1641357

22MATS21

VTU.REG.NO CS076

B.E. Second Semester Examination – July 2024

Mathematics for CSE Stream - II

Time: 3 hrs] [Maximum Marks: 100

Note: Answer any FIVE full questions, selecting atleast ONE full question from each module.

Module – I

1. a) Find the directional derivative of the function $\phi = 4xz^3 - 3x^2y^2z$ $at(2,-1,2)$ along $2i - 3j + 6k$ **(06 Marks)**

   b) If $\vec{F} = \nabla(xy^3 z^2)$ find $div\ \vec{F}$ and $curl\ \vec{F}$ at the point (1, -1, 1). **(07 Marks)**

   c) Show that $\vec{F} = (2xy^2 + yz)i + (2x^2y + xz + 2yz^2)j + (2y^2z + xy)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$. **(07 Marks)**

2. a) Express the vectors $\vec{A} = zi + 2xj + yk$, is cylindrical co-ordinates. **(06 Marks)**

   b) Find the divergence of the vector $\vec{A} = z\ \ find\ \hat{e}_\rho + z\cos\phi\hat{e}_\phi - \rho\cos\phi\hat{e}_z$ **(07 Marks)**

   c) Write a program to find the divergence of $\vec{F} = x^2yzi + y^2zxj + z^2xyk$ **(07 Marks)**

Module – II

3. a) Solve $y''' - 2y'' + 4y' - 8y = 0$ **(06 Marks)**

   b) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 4\cos^2 x$ **(07 Marks)**

   c) Solve $y'' - 4y' + 3y = 2xe^{3x}$ **(07 Marks)**

4. a) Solve $\frac{d^2y}{dx^2} + y = \sec x \tan x$ by the method of variation of parameters. **(06 Marks)**

   b) Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 9y = 3x^2 + \sin(3\log x)$ **(07 Marks)**

   c) Solve $(2x+1)^2\frac{d^2y}{dx^2} - 2(2x+1)\frac{dy}{dx} - 12y = 6x + 5$ **(07 Marks)**

Module – III

5. a) Write the vector V = (4, 2, 1) as a linear combination of the vectors $u_1 = (1, -3, 1)$, $u_2 = (0, 1, 2)$ $u_3 = (5, 1, 37)$. **(06 Marks)**

   b) Express the matrix $\begin{bmatrix} 2 & 0 \\ 4 & -5 \end{bmatrix}$ as a linear combination of the matrices **(07 Marks)**

   $A = \begin{bmatrix} 0 & -3 \\ 2 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

   c) Show that the function $T : R^3 \to R^2$ given by $T(x,y,z) = (2x - 3y, 7y + 2z)$ as linear combination. **(07 Marks)**

REDMI 12 5G 08/07/2024 15:57

6. a) Consider the following polynomials in p(t) and inner product :
$f(t)=t+2, g(t)=3t-2, \ h(t)=t^2-2t-3 \ and <f,g>=\int_0^1 f(t)g(t)dt$ **(06 Marks)**
i) Find <f,g> and <f,h> ii) Find ||f|| and ||g|| iii) Normalize f and g

b) Verify the Rank-nullity theorem for the $T:R^3 \to R^3$ define by $T(x,y,z)=(x+2y-z, y+z, x+y-2z)$ **(07 Marks)**

c) Write a program to verify Rank-nullity theorem for the linear transformation $T:R^3 \to R^3$ defined by $T(x,y,z)=(x+4y+7z, 2x+5y+8z, 3x+6y+9z)$ **(07 Marks)**

Module – IV

7. a) Show that a root of the equation $x^3+5x-11=0$ by using Newton – Raphson method, corrected to three decimal places. **(06 Marks)**

b) Use Regular Falsi method to find the real root of the equation cos x = 3x -1 corrected to three decimal places. **(07 Marks)**

c) Find the interpolating polynomial f(x) satisfying f(0) =0, f(2) = 4, f(4) = 56, f(6) = 204, f(8) = 496, f(10) = 980 and hence find f(3) using Newtons forward interpolate. **(07 Marks)**

8. a) Apply Newton's backward interpolation formula to find y at x =3. Given **(06 Marks)**

x	-4	-2	0	2	4
y	-25	1	3	29	127

b) Use Langrange's interpolation formula to find y at x = 10 given. **(07 Marks)**

x	5	6	9	11
y	12	13	14	16

c) Use Newtons divided difference interpolation formula to find f(4) given f(0) = -4, f(2) = 2, f(3) = 14 f(6) = 158, **(07 Marks)**

Module – V

9. a) Find the appropriate value of $\int_0^{\pi/2}\sqrt{\cos\theta}.d\theta$ Simpson's 1/3rd rule by taking 6 equal intervals **(06 Marks)**

b) From Taylor's series method, find y(0.1) considering upto 4th degree term, given $\frac{dy}{dx}=x-y^2, y(0)=1$ **(07 Marks)**

c) Given $\frac{dy}{dx}=3x+\frac{y}{2}$, y(0) = 1 compute y(0.2) by taking h = 0.2 using Runge-kutta method of fourth order. **(07 Marks)**

10. a) Using modified Euler's method find y(20.2) given that $\frac{dy}{dx}=\log_{10}\left(\frac{x}{y}\right)$ with y(20) = 5 taking h = 0.2. **(06 Marks)**

b) Apply Milne's method to compute y(1.4) given $\frac{dy}{dx}=x^2+\frac{y}{2}$ and data $y(1)=2, y(1.1)=2.2156, y(1.2)=2.4649, y(1.3)=2.7514$ **(07 Marks)**

c) Write program to solve $y'=e^{-x}$ with y(0) = -1 using Euler's method at x = 0.2 (0.2) 0.6. **(07 Marks)**

REDMI 12 5G 08/07/2024 15: