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Entangled Fermions

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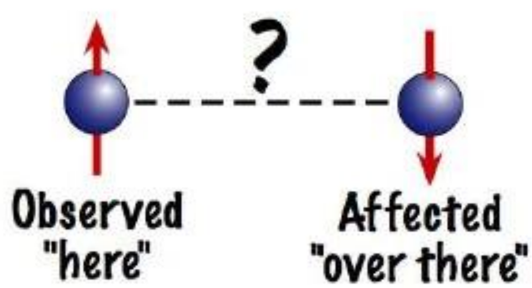

The article "Entanglement, subsystem particle numbers and topology in free fermion systems" by Sheng Li gives an insight into current research on entanglement, entanglement entropy and how these are related to the topology of the system. In the following article, the research will be presented with the goal of increasing the understanding of the reader.

As you may know, fermions are spin half particles such as quarks, leptons and many atoms. Recently, an article was published, by researchers in China, called “Entanglement, subsystem particle numbers and topology in free fermion systems” (don’t worry, this will soon be clear) with the final result of a lower-bound estimation for the entanglement entropy close to the obtained experimental values. This entanglement entropy may be used to obtain other important values such as the entanglement cost.

Since this word has come up a few times now, you could be wondering what entanglement and entanglement entropy are. Quantum entanglement or just entanglement, for short, is when the quantum state of a pair or group of particles is linked or “entangled”. Not only has this been used to characterise topological phases but this phenomenon is also a very active area in research since it is possible to have linked systems over long distances. This then means that information can be transmitted instantaneously across distances. It has also been demonstrated using photons, electrons and even larger molecules such as buckyballs. Keep in mind that large is a relative term and a buckyball is a spherical carbon system consisting of 60 atoms. The entropy of entanglement can be given by the Von Neumann entropy:

$$S_{ent} = -Tr(\rho_A \ln \rho_A) = -Tr(\rho_B \ln \rho_B),$$

where S_{ent} is the entropy of entanglement, Tr denotes the trace, A and B are two blocks of a large system and ρ is the density matrix of the quantum mechanical system. For a bipartite or two-particle, subsystem with a smooth boundary this equation can take the form

$$S_{ent} = \alpha L - S_{top}.$$

In this equation, L is the length of the boundary, α is a system-dependent coefficient and S_{top} is a universal constant called the topological entanglement entropy. In the reference paper, the reduced density matrix, ρ_A, is changed to include an entanglement Hamiltonian which allows for an eigenvalue spectrum, storing information about the system. One piece of information held in this eigenvalue spectrum is the subsystem particle number.

In a half-filled free fermion system, the discontinuity in the subsystem particle number as a function of the conserved momentum indicates whether the eigenvalue spectrum has a spectral flow. However, in the case of a half-filled quantum spin Hall (QSH) system with two dimensional symmetry, the approach has been found inadequate. The quantum spin hall state is proposed to exist in two dimensional semiconductors which have a quantised spin-Hall conductance and a vanishing charge-Hall conductance. To overcome this, spin-projected particle numbers are defined based on the spin trace indices. These are strongly related to the topological invariant of the system.

As mentioned before, a Hamiltonian was defined to model the system. This was done by modifying the tight-binding model Hamiltonian for the QSH system. This includes the nearest-neighbour hopping term, which describes conduction and spin-orbit coupling terms, which is the interaction of spin and motion of the atom. The modification from the tight-binding model is an added term representing a uniform exchange field. This exchange field explicitly violates time reversal symmetry.

As seen in the image to the right, several types of eigenvalue spectra exist, some of them leading to a gap. In case of no gap, the spectrum is called a spectral flow. In another recent article, a trace index has been defined to describe topological invariants through a subsystem particle number. In the above mentioned paper, exceptions arise for half-filled system. However, in the reference paper, by plotting the traces, it has been shown that the subsystem particle numbers can still be used to characterise the topological invariants. The same can also be done for disordered systems with the result that a topological phase transitions due to high disorder can be anticipated using the subsystem particle numbers.

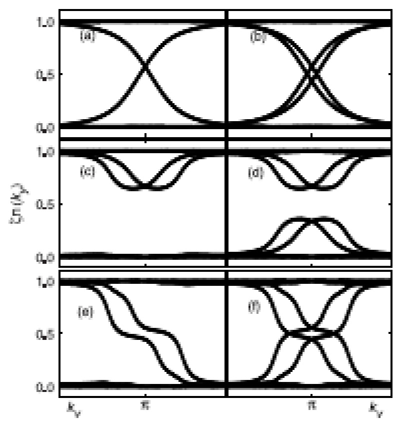

Figure 1: Single particle entanglement spectra. a) and b) are for the QSH phase in different geometries, c) and d) are in the insulator phase and e) and f) are in the quantum anomalous Hall phase.

Let us move back to entanglement: after going through work to link the subsystem particle numbers to topological properties, it can also be found that the entropy of entanglement is related to the subsystem particle fluctuation. In fact, it can be found that the Entropy would just be

$$S_{ent}(k_y) \geq (4 \ln 2\, \Delta N_A^2(k_y),$$

where ΔN is the particle fluctuation. Therefore, a lower bound for the entanglement entropy.

In conclusion, a relationship between quantum entanglement and the subsystem particle number has been found. It can also be seen that this tool does not only work for ordered system but also for disordered ones broadening the field of application.

Transcript:

After trying to get in contact with the author, eventually I had to give up but with other projects being due I realised that I did not have time to look for another article. These are the questions I would have asked but I understand if this is inadequate.

What gave you the idea to write this article?

How long did it take to write it?

What was the most frustrating about writing this article?

What future applications do you see resulting from this research?

What are your own goals after publishing this paper?