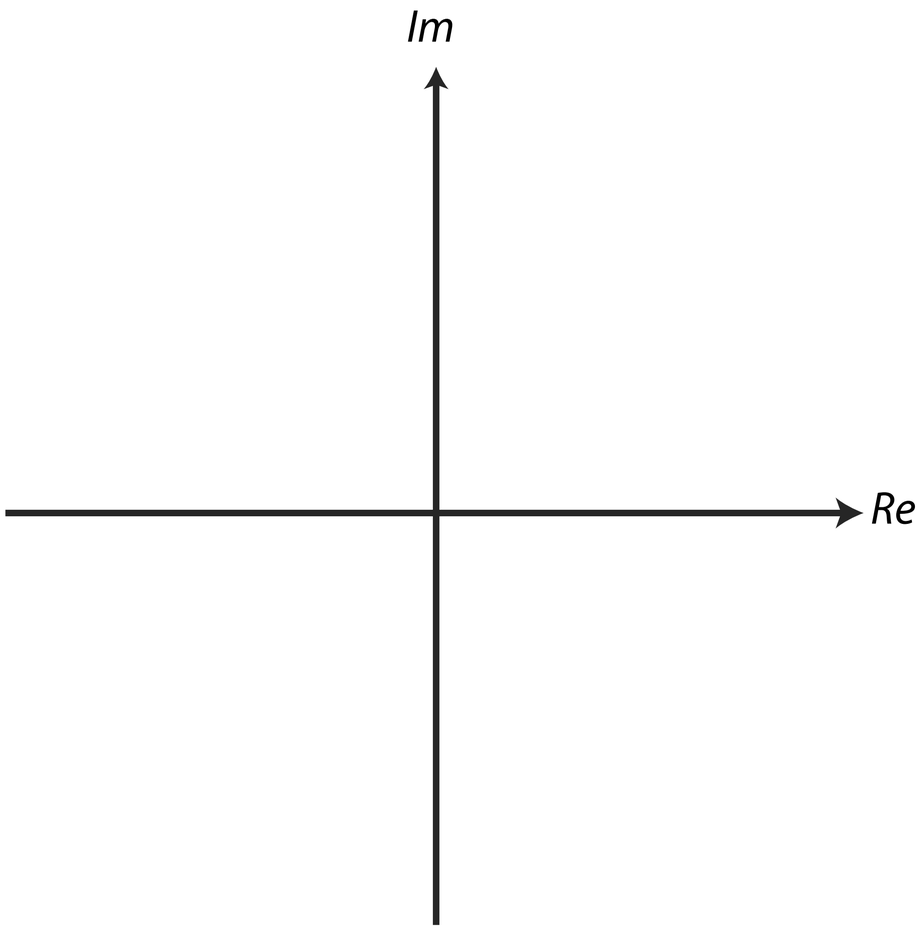
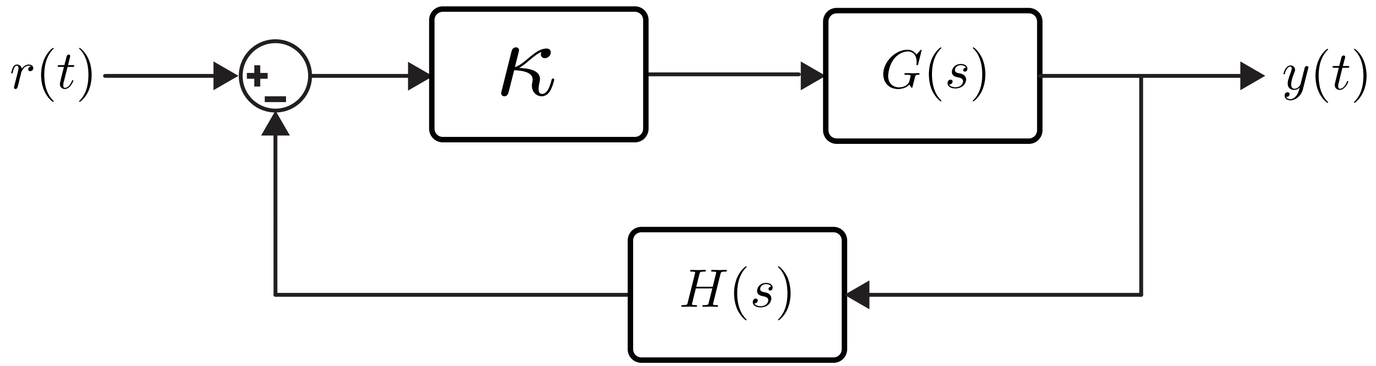

Im
Re

Gain Margin, g_m

g_m : the change in open-loop gain required to reach the Nyquist stability limit



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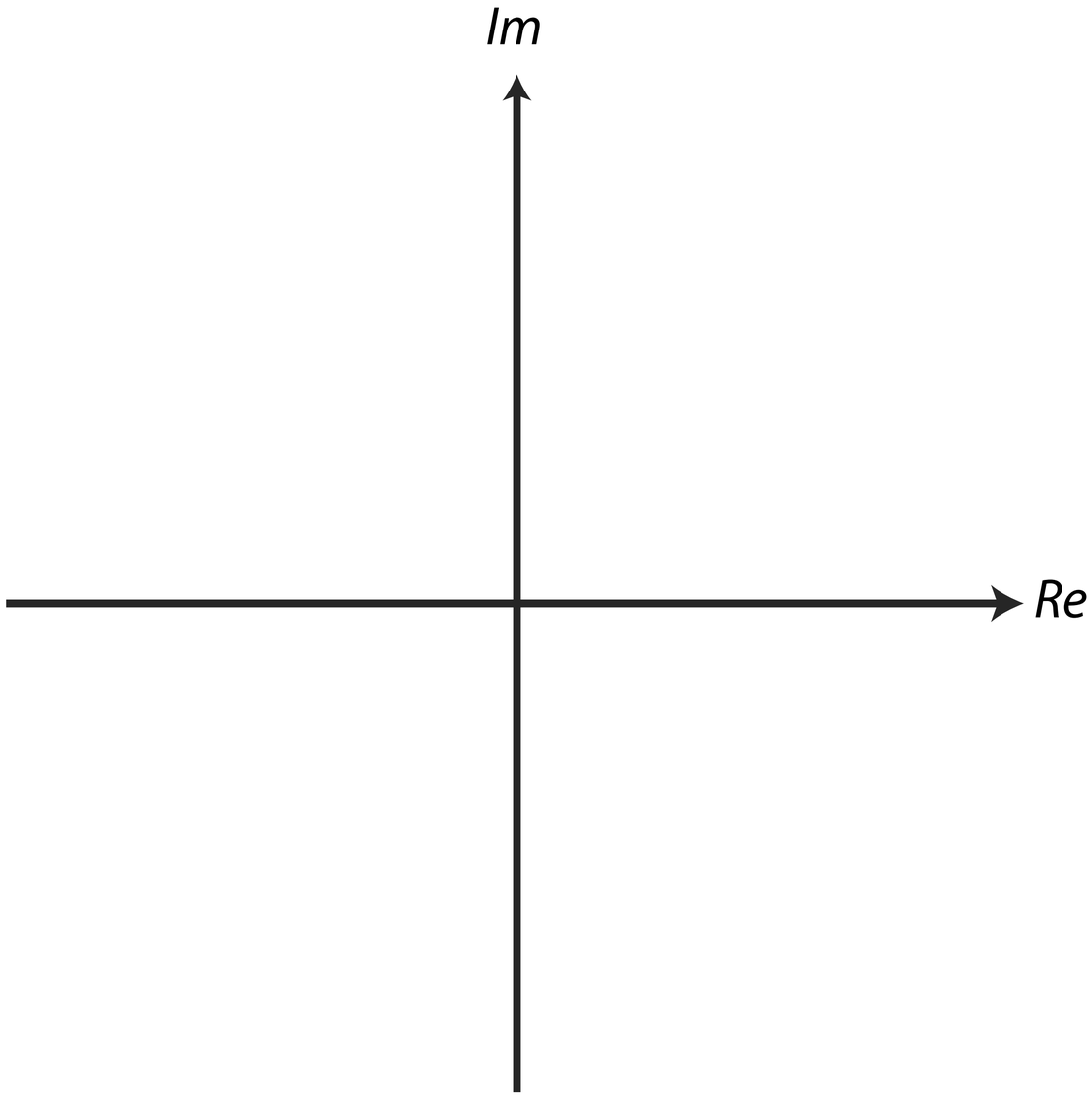

