

# The lars Package

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**Title** Least Angle Regression, Lasso and Forward Stagewise

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**Depends** R

**Description** Efficient procedures for fitting an entire lasso sequence with the cost of a single least squares

**License** GPL version 2 or newer

**URL** <http://www-stat.stanford.edu/~hastie/Papers/#LARS>

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cv.lars

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*Computes K-fold cross-validated error curve for lars*


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## Description

Computes the K-fold cross-validated mean squared prediction error for lars, lasso, or forward stage-wise.

## Usage

```
cv.lars(x, y, K = 10, fraction = seq(from = 0, to = 1, length = 100),
        trace = FALSE, plot.it = TRUE, se = TRUE, ...)
```

## Arguments

x	Input to lars
y	Input to lars
K	Number of folds
fraction	Abscissa values at which CV curve should be computed, as a fraction of the saturated lbeta1. Default is seq(from = 0, to = 1, length = 100)
trace	Show computations?
plot.it	Plot it?
se	Include standard error bands?
...	Additional arguments to lars

## Value

Invisibly returns a list with components (which can be plotted using plotCVlars)

fraction	As above
cv	The CV curve at each value of fraction
cv.error	The standard error of the CV curve

## Author(s)

Trevor Hastie

## References

Efron, Hastie, Johnstone and Tibshirani (2003) "Least Angle Regression" (with discussion) *Annals of Statistics*; see also [http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle\\_2002.pdf](http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle_2002.pdf).

## Examples

```
data(diabetes)
attach(diabetes)
cv.lars(x2,y,trace=TRUE,max.steps=80)
detach(diabetes)
```

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diabetes

*Blood and other measurements in diabetics*

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## Description

The diabetes data frame has 442 rows and 3 columns. These are the data used in the Efron et al "Least Angle Regression" paper.

## Format

This data frame contains the following columns:

**x** a matrix with 10 columns

**y** a numeric vector

**x2** a matrix with 64 columns

## Details

The x matrix has been standardized to have unit L2 norm in each column and zero mean. The matrix x2 consists of x plus certain interactions.

## Source

[http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle\\_2002.ps](http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle_2002.ps)

## References

Efron, Hastie, Johnstone and Tibshirani (2003) "Least Angle Regression" (with discussion) *Annals of Statistics*

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lars-internal	<i>Internal lars functions</i>
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## Description

Internal lars functions

## Usage

```
betabreaker(object)
backsolvbet(r,x,k = ncol(r))
cv.folds(n, folds = 10)
delcol(r, z, k = p)
downdateR (R, k = p)
error.bars(x, upper, lower, width = 0.02, ...)
nnls.lars(active, Sign, R, beta, Gram, eps = 1e-10, trace = FALSE,
          use.Gram = TRUE)
plotCVLars(cv.lars.object, se = TRUE)
updateR(xnew, R = NULL, xold, eps = .Machine$double.eps, Gram = FALSE)
.First.lib(lib, pkg)
```

## Details

These are not to be called by the user. `betabreaker` figures out if coefficients (other than lasso) pass through zero, since the L1 norm is discontinuous there, and this has an impact on predict/plot. Suggested by Yann-Ael Le Borgne. `backsolvbet` is included to make the R code compatible with the Splus code, since `backsolve` in R has a `transpose=TRUE` option already.

## Author(s)

Trevor Hastie

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lars	<i>Fits Least Angle Regression, Lasso and Infinitesimal Forward Stagewise regression models</i>
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## Description

These are all variants of Lasso, and provide the entire sequence of coefficients and fits, starting from zero, to the least squares fit.

## Usage

```
lars(x, y, type = c("lasso", "lar", "forward.stagewise", "stepwise"),
     trace = FALSE, normalize = TRUE, intercept = TRUE, Gram, eps = .Machine$double.
```