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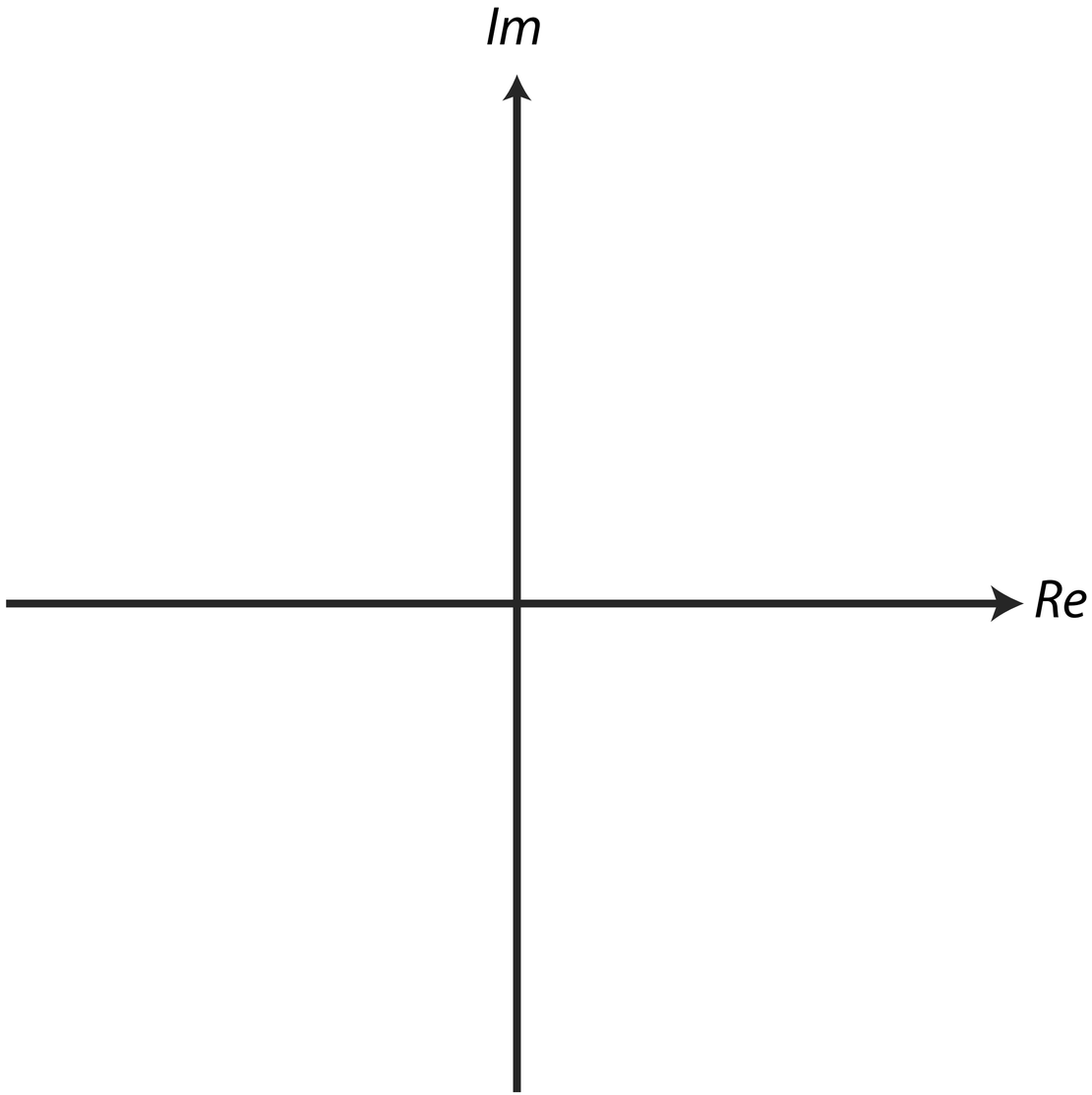
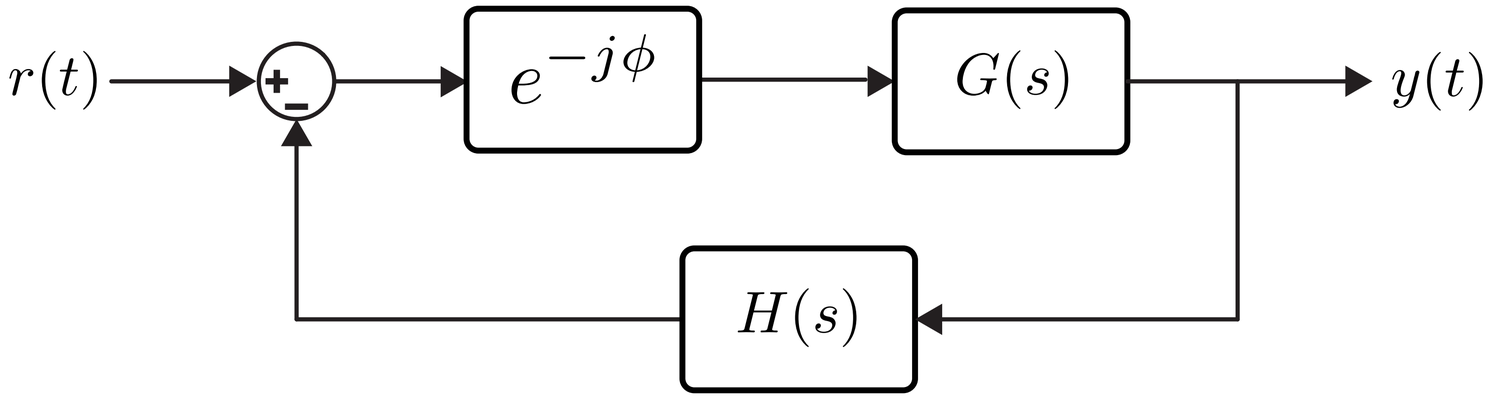


Phase Margin, ϕ_m

ϕ_m : the amount of “phase lag” required to reach the (Nyquist) stability limit



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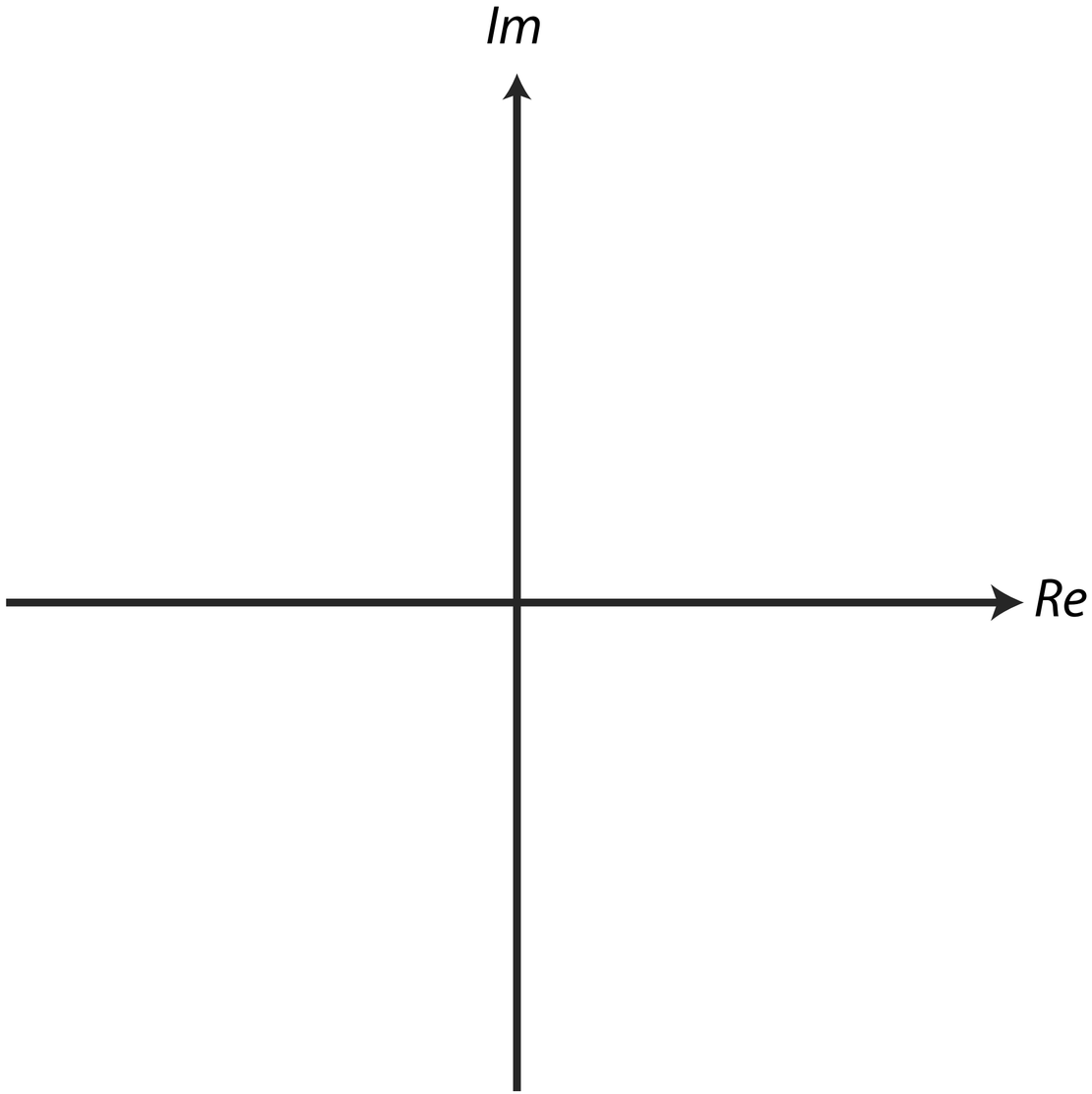

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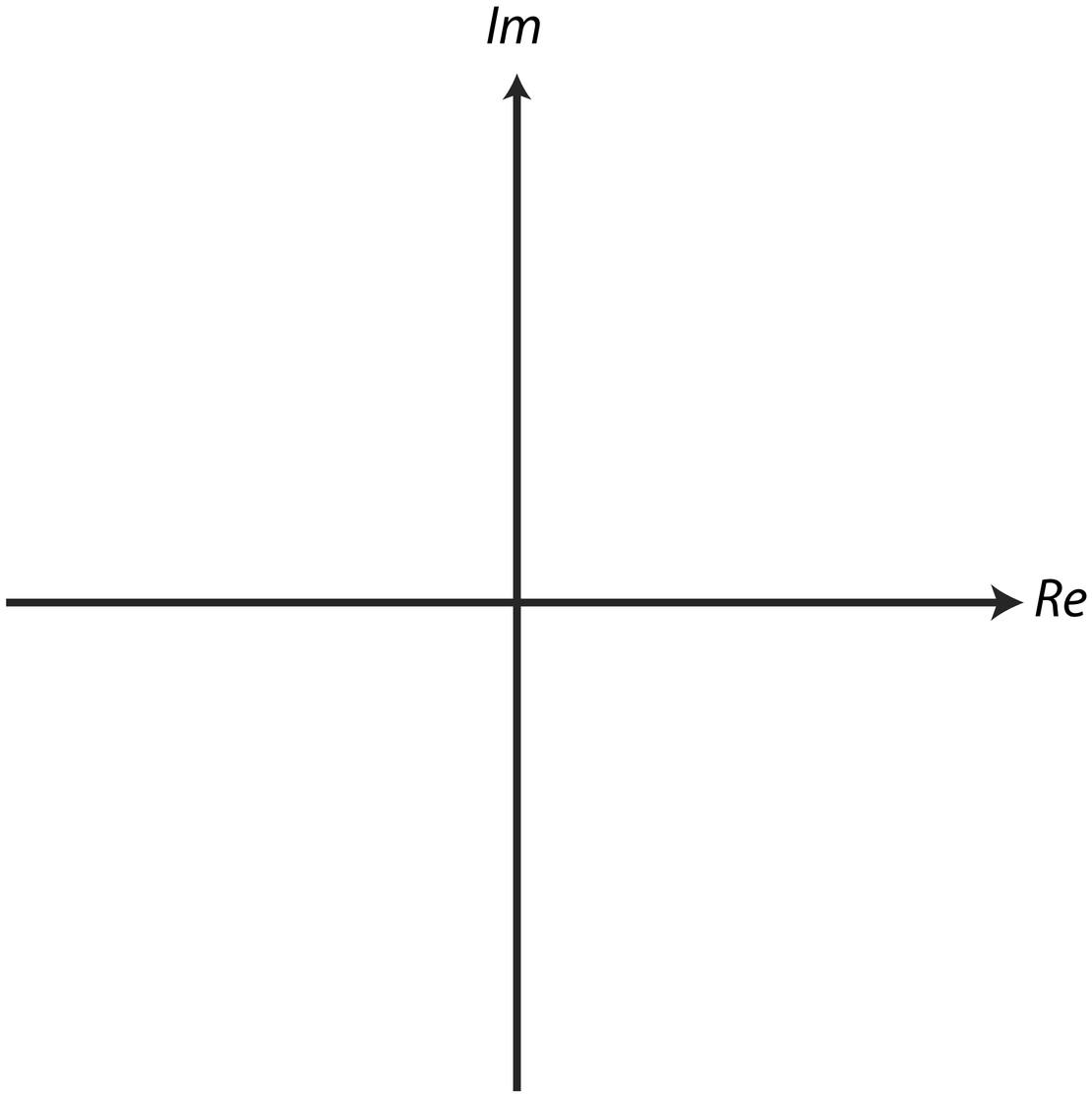
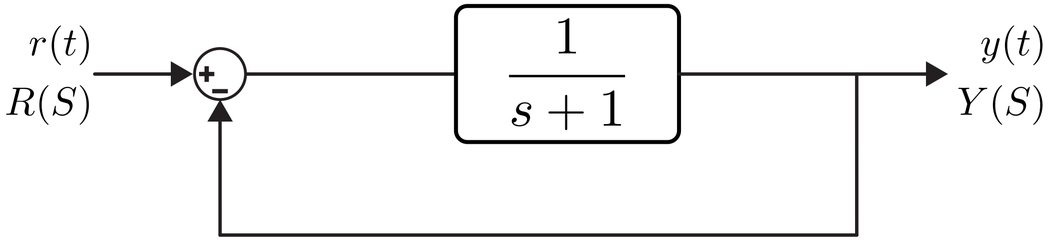
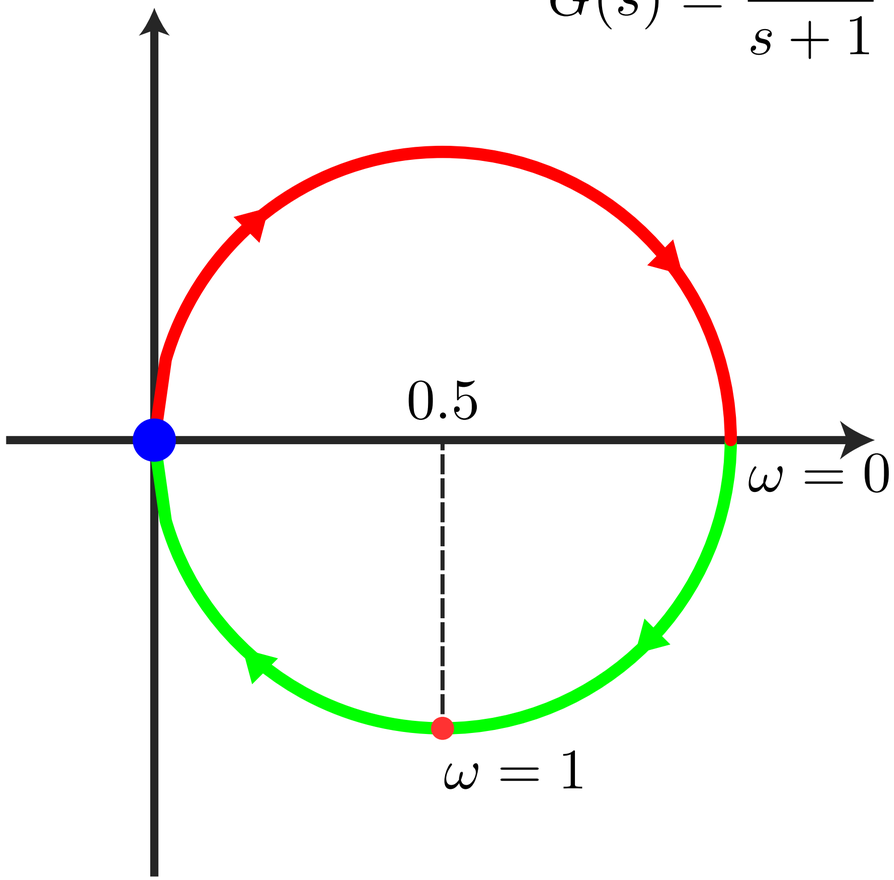