**Arguments**

<code>x</code>	matrix of predictors
<code>y</code>	response
<code>type</code>	One of "lasso", "lar", "forward.stagewise" or "stepwise". The names can be abbreviated to any unique substring. Default is "lasso".
<code>trace</code>	If TRUE, lars prints out its progress
<code>normalize</code>	If TRUE, each variable is standardized to have unit L2 norm, otherwise it is left alone. Default is TRUE.
<code>intercept</code>	if TRUE, an intercept is included in the model (and not penalized), otherwise no intercept is included. Default is TRUE.
<code>Gram</code>	The $X'X$ matrix; useful for repeated runs (bootstrap) where a large $X'X$ stays the same.
<code>eps</code>	An effective zero
<code>max.steps</code>	Limit the number of steps taken; the default is $8 * \min(m, n - \text{intercept})$ , with $m$ the number of variables, and $n$ the number of samples. For <code>type="lar"</code> or <code>type="stepwise"</code> , the maximum number of steps is $\min(m, n - \text{intercept})$ . For <code>type="lasso"</code> and especially <code>type="forward.stagewise"</code> , there can be many more terms, because although no more than $\min(m, n - \text{intercept})$ variables can be active during any step, variables are frequently dropped and added as the algorithm proceeds. Although the default usually guarantees that the algorithm has proceeded to the saturated fit, users should check.
<code>use.Gram</code>	When the number $m$ of variables is very large, i.e. larger than $N$ , then you may not want LARS to precompute the Gram matrix. Default is <code>use.Gram=TRUE</code>

**Details**

LARS is described in detail in Efron, Hastie, Johnstone and Tibshirani (2002). With the "lasso" option, it computes the complete lasso solution simultaneously for ALL values of the shrinkage parameter in the same computational cost as a least squares fit. A "stepwise" option has recently been added to LARS.

**Value**

A "lars" object is returned, for which `print`, `plot`, `predict`, `coef` and `summary` methods exist.

**Author(s)**

Brad Efron and Trevor Hastie

**References**

Efron, Hastie, Johnstone and Tibshirani (2003) "Least Angle Regression" (with discussion) *Annals of Statistics*; see also [http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle\\_2002.pdf](http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle_2002.pdf). Hastie, Tibshirani and Friedman (2002) *Elements of Statistical Learning*, Springer, NY.

**See Also**

print, plot, summary and predict methods for lars, and cv.lars

**Examples**

```
data(diabetes)
par(mfrow=c(2,2))
attach(diabetes)
object <- lars(x,y)
plot(object)
object2 <- lars(x,y,type="lar")
plot(object2)
object3 <- lars(x,y,type="for") # Can use abbreviations
plot(object3)
detach(diabetes)
```

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plot.lars

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*Plot method for lars objects*


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**Description**

Produce a plot of a lars fit. The default is a complete coefficient path.

**Usage**

```
plot.lars(x, xvar= c("norm", "df", "arc.length", "step"), breaks = TRUE, plottype =
  "Cp"), omit.zeros = TRUE, eps = 1e-10, ...)
```

**Arguments**

x	lars object
xvar	The type of x variable against which to plot. <code>xvar=norm</code> (default) plots against the L1 norm of the coefficient vector, as a fraction of the maximal L1 norm. <code>xvar=step</code> plots against the step number (which is essentially degrees of freedom for LAR; not for LASSO or Forward Stagewise). <code>xvar=arc.length</code> plots against the arc.length of the fitted vector; this is useful for a LAR object, because the L1 norm of its coefficient vector need not be monotone in the steps. <code>xvar=df</code> plots against the estimated df, which is the size of the active set at each step.
breaks	If TRUE, then vertical lines are drawn at each break point in the piecewise linear coefficient paths
plottype	Either <code>coefficients</code> (default) or <code>Cp</code> . The coefficient plot shows the path of each coefficient as a function of the norm fraction or Df. The Cp plot shows the Cp curve.
omit.zeros	When the number of variables is much greater than the number of observations, many coefficients will never be nonzero; this logical (default TRUE) avoids plotting these zero coefficients

eps	Definition of zero above, default is $1e-10$
...	Additonal arguments for generic plot. Can be used to set xlims, change colors, line widths, etc

### Details

The default plot uses the fraction of L1 norm as the xvar. For forward stagewise and LAR, coefficients can pass through zero during a step, which causes a change of slope of L1 norm vs arc-length. Since the coefficients are piecewise linear in arc-length between each step, this causes a change in slope of the coefficients.

### Value

NULL

### Author(s)

Trevor Hastie

### References

Efron, Hastie, Johnstone and Tibshirani (2003) "Least Angle Regression" (with discussion) *Annals of Statistics*; see also [http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle\\_2002.pdf](http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle_2002.pdf) Yann-Ael Le Borgne (private communication) pointed out the problems in plotting forward stagewise and LAR coefficients against L1 norm, and the solution we have implemented.