$r(t)$
$R(S)$
+
−
$\frac{1}{s+1}$
$y(t)$
$Y(S)$

$$G(s) = \frac{1}{s+1}$$



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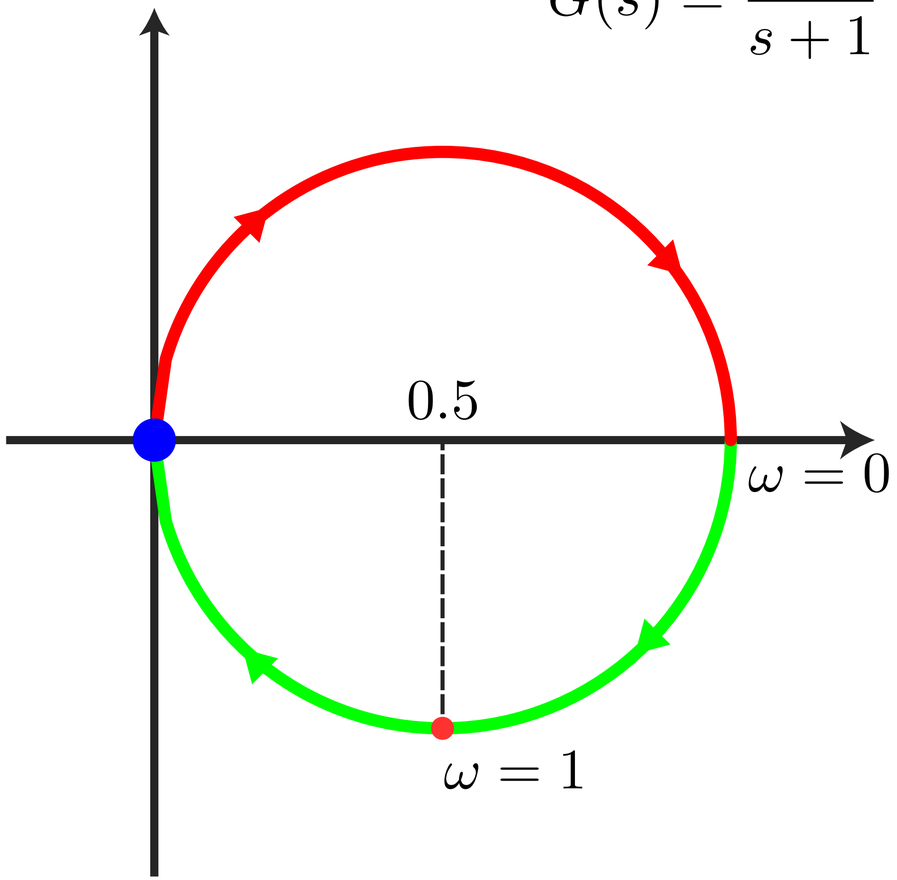
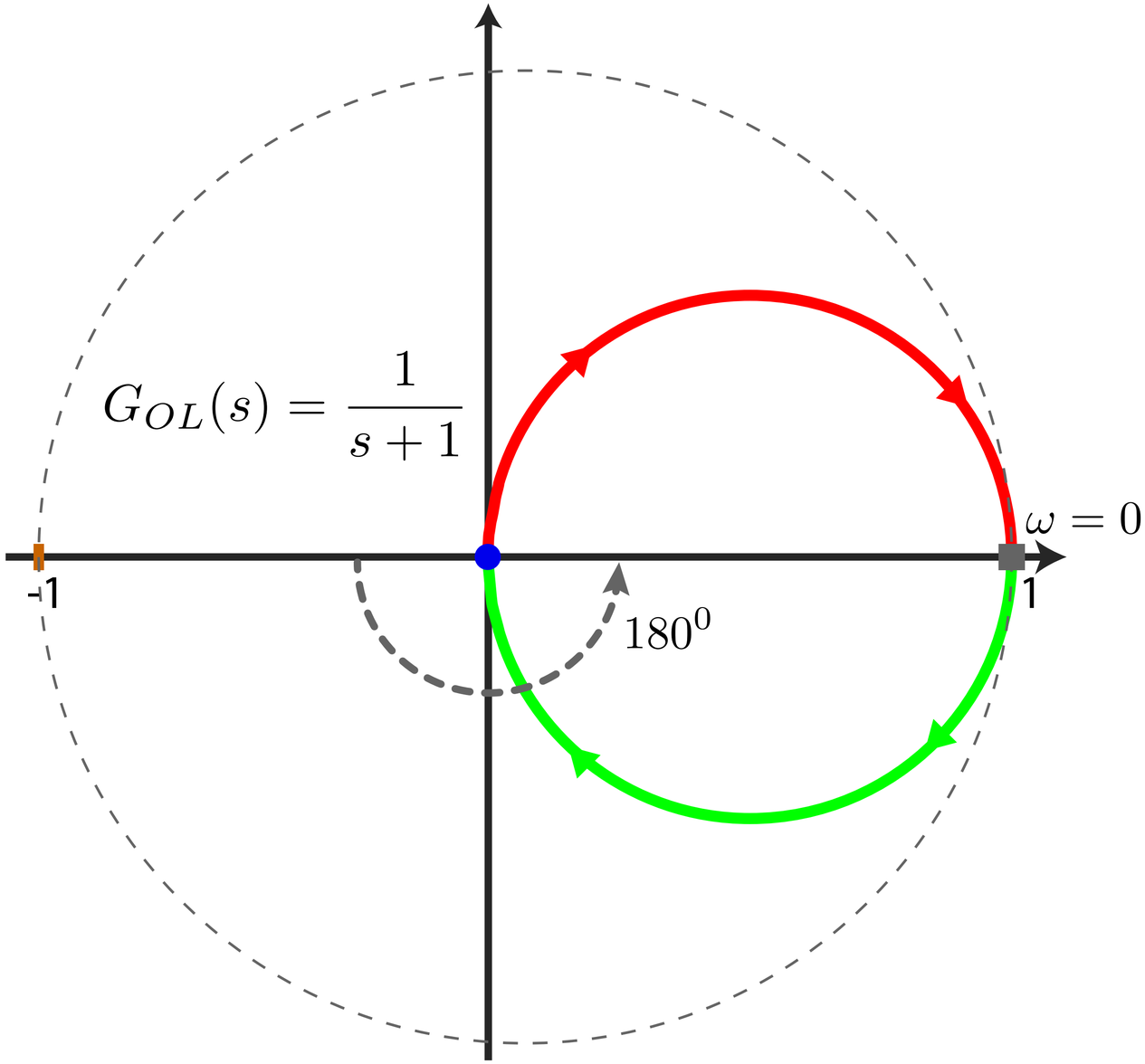
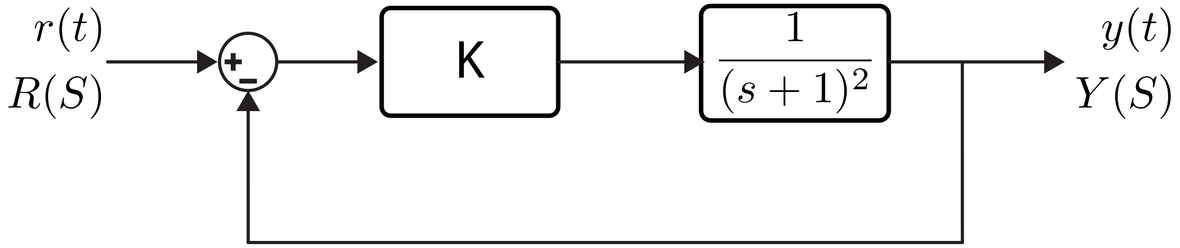
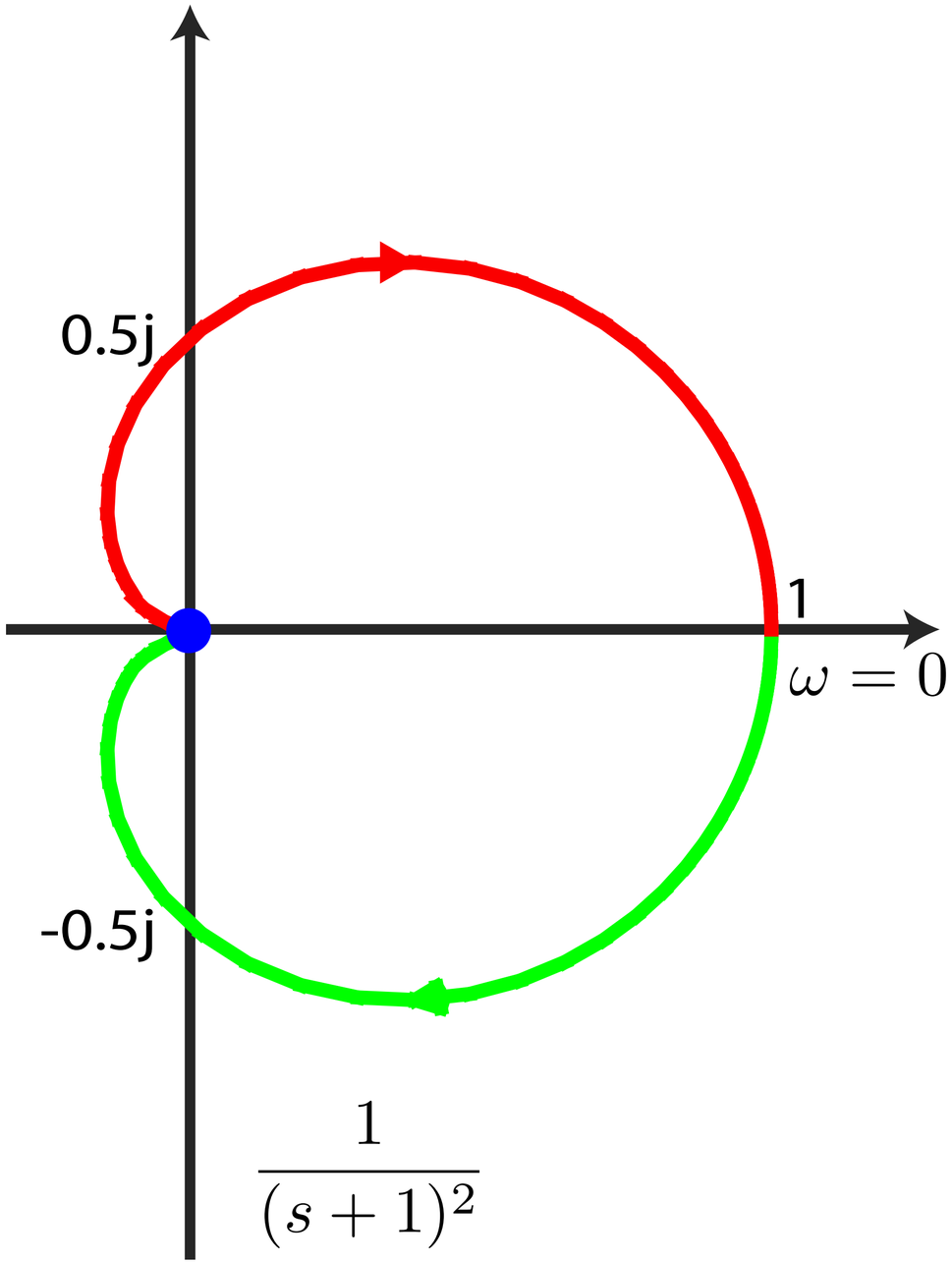
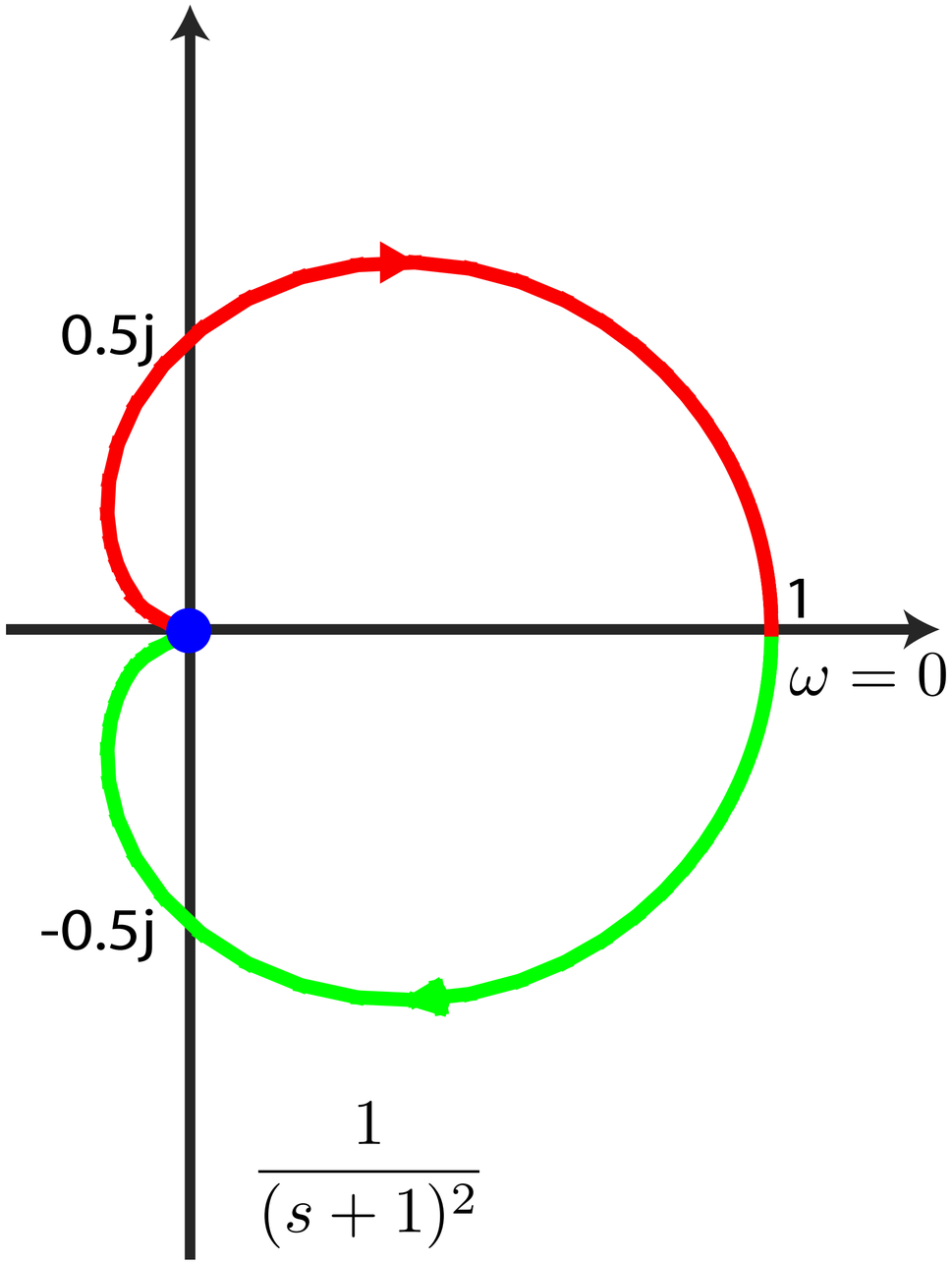


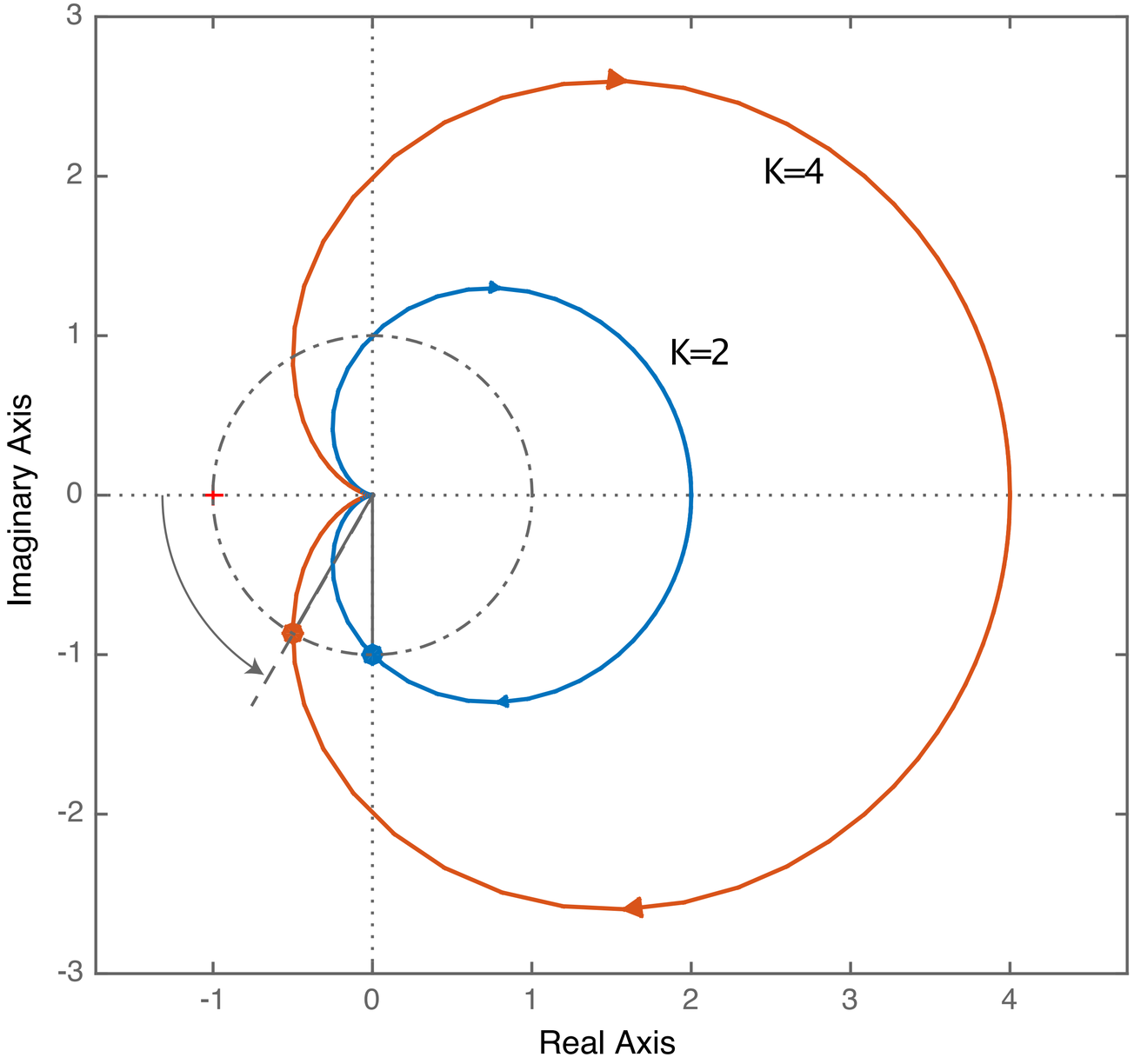


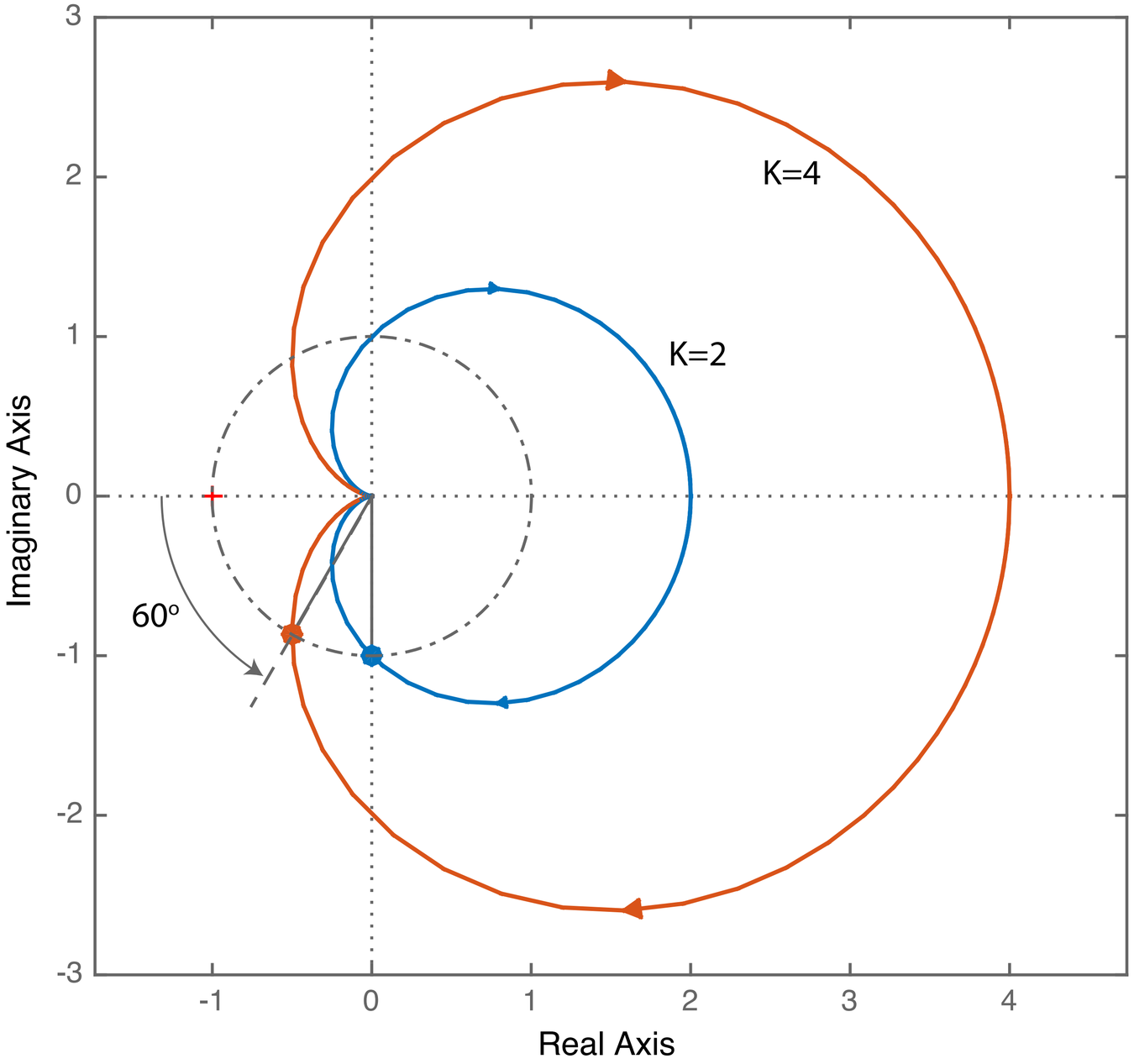


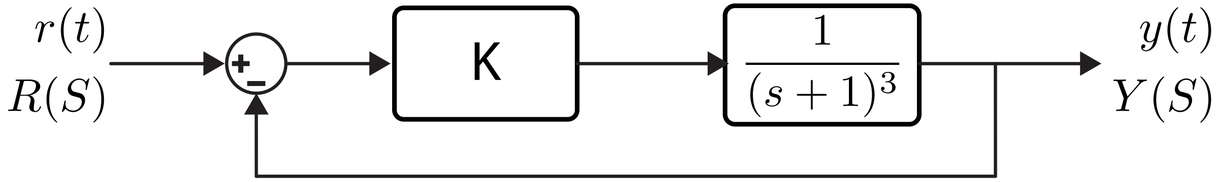
Compute the gain margin and phase margin for the following closed-loop system for K = 2 and K = 4