### Examples

```
data(diabetes)
attach(diabetes)
object <- lars(x, y)
plot(object)
detach(diabetes)
```

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predict.lars	<i>Make predictions or extract coefficients from a fitted lars model</i>
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### Description

While lars() produces the entire path of solutions, predict.lars allows one to extract a prediction at a particular point along the path.

### Usage

```
predict.lars(object, newx, s, type = c("fit", "coefficients"), mode = c("step",
  "fraction", "norm", "lambda"), ...)
coef.lars(object, ...)
```

**Arguments**

object	A fitted lars object
newx	If type="fit", then newx should be the x values at which the fit is required. If type="coefficients", then newx can be omitted.
s	a value, or vector of values, indexing the path. Its values depends on the mode= argument. By default (mode="step"), s should take on values between 0 and p (e.g., a step of 1.3 means .3 of the way between step 1 and 2.)
type	If type="fit", predict returns the fitted values. If type="coefficients", predict returns the coefficients. Abbreviations allowed.
mode	Mode="step" means the s= argument indexes the lars step number, and the coefficients will be returned corresponding to the values corresponding to step s. If mode="fraction", then s should be a number between 0 and 1, and it refers to the ratio of the L1 norm of the coefficient vector, relative to the norm at the full LS solution. Mode="norm" means s refers to the L1 norm of the coefficient vector. Mode="lambda" uses the lasso regularization parameter for s; for other models it is the maximal correlation (does not make sense for lars/stepwise models). Abbreviations allowed.
...	Any arguments for predict.lars should work for coef.lars

**Details**

LARS is described in detail in Efron, Hastie, Johnstone and Tibshirani (2002). With the "lasso" option, it computes the complete lasso solution simultaneously for ALL values of the shrinkage parameter in the same computational cost as a least squares fit.

**Value**

Either a vector/matrix of fitted values, or a vector/matrix of coefficients.

**Author(s)**

Trevor Hastie

**References**

Efron, Hastie, Johnstone and Tibshirani (2002) "Least Angle Regression" (with discussion) *Annals of Statistics*; see also [http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle\\_2002.pdf](http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle_2002.pdf). Hastie, Tibshirani and Friedman (2002) *Elements of Statistical Learning*, Springer, NY.

**See Also**

print, plot, lars, cv.lars



**Examples**

```
data(diabetes)
attach(diabetes)
object <- lars(x,y,type="lasso")
### make predictions at the values in x, at each of the
### steps produced in object
fits <- predict.lars(object, x, type="fit")
### extract the coefficient vector with L1 norm=4.1
coef4.1 <- coef(object, s=4.1, mode="norm") # or
coef4.1 <- predict(object, s=4.1, type="coef", mode="norm")
detach(diabetes)
```

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summary.lars	<i>Summary method for lars objects</i>
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**Description**

Produce an anova-type summary for a lars object.

**Usage**

```
summary.lars(object, sigma2=NULL, ...)
```

**Arguments**

object	lars object
sigma2	optional variance measure (for $p > n$ )
...	Additional arguments for summary generic

**Details**

An anova summary is produced, with Df, RSS and Cp for each step. Df is tricky for some models, such as forward stagewise and stepwise, and is not likely to be accurate. When  $p > n$ , the user is responsible for supplying sigma2.

**Value**

An anova object is returned, with rownames the step number, and with components:

Df	Estimated degree of freedom
Rss	The Residual sum of Squares
Cp	The Cp statistic

**Author(s)**

Brad Efron and Trevor Hastie

**References**

Efron, Hastie, Johnstone and Tibshirani (2003) "Least Angle Regression" (with discussion) *Annals of Statistics*; see also [http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle\\_2002.pdf](http://www-stat.stanford.edu/~hastie/Papers/LARS/LeastAngle_2002.pdf). Hastie, Tibshirani and Friedman (2002) *Elements of Statistical Learning*, Springer, NY.

**See Also**

lars, and print, plot, and predict methods for lars, and cv.lars

**Examples**

```
data(diabetes)
attach(diabetes)
object <- lars(x, y)
summary(object)
detach(diabetes)
```

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