0.5j
1
$\omega = 0$
-0.5j
$\frac{1}{(s+1)^2}$

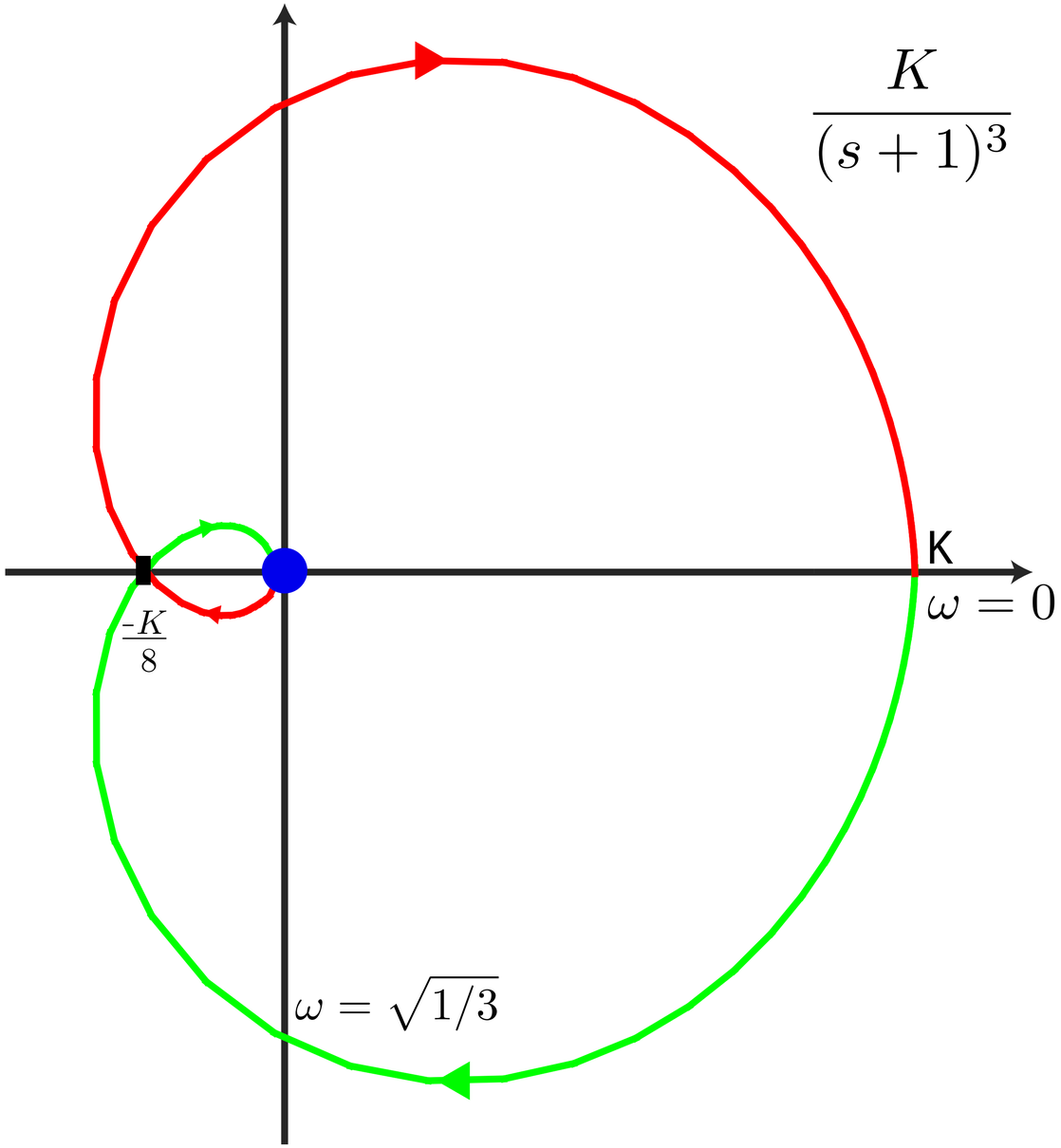

0.5j
1
$\omega = 0$
-0.5j
$\frac{1}{(s+1)^2}$

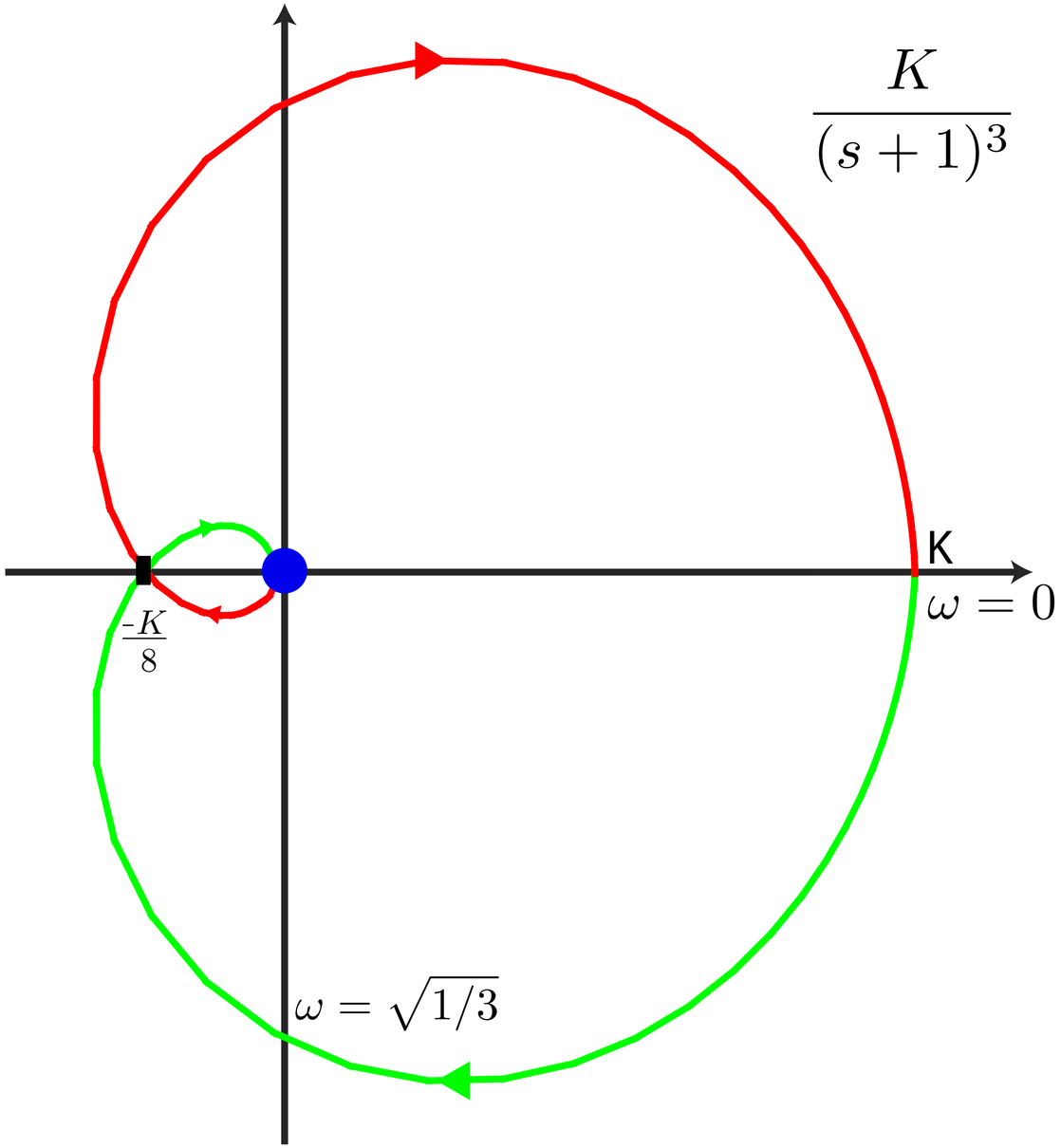

K=4
K=2
Imaginary Axis
Real Axis
3
2
1
0
-1
-2
-3
-1
0
1
2
3
4

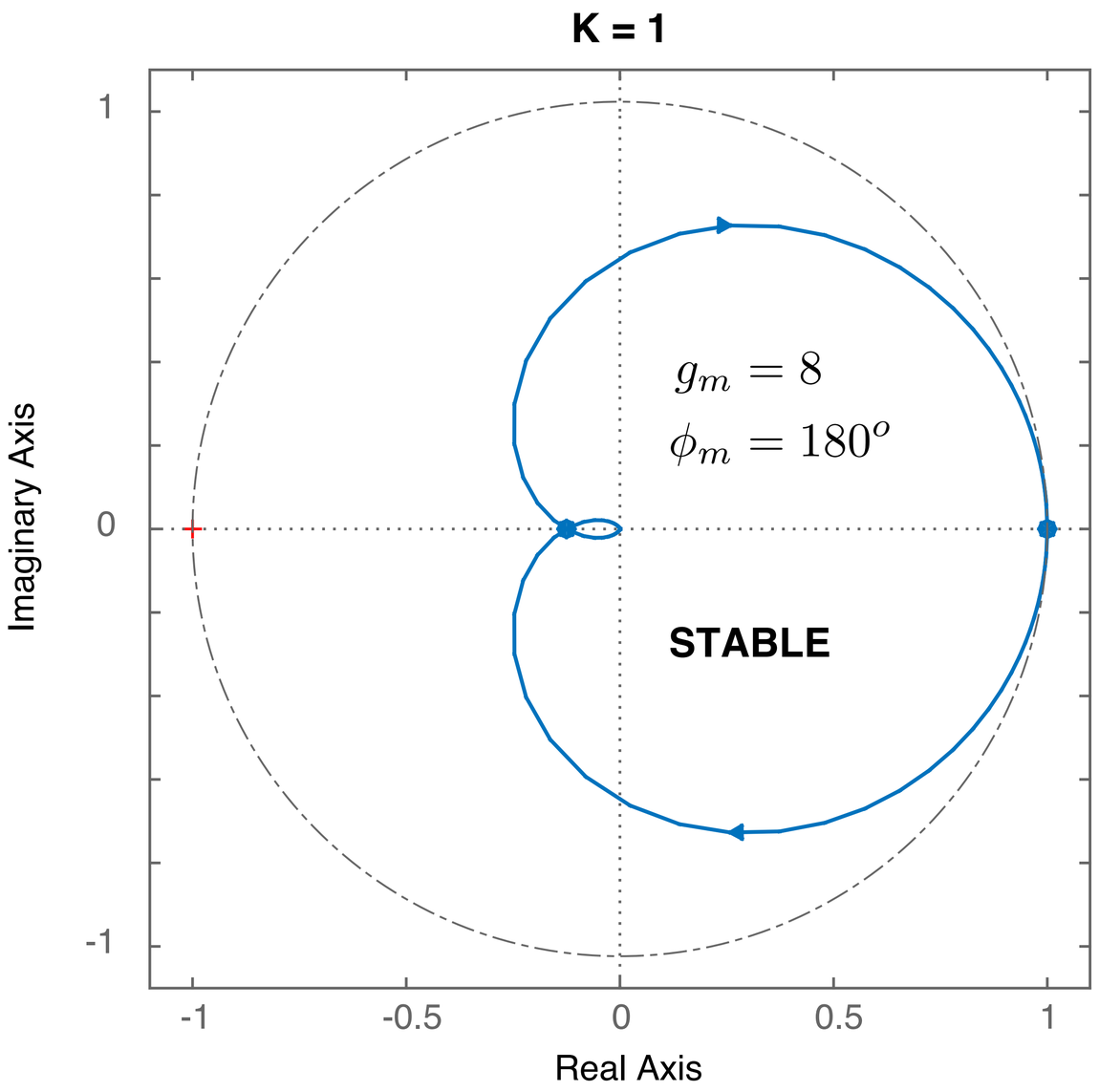

K=4
K=2
60°
Imaginary Axis
Real Axis
3
2
1
0
-1
-2
-3
-1
0
1
2
3
4

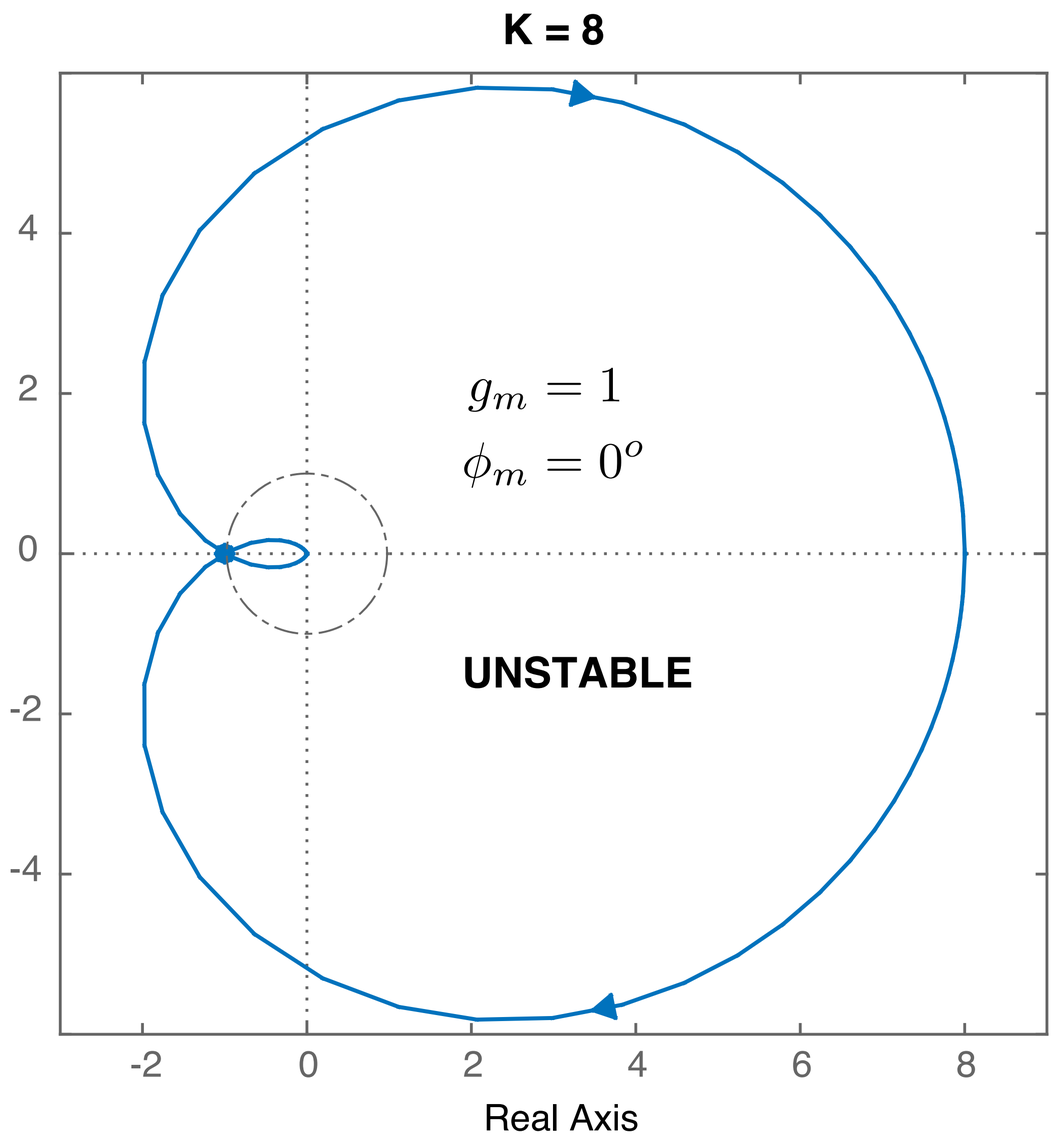


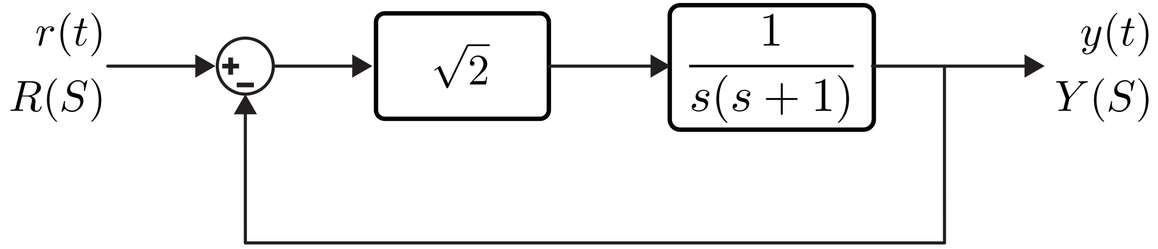
Compute the gain margin and phase margin for the following closed-loop system for K = 1 and K = 8

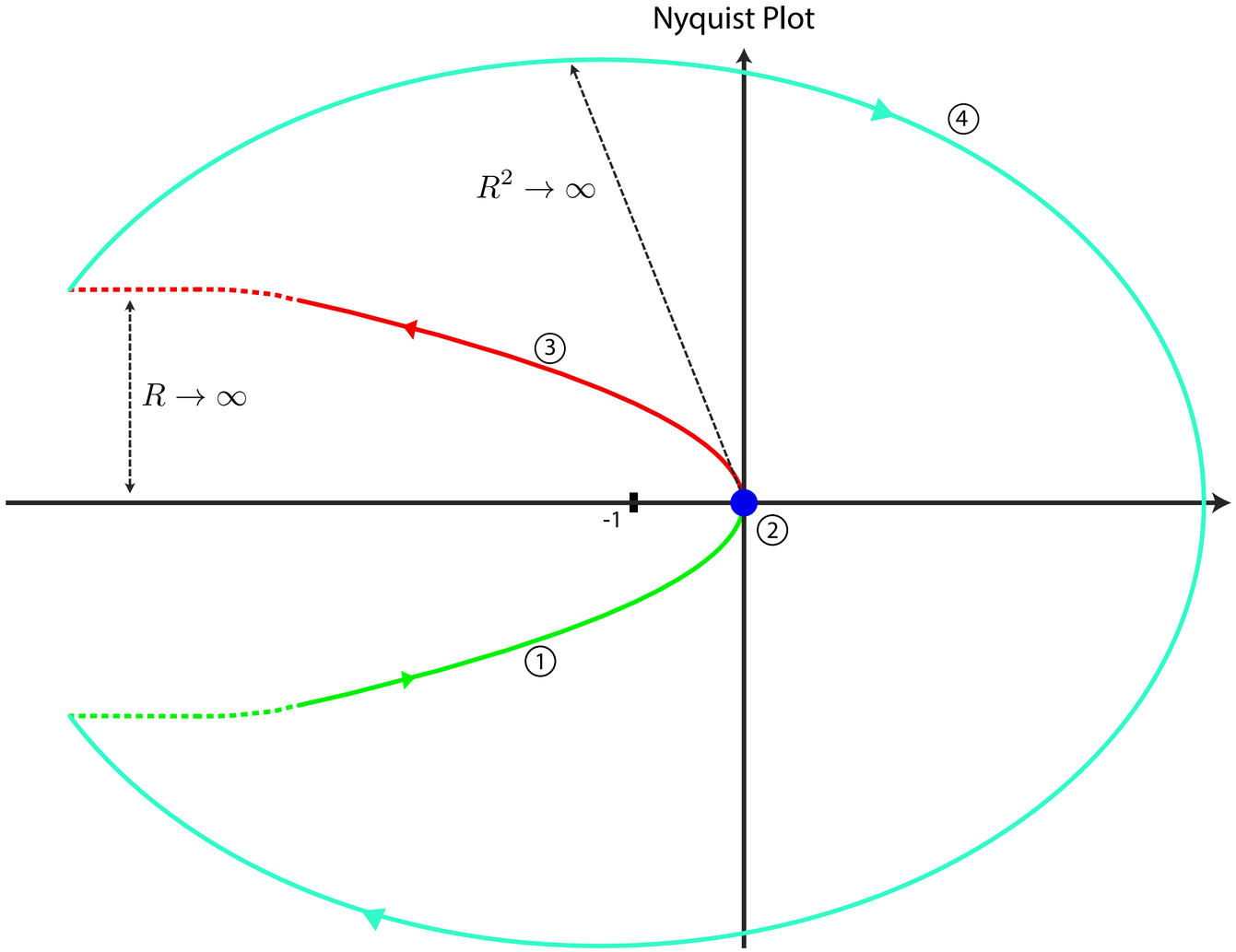

$\frac{K}{(s+1)^3}$
K
$\omega = 0$
$\frac{-K}{8}$
$\omega = \sqrt{1/3}$

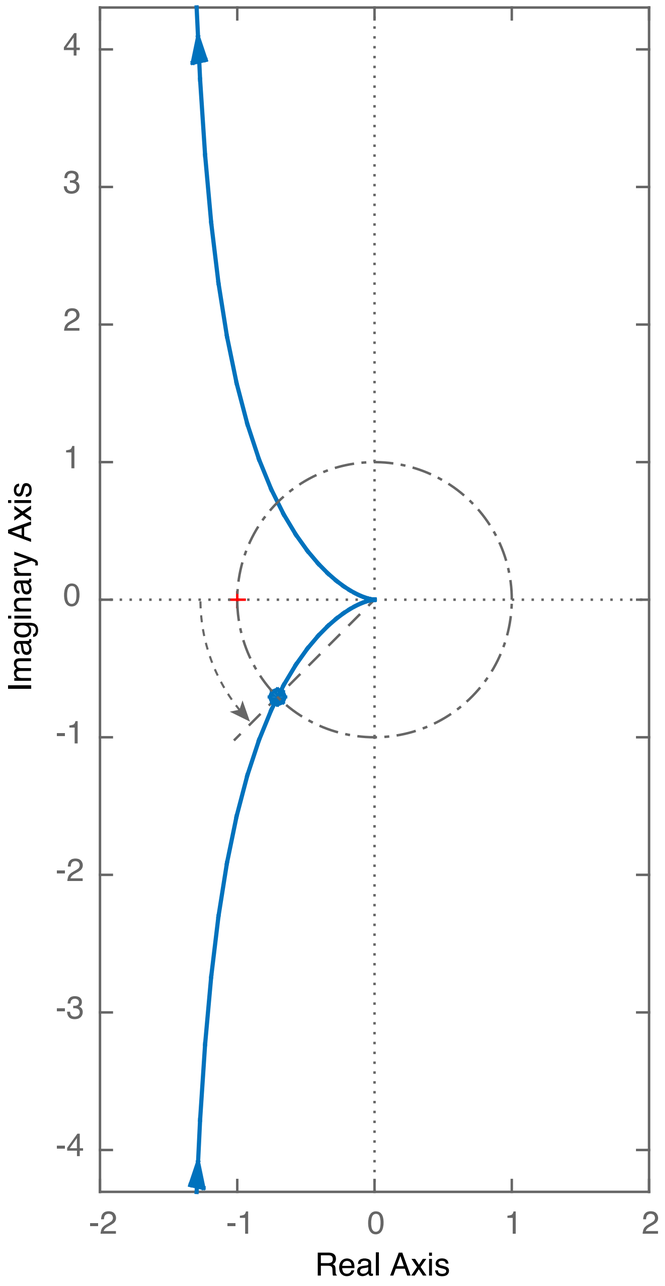

$\frac{K}{(s+1)^3}$
K
$\omega = 0$
$\frac{-K}{8}$
$\omega = \sqrt{1/3}$

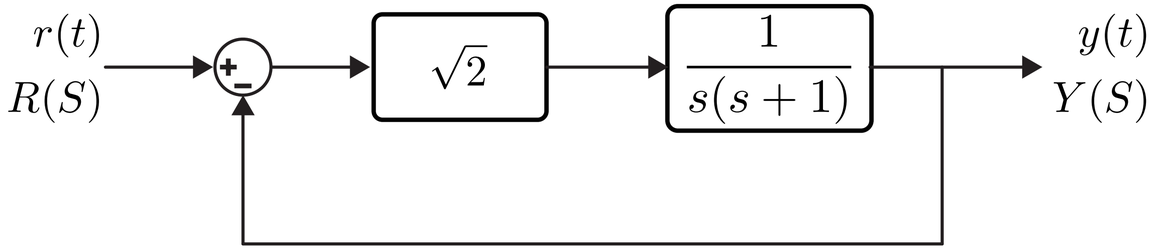

K = 1
Imaginary Axis
Real Axis
$g_m = 8$
$\phi_m = 180^o$
STABLE
1
0
-1
-1
-0.5
0
0.5
1

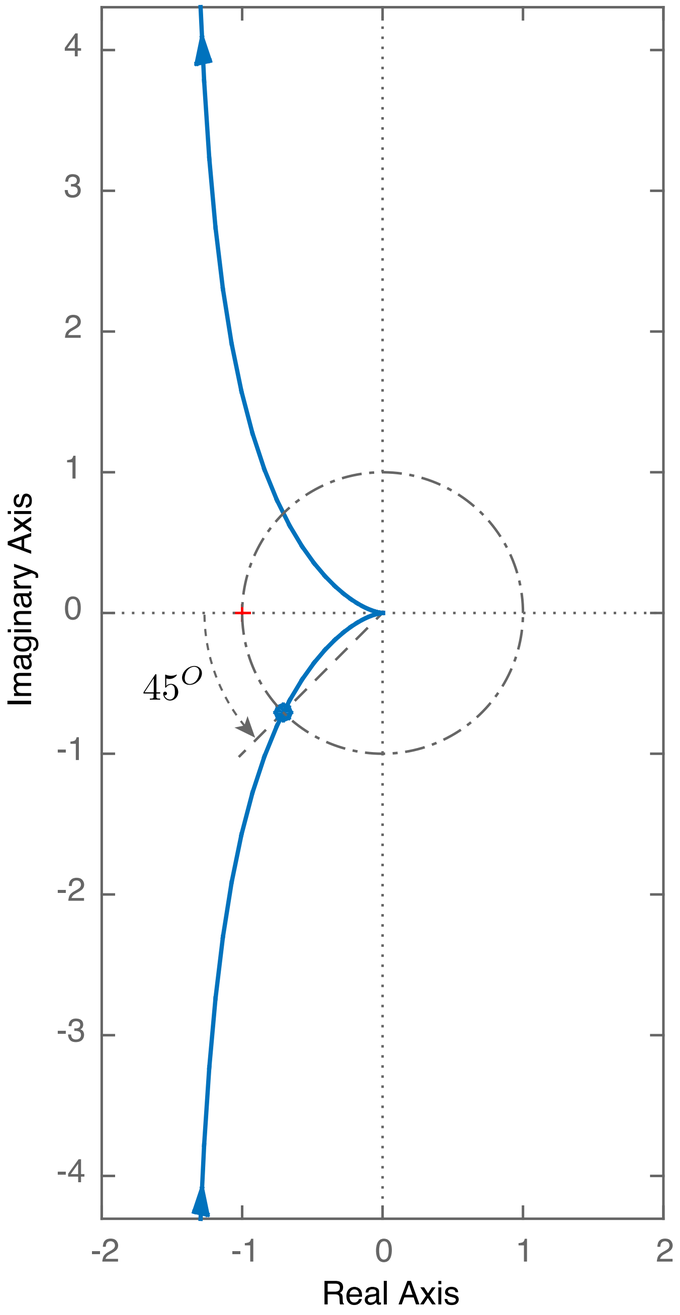

K = 8
Real Axis
$g_m = 1$
$\phi_m = 0^o$
UNSTABLE
4
2
0
-2
-4
-2
0
2
4
6
8

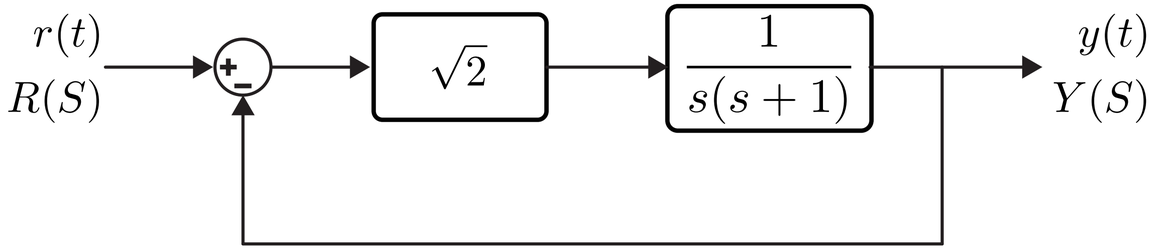

$r(t)$
$R(S)$
$\sqrt{2}$
$\frac{1}{s(s+1)}$
$y(t)$
$Y(S)$


Nyquist Plot
$R^2 \to \infty$
$R \to \infty$
-1
①
②
③
④


Imaginary Axis
Real Axis
4
3
2
1
0
-1
-2
-3
-4
-2
-1
0
1
2


$r(t)$
$R(S)$
$\sqrt{2}$
$\frac{1}{s(s+1)}$
$y(t)$
$Y(S)$


Imaginary Axis
Real Axis
4
3
2
1
0
-1
-2
-3
-4
-2
-1
0
1
2
45^O


$r(t)$
$R(S)$
$\sqrt{2}$
$\frac{1}{s(s+1)}$
$y(t)$
$Y(S)$